Module 2:

Utility Theory

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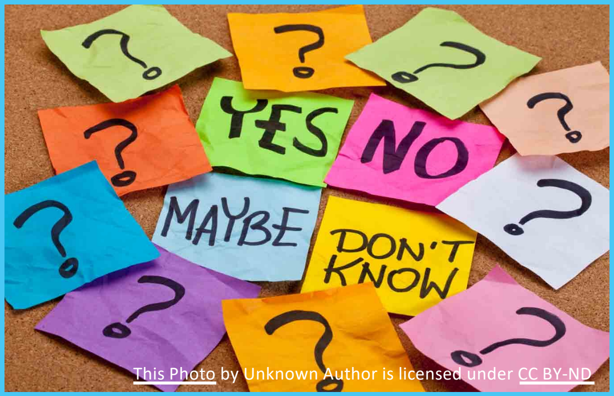
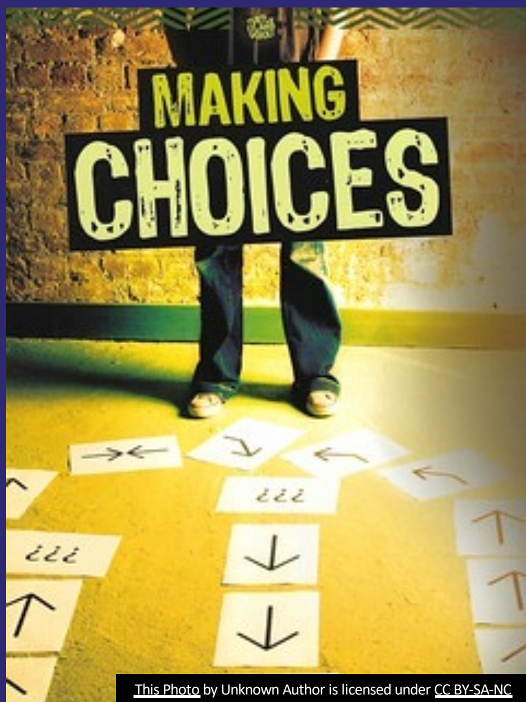


Preview

- Why do we study choices?
- Preferences and constraints
- Choice as a constrained maximization problem
- Deriving and understanding demand
- Change in prices and income



Understanding Choices

Choices arise because

- resources (e.g., time; income) are scarce
- there are competing alternatives over which we have preferences

Understanding choices is important as

- choices determine demand and thus affect markets, and market conditions in turn impact production decisions
- effective policy making for the purpose of affecting and changing behaviour hinges upon our ability to accurately predict choices

Two key ingredients to consumer choice are

- budget constraint
- preferences

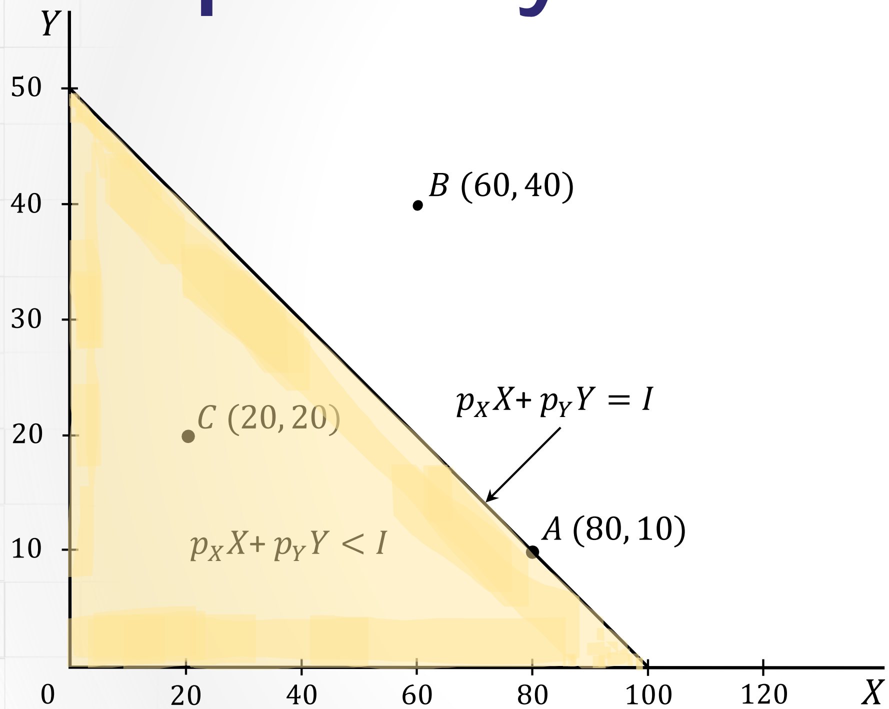
Budget Constraint

- The **budget constraint** represents the set of all combinations of goods and services a consumer can purchase at given prices and income.
- Simply put, the budget constraint states that total expenditure on goods and services cannot exceed total income.
- In an economy with two divisible goods (see note 1), we can write the budget constraint as

$$\boxed{\underbrace{p_X X + p_Y Y}_{\text{expenditure}} \leq I}$$

are the per-unit prices of the two goods, and I is income (see note 2).

Budget Constraint (BC) Graphically – 1

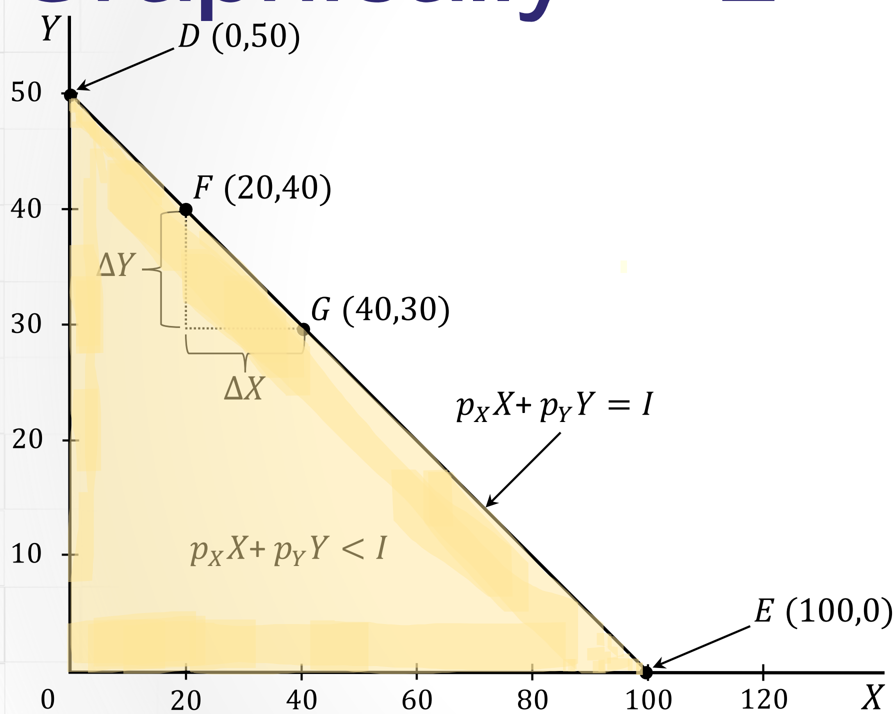


EXAMPLE: I = 100, p_X = 1, p_Y = 2

- Black line = budget line
- *BC* = black line + yellow shaded area
- In shaded area, expenditure < income
- On black line, expenditure = income
- At *A*, expenditure = 80+2(10) = 100
- At *B*, expenditure = 60+2(40) = 140 > 100
- At *C*, expenditure = 20+2(20) = 60 < 100

NOTE: Bundles (e.g., *A*) on budget line and in shaded area (e.g., *C*) are affordable, while bundles above budget line (e.g., *B*) are unaffordable.

Budget Constraint (BC) Graphically – 2



EXAMPLE: I = 100, p_X = 1, p_Y = 2

- Vertical intercept (D) = $\frac{I - p_X X}{p_Y} = \frac{100}{2} = 50$
- Horizontal intercept (E) = $\frac{I - p_Y Y}{p_X} = 100$
- Slope = $-\frac{\text{vertical intercept}}{\text{horizontal intercept}} = -\frac{50}{100} = -\frac{1}{2}$

NOTE: slope of budget line measures the opportunity cost of buying one more unit of good X, which depends on the prices of the two goods (see note).

Intercepts and Slope of Budget Line

- The equation of the budget line is

$$\boxed{p_X X + p_Y Y = I}$$

which we can rewrite as

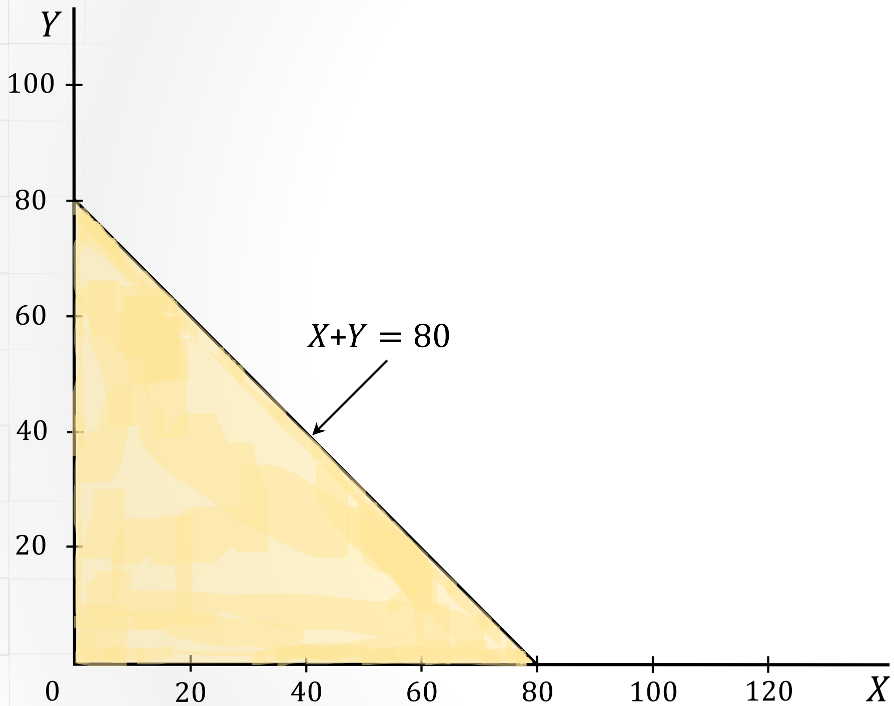
$$\boxed{Y = \frac{I - p_X X}{p_Y} = \frac{I}{p_Y} - \frac{p_X}{p_Y} X}$$

  - the vertical intercept (or Y value when $X = 0$) is $\frac{I}{p_Y}$
  - the horizontal intercept (or X value when $Y = 0$) is $\frac{I}{p_X}$
  - the slope (or amount of good Y we must give up for an additional unit of good X such that we remain on the budget line) is $-\frac{p_X}{p_Y}$

Changes in the Budget Line

- C1: An increase (decrease) in I increases (decreases) the two intercepts but does not affect the slope
- C2: An increase (decrease) in p_X increases (decreases) the slope and decreases (increases) the horizontal intercept
- C3: An increase (decrease) in p_Y decreases (increases) the slope and decreases (increases) the vertical intercept
- C4: If p_X and p_Y increase (decrease) by the same percentage, the slope remains unchanged, but the two intercepts decrease (increase)
- C5: If p_X, p_Y, and I increase (decrease) by the same percentage, the budget line remains the same – no change in the budget constraint

Changes in the Budget Constraint



INITIAL VALUES: $\boldsymbol{I}$ **= 80**, $\boldsymbol{p_X}$ **= 1**, $\boldsymbol{p_Y}$ **= 1**

Click on each change to see impact on budget line and on set of affordable bundles (the change in the set is shaded in grey)!

- C1.a: $\boldsymbol{I}$ **= 60**; C1.b: $\boldsymbol{I}$ **= 100**
- C2.a: $\boldsymbol{p_X}$ **= 0.5**; C2.b: $\boldsymbol{p_X}$ **= 2**
- C3.a: $\boldsymbol{p_Y}$ **= 0.8**; C3.b: $\boldsymbol{p_Y}$ **= 2**
- C4.a: $\boldsymbol{p_X = p_Y}$ **= 0.8**; C4.b: $\boldsymbol{p_X = p_Y}$ **= 2**
- C5: $\boldsymbol{p_X = p_Y}$ **= 0.5** and $\boldsymbol{I}$ **= 40** (prices and income fall by 50% – no change)

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Questions

Describe what happens to the budget line and set of affordable bundles when p_X increases while p_Y decreases. What happens to the opportunity cost of X?

Describe what happens to the budget line and set of affordable bundles when p_X decreases while I increases. What happens to the opportunity cost of X?

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Preferences

- Preferences determine how we rank alternatives.
- We can think of preferences as a mathematical rule that allows us to rank alternatives from most preferred to least preferred, or vice versa.
- To ensure that preferences are consistent, we make three assumptions (also referred to as axioms) about preferences:
  - ***Completeness***: given two bundles (A and B), we are always able to say that (*i*) A is preferred to B, or (*ii*) B is preferred to A, or (*iii*) A is as preferred as B
  - ***Reflexivity***: any bundle must be at least as good as an identical bundle
  - ***Transitivity***: given three bundles (A, B, and C), if A is preferred to B and B is preferred to C, then A must to be preferred to C

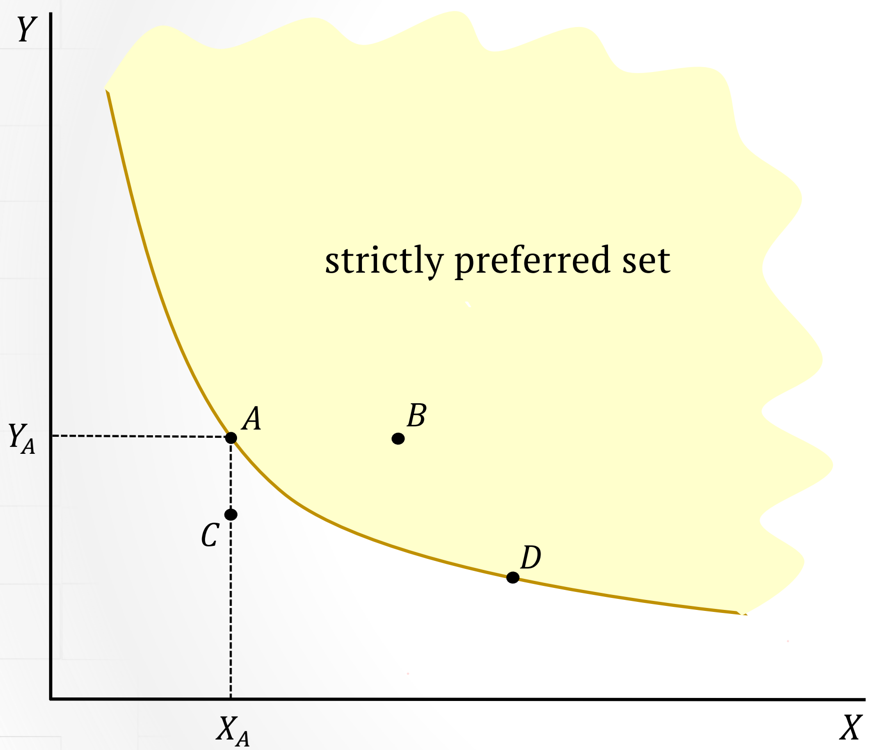
Preferences via Indifference Curves – 1

- We can represent preferences graphically using indifference curves.
- Through each consumption bundle, we can draw an indifference curve.
- An indifference curve through a bundle (say, A) is the locus of all bundles a consumer ranks the same as A and is thus indifferent to any of them.
- If we deal with goods (as opposed to bads), we can assume that more is better (this assumption is referred to as ***monotonicity***), so that
  - bundles above an indifference curve are preferred to bundles on the curve
  - bundles on an indifference curve are preferred to bundles below the curve

Preferences via Indifference Curves – 2

- Notation: given two bundles (A and B), we can write
  - $A \succ B$, if A is strictly preferred to B
  - $A \sim B$, if A is as preferred as B
  - $A \succsim B$, if A is weakly preferred to B (that is, A is at least as preferred as B)

Indifference Curves



- The indifference curve through bundle A, which consists of X_A and Y_A, gives all bundles a consumer ranks the same as A (e.g., the consumer is indifferent between A and D).
- Bundles above the indifference curve (e.g., B) are preferred to any bundle on the curve.
- Any bundle on the indifference curve is preferred to bundles below the curve (e.g., C).
- The set of bundles above the indifference curve is the "***strictly preferred***" set.
- The set of bundles on and above the curve is the "***weakly preferred***" set (click here for graph).

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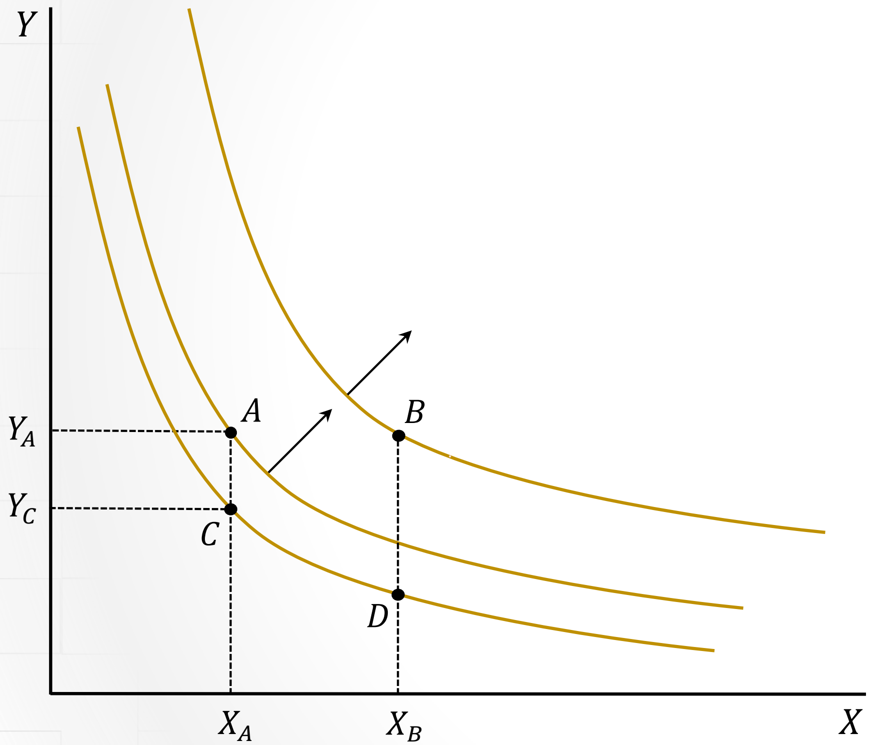
Mapping of Indifference Curves



- Through each consumption bundle, we can draw an indifference curve; hence, we can describe a set of preferences with a mapping of indifference curves.
- A consumer is indifferent to any bundle on the same indifference curve (e.g., $C \sim D$).
- Bundles on a higher indifference curve are preferred to bundles on a lower indifference curve as more is better (e.g., $B \succ A \succ C$):
  - B versus A: $X_B > X_A$ (Y is the same), so that $B \succ A$
  - A versus C: $Y_A > Y_C$ (X is the same), so that $A \succ C$
- The direction of increasing preference is up and to the right (as arrows indicate).

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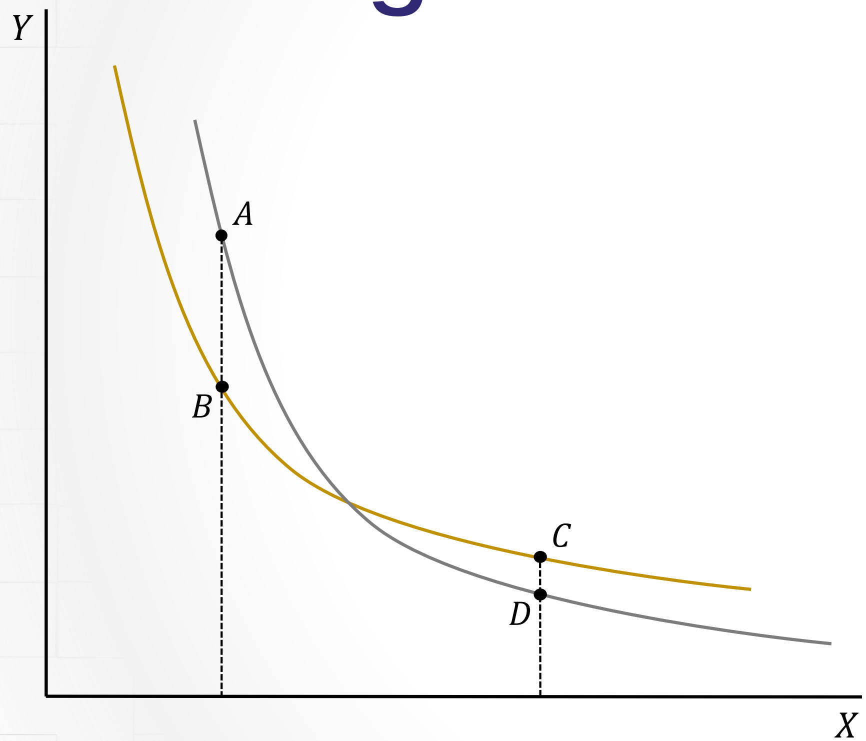
Indifference Curves: Non–Crossing



- Indifference curves from the same mapping (i.e., same set of preferences) cannot cross.
- Crossing is equivalent to a violation of the transitivity assumption.
- In the example,

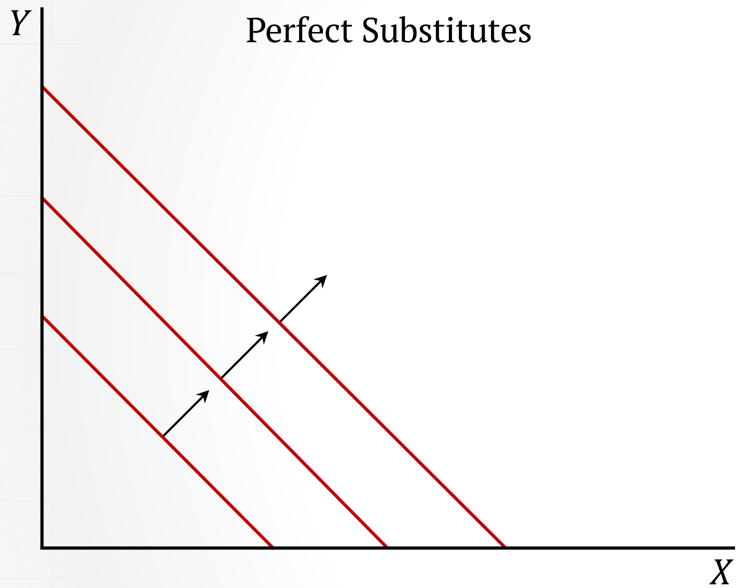
$$A \succ B \sim C \succ D$$

which, by transitivity, implies that $A \succ D$; however, as A and D lie on the same indifference curve, we know that

$$A \sim D$$

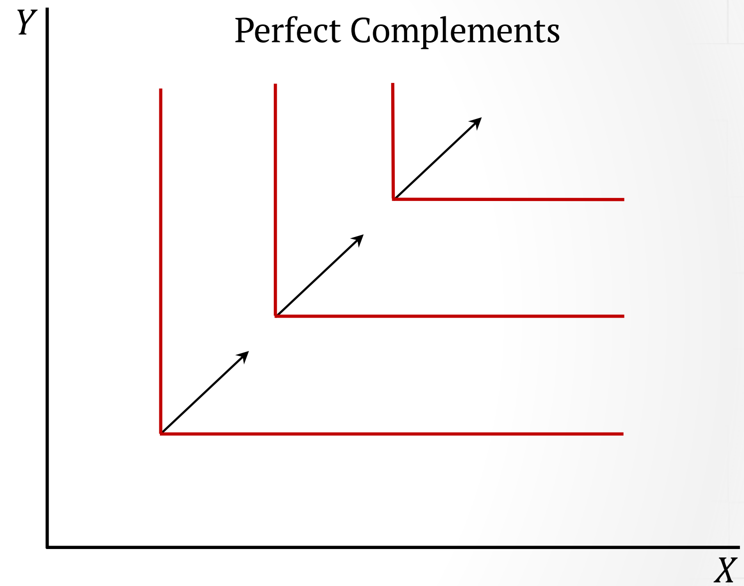
and we then have a contradiction.

Indifference Curves: Examples – 1



In this case, a consumer is willing to substitute good Y for good X at a constant rate (e.g., one for one).

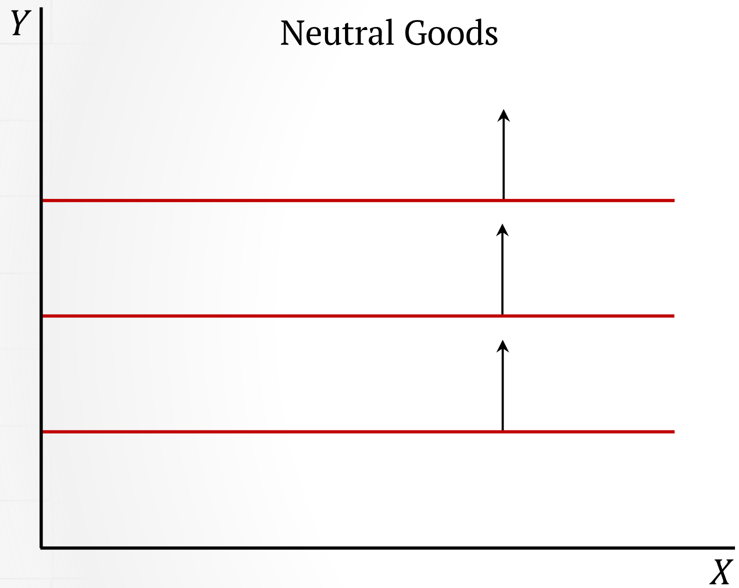
Examples: Pepsi and Coke; tea and coffee; red shirt and green shirt; butter and margarine.



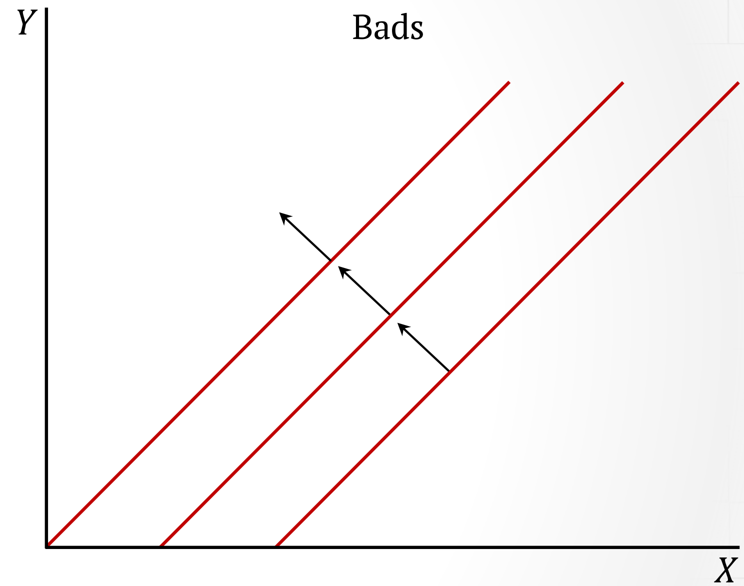
In this case, a consumer wants to consume the two goods in fixed proportions (e.g., one for one).

Examples: hockey stick and hockey puck; PlayStation and games; pencil and paper; gasoline and car.

Indifference Curves: Examples – 2



In this case, a consumer has no preferences over (is indifferent to) good *X*, and the direction of increasing preference is up (that is, in the direction of increased consumption of good *Y*).



In this case, a consumer does not like good *X*, and the direction of increasing preference is up and to the left (that is, in the direction of decreased consumption of good *X* and increased consumption of good *Y*).

Questions

Describe the shape of indifference curves when commodity Y is a neutral good and indicate the direction of increasing preference. Can you think of a neutral good according to your preferences?

Describe the shape of indifference curves when commodity Y is a bad and indicate the direction of increasing preference. Can you think of a bad according to your preferences?

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Preferences via Utility Functions

- As a mathematical rule, we can express preferences as a function that assigns a numerical value to every alternative/bundle such that more-preferred bundles have higher values than less-preferred bundles.
- The value this function, which we call utility function, assigns to each bundle is irrelevant; what is relevant, however, is that, given two bundles (A and B),
  - the value of A is greater than the value of B if A is preferred to B
  - the value of A is less than the value of B if B is preferred to A
  - the value of A is equal to the value of B if A is as preferred as B

Utility Function (U) – An Example

Bundle	$U_1 = \sqrt{XY}$	$U_2 = lnXY$	$U_3 = XY$
B1 = (5, 10)	7.07	3.91	50.00
B2 = (10, 5)	7.07	3.91	50.00
B3 = (20, 5)	10.00	4.61	100.00
B4 = (15, 10)	12.25	5.01	150.00
B5 = (25, 15)	19.36	5.93	375.00

- Regardless of the utility function, the ranking of the bundles is the same, namely,

$$B5 \succ B4 \succ B3 \succ B2 \sim B1$$

- To note is that the U_2 and U_3 functions are positive (order-preserving) transformations of the U_1 function (more simply, U_2 and U_3 are increasing functions of U_2 and U_1):

$$U_2 = 2ln(U_1) \quad \text{and} \quad U_3 = (U_1)^2$$

Utility Function and Indifference Curves

- The implication of the previous example is that positive transformations of a given utility function yield the same choices as the given utility function (we will come back to this point).
  - While the idea that we can measure satisfaction with a utility function may seem implausible, although mathematically and graphically convenient, the fact that utility assignments only matter for ranking bundles should provide some comfort.
- In a two-good economy,

$$U = U(X, Y)$$

which says that utility is a positive (so long as goods are neither neutrals nor bads) function of consumption of good X and consumption of good Y, with $U(\cdot)$ representing the general notation for function.

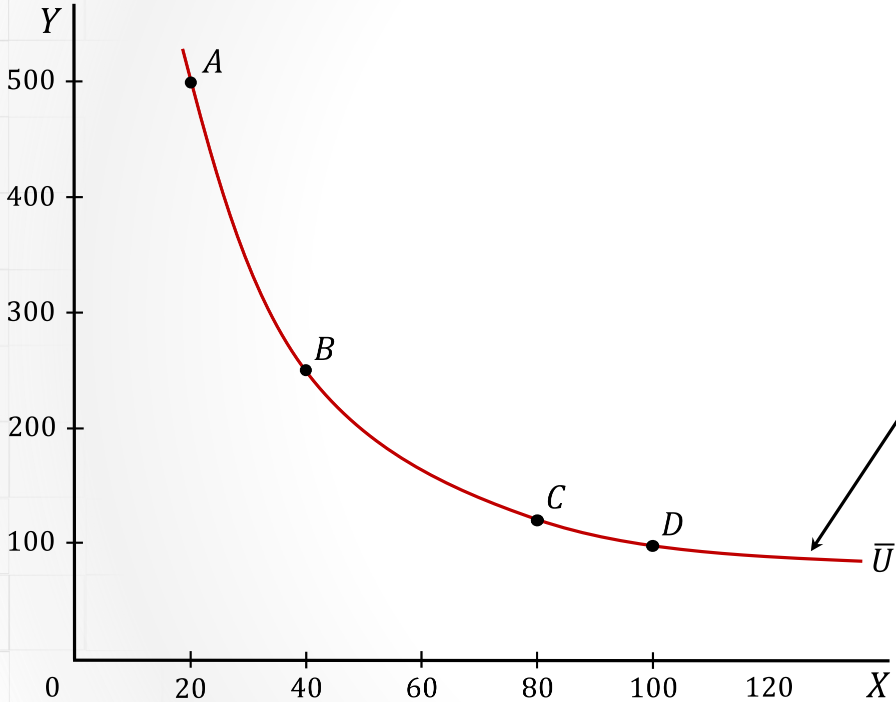
Utility Function and Indifference Curves

- If we fix the utility level, we can then look at all quantity combinations of goods X and Y that yield the same utility level.
- For example, if $U = \sqrt{XY}$ and we fix U at 100, we can write $\sqrt{XY} = 100$, or $Y = \frac{10{,}000}{X}$, which shows, for a given X consumption level, the Y consumption level we need to achieve a utility level of 100 (see note).

$\boldsymbol{X}$	$\boldsymbol{Y = \frac{10,000}{X}}$	$\boldsymbol{U = \sqrt{XY}}$
50	200	100
100	100	100
200	50	100
250	40	100

If we plot the relationship between good Y and good X as described in $Y = \frac{10{,}000}{X}$, we have a curve showing all bundles (i.e., combinations of X and Y quantities) that provide a utility level of 100; this is an indifference curve.

Utility Function and Indifference Curves



$$U_1 = \sqrt{XY} = 100 \quad \Rightarrow \quad Y = \frac{10{,}000}{X}$$

$$U_2 = ln(XY) = 9.21 \quad \Rightarrow \quad Y = \frac{e^{9.21}}{X}$$

$$U_3 = XY = 10{,}000 \quad \Rightarrow \quad Y = \frac{10{,}000}{X}$$

The $\overline{U}$ indifference curve describes each of the above utility functions, but the value of $\overline{U}$ differs across the three functions (it is equal to 100 for U_1, 9.21 for U_2, and 10,000 for U_3). Hence, the three utility functions, which are positive transformations of one another, represent the same preferences, and the actual utility level has no intrinsic meaning.

Marginal Utilities

- The marginal utility of a good is the additional utility from a small change in the amount of the given good, holding the amount of the other good (or goods in an economy with more than two goods) constant.
- Let's assume that $U = (XY)^{0.25}$. In the tables below, we list the marginal utilities of goods X and Y (MU_X and MU_Y) for a few specified changes.

X	Y	U	MU_X
16	0	0	
16	1	2.00	2.00
16	2	2.38	0.38
16	3	2.63	0.25
16	4	2.83	0.20

Y	X	U	MU_X
64	4	4.00	
64	5	4.23	0.23
64	6	4.43	0.20
64	7	4.60	0.17
64	8	4.76	0.16

Questions

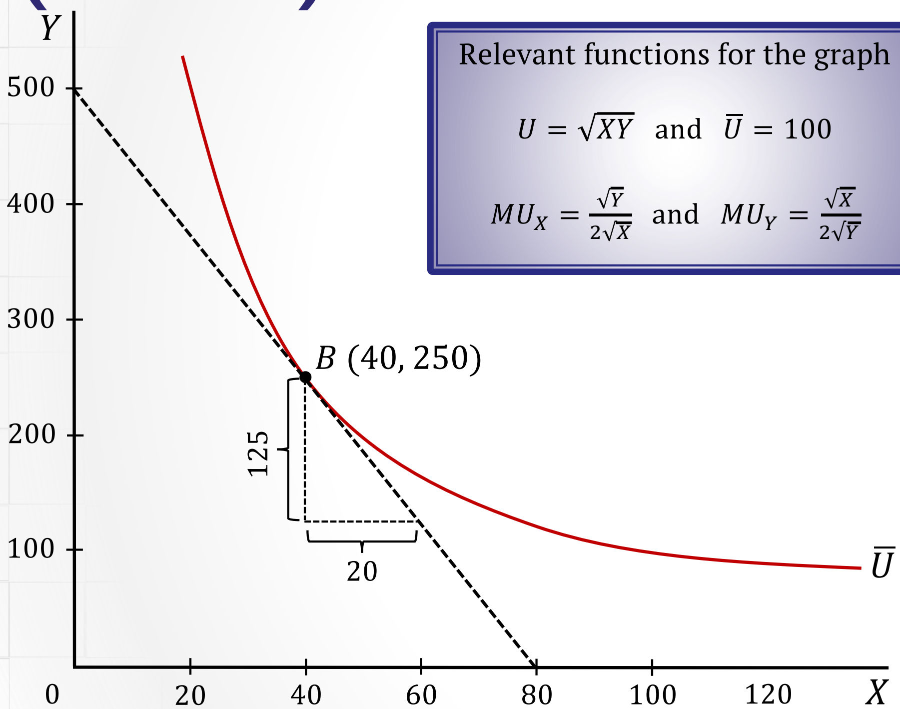
Assume that $U = \sqrt{X} + \sqrt{Y}$. Fill in the blanks for MU_X holding $Y = 1$ and MU_Y holding $X = 4$. Please note that the table does not include utility levels.

X	Y	MU_X (if $Y = 1$)	MU_Y (if X = 4)
0	0		
1	1		
2	2		
3	3		
4	4		

Marginal Utilities and MRS

- While marginal utilities do not convey behavioural information (their values depend on the underlying utility function and are thus of no particular significance), we can use them to compute the ***marginal rate of substitution*** (*MRS*).
- The MRS_{XY} is the rate at which a consumer is willing to substitute good Y for good X such that the consumer is indifferent between the new bundle and the original bundle (new and original bundles rank equally).
- Technically, MRS_{XY} is the (negative of the) slope of the indifference curve at a given bundle; it gives how much of good Y a consumer is willing to give up for a small increase in the amount of good X, holding utility constant or, equivalently, retaining the same indifference curve.

Marginal Rate of Substitution (MRS)



Relevant functions for the graph

$$U = \sqrt{XY} \quad \text{and} \quad \overline{U} = 100$$

$$MU_X = \frac{\sqrt{Y}}{2\sqrt{X}} \quad \text{and} \quad MU_Y = \frac{\sqrt{X}}{2\sqrt{Y}}$$

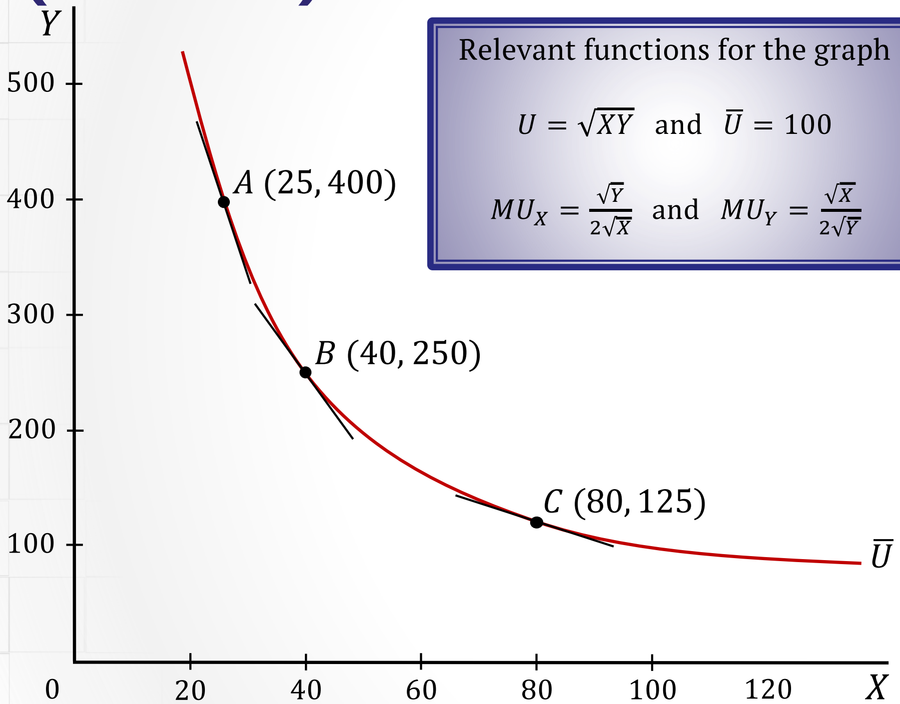
- MRS_{XY} at point B = negative of the slope of line tangent to the indifference curve at B.
- Slope of line tangent to indifference curve at B $= -\frac{500}{80} = -6.25 \left(= -\frac{125}{20}\right)$
- $MRS_{XY} = 6.25$ means that, for an additional unit of good X, a consumer is willing to give up 6.25 units of good Y to ensure that the new and original bundles are equally satisfying (i.e., they are both on the $\overline{U}$ indifference curve).
- We can show that

$$MRS_{XY} = \frac{MU_X}{MU_Y}$$

- For the given example (see note 1 and note 2),

$$\frac{MU_X}{MU_Y} = \frac{\sqrt{Y}}{2\sqrt{X}} \Big/ \frac{\sqrt{X}}{2\sqrt{Y}} = \frac{Y}{X} = \frac{250}{40} = 6.25$$

Marginal Rate of Substitution (MRS)



Relevant functions for the graph

$$U = \sqrt{XY} \quad \text{and} \quad \bar{U} = 100$$

$$MU_X = \frac{\sqrt{Y}}{2\sqrt{X}} \quad \text{and} \quad MU_Y = \frac{\sqrt{X}}{2\sqrt{Y}}$$

- For preferences to be well-behaved, MRS_{XY} must be non-increasing, that is, the slope of the tangent to a given indifference curve cannot become steeper as we move down along the curve (i.e., X increases).
- In the given example,

$$MRS_{XY} = \frac{MU_X}{MU_Y} = \frac{Y}{X}$$

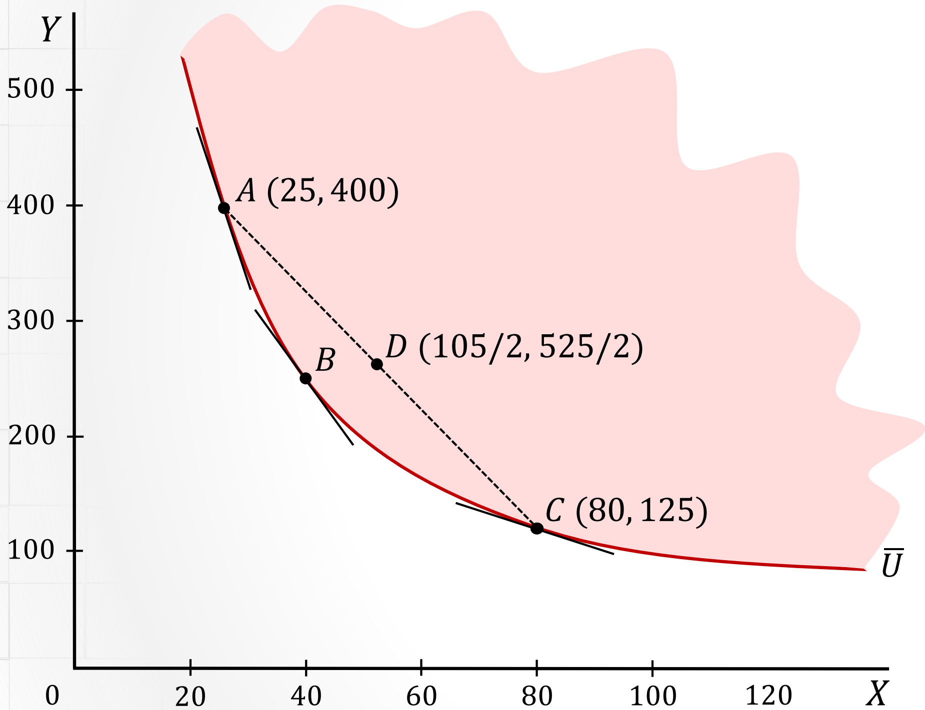
(MRS_{XY} is diminishing – tangent becomes flatter as we move down the curve, reducing Y and raising X); hence,

$$MRS^A = \frac{400}{25}; MRS^B = \frac{250}{40}; MRS^C = \frac{125}{80}$$

that is,

$$MRS^C < MRS^B < MRS^A$$

Diminishing MRS: What Does It Mean?



- Diminishing *MRS* means that a consumer prefers a more balanced consumption to a less balanced consumption.
- For example, bundle D consists of 50% of bundle A and 50% of bundle C (i.e., it is more balanced) and lies in the "strictly preferred" set, which means that

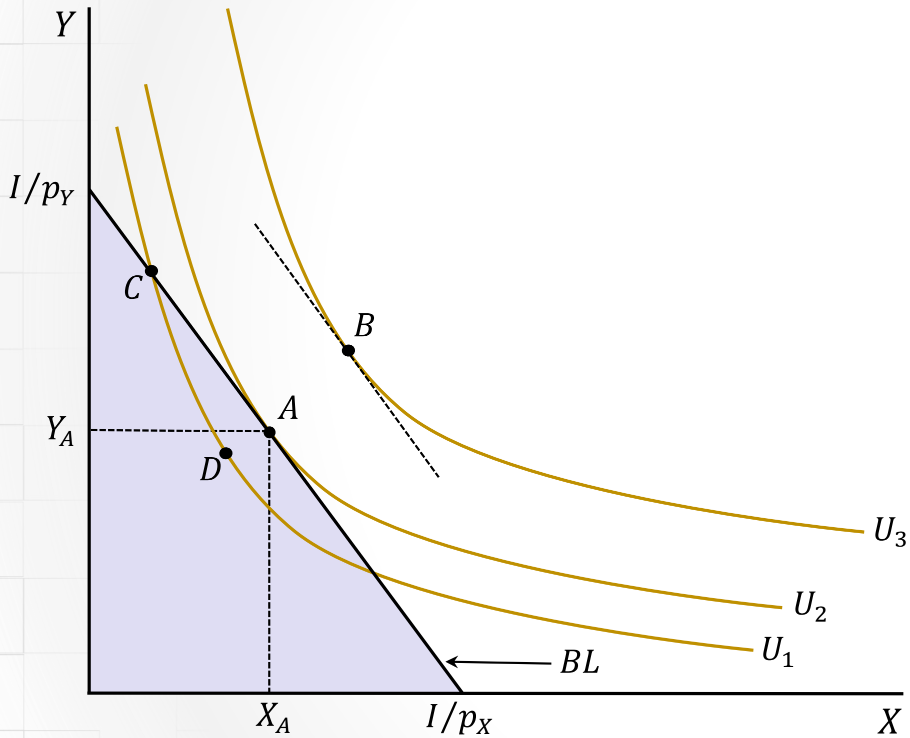
$$D \succ A \sim C$$

- Any bundle on the segment $\overline{AC}$ represents a more balanced consumption than bundle A and bundle C and is preferred to both.

Optimal Choice Graphically – 1

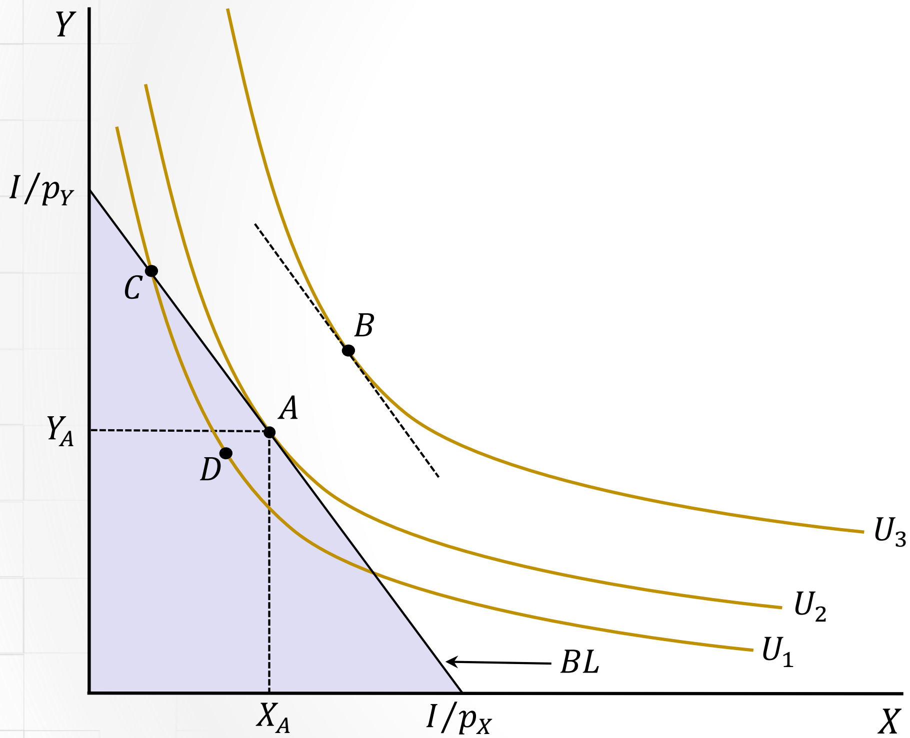
- If higher indifference curves give bundles that a consumer ranks more highly (that is, these bundles provide more utility or satisfaction), we can think of the consumer's objective in graphical terms as corresponding to choosing the highest possible indifference curve.
- However, the consumer faces a budget constraint, which means that the only relevant indifference curves are the ones that have at least one point or bundle in common with the affordable set, as the bundle of choice must be in the affordable set.
- With indifference curves that exhibit diminishing *MRS*, the highest achievable indifference curve is just tangent to the budget line, and the bundle of choice is at the tangency point.

Optimal Choice Graphically – 2



- Objective of rational consumer: achieve the highest possible indifference curve that the set of affordable bundles allows (this is equivalent to maximizing utility subject to a budget constraint).
- Given that choosing the highest possible indifference curve is the objective, the consumer will ensure to spend all of I and be on the budget line (BL).
- The consumer could choose C, but $C \sim D$, and D lies below the BL, which means that there is unspent income that the consumer can use to move to a higher indifference curve.
- The best the consumer can do is to consume bundle A, which consists of X_A and Y_A.

Optimal Choice Graphically – 3



- At bundle A, two conditions hold:
  - $MRS_{XY} = \frac{p_X}{p_Y}$ (tangency between BL and indifference curve)
  - $p_X X + p_Y Y = I$ (expenditure = income)
- The consumer would certainly prefer bundle B; the slope of the U_3 indifference curve at bundle B is also equal to the slope of the BL (the dotted line tangent to the U_3 curve at bundle B is parallel to the BL, which means that

$$MRS^A = MRS^B = \frac{p_X}{p_Y}$$

however, bundle B is not affordable (the second condition above listed is not satisfied).

The Tangency Condition Intuitively – 1

- *MRS* is the rate at which a consumer is willing to trade good *Y* for good *X* along a given indifference curve (or for a given utility level).
- The negative of the slope of the budget line, which is equal to $\frac{p_X}{p_Y}$, is the rate at which the consumer can trade good *Y* for good *X* in the market (for a given budget level).
- If $MRS_{XY} > \frac{p_X}{p_Y}$, the consumer is willing to give up more of good *Y* for a small increase in good *X* than the market requires for the purchase of the additional amount of good *X*, which means that the consumer can save some money by making the trade and use this extra money to buy more of one or both goods to move to a higher indifference curve.

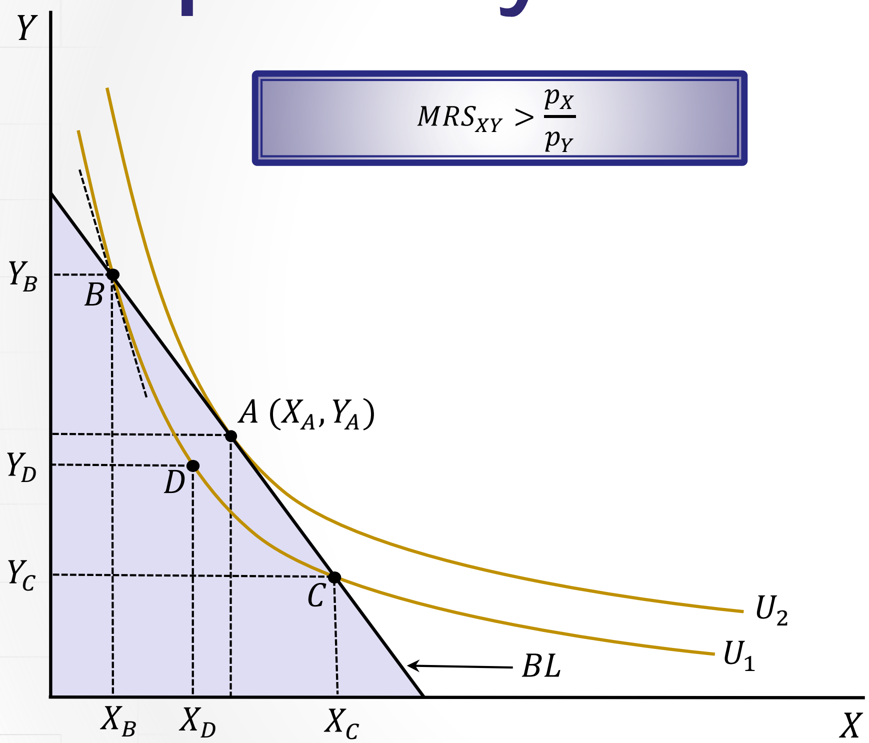
The Tangency Condition Intuitively – 2

- If $MRS_{XY} < \frac{p_X}{p_Y}$ or, equivalently, $\frac{p_X}{p_Y} > MRS_{XY}$, the consumer is willing to give up less of good Y for a small increase in good X than the market requires for the purchase of the additional amount of good X. Put differently, the market values good X relative to good Y more than the consumer does, which means that the consumer can save some money by trading good X for good Y and use this extra money to buy more of one or both goods to move to a higher indifference curve.
- We can also write the tangency condition as $\frac{MU_X}{MU_Y} = \frac{p_X}{p_Y}$, which we can re-arrange as $\frac{MU_X}{p_X} = \frac{MU_Y}{p_Y}$. We can interpret the two ratios as utilities per dollar spent on goods X and Y.

The Tangency Condition Intuitively – 3

- If $\frac{MU_X}{p_X} > \frac{MU_Y}{p_Y}$, a dollar spent on good X provides more utility (is more satisfying) than a dollar spent on good Y, prompting the consumer to trade some consumption of good Y for more consumption of good X.
- If $\frac{MU_X}{p_X} < \frac{MU_Y}{p_Y}$, a dollar spent on good X provides less utility (is less satisfying) than a dollar spent on good Y, prompting the consumer to trade some consumption of good X for more consumption of good Y.
- Consumption adjustments continue until $\frac{MU_X}{p_X} = \frac{MU_Y}{p_Y}$ or no more adjustments are feasible (i.e., there is no more consumption to trade).

The Tangency Condition Graphically – 1



- At bundle B,

$$MRS_{XY} > \frac{p_X}{p_Y} \quad \text{and} \quad p_X X + p_Y Y = I$$

  and the consumer can move to bundle D (increasing consumption of good X from X_B to X_D and decreasing consumption of good Y from Y_B to Y_D), which enjoys the same ranking as bundle B, and save some money (bundle D lies below the BL). (see note)
- The consumer can then use the saving from the trade to purchase more of both goods (increasing their consumption levels to X_A and Y_A), thus moving from the U_1 indifference curve to the U_2 indifference curve.

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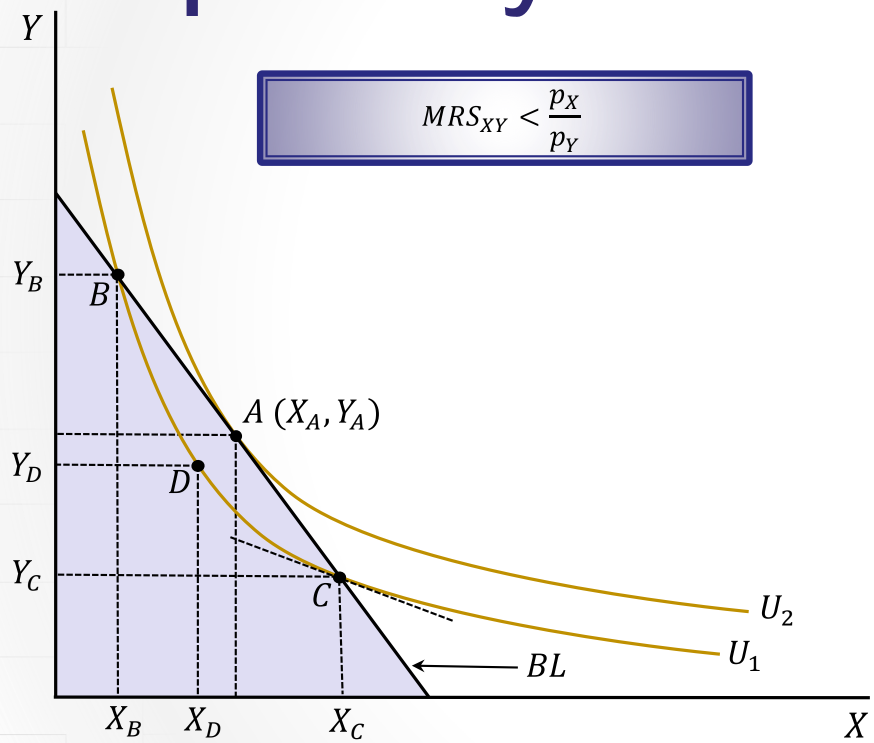
The Tangency Condition Graphically – 2

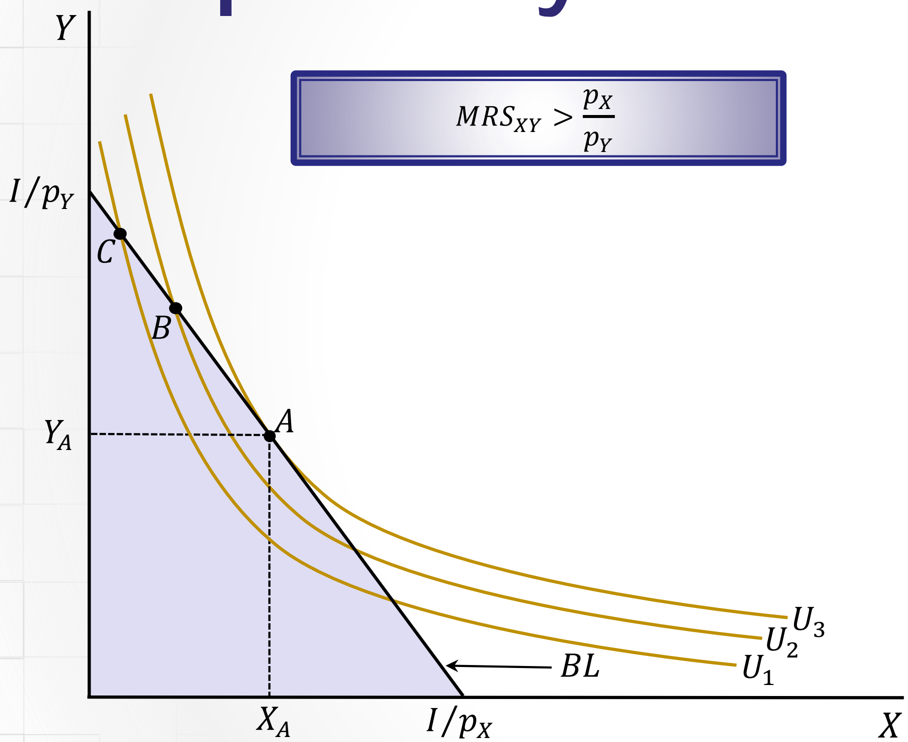


- At bundle C,

$$MRS_{XY} < \frac{p_X}{p_Y} \quad \text{and} \quad p_X X + p_Y Y = I$$

  and the consumer can move to bundle D (decreasing consumption of good X from X_C to X_D and increasing consumption of good Y from Y_C to Y_D), which enjoys the same ranking as bundle C, and save some money (bundle D lies below the BL).
- The consumer can then use the saving from the trade to purchase more of both goods (increasing their consumption levels to X_A and Y_A), thus moving from the U_1 indifference curve to the U_2 indifference curve.

The Tangency Condition Graphically – 3

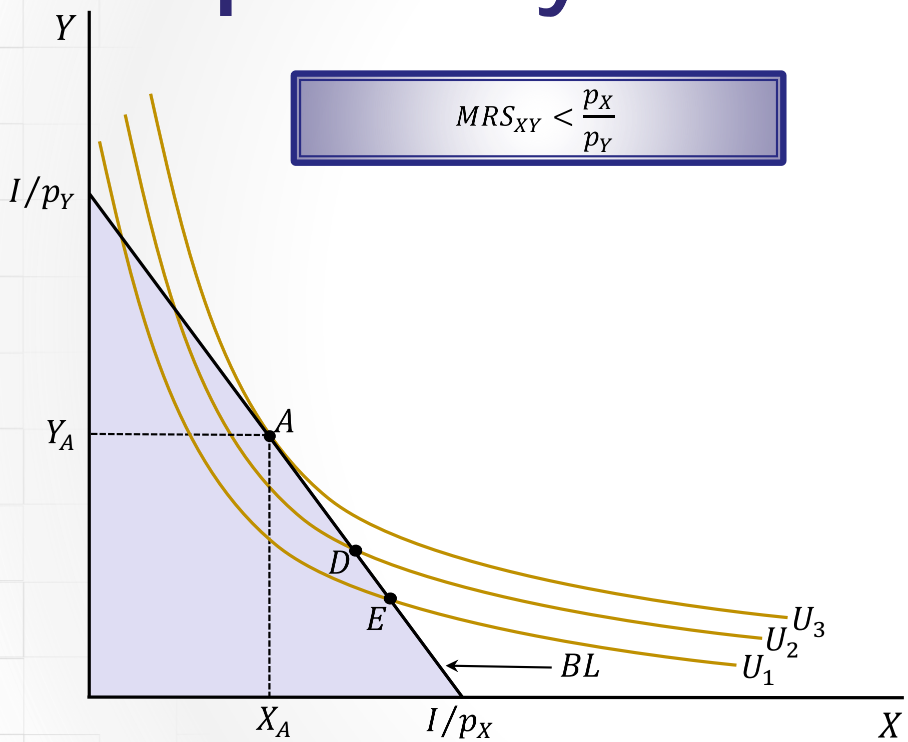


- At each of the illustrated bundles (A, B, and C), the budget constraint is satisfied.
- However,

$$MRS^C > MRS^B > MRS^A = \frac{p_X}{p_Y}$$

- At bundle C, $MRS > \frac{p_X}{p_Y}$, and the consumer can move to bundle B, which is strictly preferrable as it lies on a higher indifference curve.
- At bundle B, $MRS > \frac{p_X}{p_Y}$, and the consumer can move to bundle A, which is strictly preferrable as it lies on a higher indifference curve.

The Tangency Condition Graphically – 4

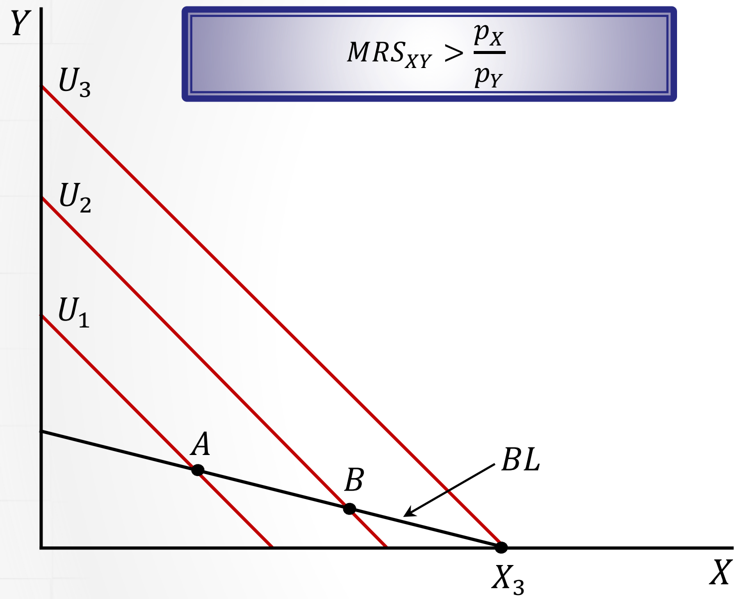


- At each of the illustrated bundles (A, D, and E), the budget constraint is satisfied.
- However,

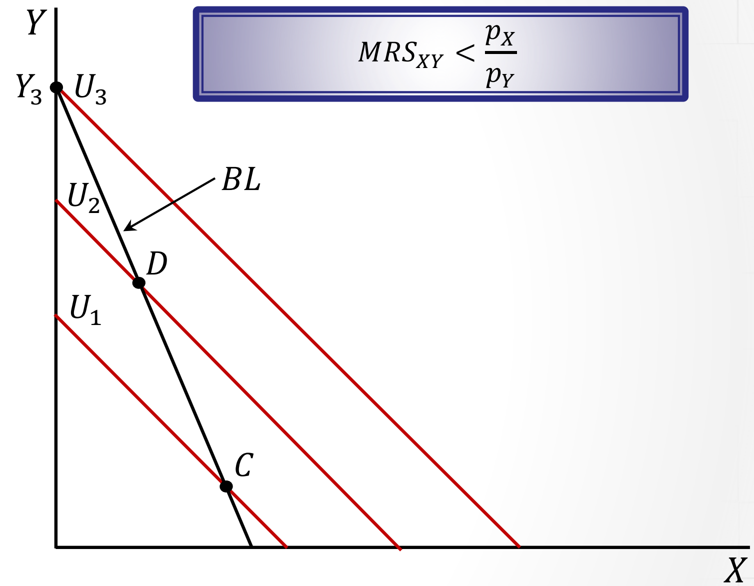
$$MRS^E < MRS^D < MRS^A = \frac{p_X}{p_Y}$$

- At bundle E, $MRS < \frac{p_X}{p_Y}$, and the consumer can move to bundle D, which is strictly preferrable as it lies on a higher indifference curve.
- At bundle D, $MRS < \frac{p_X}{p_Y}$, and the consumer can move to bundle A, which is strictly preferrable as it lies on a higher indifference curve.

Choice with Perfect Substitutes – 1



MRS is constant and always greater than the price ratio (BL is flatter than the indifference curves). A consumer will keep trading Y for X until trading is no longer feasible (i.e., $Y = 0$ and $X = X_3$).

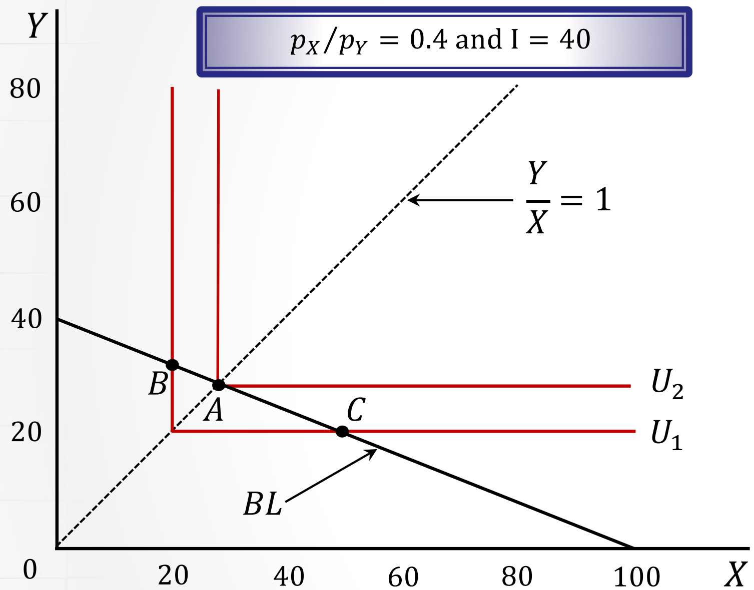


MRS is constant and always smaller than the price ratio (BL is steeper than the indifference curves). A consumer will keep trading X for Y until trading is no longer feasible (i.e., $X = 0$ and $Y = Y_3$).

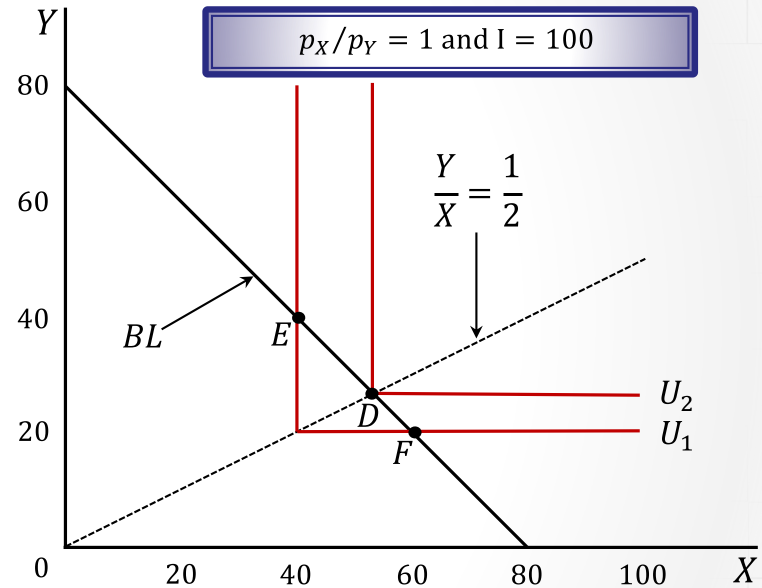
Choice with Perfect Substitutes – 2

- With perfect substitutes, optimal choice involves the consumption of only one of the two goods (***corner solution***), unless $MRS_{XY} = \frac{p_X}{p_Y}$, in which case any bundle on the budget constraint is optimal.
- In summary,
  - If $MRS_{XY} > \frac{p_X}{p_Y}$, the optimal bundle is: $X = \frac{I}{p_X}$, $Y = 0$
  - If $MRS_{XY} < \frac{p_X}{p_Y}$, the optimal bundle is: $X = 0$, $Y = \frac{I}{p_Y}$
  - If $MRS_{XY} = \frac{p_X}{p_Y}$, any bundle that satisfies $p_X X + p_Y Y = I$ is optimal

Choice with Perfect Complements – 1



In this case, we need 1 unit of good X for each unit of good Y (the slope of the ray from the origin through the corners of the L-shaped indifference curves is 1), and the optimal consumption bundle is A.

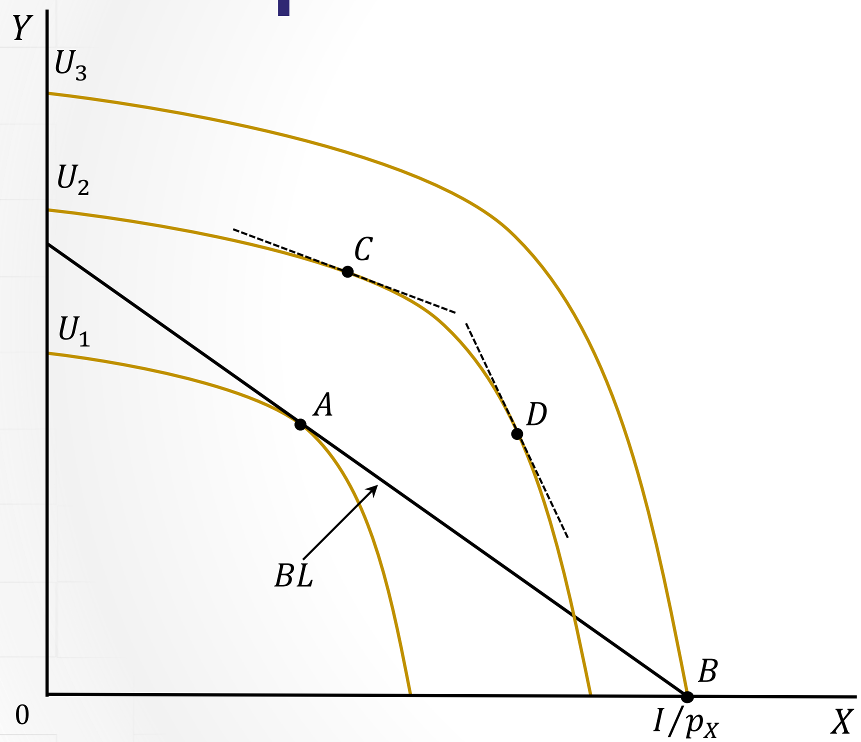


In this case, we need 1 unit of good Y for every 2 units of good X (the slope of the ray from the origin through the corners of the L-shaped indifference curves is 0.5), and the optimal consumption bundle is D.

Choice with Perfect Complements – 2

- A consumer never chooses a bundle on the vertical or horizontal segment of the L-shaped indifference curves; regardless of the price ratio, the consumer buys units of goods X and Y to satisfy the fixed proportion (e.g., 1 for 1, or 1 for 2), and the optimal choice thus lies on the ray through the origin with slope equal to the fixed proportion (and on the BL).
- Anything on the vertical (horizontal) segment of an indifference curve entails purchasing extra units of good Y (X) that the consumer cannot match with units of good X (Y):
  - In (a), at bundle B, the consumer has 20 units of good X and needs 20 units of good Y to be able to enjoy every good X unit (good Y units in excess of 20 are thus unusable).
  - In (b), at bundle F, the consumer has 20 units of good Y and needs 40 units of good X to be able to enjoy every good Y unit (good X units in excess of 40 are thus unusable).

Non–Diminishing MRS: An Example – 1

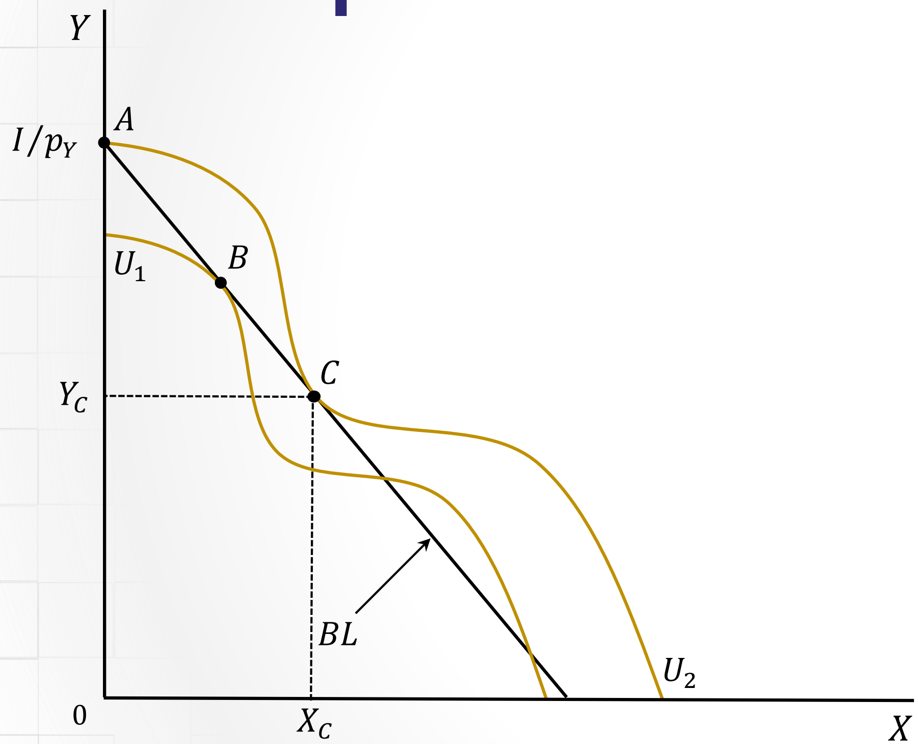


- At bundle A,

$$MRS_{XY} = \frac{p_X}{p_Y} \quad \text{and} \quad p_X X + p_Y Y = I$$

  However, bundle A is not optimal in that it does not provide the highest possible utility level.
- The highest feasible indifference curve with the given BL is U_3, and the bundle that allows a consumer to reach the U_3 indifference curve is bundle B, which consists of $X = \frac{I}{p_X}$ and $Y = 0$.
- In this case, MRS is increasing (preferences are not well-behaved); the tangent to a given indifference curve becomes steeper as we move down along the curve (e.g., $MRS^C < MRS^D$).
- The tangency condition holds at the optimal choice only if MRS is non-increasing).

Non–Diminishing MRS: An Example – 2



- At bundle B,

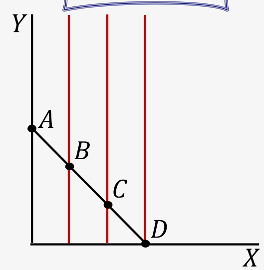
$$MRS_{XY} = \frac{p_X}{p_Y} \quad \text{and} \quad p_X X + p_Y Y = I$$

  However, bundle B is not optimal in that it does not provide the highest possible utility.
- The highest feasible indifference curve with the given BL is U_2, and there are two bundles that allow a consumer to reach the U_2 curve:
  - bundle A, which consists of $X = 0$ and $Y = \frac{I}{p_Y}$
  - bundle C, which consists of $X = X_C$ and $Y = Y_C$ (it so happens that $MRS^C = \frac{p_X}{p_Y}$, but bundles A and C are equally satisfying)
- In this case, MRS is increasing at first, then decreasing, and increasing again; preferences are not well-behaved, and we cannot rely on the tangency condition to find the optimal bundle.

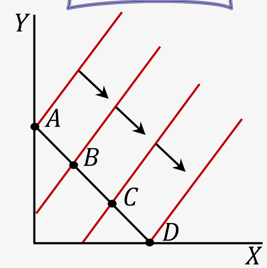
Questions

In each of the following cases, indicate the optimal choice (*BL* = black; indifference curves = red).

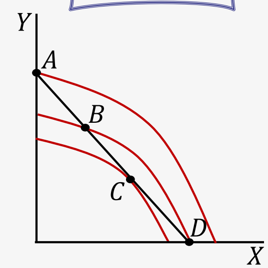
Example I



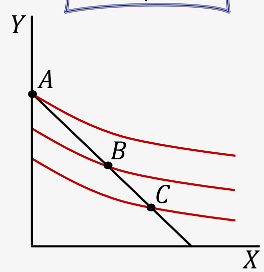
Example II



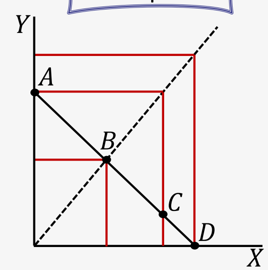
Example III



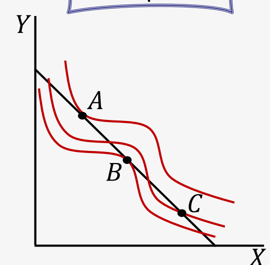
Example IV



Example V



Example VI

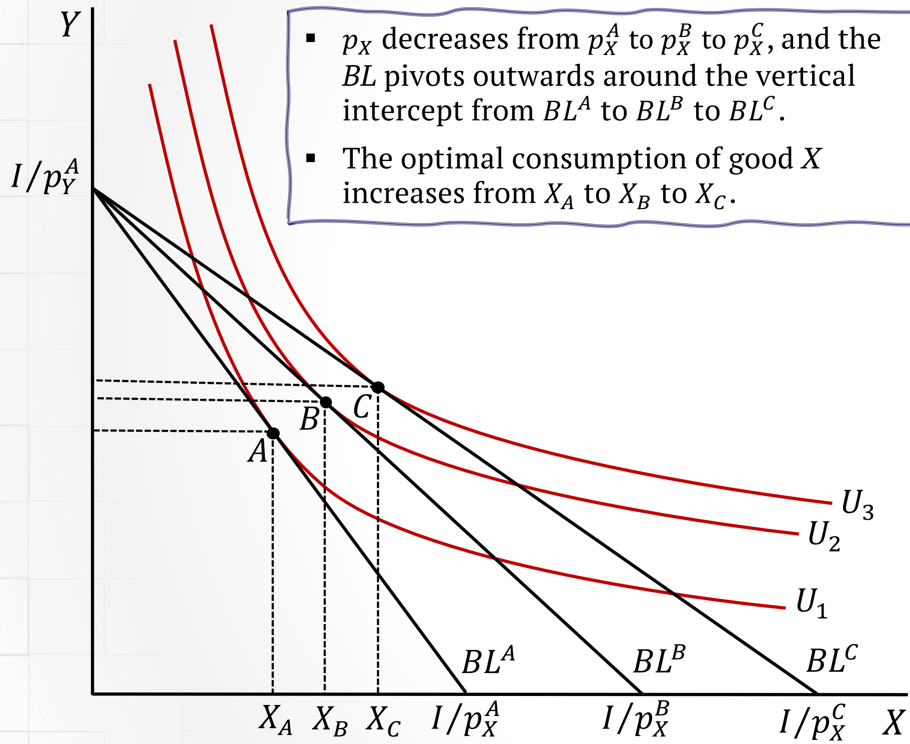


Deriving the Demand Curve – 1

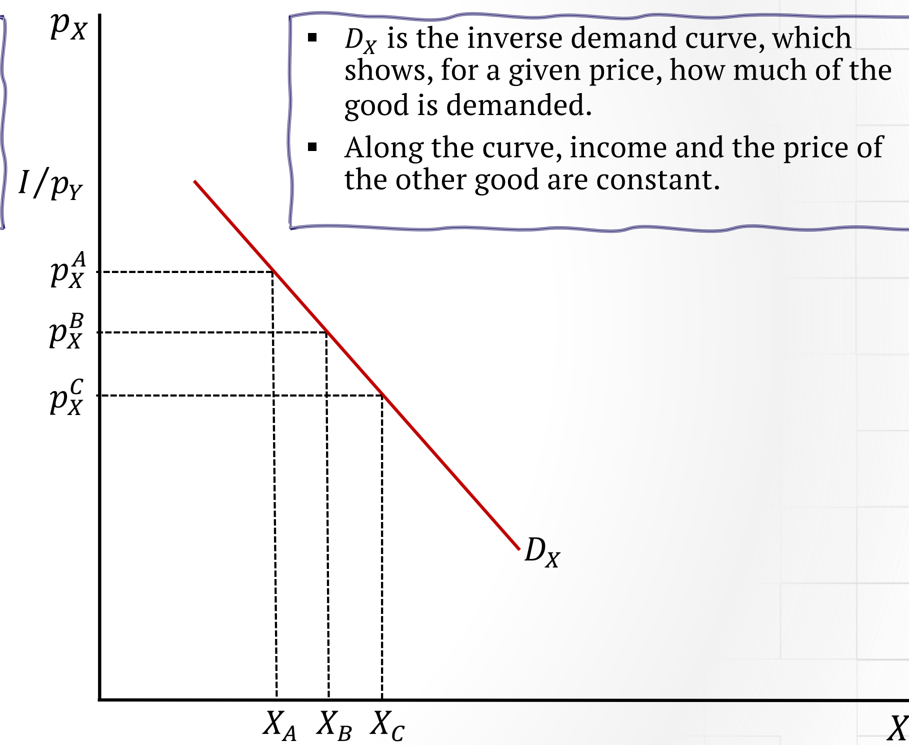
- Although the utility function may come across as a very farfetched construct, it is a useful concept to understand consumption decisions and how changes in prices, income, and preferences affect them.
- In the final section of this module, we use the (budget-)constrained utility maximization framework to derive the demand curve for a representative consumer.
- The demand curve, which we can easily estimate with available data, describes the relationship between the quantity demanded of a good and the price of the good, for a given set of preferences, income level, and prices of other goods.

Deriving the Demand Curve – 2

- p_X decreases from p_X^A to p_X^B to p_X^C, and the BL pivots outwards around the vertical intercept from BL^A to BL^B to BL^C.
- The optimal consumption of good X increases from X_A to X_B to X_C.



- D_X is the inverse demand curve, which shows, for a given price, how much of the good is demanded.
- Along the curve, income and the price of the other good are constant.



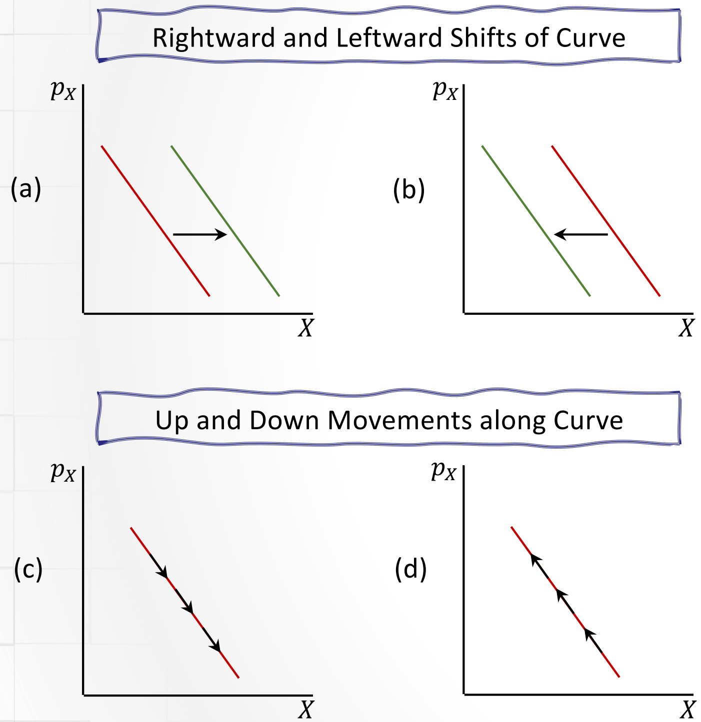
Deriving the Demand Curve – 3

- The inverse demand curve for a good records the optimal consumption level for that good as its price changes, other things being equal (*ceteris paribus*), that is, holding everything else that affects choice constant (i.e., preferences, income, and prices of other goods).
- When income increases, the *BL* shifts out in a parallel fashion, increasing the set of affordable bundles and allowing a consumer to move to a higher indifference curve and to choose a bundle that has more of at least one of the two goods:
  - a good is normal if its consumption increases when income increases, and vice versa
  - a good is inferior if its consumption decreases when income increases, and vice versa
  - in economy, at least one of the goods must be normal

Deriving the Demand Curve – 4

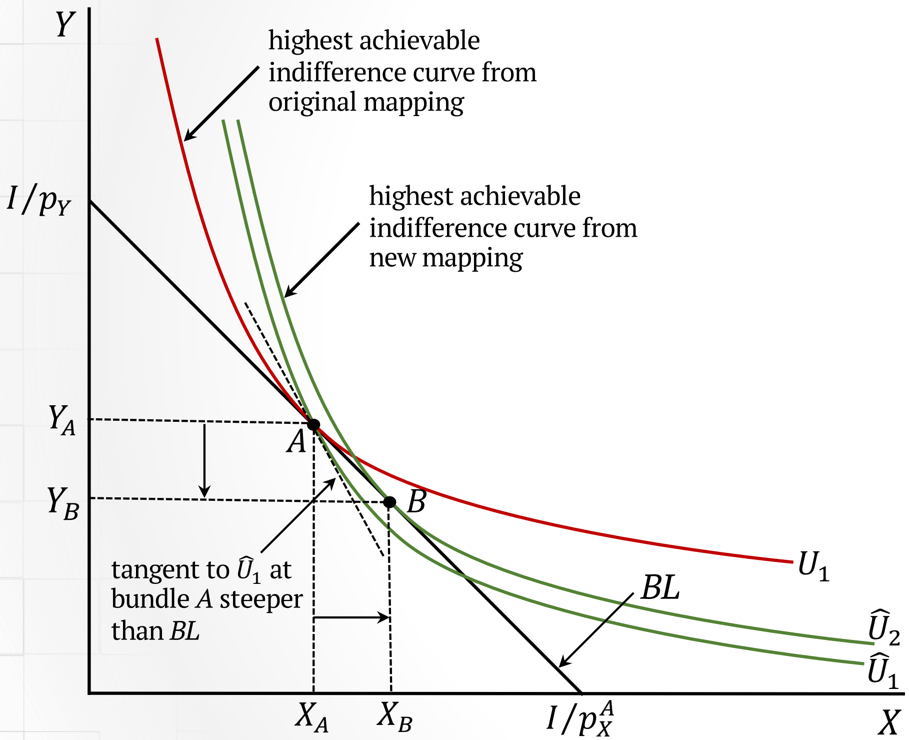
- If the price of the other good, (say, good *Y*) decreases, the *BL* pivots outward around the horizontal intercept, increasing the set of affordable bundles and allowing a consumer to move to a higher indifference curve and to choose a bundle that has at least more of good *Y*.
- What happens to the consumption of good *X* depends on whether the two goods are substitutes or complements; when p_Y decreases, good *Y* becomes cheaper relative to good *X*:
  - if the two goods are substitutes, the demand for good *X* decreases
  - if the two goods are complements, the demand for good X increases

Summary of Changes



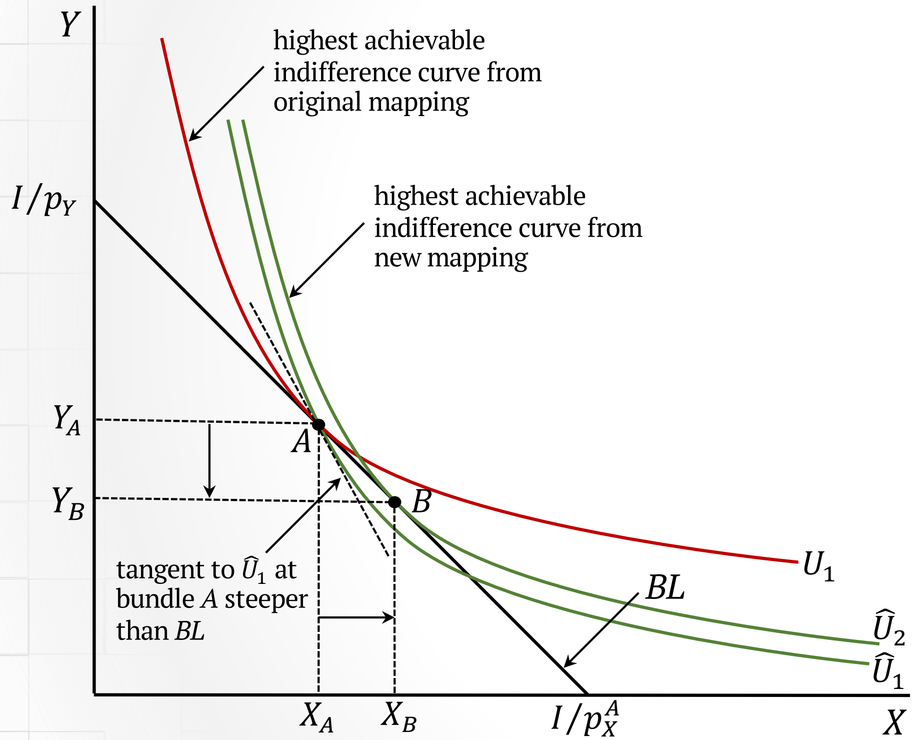
- Movements up and down the demand curve for good X result from changes in p_X; we refer to such movements as changes in quantity demanded:
  - quantity demanded increases if p_X decreases (c)
  - quantity demanded decreases if p_X increases (d)
- Shifts inward (leftward) or outward (rightward) result from changes in other parameters that affects the demand for good X but are held constant along a given curve; we refer to such shifts as changes in demand:
  - demand increases (a) if (*i*) I increases and good X is normal, (*ii*) I decreases and good X is inferior, (*iii*) preferences for good X increase, (*iv*) p_Y decreases and good Y is a complement; (*v*) p_Y increases and good Y is a substitute
  - demand decreases (b) if (*i*) I decreases and good X is normal, (*ii*) I increases and good X is inferior, (*iii*) preferences for good X decrease, (*iv*) p_Y increases and good Y is a complement; (*v*) p_Y decreases and good Y is a substitute

Changes in Preferences – 1



- Indifference curves describe preferences: if preferences change, indifference curves change, becoming steeper if preferences for good X strengthen and flatter if preferences for good X weaken; for a given level of good X (e.g., X_A),
  - larger *MRS* means that a consumer is willing to trade more of good Y for a given increase in good X, and preferences for good X are thus stronger (e.g., MRS^A of $\widehat{U}_1 > MRS^A$ of U_1)
  - lower *MRS* means that a consumer is willing to trade less of good Y for a given increase in good X, and preferences for good X are thus weaker
- In the case depicted here, preferences for good X become stronger, and the consumer chooses to raise the consumption of good X and reduce the consumption of good Y in response.

Changes in Preferences – 2



- The indifference curve from new set of preferences through the original optimal bundle (A) is steeper (tangent to the $\hat{U}_1$ curve is steeper than tangent to the U_1 curve), which means that

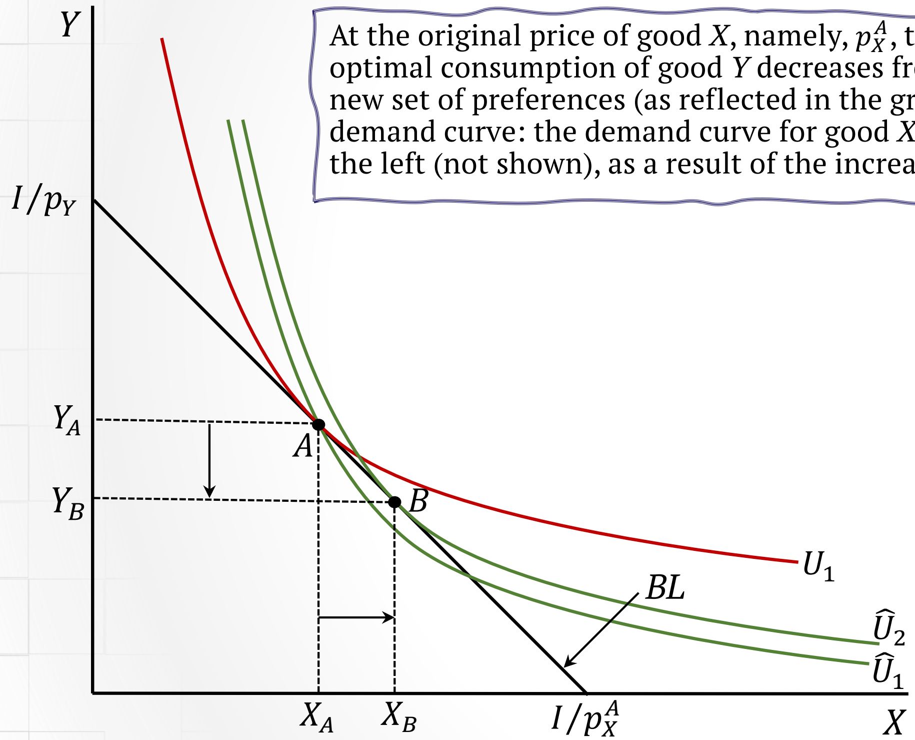
$$MRS_{XY} > \frac{p_X}{p_Y}$$

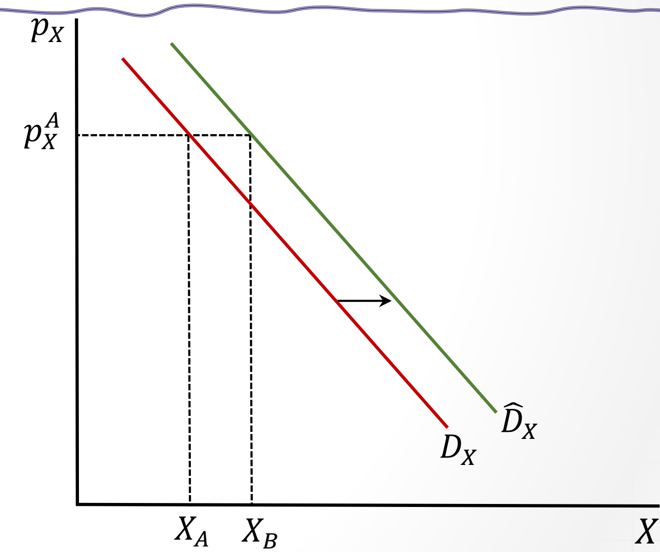
at bundle A on the $\hat{U}_1$ curve.

- The consumer can then move to the higher $\hat{U}_2$ indifference curve by trading good Y for good X; the new optimal bundle at the tangency between the $\hat{U}_2$ curve and the BL is bundle B, which consists of more of good X and less of good Y in comparison to the original optimal bundle A.

Changes in Preferences – 3

At the original price of good X, namely, p_X^A, the optimal consumption of good X increases from X_A to X_B (the optimal consumption of good Y decreases from Y_A to Y_B). The demand curve for good X corresponding to the new set of preferences (as reflected in the green indifference curves) thus lies to the right of the original demand curve: the demand curve for good X shifts to the right, while the demand curve for good Y shifts to the left (not shown), as a result of the increased preferences for good X.





Questions

Graphically show how the (inverse) demand curve for good Y changes when p_X increases and good X is a complement.

Graphically show how the (inverse) demand curve for good Y changes when preferences for good X weaken.

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Conclusion

- In this module, we provide an overview of (rational) choice as the outcome of a utility maximization problem subject to a budget constraint.
- A core principle of utility theory is that consumers with well-behaved preferences compare the relative satisfaction of a good to its relative price when choosing the optimal consumption bundle among all those that are just affordable.
- The framework is useful in deriving the demand curve, which shows how consumption responds to price changes, and in understanding how demand or choice reacts to changes in preferences, prices of substitutes and complements, and income.

Questions?

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