Solutions

1.

a) $slip_{FL} = 40\ rpm$ $\qquad slip = 30\ rpm$

$PLR = \frac{30}{40} = \mathbf{75}\%$

b) $\dot{W}_{sh} = PLR \cdot \dot{W}_{sh_{FL}} = 0.75(20\ hp) = 15\ hp$

c)

$$\tau = \frac{\dot{W}_{sh} \cdot 5252}{rpm} = \frac{15\ hp \cdot 5252}{1770\ rpm} = 44.5\ ft \cdot lb$$

d) From the efficiency graph at PLR = 75%, $\eta = 92\%$

$$\dot{W}_{el} = \frac{\dot{W}_{sh}}{\eta} = \frac{(15\ hp)(746\ W/1\ hp)}{(0.92)} = 12163\ W$$

2.

a)

$$\dot{W}_{sh} = \frac{\Delta P \cdot \dot{V}}{\eta_f} = \frac{750\ Pa \cdot 1.6\ m^3/s}{0.55} = 2182\ W$$

b)

$$\dot{W}_{el,fan} = \frac{\dot{W}_{sh}}{\eta} = \frac{2182\ W}{(0.85)} = 2567\ W$$

c)

$$\dot{Q} = \dot{W}_{el} - \dot{W}_{sh} = 2567 - 2182\ W = 385\ W$$

d)

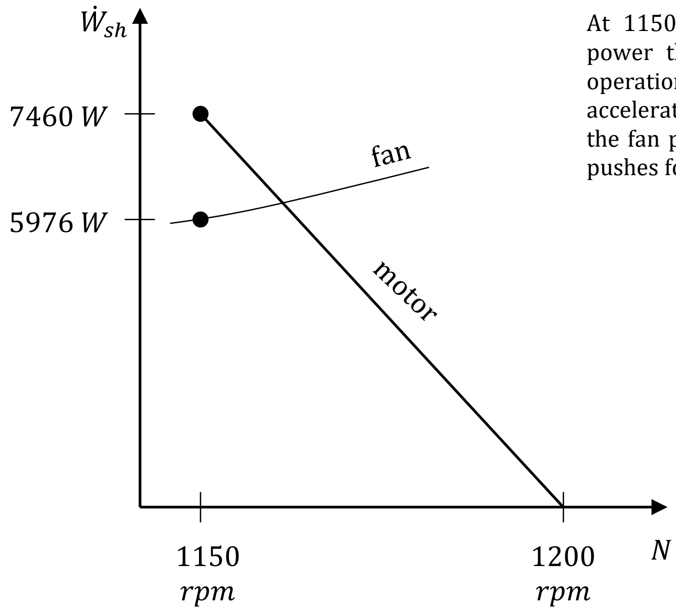
$$COP_c = \frac{\dot{Q}}{\dot{W}_{el,ac}} = \frac{20\ kW}{5.43\ kW} = 3.68$$

e)

$$\dot{W}_{el,tot} = 2.567 + 5.43 \approx 8.0\ kW$$

$$E = \dot{W}_{el,tot} \cdot \Delta t = 8\ kW(20\ h) = 160\ kWh$$

3.



At 1150 rpm, the motor delivers more power than the fan requires for steady operation. Thus, the shaft speed will accelerate until they reach balance where the fan pushes back as hard as the motor pushes forward.

motor:

$slip_{FL} = 50\ rpm$

$\dot{W}_{sh,FL} = 7460\ W$

$$\dot{W}_{sh} = \frac{slip}{slip_{FL}} \cdot \dot{W}_{sh\,FL} = \frac{41}{50} \cdot (7460\ W) = 6117\ W \qquad (at\ 1159\ rpm)$$

For fan, apply fan laws:

$N_1 = 1150\ rpm$ $\qquad \dot{W}_{sh,1} = 5976\ W$ $\qquad N_2 = 1159\ rpm$

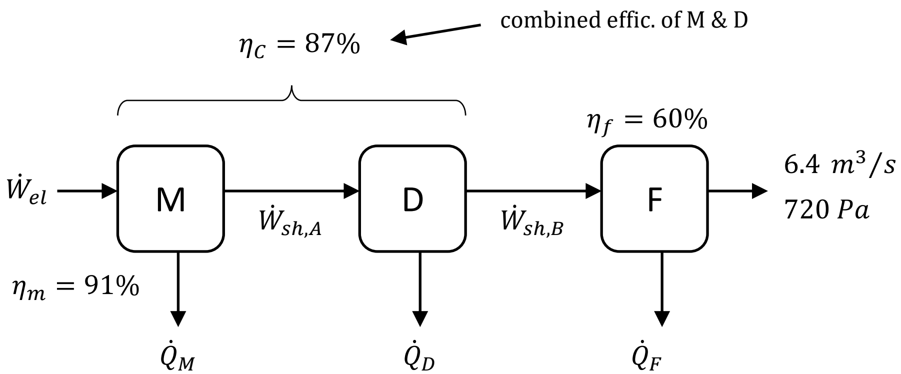
$$\dot{W}_{sh,2} = \dot{W}_{sh,1} \cdot \left(\frac{N_2}{N_1}\right)^3 = (5976\ W) \cdot \left(\frac{1159}{1150}\right)^3 \approx 6117\ W$$

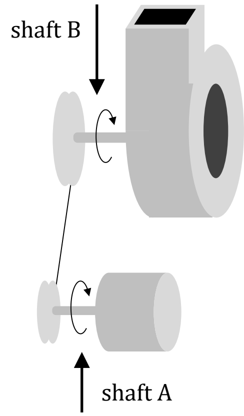
Therefore, motor output = fan input at 1159 rpm.

b)

$$\dot{W}_{el} = \frac{\dot{W}_{sh}}{\eta} = \frac{6117\ W}{0.9} = 6797\ W$$

4.





a)

$$\dot{W}_{el} = \frac{\Delta P \cdot \dot{V}}{\eta_c \cdot \eta_f} = \frac{(720\ Pa)(6.4\ m^3/s)}{(0.87)(0.60)} = \frac{4608\ W}{(0.87)(0.60)} \approx \mathbf{8828\ W}$$

b)

$$\dot{W}_{sh,A} = \dot{W}_{el} \cdot \eta_m = 8828W(0.91) = 8033\ W$$

$$\dot{Q}_M = \dot{W}_{el} - \dot{W}_{sh,A} = 8828 - 8033 = \mathbf{795\ W}$$

c)

$$\eta_C = \eta_M \cdot \eta_D \rightarrow \eta_D = \frac{\eta_C}{\eta_M} = \frac{0.87}{0.91} = \mathbf{95.6}\%$$

d)

$$\dot{V}_1 = 6.4\,m^3/s \qquad\qquad N_1 = 1150\,rpm$$

$$N_2 = N_1 \cdot \frac{\dot{V}_2}{\dot{V}_1} = (1150)\left(\frac{6.0}{6.4}\right) = \mathbf{1078\,rpm}$$

e)

At original operating speed, the fan shaft power is:

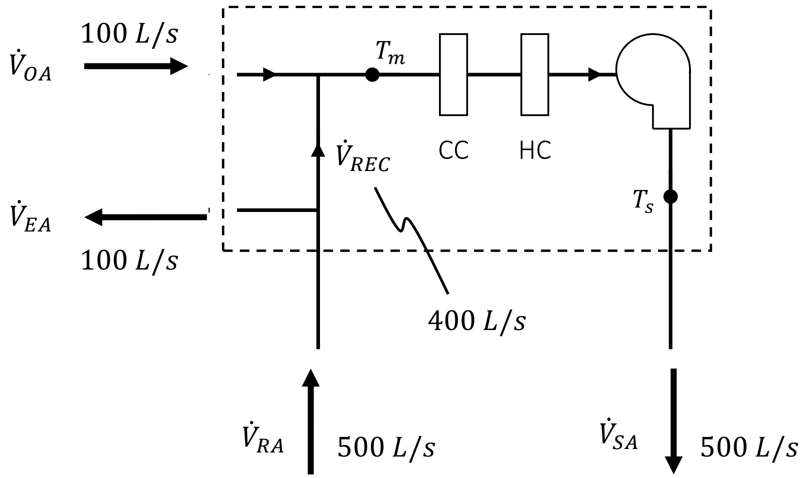
$$\dot{W}_{sh,B1} = \frac{\Delta P \cdot \dot{V}}{\eta_f} = \frac{(720\ Pa)(6.4\ m^3/s)}{(0.60)} = 7680\ W$$

At the new speed:

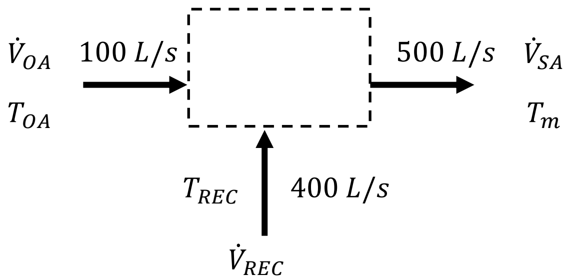
$$\dot{W}_{sh,B2} = \dot{W}_{sh,B1} \cdot \left(\frac{N_2}{N_1}\right)^3 = (7680\ W) \cdot \left(\frac{1078}{1150}\right)^3 \approx \mathbf{6326\ W}$$

5.

The airflow rates in the system are:



Air flows through the mixing section:



If the system operates steadily then:

$$\sum \dot{E}_{in} = \sum \dot{E}_{out}$$

Energy balance for the mixing section:

$$\dot{m}_{OA} \cdot h_{OA} + \dot{m}_{REC} \cdot h_{REC} = \dot{m}_{SA} \cdot h_m$$

$$\rho \cdot \dot{V}_{OA} \cdot h_{OA} + \rho \cdot \dot{V}_{REC} \cdot h_{REC} = \rho \cdot \dot{V}_{SA} \cdot h_m$$

Also: $$\dot{m}_{SA} = \dot{m}_{OA} + \dot{m}_{REC}$$

And with air density being treated as constant: $$\dot{V}_{SA} = \dot{V}_{OA} + \dot{V}_{REC}$$

So the energy balance combined with mass/volume balance:

$$\rho \cdot \dot{V}_{OA} \cdot h_{OA} + \rho \cdot \dot{V}_{REC} \cdot h_{REC} = \rho \cdot \left(\dot{V}_{OA} + \dot{V}_{REC}\right) \cdot h_m$$

$$\rho \cdot \dot{V}_{OA} \cdot h_{OA} - \rho \cdot \dot{V}_{OA} \cdot h_m = \rho \cdot \dot{V}_{REC} \cdot h_m - \rho \cdot \dot{V}_{REC} \cdot h_{REC}$$

$$\rho \cdot \dot{V}_{OA} \cdot (h_{OA} - h_m) = \rho \cdot \dot{V}_{REC}(h_m - h_{REC})$$

$$\cancel{\rho} \cdot \dot{V}_{OA} \cdot \cancel{c_p} \cdot (T_{OA} - T_m) = \cancel{\rho} \cdot \dot{V}_{REC} \cdot \cancel{c_p} \cdot (T_m - T_{REC})$$

$$\dot{V}_{OA} \cdot (T_{OA} - T_m) = \dot{V}_{REC} \cdot (T_m - T_{REC})$$

$$\dot{V}_{OA} \cdot T_{OA} - \dot{V}_{OA} \cdot T_m = \dot{V}_{REC} \cdot T_m - \dot{V}_{REC} \cdot T_{REC}$$

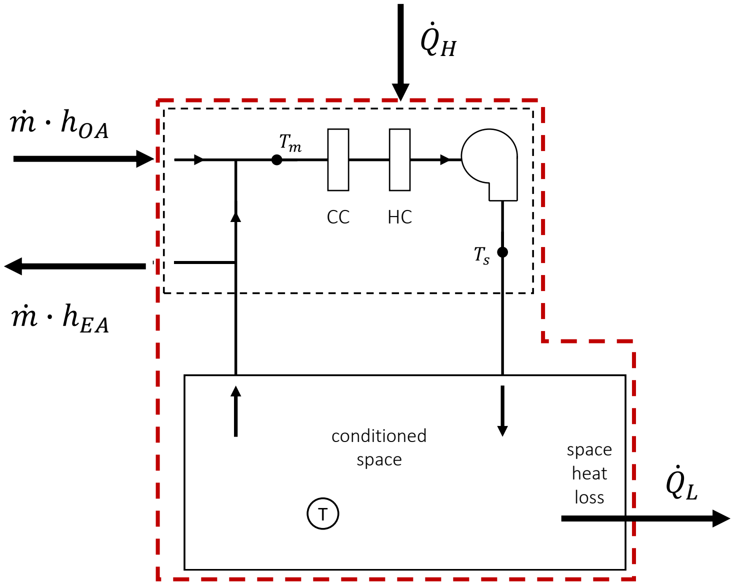
$$\dot{V}_{OA} \cdot T_{OA} + \dot{V}_{REC} \cdot T_{REC} = \dot{V}_{REC} \cdot T_m + \dot{V}_{OA} \cdot T_m$$

$$T_m = \frac{\dot{V}_{OA} \cdot T_{OA} + \dot{V}_{REC} \cdot T_{REC}}{\dot{V}_{OA} + \dot{V}_{REC}}$$

So… that was a lot of work to conclude that (with the assumptions used) the mixed air temperature is the flow weighted average of the two entering stream temperatures!

$$\boldsymbol{T_m} = \frac{(100) \cdot (-10) + (400) \cdot (21)}{100 + 400} = \mathbf{14.8}℃$$

b) Consider the overall system as a control volume:



To keep steady state, the total rate of energy flow in must match the flow out.

$$\dot{m}_{OA} \cdot h_{OA} + \dot{Q}_H = \dot{Q}_L + \dot{m}_{EA} \cdot h_{EA}$$

Both mass flow rates are the same, so:

$$\dot{Q}_H = \dot{Q}_L + \dot{m}_{OA} \cdot (h_{EA} - h_{OA}) = \dot{Q}_L + \dot{V}_{OA} \cdot \rho \cdot c_P \cdot (T_{EA} - T_{OA})$$

$T_{EA} = 21°\mathrm{C}$ (air is exhaust from the conditioned space)

$T_{OA} = -10°\mathrm{C}$

$\rho \cdot c_P = 1.23\ J/L \cdot °\mathrm{C}$

$\dot{Q}_L = 5\ kW$

$$\dot{Q}_H = \dot{Q}_L + \dot{V}_{OA} \cdot \rho \cdot c_P \cdot (T_{EA} - T_{OA})$$

$$= 5000\ W + \left(100\ \frac{L}{s}\right)\left(1.23\ \frac{J}{L \cdot °\mathrm{C}}\right)(21 - (-10)°\mathrm{C}) = 5000\ W + 3183\ W = \mathbf{8813\ W}$$

c)

$\dot{Q}_H = 8813\ W$ and the flow across the coil is 500 L/s.

$$\dot{Q}_H = \dot{V}_{SA} \cdot \rho \cdot c_P \cdot (T_{SA} - T_M)$$

$$T_{SA} = \frac{\dot{Q}_H}{\dot{V}_{SA} \cdot \rho \cdot c_P} + T_M = \frac{8813}{(500)(1.23)} + 14.8 = \mathbf{29.1}℃$$

d)

For an on-off device to provide average output 8.8 kW when its actual output rate when on is 15 kW (and zero when off).

$$f_{on} = \frac{\bar{\dot{E}}}{\dot{E}_{on}} = \frac{8.8}{15} = \mathbf{58.7}\%$$