"Building Energy Analysis" Course Notes – Fall 2021
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EA3: Aspects of Heat Transfer and Psychrometric Analysis

Module Overview

This module provides a background in selected aspects of heat transfer and psychrometric analysis and discusses some aspects of HVAC systems. The content on heat transfer will be important in the upcoming module on Building Envelope. The content on psychrometric analysis is important for the analysis of air-based HVAC systems, including in simulation analysis.

For many students, some material will be a review of concepts previously covered in other courses.

Relevant Course Intended Learning Outcomes:

• Apply basic energy calculations to a variety of components and systems impacting building energy use.

• Apply simple analysis techniques in building energy auditing and simulation.

• Recognize the interactive effects between different building components and systems as related to energy use.

Aspects Of Heat Transfer

(1.) __Thermal Transmittance and Resistance of an Assembly__

Consider steady heat transfer between two environments (at temperatures T_H and T_C) through an assembly of layers whose overall size normal to direction of heat transfer can be characterized by A (m^2). We will restrict this discussion to situations where the heat transfer can be treated as one-dimensional (1D)—that is, temperatures vary only along dimension x (which point normal to the surface).
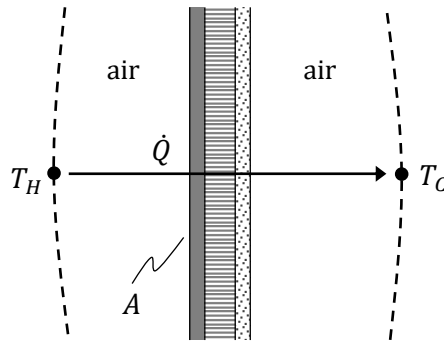
If the steady heat transfer rate is assessed using the thermal resistance method, the system might be represented by a thermal resistor separating two temperature nodes:
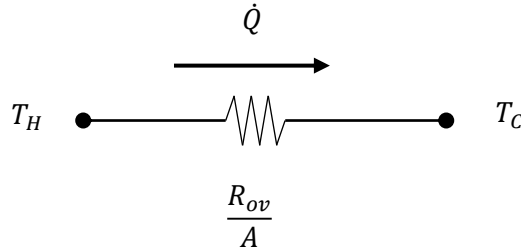
Here R_{ov} is the overall planar thermal resistance of the assembly separating T_H and T_C along the path crossing through area A.

$$\dot{Q} = \frac{A}{R_{ov}} \cdot (T_H - T_C)$$

If expressed as an overall thermal transmittance of the assembly, U, then:

$$\dot{Q} = U \cdot A \cdot (T_H - T_C)$$

U has units $W/m^2 \cdot K$ or $W/m^2 \cdot °C$, with the temperature units (i.e. K or °C) referring to the temperature <u>difference</u> between the two temperatures nodes. (U has the same value whether the units are indicated using K or °C.)

R_{ov} is the inverse of U:

$$R_{ov} = (U)^{-1}$$

Thus, the units of R_{ov} are $m^2 \cdot K/W$, and the units of R_{ov}/A are K/W.

Note that U is an indicator of how readily an assembly allows heat flow (i.e. higher U = higher heat flow), whereas R_{ov} indicates how readily an assembly resists heat flow (i.e. higher R = lower heat flow).

$\boxed{2.}$ **Thermal Resistances of Planar Layers**

<u>1-D Steady Heat Conduction through a Planar Solid</u>

Consider heat conduction occurring normal to the plane of surface area A (i.e. along x-direction only; 1-D heat transfer).

3-D View
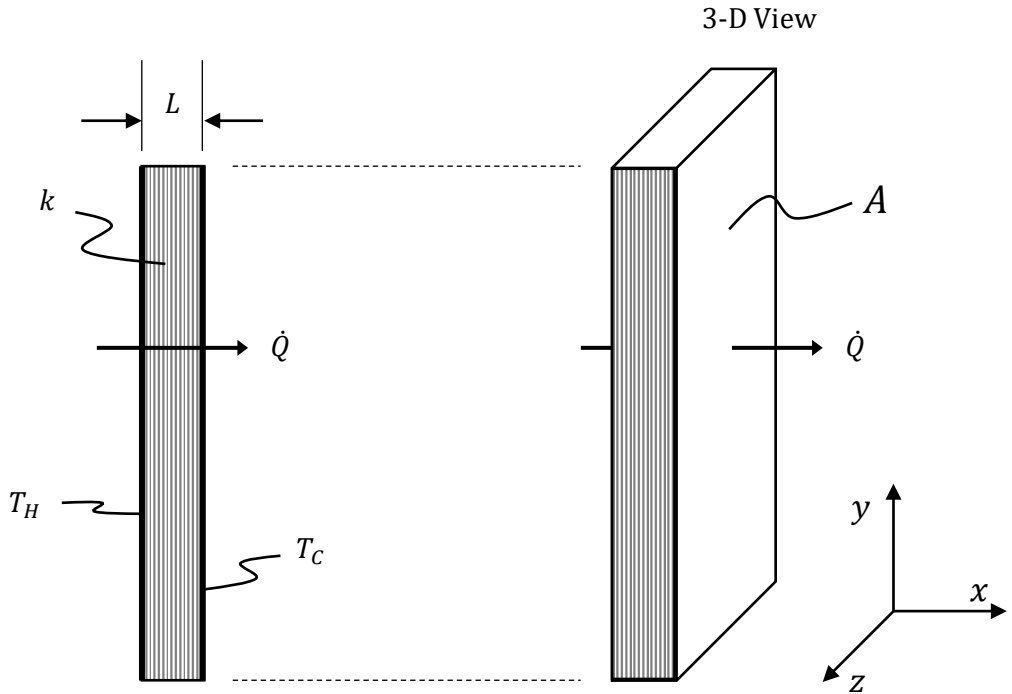
$k = thermal\ conductivity\ of\ medium\ (W/m \cdot K)$

$L = thickness\ of\ material\ (m)$

$$\dot{Q} = \frac{kA}{L} \cdot (T_H - T_C)$$

$$\therefore R = \frac{L}{k} \qquad \longleftarrow \quad \text{planar resistance of the layer}$$

e.g. R-value of layer

Example:

Consider a piece of extruded polystyrene insulation

Thickness = 25 mm

Rated Thermal Resistance: RSI-0.88

$$\text{RSI-value} \quad = 0.88 \; \frac{m^2 \cdot \,^\circ\text{C}}{W} \quad \longleftarrow \quad \frac{L}{k}$$

With this information we can calculate an estimate
of the thermal conductivity of the material.

$L \longrightarrow \;|\; \longleftarrow$

Note: The "R-value" for a material is often listed in Inch-Pound units. For example, the photo below indicates "R-20" for a 4 inch thick insulation board.

The R-20 means the rated thermal resistance is 20 $^\circ\text{F} \cdot ft^2 \cdot hr/btu$. The corresponding metric thermal resistance is called the RSI-value (where the "SI" indicates metric).

RSI = 20 / 5.678 = 3.52 $m^2 \cdot \,^\circ\text{C}/W$.   (5.678 = conversion factor)
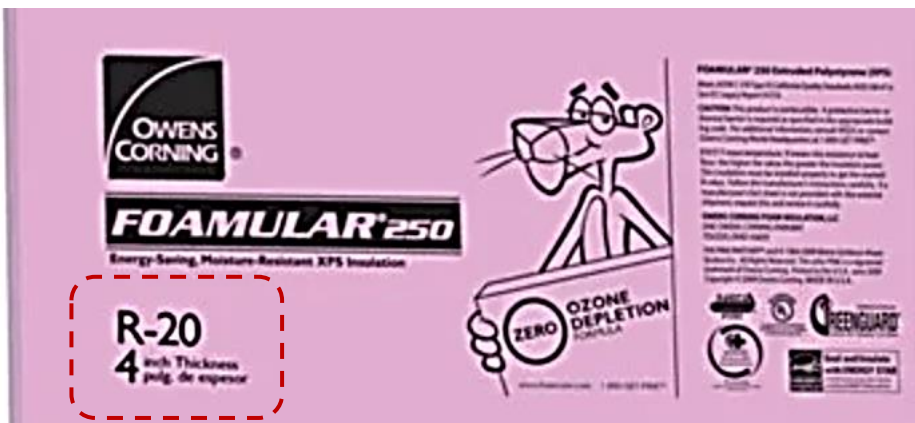
Approximate Thermal Conductivities of Selected Materials
(near "room temperature", i.e. ~300 K)

Material	Thermal conductivity, k $(W/m \cdot °C)$
Aluminum (alloy 2014)	~ 192
Steel (alloy 1080)	~ 48
Stainless steel (alloy 31)	~ 16
Concrete	~ 0.9 to 2.9 (depends on density)
Soda Lime Glass (architectural glass)	~ 1.0
PVC (vinyl)	~ 0.18
Wood (spruce)	~ 0.09
Extruded Polystyrene Insulation (XPS)	~ 0.022 to 0.030

References:

- W.D. Callister, "Materials Sciences and Engineering", 3rd Edition, 1994
- F.P. Incropera and D.P. DeWitt, "Fundamentals of Heat and Mass Transfer", 4th Edition, 1996

Combined Convection and Radiation Heat Transfer

Consider a surface with size A_s and at temperature T_s which is transferring heat to an adjacent environment through simultaneous via convection and radiation.
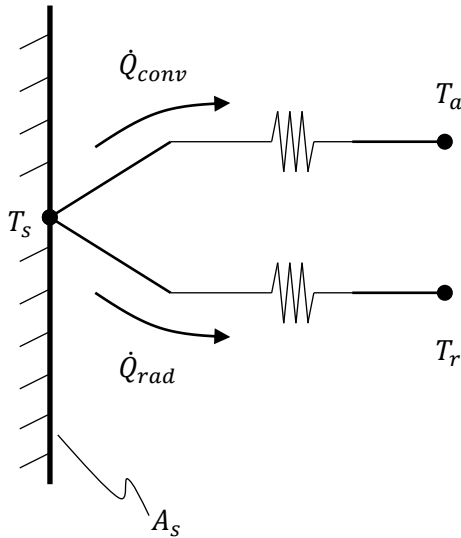
T_a = temperature of the adjacent air (at sufficient distance from wall such that to be "unaffected" by the wall).

T_r= representative temperature of the surroundings that have "line of sight" to the wall for radiant heat exchanger (e.g. hemisphere in view of wall).

The total heat transfer from the wall is:

$$\dot{Q}_T \;=\; \dot{Q}_{conv} \;+\; \dot{Q}_{rad} \;=\; h_{conv} \cdot A_s \cdot (T_s - T_a) \;+\; h_{rad} \cdot A_s \cdot (T_s - T_r)$$

where:

h_{conv} = convection heat transfer coefficient $(W/m^2 \cdot K)$

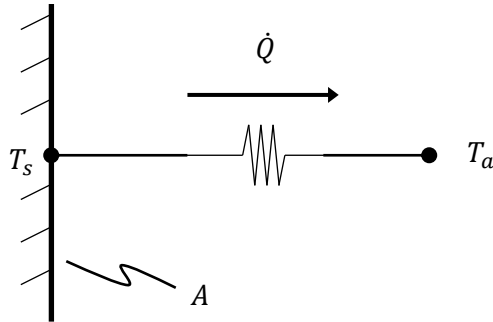h_{rad} = radiation heat transfer coefficient $(W/m^2 \cdot K)$

When assessing heat transfer from a building surface to surroundings, it's often reasonable to treat $T_r \approx T_a$ (i.e. surfaces viewed by the wall have average T close to that of the adjacent air). This can be a reasonable approximation for both interior and exterior surfaces. Applying this to the previous equation:

$$\dot{Q}_T \;=\; (h_{conv} + h_{rad}) \cdot A_s \cdot (T_s - T_a)$$

h_{comb} = combined conv. & rad. heat transfer coefficient, $W/m^2 \cdot K$

Let's now denote the total heat transfer from surface to surroundings simply as $\dot{Q}$ and denote the size of the surface as A (i.e. drop the subscripts). Then:

$$\dot{Q} = h_{comb} \cdot A \cdot (T_s - T_a)$$

The planar resistance of this convection and radiation resistance "layer" separating T_s and T_a:

$$\therefore R = \frac{1}{h_{comb}}$$

Note: Often the subscript "*comb*" is dropped from notation, and the combined heat transfer coefficient is simply denoted as h.

$$R = \frac{1}{h}$$

3. Steady 1D Heat Transmission through a Series of Planar Layers

For steady heat flow between two temperature nodes separated by a series of resistance layers, the total planar resistance is the sum of the layer planar resistances.

For example, heat flow between two environments separated by a two-layer wall:
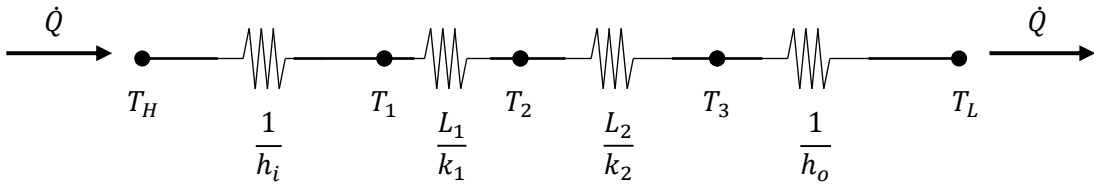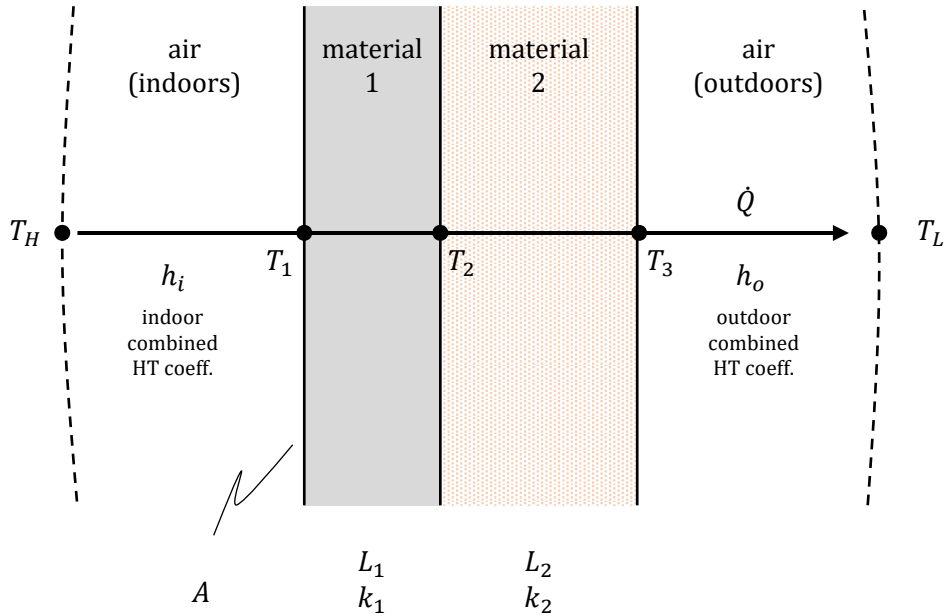
$$R_{ov} = \sum R = \frac{1}{h_i} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_o} = (U)^{-1}$$

$$\dot{Q} = \frac{A}{R_{ov}} \cdot (T_H - T_C) = UA \cdot (T_H - T_C)$$

Example:

Determine the 1-D U-value for a plywood wall (13 mm thick, k ≈ 0.12 W/m·K) if $h_i = 10\ W/m^2 \cdot K$, $h_o = 35\ W/m^2 \cdot K$. Then, estimate the steady heat transfer rate when the indoor and outdoor temperatures are $T_i = 20°C$ and $T_o = -10°C$, respectively, if $A = 50\ m^2$.

$$\frac{1}{h_i} = \frac{1}{10\ W/m^2 \cdot K} = 0.1\ \frac{m^2 \cdot K}{W}$$

$$\frac{L}{k} = \frac{0.013\ m}{0.12\ W/m \cdot K} = 0.108\ \frac{m^2 \cdot K}{W}$$

$$\sum R = R_{ov} = 0.237\ \frac{m^2 \cdot K}{W}$$

$$\frac{1}{h_o} = \frac{1}{35\ W/m^2 \cdot K} = 0.029\ \frac{m^2 \cdot K}{W}$$

$$U = \left(0.237\ \frac{m^2 \cdot K}{W} \right)^{-1} = \mathbf{4.22}\ \frac{\mathbf{W}}{\mathbf{m^2 \cdot K}}$$

$$\dot{Q} = UA \cdot (T_H - T_L) = (4.22\ W/m^2 \cdot K)(50\ m^2)(30K) = \mathbf{6330\ W}$$

4. Parallel Heat Transmission Paths

Consider a scenario where heat is transmitted between two environment through several independent parallel paths. The total heat transfer is the sum of the flows through the separate paths.

The example below shows three surfaces with different UA values. The total steady heat transmission is:

$$\dot{Q}_T = \dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3$$

$$= (UA)_1(T_H - T_L) + (UA)_2(T_H - T_L) + (UA)_3(T_H - T_L)$$

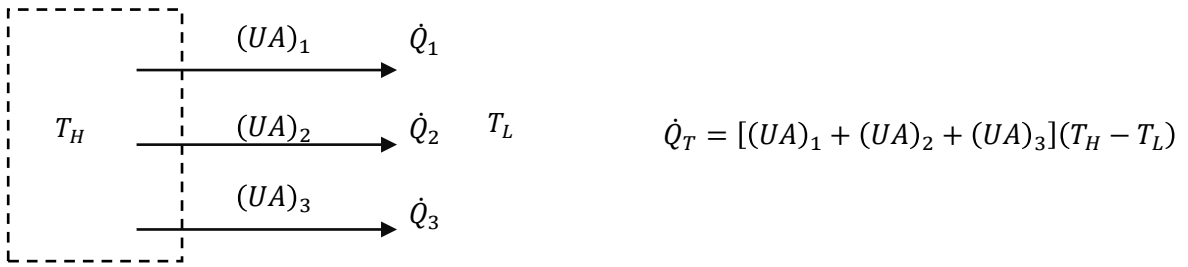$$= [(UA)_1 + (UA)_2 + (UA)_3](T_H - T_L)$$
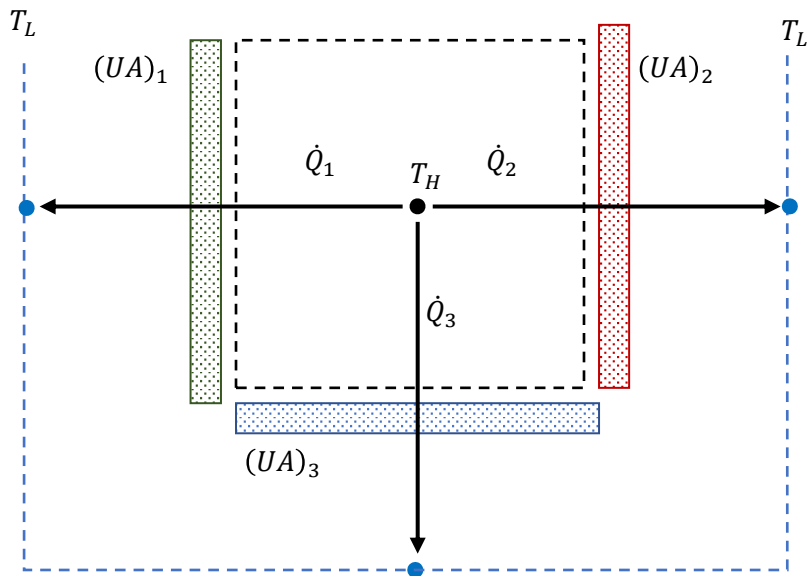
Let: $A_T = A_1 + A_2 + A_3$

An average U-value, U_T, for the overall surface area, A_T, can be determined.

$$\dot{Q}_T = U_T A_T (T_H - T_L)$$

$$U_T = \frac{\dot{Q}_T}{A_T(T_H - T_L)} = \frac{(UA)_1 + (UA)_2 + (UA)_3}{A_T} = \frac{(UA)_1 + (UA)_2 + (UA)_3}{A_1 + A_2 + A_3}$$

A simpler version of the previous diagram:

$$\dot{Q}_T = [(UA)_1 + (UA)_2 + (UA)_3](T_H - T_L)$$

Example Calculation

During a winter night, a room is maintained at 20°C while the outdoor temperature is -10°C. Heat is lost from the room through the walls, windows, and roof.

Assembly	Effective Thermal Transmittance, U (W/m²·K)	Surface Area (m²)
A) Walls	0.40	16
B) Windows	2.00	6
C) Roof	0.20	16

Determine:

1) Steady rate of heating needed to maintain the room temperature (W).

2) Assess the relative contribution of each heat loss path to the total heat loss.

$$(UA)_A = \left(0.4 \; \frac{W}{m^2 \cdot K}\right)(16 \; m^2) = 6.4 \; \frac{W}{K}$$

$$(UA)_B = \left(2.00 \; \frac{W}{m^2 \cdot K}\right)(6 \; m^2) = 12.0 \; \frac{W}{K}$$

$$(UA)_C = \left(0.20 \; \frac{W}{m^2 \cdot K}\right)(16 \; m^2) = 3.2 \; \frac{W}{K}$$

$$\dot{Q}_A = (UA)_A(T_i - T_o) = \left(6.4 \; \frac{W}{K}\right)(30K) = 192 \; W \quad (\approx \textbf{30}\%)$$

$$\dot{Q}_B = (UA)_B(T_i - T_o) = \left(12.0 \; \frac{W}{K}\right)(30K) = 360 \; W \quad (\approx \textbf{55}\%)$$

$$\dot{Q}_C = (UA)_C(T_i - T_o) = \left(3.2 \; \frac{W}{K}\right)(30K) = 96 \; W \quad (\approx \textbf{15}\%)$$

$$\dot{Q}_T = \dot{Q}_A + \dot{Q}_B + \dot{Q}_C = \textbf{648 } \textbf{\textit{W}}$$

(5.) Heat Loss through Parallel Transmission and Ventilation

Consider a room to be kept heated to a steady temperature T_i. The room loses heat due to thermal transmission to the outdoors through two surfaces (e.g, wall and window). It also experiences heat loss due air exchange with the outdoors at rate $\dot{m}$. The air inside the room is sufficiently well mixed such that it can be treated as uniform temperature.
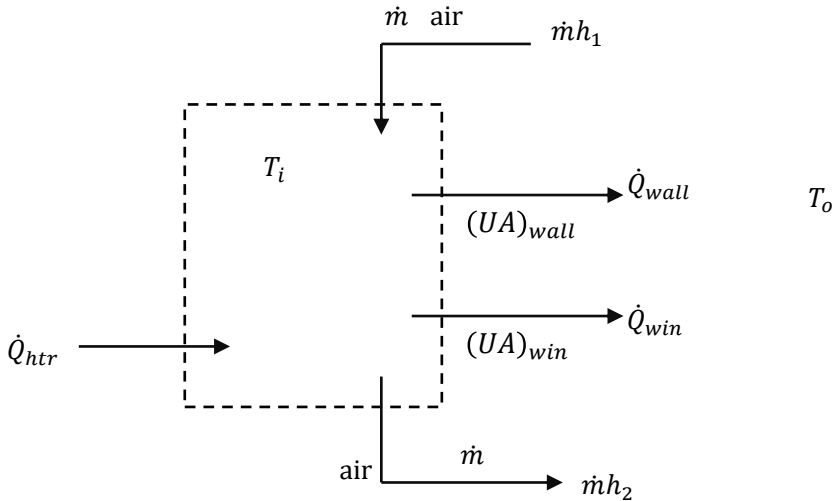
Assess the steady heating rate ($\dot{Q}_{htr}$) needed to maintain the room temperature if there are no other significant energy flows beyond those depicted.

Steady state energy balance for the CV:

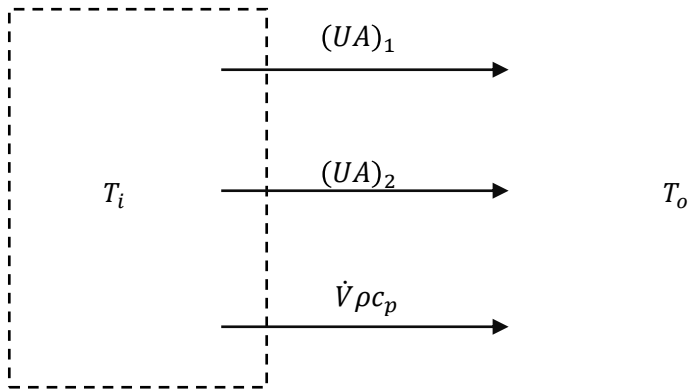$$\dot{Q}_{htr} + \dot{m}h_1 = \dot{m}h_2 + \dot{Q}_{wall} + \dot{Q}_{win}$$

Solve for the heating rate:

$$\dot{Q}_{htr} = \dot{m}(h_2 - h_1) + \dot{Q}_{wall} + \dot{Q}_{win}$$

If $\dot{m}(h_2 - h_1) \approx \dot{V}\rho c_p \cdot (T_i - T_o)$, then:

$$\dot{Q}_{htr} = \dot{V}\rho c_p \cdot (T_i - T_o) + (UA)_{wall} \cdot (T_i - T_o) + (UA)_{win} \cdot (T_i - T_o)$$

$$= \left[\dot{V}\rho c_p + (UA)_{wall} + (UA)_{win} \right] \cdot (T_i - T_o)$$

K_T = overall heat loss factor, W/K

$$\dot{Q}_T = \left[(UA)_1 + (UA)_2 + \dot{V}\rho c_p \right](T_i - T_o) \qquad \text{(total heat loss)}$$

Aspects Of Psychrometric Analysis

6. Nomenclature for Psychrometric Analysis

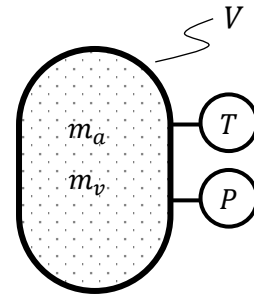Consider a mixture of ideal gases consisting of "dry air" plus water vapor, H_2O.

P = mixture total pressure (absolute)

T = temperature (drybulb) of mixture

V = total volume occupied by mixture
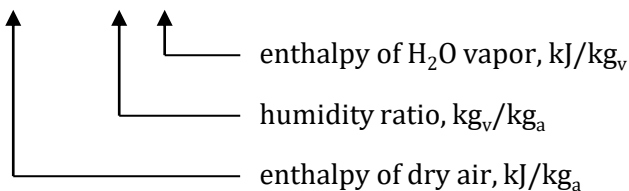
m_a = mass of "dry air" (kg)

m_v = mass of H_2O vapor (kg)

v = specific volume of mixture $\left(\dfrac{m^3}{kg_a}\right) = \dfrac{V}{m_a}$

ω = humidity ratio = $\dfrac{m_v}{m_a}$ $\left(\dfrac{kg_v}{kg_a} \ or \dfrac{g_v}{kg_a}\right)$ $\longleftarrow$ mixture enthalpy per kg_a

h = specific enthalpy of mixture $\left(\dfrac{kJ}{kg_a}\right)$

$h = h_a + \omega \cdot h_v$

— enthalpy of H_2O vapor, kJ/kg_v
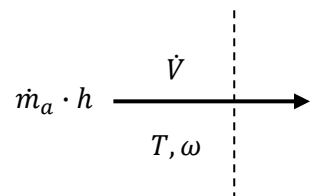
— humidity ratio, kg_v/kg_a

— enthalpy of dry air, kJ/kg_a

Consider a moist airstream flowing past a location where the mixture properties are T, ω and volumetric flowrate is $\dot{V}$.
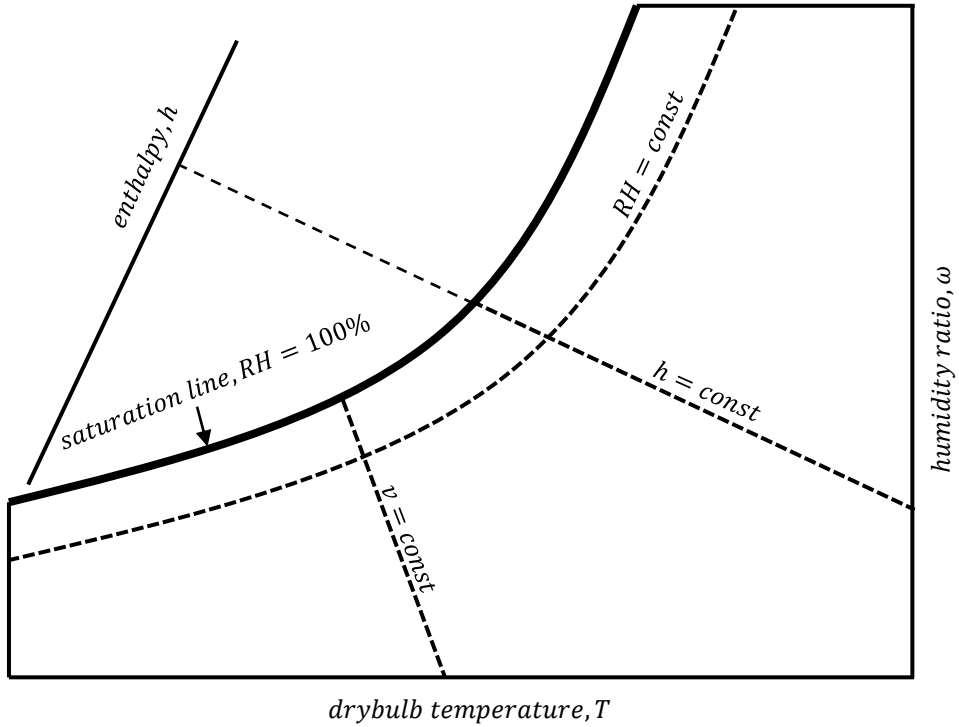
$$\dot{m}_a = \frac{\dot{V}}{v} = mass\ flow\ rate\ of\ dry\ air, kg_a/s$$

$\dot{m}_a \cdot h$

$\dot{V}$

T, ω

$$\dot{E} = \dot{m}_a \cdot h = energy\ flow\ rate\ of\ moist$$
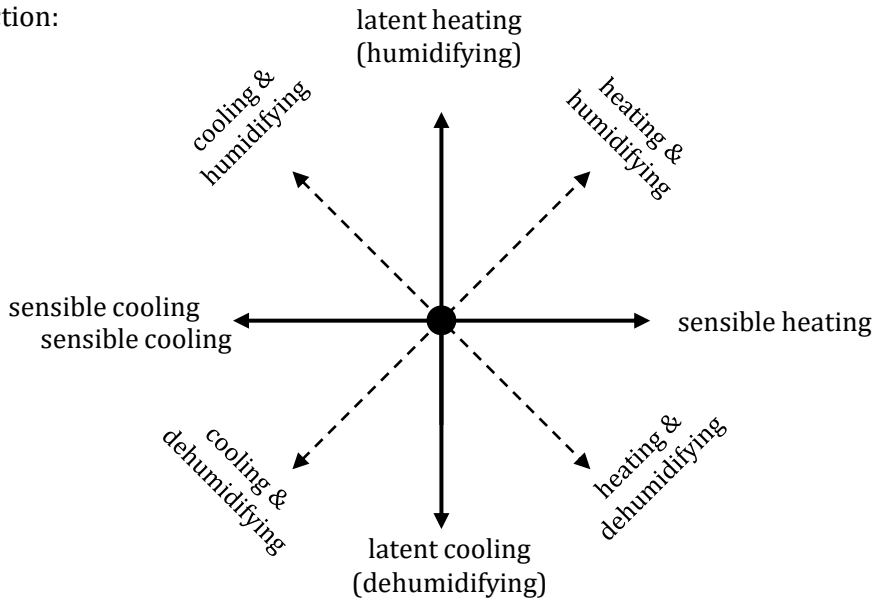$$air\ mixture, kJ/s$$

7. Psychrometric Chart

- The chart allows simple evaluation of moist-air mixture properties.
- Each chart applies to a particular total mixture pressure (i.e. barometric pressure).
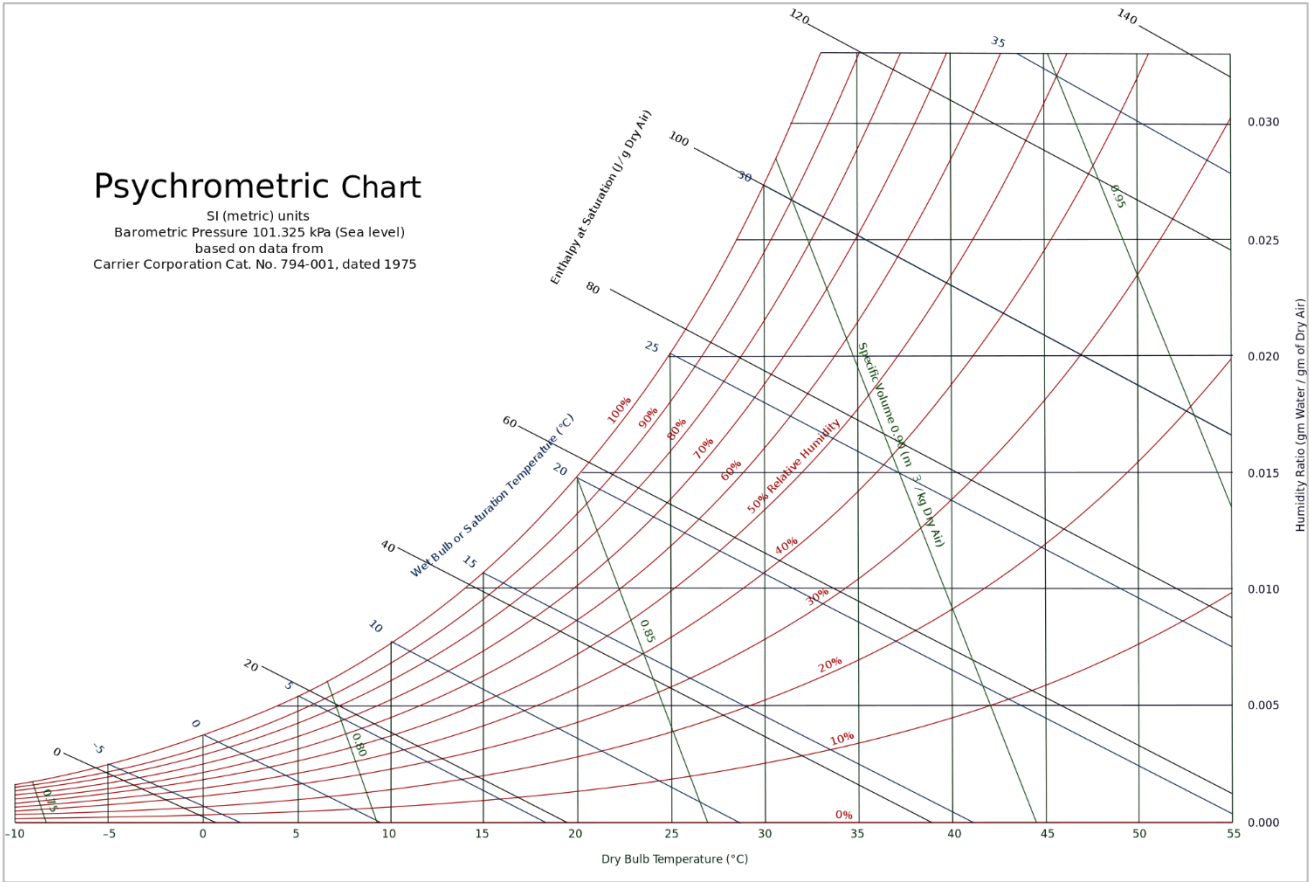

drybulb temperature, T

Process Direction:

Psychrometric Chart

SI (metric) units
Barometric Pressure 101.325 kPa (Sea level)
based on data from
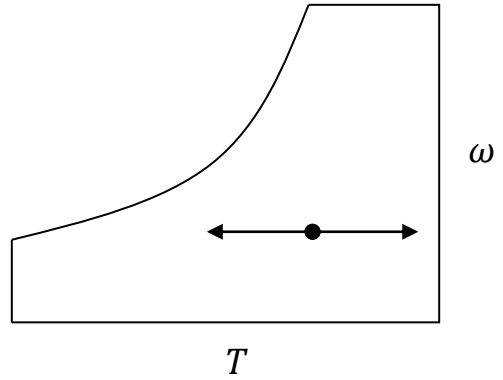Carrier Corporation Cat. No. 794-001, dated 1975

8. **Basic Air-Conditioning Processes**

Sensible heating/cooling

ω

T

Cooling with dehumidification

ω

T

Adiabatic mixing (without condensate production)

ω

T
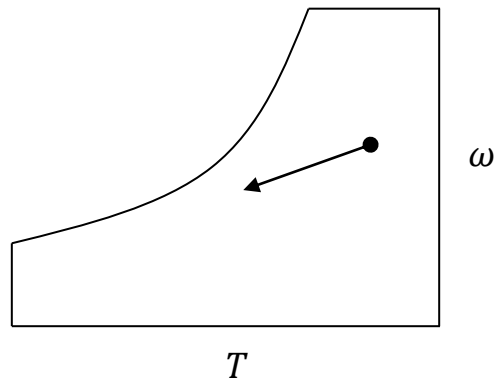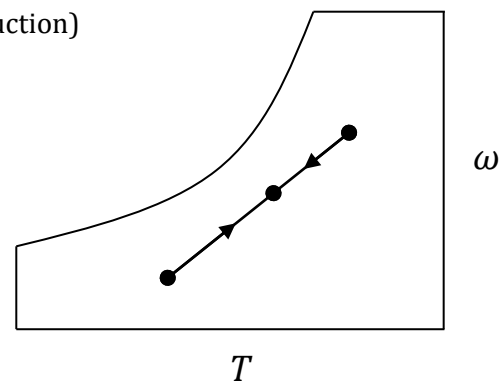
Sensible Heating/Cooling
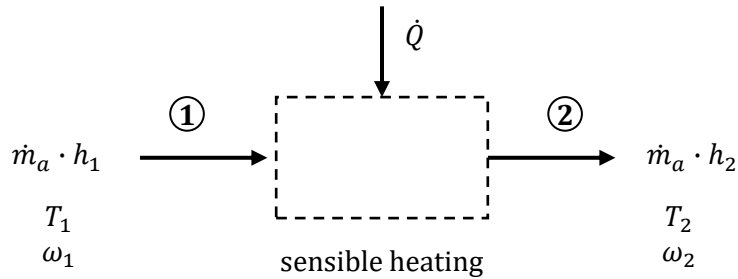
A sensible heating/cooling process means that there is no change in the amount of moisture carried by the air stream, $\omega_1 = \omega_2$.
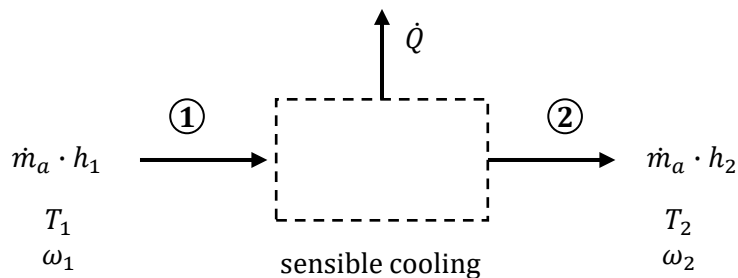
Sensible Heating $\qquad\qquad\qquad\qquad\qquad$ SS-SF, $\Delta ke, \Delta pe \approx 0$

$$\dot{Q}$$

① $\qquad$ ②

$\dot{m}_a \cdot h_1 \longrightarrow \qquad \longrightarrow \dot{m}_a \cdot h_2$

T_1 $\qquad\qquad\qquad\qquad T_2$
ω_1 $\qquad$ sensible heating $\qquad \omega_2$

E-bal: $\qquad \dot{m}_a h_1 + \dot{Q} = \dot{m}_a h_2 \qquad \rightarrow \qquad \boldsymbol{\dot{Q} = \dot{m}_a(h_2 - h_1)}$

Sensible Cooling $\qquad\qquad\qquad\qquad\qquad$ SS-SF, $\Delta ke, \Delta pe \approx 0$

$$\dot{Q}$$

① $\qquad$ ②

$\dot{m}_a \cdot h_1 \longrightarrow \qquad \longrightarrow \dot{m}_a \cdot h_2$

T_1 $\qquad\qquad\qquad\qquad T_2$
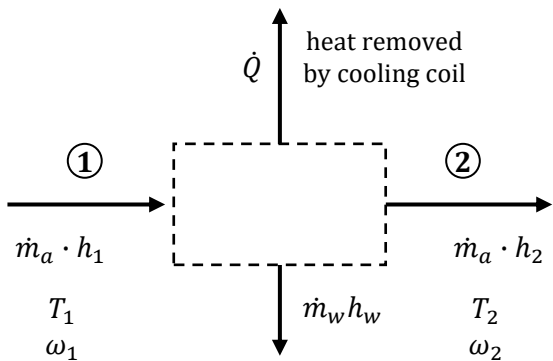ω_1 $\qquad$ sensible cooling $\qquad \omega_2$

E-bal: $\qquad \dot{m}_a h_1 = \dot{Q} + \dot{m}_a h_2 \qquad \rightarrow \qquad \boldsymbol{\dot{Q} = \dot{m}_a(h_1 - h_2)}$

Cooling with Dehumidification

A cooling process will typically result in some of the vapor being carried by the air stream to condense to liquid. Thus, the leaving air stream has lower T and ω than the entering stream.

cooling coil

air in

T_1, ω_1

air out

T_2, ω_2

condensate
out
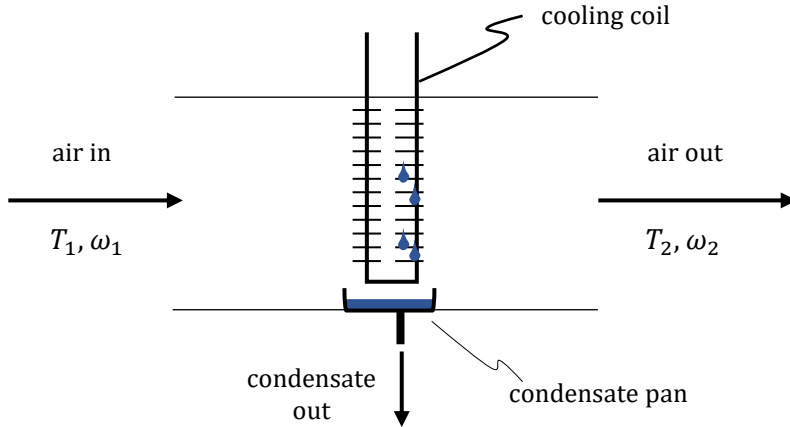
condensate pan

Energy Balance (CV is the cooled air stream):

SS-SF, $\Delta ke, \Delta pe \approx 0$

heat removed
by cooling coil

$\dot{Q}$

① $\dot{m}_a \cdot h_1$

T_1
ω_1

$\dot{m}_w h_w$

② $\dot{m}_a \cdot h_2$

T_2
ω_2

$\dot{m}_w$ = rate of condensate
production (kg$_w$/s)

h_w = enthalpy of condensate
(kJ/kg$_w$)

$$\dot{m}_a \cdot h_1 = \dot{m}_a \cdot h_2 + \dot{Q} + \dot{m}_w h_w \qquad \longrightarrow \qquad \dot{Q} = \dot{m}_a \cdot (h_1 - h_2) - \dot{m}_w h_w$$

Rate of condensate production $\longrightarrow \qquad \dot{m}_w = \dot{m}_a(\omega_1 - \omega_2)$

Typically: $\qquad |\dot{m}_a \cdot (h_1 - h_2)| \gg |\dot{m}_w h_w| \qquad\qquad \therefore \quad \dot{Q} \approx \dot{m}_a \cdot (h_1 - h_2)$

Adiabatic Mixing (without Condensate Production)

Adiabatic mixing occurs when two air streams are mixed and there is negligible heat transfer between the CV and its surroundings. Consider only conditions where the mixing causes no condensate production (i.e. all moisture contained by the entering air streams is carried out by the leaving air stream).
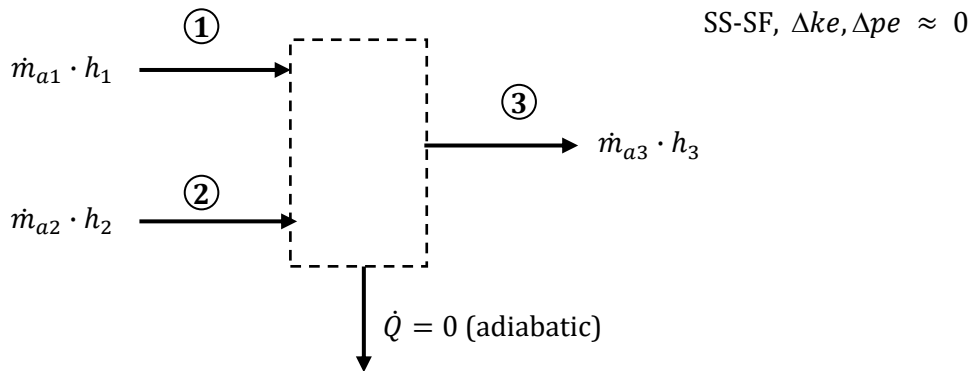
Mass balance (dry air):

$$\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$$

Mass balance (H_2O):

$$\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3} \quad \rightarrow \quad \omega_3 = \frac{\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}}$$

Energy balance (H_2O):

$$\dot{m}_{a1} \cdot h_1 + \dot{m}_{a2} \cdot h_2 = \dot{m}_{a3} \cdot h_3 \quad \rightarrow \quad h_3 = \frac{h_1 \dot{m}_{a1} + h_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}}$$

Note: On the psychrometric chart, State 3 will lie along a line connecting States 1 and 2.