Fundamentals of Business Math

FUNDAMENTALS OF BUSINESS MATH

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INTRODUCTION

The Fundamentals of Mathematics OER textbook has been developed to help students achieve success in mathematics. The OER textbook was adapted from the Foundations of Mathematics (3rd ed.) eTextbook by Thambyrajah Kugathasan, published by Vretta-Lyryx Inc. This text will provide students with learning tools that will help them to improve basic arithmetic skills and to develop reasoning and problem-solving skills.

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ACKNOWLEDGEMENTS

Land Acknowledgement

We wish to acknowledge and express gratitude to Indigenous peoples for their contributions as we work towards reconciliation by learning about the people, the land, and traditional territories on which we work and reside. Conestoga College is located on the traditional territory of the Anishnaabe, Haudenosaunee, and Neutral peoples.

Gratitude and Thanks

We wish to express our gratitude and thanks to the following list of people who supported and participated in this project.

Vretta-Lyrx Inc. has graciously granted Conestoga College permission to adapt their eTextbook *Foundations of Mathematics* (3rd ed.) by Thambyrajah Kugathasan, and release this adaptation under a CC BY-NC-SA 4.0 license. Adaptations include supplementing existing material and reordering chapters.

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Disclaimer

We have done our best to acknowledge all participants involved and with the correct job titles and credentials. In the event that we have made an error please reach out to any one of the authors to have this corrected.

PART I

CHAPTER 1 - WHOLE NUMBERS

Learning Objectives

- Identify whole numbers.
- Read, write, and round whole numbers correctly.
- Solve problems involving arithmetic operations with whole numbers.
- Determine the least common multiple (LCM) and greatest common factor (GCF).
- Perform order of operations with whole numbers.

Chapter Outline

1.1 Understanding Whole Numbers

1.2 Arithmetic Operations with Whole Numbers

1.3 Arithmetic Operations with Signed Numbers

1.4 Factors and Multiples

1.5 Order of Operations

1 Review Exercises

1 Self-Test Exercises

1 Case Study – Lightning Wholesale

Introduction

Arithmetic is the elementary branch of mathematics we use daily, such as buying, selling, estimating expenses, and checking bank balances. When we count, we use arithmetic; when we perform the simple operations of addition, subtraction, multiplication, and division, we use principles of arithmetic. Arithmetic is woven into our general interaction with the real world, and as such, it forms the basis of all science, technology, engineering, and business.

Whole numbers are simply the numbers 0, 1, 2, 3, 4,... They include all counting numbers, also known as natural numbers or positive integers (1, 2, 3, 4,...), and zero (0). All whole numbers are integers. However, whole numbers and integers differ because integers include counting numbers (positive integers) and their negatives (negative integers).
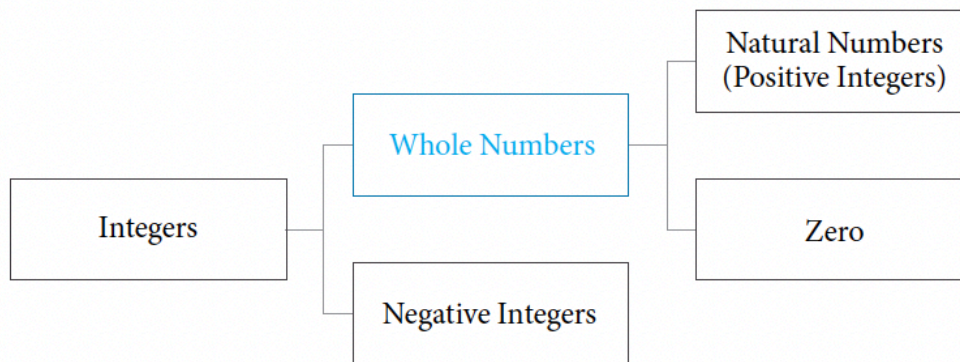
Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of Mathematics* (3rd ed.) by Thambyrajah Kugathasan, published by Vretta-Lyryx Inc., with permission. Adaptations include supplementing existing material and reordering chapters.

1.1 UNDERSTANDING WHOLE NUMBERS

Place Value of Whole Numbers

All numbers can be made up using the digits **0, 1, 2, 3, 4, 5, 6, 7, 8,** and **9**. Numbers may consist of one or more digits. When a number is written using the above digits, it is said to be in **standard form**.

For example, 7, 85, and 2,349 are examples of numbers in their standard form, where 7 is a single-(one) digit number, 85 is a two-digit number, and 2,349 is a four-digit number.

The position of each digit in a whole number determines the **place value** for the digit.

Exhibit 1.1-a illustrates the place value of each of the ten digits in the whole number: 3,867,254,129. In this whole number, 4 occupies the 'thousands' place value and represents 4 thousand (or 4,000), whereas 7 occupies the 'millions' place value and represents 7 million (or 7,000,000).

The place value of 'ones' is 10^0 (= 1), and each position has a value of 10 times the place value to its right, as shown in Table 1.1.

Exhibit 1.1-a Place Value of a Ten-Digit Whole Number
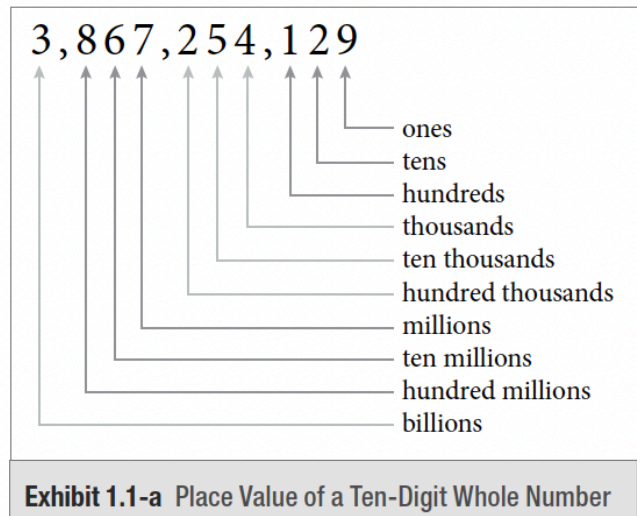
Exhibit 1.1a) Place value of a ten-digit whole number.

Table 1.1: Place Value Chart of Whole Numbers

Exponent Form	Standard Form	Place Value
10^9	1,000,000,000	Billions
10^8	100,000,000	Hundred millions
10^7	10,000,000	Ten millions
10^6	1,000,000	Millions
10^5	100,000	Hundred thousands
10^4	10,000	Ten thousands
10^3	1,000	Thousands
10^2	100	Hundreds
10^1	10	Tens
10^0	1	Ones

We read and write numbers from left to right. A number in standard form is separated into groups of three digits using commas. The vertical red lines in Table 1.1 denote the positions of the commas that separate the groups of three digits, starting from the place value for 'ones'. For example, the ten-digit number in Exhibit 1.1a is written as 3,867,254,129 in its standard form.

Digit	3	8	6	7	2	5	5	1	2	9
Place Value	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones

Numbers can also be written in **expanded form** by writing the number as the sum of what each place value represents.

For example, the number 3,867,254,129 in standard form can be written in expanded form as follows:

3,000,000,000 + 800,000,000 + 60,000,000 + 7,000,000 + 200,000 + 50,000 + 4,000 + 100 + 20 + 9

Or,

3 billion + 800 million + 60 million + 7 million + 200 thousand + 50 thousand + 4 thousand + 1 hundred + 2 tens + 9 ones

Example 1.1a: Identifying the Place Value of a Digit and the Amount it Represents

What is the place value of the digit 5 in each of the following numbers and what amount does it represent?

1. $2,543
2. $75,342
3. $6,521,890
4. $915,203,847

Solution for Example 1.1a

Dollar Amount	Place Value	Amount Represented
1. $2,**5**43	Hundreds	$500
2. $7**5**,342	Thousands	$5,000
3. $6,**5**21,890	Hundred thousands	$500,000
4. $91**5**,203,847	Millions	$5,000,000

Example 1.1b: Identifying the Digit of a Number Given its Place Value

In the number 5,320,948, identify the digit that occupies the following place values:

1. Hundred thousands
2. Ten thousands

3. Thousands
4. Tens
5. Hundreds
6. Millions

Solution for Example 1.1b

Place Value	Digit Located in Place Value
1. Hundred thousands	3
2. Ten thousands	2
3. Thousands	0
4. Tens	4
5. Hundreds	9
6. Millions	5

Example 1.1c: Identifying the Digit of a Number Given its Place Value

Write the following numbers in expanded form:

1. 698
2. 8,564
3. 49,005
4. 521,076
5. 9,865,323

6. 43,583,621

Solution for Example 1.1c

Number	Expanded Form
1. 698	600 + 90 + 8
2. 8,564	8,000 + 500 + 60 + 4
3. 49,005	40,000 + 9,000 + 5
4. 521,076	500,000 + 20,000 + 1,000 + 70 + 6
5. 9,865,323	9,000,000 + 800,000 + 60,000 + 5,000 + 300 + 20 + 3
6. 43,583,621	40,000,000 + 3,000,000 + 500,000 + 80,000 + 3,000 + 600 + 20 + 1

Reading and Writing Whole Numbers

To make it easier to read and write numbers, any number larger than three digits is separated into smaller groups of three digits, starting from the last digit of the number. Each of these groups of three digits has a name.

- The first group of three digits on the right is the "**Units**" group.
- The second group from the right is the "**Thousands**" group.
- The third group from the right is the "**Millions**" group.
- The fourth group from the right is the "**Billions**" group.
- The fifth group from the right is the "**Trillions**" group and so on, as shown in the following chart.

Trillions			Billions			Millions			Thousands			Units		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

Follow these steps to write large numbers in **word form:**

1. Start from the group furthest to the left and write the number formed by the digits in that group, followed by the group's name.
2. Moving to the next group (to the right), write the numbers formed by this next group, followed by its name. Continue to do this for each of the groups.
3. For the last group (i.e., the group furthest to the right), write the numbers formed by the group; however, for this group, do not write the name of it.

Note: When a group contains all zeros, that group is neither read nor written.

For example, 2,835,197,000,642 expressed in word form using the above rules would be as follows:

Trillions			Billions			Millions			Thousands			Units		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
		2	8	3	5	1	9	7	0	0	0	6	4	2
Two trillion,			eight hundred thirty-five billion,			one hundred ninety-seven million,						six hundred forty-two		

When writing numbers in word form, the names of the groups remain in their singular forms, irrespective of the number preceding, i.e., hundred, thousand, million, billion, trillion, etc.
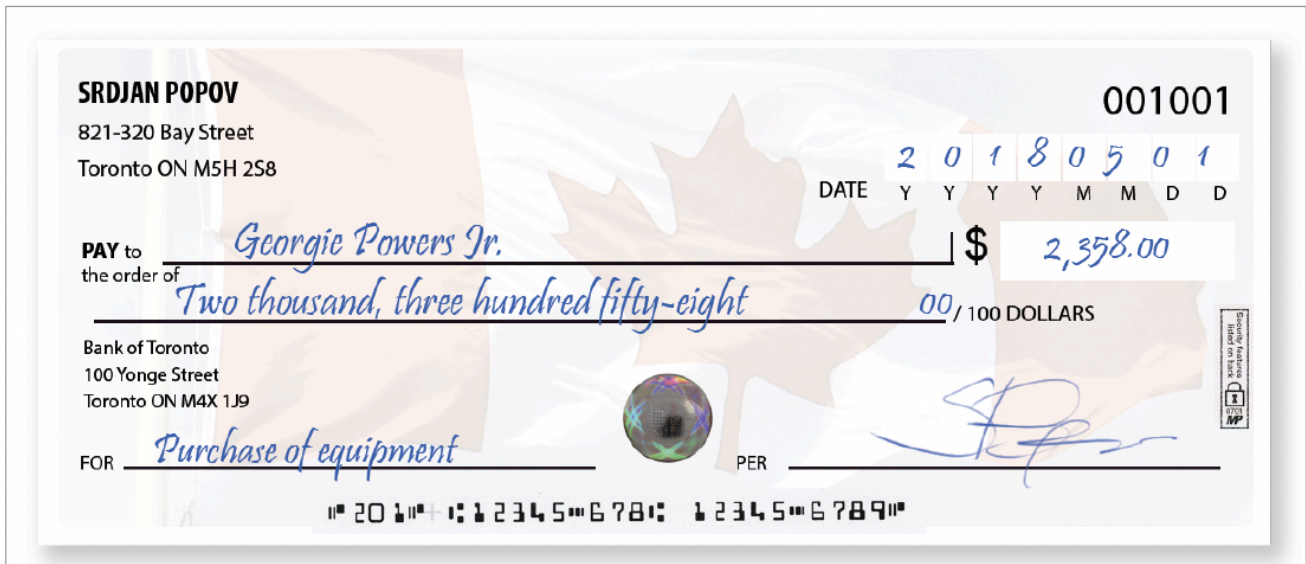
Exhibit 1.1b) Sample of a cheque showing a number expressed in it's standard form and word form.

Example 1.1d: Writing Numbers in Word Form Given their Standard Form

Write the following numbers in word form:

1. 743

2. 5,006

3. 15,017

4. 800,629

5. 6,783,251

6. 52,630,042

Solution for Example 1.1d

Number	Word Form
1. 743	Seven hundred forty-three
2. 5,006	Five thousand, six
3. 15,017	Fifteen thousand, seventeen
4. 800,629	Eight hundred thousand, six hundred twenty-nine
5. 6,783,251	Six million, seven hundred eighty-three thousand, two hundred fifty-one
6. 52,630,042	Fifty-two million, six hundred thirty thousand, forty-two

Example 1.1e: Writing Numbers In Standard Form Given their Word Form

Write the following numbers in standard form:

1. Two hundred five

2. Six thousand, four

3. Thirty-five thousand, eight hundred twenty-five

4. Eight hundred thousand, five

5. Two million, three hundred forty-two thousand, six hundred seventeen

6. Half of a million

7. One-quarter of a billion

Solution for Example 1.1e

| Number in Word Form	Number in Standard Form
1. Two hundred five	205
2. Six thousand, four	6,004
3. Thirty-five thousand, eight hundred twenty-five	35,825
4. Eight hundred thousand, five	800,005
5. Two million, three hundred forty-two thousand, six hundred seventeen	2,342,617
6. Half of a million	$1,000,000 \div 2 = 500,000$
7. One-quarter of a billion	$1,000,000,000 \div 4 = 250,000,000$

Representing Whole Numbers on a Number Line

As shown below, whole numbers can be represented graphically as a point on a horizontal line, called the number line.

The arrowhead at the end shows that the line continues indefinitely in that direction.
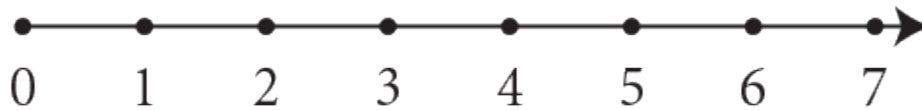
The smallest whole number is zero (0). It is not possible to find the largest whole number because, for any given number, there will always be another number greater than that number.

Writing numbers on a number line helps in comparing and identifying numbers that are smaller or larger than other numbers. Numbers that lie to the left of a number on the number line are less than (i.e., smaller than) that number, and those that lie to the right on the number line are greater than (i.e., larger than) that number.

For example,

- 6 is greater than 2 (or 2 is less than 6).
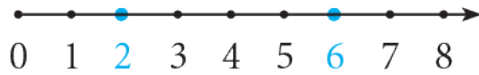
- 5 is less than 7 (or 7 is greater than 5).
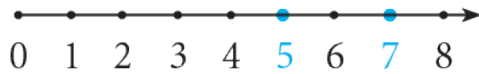
The signs used to show the relative position of two numbers (or quantities) are:

- ' > ' read as **"greater than"**, meaning that the number on the left of the sign has a value greater than that on the right.
  For example, "6 is greater than 2" is written as 6 > 2.
  This is the same as "2 is less than 6, " written as 2 < 6.

- ' < ' read as **"less than"**, meaning that the number on the left of the sign has a value less than that on the right.
  For example, "5 is less than 7" is written as 5 < 7.
  This is the same as "7 is greater than 5", written as 7 > 5.

Example 1.1f: Plotting Numbers on a Number Line and Using Signs to Show the Relative Positions of the Numbers

Plot the following numbers on a number line and place the correct sign of inequality, '>' or '<', in the space (indicated by underscore) between the numbers.

1. 7 _ 11
2. 7 _ 5

3. 11 _ 5

4. 5 _ 12

5. 3 _ 5

6. 12 _ 11

Solution

1. 7 < 11

2. 7 > 5
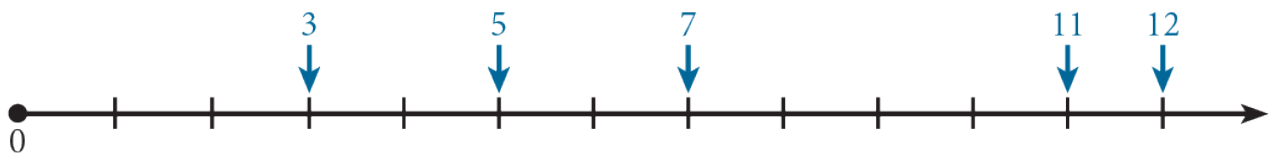
3. 11 > 5

4. 5 < 12

5. 3 > 5

6. 12 > 11

Example 1.1g: Writing a Statement to Represent '>' or '<'

Write statements using the words "greater than" or "less than" for the following expressions:

1. 24 > 22

2. 36 < 39

3. 9 > 0

4. 0 < 5

Solution for Example 1.1g

1. 24 is greater than 22, or 22 is less than 24.

2. 36 is less than 39, or 39 is greater than 36.

3. 9 is greater than 0, or 0 is less than 9.

4. 0 is less than 5, or 5 is greater than 0.

Rounding Whole Numbers

Rounding numbers makes them easier to work with and easier to remember. Rounding changes some of the digits in a number but keeps its value close to the original. It is used in reporting large quantities or values that change often, such as population, income, expenses, etc.

For example, the population of Canada is approximately 37 million, or Henry's car expense for this month is approximately $700.

Rounding numbers also makes arithmetic operations faster and easier, especially when determining the exact answer is not required.

For example, if you are required to estimate the area of a rectangular plot of land that measures 114 m by 97 m, you would have to multiply 114 × 97, resulting in 11,058 m^2. However, rounding the measurements to the nearest ten can provide a quick estimate.

Rounding Whole Numbers to the Nearest Ten, Hundred, Thousand, etc.

Rounding whole numbers refers to changing the value of the whole number to the nearest ten, hundred, thousand, etc. It is also called rounding whole numbers to a multiple of 10, 100, 1,000, etc.

- Rounding a whole number to the nearest ten is the same as rounding it to a multiple of 10.
- Rounding a whole number to the nearest hundred is the same as rounding it to a multiple of 100.
- Rounding an amount to the nearest $1,000 is the same as rounding the amount to a multiple of $1,000.

For example, rounding the measurements of the above-mentioned plot of land to the nearest ten (or multiple of 10):

- Rounding 114 to the nearest ten results in 110. 114 is closer to 110 than 120. Therefore, round down to 110.

110 ← 114   120
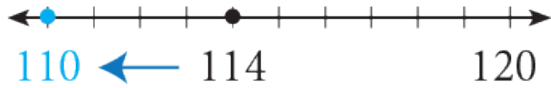
- Rounding 97 to the nearest ten results in 100. 97 is closer to 100 than 90. Therefore, round up to 100.

90   97 → 100
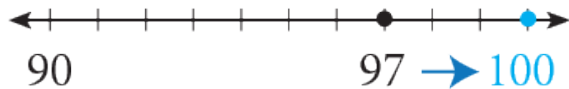
Therefore, rounding the measurements to the nearest ten results in an estimated area of 110 m × 100 m = 11,000 m^2.

Follow these steps to round whole numbers:

1. Identify the digit to be rounded (this is the place value for which the rounding is required).
2. If the digit to the immediate right of the required rounding digit is less than 5 (0, 1, 2, 3, 4), do not change the value of the rounding digit.
   If the digit to the immediate right of the required rounding digit is 5 or greater than 5 (5, 6, 7, 8, 9), increase the value of the rounding digit by one (i.e., round up by one number).
3. Change the value of all digits to the right of the rounding digit to 0.

Example 1.1-h: Rounding Numbers Using a Number Line (Visual Method)

Round the following numbers to the indicated place value using a number line:

1. 624 to the nearest ten (multiple of 10).
2. 150 to the nearest hundred (multiple of 100).
3. 1,962 to the nearest hundred (multiple of 100).

Solution

a. 624 to the nearest ten (multiple of 10). 624 is closer to 620 than it is to 630. Therefore, 624 rounded to the nearest ten is 620.

620   624   630

0
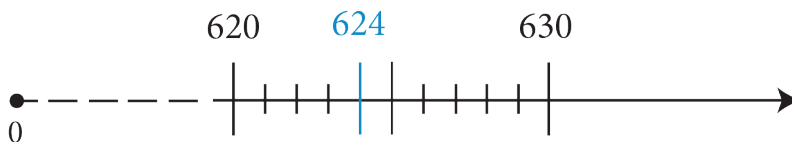
b. 150 to the nearest hundred (multiple of 100). 150 is exactly midway between 100 and 200. By convention, if a number is exactly in the middle, we round up. Therefore, 150 rounded to the nearest hundred is 200.

100   150   200

0
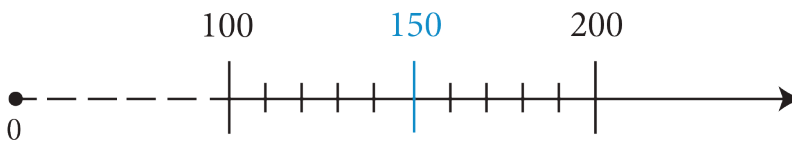
c. 1,962 to the nearest hundred (multiple of 100). 1,962 is closer to 2,000 than it is to 1,900. Therefore, 1,962 rounded to the nearest hundred is 2,000.
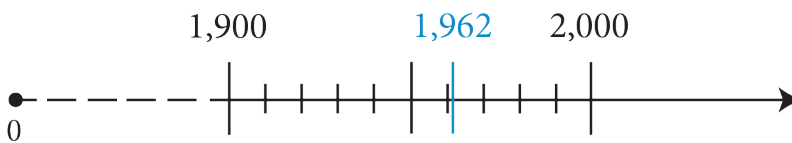
1,900   1,962   2,000

0

Follow these steps to round whole numbers:

1. Identify the digit to be rounded (this is the place value for which the rounding is required).
2. If the digit to the immediate right of the required rounding digit is less than 5 (0, 1, 2, 3, 4), do not change the value of the rounding digit.
   If the digit to the immediate right of the required rounding digit is 5 or greater than 5 (5, 6, 7, 8, 9), increase the value of the rounding digit by one (i.e., round up by one number).
3. Change the value of all digits to the right of the rounding digit to 0.

Example 1.1-i: Rounding to Indicated Place Values

Round the following numbers to the indicated place value using a number line:

1. $568 to the nearest $10.
2. $795 to the nearest $10.
3. $5,643 to the nearest $100.
4. $19,958 to the nearest $100.

Solution

a. Rounding $568 to the nearest $10:

Identify the rounding digit in the tens place: 568 (6 is the digit in the tens place).
The digit to the immediate right of the rounding digit is 8, which is greater than 5; therefore, increase the value of the rounding digit by one, from 6 to 7, and change the value of the digits that are to the right of the rounding digit to 0, which will result in 570.
Therefore, $568 rounded to the nearest $10 (or multiple of $10) is $570.

b. Rounding $568 to the nearest $10:

Identify the rounding digit in the tens place: 568 (6 is the digit in the tens place).
The digit to the immediate right of the rounding digit is 8, which is greater than 5; therefore, increase the value of the rounding digit by one, from 6 to 7, and change the value of the digits that are to the right of the rounding digit to 0, which will result in 570.
Therefore, $568 rounded to the nearest $10 (or multiple of $10) is $570.
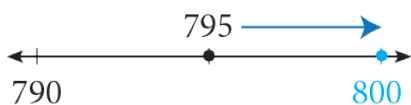
c. Rounding $568 to the nearest $10:

Identify the rounding digit in the tens place: 568 (6 is the digit in the tens place).

The digit to the immediate right of the rounding digit is 8, which is greater than 5; therefore, increase the value of the rounding digit by one, from 6 to 7, and change the value of the digits that are to the right of the rounding digit to 0, which will result in 570.

Therefore, $568 rounded to the nearest $10 (or multiple of $10) is $570.

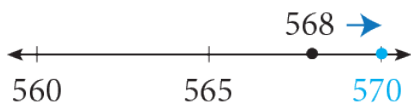d. Rounding $568 to the nearest $10:

Identify the rounding digit in the tens place: 568 (6 is the digit in the tens place).

The digit to the immediate right of the rounding digit is 8, which is greater than 5; therefore, increase the value of the rounding digit by one, from 6 to 7, and change the value of the digits that are to the right of the rounding digit to 0, which will result in 570.

Therefore, $568 rounded to the nearest $10 (or multiple of $10) is $570.
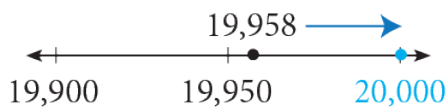
1.1 Exercises

Answers to the odd-numbered questions are available at the end of the book.

For Problems 1 to 4, (i) write the place value of the underlined digit and (ii) the value it represents.

1. a. 4,7**92**

    b. 5,**3**52

    c. 45,**7**21

2. a. 7,**6**28

    b. 4,6**87**

    c. **9**4,083

3. a. 3**1**9,526

    b. 7,**82**5,500

    c. **16**,702,555

4. a. 204,095

    b. 35,217,123

    c. 4,385,207

For Problems 5 to 10, (i) write the numbers in their expanded form and (ii) word form.

5. a. 407

    b. 2,056

6. a. 860

    b. 7,805

7. a. 29,186

    b. 464,448

8. a. 94,975

    b. 684,137

9. a. 2,604,325

    b. 15,300,604

10. a. 9,084,351

    b. 23,006,045

For Problems 11 to 16, (i) write the numbers in their standard form and (ii) word form.

11. a. 600 + 70 + 9

    b. 3,000 + 100 + 40 + 7

12. a. 400 + 50 + 6

    b. 1,000 + 900 + 30 + 2

13. a. 2,000 + 600 + 5

    b. 9,000 + 20 + 4

14. a. 5,000 + 300 + 1

    b. 7,000 + 80 + 8

15. a. 40,000 + 900 + 90

   b. 10,000 + 50 + 3

16. a. 60,000 + 700 + 80

   b. 20,000 + 100 + 4

For Problems 17 to 24, (i) write the numbers in their standard form and (ii) in expanded form.

17. a. Five hundred seventy

   b. Eight hundred three

18. a. One thousand, five

   b. Seven thousand, twenty

19. a. Eighty thousand, six hundred thirty

   b. Seventy-five thousand, twenty-five

20. a. Sixty-five thousand, two hundred forty-four

   b. Eight hundred thirty-three thousand, six hundred forty-one

21. a. Twelve million, four hundred fifty-two thousand, eight hundred thirty-two

   b. Thirty-two million, six hundred eighty-four thousand, two hundred fifty-six

22. a. Two billion, one thousand

   b. One billion, twenty-five thousand

23. a. One-eighth of a million

   b. One-quarter of a million

24. a. Half of a billion

   b. One-tenth of a billion

For Problems 25 and 26, plot the numbers on a number line.

25. a. 14, 19, 15, 7

   b. 12, 8, 17, 5

26. a. 18, 9, 6, 11

b. 4, 10, 7, 16

For Problems 27 and 28, place the correct sign '>' or '<' in the space between the numbers (indicated by underscore).

27. a. 7 _ 15

   b. 19 _ 14

   c. 0 _ 5

   d. 19 _ 0

28. a. 12 _ 17

   b. 8 _ 5

   c. 17 _ 0

   d. 0 _ 8

For Problems 29 and 30, express the relationship between the numbers using the statements (i) "less than" and (ii) "greater than".

29. a. 6 < 9

   b. 18 > 11

   c. 5 < 11

   d. 11 > 0

30. a. 4 < 7

   b. 16 > 7

   c. 10 < 16

   d. 0 < 4

For Problems 31 to 34, arrange the numbers in order from least to greatest.

31. a. 87; 108; 99; 103; 96

   b. 159; 141; 108; 139; 167

32. a. 58; 129; 147; 49; 68

b. 836; 820; 805; 873; 875

33. a. 2,067; 2,040; 2,638; 2,533

   b. 79,487; 79,534; 79,468; 78,812

34. a. 2,668; 2,630; 2,579; 2,759

   b. 68,336; 69,999; 69,067; 68,942

For Problems 35 and 36, create the least and greatest possible numbers using all the given digits.

35. a. 9, 2, 5

   b. 7, 9, 1, 8

   c. 3, 5, 4, 8

36. a. 6, 1, 7

   b. 9, 4, 8, 5

   c. 4, 7, 2, 6, 5

For Problems 37 and 38, round the numbers to the nearest ten, nearest hundred, and nearest thousand.

37.

Number		Nearest Ten	Nearest Hundred	Nearest Thousand
a.	524			
b.	1,645			
c.	53,562			
d.	235,358			

38.

Number		Nearest Ten	Nearest Hundred	Nearest Thousand
a.	895			
b.	9,157			
c.	25,972			
d.	139,835			

For Problems 39 and 40, round the numbers to nearest ten thousand, nearest hundred thousand, and nearest million.

39.

Number	Nearest Ten Thousand	Nearest Hundred Thousand	Nearest Million
a. 875,555			
b. 1,656,565			
c. 3,368,850			
d. 4,568,310			

40.

Number	Nearest Ten Thousand	Nearest Hundred Thousand	Nearest Million
a. 759,850			
b. 3,254,599			
c. 7,555,450			
d. 2,959,680			

Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of Mathematics* (3rd ed.) by Thambyrajah Kugathasan, published by Vretta-Lyryx Inc., with permission. Adaptations include supplementing existing material and reordering chapters.

1.2 ARITHMETIC OPERATIONS WITH WHOLE NUMBERS

Addition of Whole Numbers

The addition of whole numbers refers to combining two or more whole numbers to find the total.

The numbers that are added are referred to as the **addends**, and the result or answer is called the **total** or **sum**. The symbol '+' denotes addition.

For example, 9 + 5 refers to adding 9 and 5. This is read as 'nine plus five'.

Addition can be performed in any order, and the sum will be the same.

A + B = B + A

For example, 9 + 5 = 14 and 5 + 9 = 14

This is known as the **cumulative property** of addition.

Follow these steps to add whole numbers:

1. Write the numbers one under the other by aligning the place values (ones, tens, hundreds, etc.) of the numbers and draw a horizontal line underneath.
2. Starting from the ones place value, add all the numbers in the column.

    ▪ If the total is less than 10, write the total under the horizontal line in the same column.
    ▪ If the total is 10 or more, write the 'ones' digit under the horizontal line in the same column, and write the 'tens' digit above the column to the left. This is called 'carrying'.

3. Add the numbers in the tens column, followed by the hundreds column, etc., by following the same procedure for each column.

Example 1.2-a: Adding Whole Numbers

Perform the following additions:

a. 3,514 + 245
b. 8,578 + 3,982 + 564 + 92

Solution

a. 3,514 + 245

The sum of the digits in the **ones column** is 9 since 4 + 5 = 9. There are 9 ones, so we write 9 in the ones column below the horizontal line. The sum of the digits in the **tens column** is 5 since 10 + 40 = 50. There are 5 tens, so we write 5 in the tens column below the horizontal line. The sum of the digits in the **hundreds column** is 7 since 500 + 200 = 700. There are 7 hundreds, so we write 7 in the hundreds column below the horizontal line. As only 3 is in the thousands column, the sum of the digits in the **thousands column** is 3. Write 3 in the thousands column below the horizontal line. Therefore, adding 3,514 and 245 results in 3,759.

Thousands	Hundreds	Tens	Ones
3	5	1	4
+	2	4	5
3	7	5	9

b. 8,578 + 3,982 + 564 + 92

The sum of the digits in the **ones column** is 16 since 8 + 2 + 4 + 2 = 16, which is 1 ten and 6 ones. Write 6 in the ones column below the horizontal line and carry the 1 above the tens column. The sum of the digits in the **tens column** is 31 since 70 + 80 + 60 + 90 + **10** (carried from the ones column) = 310, which is 3 hundreds and 1 ten. Write 1 in the tens column below the horizontal line and carry the 3 above the hundreds column. The sum of the digits in the **hundreds column** is 22 since 500 + 900 + 500 + **300** (carried from the tens column) = 2,200, which is 2 thousands and 2 hundreds. Write 2 in the hundreds column below the horizontal line and carry the 2 above the thousands column. The sum of the digits in the **thousands column** is 13 since 8,000 + 3,000 + **2,000** (carried from the hundreds column) = 13,000, which is 1 ten thousand and 3 thousands. Write 3 in the thousands column below the horizontal line and 1 in the ten thousands column. Therefore, adding 8,578, 3,982, 564, and 92 results in 13,216.

Ten Thousands	Thousands	Hundreds	Tens	Ones
	2	3	1	
	8	5	7	8
	3	9	8	2
		5	6	4
+			9	2
1	3	2	1	6

Subtraction of Whole Numbers

Subtraction of whole numbers refers to finding the difference between numbers. This is the reverse process of addition.

The number from which another number is subtracted is referred to as the **minuend,** and the number being subtracted is referred to as the **subtrahend**. The result or answer is called the **difference**. The symbol '–' denotes subtraction.

For example, 8 – 5 refers to subtracting 5 from 8. This is read as 'eight minus five'.

$$\text{Minus sign} \longrightarrow \begin{array}{r} 8 \\ -\ 5 \\ \hline 3 \end{array} \begin{array}{l} \longleftarrow \text{Minuend} \\ \longleftarrow \text{Subtrahend} \\ \longleftarrow \text{Difference} \end{array}$$

Subtraction must be written in the written order.

A – B does not equal B – A

For example $8 - 5 = 3$ but $5 - 8 = -3$

Follow these steps to subtract a whole number from another whole number:

1. Write the numbers one under the other by aligning the place values (ones, tens, hundreds, etc.) of the numbers and draw a horizontal line underneath. Ensure that the number being subtracted from (the minuend) is in the top row and that the number being subtracted (the subtrahend) is below.

2. Starting from the ones place value, subtract the bottom number from the top number.

   ▪ If the top digit is greater than (or equal to) the bottom digit, write the difference under the horizontal line in the same column.
   ▪ If the top digit is less than the bottom digit, borrow 'one' from the digit to the left in the top number, and add 'ten' to the digit in the current place value of the top number. Then, find the difference and write it under the horizontal line. This is called 'borrowing'.

3. Subtract the numbers in the tens column, followed by the hundreds column, etc., by following the same procedure for each column.

Note: To check your answer, add the difference to the number subtracted (subtrahend); the result should be the number from which it was subtracted (minuend). e.g., 8 – 5 = 3; therefore 3 + 5 = 8

Example 1.2-b: Subtracting Whole Numbers

Perform the following subtractions:

   a. Subtract 1,314 from 3,628
   b. Subtract 789 from 8,357

Solution

a. Subtract 1,314 from 3,628

   The difference of the digits in the **ones column** is 4 since $8 - 4 = 4$. There are 4 ones, so we write 4 in the ones column below the horizontal line. The difference of the digits in the **tens column** is 1 since $20 - 10$

= 10. There is 1 ten, so we write 1 in the tens column below the horizontal line. The difference of the digits in the **hundreds column** is 3 since 600 − 300 = 300. There are 3 hundreds, so we write 3 in the hundreds column below the horizontal line. The difference of the digits in the **thousands column** is 2 since 3,000 − 1,000 = 2,000. There are 2 thousands, so we write 2 in the thousands column below the horizontal line. Therefore, subtracting 1,314 from 3,628 results in 2,314.

Thousands	Hundreds	Tens	Ones
3	6	2	8
− 1	3	1	4
2	3	1	4

b. Subtract 789 from 8,357

In the **ones column**, the ones digit on the top (7) is smaller than the ones digit on the bottom (9). Borrow one ten from the tens digit on the top row and add it to the 7 ones to get 17 ones. 17 − 9 = 8, so we write 8 in the ones column below the horizontal line. In the **tens column**, the tens digit on the top (4, after borrowing 1 for the ones column) is smaller than the tens digit on the bottom (8). Borrow one hundred from the hundreds digit on the top row and add it to the 4 tens to get 14 tens. 14 − 8 = 6, so we write 6 in the tens column below the horizontal line.In the **hundreds column**, the hundreds digit on the top (2, after borrowing 1 for the tens column) is smaller than the hundreds digit on the bottom (7). Borrow one thousand from the thousands digit on the top row and add it to the 2 hundreds to get 12 hundreds. 12 − 7 = 5, so we write 5 in the hundreds column below the horizontal line. As only 7 (after borrowing 1 for the hundreds column) is in the **thousands column**, we write 7 in the thousands column below the horizontal line. Therefore, subtracting 789 from 8,357 results in 7,568.

Thousands	Hundreds	Tens	Ones
	12	14	
7	2	4	17
8	3	5	7
−	7	8	9
7	5	6	8

Note: If a larger whole number is subtracted from a smaller whole number, the result will be a negative number. To do this, reverse the question to deduct the smaller number from the larger number following the above steps, and set the answer to be negative.

Multiplication of Whole Numbers

Multiplication is the process of finding the product of two numbers. Multiplication of whole numbers can be thought of as repeated additions. The symbol '×' denotes multiplication.

Multiplication can be performed in any order, and the product will be the same.

A x B = B x A

For example, 5 x 4 = 20 and 4 x 5 = 20

This is known as the **commutative property** of multiplication.

For example, 5 × 4 refers to repeatedly adding 5, four times. This is read as 'five times four', and can also be written as 5 · 4 or 5(4).

5 × 4 can be represented pictorially as:
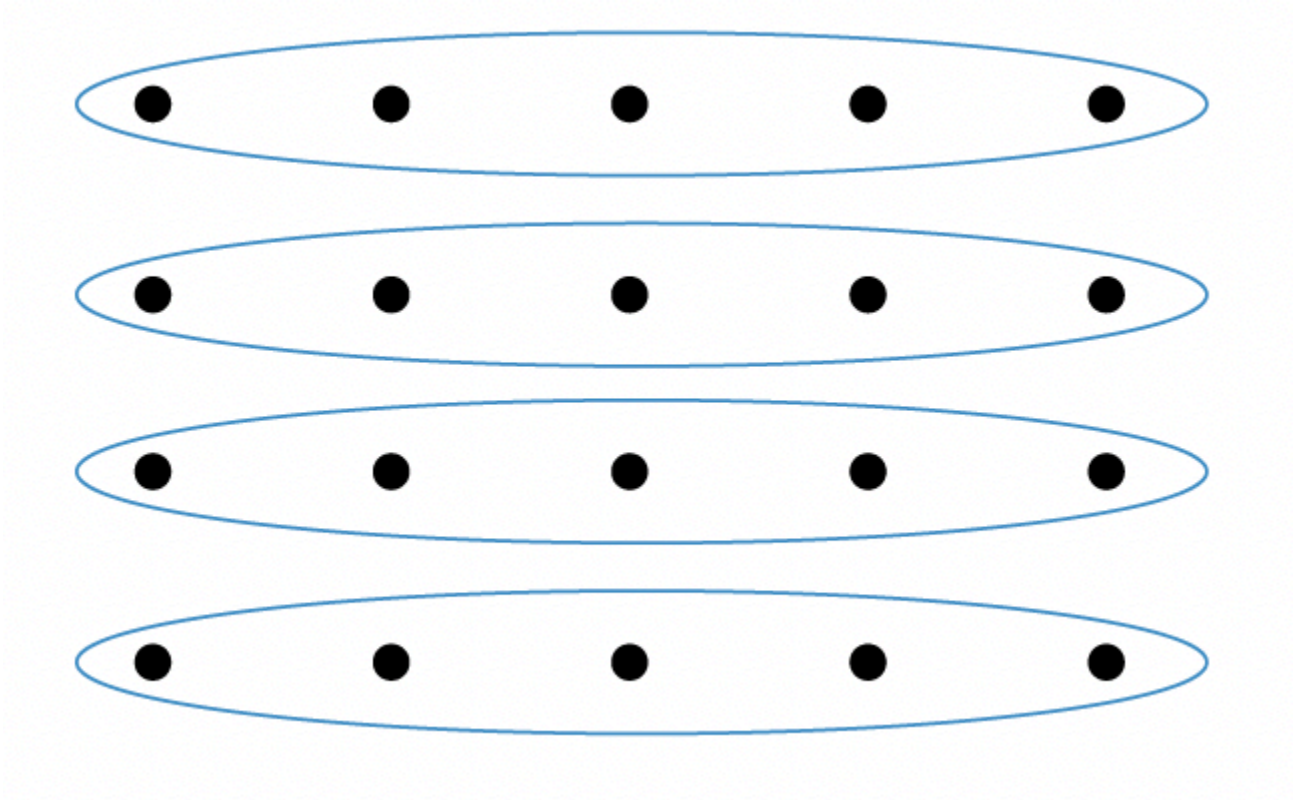
Here, the size of the set is 5, and it is repeated 4 times:

5 + 5 + 5 + 5 = 20

This can also be viewed as 4 × 5:
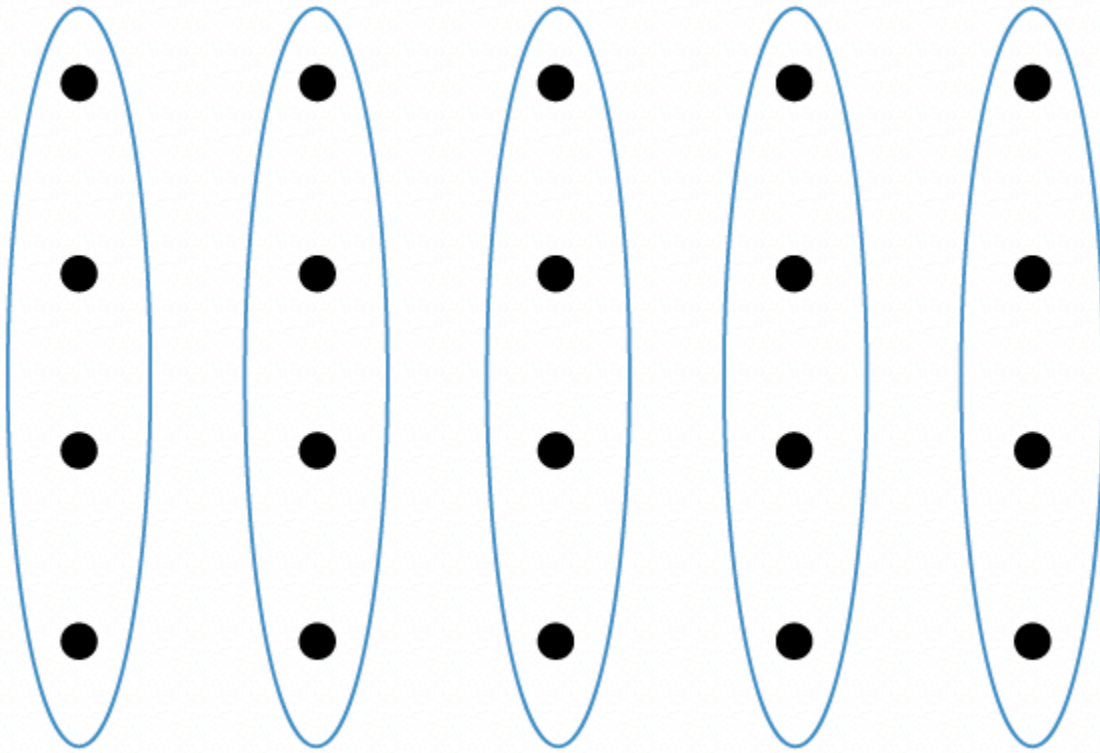
Here, the size of the set is 4, and it is repeated 5 times:

4 + 4 + 4 + 4 + 4 = 20

The numbers being multiplied are referred to as **factors,** and the result is the **product**.

In this example, 5 and 4 are factors of 20 (the product).

Follow these steps to multiply one whole number by another whole number:

1. Write the numbers one under the other by aligning the place values and drawing a horizontal line underneath, as done for addition and subtraction.
2. From the ones digit of the bottom factor, multiply the digit by each digit of the top factor, starting with the right-most digit.

- If the product is less than 10, write the product under the horizontal line.
- If the product is 10 or more, write the 'ones' digit of the product under the horizontal line, carry the 'tens' digit, and add it to the subsequent product.

3. Multiply the next digit of the bottom factor by each digit of the top factor by following the same procedure, and write the product below the first product, but one place value to the left. Continue for each digit of the bottom factor.
4. Sum all the products to obtain the answer.

The above steps for multiplying numbers are provided in detail in Example 1.2-c below.

Example 1.2-c: Multiplying Whole Numbers

Perform the following multiplications:

a. Multiply 38 by 6
b. Multiply 36 by 24
c. Multiply 263 by 425

Solution

a. Multiply 38 by 6

Multiplying 8 ones by 6 results in 48 ones. This is 4 tens and 8 ones. Write 8 in the ones column below the horizontal line and 4 above the tens column. Multiplying 3 tens by 6 results in 18 tens. Add the 4 tens from the previous step to 18 to obtain 22. Write 2 in the tens column and 2 in the hundreds column below the horizontal line. Therefore, multiplying 38 by 6 results in 228.

Hundreds	Tens	Ones
	4	
	3	8
×		6
2	2	8

b. Multiply 36 by 24.

Multiply 36 by 4 ones, as shown, to obtain 144. Multiply 36 by 2 tens. To do this, write a '0' under the horizontal line in the ones column (or leave it blank, as shown) and multiply 36 by 2 to obtain 72. Therefore, multiplying 36 by 24 results in 864.

	Hundreds	Tens	Ones
		3	6
×		2	4
	1	4	4
Add the two numbers to obtain 864. +	7	2	
	8	6	4

$$\overset{2}{3}6$$
$$\times 24$$
$$\overline{144}$$

$$\overset{1}{3}6$$
$$\times 24$$
$$\overline{72}$$

c. Multiply 263 by 425

Multiply 263 by 5 ones. Multiply 263 by 2 tens. Multiply 263 by 4 hundreds. Therefore, multiplying 263 by 425 results in 111,775.

	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
				2	6	3
			×	4	2	5
			1	3	1	5
Add the three numbers to obtain 111,775			5	2	6	
+	1	0	5	2		
	1	1	1	7	7	5

$$\overset{3\ 1}{263}$$
$$\times 425$$
$$\overline{1315}$$

$$\overset{1}{263}$$
$$\times 425$$
$$\overline{526}$$

$$\overset{2\ 1}{263}$$
$$\times 425$$
$$\overline{1052}$$

Division of Whole Numbers

The division is the process of determining how many times one number is contained in another. This is the inverse process of multiplication. When a larger number is divided by a smaller one, this division can be considered repeated subtractions. The symbol '÷' denotes division.

The division must be performed in a written order.

A ÷ B does not equal B ÷ A

For example, $20 \div 5 = 4$ but $5 \div 4 = \dfrac{1}{4}$

For example, $20 \div 5$ refers to repeatedly subtracting 5 from 20. This is read as 'twenty divided by five', and can also be written as $\dfrac{20}{5}$

$$
\begin{array}{r}
20 \\
-\ 5 \\
\hline
15 \\
-\ 5 \\
\hline
10 \\
-\ 5 \\
\hline
5 \\
-\ 5 \\
\hline
0
\end{array}
$$

5 has been subtracted 4 times

0 No remainder

In a set of 20 items, we can create four groups of 5:
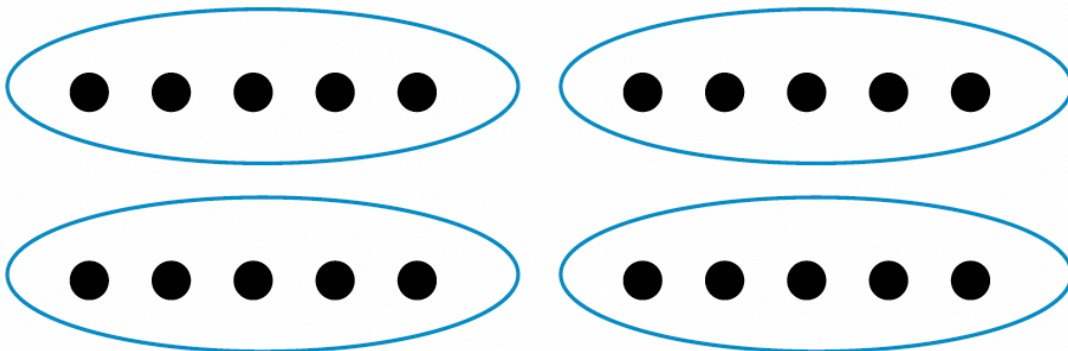
Therefore, $20 \div 5 = 4$.

The number being divided is referred to as the **dividend**, and the number by which the dividend is divided

is referred to as the divisor. The result or answer is called the **quotient**. If the divisor cannot divide the dividend evenly, the number left over is referred to as the **remainder**.

For example, consider 25 ÷ 7:

$$\begin{array}{r} 3 \quad \longleftarrow \text{Quotient} \\ \text{Divisor} \longrightarrow 7\,\overline{)25} \quad \longleftarrow \text{Dividend} \\ \underline{-21} \quad\quad\quad\quad \\ 4 \quad \longleftarrow \text{Remainder} \end{array}$$

The following relationship exists between the four components of a division problem:

Dividend = Divisor × Quotient + Remainder

25 = 7 × 3 + 4

The steps to be followed in dividing whole numbers are provided in detail in Example 1.2-d below.

Example 1.2-d: Dividing Whole Numbers

Perform the following divisions and state the quotient and remainder:

   a. Divide 76 by 3
   b. Divide 637 by 25
   c. Divide 6,543 by 12

Solution

   a. Divide 76 by 37 can be divided by 3. Therefore, determine the number of multiples of 3 there are in 7. There are two 3's in 7. Write 2 in the quotient area above 7. Multiply 2 by 3 (= 6) and subtract this from 7. Write the remainder 1. Bring down the 6 from the dividend and determine the number of multiples of 3 in 16. There are five 3's in 16. Write 5 in the quotient area above 6. Multiply 5 by 3 (= 15) and subtract this from 16 to get the final remainder of 1. Therefore, the quotient is 25 and the remainder is 1.

$$
\begin{array}{r}
25 \\
3 \overline{\smash{)}76} \\
-6 \\
\hline
16 \\
-15 \\
\hline
1
\end{array}
$$

b. Divide 637 by 256 cannot be divided by 25. Therefore, determine the number of multiples of 25 there are in 63. There are two 25's in 63. Write 2 in the quotient area above 3. Multiply 2 by 25 (= 50) and subtract this from 63. Write the remainder 13. Bring down the 7 from the dividend and determine the number of multiples of 25 there are in 137. There are five 25's in 137. Write 5 in the quotient area above 7. Multiply 5 by 25 (= 125) and subtract this from 137 to get the final remainder of 12. Therefore, the quotient is 25 and the remainder is 12.

$$
\begin{array}{r}
25 \\
25 \overline{\smash{)}637} \\
-50 \\
\hline
137 \\
-125 \\
\hline
12
\end{array}
$$

c. Divide 6,543 by 126 cannot be divided by 12. Therefore, determine the number of multiples of 12 there

are in 65. There are five 12's in 65. Write 5 in the quotient area above 5. Multiply 5 by 12 (= 60) and subtract this from 65. Write the remainder 5. Bring down the 4 from the dividend and determine the number of multiples of 12 there are in 54. There are four 12's in 54. Write 4 in the quotient area above 4. Multiply 4 by 12 (= 48) and subtract this from 54. Write the remainder 6. Bring down the 3 from the dividend and determine the number of multiples of 12 there are in 63. There are five 12's in 63. Write 5 in the quotient area above 3. Multiply 5 by 12 (= 60) and subtract this from 63 to get the final remainder of 3. Therefore, the quotient is 545 and the remainder is 3.

$$
\begin{array}{r}
545 \\
12\overline{)6543} \\
-60 \\
\hline
54 \\
-48 \\
\hline
63 \\
-60 \\
\hline
3
\end{array}
$$

Arithmetic Operations with Zero and One

Table 1.2-a: Arithmetic Operations with Zero

Operation	Description	Examples
Addition	When 0 is added to a number, or when a number is added to 0, there will be no change to that number.	25 + 0 = 25 0 + 25 = 25
Subtraction	When 0 is subtracted from a number, there will be no change to that number.	16 − 0 = 16
	When a number is subtracted from 0, the answer will be the negative value of that number.	0 − 16 = −16
Multiplication	When 0 is multiplied by a number, or when a number is multiplied by 0, the answer will be 0.	0 × 35 = 0 35 × 0 = 0
Division	When 0 is divided by a number, the answer will be 0.	0 ÷ 25 = 0
	When a number is divided by 0, the answer is undefined.	25 ÷ 0 = Undefined

Table 1.2-b: Arithmetic Operations with One

Operation	Description	Examples
Multiplication	When 1 is multiplied by a number, or when a number is multiplied by 1, there will be no change to that number.	1 × 12 = 12 12 × 1 = 12
Division	When 1 is divided by a number, the answer is the reciprocal of that number.	$1 \div 35 = \dfrac{1}{35}$
	When a number is divided by 1, there will be no change to that number.	$1 \div 35 = \dfrac{1}{35}$ = 35

Powers of Whole Numbers

We learned previously that multiplication is a shorter way to write repeated additions of a number. Similarly, when a number is multiplied by itself repeatedly, we can represent this repeated multiplication using **exponential notation.**

When 2 is multiplied 5 times, in repeated multiplication, it is represented by:

2 × 2 × 2 × 2 × 2

However, it can be tedious to represent repeated multiplication using this notation. Instead, the exponential notation can be used.

When 2 is multiplied 5 times, in exponential notation, it is represented by:

$$\text{exponent}$$

$$\text{base} \longrightarrow 2^5$$
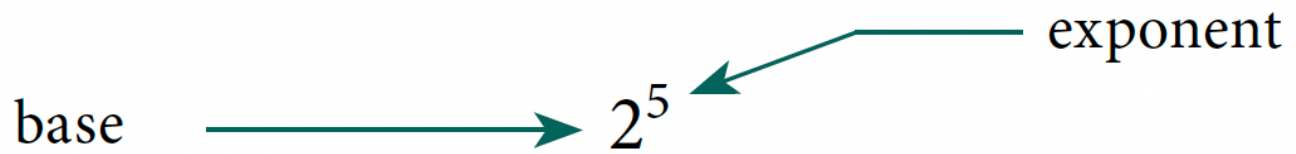
In this example, 2 is known as the **base**, 5 is known as the **exponent**, and the whole representation 2^5 is known as the **power**. The exponent is written in superscript to the right of the base and represents the number of times the base is multiplied by itself. The representation is read as "2 raised to the power of 5" or "2 to the 5$^{\text{th}}$ power".

Example 1.2-e: Converting Exponential Notation to Repeated Multiplication and Evaluating

Expand 8^2 to show the repeated multiplication and evaluate.

Solution

$8^2 = 8 \times 8 = 64$

$$\overbrace{8^2 = 8 \times 8}^{\text{Repeated Multiplication}} = 64$$

Exponential Notation

Standard Notation

Example 1.2-f: Converting Repeated Multiplication to Exponential

Notation

Express the repeated multiplication 9 × 9 × 9 × 9 × 9 × 9 in exponential notation.

Solution

The number 9 is being multiplied repeatedly 6 times.

In exponential notation, the **base** is **9** and the **exponent** is **6**.

Therefore, $9 \times 9 \times 9 \times 9 \times 9 \times 9 = 9^6$.

Example 1.2-g: Evaluating Expressions by Writing in Standard Form

Express the following in standard form and then evaluate:

a. 5×3^4
b. $2^3 \times 3^2$
c. $5^4 \times 2^2$
d. $5^2 \times 4^2$

Solution

a. $5 \times 3^4 = 5 \times [3 \times 3 \times 3 \times 3] = 5 \times 81 = 405$
b. $2^3 \times 3^2 = [2 \times 2 \times 2] \times [3 \times 3] = 8 \times 9 = 72$
c. $5^4 \times 2^2 = [5 \times 5 \times 5 \times 5] + [2 \times 2] = 625 + 4 = 629$
d. $5^2 \times 4^2 = [5 \times 5] - [4 \times 4] = 25 - 16 = 9$

Example 1.2-h: Evaluating Powers using a Different Power of the Same Base

Use the equation $6^5 = 7,776$ to evaluate the following expressions:

    a. 6^6
    b. 6^4

Solution

a. $6^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7,776$

$6^6 = 6 \times 6 \times 6 \times 6 \times 6 \times 6$

$= 6^5 \times 6$

$= 7,776 \times 6$

$= 46,656$

b. $6^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7,776$

$6^5 = 6^4 \times 6$

$7,776 = 6^4 \times 6$

$\dfrac{7,776}{6} = 6^4$

$6^4 = 1,296$

Table 1.2-c: Examples of Powers with Exponents of 2, 3, 4, and 5

Powers of 2		Powers of 3	Powers of 4	Powers of 5
$1^2 = 1$	$11^2 = 121$	$1^3 = 1$	$1^4 = 1$	$1^5 = 1$
$2^2 = 4$	$12^2 = 144$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
$3^2 = 9$	$13^2 = 169$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$
$4^2 = 16$	$14^2 = 196$	$4^3 = 64$	$4^4 = 256$	$4^5 = 1,024$
$5^2 = 25$	$15^2 = 225$	$5^3 = 125$	$5^4 = 625$	$5^5 = 3,125$
$6^2 = 36$	$16^2 = 256$	$6^3 = 216$	$6^4 = 1,296$	$6^5 = 7,776$
$7^2 = 49$	$17^2 = 289$	$7^3 = 343$	$7^4 = 2,401$	$7^5 = 16,807$
$8^2 = 64$	$18^2 = 324$	$8^3 = 512$	$8^4 = 4,096$	$8^5 = 32,768$
$9^2 = 81$	$19^2 = 361$	$9^3 = 729$	$9^4 = 6,561$	$9^5 = 59,049$
$10^2 = 100$	$20^2 = 400$	$10^3 = 1,000$	$10^4 = 10,000$	$10^5 = 100,000$

Perfect Squares and Square Roots

Any whole number base with an exponent of 2 is a perfect square. For example, 9 is the perfect square produced by 3 raised to an exponent 2. The first twenty perfect squares are shown in Table 1.2-c under Powers of 2.

Finding the **square root** of a perfect square is the inverse of raising a whole number to the power of 2. The **square root** of a number is one of two identical factors that, when multiplied together, results in that number.

For example,

- 2 is the square root of 4 because $2^2 = 4$ or $2 \times 2 = 4$
- 5 is the square root of 25 because $5^2 = 25$ or $5 \times 5 = 25$

A whole number multiplied by itself results in a perfect square. The whole number multiplied by itself to get that perfect square is called the square root of that perfect square.

For example,

- 16 is a perfect square because it is the product of two identical factors of 4; i.e., 16 = 4 × 4.
- Therefore, 4 is the square root of 16.

The radical sign $\sqrt{}$ indicates the root of a number (or expression). The square root of 16, using the radical sign, is represented by $\sqrt[2]{16}$, where 2 is the index. The index indicates which root is to be taken, and is written as a small superscript number to the left of the radical symbol. For square roots, the index 2 does not need to be written as it is understood to be there; i.e. $\sqrt[2]{16}$ is written as $\sqrt{16}$.

Note: In future chapters, we will discuss roots with indexes other than 2.

Example 1.2-i: Determining the Square Root of Perfect Squares

Determine the square root of the following:

  a. $\sqrt{36}$
  b. $\sqrt{81}$
  c. $\sqrt{144}$

Solution

  a. $\sqrt{36} = \sqrt{6 \times 6} = 6$(Using 36 = 6 × 6)
  b. $\sqrt{81} = \sqrt{9 \times 9} = 9$(Using 81 = 9 × 9)
  c. $\sqrt{36} = \sqrt{12 \times 12} = 12$(Using 144 = 12 × 12)

1.2 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

For addition Problems 1 to 8, (i) estimate the answer by rounding each number to the nearest ten and (ii) calculate the exact answer.

  1. a. 48 + 29

     b. 38 + 95

2. a. $16 + 79$

    b. $69 + 47$

3. a. $875 + 48$

    b. $574 + 79$

4. a. $459 + 27$

    b. $356 + 65$

5. a. $286 + 109 + 15$

    b. $839 + 645 + 27$

6. a. $989 + 215 + 25$

    b. $798 + 237 + 12$

7. a. $195 + 459 + 8$

    b. $996 + 816 + 6$

8. a. $896 + 642 + 9$

    b. $995 + 724 + 8$

For the addition Problems 9 to 14, estimate the answer by (i) rounding each number to the nearest hundred, and (ii) calculate the exact answer.

9. a. $745 + 668$

    b. $427 + 225$

10. a. $357 + 245$

    b. $451 + 645$

11. a. $1,883 + 5,466$

    b. $2,157 + 3,459$

12. a. $6,950 + 2,367$

    b. $3,765 + 1,992$

13. a. $2,635 + 372 + 1,524$

    b. $653 + 2,188 + 891$

14. a. $1,650 + 1,647 + 875 + 167$

   b. 3,869 + 1,967 + 550 + 745

For the subtraction Problems 15 to 22, (i) estimate the answer by rounding each number to the nearest ten, and (ii) calculate the exact answer.

15. a. 62 − 25

   b. 33 − 18

16. a. 43 − 27

   b. 71 − 59

17. a. 208 − 79

   b. 315 − 47

18. a. 327 − 28

   b. 500 − 73

19. a. 767 − 159

   b. 804 − 308

20. a. 904 − 629

   b. 584 − 167

21. a. 8,302 − 7,244

   b. 2,927 − 888

22. a. 9,185 − 6,728

   b. 5,765 − 777

For subtraction Problems 23 to 28, (i) estimate the answer by rounding each number to the nearest hundred, and (ii) calculate the exact answer.

23. a. 946 − 452

   b. 855 − 251

24. a. 868 − 74

   b. 495 − 357

25. a. 2,950 − 2,275

b. 3,961 – 1,833

26. a. 2,955 – 1,350

   b. 1,967 – 352

27. a. 3,513 – 2,846

   b. 3,981 – 1,657

28. a. 7,676 – 3,969

   b. 5,789 – 5,626

For the multiplication Problems 29 to 34, (i) estimate the answer by rounding each number to the nearest ten, and (ii) calculate the exact answer.

29. a. 58 × 75

   b. 63 × 59

30. a. 35 × 97

   b. 95 × 71

31. a. 764 × 53

   b. 799 × 68

32. a. 482 × 95

   b. 755 × 55

33. a. 1,995 × 37

   b. 2,150 × 59

34. a. 2,996 × 32

   b. 2,995 × 38

For the division Problems 35 to 44, (i) estimate the answer by rounding each number to the nearest ten, and (ii) perform the exact division and state the quotient and the remainder.

35. a. 78 ÷ 5

   b. 36 ÷ 8

36. a. 69 ÷ 7

b. 85 ÷ 9

37. a. 86 ÷ 27

   b. 78 ÷ 19

38. a. 59 ÷ 12

   b. 95 ÷ 18

39. a. 654 ÷ 14

   b. 396 ÷ 24

40. a. 777 ÷ 16

   b. 255 ÷ 19

41. a. 2,562 ÷ 18

   b. 1,225 ÷ 28

42. a. 2,097 ÷ 55

   b. 1,795 ÷ 64

43. a. 3,004 ÷ 204

   b. 6,501 ÷ 498

44. a. 3,895 ÷ 264

   b. 5,195 ÷ 255

For Problems 45 to 48, express the repeated multiplication in exponential notation.

45. a. 6 × 6 × 6 × 6

   b. 12 × 12

46. a. 5 × 5 × 5

   b. 7 × 7 × 7 × 7 × 7

47. a. 3 × 3 × 3 × 3 × 3 × 3

   b. 9 × 9 × 9 × 9

48. a. 8 × 8 × 8 × 8 × 8

   b. 4 × 4 × 4 × 4 × 4

For Problems 49 to 52, write the base and exponent for the powers.

49. a. 2^9

b. 5^7

50. a. 6^2

b. 10^9

51. a. 1^{20}

b. 8^3

52. a. 7^5

b. 12^1

For Problems 53 and 54, express the powers in standard notation and then evaluate.

53. a. 10^6

b. 3^5

54. a. 2^8

b. 5^4

55. If $3^{10} = 59,049$, evaluate a) 3^9 and b) 3^{11}

56. If $5^8 = 390,625$, evaluate a) 5^7 and b) 5^9

For Problems 57 to 62, determine the square root.

57. a. $\sqrt{64}$

b. $\sqrt{121}$

58. a. $\sqrt{25}$

b. $\sqrt{225}$

59. a. $\sqrt{1}$

b. $\sqrt{324}$

60. a. $\sqrt{100}$

b. $\sqrt{400}$

61. a. $\sqrt{8^2}$

b. $\sqrt{20^2}$

62. a. $\sqrt{12^2}$

b. $\sqrt{45^2}$

63. a. What amount is $65 less than $784?

b. What amount is $35 more than $98?

64. a. What amount is $79 less than $487?

b. What amount is $97 more than $52?

65. a. What amount is $515 increased by $847?

b. What amount is $745 decreased by $125?

66. a. What amount is $745 increased by $1,274?

b. What amount is $526 decreased by $346?

67. a. If there are 24 pens in a box, how many pens are there in 15 boxes?

b. A wire of length 420 centimetres is to be cut into 35-centimetre pieces. How many 35-centimetre pieces can be cut?

68. a. If you save $125 every month, how much will you save in one year?

b. If 8 people can be seated at a dinner table, how many tables are required to seat 280 people?

69. a. If you save $15 a week, how many weeks will it take you to save $675?

b. If there are 16 chairs in a row, how many chairs are there in 22 rows?

70. a. If you work 25 hours per week, how many weeks will it take you to complete a job that requires 450 hours?

b. If you earn $18 per hour, how much will you earn in a week in which you have worked 32 hours?

71. Mythili went to a bookstore and bought a dictionary. She gave the cashier a $100 note and received $47 as change. How much did the dictionary cost?

72. There were 744 students in a primary school. In May, after some students left to join another school, 576 students remained. How many students left for the other school?

73. Andy has $1,238 and Bill has $346 less than Andy. How much money do both of them have together?

74. A television costs $649 more than a Blu-ray player. If the Blu-ray player costs $235, how much will it cost to buy the TV and the Blu-ray player?

75. Peter earns $18 per hour. If he worked 25 hours last week, calculate his earning for that week.

76. Sam's overtime rate is $37 per hour. If he worked 29 hours overtime last week, calculate his overtime pay for that week.

77. Earl wants to save $3,150. If he saves $75 per week, how many weeks will it take for him to achieve his goal?

78. How long will it take to travel 1,190 km at 85 km per hour?

79. A large group of students visited an exhibition on Friday. The first 275 students were given one free balloon each. 487 students were disappointed that they did not receive any balloons. How many students visited the exhibition?

80. There were 2,415 boys and 1,875 girls that attended a fair. The first 2,650 visitors to the fair received gifts. How many visitors did not receive gifts?

81. There were 450 passengers on a train. Half of them were men, 125 were children, and the rest were women. How many more men than women were there on the train?

82. Out of the 3,678 visitors at an art exhibition, 1,469 were men, 1,234 were women, and the rest were children. How many more adults than children went to the exhibition?

83. Aran had some money saved up. On his birthday, Aran's grandparents gave him $125. After spending $98 on toys and $75 on clothes, he had $115 remaining. What amount did Aran have in his savings before his birthday?

84. Girija had some biscuits. She ate 19 and gave 27 to her brother and 5 to her parents. She had 12 biscuits left. How many biscuits did Girija have originally?

85. There are 19 girls in a class of 43 students. 15 of the girls and 11 of the boys are wearing eyeglasses.

    a. How many boys are there in the class?

    b. How many students do not wear eyeglasses?

86. There are 22 players on a soccer field. 10 players are wearing blue and 2 are wearing green. The rest of the players are wearing red. How many players are wearing red?

1.3 ARITHMETIC OPERATIONS WITH SIGNED NUMBERS

Positive real numbers can be represented by points on a number line from zero to the right of the zero. That is, whole numbers and positive rational and irrational numbers can be represented on a number line from zero to the right of zero.

Every positive number has a negative number known as its opposite, which lies to the left of zero on the number line. We use the negative sign '−' to represent negative numbers and the positive sign '+' to represent positive numbers. Zero, '0', is neither positive nor negative.

For example, plotting the positive numbers $\frac{3}{4}$, 4, and 6.5 and their opposites on the number line:
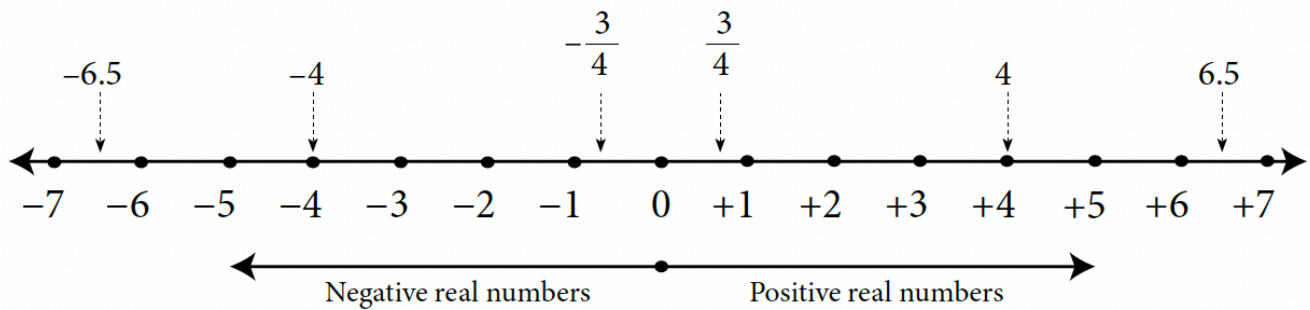
The arrowhead on either end shows that the number line continues indefinitely in both the positive and negative directions.

Positive and negative numbers are collectively called **signed numbers**. Since numbers are naturally positive, we usually omit the word 'positive' or the positive sign (+) when we read or write positive numbers. However, we must read or write the number as 'negative' or include the negative sign (−) when the number is negative. For example, **'+7'** is read as **'seven'** and written as '7'. However, **'−7'** is read as **'negative seven'** and written with the negative sign as **'−7'**.

Any positive number and its negative (opposite) will equal zero (origin) on the number line.

Origin
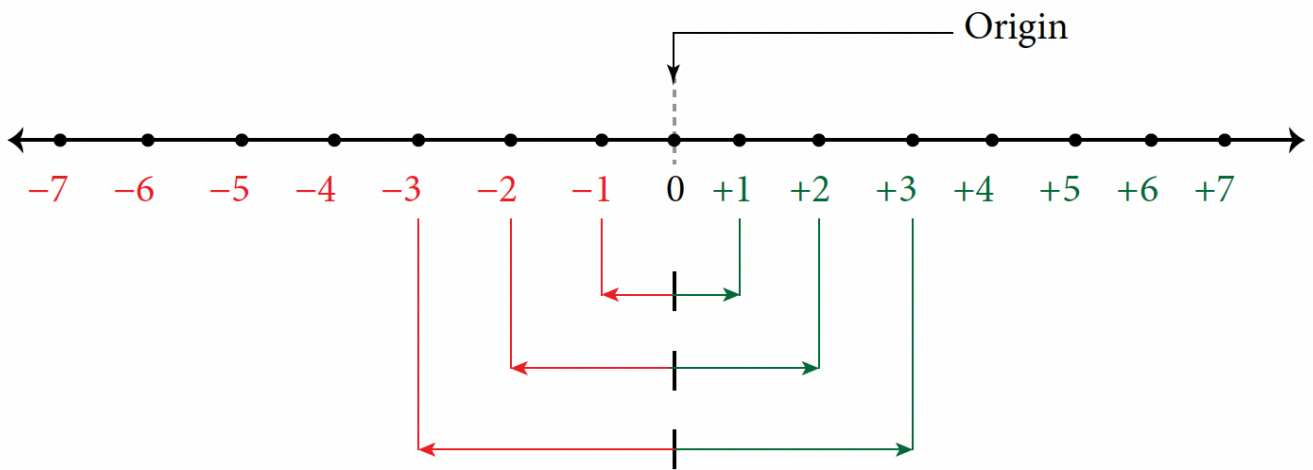
Numbers that lie to the left of a number on the number line are **less than** that number, and those that lie to the right on a number line are **greater than** that number.

For example,

- 3 is greater than −2,   i.e., 3 > −2
- −3 is greater than −5,   i.e., −3 > −5
- −5 is less than −4,   i.e., −5 < −4
- −1 is less than 2,   i.e., −1 < 2

Addition and Subtraction of Signed Numbers

- The answer is always positive (+) when adding two positive numbers. For example, Adding +5 and +3: (+5) + (+3) = 5 + 3 = **8.** This is the same as **+8.**

+5   +3

0   +5   +8
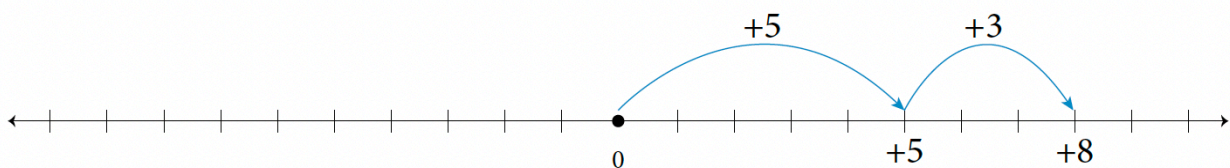
- The answer is always negative when adding two negative numbers (−). For example, Adding −4 and −3: (−4) + (−3) = −4 − 3 = **−7**
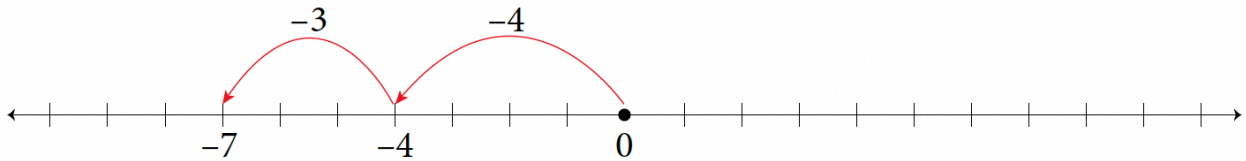
- When adding numbers with different signs, subtract the smaller absolute value (positive value) from, the larger absolute value (positive value), and keep the sign of the number with the larger absolute value. For example,

   a. Adding $+8$ and -12 (or -12 and $+8$):$(+8) + (-12) = 8 - 12 = \mathbf{-4}$
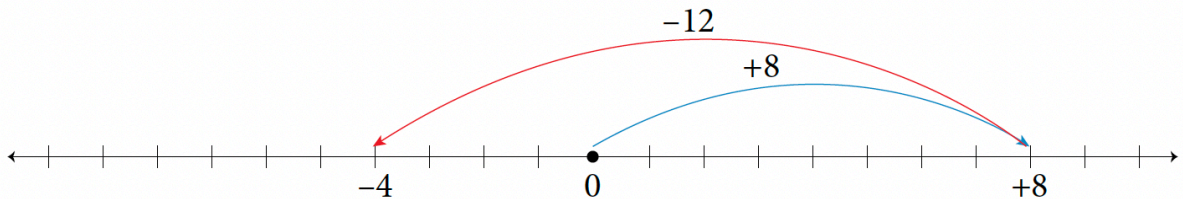
   b. Adding -5 and $+8$ (or $+8$ and -5):$(-5) + (+8) = -5 + 8 = \mathbf{3\ (or\ +3)}$
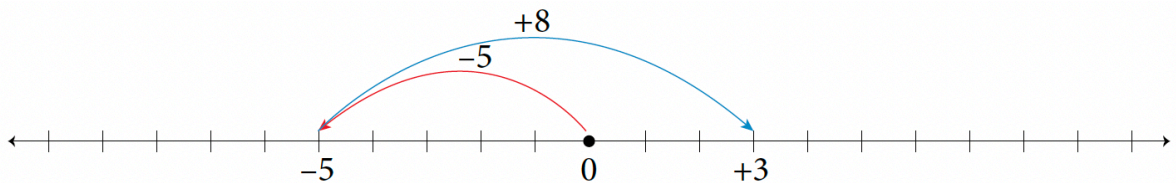
- When subtracting negative numbers, change all the subtraction problems to addition problems by adding the opposite, and then follow the rules for adding signed numbers. For example,

   a. Subtracting 12 from 18:$18 - 12 = 18 + (-12) = 6$
   b. Subtracting -12 from 18:$18 - (-12) = 18 + 12 = 30$
   c. Subtracting 12 from -18:$-18 - 12 = -18 + (-12) = -30$
   d. Subtracting -12 from -18:$-18 - (-12) = -18 + 12 = -6$

Multiplication and Division of Signed Numbers

When multiplying two signed numbers:

- The product of two numbers with the **same sign** is **positive**. For example,

a. $(+5)(+4) = +20$
b. $(-5)(-4) = +20$

▪ The product of two numbers with **different signs** is **negative**. For example,

a. $(+5)(-4) = -20$
b. $(-5)(+4) = -20$

When dividing two signed numbers:

▪ The quotient of two numbers with the **same sign** is **positive**. For example,

a. $\dfrac{+12}{+8} = \dfrac{3}{2}$
b. $\dfrac{-12}{-8} = \dfrac{3}{2}$

▪ The quotient of two numbers with **different signs** is **negative**. For example,

a. $\dfrac{-12}{+8} = -\dfrac{3}{2}$
b. $\dfrac{+12}{-8} = -\dfrac{3}{2}$

When multiplying or dividing more than two signed numbers, group them into pairs to determine the sign using the rules for multiplication and division of signed numbers.
   For example,

a. $(-3)(-2)(+4)(-1)(-5) = (6)(-4)(-5) = (-24)(-5) = 120$

b. $\dfrac{(-15)(+8)(-50)}{(-25)(14)} = \dfrac{-(15 \times 8)(-50)}{-(25 \times 14)} = \dfrac{+(15 \times 8 \times 50)}{-(25 \times 14)}$

$= \dfrac{+(15 \times 4 \times 2)}{-(1 \times 7)} = -\dfrac{15 \times 4 \times 2}{7} = -\dfrac{120}{7}$

Powers with Negative Bases

When a power has a negative base, there are four possible scenarios, as outlined in the following table:

Table 1.3-a: Powers with Negative Bases

Sign and Parity of Exponent	Example	Sign of Answer
Positive and Even	$(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$	+
Positive and Odd	$(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$	–
Negative and Even	$(-2)^{-6} = \dfrac{1}{(-2)^6} = \dfrac{1}{(-2)(-2)(-2)(-2)(-2)(-2)} = \dfrac{1}{64} = 0.015625$	+
Negative and Odd	$(-2)^{-5} = \dfrac{1}{(-2)^5} = \dfrac{1}{(-2)(-2)(-2)(-2)(-2)} = \dfrac{1}{-32} = -0.03125$	–

From the above scenarios, you will note:

- A negative base with an even exponent produces a positive result (because pairs of negatives become positive).
- A negative base with an odd exponent produces a negative result (because after negatives are paired, one negative will be left over).

A negative base of a power expressed within a bracket, as in $(-a)^n$, results in a different answer than a negative base expressed without a bracket, as in $-a^n$.

In $(-a)^n$, the exponent applies to both the negative sign and a.

In $-a^n$, the exponent applies only to a and the negative sign is applied to the answer.

For example,

a. In $(-5)^4$, (-5) is multiplied 4 times; i.e., $(-5)^4 = (-5)(-5)(-5)(-5) = 625$
b. In $(-5)^3$, (-5) is multiplied 3 times; i.e., $(-5)^3 = (-5)(-5)(-5) = -125$
c. In -5^4, only 5 is multiplied 4 times and the answer is negative; i.e.,
$$-5^4 = -[5 \times 5 \times 5 \times 5] = -625$$
d. In -5^3, only 5 is multiplied 3 times and the answer is negative; i.e.,
$$-5^3 = -[5 \times 5 \times 5] = -125$$

Example 1.3-a: Evaluating Expressions with Negative Bases using the Product Rule

Evaluate the following expressions:

a. $(-5)^4 \times (-5)^{-1}$
b. $(-2)^5 \times (-2)^2 \times (-2)^0 \times 2$

Solution

a. $(-5)^4 \times (-5)^{-1} = (-5)^{(4-1)} = (-5)^3 = -125$

b. $(-2)^5 \times (-2)^2 \times (-2)^0 \times 2 = (-2)^{(5+2+0)} \times 2 = (-2)^7 \times 2 = -128 \times 2 = -256$

Example 1.3-b: Evaluating Expressions with Negative Bases using the Quotient Rule

Evaluate the following expressions:

a. $(-3)^7 \div (-3)^2$
b. $(-5)^3 \div (-5)^0$

Solution

a. $(-3)^7 \div (-3)^2 = (-3)^{(7-2)} = (-3)^5 = -243$

b. $(-5)^3 \div (-5)^0 = (-5)^{(3-0)} = (-5)^3 = -125$

Example 1.3-c: Evaluating Expressions with Negative Bases using the Power of a Product Rule

Evaluate the following expressions:

a. $(-5 \times 2)^3$

b. $(-3 \times 2)^{-2}$

Solution

a. $(-5 \times 2)^3 = (-5)^3 \times 2^3 = -125 \times 8 = -1,000 \text{ or} (-5 \times 2)^3 = (-10)^3 = -1,000$

b. $(-3 \times 2)^{-2} = (-3)^{-2} \times 2^{-2} = \dfrac{1}{(-3)^2} \times \dfrac{1}{2^2} = \dfrac{1}{9} \times \dfrac{1}{4} = \dfrac{1}{36} \text{ or}$

$(-3 \times 2)^{-2} = (-6)^{-2} = \dfrac{1}{(-6)^2} = \dfrac{1}{36}$

Example 1.3-d: Evaluating Expressions with Negative Bases using the Power of a Quotient Rule

Evaluate the following expressions:

a. $(-2 \div 3)^{-2}$

b. $(3 \div (-2))^{-3}$

Solution

a. $(-2 \div 3)^{-2} = \left(\dfrac{-2}{3}\right)^{-2} = \left(\dfrac{3}{-2}\right)^{2} = \dfrac{3^2}{(-2)^2} = \dfrac{9}{4}$

b. $(3 \div (-2))^{-3} = \left(\dfrac{3}{-2}\right)^{-3} = \left(\dfrac{-2}{3}\right)^{3} = \dfrac{(-2)^3}{3^3} = \dfrac{-8}{27} = -\dfrac{8}{27}$

Example 1.3-e: Evaluating Expressions with Negative Bases using the Power of a Power Rule

Evaluate the following expressions:

   a. $(-2^3)^3$
   b. $(-3^3)^2$

Solution

a. $(-2^3)^3 = (-2)^{3 \times 3} = (-2)^9 = -512$

b. $(-3^3)^2 = (-3)^{3 \times 2} = (-3)^6 = 729$

Principal Roots

Roots of Positive Numbers

- When the index of the root is **even**, any positive number will have two real number solutions, with one being the negative of the other. The positive solution is known as its **principal root**.
  - For example, $\sqrt{9}$ has two roots: +3 and –3, because $(3) \times (3) = 9$ and $(-3) \times (-3) = 9$ This is usually written as ±3 and read as "plus or minus 3", and the principal root is 3.
  - $\sqrt[4]{16}$ has two roots: +2 and –2, because $(2) \times (2) \times (2) \times (2) = 16$ and $(-2) \times (-2) \times (-2) \times (-2) = 16$ i.e., the roots are ±2 and the principal root is 2.

- When the index of the root is **odd**, there is only one real number solution and it is positive. This positive solution is the **principal root**.
  - For example, $\sqrt[3]{27} = 3$ because (3) × (3) × (3) = 27; i.e., the principal root is 3.
  - Similarly, $\sqrt[5]{32} = 2$ because (2) × (2) × (2) × (2) × (2) = 32; i.e., the principal root is 2.

Roots of Negative Numbers

- When the index of the root is **even**, there is **no real number solution** to any negative number.
  - For example, $\sqrt[4]{-81}$ has no real roots.
- When the index of the root is **odd**, there is only one real number solution to any negative number and it is negative. This negative solution is the **principal root**.
  - For example, $\sqrt[5]{-32} = -2$ because $(-2)(-2)(-2)(-2)(-2) = -32$; i.e., the principal root is –2.

Real Numbers

Real numbers include all positive numbers and negative numbers. A summary of the types of real numbers, which we have learned about in the last three chapters, is provided below:

Table 1.3-b: Types of Real Numbers

Type	Description	Examples
Natural Numbers	Counting numbers (numbers starting from 1).	
Whole Numbers	Natural numbers, including zero.	Counting Numbers or Natural Numbers: 0 1 2 3 4 ... Whole Numbers
Integers	Natural numbers (positive integers), their negatives (negative integers), and zero.	$-4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4$ Negative Integers, Positive Integers. Zero is neither positive nor negative
Rational Numbers	Numbers that can be expressed as one integer divided by another non-zero integer; i.e., numbers that can be written as a quotient of integers with non-zero divisors.	$-\dfrac{5}{2}, 0.75, \dfrac{3}{2}$
Irrational Numbers	Numbers that cannot be expressed as a rational number.	$sqrt2, \pi, 2.718281...$

Note: Terminating decimals (decimals that end) and repeating decimals (decimals that do not end but show a repeating pattern) are also rational numbers because they can be expressed as a quotient of integers.

For example,

- 0.375 *can be expressed as* $\dfrac{3}{8}$.

- $0.185185...$ *is usually written as* $0.\overline{185}$ *and can be expressed as* $\dfrac{5}{27}$.
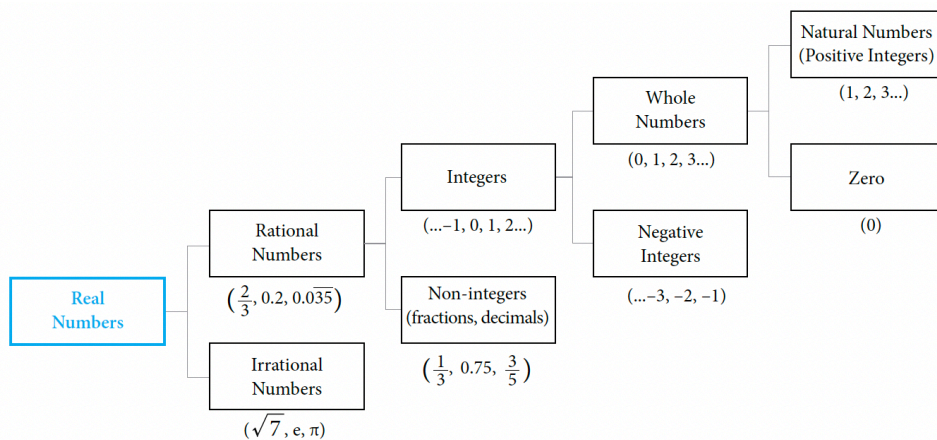
Exhibit 1.3 Real Number System With Examples

1.3 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

For problems 1 to 6, place the correct sign '<' or '>' in the space between the following pairs of numbers indicated by the underscore.

1. a. –5 _ 0   b. –2 _ +6
2. a. 0 _ –3   b. –5 _ +3
3. a. +8 _ –3   b. +1 _ –2
4. a. –5 _ +4   b. +3 _ –7
5. a. –6 _ –8   b. –5 _ –2
6. a. –7 _ –9   b. –8 _ –4

For problems, 7 to 10, arrange the numbers from least to greatest.

7. a. 5, –6, 8, –8, –5, 2   b. –8, 4, –6, 3, –9, 7
8. a. –2, –3, 5, 2, –1, 4   b. 15, –14, 17, 4, –5, –7
9. a. 9, –5, –8, 3, 7, 10   b. 12, –13, 15, 2, –8, –3
10. a. –3, 6, 1, –7, –1, 7   b. –12, 0, 12, –16, 15, –5]

Evaluate problems 11 to 30.

11. a. $-8 + (-5 - 7)$
    b. $2 - (-3) + 1$
12. a. $-9 + (-3 - 8)$
    b. $5 - (-7) + 8$

13. a. $-3-(-7)+8$

    b. $(-4+9)-(-3-6)$

14. a. $-7-(-9)-1$

    b. $(-5+3)+(-4-9)$

15. a. $4+(-3)-[5+(-11)]$

    b. $-6+(-4)-[-(15-8)]$

16. a. $5+(-4)-[7+(-9)]$

    b. $-8+(-15)-[-(6-7)]$

17. a. $2(-3)(-5)$

    b. $-4(-3)(-2)$

18. a. $6(-2)(-4)$

    b. $-5(-3)(-2)$

19. a. $-64 \div (-8)$

    b. $45 \div (-5)$

20. a. $-48 \div (-6)$

    b. $36 \div (-4)$

21. a. $-5+(-2)(-5)-(6-3)$

    b. $7(2-3)-4(-7+1)$

22. a. $-8(5-6)-3(-6+2)$

    b. $-7+(-3)(-4)-(-8-3)$

23. a. $(5+7)^2 - 5^2 - 7^2$

    b. $2^2 - 2^4 - 20 \times 3$

24. a. $(9-12)^2 - 9^2 - 12^2$

    b. $7 \times 8 - 3^2 - 2^3$

25. a. $(20 \times 4 - 8^2)^2 + 9$

    b. $(3 \times 9 - 3^2) + 12$

26. a. $-12 \div 4 - 3 \times 2^4$

    b. $-5 \times 3^3 \div 9 + 5^2$

27. $-11^2 - 4 \times 54 \div (5-2)^3 - 3$

28. $[(1+12)(1-5)]^2 \div (5 - 3 \times 2^2 - 4)$

29. $-31 - [(15 \div 3) \times 32] \div \left(2^2 \times \dfrac{23}{46}\right)$

30. $3(7^2 + 2 \times 15 \div 3) - (1 - 3 \times 4)^2$

Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of Mathematics* (3rd ed.) by Thambyrajah Kugathasan, published by Vretta-Lyryx Inc., with permission. Adaptations include supplementing existing material and reordering chapters.

1.4 FACTORS AND MULTIPLES

Factors of a number are whole numbers that can divide the number evenly (i.e., with no remainder).

1 is a factor of every number and every number is a factor of itself.

For example, to find factors of 12, divide the number 12 by 1, 2, 3, 4,...; the numbers that divide 12 evenly are its factors.

$12 \div 1 = 12$

$12 \div 2 = 6$

$12 \div 3 = 4$

$12 \div 4 = 3$

$12 \div 6 = 2$

$12 \div 12 = 1$

Therefore, 1, 2, 3, 4, 6, and 12 are factors of 12.

Note: 5, 7, 8, 9, 10, and 11 do not divide 12 evenly. Therefore, they are not factors of 12.

A number is **divisible** by another number if it divides evenly with no remainder.

We can also express the factors of a number by showing how the product of two factors results in the number.

$12 = 1 \times 12 \quad \text{or} \quad 12 \times 1$

$12 = 2 \times 6 \quad \text{or} \quad 6 \times 2$

$12 = 3 \times 4 \quad \text{or} \quad 4 \times 3$

Multiples of a number are the products of the number and the natural numbers (1, 2, 3, 4,...).

For example, multiples of 10:

$12 = 1 \times 12 \quad \text{or} \quad 12 \times 1$

$12 = 2 \times 6 \quad \text{or} \quad 6 \times 2$

$12 = 3 \times 4 \quad \text{or} \quad 4 \times 3$

Multiples of a number are the products of the number and the natural numbers (1, 2, 3, 4,...).

For example, multiples of 10:

$10 \times 1 = 10 \quad 10 \times 2 = 20 \quad 10 \times 3 = 30 \quad 10 \times 4 = 40 \quad 10 \times 5 = 50$

Therefore, multiples of 10 are 10, 20, 30, 40, 50, etc.

Note: Multiples of a number can be divided by the number with no remainder.

Prime Numbers and Composite Numbers

A **prime number** is a whole number with only two factors: 1 and the number itself; i.e., prime numbers can be divided evenly only by 1 and the number itself.

For example, 7 is a prime number because it only has two factors: 1 and 7.

A **composite number** is a whole number with at least one factor other than 1 and the number itself; i.e., all numbers that are not prime numbers are composite numbers.

For example, 8 is a composite number because it has more than two factors: 1, 2, 4, and 8.

Note: 0 and 1 are neither prime numbers nor composite numbers.

Example 1.4-a: Identifying Prime Numbers

Identify all the prime numbers less than 25.

Solution

All the prime numbers less than 25 are:

2, 3, 5, 7, 11, 13, 17, 19, and 23.

Example 1.4-b: Identifying Composite Numbers

Identify all the composite numbers less than 25.

Solution

All the composite numbers less than 25 are:

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, and 24.

Example 1.4-c: Finding Factors of Prime Numbers

Find all the factors of 13.

Solution

1 and 13 are the only factors of 13.

Example 1.4-d: Finding All Factors of Composite Numbers

Find all the factors of:

    a. 16
    b. 20

Solution

    a.   The factors of 16 are 1, 2, 4, 8, and 16.
    b.   The factors of 20 are 1, 2, 4, 5, 10, and 20.

Example 1.4-e: Finding the Prime Factors of Composite Numbers

Find all the prime factors of 24.

Solution

All the factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.
    Of the above factors, only 2 and 3 are prime numbers.
    Therefore, the prime factors of 24 are 2 and 3.

Factor Tree

A factor tree helps to find all of the prime factors of a number. It also shows the number of times each prime factor appears when writing that number as a product of its prime factors.
    The following steps illustrate creating a factor tree for the number 24:

Step 1: Write 24. As shown, draw two short lines down from the number at diverging angles.

$$24$$
$$\wedge$$

Step 2: 24 is divisible by the first prime number 2; i.e. $24 = 2 \times 12$.
    Write these factors at the end of the two lines. Now 24 is at the top, and 2×12 is the 2nd layer below it, as shown.

$$24$$
$$\wedge$$
$$2 \times 12$$
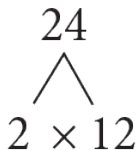
Step 3: Now, 12 is divisible by the prime number 2, i.e. $12 = 2 \times 6$.
    Write these factors below 12; i.e., the 3rd layer is $2 \times 2 \times 6$, as shown.

$$24$$
$$2 \times 12$$
$$2 \times 2 \times 6$$
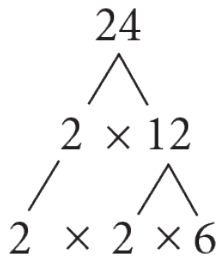
Step 4: Next, 6 is divisible by the prime number 2; i.e. 6 = 2 × 3.
Write these factors below 6; i.e., the 4th layer is 2 × 2 × 2 × 3, as shown.

Step 5: The factors at the 4th layer are all prime numbers and cannot be factored any further.
Therefore, writing 24 as a product of its prime factors: 24 = 2 × 2 × 2 × 3.

$$24$$
$$2 \times 12$$
$$2 \times 2 \times 6$$
$$2 \times 2 \times 2 \times 3$$
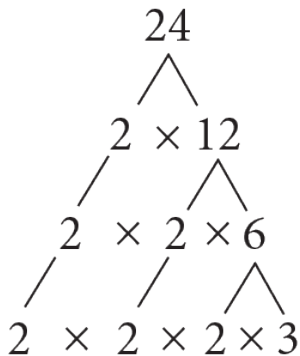
Note: Doing every step starting with a prime number is unnecessary. You may start with two factors that multiply to get the number. For example, 24 = 4 × 6. Then continue factoring until you are left with only prime numbers on the bottom layer, as shown. The answer will be the same.

$$24$$
$$4 \times 6$$
$$2 \times 2 \times 2 \times 3$$
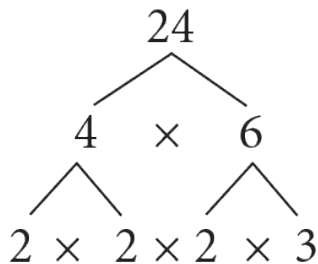
Least or Lowest Common Multiple (LCM)

The **Least Common Multiple (LCM)** of two or more whole numbers is the smallest multiple that is common to those numbers. LCM is the smallest integer, a common multiple of two or more numbers. The LCM can be determined using one of the following methods:

Method 1:

First, select the largest number and check to see if it is divisible by all the other numbers. If it divides, then the largest number is the LCM. For example, in finding the LCM of 2, 3, and 12, the largest number 12 is divisible by the other numbers 2 and 3. Therefore, the LCM of 2, 3, and 12 is 12.

If none of the numbers have a common factor, then the LCM of the numbers is the product of all the numbers. For example, none of these numbers have a common factor in finding the LCM of 2, 5, and 7. Therefore, the LCM of 2, 5, and 7 is 2 × 5 × 7 = 70.

If the largest number is not divisible by the other numbers and there is a common factor between some of the numbers, then find a multiple of the largest number that is divisible by all the other numbers. For example, in finding the LCM of 3, 5, and 10, the largest number 10 is not divisible by 3, and 5 and 10 have a common factor of 5. Multiples of 10 are 10, 20, 30, 40, etc. 30 are divisible by 3 and 5. Therefore, the LCM of 3, 5, and 10 is 30.

Method 2:

Step 1: Find the prime factors of each number using a factor tree and list the different prime numbers.

Step 2: Count the times each prime number appears in each factor tree.

Step 3: Find the largest of these counts for each prime number.

Step 4: List each prime number as many times as you counted it in Step 3. The LCM is the product of all the prime numbers listed.

Tip:

- A number is divisible by 2 if the last digit of the number ends in a 0, 2, 4, 6, or 8.
- A number is divisible by 3 if the sum of all the digits of a number is divisible by 3.
- A number is divisible by 5 if the last digit is 0 or 5.

Example 1.4-f: Finding the Least Common Multiple

Find the LCM of 9 and 15.

Solution

Method 1:

The largest number, 15, is **not** divisible by 9.

Multiples of 15 are: 15, 30, 45...

45 is divisible by 9.

Therefore, 45 is the LCM of 9 and 15.

Method 2:

Step 1 – Draw a factor tree for 9 and 15

```
   15
  /  \
 3 x 5

    9
  /  \
 3 x 3
```

Step 2 – Count the number of times each prime number appears in the factor trees.

Number of 3's = 2

Number of 5's = 1

Number of 3's = 1

Step 3 – find the largest count for each prime number.

- Largest count for the prime number 3 = 2
- Largest count for the prime number 5 = 1

Step 4 – List each prime number as many times as you counted it in Step 3.

LCM = 3x3x5 = 45

Example 1.4-g: Finding the Least Common Multiple

Find the LCM of 3, 5, and 8.

Solution

Method 1:

The largest number, 8, is **not** divisible by 3 and 5.

Since 3, 5, and 8 have no common factors, the LCM is the product of the three numbers:
3 × 5 × 8 = 120.

Therefore, 120 is the LCM of 3, 5, and 8.

Method 2:

Step 1 – Draw a factor tree for 3, 5 and 8

3

5

```
        8
       / \
      2 x 4
         / \
        2 x 2
```

Step 2 – Count the number of times each prime number appears in the factor trees.

Number of 3's = 1
Number of 5's = 1

Number of 2's = 3

Step 3 – find the largest count for each prime number.

- Largest count for the prime number 3 = 1
- Largest count for the prime number 5 = 1
- Largest count for the prime number 2 = 3

Step 4 – List each prime number as many times as you counted it in Step 3.

LCM = 2x2x2x3x5 = 120

Example 1.4-h: Finding the Least Common Multiple

Find the LCM of 3, 6, and 18.

Solution

Method 1:

The largest number, 18, is divisible by both 6 and 3.

 Therefore, 18 is the LCM of 3, 6, and 18.

Method 2:

3

$$
6
$$
$$
/ \quad \backslash
$$
$$
2 \times 3
$$

$$
18
$$
$$
/ \quad \backslash
$$
$$
2 \times 9
$$
$$
/ \quad \backslash
$$
$$
3 \times 3
$$

Step 2 – Count the number of times each prime number appears in the factor trees.

Number of 3's = 1

Number of 2's = 1
 Number of 3's = 1

Number of 2's = 1
 Number of 3's = 2

Step 3 – find the largest count for each prime number.

- Largest count for the prime number 2 = 1
- Largest count for the prime number 3 = 2

Step 4 – List each prime number as many times as you counted it in Step 3.

LCM = 2x3x3 = 18

Example 1.4-i: Finding the Least Common Multiple

Find the LCM of 24, 36, and 48.

Solution

Method 1:

The largest number, 48, is divisible by 24 but **not** 36.

Multiples of 48 are 48, 96, 144,...

144 is divisible by both 24 and 36.

Therefore, 144 is the LCM of 24, 36, and 48.

Method 2:

Step 1 – Draw a factor tree for 24, 36 and 48

```
 24
 / \
2 x 12
    / \
   2 x 6
       / \
      2 x 3
```

```
 36
 / \
2 x 18
    / \
   2 x 9
       / \
      3 x 3
```

```
      48
     /  \
    2 x 24
       /  \
      2 x 12
         /  \
        2 x 6
           /  \
          2 x 3
```
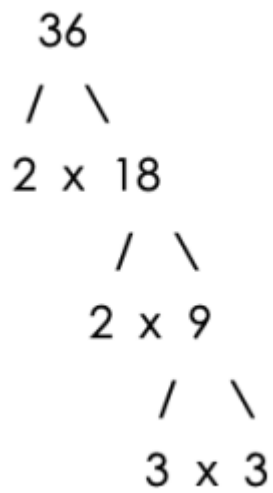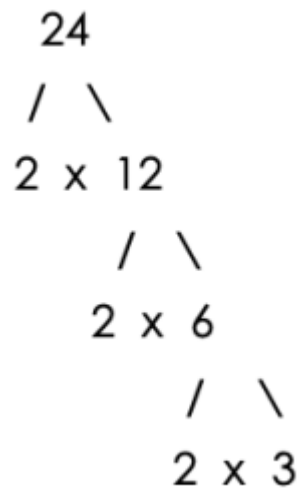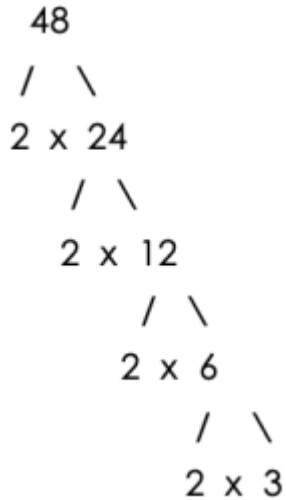
Step 2 – Count the number of times each prime number appears in the factor trees.

Number of 2's = 3
  Number of 3's = 1

Number of 2's = 2
  Number of 3's = 2

Number of 2's = 4
  Number of 3's = 1

Step 3 – find the largest count for each prime number.

- Largest count for the prime number 2 = 4
- Largest count for the prime number 3 = 2

Step 4 – List each prime number as many times as you counted it in Step 3.

LCM = 2x2x2x2x3x3 = 144

Example 1.4-j: Finding the Least Common Multiple to Solve a Word Problem

Two flashing lights are turned on at the same time. One light flashes every 16 seconds, and the other flashes every 20 seconds. How often will they flash together?

Solution

In this example, we are required to find the **least common** interval for both lights to flash together. Both lights will continue to flash together at this interval (**multiple**).

Method 1:

The largest number, 20, is **not** divisible by 16.

Multiples of 20 are 20, 40, 60, 80,...

80 is divisble by 16.

Therefore, 80 is the LCM of 16 and 20.

Method 2:

Step 1 – Draw a factor tree for 16 and 20

```
    16
   /  \
  2 x 8
     /  \
    2 x 4
       /  \
      2 x 2
```

```
    20
   /  \
  2 x 10
      /  \
     2 x 5
```

Step 2 – Count the number of times each prime number appears in the factor trees.

Number of 2's = 4

Number of 2's = 2
   Number of 5's = 1

Step 3 – find the largest count for each prime number.

- Largest count for the prime number 2 = 4
- Largest count for the prime number 5 = 1

Step 4 – List each prime number as many times as you counted it in Step 3.

LCM = 2x2x2x2x5 = 80

Therefore, the two flashing lights will flash together every 80 seconds.

Greatest Common Factor (GCF)

The factors that are common to two or more numbers are called **common factors** of those numbers.

The **Greatest Common Factor (GCF)** of two or more numbers is the largest common number that divides the numbers with no remainder. In other words, the GCF is the largest of all the common factors.

The GCF can be determined using one of the following methods:

Method 1:

First list all the factors of all the numbers. Then select all the common factors of the numbers. The highest value of the common factors is the GCF.

For example, in finding the GCF of 12 and 18:

- The factors of 12 are 1, **2**, **3**, 4, **6**, and 12.
- The factors of 18 are 1, **2**, **3**, **6**, 9, and 18.

The common factors are **2**, **3**, and **6**.

Therefore, the GCF is 6.

Note: 1 is a factor common to all numbers, so we do not bother to include it in the list of common factors. If there are no common factors other than 1, then 1 is the greatest common factor.

For example, 1 is the only common factor of 3, 5, 7, and 9, and therefore the GCF = 1.

Method 2:

First, express each number as a product of prime factors. Then identify the set of prime factors common to **all** the numbers (including repetitions). The product of these prime factors is the GCF.

For example, in finding the GCF of 12 and 18:

```
        12
       /  \
      2 x 6
          /  \
         2 x 3


        18
       /  \
      2 x 9
          /  \
         3 x 3
```

The Prime factors are:

- 12 = 2x2x**3**
- 18 = **2**x3x3

The set of prime factors common to 12 and 18 is one 2 and one 3.

Therefore, the GCF is 2 × 3 = 6.

Example 1.4-k: Finding the Greatest Common Factor

Find the GCF of 36 and 60.

Solution

Method 1:

Factors of 36 are 1, **2**, **3**, **4**, **6**, 9, **12**, 18, and 36.

Factors of 60 are 1, **2**, **3**, **4**, 5, **6**, 10, **12**, 15, 20, 30, and 60.

The common factors are **2**, **3**, **4**, **6**, and **12**.

Therefore, the GCF is 12.

Method 2:

First, express each number as a product of prime factors. Then identify the set of prime factors common to **all** the numbers (including repetitions). The product of these prime factors is the GCF.

For example, in finding the GCF of 36 and 60:

```
   36
  / \
 2 x 18
    / \
   2 x 9
      / \
     3 x 3
```

```
        60
       /  \
      2 x  30
          /  \
         2 x  15
             /  \
            3 x  5
```

The Prime factors are:

- 36 = **2×2**x3x3
- 60 = 2x2x**3**x5

The set of prime factors common to 36 and 60 is two 2's and one 3.
Therefore, the GCF is 2 × 2 × 3 = 12.

Example 1.4-l: Finding the Greatest Common Factor

Find the GCF of 72, 126, and 216.

Solution

Method 1:

Factors of 72 are 1, **2**, **3**, 4, **6**, 8, **9**, 12, **18**, 24, 36, and 72.
   Factors of 126 are 1, **2**, **3**, **6**, 7, **9**, 14, **18**, 21, 42, 63, and 126.
   Factors of 216 are 1, **2**, **3**, 4, **6**, 8, **9**, 12, **18**, 24, 27, 36, 54, 72, 108, and 216.

The common factors are **2**, **3**, **6**, **9**, and **18**.

Therefore, the GCF is 18.

Method 2:

First, express each number as a product of prime factors. Then, identify the set of prime factors common to **all** the numbers (including repetitions). The product of these prime factors is the GCF.

For example, in finding the GCF of 72, 126 and 216

```
72
/ \
2 x 36
   / \
  2 x 18
     / \
    2 x 9
       / \
      3 x 3
```

```
126
/ \
2 x 63
   / \
  3 x 21
     / \
    3 x 7
```

```
  216
  / \
 2 x 108
    / \
   2 x  54
      / \
     2 x 27
        / \
       3 x 9
          / \
         3 x 3
```

The prime factors are:

- 72=2x2x2x**3×3**
- 126=2x2x2x**3×3**
- 216=2x2x2x3x3x3

The set of prime factors which is common to 72, 126 and 216 is one 2 and two 3's.

Therefore, the GCF is 2 × 3 x 3 = 18

Example 1.4-m: Finding the Greatest Common Factor to Solve a Word Problem

Three pieces of timber with lengths 48 cm, 72 cm, and 96 cm are to be cut into smaller pieces of equal length without remainders.

    a. What is the greatest possible length of each piece?
    b. How many pieces of such equal lengths are possible?

Solution

Method 1:

Factors of 48 are: 1, **2**, **3**, **4**, **6**, **8**, **12**, 16, **24**, and 48.

Factors of 72 are: 1, **2**, **3**, **4**, **6**, **8**, 9, **12**, 18, **24**, 36, and 72.

Factors of 96 are: 1, **2**, **3**, **4**, **6**, **8**, **12**, 16, **24**, 32, 48, and 96.

The common factors are: **2**, **3**, **4**, **6**, **8**, **12**, and **24**.

Therefore, the GCF is 24.

Method 2:

First, express each number as a product of prime factors. Then, identify the set of prime factors common to **all** the numbers (including repetitions). The product of these prime factors is the GCF.

For example, in finding the GCF of 48, 72, 96

```
   48
  /  \
 2 x 24
     /  \
    2 x 12
        /  \
       2 x 6
           /  \
          2 x 3
```

```
72
/  \
2 x 36
    /  \
    2 x 18
        /  \
        2 x 9
            /  \
            3 x 3
```

```
96
/  \
2 x 48
    /  \
    2 x  24
        /  \
        2 x 12
            /  \
            2 x 6
                /  \
                2 x 3
```

The prime factors are:

- 48=**2x2x2x3**
- 72=2x2x2x3x3
- 90=2x2x2x2x2x3

The set of prime factors which is common to 48, 72 and 96 is three 2's and one 3.

Therefore, the GCF = 2 × 2 × 2 × 3 = 24.

a. Therefore, the greatest possible length of each piece is 24 cm.
b. The total number of equal pieces is the number of multiples of 24 cm in each piece:

$48 = 24 \times 2$

$72 = 24 \times 3$

$96 = 24 \times 4$

Therefore, the total number of equal pieces of 24 cm possible is $2 + 3 + 4 = 9$.

1.4 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

1. List all the prime numbers less than 20.

2. List all the prime numbers between 20 and 40.

3. List all the composite numbers greater than 10 and less than 30.

4. List all the composite numbers greater than 30 and less than 50.

5. Identify the prime numbers in the given sets of numbers:

   a. 13, 19, 36, 47, 49

   b. 11, 14, 29, 35, 43

6. Identify the prime numbers in the given sets of numbers:

   a. 31, 39, 41, 59, 63

   b. 23, 37, 45, 51, 53

For Problems 7 to 14, (i) find all the factors and (ii) list the prime factors of the numbers.

7. a. 15

   b. 34

8. a. 18

   b. 35

9. a. 64

   b. 54

10. a. 56

    b. 60

11. a. 21

    b. 25

12. a. 30

    b. 42

13. a. 36

    b. 65

14. a. 40

    b. 49

For Problems 15 to 18, find the first six multiples of the numbers.

15. a. 6

    b. 8

16. a. 5

    b. 12

17. a. 9

    b. 10

18. a. 7

    b. 15

For Problems 19 to 24, find the least common multiple (LCM) of each pair of the numbers.

19. a. 10, 15

    b. 15, 18

20. a. 12, 16

    b. 21, 36

21. a. 18, 24

    b. 35, 45

22. a. 10, 25

   b. 45, 60

23. a. 16, 64

   b. 8, 60

24. a. 12, 90

   b. 25, 30

For Problems 25 to 32, find the least common multiple (LCM) of the sets of numbers.

25. a. 2, 3, 8

   b. 4, 9, 10

26. a. 8, 5, 12

   b. 5, 15, 20

27. a. 10, 15, 25

   b. 6, 27, 36

28. a. 4, 8, 40

   b. 3, 16, 2

29. a. 14, 21, 28

   b. 3, 18, 27

30. a. 4, 6, 21

   b. 12, 28, 42

31. a. 24, 36, 12

   b. 6, 15, 18

32. a. 5, 12, 15

   b. 12, 40, 48

For Problems 33 to 38, find the (i) factors, (ii) common factors, and (iii) greatest common factor (GCF) of each pair of numbers.

33. a. 15, 25

b. 18, 32

34. a. 14, 35

    b. 8, 36

35. a. 18, 48

    b. 32, 60

36. a. 16, 30

    b. 36, 42

37. a. 25, 80

    b. 40, 120

38. a. 35, 75

    b. 24, 64

For Problems 39 to 44, find the (i) factors, (ii) common factors, and (iii) greatest common factor (GCF) of the sets of numbers.

39. a. 8, 12, 15

    b. 6, 15, 20

40. a. 6, 8, 10

    b. 10, 15, 25

41. a. 12, 18, 24

    b. 12, 30, 42

42. a. 24, 36, 40

    b. 12, 36, 48

43. a. 40, 50, 80

    b. 30, 75, 90

44. a. 50, 75, 125

    b. 60, 90, 120

45. Two 96 cm and 160 cm wires are to be cut into pieces of equal length without wastage. Find the greatest

possible length of each piece.

46. Two ribbons 112 cm and 154 cm lengths are to be cut into pieces of equal length, without wastage. Find the greatest possible length of each piece.

47. Tahrell has music lessons every 6th day and swimming lessons every 8th day. If he had music and swimming lessons on February 4th, on which date would he have both lessons again?

48. Enea has skating lessons every 8th day and ballet lessons every 10th day. If she had skating and ballet lessons on March 3rd, on which date would she have both lessons again?

49. Three wires measuring 18 m, 45 m, and 36 m will be cut into pieces of equal length without wastage. What is the maximum possible length of each piece?

50. A store has 54 green marbles, 72 yellow marbles, and 90 red marbles. The owner packages all the marbles into bags, each containing the same number. As well, each bag has to contain marbles of the same colour. Find the maximum possible number of marbles in each bag.

51. Three red, blue, and green lights flash at intervals of 15, 18, and 40 seconds, respectively. If they begin flashing at the same time, how long will it take (in minutes) until all three flashes at the same time again?

52. Three bells ring simultaneously. If they ring at intervals of 24, 36, and 40 seconds, respectively, how long will it take

(in minutes) until they ring together again?

1.5 ORDER OF OPERATIONS

In this chapter, we have learned how to perform addition, subtraction, multiplication, division, powers, and square roots with whole numbers. In this section, we will learn the correct order (or sequence) for performing the combined arithmetic operations of whole numbers.

When there are no groupings (operations within brackets or a radical sign), the six arithmetic operations are performed in the following sequence:

1. Exponents (Powers) and Roots
2. Division and Multiplication, in order from left to right
3. Addition and Subtraction, in order from left to right

Example 1.5-a: Evaluating Expressions with Mixed Arithmetic Operations

Evaluate the following arithmetic expressions:

a. $16 \div 2^2 + 44 - 3^3$
b. $12 \div 3 \times 2 + 5^2$

Solution

a. $16 \div 2^2 + 44 - 3^3$

Evaluating the exponents,

$= \mathbf{16 \div 4} + 44 - 27$

Dividing,

$= \mathbf{4} + \mathbf{44} - 27 = \mathbf{48} - \mathbf{27}$

Adding and subtracting from left to right,

$= 21$

b. $12 \div 3 \times 2 + 5^2$

Evaluating the exponent,

$= \mathbf{12} \div \mathbf{3} \times 2 + 25 = \mathbf{4} \times \mathbf{2} + 25$

Dividing and multiplying from left to right,

$= \mathbf{8} + \mathbf{25}$

Adding,

$= 33$

When there are groupings, the arithmetic operations within the groupings are to be evaluated first. Common symbols used for groupings are brackets ( ), [ ], { }, and the radical sign √ .

When there are groupings with more than one bracket, start by evaluating the innermost bracket, and move outwards to evaluate all expressions within the brackets by following the order of operations explained earlier.

In summary, arithmetic expressions that contain multiple operations, with brackets, exponents, divisons, multiplications, additions, and subtractions, are performed in the following sequence:

1. Brackets
2. Exponents (Powers) and Roots
3. Division and Multiplication, in order from left to right
4. Addition and Subtraction, in order from left to right

The above order of operations, **B**rackets, **E**xponents, **D**ivision and **M**ultiplication, and **A**ddition and **S**ubtraction, can be remembered by the following acronym:

B E D M A S

Performed together, in order from left to right

Performed together, in order from left to right

Example 1.5-b: Evaluating Expressions with Square Roots

Evaluate the following expressions:

a. $\sqrt{49} + \sqrt{25}$
b. $\sqrt{64} - \sqrt{16}$
c. $\sqrt{4} \times \sqrt{9}$

Solution

a. $\sqrt{49} + \sqrt{25}$

Evaluating the square roots,

$= 7 + 5$

Adding,

$= 12$

b. $\sqrt{64} - \sqrt{16}$

Evaluating the square roots,

= 8 − 4

Subtracting,

= 4

c. $\sqrt{4} \times \sqrt{9}$

Evaluating the square roots,

= 2 × 3

Multiplying,

= 6

Example 1.5-c: Evaluating Expressions with Square Roots

Evaluate the following expressions:

   a. $5 \times \sqrt{121}$
   b. $\sqrt{100} \div 5$
   c. $35 + \sqrt{49} - \sqrt{9}$

Solution

a. $5 \times \sqrt{121}$

Evaluating the square root,

= 5 × 11

Multiplying,

= 55

b. $\sqrt{100} \div 5$

Evaluating the square root,

= 10 ÷ 5

Dividing,

= 2

c. $35 + \sqrt{49} - \sqrt{9}$

Evaluating the square roots,

= 35 + 7 – 3

Adding and subtracting from left to right,

= 39

Example 1.5-d: Evaluating Expressions with Groupings

Evaluate the following expressions:

   a. $4 \times 50 \div (8 - 3)^2 - 1$
   b. (100 – 3 × 24) + 8(6 – 3) ÷ 6
   c. $\sqrt{3^2 + 4^2} \times (7 - 4) + 2$
   d. $35 \div 7 + \sqrt{4^2 + 9}$
   e. $4^2 - \sqrt{13^2 - 5^2} + 3^2\sqrt{25}$

Solution

a. $4 \times 50 \div (8 - 3)^2 - 1$

Evaluating the operation within the bracket,

$= 4 \times 50 \div 5^2 - 1$

Evaluating the exponent,

$= 4 \times 50 \div 25 - 1 = 200 \div 25 - 1$

Dividing and multiplying from left to right,

$= 8 - 1$

Subtracting,

$= 7$

b. $(100 - 3 \times 24) + 8(6 - 3) \div 6$

Evaluating the operations within the brackets in the order of operations,

$= (100 - 72) + 8(3) \div 6$

$= 28 + 8(3) \div 6 = 28 + 24 \div 6$

Dividing and multiplying from left to right,

$= 28 + 4$

Adding,

$= 32$

c. $\sqrt{3^2 + 4^2} \times (7 - 4) + 2$

Evaluating the operations within the groupings (radical sign and bracket) in the order of operations,

$= \sqrt{9 + 16} \times 3 + 2$

$= \sqrt{25} \times 3 + 2$

Evaluating the square root,

$= 5 \times 3 + 2$

Multiplying,

$= 15 + 2$

Adding,

$= 17$

d. $35 \div 7 + \sqrt{4^2 + 9}$

Evaluating the operations within the radical sign in the order of operations,

$= 35 \div 7 + \sqrt{16 + 9}$

$= 35 \div 7 + \sqrt{25}$

Evaluating the square root,

$= 35 \div 7 + 5$

Dividing,

$= 5 + 5$

Adding,

$= 10$

e. $4^2 - \sqrt{13^2 - 5^2} + 3^2\sqrt{25}$

Evaluating the operations within the radical sign in the order of operations,

$= 4^2 - \sqrt{169 - 25} + 3^2\sqrt{25}$

$= 4^2 - \sqrt{144} + 3^2\sqrt{25}$

Evaluating the exponents and square roots,

= 16 − 12 + **9** × **5**

Multiplying,

= **16 − 12** + 45 = **4 + 45**

Adding and subtracting from left to right,

= 49

Example 1.5-e: Evaluating Expressions with More than One Bracket

Evaluate the following arithmetic expressions:

a. $4 \times 50 \div \left[(8 - 3)^2 - 5 \right]$

b. $100 - 3 \left[24 \div 2(6 - 3) \right] \div 2$

c. $\left[10^2 \times 4 + 50 \right] \div \left[(8 - 3)^2 - 4^2 \right]$

Solution

Solutions to Example 1.5-e

Question	Formulae	Description
a.	$4 \times 50 \div [(8 - 3)^2 - 5]$	Evaluating the operation within the inner bracket,
	$= 4 \times 50 \div [5^2 - 5]$	Evaluating the operations within the outer bracket in the order of operations,
	$= 4 \times 50 \div [25 - 5]$	
	$= 4 \times 50 \div 20 = 200 \div 20$	Dividing and multiplying from left to right,
	$= 10$	
b.	$100 - 3\,[24 \div 2(6 - 3)] \div 2$	Evaluating the operation within the inner bracket,
	$= 100 - 3[24 \div 2 \times 3] \div 2$	Dividing and multiplying from left to right within the outer bracket,
	$= 100 - 3[12 \times 3] \div 2$	
	$= 100 - 3 \times 36 \div 2 = 100 - 108 \div 2$	Dividing and multiplying from left to right,
	$= 100 - 54$	Subtracting,
	$= 46$	
c.	$[10^2 \times 4 + 50] \div [(8 - 3)^2 - 4^2]$	
	$= [10^2 \times 4 + 50] \div [5^2 - 4^2]$	Evaluating the exponents within the brackets,
	$= [100 \times 4 + 50] \div [25 - 16]$	Dividing and multiplying from left to right within the outer bracket,
	$= [400 + 50] \div 9$	
	$= 450 \div 9$	Dividing,
	$= 50$	

1.5 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

For Problems 1 to 12, evaluate the expressions with mixed arithmetic operations.

1. a. $5 \times 4 + 25 \div 25$

   b. $64 \div 8 \times 2$

2. a. $7 \times 5 + 20 \div 4$

   b. $36 \div 4 \times 9$

3. a. $100 \div 25 \times 4$

   b. $18 \div 2 \times 3 + 5$

4. a. $80 \div 10 \times 8$

   b. $50 \times 2 \times 5 + 10$

5. a. $32 \div 4 \div 2 \times 4$

   b. $96 \div 12 \times 2 + 4$

6. a. $20 \div 4 \times 5 + 2$

   b. $56 \div 4 \div 2 \times 5$

7. a. $4^2 \times 2^4$

   b. $3^2 \times 2^3$

8. a. $5^2 \times 2^3$

   b. $4^2 \times 3^3$

9. a. $2^3 + 3^3$

   b. $5^2 - 4^2$

10. a. $5^2 + 6^2$

    b. $8^2 - 6^2$

11. a. $6^2 \times 2 - 2$

    b. $100 - 5^2 \times 3$

12. a. $5^2 \times 3 - 15$

    b. $144 - 3^3 \times 4$

For Problems 13 to 22, evaluate the expressions with square roots.

13. a. $\sqrt{100} + \sqrt{25}$

    b. $\sqrt{81} - \sqrt{16}$

14. a. $\sqrt{121} + \sqrt{36}$

    b. $\sqrt{144} - \sqrt{9}$

15. a. $\sqrt{9} \times \sqrt{16}$

    b. $\sqrt{36} \times \sqrt{49}$

16. a. $\sqrt{25} \times \sqrt{64}$

    b. $\sqrt{81} \times \sqrt{121}$

17. a. $\sqrt{40} - \sqrt{24}$

    b. $\sqrt{75} - \sqrt{11}$

18. a. $\sqrt{125} - \sqrt{76}$

    b. $\sqrt{48} - \sqrt{23}$

19. a. $\sqrt{3^2} + \sqrt{4^2}$

    b. $\sqrt{13^2} - \sqrt{5^2}$

20. a. $\sqrt{5^2} - \sqrt{4^2}$

    b. $\sqrt{12^2} + \sqrt{5^2}$

21. a. $\sqrt{100 \div 25}$

    b. $\sqrt{196 \div 4}$

22. a. $\sqrt{256 \div 4}$

    b. $\sqrt{225 \div 25}$

For Problems 23 to 46, evaluate the expressions with groupings.

23. a. $(7 - 4)^2$

    b. $(3 + 2)^3$

24. a. $(8 - 5)^3$

    b. $(4 + 1)^2$

25. a. $5^2 \sqrt{16} + 10 - 2$

    b. $19 - 2^2 \sqrt{9} + 3$

26. a. $40 - 3^2 \sqrt{16} + 1$

    b. $6^2 \sqrt{25} + 16 - \sqrt{100}$

27. a. $(4 + 3)^2 - 5^2 + 2^3$

    b. $6^2 + 2^3 - (12 - 8)^2$

28. a. $(9 - 6)^2 - 2^2 + 3^3$

    b. $3^2 + 2^4 - (15 - 9)^2$

29. $12^2 - 5 \times 27 \div (5 - 2)^2 - 3$

30. $16 \div 8 \times 10^2 + (12 - 7)^2 \times 2$

31. $3^2[(9 - 6)^2 \div 9 + 7 - 4]$

32. $3^2[(12^2 + 8)^2 \div 4 + 6 - 2]$

33. $7 + (3\sqrt{49} - 1)^2$

34. $15 + (5\sqrt{10^2} - 8)^2$

35. $16 \div \sqrt{64} + \sqrt{10^2 - 6^2}$

36. $\sqrt{12^2 + 5^2} - 27 \div \sqrt{81}$

37. $[(20 \div 5) \times 8] \div (2^2 + 4)$

38. $[(6^2 \div 4) \times 5] \div (2^2 + 5)$

39. $(64 \div 8 \div 4)^2 + (3^2 + 6^0)$

40. $(81 \div 9 \div 3)^2 + (9^0 + 2^2)$

41. $[(11 - 2)^2 + 3] + (16 \div 2)^2$

42. $(20 \div 2)^2 + [(13 - 6) + 4^2]$

43. $(45 - \sqrt{9^2 + 12^2}) \div (4^2 - \sqrt{36})$

44. $(27 + \sqrt{15^2 - 12^2}) \div (5^2 - \sqrt{49})$

45. $\sqrt{144} - 3^2 + (64 \div 4^2)^2 \div (18 \div 3^2)$

46. $\sqrt{81} + 2^2 - (36 \div 3^2)^2 \div (16 \div 2^2)$

1. REVIEW EXERCISES

Answers to the odd-numbered problems are available at the end of the textbook.

For Problems 1 and 2, write the numbers in expanded form and word form.

1. a. 7,502   b. 25,047   c. 620,025   d. 3,054,705

2. a. 9,024   b. 38,024   c. 405,037   d. 2,601,071

For Problems 3 and 4, write the the numbers in standard form and expanded form.

3. a. Five thousand, six hundred seven

   b. Thirty-seven thousand, forty

   c. Four hundred eight thousand, one hundred five

   d. One million, seventy thousand, fifty-five

4. a. Nine thousand, nine hundred three

   b. Fifty-nine thousand, three hundred three

   c. Seven hundred thousand, eight hundred eighty-eight

   d. Seven million, seventy-six thousand, fifty-five

For Problems 5 and 6, insert the correct sign '>' or '<' in the space between the numbers (indicated by underscore).

5. a. 167 _ 176

   b. 2,067 _ 2,097

   c. 79,084 _ 79,087

    d. 162,555 _ 162,507

6. a. 159 _ 139

    b. 1,838 _ 1,868

    c. 52,109 _ 51,889

    d. 379,847 _ 397,487

For Problems 7 and 8, estimate the answer by rounding each number to the nearest hundred, and calculate the exact answer.

7. a. $3,495 + 276 + 85$ b. $5,555 + 157 + 60$

    c. $7,836 - 655$ d. $6,405 - 2,769$

8. a. $8,655 + 348 + 75$ b. $3,450 + 645 + 50$

    c. $5,245 - 876$ d. $2,056 - 444$

For Problems 9 and 10, perform the arithmetic operation.

9. a. 465×23 b. 365×24 c. $315 \div 5$ d. $2,532 \div 12$

10. a. 345×34 b. 237×25 c. $276 \div 6$ d. $4,785 \div 15$

Evaluate problems 11 to 22 and express the answers rounded to two decimal places, wherever applicable.

11. a. $\dfrac{16 + 4(-3)}{10 - 4 + 1} + \dfrac{(16 + 4) - 3}{10 - (4 + 1)} -$

    b. $14 - 3[(6 - 9)(-4) + 12] \div (-2)$

12. a. $\dfrac{2(-6) + 4}{24 - (7 + 3)} + \dfrac{2(-6 + 4)}{24 - 7 + 3} -$

    b. $5(-4) - 3[(-9 + 6) + (-3) - 4]$

13. a. $[(1 + 12)(1 - 5)]^2 \div [(5 + 3) \times 2^2 - (-2)^2]$

    b. $2^2 [(9 - 7) \div 2 + 9 - 4]$

14. a. $8 \div 4 + (4 - 6^2) \div (13 - 5) \times (-2)^6$

    b. $6 \div [4 \times (2 - 8) \div (3^2 + 3)] \div 4$

15. a. $64 \div (-2)^4 + 4(-3^2) \div 2 - 5$

    b. $(-6)^2 - 9^2 \div 3^3 - (-3)(-2)$

16. a. $8 \div (-2)^3(-9) + 6(-5)^3 \div (-5)^2$

    b. $(-8)^2 - 4^3 \div 2^2 - (-6)(-2)$

17. a. $-15 - (-15)$

    b. $-14 - (-7)$

18. a. $13 - (-11) + 0$

    b. $22 - (-4) - 6$

For Problems 19 and 20, find the LCM of the numbers.

19. a. 12 and 20   b. 16 and 72   c. 16, 18, and 33

20. a. 16 and 40   b. 36 and 54   c. 8, 24, and 32

For Problems 21 and 22, find the GCF of the numbers.

21. a. 8 and 12   b. 42 and 48   c. 24, 30, and 32

22. a. 4 and 9   b. 40 and 72   c. 12, 16, and 60

For Problems 23 to 36, perform the arithmetic operations.

23. a. $6 + 8 - 6 \times 2 \div 4$

 b. $15 - (7 - 5) \div 2$

24. a. $9 + 2 - 4 \times 3 \div 2$

 b. $10 - (7 - 4) \div 3$

25. a. $12 - 2(9 - 6) + 10 \div 5 + 5$

 b. $9 - 8(7 - 5) \div (6 + 2)$

26. a. $8 - 4(6 - 4) + 16 \div 4 + 4$

 b. $10 - 4(9 - 7) \div (5 + 3)$

27. a. $8(7 + 3) + 6^2 \div 4$

 b. $8^2 \div 4 - 6(5 - 3)$

28. a. $7(6 + 4) + 4^2 \div 2$

 b. $9^2 \div 3(8 - 5) - 4(5 + 3)$

29. a. $24 \div 2^2 \times 3 + (5 - 2)^2$

 b. $8(7 - 3)^2 \div 4 - 5$

30. a. $64 \div (8 - 4)^2 + 5^2$

 b. $9(8 - 5) \div 3 + (7 - 4)^2$

31. a. $(16 + 4 \times 2) \div (4^2 - 8)$

 b. $6^2 - 2[(6 - 3)^2 + 4]$

32. a. $(6 + 3 \times 2) \div (2^2 - 1)$

    b. $8^2 - 3[(7 - 3)^2 + 2]$

33. a. $\sqrt{9} - (8 - 5) + 10 \div 5 + 7$

    b. $15 - 15(8 - 6) \div \sqrt{36} + 15$

34. a. $\sqrt{49} - 7(6 - 4) \div (5 - 3)$

    b. $\sqrt{16} + (10 - 7) + 20 \div 4 - 3$

35. a. $6^2 \div 9 + 6(5^2 - 2^2)$

    b. $3[(7 - 4)^2 + 4] - (2 + 3)^2$

36. a. $5(12^2 - 2^2) + 48 \div 4^2$

    b. $(6 + 2)^2 - 4[(12 - 9)^2 + 3]$

37. After Martha gave 175 stamps to her brother, she had 698 stamps left. How many stamps did she have at the beginning?

38. Amy spent $349 and had $167 left. How much did she have at the beginning?

39. Each ticket for a concert costs $25. A total of $35,000 was collected from ticket sales for Saturday and Sunday. If 550 tickets were sold on Saturday, how many were sold on Sunday?

40. A company manufactured printers for $40 a unit. Over two weeks, $46,000 was spent on manufacturing printers. If 500 printers were manufactured in the first week, how many printers were manufactured in the second week?

41. At a concert, 245 tickets were sold for $125 each and 325 tickets were sold for $68 each. How much money was collected altogether?

42. Susie held a bake sale. She sold 45 cookies for $2 each and 63 brownies for $3 each. How much money did she make altogether?

43. Allan and Babar have a total of $2,550. Allan has $800 more than Babar. How much money does each of them have?

44. Ayesha saved $5,500 more than Beth. If they saved $32,450 together, how much did each of them save?

45. An elevator can carry a maximum of 540 kg. Two workers want to move 20 boxes of tiles, each weighing 24 kg. One of the workers weighs 72 kg and the other weighs 65 kg. What number of boxes can be carried in the elevator if both workers are in it?

46. A delivery truck can carry a maximum of 2,000 kg. Two workers want to move 100 planks of wood weighing 30 kg. One of the workers weighs 85 kg and the other weighs 77 kg. What is the largest number of planks that can be carried by the truck if both workers are in the truck?

47. Three balls of yarn measuring 24 metres, 60 metres, and 36 metres are to be cut into pieces of equal lengths, without wastage. What is the maximum possible length of each piece?

48. A store has 32 oranges, 48 bananas, and 72 apples. The owner decides to make fruit baskets each containing an equal number of fruits, without any left over. Each basket was required to have only one type of fruit in it. Find the maximum possible number of fruits in each basket.

49. Amy, Bob, and Cathy go for a swim every 3rd, 7th, and 14th day, respectively. If they met each other on a particular day at the pool, how many days later would they meet again?

50. Three gentlemen decided to go for a walk around a circular park. The first man takes 6 minutes, the second takes 10 minutes, and the third takes 8 minutes. If they start together, when will they meet again?

1. SELF-TEST

Answers to all problems are available at the end of the textbook.

For Problem 1, estimate the answer by rounding each number to the nearest hundred, and calculate the exact answer.

1. a. $5,475 + 1,260 + 179 - 50$

    b. $1,274 \times 350$

    c. $6,720 \div 112$

For Problems 2 – 5, evaluate the expression.

2. a. $5(3^2 - 4) + (5^2 - 4^2)$

    b. $(5 + 4)^2 \div (5 - 2)^3$

    c. $4[12 - (6 - 3)(2)] \div 12$

3. a. $8 + (8 \times 5 + 4) + 11 - 7$

    b. $(5 + 1)^2[(11 - 9)^2 - 6 \div 2]$

    c. $100 \div [5 \times 4 + (8 - 3)^2 + 5]$

4. a. $\sqrt{(4^2 \times 5 + 1)} - \sqrt{6^2 \div 3^2}$

    b. $16 \times 8 - (4^2 + 8) + \sqrt{(16^2 \div 4)^2}$

    c. $5(5^2 - 3^2) \div (5 - 3)$

5. a. $(4 \times 8 - 4^2) \div \sqrt{10^2 \div 6^2}$

    b. $[(12^2 - 4 \times 3) + 8] \div 7$

c. $12^2 \div 4 - 3 \times 3^2$

6. Bob and Hari saved a total of $7,650. If Bob saved five times as much as Hari, how much money do each of them have?

7. A truck can hold 1,275 cartons of milk. Each carton weighs 18 -kg and the empty truck weighs 3,045 kg. Calculate the total weight of the truck when it is fully loaded with milk.

8. Caroline's take home pay for last year was $48,880. Calculate her pay for each week. (Assume 1 year = 52 weeks).

9. 4,256 people visited the Toronto Zoo on Sunday. 1,968 were adults and the rest were children. How many more children than adults visited the zoo?

10. Four lights, red, blue, green, and yellow, flash at intervals of 12, 16, 18, and 21 seconds, respectively. If they begin flashing at the same time, when will they flash together again?

11. Four wires measuring 12 m, 18 m, 24 m, and 42 m are to be cut into pieces of equal lengths, without wastage. What is the maximum possible length of each piece?

1. CASE STUDY - LIGHTING WHOLESALE

Lightning Wholesale is a mid-to large-size company that works directly with manufacturers to bring sporting goods and equipment to mid to small-sized retailers. The company is in Southern Ontario, Canada. Lightning Wholesale acquires the goods, stores them in its warehouses, and distributes them to retailers, who then sell them to sporting enthusiasts across the province. Suppliers are global, including those from the United States.

Lightning Wholesale is a dynamic office with global networking, communications, customer service, and complex workflows. Lighting Wholesale depends on its office professionals to support staff and management. Office Professional tasks include clerical support, bookkeeping, customer service, data analysis, and document processing.

Lightning Wholesale cares about its employees and retailers. Where managers can assist their employees, they try to. Where retailers need guidance, Lightning Wholesales tries to fill this gap as best they can. In working with its small retailers, many of which are owned and operated by sole proprietors, Lightning Wholesale assists the owners with their pricing to help develop promotional pricing plans and the amounts of the payments that can be advertised. Lightning Wholesale freely offers help since it means more sales and increased profits for the company.

There are currently a few main tasks for Lightning Wholesale management:

- The payroll needs review for the past fiscal year.
- Costs and expenses need forecasting for the next fiscal year.
- Detailed analysis of office supply orders from quarter to quarter with recommendations for the next fiscal year.
- Forklifts need replacing in the warehouse.
- The next round of co-op students needs a computer.
- The employee appreciation event needs planning for next year.
- Travel expenses for the mobile team need to be summarized for the last fiscal year.

With year-end approaching, Lighting Wholesale has many processes they are wrapping up. It is important that all departments execute their responsibilities as accurately and seamlessly as possible.

Questions

1. The sales team is presenting Q4 results to the management team. The revenues for Q4 were 1,347,201.

In the presentation to the team, they would like to round the nearest ten thousand. What is the rounded revenue amount? Why would the management team use a rounded number for the presentation?

2. The budget for office supplies for the product management team is $150 monthly. How much is the budget for 2 quarters (a quarter is 3 months)?

3. Fifteen office professionals are working at Lightning Wholesale. The sales team has 3 office professionals, the marketing team has 1 office professional, the human resources team has 2 office professionals, and finance has 2 office professionals. The rest of the office professionals work in customer service. How many office professionals work in customer service?

PART II

CHAPTER 2 - DECIMALS AND FRACTIONS

OBJECTIVES

- Read, write, and round decimal numbers.
- Identify the types of fractions and perform computations with them.
- Determine the least common denominator (LCD).
- Solve problems involving fractions and decimal numbers.
- Determine the relationship between fractions and decimal numbers.
- Perform arithmetic operations combined with fractions and decimal numbers.

CHAPTER OUTLINE

Introduction

In the previous chapter, we learned about whole numbers and how to perform basic arithmetic operations with whole numbers. However, measurements and calculations of quantities, values, amounts, etc., cannot always be represented by whole numbers. Most of these involve portions of whole numbers, represented by **fractions** and **decimal numbers**.

Fractions and decimal numbers express values that are a portion of a whole number. Fractions are widely used throughout mathematics, including measurement, probability, and data applications. Decimal numbers are a special type of fraction that express numbers as a portion of powers of 10 (10, 100, 1,000, etc.).

Fractions and decimal numbers have different benefits. Fractions can be more precise than decimal numbers; for example, it is impossible to exactly represent the fraction $\dfrac{1}{3}$ as a decimal number.

However, reading, writing, and performing arithmetic operations with decimal numbers is easier than with fractions. In addition, it is easier to determine the magnitude of numbers when they are expressed as decimal numbers rather than as fractions. For example, it is easier to recognize that the decimal number 7.75, as opposed to its fractional form $\dfrac{31}{4}$, lies between the whole numbers 7 and 8.

Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of Mathematics* (3rd ed.) by Thambyrajah Kugathasan, published by Vretta-Lyryx Inc., with permission. Adaptations include supplementing existing material and reordering chapters.

2.1 DECIMAL NUMBERS

Decimal numbers, also known as **decimals**, represent a part or a portion of a whole.

Decimal numbers are used in situations requiring more precision than whole numbers. We use decimal numbers frequently in our daily lives; a good example of this is money. For example, a nickel is worth 5¢, equal to $0.05, and the bus fare for a city may be $3.25; 0.05 and 3.25 are examples of decimal numbers.

A decimal number contains a whole number portion and a decimal portion. The decimal point (.) is used to separate these two portions: the whole number portion is comprised of the digits to the left of the decimal point, and the decimal portion is comprised of the digits to the right of the decimal point. The decimal portion represents a value less than 1.

For example,
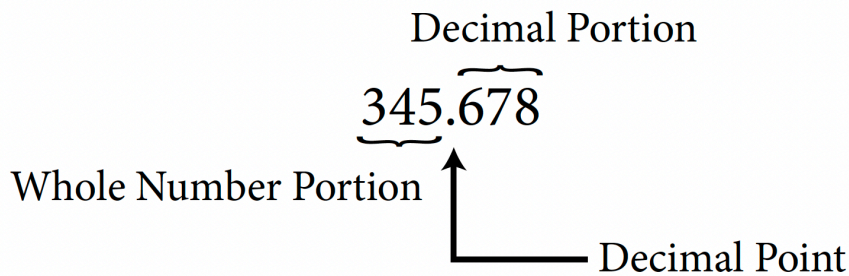
The decimal portion of a decimal can be represented as a fraction with a denominator that is a power of 10 (i.e., 10, 100, 1,000, etc.). These fractions are referred to as **decimal fractions**.

For example,

- The decimal number 0.3 is $\dfrac{3}{10}$ as a decimal fraction.

- The decimal number 0.07 is $\dfrac{7}{100}$ as a decimal fraction.

- The decimal portion of the decimal number 345.678 is $\dfrac{678}{1,000}$ as a decimal fraction.

When decimal numbers are expressed as a decimal fraction with a denominator that is a power of 10, we do not reduce to their lowest terms.

For example, $\dfrac{678}{1,000}$ if reduced to $\dfrac{339}{500}$ is no longer expressed with a denominator that is a power of 10, and therefore is not a decimal fraction.

Similarly,

- $1.2 = 1\dfrac{2}{10}$
- $23.45 = 23\dfrac{45}{100}$
- $75.378 = 75\dfrac{378}{1,000}$

Every whole number can be written as a decimal number by placing a decimal point to the right of the unit's digits.

For example, the whole number 5 written as a decimal number is 5. or 5.0 or 5.00, etc.

The number of decimal places in a decimal number is the number of digits written to the right of the decimal point.

For example,

- 5.   No decimal places
- 5.0   One decimal place
- 5.00   Two decimal places
- 1.250   Three decimal places
- 2.0050   Four decimal places

Types of Decimal Numbers

There are three different types of decimal numbers.

1. Non-repeating, terminating decimals numbers:
   For example,   0.2, 0.3767, 0.86452
2. Repeating, non-terminating decimal numbers:
   For example,   0.222222.... (0.2), 0.255555.... (0.25), 0.867867.... (0.867)
3. Non-repeating, non-terminating decimal numbers:
   For example,   0.453740...., π (3.141592...), e (2.718281...)

Place Value of Decimal Numbers

The position of each digit in a decimal number determines the place value of the digit. Exhibit 2.1 illustrates the place value of the five-digit decimal number: 0.35796.

The place value of each digit as you move right from the decimal point is found by decreasing powers of

10. The first place value to the right of the decimal point is the tenths place; the second place value is the hundredths place, and so on, as shown in Table 2.1-b.
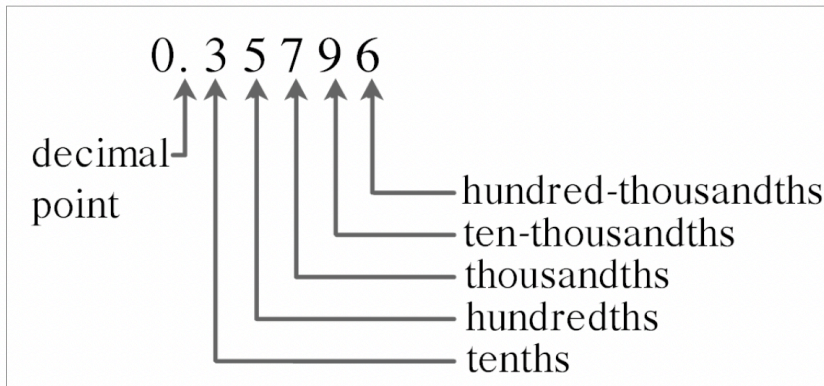
Exhibit 2.1 Place Value of a Five-Digit Decimal Number

Place Value Chart of Decimal Numbers

Table 2.1-a: Place Values Chart

Exponent and Fractional Equivalent	Decimal Form	Place Value
$10^{-1} = \dfrac{1}{10}$	0.1	Tenths
$10^{-2} = \dfrac{1}{100}$	0.01	Hundredths
$10^{-3} = \dfrac{1}{1,000}$	0.001	Thousandths
$10^{-4} = \dfrac{1}{10,000}$	0.0001	Ten-thousandths
$10^{-5} = \dfrac{1}{100,000}$	0.00001	Hundred-thousandths

The five-digit number in Exhibit 2.1 is written as 0.35796 in its **standard form**.

Table 2.1-b: Exhibit 2.1 with Place Values Identified

Digit	Place Value
0.	The whole number in front of the decimal point.
3	Tenths
5	Hundredths
7	Thousandths
9	Ten-thousandths
6	Hundred-thousandths

The decimal number 0.35796 can also be written in **expanded form** as follows:

$$0.3 + 0.05 + 0.007 + 0.0009 + 0.00006$$

Or,

3 tenths + 5 hundredths + 7 thousandths + 9 ten-thousandths + 6 hundred-thousandths

Or,

$$\frac{3}{10} + \frac{5}{100} + \frac{7}{1,000} + \frac{9}{10,000} + \frac{6}{100,000}$$

(0.35796 as a decimal fraction is $\dfrac{35,796}{100,000}$)

Reading and Writing Decimal Numbers

Follow these steps to read and write decimal numbers in **word form**:

1. Read or write the number to the left of the decimal point as a whole number.
2. Read or write the decimal point as "and".
3. Read or write the number to the right of the decimal point as a whole number, followed by the name of the place value occupied by the right-most digit.

For example, 745.023 is written in word form as:

Seven hundred forty-five and twenty-**three** thousandths

Whole Number Portion — Decimal Point — Decimal Portion

The last digit, three, ends in the thousandths place. Therefore, the decimal fraction is $\dfrac{23}{1,000}$.

As noted below, there are other ways of reading and writing decimal numbers.

- Use "point" to indicate the decimal point and read or write each digit individually. For example, 745.023 can be read or written as seven hundred forty-five point zero, two, three.
- Ignore the decimal point of the decimal number and read or write the number as a whole number followed by the name of the place value occupied by the right-most digit of the decimal portion. For example, 745.023 can also be read or written as seven hundred forty-five thousand, twenty-three thousandths.

  (i.e., $\dfrac{745,023}{1,000}$).

Note: The above two representations are not used in this chapter's examples and exercise questions.

Use of Hyphens to Express Decimal Numbers in Word Form

- A hyphen (–) is used to express the two-digit numbers 21 to 29, 31 to 39, 41 to 49, ... 91 to 99 in each group in their word form.
- A hyphen (–) is also used while expressing the place value portion of a decimal number, such as ten-thousandths, hundred-thousandths, ten-millionths, hundred-millionths, and so on.

The following examples illustrate the use of hyphens to express numbers in their word form:

- 0.893   Eight hundred ninety-three thousandths
- 0.0506   Five hundred six ten-thousandths
- 0.00145   One hundred forty-five hundred-thousandths

Example 2.1-a: Writing Decimal Numbers in Standard Form

Write the following decimal numbers in standard form:

  a. Two hundred and thirty-five hundredths
  b. Three and seven tenths
  c. Eighty-four thousandths

Solution

a. 200 (whole number portion)

and $\dfrac{35}{100} = 0.35$ (decimal portion)

Therefore, the number is written in standard form as 200.35.

b. 3 (whole number portion)

and $\dfrac{7}{10} = 0.7$ (decimal portion)

Therefore, the number is written in standard form as 3.7.

c. 0 (whole number portion)

and $\dfrac{84}{1,000} = 0.084$ (decimal portion)

Therefore, the number is written in standard form as 0.084.

Example 2.1-b: Writing Decimal Numbers in Word Form

Write the following decimal numbers in word form:

a. 23.125
b. 7.43
c. 20.3
d. 0.2345

Solution

a. 23.125

The last digit, 5, is in the thousandths place.

$$= 23\dfrac{125}{1,000}$$

Twenty-three and one hundred twenty-five thousandths

b. 7.43

The last digit, 3, is in the hundredths place.

$$= 7\frac{43}{100}$$

Seven and forty-three hundredths

c. 20.3

The last digit, 3, is in the tenths place.

$$= 20\frac{3}{10}$$

Twenty and three tenths

d. 0.2345

The last digit, 5, is in the ten-thousandths place.

$$= \frac{2,345}{10,000}$$

Two thousand, three hundred forty-five ten-thousandths

Rounding Decimal Numbers

Rounding Decimal Numbers to the Nearest Whole Number, Tenth, Hundredth, etc.

Rounding decimal numbers refers to changing the value of the decimal number to the nearest whole number, tenth, hundredth, thousandth, etc. It is also called rounding to a specific number of decimal places, indicating the number of decimal places left when the rounding is complete.

- Rounding to the nearest whole number is the same as rounding without decimals.
- Rounding to the nearest tenth is the same as rounding to one decimal place.

- Rounding to the nearest hundredth is the same as rounding to two decimal places.
- Rounding to the nearest cent refers to rounding the amount to the nearest hundredth, the same as rounding to two decimal places.

Follow these steps to round decimal numbers:

1. Identify the digit to be rounded (this is the place value for which the rounding is required).
2. If the digit to the immediate right of the identified rounding digit is less than 5 (0, 1, 2, 3, 4), do not change the value of the rounding digit.
   If the digit to the immediate right of the identified rounding digit is 5 or greater than 5 (5, 6, 7, 8, 9), increase the value of the rounding digit by one (i.e., round up by one number).
3. Drop all digits to the right of the rounding digit.

Example 2.1-c: Rounding Decimal Numbers

Round the following decimal numbers to the indicated place value:

   a. 268.143 to the nearest tenth
   b. 489.679 to the nearest hundredth
   c. $39.9985 to the nearest cent

Solution

a. Rounding 268.143 to the nearest tenth:

   1 is the rounding digit in the tenths place: 268.143.

   The digit to the immediate right of the rounding digit is less than 5; therefore, do not change the value of the rounding digit. Drop all of the digits to the right of the rounding digit. This will result in 268.1.

   Therefore, 268.143 rounded to the nearest tenth is 268.1.

b. Rounding 489.679 to the nearest hundredth:

7 is the rounding digit in the hundredths place: 489.679.

The digit to the immediate right of the rounding digit is greater than 5; therefore, increase the value of the rounding digit by one, from 7 to 8, and drop all of the digits to the right of the rounding digit. This will result in 489.68.

Therefore, 489.679 rounded to the nearest hundredth is 489.68.

c. Rounding $39.9985 to the nearest cent:

9 is the rounding digit in the hundredths place: $39.9985.

The digit to the immediate right of the rounding digit is greater than 5; therefore, increase the value of the rounding digit by one, from 9 to 10, by replacing the rounding digit 9 with 0 and carrying the one to the tenths place, then to the ones, and then to the tens, to increase the digit 3 to 4. Finally, drop all digits that are to the right of the rounding digit. This will result in $40.00.

Therefore, $39.9985 rounded to the nearest cent is $40.00.

2.1 Exercises

Answers to odd-numbered problems are available at the end of the textbook.

For Problems 1 to 8, express the decimal fractions as decimal numbers.

1. a. $\dfrac{6}{10}$

   b. $\dfrac{7}{1,000}$

2. a. $\dfrac{9}{10,000}$

   b. $\dfrac{41}{1,000}$

3. a. $\dfrac{12}{100}$

b. $\dfrac{29}{1,000}$

4. a. $\dfrac{75}{100}$

b. $\dfrac{3}{10}$

5. a. $7\dfrac{5}{10}$

b. $9\dfrac{503}{1,000}$

6. a. $9\dfrac{3}{10}$

b. $6\dfrac{207}{1,000}$

7. a. $\dfrac{367}{100}$

b. $\dfrac{2,567}{1,000}$

8. a. $\dfrac{475}{10}$

b. $\dfrac{2,972}{1,000}$

For Problems 9 to 24, write the numbers in (i) standard and (ii) expanded forms.

9. Eighty-seven and two tenths

10. Thirty-five and seven tenths

11. Three and four hundredths

12. Nine and seven hundredths

13. Four hundred one ten-thousandths

14. Fifty-two and three hundred five thousandths

15. Eighty-nine and six hundred twenty-five ten-thousandths

16. Two hundred eight thousandths

17. One thousand, seven hundred eighty-seven and twenty-five thousandths

18. Seven thousand, two hundred sixty and fifteen thousandths

19. Four hundred twelve and sixty-five hundredths

20. Nine hundred eighty-seven and twenty hundredths

21. One million, six hundred thousand and two hundredths

22. Six million, two hundred seventeen thousand and five hundredths

23. Twenty-three and five-tenths

24. Twenty-nine hundredths

For Problems 25 to 32, express the decimal numbers in their word form.

25. a. 42.55

    b. 734.125

26. a. 7.998

    b. 12.77

27. a. 0.25

    b. 9.5

28. a. 0.987

    b. 311.2

29. a. 7.07

    b. 15.002

30. a. 11.09

    b. 9.006

31. a. 0.062

    b. 0.054

32. a. 0.031

    b. 0.073

Arrange the following decimal numbers from least to greatest.

33. 0.034, 0.403, 0.043, 0.304

Arrange the following decimal numbers from least to greatest.

34. 1.014, 1.011, 1.104, 1.041

For Problems 35 to 42, round the numbers to one decimal place (nearest tenth).

35. 415.1654

36. 7.8725

37. 264.1545

38. 25.5742

39. 24.1575

40. 112.1255

41. 10.3756

42. 0.9753

For Problems 43 to 50, round the numbers to two decimal places (nearest hundredth).

43. 14.3585

44. 19.6916

45. 181.1267

46. $10.954

47. $16.775

48. $24.995

49. $9.987

2.2 ARITHMETIC OPERATIONS WITH DECIMAL NUMBERS

Addition of Decimal Numbers

Adding decimal numbers means combining decimal numbers to find the total or sum. It is similar to adding whole numbers.

Follow these steps to add decimal numbers:

1. Write the numbers one under the other by aligning the decimal points of the numbers.
2. Add zeros to the end of any decimal number with fewer decimal places, if necessary, to ensure that each number has the same number of decimal places. Draw a horizontal line underneath.
3. Add all the numbers in that column starting from the right-most place value.

   - If the total is less than 10, write the total under the horizontal line in the same column.
   - If the total is 10 or more, write the 'ones' digit under the horizontal line in the same column, and write the 'tens' digit above the column to the left.

4. Follow this procedure for each column going from right to left. Write the decimal point in the answer, aligned with the other decimal points in the sum.

Example 2.2-a: Adding Decimal Numbers

Perform the following additions:

    a. $25.125 + 7.14$
    b. $741.87 + 135.456$
    c. $127 + 68.8 + 669.95$

Solution

Tip – don't forget to add a zero to match the number of decimal places.

a. $25.125 + 7.14$

$$
\begin{aligned}
2\,5\,.\,1\,2\,5 \\
\underline{+\,7\,.\,1\,4\,0} \\
3\,2\,.\,2\,6\,5
\end{aligned}
$$

Therefore, adding 25.125 and 7.14 results in 32.265.

b. $741.87 + 135.456$

$$
\begin{aligned}
741.870 \\
\underline{+135.456} \\
877.326
\end{aligned}
$$

Therefore, adding 741.87 and 135.456 results in 877.326.

c. $127 + 68.8 + 669.95$

$$
\begin{aligned}
127.00 \\
68.80 \\
\underline{+669.95} \\
865.75
\end{aligned}
$$

Therefore, adding 127, 68.8, and 669.95 results in 865.75.

Subtraction of Decimal Numbers

Subtraction of decimal numbers refers to finding the difference between decimal numbers. It is similar to subtracting whole numbers.

Follow these steps to subtract a decimal number from another decimal number:

1. Write the numbers one under the other by aligning the decimal points of the numbers. Ensure that the number from which subtraction is indicated (the minuend) is in the top row and that the number being subtracted (the subtrahend) is below.
2. Add zeros to the end of any decimal number with fewer decimal places, if necessary, to ensure that each number has the same number of decimal places. Draw a horizontal line underneath.
3. Starting from the right-most place value, subtract the bottom number from the top number.

▪ If the top digit is greater than (or equal to) the bottom digit, write the difference under the horizontal line in the same column.

▪ If the top digit is less than the bottom digit, borrow 'one' from the digit to the left in the top number, and add 'ten' to the digit in the current place value of the top number. Then, find the difference and write it under the horizontal line.

4. Follow this procedure for each column going from right to left. Write the decimal point in the answer, aligned with the other decimal points in the difference.

Example 2.2-b: Subtracting Decimal Numbers

Perform the following subtractions:

   a. Subtract 29.02 from 135.145

   b. Subtract 38.7 from 457

Solution

a. Subtract 29.02 from 135.145

$$1 \overset{2}{\cancel{3}} \overset{15}{\cancel{5}}.1\,4\,5 \\ \underline{-\ 2\,9\,.\,0\,2\,0} \\ 1\,0\,6\,.\,1\,2\,5$$

Therefore, subtracting 29.02 from 135.145 results in 106.125.

b. Subtract 38.7 from 457

Therefore, subtracting 38.7 from 457 results in 418.3

Multiplication of Decimal Numbers

Multiplication of decimal numbers refers to finding the **product** of two decimal numbers.

Follow these steps to multiply one decimal number by another decimal number:

1. Line up the numbers on the right without aligning the decimal points.
2. Multiply the number assuming there are no decimal points; i.e., multiply each digit in the top number by each digit in the bottom number and add the products, just like the process for multiplying whole numbers.
3. Count the number of decimal places in the numbers being multiplied (the factors).
4. The number obtained in Step 3 is equal to the number of decimal places in the answer. Starting at the right of the answer, move towards the left by the total number of decimal places counted, and place the decimal point there.

Example 2.2-c: Multiplying Decimal Numbers

Multiply 12.56 and 1.8.

Solution

$$\begin{align*} \renewcommand{\ULdepth}{1.8pt} 1\ 2.\ 5\ 6\\ \underline{\times 1\ .8\ } \\ 1\ 0\ 0\ 4\ 8\\ \underline{1\ 2\ 5\ 6\ 0}\\ 2\ 2.\ 6\ 0\ 8 \end{align*}$$

The number of decimal places in the numbers being multiplied is 3, so the answer has 3 decimal places. Therefore, multiplying 12.56 and 1.8 results in 22.608.

Division of Decimal Numbers

Division of decimal numbers determines how many times one decimal number is contained in another decimal number.

Follow these steps to divide a decimal number:

1. If the divisor is not a whole number, convert it to a whole number by moving the decimal point to the right. Move the decimal point in the dividend by the same number of places.
2. Divide by following a similar process to the process of dividing whole numbers. Add zeros to the right of the last digit of the dividend and keep dividing until there is no remainder or a repeating pattern shows up in the quotient.

Example 2.2-d: Dividing Decimal Numbers

Perform the following divisions:

    a. Divide 8.25 by 0.6
    b. Divide: 0.166 by 0.03

Solution

a. Step 1:

Since the denominator contains one decimal place, move the decimal point one decimal place to the right for both the numerator and the denominator.

$$8.25 \div 0.6 = \frac{8.25}{0.6} = \frac{82.5}{6}$$

This is the same as multiplying both the numerator and denominator by 10.

$$8.25 \div 0.6 = \frac{8.25 \times 10}{0.6 \times 10} = \frac{82.5}{6}$$

Step 2:

Position the decimal point within the quotient directly above the decimal point within the dividend.

\begin{array}{r} 13.75\\ 6\enclose{longdiv}{82.50}\\ -6 &&\hbox{bring the 2 down} \\ \hline 22 \\ -18 &&\hbox{bring the 5 down}\\ \hline 45\\ -42\\ \hline 30 &&\hbox{add a zero}\\ -30 \\ \hline 0\\ \end{array}

Therefore, when 8.25 is divided by 0.6, the quotient is 13.75.

b. Step 1:

Since the denominator contains two decimal places, move the decimal point two decimal places to the right for both the numerator and the denominator.

$$0.166 \div 0.03 = \frac{0.166}{0.03} = \frac{16.6}{3}$$

This is the same as multiplying both the numerator and denominator by 100.

$$0.166 \div 0.03 = \frac{0.166 \times 100}{0.03 \times 100} = \frac{16.6}{3}$$

Step 2:

Position the decimal point within the quotient directly above the decimal point within the dividend.

\begin{array}{r} 5.533\\ 3\enclose{longdiv}{16.600}\\ -15 &&\hbox{bring the 6 down}\\ \hline 16 \\ -15 \\ \hline 10 &&\hbox{add a zero}\\ -9\\ \hline 10 &&\hbox{add a zero}\\-9 \\ \hline 1\\ \end{array}

Therefore, when 0.166 is divided by 0.03, the quotient is 5.53.

Powers and Square Roots of Decimal Numbers

Similar to fractions, powers of decimal numbers are usually written within brackets.

For example, $(0.12)^3$ is read as "twelve hundredths raised to the power of three".

- This means that 0.12 is used as a factor three times.
- i.e., $(0.12)^3 = (0.12)(0.12)(0.12) = 0.001728$

Example 2.2-e: Evaluating Powers of Decimal Numbers

Evaluate the power: $(1.25)^3$

Solution

$(1.25)^3$

Expanding by using 1.25 as a factor three times,

$$= (1.25)(1.25)(1.25) = 1.953125$$

Determining square roots of decimal numbers is simple if the decimal number can first be converted to a decimal fraction with an even power of ten as the denominator (i.e., $10^2 = 100$, $10^4 = 10,000$, etc.). Then, follow the procedure for evaluating the square root of a fraction.

For example, $\sqrt{0.25} = \sqrt{\dfrac{25}{100}} = \dfrac{\sqrt{25}}{\sqrt{100}} = \dfrac{5}{10} = 0.5$

Example 2.2-f: Evaluating Square Roots of Decimal Numbers

Evaluate the square root: $\sqrt{0.49}$

Solution

$\sqrt{0.49}$

Converting the decimal number into a decimal fraction,

$$= \sqrt{\frac{49}{100}}$$

Determining the square root of the numerator and denominator separately,

$$= \frac{\sqrt{49}}{\sqrt{100}} = \frac{7}{10} = 0.7$$

2.2 Exercises

Answers to odd-numbered problems are available at the end of the textbook.

For Problems 1 to 8, perform the additions.

1. $927.896 + 659.50 + 128.649$

2. $619.985 + 52.82 + 3.187$

3. $74 + 129.258 + 0.32 + 666.015$

4. $17 + 3.48 + 0.278 + 78.24$

5. $292.454 + 121.69 + 65.3$

6. $396.716 + 191.68 + 90.6$

7. $948.684 + 15.17 + 0.717$

8. $625.365 + 27.97 + 0.613$

9. Calculate the sum of the following numbers:

   Twenty and ninety-five hundredths; Two hundred and seventy-two thousandths; Nineteen and nine-tenths.

10. Calculate the sum of the following numbers:

    Six and thirty-nine thousandths; Eighty and fourteen hundredths; Sixteen and eight tenths.

For Problems 11 to 18, perform the subtractions.

11. $423.92 - 185.728$

12. $9.555 - 7.18$

13. $29.28 - 13.4$

14. $15.7 - 7.92$

15. $539.64 - 258.357$

16. $848.62 - 495.476$

17. $409.5 - 179.832$

18. $475.3 - 281.375$

19. Subtract three hundred five and thirty-nine hundredths from seven hundred twenty and four-tenths.

20. Subtract eight hundred twenty and four hundredths from one thousand, one hundred one and six tenths.

For Problems 21 to 28, perform the multiplications

21. 137.89×5.4

22. 189.945×6.3

23. 62.095×4.18

24. 92.74×3.25

25. 0.43×0.8

26. $25.0.59 \times 0.9$

27. 109.78×2.91

28. 145.75×3.74

For Problems 29 to 36, perform the divisions

29. $67.78 \div 9$

30. $261.31 \div 7$

31. $732.6 \div 8$

32. $413.9 \div 6$

33. $14.6 \div 0.6$

34. $9.155 \div 0.7$

35. $3.1 \div 0.25$

36. $2.7 \div 0.15$

For Problems 37 to 40, evaluate the powers of the decimal numbers.

37. a. $(0.1)^3$

   b. $(0.3)^2$

38. a. $(1.1)^3$

   b. $(1.2)^3$

39. a. $(0.4)^2$

   b. $(0.02)^3$

40. a. $(0.9)^2$

   b. $(0.05)^3$

For Problems 41 to 46, evaluate the square roots of the decimal numbers.

41. a. $\sqrt{0.25}$

   b. $\sqrt{0.49}$

42. a. $\sqrt{0.36}$

   b. $\sqrt{0.64}$

43. a. $\sqrt{1.21}$

   b. $\sqrt{1.69}$

44. a. $\sqrt{2.56}$

   b. $\sqrt{1.44}$

45. a. $\sqrt{0.01}$

   b. $\sqrt{0.0049}$

46. a. $\sqrt{0.09}$

   b. $\sqrt{0.0004}$

For Problems 47 to 54, formulate arithmetic expressions and evaluate them.

47. Find the amount that is $248.76 less than $627.40.

48. Find the amount that is $45.27 less than $90.75.

49. Find the difference in the amounts $30.75 and $15.89.

50. Find the difference in the amounts $235.62 and $115.75.

51. Find the sum of $52.43 and $23.95.

52. Find the sum of $252.34 and $297.90.

53. Find the amount that is $38.89 more than $25.67.

54. Find the amount that is $412.78 more than $634.25.

55. The cost of an item is $88.46. If you gave $90.00 to the cashier, how much change would you receive?

56. The cost of an item is $125.69. If Arun gave $150.00 to the cashier, how much change would Arun receive?

57. Bill saved $578.50 this week. He saved $124.85 more last week than this week. How much did Bill save during the two-week period?

58. Last week Carol spent $96.75 more on food than on transportation. She spent $223.15 on transport. How much did Carol spend on both food and transportation last week?

59. The normal selling price of an item is $237.75. When this item was on sale Dave paid $49.89 less for it. How much did Dave pay for that item?

60. A car driver filled gas when the odometer reading was 35,894.9 km. The odometer reading now is 39,894.4 km. How many kilometres did the driver travel, rounded to the nearest kilometre?

61. After spending $38.96 on toys and $1.75 on wrapping paper, Ann still had $45.75. How much money did Ann have initially?

62. After paying $515.09 for a car lease and $379.92 for property tax, Elisa's bank balance was $675.45. How much money did Elisa have initially?

63. Simon bought a camera that was on sale for $799.99. He agreed to pay $70.35 every month for 12 months. How much more money than the sale price did Simon pay for the camera?

64. Andy bought a TV that was on sale for $2,249.95. He agreed to pay $130.45 every month for 18 months. How much more money than the sale price did Andy pay for the TV?

65. A salesperson earns a salary of $725.35 every week. During the past three weeks, he also received commissions of $375.68, $578.79, and $338.57. Calculate his total income for the past three weeks.

66. Danny leased a car on a four-year term at $694.38 per month. At the end of the lease period, she paid an additional $18,458.74 to purchase the car. Calculate the total amount Danny paid for the car.

67. John bought two shirts at $20.95 each and three pairs of pants at $34.55 each. He gave $200 to the cashier. Calculate the balance he should receive from the cashier.

68. Taylor bought 3 kg of walnuts at $8.69 per kg and 4 kg of almonds at $7.72 per kg. He gave the cashier a $100 bill. How much change should Taylor receive from the cashier?

69. A string that measured 0.875 m was cut into pieces of 0.0625 m each. How many pieces were there?

70. A cake that weighed 0.82 kg was cut into slices that weighed 0.1025 kg each. How many slices were there?

71. Marion bought three dresses at $22.49 per dress and two pairs of shoes at $14.99 per pair. She gave a $100 bill to the cashier. What change should she expect to receive from the cashier?

72. Gilbert bought 2 kg of grapes at $3.29 per kg and 1.5 kg of strawberries at $5.99 per kg. He gave a $20 bill to the cashier. How much should he expect to receive in change from the cashier?

2.3 FRACTIONS

If we divide one whole unit into several equal portions, then one or more of these equal portions can be represented by a **fraction**.

A fraction is composed of the following three parts:

1. **Numerator:** the number of equal parts of a whole unit.
2. **Fraction bar:** the division sign, meaning 'divided by'.
3. **Denominator:** the total number of equal parts into which the whole unit is divided.

For example, $\dfrac{3}{8}$ is a fraction.

fraction bar $\longrightarrow$ $\dfrac{3}{8}$ $\longleftarrow$ numerator $\longleftarrow$ denominator

The **numerator '3'** indicates that the fraction represents 3 equal parts of a whole unit, and the **denominator '8'** indicates that the whole unit is divided into 8 equal parts, as shown above. The **fraction bar** indicates that the numerator '3' is divided by the denominator '8'.

$\dfrac{3}{8}$ is read as "three divided by eight", "three-eighths", or "three over eight". All of these indicate that 3 is the numerator, 8 is the denominator, and the fraction represents 3 of 8 pieces of the whole. The numerator and denominator are referred to as the terms of the fraction.

Note: The denominator of a fraction cannot be zero since a number cannot be divided into zero equal parts.

Any number that can be represented as a fraction is known as a **rational number**.

For example,

- $\dfrac{2}{3}, \dfrac{5}{2}$, and $\dfrac{7}{1}$ are rational numbers.
- The whole number 7 can be written as the fraction $\dfrac{7}{1}$; therefore, 7 is also a rational number.

Fractions can be represented on a number line. They are plotted in-between whole numbers.

For example, $\frac{1}{2}$ is represented on a number line as:

When one unit is divided into two equal portions, each portion represents one-half of that unit. One of such equal portions is written as $\frac{1}{2}$.
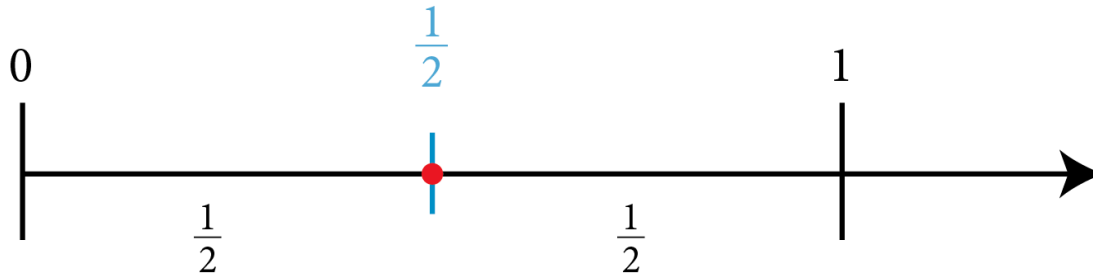
Example 2.3-a: Representing Fractions on a Number Line

Represent the following fractions on a number line:

a. $\dfrac{2}{3}$

b. $\dfrac{3}{5}$

Solution

a. When one unit is divided into three equal portions, each portion represents one-third of that unit. Two of such equal portions is two-thirds and is written as $\dfrac{2}{3}$.
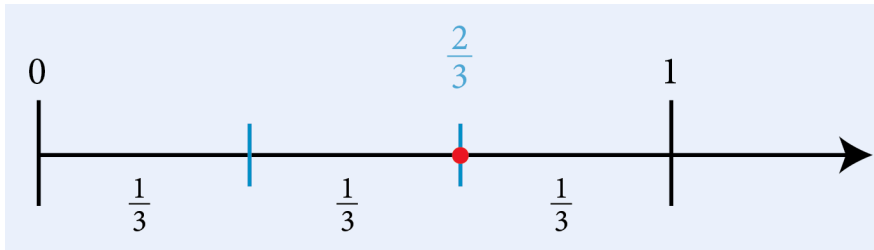
b. When one unit is divided into five equal portions, each portion represents one-fifth of that unit. Three of such equal portions is three-fifths and is written as $\dfrac{3}{5}$.
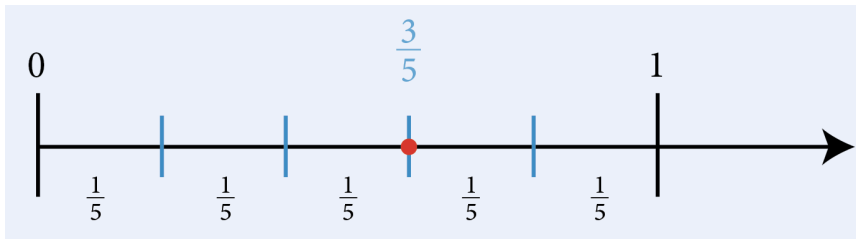
Types of Fractions

Proper Fractions

A **proper fraction** is a fraction in which the numerator is less than the denominator.
For example,

- $\dfrac{3}{8}$ is a proper fraction because the numerator, 3, is less than the denominator, 8.
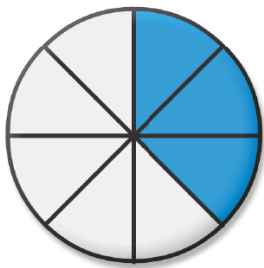
three-eighths

Improper Fractions

An **improper fraction** is a fraction in which the numerator is greater than or equal to the denominator; i.e., the value of the entire fraction is more than 1.

For example,

- $\dfrac{7}{4}$ is an improper fraction because the numerator 7 is greater than the denominator 4

  (i.e., $7 > 4$, or $\dfrac{7}{4} > 1$).
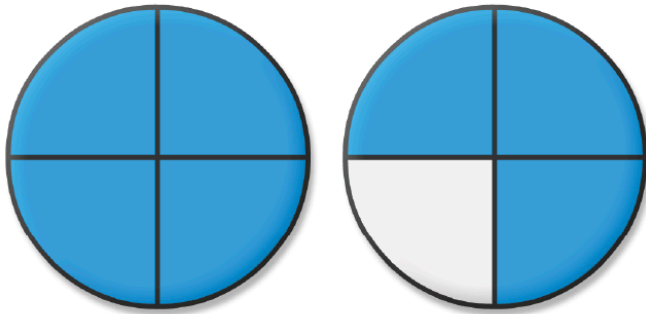
seven-quarters

Mixed Numbers (or Mixed Fractions)

A **mixed number** consists of both a whole number and a proper fraction, written side-by-side, which implies that the whole number and the proper fraction are added.

For example,

- $3\dfrac{5}{8}$ is a mixed number, where 3 is the whole number, and $\dfrac{5}{8}$ is the proper fraction.

  $3\dfrac{5}{8}$ implies $3 + \dfrac{5}{8}$

three   five-eighths
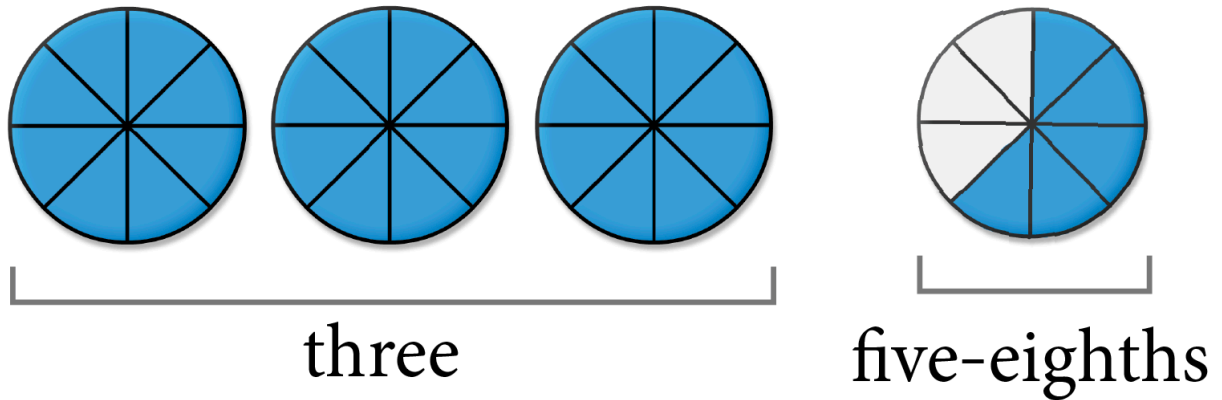
Relationship Between Mixed Numbers and Improper Fractions

Converting a Mixed Number to an Improper Fraction

Follow these steps to convert a mixed number to an improper fraction:

Step 1: Multiply the whole number by the fraction's denominator and add this value to the fraction's numerator.

Step 2: The resulting answer will be the numerator of the improper fraction.

Step 3: The denominator of the improper fraction is the same as the denominator of the original fraction in the mixed number.

Example: Convert $3\dfrac{5}{8}$ to an improper fraction.

Step 1: 3(8) = 24

Step 2: 24 + 5 = 29. The numerator will be 29.

Step 3: The denominator will be 8.

Therefore, $3\dfrac{5}{8} = \dfrac{29}{8}$

Note: You can perform the above steps in a single line of arithmetic as follows:

$$3\frac{5}{8} = \frac{3(8)+5}{8} = \frac{24+5}{8} = \frac{29}{8}$$

Four circles are divided into 8 equal pieces. Three of the four circles are completely shaded, and five pieces of the last circle are shaded for 29 pieces.
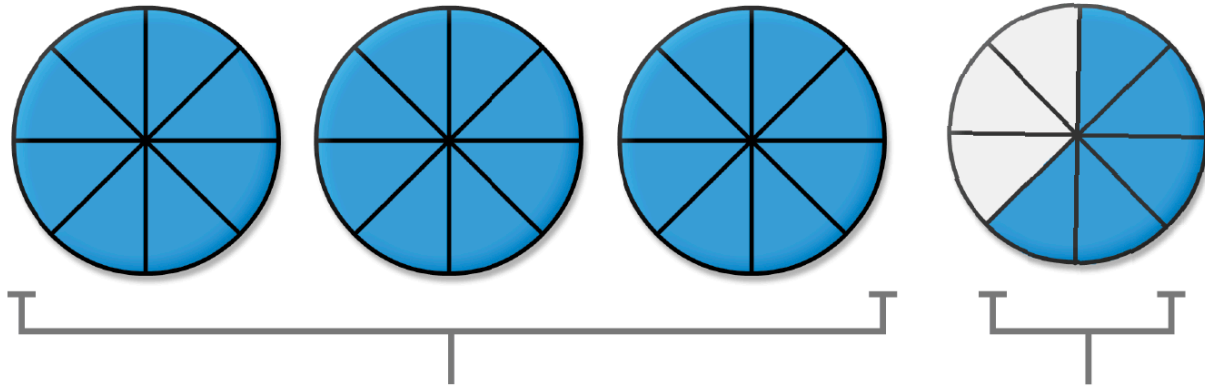
$$3 \times 8 = 24 \text{ pieces} \quad 5 \text{ pieces}$$

Converting an Improper Fraction to a Mixed Number

Follow these steps to convert an improper fraction to a mixed number:

Step 1: Divide the numerator by the denominator.

Step 2: The quotient becomes the whole number, and the remainder becomes the numerator of the fraction portion of the mixed number.

Step 3: The denominator of the fraction portion of the mixed number is the same as the denominator of the original improper fraction.

Example: Convert $\dfrac{29}{8}$ to an improper fraction.

Step 1: Divide 29 by 8.

Step 2: The quotient (3) becomes the whole number, and the remainder (5) becomes the numerator of the fraction portion of the mixed number.

Step 3: The denominator of the fraction portion of the mixed number (8) is the same as the denominator of the original improper fraction (8).

$$\begin{array}{r} 3 && \hbox{Quotient}\\ 8\enclose{longdiv}{29}\\ -24 \\ \hline 5 && \hbox{Remainder}\\ \end{array}$$

Therefore, $\dfrac{29}{8} = 3\dfrac{5}{8}$

Equivalent Fractions

When a fraction's numerator and denominator are either multiplied by the same number or divided by the

same number, the result is a new fraction known as an **equivalent fraction**. Equivalent fractions have the same value.

That is, the same part (or portion) of a whole unit can be represented by different fractions.

For example,
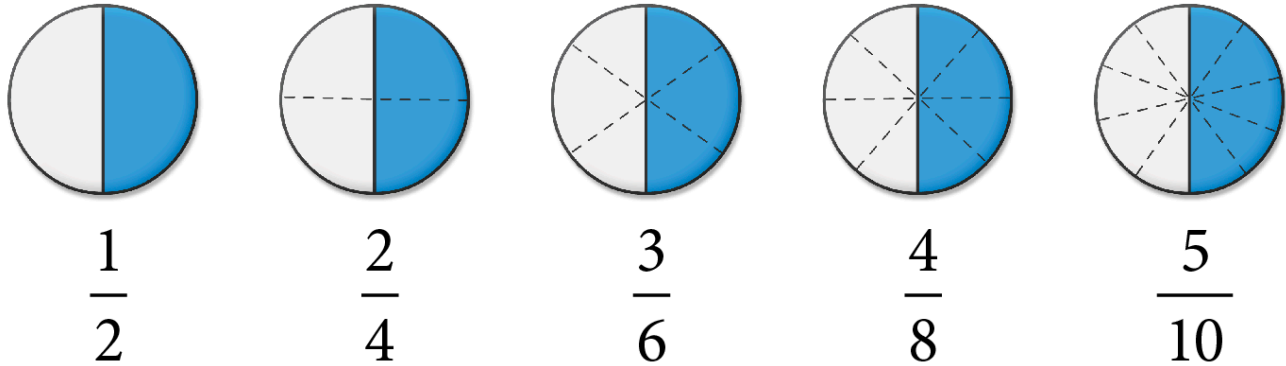
$$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots \text{ are equivalent fractions.}$$

Example 2.3-b: Finding Equivalent Fractions by Raising to Higher Terms

Find two equivalent fractions of $\frac{2}{5}$ by raising to higher terms.

Solution

To find an equivalent fraction of $\frac{2}{5}$, multiply both the numerator and denominator by 2. The resulting equivalent fraction is $\frac{4}{10}$.

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$
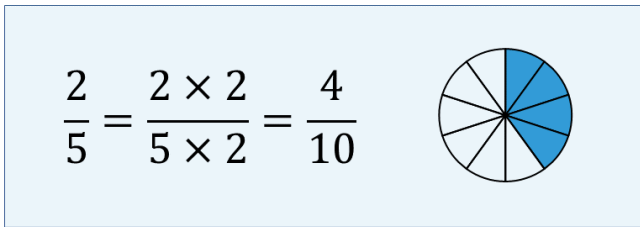
To find an equivalent fraction of $\frac{2}{5}$, multiply both the numerator and denominator by 3. The resulting equivalent fraction is $\frac{6}{15}$.

$$\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$
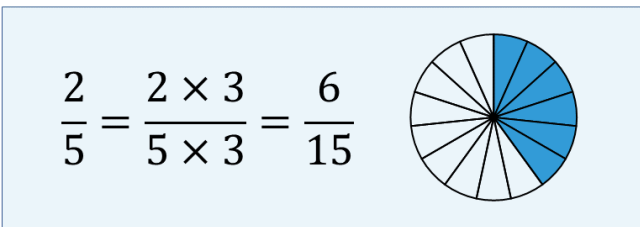
Therefore, $\frac{4}{10}$ and $\frac{6}{15}$ are equivalent fractions of $\frac{2}{5}$.

Example 2.3-c: Finding Equivalent Fractions by Reducing to Lower Terms

Find two equivalent fractions of $\frac{12}{30}$ by reducing to lower terms.

Solution

To find an equivalent fraction of $\frac{12}{30}$, divide both the numerator and denominator by 3. The resulting equivalent fraction is $\frac{4}{10}$.

$$\frac{12}{30} = \frac{12 \div 3}{30 \div 3} = \frac{4}{10}$$

To find another equivalent fraction, divide the numerator and denominator by 2. The resulting equivalent fraction is $\frac{2}{5}$.

$$\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$$

Or, the lowest terms can be found by dividing both the numerator and denominator of the original fraction, $\frac{12}{30}$, by 6.

$$\frac{12}{30} = \frac{12 \div 6}{30 \div 6} = \frac{2}{5}$$

Therefore, $\frac{4}{10}$ and $\frac{2}{5}$ are equivalent fractions of $\frac{12}{30}$.

Note: The fraction $\frac{2}{5}$ cannot be further reduced, as 2 and 5 do not have any common factors.

Therefore, $\frac{2}{5}$ is a fraction in its lowest (or simplest) terms. We will learn more about this further in the section.

Identifying Equivalent Fractions Using Cross Products

If the **cross products** of two fractions are equal, then the two fractions are equivalent fractions, and vice versa (i.e., if the fractions are equivalent, then their cross products are equal).

That is, if $\frac{a}{b} = \frac{c}{d}$, then $a \times d = b \times c$

$a \times d$ and $b \times c$ are known as cross products

For example, $\dfrac{3}{5}$ and $\dfrac{12}{20}$ are equivalent fractions because their cross products, 3 × 20 and 5 × 12, are equal:

3 × 20 = 60

5 × 12 = 60

Example 2.3-d: Classifying Fractions as Equivalent or Not Equivalent

Classify the pair of fractions as 'equivalent' or 'not equivalent' by using their cross products.

    a. $\dfrac{2}{5}$ and $\dfrac{12}{20}$

    b. $\dfrac{5}{4}$ and $\dfrac{20}{12}$

    c. $\dfrac{3}{8}$ and $\dfrac{9}{24}$

Solution

  a. $\dfrac{2}{5}$ and $\dfrac{12}{20}$

The cross products are 2 × 30 and 5 × 12.

2 × 30 = 60

  5 × 12 = 60

Therefore, the two fractions are equivalent.

The cross-products are equal.

  b. $\dfrac{5}{4}$ and $\dfrac{20}{12}$

The cross products are 5 × 12 and 4 × 20.

$5 \times 12 = 60$

$4 \times 20 = 80$

Therefore, the two fractions are not equivalent.

The cross products are not equal.

c. $\dfrac{3}{8}$ and $\dfrac{9}{24}$

The cross products are 3×24 and 8×9.

$3 \times 24 = 72$

$8 \times 9 = 72$

Therefore, the two fractions are equivalent.

The cross-products are equal.

Fractions in Lowest (or Simplest) Terms

Dividing a fraction's numerator and denominator by the same number, which results in an equivalent fraction, is known as **reducing** or **simplifying** the fraction.

For example, we saw in Example 2.1-c that $\dfrac{4}{10}$ and $\dfrac{2}{5}$ are reduced fractions of $\dfrac{12}{30}$.

A fraction in which the numerator and denominator have no factors in common (other than 1) is said to be a fraction in its **lowest (or simplest) terms**.

Any fraction can be **fully reduced** to its lowest terms by one of the following two methods:

Method 1: Dividing the numerator and denominator by the greatest common factor (GCF).

Method 2: Writing the numerator and denominator as products of prime factors and reducing by the common prime factors.

Example 2.3-e: Reducing Fractions to their Lowest Terms

Reduce the following fractions to their lowest terms.

a. $\dfrac{40}{45}$

b. $\dfrac{54}{24}$

Solution

a. $\dfrac{40}{45}$

Method 1: Factors of 40 are: 1, 2, 4, **5**, 8, 10, 20, and 40. Factors of 45 are: 1, 3, **5**, 9, 15, and 45. The GCF

is 5. Dividing the numerator and denominator by the GCF, 5,

$$\frac{40}{45} = \frac{40 \div 5}{45 \div 5} = \frac{8}{9}$$

Method 2: Prime factors of 40 are: 2 × 2 × 2 × 5. Prime factors of 45 are: 3 × 3 × 5. The common prime

factor is one 5. Therefore, $\dfrac{40}{45}$ is equal to $\dfrac{8}{9}$ reduced to its lowest terms.

b. $\dfrac{54}{24}$

Method 1: Factors of 54 are: 1, 2, 3, **6**, 9, 18, 27, and 54. Factors of 24 are: 1, 2, 3, 4, **6**, 8, 12, and 24. The

GCF is 6. Dividing the numerator and denominator by the GCF, 6,

$$\frac{54}{24} = \frac{54 \div 6}{24 \div 6} = \frac{9}{4}$$

Method 2: Prime factors of 54 are: 2 × 3 × 3 × 3. Prime factors of 24 are: 2 × 2 × 2 × 3. The common prime

factors are one 2 and one 3.Therefore, $\dfrac{54}{24}$ is equal to $\dfrac{9}{4}$ reduced to its lowest terms.

Reciprocals of Fractions

Two numbers whose product equals 1 are known as **reciprocals** of each other. Every non-zero real number has a reciprocal.

For example,

- $\dfrac{2}{3}$ and $\dfrac{3}{2}$ are reciprocals of each other because $\dfrac{2}{3} \times \dfrac{3}{2} = 1$

When the numerator and denominator of a fraction are interchanged, the resulting fraction is the reciprocal of the original fraction.

For example,

- 5 and $\dfrac{1}{5}$ are reciprocals (5 can also be written as $\dfrac{5}{1}$).
- Similarly, the reciprocal of $\dfrac{-2}{5}$ is $\dfrac{5}{-2} = -\dfrac{5}{2}$

The reciprocal of a positive number is always positive and the reciprocal of a negative number is always negative.

Note:

- *The reciprocal of a number is not the negative of that number. (The reciprocal of $3 \neq -3$. The reciprocal of $3 = \dfrac{1}{3}$.)*
- *The reciprocal of a fraction is not an equivalent fraction of that fraction. (The reciprocal of $\dfrac{2}{5} \neq \dfrac{4}{10}$. The reciprocal of $\dfrac{2}{5} = \dfrac{5}{2}$.)*

Table 2.3-a: Examples of Numbers with their Negatives and Reciprocals

Examples of Numbers with Their Negatives and Reciprocals

Number	5	-3	$\dfrac{2}{3}$	$-\dfrac{3}{8}$
Negative of the Number	-5	3	$-\dfrac{2}{3}$	$\dfrac{3}{8}$
Reciprocal of the Number	$\dfrac{1}{5}$	$-\dfrac{1}{3}$	$\dfrac{3}{2}$	$-\dfrac{8}{3}$

2.3 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

For problems 1 to 6, classify the fractions as proper fractions, improper fractions, or mixed numbers.

1. a. $\dfrac{16}{35}$

    b. $3\dfrac{2}{9}$

2. a. $15\dfrac{12}{13}$

    b. $\dfrac{29}{30}$

3. a. $\dfrac{19}{16}$

    b. $9\dfrac{7}{8}$

4. a. $\dfrac{21}{22}$

    b. $\dfrac{52}{25}$

5. a. $4\dfrac{2}{5}$

    b. $\dfrac{7}{3}$

6. a. $6\dfrac{1}{2}$

    b. $\dfrac{20}{75}$

For problems 7 to 10, convert the mixed numbers to improper fractions.

7. a. $2\dfrac{2}{7}$

    b. $3\dfrac{1}{8}$

8. a. $3\dfrac{2}{5}$

    b. $7\dfrac{5}{8}$

9. a. $5\dfrac{4}{5}$

    b. $6\dfrac{3}{4}$

10. a. $4\dfrac{3}{7}$

    b. $9\dfrac{5}{6}$

For problems 11 to 14, convert the improper fractions to mixed numbers.

11. a. $\dfrac{19}{7}$

    b. $\dfrac{45}{8}$

12. a. $\dfrac{23}{7}$

    b. $\dfrac{34}{3}$

13. a. $\dfrac{23}{3}$

    b. $\dfrac{31}{6}$

14. a. $\dfrac{26}{4}$

    b. $\dfrac{29}{5}$

For problems 15 to 22, classify the pair of fractions as 'equivalent' or 'not equivalent' by first converting the mixed numbers to improper fractions.

15. $\dfrac{44}{5}$ and $4\dfrac{4}{5}$

16. $\dfrac{47}{8}$ and $5\dfrac{7}{8}$

17. $11\dfrac{5}{7}$ and $7\dfrac{5}{7}$

18. $\dfrac{41}{4}$ and $10\dfrac{3}{4}$

19. $\dfrac{54}{7}$ and $7\dfrac{5}{7}$

20. $\dfrac{17}{8}$ and $2\dfrac{3}{8}$

21. $\dfrac{37}{9}$ and $4\dfrac{1}{9}$

22. $\dfrac{45}{11}$ and $4\dfrac{3}{11}$

For problems 23 to 30, classify the pair of fractions as 'equivalent' or 'not equivalent' by first converting the improper fractions to mixed numbers.

23. $\dfrac{15}{4}$ and $3\dfrac{1}{4}$

24. $\dfrac{43}{6}$ and $7\dfrac{5}{6}$

25. $\dfrac{18}{5}$ and $3\dfrac{3}{5}$

26. $\dfrac{45}{7}$ and $6\dfrac{3}{7}$

27. $3\dfrac{8}{9}$ and $\dfrac{35}{9}$

28. $\dfrac{34}{8}$ and $4\dfrac{1}{8}$

29. $\dfrac{41}{12}$ and $3\dfrac{5}{12}$

30. $7\dfrac{3}{9}$ and $\dfrac{67}{9}$

For problems 31 to 36, (i) reduce the fractions to their lowest terms and (ii) write their reciprocals.

31. a. $\dfrac{30}{20}$

    b. $\dfrac{48}{84}$

32. a. $\dfrac{44}{12}$

    b. $\dfrac{42}{70}$

33. a. $\dfrac{56}{48}$

    b. $\dfrac{84}{21}$

34. a. $\dfrac{75}{105}$

    b. $\dfrac{144}{48}$

35. a. $\dfrac{36}{63}$

    b. $\dfrac{60}{96}$

36. a. $\dfrac{131}{84}$

    b. $\dfrac{54}{126}$

For problems 37 to 44, classify the pair of fractions as 'equivalent' or 'not equivalent'.

37. $\dfrac{6}{12}$ and $\dfrac{15}{30}$

38. $\dfrac{6}{10}$ and $\dfrac{9}{15}$

39. $\dfrac{8}{10}$ and $\dfrac{15}{12}$

40. $\dfrac{12}{18}$ and $\dfrac{18}{27}$

41. $\dfrac{15}{12}$ and $\dfrac{36}{45}$

42. $\dfrac{35}{15}$ and $\dfrac{28}{12}$

43. $\dfrac{20}{25}$ and $\dfrac{24}{30}$

44. $\dfrac{16}{24}$ and $\dfrac{25}{30}$

For problems 45 to 50, determine the missing values.

45. a. $\dfrac{4}{9} = \dfrac{?}{27}$

    b. $\dfrac{4}{9} = \dfrac{20}{?}$

46. a. $\dfrac{42}{36} = \dfrac{14}{?}$

    b. $\dfrac{42}{36} = \dfrac{?}{30}$

47. a. $\dfrac{9}{12} = \dfrac{18}{?}$

    b. $\dfrac{9}{12} = \dfrac{?}{4}$

48. a. $\dfrac{45}{75} = \dfrac{?}{25}$

    b. $\dfrac{45}{75} = \dfrac{18}{?}$

49. a. $\dfrac{3}{2} = \dfrac{12}{?}$

    b. $\dfrac{3}{2} = \dfrac{?}{12}$

50. a. $\dfrac{25}{15} = \dfrac{?}{3}$

    b. $\dfrac{25}{15} = \dfrac{35}{?}$

For problems 51 to 58, express the answer as a fraction reduced to its lowest terms.

51. What fraction of 1 year is 4 months?

52. What fraction of 1 hour is 25 minutes?

53. Karen cut a pizza into 16 equal slices and served 12 slices to her friends. What fraction of the pizza was served?

54. Out of 35 students in a math class, 15 received an 'A' in their final exam. What fraction of the students in the class received an 'A' grade?

55. In a survey of 272 people, 68 people responded 'yes' and the remaining responded 'no'. What fraction of the people responded 'no'?

56. In a finance math course with 490 students, 70 students failed the final exam. What fraction of the students passed the final exam in this course?

57. Out of the 480 units in a condominium tower, 182 are rented. What fraction of the units are not rented?

58. In a community of 6,000 people, 1,800 were 60 years or older. What fraction of the people were below the age of 60 years?

2.4 ARITHMETIC OPERATIONS WITH FRACTIONS

Least or Lowest Common Denominator (LCD)

The **Least Common Denominator (LCD)** of a set of two or more fractions is the smallest whole number divisible by each denominator. It is the least common multiple (LCM) of the denominators of the fractions. Two methods of finding the LCM are explained in Chapter 1, Section 1.3.

In performing addition and subtraction of fractions, it is necessary to determine equivalent fractions with common denominators. The LCD is the best choice for a common denominator because it makes further simplification easier.

Example 2.4-a: Determining the Least Common Denominator

Determine the LCD of $\dfrac{4}{9}$ and $\dfrac{7}{15}$.

Solution

The LCD of the fractions $\dfrac{4}{9}$ and $\dfrac{7}{15}$ is the same as the LCM of the denominators 9 and 15.

Using one of the methods from Chapter 1, Section 1.3:

- The largest number, 15, is **not** divisible by 9.
- Multiples of 15 are: 15, 30, 45, ...
- 45 is divisible by 9.
- Thus, the LCM of 9 and 15 is 45.

Therefore, the LCD of $\dfrac{4}{9}$ and $\dfrac{7}{15}$ is 45.

Comparing Fractions

Fractions can easily be compared when they have the same denominator. If they do not have the same denominator, first determine the LCD of the fractions and convert them into equivalent fractions with the LCD as the denominator.

Example 2.4-b: Comparing Fractions

Which of the fractions, $\dfrac{5}{12}$ or $\dfrac{3}{8}$, is greater?

Solution

Step 1: Since the fractions do not have the same denominator, we need to first determine the LCD of the fractions, which is the same as the LCM of the denominators. The LCM of 12 and 8 is 24.

Step 2: Convert each fraction to its equivalent fraction with 24 as the denominator.

To convert $\dfrac{5}{12}$ to its equivalent fraction with 24 as the denominator, multiply the denominator by 2 to obtain the LCD of 24, and multiply the numerator by 2 as well to maintain an equivalent fraction.

$$\frac{5}{12} = \frac{5 \times 2}{12 \times 2} = \frac{10}{24}$$

5 portions of 12 equal parts of a whole are equal to 10 portions of 24 equal parts of that whole.

Similarly, convert $\dfrac{3}{8}$ to an equivalent fraction with 24 as the denominator:

$$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

3 portions of 8 equal parts of a whole is equal to 9 portions of 24 equal parts of that whole.

Step 3: Since the denominators are the same, we can compare the fractions' numerators to identify the greater fraction.

Since 10 > 9, it implies: $\dfrac{10}{24} > \dfrac{9}{24}$.

   Therefore, $\dfrac{5}{12} > \dfrac{3}{8}$.

Addition of Fractions

The denominator of a fraction indicates the number of parts into which an item is divided. Therefore, adding fractions requires that every fraction's denominators be the same. If the denominators are different, they must first be made the same by determining the LCD and changing each fraction to its equivalent fraction with the LCD as the denominator.

   When the fractions have the same denominator, the numerators of each of the fractions may be added. The numerator of the resulting fraction is equal to this sum, and the denominator is equal to the common denominator of the fractions being added.

   Express the final answer reduced to the lowest terms and as a mixed number, where applicable.

Example 2.4-c: Adding Fractions that Have the Same Denominator

Add $\dfrac{2}{9}$ and $\dfrac{5}{9}$.

Solution

$$\frac{2}{9} + \frac{5}{9}$$

The denominators of the fractions are the same. Adding the numerators and keeping the common denominator,

$$= \frac{2+5}{9}$$
$$= \frac{7}{9}$$
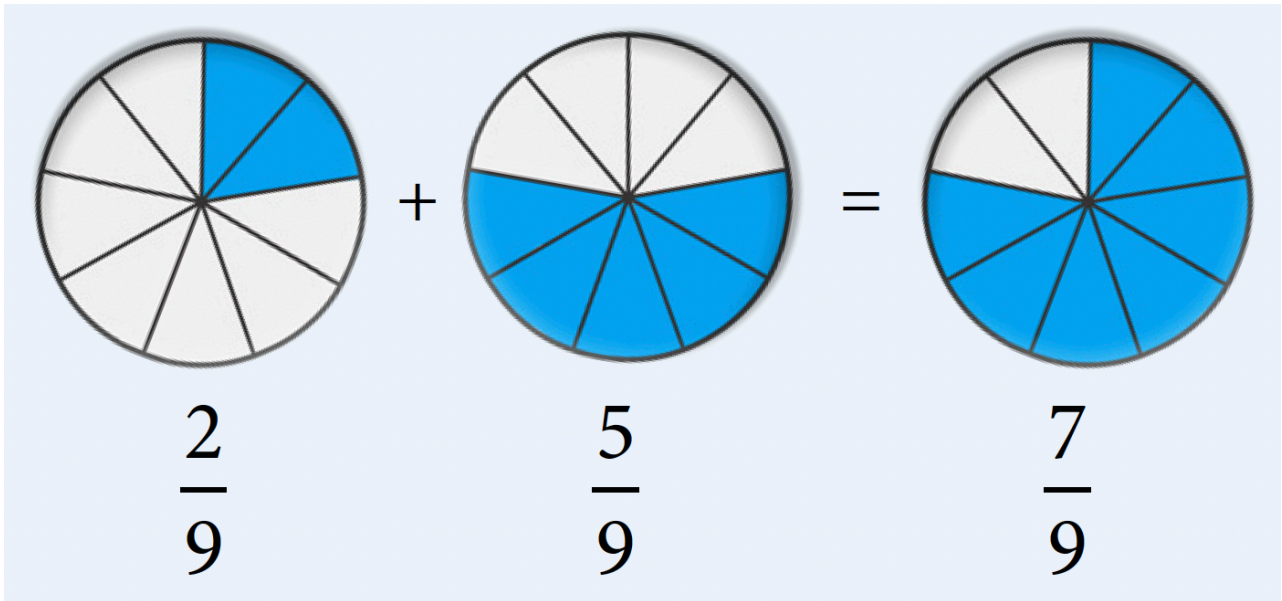
Therefore, the result from adding $= \dfrac{2}{9}$ and $= \dfrac{5}{9}$ is $\dfrac{7}{9}$.

Example 2.4-d: Adding Fractions that have Different Denominators

Add $\dfrac{3}{4}$ and $\dfrac{2}{3}$.

Solution

$$\dfrac{3}{4} + \dfrac{2}{3}$$

LCM of 4 and 3 is 12 (i.e., LCD = 12). Determining the equivalent fractions with a denominator of 12,

$$= \dfrac{9}{12} + \dfrac{8}{12}$$

$$= \dfrac{9 + 8}{12}$$

Adding the numerators and keeping the common denominator,

$$= \frac{17}{12}$$

$$= 1\frac{5}{12}$$

Converting the improper fraction to a mixed number,

Therefore, the result from adding $= \frac{3}{4}$ and $= \frac{2}{3}$ is $1\frac{5}{12}$.

Example 2.4-e: Adding a Mixed Number and a Proper Fraction

Add $3\frac{5}{6}$ and $\frac{4}{9}$.

Solution

Method 1:

$$3\frac{5}{6} + \frac{4}{9}$$

Converting the mixed number to an improper fraction,

$$= \frac{(3 \times 6) + 5}{6} + \frac{4}{9} = \frac{23}{6} + \frac{4}{9}$$

LCM of 6 and 9 is 18 (i.e., LCD = 18). Determining the equivalent fractions with a denominator of 18,

$$= \frac{69}{18} + \frac{8}{18}$$

Adding the numerators and keeping the common denominator,

$$= \frac{77}{18}$$

Converting the improper fraction to a mixed number,

$$= 4\frac{5}{18}$$

Method 2:

$$3\frac{5}{6} + \frac{4}{9}$$

Separating the whole number and the fractions,

$$= 3 + \left(\frac{5}{6} + \frac{4}{9}\right)$$

LCM of 6 and 9 is 18 (i.e., LCD = 18). Determining the equivalent fractions with a denominator of 18,

$$= 3 + \left(\frac{15}{18} + \frac{8}{18}\right)$$

Adding the numerators and keeping the common denominator,

$$= 3 + \frac{23}{18}$$

Converting the improper fraction to a mixed number,

$$= 3 + 1\frac{5}{18}$$

$$= 3 + 1 + \frac{5}{18}$$

Adding the whole numbers and then the fraction,

$$= 4 + \frac{5}{18}$$

$$= 4\frac{5}{18}$$

Therefore, the result from adding $3\dfrac{5}{6}$ and $\dfrac{4}{9}$ is $4\dfrac{5}{18}$.

Example 2.4-f: Adding Mixed Numbers

Add:

a. $2\dfrac{1}{6}$ and $4\dfrac{3}{4}$

b. $15\dfrac{2}{3}$ and $3\dfrac{3}{5}$

Solution

a. $2\dfrac{1}{6} + 4\dfrac{3}{4}$

LCM of 6 and 4 is 12 (i.e., LCD = 12). Determining the equivalent mixed numbers with a denominator of 12,

$$= 2\dfrac{2}{12} + 4\dfrac{9}{12}$$

Separating the whole numbers and the fractions,

$$= (2 + 4) + \left(\dfrac{2}{12} + \dfrac{9}{12}\right)$$

Adding the whole numbers and the fractions,

$$= 6\dfrac{11}{12}$$

Therefore, the result from adding $2\dfrac{1}{6}$ and $4\dfrac{3}{4}$ is $6\dfrac{11}{12}$.

b. $15\dfrac{2}{3} + 3\dfrac{3}{5}$

LCM of 3 and 5 is 15 (i.e., LCD = 15). Determining the equivalent mixed numbers with a denominator of 15,

$$15\dfrac{10}{15} + 3\dfrac{9}{15}$$

Separating the whole numbers and the fractions,

$$= (15 + 3) + (\dfrac{10}{15} + \dfrac{9}{15})$$

Adding the whole numbers and the fractions,

$$= 18 + \dfrac{19}{15}$$

Converting the improper fraction to a mixed number,

$$= 18 + 1\dfrac{4}{15}$$

Adding the whole numbers and then the fraction,

$$= 19\dfrac{4}{15}$$

Therefore, the result from adding $15\dfrac{2}{3}$ and $3\dfrac{3}{5}$ is $19\dfrac{4}{15}$.

Subtraction of Fractions

The process for the subtraction of fractions is the same as that of the addition of fractions. First, determine a common denominator and change each fraction to its equivalent fraction with the common denominator. When the fractions have the same denominator, the numerators of the fractions may be subtracted. The numerator of the resulting fraction is equal to this difference, and the denominator is equal to the common denominator of the fractions being subtracted.

Express the final answer reduced to the lowest terms and as a mixed number, where applicable.

Example 2.4-g: Subtracting Fractions that have the Same Denonimator

Subtract $\dfrac{3}{8}$ from $\dfrac{7}{8}$.

Solution

$$\frac{7}{8} - \frac{3}{8}$$

The denominators of the fractions are the same. Subtracting the numerators and keeping the common denominator,

$$= \frac{7 - 3}{8}$$

$$= \frac{4}{8} = \frac{1}{2}$$

Reducing to the lowest terms,

$$= \frac{1}{2}$$

$$\frac{7}{8} \quad - \quad \frac{3}{8} \quad = \quad \frac{4}{8} = \frac{1}{2}$$
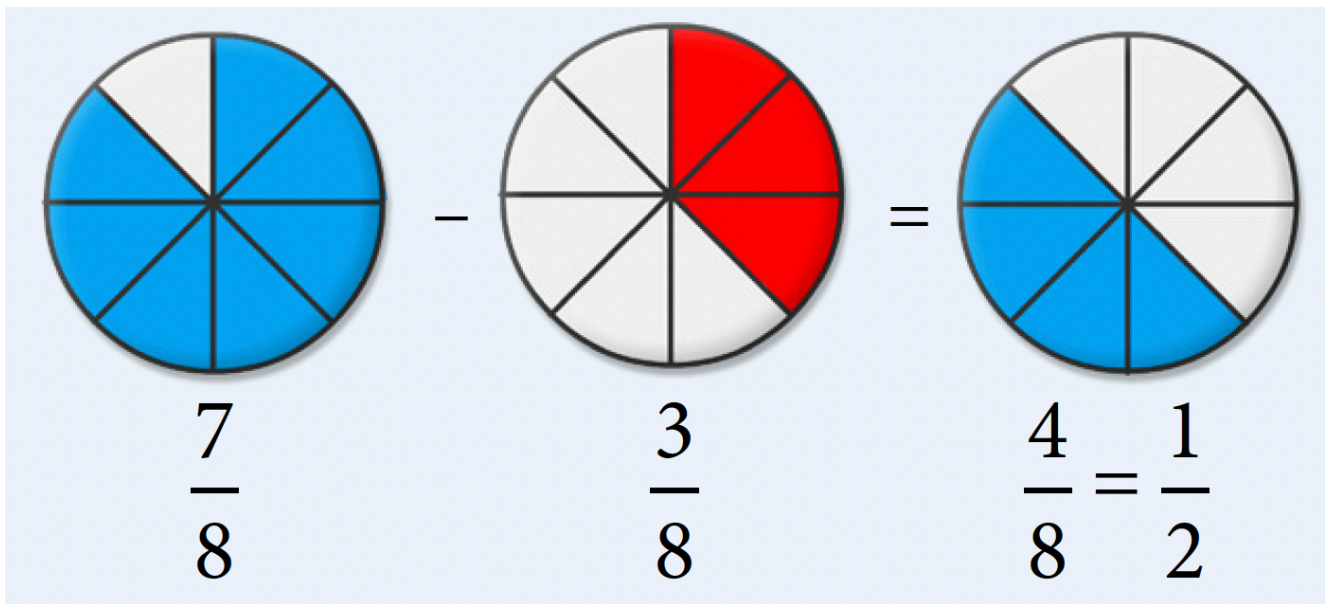
Therefore, the result from subtracting $\frac{3}{8}$ from $\frac{7}{8}$ is $\frac{1}{2}$.

Example 2.4-h: Subtracting Fractions that Have Different Denominators

Subtract $\frac{2}{8}$ from $\frac{7}{10}$.

Solution

$$\frac{7}{10} - \frac{2}{8}$$

LCM of 8 and 10 is 40 (i.e., LCD = 40). Determining the equivalent fractions with a denominator of 40,

$$\frac{28}{40} - \frac{10}{40}$$

Subtracting the numerators and keeping the common denominator,

$$\frac{28 - 10}{40}$$

$$\frac{18}{40} = \frac{9}{20}$$

Reducing to the lowest terms,

$$\frac{9}{20}$$

Therefore, the result from subtracting $\frac{2}{8}$ from $\frac{7}{10}$ is $\frac{9}{20}$.

Example 2.4-i: Subtracting Mixed Numbers

Subtract $7\frac{2}{3}$ from $12\frac{1}{2}$.

Solution

Method 1:

$$12\frac{1}{2} - 7\frac{2}{3}$$

Converting the mixed numbers to improper fractions,

$$= \frac{(12 \times 2) + 1}{2} - \frac{(7 \times 3) + 2}{3}$$

$$= \frac{25}{2} - \frac{23}{3}$$

LCM of 2 and 3 is 6 (i.e., LCD = 6). Determining the equivalent fractions with a denominator of 6,

$$= \frac{75}{6} - \frac{46}{6}$$

Subtracting the numerators and keeping the common denominator,

$$= \frac{29}{6}$$

Converting the improper fraction to a mixed number,

$$= 4\frac{5}{6}$$

Method 2:

$$12\frac{1}{2} - 7\frac{2}{3}$$

LCM of 2 and 3 is 6 (i.e., LCD = 6). Determining the equivalent mixed numbers with a denominator of 6,

$$= 12\frac{3}{6} - 7\frac{4}{6}$$

The fraction $\frac{4}{6}$ is greater than $\frac{3}{6}$. Therefore, we have to regroup the mixed number $12\frac{3}{6}$ by borrowing 1 from 12:

$$12\frac{3}{6} = 11 + 1 + \frac{3}{6} = 11 + \frac{6}{6} + \frac{3}{6} = 11\frac{9}{6}$$

$$= 11\frac{9}{6} - 7\frac{4}{6}$$

Subtracting the whole numbers and then the fractions,

$$= 4\frac{(9-4)}{6}$$

$$= 4\frac{5}{6}$$

Therefore, the result from subtracting $7\frac{2}{3}$ from $12\frac{1}{2}$ is $4\frac{5}{6}$.

Multiplication of Fractions

First, convert any mixed number to its improper fraction to multiply two or more fractions. Then, multiply the numerators together to get the new numerator and multiply the denominators together to get the new denominator.

When multiplying fractions, you can reduce any numerator term with any denominator term. Reduce as much as possible before multiplying the numerators and the denominators together to keep the numbers as simple as possible.

Express the final answer reduced to the lowest terms and as a mixed number, where applicable.

Note: When multiplying mixed numbers, it is incorrect to multiply the whole number parts separately from the fractional parts to arrive at the answer.

Example 2.4-j: Multiplying Fractions

Multiply:

a. $\dfrac{3}{2} \times \dfrac{4}{11}$

b. $15 \times \dfrac{2}{5}$

c. $3\dfrac{1}{8} \times 2\dfrac{4}{5}$

Solution

a. $\dfrac{3}{2} \times \dfrac{4}{11} = \dfrac{3}{1} \times \dfrac{2}{11}$

Reducing the fractions,

$= \dfrac{3}{1} \times \dfrac{2}{11}$

Multiplying the numerators together and denominators together,

$$= \frac{6}{11}$$

Therefore, the result of $\frac{3}{2} \times \frac{4}{11}$ is $\frac{6}{11}$.

b. $15 \times \dfrac{2}{5} = \dfrac{3}{1} \times \dfrac{2}{1}$

Reducing the fractions,

$$= \frac{3}{1} \times \frac{2}{1}$$

Multiplying the numerators together and denominators together,

$$= \frac{6}{1} = 6$$

Therefore, the result of $15 \times \dfrac{2}{5}$ is 6.

c. $3\dfrac{1}{8} \times 2\dfrac{4}{5}$

Converting the mixed numbers to improper fractions,

$$= \frac{(3 \times 8) + 1}{8} \times \frac{(2 \times 5) + 4}{5}$$

$$= \frac{25}{8} \times \frac{14}{5} = \frac{5}{4} \times \frac{7}{1}$$

Reducing the fractions,

$$= \frac{5}{4} \times \frac{7}{1}$$

Multiplying the numerators together and denominators together,

$$= \frac{35}{4}$$

Converting the improper fraction to a mixed number,

$$8\frac{3}{4}$$

Therefore, the result of $3\frac{1}{8} \times 2\frac{4}{5}$ is $8\frac{3}{4}$

Division of Fractions

When dividing fractions, as in multiplication, first convert any mixed number to its improper fraction. The division of fractions is done by multiplying the first fraction by the reciprocal of the second fraction. Then, follow the procedure used in multiplication to arrive at the final result.

Express the final answer reduced to the lowest terms and as a mixed number, where applicable.

Note:

Dividing by 2 is the same as multiplying by the reciprocal of 2, which is $\frac{1}{2}$.

When multiplying or dividing mixed numbers, it is incorrect to multiply or divide the whole number parts separately from the fractional parts to arrive at the answer.

Example 2.4-k: Dividing Fractions

Divide $\frac{15}{16}$ by $\frac{9}{20}$.

Solution

$$\frac{15}{16} \div \frac{9}{20}$$

Multiplying $\frac{15}{16}$ by the reciprocal of $\frac{9}{20}$, which is $\frac{20}{9}$,

$$= \frac{15}{16} \times \frac{20}{9} = \frac{5}{4} \times \frac{5}{3}$$

Reducing the fractions,

$$= \frac{5}{4} \times \frac{5}{3}$$

Multiplying the numerators together and denominators together,

$$= \frac{25}{12}$$

Converting the improper fraction to a mixed number,

$$= 2\frac{1}{12}$$

Therefore, the result of $\frac{15}{16}$ divided by $\frac{9}{20}$ is $2\frac{1}{12}$.

Example 2.4-l: Dividing Mixed Numbers

Divide $3\frac{3}{20}$ by $1\frac{4}{5}$.

Solution

$$3\frac{3}{20} \div 1\frac{4}{5}$$

Converting the mixed numbers to improper fractions,

$$= \frac{(3 \times 20) + 3}{20} \div \frac{(1 \times 5) + 4}{5}$$

$$= \frac{63}{20} \div \frac{9}{5}$$

Multiplying $\dfrac{63}{20}$ by the reciprocal of $\dfrac{9}{5}$, which is $\dfrac{5}{9}$,

$$= \frac{63}{20} \times \frac{5}{9} = \frac{7}{4} \times \frac{1}{1}$$

Reducing the fractions,

$$= \frac{7}{4}$$

Converting the improper fraction to a mixed number,

$$= 1\frac{3}{4}$$

Therefore, the result of $3\dfrac{3}{20}$ divided by $1\dfrac{4}{5}$ is $1\dfrac{3}{4}$.

Complex Fractions

A **complex fraction** is a fraction in which one or more fractions are found in the numerator or denominator. For example,

- $\dfrac{1}{\left(\dfrac{5}{8}\right)}$ is a complex fraction because it has a fraction in the denominator.

- $\dfrac{\left(\dfrac{2}{3}\right)}{6}$ is a complex fraction because it has a fraction in the numerator.

- $\dfrac{\left(\dfrac{2}{5} + \dfrac{1}{4}\right)}{3}$ is a complex fraction because it has two fractions in the numerator.

- $\dfrac{\left(\dfrac{5}{6}\right)}{\left(\dfrac{1}{8}\right)}$ is a complex fraction because it has a fraction in both the numerator and the denominator.

A complex fraction can be simplified by dividing the numerator by the denominator and then, following the rule(s) for dividing fractions.

$$\frac{1}{\left(\frac{5}{8}\right)} = 1 \div \frac{5}{8} = 1 \times \frac{5}{5} = \frac{8}{5}$$

$$\frac{\left(\frac{2}{3}\right)}{6} = \frac{2}{3} \div 6 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

Example 2.4-m: Simplifying Complex Fractions

Add:

a. $\dfrac{\left(\dfrac{7}{2}\right)}{5}$

b. $\dfrac{\dfrac{6}{9}}{\left(\dfrac{9}{8}\right)}$

Solution

a. $\dfrac{\left(\dfrac{7}{2}\right)}{5} = \dfrac{7}{2} \div 5 = \dfrac{7}{2} \times \dfrac{1}{5} = \dfrac{7}{10}$

b. $\dfrac{\dfrac{6}{9}}{\left(\dfrac{9}{8}\right)} = 6 \div \dfrac{9}{8} = \dfrac{2}{1} \times \dfrac{8}{3} = 5\dfrac{1}{3}$

Powers and Square Roots of Fractions

Powers of fractions are expressed the same way as whole numbers. When the base of a power is a fraction, it is written within brackets.

For example, $\left(\dfrac{2}{3}\right)^2$ is read as "two-thirds squared".

- This means that $\dfrac{2}{3}$ is used as a factor two times.

- i.e., $\left(\dfrac{2}{3}\right)^2 = \dfrac{2}{3} \times \dfrac{2}{3} = \dfrac{4}{9}$

A mixed number that is raised to a power is evaluated by first converting it into an improper fraction and then following the same procedure explained above.

For example, $\left(1\dfrac{2}{3}\right)^4$ is evaluated by first converting $1\dfrac{2}{3}$ into an improper fraction.

$$\left(1\dfrac{2}{3}\right)^4 = \left(\dfrac{1(3)+2}{3}\right)^4 = \left(\dfrac{5}{3}\right)^4 = \left(\dfrac{5}{3}\right)\left(\dfrac{5}{3}\right)\left(\dfrac{5}{3}\right)\left(\dfrac{5}{3}\right) = \dfrac{625}{81} = 7\dfrac{58}{81}$$

Example 2.4-n: Evaluating Powers of Fractions

Evaluate the following powers:

a. $\left(\dfrac{4}{5}\right)^4$

b. $\left(1\dfrac{1}{2}\right)^5$

Solution

a. $\left(\dfrac{4}{5}\right)^4$

Expanding by using $\dfrac{4}{5}$ as a factor four times,

$$= \left(\dfrac{4}{5}\right)\left(\dfrac{4}{5}\right)\left(\dfrac{4}{5}\right)\left(\dfrac{4}{5}\right) = \dfrac{256}{625}$$

b. $\left(1\dfrac{1}{2}\right)^5$

Converting the mixed number into an improper fraction,

$$= (\frac{1(2) + 1}{2})^5 = (\frac{3}{2})^5$$

Expanding by using $\frac{3}{2}$ as a factor five times,

$$= (\frac{3}{2})(\frac{3}{2})(\frac{3}{2})(\frac{3}{2})(\frac{3}{2}) = \frac{243}{32}$$

Converting back to a mixed number,

$$7\frac{19}{32}$$

Square roots of fractions are calculated the same way as square roots of whole numbers, but the numerators and denominators are evaluated separately.

For example, $\sqrt{\frac{9}{16}}$ is the same as $\frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$.

Example 2.4-o: Evaluating Square Roots of Fractions

Evaluate the square root: $\sqrt{\frac{25}{144}}$

Solution

$$\sqrt{\frac{25}{144}}$$

Determining the square root of the numerator and denominator separately,

$$\frac{\sqrt{25}}{\sqrt{144}} = \frac{5}{12}$$

2.4 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

For Problems 1 to 8, identify the greater fraction in each pair.

1. $\dfrac{2}{5}$ or $\dfrac{3}{8}$

2. $\dfrac{4}{3}$ or $\dfrac{6}{5}$

3. $\dfrac{12}{15}$ or $\dfrac{35}{45}$

4. $\dfrac{5}{4}$ or $\dfrac{7}{6}$

5. $\dfrac{8}{7}$ or $\dfrac{13}{12}$

6. $\dfrac{5}{13}$ or $\dfrac{16}{39}$

7. $\dfrac{8}{9}$ or $\dfrac{39}{45}$

8. $\dfrac{3}{8}$ or $\dfrac{25}{48}$

For Problems 9 to 16, perform the addition, reduce to lowest terms, and express the answer as a mixed number, whenever possible.

0. a. $\dfrac{5}{8} + \dfrac{7}{8}$

   b. $\dfrac{7}{12} + \dfrac{3}{4}$

1. a. $\dfrac{5}{9} + \dfrac{7}{9}$

   b. $\dfrac{7}{10} + \dfrac{9}{20}$

2. a. $\dfrac{4}{3} + \dfrac{5}{6}$

    b. $12\dfrac{3}{4} + 5\dfrac{1}{3}$

3. a. $\dfrac{23}{12} + \dfrac{1}{3}$

    b. $18\dfrac{5}{7} + 2\dfrac{2}{5}$

4. a. $9\dfrac{3}{4} + 6\dfrac{1}{6}$

    b. $8\dfrac{2}{3} + 5\dfrac{3}{4}$

5. a. $11\dfrac{1}{4} + 5\dfrac{2}{3}$

    b. $7\dfrac{1}{12} + 5\dfrac{3}{4}$

6. a. $\dfrac{1}{10} + \dfrac{17}{100} + \dfrac{39}{1,000}$

    b. $\dfrac{3}{5} + \dfrac{7}{10} + \dfrac{9}{15}$

7. a. $\dfrac{3}{10} + \dfrac{47}{100} + \dfrac{241}{1,000}$

    b. $\dfrac{2}{3} + \dfrac{3}{4} + \dfrac{5}{8}$

For Problems 17 to 22, perform the subtraction, reduce to lowest terms, and express the answer as a mixed number, whenever possible.

0. a. $\dfrac{2}{3} - \dfrac{1}{9}$

    b. $\dfrac{9}{12} - \dfrac{3}{5}$

1. $\dfrac{1}{6} - \dfrac{1}{8}$

    b. $\dfrac{19}{20} - \dfrac{3}{10}$

2. a. $\dfrac{5}{3} - \dfrac{3}{8}$

b. $16\dfrac{1}{8} - 1\dfrac{1}{2}$

3.　a. $\dfrac{17}{9} - \dfrac{5}{6}$

　　b. $5\dfrac{2}{3} - 1\dfrac{5}{12}$

4.　a. $8\dfrac{5}{6} - 5\dfrac{3}{9}$

　　b. $9\dfrac{2}{5} - 7\dfrac{3}{10}$

5.　a. $8\dfrac{5}{12} - 4\dfrac{3}{6}$

　　b. $5\dfrac{5}{8} - 4\dfrac{5}{6}$

For Problems 23 and 24, perform the mixed additions and subtractions, reduce to lowest terms, and express the answer as a mixed number, whenever possible

0.　a. $\dfrac{32}{100} - \dfrac{8}{1,000} + \dfrac{3}{25}$

　　b. $\dfrac{5}{8} + \dfrac{13}{16} - \dfrac{3}{4}$

1.　a. $\dfrac{3}{10} - \dfrac{4}{1,000} + \dfrac{5}{100}$

　　b. $\dfrac{7}{12} + \dfrac{5}{6} - \dfrac{2}{3}$

For Problems 25 to 30, perform the multiplication, reduce to lowest terms, and express the answer as a mixed number, whenever possible.

0.　a. $\dfrac{16}{5} \times \dfrac{5}{4}$

　　b. $3 \times \dfrac{7}{9}$

1.　a. $\dfrac{12}{5} \times \dfrac{25}{3}$

b. $\dfrac{6}{9} \times \dfrac{19}{12}$

2. a. $\dfrac{3}{8} \times \dfrac{5}{11}$

b. $9\dfrac{3}{5} \times 1\dfrac{29}{96}$

3. a. $\dfrac{4}{5} \times \dfrac{23}{9}$

b. $11\dfrac{3}{4} \times 1\dfrac{1}{74}$

4. a. $\dfrac{9}{38} \times \dfrac{19}{63}$

b. $2\dfrac{2}{9} \times 1\dfrac{1}{2}$

5. a. $\dfrac{15}{27} \times \dfrac{18}{45}$

b. $2\dfrac{3}{7} \times \dfrac{25}{45}$

For Problems 31 to 36, perform the division, reduce to the lowest terms, and express the answer as a mixed number whenever possible.

0. a. $\dfrac{2}{3} \div \dfrac{4}{9}$

b. $\dfrac{3}{8} \div 4$

1. a. $\dfrac{3}{5} \div \dfrac{3}{4}$

b. $\dfrac{1}{7} \div \dfrac{3}{5}$

2. a. $\dfrac{10}{15} \div \dfrac{3}{7}$

b. $23\dfrac{1}{2} \div 8\dfrac{13}{16}$

3. a. $\dfrac{8}{12} \div \dfrac{2}{4}$

b. $10\dfrac{1}{4} \div 2\dfrac{27}{48}$

4. a. $5\dfrac{1}{5} \div 13$

b. $18 \div 4\dfrac{4}{5}$

5. a. $5\dfrac{1}{4} \div 7$

b. $15 \div 3\dfrac{1}{3}$

For Problems 37 and 38, express the complex fractions as a proper fraction or a mixed number, where possible.

0. a. $\dfrac{\dfrac{1}{9}}{\left(\dfrac{9}{4}\right)}$

b. $\dfrac{4\frac{3}{4}}{\left(\dfrac{3}{8}\right)}$

1. a. $\dfrac{\dfrac{1}{11}}{\left(\dfrac{11}{2}\right)}$

b. $\dfrac{8\frac{4}{9}}{\left(\dfrac{11}{2}\right)}$

For Problems 39 to 44, evaluate the powers of the fractions.

0. a. $\left(\dfrac{3}{5}\right)^2$

b. $\left(\dfrac{6}{7}\right)^2$

1. a. $\left(\dfrac{3}{4}\right)^2$

b. $\left(\dfrac{2}{9}\right)^2$

2. a. $\left(\dfrac{3}{4}\right)^3$

    b. $\left(\dfrac{5}{3}\right)^4$

3. a. $\left(\dfrac{2}{7}\right)^3$

    b. $\left(\dfrac{6}{5}\right)^4$

4. a. $\left(1\dfrac{1}{3}\right)^2$

    b. $\left(3\dfrac{1}{2}\right)^3$

5. a. $\left(2\dfrac{1}{4}\right)^2$

    b. $\left(1\dfrac{2}{3}\right)^3$

For Problems 45 to 50, evaluate the square roots of the fractions.

0. a. $\sqrt{\dfrac{1}{9}}$

    b. $\sqrt{\dfrac{1}{49}}$

1. a. $\sqrt{\dfrac{1}{16}}$

    b. $\sqrt{\dfrac{1}{10,000}}$

2. a. $\sqrt{\dfrac{4}{25}}$

    b. $\sqrt{\dfrac{81}{16}}$

3. a. $\sqrt{\dfrac{36}{100}}$

    b. $\sqrt{\dfrac{144}{81}}$

4. a. $\sqrt{3\dfrac{1}{16}}$

    b. $\sqrt{6\dfrac{1}{4}}$

5. a. $\sqrt{1\dfrac{11}{25}}$

    b. $\sqrt{1\dfrac{21}{100}}$

For Problems 51 to 72, express your answers as a proper fraction or a mixed number, where appropriate:

0. Peter spent $\dfrac{5}{12}$ of his money on rent and $\dfrac{1}{4}$ on food. What fraction of his money did he spend on rent and food?

1. Alan walked $\dfrac{3}{5}$ km to his friend's house and from there, he walked another $\dfrac{3}{4}$ km to his school. How far did Alan walk?

2. Last night, Amy spent $3\dfrac{1}{6}$ hours on her math project and $2\dfrac{3}{10}$ hours on her design project. How much time did she spend on both projects altogether?

3. A bag contains $2\dfrac{3}{5}$ kg of red beans and $1\dfrac{1}{8}$ kg of green beans. What is the total weight of the bag?

4. Thomas baked a $2\dfrac{1}{2}$ pound cake. He gave $1\dfrac{5}{8}$ pounds of it to his friend Yan. How much was left?

5. Alexander bought $4\dfrac{2}{5}$ litres of milk and drank $1\dfrac{2}{3}$ litres of it. How much milk was left?

6. Sarah had $\dfrac{3}{4}$ kg of cheese. She used $\dfrac{2}{7}$ kg of the cheese while baking. How many kilograms of cheese was left?

7. Cassidy bought $\dfrac{5}{8}$ litres of olive oil and used $\dfrac{1}{3}$ litre of the oil while cooking. What quantity of olive oil was left?

8. David spent $\dfrac{7}{10}$ of his money on toys and $\dfrac{1}{3}$ of the remainder on food. What fraction of his money

   was spent on food?

9. Mary spent $\dfrac{2}{5}$ of her money on a school bag. She then spent $\dfrac{1}{3}$ of the remainder on shoes. What

   fraction of her money was spent on shoes?

10. After selling $\dfrac{2}{5}$ of its textbooks, a bookstore had 810 books left. How many textbooks were in the

    bookstore initially?

11. Rose travelled $\dfrac{3}{5}$ of her journey by car and the remaining 20 km by bus. How far did she travel by car?

12. Cheng can walk $5\dfrac{1}{4}$ km in $1\dfrac{1}{2}$ hours. How many kilometres can he walk in 1 hour?

13. $2\dfrac{3}{4}$ litres of juice weighs $4\dfrac{2}{3}$ kg. Determine the weight (in kilograms) of 1 litre of juice.

14. A chain of length $\dfrac{7}{8}$ metres is cut into pieces measuring $\dfrac{1}{16}$ metres each. How many pieces are there?

15. A cake that weighs $\dfrac{2}{3}$ kg is cut into slices weighing $\dfrac{1}{12}$ kg each. How many slices are there?

16. A bottle of medicine contains 80 mg of medicine. Each dose of the medicine is $\dfrac{2}{5}$ mg. How many doses

    are there in the bottle?

17. A box of cereal contains 917 grams of cereal. How many bowls of cereal will there be if each serving is

    $32\dfrac{3}{4}$ grams?

18. Out of 320 bulbs, $\dfrac{1}{20}$ of the bulbs are defective. How many of them are not defective?

19. If $\dfrac{4}{15}$ of the 1,800 students in a school enrolled for a mathematics course, how many students did not

    enroll for the course?

20. The product of two numbers is 9. If one number is $3\dfrac{3}{4}$, what is the other number?

21. If a wire that is $42\dfrac{3}{4}$ cm long is cut into several $2\dfrac{1}{4}$ cm equal pieces, how many pieces would exist?

2.5 CONVERTING BETWEEN FRACTIONS AND DECIMAL NUMBERS AND COMBINED ORDER OF OPERATIONS

Converting Decimal Numbers to Fractions

It is possible to convert **terminating** decimal numbers (e.g., 0.275) and **repeating, non-terminating** decimal numbers (e.g., 0.333333...) to fractions. However, there is no exact equivalent fraction for **non-repeating, non-terminating** decimal numbers (e.g., 0.837508...).

Converting Terminating Decimal Numbers to Fractions

Any non-repeating, terminating decimal number can be converted to a fraction by following these steps:

1. Count the number of decimal places in the decimal number.
2. Move the decimal point by that many places to the right, making it a whole number, and divide this whole number by 10 raised to the power of the number of decimal places counted in Step 1; for example, if there were 2 decimal places in the original number, we divide by $10^2 = 100.$
3. Simplify (or reduce) the fraction.

Example 2.5-a: Converting Terminating Decimal Numbers to Fractions

Convert the following decimal numbers to their fractional equivalents:

a. 3.75
b. 0.015

Solution

a. Converting 3.75 to its fractional equivalent:

3.75

3.75 contains two decimal places. Therefore, move the decimal point two places to the right and divide by $10^2 = 100$.

$$= \frac{3.75}{100} = \frac{375}{100}$$

(This is the same as multiplying by 100 and dividing by 100: $\frac{3.75 \times 100}{100} = \frac{375}{100}$)

$$= \frac{375}{100} = \frac{375 \div 25}{100 \div 25}$$

Dividing by 25 to simplify,

$$= \frac{15}{4} = 3\frac{3}{4}$$

Therefore, 3.75 converted to its fractional equivalent is $\frac{15}{4}$ or $3\frac{3}{4}$.

b. Converting 0.015 to its fractional equivalent:

0.015

0.015 contains three decimal places. Therefore, move the decimal point three places to the right and divide by $10^3 = 1,000$.

$$= \frac{0.015}{1,000} = \frac{15}{1,000}$$

(This is the same as multiplying by 1,000 and dividing by 1,000: $\frac{0.015 \times 1,000}{1,000} = \frac{15}{1,000}$)

$$= \frac{15}{1,000} = \frac{15 \div 5}{1,000 \div 5}$$

Dividing by 5 to simplify,

$$= \frac{3}{200}$$

Therefore, 0.015 converted to its fractional equivalent is $\dfrac{3}{200}$.

Converting Repeating Decimal Numbers to Fractions

Any repeating, non-terminating decimal number can be converted to a fraction by following the procedure illustrated in the following examples.

Example 2.5-b: Converting Repeating Decimal Numbers to Fractions

Convert 0.777777... to a fraction.

Solution

Let 0.777777... be equal to a fraction x.

Therefore,

a. $x = 0.777777... = 0.\overline{7}$

Multiplying both sides by 10,

b. $10x = 7.777777... = 7.\overline{7}$

We now have two repeating decimal numbers, each with the same decimal portion.

Subtracting **a** from **b**,

$$10x - x = 7.\overline{7} - 0.\overline{7}$$

$$9x = 7$$

Dividing both sides by 9,

$$x = \dfrac{7}{9}$$

Therefore, $0.777777... = \dfrac{7}{9}$.

Example 2.5-c: Converting Repeating Decimal Numbers to Fractions

Convert 0.655555... to a fraction.

Solution

Let 0.655555... be equal to a fraction x.

Therefore,

a. $x = 0.655555... = 0.6\overline{5}$

Multiplying both sides by 10,

b. $10x = 6.555555... = 6.\overline{5}$

The decimal portions of the two decimal numbers are different. Multiplying both sides by 10 again,

c. $100x = 65.555555... = 65.\overline{5}$

We now have two repeating decimal numbers with the same decimal portion (**b** and **c**).

Subtracting **b** from **c**,

$$100x - 10x = 65.\overline{5} - 6.\overline{5}$$

$$90x = 59$$

Dividing both sides by 90,

$$x = \frac{59}{90}$$

Therefore, $0.655555... = \dfrac{59}{90}$.

Example 2.5-d: Converting Repeating Decimal Numbers to Fractions

Convert 0.353535... to a fraction.

Solution

Let 0.353535... be equal to a fraction x.

Therefore,

a. $x = 0.353535... = 0.\overline{35}$

Multiplying both sides by 10,

b. $10x = 3.535353... = 3.\overline{53}$

The decimal portions of the two decimal numbers are different. Multiplying both sides by 10 again,

c. $100x = 35.353535... = 35.\overline{35}$

We now have two repeating decimal numbers with the same decimal portion (**a** and **c**).

Subtracting **a** from **c**,

$100x - x = 35.\overline{35} - 0.\overline{35}$

$99x = 35$

Dividing both sides by 99,

$x = \dfrac{35}{99}$

Therefore, $0.353535... = \dfrac{35}{99}$.

From Examples 2.5-a to 2.5-d, we have learned that it is possible to convert terminating decimal numbers (e.g.,

0.015) and repeating, non-terminating decimal numbers (e.g., 0.65) into fractions. Therefore, such decimal numbers are **rational numbers**.

However, it is not possible to convert non-repeating, non-terminating decimal numbers (e.g., $\sqrt{2}, \pi, 5.81271...$) to fractions. Such decimal numbers are referred to as **irrational numbers**.

These rational numbers and irrational (non-rational) numbers together form the **real numbers** in the number system.

Converting Fractions to Decimal Numbers

Converting Proper and Improper Fractions to Decimal Numbers

A proper or improper fraction can be converted to its equivalent decimal number by dividing the numerator by the denominator, as shown in the following examples.

Example 2.5-e: Converting Proper and Improper Fractions to Decimal Numbers

Convert the following fractions to their decimal number equivalents:

a. $\dfrac{3}{8}$

b. $\dfrac{15}{11}$

Solution

a. $\dfrac{3}{8} = 3 \div 8 = 0.375$

Therefore, 0.375 is the decimal equivalent of $\dfrac{3}{8}$.

b. $\dfrac{15}{11} = 15 \div 11 == 1.363636.... = 1.\overline{36}$

Therefore, 1.36 is the decimal equivalent of $\dfrac{15}{11}$.

Converting Mixed Numbers to Decimal Numbers

A mixed number can be converted to its decimal form by first converting it to an improper fraction, then dividing the numerator by the denominator, as shown in the following example.

Example 2.5-f: Converting Mixed Numbers to Decimal Numbers

Convert the following mixed numbers to their decimal number equivalents:

a. $3\dfrac{2}{5}$

b. $11\dfrac{3}{7}$

Solution

a. $3\dfrac{2}{5}$

Coverting to an improper fraction,

$$= 3\dfrac{3(5) + 2}{5} = \dfrac{17}{5}$$

Dividing the numerator by the denominator,

$$= 3.4$$

Therefore, the decimal number equivalent of $3\dfrac{2}{5}$ is $= 3.4$.

b. $11\dfrac{3}{7}$

Coverting to an improper fraction,

$$= 3\dfrac{11(7)+3}{7} = \dfrac{80}{7}$$

Dividing the numerator by the denominator,

$$= 11.428571... = 11.43$$

Therefore, the decimal number equivalent of $11\dfrac{3}{7}$, rounded to two decimal places, is 11.43.

Combined Order of Operations

The Order of Operations (**BEDMAS**), as learned in Chapter 1, Section 1.4, is also used in evaluating expressions with fractions and decimal numbers.

Arithmetic expressions with fractions and decimal numbers that contain multiple operations are performed in the following sequence:

1. Evaluate the expressions within grouping symbols (i.e., **B**rackets and radical signs).
2. Evaluate powers (i.e., **E**xponents) and roots.
3. Perform **D**ivision and **M**ultiplication, in order from left to right.
4. Perform **A**ddition and **S**ubtraction, in order from left to right.

Note: For multiplication, division, powers, and roots of mixed numbers, they must first be converted to improper fractions before proceeding with the Order of Operations.

Example 2.5-g: Evaluating Expressions Using Order of Operations (BEDMAS)

Evaluate the following expressions:

a. $\left(1\dfrac{1}{3}\right)^2 + \sqrt{\dfrac{5}{16} + \dfrac{20}{16}}$

b. $\left(\dfrac{2}{3}\right)^2 + \dfrac{1}{2}\left(4\dfrac{1}{2}\right)^2 \div \sqrt{81}$

c. $\left(\dfrac{4}{5}\right)^2 + \left(\dfrac{11}{9} + \sqrt{\dfrac{49}{81}}\right) \times \dfrac{3}{25}$

d. $\sqrt{1\dfrac{69}{100}} + \sqrt{0.09} + \sqrt{\dfrac{64}{25}}$

e. $\left(1 + \dfrac{0.08}{4}\right)^2 - 1$

Solution

a. $\left(1\dfrac{1}{3}\right)^2 + \sqrt{\dfrac{5}{16} + \dfrac{20}{16}} = \left(\dfrac{4}{3}\right)^2 + \sqrt{\dfrac{25}{16}} = \left(\dfrac{4}{3}\right)\left(\dfrac{4}{3}\right) + \dfrac{\sqrt{25}}{\sqrt{16}}$

$= \dfrac{16}{9} + \dfrac{5}{4} = \dfrac{64}{36} + \dfrac{45}{36} = \dfrac{109}{36} = 3\dfrac{1}{36}$

b. $\left(\dfrac{2}{3}\right)^2 + \dfrac{1}{2}\left(4\dfrac{1}{2}\right)^2 \div \sqrt{81} = \left(\dfrac{2}{3}\right)\left(\dfrac{2}{3}\right) + \dfrac{1}{2}\left(\dfrac{9}{2}\right)\left(\dfrac{9}{2}\right) \div 9 = \dfrac{4}{9} + \dfrac{81}{8} \times \dfrac{1}{9}$

$= \dfrac{4}{9} + \dfrac{9}{8} = \dfrac{32}{72} + \dfrac{81}{72} = \dfrac{113}{72} = 1\dfrac{41}{72}$

c. $\left(\dfrac{4}{5}\right)^2 + \left(\dfrac{11}{9} + \sqrt{\dfrac{49}{81}}\right) \times \dfrac{3}{25} = \left(\dfrac{4}{5}\right)^2 + \left(\dfrac{11}{9} + \dfrac{\sqrt{49}}{\sqrt{81}}\right) \times \dfrac{3}{25} = \left(\dfrac{4}{5}\right)^2 + \left(\dfrac{11}{9} + \dfrac{7}{9}\right) \times \dfrac{3}{25}$

$= \left(\dfrac{4}{5}\right)^2 + \dfrac{18}{9} \times \dfrac{3}{25} = \left(\dfrac{4}{5}\right)\left(\dfrac{4}{5}\right) + 2 \times \dfrac{3}{25} = \dfrac{16}{25} + \dfrac{6}{25} = \dfrac{22}{25}$

d. $\sqrt{1\dfrac{69}{100}} + \sqrt{0.09} + \sqrt{\dfrac{64}{25}} = \sqrt{\dfrac{169}{100}} + \sqrt{\dfrac{9}{100}} + \sqrt{\dfrac{64}{25}} = \dfrac{\sqrt{169}}{\sqrt{100}} + \dfrac{\sqrt{9}}{\sqrt{100}} + \dfrac{\sqrt{64}}{\sqrt{25}}$

$= \dfrac{13}{10} + \dfrac{3}{10} + \dfrac{8}{5} = \dfrac{13}{10} + \dfrac{3}{10} + \dfrac{16}{10} = \dfrac{32}{10} = \dfrac{16}{5} = 3\dfrac{1}{5}$

e. $\left(1 + \dfrac{0.08}{4}\right)^2 - 1 = (1 + 0.02)^2 - 1 = (1.02)^2 - 1$

$= (1.02)(1.02) - 1 = 1.0404 - 1 = 0.0404$

Example 2.5-h: Evaluating Expressions by Using the Order of Operations (BEDMAS)

Evaluate: $\dfrac{4}{2^3}[(0.5 \times 5^2 + 2.5)^2 \div 3^2] + \sqrt{25}$

Solution

$$\dfrac{4}{2^3}[(0.5 \times 5^2 + 2.5)^2 \div 3^2] + \sqrt{25}$$

$$= \dfrac{4}{2^3}[(0.5 \times 25 + 2.5)^2 \div 3^2] + \sqrt{25}$$

$$= \dfrac{4}{2^3}[(12.5 + 2.5)^2 \div 3^2] + \sqrt{25}$$

$$= \dfrac{4}{2^3}[15^2 \div 3^2] + \sqrt{25}$$

$$= \dfrac{4}{2^3}[225 \div 9] + \sqrt{25}$$

$$= \dfrac{4}{2^3} \times 25 + \sqrt{25}$$

$$= \dfrac{4}{8} \times 25 + 5$$

$$= 0.5 \times 25 + 5$$

$$= 12.5 + 5$$

$$= 17.5$$

2.5 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

For problems 1 to 8, convert the decimal numbers to proper fractions and the proper fractions to decimal numbers indicated by the question marks in the tables below.

1.

Question	Decimal Number	Proper Fraction
a.	0.2	?
b.	?	$\dfrac{3}{4}$
c.	0.06	?

2.

Question	Decimal Number	Proper Fraction
a.	0.26	?
b.	?	$\dfrac{41}{50}$
c.	0.92	?

3.

Question	Decimal Number	Proper Fraction
a.	?	$\dfrac{9}{25}$
b.	0.004	?
c.	?	$\dfrac{7}{50}$

4.

Question	Decimal Number	Proper Fraction
a.	?	$\dfrac{16}{25}$
b.	0.225	?
c.	?	$\dfrac{19}{20}$

5.

Question	Decimal Number	Proper Fraction
a.	?	$\dfrac{1}{2}$
b.	0.4	?
c.	?	$\dfrac{3}{50}$

6.

Question	Decimal Number	Proper Fraction
a.	?	$\dfrac{13}{50}$
b.	0.425	?
c.	?	$\dfrac{14}{25}$

7.

Question	Decimal Number	Proper Fraction
a.	0.005	?
b.	?	$\dfrac{9}{25}$
c.	0.01	?

8.

Question	Decimal Number	Proper Fraction
a.	0.66	?
b.	?	$\dfrac{43}{50}$
c.	0.78	?

For problems 9 to 12, convert the decimal numbers to improper fractions and the improper fractions to decimal numbers.

9.

Question	Decimal Number	Improper Fraction
a.	3.5	?
b.	?	$\dfrac{8}{5}$
c.	5.6	?

10.

Question	Decimal Number	Improper Fraction
a.	7.2	?
b.	?	$\dfrac{37}{5}$
c.	8.4	?

11.

Question	Decimal Number	Improper Fraction
a.	?	$\dfrac{101}{20}$
b.	6.8	?
c.	?	$\dfrac{11}{4}$

12.

Question	Decimal Number	Improper Fraction
a.	?	$\dfrac{107}{50}$
b.	4.8	?
c.	?	$\dfrac{23}{4}$

For problems 13 to 16, convert the decimal numbers to mixed numbers and the mixed numbers to decimal numbers.

13.

Question	Decimal Number	Mixed Number
a.	2.25	?
b.	?	$1\dfrac{3}{4}$
c.	4.02	?

14.

Question	Decimal Number	Mixed Number
a.	5.04	?
b.	?	$12\dfrac{3}{5}$
c.	14.025	?

15.

Question	Decimal Number	Mixed Number
a.	?	$8\dfrac{7}{20}$
b.	16.005	?
c.	?	$15\dfrac{1}{2}$

16.

Question	Decimal Number	Mixed Number
a.	?	$3\dfrac{5}{8}$
b.	4.75	?
c.	?	$5\dfrac{9}{20}$

For problems 17 to 20, convert the repeating decimal numbers to proper fractions and the proper fractions to repeating decimal numbers.

17.

Question	Decimal Number	Proper Fraction
a.	$0.\overline{6}$	?
b.	?	$\dfrac{23}{90}$
c.	$0.\overline{25}$	?

18.

Question	Decimal Number	Proper Fraction
a.	$0.\overline{27}$	?
b.	?	$\dfrac{7}{4}$
c.	$0.\overline{83}$	?

19.

Question	Decimal Number	Proper Fraction
a.	?	$\dfrac{5}{11}$
b.	$0.\overline{2}$	?
c.	?	$\dfrac{2}{7}$

20.

Question	Decimal Number	Proper Fraction
a.	?	$\dfrac{4}{99}$
b.	$0.\overline{75}$	?
c.	?	$\dfrac{11}{15}$

For problems 21 to 52, evaluate the expressions.

21. a. $\left(\dfrac{3}{5}\right)^2 \left(\dfrac{2}{3}\right)^3$

b. $\left(\dfrac{3}{4}\right)^3 \left(\dfrac{1}{6}\right)^2$

22. a. $\left(\dfrac{5}{2}\right)^3 \left(\dfrac{1}{3}\right)^2$

b. $\left(\dfrac{3}{8}\right)^2 \left(\dfrac{4}{3}\right)^3$

23. a. $\left(\dfrac{1}{4}\right)^2 \div \left(\dfrac{2}{3}\right)^3$

b. $\left(\dfrac{5}{3}\right)^2 \div \left(\dfrac{10}{9}\right)^2$

24. a. $\left(\dfrac{1}{2}\right)^2 \div \left(\dfrac{1}{3}\right)^2$

b. $\left(\dfrac{2}{3}\right)^2 \div \left(\dfrac{4}{9}\right)^2$

25. a. $\sqrt{\dfrac{5}{9} + \dfrac{4}{9}}$

b. $\sqrt{\dfrac{15}{36} + \dfrac{5}{18}}$

26. a. $\sqrt{\dfrac{2}{25} + \dfrac{14}{25}}$

b. $\sqrt{\dfrac{1}{16} + \dfrac{1}{2}}$

27. a. $\left(2\dfrac{1}{6} + 1\dfrac{2}{3}\right) \div 5\dfrac{3}{4}$

b. $8\dfrac{1}{2} \div \left(2\dfrac{2}{5} + 2\right)$

28. a. $\left(5\dfrac{1}{4} + 2\dfrac{5}{6}\right) \div 1\dfrac{1}{2}$

b. $2\dfrac{2}{3} \div \left(1\dfrac{7}{15} + \dfrac{2}{3}\right)$

29. a. $\left(\dfrac{3}{5}\right)^2 + \left(1\dfrac{1}{5}\right)\left(\sqrt{144}\right)$

b. $\left(\dfrac{2}{5}\right)^2 + \left(\dfrac{3}{2}\right)^3$

30. a. $\left(\dfrac{4}{7}\right)^2 + \sqrt{\dfrac{3}{9} + \dfrac{1}{9}}$

    b. $\left(\dfrac{3}{8}\right)^2 + \left(\dfrac{1}{2}\right)^3$

31. a. $\sqrt{4\dfrac{21}{25}} \times \left(\dfrac{5}{3}\right)^2$

    b. $\left(\dfrac{1}{4}\right)^2 \times \left(\dfrac{1}{8}\right)^2$

32. a. $\sqrt{1\dfrac{9}{16}} \times \left(\dfrac{4}{5}\right)^2$

    b. $\left(\dfrac{1}{3}\right)^2 \div \left(\dfrac{1}{6}\right)^2$

33. a. $(1.3)^2 \times \sqrt{0.04}$

    b. $(0.1)^3 \div \sqrt{\dfrac{1}{100}}$

34. a. $(0.01)^2 \times \sqrt{0.09}$

    b. $(0.5)^3 \div \sqrt{\dfrac{1}{100}}$

35. $\left(\dfrac{5}{8}\right)^2 + \dfrac{3}{16} + \dfrac{5}{12} + 1\dfrac{2}{3}$

36. $\left(\dfrac{6}{9}\right)^2 + 1\dfrac{5}{9} + \dfrac{5}{6} + 4\dfrac{1}{2}$

37. $\sqrt{\dfrac{7}{9} - \dfrac{2}{3}} \div \left(\dfrac{1}{12} + \dfrac{1}{9}\right)$

38. $\left(\dfrac{5}{12} + \dfrac{3}{8}\right) \div \sqrt{\dfrac{4}{18} + \dfrac{1}{36}}$

39. $10\dfrac{1}{2} \div 4\dfrac{1}{5} + \dfrac{9}{10} \times 2\dfrac{2}{5} - \dfrac{3}{7}$

40. $7\dfrac{2}{3} \div 2\dfrac{1}{3} + \dfrac{3}{5} \times 4\dfrac{2}{3} - \dfrac{2}{5}$

41. $13\dfrac{3}{7} \times \dfrac{5}{94} + 6\dfrac{4}{5} \div \dfrac{4}{15} + 7\dfrac{1}{5}$

42. $2\dfrac{1}{6} \div \dfrac{26}{45} + 3\dfrac{1}{4} \times 4\dfrac{1}{2} + \dfrac{3}{8}$

43. $[0.8 - (7.2 - 6.5)] \div [3 \div (3.4 - 0.4)]$

44. $(9.9 \div 1.1) \div (8.1 \div 1.5) + (9.2 - 7.7 + 1.5)$

45. $(9.2 + 2.8)0.25 \div (5.6 - 2.3 + 1.7)$

46. $(9.1 - 7.3)0.5 \div (5.8 + 8.6 - 5.4)$

47. $\sqrt{49} + \dfrac{8}{4^2}(0.4 \times 6^2 \div 1.2)^2$

48. $\sqrt{81} + \dfrac{12.5}{5^2}(0.3 \times 5^2 \div 1.5)^2$

49. $\sqrt{2\dfrac{25}{100}} + \sqrt{0.16} + \sqrt{\dfrac{49}{16}}$

50. $\sqrt{1\dfrac{21}{100}} + \sqrt{0.25} + \sqrt{\dfrac{25}{36}}$

51. $\dfrac{9}{50}\left(\sqrt{\dfrac{49}{81}} + 1\dfrac{4}{9}\right) + \left(1\dfrac{3}{5}\right)^2$

52. $\dfrac{16}{25}\left(\sqrt{\dfrac{25}{16}} + 2\dfrac{1}{4}\right) + \left(1\dfrac{2}{5}\right)^2$

2. REVIEW EXERCISES

Answers to the odd-numbered problems are available at the end of the textbook.

For problems 1 to 4, express the decimal numbers in their word form.

1. a. 0.5 b. 0.007 c. 0.12 d. 0.029

2. a. 0.75 b. 0.3 c. 0.008 d. 0.04

3. a. 32.04 b. 200.2 c. 45,005.001 d. 1,005,071.25

4. a. 27.602 b. 470.5 c. 32,010.07 d. 3,500,007.45

For problems 5 and 6, perform the indicated arithmetic operations.

5. a. $478.82 + 85.847$ b. $65.09 - 24.987$ c. 54.37×1.46 d. $77.09 \div 8$

6. a. $716.03 + 49.936$ b. $15.71 - 3.509$ c. 15.71×3.26 d. $39.83 \div 9$

7. What is the difference between the least and the greatest numbers of the following? 0.012, 0.201, 0.02, 0.102

8. What is the sum of the least and the greatest numbers of the following? 0.041, 0.011, 0.014, 0.01

9. Which of the following is closest to 2? 2.011, 2.005, 1.996, 1.995

10. Which of the following is closest to 2? 2.011, 2.004, 1.997, 1.996

For problems 11 and 12, determine the missing values.

11. a. $\dfrac{6}{12} = \dfrac{?}{6} = \dfrac{24}{?}$ b. $\dfrac{12}{45} = \dfrac{?}{15} = \dfrac{16}{?}$

    c. $\dfrac{20}{25} = \dfrac{?}{5} = \dfrac{12}{?}$ d. $\dfrac{36}{48} = \dfrac{?}{36} = \dfrac{18}{?}$

12. a. $\dfrac{9}{15} = \dfrac{?}{10} = \dfrac{15}{?}$ b. $\dfrac{18}{27} = \dfrac{?}{18} = \dfrac{10}{?}$

    c. $\dfrac{21}{35} = \dfrac{?}{25} = \dfrac{12}{?}$ d. $\dfrac{12}{28} = \dfrac{?}{70} = \dfrac{15}{?}$

For problems 13 and 14, place the appropriate symbol (<, >, or =) between each of the fractions.

13. a. $\dfrac{24}{21} \square \dfrac{11}{5}$ b. $\dfrac{20}{44} \square \dfrac{6}{15}$ c. $\dfrac{18}{45} \square \dfrac{16}{4}$ d. $\dfrac{15}{25} \square \dfrac{63}{100}$

14. a. $\dfrac{15}{18} \square \dfrac{30}{42}$ b. $\dfrac{21}{24} \square \dfrac{35}{40}$ c. $\dfrac{40}{48} \square \dfrac{35}{42}$ d. $\dfrac{8}{38} \square \dfrac{12}{57}$

For problems 15 and 16, convert the improper fractions into mixed numbers in the simplest form.

15. a. $\dfrac{15}{10}$ b. $\dfrac{39}{26}$ c. $\dfrac{88}{12}$ d. $\dfrac{102}{9}$

16. a. $\dfrac{18}{8}$ b. $\dfrac{98}{12}$ c. $\dfrac{88}{10}$ d. $\dfrac{48}{15}$

For problems 17 and 18, reduce the fractions to their lowest terms.

17. a. $\dfrac{75}{345}$ b. $\dfrac{124}{48}$ c. $\dfrac{70}{14}$ d. $\dfrac{292}{365}$

18. a. $\dfrac{36}{144}$ b. $\dfrac{68}{10}$ c. $\dfrac{80}{12}$ d. $\dfrac{61}{366}$

For problems 19 and 20, convert the decimal numbers to proper fractions in lowest terms and the fractions to decimal numbers.

19.

Question	Decimal Number	Proper Fraction
a.	0.025	?
b.	?	$\dfrac{5}{8}$
c.	0.08	?
d.	?	$\dfrac{7}{25}$
e.	0.002	?
f.	?	$\dfrac{39}{50}$

20.

Question	Decimal Number	Proper Fraction
a.	0.06	?
b.	?	$\dfrac{23}{50}$
c.	0.075	?
d.	?	$\dfrac{27}{40}$
e.	0.004	?
f.	?	$\dfrac{17}{25}$

21. Find the total weight of three items that weigh $3\dfrac{2}{3}$ kg, $4\dfrac{1}{2}$ kg, and $5\dfrac{3}{8}$ kg.

22. Alex used $2\dfrac{1}{5}$ litres of paint for his bedroom, $1\dfrac{1}{4}$ litres for his study room, and $7\dfrac{1}{3}$ litres for his living room. How many litres of paint did he use for the three rooms?

23. Henry is $8\dfrac{1}{4}$ years old. Amanda is $2\dfrac{5}{12}$ years old. How many years younger is Amanda than Henry?

24. Harsha purchased $12\dfrac{1}{3}$ hectares of land and sold $4\dfrac{2}{5}$ hectares. How many hectares does she now own?

25. A car can travel $8\dfrac{1}{4}$ km with one litre of gas. How many kilometres can it travel using $45\dfrac{3}{5}$ litres of gas?

26. Samantha can walk $5\dfrac{3}{8}$ km in one hour. How far can she walk in $4\dfrac{1}{3}$ hours?

27. Alisha and Beyonce saved a total of $580. Two-fifths of Alisha's savings equals $144. How much did each of them save?

28. Andy and Bob had $128. One-third of Bob's amount is $25. How much did each of them have?

29. Lakshmi had $2,675.68 in her chequing account. She deposited 2 cheques in the amounts of $729.27 and $72.05 and withdrew $1,275.60. How much did she have in her account after the withdrawal?

30. George's car had 12.47 litres of gas at the start of his trip to the United States. He added the following quantities of gas during his trip: 34.25 litres, 15.2 litres, and 20.05 litres. At the end of the trip, there were 7.9 litres of gas left in the car. How much gas was used during the trip?

31. Barbie's hourly pay is $23.07. If Barbie worked 37.75 hours last week, calculate her gross pay for last week.

32. Carol's overtime rate of pay is $57.45 per hour. If Carol worked 12.5 hours of overtime last week, calculate her gross overtime pay for last week.

33. Three-fourths of the number of boys at a school is equal to half of the number of girls. The school has 480 students in total. How many more girls are there than boys?

34. There are 3,400 spectators at a soccer match. Three-fifths of the number of men equals six-sevenths of the number of women. How many spectators are women?

For problems 35 to 46, evaluate the expressions.

35. $\left(5 + \dfrac{6}{10}\right)^3 + \sqrt{6^2 + 8^2}$

36. $\left(\dfrac{5}{7}\right)^2 \div 5 + \sqrt{3^2 + 4^2}$

37. $\sqrt{1.21} - (0.5)^2 + \sqrt{\dfrac{4}{25}}$

38. $\sqrt{0.81} - (0.2)^2 + \sqrt{\dfrac{9}{36}}$

39. $\left(\dfrac{1}{5}\right)^3 + \left(\dfrac{4}{25}\right)^2$

40. $\left(\dfrac{2}{3}\right)^3 + \left(\dfrac{2}{9}\right)^2$

41. $\left(\dfrac{5}{12}\right)^3 \left(\dfrac{4}{5}\right)^2$

42. $\left(\dfrac{4}{7}\right)^2 \left(\dfrac{1}{4}\right)^3$

43. $\left(2\dfrac{4}{5} - \dfrac{7}{10}\right) \div 2\dfrac{4}{52}$

44. $\left(2\dfrac{5}{8} + 1\dfrac{5}{12}\right) \div 1\dfrac{1}{2}$

45. $\left(3\dfrac{1}{5}\right) \div \left(1\dfrac{1}{5} + 1\right)$

46. $\left(2\dfrac{2}{3}\right) \div \left(2\dfrac{2}{5} + 6\right)$

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2. SELF-TEST

Self-Test

Answers to the Self-Test can be found the end of the textbook.

1. Determine the missing values:

   a. $\dfrac{5}{5} = \dfrac{?}{20} = \dfrac{24}{?}$

   b. $\dfrac{12}{22} = \dfrac{6}{?} = \dfrac{?}{55}$

   c. $\dfrac{48}{72} = \dfrac{12}{?} = \dfrac{?}{144}$

   d. $\dfrac{11}{12} = \dfrac{?}{72} = \dfrac{22}{?}$

2. Reduce the following fractions to their lowest terms:

   a. $\dfrac{225}{30}$

   b. $\dfrac{156}{18}$

   c. $\dfrac{256}{144}$

   d. $\dfrac{135}{825}$

3. Perform the indicated operations:

   a. $7\dfrac{1}{2} + 6\dfrac{1}{4}$

   b. $5\dfrac{1}{3} - 3\dfrac{7}{15}$

   c. $\dfrac{3}{4} \times \dfrac{26}{27} \times \dfrac{9}{13}$

   d. $2\dfrac{1}{4} \div \dfrac{3}{8}$

4. Perform the indicated operations:

a. 0.165 + 10.8478 + 14.7 + 2.19

b. 34.09 – 25.957

c. 0.524 × 4.08d

d. 6.893 ÷ 3

5. Express the following numbers in word form

a. 0.004

b. 6.05

c. 300.02

d. 7.071

6. Convert the decimal numbers to fractions in lowest terms and fractions to decimal numbers:

Question	Decimal Number	Fraction
a.	0.625	?
b.	3.2	?
c.	$0.\overline{72}$	?
d.	?	$\dfrac{7}{20}$
e.	?	$1\dfrac{4}{5}$
f.	?	$\dfrac{16}{15}$

7. On a certain map, 1 cm represents 125 km. How many km are represented by 4.75 cm? How many cm on the map will represent a distance of 4,725 km?

8. Kyle had $4,000 and gave half of it to Bob. Bob spent a quarter of the money he received from Kyle. How much money does Bob have left?

For problems 9 and 10, evaluate the expressions.

9.

a. $\sqrt{\dfrac{81}{64}} - \left(\dfrac{5}{4}\right)^3 \div \left(\dfrac{25}{16}\right)^2 + \sqrt{7^2 + 24^2}$

b. $\left(\dfrac{3}{5}\right)^3 \left(\dfrac{25}{6}\right)^2 + (\sqrt{0.25})\sqrt{100}$

c. $\sqrt{5^2 + 12^2} - (\sqrt{0.49})\sqrt{\dfrac{25}{144}}$

10.

a. $\left(4\dfrac{9}{10} \div \dfrac{7}{15} \times 1\dfrac{3}{5}\right) \div 1\dfrac{9}{10}$

b. $3\dfrac{1}{2} \div 1\dfrac{2}{5} + \dfrac{9}{10} \times 2\dfrac{2}{5} - \dfrac{3}{4}$

c. $\left(3\dfrac{4}{15} + 2\dfrac{3}{5}\right) \times \left(\dfrac{5}{6} + \dfrac{1}{9}\right) - 1\dfrac{1}{2}$

11. Henry took 3 days to make 69 deliveries. On the first day, he completed one-third of the deliveries. On the second day, he made 10 more deliveries than on the first day. How many deliveries did he make on the third day?

12. Niveda spent one-third of her money on a handbag and half of the remainder on shoes. What fraction of her money did she spend on shoes? If she has $40 left, how much did she spend?

13. The cost per day to rent a car is $24.45 plus $0.37 per kilometre driven. What would be the cost to rent a car for 5 days if Sally plans to drive 325.50 km?

14. Ana walked for $\dfrac{3}{4}$ hour at $5\dfrac{1}{2}$ km per hour and jogged for $\dfrac{1}{2}$ hour at 10 km per hour. What was the total distance that she covered?

2. CASE STUDY - LIGHTNING WHOLESALE

Lightning Wholesale is a mid-to large-size company that works directly with manufacturers to bring sporting goods and equipment to mid to small-sized retailers. The company is in Southern Ontario, Canada. Lightning Wholesale acquires the goods, stores them in its warehouses, and distributes them to retailers, who then sell them to sporting enthusiasts across the province. Suppliers are global, including those from the United States.

Lightning Wholesale is a dynamic office with global networking, communications, customer service, and complex workflows. Lighting Wholesale depends on its office professionals to support staff and management. Office Professional tasks include clerical support, bookkeeping, customer service, data analysis, and document processing.

Lightning Wholesale cares about its employees and retailers. Where managers can assist their employees, they try to. Where retailers need guidance, Lightning Wholesales tries to fill this gap as best they can. In working with its small retailers, many of which are owned and operated by sole proprietors, Lightning Wholesale assists the owners with their pricing to help develop promotional pricing plans and the amounts of the payments that can be advertised. Lightning Wholesale freely offers help since it means more sales and increased profits for the company.

There are currently a few main tasks for Lightning Wholesale management:

- The payroll needs review for the past fiscal year.
- Costs and expenses need forecasting for the next fiscal year.
- Detailed analysis of office supply orders from quarter to quarter with recommendations for the next fiscal year.
- Forklifts need replacing in the warehouse.
- The next round of co-op students needs a computer.
- The employee appreciation event needs planning for next year.
- Travel expenses for the mobile team need to be summarized for the last fiscal year.

With year-end approaching, Lighting Wholesale has many processes they are wrapping up. It is important that all departments execute their responsibilities as accurately and seamlessly as possible.

Questions

1. The office supply order has arrived. $1\frac{1}{2}$ boxes are for the marketing department, $2\frac{1}{4}$ are for the sales

department, and $3\frac{2}{5}$ are for the customer service department. How many boxes in total were ordered? Express your answer as a mixed number, if applicable.

2. Jaspreet has been working with Lightning Wholesale for $2\frac{5}{6}$ years, and Samantha has been with the company for $5\frac{2}{3}$. How much longer has Samantha been with the company than Jaspreet? Express your answer as a mixed number, if applicable.

3. Lightning Wholesale just received a gift from one of it's top customers. The gift includes $5\frac{2}{3}$ boxes of apparel. If the boxes are to be divided by 7 groups, how many boxes will each group receive? The boxes do not have to be full. Express your answer as a fraction.

4. George and Harvinder have had some great reviews from customers. They will each be getting a small bonus for their hard work. George will receive $\frac{2}{3}$ of $250 and Harvinder will receive $\frac{3}{5}$ of $250. How much will George and Harvinder receive? Round your final answer to 2 decimal places.

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PART III

CHAPTER 3 - OPERATIONS WITH EXPONENTS AND INTEGERS

OBJECTIVES

- Identify the types and properties of exponents.
- Perform arithmetic operations with exponents.
- Identify types of roots.
- Perform computations with roots and fractional exponents.

CHAPTER OUTLINE

3.1 Exponents and Properties (Rules) of Exponents

3.2 Roots, Fractional Exponents, and Negative Exponents

3 Review Exercises

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Introduction

In Chapters 1 and 2, we learned that **exponents** indicate that a number or expression is to be multiplied by a certain number of times. Exponents allow us to perform arithmetic operations more simply than using the standard form of a number. A practical application of exponents is used in calculating compound interest on loans and investments. In this chapter, we will expand upon our knowledge of exponents and look further into the properties of exponents and the relationship between exponents and roots.

Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of Mathematics* (3rd ed.) by Thambyrajah Kugathasan, published by Vretta-Lyryx Inc., with permission. Adaptations include supplementing existing material and reordering chapters.

3.1 EXPONENTS AND PROPERTIES (RULES) OF EXPONENTS

In Chapters 1 and 2, we learned about powers of whole numbers, fractions, and decimal numbers.

Recall that when a number is raised to a whole number exponent, we can think of it as repeated multiplication. Powers are a simpler way to indicate repeated multiplication, similar to how multiplication is a simpler way to indicate repeated addition. Powers are expressed using exponential notation.

For example, the whole number 2 multiplied by itself 5 times is written in exponential notation as:

base $\longrightarrow$ 2^5 $\longleftarrow$ exponent

2 is the base, 5 is the exponent, and the whole representation 2^5 is known as the power.

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

| Exponential Notation | Repeated Multiplication | Standard Notation |

Similarly, the fraction $\dfrac{3}{4}$ multiplied by itself 4 times is written in exponential notation as:

base $\longrightarrow$ $\left(\dfrac{3}{4}\right)^4$ exponent

$\dfrac{3}{4}$ is the base, 4 is the exponent, and the whole representation $\left(\dfrac{3}{4}\right)^4$ is the power.

$$\left(\frac{3}{4}\right)^4 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{256}$$

| Exponential Notation | Repeated Multiplication | Standard Notation |

Similarly, the decimal number 1.2 multiplied by itself 3 times is written in exponential notation as:

base $\longrightarrow (1.2)^{3}\leftarrow$ exponent

1.2 is the base, 3 is the exponent, and the whole representation $(1.2)^3$ is the power.

$$(1.2)^3 = 1.2 \times 1.2 \times 1.2 = 1.728$$

Exponential Notation   Repeated Multiplication   Standard Notation

Properties (Rules) of Exponents

The following properties of exponents, known as the rules or laws of exponents, are used to simplify expressions that involve exponents.

Product of Powers (Product Rule)

To multiply powers with the same base, add their exponents.

For example,

$7^5 \times 7^3 = (7 \times 7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7)$

$= (7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7)$

$= 7^8$, which is the same as $= 7^{(5+3)}$

You will note that the resulting exponent, 8, can be obtained by adding the exponents 5 and 3 since the powers have the same base.

In general, for powers with base 'a' and exponents 'm' and 'n',

$$a^m \times a^n = a^{(m+n)}$$

Note: $a^m + a^n \neq a^{(m+n)}$

Example 3.1-a: Simplifying Exponential Expressions Using the Product Rule

Express the following as a single power:

a. $2^3 \times 2^4 \times 2^2$

b. $\left(\dfrac{3}{5}\right)^6 \times \left(\dfrac{3}{5}\right)^2$

c. $(0.2)^3 \times (0.2)^2$

Solution

a. $2^3 \times 2^4 \times 2^2 = 2^{(3+4+2)} = 2^9$

b. $\left(\dfrac{3}{5}\right)^6 \times \left(\dfrac{3}{5}\right)^2 = \left(\dfrac{3}{5}\right)^{(6+2)} = \left(\dfrac{3}{5}\right)^8$

c. $(0.2)^3 \times (0.2)^2 = (0.2)^{3+2} = (0.2)^5$

Quotient of Powers (Quotient Rule)

To divide two powers with the same base, subtract their exponents.

For example,

$$4^7 \div 4^2 = \frac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4}$$

$$= \frac{4 \times 4}{4 \times 4} \times 4 \times 4 \times 4 \times 4 \times 4$$

$$= 1 \times 4 \times 4 \times 4 \times 4 \times 4$$

$= 4^5$, which is the same as $= 4^{(7-2)}$

You will note that the resulting exponent, 5, can be obtained by subtracting the denominator's exponent from the numerator's exponent $(7 - 2 = 5)$ since the powers have the same base.

In general, for powers with a non-zero base 'a' and exponents 'm' and 'n',

$$\frac{a^m}{a^n} = a^{(m-n)}, \text{ where } a \neq 0$$

Note: $a^m - a^n \neq a^{(m-n)}$

Example 3.1-b: Simplifying Exponential Expressions Using the Quotient Rule

Express the following as a single power:

a. $3^9 \div 3^5$

b. $\left(\dfrac{2}{3}\right)^6 \div \left(\dfrac{2}{3}\right)^4$

c. $(1.15)^5 \div (1.15)^2$

Solution

a. $3^9 \div 3^5 = 3^{(9-5)} = 3^4$

b. $\left(\dfrac{2}{3}\right)^6 \div \left(\dfrac{2}{3}\right)^4 = \left(\dfrac{2}{3}\right)^{6-4} = \left(\dfrac{2}{3}\right)^2$

c. $(1.15)^5 \div (1.15)^2 = (1.15)^{(5-2)} = (1.15)^3$

Power of a Product (Power of a Product Rule)

Each product's factor is raised to the indicated power to determine a product's power.

For example,

$$(3 \times 5)^4 = (3 \times 5)(3 \times 5)(3 \times 5)(3 \times 5)$$
$$= 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$$
$$= 3^4 \times 5^4$$

You will note from the result that each factor of the product is raised to the power of 4.

In general, for any product of factors '*a*' and '*b*' that is raised to the power '*n*',

$$(a \times b)^n = a^n \times b^n$$

Example 3.1-c: Expanding Exponential Expressions Using the Power of a Product Rule

Express the following in expanded form using the Power of a Product Rule:

a. $(8 \times 6)^3$

b. $\left(\dfrac{3}{5} \times \dfrac{2}{7} \right)^3$

c. $(1.12 \times 0.6)^3$

Solution

a. $(8 \times 6)^3 = 8^3 \times 6^3$

b. $\left(\dfrac{3}{5} \times \dfrac{2}{7} \right)^3 = \left(\dfrac{3}{5} \right)^3 \times \left(\dfrac{2}{7} \right)^3$

c. $(1.12 \times 0.6)^3 = (1.12)^3 \times (0.6)^3$

Power of a Quotient (Power of a Quotient Rule)

The Power of a Quotient Rule is similar to the Power of a Product Rule. To determine the power of a quotient, raise the numerator to the indicated power and divide by the denominator raised to the indicated power.

For example,

$$\left(\frac{5}{8}\right)^3 = \left(\frac{5}{8}\right) \times \left(\frac{5}{8}\right) \times \left(\frac{5}{8}\right)$$

$$\frac{5 \times 5 \times 5}{8 \times 8 \times 8} = \frac{5^3}{8^3}$$

You will note from the result that the numerator and the denominator of the expression is raised to the power of 3.

In general, for any quotient with numerator *'a'* and non-zero denominator *'b'* that is raised to the power *'n'*,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \text{where } b \neq 0$$

Example 3.1-d: Expanding Exponential Expressions Using the Power of a Quotient Rule

Express the following in expanded form using the Power of a Quotient Rule:

a. $(\frac{7}{4})^3$

b. $\left[\dfrac{\left(\dfrac{2}{3}\right)}{\left(\dfrac{3}{5}\right)}\right]^4$

c. $\left(\dfrac{1.05}{0.05}\right)^3$

Solution

a. $(\frac{7}{4})^3 = \frac{7^3}{4^3}$

b. $\left[\dfrac{\left(\dfrac{2}{3} \right)}{\left(\dfrac{3}{5} \right)} \right]^4 = \left(\dfrac{2}{3} \div \dfrac{3}{5} \right)^4 = \left(\dfrac{2}{3} \right)^4 \div \left(\dfrac{3}{5} \right)^4 = \dfrac{2^4}{3^4} \div \dfrac{3^4}{5^4}$

c. $\left(\dfrac{1.05}{0.05} \right)^3 = \dfrac{(1.05)^3}{(0.05)^3}$

Power of a Power (Power of a Power Rule)

To determine the power of a power of a number, multiply the two exponents of the powers together to obtain the new exponent of the power.

For example,

$(9^3)^2 = (9^3) \times (9^3)$

$= (9 \times 9 \times 9) \times (9 \times 9 \times 9)$

$= 9 \times 9 \times 9 \times 9 \times 9 \times 9$

$= 9^6$, which is the same as $9^{(3 \times 2)}$

You will note that the resulting exponent, 6, can be obtained by multiplying the exponents 3 and 2.

In general, to raise the power of a number 'a' to a power 'm', and then raise it to a power 'n',

$(a^m)^n = a^{(m \times n)}$

Example 3.1-e: Simplifying Exponential Expressions Using the Power of a Power Rule

Express the following as a single power:

a. $(5^4)^3$

b. $\left[\left(\dfrac{3}{8} \right)^3 \right]^2$

c. $[(1.04)^4]^2$

Solution

a. $(5^4)^3 = 5^{(4 \times 3)} = 5^{12}$

b. $\left[\left(\dfrac{3}{8}\right)^3\right]^2 = \left(\dfrac{3}{8}\right)^{(3 \times 2)} = \left(\dfrac{3}{8}\right)^6$

c. $[(1.04)^4]^2 = (1.04)^{(4 \times 2)} = (1.04)^8$

Example 3.1-f: Solving Expressions Using the Power of a Power Rule, Product Rule, and Quotient Rule

Solve the following:

a. $(2^2)^3 \times 2^7 \div 2^9$
b. $7^5 \div (7^3)^2 \times 7^2$

Solution

a. $(2^2)^3 \times 2^7 \div 2^9 = 2^6 \times 2^7 \div 2^9 = 2^{(6+7-9)} = 2^4 = 16$

b. $7^5 \div (7^3)^2 \times 7^2 = 7^5 \div 7^6 \times 7^2 = 7^{(5-6+2)} = 7^1 = 7$

Table 3.1-a summarizes the properties (rules) of exponents.

Table 3.1-a: Properties (Rules) of Exponents

Property (Rule)	Rule in Exponential Form	Example
Product Rule	$a^m \times a^n = a^{(m+n)}$	$3^5 \times 3^4 = 3^{(5+4)}$
Quotient Rule	$\dfrac{a^m}{a^n} = a^{(m-n)}$	$\dfrac{3^7}{3^4} = 3^{(7-4)}$
Power of a Product Rule	$(a \times b)^n = a^n \times b^n$	$(3 \times 5)^2 = 3^2 \times 5^2$
Power of a Quotient Rule	$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$\left(\dfrac{3}{5}\right)^3 = \dfrac{3^3}{5^3}$
Power of a Power Rule	$(a^m)^n = a^{(m \times n)}$	$(3^2)^3 = 3^{(2 \times 3)}$

Properties of Exponents and Bases of One and Zero

Table 3.1-b summarizes the properties of powers with exponents and bases of one and zero.

Exponents and Bases of One (1) and Zero (0)

Property (Rule)	Description	Rule in Exponential Form	Example
Base '*a*' Exponent 1	Any base '*a*' raised to the exponent '1' equals the base itself.	$a^1 = a$	$8^1 = 8$
Base '*a*' Exponent 0	Any non-zero base '*a*' raised to the exponent '0' equals 1.	$a^0 = 1$	$8^0 = 1$
Base '1' Exponent '*n*'	A base of '1' raised to any exponent '*n*' equals 1.	$1^n = 1$	$1^5 = 1$
Base '0' Exponent '*n*'	A base of '0' raised to any positive exponent '*n*' equals 0.	$0^n = 0$	$0^5 = 0$
Base '0' Exponent '0'	A base of '0' raised to the exponent '0' is indeterminate.	$0^0 = indeterminate$	

Addition and Subtraction of Powers

There is no special rule for powers with the same or different bases for adding and subtracting powers. Evaluate each operation separately and then perform the addition or subtraction.

For example,

- Addition of exponential expressions with the same base: $2^3 + 2^4$

  ◦ Evaluating 2^3 and 2^4 separately and then adding, $8 + 16 = 24$

- Addition of exponential expressions with different bases: $2^2 + 3^3$

  ◦ Evaluating 2^2 and 3^3 separately and then adding, $4 + 27 = 31$

- Subtraction of exponential expressions with the same base: $5^3 - 5^2$

  ◦ Evaluating 5^3 and 5^2 separately and then subtracting, $125 - 25 = 100$

- Subtraction of exponential expressions with different bases: $4^3 - 2^3$

  ◦ Evaluating 4^3 and 2^3 separately and then subtracting, $64 - 8 = 56$

Example 3.1-g: Adding and Subtracting Powers

Evaluate the following:

a. $3^3 + 3^2$
b. $5^2 + 3^2$
c. $5^4 - 5^2$
d. $6^2 - 4^2$

Solution

a. $3^3 + 3^2 = 27 + 9 = 36$

   ◦ *Note:* $a^m + a^n \neq a^{(m+n)}\, 3^3 + 3^2 \neq 3^5$

b. $5^2 + 3^2 = 25 + 9 = 34$

   ◦ *Note:* $a^m + b^m \neq (a+b)^m\, 5^2 + 3^2 \neq 8^2$

c. $5^4 - 5^2 = 625 - 25 = 600$

   ◦ *Note:* $a^m - a^n \neq a^{(m-n)}\, 5^4 - 5^2 \neq 5^2$

d. $6^2 - 4^2 = 36 - 16 = 20$

   ◦ *Note:* $a^m - b^m \neq (a-b)^m\, 6^2 - 4^2 \neq 2^2$

Multiplication and Division of Powers with Different Bases

There is also **no** special rule for the product or quotient of powers having different bases. Evaluate each operation separately and then perform the multiplication or division.

   For example,

- Product of exponential expressions with different bases: $2^4 \times 3^2$

   ◦ Evaluating 2^4 and 3^2 separately and then multiplying,$= 16 \times 9 = 144$

- Quotient of exponential expressions with different bases: $\dfrac{3^3}{2^4}$

   ◦ Evaluating 3^3 and 2^4 separately and then dividing,$= \dfrac{27}{16} = 1.6875$

Example 3.1-h: Multiplying and Dividing Powers that Have Different Bases

Evaluate the following:

    a. $5^3 \times 4^2$
    b. $5^3 \div 3^2$
    c. $9^2 \times 5^1$
    d. $4^2 \times 5^0$
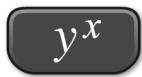
Solution

a. $5^3 \times 4^2 = 125 \times 16 = 2,000$

b. $5^3 \div 3^2 = 125 \div 9 = \dfrac{125}{9} = 13\dfrac{8}{9}$
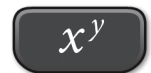
c. $9^2 \times 5^1 = 81 \times 5 = 405$

d. $4^2 \times 5^0 = 16 \times 1 = 16$

Multiplication and Division of Powers with Different Bases

The exponent key on different calculators can be identified by symbols such as $\boxed{\wedge}$, $\boxed{x^y}$,

$\boxed{y^x}$, etc.

In the following examples ' $\boxed{\wedge}$ ' will be used to represent the exponent key.
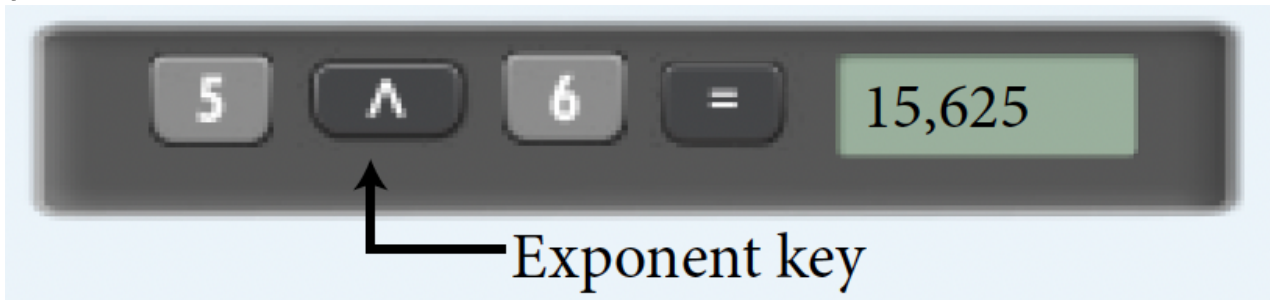
Example 3.1-i: Evaluating Exponential Expressions using a Calculator
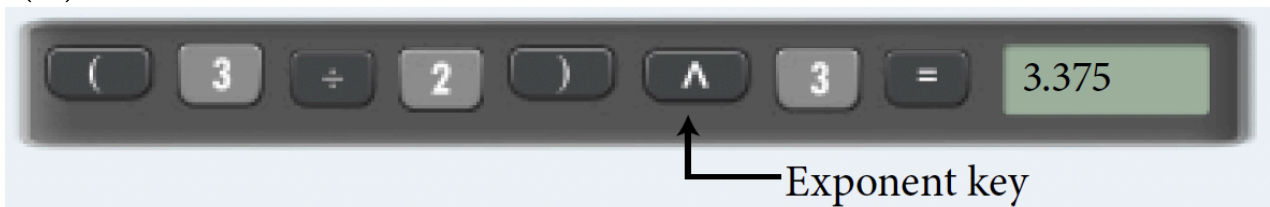
Evaluate the following using a calculator:

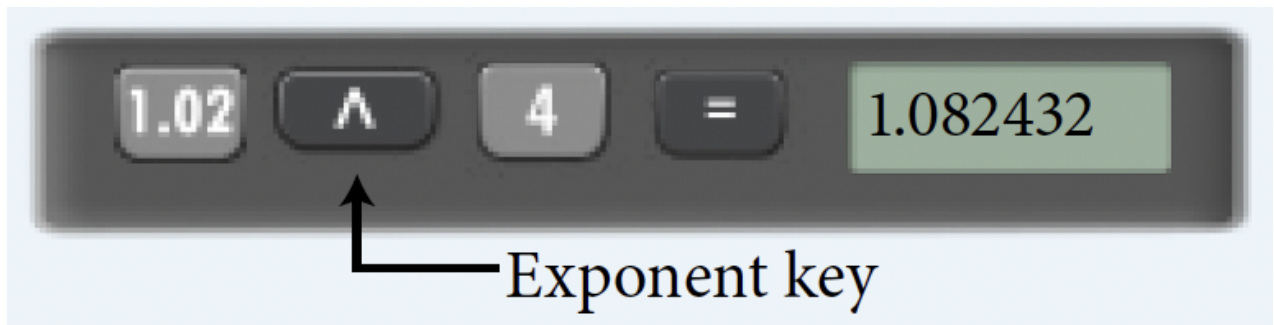a. 5^6

b. $\left(\dfrac{3}{2}\right)^3$

c. $(1.02)^4$

Solution

a. 5^6

Exponent key

b. $\left(\dfrac{3}{2}\right)^3$

Exponent key

c. $(1.02)^4$

Exponent key

3.1 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

For problems 1 and 2, identify the values in the empty cells for Repeated Multiplication, Base, Exponent, and Power (Exponential Notation).

1.

Question	Repeated Multiplication	Base	Exponent	Power (Exponential Notation)
a.	$7 \times 7 \times 7 \times 7$			
b.				9^5
c.		3	4	
d.	$\dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5}$			
e.				$\left(\dfrac{5}{7}\right)^5$
f.		$\left(\dfrac{4}{7}\right)$	3	
g.	$(1.15) \times (1.15) \times (1.15) \times (1.15)$			
h.				$(1.6)^3$
i.		(1.25)	5	

2.

Question	Repeated Multiplication	Base	Exponent	Power (Exponential Notation)
a.	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$			
b.				6^7
c.		5	3	
d.	$\dfrac{2}{7} \times \dfrac{2}{7} \times \dfrac{2}{7} \times \dfrac{2}{7} \times \dfrac{2}{7}$			
e.				$\left(\dfrac{3}{8}\right)^4$
f.		$\left(\dfrac{2}{9}\right)$	4	
g.	$(2.5) \times (2.5) \times (2.5) \times (2.5) \times (2.5)$			
h.				$(1.1)^5$
i.		(0.75)	4	

Express problems 3 to 22 as a single power and then evaluate using a calculator. Round the answer to two decimal places, wherever applicable.

3. $4^3 \times 4^6$

4. $5^5 \times 5^6$

5. $\left(\dfrac{1}{2}\right)^4 \left(\dfrac{1}{2}\right)^3$

6. $\left(\dfrac{2}{3}\right)^2 \left(\dfrac{2}{3}\right)^3$

7. $\left(\dfrac{5}{2}\right)^2 \left(\dfrac{5}{2}\right)^3$

8. $\left(\dfrac{5}{3}\right)^3 \left(\dfrac{5}{3}\right)^2$

9. $(3.25)^4 (3.25)^2$

10. $(0.75)^3 (0.75)^4$

11. $6^8 \div 6^3$

12. $3^7 \div 3^5$

13. $\left(\dfrac{2}{5}\right)^3 \div \left(\dfrac{2}{5}\right)^1$

14. $\left(\dfrac{3}{2}\right)^4 \div \left(\dfrac{3}{2}\right)^3$

15. $(1.4)^5 \div (1.4)^2$

16. $(3.25)^6 \div (3.25)^5$

17. $\left[(6)^2\right]^3$

18. $\left[(5)^3\right]^2$

19. $\left[\left(\dfrac{2}{3}\right)^4\right]^3$

20. $\left[\left(\dfrac{3}{4}\right)^4\right]^2$

21. $\left[(2.5)^2\right]^3$

22. $\left[(1.03)^3\right]^2$

Express problems 23 to 34 as a power of the indicated base value.

23. 4^5 as a power of 2

24. 9^6 as a power of 3

25. $9(27)^2$ as a power of 3

26. $8(16)^2$ as a power of 2

27. $\dfrac{3^9 \times 3^2}{3^5}$ as a power of 3

28. $\dfrac{2^9 \times 2^1}{2^5}$ as a power of 2

29. $\dfrac{(2^5)^4}{4^6}$ as a power of 2

30. $\dfrac{(2^5)^4}{16^3}$ as a power of 2

31. $\dfrac{10^6}{10^0}$ as a power of 10

32. $\dfrac{3^7}{27}$ as a power of 3

33. $\dfrac{8^5}{8^3}$ as a power of 2

34. $\dfrac{5^6}{125}$ as a power of 5

Evaluate problems 35 to 70.

35. $5^2 + 5^3$

36. $4^3 + 4^2$

37. $5^4 - 2^4$

38. $6^3 - 4^3$

39. $4^4 - 4^2$

40. $6^3 - 6^1$

41. $7^2 + 3^2$

42. $6^2 - 4^2$

43. $3^5 + 5^3$

44. $4^3 + 3^4$

45. $2^5 - 5^2$

46. $2^6 - 6^2$

47. $5^4 - 4^2$

48. $10^3 - 7^2$

49. $4^0 + 4^4$

50. $3^0 + 3^4$

51. $(5 \times 4)^3$

52. $(10 \times 2)^4$

53. $(1.25 \times 4)^4$

54. $(5 \times 0.8)^3$

55. $\left(\dfrac{2}{3} + 6\right)^5$

56. $\left(\dfrac{3}{5} + 1\right)^3$

57. $3^4 + 3^2 + 3^0$

58. $4^2 + 4^4 + 4^0$

59. $2^4 + 3^4 - 1^4$

60. $3^3 + 2^3 - 1^3$

61. $\left(\dfrac{1}{2}\right)^3 + \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^0$

62. $\left(\dfrac{1}{5}\right)^2 + \left(\dfrac{1}{5}\right)^0 + \left(\dfrac{1}{5}\right)^4$

63. $(2.1)^2 + (2.1)^0$

64. $(3.2)^2 + (3.2)^1$

65. $4^3 \times 3^4$

66. $7^2 \times 2^2$

67. $6^4 \div 5^4$

68. $8^2 \div 7^2$

69. $\left(2 \times 3^2\right)^4$

70. $\left(5 \times 2^2\right)^3$

3.2 ROOTS, FRACTIONAL EXPONENTS, AND NEGATIVE EXPONENTS

Roots

Roots are the inverse operation of exponents.

In the previous chapters, we learned that taking the square root of a number is the inverse of raising a number to the power of 2.

For example,

- The square of 2 is $2^2 = 4$. Therefore, the square root of 4 is $\sqrt{4} = 2$.
- The square of 3 is $3^2 = 9$. Therefore, the square root of 9 is $\sqrt{9} = 3$.

We can also use roots to express the inverse of raising a number to a power greater than 2.

We know that,

$$9 = 3 \times 3 = 3^2$$

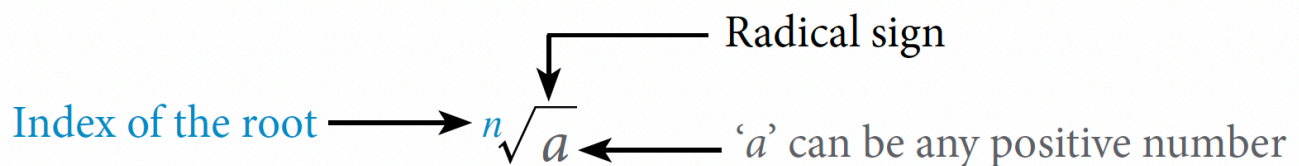Therefore, the square root (2nd root) of 9 is 3.

Similarly,

$$125 = 5 \times 5 \times 5 = 5^3$$

Therefore, the cube root (3rd root) of 125 is 5.

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$

Therefore, the 4th root of 16 is 2.

The following notation is used to represent roots, also known as **radicals**

Radical sign

Index of the root $\longrightarrow$ $\sqrt[n]{a}$ $\longleftarrow$ 'a' can be any positive number

The index is written as a small number to the left of the radical sign. It indicates which root is to be taken.

For example,

- $\sqrt[3]{125}$ indicates the cube root (3rd root) of 125.
- $\sqrt[4]{16}$ indicates the 4th root of 16.

Note: Recall that for square roots, the index 2 does not need to be written as it is understood to be there; i.e., $\sqrt[2]{9} = \sqrt{9}$.

Perfect Roots

Roots of a whole number may not be a whole number. A whole number is a perfect root if its root is also a whole number.

For example,

- 4 is a perfect square root of 16 because $4^2 = 16$; i.e., $\sqrt{16} = 4$
- 3 is a perfect cube root of 27 because $3^3 = 27$; i.e., $\sqrt[3]{27} = 3$

Table 3.2-a: Examples of Perfect Roots

Roots	Square Roots	Cube Roots	Fourth Roots
1	$\sqrt{1}$	$\sqrt[3]{1}$	$\sqrt[4]{1}$
2	$\sqrt{4}$	$\sqrt[3]{8}$	$\sqrt[4]{16}$
3	$\sqrt{9}$	$\sqrt[3]{27}$	$\sqrt[4]{81}$
4	$\sqrt{16}$	$\sqrt[3]{64}$	$\sqrt[4]{256}$
5	$\sqrt{25}$	$\sqrt[3]{125}$	$\sqrt[4]{625}$
6	$\sqrt{36}$	$\sqrt[3]{216}$	$\sqrt[4]{1,296}$
7	$\sqrt{49}$	$\sqrt[3]{343}$	$\sqrt[4]{2,401}$
8	$\sqrt{64}$	$\sqrt[3]{512}$	$\sqrt[4]{4,096}$
9	$\sqrt{81}$	$\sqrt[3]{729}$	$\sqrt[4]{6,561}$
10	$\sqrt{100}$	$\sqrt[3]{1,000}$	$\sqrt[4]{10,000}$

Simplifying Roots Using Perfect Roots

Just like we can simplify a power of a product, we can simplify a root of a product as follows:

$$\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$$

For example, $\sqrt{10} = \sqrt{2 \times 5} = \sqrt{2} \times \sqrt{5}$.

We can use this rule and our knowledge of perfect roots to simplify square and cube roots.

For example,

- To simplify $\sqrt{12}$, we can write 12 as the product 4×3, where one of the factors, 4, is a perfect square. Therefore, $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$
- To simplify $\sqrt[3]{54}$, we can write 54 as the product 27×2, where one of the factors, 27, is a perfect cube. Therefore, $\sqrt[3]{54} = \sqrt[3]{27 \times 2} = \sqrt[3]{27} \times \sqrt[3]{2} = 3\sqrt[3]{2}$

Example 3.2-a: Simplifying Using Perfect Roots

Simplify using perfect roots of a number:

a. $\sqrt{72}$

b. $\sqrt[3]{40}$

Solution

a. $\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

b. $\sqrt[3]{40} = \sqrt[3]{8 \times 5} = \sqrt[3]{8} \times \sqrt[3]{5} = 2\sqrt[3]{5}$

Fractional Exponents

A radical can also be represented using **fractional exponents**. Fractional exponents are often easier to write and perform operations with than radicals.

$$\text{Index of the Root} \longrightarrow \sqrt[n]{a} \;=\; a^{\frac{1}{n}} \longleftarrow \text{Fractional Exponent}$$

For example,

- The **square (2nd) root** of a number can be written as the number raised to the power of $\dfrac{1}{2}$ .e.g.,

$$\sqrt{5} = 5^{\frac{1}{2}}$$

- The **cube (3rd) root** of a number can be written as the number raised to the power of $\dfrac{1}{3}$ .e.g.,

$$\sqrt[3]{8} = 8^{\frac{1}{3}}$$

- The **fourth (4th) root** of a number can be written as the number raised to the power of $\dfrac{1}{4}$ .e.g.,

$$\sqrt[4]{5^3} = (5^3)^{\frac{1}{4}} = 5^{3\times\frac{1}{4}} = 5^{\frac{3}{4}}$$

An appropriate radical will "undo" an exponent.

For example,

- $\sqrt{5^2} = (5^2)^{\frac{1}{2}} = 5^{2\times\frac{1}{2}} = 5$
- $\sqrt[3]{7^3} = (7^3)^{\frac{1}{3}} = 7^{3\times\frac{1}{3}} = 7$

Example 3.2-b: Expressions in Radical Form

Express the following in radical form:

a. $2^{\frac{5}{6}}$

b. $3^{\frac{2}{5}}$

c. $\left(\dfrac{2}{3}\right)^{\frac{3}{4}}$

Solution

a. $2^{\frac{5}{6}} = \sqrt[6]{2^5}$

b. $3^{\frac{2}{5}} = \sqrt[5]{3^2}$

c. $\left(\dfrac{2}{3}\right)^{\frac{3}{4}} = \sqrt[4]{\left(\dfrac{2}{3}\right)^3}$

Evaluating Expressions with Fractional Exponents Using a Calculator

When entering a fractional exponent in a calculator, **brackets** must be used.

For example, to evaluate $25^{\frac{2}{5}}$ using a calculator, enter it as follows:

| 25 | ∧ | (| 2 | ÷ | 5 |) | = |

3.623898...

Exponent key

Note: Without the brackets, the operation will mean $(25)^2 \div 5$, which is incorrect.

Example 3.2-c: Evaluating Expressions with Fractional Exponents Using a Calculator

Evaluate the following using a calculator:

a. $15^{\frac{3}{2}}$

b. $\left(\dfrac{3}{5}\right)^{\frac{1}{4}}$

c. $(2.5)^{\frac{3}{7}}$

Solution

a. $15^{\frac{3}{2}} = 58.09$

```
[15] [∧] [(] [3] [÷] [2] [)] [=]
```

```
58.094750...
```

b. $\left(\dfrac{3}{5}\right)^{\frac{1}{4}} = 0.88$

```
[(] [3] [÷] [5] [)] [∧] [(] [1] [÷] [4] [)] [=]
```

```
0.880111...
```

c. $(2.5)^{\frac{3}{7}} = 1.48$

```
[(] [2.5] [)] [∧] [(] [3] [÷] [7] [)] [=]
```

```
1.480968...
```

Arithmetic Operations with Fractional Exponents

All the rules of exponents, Product Rule, Quotient Rule, Power of a Product Rule, Power of a Quotient Rule, Power of a Power Rule, etc., learned in Section 3.1 and as outlined in Tables 3.1-a and 3.1-b are applicable to fractional exponents.

Example 3.2-d: Evaluating Expressions with Fractional Exponents using the Product Rule

Simplify the following using the Product Rule to express as a single power, and then evaluate to two decimal places, where applicable.

a. $2^{\frac{1}{2}} \times 2^{\frac{1}{3}}$

b. $3^{\frac{3}{4}} \times 3^{\frac{9}{4}} \times 3^0$

c. $\left(\dfrac{3}{5}\right)^{\frac{7}{3}} \times \left(\dfrac{3}{5}\right)^{\frac{2}{3}}$

Solution

a. $2^{\frac{1}{2}} \times 2^{\frac{1}{3}} = 2^{\left(\frac{1}{2}+\frac{1}{3}\right)} = 2^{\left(\frac{3+2}{6}\right)} = 2^{\frac{5}{6}} = 1.781797... = 1.78$

b. $3^{\frac{3}{4}} \times 3^{\frac{9}{4}} \times 3^0 = 3^{\left(\frac{3}{4}+\frac{9}{4}+0\right)} = 3^{\frac{12}{4}} = 3^3 = 27$

c. $\left(\dfrac{3}{5}\right)^{\frac{7}{3}} \times \left(\dfrac{3}{5}\right)^{\frac{2}{3}} = \left(\dfrac{3}{5}\right)^{\left(\frac{7}{3}+\frac{2}{3}\right)} = \left(\dfrac{3}{5}\right)^{\frac{9}{3}} = \left(\dfrac{3}{5}\right)^3 = 0.216 = 0.22$

Example 3.2-e: Evaluating Expressions with Fractional Exponents using the Quotient Rule

Simplify the following using the Quotient Rule to express as a single power, and then evaluate to two decimal places, where applicable.

a. $2^{\frac{4}{3}} \div 2^{\frac{2}{3}}$

b. $(1.2)^{\frac{5}{2}} \div (1.2)^{\frac{1}{2}}$

c. $\left(\dfrac{1}{3}\right)^{\frac{6}{4}} \div \left(\dfrac{1}{3}\right)^{\frac{3}{4}}$

Solution

a. $2^{\frac{4}{3}} \div 2^{\frac{2}{3}} = 2^{\left(\frac{4}{3} - \frac{2}{3}\right)} = 2^{\frac{2}{3}} = 1.587401... = 1.59$

b. $(1.2)^{\frac{5}{2}} \div (1.2)^{\frac{1}{2}} = (1.2)^{\left(\frac{5}{2} - \frac{1}{2}\right)} = (1.2)^{\frac{4}{2}} = (1.2)^2 = 1.44$

c. $\left(\dfrac{1}{3}\right)^{\frac{6}{4}} \div \left(\dfrac{1}{3}\right)^{\frac{3}{4}} = \left(\dfrac{1}{3}\right)^{\left(\frac{6}{4} - \frac{3}{4}\right)} = \left(\dfrac{1}{3}\right)^{\frac{3}{4}} = 0.438691... = 0.44$

Example 3.2-f: Evaluating Expressions with Fractional Exponents using the Power of a Product Rule

Simplify the following using the Power of a Product Rule, and then evaluate to two decimal places, where applicable.

a. $(4^2 \times 3^2)^{\frac{1}{2}}$

b. $\left(7^2 \times \dfrac{1}{3^2}\right)^{\frac{1}{2}}$

c. $(2^6 \times 3^2)^{\frac{3}{2}}$

Solution

a. $(4^2 \times 3^2)^{\frac{1}{2}} = (4^2)^{\frac{1}{2}} \times (3^2)^{\frac{1}{2}} = 4 \times 3 = 12$

b. $\left(7^2 \times \dfrac{1}{3^2}\right)^{\frac{1}{2}} = (7^2)^{\frac{1}{2}} \times \left(\dfrac{1}{3^2}\right)^{\frac{1}{2}} = \dfrac{7}{3} = 2.333333... = 2.33$

c. $(2^6 \times 3^2)^{\frac{3}{2}} = (2^6)^{\frac{3}{2}} \times (3^2)^{\frac{3}{2}} = 2^{\left(6 \times \frac{3}{2}\right)} \times 3^{\left(2 \times \frac{3}{2}\right)} = 2^9 \times 3^3 = 512 \times 27 = 13,824$

Example 3.2-g: Evaluating Expressions with Fractional Exponents using the Power of a Quotient Rule

Simplify the following using the Power of a Quotient Rule, and then evaluate to two decimal places, where applicable.

a. $\left(\dfrac{4^2}{3^2}\right)^{\frac{1}{2}}$

b. $\left(\dfrac{5^3}{2^6}\right)^{\frac{1}{3}}$

Solution

a. $\left(\dfrac{4^2}{3^2}\right)^{\frac{1}{2}} = \dfrac{(4^2)^{\frac{1}{2}}}{(3^2)^{\frac{1}{2}}} = \dfrac{4}{3} = 1.333333... = 1.33$

b. $\left(\dfrac{5^3}{2^6}\right)^{\frac{1}{3}} = \dfrac{(5^3)^{\frac{1}{3}}}{(2^6)^{\frac{1}{3}}} = \dfrac{5}{2^{\frac{6}{3}}} = \dfrac{5}{2^2} = \dfrac{5}{4} = 1.25$

Example 3.2-h: Evaluating Expressions with Fractional Exponents using the Power of a Power Rule

Simplify the following using the Power of a Power Rule to express as a single power, and then evaluate to two decimal places, where applicable.

a. $\left(6^{\frac{1}{2}}\right)^3$

b. $\left(18^{\frac{1}{3}}\right)^{\frac{1}{4}}$

c. $\left[\left(\dfrac{2}{3}\right)^3\right]^2$

Solution

a. $\left(6^{\frac{1}{2}}\right)^3 = 6^{\left(\frac{1}{2}\times 3\right)} = 6^{\frac{3}{2}} = 14.696938... = 14.70$

b. $\left(18^{\frac{1}{3}}\right)^{\frac{1}{4}} = 18^{\left(\frac{1}{3}\times\frac{1}{4}\right)} = 18^{\frac{1}{12}} = 1.272348... = 1.27$

c. $\left[\left(\dfrac{2}{3}\right)^3\right]^2 = \left(\dfrac{2}{3}\right)^{3\times 2} = \left(\dfrac{2}{3}\right)^6 = \dfrac{2^6}{3^6} = \dfrac{64}{729} = 0.087791... = 0.09$

Example 3.2-i: Evaluating Expressions with Fractional Exponents and Different Bases

Evaluate the following to two decimal places, where applicable.

a. $16^{\frac{1}{2}} + 8^{\frac{1}{2}}$

b. $25^{\frac{1}{2}} - 27^{\frac{1}{3}}$

c. $\left(\dfrac{7}{8}\right)^{\frac{1}{4}} - \left(\dfrac{2}{3}\right)^{\frac{1}{3}}$

d. $5^{\frac{1}{2}} \times 3^{\frac{1}{2}}$

e. $2^{\frac{3}{4}} \div 3^{\frac{1}{2}}$

f. $5^{2\frac{3}{4}}$

Solution

a. $16^{\frac{1}{2}} + 8^{\frac{1}{2}} = 4 + 2.828427... = 6.828427... = 6.83$

b. $25^{\frac{1}{2}} - 27^{\frac{1}{3}} = 5 - 3 = 2$

c. $\left(\dfrac{7}{8}\right)^{\frac{1}{4}} - \left(\dfrac{2}{3}\right)^{\frac{1}{3}} = 0.967168... - 0.873580... = 0.093587... = 0.09$

d. $5^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 2.236067... \times 1.732050... = 3.872983... = 3.87$

e. $2^{\frac{3}{4}} \div 3^{\frac{1}{2}} = 1.681792... \div 1.732050... = 0.970983... = 0.97$

f. $5^{2\frac{3}{4}} = 5^{\frac{11}{4}} = 83.592538... = 83.59$

Negative Exponents

In exponential notation, the base of the number may be raised to a negative exponent. We can represent this as a^{-n}. A power with a negative exponent is the **reciprocal** of the power with the positive exponent.
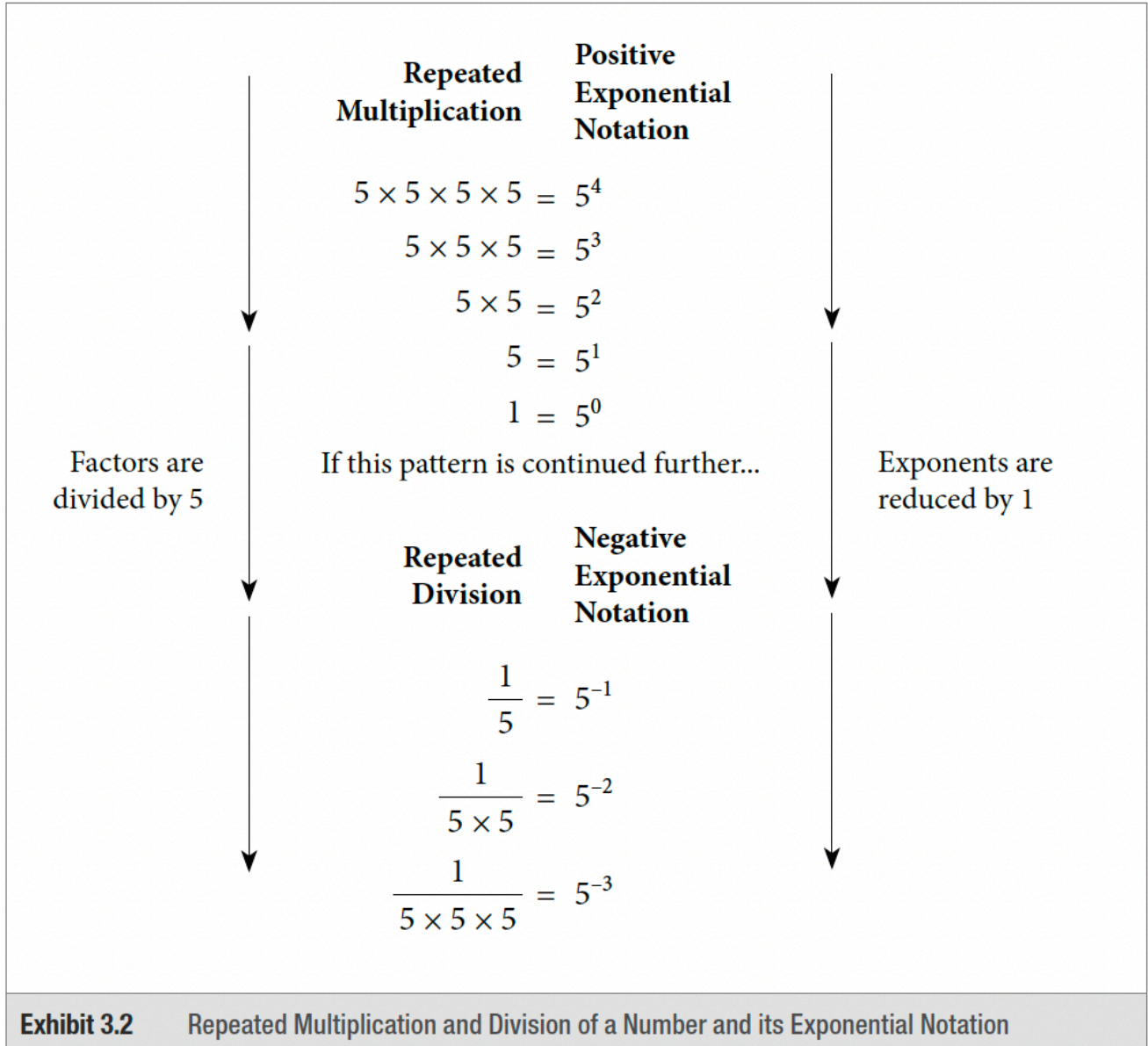
Positive Exponent:

$a^n = a \times a \times a \times \ldots \times a$

(multiplication of $'n'$ factors of $'a'$)

Negative Exponent:

$$a^{-n} = \frac{1}{a^n} = \frac{1}{a \times a \times a \times \ldots \times a}$$

(division of $'n'$ factors of $'a'$)

Repeated Multiplication | Positive Exponential Notation

$$5 \times 5 \times 5 \times 5 = 5^4$$

$$5 \times 5 \times 5 = 5^3$$

$$5 \times 5 = 5^2$$

$$5 = 5^1$$

$$1 = 5^0$$

If this pattern is continued further...

Factors are divided by 5

Exponents are reduced by 1

Repeated Division | Negative Exponential Notation

$$\frac{1}{5} = 5^{-1}$$

$$\frac{1}{5 \times 5} = 5^{-2}$$

$$\frac{1}{5 \times 5 \times 5} = 5^{-3}$$

Exhibit 3.2   Repeated Multiplication and Division of a Number and its Exponential Notation

---

$$a^{-n} = \frac{1}{a^n}, \text{ and } \frac{1}{a^{-n}} = a^n$$

Therefore, a^n and a^{-n} are reciprocals.

Any positive base with a **negative** exponent will always result in a **positive** answer.

Table 3.2-b: Examples of Negative and Positive Exponential Notation

Negative Exponential Notation	Positive Exponential Notation	Repeated Division	Standard Notation
5^{-1}	$\dfrac{1}{5^1}$	$\dfrac{1}{5}$	$\dfrac{1}{5}$
5^{-2}	$\dfrac{1}{5^2}$	$\dfrac{1}{5 \times 5}$	$\dfrac{1}{25}$
5^{-3}	$\dfrac{1}{5^3}$	$\dfrac{1}{5 \times 5 \times 5}$	$\dfrac{1}{125}$

The properties (rules) of exponents in Section 3.1 of this chapter (summarized in Table 3.1-a) also apply to all negative exponents. We can use these properties to simplify powers with negative exponents and then convert negative exponents to positive exponents.

Example 3.2-j: Multiplying and Dividing Powers with Negative Exponents

Simplify the following and express the answer in exponential form with positive exponents:

a. $2^{-2} \times 2^{-3}$

b. $\dfrac{3^{-4}}{3^{-2}}$

c. $(3 \times 5)^{-2}$

d. $(3^{-2})^3$

Solution

a. $2^{-2} \times 2^{-3} = 2^{-2+(-3)} = 2^{-2-3} = 2^{-5} = \dfrac{1}{2^5}$

b. $\dfrac{3^{-4}}{3^{-2}} = 3^{-4-(-2)} = 3^{-4+2} = 3^{-2} = \dfrac{1}{3^2}$

c. $(3 \times 5)^{-2} = 3^{-2} \times 5^{-2} = \dfrac{1}{3^2} \times \dfrac{1}{5^2}$

d. $(3^{-2})^3 = 3^{-2(3)} = 3^{-6} = \dfrac{1}{3^6}$

Fractions with Negative Exponents

When a fraction has a negative exponent, change the fraction to its reciprocal and drop the negative sign from the exponent. After this change, the exponent indicates the number of times the numerator and denominator should be multiplied.

$$\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^{n}$$

For example,

$$\left(\dfrac{2}{5}\right)^{-3} = \left(\dfrac{5}{2}\right)^{3} = \left(\dfrac{2}{5}\right)\left(\dfrac{2}{5}\right)\left(\dfrac{2}{5}\right) = \dfrac{5 \times 5 \times 5}{2 \times 2 \times 2} = \dfrac{125}{8}$$

Note: The reciprocal of $\dfrac{2}{5}$ *is* $\dfrac{5}{2}$.

Example 3.2-k: Evaluating Fractions with Negative Exponents

Evaluate the following to two decimal places, where applicable.

a. $\left(\dfrac{5}{4}\right)^{-2} \times \left(\dfrac{2}{3}\right)^{-3}$

b. $\left(\dfrac{3}{5}\right)^{-3} \div \left(\dfrac{2}{5}\right)^{-2}$

Solution

a. $\left(\dfrac{5}{4}\right)^{-2} \times \left(\dfrac{2}{3}\right)^{-3} = \left(\dfrac{4}{5}\right)^{2} \times \left(\dfrac{3}{2}\right)^{3} = \dfrac{4^2}{5^2} \times \dfrac{3^3}{2^3}$

$= \dfrac{16}{25} \times \dfrac{27}{8} = \dfrac{2}{25} \times \dfrac{27}{1} = \dfrac{54}{25} = 2\dfrac{4}{25} = 2.16$

b. $\left(\dfrac{3}{5}\right)^{-3} \div \left(\dfrac{2}{5}\right)^{-2} = \left(\dfrac{5}{3}\right)^{3} \div \left(\dfrac{5}{2}\right)^{2} = \dfrac{5^3}{3^3} \div \dfrac{5^2}{2^2}$

$= \dfrac{125}{27} \div \dfrac{25}{4} = \dfrac{125}{27} \times \dfrac{4}{25} = \dfrac{5}{27} \times \dfrac{4}{1} = \dfrac{20}{27} = 0.740740... = 0.74$

3.2 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

Express Problems 1 to 4 in their radical form and evaluate.

1. a. $64^{\frac{1}{2}}$ b. $\left(\dfrac{25}{16}\right)^{\frac{1}{2}}$

2. a. $81^{\frac{1}{2}}$ b. $\left(\dfrac{8}{25}\right)^{\frac{1}{2}}$

3. a. $8^{\frac{1}{3}}$ b. $\left(\dfrac{27}{64}\right)^{\frac{1}{3}}$

4. a. $64^{\frac{1}{3}}$ b. $\left(\dfrac{125}{8}\right)^{\frac{1}{3}}$

Express problems 5 to 10 in their fractional exponent form and then evaluate, round to two decimal places, wherever applicable.

5. a. $\sqrt{144}$ b. $\sqrt[3]{64}$

6. a. $\sqrt{81}$ b. $\sqrt[3]{125}$

7. a. $\sqrt{2^6}$ b. $\sqrt{40}$

8. a. $\sqrt{3^4}$ b. $\sqrt{50}$

9. a. $\sqrt{8} \times \sqrt{12}$ b. $\sqrt{7} \times \sqrt{14}$

10. a. $\sqrt{12} \times \sqrt{10}$ b. $\sqrt{9} \times \sqrt{27}$

Express problems 11 and 12 in their fractional exponent form and then evaluate. Express your answer as a simplified fraction.

11. a. $\sqrt{\dfrac{25}{49}}$ b. $\sqrt{\dfrac{64}{9}}$

12. a. $\sqrt{\dfrac{36}{64}}$ b. $\sqrt{\dfrac{169}{16}}$

Express problems 13 and 14 in their fractional exponent form with a single base, and then evaluate, round to two decimal places, wherever applicable.

13. a. $\sqrt[4]{2^2 \times 2}$ b. $\sqrt[4]{5^2 \times 25^2}$

14. a. $\sqrt{3^4 \times 3^2}$ b. $\sqrt[6]{9^3 \times 27^4}$

Simplify problems 15 to 24 by expressing the powers using a single exponent and then evaluate, round to two decimal places, wherever applicable.

15. a. $5^{\frac{1}{2}} \times 5^{\frac{3}{4}}$ b. $3^{\frac{7}{8}} \times 3^{\frac{5}{9}}$

16. a. $3^{\frac{1}{2}} \times 3^{\frac{1}{4}}$ b. $11^{\frac{3}{4}} \times 11^{\frac{2}{3}}$

17. a. $8^{\frac{4}{5}} \times 8^{\frac{2}{5}} \times 8^{\frac{1}{5}}$ b. $5^{\frac{1}{3}} \times 5^{\frac{1}{2}} \times 5^0$

18. a. $5^{\frac{4}{7}} \times 5^{\frac{4}{7}} \times 5^{\frac{6}{7}}$ b. $9^{\frac{5}{8}} \times 9^{\frac{2}{3}} \times 9^0$

19. a. $8^{\frac{1}{3}} \times 8^{\frac{2}{3}} \times 8^1$ b. $\dfrac{3^{\frac{8}{3}}}{3^2}$

20. a. $2^{\frac{2}{3}} \times 2^{\frac{1}{2}} \times 2^1$ b. $\dfrac{6^{\frac{7}{2}}}{6^2}$

21. a. $\dfrac{4^{\frac{5}{7}}}{4^{\frac{2}{7}}}$ b. $\left(3^2\right)^{\frac{1}{3}}$

22. a. $\dfrac{2^{\frac{4}{5}}}{2^{\frac{3}{5}}}$ b. $(10^3)^0$

23. a. $\left(12^{\frac{1}{2}}\right)^4$ b. $\left(7^{\frac{1}{4}}\right)^8$

24. a. $\left(5^{\frac{2}{3}}\right)^6$ b. $\left(4^{\frac{2}{3}}\right)^6$

Evaluate problems 25 to 32, round to two decimal places, wherever applicable.

25. a. $5^{\frac{1}{2}} + 7^{\frac{1}{2}}$ b. $16^{\frac{1}{2}} - 9^{\frac{1}{2}}$

26. a. $125^{\frac{1}{3}} + 64^{\frac{1}{3}}$ b. $50^{\frac{1}{2}} - 40^{\frac{1}{2}}$

27. a. $5 \times 3^{\frac{1}{2}} + 2^{\frac{1}{2}}$ b. $(2^5)^{\frac{1}{2}} - (5^2)^{\frac{1}{2}}$

28. a. $12 \times 10^{\frac{1}{2}} + 5^{\frac{1}{2}}$ b. $(3^4)^{\frac{1}{3}} - (4^3)^{\frac{1}{4}}$

29. a. $8^{\frac{1}{2}} \times 9^{\frac{1}{2}}$ b. $45^{\frac{1}{2}} \times 60^{\frac{1}{2}}$

30. a. $6^{\frac{1}{2}} \times 3^{\frac{1}{2}}$ b. $24^{\frac{1}{2}} \times 75^{\frac{1}{2}}$

31. a. $\dfrac{5 + 4^{\frac{1}{2}}}{36^{\frac{1}{2}}}$ b. $\dfrac{10^{\frac{1}{2}} - 5^{\frac{1}{2}}}{25^{\frac{1}{2}}}$

32. a. $\dfrac{6^{\frac{1}{2}} + 6^{\frac{1}{2}}}{9^{\frac{1}{2}}}$ b. $\dfrac{7 - 7^{\frac{1}{2}}}{4^{\frac{1}{2}}}$

Simplify problems 33 to 42 by expressing the powers using a single exponent and as a radical (where applicable), and then evaluate. Round to two decimal places, wherever applicable.

33. a. $6^{-\frac{5}{4}} \times 6^{\frac{3}{4}}$ b. $7^{\frac{4}{3}} \times 7^{-\frac{2}{3}}$

34. a. $5^{\frac{4}{9}} \times 5^{-\frac{2}{9}}$ b. $3^{-\frac{6}{7}} \times 3^{\frac{2}{7}}$

35. a. $\dfrac{10^{-\frac{3}{5}} \times 10^{\frac{4}{5}}}{10^{\frac{2}{5}}}$ b. $\dfrac{2^{\frac{5}{7}} \times 2^{-\frac{6}{7}}}{2^{-\frac{8}{7}}}$

36. a. $\dfrac{5^{\frac{2}{7}} \times 5^{\frac{4}{7}}}{5^{-\frac{6}{7}}}$ b. $\dfrac{3^{\frac{2}{3}} - 3^{-\frac{4}{3}}}{3^{\frac{5}{3}}}$

37. a. $\dfrac{6^{-\frac{5}{9}} \times 6^0}{6^{\frac{7}{9}}}$ b. $\dfrac{7^{\frac{7}{8}} \times 7^{\frac{8}{3}}}{7^2}$

38. a. $\dfrac{9^{\frac{2}{5}} \times 9^0}{9^{\frac{3}{5}}}$ b. $\dfrac{5^{\frac{5}{6}} \times 5^{\frac{2}{3}}}{5^2}$

39. a. $\left(5^{-2}\right)^{\frac{4}{3}}$ b. $\left(6^{-\frac{1}{2}}\right)^{-6}$

40. a. $\left(4^{-2}\right)^{\frac{5}{2}}$ b. $\left(2^{-\frac{4}{5}}\right)^{-5}$

41. a. $\left(8^{-\frac{2}{3}}\right)^{-6}$ b. $\left(7^{-\frac{1}{3}}\right)^{9}$

42. a. $\left(6^{-\frac{2}{3}}\right)^{-3}$ b. $\left(3^{-\frac{4}{9}}\right)^{0}$

Evaluate problems 43 to 48, round to two decimal places, wherever applicable.

43. a. $\dfrac{3^{-1}}{2^{-1}}$ b. $3^{-1} + 2^{-1}$

44. a. $\dfrac{2^{-2}}{3^{-1}}$ b. $2^{-2} + 3^{-1}$

45. a. $3^{-1} \times 3^2 \times 3^{-2}$ b. $\left[2^{-3}\right]^{-1}$

46. a. $5^{-2} \times 5^2 \times 5^3$ b. $\left[5^{-2}\right]^{-2}$

47. a. $\dfrac{2^{\frac{2}{5}} + 3^0 \times 2^{-1}}{\left(\frac{1}{2}\right)^{-1}}$ b. $2^{-2} + \dfrac{1}{2^{-1}}$

48. a. $\dfrac{3^{-1} + 2 \times 3^{-1}}{\left(\frac{1}{3}\right)^{-1}}$ b. $3^{-2} + \dfrac{1}{3^{-1}}$

3. REVIEW EXERCISES

Answers to the odd-numbered questions are available at the end of the book.

1. Calculate the difference between 2^5 and 5^2.

2. Calculate the difference between 3^4 and 4^3.

3. Express 243 as a power of 3 and then evaluate $243\frac{3}{5}$.

4. Express 512 as a power of 2 and then evaluate $512\frac{4}{9}$.

Express Problems 5 to 8 as a single power and then evaluate.

5. a. $\left(2^6\right)^{\frac{1}{3}}$

    b. $\left(5^{15}\right)^{\frac{1}{5}}$

6. a. $\left(\dfrac{3^9}{3^3}\right)^{\frac{1}{3}}$

    b. $\left(\dfrac{2^{12}}{2^4}\right)^{\frac{1}{4}}$

7. a. $\left(3^2\right)^{\frac{1}{2}} \times \left(3^3\right)^{\frac{2}{3}}$

    b. $\left(6^2\right)^{\frac{1}{3}} \times \left(6^3\right)^{\frac{1}{9}}$

8. a. $\left(2^2\right)^{\frac{1}{4}} \times \left(2^5\right)^{\frac{3}{10}}$

    b. $\left(5^3\right)^{\frac{2}{3}} \times \left(5^2\right)^{\frac{1}{2}}$

For Problems 9 to 16, simplify using laws of exponents and then evaluate.

9. a. $\dfrac{2^3 \times 3^4 \times 2^2}{3 \times 2^5}$

    b. $\dfrac{\left(5^2\right) \times 5^4}{5^7}$

10. a. $\dfrac{5^2 \times 7^3 \times 54}{7 \times 5^6}$

    b. $\dfrac{(2^5) \times 2^2}{2^{17}}$

11. a. $(-5)^2 \times (4)^2$

    b. $-10^4 \times 10^3$

12. a. $(-2)^2 \times (3)^2$

    b. $-2^4 \times 2^2$

13. a. $(125)^{-\frac{1}{3}}$

    b. $(49)^{-\frac{1}{2}}$

    c. $\sqrt{\dfrac{64}{81}}$

14. a. $(16)^{-\frac{1}{4}}$

    b. $(27)^{-\frac{1}{3}}$

    c. $\sqrt{\dfrac{25}{49}}$

15. a. $\sqrt{7^4}$

    b. $\sqrt{\dfrac{25}{36}}$

    c. $\sqrt[3]{\dfrac{216}{125}}$

16. a. $\sqrt{5^6}$

    b. $\sqrt{\dfrac{49}{64}}$

    c. $\sqrt[3]{\dfrac{64}{27}}$

Evaluate Problems 17 to 22 and express the answers rounded to two decimal places, wherever applicable.

17. a. $6,000\left(1 + \dfrac{0.06}{12}\right)^{36}$

    b. $2,000(1 + 0.004)^{-24}$

18. a. $4,000\left(1 + \dfrac{0.075}{12}\right)^{60}$

    b. $5,000(1 + 0.003)^{-48}$

19. $\dfrac{3,000\left[(1.06)^{25} - 1\right]}{0.06}$

20. $\dfrac{1,400\left[(1.03)^{30} - 1\right]}{0.03}$

21. $\dfrac{950\left[1 - (1.03)^{15}\right]}{0.03}$

22. $\dfrac{1,200\left[1 - (1.04)^{20}\right]}{0.04}$

3. SELF-TEST

Answers to the questions are available at the end of the book.

1. Express the following as a power of the indicated bases:

    0. 729 as a power of 3.

    a. 128 as a power of 2.

    b. $(9)^{\frac{3}{2}}$ as a power of 3.

    c. $(16)^{\frac{3}{4}}$ as a power of 2.

2. Express the following as a single power:

    0. $3^4 \times 3^{(4+2)}$

    a. $10^4 \times 10^{(3+2)}$

3. Simplify using the laws of exponents and then evaluate, rounding to two decimal places, wherever applicable.

    0. $\dfrac{2^3 \times (3)^3 \times 3^4}{2^3 \times (2^3)^2 \times 3^5}$

    a. $\dfrac{2^4 \times (3^2)^4 \times 2^2}{2^5 \times (3^3)^2 \times 2^0}$

Evaluate problems 4 to 12, round to two decimal places, wherever applicable.

4. a. $(2^2 \times 3^3 \times 5^0)^{-1}$

    b. $(3^2 \times 2^{-2} \times 5)^0$

5. a. $\left(\dfrac{3}{2}\right)^2 \times \dfrac{3}{8}$

b. $4 \times \dfrac{8}{5} \div \dfrac{4}{3} + \sqrt{4} - 1$

6. a. $\dfrac{2}{5} \times \sqrt{100} + 2^4 \div \dfrac{5}{3}$

   b. $\sqrt{36} \times \dfrac{4}{3} \div \dfrac{8}{6} - 7 + 2$

7. a. $\left(\dfrac{2}{3} + \dfrac{4}{3} \right)^5$

   b. $\left(\dfrac{9^{\frac{2}{5}}}{9^{\frac{3}{5}}} \right) \times 3^2$

8. a. $-5^3 (-25)^3$

   b. $3^{-2} \times 3^3$

9. a. $(-3)^3 - (-1)^3$

   b. $(-2)^5 \times (4 - 5)^3 - (-10)$

10. a. $(-5)^3 - (-4)^3$

   b. $(-4)^5 \div (-2)^6$

PART IV
CHAPTER 4 - AVERAGES

The following chapter has been adapted from Business Math: A Step-by-Step Handbook, 2021A version, Jean-Paul Olivier.

OBJECTIVES

- Calculate the simple average of a set of values.
- Calculate the weighted average of a set of values.

CHAPTER OUTLINE

4.1 Simple and Weighted Averages

4. Review Exercises

Introduction

Why Do We Need to Know about Averages?

No matter where you go or what you do, averages are everywhere. Let's look at some examples:

- Three-quarters of your student loan is spent. Unfortunately, only half of the first semester has passed, so you resolve to squeeze the most value out of the money that remains. But have you noticed that many grocery products are difficult to compare in terms of value because they are packaged in different-sized containers with different price points? For example, one tube of toothpaste sells in a 125 mL size for $1.99 while a comparable brand sells for $1.89 for 110 mL. Which is the better deal? A fair comparison requires you to calculate the average price per millilitre.
- Your local transit system charges $2.25 for an adult fare, $1.75 for students and seniors, and $1.25 for children. Is this enough information for you to calculate the average fare, or do you need to know how many riders of each kind there are?
- Five years ago you invested $8,000 in Roller Coasters Inc. The stock value has changed by 9%, −7%, 13%, 4%, and −2% over these years, and you wonder what the average annual change is and whether your investment kept up with inflation.
- If you participate in any sport, you have an average of some sort: bowlers have bowling averages; hockey or soccer goalies have a Goals Against Average (GAA); and baseball pitchers have an earned run average (ERA).
- Averages generally fall into three categories.
- This chapter explores **simple, weighted, and geometric averages.**

Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of Mathematics* (3rd ed.) by Thambyrajah Kugathasan, published by Vretta-Lyryx Inc., with permission. Adaptations include supplementing existing material and reordering chapters.

4.1 SIMPLE AND WEIGHTED AVERAGES

Simple Averages

An **average** is a single number that represents the middle of a data set. It is commonly interpreted to mean the "typical value." Calculating averages facilitates easier comprehension of and comparison between different data sets, particularly if there is a large amount of data.

For example, what if you want to compare year-over-year sales? One approach would involve taking company sales for each of the 52 weeks in the current year and comparing these with the sales of all 52 weeks from last year. This involves 104 weekly sales figures with 52 points of comparison. From this analysis, could you concisely and confidently determine whether sales are up or down? Probably not. An alternative approach involves comparing last year's average weekly sales against this year's average weekly sales. This involves the direct comparison of only two numbers, and the determination of whether sales are up or down is very clear.

In a **simple average**, all individual data share the same level of importance (weight) in determining the typical value. Each individual data point also has the same frequency, meaning that no one piece of data occurs more frequently than another. Also, the data do not represent a percent change. To calculate a simple average, you require two components:

1. The data itself—you need the value for each piece of data.
2. The quantity of data—you need to know how many pieces of data are involved (the count), or the total quantity used in the calculation.

The Formula

To calculate the simple average you add together all of the pieces of data then take that total and divide it by the quantity.

$$\text{Simple Average} = \frac{\text{Sum of the Values}}{\text{Total Number of Items}}$$

Specifically, you can calculate the simple average of $x_1, x_2, \ldots, x_n$ as follows:

$$\text{simple average} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

How It Works

The steps required to calculate a simple average are as follows:

Step 1: Sum every piece of data.

Step 2: Determine the total quantity involved.

Step 3: Calculate the simple average using the Simple Average Formula.

Example

Assume you want to calculate an average on three pieces of data: 95, 108, and 97.

Note that the data are equally important and each appears only once, thus having the same frequency. You require a **simple average.**

There are three pieces of data: $x_1 = 95$, $x_2 = 108$ and $x_3 = 97$, and $n = 3$.

- Step 1: Find the sum = 95 + 108 + 97 = 300.
- Step 2: There are three pieces of data, or n = 3.
- Step 3: Apply Simple Average Formula $\text{Simple Average} = \dfrac{300}{3} = 100$
- Final Answer: The simple average of the data set is 100.

Important Notes

Although mentioned earlier, it is critical to stress that a simple average is calculated only when all of the following conditions are met:

1. All of the data shares the same level of importance toward the calculation.
2. All of the data appear the same number of times.

3. The data does not represent percent changes or a series of numbers intended to be multiplied with each other.

If any of these three conditions are not met, then either a weighted or geometric average is used depending on which of the above criteria failed. We discuss this later when each average is introduced.

Give It Some Thought

It is critical to recognize if you have potentially made any errors in calculating a simple average. Review the following situations and, without making any calculations, determine the best answer.

1. The simple average of 15, 30, 40, and 45 is:
   a. lower than 20
   b. between 20 and 40, inclusive.
   c. higher than 40.
2. If the simple average of three pieces of data is 20, which of the following data do not belong in the data set? Data set: 10, 20, 30, 40
   a. 10
   b. 20
   c. 30
   d. 40

Solution

1. The answer is **b** – a simple average should fall in the middle of the data set, which appears spread out between 15 and 45, so the middle would be around 30
2. The answer is **d** – if the number 40 is included in any average calculation involving the other numbers, it is impossible to get a low average of 20

Example 4.1 A: Comparing Average Sales

First quarter sales for Buzz Electronics are as indicated in the table below.

First Quarter Sales for Buzz Electronics 2013 and 2014

Month	2013 Sales	2014 Sales
January	$413,200	$455,875
February	$328,987	$334,582
March	$350,003	$312,777

Martha needs to prepare a report for the board of directors comparing year-over-year quarterly performance. To do this, she needs you to do the following:

- Calculate the average sales in the quarter for each year.

SOLUTION:

We will need to calculate a simple average for the first quarter in each of 2013 and 2014. Then convert the numbers into a percentage.

WHAT WE ALREADY KNOW:

You know the three quarters annually:

2013: $x_{1}=\$413,200,\: x_{2}=\$328,986,\: x_{3}=\$350,003$

2014: $x_1 = \$455,876, \ x_2 = \$334,582, \ x_3 = \$312,777$

STEPS:

For each year, perform steps 1 to 3:

- **Step 1:** Sum the data.
- **Step 2:** Count the total quantity of data involved.
- **Step 3:** Calculate the simple average using Formula 3.3.

PERFORM:

Step 1

- total of 2013 = $413,200 + $328,986 + $350,003 = $1,092,189

- total of 2014 = $455,876 + $334,582 + $312,777 = $1,103,235

Step 2

- for both years, n=3

Step 3

- For 2013, $\text{Simple Average} = \dfrac{\$1,092,189}{3} = \$364,063$
- For 2014, $\text{Simple Average} = \dfrac{\$1,103,235}{3} = \$367,745$

FINAL ANSWER:

The average monthly sales in 2013 were $364,063 compared to sales in 2014 of $367,745.

Weighted Averages

Have you considered how your grade point average (GPA) is calculated? Your business program requires the successful completion of many courses. Your grades in each course combine to determine your GPA; however, not every course necessarily has the same level of importance as measured by your course credits.

Perhaps your math course takes one hour daily while your communications course is only delivered in one-hour sessions three times per week. Consequently, the college assigns the math course five credit hours and the communications course three credit hours. If you want an average, these different credit hours mean that the two courses do not share the same level of importance, and therefore a simple average cannot be calculated.

In a **weighted average**, not all pieces of data share the same level of importance or they do not occur with the same frequency. The data cannot represent a percent change or a series of numbers intended to be multiplied with each other. To calculate a weighted average, you require two components:

1. The data itself—you need the value for each piece of data.
2. The weight of the data—you need to know how important each piece of data is to the average. This is either an assigned value or a reflection of the number of times each piece of data occurs (the frequency).

The Formula

To calculate the weighted average you add the products of the weights and data for the entire data set and then divide this total by the total of the weights.

Using these pieces of information, you can calculate the weighted average of $x_1, x_2, \ldots, x_n$ as follows:

$$\text{weighted average} = \frac{w_1 \cdot x_1 + w_2 \cdot x_2 + \ldots + w_n \cdot x_n}{w_1 + w_2 + \ldots + w_n}$$

where $w_1, w_2, \ldots, w_n$ is the weight of the specific data points $x_1, x_2, \ldots, x_n$

How it Works:

The steps required to calculate a weighted average are:

- **Step 1**: Sum every piece of data multiplied by its associated weight.
- **Step 2**: Sum the total weight.
- **Step 3**: Calculate the weighted average using the formula above.

Note that the simple average is just a special case of a weighted average where the weights are all equal and of value 1. Then $w_i x_i = x_i$ for all i from 1 to n, and $w_1 + w_2 + \ldots + w_n = n$.

Let's stay with the illustration of the math and communications courses and your GPA. Assume that these are the only two courses you are taking. You finish the math course with an A, translating into a grade point of 4.0. In the communications course, your C+ translates into a 2.5 grade point. These courses have five and three credit hours, respectively. **Since they are not equally important, you use a weighted average.**

Step 1: In the numerator, sum the products of each course's credit hours (the weight) and your grade point (the data). This means (math credit hours × math grade point) + (communications credit hours × communications grade point). Numerically, this is = (5 × 4) + (3 × 2.5) = 27.5.

Step 2: In the denominator, sum the weights. These are the credit hours. You have 5 + 3 = 8.

Step 3: Apply the weighted average formula to calculate your GPA.

$$\text{Weighted Average} = \frac{27.5}{8} = 3.44 \text{ (GPAs have two decimals).}$$

Note that your GPA is higher than if you had just calculated a simple average of 4 + 2.5 = 3.25. This happens because your math course, in which you scored a higher grade, was more important in the calculation.

Things To Watch Out For

The most common error in weighted averages is to confuse the data with the weight. If you have the two backward, your numerator is still correct; however, your denominator is incorrect. To distinguish the data from the weight, notice that the data forms a part of the question. In the above example, you were looking to calculate your *grade point* average; therefore, grade point is the data. The other information, the credit hours, must be the weight.

Paths To Success

The formula used for calculating a simple average is a simplification of the weighted average formula. In a simple average, every piece of data is equally important. Therefore, you assign a value of 1 to the weight for each piece of data. Since any number multiplied by 1 is the same number, the simple average formula omits the weighting in the numerator as it would have produced unnecessary calculations. In the denominator, the sum of the weights of 1 is no different from counting the total number of pieces of data. In essence, you can use a weighted average formula to solve simple averages.

Give It Some Thought

In each of the following, determine which information is the data and which is the weight.

1. Rafiki operates a lemonade stand during his garage sale today. He has sold 13 small drinks for $0.50, 29 medium drinks for $0.90, and 21 large drinks for $1.25. What is the average price of the lemonade sold?
2. Natalie received the results of a market research study. In the study, respondents identified how many times per week they purchased a bottle of Coca-Cola. Calculate the average number of purchases made per week.

Results of a Market Research Study

Purchases per Week	# of People
1	302
2	167
3	488
4	256

Solution:

1. The price of the drinks is the data, and the number of drinks is the weight.

2. The purchases per week is the data, and the number of people is the weight.

Example 4.2 Calculating Your Weighted Grade Point Average

Here is a mark transcript received by a student at a local college. The second table below shows how the grade translates into a grade point.

Student Grades and Number of Credit Hours by Course

Course	Grade	Credit Hours
Economics 100	B	4
Math 100	A	5
Marketing 100	B+	3
Communications 100	C	4
Computing 100	A+	3
Accounting 100	D	4

Grades and Associated Grade Points

Grade	Grade Point
A+	4.5
A	4.0
B+	3.5
B	3.0
C+	2.5
C	2.0
D	1.0
F	0.0

Calculate the student's grade point average (GPA). Round your final answer to two decimals.

SOLUTION:

PLAN:

The courses do not carry equal weights as they have different credit hours. Therefore, to calculate the GPA you must find a weighted average.

UNDERSTAND:

What You Already Know

Since the question asked for the grade point average, the grade points for each course are the data, or x. The corresponding credit hours are the weights, or w.

How You Will Get There

You need to convert the grade for each course into a grade point using the secondary table. Then perform the following steps:

Step 1: Sum every piece of data multiplied by its associated weight.

Step 2: Sum the total weight.

Step 3: Calculate the weighted average using the formula

PERFORM:

Convert the grades.

Grades, Grade Points, and Credit Hours by Course

Course	Grade	Grade Point	Credit Hours
Economics 100	B	3.0	4
Math 100	A	4.0	5
Marketing 100	B+	3.5	3
Communications 100	C	2.0	4
Computing 100	A+	4.5	3
Accounting 100	D	1.0	4

Step 1:

Find the sum of weights x grade =

$(4 \times 3.0) + (5 \times 4.0) + (3 \times 3.5) + (4 \times 2.0) + (3 \times 4.5) + (4 \times 1.0) = 68$

Step 2:

Find the sum of the weights = $4 + 5 + 3 + 4 + 3 + 4 = 23$

Step 3:

$$Weighted Average = \frac{68}{23} = 2.96$$

4. REVIEW EXERCISES

Answers to the odd-numbered problems are available at the end of the book.

1. Calculate a simple average for the following series of numbers: 8, 17, 6, 33, 15, 12, 13, 16
2. Calculate a simple average for the following series of dollar amounts: $1,500, $2,000, $1,750, $1,435, $2,210
3. Calculate a weighted average for the following series of numbers: 4,   4,   4,   4,   12,   12,   12,   12,   12,   12,   12,   15,   15
4. Calculate a weighted average for the data and associated weights outlined in the table below.

Data and Associated Weights

Data	Weight
$3,600	2
$3,300	5
$3,800	3
$2,800	6
$5,800	4

Applications

5. If a 298 mL can of soup costs $2.39, what is the average price per millilitre?
6. Kerry participated in a fundraiser for the Children's Wish Foundation yesterday. She sold 115 pins for $3 each, 214 ribbons for $4 each, 85 coffee mugs for $7 each, and 347 baseball hats for $9 each. Calculate the average amount Kerry raised per item.
7. In determining the hourly wages of its employees, a company uses a weighted system that factors in local, regional, and national competitor wages. Local wages are considered most important and have been assigned a weight of 5. Regional and national wages are not as important and have been assigned weights of 3 and 2, respectively. If the hourly wages for local, regional, and national competitors are $16.35, $15.85, and $14.75, what hourly wage does the company pay?
8. Canadian Tire is having an end-of-season sale on barbecues, and only four floor models remain, priced at $299.97, $345.49, $188.88, and $424.97. What is the average price for the barbecues?
9. Calculate the grade point average (GPA) for the following student. Round your answer to two decimals.

Grades and Credit Hours by Course

Course	Grade	Credit Hours
Economics 100	D	5
Math 100	B	3
Marketing 100	C	4
Communications 100	A	2
Computing 100	A+	3
Accounting 100	B+	4

Grades and Associated Grade Points

Grade	Grade Point
A+	4.5
A	4.0
B+	3.5
B	3.0
C+	2.5
C	2.0
D	1.0
F	0.0

10. An accountant needs to report the annual average age (the length of time) of accounts receivable (AR) for her corporation. This requires averaging the monthly AR averages, which are listed below. Calculate the annual AR average.

Accounts Receivable (AR) Averages by Month

Month	Monthly AR Average
January	$45,000
February	$70,000
March	$85,000
April	$97,000
May	$145,000
June	$180,000
July	$260,000
August	$230,000
September	$185,000
October	$93,000
November	$60,000
December	$50,000

Challenge, Critical Thinking, & Other Applications

11. Gabrielle is famous for her trail mix recipe. By weight, the recipe calls for 50% pretzels, 30% Cheerios, and 20% peanuts. She wants to make a 2 kg container of her mix. If pretzels cost $9.99/kg, Cheerios cost $6.99/kg, and peanuts cost $4.95/kg, what is the average cost per 100 g rounded to four decimals?

12. Caruso is the marketing manager for a local John Deere franchise. He needs to compare his average farm equipment sales against his local Case IH competitor's sales. In the past three months, his franchise has sold six $375,000 combines, eighteen $210,000 tractors, and fifteen $120,000 air seeders. His sales force estimates that the Case IH dealer has sold four $320,000 combines, twenty-four $225,000 tractors, and eleven $98,000 air seeders. Express the Case IH dealer's average sales as a percentage of the John Deere dealer's average sales.

13. You are shopping for a Nintendo WiiU gaming console visit a shopping website, which finds online sellers and lists their prices for comparison. Based on the following list, what is the average price for a gaming console (rounded to two decimals)?

Price of WiiU by Online Seller

Online Seller	Price
NothingButSoftware	$274.99
eComElectronics	$241.79
NextDayPC	$241.00
Ecost	$249.99
Amazon	$169.99
eBay	$165.00
Buy	$199.99
HSN	$299.95
Gizmos for Life	$252.90
Toys 'R' Us	$169.99
Best Buy	$169.99
The Bay	$172.69
Walmart	$169.00

4. SELF-TEST

Answers to all problems are available at the end of the textbook.

1. John had test scores of 84, 90, 95, 98 and 88. Find the average of his test scores.

2. Cheryl's tips for the week were as follows: $86, $103, $118, $117, $126 and $149. Find the average of her tips.

3. The monthly rain fall for the last 5 months was: 31mm, 15mm, 28mm, 8mm, 41mm. Calculate the average rainfall over the 5 months.

4. The average regular season home attendance of the Toronto Blue Jays from 2015 to 2022 is listed in the chart below. Calculate the average attendance over this time. Round to the nearest person. (Note: 2020 there was no baseball, so it was not included).

Year and Attendance Data for Toronto Blue Jays Regular Home Season

Year	Attendance
2015	34,504
2016	41,880
2017	39,554
2018	29,066
2019	21,606
2021	10,119
2022	32,763

5. Find the weighted average. Round to the nearest hundredth.

Data for Question 5: Number of Customers and Frequency

Customers each Hour	Frequency
9	2
11	15
17	5
25	2

6. An independent research company conducted tests to determine the fuel consumption (L/100km) of a fleet of cars. Find the weighted average of L/100km for the fleet of cars. Round to the nearest tenth.

Mileage and Number of Cars per Fleet

L/100km	Number of Cars
10.4	5
9.2	6
8.7	10
8.5	14
8.1	5
7.8	6
7.2	4

7. Kim took Math1820 and received the following grades. Calculate her final grade.

Kim's Grades in Math1820

Assessment	Kim's Marks	Weight of the Final Grade
Lab #1	70%	5%
Lab #2	75%	5%
Lab #3	60%	5%
Lab #4	80%	5%
Test #1	72%	20%
Test #2	78%	20%
Test #3	55%	20%
Test #4	88%	20%

4. CASE STUDY - LIGHTNING WHOLESALE

Lightning Wholesale is a mid-to large-size company that works directly with manufacturers to bring sporting goods and equipment to mid to small-sized retailers. The company is in Southern Ontario, Canada. Lightning Wholesale acquires the goods, stores them in its warehouses, and distributes them to retailers, who then sell them to sporting enthusiasts across the province. Suppliers are global, including those from the United States.

Lightning Wholesale is a dynamic office with global networking, communications, customer service, and complex workflows. Lighting Wholesale depends on its office professionals to support staff and management. Office Professional tasks include clerical support, bookkeeping, customer service, data analysis, and document processing.

Lightning Wholesale cares about its employees and retailers. Where managers can assist their employees, they try to. Where retailers need guidance, Lightning Wholesales tries to fill this gap as best they can. In working with its small retailers, many of which are owned and operated by sole proprietors, Lightning Wholesale assists the owners with their pricing to help develop promotional pricing plans and the amounts of the payments that can be advertised. Lightning Wholesale freely offers help since it means more sales and increased profits for the company.

There are currently a few main tasks for Lightning Wholesale management:

- The payroll needs review for the past fiscal year.
- Costs and expenses need forecasting for the next fiscal year.
- Detailed analysis of office supply orders from quarter to quarter with recommendations for the next fiscal year.
- Forklifts need replacing in the warehouse.
- The next round of co-op students needs a computer.
- The employee appreciation event needs planning for next year.
- Travel expenses for the mobile team need to be summarized for the last fiscal year.

Lightning Wholesale needs to predict dollar sales for each month in 2022. A variety of techniques are used in business to forecast sales. One technique examines the year-to-year change in sales. Thus, the management at Lightning Wholesale has decided to perform a historical three-year analysis of both average monthly sales and average yearly sales to grasp how these figures have changed. Once the change is understood, the averages will be used to project the sales for each month in 2022.

Lighting Wholesale Yearly Sales by Month

Yearly Sales	January	February	March	April	May	June	July	August	September	October	November	Dec
2019 Sales	$4,345	$1,562	$2,345	$3,589	$2,885	$8,896	$5,236	$4,236	$4,226	$5,300	$11,596	$8,1
2020 Sales	$1,345	$4,522	$6,785	$4,580	$9,885	$2,876	$5,136	$8,236	$3,216	$3,369	$12,500	$5,4
2021 Sales	$4,345	$1,562	$2,345	$3,589	$2,885	$1,896	$4,236	$9,236	$5,569	$4,369	$13,590	$12

Questions

1. Using the sales data from the above table for the years 2019, 2020, and 2021:
   a. Calculate the average sales for each month.
   b. Calculate the average sales for 2019, 2020, and 2021.
   c. Based on the information from a and b, make sale predictions for 2022 for each month and overall.
2. Lightning Wholesale covers the cost of customer service representatives monthly cellphone costs. After reviewing last year's expenses, management would like to evaluate the average monthly cost per employee. Here is the list of plans and monthly expenses for the customer service reps.

Cellphone Provider Data for Lighting Wholesale

Providers	# of employees	Monthly Cost
Cellphone Provider #1	2	$95.99
Cellphone Provider #2	3	$109.95
Cellphone Provider #3	10	$129.99
Cellphone Provider #4	5	$85.99

   a. Calculate the average monthly cost of the cellphone costs per employee.
   b. What recommendations would you give to management based on the data?

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PART V

CHAPTER 5 - RATIOS, PROPORTIONS, AND APPLICATIONS

OBJECTIVES

- Identify ratios and rates to compare quantities.
- Set up ratios and use them to solve problems involving the allocation and sharing of quantities.
- Solve problems by determining unknown quantities using proportions as equivalent sets of ratios.

CHAPTER OUTLINE

Introduction

We use mathematics daily by comparing numbers and quantities of two or more items. Numbers and quantities are more meaningful and easier to work with when relevant comparisons can be made between them. A **ratio** is a comparison or relationship between two or more quantities. An example of how ratios can be used in our daily lives is in grocery shopping: if a 260 gram box of cereal costs $2.67, and a 400 gram box of the same cereal costs $3.99, we can use ratios to calculate the unit prices and determine which box of cereal is more economical.

When two sets of ratios are equal, we say that they are **proportional** to each other. We can use proportions to calculate unknown quantities that would otherwise be difficult to estimate. For example, if you wanted to calculate the amount of gas needed to travel 375 km, knowing that your car's fuel efficiency is 9.8 litres per 100 km, then you could set up a proportion equation to determine the amount of gas needed for the trip.

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5.1 RATIOS

A **ratio** is a comparison or relationship between two or more quantities with the same units. Therefore, ratios are not expressed with units.

For example, if Andy (A) invested $5,000 and Barry ($B$) invested $4,000 in a business, the comparison of A's investment to B's investment in the same order is known as the ratio of their investments.

Expressing Ratios

Expressing a Ratio of Two Quantities

When comparing two quantities, there are different ways to express the ratio. In the example above, the ratio of A's investment to B's may be expressed in any of the following forms:

5,000 to 4,000   (separate the quantities using the word 'to')

5,000 : 4,000   (using a colon and read as '5,000 is to 4,000')

$\dfrac{5,000}{4,000}$ (as a fraction and read as '5,000 over 4,000')

In the above example, the decimal equivalent of the fraction $\dfrac{5,000}{4,000}$ is 1.25; therefore, we can state that "A's invesment is 1.25 times B's investment".

Note: When representing a ratio as a fraction, if the denominator is 1, the denominator (1) must still be written. For example, if the ratio of two quantities is $\dfrac{3}{1}$, then it is incorrect to say that the ratio is 3. It should be stated as $\dfrac{3}{1}$ or 3 : 1.

Expressing a Ratio of More than Two Quantities

When comparing more than two quantities, the ratio is expressed using colons.

For example, if A's investment is $5,000, B's investment is $4,000, and C's investment is $1,000 in a business, then the ratio of their investments, respectively, is expressed as:

5,000 : 4,000 : 1,000

Terms of a Ratio

The quantities in a ratio are known as the **terms** of the ratio.

For example, the terms of the ratio 5 : 7 : 19 are 5, 7, and 19.

Equivalent Ratios

When all the terms of the ratio are multiplied by the same number or divided by the same number, the result will be an **equivalent ratio**.

For example, when the terms of the ratio 12 : 15 are multiplied by 2, we obtain an equivalent ratio of 24 : 30.

12 : 15

12 × **2** : 15 × **2**

24 : 30

When the terms of the ratio 12 : 15 are divided by the common factor 3, we obtain the equivalent ratio of 4 : 5.

12 : 15

12 ÷ **3** : 15 ÷ **3**

4 : 5

Therefore, the ratios 12 : 15, 24 : 30, and 4 : 5 are equivalent ratios.
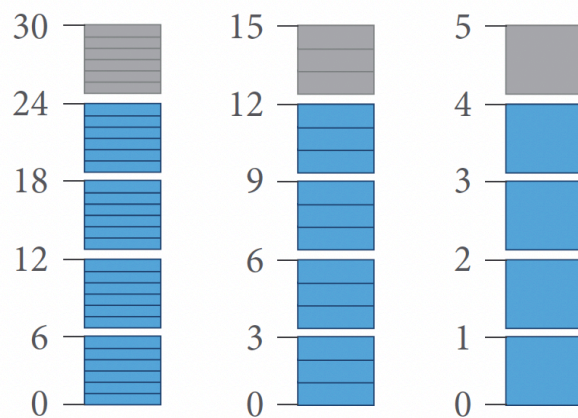
$$24 : 30 = 12 : 15 = 4 : 5$$
$$\frac{24}{30} = \frac{12}{15} = \frac{4}{5}$$

Reducing a Ratio to its Simplest or Lowest Terms

Comparisons are easier when ratios are reduced to their **lowest terms**. When all the terms of a ratio are integers, the ratio can be reduced to its lowest terms by dividing all the terms by their common factors.

For example, if A earns \$3,000, B earns \$4,500, and C earns \$6,000, then the equivalent ratio of their earnings reduced to its lowest terms is calculated as follows:

$A:B:C$

$3,000:4,500:6,000$ Dividing each term by the common factor 100,

$30:45:60$ Dividing each term by the common factor 15,

$2:3:4$ The ratio is now in its lowest terms.

By reducing the ratio to lowest terms, we can say that the earnings of A, B, and C are in the ratio of $2:3:4$.

When two ratios are equivalent, they result in the same answer when reduced to lowest terms.

Example 5.1-a: Determining Equivalent Ratios

Determine whether the given pairs of ratios are equivalent by reducing them to lowest terms.

    a.   18 : 12 and 12 : 8
    b.   20 : 24 and 15 : 20

Solution

   a. $18:12 \quad 18 \div 6:12 \div 6 \quad 3:2 \qquad 12:8 \quad 12 \div 4:8 \div 4 \quad 3:2$

       Therefore, the ratios 18 : 12 and 12 : 8 are equivalent.

   b. $20:24 \quad 20 \div 4:24 \div 4 \quad 5:6 \qquad 15:20 \quad 15 \div 5:20 \div 5 \quad 3:4$

       Therefore, the ratios 20 : 24 and 15 : 20 are **not** equivalent.

Reducing Ratios When One or More of the Terms of the Ratio Are Fractions

To reduce a ratio with fractions, first convert all the terms to integers by multiplying all the terms by the least common denominator, and then reduce to lowest terms.

For example,

$$\frac{2}{3} : \frac{4}{5} : 2 \qquad \text{Multiplying each term by the least common denominator 15,}$$

$10 : 12 : 30$

Dividing each term by the common factor 2,

$5 : 6 : 15$

Reducing Ratios When One or More of the Terms of the Ratio Are Decimal Numbers

To reduce a ratio with decimal numbers, first convert all the terms to integers by moving the decimal point of all the terms to the right by the same number of places, and then reduce to lowest terms.

For example,

$2.25 : 3.5 : 5$

Moving the decimal point of each term two places to the right,

$225 : 350 : 500$

Dividing each term by the common factor 25,

$9 : 14 : 20$

Reducing Ratios When the Terms of the Ratio Are a Combination of Fractions and Decimal Numbers

To reduce a ratio with both fractions and decimal numbers, first convert all the fractional terms to integers, and then convert all the decimal terms to integers. Finally, reduce to lowest terms.

For example,

$$5.8 : \frac{9}{2} : 4 \qquad \text{Multiplying each term by 2,}$$

$11.6 : 9 : 8$

Moving the decimal point of each term one place to the right,

$116 : 90 : 80$

Dividing each term by the common factor 2,

$58 : 45 : 40$

Reducing Ratios to an Equivalent Ratio Where the Smallest Term is 1

To make the comparison of quantities easier, we can also reduce a ratio to its equivalent ratio where the smallest term is equal to 1, by dividing all the terms by the smallest term.

For example, if the investment amounts of three partners A, B, and C are \$35,000, \$78,750, and \$59,500, respectively, then the equivalent ratio of their investments, where the smallest term is 1, is calculated as follows:

$A : B : C$

$35,000 : 78,750 : 59,500$

Dividing each term by the smallest term, 35,000,

$1 : 2.25 : 1.7$

The ratio is now reduced to its equivalent ratio with the smallest term equal to 1.

By reducing the ratio so that the smallest term is equal to 1, we can state: (i) B's investment is 2.25 times A's investment, and (ii) C's investment is 1.7 times A's investment.

Example 5.1-b: Reducing Ratios to Lowest Terms

Express the following ratios as equivalent ratios in their lowest terms (as whole numbers) and then reduce them to ratios where the smallest term is 1.

a. $2\dfrac{7}{9} : 3\dfrac{1}{3} : 5$

b. $2.5 : 1.75 : 0.625$

c. $1.25 : \dfrac{5}{6} : 2$

Solution

a. $2\dfrac{7}{9} : 3\dfrac{1}{3} : 5$

Converting the terms with mixed numbers to improper fractions, $\dfrac{25}{9} : \dfrac{10}{3} : 5$

Multiplying each term by the least common denominator 9, $25 : 30 : 45$

Dividing each term by the common factor 5, $5 : 6 : 9$

Dividing each term by the smallest term 5, $1 : 1.2 : 1.8$

Therefore, $2\dfrac{7}{9} : 3\dfrac{1}{3} : 5$ reduced to its lowest terms is $5 : 6 : 9$ and the equivalent ratio where the smallest term is 1 is $1 : 1.2 : 1.8$.

b. $2.5 : 1.75 : 0.6252.5 : 1.75 : 0.625$

Moving the decimal point of each term three places to the right, $2{,}500 : 1{,}750 : 625$

Dividing each term by the common factor 125, $20 : 14 : 5$

Dividing each term by the smallest term 5, $4 : 2.8 : 1$

Therefore, $2.5 : 1.75 : 0.625$ reduced to its lowest terms is $20 : 14 : 5$ and the equivalent ratio where the smallest term is 1 is $4 : 2.8 : 1$.

c. $1.25 : \dfrac{5}{6} : 2$

Multiplying each term by 6, $7.5 : 5 : 12$

Moving the decimal point of each term one place to the right, $75 : 50 : 120$

Dividing each term by the common factor 5, $15 : 10 : 24$

Dividing each term by the smallest term 10, $1.5 : 1 : 2.4$

Therefore, $1.25 : \dfrac{5}{6} : 2$ reduced to its lowest terms is $15 : 10 : 24$ and the equivalent ratio where the smallest term is 1 is $1.5 : 1 : 2.4$.

Order of a Ratio

The order of presenting terms in a ratio is important. For example, if A saves \$800, B saves \$1,500, and C saves \$1,200, then the ratio of the savings of A to B to C is:

$$A : B : C$$

$800 : 1{,}500 : 1{,}200$ Dividing each term by the common factor 100,

$$8 : 15 : 12$$
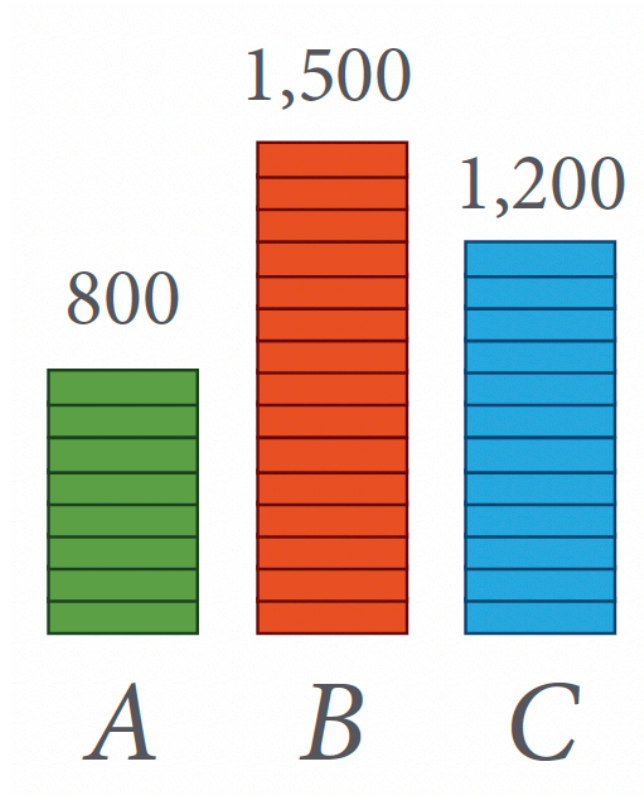
In the previous example, the ratio of the savings of C to B to A is:

$$C:B:A$$

$1,200 : 1,500 : 800$ Dividing each term by the common factor 100,

$$12 : 15 : 8$$
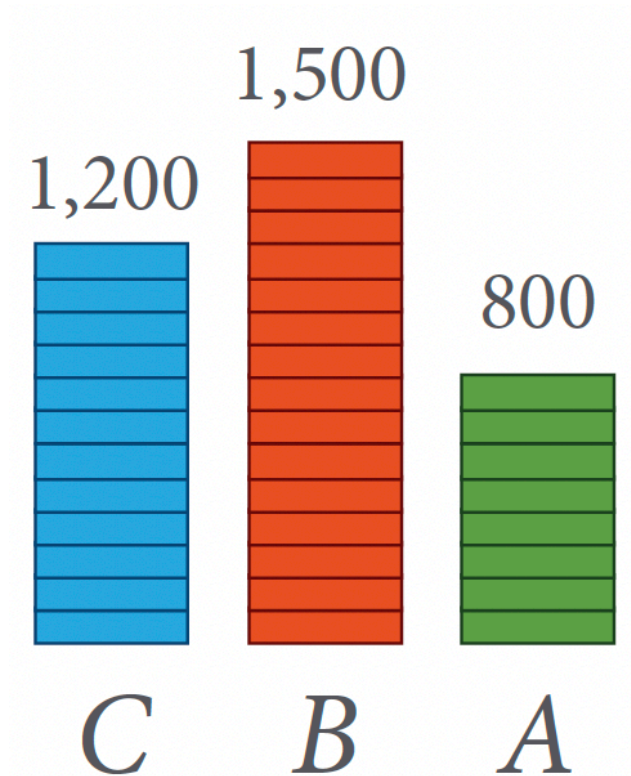
Notice that $C : B : A$ is not the same as $A : B : C$.

Ratios compare the numbers in order. The ratio 12 : 15 : 8 expresses a different comparison than the ratio 8 : 15 : 12.

Comparing Quantities

When using ratios to compare quantities of items that have the **same kind of measure**, the units have to be the same.

For example, the ratio of 45 minutes to 2 hours is not 45 : 2. We are comparing 'time' in both these cases; therefore, the units used have to be the same.

45 minutes : 2 hours

Converting 2 hours to minutes using 1 hour = 60 minutes,

45 minutes : 120 minutes

Dividing by the common factor 15,

3 : 8

Similarly, we determine the ratio of 2.5 kilometres to 3,000 metres as follows:

2.5 km : 3,000 m

Converting km to m using 1 km = 1,000 m,

2,500 m : 3,000 m

Dividing by the common factor 100,

   25 : 30

Dividing by the common factor 5,

   5 : 6

Example 5.1-c: Comparing Quantities that have the Same Kind of Measure but Different Units

Express each of the following ratios in its simplest form:

   a. 1.2 L to 800 mL
   b. 16 weeks to 2 years

Solution

  a. 1.2 L to 800 mL

     Converting 1.2 L to mL using 1 L = 1,000 mL,1,200 mL : 800 mL

     Dividing both terms by the common factor 400, 3 : 2

     Therefore, the ratio of 1.2 L to 800 mL is 3 : 2.

  b. 16 weeks to 2 years

     Converting 2 years to weeks using 1 year = 52 weeks, 16 weeks : 104 weeks

     Dividing both terms by the common factor 8,2 : 13

     Therefore, the ratio of 16 weeks to 2 years is 2 : 13.

When using ratios to compare quantities of items that have **different kinds of measure**, the units of measurement of each quantity must be included in the ratio.

   For example, when baking a cake, Maggie uses 4 kilograms of flour, 2 litres of water, and 6 eggs.

Therefore, the ratio of flour to water to eggs is,

   flour (kg) : water (L) : eggs (number)

   4 : 2 : 6

Dividing each term by the common factor 2,

2 : 1 : 3

i.e., 2 kg flour : 1 L water : 3 eggs

Rate, Unit Rate, and Unit Price

Rate

A **rate** is a special ratio that is used to compare two quantities or amounts that have different kinds of measure. The units of measurement of both quantities are included when expressing a rate.

For example, if a car travels 100 km using 9 L of gas, then the rate of kilometres per litres is 100 km : 9 L. The first term of the ratio is measured in kilometres and the second term is measured in litres. The word 'per' indicates that it is a rate and is usually denoted by a slash "/". Therefore, 100 km : 9 L is usually written as 100 km per 9 L or 100 km/9 L.

Rates are used in our day-to-day activities such as travelling, working, shopping, etc.

For example,

- travelling 90 kilometres in 1.5 hours: 90 km/1.5 hours
- working 75 hours in 2 weeks: 75 hours/2 weeks
- paying $4.80 for 4 litres of milk: $4.80/4 L

Example 5.1-d: Expressing Rate as a Ratio of Different Units of Measurement

A laser printer printed 88 pages in 6 minutes. Express the rate in lowest terms.

Solution

The unit of the first term is in number of pages and the unit of the second term is in minutes.

88 pages : 6 minutes

Dividing each term by the common factor 2,

44 pages : 3 minutes

This is expressed as a rate as 44 pages/3 minutes.

Therefore, the printing rate is 44 pages/3 minutes.

Unit Rate

Unit rate represents the number of units of the first quantity (or measurement) that corresponds to one unit of the second quantity (or measurement). That is, unit rate is a rate which has a denominator of 1.

Rate can be converted to unit rate simply by dividing the first term by the second term.

For example, a rate of 90 kilometres in 1.5 hours, converted to unit rate:

90 km/1.5 hours

$$= \frac{90km}{1.5hours}$$

= 60 km/hour

Similarly, a rate of 75 hours in 2 weeks, converted to unit rate:

75 hours/2 weeks

$$= \frac{75hours}{2weeks}$$

= 37.5 hours/week

Example 5.1-e: Calculating Unit Rate

A car travelled 300 kilometres in 5 hours. Calculate its average speed in km/h.

Solution

The rate of speed is 300 kilometres in 5 hours.

$$= \frac{300km}{5h}$$

Dividing the first term by the second term to obtain the **unit rate**,

= 60 km/h

Therefore, the average rate of speed is 60 kilometres per hour, or 60 km/h.

Example 5.1-f: Calculating Hourly Rate of Pay

Peter worked 9 hours and earned $247.50. Calculate his hourly rate of pay.

Solution

The rate of pay is $247.50 for 9 hours.

$$= \frac{\$247.50}{9h}$$

Dividing the first term by the second term to obtain the **unit rate**,

= $27.50/h

Therefore, his hourly rate of pay is $27.50 per hour, or $27.50/h.

Example 5.1-g: Using Unit Rates to Solve a World Problem

A car travels 90 km in 1.5 hours. At this rate, how many kilometres will it travel in 5 hours?

Solution

This question can be solved by first determining the unit rate.

The rate of speed is 90 kilometres in 1.5 hours.

Therefore, the number of km travelled per hour $= \dfrac{90km}{1.5h}$

= 60 km/h

That is, the distance travelled in 1 hour = 60 km.

Therefore, the distance travelled in 5 hours = 60 × 5 = 300 km.

Example 5.1-h: Comparing Unit Rates

Car A requires 8.9 litres of gas to travel 100 km. Car B requires 45 litres of gas to travel 475 km. Which car has the better fuel economy?

Solution

Car A : 100 km requires 8.9 litres of gas.

Therefore, the number of km travelled per litre $= \dfrac{100km}{8.9L}$

= 11.235955... = 11.24 km/L of gas

Car B : 475 km requires 45 litres of gas.

Therefore, the number of km travelled per litre $= \dfrac{475km}{45L}$

= 10.555555... = 10.56 km/L of gas

Therefore, Car A has the better fuel economy.

Unit Price

Unit price is the unit rate that expresses the cost of an item (in dollars, cents, etc.) per one unit of that item. The price is always the numerator and the unit quantity is the denominator. That is, price is expressed per quantity of 1.

Examples of unit price are:

- Price of gas is $1.36 per litre ($1.36/L)
- Price of grapes is $2 per kilogram ($2/kg)
- Price of juice is $0.75 per can

If the total price for a given quantity of an item is known, to determine its unit price, divide the total price of the item by its quantity.

Unit price is used in comparing and making decisions in purchasing items when various options are

available. We save money when we compare the unit prices of the same item in different sized containers or different packages to determine the cheaper price per unit for our purchases.

Example 5.1-i: Calculating the Unit Price of an Item

If 4 litres of milk cost $4.80, then what is the unit price of milk?

Solution

To determine the unit price of milk, divide the total price by the given quantity.

That is, divide $4.80 by 4 litres.

$$\text{Unit price} = \frac{\$4.80}{4L} = \$1.20/L$$

Therefore, the unit price of milk is $1.20 per litre ($1.20/L).

Example 5.1-j: Comparing Unit Prices

5 kg of almonds cost $43.50 and 4 kg of almonds cost $34.20. Which is cheaper to buy based on its unit price?

Solution

5 kg of almonds cost $43.50

$$\text{Therefore, unit price} = \frac{\$43.50}{5kg} = \$8.70/kg$$

4 kg of almonds cost $34.20

Therefore, unit price $= \dfrac{\$34.20}{4kg} = \$8.55/\text{kg}$

Therefore, based on unit price, buying 4 kg of almonds for $34.20 is cheaper than buying 5 kg of almonds for $43.50.

Note: Unit rate and unit price problems can also be solved using the method of proportions as demonstrated in the next section.

Sharing Quantities Using Ratios

Sharing quantities refers to the allocation or distribution of a quantity into two or more portions (or units) based on a given ratio.

For example, to allocate last year's $1,000 profit among persons A, B, and C in the ratio of 2 : 3 : 5, first add the terms of the ratio (i.e., 2, 3, and 5), which results in a total of 10 units. These 10 units represent the total profit of $1,000, where A's share constitutes 2 units, B's 3 units, and C's 5 units, as shown in the diagram below.

Each person's share can then be calculated, as follows:

A's share $\dfrac{2}{10} \times 1,000.00 = \$200.00$

B's share $\dfrac{3}{10} \times 1,000.00 = \$300.00$

C's share $\dfrac{5}{10} \times 1,000.00 = \$500.00$

10 Units = $1,000

A (2 Units) B (3 Units) C (5 Units)

- The total of A, B, and C's shares is equal to the profit amount of $1,000. That is, the shares of $A + B + C$ = 200 + 300 + 500 = 1,000.
- If we reduce the ratio of the amounts shared by A, B, and C to its lowest terms, the result would be the

original ratio. That is, 200 : 300 : 500 reduced to lowest terms is 2 : 3 : 5.

If this year, the ratio of A's share : B's share : C's share is changed to 5 : 3 : 2 (instead of last year's 2 : 3 : 5), and the profit of $1,000 remained the same, then their individual shares will change.

Their shares are recalculated as follows:

A's share $\dfrac{5}{10} \times 1,000.00 = \$500.00$

B's share $\dfrac{3}{10} \times 1,000.00 = \$300.00$

C's share $\dfrac{2}{10} \times 1,000.00 = \$200.00$

10 Units = $1,000

A
(5 Units)

B
(3 Units)

C
(2 Units)

- The total of A, B, and C's shares this year is still equal to the profit amount of $1,000. That is, the shares of $A + B + C = 500 + 300 + 200 = 1,000$.
- If we reduce the ratio of the amounts shared by A, B, and C to its lowest terms, the result would be the original ratio. That is, 500 : 300 : 200 reduced to lowest terms is 5 : 3 : 2.

Example 5.1-k: Sharing Quantities Using Ratios

A, B, and C start a business and invest $3,500, $2,100, and $2,800, respectively. After a few months,

C decides to sell his share of the business to A and B. How much would A and B have to pay for C's shares if A and B want to maintain their initial investment ratio?

Solution

The investments of A, B, and C are in the ratio of 3,500 : 2,100 : 2,800, which can be reduced to 5 : 3 : 4.

If A and B want to maintain their initial investment ratio of 5 : 3, then C's share (of $2,800) has to be paid for by A and B in the same ratio, 5 : 3.

By adding the terms of A and B, we know that C's share is to be divided into a total of 8 units, as illustrated:

$$A \text{ would have to pay } C : \frac{5}{8} \times 2,800.00 = \$1,750.00$$

$$B \text{ would have to pay } C : \frac{3}{8} \times 2,800.00 = \$1,050.00$$

8 Units = $2,800.00

A
(5 Units)

B
(3 Units)
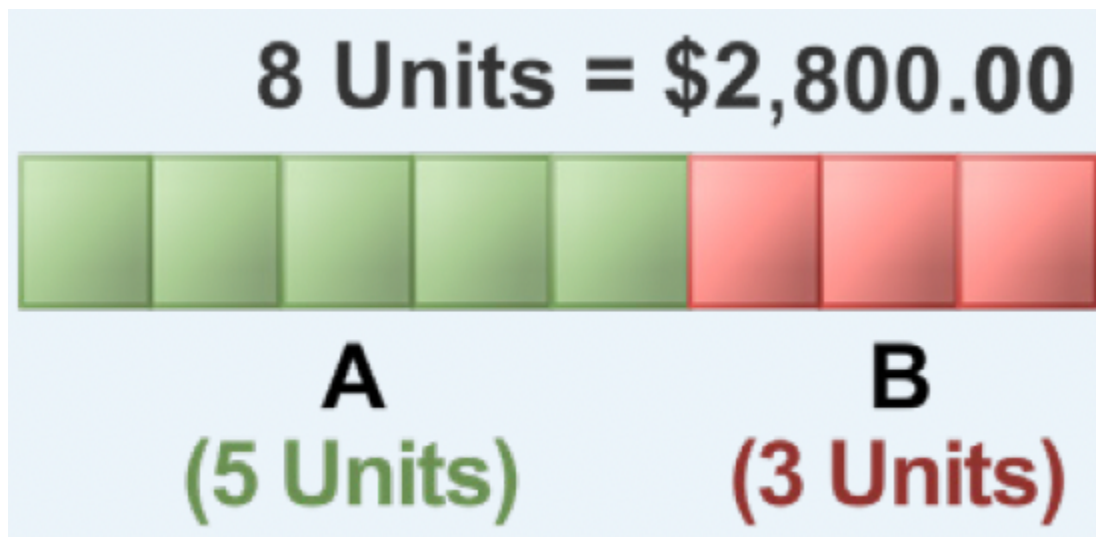
Therefore, A would have to pay C $1,750 and B would have to pay C $1,050 in order to maintain their initial investment ratio.

Example 5.1-l: Sharing Quantities Using Fractional Ratios

The sum of all three angles of a triangle is 180 degrees. If the three angles of a triangle are in the ratio of $\dfrac{1}{6} : \dfrac{1}{3} : \dfrac{1}{4}$, calculate each angle of the triangle.

Solution

First, convert all the terms in the ratio to integers.

$$\frac{1}{6} : \frac{1}{3} : \frac{1}{4}$$

Multiplying each term by the least common denominator 12,

$$2 : 4 : 3$$

This ratio is in lowest terms.

By adding all the terms, we know that the 180 degrees is to be divided into a total of $2 + 4 + 3 = 9$ units.

1st angle: $\dfrac{2}{9} \times 180° = 40°$

2nd angle: $\dfrac{4}{9} \times 180° = 80°$

3rd angle: $\dfrac{3}{9} \times 180° = 60°$

9 units = 180°

1st angle **2nd angle** **3rd angle**
(2 Units) **(4 Units)** **(3 Units)**
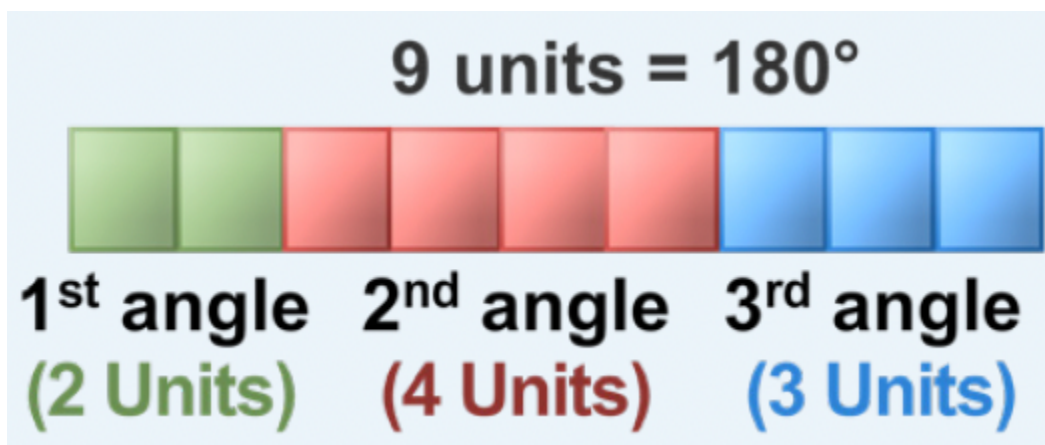
Therefore, the three angles of the triangle are 40°, 80°, and 60°, which sum to 40° + 80° + 60° = 180°.

Example 5.1-m: Application Using Equivalent Ratios

A fashion boutique is split into three departments: Clothing, Beauty products, and Accessories. Each department takes up a different area in the boutique. The ratio of the areas of Accessories to Beauty is 2 : 3, and that of Beauty to Clothing is 4 : 5. What is the ratio of the department areas of Accessories : Beauty : Clothing?

Solution

$A : B = 2 : \mathbf{3}$ and $B : C = \mathbf{4} : 5$

Determine the equivalent ratios for $A : B$ and $B : C$ such that the number of units in B is the same in both ratios.

$A : B = 2 : 3$

Multiplying each term by 4,

$= 8 : \mathbf{12}$

$B : C = 4 : 5$

Multiplying each term by 3,

$= \mathbf{12} : 15$
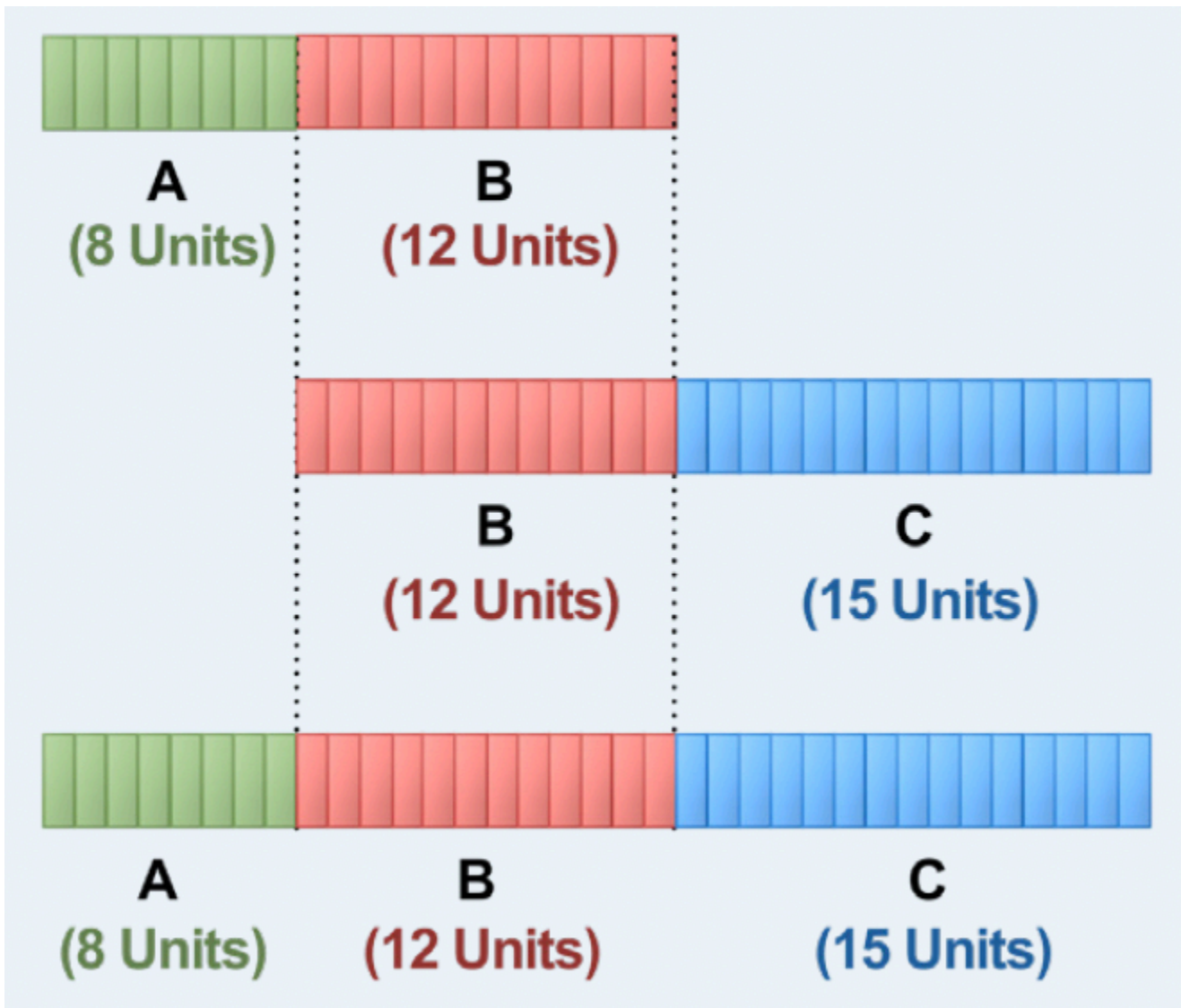
Therefore, the ratio of the department areas of Accessories : Beauty : Clothing in the boutique is 8 : 12 : 15.

5.1 Exercises

Answers to the odd-numbered questions are available at the end of the book.

For problems 1 and 2, determine whether the given pairs of ratios are equivalent.

1. a. 4 : 6 and 6 : 10

    b. 8 : 10 and 28 : 35

    c. 6 : 8 and 27 : 32

  d. 16 : 22 and 64 : 88

 2. a. 16 : 20 and 24 : 30

     b. 10 : 12 and 35 : 42

     c. 12 : 14 and 30 : 42

     d. 12 : 26 and 30 : 65

Determine if the following ratios are equivalent or not equivalent to the ratio 6 : 9 : 12.

 3. a. 4 : 6 : 8

     b. 2 : 3 : 4

     c. 1 : 3 : 2

     d. 8 : 12 : 16

Determine if the following ratios are equivalent or not equivalent to the ratio 6 : 9 : 12.

 4. a. 20 : 30 : 15

     b. 8 : 12 : 6

     c. 28 : 42 : 21

     d. 24 : 36 : 18

For problems 5 to 8, express the ratios as (i) equivalent ratios in their lowest terms (as whole numbers), and (ii) equivalent ratios where the smallest term is 1.

 5. a. 18 : 48 : 30

     b. 175 : 50 : 125

     c. 0.45 : 1 : 2

6. a. $27 : 45 : 72$

    b. $180 : 60 : 150$

    c. $2.4 : 0.75$

7. a. $\dfrac{2}{3} : \dfrac{1}{5}$

    b. $12 : \dfrac{5}{3} : 3$

    c. $1.7 : 8.5 : \dfrac{34}{3}$

8. a. $\dfrac{3}{2} : \dfrac{2}{5}$

    b. $65 : 91 : \dfrac{13}{2}$

    c. $12 : 1.5 : \dfrac{3}{2}$

For problems 9 and 10, express the ratios in lowest terms.

9. a. 9 months to 2 years

    b. 750 g to 3 kg

    c. 30 minutes to 1 hour and 15 minutes

10. a. 3 weeks to 126 days

    b. 120 g to 2 kg

    c. 55 minutes to 2 hours and 45 minutes

11. Emily was planning to make an authentic Indian dish for her guests. She planned to use 36 eggs, 6 litres of water, 3 tablespoons of chilli powder, and 12 tomatoes.

     0. What is the ratio of the ingredients in her recipe, expressed in lowest terms?

a. If she decides to reduce the quantity of chilli powder to $1\frac{1}{2}$ tablespoons, calculate the new ratio of

the ingredients in her recipe, expressed in lowest terms.

12. In Murphy's battery manufacturing company, 600 kg of lead, 45 kg of carbon, 30 litres of battery acid,

and 120 kg of rubber are used per day to make batteries.

0. What is the ratio of the raw materials used per day to make batteries, expressed in lowest terms?

a. If they alter the quantity of carbon used in the batteries and utilize 30 kg of carbon per day,

calculate the new ratio of raw materials used per day, expressed in lowest terms.

For problems 13 to 20, determine the unit rate.

13. 525 kilometres in 7 hours = ? km/h

14. 680 kilometres in 8 hours = ? km/h

15. 154 kilometres to 14 litres = ? km/L

16. 228 kilometres to 19 litres = ? km/L

17. 450 words typed in 6 minutes = ? words/min

18. 496 words typed in 8 minutes = ? words/min

19. 261 pages in 9 minutes = ? pages/min

20. 192 pages in 8 minutes = ? pages/min

For problems 21 to 26, identify the option with the lower cost, based on unit price.

21. 2 kg of flour for $3.30, or 5 kg of flour for $8.40.

22. 3 kg of sugar for $3.90, or 5 kg of sugar for $6.25.

23. 12 pencils for $4.44, or 8 pencils for $2.88.

24. 6 litres of paint for $45.60, or 5 litres of paint for $37.25.

25. 1.2 litres of juice for $2.16, or 0.8 litres of juice for $1.40.

26. 2.2 kg of jam for $11.00, or 1.5 kg of jam for $7.20.

27. In a race, participants are required to swim 3,850 metres and bike 7 kilometres. Calculate the ratio of the distance covered by swimming to the distance covered by biking, in its lowest terms.

28. Adam, a hardware engineer, wants to install a microchip that is 38.2 mm in length into his laptop. The length of the installation space provided in his laptop is 4.8 cm. Calculate the ratio of the length of the microchip to the installation space, in its lowest terms.

29. An aircraft travels a distance of 3,105 km in 5 hours and 45 minutes. Calculate the ratio of the distance traveled to the time taken, reduced to a rate of kilometres per hour.

30. Speed is defined as the ratio of the distance travelled to the time taken. If Mary, who lives in Toronto, took 6 hours and 15 minutes to reach her parent's home in Montreal, which is 575 km away, calculate the average speed at which she was travelling in kilometres per hour.

31. If A earns $196 for working 8 hours and B earns $98 for working 5 hours, whose hourly rate is higher?

32. If Amanda travelled 325 km in 4 hours and 15 minutes and Ashton travelled 290 km in 3 hours and 30 minutes, whose average speed was greater?

33. Kate earned $210 for 7.5 hours of work and Susan earned $249.75 for 9 hours of work.

    a. Calculate their hourly rates.

    b. Whose hourly rate was higher and by how much?

34. Jack's monthly pay is $4,200. Steve's weekly pay is $975.

    a. Calculate their annual salaries. (1 year = 12 months = 52 weeks)

    b. Whose annual salary is more and by how much?

35. A TV cable bill of \$90 is shared between two housemates, Mike and Sarah, in the ratio of $2 : 2\frac{1}{2}$. How much did each person pay?

36. Alexander and Alyssa invested a total of \$10,000 in a web-design business. If the ratio of their investments is 3 : 5, what were their investments?

37. Amy and two of her friends received the first prize for a marketing case competition. They received an amount of \$7,500; however, they decided to share the prize in the ratio of the amount of time each of them spent on the marketing case. If Amy spent 5 hours, Gary spent 8 hours, and Andrew spent 2 hours, what would be each person's share of the prize?

38. Three friends, Andy, Berry, and Cassandra, jointly insured a commercial property in the ratio of 10 : 9 : 6, respectively. How will an annual premium of \$8,000 be distributed among the three of them?

39. The lengths of the three sides of a triangle are in the ratio of $\frac{3}{4} : \frac{2}{5} : \frac{1}{2}$. If the perimeter of the triangle (i.e., the sum of the three sides lengths) is 66 cm, calculate the length of each side.

40. The lengths of the three sides of a triangle are in the ratio of $\frac{2}{3} : \frac{5}{6} : \frac{4}{5}$. If the perimeter of the triangle (i.e., the sum of the three sides lengths) is 128 cm, calculate the length of each side.

41. Three friends, Alex, Brooks, and Charlie, have decided to invest \$4,000, \$6,000, and \$2,000, respectively, to start a software development business. If Charlie decided to leave the business, how much would Alex and Brooks have to pay for Charlie's share if they want to maintain their initial investment ratio?

42. Chuck decided to build a yacht with his two friends Rob and Bob; they invested \$9,000, \$11,000, and \$6,500, respectively. After the yacht was built, Bob decided to sell his share of the investment to Chuck and Rob. How much would each of them have to pay if they want to maintain the same ratio of their investments in the yacht?

43. A box contains red, green, and blue marbles. $\frac{1}{3}$ of the marbles are red, $\frac{2}{5}$ of the marbles are green, and

the remaining marbles are blue.

    0. What fraction of the marbles in the box are blue?

    a. Calculate the ratio of red to green to blue ($R : G : B$) marbles in the box, expressed in its lowest terms (as whole numbers).

    b. If there are 75 marbles in the box, how many of each colour are there?

44. A box contains black, white, and yellow marbles. $\frac{1}{4}$ of the marbles are black, $\frac{1}{3}$ of the marbles are white, and the remaining marbles are yellow.

    0. What fraction of the marbles in the box are yellow?

    a. Calculate the ratio of black to white to yellow ($B : W : Y$) marbles in the box, expressed in its lowest terms (as whole numbers).

    b. If there are 72 marbles in the box, how many of each colour are there?

45. Abey and Baxter invested equal amounts of money in a business. A year later, Abey withdrew $7,500, making the ratio of their investments 5 : 9. How much money did each of them invest in the beginning?

46. Jessica and Russel invested equal amounts to start a business. Two months later, Jessica invested an additional $3,000 in the business, making the ratio of their investments 11 : 5. How much money did each of them invest in the beginning?

47. If $A : B = 4 : 3$ and $B : C = 6 : 5$, determine $A : B : C$.

48. If $X : Y = 5 : 2$ and $Y : Z = 7 : 6$, determine $X : Y : Z$.

Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of*

5.2 PROPORTIONS

The Proportion Equation

When two ratios are equivalent, we say that they are **proportional** to each other. In the proportion equation, the ratio on the left side of the equation is equivalent to the ratio on the right side of the equation.

Consider an example where $A : B$ is $50 : 100$ and $C : D$ is $30 : 60$.

Reducing the ratios to lowest terms, we obtain the ratio of $A : B$ as $1 : 2$ and the ratio of $C : D$ as $1 : 2$.

Since these ratios are equivalent, they are proportional to each other, and their proportion equation is:

$$A : B = C : D$$

The proportion equation can also be formed by representing the ratios as fractions.

Equating the fraction obtained by dividing the first term by the second term on the left side, to the one obtained by dividing the first term by the second term on the right side, we obtain:

$$\frac{A}{B} = \frac{C}{D}$$

This proportion equation can be simplified by multiplying both sides of the equation by the product of both denominators, which is $B \times D$.

$$\frac{A}{B} = \frac{C}{D} \qquad \text{Multiplying both sides by } (B \times D),$$

$$\frac{A}{B}(B \times D) = \frac{C}{D}(B \times D) \qquad \text{Simplifying,}$$

$$AD = BC$$

The same result can be obtained by equating the product of the numerator of the first ratio and the denominator of the second ratio with the product of the denominator of the first ratio and the numerator of the second ratio. This is referred to as cross-multiplication and is shown below:
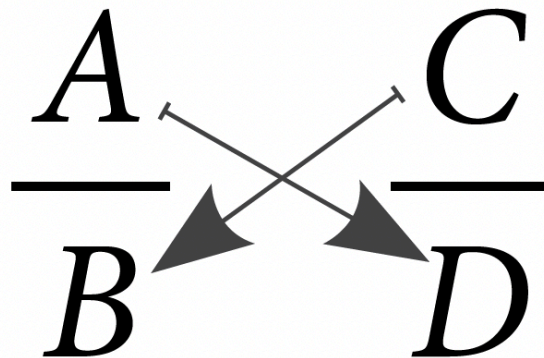
$$\frac{A}{B} \diagdown\!\!\!\!\diagup \frac{C}{D}$$

Cross-multiplying,

$$AD = BC$$

Notice that the cross-multiplication of $\dfrac{A}{C} = \dfrac{B}{D}$ also gives the same result of $AD = BC$. Therefore,

$A : B = C : D$ is equivalent to both $\dfrac{A}{B} = \dfrac{C}{D}$ and $\dfrac{A}{C} = \dfrac{B}{D}$. If any three terms of the proportion

equation are known, the fourth term can be calculated.

Proportion Equation With Sets of Ratios Having More Than Two Terms

Consider the proportion equation $A : B : C = D : E : F$.

This proportion equation can also be expressed using fractions: $\dfrac{A}{B} = \dfrac{D}{E}, \dfrac{B}{C} = \dfrac{E}{F}$, and $\dfrac{A}{C} = \dfrac{D}{F}$

Cross-multiplying leads to: $AE = BD, BF = CE$, and $AF = CD$

The proportion $A : B : C = D : E : F$ can be illustrated by the table below,

Example Proportion Breakdown

1st Term	2nd Term	3rd Term
A	B	C
D	E	F

and expressed as, $A : D = B : E = C : F$.

Expressing using fractions, $\dfrac{A}{D} = \dfrac{B}{E} = \dfrac{C}{F}$

Cross-multiplying leads to the same result as shown above: $AE = BD, BF = CE, and\, AF = CD$

.

Therefore, $A : B : C = D : E : F$ is equivalent to both

- $\dfrac{A}{B} = \dfrac{D}{E}, \dfrac{B}{C} = \dfrac{E}{F}, \dfrac{A}{C} = \dfrac{D}{F}$, and
- $\dfrac{A}{D} = \dfrac{B}{E} = \dfrac{C}{F}$

Example 5.2-a: Solving for the Unknown Quantity in Proportions

Determine the missing term in the following proportions:

a. $4 : 5 = 8 : x$

b. $6 : x = 10 : 25$

c. $x : 1.9 = 2.6 : 9.88$

d. $3 : 3\dfrac{3}{4} = x : 5\dfrac{1}{4}$

Solution

a. $4 : 5 = 8 : x$

Using fractional notation, $\dfrac{4}{5} = \dfrac{8}{x}$ **or** $\dfrac{4}{8} = \dfrac{5}{x}$.

Cross-multiplying, $4x = 40$.

Simplifying, $x = \dfrac{40}{4}$.

Therefore, $x = 10$.

Solution Breakdown for Example 5.2-a A

1st Term	2nd Term
4	5
8	x

b. $6 : x = 10 : 25$

Using fractional notation, $\dfrac{6}{x} = \dfrac{10}{25}$ **or** $\dfrac{6}{10} = \dfrac{x}{25}$.

Cross-multiplying, $150 = 10x$.

Simplifying, $x = \dfrac{150}{10}$.

Therefore, $x = 15$.

Solution Breakdown for Example 5.2-a B

1ˢᵗ Term	2ⁿᵈ Term
6	x
10	25

c. $x : 1.9 = 2.6 : 9.88$

Using fractional notation, $\dfrac{x}{1.9} = \dfrac{2.6}{9.88}$ **or** $\dfrac{x}{2.6} = \dfrac{1.9}{9.88}$.

Cross-multiplying, $9.88x = 4.94$.

Simplifying, $x = \dfrac{4.94}{9.88}$.

Therefore, $x = 0.5$.

Solution Breakdown for Example 5.2-a C

1ˢᵗ Term	2ⁿᵈ Term
x	1.9
2.6	9.88

d. $3 : 3\dfrac{3}{4} = x : 5\dfrac{1}{4}$

Rewriting as an improper fraction, $3 : \dfrac{15}{4} = x : \dfrac{21}{4}$.

Multiplying both sides by 4, $12 : 15 = 4x : 21$.

Using fractional notation, $\dfrac{12}{15} = \dfrac{4x}{21}$ **or** $\dfrac{12}{4x} = \dfrac{15}{21}$.

Cross-multiplying, $252 = 60x$.

Simplifying, $x = \dfrac{252}{60}$.

Therefore, $x = \dfrac{21}{5} = 4\dfrac{1}{5}$.

Solution Breakdown for Example 5.2-a D

1ˢᵗ Term	2ⁿᵈ Term
12	15
$4x$	21

Example 5.2-b: Solving Word Problems Using Proportions

Express each of the following ratios in its simplest form:

a. The distance (in km) that Ben can walk in $3\frac{1}{2}$ hours.

b. How long (in hours and minutes) will it take him to walk 15 km?

Solution

a. Calculating the distance (in km) he can walk in $3\frac{1}{2}$ hours. $\mathbf{km : h = km : h}$ $9 : 2 = x : 3\frac{1}{2}$.

Rewriting as an improper fraction, $9 : 2 = x : \dfrac{7}{2}$.

Multiplying both sides by 2, $18 : 4 = 2x : 7$.

Using fractional notation, $\dfrac{18}{4} = \dfrac{2x}{7}$ **or** $\dfrac{18}{2x} = \dfrac{4}{7}$.

Cross-multiplying, $126 = 8x$.

Simplifying, $x = \dfrac{126}{8}$ $x = 15.75 km$.

Therefore, Ben can walk a distance of 15.75 km in $3\frac{1}{2}$ hours.

Solution Breakdown for Example 5.2-b A

1st Term	2nd Term
18	4
$2x$	7

b. Calculating the time (in hours and minutes) that it will take him to walk 15 km: $\mathbf{km : h = km : h}$

$9 : 2 = 15 : x$

Using fractional notation, $\dfrac{9}{2} = \dfrac{15}{x}$ **or** $\dfrac{9}{15} = \dfrac{2}{x}$.

Cross-multiplying, $9x = 30$.

Simplifying, $x = \dfrac{30}{9} = 3\dfrac{1}{3}x = 3h + \left(\dfrac{1}{3} \times 60\right) \, minx = 3h20min.$

Therefore, Ben can walk 15 km in 3 hours and 20 minutes.

Solution Breakdown for Example 5.2-b B

1st Term	2nd Term
9	2
15	x

Example 5.2-c: Sharing Using Proportions

Andrew (A), Brandon (B), and Chris (C) decide to form a partnership to start a snow removal business together. A invests $31,500, B invests $42,000, and C invests $73,500. They agree to share the profits in the same ratio as their investments.

a. What is the ratio of their investments, expressed in lowest terms?
b. In the first year of running the business, A's profit was $27,000. What were B's and C's profits?

Solution

a. Ratio of their investments $:A : B : C$ 31,500 : 42,000 : 73,500.

Dividing each term by the common factor of 100,315 : 420 : 735.

Dividing each term by the common factor of 5,63 : 84 : 147.

Dividing each term by the common factor of 7,9 : 12 : 21.

Dividing each term by the common factor of 3,3 : 4 : 7.

Therefore, the ratio of their investments is 3 : 4 : 7.

b. A's profit was \$27,000. B's and C's profits are calculated using one of the two methods, as follows:

Method 1: Ratio of Investment = Ratio of Profit $A : B : C = A : B : C$

Substituting terms, $3 : 4 : 7 = 27,000 : x : y.$

Using fractional notation, $\dfrac{3}{4} = \dfrac{27,000}{x}$ and $\dfrac{3}{7} = \dfrac{27,000}{y}.$

Cross-multiplying, $3x = 108,000 \quad 3y = 189,000.$

Simplifying, $x = \$36,000.00 \quad y = \$63,000.00.$

Method 2: Ratio of Investment = Ratio of Profit $A : B : C = A : B : C$ Substituting terms,

$3 : 4 : 7 = 27,000 : x : y.$ Using fractional notation, $\dfrac{3}{27,000} = \dfrac{4}{x} = \dfrac{7}{y}.$ Hence,

$\dfrac{3}{27,000} = \dfrac{4}{x}$ and $\dfrac{3}{27,000} = \dfrac{7}{y}.$ Cross-multiplying, $3x = 108,000 \quad 3y = 189,000$

.Simplifying, $x = \$36,000.00 \quad y = \$63,000.00.$ Therefore, B's profit is \$36,000, and C's profit

is \$63,000.

Solution Breakdown for Example 5.2-c B

1st Term	2nd Term	3rd Term
3	4	7
27,000	x	y

5.2 Exercises

Answers to the odd-numbered questions are available at the end of the book.

For problems 1 and 2, determine whether the ratio pairs are in proportion.

1. a. $6 : 9$ and $14 : 21$

b. $5 : 15$ and $2 : 8$

c. $18 : 24$ and $12 : 16$

d. $12 : 60$ and $6 : 24$

2. a. $9 : 12$ and $4 : 3$

   b. $10 : 30$ and $8 : 24$

   c. $14 : 20$ and $28 : 42$

   d. $15 : 12$ and $24 : 30$

For problems 3 to 6, solve the proportion equations for the unknown value.

3. a. $x : 4 = 27 : 36$

   b. $24 : x = 6 : 9$

   c. $5 : 9 = x : 3$

   d. $1 : 2 = 5 : x$

4. $x : 8 = 6 : 24$

5. $3 : x = 18 : 42$

6. $15 : 5 = x : 15$

7. $28 : 35 = 4 : x$

5. a. $x : 18\dfrac{1}{4} = 8 : 11\dfrac{3}{4}$

   b. $7\dfrac{1}{5} : x = 5\dfrac{3}{4} : 3\dfrac{2}{5}$

   c. $1 : 4\dfrac{1}{2} = x : 2\dfrac{3}{4}$

   d. $\displaystyle 1\frac{1}{2} : 2\frac{1}{4} = 1\frac{3}{4} : x$

6. a. $x : 3.65 = 5.5 : 18.25$

b. $2.2 : x = 13.2 : 2.5$

c. $4.25 : 1.87 = x : 2.2$

e. $2.4 : 1.5 = 7.2 : x$

7. A truck requires 96 litres of gas to cover 800 km. How many litres of gas will it require to cover 1,500 km? br

8. Based on Alvin's past experience, it would take his team five months to complete two projects. How long would his team take to complete eight similar projects?

9. Eric paid a property tax of $3,600 for his land that measures 330 square metres. Using the same tax rate, what would his neighbour's property tax be if the size of the house is 210 square metres and is taxed at the same rate?

10. The city of Brampton charges $1,750 in taxes per year for a 2,000 square metre farm. How much would Maple Farms have to pay in taxes if they had a 12,275 square metre farm in the same area?

11. On a map, 4 cm represents 5.0 km. If the distance between Town A and Town B on the map is 9.3 cm, how many kilometres apart are these towns?

12. On a house plan, 1.25 cm represents 3 metres. If the actual length of a room is 5.4 metres, how will this length be represented in the plan in cm?

13. Steve invested his savings in a GIC, mutual funds, and a fixed deposit in the ratio of 5 : 4 : 3, respectively. If he invested $10,900 in mutual funds, calculate his investments in the GIC and the fixed deposit.

14. The ratio of the distance from An's house to Mark, Jeff, and Justin's houses is 3 : 5.25 : 2, respectively. If the distance from Ann's house to Mark's is 9.50 km, calculate the distance from Ann's house to Justin's and Ann's house to Jeff's.

15. *A, B,* and *C,* started a business with investments in the ratio of 5 : 4 : 3, respectively. A invested $25,000,

and all three of them agreed to share profits in the ratio of their investments.

    a. Calculate B and C's investments.

    b. If A's profit was \$30,000 in the first year, calculate B and C's profits.

16. A, B, and C formed a partnership and invested in the ratio of $7:9:5$, respectively. They agreed to share

    the profit in the ratio of their investments. A invested \$350,000.

    a. Calculate B and C's investments in the partnership.

    b. In the first year, if A made \$38,500 in profit from the partnership, how much did B and C make?

5. REVIEW EXERCISES

Answers to the odd-numbered questions are available at the end of the book.

For problems 1 and 2, determine whether the given pairs of ratios are equivalent.

1. a. 6 : 8 and 18 : 24

    b. 30 : 25 and 36 : 48

    c. 10 : 35 and 14 : 49

    d. 24 : 30 and 12 : 18

2. a. 16 : 20 and 18 : 30

    b. 4 : 10 and 10 : 24

    c. 35 : 50 and 21 : 36

    d. 20 : 16 and 30 : 24

3. What is the ratio of a Canadian quarter (25¢) to a Canadian $5 bill, reduced to lowest terms?

4. What is the ratio of 12 minutes to 2 hours, reduced to lowest terms?

5. Ali can run 12 km in 40 minutes.

    a. Calculate his speed in km/hour.

    b. At this speed, how far can he run in 1.5 hours?

6. A car can travel 486 km using 45 litres of gas.

    a. Calculate the fuel efficiency of the car in km/litre.

    b. At this rate, how many litres of gas is required for a trip of 810 km?

7. Jeffrey and Gina were classmates who graduated together from college. Jeffrey found a job as a banker that pays him $189 for 9 hours and Gina found a job as a freelance artist that pays her $174 for 8 hours of work. Who is being paid a higher hourly rate and by how much?

8. Gregory purchased a racing motorbike, and Chris purchased a cruising motorbike. Jeffery travelled 765

km from Toronto to New York City in 8 hours and 20 minutes. Chris travelled 165 km from Toronto to Buffalo in 2 hours and 10 minutes. Based on this information, whose average speed was greater and by how much (in km/h)?

9. Which is the better buy based on the unit price: 360 grams for $2.99 or 480 grams for $3.75?

10. Which is the better buy based on the unit price: 125 grams for $4.75 or 175 grams for $5.95?

For problems 11 and 12, calculate the unit prices to determine which offer is best.

11. a. 3 kg of oranges for $7.99
     b. 4 kg of oranges for $9.99
     c. 5 kg of oranges for $11.99

12. a. 5 kg of rice for $4.99

     b. 8 kg of rice for $7.99

     c. 10 kg of rice for $9.49

13. If the ratio of sugar to flour in a pie is 3 : 5 and that of flour to eggs is 3 : 1, calculate the ratio of sugar : flour : eggs in the pie.

14. If the ratio of salespeople to marketing people in an organization is 5 : 4 and the ratio of marketing people to finance people is 5 : 2, what is the ratio of sales people : marketing people : finance people in the organization?

For problems 15 and 16, solve the proportion equations for the unknown value.

0. a. $x : 9 = 26 : 39$
    b. $16 : 24 = 12 : x$
    c. $x : 0.45 = 0.16 : 1.20$

0. a. $x : 15 = 24 : 36$
    b. $8 : 14 = x : 35$
    c. $12.5 : 70 = x : 1.4$

17. A car can travel 486 km using 45 litres of gas.

    a. Calculate the fuel efficiency of the car in km/litre.

    b. At this rate, how many litres of gas is required for a trip of 810 km?

18. A 450 gram loaf of bread costs $3.15 and has 15 slices.

    a. Determine the cost per 100 grams of bread..

    b. Determine the cost per slice of cheese.

19. The sales tax on an item costing $350.00 is $45.50. What will be the sales tax on an item costing $1,250.00?

20. Peter works 5.5 hours per day and his salary per day is $112.75. At this rate, how much will he receive if he works 7.5 hours per day?

21. Alexander, an investment banker, invests all his yearly earnings in stocks of high-tech, mining, and real-estate in the ratio of 4 : 5 : 3, respectively. Calculate his investment in mining stocks if his investment in high-tech stocks was $10,900.

22. The ratio of the driving distance from London to Hamilton, Mississauga, and Toronto is 3 : 4 : 5, respectively. If the distance from London to Hamilton is 125 km, calculate the distance from London to Mississauga and from London to Toronto.

23. Three college classmates, Khan, Thomas, and Lee decided to start a small business and invested $1,000, $2,500, and $3,500, respectively. If Lee decided to leave the business, how much would Khan and Thomas have to pay for Lee's shares if they wanted to maintain their initial investment ratio?

24. Calvin decided to build a shopping complex with his two brothers, Kevin and Alex. They invested $200,000, $350,000, and $450,000, respectively. After the complex was built, Kevin decided to sell his share of the investment to Calvin and Alex. How much would each of them have to pay if they wanted to maintain the same ratio of their investments in the complex?

25. *A*, *B*, and *C* invested a total of $900 in the ratio of 3 : 4 : 5, respectively, to purchase a billiards table for their club house. After the table was delivered, each invested an additional $200 to purchase balls and cue sticks. Calculate their new investment ratio after their additional investments.

26. Samuel, his wife, and his mother jointly purchased an estate for $1,350,000. Their individual investments in the estate were in the ratio of 5 : 3 : 1 , respectively Each of them decided to invest an additional $250,000 to develop the estate into a small family resort. Calculate their new investment ratio after the additional investments.

5. SELF-TEST

Answers to the questions are available at the end of the book.

1. Which is cheaper: a 240-gram box of cereal for $3.69, or a 360-gram box of cereal for $4.89?

2. A 500 km trip by car took 6 hours and 45 minutes. Calculate the average speed of the car in km/h.

3. The average speed of a car is 75 km/h. At this speed, how many hours will it take to travel 700 km?

For problems 4 and 5, solve the proportion equations for the unknown value.

4.
   0. $35 : 7 = 60 : x$

   a. $45 : x = 70 : 63$

5.
   0. $25 : 9 = x : 2.7$

   a. $x : 8 = 2\dfrac{1}{4} : 2\dfrac{1}{2}$

6. The scale on a map is 3 cm = 50 km. Calculate the actual distance between two cities that are 12.75 cm apart on the map.

7. An investment of $1,500 earned $820 in interest in one year. What amount should be invested at this rate to earn $1,459 in interest?

8. A car travelled 250 km. If the car's fuel efficiency is 9 litres per 100 km, calculate the amount of gas used for the trip.

9. Which offer is the best based on the unit price?

   a. 75 grams of chocolate for $1.49

   b. 100 grams of chocolate for $1.99

   c. 125 grams of chocolate for $2.25

10. Andrew's earnings to Bill's was in the ratio of 3 : 5 and Bill's to Cathy's was in the ratio of 4 : 6. What was the ratio of the earnings of Andrew : Bill : Cathy?

5. CASE STUDY - LIGHTNING WHOLESALE

Lightning Wholesale is a mid-to large-size company that works directly with manufacturers to bring sporting goods and equipment to mid to small-sized retailers. The company is in Southern Ontario, Canada. Lightning Wholesale acquires the goods, stores them in its warehouses, and distributes them to retailers, who then sell them to sporting enthusiasts across the province. Suppliers are global, including those from the United States.

Lightning Wholesale is a dynamic office with global networking, communications, customer service, and complex workflows. Lighting Wholesale depends on its office professionals to support staff and management. Office Professional tasks include clerical support, bookkeeping, customer service, data analysis, and document processing.

Lightning Wholesale cares about its employees and retailers. Where managers can assist their employees, they try to. Where retailers need guidance, Lightning Wholesales tries to fill this gap as best they can. In working with its small retailers, many of which are owned and operated by sole proprietors, Lightning Wholesale assists the owners with their pricing to help develop promotional pricing plans and the amounts of the payments that can be advertised. Lightning Wholesale freely offers help since it means more sales and increased profits for the company.

There are currently a few main tasks for Lightning Wholesale management:

- The payroll needs review for the past fiscal year.
- Costs and expenses need forecasting for the next fiscal year.
- Detailed analysis of office supply orders from quarter to quarter with recommendations for the next fiscal year.
- Forklifts need replacing in the warehouse.
- The next round of co-op students needs a computer.
- The employee appreciation event needs planning for next year.
- Travel expenses for the mobile team need to be summarized for the last fiscal year.

Questions

1. Management would like to analyze the data on dollars spent on employee professional development across the 3 departments – Office Professionals, Customer Service Representatives, and Project Managers. Office Professionals spent $5000, Customer Service Representatives spent $8000, and Project Managers spent $1500. Express the ratio of dollars spent on professional development in the 3 departments in its lowest terms.

2. Lightning Wholesale is analyzing its Canadian sales by region: 23% of sales were on the West Coast; 18% were in the Prairies, 45% were in central Canada, and 14% were in Atlantic Canada. Express and reduce the ratio of Lightning Wholesale's respective sales to the smallest term of one. Round to two decimals.

3. The bonus structure for Lightning Wholesale is 2% for Office Professionals, 3% for Customer Service Representatives, and 5% for Project Managers. If the Office Professionals received a bonus of $500, calculate how much Customer Service Representatives and Project Managers receive.

4. Recently, Lightning Wholesale has hired five additional staff. Management would like an estimate on how this will affect the budget for office supplies. Last year for 30 employees, $2,500 was spent on office supplies. What amount will be spent with a total of 35 employees?

PART VI
CHAPTER 6: PERCENTS

OBJECTIVES

- Convert percents to equivalent fractions and decimal numbers.
- Solve percent problems using different methods.
- Calculate bases, rates, or portions of quantities expressed in percents.

CHAPTER OUTLINE

6.1 Percents

6. Review Exercises

6. Self-Test Exercises

6. Case Study – Lightning Wholesale

Introduction

In Chapter 2, we learned that fractions and decimal numbers represent portions of a whole number or quantity. In this chapter, we will learn about another form used to represent a part of a whole: **percents**.

Percents are used to represent the number of parts per one hundred. Due to their simplicity, percents are a widely accepted measure for expressing fractions and decimal numbers. They make comparisons easy and are used often in our daily lives and in making business decisions; some examples of the use of percents include the interest rate charged by a bank on a loan, the discount offered at a store, the sales tax charged on items purchased, and the commission received by a sales representative.

In this chapter, we will learn the relationship between fractions, decimal numbers, and percents and learn to solve problems related to percents, including business applications.

Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of Mathematics* (3rd ed.) by Thambyrajah Kugathasan, published by Vretta-Lyryx Inc., with permission. Adaptations include supplementing existing material and reordering chapters.

6.1 PERCENTS

> **Percent** (per cent or per hundred in the literal meaning) is used to express a quantity out of 100 units and is represented by the symbol '%'.

For example, 5% means 5 **per** hundred, or 5 out of 100, or $\dfrac{5}{100}$ $\left(5\% = \dfrac{5}{100} = 0.05\right)$.

- 100% means 100 out of 100 (i.e, the whole quantity)
- 75% means 75 out of 100 (i.e., $\dfrac{75}{100} = \dfrac{3}{4}$ or 0.75)
- 50% means 50 out of 100 (i.e., $\dfrac{50}{100} = \dfrac{1}{2}$ or 0.50)
- 25% means 25 out of 100 (i.e., $\dfrac{25}{100} = \dfrac{1}{4}$ or 0.25)

Note: A percent greater than 100 is typically expressed as a comparison to the original amount, rather than as parts of a whole.

For example, 200% means 2 times (or double, or twice) the whole quantity; although technically correct, it is not meaningful to say 200% means 200 out of 100 (since the total number of parts exceeds the whole). Similarly, 350% means $3\dfrac{1}{2}$ times the whole quanity.

Relationship Among Percents, Fractions, and Decimal Numbers

Fractions and decimal numbers can be converted to percents and vice-versa. For example, 3 out of 4 equal parts of a quantity can be represented as a fraction, decimal number, or in percent form, as follows:

$$\frac{3}{4}$$

| $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
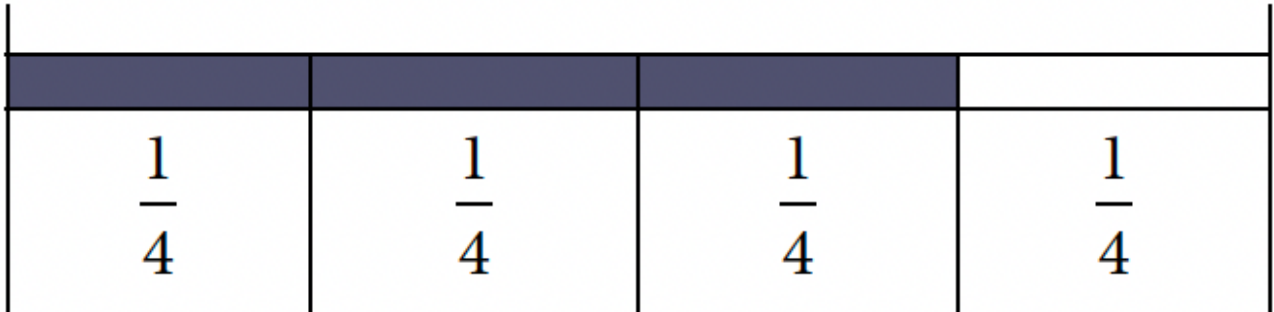
As a fraction, it is $\frac{3}{4}$.

$$0.75$$

| 0.25 | 0.25 | 0.25 | 0.25 |
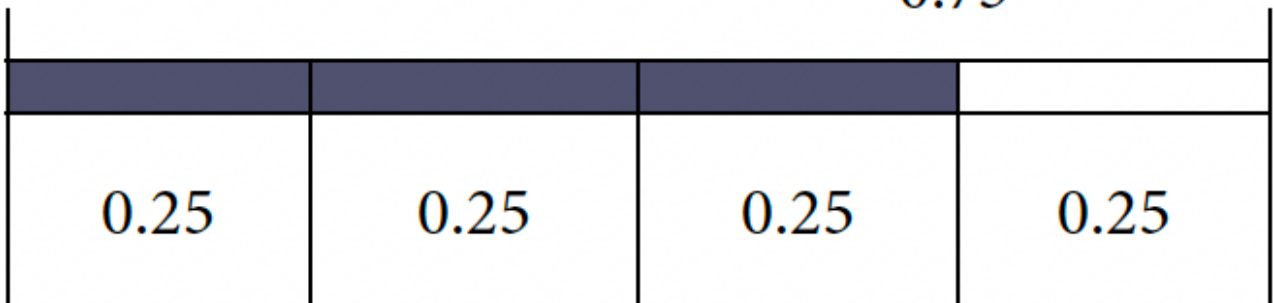
As a decimal, it is 0.75.

$$75\%$$

| 25% | 25% | 25% | 25% |
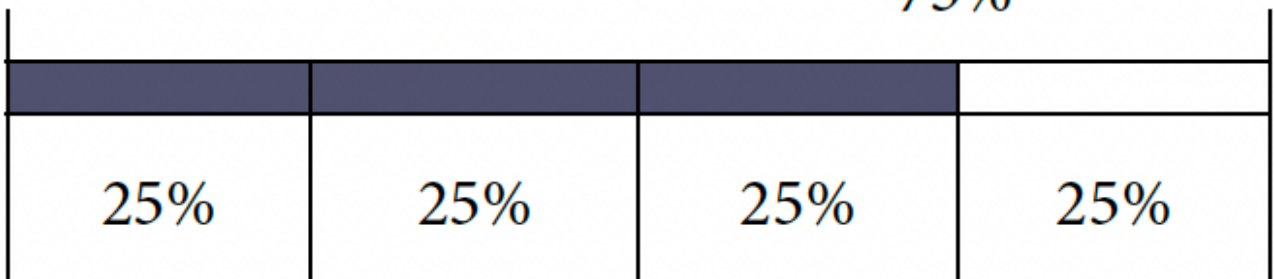
As a percent, it is 75%.

In day-to-day business, percents are commonly used to represent interest rates, sales, discounts, commissions, comparison of changes in quantity, etc. However, in actual calculations, fractions or decimal equivalents are used. Therefore, it is necessary to know the methods for converting from one form to the other.

Converting Percents to Decimal Numbers

If the percent is a whole number or a decimal number, remove the '%' sign and move the decimal point two places to the left. This is the same as dividing the number by 100 and removing the '%' sign.

For example,

- To convert 45% to a decimal number, 45% = 45.0. Remove the % sign and move the decimal point two places to the left.= 0.45. This is the same as $\dfrac{45\%}{100\%} = 0.45$. Therefore 45% = 0.45.

- To convert 0.38% to a decimal number, 0.38% = 0.38. Remove the % sign and move the decimal point two places to the left. = 0.0038. This is the same as $\dfrac{0.38\%}{100\%} = 0.0038$. Therefore 0.38% = 0.0038.

If the percent is a fraction or a mixed number, change it to its decimal equivalent and follow the steps as shown above.

For example,

- To convert $2\dfrac{1}{2}\%$ to a decimal number, $2\dfrac{1}{2}\% = 2.5\%$. Convert the mixed number to a decimal number.= 2.5. Remove the % sign and move the decimal point two places to the left = 0.025. This is the same as $\dfrac{2.5\%}{100\%} = 0.025$. Therefore $2\dfrac{1}{2}\% = 0.025$.

Example 6.1-a: Converting Percents to Decimal Numbers

Convert each percent to its equivalent decimal number.

a. 85%

b. $5\dfrac{1}{4}\%$

c. 20.75%

d. 225%

e. $\dfrac{2}{3}\%$

Solution

a. $85\% = 85.0\% = 85.0 = 0.85$

b. $5\dfrac{1}{4}\% = 5.25\% = 5.25 = 0.0525$

c. $20.75\% = 20.75 = 0.2075$

d. $225\% = 225.0\% = 225.0 = 2.25$

e. $\dfrac{2}{3}\% = 0.666666...\% = 0.\overline{6}\% = 0.\overline{6} = 0.00\overline{6}$

Converting Decimal Numbers to Percents

To convert a decimal number or a whole number to a percent, move the decimal point two places to the right and insert the '%' sign. This is the same as multiplying the number by 100 and inserting the '%' sign.

For example,

- To convert 0.35 to a percent, 0.35 = 0.35%. Move the decimal point two places to the right and insert the % sign.= 35%. This is the same as $0.35 \times 100\% = 35\%$. Therefore 0.35 = 35%.
- To convert 5 to a percent, 5 = 5.00%. Move the decimal point two places to the right and insert the % sign.= 500%. This is the same as $5 \times 100\% = 500\%$. Therefore, 5 = 500%.

Example 6.1-b: Converting Decimal Numbers to Percents

Convert each decimal number to its equivalent percent.

a. 0.45
b. 0.03
c. 5.25
d. 0.002

Solution

a. $0.45 = 0.45\% = 45\%$
b. $0.03 = 0.03\% = 3\%$
c. $5.25 = 5.25\% = 525\%$
d. $0.002 = 0.002\% = 0.2\%$

Example 6.1-c: Application of Converting from Decimal to Percent

If Alexander earns three times the amount that Emma earns, what percent is Alexander's earnings compared to Ema's earnings?

Solution

Here, '3 times' when converted to a percent $= 3 \times 100\% = 300\%$.

Therefore, Alexander's earnings are 300% of Emma's.

*Note: This also means that Alexander earns 200% **more than** Emma. It is important not to confuse these two statements!*

Converting Percents to Fractions

> **If the percent is a whole number**, remove the percent sign and divide by 100 (or multiply by $\frac{1}{100}$). Reduce the resulting fraction to its lowest terms.

For example, to convert 60% to a fraction in its lowest terms:

$$60\% = \frac{60}{100}$$

Reduce the fraction to its lowest terms.

$$= \frac{3}{5}$$

Therefore, $60\% = \frac{3}{5}$.

> **If the percent is a decimal number**, remove the percent sign and divide by 100; eliminate the decimal in the numerator of the resulting fraction by multiplying both the numerator and denominator by an appropriate power of 10. Reduce the fraction to its lowest terms.

For example, to convert 42.5% to a fraction:

$$42.5\% = \frac{42.5}{100}$$

Eliminate the decimal in the numerator by multiplying both the numerator and the denominator by 10.

$$= \frac{425}{1,000}$$

Reduce the fraction to its lowest terms.

$$= \frac{17}{40}$$

Therefore, $42.5\% = \dfrac{17}{40}.$

> **If the percent is a fraction or a mixed number**, either convert it to its decimal equivalent and follow the steps outlined above, or convert it to an improper fraction (if required), remove the percent sign, and multiply the fraction by $\dfrac{1}{100}$.

For example, to convert $6\dfrac{1}{2}\%$ to a fraction,

Method 1:

By converting it to its decimal equivalent.

$$6\frac{1}{2}\% = 6.5\%$$

Remove the percent sign and divide by 100.

$$= \frac{6.5}{100}$$

Eliminate the decimal in the numerator by multiplying both the numerator and the denominator by 10.

$$= \frac{65}{1,000}$$

Reduce the fraction to its lowest terms.

$$= \frac{13}{200}$$

Method 2:

By converting it to its improper fractional equivalent.

$$6\frac{1}{2}\% = \frac{13}{2}\%$$

Remove the percent sign and multiply the fraction by $\dfrac{1}{100}.$

$$= \frac{13}{2} \times \frac{1}{100}$$

Simplify.

$$= \frac{13}{200}$$

Therefore, $6\frac{1}{2}\% = \frac{13}{200}$.

Example 6.1-d: Converting Percents to Fractions

Convert each percent to its equivalent fraction or mixed number and simplify to lowest terms.

    a. 45%

    b. $8\frac{1}{3}\%$

    c. 6.25%

    d. 175%

    e. $\frac{1}{5}\%$

Solution

a. $45\% = \dfrac{45}{100} = \dfrac{9}{20}$

b. $8\dfrac{1}{3}\% = \dfrac{25}{3}\% = \dfrac{25}{3} \times \dfrac{1}{100} = \dfrac{1}{3} \times \dfrac{1}{4} = \dfrac{1}{12}$

c. $6.25\% = \dfrac{6.25}{100} = \dfrac{6.25}{100} \times \dfrac{100}{100} = \dfrac{625}{10,000} = \dfrac{1}{16}$

d. $175\% = \dfrac{175}{100} = \dfrac{7}{4} = 1\dfrac{3}{4}$

e. $\dfrac{1}{5}\% = \dfrac{1}{5} \times \dfrac{1}{100} = \dfrac{1}{500}$

Converting Fractions or Mixed Numbers to Percents

To convert a fraction or a mixed number to a percent, first convert the fraction or the mixed number to a decimal number. Then, convert the decimal number to a percent by moving the decimal point two places to the right and inserting the '%' sign. This is the same as multiplying the decimal by 100 and inserting the '%' sign.

For example,

- To convert $\dfrac{3}{8}$ to a percent, $\dfrac{3}{8} = 0.375$. Convert the fraction to its decimal equivalent. $= 0.375\%$.
  Convert the decimal to percent, by moving the decimal point two places to the right and inserting the %
  sign. $= 37.5\%$. This is the same as $0.375 \times 100\% = 37.5\%$. Therefore $\dfrac{3}{8} = 37.5\%$.

- To convert $5\dfrac{1}{2}$ to a percent, $5\dfrac{1}{2} = 5.50$. Convert the fraction to its decimal equivalent. $= 5.50\%$.
  Convert the decimal to percent, by moving the decimal point two places to the right and inserting the %
  sign. $= 550\%$. This is the same as $5.50 \times 100\% = 550\%$. Therefore $5\dfrac{1}{2} = 550\%$.

Example 6.1-e: Converting Fractions or Mixed Numbers to Percents

Convert each of the following fractions to its equivalent percent.

a. $\dfrac{3}{25}$

b. $5\dfrac{1}{4}$

c. $\dfrac{18}{5}$

d. $\dfrac{1}{200}$

Solution

a. $\dfrac{3}{25} = 0.12 = 0.12\% = 12\%$

b. $5\dfrac{1}{4} = 5.25 = 5.25\% = 525\%$

c. $\dfrac{18}{5} = 3.60 = 3.60\% = 360\%$

d. $\dfrac{1}{200} = 0.005 = 0.005\% = 0.5\%$

Example 6.1-f: Application of Converting from Fractions to Percents

Peter and Angela study Business Mathematics but at different colleges. Peter scored 46 out of 60 on his final exam, while Angela scored 63 out of 75 on her exam. Who received a better grade on their exam?

Solution

It is not possible to answer the question by simple observation, because Peter's score is expressed out of 60, while Angela's score is expressed out of 75. To compare their scores, we can convert them to their percent equivalents, as shown below:

Peter's score: $\dfrac{46}{60} = 0.766666\ldots = 76.67\%$

Angela's score: $\dfrac{63}{75} = 0.84 = 84\%$

Therefore, Angela received a better grade than Peter on the exam.

Example 6.1-g: Comparing Fractions and Percents

Out of 54 students in Class A, 43 students passed the final exam. In Class B, 78% of the students passed the final exam. Which class had the better pass rate?

Solution

Class A:

$$\text{Fraction of students passed} = \frac{43}{54}$$

$$\text{Percent of students passed} = \frac{43}{54} \times 100\%$$

$$= 0.796296... \times 100\%$$

$$= 79.63\%$$

Class B:

$$\text{Percent of students passed} = 78\%$$

Therefore, Class A had a better pass rate.

Solving Percent Problems

Many methods may be used to solve percent problems. Described below are three common methods:

Method 1: Formula Method

Every percent problem will contain three variables: B, P, and R.

Base (B): Whole quantity or value (100%). It usually follows the word 'of', or 'percent of'.

Portion (P): Portion of the whole quantity or value (i.e., portion of the base).

Rate (R): Relationship between the base and portion, usually expressed as a percent. It usually carries the percent sign (%) or the word '*percent*'.

Every percent statement can be expressed as: **P is R% of B.** The value of R is used as a decimal or fractional equivalent in calculations.

The relationship between these variables can be expressed as follows:

Portion = *Rate* × *Base*

Or

Formula 6.1: Portion

$$P = R \times B$$

Rearranging, we obtain $R = \dfrac{P}{B}$ and $B = \dfrac{P}{R}$.

Therefore, if any two of these quantities are known, then the third quantity can be calculated.

P, R, B triangle

Here is a triangle that can be used to help in rearranging the formula $P = R \times B$ to solve for the variable R or B.

Variables beside each other at the bottom are multiplied, ($R \times B$, as shown).

Variable P is divided by the variables at the bottom.

Cover the variable for which you want to solve to see the new formula.

For example, if you want to solve for R, the formula can be found by covering R and reading the remaining variables in the triangle, to obtain $R = \dfrac{P}{B}$
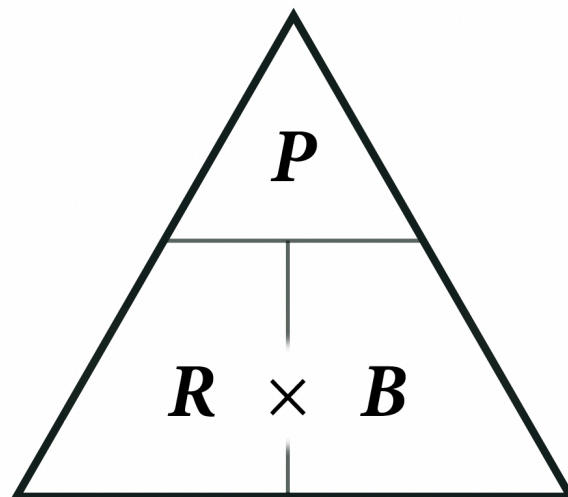
Method 2: Algebraic Method

Solving problems using algebraic methods involve forming an equation for an unknown 'x' and then solving for 'x'. In this method, we assume that the unknown is 'x', and form equation(s) for the given problem (or statement) using key words to represent certain arithmetic operations.

The following table shows key words and phrases that are commonly used to indicate various arithmetic operations.

Operations and Corresponding Keywords or Phrases

Operation	Key Words or Phrases
Addition (+)	Add, sum, total, and, plus, more than, increased by, appreciate, rise
Subtraction (−)	Subtract, difference, minus, less than, decreased by, depreciate, fall
Multiplication (× or () or ·)	Multiply, product, times, of
Division (÷ or /)	Divide, ratio, divided by, quotient, per
Equal (=)	Is, was, gives, given by
Unknown value (denoted by a variable, such as 'x')	What, how much

For example, we can use these key words to form the algebraic equations for the following:

- What percent of 200 is 60? $x\% \times 200 = 60$
- $52 is 13% of what amount? $\$52 = 13\%(x)$
- 40% of what amount is $280? $40\%(x) = \$280$
- How much is 2% of 75? $x = 2\% \times 75$

Method 3: Ratio-Proportion Method

The whole amount or quantity is represented by 100% and is known as the **base (B)**. The **portion (P)** is a part of the base and it forms a percent, or **rate (R%)**, of the base. Thus, in the ratio proportion method, we first identify the base, portion, and rate in the problem, then solve for the unknown using the following proportion equation:

Portion : Base = Rate : 100%

$$P : B = R\% : 100\%$$
$$\frac{P}{B} = \frac{R\%}{100\%}$$

$$\frac{P}{B} = \frac{R}{100}$$

Example 6.1-h: Calculating the Portion of a Whole Quantity

What is 75% of $250? (Or, 75% of $250 is how much?)

Solution

Method 1: Using the Formula Method

$R\%$ = 75% (value with % sign); i.e., R = 0.75

B = $250 (value that follows the word 'of')

P = ? (the other value)

Using Formula 6.1,

$P = R \times B$

Substituting values for R and B, P = 0.75 × 250.00

Simplifying,

= $187.50

Method 2: Using the Algebraic Method

What is 75% of $250?

x = 75% × 250.00

Expressing the percent as a decimal,

x = 0.75 × 250.00

Simplifying,

= $187.50

Method 3: Using the Ratio-Proportion Method

Here, the whole amount of $250 is the base and is represented by 100%. 75% is a portion of the base, as illustrated:

Portion : Base = Rate : 100%

$$P : B = R\% : 100\%$$

$$P : \$250 = 75\% : 100\%$$

$$\frac{P}{\$250} = \frac{75\%}{100\%}$$

$$\frac{P}{250.00} = \frac{75}{100}$$

Cross-multiplying,

$$100P = 250.00 \times 75$$

Solving for P,

$$P = \frac{250.00 \times 75}{100}$$

Simplifying,

$$= \$187.50$$

Therefore, 75% of $250.00 is $187.50.

Note: The following examples use the formula method; however, you can solve them using any of the three methods described above.

Example 6.1-i: Calculating Portion When Rate is More Than 100%

What is 150% of 200?

Solution

$R\%$ = 150% (value with % sign); i.e., $R = 1.50$

$B = 200$ (value that follows the word 'of')

$P = ?$ (the other value)

Using Formula 6.1,

$P = R \times B$

Substituting values for R and B,

$P = 1.50 \times 200$

Simplifying,

$= 300$

Therefore, 150% of 200 is 300.

Example 6.1-j: Calculating Rate When Portion and Base are Known

What percent of $775 is $1,250? (Or, $1,250 is what percent of $775?)

Solution

B = $775 (value that follows the word 'of')

P = $1,250 (the other value)

R = ? (value for the word 'percent')

Using Formula 6.1,

$P = R \times B$

Rearranging for R,

$$R = \frac{P}{B}$$

Substituting values for P and B,

$$R = \frac{1,250.00}{775.00}$$

Simplifying,

$R = 1.612903...$

Converting the decimal to a percent,

$R\% = 1.612903... \times 100\%$

$= 161.29\%$

Therefore, 161.29% of $775.00 is $1,250.00.

Note: It is tempting to treat $1,250 as the base since it is the larger value. However, in this question $775 is the base since it follows the word 'of'. This illustrates the importance of reading these types of questions carefully!

Example 6.1-K: Calculating Base When Portion and Rate are Known

50% of what number is 200?

Solution

$R\% = 50\%$ (value with % sign); i.e., $R = 0.50$

$P = 200$ (the other value)

$B = ?$ (value that follows the word 'of')

Using Formula 6.1,

$P = R \times B$

Rearranging for B,

$$B = \frac{P}{R}$$

Substituting values for P and R,

$$B = \frac{200}{0.50}$$

Simplifying,

$$= 400$$

Therefore, 50% of 400 is 200.

Example 6.1-l: Application Problem with a Fractional Rate

Sandra owns $25\frac{1}{3}\%$ of a web development company. If the company is valued at $180,000 what is the value of Sandra's ownership in the company?

Solution

$R\% = 25\dfrac{1}{3}\%$ (value with % sign) $= 25.333333...$; i.e., $R = 0.253333...$

$B = \$180,000$ (the whole value)

P? (the other value)

Using Formula 6.1,

$P = R \times B$

Substituting values for R and B,

$= 0.253333... \times 180,000.00$

Simplifying,

$= \$45,600.00$

Alternatively, instead of performing calculations with repeating decimals, this question can be solved by converting the percent into a fraction:

$R\% = 25\dfrac{1}{3}\%$

Converting the mixed number into an improper fraction,

$= \dfrac{76}{3}\%$

Converting the percent into a fraction,

$R = \dfrac{76}{3} \times \dfrac{1}{100}$

$= \dfrac{76}{300}$

Using Formula 6.1,

$P = R \times B$

Substituting values for R and B,

$= \dfrac{76}{300} \times 180,000.00$

Simplifying,

$= \$45,600.00$

Therefore, the value of Sandra's ownership in the company is $45,600.00.

6.1 Exercises

For problems 1 to 18, calculate the missing values (indicated with a question mark) in the tables below.

1.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	75%	?	?
b.	?	0.30	?
c.	?	?	$\dfrac{1}{4}$

2.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	50%	?	?
b.	?	0.70	?
c.	?	?	$\dfrac{1}{5}$

3.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	5%	?	?
b.	?	0.2	?
c.	?	?	$\dfrac{3}{5}$

4.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	2%	?	?
b.	?	0.8	?
c.	?	?	$\dfrac{3}{8}$

5.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	150%	?	?
b.	?	0.175	?
c.	?	?	$\dfrac{12}{25}$

6.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	225%	?	?
b.	?	0.225	?
c.	?	?	$\dfrac{7}{20}$

7.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	12.5%	?	?
b.	?	0.05	?
c.	?	?	$4\dfrac{1}{2}$

8.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	7.5%	?	?
b.	?	0.03	?
c.	?	?	$3\dfrac{3}{4}$

9.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	0.6%	?	?
b.	?	0.005	?
c.	?	?	$4\dfrac{3}{5}$

10.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	0.8%	?	?
b.	?	0.003	?
c.	?	?	$1\dfrac{1}{5}$

11.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	0.05%	?	?
b.	?	0.0025	?
c.	?	?	$1\dfrac{1}{8}$

12.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	0.08%	?	?
b.	?	0.075	?
c.	?	?	$2\dfrac{3}{8}$

13.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	$\dfrac{3}{5}\%$	?	?
b.	?	1.08	?
c.	?	?	$\dfrac{3}{55}$

14.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	$\dfrac{3}{8}\%$	?	?
b.	?	2.04	
c.	?	?	$\dfrac{22}{75}$

15.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	$1\dfrac{3}{4}\%$	?	?
b.	?	2.025	?
c.	?	?	$\dfrac{1}{400}$

16.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	$2\dfrac{3}{4}\%$	?	?
b.	?	1.075	?
c.	?	?	$\dfrac{1}{250}$

17.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	$6\dfrac{1}{2}\%$	?	?
b.	?	2.5	?
c.	?	?	$\dfrac{4}{25}$

18.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	$10\dfrac{3}{5}\%$	?	?
b.	?	7.5	?
c.	?	?	$\dfrac{13}{50}$

For the following problems, express the answers rounded to two decimal places, wherever applicable.

For problems 19 to 48, calculate the value.

19. a. 20% of 350

   b. 12.5% of 800

20. a. 45% of 180

   b. 2.5% of 960

21. a. 0.25% of 75

   b. $\dfrac{1}{4}\%$ of 200 km

22. a. 0.755% of 120

    b. $\dfrac{1}{8}\%$ of 450 km

23. a. 130% of 40

    b. $5\dfrac{1}{3}\%$ of $1,000

24. a. 285% of 110

    b. $12\dfrac{3}{4}\%$ of $1,260

25. What is 2.5% of 80?

26. What is 40% of 160?

27. $8\dfrac{1}{4}\%$ of $200 is how much?

28. $25\dfrac{3}{4}\%$ of $2,680 is how much?

29. How much is $\dfrac{1}{4}\%$ of $108?

30. How much is $\dfrac{3}{4}\%$ of 350 kg?

31. What number is 125% of 6?

32. What number is 250% of 12?

33. 12 is what percent of 30?

34. 18 is what percent of 40?

35. What percent of 4 is 16?

36. What percent of 9 is 45?

37. What percent of 220 is 100?

38. What percent of 22.10 is 110.50?

39. 280 metres is what percent of a kilometre?

40. 180 grams is what percent of 3 kilograms?

41. 400 is 50% of what number?

42. 225 is 25% of what number?

43. 15% of what amount is $27.90?

44. 30% of what amount is 708?

45. 120% of what amount is 156?

46. 215% of what amount is 258?

47. $16.50 is 0.75% of what amount?

48. $16.40 is 0.5% of what amount?

49. How much tax was charged on a table that costs $250, if the tax rate is 13%?

50. The monthly gross salary of an employee is $6,250. 26% of the salary was deducted for taxes. How much money was deducted for taxes?

51. 5% of the commission on sales was $1,250. How much was the sales amount?

52. 3% interest on a loan was $210. How much was the loan?

53. In a survey of 450 people, 117 responded 'yes'. What percent of the people surveyed responded 'no'?

54. 144 out of 600 students took Business Mathematics. What percent of students did not take the course?

55. A company that makes games sets sales targets at $280,000 per year for each of its salespeople. If Amanda, an excellent salesperson, achieved 250% of her target this year, calculate her sales for the year.

56. If the population of Canada was estimated to be 36,910,000 in May 2018, and the population of Ontario was estimated to be $38\frac{1}{4}\%$ of Canada's population, calculate the estimated population of Ontario.

57. When there was a boom in the real estate market, Lucy sold her property for $410,440, which was 130% of the amount she originally paid. Calculate the amount she originally paid for the property.

58. Ronald, an investment banker sold his shares for $18,568.50 when there was a boom in the stock market. Calculate the amount he paid for the shares if his selling price was 180.65% of the amount he paid for the shares originally.

59. Evan, a business development representative of a leading pharmaceutical firm, took his client out for a dinner that cost $180.75 before taxes. If the tax was $23.50, calculate the tax rate.

60. A leading information technology company donated $87,790 out of its 2017 fiscal revenue of $17,558,643 towards socially-responsible causes. What percent of its revenue did it contribute towards these causes?

61. Neel Plastics Manufacturing Corporation targets to obtain $120,000 of funding from their investors to purchase new machinery. If they were only able to obtain 25.5% of their total target, calculate the amount of money that is yet to be received.

62. Pamela and Martha run a business that made a profit of $12,750. As Pamela invested a higher amount in the business, she received 57.5% of the profits and Martha received the remaining. Calculate Martha's share of the profit, in dollars.

6. REVIEW EXERCISES

Answers to the odd-numbered questions are available at the end of the book.

For problems 1 and 2, calculate the missing values (indicated with a question mark) in the tables below.

1.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	80%	?	?
b.	?	0.25	?
c.	?	?	$1\frac{1}{2}$
d.	$6\frac{1}{3}\%$	?	?
e.	?	0.048	?
f.	?	?	$\frac{2}{25}$

2.

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	2%	?	?
b.	?	0.245	?
c.	?	?	$\frac{5}{12}$
d.	$12\frac{2}{5}\%$	?	?
e.	?	1.075	?
f.	?	?	$\frac{3}{80}$

For the following problems, express the answers rounded to two decimal places, wherever applicable.

For problems 3 and 4, calculate the value.

3. a. 125% of what number is 45?

   b. What percent of $180 is $36?

   c. How much is $\frac{3}{8}$% of $60?

4. a. 225% of what number is 180?

   b. What percent of $750 is $300?

   c. How much is $\frac{2}{5}$% of $30?

5. Paul sold a property for $575,000, which was 125% of the purchase price. Calculate the purchase price.

6. Peter sold his shares for $14,437.50. Calculate the amount he paid for the shares if his selling price was 275% of the amount he paid for the shares.

7. Lian scored 45 out of 60 on a math test. What was his percent grade on the test?

8. There were 48 questions in a test. Ann answered 40 questions correctly. What percent of the questions did she answer correctly?

9. The total expenditure for the construction of a highway was $1,280,000. If this is 111% of the budgeted amount, calculate the amount budgeted to build the highway.

10. If Henry's business expenditures were $14,480 in March and $14,806.50 in April, which were 112% and 122% of his budgeted expenditure for March and April respectively, calculate his total budgeted expenditure for the two months.

11. Assume that out of the 350,000 people who immigrated to Canada in 2016, 12.25% were from China, 9.75% were from the Philippines, and the rest were from other countries.

    a. Calculate the number of people who immigrated to Canada from China.

    b. If the combined number of immigrants from China and the Philippines constituted 0.2% of the population of Canada, calculate the population of Canada in 2016. (Round the answer up to the

next whole number.)

12. Dinner at a restaurant cost you $27.80 and you tipped the waiter 15% of the cost.

   a. What was the value of the tip?

   b. If the tip you gave the waiter was 2% of all the money he made from tips that night, calculate the

      amount that the waiter earned from tips that night.

6. SELF-TEST

Answers to the questions are available at the end of the book.

1. Calculate the missing values indicated by a question mark in the table below:

Question	Percents	Decimals	Fractions (or mixed numbers) in lowest terms
a.	$10\dfrac{3}{5}\%$	?	?
b.	?	2.25	?
c.	?	?	$\dfrac{1}{400}$
d.	$\dfrac{1}{2}\%$	?	?
e.	?	0.002	?
f.	?	?	$\dfrac{53}{200}$

2. a. What percent of $30 is $3.75?

   b. 22.5% of $1,500 is how much?

   c. $75 is 15% of what number?

3. Colton's store expenses for the month of July and August were $33,480 and $36,580, which were 110% and 90% of the budgeted expenditure for the months of July and August, respectively. Calculate the total budgeted expenses for the two months.

4. Sandra earns an annual salary of $60,000. Every month, she spends $1,400 on rent, $600 on car expenses, $200 on loan repayment, $800 on miscellaneous expenses, and saves $2,000.

   a. What percent of her annual salary is her annual expenses?

   b. If she invests 25% of the $2,000 savings in a mutual fund every month, what percent of the annual

salary is invested in the fund over the year?

6. CASE STUDY - LIGHTNING WHOLESALE

Lightning Wholesale is a mid-to large-size company that works directly with manufacturers to bring sporting goods and equipment to mid to small-sized retailers. The company is in Southern Ontario, Canada. Lightning Wholesale acquires the goods, stores them in its warehouses, and distributes them to retailers, who then sell them to sporting enthusiasts across the province. Suppliers are global, including those from the United States.

Lightning Wholesale is a dynamic office with global networking, communications, customer service, and complex workflows. Lighting Wholesale depends on its office professionals to support staff and management. Office Professional tasks include clerical support, bookkeeping, customer service, data analysis, and document processing.

Lightning Wholesale cares about its employees and retailers. Where managers can assist their employees, they try to. Where retailers need guidance, Lightning Wholesales tries to fill this gap as best they can. In working with its small retailers, many of which are owned and operated by sole proprietors, Lightning Wholesale assists the owners with their pricing to help develop promotional pricing plans and the amounts of the payments that can be advertised. Lightning Wholesale freely offers help since it means more sales and increased profits for the company.

There are currently a few main tasks for Lightning Wholesale management:

- The payroll needs review for the past fiscal year.
- Costs and expenses need forecasting for the next fiscal year.
- Detailed analysis of office supply orders from quarter to quarter with recommendations for the next fiscal year.
- Forklifts need replacing in the warehouse.
- The next round of co-op students needs a computer.
- The employee appreciation event needs planning for next year.
- Travel expenses for the mobile team need to be summarized for the last fiscal year.

Questions

1. If an employee earns a gross salary of $50,000 annually and the federal tax withholding rate is 20%, how much will be withheld for federal taxes from their monthly paycheck?

2. An employee's gross salary is $60,000 per year. If they contribute 8% of their salary to their RRSP

(retirement savings), how much money will they contribute to the plan annually?

3. Employees are eligible for a performance bonus of 5% of their annual salary. If the bonus amount is $2700, what is their annual salary?

4. An employee's monthly health insurance premium is $150. Their employer covers 75% of the premium cost. How much will the employee pay monthly for their health insurance?

5. Lighting Wholesale needs to purchase a new forklift for the warehouse. The forklift they purchased 5 years ago was $170,000. A new forklift bought today is priced at 134.87% of the price of the forklift purchased 5 years ago. How much is the price of the new forklift? If they can sell the old forklift for $90,000, how much will they need to purchase the new forklift?

PART VII
CHAPTER 7 - BASIC ALGEBRA

OBJECTIVES

- Perform basic arithmetic operations on algebraic expressions.
- Set up basic linear equations with one variable.
- Solve linear equations with one variable using various arithmetic operations.
- Create, rearrange, and use equations to solve for unknown variables.

CHAPTER OUTLINE

Introduction

Algebra is a branch of mathematics used to analyze and solve practical problems using letters and symbols, known as variables, to represent numbers. These variables, numbers, and operations can be combined to create algebraic expressions and equations. Algebra provides a framework from which formulas are derived to solve general problems and helps develop logical thinking and problem-solving skills systematically and analytically. Algebra is required in any occupational field, including business and sciences.

This chapter will teach us how to evaluate and perform arithmetic with algebraic expressions, form and solve algebraic equations, and rearrange equations and formulas to isolate a particular variable.

Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of Mathematics* (3rd ed.) by Thambyrajah Kugathasan, published by Vretta-Lyryx Inc., with permission. Adaptations include supplementing existing material and reordering chapters.

7.1 ALGEBRAIC EXPRESSIONS

In arithmetic, we use numbers and operations in expressions. The following are examples of arithmetic expressions:

$$25 + 15, \quad 75 - 22, \quad 8 \times 9, \quad \frac{9}{5}$$

In algebra, we use numbers, operations, and **variables** (letters and symbols that represent various numbers) in expressions. The following are examples of **algebraic expressions**:

$$2x + 5, \quad 30 - 5y, \quad 6(2a + 5), \quad \frac{b + 3}{2}$$

We can use algebra to translate a statement (using words) into an expression (using numbers, operations, and variables). The following keywords will help in translating written statements into algebraic expressions:

Table 7.1: Arithmetic Operations and their Meanings

Operation	Key Words
Addition (+)	add, sum, total, and, plus, more than, increased by, appreciate, rise
Subtraction (−)	subtract, difference, minus, less than, decreased by, depreciate, fall
Multiplication (×), (·)	multiply, product, times, of
Division (÷)	divide, ratio, divided by, quotient, per

Examples of Terminology and Corresponding Algebraic Equations

Expressed In Words	Expressed as an Algebraic Expression
Ten more than a number	$x + 10$
A number more than ten	$10 + x$
Twenty less than a number	$x - 20$
A number less than twenty	$20 - x$
Product of five and a number	$5x$
Divide a number by twenty	$\dfrac{x}{20}$
Divide twenty by a number	$\dfrac{20}{x}$
Half of a number	$\dfrac{1}{2}x$ or $\dfrac{x}{2}$
Twice a number	$2x$
Ten more than the product of two numbers	$xy + 10$
'x' less than 'y' or 'y' minus 'x'	$y - x$
'y' less than 'x' or 'x' minus 'y'	$x - y$
Seventy decreased by three times a number	$70 - 3x$
'm' subtracted from 'n'	$n - m$

Table 7.2 Terminology Used in Algebraic Expressions

Terminology	Description	Examples
Variable	A number, variable, or a combination of numbers and variables which are multiplied or divided together.	x, y, a, b In the expression $2m + 5n - 6$, m and n are variables.
Term	A letter or symbol used in expressions to represent a varying or unknown quantity.	$5, x, 5x^2y, 2xy, \dfrac{4}{a}, \dfrac{b}{3}$ are all single terms. The expression $5x + y$ has 2 terms. The expression $\dfrac{x}{4} - y^2 + \dfrac{x}{y} - \dfrac{1}{x}$ has 4 terms.
Constant	A term that only has a number with no variables.	In the expression $2x + 3y + 5$ the 3rd term, +5, is a constant. In the expression $5x^2 - 8$ the 2nd term, –8, is a constant.
Coefficient	The product of all numerical factors in a single term involving a variable. If a coefficient is not present, its value is 1.	In the expression $x3 + 5x2 - 3(2y3)$, the coefficient of the 1st term is 1, the coefficient of the 2nd term is 5, and the coefficient of the 3rd term is –6.
Expression	A mathematical phrase made up of a combination of terms and operations.	Expressions with one variable: $(2x + 5), (9x - 3)$ Expressions with two variables: $(5x - 7y + 5)$, $(xy + 3x + 7)$
Like terms	Terms that have the same variables and exponents. They differ only in their numerical coefficient. Constant terms are like terms.	$5x$ and $5x$ are like terms. $30a^2, -4a^2$, and $9a^2$ are like terms. 5 and -9 are like terms.
Unlike terms	Terms that have different variables or the same variables with different exponents.	$12y$ and $3y^2$ are unlike terms. x, y, and 1 are unlike terms. $5xy, -3x^2y$, and $7xy^2$ are unlike terms.
Factors	Refer to each of the combinations of variables and/or numbers multiplied together in a term.	5 and x are factors of the term $5x$. $3, x$, and y are factors of the term $3xy$.
Monomial	An algebraic expression that has only one term.	$8, 7x, 4y$, and $2xy$ are monomials.
Polynomial	An algebraic expression that has two or more terms.	$(8x^2 - 5x + 3)$ is a polynomial with 3 terms, where the 1st term is $8x^2$, the 2nd term is $-5x$, and the 3rd term is 3. The coefficient of the 1st term is 8, and the coefficient of the 2nd term is –5. The 3rd term is a constant.

Terminology	Description	Examples
Binomial	A polynomial with 2 terms.	$(4x - 3y), (x - 5), (4xy + 7x)$ are binomials.
Trinomial	A polynomial with 3 terms.	$(2x + 3y + 5), (xy + x - 2), (2x + xy + 3z)$ are trinomials.

Example 7.1-a: Identifying Components of an Algebraic Expression

Given: $7x^2 - 4x + 7 - 12x$

Identify the following (if applicable):

a. the expression
b. the terms
c. any like terms
d. the coefficients
e. the constant

Solution

$$\overbrace{7x^2 - 4x + 7 - 12x}^{\text{the expression}}$$

1st term
Coefficient: 7

3rd term
Constant

2nd term
Coefficient: −4

4th term
Coefficient: −12

like terms

Evaluating Algebraic Expressions

In an algebraic expression, replacing all the variables with numerical values and simplifying the expression is referred to as evaluating the algebraic expression. The simplified answer is the value of the expression.

Example 7.1-b: Evaluating Algebraic Expressions

Evaluate the following expressions:

a. $2x + y$, where $x = 10$ and $y = 5$

b. $\dfrac{3xy + 3x}{2y + 5}$, where $x = 3$ and $y = 2$

Solution

a. $2x + y$ Substituting $x = 10$ and $y = 5, = 2(10) + 5$ [$2x$ means $2(x)$] $= 20 + 5 = 25$

b. $\dfrac{3xy + 3x}{2y + 5}$ Substituting $x = 3$ and $y = 2, = \dfrac{3(3)(2) + 3(3)}{2(2) + 5} = \dfrac{18 + 9}{4 + 5} = \dfrac{27}{9} = 3$

Example 7.1-c: Evaluating Algebraic Expressions with Exponents

Evaluate the following expressions:

a. $\dfrac{(5x)^2 \times 4y}{50}$, where $x = 2$ and $y = 3$

b. $2(x^2 + 3x) - 5y$, where $x = 4$ and $y = -3$

Solution

a. $\dfrac{(5x)^2 \times 4y}{50}$ Substituting $x = 2$ and $y = 3, = \dfrac{[5(2)]^2 \times 4(3)}{50}$

$= \dfrac{10^2 \times 12}{50} = \dfrac{100 \times 12}{50} = 24$

b. $2(x^2 + 3x) - 5y$ Substituting $x = 4$ and $y = -3, = 2[(4)^2 + 3(4)] - 5(-3)$

$= 2(16 + 12) + 15 = 56 + 15 = 71$

7.1 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

For problems 1 and 2, translate the written statements into algebraic expressions.

1. a. Three less than twice a number.

    b. Two times a number divided by five.

    c. Twenty-five increased by three times a number.

2. a. Seven less than four times a number.

    b. Fifteen divided by three times a number.

    c. Twenty increased by twice a number.

For problems 3 to 6, (a) identify the number of terms in each expression, (b) identify the constant term (if applicable), and (c) state the coefficients of each of the variable terms.

3. a. $3x^2 + 7xy - 4y$

    b. $x^2 - 5x$

    c. $9xy + 7x - 6y + 2$

4. a. $-x^2 + 9xy + y + 7$

    b. $7xy$

    c. $10x^2 + 5xy - 7y^2$

5. a. $5x^2 - 3xy + 5$

    b. $-2y^2 + 3x + 1$

    c. $-2xy^2 - 2x^2y + 7$

6. a. $-2y^2 + 3y - 4$

    b. $y^5 - 2y^7 - 2$

    c. $2x^3 - 3x^2 + 1$

For problems, 7 to 10, identify any like terms in the expressions.

7. a. $12A + 4B - 7A - B$

    b. $6x + 8y - 5x - 3y + 7$

8. a. $6B + 8A - A - 2B$

b. $14 - 3x + 10y + 4y$

9. a. $-2x + 5x - 12x + 8 + 7y - 3$

b. $6xy^2 - 2x^2y - 4x^2 + 2xy^2 + 3x^2y + 2x^2 + 4$

10. a. $3 + 6x - 20x + 8y + 8y + 5x + 12$

b. $3x^2y - 12xy^2 - 6x^2y - 5xy - 2xy - 4xy^2$

For problems, 11 to 14, evaluate the expressions.

11. $2a + 14b - 12c + 2$, where $a = 3, b = 2, c = 1$

12. $x + 2y - 13z + 1$, where $x = 5, y = 8, z = 2$

13. $3x^2 - 2x$, where $x = 5$

14. $2a^2 + a$, where $a = 15$

For problems, 15 to 18, evaluate the expressions, given **x = 2** and **y = 3**.

15. a. $\dfrac{19x - 5y}{9}$

b. $x^2 + 6x + 8$

16. a. $\dfrac{7x - 5y}{3}$

b. $-x^2 + 10x + 7$

17. a. $\dfrac{(3x)^2(5y)}{6y}$

b. $-2x^2 + 3x + 8y$

18. a. $\dfrac{(2x)^2(2y)}{5y}$

b. $4x^2 + 10x - 4y$

7.2 ARITHMETIC OPERATIONS WITH ALGEBRAIC EXPRESSIONS

All arithmetic operations can be applied to algebraic expressions by following the rules that we have learned thus far, including the order of operations (BEDMAS), properties of exponents, and operations with signed numbers.

Addition and Subtraction

Addition and Subtraction of Monomials

Addition and subtraction of monomials can be performed by adding and subtracting the coefficients of like terms, according to the rules of signed numbers.

Note: Recall that if a coefficient of a term is not written, it is 1.

Example 7.2-a: Adding and Subtracting Monomials

    a. Add $6x$ and $3x$
    b. Add $4x^2y$ and x^2y
    c. Subtract $5x^3$ from $7x^3$
    d. Subtract $8x$ from the sum of $7x$ and $4x$
    e. Add $5x$ and $6y$
    f. Subtract $2y^2$ from $7y^3$

Solution

    a. $6x + 3x$
       Adding like terms, $= 9x$

b. $4x^2y + x^2y$

Adding like terms, $= 5x^2y$

c. $7x^3 - 5x^3$

Subtracting like terms, $= 2x^3$

d. $(7x + 4x) - (8x)$

Adding like terms inside the brackets, $= 11x - 8x$

Subtracting like terms, $= 3x$

e. $5x + 6y$

Since these are not like terms, we cannot simplify the expression at all.

f. $7y^3 - 2y^2$

Since these are not like terms, we cannot simplify the expression at all.

Addition and Subtraction of Polynomials

When adding or subtracting algebraic expressions, first collect the like terms and group them, then add or subtract the coefficients of the like terms.

- For the **addition of polynomials** (indicated by a plus '+' sign outside the brackets), brackets can be removed without changing any of the signs of the terms within the brackets.
- For the **subtraction of polynomials** (indicated by a negative '–' sign outside the brackets), brackets can be removed by distributing the negative sign to the terms within the brackets; this is equivalent to multiplying every term within the brackets by –1, resulting in the signs changing on each term within the brackets.

Example 7.2-b: Adding and Subtracting Algebraic Expressions

Evaluate the following expressions:

a. Add $(3x + 7)$ and $(5x + 3)$
b. Add $(4y^2 - 8y - 9)$ and $(2y^2 + 6y - 2)$
c. Subtract $(x^2 + 5x - 7)$ from $(2x^2 - 2x + 3)$

d. Subtract $[5x - (x + 8)]$ from $(x - 3)$

Solution

a. $(3x + 7) + (5x + 3)$

Removing the brackets,$= 3x + 7 + 5x + 3$

Grouping like terms, $= 3x + 5x + 7 + 3$

Adding like terms,$= 8x + 10$

b. $(4y^2 - 8y - 9) + (2y^2 + 6y - 2)$

Removing the brackets, $= 4y^2 - 8y - 9 + 2y^2 + 6y - 2$

Grouping like terms, $= 4y^2 + 2y^2 - 8y + 6y - 9 - 2$

Adding and subtracting like terms, $= 6y^2 - 2y - 11$

c. $(2x^2 - 2x + 3) - (x^2 + 5x - 7)$

Removing the brackets by distributing the negative sign to all the terms within the bracket,

$= 2x^2 - 2x + 3 - x^2 - 5x + 7$

Grouping like terms, $= 2x^2 - x^2 - 2x - 5x + 3 + 7$

Adding and subtracting like terms,$= x^2 - 7x + 10$

d. $(x - 3) - [5x - (x + 8)]$

Removing the brackets by distributing the negative sign to all the terms within the bracket,

$= x - 3 - [5x - x - 8] = x - 3 - 5x + x + 8$

Grouping like terms, $= x - 5x + x - 3 + 8$

Adding and subtracting like terms, $= -3x + 5$

Multiplication

Multiplying a Monomial by a Monomial

Multiplying a monomial by another monomial is just simplifying an algebraic term, as we did in the previous section. Where applicable, multiply the coefficients.

Evaluate the following expressions:

a. Multiply $6x^2y$ and 5
b. Multiply $(3a^3)$, (-4) and (2)

Solution

a. $(6x^2y)(5) = (6)(5)(x^2)(y) = 30x^2y$
b. $(3a^3)(-4)(2) = (3)(-4)(2)(a^3) = -24a^3$

Multiplying a Polynomial by a Monomial

When multiplying a polynomial by a monomial, multiply the monomial by each term of the polynomial. This is also known as the distributive property of multiplication, as shown below.

$a(b + c) = ab + ac$

Then, group the like terms and simplify using addition and subtraction.

Example 7.2-d: Multiplying Polynomials by Monomials

a. Multiply: 2 and $(3x^2 + 2x - 5)$
b. Expand and simplify: $8(x + 3) + 4(x - 4)$

Solution

a. $2(3x^2 + 2x - 5)$
   Expanding, by following the Distributive Property, $= 6x^2 + 4x - 10$
b. $8(x + 3) + 4(x - 4)$
   Expanding, $= 8x + 24 + 4x - 16$

Grouping like terms, $= 8x + 4x + 24 - 16$

Adding and subtracting like terms, $= 12x + 8$

Division

Dividing a Monomial by a Monomial

Dividing a monomial by another monomial simplifies an algebraic term, as we did in the previous section. Divide the coefficients, where applicable.

Example 7.2-i: Dividing Monomials by Monomials

    a. Divide $8x^2y$ by 6

    b. Divide $-9x^2$ by 3

Solution

a. $\dfrac{8x^2y}{6} = \dfrac{4x^2y}{3}$

b. $\dfrac{-9x^2}{3} = -3x^2$

7.2 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

For problems 1 to 8, simplify and evaluate the expressions.

1. $6y + 4y - 7y$, where $y = 10$

2. $3x + 5x - 8x$, where $x = 4$

3. $2z - z + 7z$, where $z = 7$

4. $3A - A + 6A$, where $A = 10$

5. $(6x)(3x) - (5x)(4x)$, where $x = 3$

6. $(10x \times 4.5x) - (11x \times 4x)$, where $x = 50$

7. $(2x)(0.5x + 4x)(5x + x)$, where $x = 5$

8. $(4x)(12x + 0.25x)(0.5x + x)$, where $x = 3$

For problems 9 to 28, simplify the expressions.

9. $13x^2 + 8x - 2x^2 + 9x$

10. $7x + 12x^2 - 4x + 5x^2$

11. $-18y - 5y^2 + 19y - 2y^2$

12. $-14y - 2y^2 + 7y + 7y^2$

13. $6x - 3x + 2y^2 + y^2$

14. $9x^2 - 6x^2 + 7y - 6y$

15. $4xy^2 - x^2y^2 - 3xy^2 + 2x^2y^2$

16. $3x^2y^2 - 2xy^2 - 8x^2y^2 + xy^2$

17. $3[(5 - 3)(4 - x)] - 2 - 5[3(5x - 4) + 8] - 9x$

18. $(5 - 14)x - 8[3 - 5(2x - 3) + 3x] - 3$

19. $6[4(8 - y) - 5(3 + 3y)] - 21 - 7[3(7 + 4y) - 4] + 198y$

20. $\dfrac{1}{2}\{y - 15[2 - 3(3y - 2) - 7y] - 4\}$

21. $y - \{4x - [y - (2y - 9) - x] + 2\}$

22. $2y + \{-6y - [3x + (-4x + 3)] + 5\}$

23. $(x - 1) - \{[x - (x - 3)] - x\}$

24. $9x - \{3y + [4x - (y - 6x)] - (x + 7y)\}$

25. $5\{-2y + 3[4x - 2(3 + x)]\}$

26. $4\{-7y + 8[5x - 3(4x + 6)]\}$

27. $2y + \{8[3(2y - 5) - (8y + 9) + 6]\}$

28. $7x - \{5[4(3x - 8) - (9x + 10)] + 14\}$

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7.3 SIMPLE ALGEBRAIC EQUATIONS AND WORD PROBLEMS

An **algebraic equation** is a mathematical sentence expressing equality between two algebraic expressions (or an algebraic expression and a number).

When two expressions are joined by an equal (=) sign, it indicates that the expression to the left of the equal sign is identical in value to the expression to the right of the equal sign.

For example, when two algebraic expressions, such as $5x + 7$ and $x + 19$, are equal, the two expressions are joined by an equal (=) sign and the equation is written as:

$5x + 7 = x + 19$

'Left side' (LS) = 'Right side' (RS)

The **solution** to the equation is the value of the variable that makes the left side (LS) evaluate to the same number as the right side (RS).

Note: You need an equation to solve for an unknown variable – you cannot solve for a variable in an algebraic expression that is not part of an equation.

- *If you have an **expression**, it needs to be **simplified**.*
- *If you have an **equation**, it needs to be **solved**.*

In algebra, there are a variety of equations. In this section, we will learn one equation category, **linear equations with one variable**.

Examples of linear equations with one variable are:

$2x = 8, \quad 3x + 5 = 14, \quad 5x + 7 = x + 19$

An equation is either true or false depending on the value of the variable.

For example, consider the equation $2x = 8$:

- If $x = 4$,   LS = 2(4) = 8, RS = 8; therefore, the equation is true.
- If $x = 3$,   LS = 2(3) = 6, RS = 8; therefore, the equation is false.

Equations may be classified into the following three types:

1. **Conditional equation:** these equations are only true when the variable has a specific value. For example, $2x = 8$ is a conditional equation, true if and only if $x = 4$.
2. **Identity:** these equations are true for any value for the variable. For example, $2x + 10 = 2(x + 5)$ is

an identity, true for any value of x.

3. **Contradiction:** these equations are not true for any value of the variable. For example,
$x + 5 = x + 4$ is a contradiction, not true for any value of x.

Equivalent Equations

Equations with the same solutions are called **equivalent equations**.

For example, $2x + 5 = 9$ and $2x = 4$ are equivalent equations because the solution $x = 2$ satisfies each equation.

Similarly, $3x - 4 = 5$, $2x = x + 3$, and $x + 1 = 4$ are equivalent equations because the solution $x = 3$ satisfies each equation.

Properties of Equality

If $a = b$, then,

Properties of Equality with Corresponding Expressions

Expression	Property	Meaning
$a = b$	Symmetric Property	Interchanging LS and RS.
$a + c = b + c$	Addition Property	Adding the same quantity to both sides.
$a - c = b - c$	Subtraction Property	Subtracting the same quantity from both sides.
$a \cdot c = b \cdot c$	Multiplication Property	Multiplying by the same quantity on both sides.
$\dfrac{a}{c} = \dfrac{b}{c}$	Division Property, $c \neq 0$	Dividing by the same quantity on both sides.

These properties are used to solve equations.

Equations with Fractional Coefficients

If an equation contains fractional coefficients, then the fractional coefficients can be changed to whole numbers by multiplying both sides of the equation by the least common denominator (LCD) of all the fractions, using the Multiplication Property.

For example,

$$\frac{2}{3}x = \frac{5}{2} + 4$$

Since the LCD of the denominators 3 and 2 is 6, multiply both sides of the equation by 6.

$$6\left(\frac{2}{3}x\right) = 6\left(\frac{5}{2} + 4\right)$$

This is the same as multiplying each term by the LCD of 6.

$$6\left(\frac{2}{3}x\right) = 6\left(\frac{5}{2}\right) + 6(4)$$

Simplifying, $4x = 15 + 24$

Now, the equation has only whole number coefficients $4x = 39$.

Equations with Decimal Coefficients

If an equation contains decimal coefficients, then the decimal coefficients can be changed to whole numbers by multiplying both sides of the equation by an appropriate power of 10, using the Multiplication Property.

For example,

$$\underline{1.25}x = \underline{0.2} + 4$$

Since there is at most 2 decimal places in any of the coefficients or constants, multiply both sides of the equation by $10^2 = 100$.

$$100(1.25x) = 100(0.2 + 4)$$

This is the same as multiplying each term by 100. $100(1.25x) = 100(0.2) + 100(4)$

Simplifying, $125x = 20 + 400$

Now, the equation has only whole number coefficients $125x = 420$.

Steps to Solve Algebraic Equations with One Variable

Step 1:

If the equation contains fraction and/or decimal coefficients, it is possible to work with them as they are – in that case, proceed to Step 2. Alternatively, as explained earlier, the equation may be rewritten in whole numbers to make calculations and rearrangements easier.

Step 2:

If present, expand and clear brackets in the equation by following the order of arithmetic operations (BEDMAS).

Step 3:

Use the addition and subtraction properties to collect and group all **variable** terms on the **left side** of the equation and all **constants** on the **right side**. Then, simplify both sides.

Note: If it is more convenient to gather all the variable terms on the right side and the constants on the left side,

you may do so, and then use the symmetric property and switch the sides of the equation to bring the variables over to the left side and the constants to the right side.

Step 4:

Use the division and multiplication properties to ensure that the coefficient of the variable is +1.

Step 5:

After completing Step 4, there should be a single variable with a coefficient of +1 on the left side and a single constant term on the right side – that constant term is the solution to the equation.

Step 6:

Verify the answer by substituting the solution from Step 5 back into the original problem.

Step 7:

State the answer.

Example 7.3-a: Solving Equations Using the Addition and Subtraction Properties

Solve the following equations and verify the solutions:

a. $x - 11 = 4$

b. $8 + x = 20$

Solution

a. $x - 11 = 4$

Adding **11** to both sides, $x - 11 + 11 = 4 + 11$

$x = 15$

Verify by substituting $x = 15$:

$\text{LS} = x - 11 = 15 - 11 = 4 \quad \text{RS} = 4$

$\text{LS} = \text{RS}$

Therefore, the solution is $x = 15$.

b. $8 + x = 20$

Subtracting **8** from both sides, $8 - 8 + x = 20 - 8$

$x = 12$

Verify by substituting $x = 12$:

LS $= 8 + x = 8 + 12 = 20$ RS $= 20$

LS = RS

Therefore, the solution is $x = 12$.

Example 7.3-b: Solving Equations Using the Multiplication and Division Properties

Solve the following equations and verify the solutions:

 a. $5x = 20$

 b. $\dfrac{3}{8}x = 12$

Solution

 a. $5x = 20$

 Dividing both sides by **5**, $\dfrac{5x}{5} = \dfrac{20}{5}$

 $x = 4$

 Verify by substituting $x = 4$:

 LS $= 5x = 5(4) = 20$ RS $= 20$

 LS = RS

 Therefore, the solution is $x = 4$.

 b. $\dfrac{3}{8}x = 12$

 Multiplying both sides by $\dfrac{8}{3}$ (the reciprocal of $\dfrac{3}{8}$), $\left(\dfrac{8}{3}\right) \cdot \dfrac{3}{8}x = \left(\dfrac{8}{3}\right) \cdot 12$

 $x = 8 \times 4$

 $x = 32$ or $\dfrac{3}{8}x = 12$

 Multiplying both sides by **5**, $(8) \cdot \dfrac{3}{8}x = (8) \cdot 12$

$3x = 96$ Dividing both sides by **3**, $\dfrac{3x}{3} = \cdot\dfrac{96}{3}x = 32$ Verify by substituting $x = 32$:LS

$= \dfrac{3}{8}x = \dfrac{3}{8} \times 32 = 12$RS $= 12$LS = RSTherefore, the solution is $x = 32$.

Example 7.3-c: Solving Equations with Variables on Both Sides

Solve the following equations and verify the solutions:

a. $3x - 8 = 12 - 2x$
b. $15 + 6x - 4 = 3x + 31 - x$

Solution

a. $3x - 8 = 12 - 2x$

Adding **2x** to both sides, $3x + 2x - 8 = 12 - 2x + 2x$

$5x - 8 = 12$

Adding **8** to both sides, $5x - 8 + 8 = 12 + 8$

$5x = 20$

Dividing both sides by **5**, $\dfrac{5x}{5} = \dfrac{20}{5}$

$x = 4$

Verify by substituting $x = 4$:

LS $= 3x - 8 = 3(4) - 8 = 12 - 8 = 4$RS $= 12 - 2x = 12 - 2(4) = 12 - 8 = 4$LS = RSTherefore, the solution is $x = 4$.

b. $15 + 6x - 4 = 3x + 31 - x$

Combining like terms (LS: $15 - 4 = 11$, and RS: $3x - x = 2x$), $11 + 6x = 2x + 31$

Subtracting $2x$ from both sides, $11 + 6x - 2x = 2x - 2x + 31$

$11 + 4x = 31$

Subtracting **11** from both sides, $11 - 11 + 4x = 31 - 11$

$4x = 20$

Dividing both sides by **4**, $\dfrac{4x}{4} = \dfrac{20}{4}$

$x = 5$

Verify by substituting $x = 5$ back into the original equation:LS

$$= 15 + 6x - 4 = 15 + 6(5) - 4 = 15 + 30 - 4 = 41 \text{RS}$$

$$= 3x + 31 - x = 3(5) + 31 - 5 = 15 + 31 - 5 = 41 \text{LS = RSTherefore, the solution is}$$

$x = 5.$

Example 7.3-d: Solving Equations with Fractions

Solve the following equation and verify the solution:

$$\frac{x}{3} - \frac{1}{12} = \frac{1}{6} + \frac{x}{4}$$

Solution

$$\frac{x}{3} - \frac{1}{12} = \frac{1}{6} + \frac{x}{4}$$

LCD of 3, 4, 6, and 12 is 12.

Multiplying each term by **12**,

$$12\left(\frac{x}{3}\right) - 12\left(\frac{1}{12}\right) = 12\left(\frac{1}{6}\right) + 12\left(\frac{x}{4}\right)$$

$$4x - 1 = 2 + 3x$$

Subtracting $3x$ from both sides,

$$4x - 3x - 1 = 2 + 3x - 3x$$

$$x - 1 = 2$$

Adding **1** to both sides,

$$x - 1 + 1 = 2 + 1$$

$$x = 3$$

Verify by substituting $x = 3$ back into the original equation:

$$\text{LS} = \frac{x}{3} - \frac{1}{12} = \frac{3}{3} - \frac{1}{12} = \frac{12}{12} - \frac{1}{12} = \frac{11}{12}$$

$$\text{RS} = \frac{1}{6} + \frac{x}{4} = \frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{11}{12}$$

LS = RS

Therefore, the solution is $x = 3$.

Example 7.3-e: Solving Equations with Decimals

Solve the following equation and verify the solution:

$$0.15x + 1.2 = 0.4x - 0.8$$

Solution

$$0.15x + 1.2 = 0.4x - 0.8$$

Greatest number of decimal places is 2 (i.e., hundredths).

Multiplying all the terms by $10^2 = 100$,

$$100(0.15x) + 100(1.2) = 100(0.4x) - 100(0.8)$$

$$15x + 120 = 40x - 80$$

Interchanging the LS and RS using the Symmetric Property to have the larger x term on the LS,

$$40x - 80 = 15x + 120$$

Subtracting $15x$ from both sides,

$$40x - 15x - 80 = 15x - 15x + 120$$

$$25x - 80 = 120$$

Adding **80** to both sides,

$$25x - 80 + 80 = 120 + 80$$

$$25x = 200$$

Dividing both sides by **25**,

$$\frac{25x}{25} = \frac{200}{25}$$

$$x = 8$$

Verify by substituting $x = 8$ back into the original equation:

LS $= 0.15x + 1.2 = 0.15(8) + 1.2 = 1.2 + 1.2 = 2.4$

RS $= 0.4x - 0.8 = 0.4(8) - 0.8 = 3.2 - 0.8 = 2.4$

LS = RS

Therefore, the solution is $x = 8$.

Note: For the rest of the examples in this section, we will not show the verification by substitution step.

Example 7.3-f: Solving Equations Using All the Properties

Solve the following equations by using the properties of equality, and express the answer as a fraction in its lowest terms, or as a mixed number, wherever applicable:

a. $8x + 7 - 3x = -6x - 15 + x$

b. $2(3x - 7) = 28 - 3(x + 1)$

c. $\dfrac{1}{4}(x + \dfrac{2}{3}) = \dfrac{1}{2}(x - 3) + x$

d. $0.45(2x + 3) - 2.55 = 0.6(3x - 5)$

e. $\dfrac{x + 2}{3} = \dfrac{5 - 2x}{7}$

Solution

a. $8x + 7 - 3x = -6x - 15 + x$

Grouping like terms on both sides, $8x - 3x + 7 = -6x + x - 15$

$5x + 7 = -5x - 15$

Adding **5** to both sides, $5x + 5x + 7 = -5x + 5x - 15$

$10x + 7 = -15$

Subtracting **7** from both sides, $10x + 7 - 7 = -15 - 7$

$10x = -22$

Dividing both sides by **10**, $\dfrac{10x}{10} = -\dfrac{22}{10}$

$x = -\dfrac{11}{5} = -2\dfrac{1}{5}$

b. $2(3x - 7) = 28 - 3(x + 1)$

Expanding both sides, $6x - 14 = 28 - 3x - 3$ Grouping like terms,

$6x - 14 = 28 - 3 - 3x$

$6x - 14 = 25 - 3x$

Adding $3x$ to both sides, $6x + 3x - 14 = 25 - 3x + 3x$

$9x - 14 = 25$

Adding **14** to both sides, $9x - 14 + 14 = 25 + 14$

$9x = 39$

Dividing both sides by **9**, $\dfrac{9x}{9} = \dfrac{39}{9}$

$x = \dfrac{13}{3} = 4\dfrac{1}{3}$

c. $\dfrac{1}{4}(x + \dfrac{2}{3}) = \dfrac{1}{2}(x - 3) + x$

Expanding both sides, $\dfrac{1}{4}x + \dfrac{1}{6} = \dfrac{1}{2}x - \dfrac{3}{2} + x$

Multiplying each term by the LCD **12**,

$$12\left(\dfrac{1}{4}x\right) + 12\left(\dfrac{1}{6}\right) = 12\left(\dfrac{1}{2}x\right) - 12\left(\dfrac{3}{2}\right) + 12(x)$$

$3x + 2 = 6x - 18 + 12x$

Grouping like terms, $3x + 2 = 6x + 12x - 18$

$3x + 2 = 18x - 18$

Interchanging the LS and RS using the Symmetric Property to have the larger x term on the LS,

$18x18 = 3x + 2$ ⠀⠀ Subtracting $3x$ from both sides, $18x - 3x - 18 = 3x - 3x + 2$

$15x - 18 = 2$

Adding **18** to both sides, $15x - 18 + 18 = 2 + 18$

$15x = 20$

Dividing both sides by **15**, $\dfrac{15x}{15} = \dfrac{20}{15} x = \dfrac{4}{3} = 1\dfrac{1}{3}$

d. $0.45(2x + 3) - 2.55 = 0.6(3x - 5)$

Expanding both sides, $0.90x + 1.35 - 2.55 = 1.8x - 3.0$

Greatest number of decimal places is 2 (i.e., hundredths).

Multiplying all the terms by $10^2 = 100$,

$100(0.90x) + 100(1.35) - 100(2.55) = 100(1.8x) - 100(3.0)$

$90x + 135 - 255 = 180x - 300$

Grouping like terms, $90x120 = 180x300$

Interchanging the LS and RS using the Symmetric Property to have the larger x term on the LS,

$180x - 300 = 90x - 120$

Subtracting $90x$ from both sides, $180x - 90x - 300 = 90x - 90x - 120$

$90x - 300 = -120$

Adding **300** to both sides, $90x - 300 + 300 = -120 + 300$

$90x = 180$

Dividing both sides by **90**, $\dfrac{90x}{90} = \dfrac{180}{90}$

e.
$$x = 2$$
$$\frac{x + 2}{3} = \frac{5 - 2x}{7}$$

Cross-multiplying, $7(x + 2) = 3(5 - 2x)$

Expanding both sides, $7x + 14 = 15 - 6x$

Adding $6x$ to both sides, $7x + 6x + 14 = 15 - 6x + 6x$

$13x + 14 = 15$

Subtracting 14 from both sides, $13x + 14 - 14 = 15 - 14$

$13x = 1$

Dividing both sides by 13, $\dfrac{13x}{13} = \dfrac{1}{13}$

$$x = \frac{1}{13}$$

Steps to Solve Word Problems

Step 1:

Read the entire problem and ensure you understand the situation.

Step 2:

Identify the given information and the question to be answered.

Step 3:

Look for keywords. Some words indicate certain mathematical operations (see Table 7.1).

Step 4:

Choose a variable to represent the unknown(s) and state what that variable represents, including the unit of measure.

Note: For now, if there is more than one unknown, try to identify all the unknowns in terms of one variable, as all the questions in this chapter can be solved with only one variable.

Step 5:

Where necessary, draw a simple sketch to identify the information. This helps with envisioning the question more clearly.

Step 6:

Create an equation (or set of equations) to describe the relationship between the variables and the constants in the question.

Step 7:

Group like terms, isolate the variable and solve for the unknown(s).

Step 8:

State the solution to the given problem.

Example 7.3-g: Solving a Word Problem Using Algebraic Equations

If Harry will be 65 years old in 5 years, how old is he today?

Solution

Let Harry's age today be x years.

Therefore, in 5 years, Harry's age will be:

$x + 5 = 65$ Solving for x,

$x = 65 - 5 = 60$

Therefore, Harry is 60 years old today.

Example 7.3-h: Solving a Geometry Problem Using Algebraic Equations

The perimeter of a rectangular garden is 50 metres. The length is 5 metres more than the width. Find the dimensions of the garden.

Hint: Perimeter = 2(length) + 2(width)

Solution

Let the width be w metres.

Therefore, the length is (w + 5) metres.

Perimeter = 2(length) + 2(width)

$50 = 2(w + 5) + 2w$

$50 = 2w + 10 + 2w$

$$2w + 10 + 2w = 50$$
$$4w + 10 = 50$$
$$4w = 50 - 10$$
$$4w = 40$$
$$w = \frac{40}{4}$$
$$w = 10$$

Therefore, the width of the garden is 10 metres and the length is $(10 + 5) = 15$ metres.

Example 7.3-i: Solving a Finance Problem Using Algebraic Equations

A TV costs $190 more than a Blu-ray player. The total cost of the TV and the Blu-ray player is $688. Calculate the cost of the TV and the cost of the Blu-ray player.

Solution

Let the cost of the Blu-ray player be $\$x$.

Therefore, the cost of the TV is $\$(x + 190.00)$.

The total cost is $688.00.

$$x + (x + 190.00) = 688.00$$
$$x + x + 190.00 = 688.00$$
$$2x + 190.00 = 688.00$$
$$2x = 688.00 - 190.00$$
$$2x = 498.00$$
$$x = \frac{498.00}{2}$$
$$x = \$249.00$$

Therefore, the cost of the Blu-ray player is $249.00 and the cost of the TV is $(249.00 + 190.00) = \$439.00$.

Example 7.3-j: Solving a Mixture Problem Using Algebraic Equations

How many litres of water need to be added to 30 litres of a 15% saline solution to make a saline solution that is 10% saline?

Solution

Example 7.3-j Solution Table

Solution Ingredients	# of Litres	% Saline	Total Litres of Saline
Water	x	0	0
15% Saline Solution	30	0.15	$0.15 \times 30 = 4.5$
10% Saline Solution	$30 + x$	0.10	$0.10 \times (30 + x)$

From the last column, we get the equation for the saline mix. The number of litres of saline in the 15% solution must be the same as the number of litres in the final 10% solution, as only water is being added, which does not contribute any additional saline to the solution. Therefore,

$$4.5 = 0.10 \times (30 + x)$$
$$4.5 = 3 + 0.10x$$
$$1.5 = 0.10x$$
$$x = 15$$

Therefore, 15 litres of water need to be added to the 15% saline solution to make the solution 10% saline.

7.3 Exercises

Answers to the odd-numbered problems are available at the end of the textbook.

For problems 1 to 8, simplify and evaluate the expressions.

1. The sum of a number and six is ten.

2. A number decreased by fifteen is five.

3. Six times a number is seventy-two.

4. The product of a number and four is twenty-eight.

5. A number divided by five is four.

6. A number divided by three is three.

7. Two-thirds of a number is twelve.

8. Two-fifths of a number is six.

For problems 9 to 30, solve the algebraic equations using the properties of equality, and express the answer as a fraction in its lowest terms or as a mixed number, wherever applicable.

9. $x - 20 = 10$

10. $x - 25 = 17$

11. $22 = 40 - x$

12. $54 = 23 - x$

13. $21 + x = 4$

14. $50 + x = 45$

15. $16 + x = 22$

16. $12 + x = 38$

17. $11x + 4 = 17$

18. $7x - 16 = 22$

19. $x - \dfrac{4}{5} = \dfrac{3}{5}$

20. $x - \dfrac{1}{6} = 1$

21. $\dfrac{10}{15} = x - \dfrac{4}{3}$

22. $\dfrac{x}{7} + 15 = 24$

23. $x + \dfrac{2}{5} = \dfrac{1}{4}$

24. $2x - \dfrac{2}{3} = \dfrac{5}{6}$

25. $5x = 20$

26. $4x = 24$

27. $\dfrac{2x}{3} + 1 = \dfrac{5x}{8} + 2$

28. $\dfrac{x}{2} - \dfrac{1}{6} = \dfrac{1}{3} + \dfrac{3x}{5}$

29. $\dfrac{7x}{8} - 4 = \dfrac{x}{4} + 6$

30. $\dfrac{8x}{3} - 5 = \dfrac{x}{3} + 2$

For problems 31 to 54, solve the algebraic equations using the properties of equality, and round the answer to 2 decimal places, wherever applicable.

31. $10y - 0.09y = 17$

32. $x + 0.13x = 70$

33. $0.3x - 3.2 = 0.4 - 0.6x$

34. $4 + 0.2x = 0.7x - 0.5$

35. $0.4x - 1.38 = 0.3x - 1.2$

36. $1.2 - 0.7x = 2.7 - 0.5x$

37. $0.43x + 0.25 = 0.29x - 0.03$

38. $0.6x - 1.2 = 0.9 - 1.5x$

39. $8x + 7 - 3x = -6x - 15 + x$

40. $x - 2 - 4x = -3x - 8 + 5x$

41. $2(3x - 7) = 28 - 3(x + 1)$

42. $4(2x - 5) = 32 - 4(x - 2)$

43. $(4 + 6)(2 + 4x) = 45 - 2.5(x + 3)$

44. $(5 + 0.5x)(1 + 3) = -1.2(2x + 4) + 25$

45. $15 + 5(x - 10) = 3(x - 1)$

46. $2(x - 3) + 3(x - 5) = 4$

47. $4(y + 7) - 2(y - 4) = 3(y - 2)$

48. $8(2y + 4) - 6(3y + 7) = 3y$

49. $\dfrac{x-7}{2} + \dfrac{x+2}{3} = 41$

50. $\dfrac{7}{12}(2x+1) + \dfrac{3}{4}(x+1) = 3$

51. $\dfrac{5}{y+4} = \dfrac{3}{y-2}$

52. $\dfrac{3}{x+1} = \dfrac{2}{x-3}$

53. $\dfrac{7}{5x-3} = \dfrac{5}{4x}$

54. $\dfrac{5}{y+2} = \dfrac{3}{y}$

For problems 55 to 76, solve the word problems using algebraic equations.

55. If three times a number plus twenty is seven times that number, what is the number?

56. Fifteen less than three times a number is twice that number. What is the number?

57. A 25-metre-long wire is cut into two pieces. One piece is 7 metres longer than the other. Find the length of each piece.

58. A 9-metre-long pipe is cut into two pieces. One piece is twice the length of the other piece. Find the length of each piece.

59. $500 is shared between Andy and Becky. Andy's share is $150 less than Becky's share. Calculate the amount of each of their shares.

60. $200 is shared between Bill and Ann. Ann's share is $50 more than Bill's share. Calculate the size of each of their shares.

61. Movie tickets that were sold to each child were $3 cheaper than those sold to each adult. If a family of two adults and two children paid $34 to watch a movie at the cinema, what was the price of each adult ticket and each child ticket?

62. Giri had twice the number of quarters (25 cents) in his bag than dimes (10 cents). If he had a total of 54 coins, how many of them were quarters? What was the total dollar value of these coins?

63. A square garden, with sides of length x metres, is widened by 4 metres and lengthened by 3 metres. Write the equation for the area (A) of the expanded garden. If each side was originally 10 metres in length, find

the new area. *(Hint: Area of a Rectangle = Length × Width)*

64. A square garden, with sides of length x metres, has had its width reduced by 4 metres and its length reduced by 2 metres. Write the equation for the Area (A) of the smaller garden. If each side was originally 20 metres in length, find the new area.

65. Aran bought a shirt and a pair of pants for $34.75. The pair of pants cost $9.75 more than the shirt. Calculate the cost of the shirt.

66. Mythili bought a schoolbag and a toy for $30.45. The school bag cost $5.45 more than the toy. Calculate the cost of the school bag.

67. Sam is paid $720 a week. He worked 9 hours of overtime last week and he received $954. Calculate his overtime rate per hour.

68. Lisa is paid $840 a week. Her overtime rate is $28 per hour. Last week she received $1,036. How many hours of overtime did she work last week?

69. The sum of the three angles of any triangle is 180°. If $3x$, $7x$, and $8x$ are the measures of the three angles of a triangle, calculate the measure of each angle of the triangle.

70. The sum of the three angles of any triangle is 180°. If $3x$, $4x$, and $5x$ are the measures of the three angles of a triangle, calculate the measure of each angle of the triangle.

71. The perimeter of a triangle is the sum of the lengths of the three sides of the triangle. The perimeter of a triangle with sides x cm, $(x + 10)$ cm, and $2x$ cm is 70 cm. Calculate the length of each side of the triangle.

72. The perimeter of a triangle is the sum of the lengths of the three sides of the triangle. The perimeter of a triangle with sides $x + 10, 2x + 10$, and $3x$ is 110 cm. Calculate the length of each side of the triangle.

73. After completing a weight-loss program, a patient weighs 160 lb. His dietician observes that the patient has lost 15% of his original weight. What was the patient's starting weight?

74. A beaker in a chemistry lab contains 3 litres of water. While conducting an experiment, the chemistry professor removes three-fifths of the water from the beaker. He then adds three-fifths of the remaining volume to the beaker. How much water is left in the beaker at the end of the experiment?

75. A researcher wants to make 4 L of a 7% acid solution. She has a beaker of 15% acid solution in stock.

How much of the 15% solution does she need to use and how much water must she add in order to prepare her desired solution?

76. A chemist wants to make a 10% acid solution. She has 5 L of 25% acid solution. How many litres of water should she add to the 25% solution in order to prepare her desired solution?

7. REVIEW EXERCISES

Answers to the odd-numbered questions are available at the end of the book.

For problems 1 to 4, write the algebraic expression.

1. a. Twelve increased by three times a number.

   b. The difference between a number and five.

2. a. Eight decreased by twice a number.

   b. Six less than the total of a number and ten.

3. a. The product of three more than a number and the number.

   b. Sum of ten times a number and fifteen.

4. a. Sum of fifteen and half of a number.

   b. Product of two times a number and seven.

For problems 5 to 10, simplify the expressions, then evaluate for the given value of the variables in the brackets.

5. a. $-4x^2 + 3x - 5 + 7x^2 - 2x + 3 \quad (x = 2)$

   b. $4x^2 - 5 + 7x - 2x^2 - x - 3 \quad (x = -1)$

6. a. $3x^2 - x + 2 + x^2 - 5x - 2 \quad (x = 3)$

   b. $-5y^2 - 7y + 3 + y^2 - 5y + 2 \quad (y = -2)$

7. a. $-y^2 + 4xy + x^2 - 6y^2 - xy - 11x^2 \quad (x = 1, y = 2)$

   b. $(x - 4)(x + 2) + 3(x + 2) \quad (x = 3)$

8. a. $-4x^2 + 6xy - 6y^2 + 6x^2 - 2xy + 3y^2 \quad (x = 2, y = 1)$

   b. $(y - 2)(y - 3) + 2(y - 2) \quad (y = 4)$

9. a. $(2x - 3)^2 - (x + 3)^2 \quad (x = 4)$

b. $(5 + x)^2 + (4 - x)(4 + x)$ $(x = 5)$

10. a. $(2x + 1)^2 - (x - 2)^2$ $(x = 1)$

b. $(3 - x)^2 + (x - 3)(x + 3)$ $(x = 2)$

For problems 11 to 14, write the algebraic equation and solve.

11. a. Seventeen more than five times a number is forty-two.

b. A number divided by fifteen is forty-five.

12. a. The product of five and a number is seventy-five.

b. Three more than two times a number is nine.

13. a. The difference between a number and ten is ten

b. The product of four times a number and three is thirty-six.

14. a. The sum of two times a number and eight is one hundred.

b. A number divided by three is seven.

For problems 15 to 18, solve for the unknown variable, x, using the principles of equality.

15. a. $5x - 5 = 10$

b. $\dfrac{x}{3} + 4 = 10$

16. a. $3x - 5 = -17$

b. $\dfrac{x}{4} - 2 = 1$

17. a. $12 - 3x = 3 - 4x$

b. $4(x + 4) = 24$

18. a. $4x - 2 = 13 - 6x$

b. $3(2x - 5) = 3$

Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of*

7. SELF-TEST

Answers to the questions are available at the end of the book.

For problems 1 and 2, write the algebraic expression.

1. a. Twenty-five less than three times a number.

    b. A number increased by eighteen.

2. a. The difference between twice a number and six.

    b. A number divided by three.

For problems 3 and 4, simplify the expressions, then evaluate for the given value of the variables in the brackets.

3. a. $-3x^2 + 2x + 2x^2 - 8x + 10 \quad (x = -3)$

    b. $5(2x - 3y) - 2(3x - 2y) + 7 \quad (x = 2 \, and \, y = 1)$

4. a. $(x + 3)^2 - (x + 2)(x - 2) \quad (x = 3)$

    b. $(2x + 5)^2 - (3x - 1)^2 \quad (x = 1)$

For problems 5 and 6, write the algebraic equation and solve.

5. a. Nine less than twice a number is twenty-one.

    b. Twenty-two is five times a number less than three.

6. a. Four times eight is sixteen times a number.

    b. Thirty is a product of six and a number.

For problems 7 and 8, solve for the unknown variable, x, using the principles of equality.

7. a. $24 - 5x = 4$

    b. $\dfrac{x}{3} - 2 = 4$

8. a. $8 + 2x = 4 - 5x$

    b. $3(3x - 3) = 33$

7. CASE STUDY - LIGHTNING WHOLESALE

Lightning Wholesale is a mid-to large-size company that works directly with manufacturers to bring sporting goods and equipment to mid to small-sized retailers. The company is in Southern Ontario, Canada. Lightning Wholesale acquires the goods, stores them in its warehouses, and distributes them to retailers, who then sell them to sporting enthusiasts across the province. Suppliers are global, including those from the United States.

Lightning Wholesale is a dynamic office with global networking, communications, customer service, and complex workflows. Lighting Wholesale depends on its office professionals to support staff and management. Office Professional tasks include clerical support, bookkeeping, customer service, data analysis, and document processing.

Lightning Wholesale cares about its employees and retailers. Where managers can assist their employees, they try to. Where retailers need guidance, Lightning Wholesales tries to fill this gap as best they can. In working with its small retailers, many of which are owned and operated by sole proprietors, Lightning Wholesale assists the owners with their pricing to help develop promotional pricing plans and the amounts of the payments that can be advertised. Lightning Wholesale freely offers help since it means more sales and increased profits for the company.

There are currently a few main tasks for Lightning Wholesale management:

- The payroll needs review for the past fiscal year.
- Costs and expenses need forecasting for the next fiscal year.
- Detailed analysis of office supply orders from quarter to quarter with recommendations for the next fiscal year.
- Forklifts need replacing in the warehouse.
- The next round of co-op students needs a computer.
- The employee appreciation event needs planning for next year.
- Travel expenses for the mobile team need to be summarized for the last fiscal year.

Questions

1. The Customer Service Representatives and the Office Professionals will receive a Q1 bonus due to a better-than-expected quarter revenue result. The Customer Service Representatives will receive $50 more than the Office Professionals. If there are 5 Customer Service Representatives and 3 Office Professionals, how much will everyone get if a total of $1850 is allocated for the bonus?
2. The manager of Lightning Wholesale decided to treat everyone to lunch to celebrate the upcoming

holidays. She decided to tip the server 20%, which increased the meal cost to $625.75. What was the cost of the meal before the tip?

3. The management team at Lightning Wholesale is in the process of analyzing the payroll for all employees. If the Director of Finance has an annual salary that is 3 times the amount of the Office Professionals. How much do they earn if their salaries are $200,000?

PART VIII

CHAPTER 8 - GRAPHS AND SYSTEMS OF LINEAR EQUATIONS

OBJECTIVES

- Identify the basic terminology of rectangular coordinate systems.
- Express linear equations in standard form and slope-intercept form.
- Determine the slope and y-intercept of a line from its equation.
- Graph a linear equation using the table of values, slope-intercept, and x- and y-intercepts.
- Determine the equation of a line from a graph.
- Determine the equations of parallel and perpendicular lines.

CHAPTER OUTLINE

Introduction

A linear equation describes a relationship in which one variable's value depends on another's value. The solutions to a linear equation are ordered pairs of numbers in which each number replaces one of the variables in the equation. A straight-line graph on a two-dimensional coordinate axis system is an illustrative way to depict a linear equation with two variables.

Unless otherwise indicated, this chapter is an adaptation of the eTextbook *Foundations of Mathematics* (3rd ed.) by Thambyrajah Kugathasan, published by Vretta-Lyryx Inc., with permission. Adaptations include supplementing existing material and reordering chapters.

8.1 RECTANGULAR COORDINATE SYSTEM

Graphs drawn on a **rectangular coordinate system**, known as the Cartesian coordinate system (invented by René Descartes), help provide information in a visual form. Understanding the rectangular coordinate system is crucial to be able to read and draw graphs, which is essential in many branches of mathematics.

The rectangular coordinate system uses a horizontal and a vertical number line, each known as an axis. These two perpendicular axes cross at the point (O), known as the **origin**.

The horizontal number line (moving to the left or the right) is called the **X-axis,** and the vertical number line (moving up or down) is called the **Y-axis**, as illustrated in Exhibit 8.1-a.
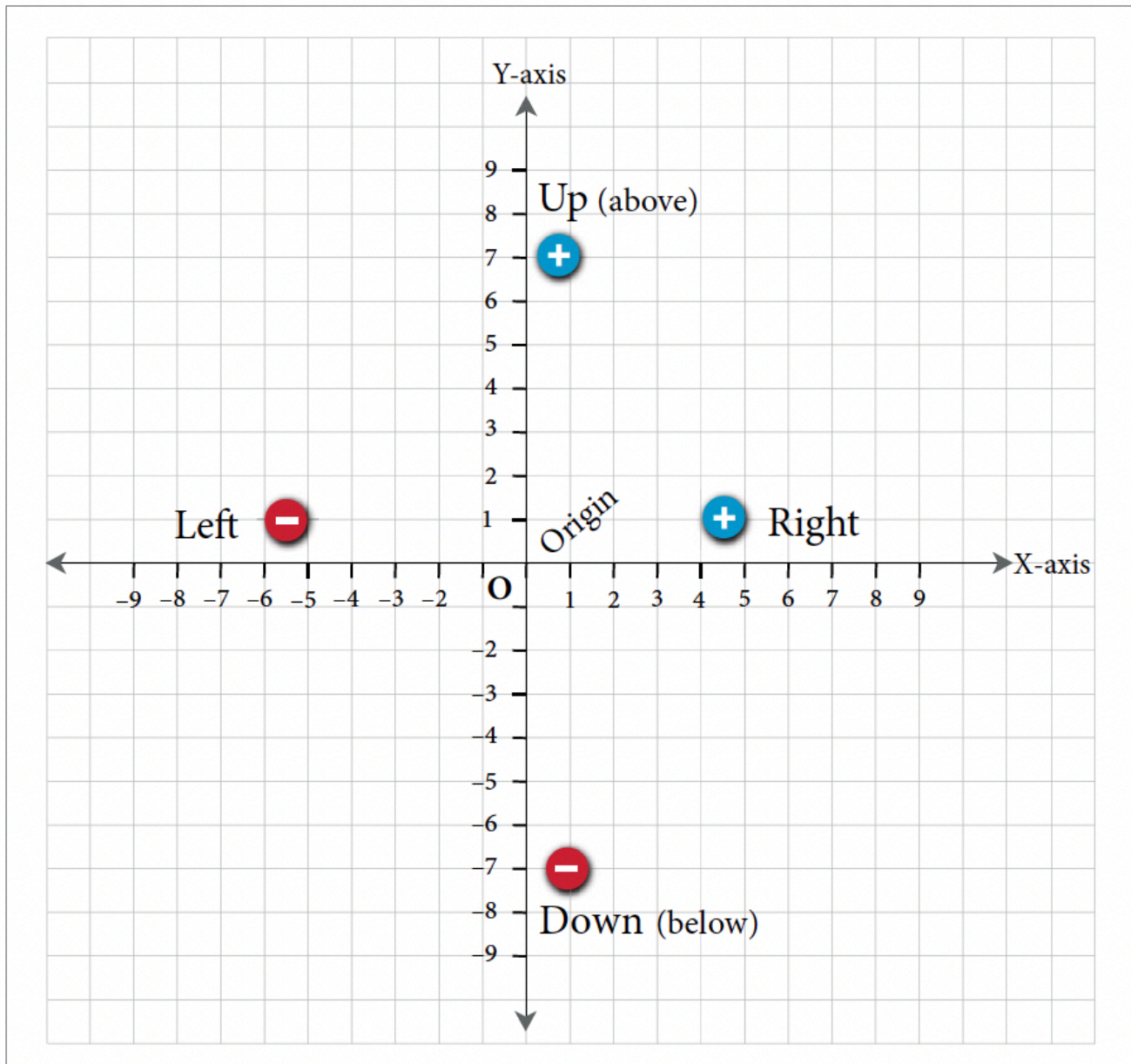
Exhibit 8.1-a Rectangular Coordinate System

The numbers to the **right** of the origin along the X-axis are **positive (+)**, and those to the **left** are **negative** (−). The numbers **above** the origin along the Y-axis are **positive (+)**, and those **below** are **negative** (−).

The purpose of the rectangular coordinate system and the sign convention is to locate a point relative to the X- and Y-axes and in reference to the origin 'O'.

Points in the Rectangular Coordinate System

A **point** in the Cartesian coordinate system is a location in the plane, represented as an **ordered pair** of numbers inside a set of brackets called **coordinates**. The first number is called the **x-coordinate**, representing its horizontal position with respect to the origin, and the second is called the **y-coordinate**, representing its vertical position with respect to the origin. The ordered pair of coordinates for a given point P is written as P(x, y) or (x, y). For example, the origin (i.e., the point where the x-axis and y-axis intersect) is identified by the coordinates (0, 0) since its x and y coordinates are 0.

As illustrated in Exhibit 8.1-b, the ordered pair (2, 3) refers to the coordinates of point P, which is 2 units to the right and 3 units above, in reference to the origin.

It is important to identify the coordinate numbers in their order. They are called ordered pairs because the order in which they appear determines their position on the graph. Changing the order of the coordinates will result in a different point.

For example, (2, 3) and (3, 2) are different points.

- (2, 3) refers to a point 'P', 2 units to the right of the origin and 3 units above the origin.
- (3, 2) refers to a point 'Q', 3 units to the right of the origin and 2 units above the origin.
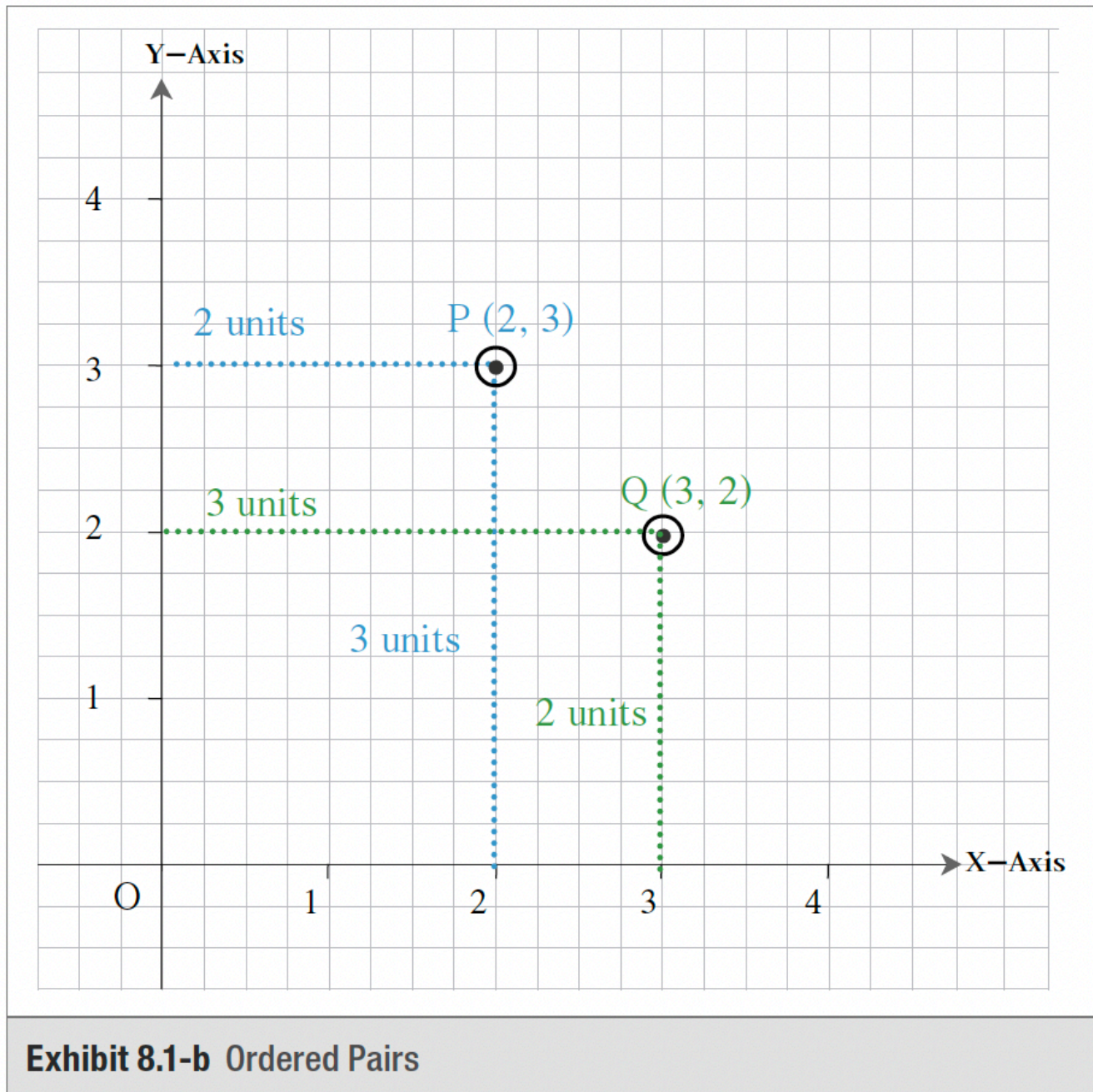
Exhibit 8.1-b Ordered Pairs

It is called a rectangular coordinate system because the x- and y-coordinates form a rectangle with the X- and Y-axes, as seen in the exhibit above.

Quadrants

The X- and Y-axes divide the coordinate plane into four regions called **quadrants**. Quadrants are numbered counter-clockwise from one (I) to four (IV), starting from the upper-right quadrant, as illustrated in Exhibit 8.1-c.

The upper-right quadrant is Quadrant I, the upper-left quadrant is Quadrant II, the lower-left quadrant is Quadrant III, and the lower-right quadrant is Quadrant IV. Table 8.1 shows the sign convention of coordinates in each quadrant, with examples plotted on the graph in Exhibit 8.1-d.

Table 8.1: Sign Convention of Coordinates in Different Quadrants, Axes, and Origin

Quadrant, Axis, Origin	Sign of x-coordinate	Sign of y-coordinate	Example (plotted in Exhibit 8.1-d)
Quadrant I	Positive (+)	Positive (+)	A (3, 2)
Quadrant II	Negative (−)	Positive (+)	B (−3, 4)
Quadrant III	Negative (−)	Negative (−)	C (−5, −2)
Quadrant IV	Positive (+)	Negative (−)	D (5, −3)
X−Axis	Positive (+) or Negative (−)	Zero (0)	E (4, 0), F (−2, 0)
Y−Axis	Zero (0)	Positive (+) or Negative (−)	G (0, 3), H (0, −4)
Origin	Zero (0)	Zero (0)	0 (0, 0)
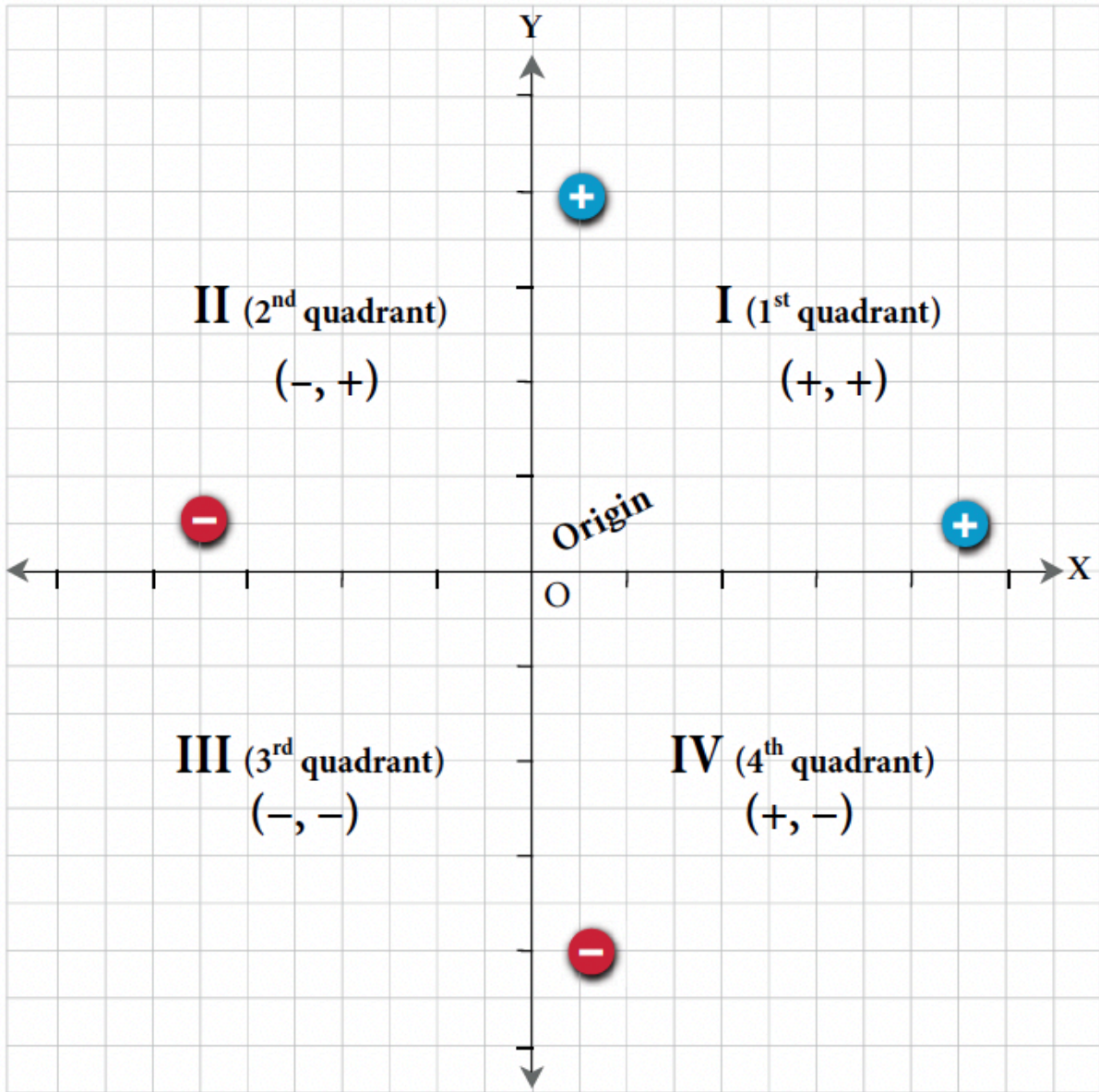
Exhibit 8.1-c The Quadrants
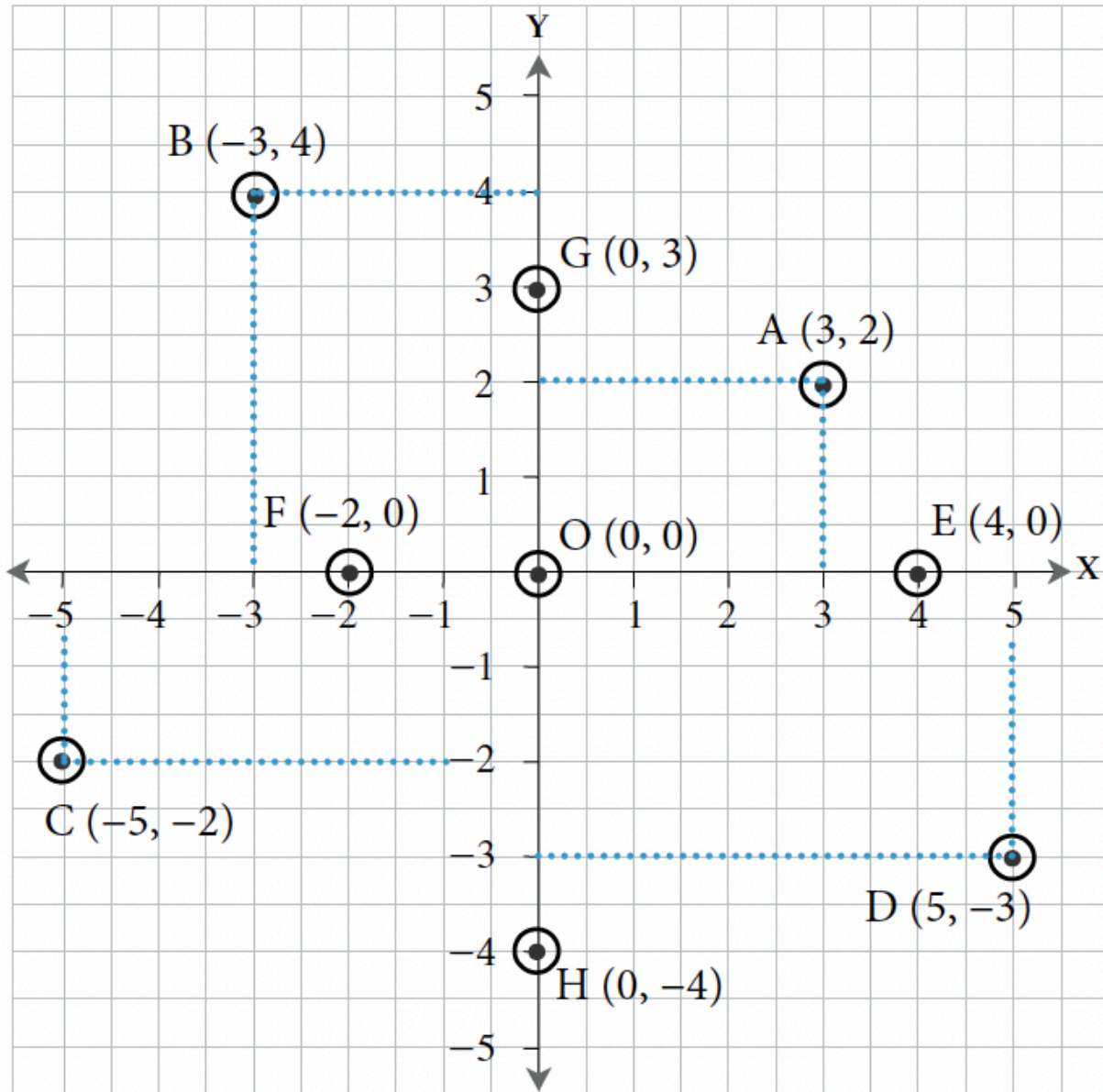
Exhibit 8.1-d Coordinates in Different Quadrants

Example 8.1-a: Identifying x– and y-Coordinates

Determine the x– and y-coordinates of the points A, B, C, D, E, F, G, and H labelled in the graph.
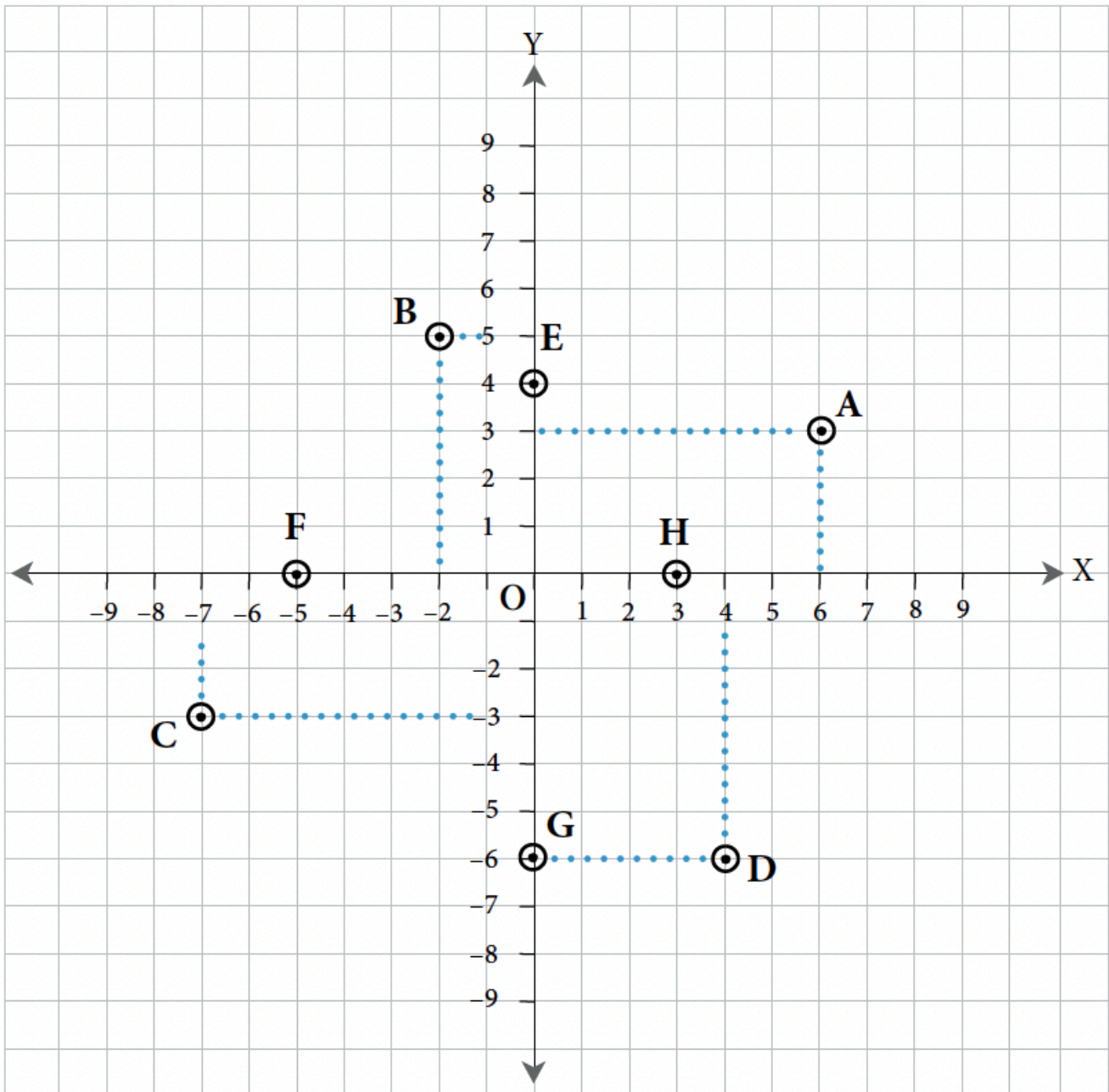
Solution

A: (6, 3) 6 units along the x-axis to the right, 3 units up the y-axis.

B: (−2, 5) 2 units to the left along the x-axis, 5 units up the y-axis.

C: (−7, −3) 7 units to the left along the x-axis, 3 units down the y-axis.

D: (4, −6) 4 units to the right along the x-axis, 6 units down the y-axis.

E: (0, 4) 0 units along the x-axis, 4 units up the y-axis.

F: (−5, 0) 5 units to the right along the x-axis, 0 units up or down the y-axis.

G: (0, −6) 0 units along the x-axis, 6 units down the y-axis.

H: (3, 0) 3 units to the left along the x-axis, 0 units up or down the y-axis.

Example 8.1-b: Identifying the Quadrant or the Axis

Identify the quadrant or the axis in which the following points are located:

a. A (−15, 20)
b. B (20, 5)
c. C (9, 0)
d. D (0, 20)
e. E (12, −18)
f. F (0, −6)
g. G (−30, −15)
h. H (−1, 0)

Solution

a. $A\,(-15, 20) \longrightarrow (-, +) = 2^{nd}$ Quadrant
b. $B\,(20, 5) \longrightarrow (+, +) = 1^{st}$ Quadrant
c. $C\,(9, 0) \longrightarrow (+, 0) =$ X-Axis (Right)
d. $D\,(0, 20) \longrightarrow (0, +) =$ Y-Axis (Up)
e. $E\,(12, -18) \longrightarrow (+, -) = 4^{th}$ Quadrant
f. $F\,(0, -6) \longrightarrow (0, -) =$ Y-Axis (Down)
g. $G\,(-30, -15) \longrightarrow (-, -) = 3^{rd}$ Quadrant

h. $H(-1, 0) \longrightarrow (-, 0) = \text{X-Axis (Left)}$

Example 8.1-c: Plotting Coordinates to Form a Rectangle

Three vertices of a rectangle ABCD have points A (–3, 3), B (4, 3), and C (4, –2). Find the coordinates of the 4th vertex D.

Solution
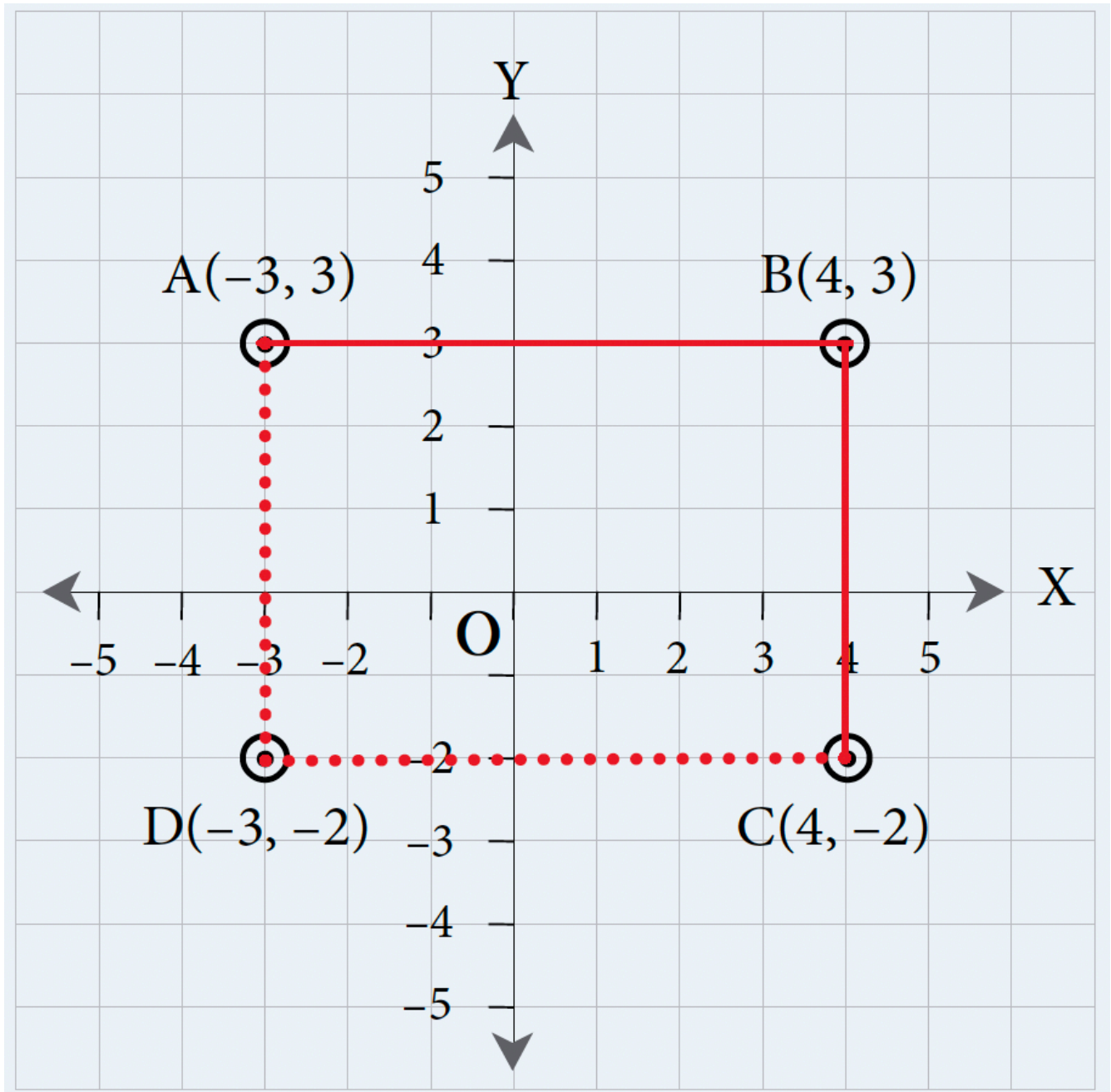
Plotting points A, B, and C:

A (−3, 3): 3 units to the left of the origin and 3 units above the origin

B (4, 3): 4 units to the right of the origin and 3 units above the origin

C (4, −2): 4 units to the right of the origin and 2 units below the origin

Connecting point A to point B results in a horizontal line (since they share the same *y*-coordinate), and connecting point B to point C results in a vertical line (since they share the same *x*-coordinate).

The 4th vertex of the rectangle, D, will have the same x-coordinate as point A and the same y-coordinate as point C.

Therefore, the coordinates for the 4th vertex are D (−3, −2).

Example 8.1-d: Plotting Coordinates to Form a Vertical Line

A vertical line has a length of 3 units, and the coordinates at one end of the line are P (−2, 1). Find the possible coordinates of the other end of the line, Q.

Solution
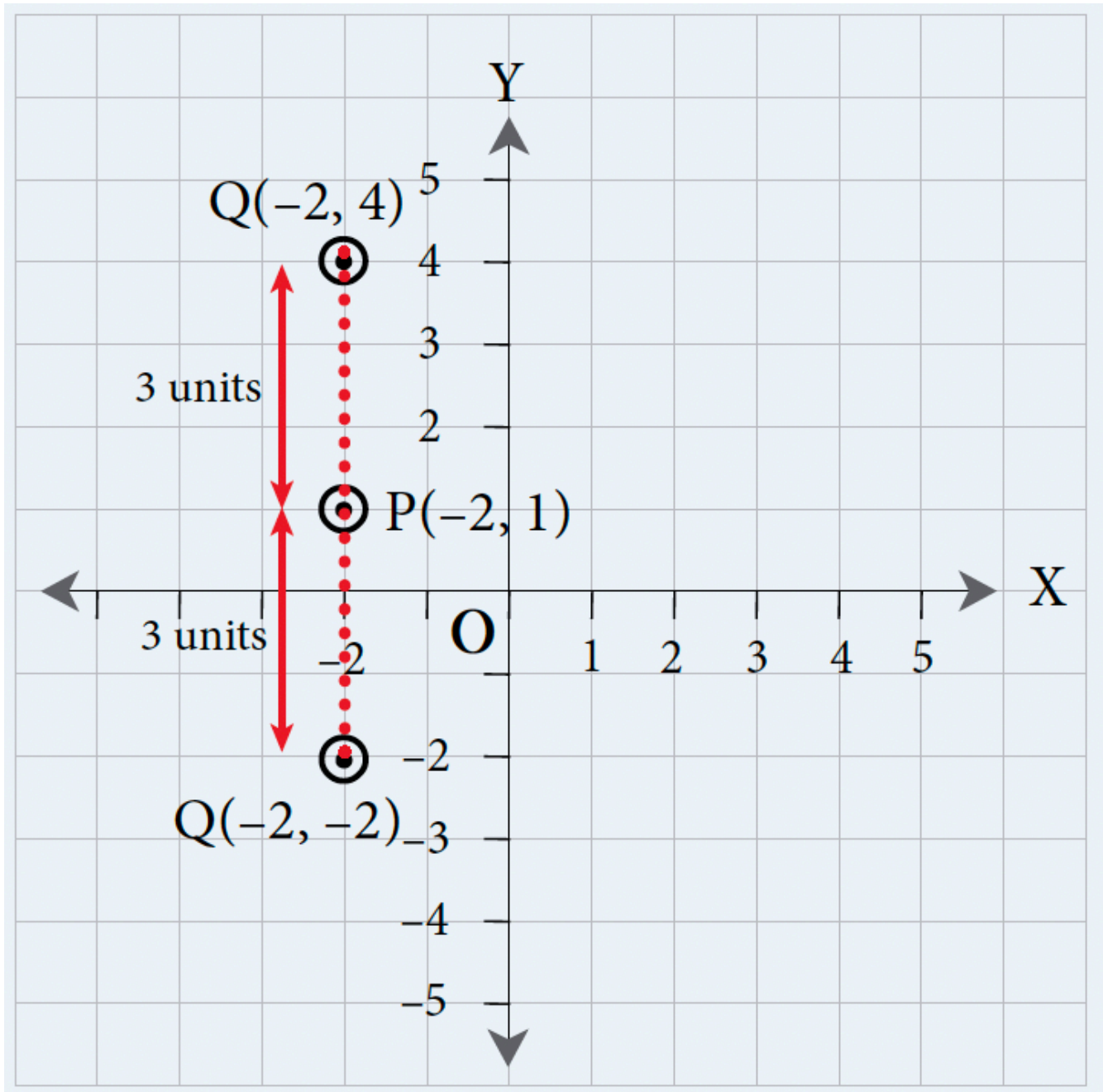
Plotting point P:

P (–2, 1): 2 units to the left of the origin and 1 unit above the origin

Since we are drawing a vertical line, point Q will have the same x-coordinate as point P.

One possible set of coordinates for the other end of the line is 3 units **above** point P, i.e., Q (–2, 4).

The other possible set of coordinates for the other end of the line is 3 units **below** point P, i.e., Q (–2, –2).

8.1 Exercises

Answers to the odd-numbered questions are available at the end of the book.

For problems 1 to 4, plot the points on a graph.

1. a. A $(-3, 5)$
    b. B $(5, -3)$
    c. C $(0, -4)$
2. a. A $(-6, 0)$
    b. B $(4, -2)$
    c. C $(0, -7)$
3. a. D $(6, 0)$
    b. E $(-2, 4)$
    c. F $(5, 2)$
4. a. D $(8, 0)$
    b. E $(-3, -5)$
    c. F $(5, 5)$

For problems 5 to 8, determine the quadrant or axis in which the points lie.

5. a. A $(-1, 2)$
    b. B $(5, -1)$
    c. C $(3, 5)$
6. a. A $(1, 6)$
    b. B $(4, -3)$
    c. C $(-7, 3)$
7. a. D $(-4, 0)$
    b. E $(-2, -7)$
    c. F $(0, 5)$
8. a. D $(6, 0)$
    b. E $(-1, -13)$
    c. F $(0, -7)$

For problems 9 to 12, plot the pairs of points on a graph and calculate the length of each horizontal line joining the pair of points.

9. a. $(3, 4)$ and $(5, 4)$

b. (−7, 1) and (2, 1)

10. a. (2, −6) and (7, −6)

b. (−5, −4) and (0, −4)

11. a. (−5, 3) and (0, 3)

b. (−2, −2) and (6, −2)

12. a. (−6, 8) and (−1, 8)

b. (7, −5) and (2, −5)

For problems 13 to 16, plot the pairs of points on a graph and calculate the length of each vertical line joining the pair of points.

13. a. (3, 6) and (3, 1)

b. (5, 2) and (5, −5)

14. a. (−3, −5) and (−3, −9)

b. (−3, 0) and (−3, 6)

15. a. (5, 6) and (5, 2)

b. (7, 2) and (7, −4)

16. a. (−3, 5) and (−3, −4)

b. (−3, 5) and (−3, 0)

17. Three vertices of a square ABCD have points A (−3, 3), B (1, 3), and C (1, −1). Find the coordinates of the 4th vertex D.

18. Three vertices of a square EFGH have points E (−1, −2), F (6, −2), and G (6, 5). Find the coordinates of the 4th vertex H.

19. Three vertices of a rectangle PQRS have points P (−3, 4), Q (6, 4), and R (6, −1). Find the coordinates of the 4th vertex S.

20. Three vertices of a rectangle TUVW have points T (−4, 7), U (5, 7), and V (5, 4). Find the coordinates of the 4th vertex W.

21. A vertical line has a length of 7 units, and the coordinates of one end of the line are (1, 5). Find the possible coordinates of the other end of the line.

22. A vertical line has a length of 5 units, and the coordinates of one end of the line are (−3, 1). Find the possible coordinates of the other end of the line.

23. A horizontal line has a length of 6 units, and the coordinates of one end of the line are (−1, 3). Find the possible coordinates of the other end of the line.

24. A horizontal line has a length of 8 units, and the coordinates of one end of the line are (−1, −2). Find the possible coordinates of the other end of the line.

8.2 GRAPHING LINEAR EQUATIONS

A linear equation is an algebraic equation with one or two variables (each with one exponent), producing a straight line when plotted on a graph.

Examples of linear equations with one variable in the rectangular coordinate system are:

$$3x - 5 = 0, \quad x = 3, \quad y + 2 = 0, \quad y = -\frac{7}{5}$$

Finding the value of the variables for which the equation is true is known as solving the equation. A linear equation with two variables has infinitely many pairs of values as solutions; therefore, it is often convenient to represent these solutions by drawing a graph.

Linear equations with two variables are generally represented by the variables x and y and are expressed either in the form of $Ax + By = C$ (where A, B, and C are integers and A is positive), known as **standard form**, or in the form of $y = mx + b$ (where m and b are integers or fractions), known as **slope-intercept form**.

Linear Equations in Standard Form

The 'standard' form for a linear equation with two variables, x, and y, is written as $\boldsymbol{Ax + By = C}$, where A, B, and C are integers, v is positive, and A, B, and C have no common factors other than 1.

For example, consider the following simple linear equation with two variables: $2x - y = -3$

The equation $Ax + By = C$, where $A = 2$, $B = -1$, and $C = -3$.

- If a given equation has fractions, multiply each term by the least common denominator (LCD), divide each term by any common factors, and rearrange the equation into standard form.
- If a given equation has decimals, multiply each term by an appropriate power of 10 to eliminate the decimals, divide each term by common factors, and rearrange the equation into standard form.
- If a given equation has no fractions or decimals, divide each term by any common factors and rearrange it into standard form.

Example 8.2-a: Writing Linear Equations in Standard Form

Write the following linear equations in standard form:

a. $\dfrac{2}{3}x + \dfrac{1}{2}y - 3 = 0$

b. $0.3x = 1.25y - 2$

c. $y = \dfrac{2}{3}x - 5$

Solution

a. $\dfrac{2}{3}x + \dfrac{1}{2}y - 3 = 0$

Multiplying each term by the LCD of 6 and simplifying, $4x + 3y - 18 = 0$

Rearranging, $4x + 3y = 18$

This is in the form $Ax + By = C$.

Therefore, the equation $\dfrac{2}{3}x + \dfrac{1}{2}y - 3 = 0$, in standard form, is $4x + 3y = 18$

b. $0.3x = 1.25y - 2$

Multiplying each term by 100, $30x = 125y - 200$

Dividing each term by the common factor 5 and simplifying, $6x = 25y - 40$

Rearranging, $6x - 25y = -40$

This is in the form $Ax + By = C$.

Therefore, the equation $0.3x = 1.25y - 2$, in standard form, is $6x - 25y = -40$

c. $y = \dfrac{2}{3}x - 5$

Multiplying each term by 3 and simplifying, $3y = 2x - 15$

Rearranging, $-2x + 3y = -15$

Multiplying each term by –1 to make A positive, $2x - 3y = 15$

This is in the form $Ax + By = C$.

Therefore, the equation $y = \dfrac{2}{3}x - 5$, in standard form, is $2x - 3y = 15$

Linear Equations in Slope-Intercept Form

The 'slope-intercept' form for a linear equation with two variables, x, and y, is written as $y = mx + b$, where m and b are either integers or fractions. **'m'** represents the slope, and **'b'** represents the y-coordinate of the y-intercept.

The **slope** is the steepness of the line relative to the X-axis.

The **y-intercept** is the point at which the line crosses the Y-axis and where the x-coordinate is zero.

For example, consider a simple linear equation such as $y = 2x + 3$. This equation is in the slope-intercept form of $y = mx + b$, where $m = 2$ and $b = 3$; hence, the slope is 2, and the y-intercept is $(0, 3)$.

Example 8.2-b: Writing Linear Equations in Slope-Intercept Form

Write the following linear equations in slope-intercept form and identify the slope and the y-intercept.

a. $4x + 3y = 18$

b. $x = \dfrac{25}{6}y + \dfrac{20}{3}$

Solution

a. $4x + 3y = 18$

Rearranging the equation with y on the left, $3y = -4x + 18$

Dividing each term by 3 and simplifying, $y = -\dfrac{4}{3}x + 6$

This is in the form $y = mx + b$.

Therefore, the slope is $m = -\dfrac{4}{3}$ and the y-intercept is the point $(0, 6)$.

b. $x = \dfrac{25}{6}y + \dfrac{20}{3}$

Multiplying each term by the LCD of 6 and simplifying, $6x = 25y + 40$

Rearranging the equation with y on the left, $-25y = -6x + 40$

Multiplying each term by -1, $25y = 6x - 40$

Dividing each term by 25 and simplifying, $y = \dfrac{6}{25}x - \dfrac{8}{5}$

This is in the form $y = mx + b$.

Therefore, the slope is $m = -\dfrac{6}{25}$ and the y-intercept is the point $\left(0, -\dfrac{8}{5}\right)$.

Determining the Solution Set of a Linear Equation

If a linear equation has two variables, x, and y, then there are infinitely many solutions to the equation, and it is not possible to solve the equation for a single value of each variable. However, it is possible to create a **set** of solutions by replacing one variable (either x or y) with any value and then computing the value of the other variable.

For example, consider the equation: $y = 2x + 3$

Choosing $x = 1$ and substituting $x = 1$ into the equation:

$y = 2x + 3$

$y = 2(1) + 3 = 5$

Therefore, $x = 1$ and $y = 5$ is one of the infinitely many solutions to the equation; i.e., the ordered pair (1, 5) is a point that satisfies the equation.

Choosing $x = 2$ and substituting $x = 2$ into the equation:

$y = 2x + 3$

$y = 2(2) + 3 = 7$

Therefore, $x = 2$ and $y = 7$ is another solution to the equation; i.e., the ordered pair (2, 7) is another point that satisfies the equation.

Similarly, we can obtain any number of points that satisfy the equation by choosing different values for x, and computing the corresponding value for y.

We can represent the full solution set by graphing the linear equation. The graph of the linear equation will be a line formed by all the solutions to the linear equation. Conversely, any point on the line is a solution to the linear equation.

Graphing Linear Equations Using a Table of Values

Follow these steps to graph a linear equation using a table of values:

Step 1: Create a table of values by choosing any value for the variable x (0 is often a good first choice).

Step 2: Compute the corresponding value for the variable y (this is easiest if the equation is in slope-intercept form).

Step 3: Form the ordered pair (x, y).

Step 4: Repeat Steps 1 to 3 at least two more times to create at least three ordered pairs.

Step 5: Plot the ordered pairs (points) on the coordinate system using an appropriate scale.

Step 6: Join the points in a straight line, continuing the line indefinitely in both directions using arrows.

Step 7: Label the graph with the equation of the line.

For example, consider the linear equation $y = 2x + 3$. We will first determine the coordinates of four ordered pairs on this line by choosing values for x and finding the corresponding values for y; then, we will draw the graph.

- Choosing $x = 0, y = 2x + 3 = 2(0) + 3 = 0 + 3 = 3, (0, 3)$ is a point on the line.
- Choosing $x = 1, y = 2x + 3 = 2(1) + 3 = 2 + 3 = 5, (1, 5)$ is a point on the line.
- Choosing $x = 2, y = 2x + 3 = 2(2) + 3 = 4 + 3 = 7, (2, 7)$ is a point on the line.
- Choosing $x = 3, y = 2x + 3 = 2(3) + 3 = 6 + 3 = 9, (3, 9)$ is a point on the line.

Table of Values for $y = 2x + 3$

X Values	Y Values	Corresponding Coordinates
x	y	(x, y)
0	3	$(0, 3)$
1	5	$(1, 5)$
2	7	$(2, 7)$
3	9	$(3, 9)$
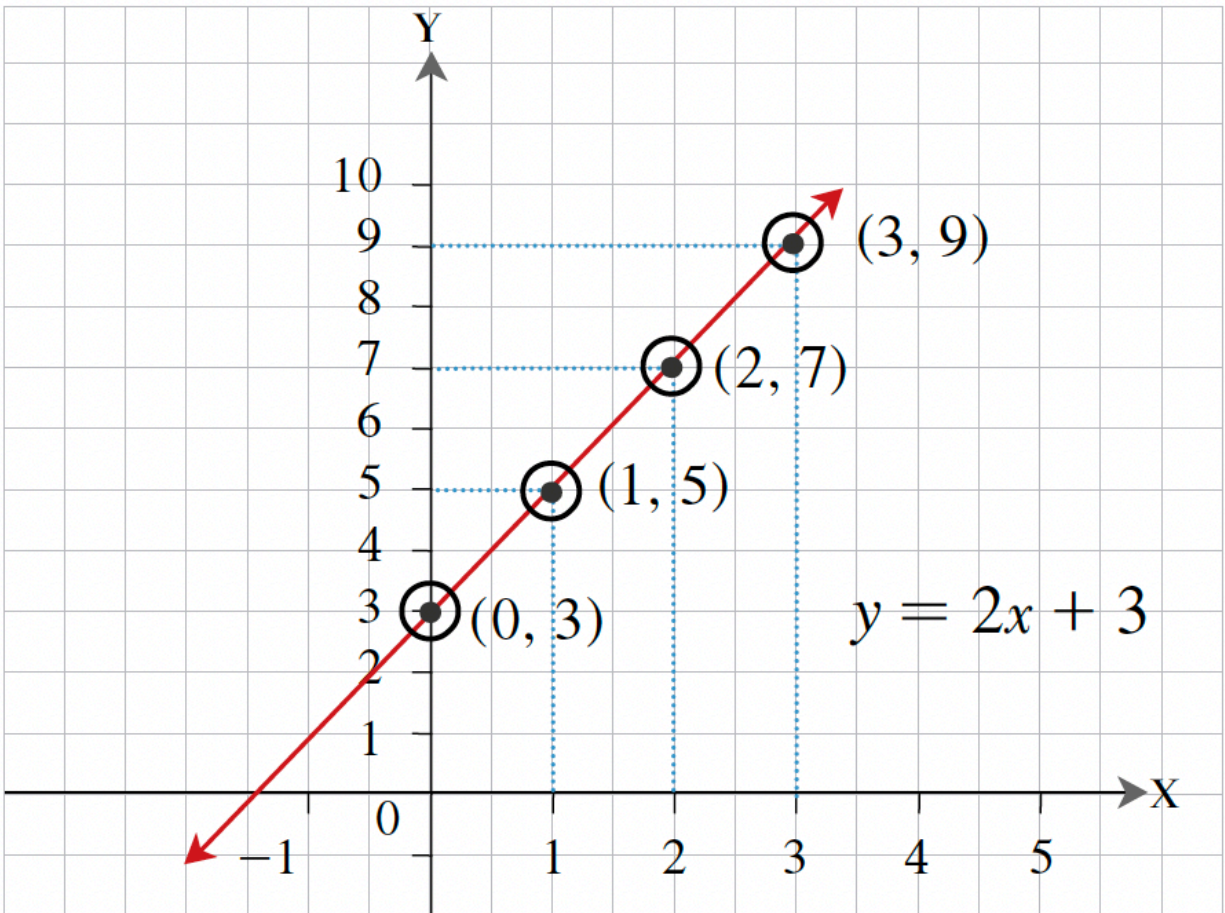
Exhibit 8.2-a Graphing a Linear Equation Using a Table of Values

Since all the points in the Table of Values fall on a line when joined, it verifies that the plotted line represents the equation.

Graphing Linear Equations Using the *x*-intercept and the *y*-intercept

Recall that the **y-intercept** is the point at which the line crosses the Y-axis and where the x-coordinate is zero. Similarly, the **x-intercept** is the point at which the line crosses the X-axis and where the y-coordinate is zero.

If the intercepts are not at the origin (0, 0), we may use the *x*-intercept and *y*-intercept as two points to draw a linear graph and use a third point to test the drawn line.

If the intercepts are at the origin (0, 0), we need to compute another ordered pair to use as the second point (along with the origin) to draw the line and a third point to test the drawn line.

For example, consider a linear equation, $3x - y = -9$, where we will find the x-intercept, y-intercept, and another ordered pair to draw the graph.

Finding the x-intercept:

- Substituting $y = 0$ into the given equation and solving for x,
- $3x - 0 = -9$, thus, $x = -3$. Therefore, $(-3, 0)$ is the x-intercept.

Finding the y-intercept:

- Substituting $x = 0$ into the given equation and solving for y,
- $3(0) - y = -9$, thus, $y = 9$. Therefore, $(0, 9)$ is the y-intercept.

Finally, finding another ordered pair as a third point on the line to use as a test point to verify the plotted line joining the x-intercept and y-intercept:

- Choosing $x = -1$, substituting this in the given equation, and solving for y,
- $3(-1) - y = -9$, thus, $y = 6$. Therefore, $(-1, 6)$ is a point on the line $3x - y = -9$.

Now that we have the x-intercept, the y-intercept, and a test point:

- Plot the ordered pairs on the coordinate system using an appropriate scale.
- Draw a line to join the x-intercept and y-intercept.
- Verify that the test point falls on the graph of the plotted line.

Table of Values for $3x - y = -9$

X Values	Y Values	Corresponding Coordinates	Point Identification
x	y	(x, y)	none
-3	0	$(-3, 0)$	x-intercept
0	9	$(0, 9)$	y-intercept
-1	6	$(-1, 6)$	Test point

Exhibit 8.2-b Graphing a Linear Equation Using the *x*-intercept and the *y*-intercept

Since the point (–1, 6) falls on the line when plotted on the graph, it verifies that the plotted line represents the linear equation.

Graphing Linear Equations Using the Slope and the *y*-intercept

Recall that a linear equation in the form of $y = mx + b$ is the equation in slope-intercept form, where '*m*' is the slope and '*b*' is the *y*-coordinate of the *y*-intercept.

If the equation is in the standard form $Ax + By = C$, it can be rearranged to represent the slope-intercept form as follows:

$$Ax + By = C$$
$$By = -Ax + C$$
$$y = -\frac{A}{B}x + \frac{C}{B}$$

This is in the form $y = mx + b$, where the slope $m = -\dfrac{A}{B}$, and the y-coordinate of the *y*-intercept $b = \dfrac{C}{B}$.

The Slope and *y*-intercept of a Line

The slope (m) is the steepness of the line relative to the X-axis. It is the ratio of the change in the value of *y* (called the 'rise' and denoted Δy) to the corresponding change in the value of *x* (called the 'run' and denoted Δx).

If P (x_1, y_1) and Q (x_2, y_2) are two different points on a line, then the slope of the line PQ between the two points is:

$$m = \frac{Rise}{Run} = \frac{Change\ in\ y\ value}{Change\ in\ x\ value} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

This is illustrated in Exhibits 8.2-c and 8.2-d:

Exhibit 8.2-c Coordinates of Points P and Q
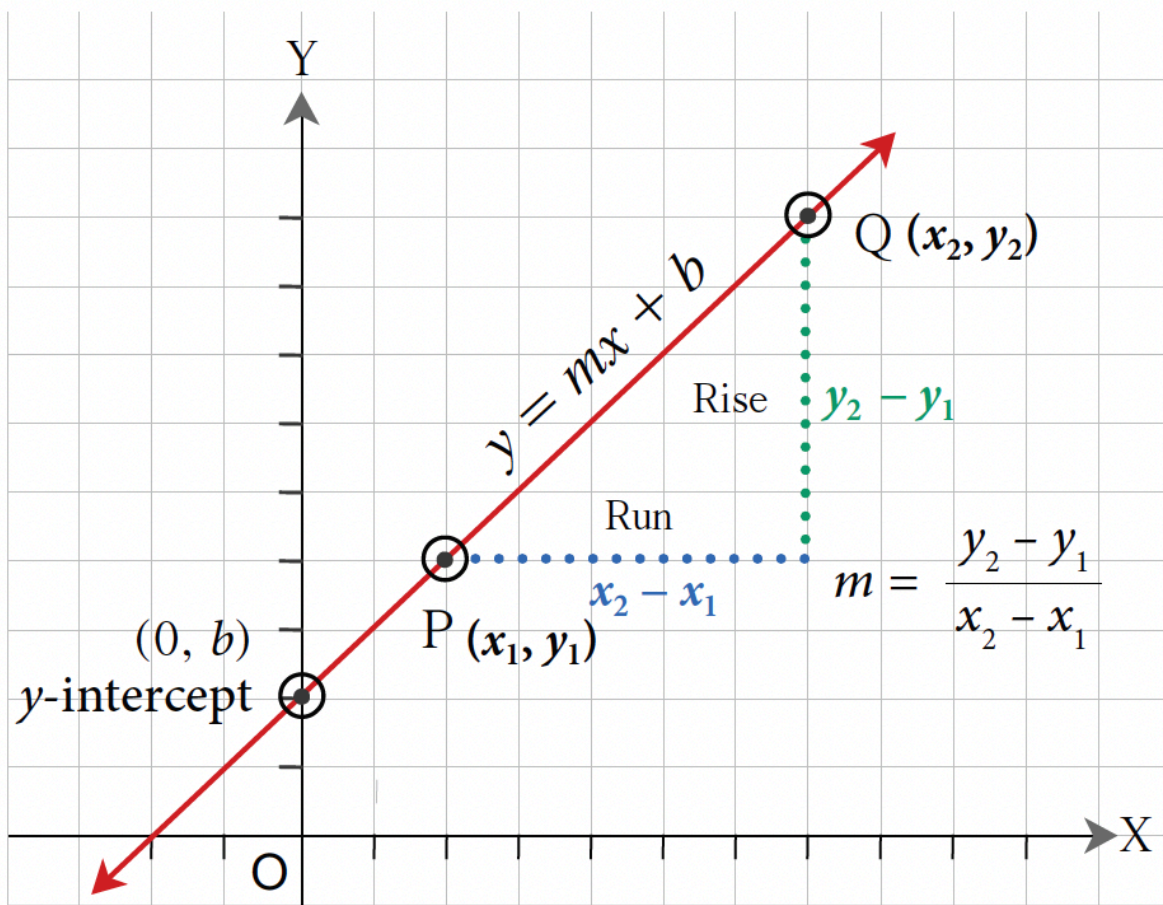
Exhibit 8.2-d Rise and Run Between Points P and Q

Example 8.2-c: Finding the Slope and y-intercept of a Linear Equation and Graphing the Equation

Find the slope and y-intercept of the linear equation $-2x + 3y - 12 = 0$ and graph the equation.

Solution

$4x + 3y = 18$

Rearranging the equation with y on the left,

$3y = -4x + 18$

Dividing each term by 3 and simplifying,

$y = -\dfrac{4}{3}x + 6$

This is in the form $y = mx + b$.

Therefore, the slope is $m = -\dfrac{4}{3}$ and the y-intercept is the point $(0, 6)$.

Therefore, $(0, 4)$ is a point on the line and the slope, $m = \dfrac{Rise}{Run} = \dfrac{Change\ in\ y\ value}{Change\ in\ x\ value} = \dfrac{2}{3}$,

represents an increase of 2 in the vertical direction ('rise') for every increase of 3 in the horizontal direction ('run').

Representing this on a graph:

a. First, plot the y-intercept $(0, 4)$.
b. From this point, move 3 units to the right and then move 2 units up to locate the new point $(3, 6)$.
   Note: This is the same as moving 2 units up and then 3 units to the right.
c. Similarly, from the point $(3, 6)$, move 3 units to the right and 2 units up to locate another point $(6, 8)$.
d. Draw the line through these points to graph the equation.

Or,

a. First, plot the y-intercept $(0, 4)$.
b. From this point, move 3 units to the left and then move 2 units down to locate the new point $(-3, 2)$.
c. Similarly, from the point $(-3, 2)$, move 3 units to the left and 2 units down to locate another point $(-6, 0)$.
d. Draw the line through these points to graph the equation.
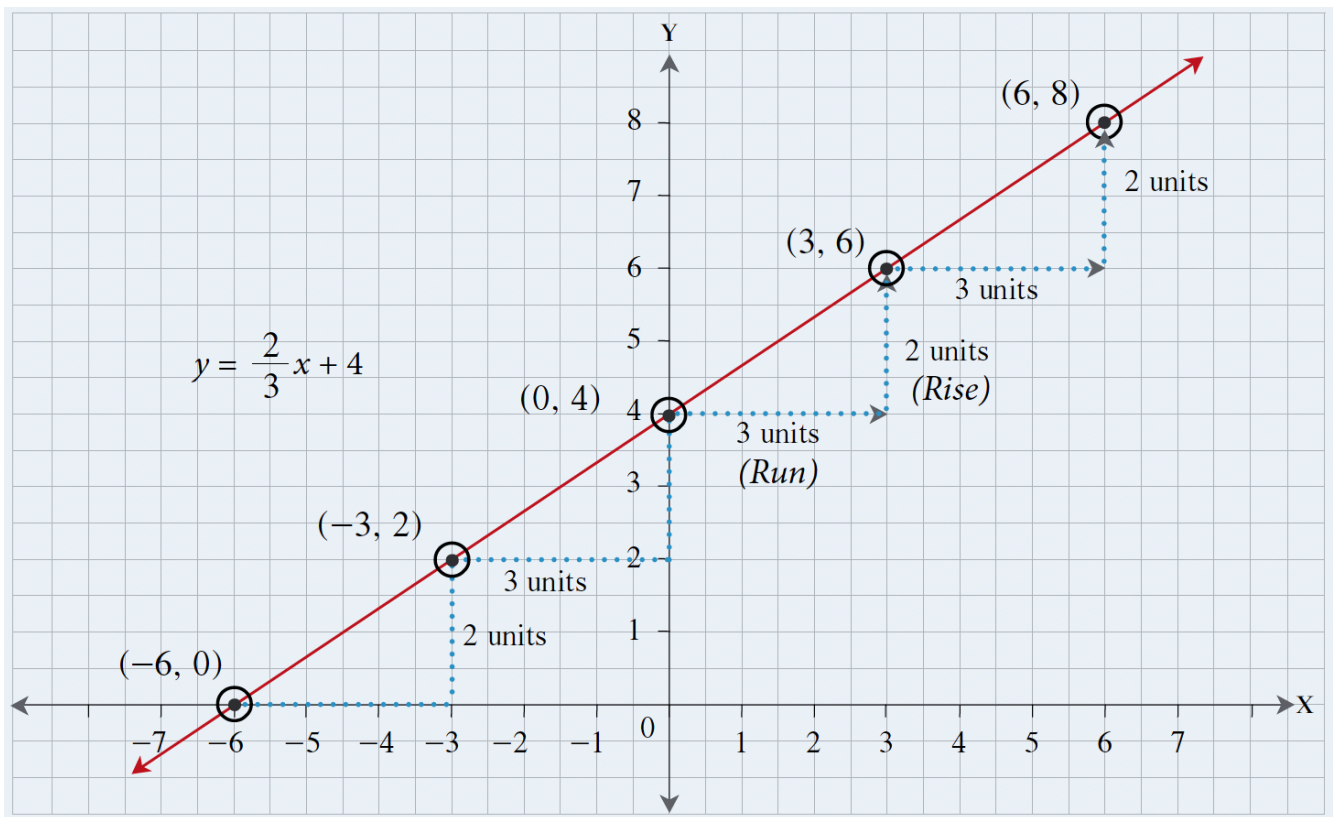
Note: All the points must lie on the same line, or a mistake has been made.

Example 8.2-d: Graphing a Linear Equation in the Slope-Intercept Form

Graph the equation $y = -\dfrac{3}{4}x - 2$.

Solution

The equation is $y = mx + b$.

Therefore, $m = -\dfrac{3}{4}$ and $b = -2$, which means the y-intercept is the point $(0, -2)$ and the slope is

$$m = \frac{Rise}{Run} = \frac{Change\ in\ y\ value}{Change\ in\ x\ value} = \frac{-3}{4} \text{ or } \frac{3}{-4}$$

First, plot the point (0, –2). Then, using the slope, $m = \dfrac{-3}{4}$, from the point (0, –2), move 4 units to the right and 3 units down to locate the new point, (4, –5).

Alternatively, using the slope, $m = \dfrac{3}{-4}$, from the point (0, –2), move 4 units to the left and 3 units up to locate another point on the line, (–4, 1).

Draw a line through these points to graph the equation.
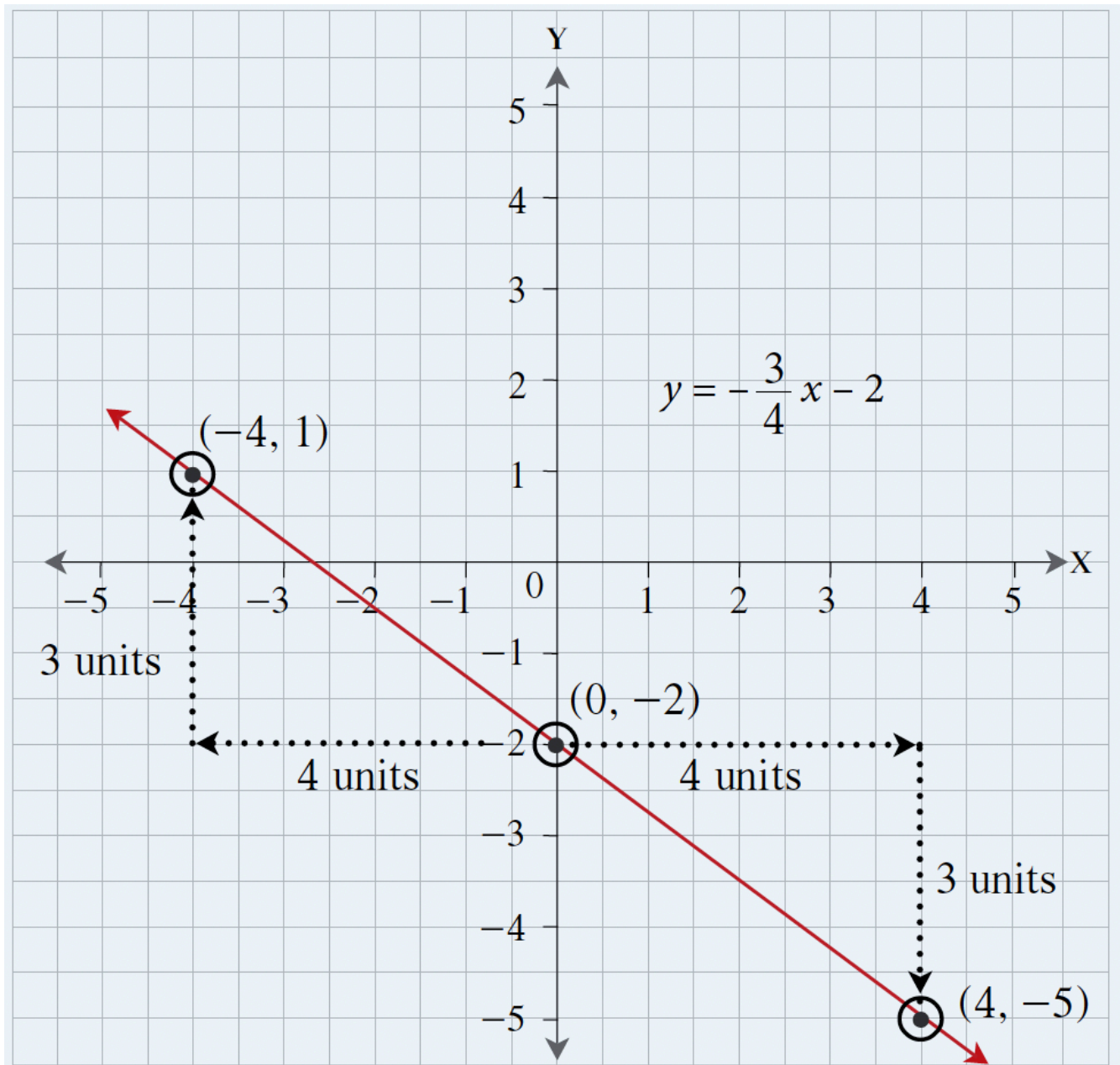
Slope and Direction of a Line

A line's slope, m, is a number that describes the line's direction and steepness.

The direction of the line either a. slopes upwards to the right, b. slopes downwards to the right, c. is horizontal, or d is vertical, as discussed below:

If the sign of 'm' is positive, the line slopes upwards to the right (i.e., the line rises from left to right), as illustrated in Exhibit 8.2-e.

Example: $y = \dfrac{1}{2}x + 1$

Slope, $m = \dfrac{1}{2}$

m is positive

(0, 1)

1 unit

2 units
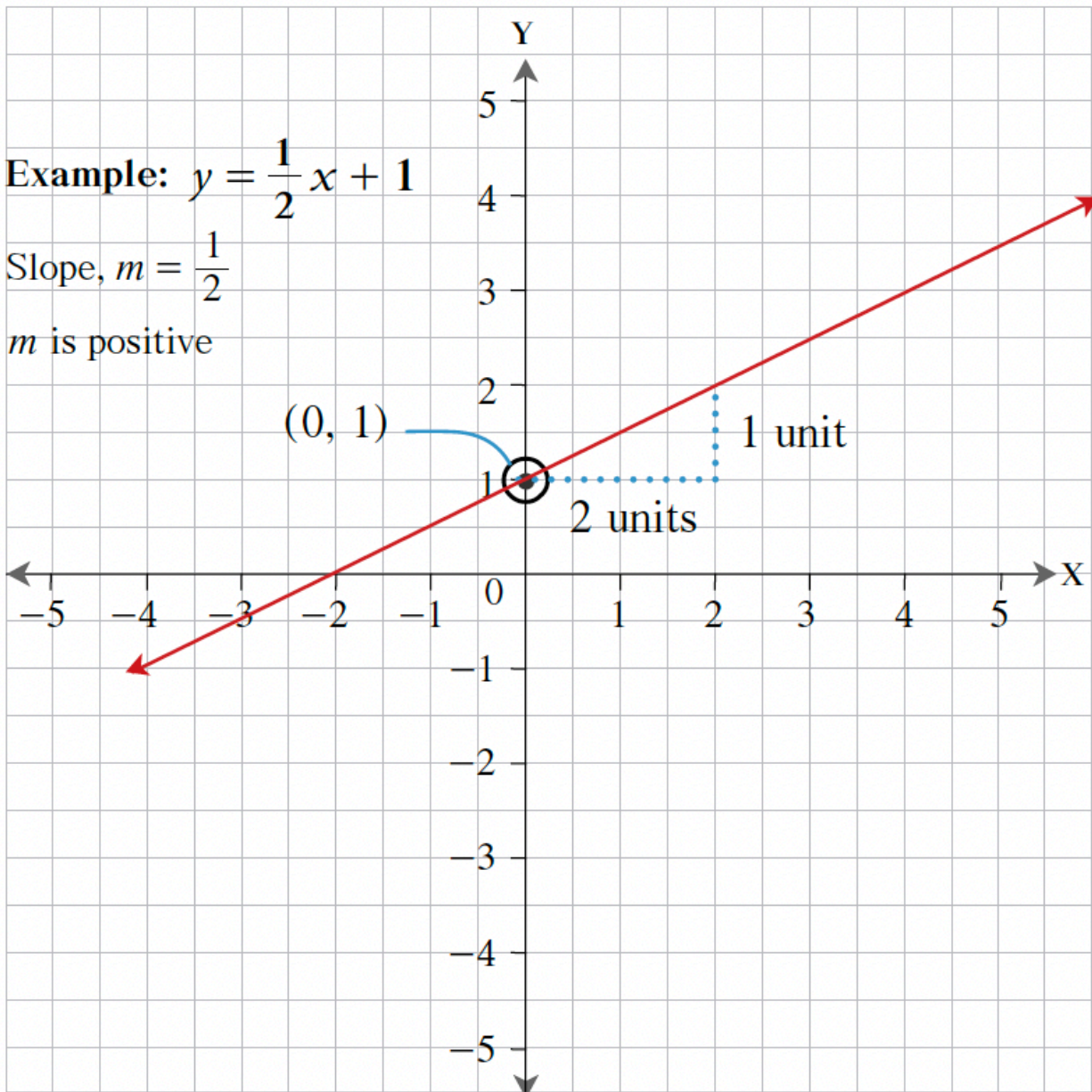
Exhibit 8.2-e The Graph of a Line When 'm' is Positive

If the sign of '*m*' is negative, the line slopes downwards to the right (i.e., the line falls from left to right), as illustrated in Exhibit 8.2-f.
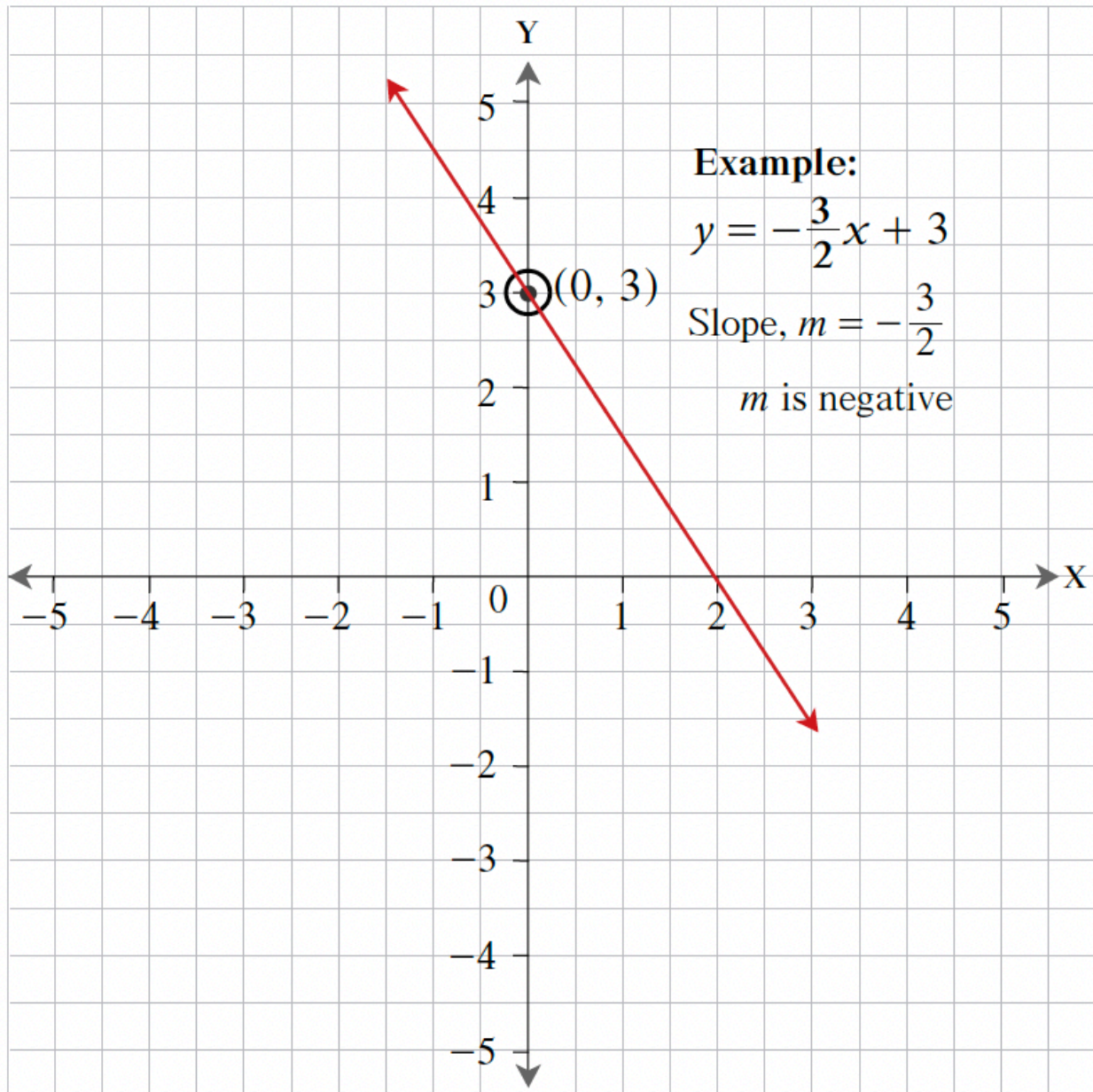
Exhibit 8.2-f The Graph of a Line When '*m*' is Negative

If '*m*' is zero, the line is horizontal (parallel to the X-axis). A slope of zero means that when the *x*-coordinate increases or decreases, the *y*-coordinate does not change (i.e., 'rise' = 0). This is a special case of the linear equation $Ax + By = C$, where the value $A = 0$. Therefore, the equation of a horizontal line will be in the form *y* = b, and the *y*-intercept of the line is (0, b). For example, in the equation, $y = 3$, (i.e., $y = 0x + 3$),

the slope is zero, and the value of the *y*-coordinate is 3 for all values of *x*. Therefore, the line is horizontal and passes through the *y*-intercept (0, 3), as illustrated in Exhibit 8.2-g.
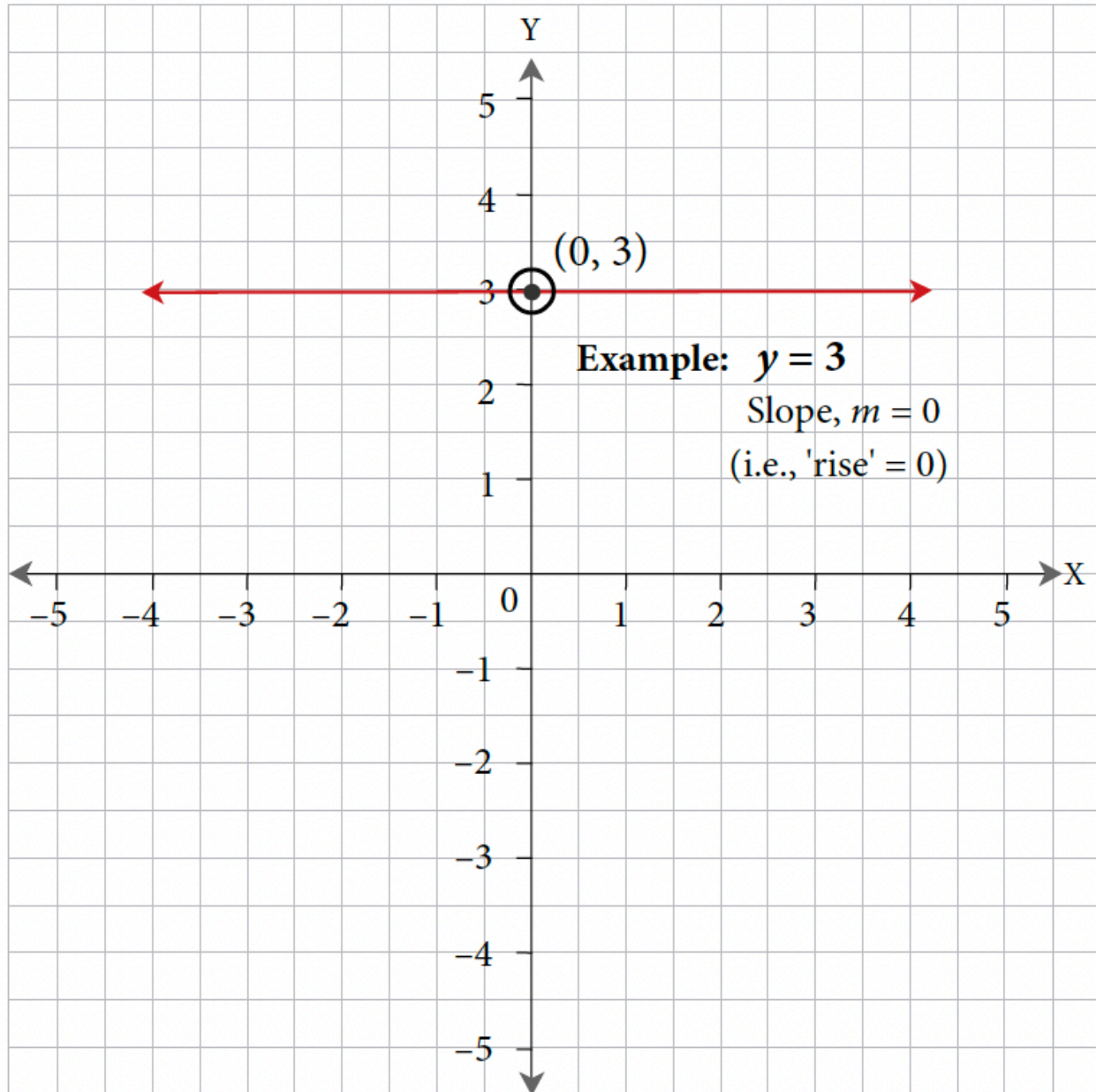
Exhibit 8.2-g The Graph of a Line When '*m*' is Zero

If '*m*' is undefined, the line is vertical (parallel to the Y-axis). An undefined slope means that when the *y*-coordinate increases or decreases, the *x*-coordinate does not change (i.e., 'run' = 0). This is a special case of the linear equation $Ax + By = C$, where the value $B = 0$. An equation in this form cannot be rearranged

into slope-intercept form, as it would require dividing by 0. As such, rather than isolating for y, we isolate for the variable x. Therefore, the equation of a vertical line will be in the form $x = a$, and the x-intercept of the line is $(a, 0)$. For example, in the equation, $x = 2$, the slope is undefined, and the value of the x-coordinate is 2 for all values of y. Therefore, the line is vertical and passes through the x-intercept $(2, 0)$, as illustrated in Exhibit 8.2-h.

The graph shows a vertical line at $x = 2$ with the label:

Example: $x = 2$
Slope undefined
(i.e., 'run' = 0)

$(2, 0)$
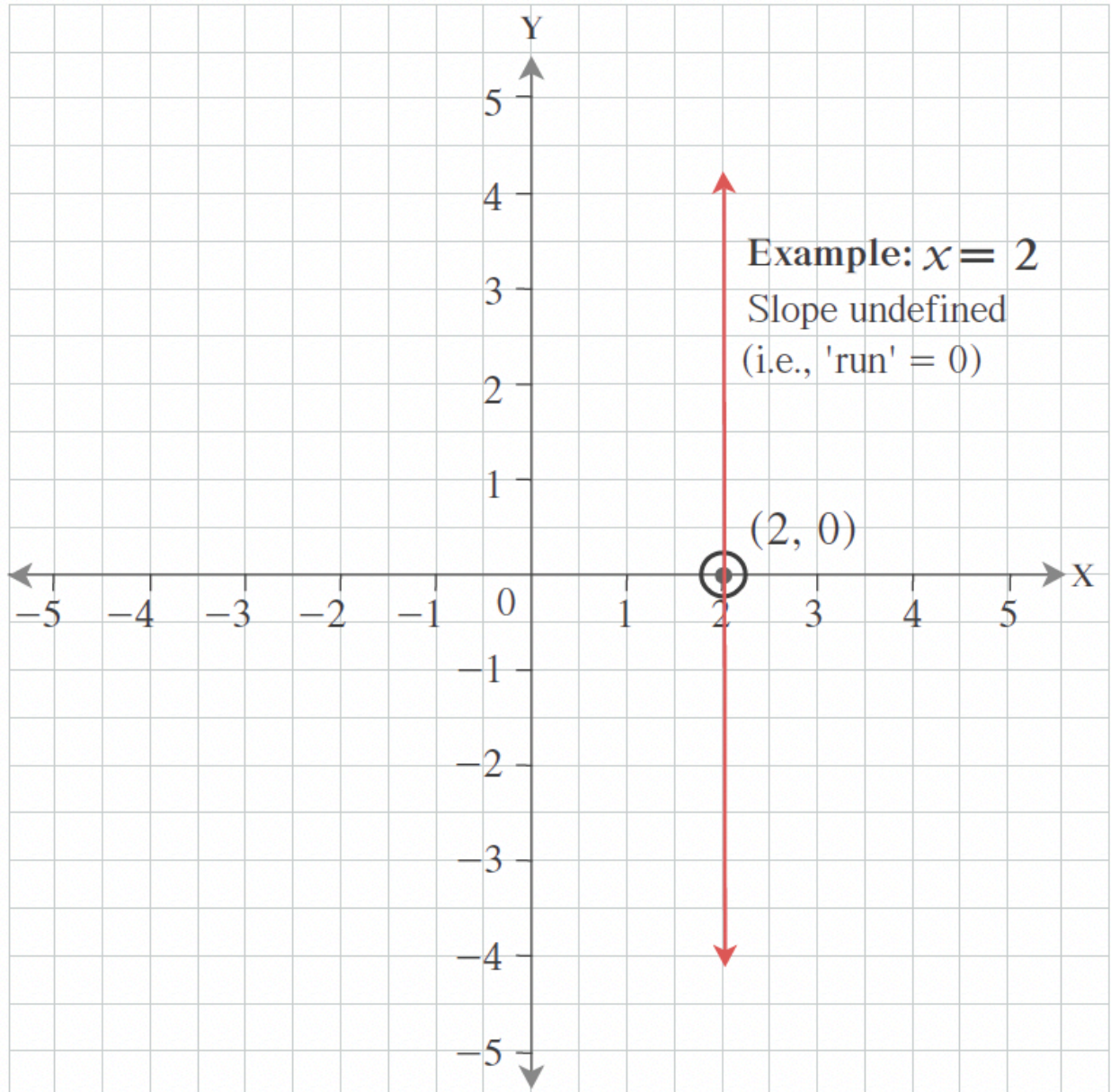
Exhibit 8.2-h The Graph of a Line When 'm' is Undefined

Slope and Steepness of a Line

The steepness of the line is also measured by the slope ('*m*') of the line. The farther the coefficient '*m*' is away from zero, the steeper the slope. The closer the coefficient '*m*' is to zero, the flatter the slope.

- A greater value for a positive slope indicates a steeper rise.

Slope, $m = \dfrac{1}{1} = 1$

SLOPE OF 1

Slope, $m = \dfrac{4}{2} = 2$

STEEPER RISE

Slope, $m = \dfrac{1}{4}$
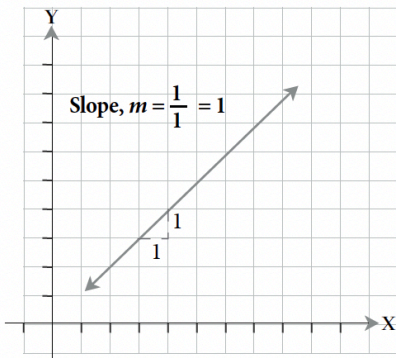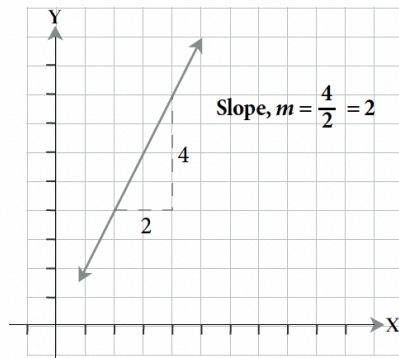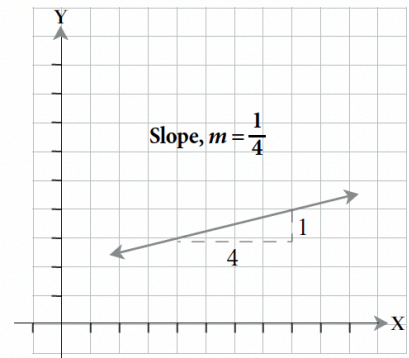
FLATTER RISE

Positive slopes

- A lesser value for a negative slope indicates a steeper fall.

Slope, $m = \dfrac{-1}{1} = -1$

SLOPE OF −1

Slope, $m = \dfrac{-4}{2} = -2$

STEEPER FALL

Slope, $m = \dfrac{-1}{4} = -\dfrac{1}{4}$
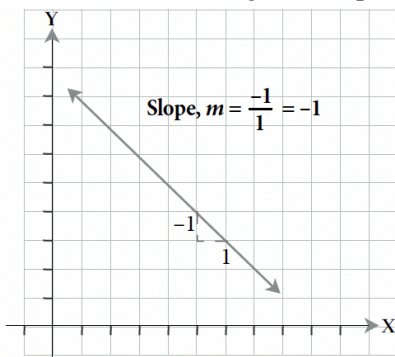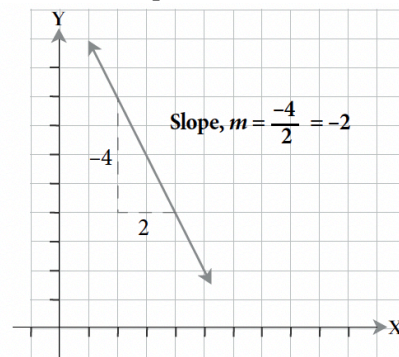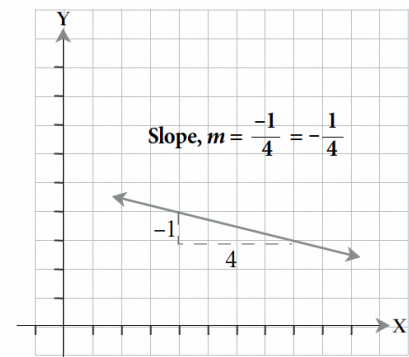
FLATTER FALL

Negative slopes

- As discussed earlier, a slope of 0 indicates a horizontal line (no steepness), and an undefined slope indicates a vertical line (infinite steepness).

SLOPE OF 0 - NO STEEPNESS

UNDEFINED SLOPE - INFINITE STEEPNESS
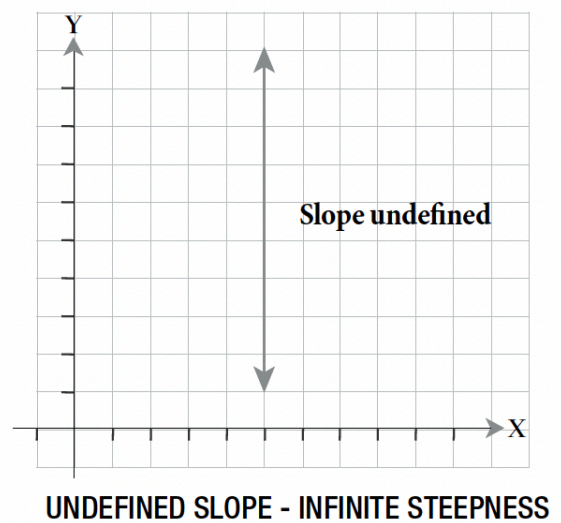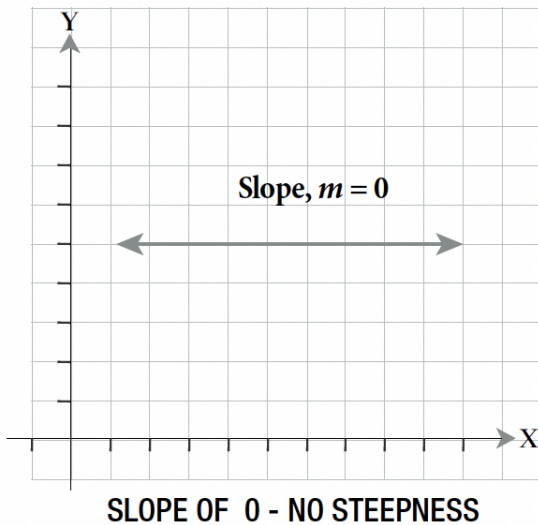
Special slopes

Lines Passing Through the Origin

A line passing through the origin means the point $(0, 0)$ is on the line. The x- and y-intercept coordinates are $(0, 0)$. Hence, the equation of a line passing through the origin will be in the form $y = mx$, with the exception of two special cases:

- $y = 0$ – **the equation of the X-axis** (i.e., the horizontal line passing through the origin)
- $x = 0$ – **the equation of the Y-axis** (i.e., the vertical line passing through the origin)

For example, in the equation, $y = 2x$, $m = 2$, and $b = 0$; hence the slope is 2, and the y-intercept is the point $(0, 0)$.

Similarly, in the equation $y = -x$, $m = -1$, and $b = 0$; hence the slope is –1 and the y-intercept is the point $(0, 0)$.

Therefore, both lines above pass through the origin $(0, 0)$, as illustrated in Exhibit 8.2-i.
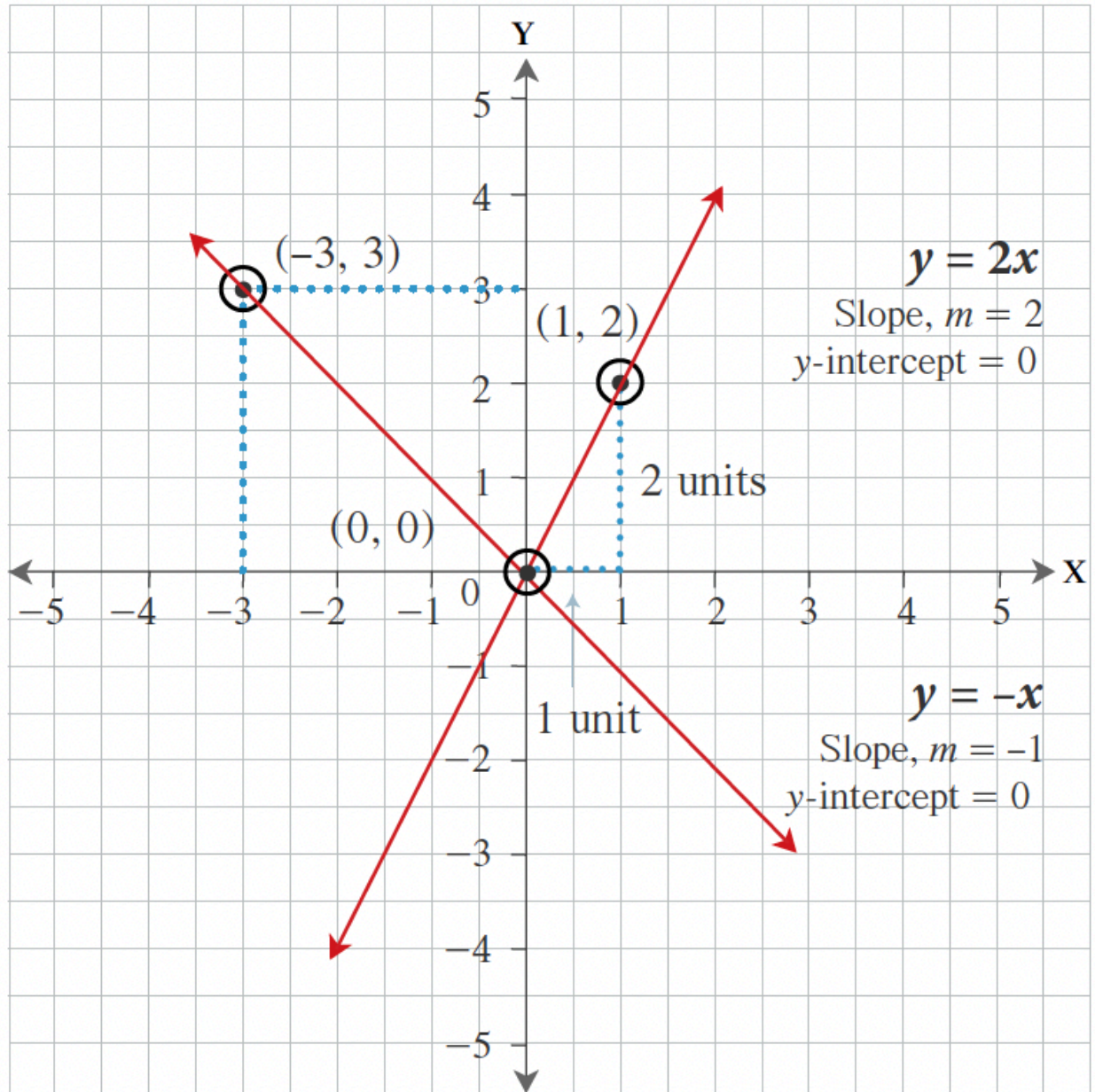
Exhibit 8.2-i Graphs of Lines Passing Through the Origin

8.2 Exercises

Answers to the odd-numbered questions are available at the end of the book.

1. For the equation $2x + 3y = 18$, find the missing values in the following ordered pairs:

   a. $(3, ?)$

b. (-6, ?)

c. (0, ?)

d. (?, 0)

e. (?, -4)

f. (?, 2)

2. For the equation $x + 5y = 20$, find the missing values in the following ordered pairs:

a. (0, ?)

b. (-15, ?)

c. (5, ?)

d. (?, 6)

e. (?, -3)

f. (?, 0)

3. For the equation $y = -\dfrac{2}{3}x + 1$, find the missing values in the following ordered pairs:

a. (6, ?)

b. (−3, ?)

c. (0, ?)

d. (?, 0)

e. (?, −3)

f. (?, 5)

4. For the equation $y = -\dfrac{2}{3}x + 1$, find the missing values in the following ordered pairs:

a. (2, ?)

b. (−3, ?)

c. (0, ?)

d. (?, 0)

e. (?, −3)

f. (?, 9)

For problems 5 to 10, write the equations in standard form.

5. $y = \dfrac{5}{2}x + 1$

6. $y = \dfrac{2}{5}x - 1$

7. $y = -\dfrac{3}{4}x - 3$

8. $y = -\dfrac{4}{3}x + 4$

9. $y = \dfrac{1}{2}x + \dfrac{3}{2}$

10. $y = \dfrac{3}{2}x - \dfrac{1}{4}$

For problems 11 to 16, write the equations in slope-intercept form.

11. $4y + 6x = -3$
12. $9y + 2x = 18$
13. $3y - 2x = 15$
14. $5y - 2x = -20$
15. $\dfrac{x}{2} + \dfrac{y}{3} = 1$
16. $\dfrac{x}{4} + \dfrac{y}{5} = 2$

For problems 17 to 24, graph the equations using a table of values.

17. $y = x + 3$
18. $y = 3x + 2$
19. $y = -5x + 1$
20. $y = -2x + 3$
21. $2x + y + 1 = 0$
22. $4x + y + 2 = 0$
23. $2x - y - 3 = 0$
24. $x - y - 1 = 0$

For problems 25 to 30, determine the x- and y-intercepts and graph the equations.

25. $3x + y = -2$
26. $5x + y = -3$
27. $x + y - 3 = 4$
28. $x + y - 4 = 7$
29. $y = 4x + 1$
30. $4x + y + 2 = 0$

For problems 31 to 34, determine the slopes and y-intercepts of the equations and graph the equations.

31. $2x - 3y - 18 = 0$

32. $5x - 2y + 10 = 0$
33. $-4x + 7y - 21 = 0$
34. $-7x + 8y - 32 = 0$

35. Point 'A' is in the 3rd quadrant and Point 'B' is in the 1st quadrant. Determine the sign of the slope of the line AB.

36. Point 'C' is in the 4th quadrant and Point 'D' is in the 2nd quadrant. Determine the sign of the slope of the line CD.

For problems 37 to 40, determine the slopes of the lines passing through:

37. $(2, 1)$ and $(6, 1)$
38. $(-6, 4)$ and $(2, 4)$
39. $(-5, 4)$ and $(3, -1)$
40. $(5, 6)$ and $(5, -4)$

8.3 FINDING THE EQUATION OF A LINE

In order to determine the equation of a line, we need both the slope and the y-intercept. If we do not know this information, we can determine it by following these steps:

Step 1:

If the slope is unknown, determine it by computing the rise and run between two points on the graph.

Step 2:

Replace the unknown m in the slope-intercept form of the equation with the slope value from Step 1.

Step 3:

If the y-intercept is unknown, determine it by substituting the coordinates of a point on the line for the x and y values in the slope-intercept form of the equation from Step 2, and solve for b.

Step 4:

Replace the unknown b in the slope-intercept form of the equation from Step 2 with the y-intercept value from Step 3 to arrive at the final equation of the line.

> ## Example 8.3-a: Finding the Equation of a Line Given the Slope and One Point
>
> Find the equation of a line having a slope of −2 and passing through the point (3, 5).

Solution

Step 1:

In this case, the slope is already known: $m = -2$.

Step 2:

Replace m in the equation $y = mx + b$ with the value given.

Substituting for m in the slope-intercept equation $y = mx + b$, we obtain $y = -2x + b$.

Step 3:

Substitute the coordinates of the given point into the equation to solve for b.

Substituting the x– and y-coordinates of the point $(3, 5)$ into the above equation, we obtain,

$5 = -2(3) + b$

$b = 5 + 6 = 11$

Step 4:

Write the equation in slope-intercept form, $y = mx + b$, by substituting for the values of m and b determined in the steps above.

Therefore, the equation of the line is $y = -2x + 11$.

Example 8.3-b: Finding the Slope and the Equation of a Line Given Two Points

Find the equation of a line that passes through the points $(3, 2)$ and $(4, 5)$.

Solution

Step 1:

Calculate the slope.

$$m = \frac{Change\ in\ y\ value}{Change\ in\ x\ value} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{4 - 3} = \frac{3}{1} = 3$$

Step 2:

Replace m in the equation $y = mx + b$ with the calculated slope.

Substituting for m in the slope-intercept equation $y = mx + b$, we obtain $y = 3x + b$.

Step 3:

Substitute the coordinates of one of the given points into the equation to solve for b.

Substituting the x– and y-coordinates of the point $(3, 2)$ into the above equation, we obtain,

$2 = 3(3) + b$

$b = 2 - 9 = -7$

Step 4:

Write the equation in slope-intercept form, $y = mx + b$, by substituting for the values of m and b determined in the steps above.

Therefore, the equation of the line is $y = 3x - 7$.

Note: A good check to validate the equation is substituting the coordinates of the other point into the equation to

ensure that it is a solution.

That is, substituting the x- and y-coordinates of the point (4, 5) into the equation,

$$y = 3x - 7$$
$$5 = 3(4) - 7$$
$$= 12 - 7$$
$$= 5 \, (\text{True})$$

Example 8.3 -c: Finding the Equation of a Line Given a Graph

Determine the equation of the line in standard form that is plotted on the graph shown:
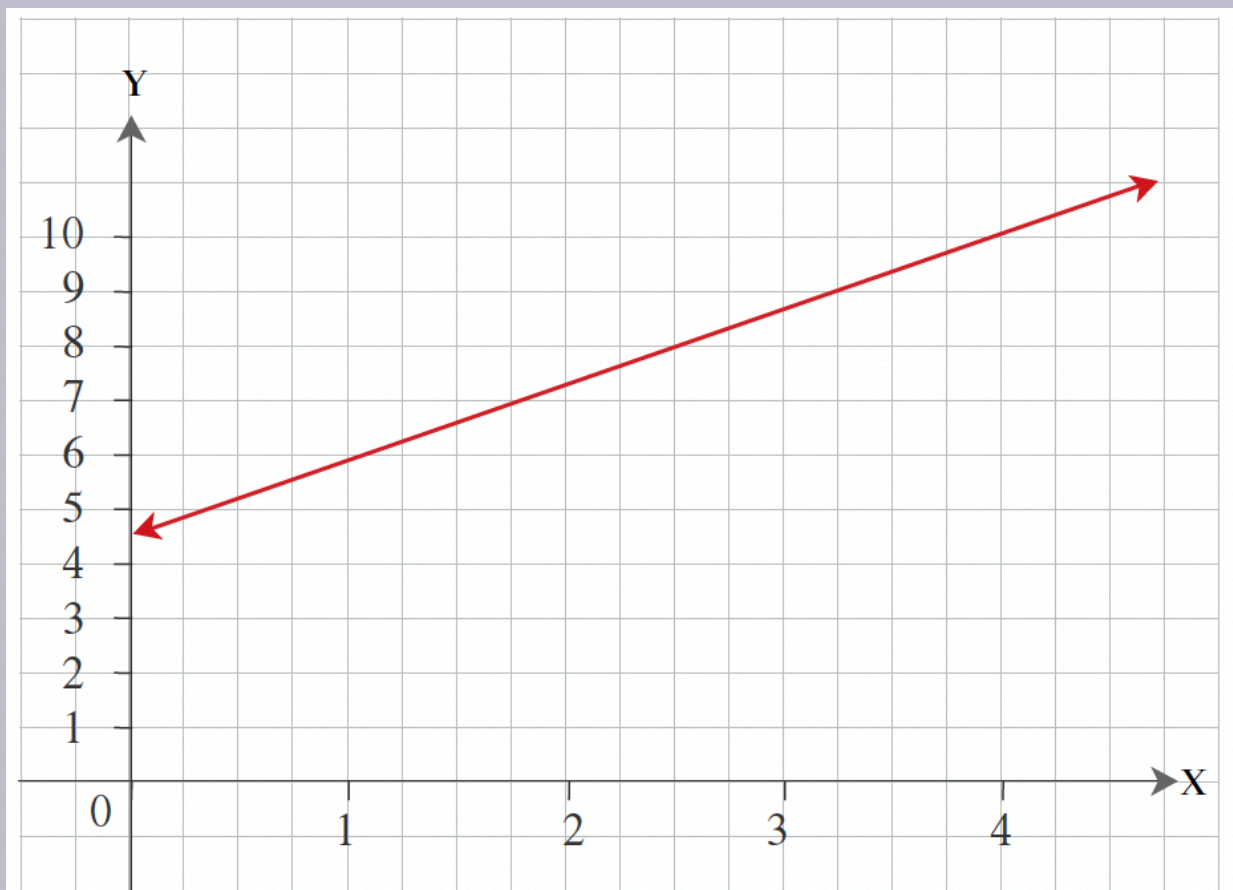
Solution

Step 1:

Start by choosing any two points (with integer coordinates) on the line: e.g., $(1, 6)$ and $(4, 10)$.
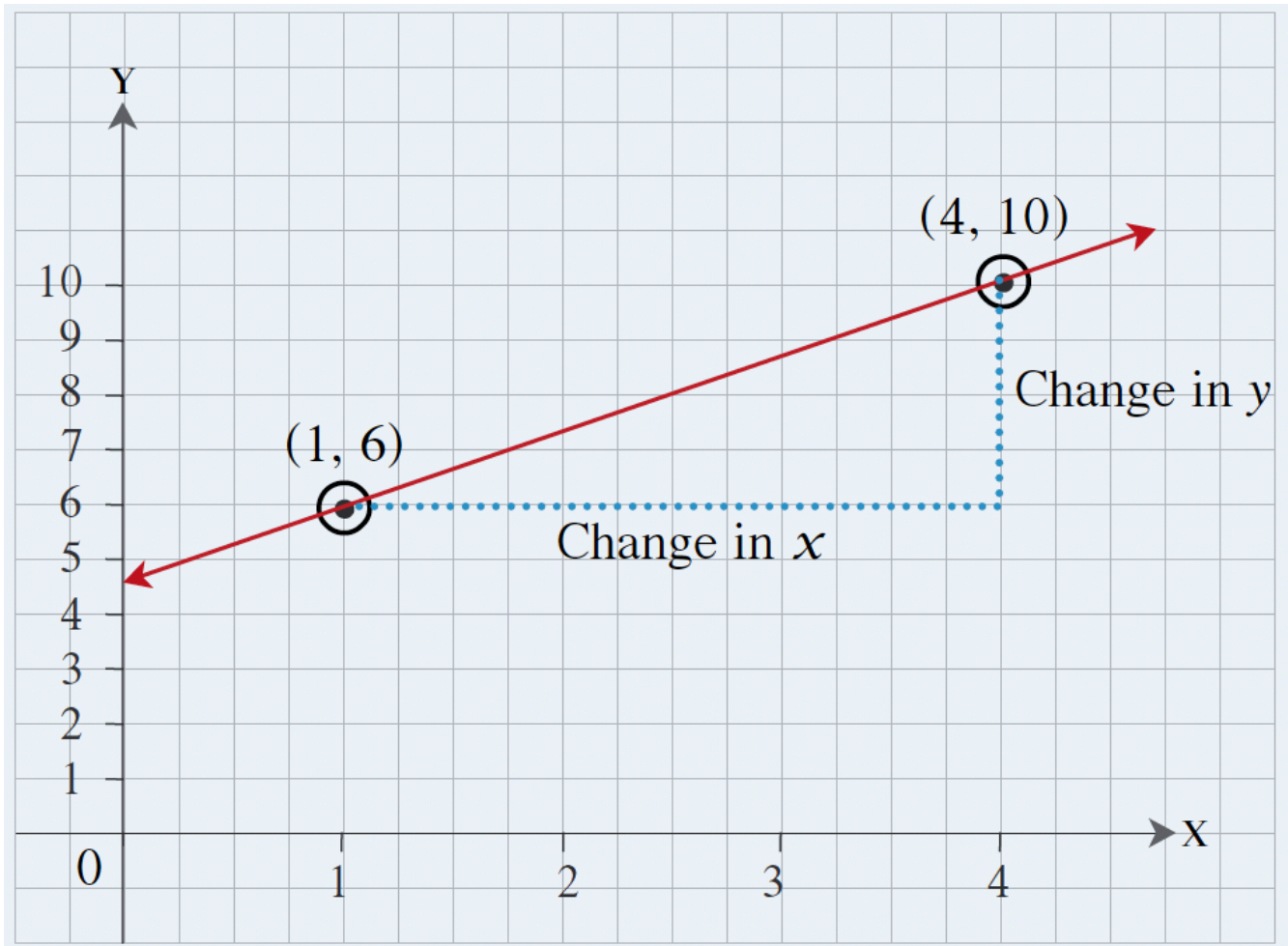
The slope of the line is $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{10 - 6}{4 - 1} = \dfrac{4}{3}.$

Step 2:

Let the equation of the line be $\quad y = mx + b$

Substituting $\quad m = \dfrac{4}{3},$

Therefore, $\quad y = \dfrac{4}{3}x + b$

Step 3:

Substituting the coordinates of one of the points $(1, 6)$ into the above equation and solving for b,

$$y = \dfrac{4}{3}x + b$$

$$6 = \frac{4}{3}(1) + b$$

$$b = 6 - \frac{4}{3} = \frac{18 - 4}{3} = \frac{14}{3}$$

Step 4:

Therefore, the equation of the line in slope-intercept form is: $y = \frac{4}{3}x + \frac{14}{3}$

$y = \frac{4}{3}x + \frac{14}{3}$ Multiplying each term by 3 and simplifying,

$3y = 4x + 14$ Rearranging, ensuring that the coefficient of the x term remains positive,

$4x - 3y = -14$

Therefore, the equation of the line in standard form is: $4x - 3y = -14$.

8.3 Exercises

Answers to the odd-numbered questions are available at the end of the book.

For problems 1 to 4, determine the equations of the lines in slope-intercept form that pass through the points.

1. $(1, 2)$ and $(5, 2)$
2. $(5, 0)$ and $(4, 5)$
3. $(-3, -5)$ and $(3, 1)$
4. $(-4, -7)$ and $(5, 2)$

For problems 5 to 14, determine the equations of the lines in slope-intercept form that:

5. Have a slope of -2 and pass through $(2, 6)$.
6. Have a slope of 3 and pass through $(-3, -2)$.
7. Have a slope of $\frac{2}{3}$ and pass through the origin.
8. Have a slope of $-\frac{4}{5}$ and pass through the origin.
9. Have an x-intercept = 4 and a y-intercept = 6.
10. Have an x-intercept = -4 and a y-intercept = 2.
11. The slope of a line is 3. The line passes through $A(4, y)$ and $B(6, 8)$. Find y.
12. The slope of a line is 2. The line passes through $A(x, 8)$ and $B(2, 4)$. Find x.
13. Points $A(2, 3)$, $B(6, 5)$, and $C(10, y)$ are on a line. Find y.
14. Points $D(3, 2)$, $E(6, 5)$, and $F(x, 1)$ are on a line. Find x.

For problems 15 to 18, determine the equation of the line (in standard form) for the graphs.
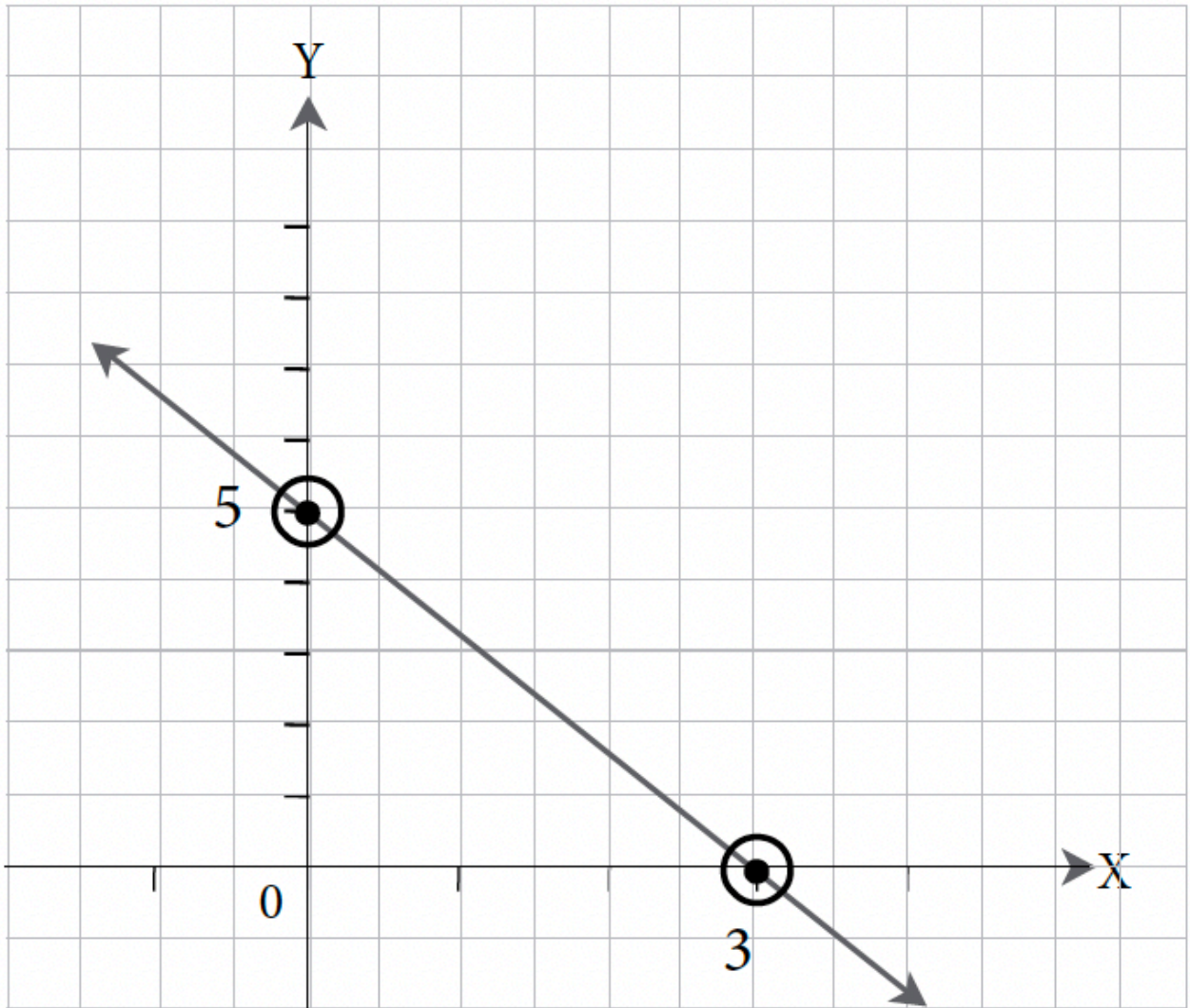
15.
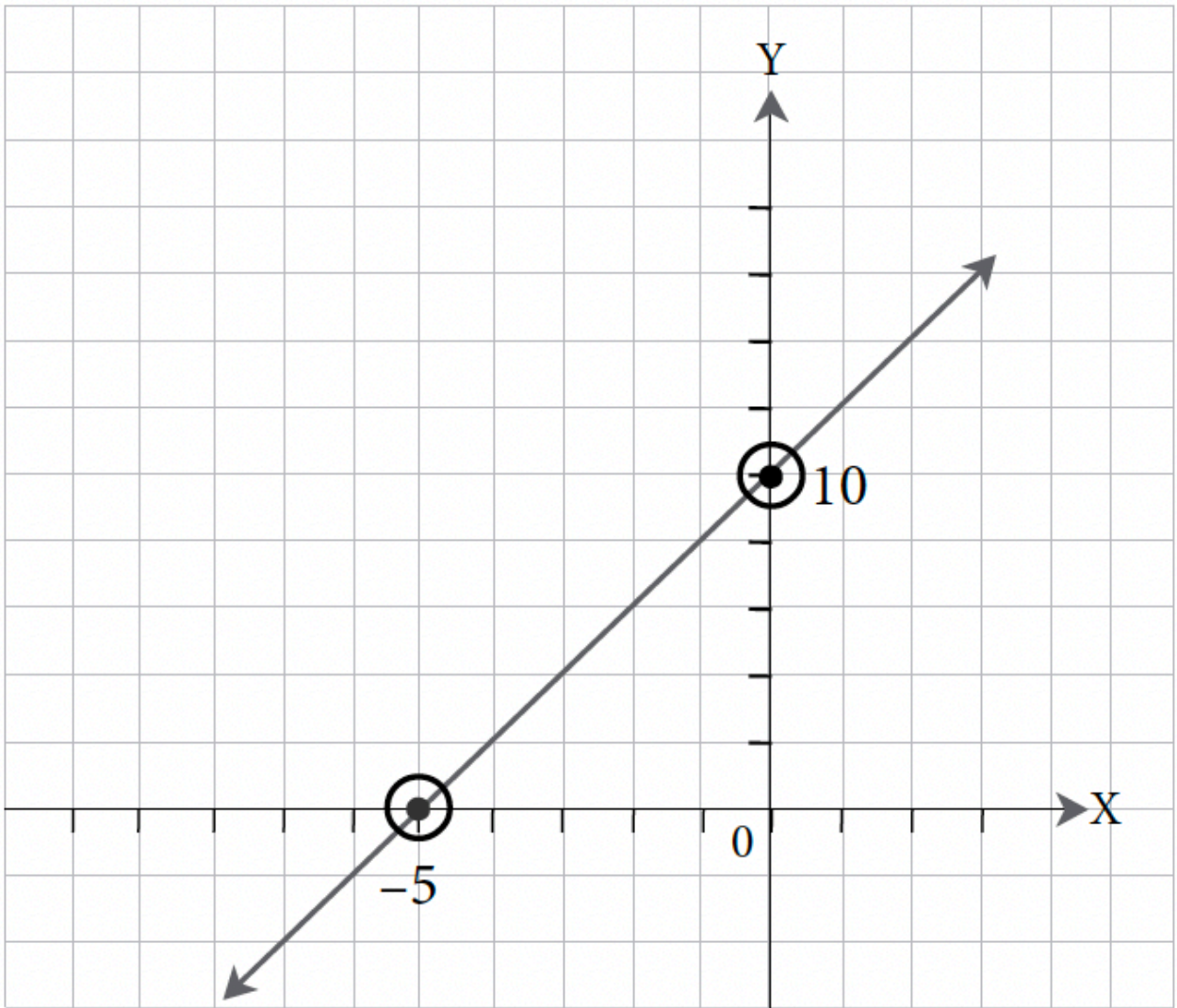
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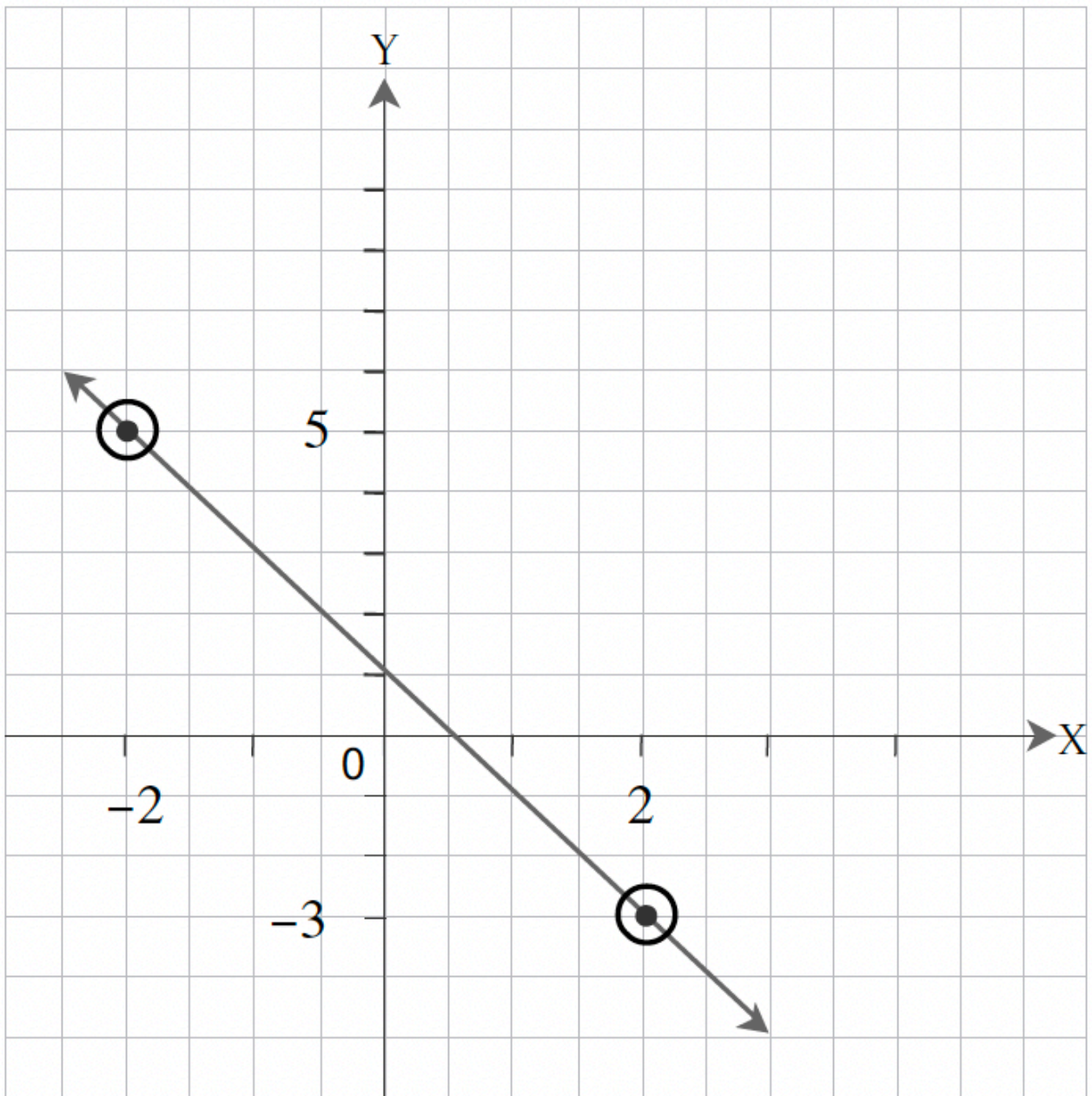
17.
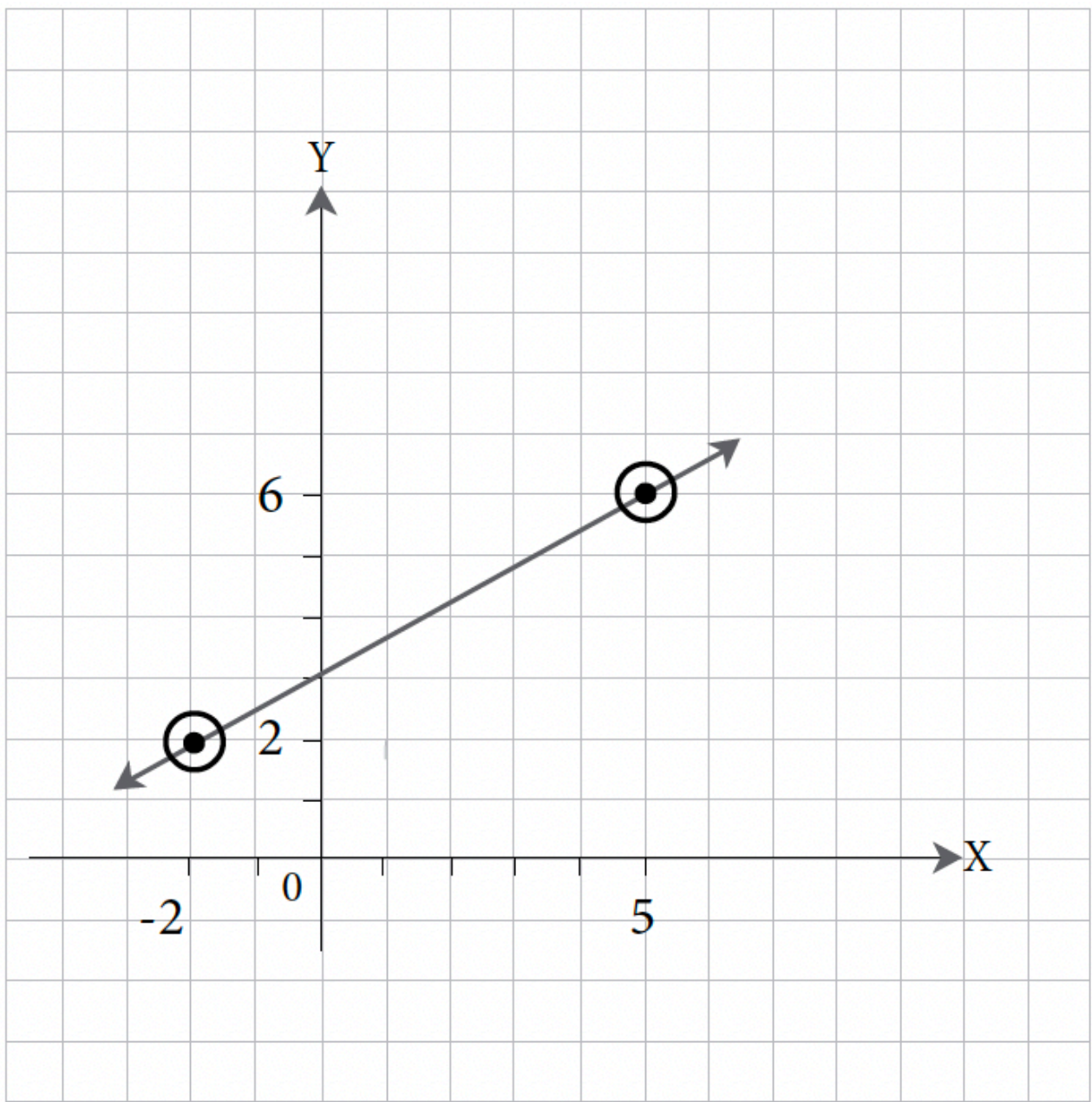
18.

8.4 PARALLEL AND PERPENDICULAR LINES

Parallel Lines

Lines that never intersect are called parallel lines. All lines with the same slope are parallel to each other.

Lines represented by the equations $y = \dfrac{3}{2}x + 6$, $y = \dfrac{3}{2}x + 3$, and $y = \dfrac{3}{2}x - 6$ have the same slope

(equal to $\dfrac{3}{2}$). Therefore, they are all parallel to each other.
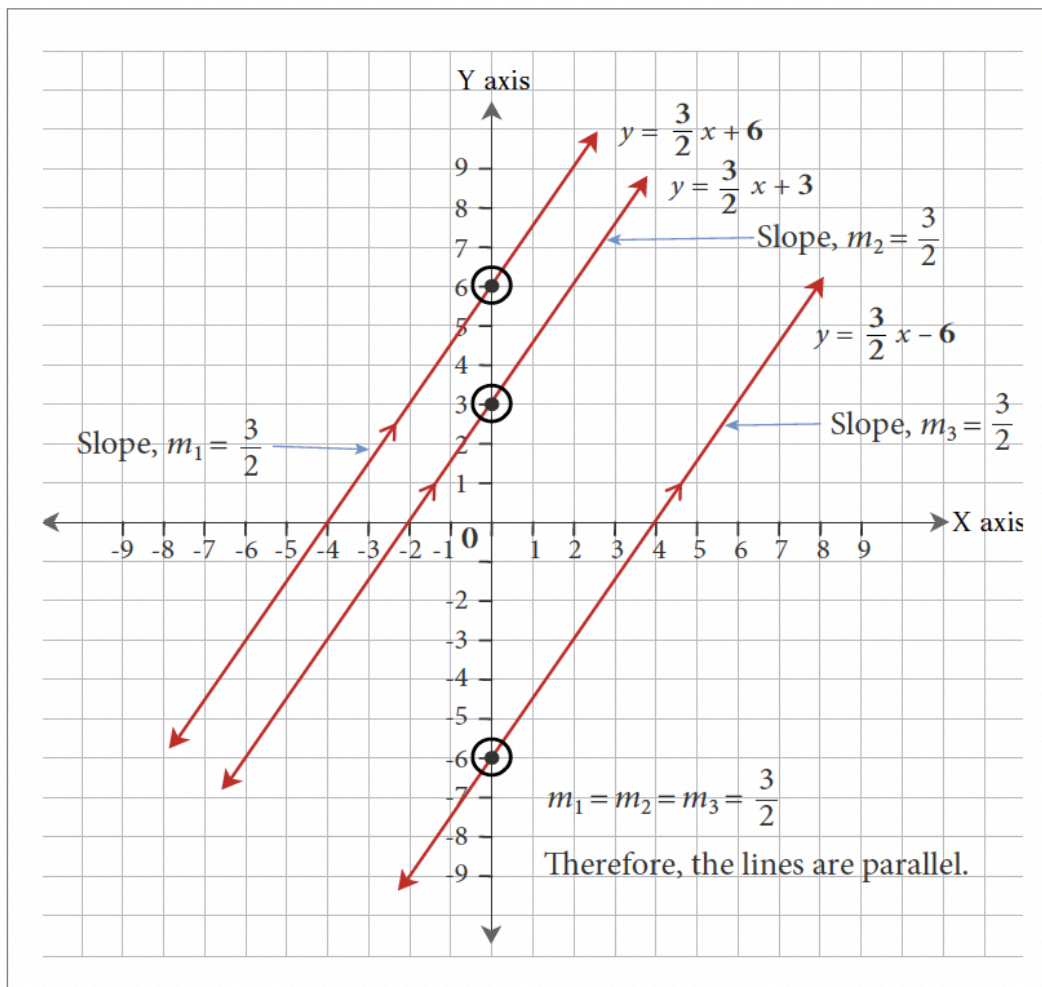
Exhibit 8.4-a
Parallel Lines

All horizontal lines are parallel to each other and all vertical lines are parallel to each other. For example, lines represented by the equations $y = 4$, $y = 2$, and $y = -2$ are horizontal lines and have slopes equalling zero.

Therefore, they are all parallel to each other. Horizontal lines are also parallel to the X-axis (which has the equation $y = 0$).
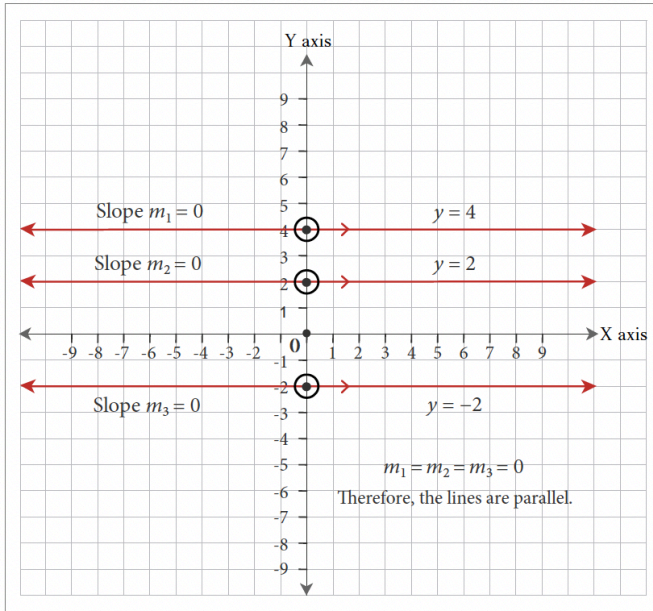
Lines represented by the equations $x = -4$, $x = 2$, and $x = 5$ are vertical lines and have undefined slopes. Therefore, they are all parallel to each other. Vertical lines are also parallel to the Y-axis (which has the equation $x = 0$).

Exhibit 8.4-c
Vertical Lines are
Parallel

Exhibit 8.4-b Horizontal Lines are Parallel
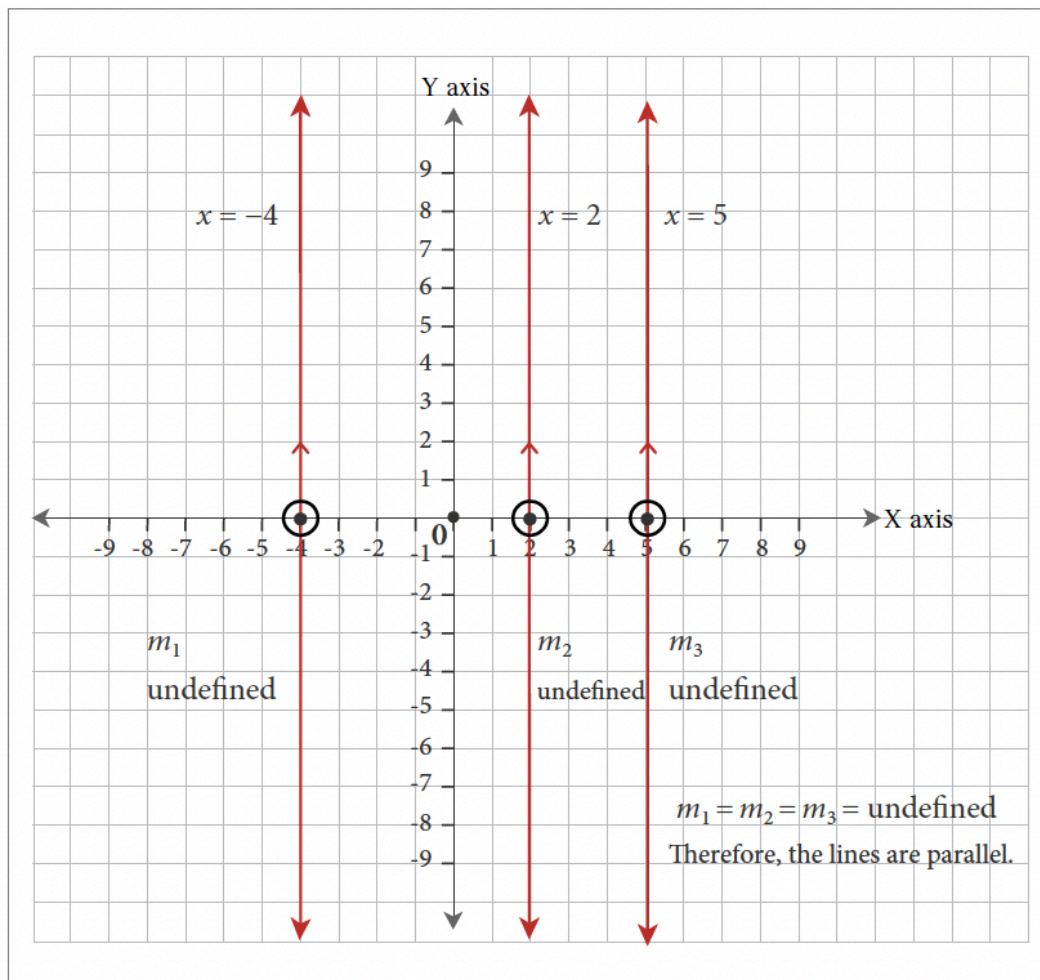
Example 8.4-a: Writing the Equation of a Line Parallel to a Given Line and Passing Through a Given Point

Write the equation of the line that is parallel to $3x + y = 5$ and passes through the point P(2, 4).

Solution

$3x + y = 5$

Rearranging to slope-intercept form,

$y = -3x + 5$

Therefore, the slope is $m = -3$.

The slope of the line parallel to this will have the same slope, $m = -3$.

Let the equation of the line parallel to $3x + y = 5$ be $y = mx + b$.

It passes through (2, 4) and has a slope of $m = -3$.

$y = mx + b$

Substituting the slope $m = -3$,

$y = -3x + b$

Substituting the coordinates of the given point (2, 4),

$4 = -3(2) + b$

Solving for b,

$4 = -6 + b$

$b = 4 + 6 = 10$

Therefore, the equation of the desired line is $y = -3x + 10$, or $3x + y = 10$ in standard form.

Note: For parallel lines in standard form $Ax + By = C$, the values of A and B will always be proportional, and often will be equal.

Perpendicular Lines

Two lines that meet at a right angle are known as **perpendicular lines**. If the product of the slopes of two lines is -1, then the two lines are perpendicular. This is the same as stating that if the slope of one line is the **negative reciprocal** of the other, the two lines are perpendicular.

Lines represented by the equations $y = 2x + 4$ and $y = -\dfrac{1}{2}x + 1$ are perpendicular to each other because their slopes are negative reciprocals of each other. $m_1 = 2, \quad m_2 = -\dfrac{1}{2}$

$$m_1 \cdot m_2 = 2\left(\dfrac{1}{2}\right) = -1.$$

Two lines are perpendicular if the product of their slopes is –1.
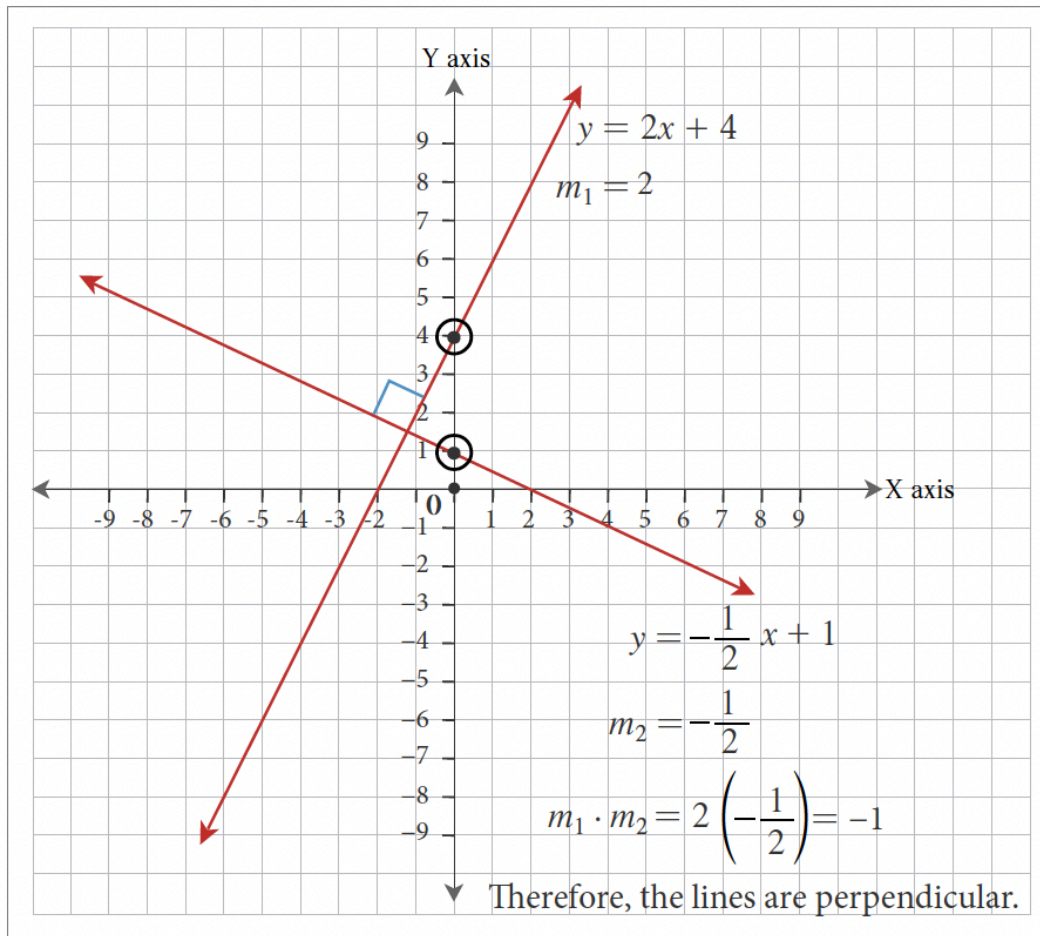
Exhibit 8.4-d
Perpendicular
Lines

All vertical lines (slope is undefined) and horizontal lines (slope is zero) are perpendicular.
Lines represented by the equations y = 2 and x = 4 are perpendicular.
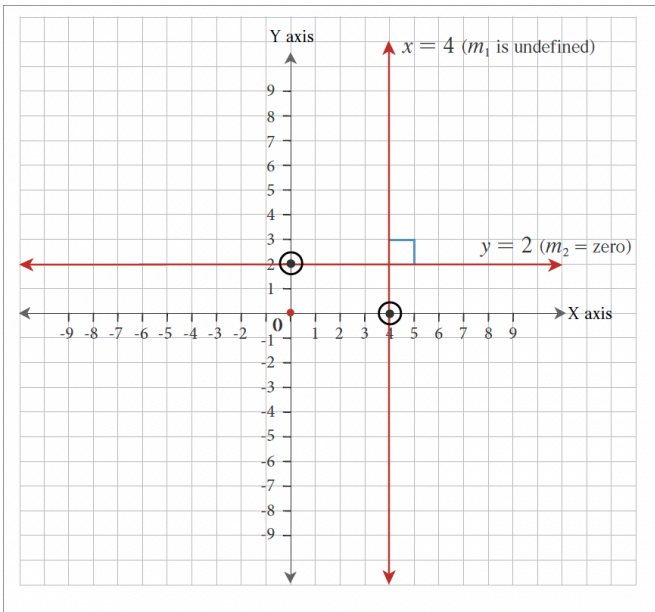
Exhibit 8.4-e Vertical and Horizontal Lines are Perpendicular

Example 8.4-b: Writing the Equation of a Line Perpendicular to a Given Line and Passing Through a Given Point

Write the equation of the line that is perpendicular to $x + 3y = 9$ and passes through the point P(-4, 2).

Solution

$x + 3y = 9$

Rearranging to slope-intercept form,

$3y = -x + 9$

$3y = -\dfrac{1}{3}x + 3$

Therefore, the slope is $m = -\dfrac{1}{3}$.

The slope of the line perpendicular to this will be the negative reciprocal of $m = -\dfrac{1}{3}$, which is 3.

Let the equation of the line perpendicular to $x + 3y = 9$ be $y = mx + b$. It passes through the point $(-4, 2)$ and has a slope of $m = 3$.

$y = mx + b$

Substituting the slope $m = 3$,

$y = 3x + b$

Substituting the coordinates of the given point $(-4, 2)$,

$2 = 3(-4) + b$

Solving for b,

$2 = -12 + b$

$2 = -12 + b$

Therefore, the equation of the desired line is $y = 3x + 14$, or $3x - y = -14$ in standard form.

8.4 Exercises

Answers to the odd-numbered questions are available at the end of the book.

For problems 1 to 7, use the slope property of parallel and perpendicular lines to identify whether each of the pairs of lines is parallel, perpendicular, or neither.

1. $y = x + 14$ $x + 4y = -1$
2. $6x - 5y = 10$ $y = -\dfrac{6}{5}x - 12$
3. $x - 3y = -60$ $y = \dfrac{1}{3}x - 4$
4. $3x - 2y = -12$ $2x + 3y = -12$
5. $2x + 5y = -5$ $y = \dfrac{5}{2}x - 4$
6. $7x + 4y = 16$ $y = -\dfrac{4}{7}x + 3$
7. $3x - 2y = -6$ $y = \dfrac{3}{2}x - 16$

For problems 8 to 15, determine the equations for the lines in slope-intercept form.

8. A line parallel to $3y - 2x = 6$ and passing through the point P(2, −3).
9. A line parallel to $y = 3x - 1$ and passing through the point P(−2, −4).
10. A line parallel to $3x - 9y = -2$ and passing through the y-intercept of the line $5x - y = 20$.
11. A line parallel to $3x + y = -2$ and passing through the x-intercept of the line $2x + 3y - 4 = 0$.
12. A line perpendicular to $x + y = 3$ and passing through the point P(−2, 5).
13. A line perpendicular to $4x + y + 1 = 0$ and passing through the point P(3, 4).
14. A line perpendicular to $2x - y = 5$ and passing through the x-intercept of the line $3x + 2y - 6 = 0$.
15. A line perpendicular to $3x + y + 9 = 0$ and passing through the y-intercept of the line $2x + 3y - 10 = 0$.

8. REVIEW EXERCISES

Answers to the odd-numbered questions are available at the end of the book.

1. In which quadrant or on which axis do the following points lie?

    a. A $(5, -1)$

    b. B $(-2, 3)$

    c. C $(3, 0)$

    d. D $(4, -2)$

    e. E $(2, 0)$

    f. F $(0, 4)$

2. In which quadrant or on which axis do the following points lie?

    a. A $(4, -1)$

    b. B $(-5, 0)$

    c. C $(-2, -7)$

    d. D $(0, -3)$

    e. E $(6, 6)$

    f. F $(5, 4)$

3. Plot the following points and join them in the order A, B, C, D. Identify the type of quadrilateral and find its area and perimeter.

    a. A $(6, -3)$

    b. B $(6, -6)$

    c. C $(-2, -6)$

    d. D $(-2, -3)$

4. Plot the following points and join them in the order P, Q, R, S. Identify the type of quadrilateral and find its area and perimeter.

    a. P $(-2, 4)$

b. Q $(-8, 4)$

c. R $(-8, -2)$

d. S $(-2, -2)$

For problems 5 to 10, graph the equations using a table of values with four points.

5. $4x - y = 2$

6. $2x + 3y = 12$

7. $x + y - 4 = 0$

8. $x + 2y - 4 = 0$

9. $y = \dfrac{1}{2}x + 2$

10. $y = -\dfrac{1}{2}x - 2$

For problems 11 to 16, graph the equations using the x-intercept, y-intercept, and another point.

11. $3x - 4y = 12$

12. $x - 2y = -1$

13. $x - 2y - 6 = 0$

14. $3x + y - 4 = 0$

15. $y = 4x$

16. $x = 2y$

For problems 17 to 22, graph the equations using the slope and y-intercept method.

17. $y = 4x + 6$

18. $y = 5x + 4$

19. $3x + 2y - 12 = 0$

20. $2x + 3y + 6 = 0$

21. $y = -\dfrac{3}{4}x - 1$

22. $y = -\dfrac{1}{3}x - 1$

For problems 23 to 28, determine the equation of the line in slope-intercept form that passes through the following points:

23. $(3, 2)$ and $(7, 5)$

24. $(4, 6)$ and $(2, 4)$

25. $(5, -4)$ and $(-1, 4)$

26. $(0, -7)$ and $(-6, -1)$

27. $(1, -2)$ and $(4, 7)$

28. $(3, -4)$ and $(-1, 4)$

29. Write the equation of a line parallel to $3x - 4y = 12$ and that passes through the point P$(-2, 3)$.

30. Write the equation of a line parallel to $x - 3y = 9$ and that passes through the point P$(2, -3)$.

31. Write the equation of a line perpendicular to $2y = x + 4$ and that passes through the point P$(-2, 5)$.

32. Write the equation of a line perpendicular to $3x + 4y + 6 = 0$ and that passes through the point P$(4, -1)$.

8. SELF-TEST

Answers to the questions are available at the end of the book.

1. Three vertices of a rectangle ABCD have the points A $(-3, 4)$, B $(5, 4)$, and C $(5, -1)$. Find the coordinates of the 4th vertex, and determine the area of the rectangle. *(Hint: Area of a rectangle = length × width)*

2. Write the following equations in standard form:

   a. $y = \dfrac{2}{3}x - 2$

   b. $6 - 2x + \dfrac{1}{4} = 0$

3. Write the following equations in slope-intercept form:

   a. $2x - 3y + 6 = 0$

   b. $3x + 4y - 5 = 0$

4. Graph the equation $2x - 3y = 9$ using a table of values with 4 points.

5. Graph the equation $3y + 4x = 0$ using the x-intercept, y-intercept, and another point on the line.

6. Use the following slopes (m) and y-intercepts (b) to graph the equations:

   a. $m = -\dfrac{1}{2}, b = -4$

   b. $m = \dfrac{2}{3}, b = -2$

7. Determine the equation of the line, in standard form, that passes through the points P $(-4, 5)$ and Q $(1, 1)$.

8. Determine the equation of the line, in standard form, having an x-intercept equal to 5 and a y-intercept equal to -3.

9. Determine the equation of a line, in standard form that is parallel to $3x - 2y + 9 = 0$ and that passes through the point $(-6, -3)$.

10. Determine the equation of the line, in standard form, that passes through the origin and is

perpendicular to the line passing through the points P(–3, 5) and Q(5, –1).

8. CASE STUDY - LIGHTNING WHOLESALE

Lightning Wholesale is a mid-to large-size company that works directly with manufacturers to bring sporting goods and equipment to mid to small-sized retailers. The company is in Southern Ontario, Canada. Lightning Wholesale acquires the goods, stores them in its warehouses, and distributes them to retailers, who then sell them to sporting enthusiasts across the province. Suppliers are global, including those from the United States.

Lightning Wholesale is a dynamic office with global networking, communications, customer service, and complex workflows. Lighting Wholesale depends on its office professionals to support staff and management. Office Professional tasks include clerical support, bookkeeping, customer service, data analysis, and document processing.

Lightning Wholesale cares about its employees and retailers. Where managers can assist their employees, they try to. Where retailers need guidance, Lightning Wholesales tries to fill this gap as best they can. In working with its small retailers, many of which are owned and operated by sole proprietors, Lightning Wholesale assists the owners with their pricing to help develop promotional pricing plans and the amounts of the payments that can be advertised. Lightning Wholesale freely offers help since it means more sales and increased profits for the company.

There are currently a few main tasks for Lightning Wholesale management:

- The payroll needs review for the past fiscal year.
- Costs and expenses need forecasting for the next fiscal year.
- Detailed analysis of office supply orders from quarter to quarter with recommendations for the next fiscal year.
- Forklifts need replacing in the warehouse.
- The next round of co-op students needs a computer.
- The employee appreciation event needs planning for next year.
- Travel expenses for the mobile team need to be summarized for the last fiscal year.

The equation $y = \dfrac{1}{4}x + 1.25$ represents the relationship between the number of employees and office supply expenses.

- y represents the office supply expenses
- x represents the number of employees
- m is the slope of the line, representing the rate of change in expenses per employee
- b is the y-intercept, representing the fixed expenses with zero employees

Questions

1. Graph the linear equation $y = \frac{1}{4}x + 1.25$. Copy the following graph on your paper, and label the x and y axis and the line representing the relationship between the number of employees and office supply expenses.
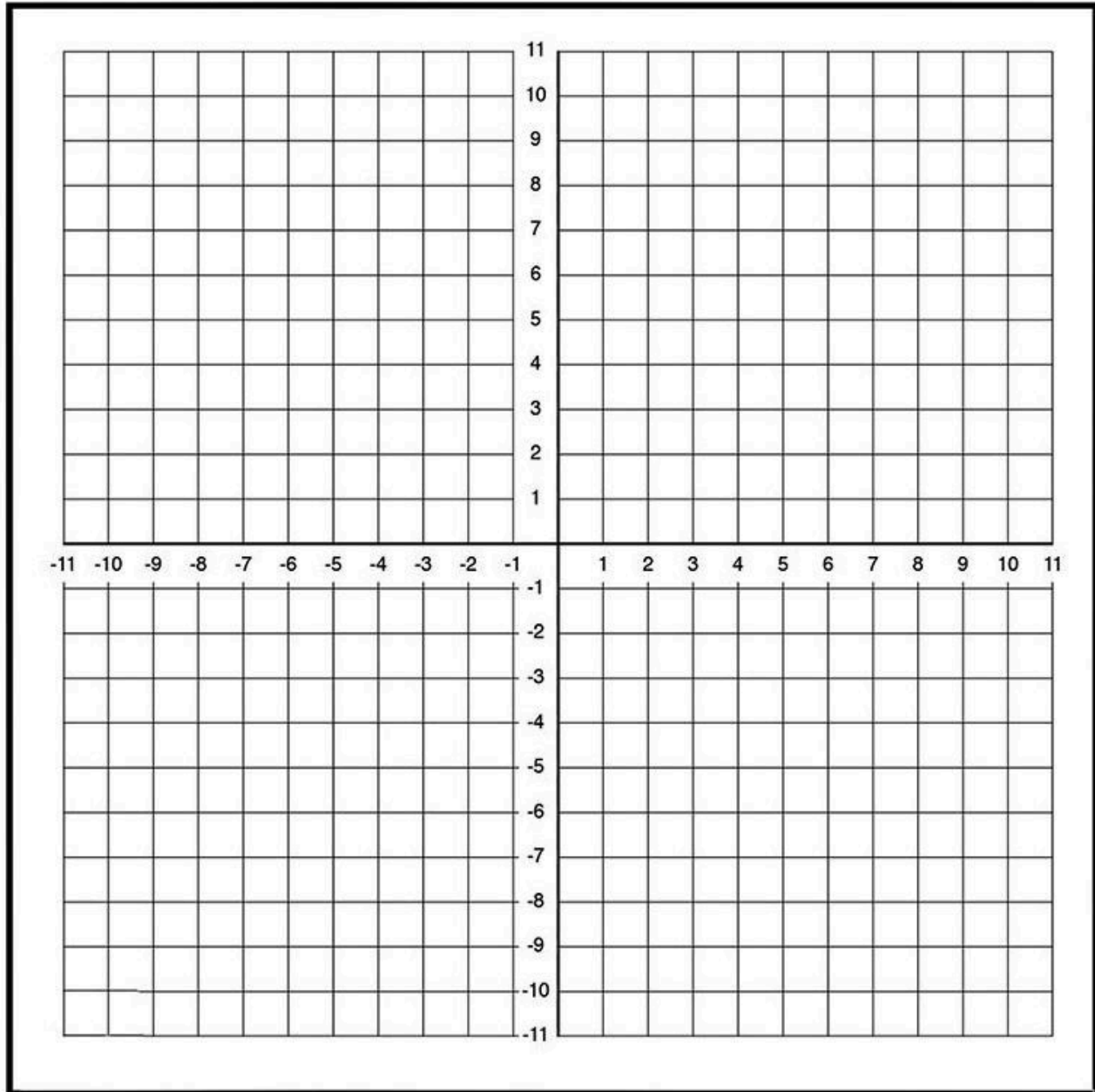
2. Is the slope positive or negative? What does the slope tell us about the relationship between the number of employees and the office supply expense?

3. What happens with the office supply expense if Lightning Wholesale hires more employees?

4. Lightning Wholesale would like to lower the office supply expense for next year. If it is decided that the amount of office expense per employee will be lowered, will the linear equation change – will the slope change?

5. After reviewing past and current expenses, the management team has decided to decrease the office supply expense per employee. The new linear equation represents the relationship between the number of employees and office supply expenses: $y = \dfrac{1}{2}x + 1.25$. Draw the new linear equation on your graph and compare the 2 lines.

- What happens to y when x increases?
- Why are the slopes different? What does this indicate?

PART IX
ANSWER KEY TO CHAPTER QUESTIONS

ANSWER KEY - CHAPTER 1

Exercise 1.1

1.　　a. (i) Tens　　(ii) 90
　　　　b. (i) Hundreds　　(ii) 300
　　　　c. (i) Hundreds　　(ii) 700

3.　　a. (i) Ten thousands　　(ii) 10,000
　　　　b. (i) Ten thousands　　(ii) 20,000
　　　　c. (i) Millions　　(ii) 6,000,000

5.　　a. (i) Expanded form: 400 + 7　　(ii) Word form: Four hundred seven
　　　　b. (i) Expanded form: 2,000 + 50 + 6　　(ii) Word form: Two thousand, fifty-six

7.　　a. (i) Expanded form: 20,000 + 9,000 + 100 + 80 + 6
　　　　　　(ii) Word form: Twenty-nine thousand, one hundred eighty-six
　　　　b. (i) Expanded form: 400,000 + 60,000 + 4,000 + 400 + 40 + 8
　　　　　　(ii) Word form: Four hundred sixty-four thousand, four hundred forty-eight

9.　　a. (i) Expanded form: 2,000,000 + 600,000 + 4,000 + 300 + 20 + 5
　　　　　　(ii) Word form: Two million, six hundred four thousand, three hundred twenty-five
　　　　b. (i) Expanded form: 10,000,000 + 5,000,000 + 300,000 + 600 + 4
　　　　　　(ii) Word form: Fifteen million, three hundred thousand, six hundred four

11.　　a. (i) Standard form: 679　(ii) Word form: Six hundred seventy-nine
　　　　b. (i) Standard form: 3,147　(ii) Word form: Three thousand, one hundred forty-seven

13.　　a. (i) Standard form: 2,605　(ii) Word form: Two thousand, six hundred five
　　　　b. (i) Standard form: 9,024　(ii) Word form: Nine thousand, twenty-four

15.　　a. (i) Standard form:40,990　(ii) Word form: Forty thousand, nine hundred ninety
　　　　b. (i) Standard form:10,053　(ii) Word form: Ten thousand, fifty-three

17.　　a. (i) Standard form: 570　(ii) Expanded form: 500 + 70

    b. (i) Standard form: 803 (ii) Expanded form: 800 + 3

19.   a. (i) Standard form: 80,630 (ii) Expanded form: 80,000 + 600 + 30

           b. (i) Standard form: 75,025 (ii) Expanded form: 70,000 + 5,000 + 20 + 5

21.   a. (i) Standard form: 12,452,832
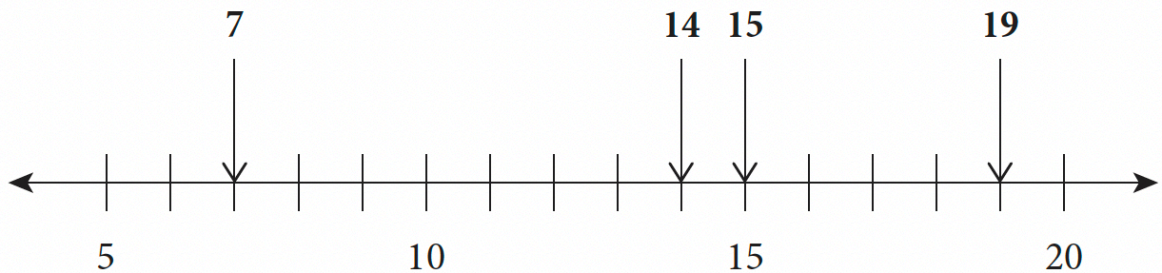
              (ii) Expanded form:10,000,000 + 2,000,000 + 400,000 + 50,000 + 2,000 + 800 + 30 + 2
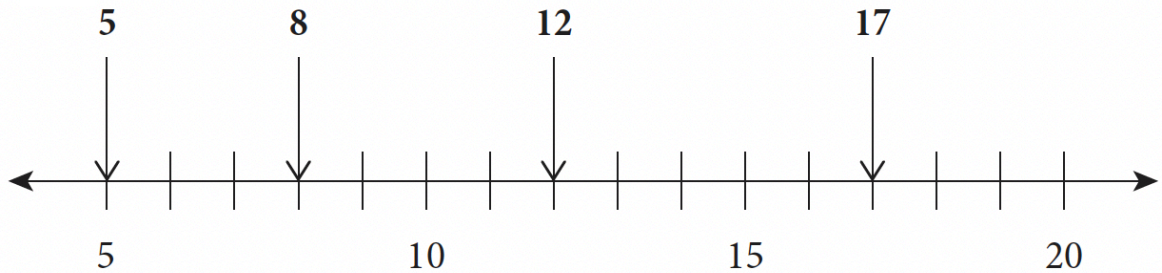
           b. (i) Standard form: 32,684,256

              (ii) Expanded form: 30,000,000 + 2,000,000 + 600,000 + 80,000 + 4,000 + 200 + 50 + 6

23.   a. (i) Standard form: 125,000 (ii) Expanded form: 100,000 + 20,000 + 5,000

           b. (i) Standard form: 250,000 (ii) Expanded form: 200,000 + 50,000

25.   a.

        b.

27.   a. 7 < 15

           b. 19 > 14

           c. 0 < 5

           d. 19 > 0

29.   a. (i) 6 is less than 9   (ii) 9 is greater than 6

           b. (i)11 is less than 18   (ii)18 is greater than 11

           c. (i) 5 is less than 11   (ii) 11 is greater than 5

           d. (i) 0 is less than 11   (ii) 11 is greater than 0

31.   a. 87, 96, 99, 103, 108

b. 108, 139, 141, 159, 167

33. a. 2,040; 2,067; 2,533; 2,638

b. 78,812; 79,468; 79,487; 79,534

35. a. (i) 259   (ii) 952

b. (i) 1,789   (ii) 9,871

c. (i) 3,458   (ii) 8,543

37. a. (i) 520   (ii) 500   (iii) 1,000

b. (i) 1,650   (ii) 1,600   (iii) 2,000

c. (i) 53,560   (ii) 53,600   (iii) 54,000

d. (i) 235,360   (ii) 235,400   (iii) 235,000

39. a. (i) 880,000   (ii) 900,000   (iii) 1,000,000

b. (i) 1,660,000   (ii)1,700,000   (iii) 2,000,000

c. (i) 3,370,000   (ii) 3,400,000   (iii) 3,000,000

d. (i) 4,570,000   (ii) 4,600,000   (iii) 5,000,000

Exercise 1.2

1. a. (i) 80   (ii) 77

b. (i) 140   (ii) 133

3. a. (i) 930   (ii) 923

b. (i) 650   (ii) 653

5. a. (i) 420   (ii) 410

b. (i) 1,520   (ii) 1,511

7. a. (i) 670   (ii) 662

b. (i) 1,830   (ii) 1,818

9. a. (i) 1,400   (ii) 1,413

b. (i) 600   (ii) 652

11. a. (i) 7,400   (ii) 7,349

b. (i) 5,700　　(ii) 5,616

13.　　　a. (i) 4,500　　(ii) 4,531
　　　　　b. (i) 3,800　　(ii) 3,732

15.　　　a. (i) 30　　(ii) 37
　　　　　b. (i) 10　　(ii) 15

17.　　　a. (i) 130　　(ii) 129
　　　　　b. (i) 270　　(ii) 268

19.　　　a. (i) 610　　(ii) 608
　　　　　b. (i) 490　　(ii) 496

21.　　　a. (i) 1,060　　(ii) 1,058
　　　　　b. (i) 2,040　　(ii) 2,039

23.　　　a. (i) 400　　(ii) 494
　　　　　b. (i) 600　　(ii) 604

25.　　　a. (i) 700　　(ii) 675
　　　　　b. (i) 2,200　　(ii) 2,128

27.　　　a. (i) 700　　(ii) 667
　　　　　b. (i) 2,300　　(ii) 2,324

29.　　　a. (i) 4,800　　(ii) 4,350
　　　　　b. (i) 3,600　　(ii) 3,717

31.　　　a. (i) 38,000　　(ii) 40,492
　　　　　b. (i) 56,000　　(ii) 54,332

33.　　　a. (i) 80,000　　(ii) 73,815
　　　　　b. (i) 129,000　　(ii) 126,850

35.　　　a. (i) 8　　(ii) Quotient: 15 Reminder: 3
　　　　　b. (i) 4　　(ii) Quotient: 4 Reminder: 4

37.   a. (i) 3   (ii) Quotient: 3 Reminder: 5
        b. (i) 4   (ii) Quotient: 4 Reminder: 2

39.   a. (i) 65   (ii) Quotient: 46 Reminder: 10
        b. (i) 20   (ii) Quotient: 16 Reminder: 12

41.   a. (i) 128   (ii) Quotient: 142 Reminder: 6
        b. (i) 41   (ii) Quotient: 43 Reminder: 21

43.   a. (i) 15   (ii) Quotient: 14 Reminder: 148
        b. (i) 13   (ii) Quotient: 13 Reminder: 27

45.   a. 6^4
        b. 12^2

47.   a. 3^6
        b. 9^4

49.   a. Base: 2 Exponent: 9
        b. Base: 5 Exponent: 7

51.   a. Base: 1 Exponent: 20
        b. Base: 8 Exponent: 3

53.   a. $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$
        b. $3 \times 3 \times 3 \times 3 \times 3 = 243$

55.   a. $3^9 = 19,683$
        b. $3^{11} = 177,147$

57.   a. 8
        b. 11

59.   a. 1
        b. 18

61.   a. 8
        b. 20

63.   a. $719

              b. $133

65.   a. $1,362

              b. $620

67.   a. 360 pens

              b. 12 pieces

69.   a. 45 weeks

              b. 352 chairs

71. $53

73. $2,130

75. $450

77. 42 weeks

79. 762 students

81. 125

83. $163

85.   a. 24 boys

              b. 17 students

Exercise 1.3

1.   a. $-5 < 0$

              b. $-2 < 6$

3.   a. $8 > -3$

              b. $1 > -2$

5.   a. $-6 > -8$

b. $-5 < -2$

7. a. $-8, -6, -5, 2, 5, 8$

b. $-9, -8, -6, 3, 4, 7$

9. a. $-8, -5, 3, 7, 9, 10$

b. $-13, -8, -3, 2, 12, 15$

11. a. -20

b. 6

13. a. 12

b. 14

15. a. 7

b. -3

17. a. 30

b. -24

19. a. 8

b. -9

21. a. 2

b. 17

23. a. 70

b. -72

25. a. 265

b. 30

27. -132

29. -111

Exercise 1.4

1. 2, 3, 5, 7, 11, 13, 17 and 19

3. 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27 and 28

5.   a. 13, 19 and 47
        b. 11, 29 and 43

7.   a. (i) Factors: 1, 3, 5 and 15   (ii) Prime factors: 3 and 5
        b. (i) Factors: 1, 2, 17 and 34   (ii) Prime factors: 2 and 17

9.   a. (i) Factors: 1, 2, 4, 8, 16, 32 and 64   (ii) Prime factors: 2
        b. (i) Factors: 1, 2, 3, 6, 9, 18, 27 and 54   (ii) Prime factors: 2 and 3

11.   a. (i) Factors: 1, 3, 7 and 21   (ii) Prime factors: 3 and 7
        b. (i) Factors: 1, 5, 25   (ii) Prime factors: 5

13.   a. (i) Factors: 1, 2, 3, 4, 6, 9, 12, 18 and 36   (ii) Prime factors: 2 and 3
        b. (i) Factors: 1, 5, 13 and 65   (ii) Prime factors: 5 and 13

15.   a. 6, 12, 18, 24, 30 and 36
        b. 8, 16, 24, 32, 40 and 48

17.   a. 9, 18, 27, 36, 45 and 54
        b. 10, 20, 30, 40, 50 and 60

19.   a. 30
        b. 90

21.   a. 72
        b. 315

23.   a. 64
        b. 120

25.   a. 24
        b. 180

27.   a. 150

           b. 108

29.   a. 84

           b. 54

31.   a. 72

           b. 90

33.   a. (i) Factors of 15: 1, 3, 5 and 15 Factors of 25: 1, 5 and 25

               (ii) Common factors: 1 and 5

               (iii) GCF: 5

           b. (i) Factors of 18: 1, 2, 3, 6, 9 and 18 Factors of 32: 1, 2, 4, 8, 16 and 32

               (ii) Common factors: 1 and 2

               (iii) GCF: 2

35.   a. (i) Factors of 18: 1, 2, 3, 6, 9 and 18 Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48

               (ii) Common factors: 1, 2, 3 and 6

               (iii) GCF: 6

           b. (i) Factors of 32: 1, 2, 4, 8, 16 and 32 Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60

               (ii) Common factors: 1, 2 and 4

               (iii) GFC: 4

37.   a. (i) Factors of 25: 1, 5 and 25 Factors of 80: 1, 2, 4, 5, 8, 10, 16, 20, 40 and 80

               (ii) Common factors: 1 and 5

               (iii) GFC: 5

           b. (i) Factors of 40: 1, 2, 4, 5, 8, 10, 20 and 40 Factors of 120: 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60 and 120

               (ii) Common factors: 1, 2, 4, 5, 8, 10, 20 and 40

               (iii) GFC: 40

39.   a. (i) Factors of 8: 1, 2, 4 and 8 Factors of 12: 1, 2, 3, 4, 6 and 12 Factors of 15: 1, 3, 5 and 15

               (ii) Common factor: 1

               (iii) GFC: 1

           b. (i) Factors of 6: 1, 2, 3 and 6 Factors of 15: 1, 3, 5 and 15 Factors of 20: 1, 2, 4, 5, 10 and 20

               (ii) Common factor: 1

               (iii) GFC: 1

41.   a. (i) Factors of 12: 1, 2, 3, 4, 6 and 12 Factors of 18: 1, 2, 3, 6, 9 and 18 Factors of 24: 1, 2, 3, 4, 6, 8, 12 and 24

(ii) Common factors: 1, 2, 3 and 6

(iii) GFC: 6

    b. (i) Factors of 12: 1, 2, 3, 4, 6 and 12 Factors of 30: 1, 2, 3, 5, 6, 10, 15 and 30 Factors of 42: 1, 2, 3, 6, 7, 14, 21 and 42

(ii) Common factors: 1, 2, 3 and 6

(iii) GFC: 6

43.   a. (i) Factors of 40: 1, 2, 4, 5, 8, 10, 20 and 40 Factors of 50: 1, 2, 5, 10, 25 and 50 Factors of 80: 1, 2, 4, 5, 8, 10, 16, 20, 40 and 80

(ii) Common factors: 1, 2, 5 and 10

(iii) GFC: 10

    b. (i) Factors of 30: 1, 2, 3, 5, 6, 10, 15 and 30 Factors of 75: 1, 3, 5, 15, 25 and 75 Factors of 90: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45 and 90

(ii) Common factors: 1, 3, 5 and 15

(iii) GFC: 15

45. 32 cm

47. Feb 28$^{\text{th}}$

49. 9 m

51. 6 minutes

Exercise 1.5

1.   a. 21

    b. 16

3.   a. 16

    b. 32

5.   a. 16

    b. 20

7.   a. 256
        b. 72

9.   a. 35
        b. 9

11.   a. 70
        b. 25

13.   a. 15
        b. 5

15.   a. 12
        b. 42

17.   a. 4
        b. 8

19.   a. 5
        b. 12

21.   a. 2
        b. 7

23.   a. 9
        b. 125

25.   a. 108
        b. 10

27.   a. 32
        b. 28

29. 126

31. 36

33. 407

35. 10

37. 4

39. 14

41. 148

43. 3

45. 11

Review Exercises 1

1.   a. (i) 7,000 + 500 + 2   (ii) Seven thousand, five hundred two
        b. (i) 20,000 + 5,000 + 40 + 7   (ii) Twenty-five thousand, forty-seven
        c. (i) 600,000 + 20,000 + 20 + 5   (ii) Six hundred twenty thousand, twenty-five
        d. (i) 3,000,000 + 50,000 + 4,000 + 700 + 5   (ii) Three million, fifty-four thousand, seven hundred
            five

3.   a. (i) 5,607   (ii) 5,000 + 600 + 7
        b. (i) 37,040   (ii) 30,000 + 7,000 + 40
        c. (i) 408,105   (ii) 400,000 + 8,000 + 100 + 5
        d. (i) 1,070,055   (ii) 1,000,000 + 70,000 + 50 + 5

5.   a. 167 < 176
        b. 2,067 < 2,097
        c. 79,084 < 79,087
        d. 162,555 > 162,507

7.   a. (i) 3,900   (ii) 3,856
        b. (i) 5,900   (ii) 5,772
        c. (i) 7,100   (ii) 7,181
        d. (i) 3,600   (ii) 3,636

9.   a. 10,695
        b. 8,760

      c. 63

      d. 211

11.   a. 3.98

      b. 50

13.   a. 96.57

      b. 24

15.   a. −19

      b. 27

17.   a. 7,180.08

      b. 1,817.28

19.   a. 164,593.54

      b. −17,668.97

21.   a. 0

      b. −7

23.   a. 60

      b. 144

      c. 1,584

25.   a. 4

      b. 6

      c. 2

27.   a. 11

      b. 14

29.   a. 13

      b. 7

31.   a. 89

      b. 4

33.   a. 27
           b. 27

35.   a. 3
           b. 10

37.   a. 9
           b. 25

39.   a. 130
           b. 14

41. 873 stamps

43. 850 tickets

45. $52,725

47. Babar: $875, Allan: $1,675

49. 16 boxes

51. 12 metres

53. 42 days

Self-Test Exercises 1

1.   a. (i) 6900 (ii) 6,864
           b. (i) 520,000 (ii) 445,900
           c. (i) 67 (ii) 60

2.   a. 34
           b. 3
           c. 3

3.   a. 56
           b. 36

    c. 2

4.   a. 7

        b. 168

        c. 40

5.   a. 2

        b. 20

        c. 9

6. Hari: $1,275   Bob: $6,375

7. 25,995 kg

8. $940

9. 320

10. 1,008 sec or 16 min 48 sec

11. 6 metres

ANSWER KEY - CHAPTER 2

Exercise 2.1

1.   a. 0.6
            b. 0.007

3.   a. 0.12
            b. 0.029

5.   a. 7.5
            b. 9.503

7.   a. 3.67
            b. 2.567

9. (i) 87.2   (ii) 80 + 7 + 0.2

11. (i) 3.04   (ii) 3 + 0.04

13. (i) 0.0401   (ii) 0.04 + 0.0001

15. (i) 89.0625   (ii) 80 + 9 + 0.06 + 0.002 + 0.0005

17. (i) 1,787.025   (ii) 1,000 + 700 + 80 + 7 + 0.02 + 0.005

19. (i) 412.65   (ii) 400 + 10 + 2 + 0.6 + 0.05

21. (i) 1,600,000.02   (ii) 1,000,000 + 600,000 + 0.02

23. (i) 23.5   (ii) 20 + 3 + 0.5

25.   a. Forty-two and fifty-five hundredths
           b. Seven hundred thirty-four and one hundred twenty-five thousandths

27.   a. Twenty-five hundredths
          b. Nine and five-tenths

29.   a. Seven and seven hundredths
          b. Fifteen and two thousandths

31.   a. Sixty-two thousandths
          b. Fifty-four thousandths

33. 0.034, 0.043, 0.304, 0.403

35. 415.2

37. 264.2

39. 24.2

41. 10.4

43. 14.36

45. 181.13

47. $16.78

49. $9.99

Exercise 2.2

1. 1,716.045

3. 869.593

5. 479.444

7. 964.571

9. 240.922

11. 238.192

13. 15.88

15. 281.283

17. 229.668

19. 415.01

21. 744.606

23. 259.5571

25. 0.344

27. 319.4598

29. $7.53\overline{1}$

31. 91.575

33. $24.\overline{3}$

35. 12.4

37.   a. 0.001
        b. 0.09

39.   a. 0.16
        b. 0.000008

41.   a. 0.5
        b. 0.7

43.   a. 1.1
        b. 1.3

45.   a. 0.1
       b. 0.07

47. $378.64

49. $14.86

51. $76.38

53. $64.56

55. $1.54

57. $1,281.85

59. $187.86

61. $86.46

63. $44.21

65. $3,469.09

67. $54.45

69. 14 pieces

71. $2.55

Exercise 2.3

1.   a. Proper fraction
      b. Mixed number

3.   a. Improper fraction
      b. Mixed number

5.   a. Mixed number

    b. Improper fraction

7.   a. $\dfrac{16}{7}$

        b. $\dfrac{25}{8}$

9.   a. $\dfrac{29}{5}$

        b. $\dfrac{27}{4}$

11.   a. $2\dfrac{5}{7}$

        b. $5\dfrac{5}{8}$

13.   a. $7\dfrac{2}{3}$

        b. $5\dfrac{1}{6}$

15. Not equivalent

17. Not equivalent

19. Equivalent

21. Equivalent

23. Not equivalent

25. Equivalent

27. Equivalent

29. Equivalent

31.   a. (i) $\dfrac{3}{2}$

(ii) $\dfrac{2}{3}$

b. (i) $\dfrac{4}{7}$

(ii) $\dfrac{7}{4}$

33.   a. (i) $\dfrac{7}{6}$

(ii) $\dfrac{6}{7}$

b. (i) 4

(ii) $\dfrac{1}{4}$

35.   a. (i) $\dfrac{4}{7}$

(ii) $\dfrac{7}{4}$

b. (i) $\dfrac{5}{8}$

(ii) $\dfrac{8}{5}$

37. Equivalent

39. Not equivalent

41. Not equivalent

43. Equivalent

45.   a. 12

    b. 45

47.   a. 24

    b. 3

49.   a. 8

    b. 18

51. $\dfrac{1}{3}$

53. $\dfrac{3}{4}$

55. $\dfrac{3}{4}$

57. $\dfrac{149}{240}$

Exercise 2.4

1. $\dfrac{2}{5}$ 、

3. $\dfrac{12}{15}$

5. $\dfrac{8}{7}$

7. $\dfrac{8}{9}$

9.   a. $1\dfrac{1}{2}$

        b. $1\dfrac{1}{3}$

11.   a. $2\dfrac{1}{6}$

        b. $18\dfrac{1}{12}$

13.   a. $15\dfrac{11}{12}$

        b. $14\dfrac{5}{12}$

15.   a. $\dfrac{309}{1000}$

         b. $1\dfrac{9}{10}$

17.   a. $\dfrac{5}{9}$

         b. $\dfrac{3}{20}$

19.   a. $1\dfrac{7}{24}$

         b. $14\dfrac{5}{8}$

21.   a. $3\dfrac{1}{2}$

         b. $2\dfrac{1}{10}$

23.   a. $\dfrac{54}{125}$

         b. $\dfrac{11}{16}$

25.   a. 4

         b. $2\dfrac{1}{3}$

27.   a. $\dfrac{15}{88}$

         b. $12\dfrac{1}{2}$

29.   a. $\dfrac{1}{14}$

         b. $3\dfrac{1}{3}$

31.   a. $1\dfrac{1}{2}$

b. $\dfrac{3}{32}$

33.   a. $1\dfrac{5}{9}$

       b. $2\dfrac{2}{3}$

35.   a. $\dfrac{2}{5}$

       b. $3\dfrac{3}{4}$

37.   a. $\dfrac{4}{9}$

       b. $12\dfrac{2}{3}$

39.   a. $\dfrac{9}{25}$

       b. $\dfrac{36}{49}$

41.   a. $\dfrac{27}{64}$

       b. $\dfrac{625}{81} = 7\dfrac{58}{81}$

43.   a. $\dfrac{16}{9} = 1\dfrac{7}{9}$

       b. $\dfrac{625}{81} = 7\dfrac{58}{81}$

45.   a. $\dfrac{1}{3}$

       b. $\dfrac{1}{7}$

47.   a. $\dfrac{2}{5}$

       b. $\dfrac{9}{4} = 2\dfrac{1}{4}$

49.　　a. $\dfrac{7}{4} = 1\dfrac{3}{4}$

　　　　b. $\dfrac{5}{2} = 2\dfrac{1}{2}$

51. $\dfrac{2}{3}$

53. $5\dfrac{7}{15}$ hours

55. $\dfrac{7}{8}$ pounds

57. $\dfrac{13}{28}$ kg

59. $\dfrac{1}{10}$

61. 1,350 textbooks

63. $3\dfrac{1}{2}$ km

65. 14 pieces

67. 200 doses

69. 304 bulbs

71. $2\dfrac{2}{5}$

Exercise 2.5

1.　　a. $\dfrac{1}{5}$

　　　　b. 0.75

　　　　c. $\dfrac{3}{50}$

3.   a. 0.36
       b. $\dfrac{1}{250}$
       c. 0.14

5.   a. 0.5
       b. $\dfrac{2}{5}$
       c. 0.06

7.   a. $\dfrac{1}{200}$
       b. 0.36
       c. $\dfrac{1}{100}$

9.   a. $\dfrac{7}{2}$
       b. 1.6
       c. $\dfrac{28}{5}$

11.   a. 5.05
       b. $\dfrac{34}{5}$
       c. 2.75

13.   a. $2\dfrac{1}{4}$
       b. 1.75
       c. $4\dfrac{1}{50}$

15.   a. 8.35
       b. $16\dfrac{1}{200}$
       c. 15.5

17.   a. $\dfrac{2}{3}$
       b. $0.2\overline{5}$

c. $\dfrac{25}{99}$

19. a. $0.\overline{45}$

b. $\dfrac{2}{9}$

c. $0.\overline{285714}$

21. a. $\dfrac{8}{75}$

b. $\dfrac{3}{256}$

23. a. $\dfrac{27}{128}$

b. $\dfrac{9}{4} = 2\dfrac{1}{4}$

25. a. 1

b. $\dfrac{5}{6}$

27. a. $\dfrac{2}{3}$

b. $\dfrac{85}{44} = 1\dfrac{41}{44}$

29. a. $\dfrac{369}{25} = 14\dfrac{19}{25}$

b. $\dfrac{707}{200} = 3\dfrac{107}{200}$

31. a. $\dfrac{55}{9} = 6\dfrac{1}{9}$

b. $\dfrac{1}{1024}$

33. a. $\dfrac{169}{500} = 0.338$

b. $\dfrac{1}{100} = 0.01$

35. $\dfrac{511}{192} = 2\dfrac{127}{192}$

37. $\dfrac{12}{7} = 1\dfrac{5}{7}$

39. $\dfrac{391}{100} = 3\dfrac{91}{100}$

41. $\dfrac{2339}{70} = 33\dfrac{29}{70}$

43. $\dfrac{1}{10} = 0.1$

45. $\dfrac{3}{5} = 0.6$

47. **79**

49. $\dfrac{73}{20} = 3\dfrac{13}{20} = 3.65$

51. $\dfrac{74}{25} = 2\dfrac{24}{25} = 2.96$

Review Exercises 2

1.
   a. Five tenths
   b. Seven thousandths
   c. Twelve hundredths
   d. Twenty-nine thousandths

3.
   a. Thirty-two and four hundredths
   b. Two hundred and two tenths
   c. Forty-five thousand, five and one thousandth
   d. One million, five thousand, seventy-one and twenty-five hundredths

5.
   a. 564.667
   b. 40.103

      c. 79.3802

      d. 9.63625

7. 0.189

9. 1.996

11.   a. 3, 48

      b. 4, 60

      c. 4, 15

      d. 27, 24

13.   a. $\dfrac{24}{21} < \dfrac{11}{5}$

      b. $\dfrac{20}{44} > \dfrac{6}{15}$

      c. $\dfrac{18}{45} < \dfrac{16}{4}$

      d. $\dfrac{15}{25} = \dfrac{63}{105}$

15.   a. $1\dfrac{1}{2}$

      b. $1\dfrac{1}{2}$

      c. $7\dfrac{1}{3}$

      d. $11\dfrac{1}{3}$

17.   a. $\dfrac{5}{23}$

      b. $\dfrac{31}{12}$

      c. 5

      d. $\dfrac{4}{5}$

19.   a. $\dfrac{1}{40}$

      b. 0.625

c. $\dfrac{2}{25}$

d. 0.28

e. $\dfrac{1}{500}$

f. 0.78

21. $13\dfrac{13}{24}$ kg

23. $5\dfrac{5}{6}$ years

25. $376\dfrac{1}{5}$ km

27. Alisha: \$360, Beyonce: \$220

29. \$2,201.40

31. \$870.89

33. 96 girls

35. $15\dfrac{27}{125}$

37. $1\dfrac{1}{4} = 1.25$

39. $\dfrac{21}{625}$

41. $\dfrac{5}{108}$

43. $\dfrac{3}{4}$

45. $1\dfrac{5}{11}$

Self-Test Exercises 2

1. a. $20, 24$
    b. $11, 30$
    c. $18, 96$
    d. $66, 24$

2. a. $\dfrac{15}{2} = 7\dfrac{1}{2}$

    b. $\dfrac{26}{3} = 8\dfrac{2}{3}$

    c. $\dfrac{16}{9} = 1\dfrac{7}{9}$

    d. $\dfrac{9}{55}$

3. a. $\dfrac{55}{4} = 13\dfrac{3}{4}$

    b. $\dfrac{28}{15} = 1\dfrac{13}{15}$

    c. $\dfrac{1}{2}$

    d. 6

4. a. 27.9028
    b. 8.133
    c. 2.13792
    d. $2.297\overline{6}$

5. a. Four thousandths
    b. Six and five hundredths
    c. Three hundred and two hundredths
    d. Seven and seventy-one thousandths

6. a. $\dfrac{5}{8}$

    b. $\dfrac{16}{5} = 3\dfrac{1}{5}$

    c. $\dfrac{8}{11}$

    d. 0.35

    e. 1.8

    f. $1.0\overline{6}$

7. 593.75km; 37.8 cm

8. $1,500

9.   a. $25\dfrac{13}{40}$

    b. $8\dfrac{3}{4}$

    c. $12\dfrac{17}{24} = 12.708\overline{3}$

10.   a. $8\dfrac{16}{19}$

    b. $3\dfrac{91}{100}$

    c. $4\dfrac{11}{270}$

11. 13 deliveries

12. $\dfrac{1}{3}$; $80

13. $242.69

14. $9\dfrac{1}{8}$ km

ANSWER KEY - CHAPTER 3

Exercise 3.1

1. a. Base: 7; Exponent: 4; Power: 7^4

   b. Repeated multiplication: $9 \times 9 \times 9 \times 9 \times 9$; Base: 9; Exponent: 5

   c. Repeated multiplication: $3 \times 3 \times 3 \times 3$; Base 3^4

   d. Base $\dfrac{2}{5}$; Exponent: 6; Power: $\left(\dfrac{2}{5}\right)^6$

   e. Repeated multiplication: $\dfrac{5}{7} \times \dfrac{5}{7} \times \dfrac{5}{7} \times \dfrac{5}{7} \times \dfrac{5}{7}$; Base: $\dfrac{5}{7}$; Exponent: 5

   f. Repeated multiplication: $\dfrac{4}{7} \times \dfrac{4}{7} \times \dfrac{4}{7}$; Power: $\left(\dfrac{4}{7}\right)^3$

   g. Base: 1.15; Exponent: 4; Power: $(1.15)^4$

   h. Repeated multiplication: $1.6 \times 1.6 \times 1.6$; Base: 1.6; Exponent: 3

   i. Repeated multiplication: $1.25 \times 1.25 \times 1.25 \times 1.25 \times 1.25$; Power: $(1.25)^5$

3. $4^9 = 262,144$

5. $\left(\dfrac{1}{2}\right)^7 = 0.01$

7. $\left(\dfrac{5}{2}\right)^5 = 97.66$

9. $(3.25)^6 = 1,178.42$

11. $6^5 = 7,776$

13. $\left(\dfrac{2}{5}\right)^2 = 0.16$

15. $(1.4)^3 = 2.74$

17. $6^6 = 46,656$

19. $\left(\dfrac{2}{3}\right)^{12} = 0.01$

21. $(2.5)^6 = 244.14$

23. 2^{10}

25. 3^8

27. 3^6

29. 2^8

31. 10^6

33. 2^6

35. 150

37. 609

39. 240

41. 58

43. 368

45. 7

47. 609

49. 257

51. 8,000

53. 625

55. 13,168.72

57. 91

59. 96

61. $1\dfrac{3}{8} = 1.375$

63. 5.41

65. 5,184

67. 2.07

69. 104,976

Exercise 3.2

1. a. $\sqrt{64} = 8$

   b. $\sqrt{\dfrac{25}{16}} = \dfrac{5}{4} = 1.25$

3. a. $\sqrt[3]{8} = 2$

   b. $\sqrt[3]{\dfrac{27}{64}} = \dfrac{3}{4} = 0.75$

5. a. $(144)^{\frac{1}{2}} = 12$

   b. $(64)^{\frac{1}{3}} = 4$

7. a. $(2)^{\frac{6}{2}} = 8$

   b. $(40)^{\frac{1}{2}} = 6.32$

9. a. $(8)^{\frac{1}{2}} \times (12)^{\frac{1}{2}} = 9.80$

   b. $(7)^{\frac{1}{2}} \times (14)^{\frac{1}{2}} = 9.90$

11. a. $\left(\dfrac{25}{49}\right)^{\frac{1}{2}} = 0.71$

    b. $\left(\dfrac{64}{9}\right)^{\frac{1}{2}} = 2.67$

13. a. $(2)^{\frac{3}{4}} = 1.68$

    b. $(5)^{\frac{3}{2}} = 11.18$

15. a. $(5)^{\frac{5}{4}} = 7.48$

    b. $(3)^{\frac{103}{72}} = 4.81$

17. a. $(8)^{\frac{7}{5}} = 18.38$

    b. $(5)^{\frac{5}{6}} = 3.82$

19. a. $8^2 = 64$

    b. $(3)^{\frac{2}{3}} = 2.08$

21. a. $(4)^{\frac{3}{7}} = 1.81$

    b. $(3)^{\frac{2}{3}} = 2.08$

23. a. $12^2 = 144$

    b. $7^2 = 49$

25. a. 4.88
    b. 1

27. a. 10.07
    b. 0.66

29. a. 8.49
    b. 51.96

31. a. $\dfrac{7}{6} = 1.17$
    b. 0.19

33. a. $6^{-\frac{1}{2}} = \dfrac{1}{\sqrt{6}} = 0.41$
    b. $7^{\frac{2}{3}} = \sqrt[3]{7^2} = 3.66$

35. a. $10^{-\frac{1}{5}} = \sqrt[5]{\dfrac{1}{10}} = 0.63$
    b. $2^{-\frac{9}{7}} = \sqrt[7]{\left(\dfrac{1}{2}\right)^9} = 0.41$

37. a. $6^{-\frac{4}{3}} = \sqrt[3]{\left(\dfrac{1}{6}\right)^4} = 0.09$
    b. $7^{\frac{37}{24}} = \sqrt[24]{7^{37}} = 20.08$

39. a. $5^{-\frac{8}{3}} = \sqrt[3]{\left(\dfrac{1}{5}\right)^8} = 0.01$
    b. $6^3 = 216$

41. a. $8^4 = 4,096$
    b. $7^{-3} = 0.003$

43. a. 0.67
    b. 0.83

45. a. 0.33

b. 8

47. a. 0.91

b. 2.25

Review Exercises 3

1. 7

3. $(3^5)^{\frac{3}{5}} = 27$

5. a. 2^2 = 4

b. 5^3 = 125

7. a. 3^3 = 27

b. 6^1 = 6

9. a. 27

b. 0.2

11. a. 400

b. −10,000,000

13. a. $\dfrac{1}{5} = 0.2$

b. $\dfrac{1}{7} = 0.\overline{142857}$

c. $\dfrac{8}{9} = 0.\overline{8}$

15. a. 49

b. $\dfrac{5}{6} = 0.8\overline{3}$

c. $\dfrac{6}{5} = 1.2$

17. a. 7,180.08

b. 1,817.28

19. 164,593.54

21. −17,668.97

Self-Test Exercises 3

1. a. 3^6

   b. 2^7

   c. 3^3

   c. 2^3

2. a. 3^{10}

   b. 10^9

3. a. $\dfrac{3^2}{2^6} = 0.14$

   b. $2 \times 3^2 = 18$

4. a. 0.01

   b. 1

5. a. 0.84

   b. 5.80

6. a. 13.6

   b. 1

7. a. 32

   b. 5.80

8. a. 1,953,125

   b. 3

9. a. –26

   b. 42

10. a. –61

    b. –16

ANSWER KEY - CHAPTER 4

Review Exercises 4

1. 15

   3. 10

   5. $0.00802/ml

   7. $15.88

   9. 2.74

   11. $0.8082

   13. $213.64

Self-Test Exercises 4

1. 91

   2. $122.43

   3. 24.6mm

   4. 29,927

   5. 13.25

   6. 8.6

   7. 72.85

ANSWER KEY - CHAPTER 5

Exercise 5.1

1.
    a. Not equivalent
    b. Equivalent
    c. Not equivalent
    d. Equivalent

3.
    a. Equivalent
    b. Equivalent
    c. Not equivalent
    d. Equivalent

5.
    a. (i) $3 : 8 : 5$ (ii) $1 : 2.67 : 1.67$
    b. (i) $7 : 2 : 5$ (ii) $3.5 : 1 : 2.5$
    c. (i) $9 : 20 : 40$ (ii) $1 : 2.22 : 4.44$

7.
    a. (i) $10 : 3$ (ii) $3.33 : 1$
    b. (i) $36 : 5 : 9$ (ii) $7.2 : 1 : 1.8$
    c. (i) $3 : 15 : 20$ (ii) $1 : 5 : 6.67$

9.
    a. $3 : 8$
    b. $1 : 4$
    c. $2 : 5$

11.
    a. $12 : 2 : 1 : 4$
    b. $24 : 4 : 1 : 8$

13. 75 km/h

15. 11 km/L

17. 75 words/min

19. 29 pages/min

21. 2 kg of flour for $3.30

23. 8 pencils for $2.88

25. 0.8 litres of juice for $1.40

27. 11:20

29. 540 km/h

31. A has a higher hourly rate

    a. Kate's rate: $28/h; Susan's rate: $27.75/h
    b. Kate's rate is higher by $0.25 per hour.

35. Mike pays $40; Sarah pays $50.

37. Amy's share: $2,500; Gary's share: $4,000; Andrew's share: $1,000.

39. 30 cm, 16 cm, 20 cm

41. Alex: $800, Brooks: $1,200.

43.   a. $\dfrac{4}{15}$
    b. $5 : 6 : 4$
    c. 25 red, 30 green, 20 blue

45. $16,875

47. $8 : 6 : 5$

Exercise 5.2

1. a. Yes
   b. No

    c. Yes

    d. No

3.   a. $x = 3$

    b. $x = 36$

    c. $\dfrac{5}{3}$

    d. $x = 10$

5.   a. $x = 12\dfrac{20}{47}$

    b. $x = 3\dfrac{411}{475}$

    c. $x = \dfrac{11}{18}$

    d. $x = 2\dfrac{5}{8}$

7.   180 L

9. $2,290.91

11.   11.63 km

13.   GIC: $13,625, Fixed deposit: $8,175

15.   a. B's investment: $20,000 C's investment: $15,000

     b. B's profit: $24,000, C's profit: $18,000

Review Exercises 5

1.   a. Equivalent

    b. Not equivalent

    c. Equivalent

    d. Not equivalent

3.   1 : 20

5.   a. 18 km/hr

b. 27 km

7. Gina by $0.75per hour

9. 480 grams for $3.75

11. a. $2.66/kg
   b. $2.50/kg
   c. $2.40/kg
   d. Best offer: c

13. 9 : 15 : 5

15. a. 6
   b. 18
   c. 0.66

   a. $0.7
   b. $0.21

19. $1,057.69

21. $162.5

23. $13,625

25. Khan: $1,000; Thomas: $2,500

Self-Test Exercises 5

1. 360 gram for $4.89

2. 74.07 km/hr

3. 9 hours 20 minutes

4. a. 12

   b.40.5

5. a. 7.5

   b. $7\dfrac{1}{5}$

6. 212.5 km

7. $2,668.90

8. 22.5L

9. Best offer: c

10. 6 : 10 : 15

ANSWER KEY - CHAPTER 6

Exercise 6.1

1.   a. $0.75, \dfrac{3}{4}$

         b. $30\%, \dfrac{3}{10}$

         c. $25\%, 0.25$

3.   a. $0.05, \dfrac{1}{20}$

         b. $20\%, \dfrac{1}{5}$

         c. $60\%, 0.60$

5.   a. $1.5, 1\dfrac{1}{2}$

         b. $17.5\%, \dfrac{7}{40}$

         c. $48\%, 0.48$

7.   a. $0.125, \dfrac{1}{8}$

         b. $5\%, \dfrac{1}{20}$

         c. $450\%, 4.5$

9.   a. $0.006, \dfrac{3}{500}$

         b. $0.5\%, \dfrac{1}{200}$

         c. $460\%, 4.6$

11.   a. $0.0005, \dfrac{1}{2,000}$

b. $0.25\%, \dfrac{1}{400}$

c. $112.5\%, 1.125$

13.   a. $0.0005, \dfrac{1}{2,000}$

b. $108\%, 1\dfrac{2}{25}$

c. $5.\overline{45}\%, 0.0\overline{54}$

15.   a. $0.0175, \dfrac{7}{400}$

b. $202.5\%, 2\dfrac{1}{40}$

c. $0.25\%, 0.0025$

17.   a. $0.065, \dfrac{13}{200}$

b. $250\%, 2\dfrac{1}{2}$

c. $16\%, 0.16$

19.   a. 70

b. 100

21.   a. 0.19

b. 0.5 km

23.   a. 52

b. $53.33

25. 2

27. $16.50

29. $0.27

31. 7.5

33. 40%

35. 400%

37. 45.45%

39. 28%

41. 800

43. $186

45. 130

47. $2,200

49. $32.50

51. $25,000

53. 74%

55. $700,000

57. $315,723.08

59. 13%

61. $89,400

Review Exercises 6

1.
    a. $0.80, \dfrac{4}{5}$

    b. $25\%, \dfrac{1}{4}$

    c. $150\%, 1.50$

d. $0.06\overline{3}, \dfrac{19}{300}$

e. $4.8\%, \dfrac{6}{125}$

f. $8\%, 0.08$

3.
  a. 36

  b. 20%

  c. $0.23

5. $460,000

7. 75%

9. $1,153,153.15

11.
  a. 42,875
  b. 38,500,000

Self-Test Exercises 6

1.
  a. $0.106, \dfrac{53}{500}$

  b. $225\%, 2\dfrac{1}{4}$

  c. $0.25\%, 0.0025$

  d. $0.005, \dfrac{1}{200}$

  e. $0.2\%, \dfrac{1}{500}$

  f. $26.5\%, 0.265$

3.
  a. 12.5%

  b. $337.5

  c. $500

ANSWER KEY - CHAPTER 7

Exercise 7.1

1. a. $2x - 3$

    b. $\dfrac{2x}{5}$

    c. $25 + 3x$

3. a. i. 3   ii. 0   iii. 3, 7, −4

    b. i. 2   ii. 0   iii. 1, −5

    c. i. 4   ii. 2   iii. 9, 7, −6

5. a. i. 3   ii. 5   iii. 5, −3

    b. i. 3   ii. 1   iii. −2, 3

    c. i. 3   ii. 7   iii.−2, −2

7. a. $12A$ and $-7A$; $4B$ and $-B$

    b. $6x$ and $-5x$; $8y$ and $-3y$

9. a. $-2x, 5x$ and $-12x$; 8 and -3

    b. $6xy^2$ and $2xy^2$; $-2x^2y$ and $3x^2y$; $-4x^2$ and $2x^2$

11. 24

13. 65

15. a. $\dfrac{23}{9}$

    b. 24

17. a. 30

    b. 22

Exercise 7.2

1. 30

3. 56

5. –6

7. 6,750

9. $11x^2 + 17x$

11. $-7y^2 + y$

13. $3y^2 + 3x$

15. $x^2y^2 + xy^2$

17. $-90x + 42$

19. –38

21. $-5x + 7$

23. $2x - 4$

25. $30x - 10y - 90$

27. $-14y - 144$

Exercise 7.3

1. $x + 6 = 10; x = 4$

3. $6x = 72; x = 12$

5. $\dfrac{x}{5} = 4; x = 20$

7. $\dfrac{2}{3}x = 12; x = 18$

9. $x = 30$

11. $x = 18$

13. $x = -17$

15. $x = 6$

17. $x = 1\dfrac{2}{11}$

19. $x = 1\dfrac{2}{5}$

21. $x = 2$

23. $x = -\dfrac{3}{20}$

25. $x = 4$

27. $x = 24$

29. $x = 16$

31. $y = 1.72$

33. $x = 4$

35. $x = 1.8$

37. $x = -2$

39. $x = -2.2$

41. $x = 4.33$

43. $x = 0.41$

45. $x = 16$

47. $y = 42$

49. $x = 52.6$

51. $y = 11$

53. $x = -5$

55. 5

57. 9-metre and 16-metre long wire

59. Becky's share: $325; Andy's share: $175

61. Adult ticket: $10; Child ticket: $7

63. $A = (x + 4)(x + 3)$; $A = 182$ square metres

65. $12.5

67. $26

69. 30°, 70°, and 80°

71. 15 cm, 25 cm and 30 cm

73. 188.24 lb

75. Water: 2.13 L; 15% Solution: 1.87 L

Review Exercises 7

1.   a. $3x + 12$

            b. $x - 5$

3.   a. $(3 + x)x$

            b. $10x + 15$

5.   a. $\dfrac{1}{x^2}$

            b. $-x^7$

7.   a. x^4

            b. $\dfrac{1}{x^2}$

9.   a. $x^{\frac{3}{2}} y$

            b. x^2

11.   a. $5x + 17 = 42; x = 5$

            b. $\dfrac{x}{15} = 45; x = 675$

13.   a. $x - 10 = 10; x = 20$

            b. $3(4x) = 36; x = 3$

15.   a. $x = 3$

            b. $x = 18$

17.   a. $x = -9$

            b. $x = 2$

Self-Test Exercises 7

1. a. $3x - 25$

      b. $x + 18$

2. a. $2x - 6$

   b. $\dfrac{x}{3}$

3. a. $-x^2 - 6x + 10; 19$

   b. $4x - 11y + 7; 4$

4. a. $6x + 13; 31$

   b. $-5x^2 + 26x + 24; 45$

5. a. $2x - 9 = 21; x = 15$

   b. $3 - 5x = 22; x = -\dfrac{19}{5} = -3.8$

6. a. $(4)(8) = 16x; x = 2$

   b. $6x = 30; x = 5$

7. a. $x = 4$

   b. $x = 18$

8. a. $-\dfrac{4}{7}$

   b. $4\dfrac{2}{3}$

ANSWER KEY - CHAPTER 8

Exercise 8.1

1.

3.
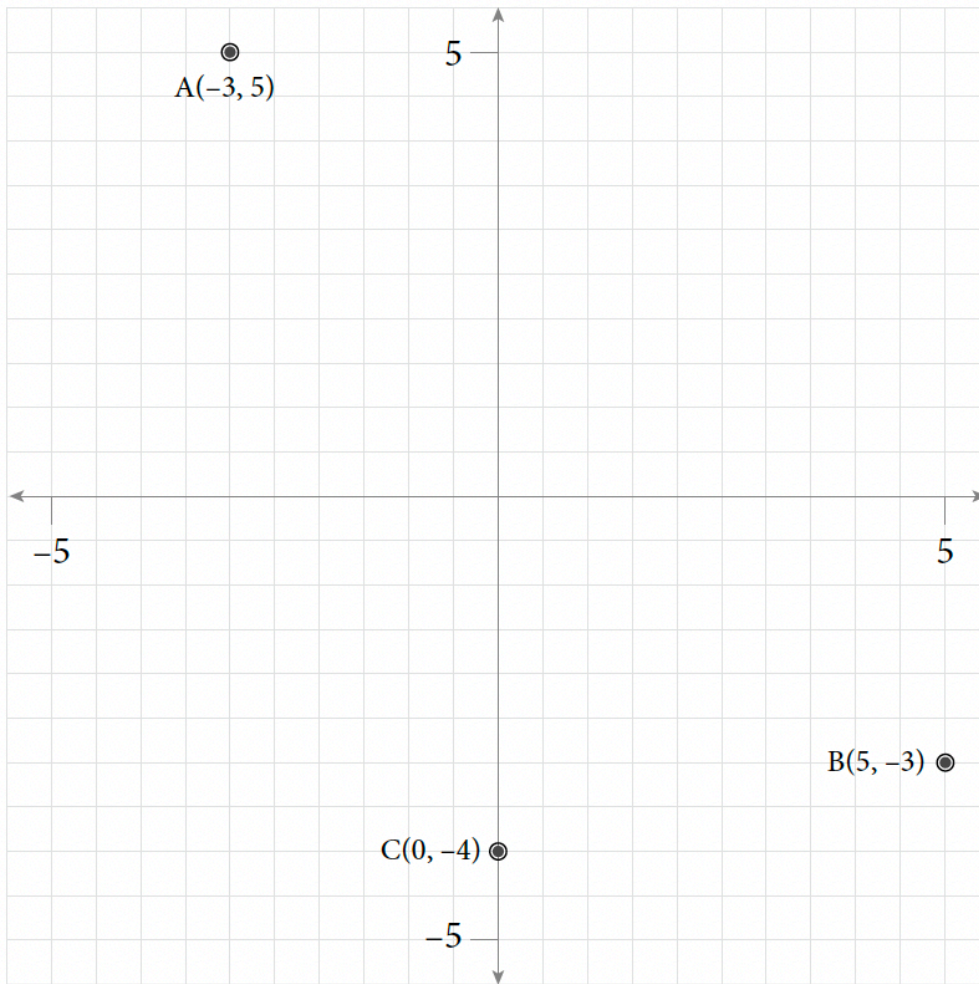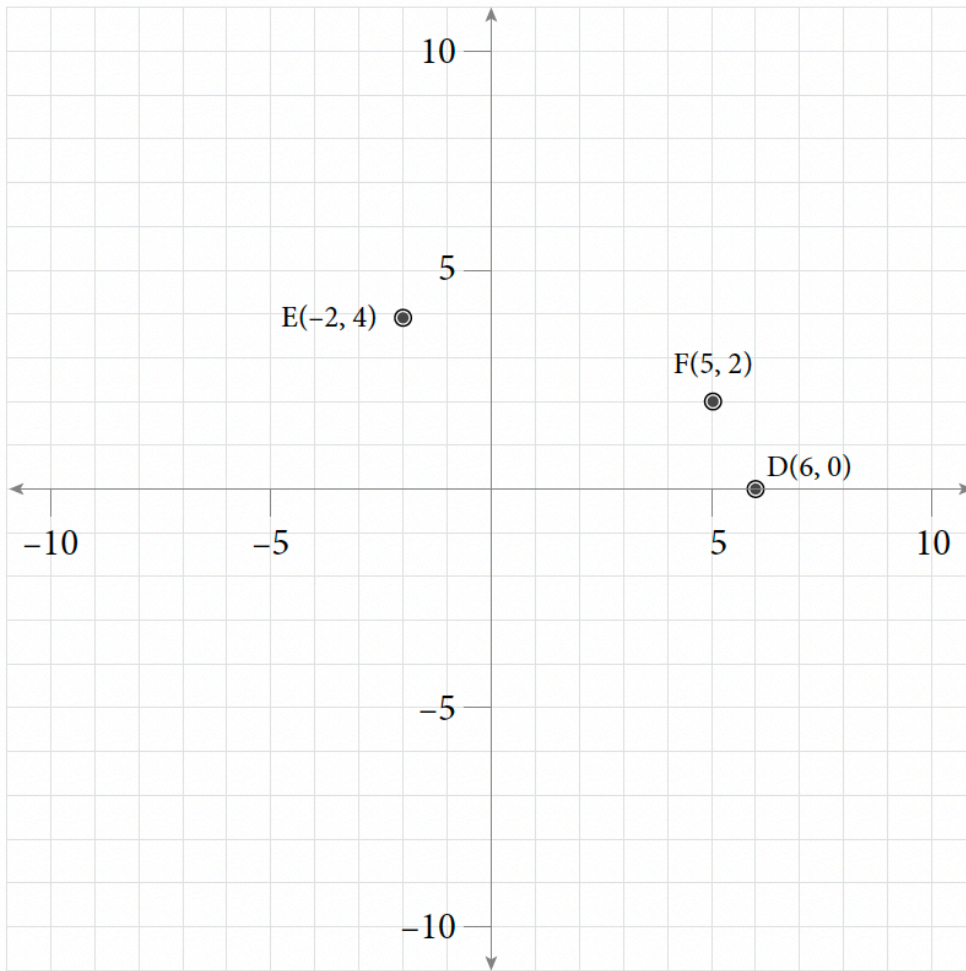
5. a. Quadrant II
    b. Quadrant IV
    c. Quadrant I

7. a. X-axis (left)
    Quadrant III
    Y-axis (up)

9. a. 2 units
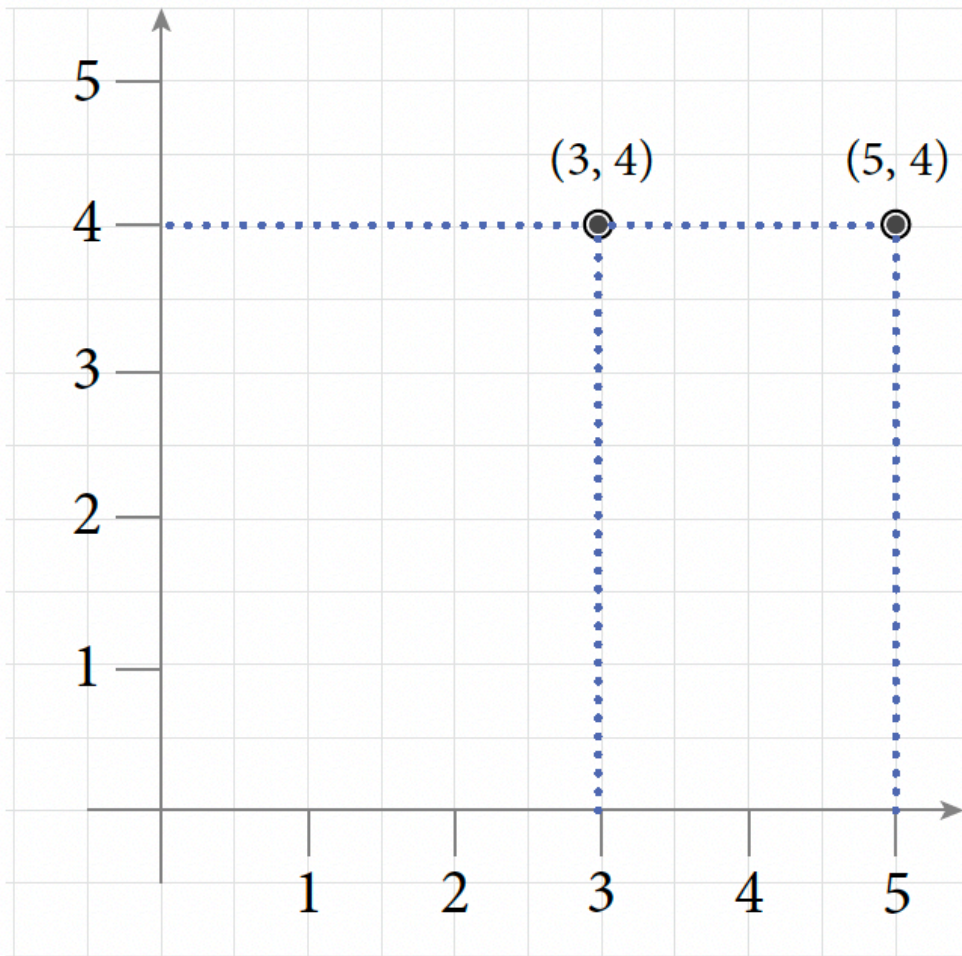
b. 9 units
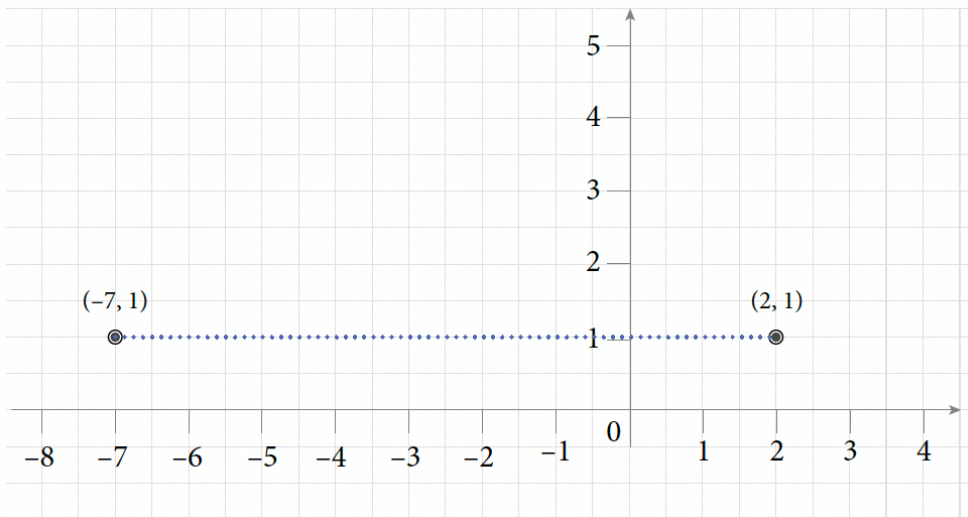
11. a. 5 units
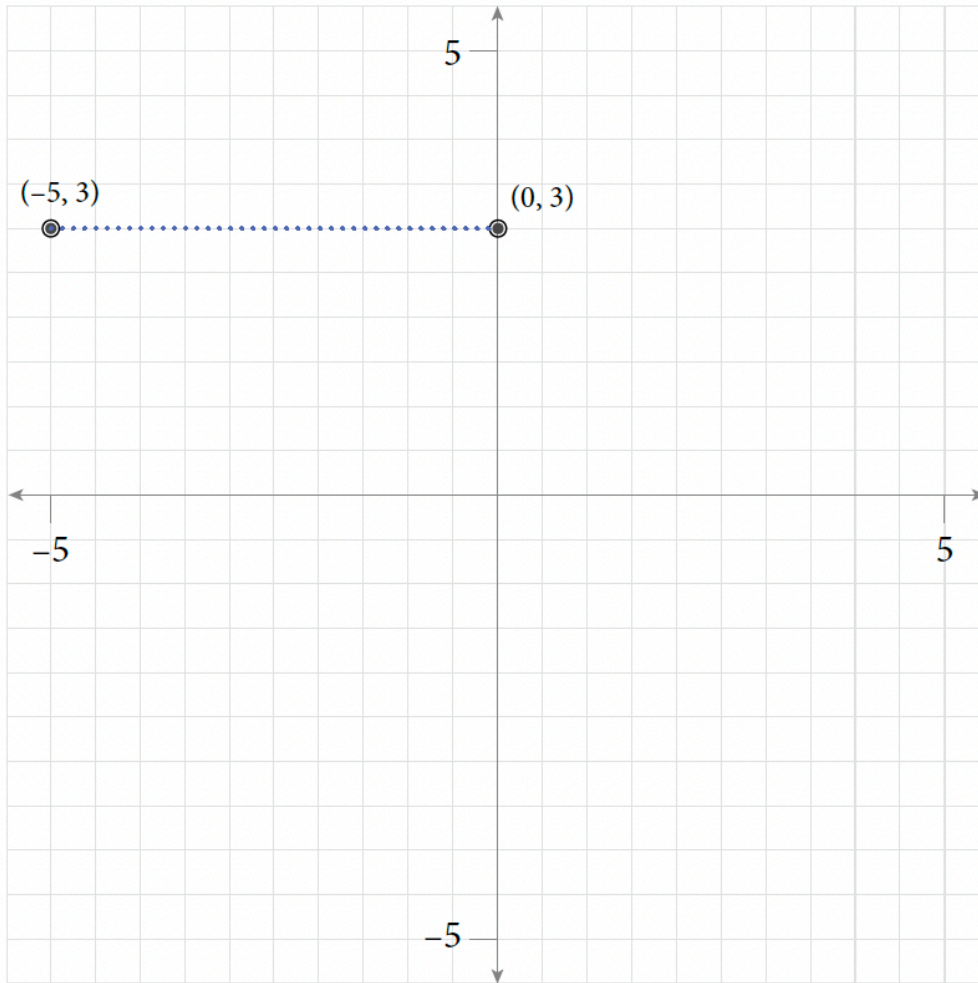
b. 8 units
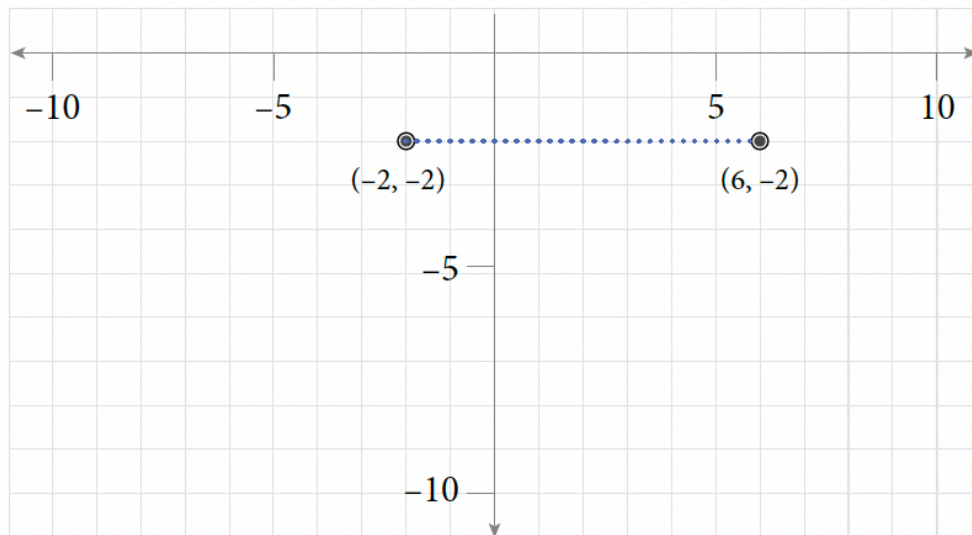
13. a. 5 units
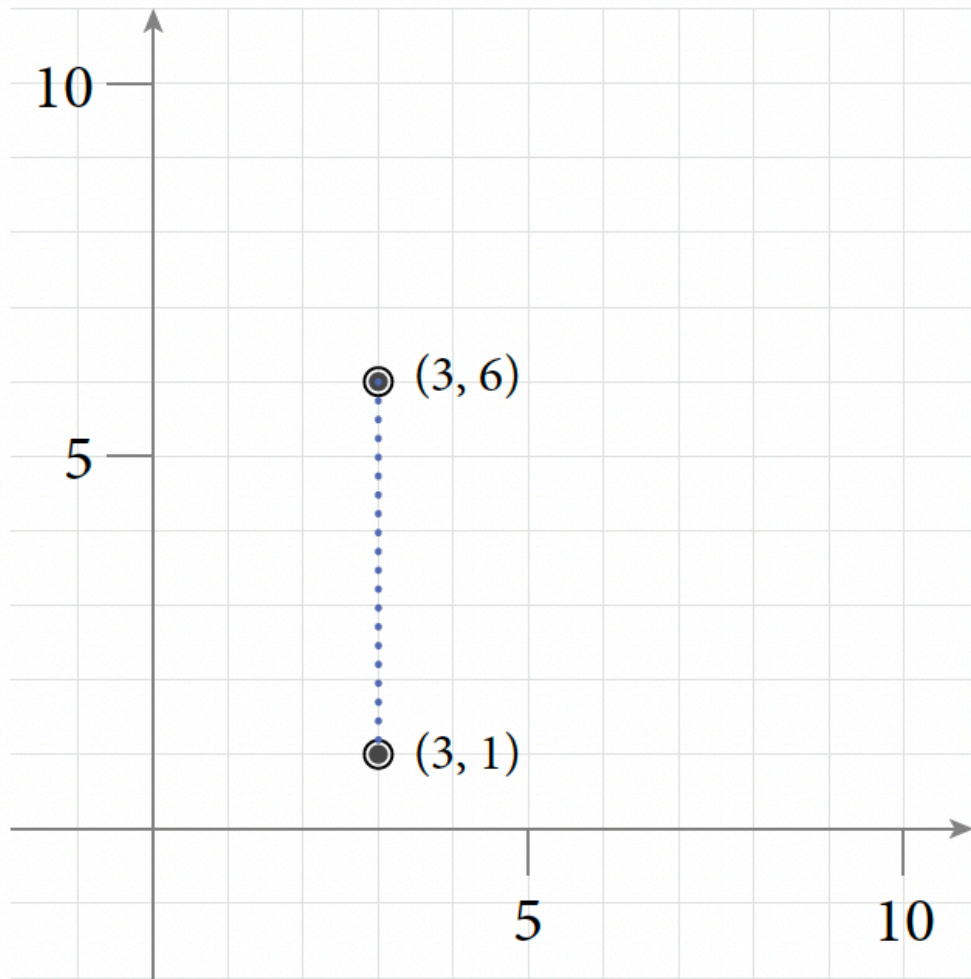
b. 7 units
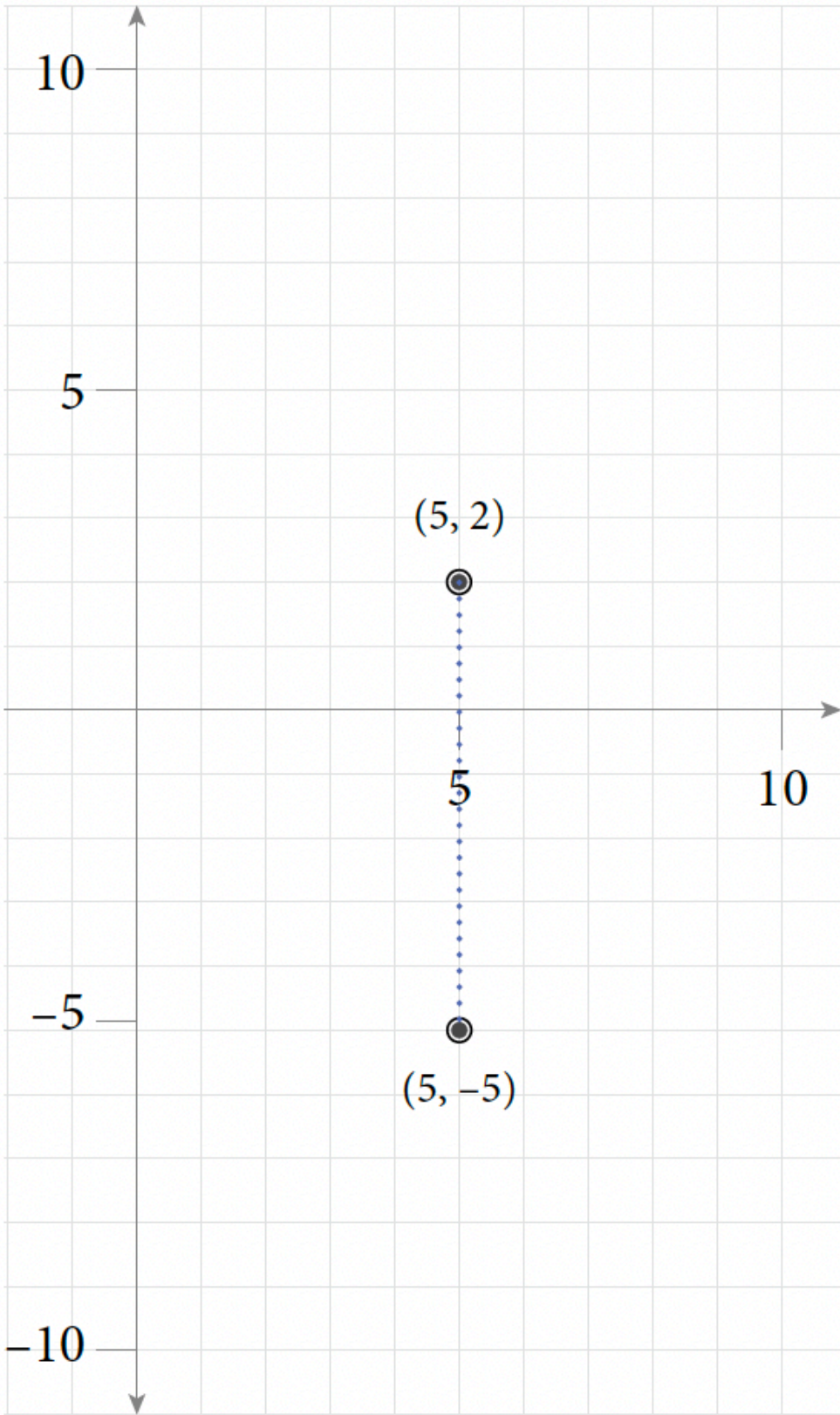
15. a. 4 units
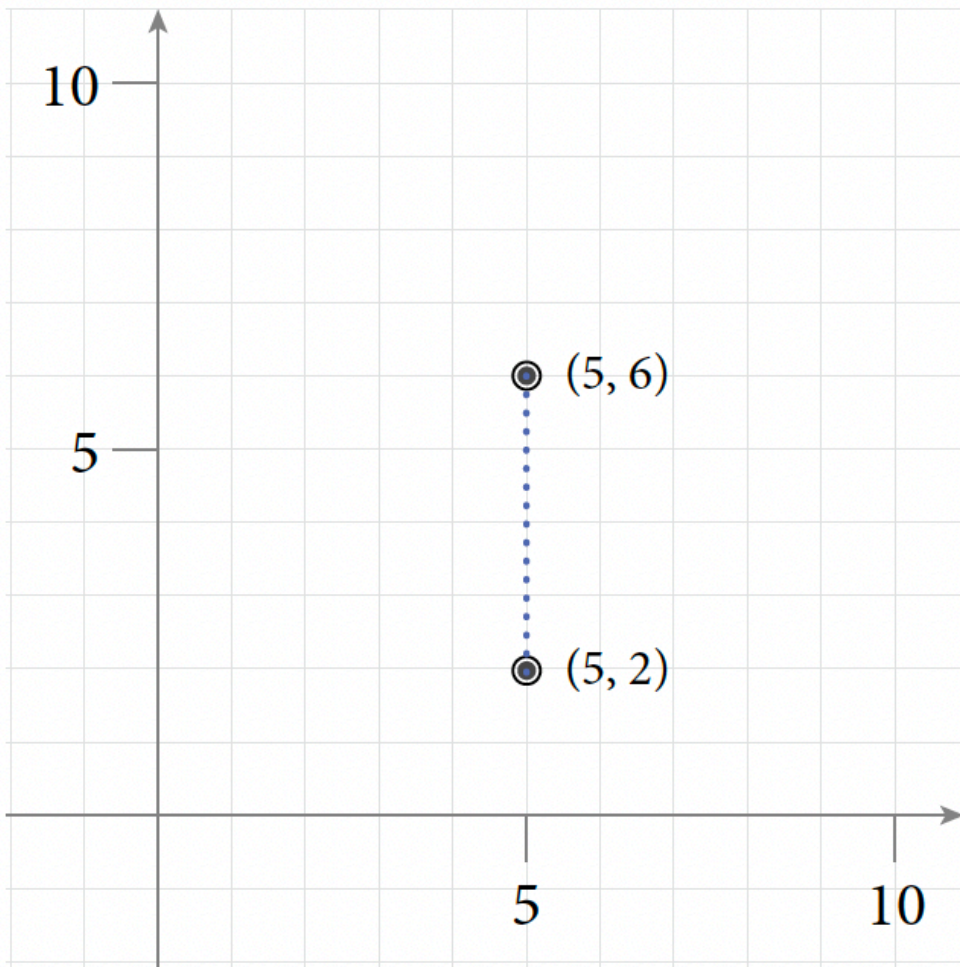
b. 6 units
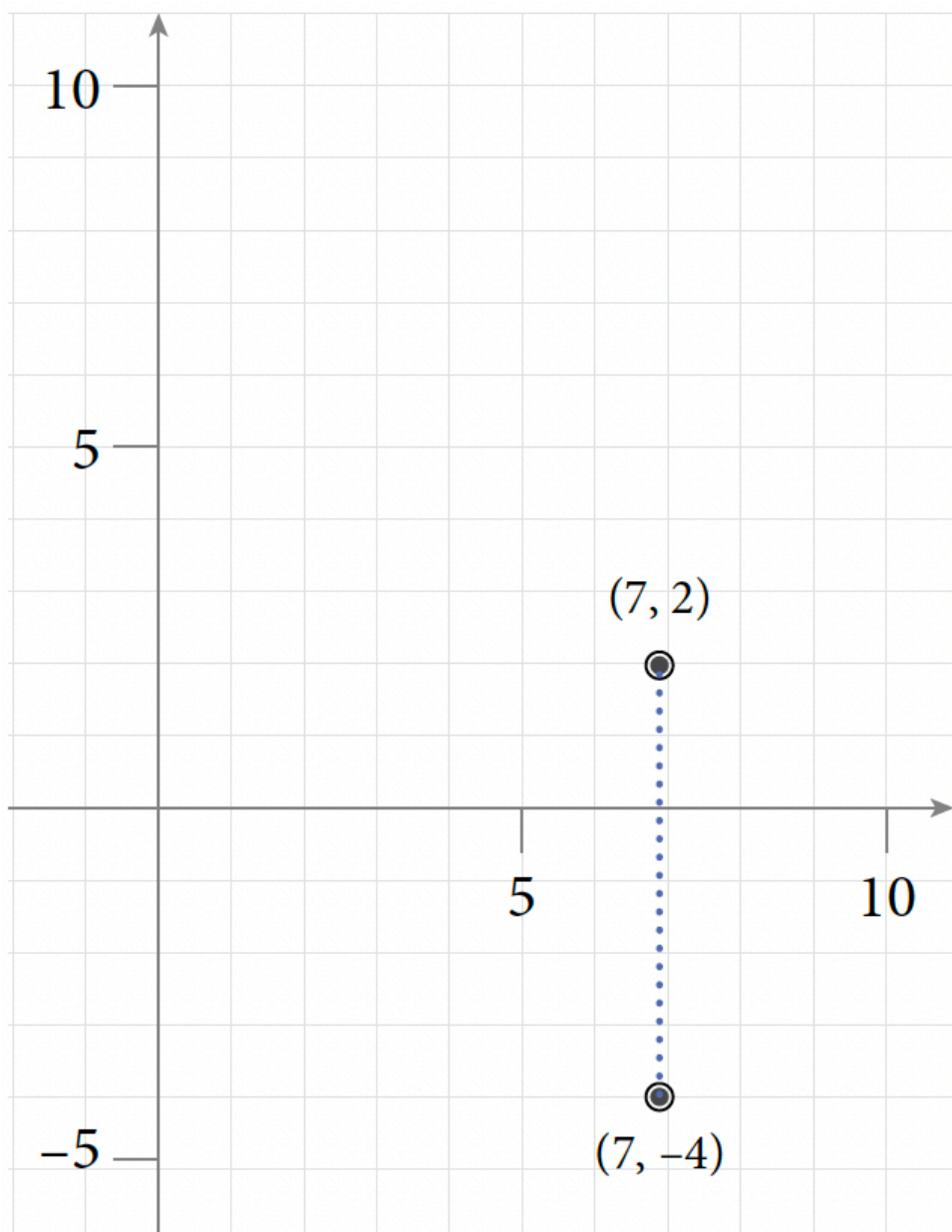
17. D (−3, −1)

19. S(−3, −1)

21. (1, 12) and (1, −2)

23. (−7, 3) and (5, 3)

Exercise 8.2

1. a. 4

   b. 10

   c. 6

   d. 9

   e. 15

   f. 6

3. a. –3

   b. 3

   c. 1

   d. $\dfrac{3}{2}$

   e. 6

   f. –6

5. $5x - 2y = -2$

7. $3x + 4y = -12$

9. $x - 2y = -3$

11. $y = -\dfrac{3}{2}x - \dfrac{3}{4}$

13. $y = \dfrac{3}{2}x + 5$

15. $y = -\dfrac{3}{2}x + 3$

17.

Table of Values for Exercise 17

x	y	(x, y)
0	3	$(0, 3)$
1	4	$(1, 4)$
2	5	$(2, 5)$
3	6	$(3, 6)$

19.
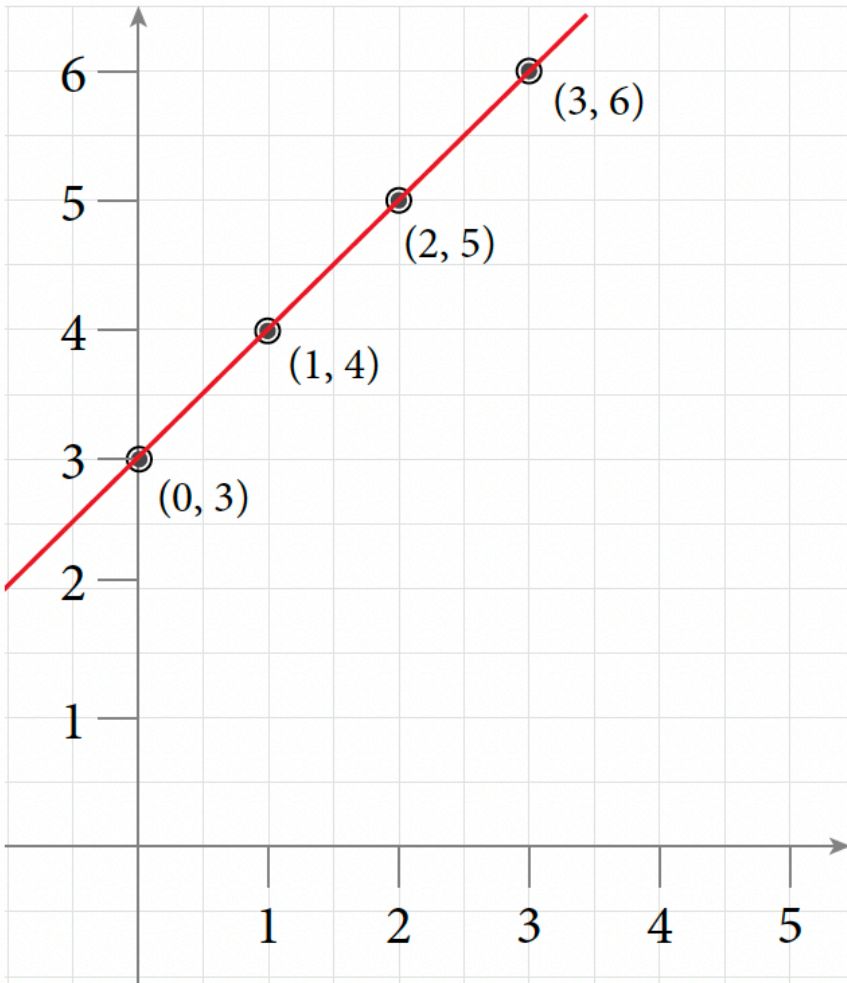
Table of Values for Exercise 19

x	y	(x, y)
0	1	$(0, 1)$
1	-4	$(1, -4)$
2	-9	$(2, -9)$
3	-14	$(3, -14)$
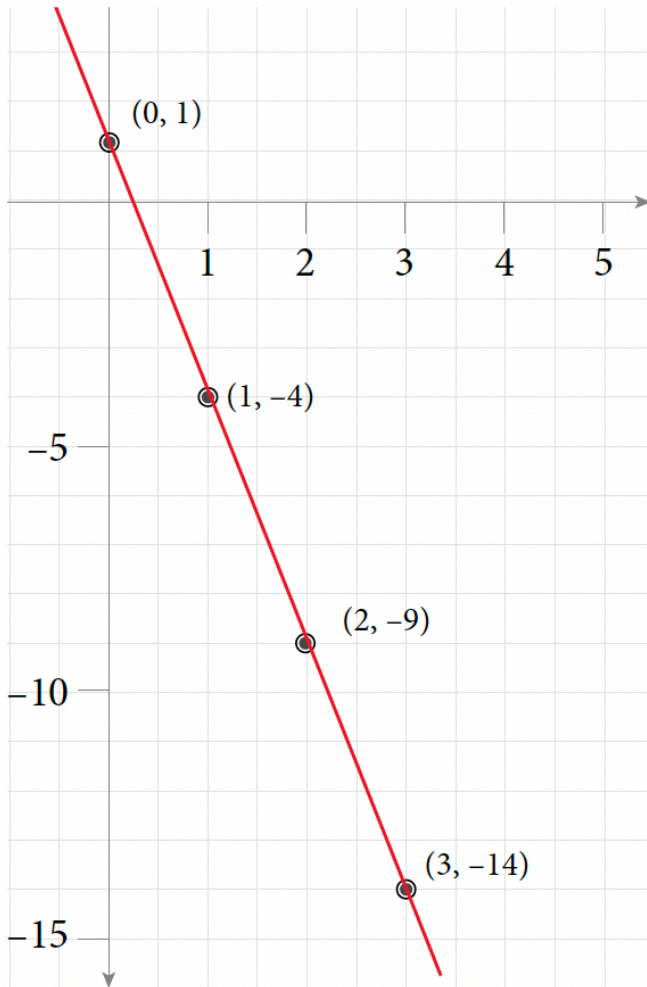
21.

Table of Values for Exercise 21

x	y	(x, y)
0	-1	$(0, -1)$
1	-3	$(1, -3)$
2	-5	$(2, -5)$
3	-7	$(3, -7)$
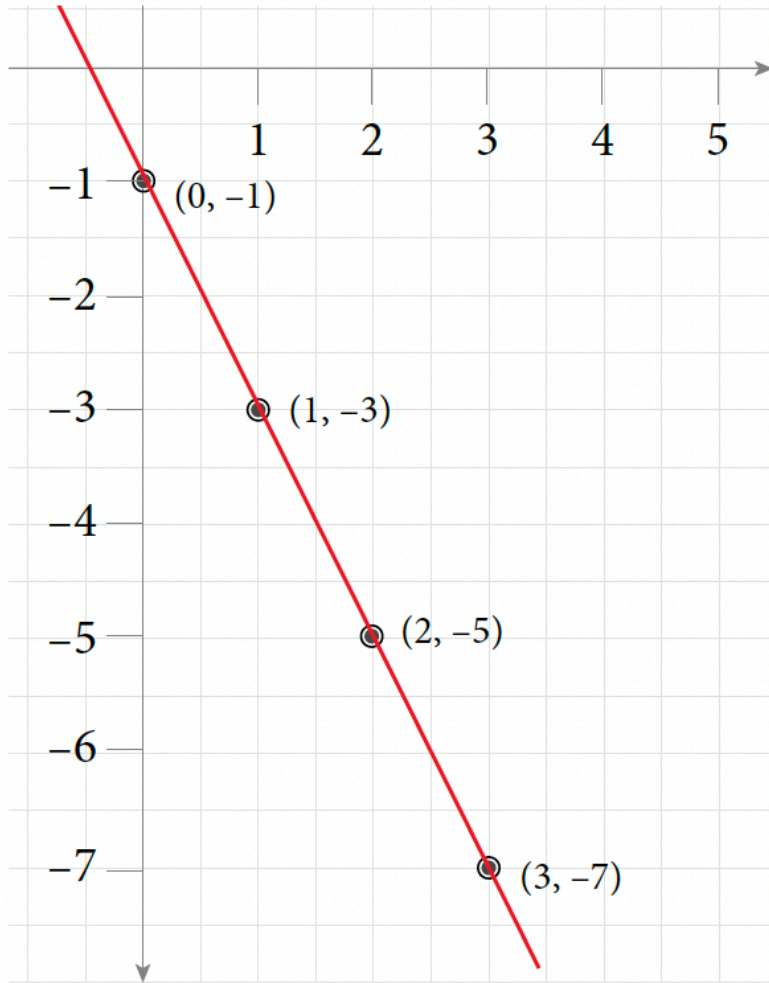
23.

Table of Values for Exercise 23

x	y	(x, y)
0	-3	$(0, -3)$
1	-1	$(1, -1)$
2	1	$(2, 1)$
3	3	$(3, 3)$
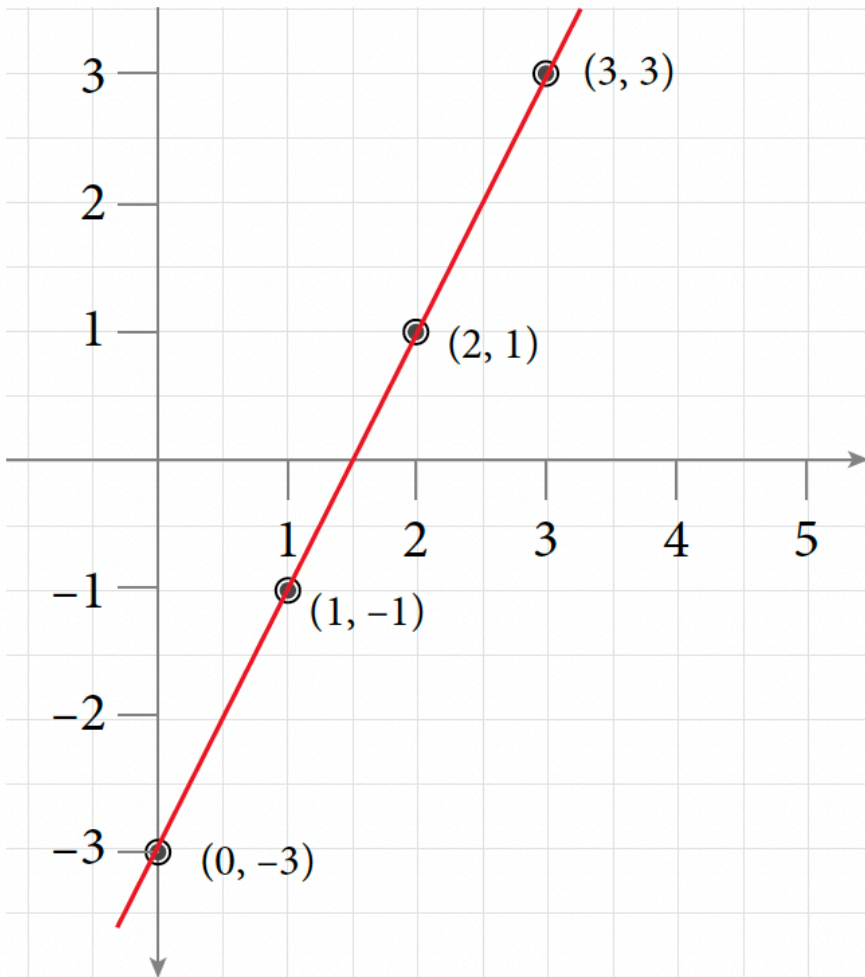
25. x-intercept: $\left(-\dfrac{2}{3}, 0\right)$; y-intercept: $(0, -2)$
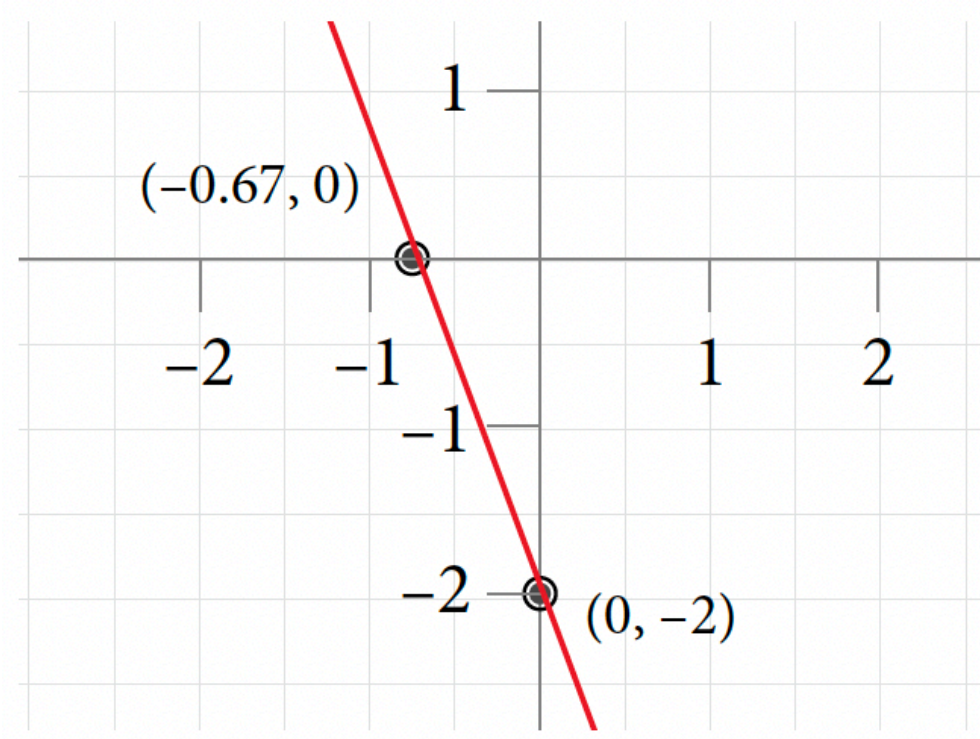
27. *x*-intercept: $7, 0$; *y*-intercept: $0, 7$
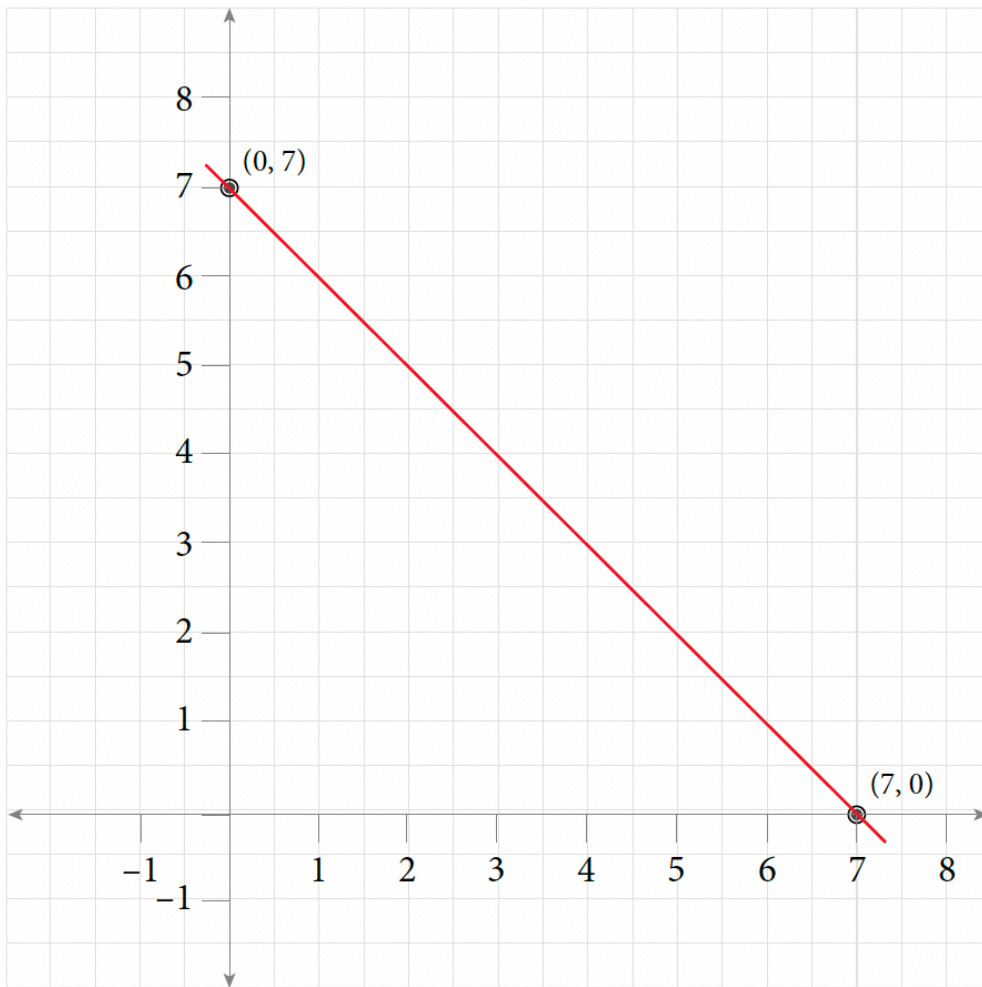
29. x-intercept: $-2, 0$; y-intercept: $0, 4$
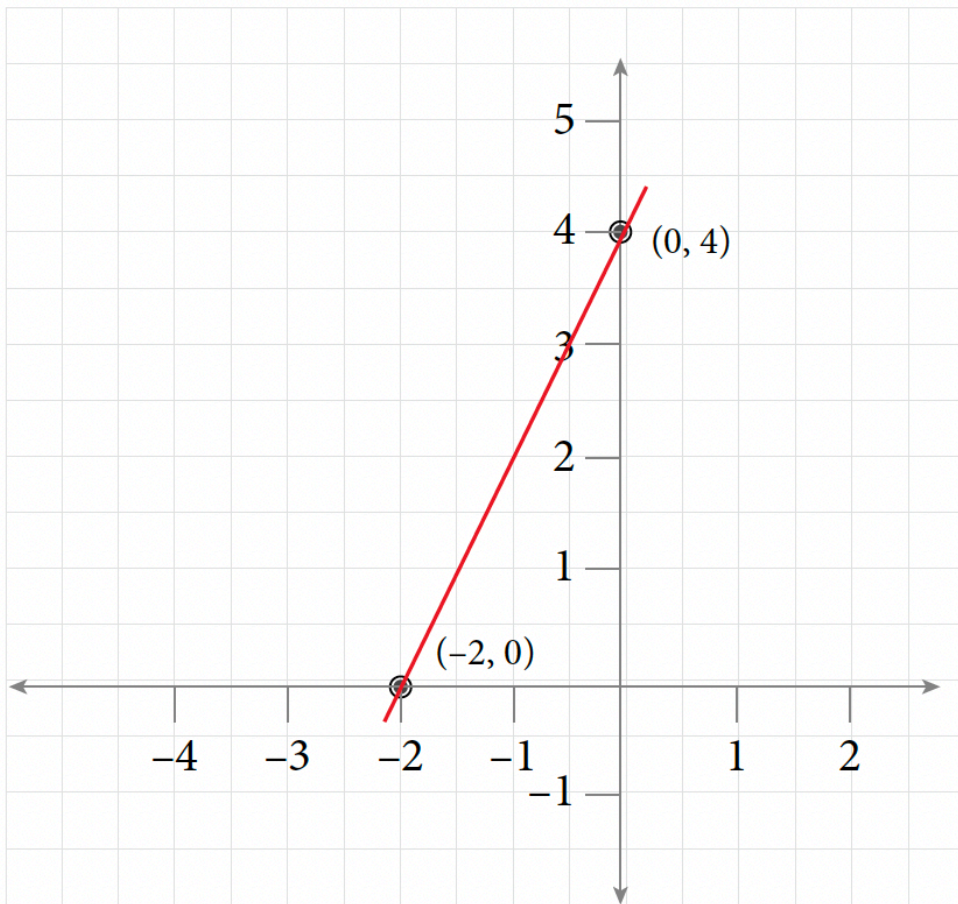
31. $m = \dfrac{2}{3}; b = -6$
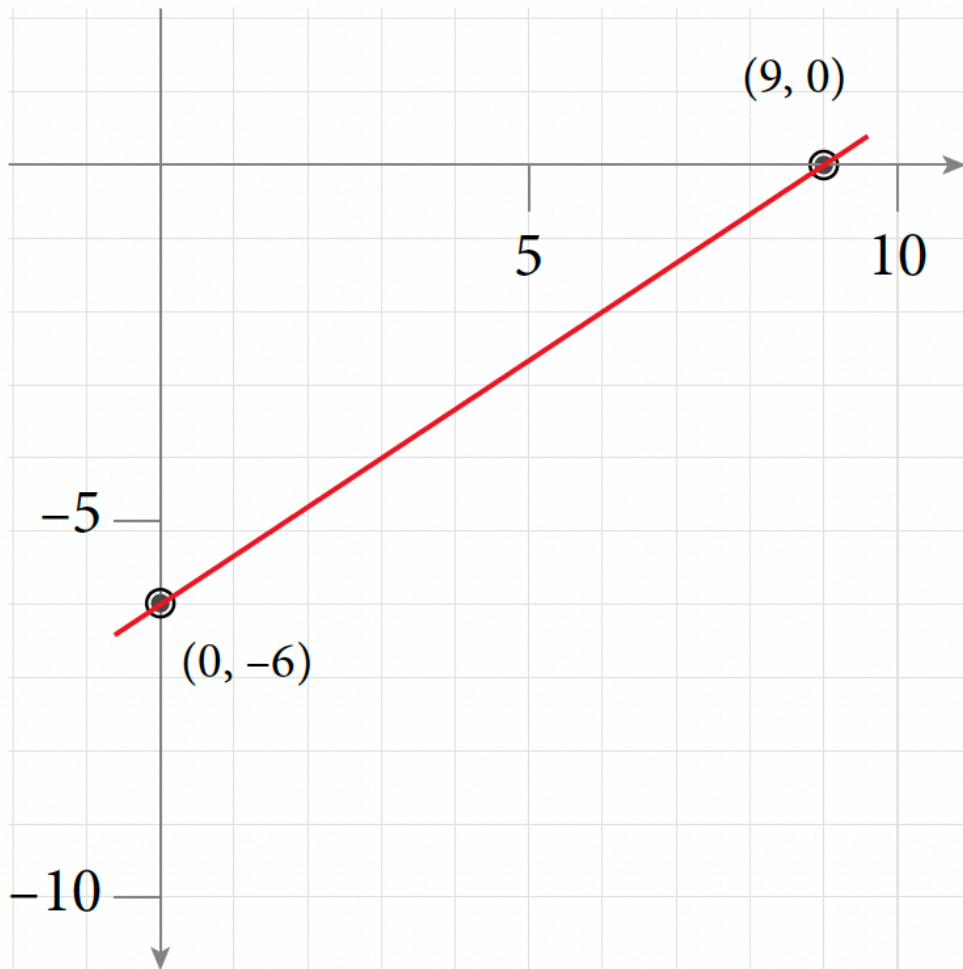
33. $m = \dfrac{4}{7}; b = 3$
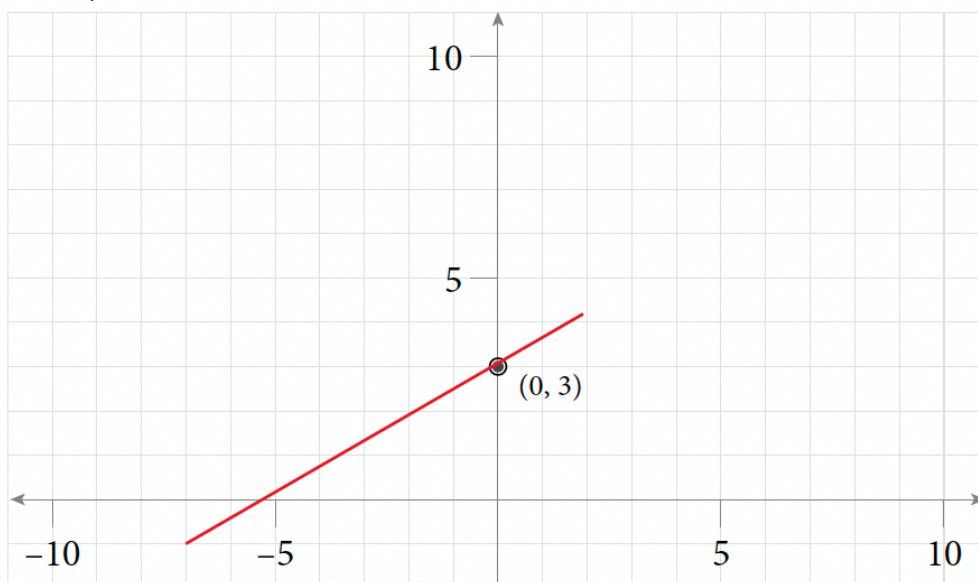
35. Positive slope

37. 0

39. $-\dfrac{5}{8}$

8.3 Exercise

1. $y = 2$

3. $y = x - 2$

5. $y = -2x + 10$

7. $y = \dfrac{2}{3}x$

9. $y = -\dfrac{3}{2}x + 6$

11. 2

13. 7

15. $5x + 3y = 15$

17. $2x + y = 1$

8.4 Exercise

1. Perpendicular

3. Parallel

5. Perpendicular

7. Neither

9. $y = \dfrac{2}{3}x - \dfrac{13}{3}$

11. $y = \dfrac{1}{3}x - 20$

13. $y = x + 7$

15. $y = -\dfrac{1}{2}x + 1$

Review Exercises 8

1. a. Quadrant IV
   b. Quadrant II
   c. X-axis (right)
   d. Quadrant IV
   e. X-axis (right)
   f. Y-axis (up)

3. RectanglePerimeter = 22 units; Area=24 square units

5.
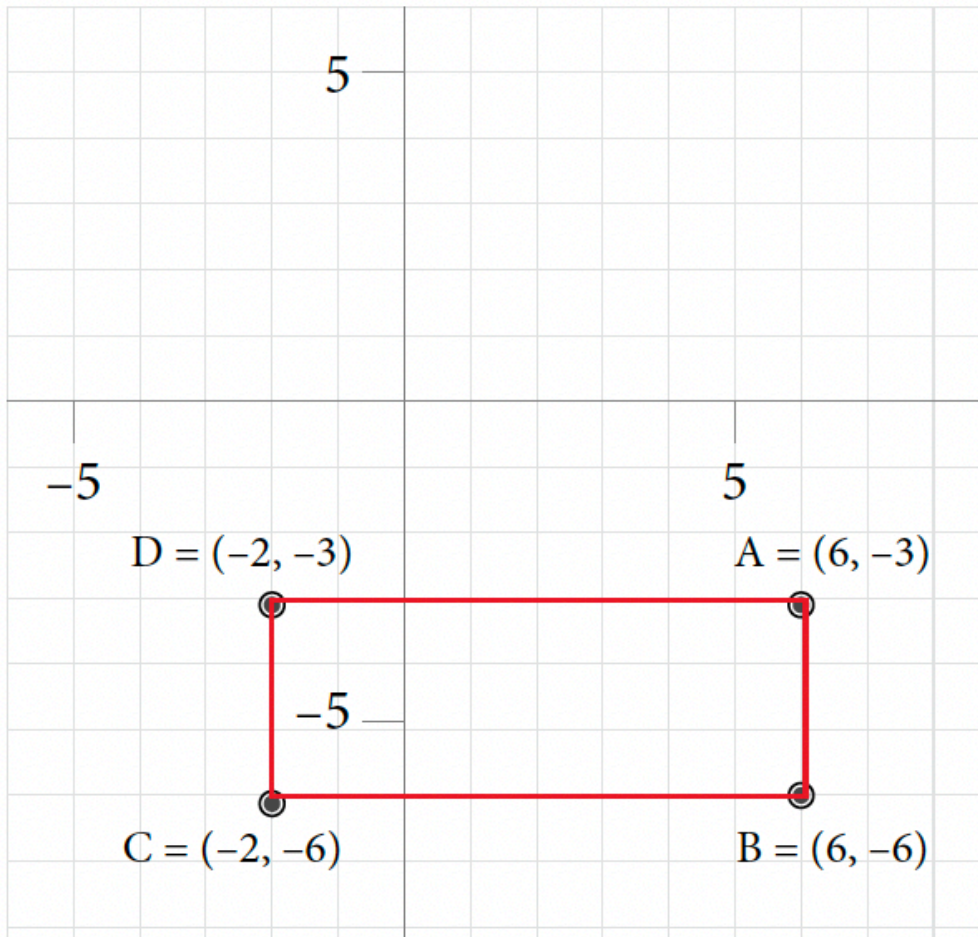
Table of Values for Exercise 5

x	y	(x, y)
0	-2	$(0, -2)$
1	2	$(1, 2)$
2	6	$(2, 6)$
3	10	$(3, 10)$

7.
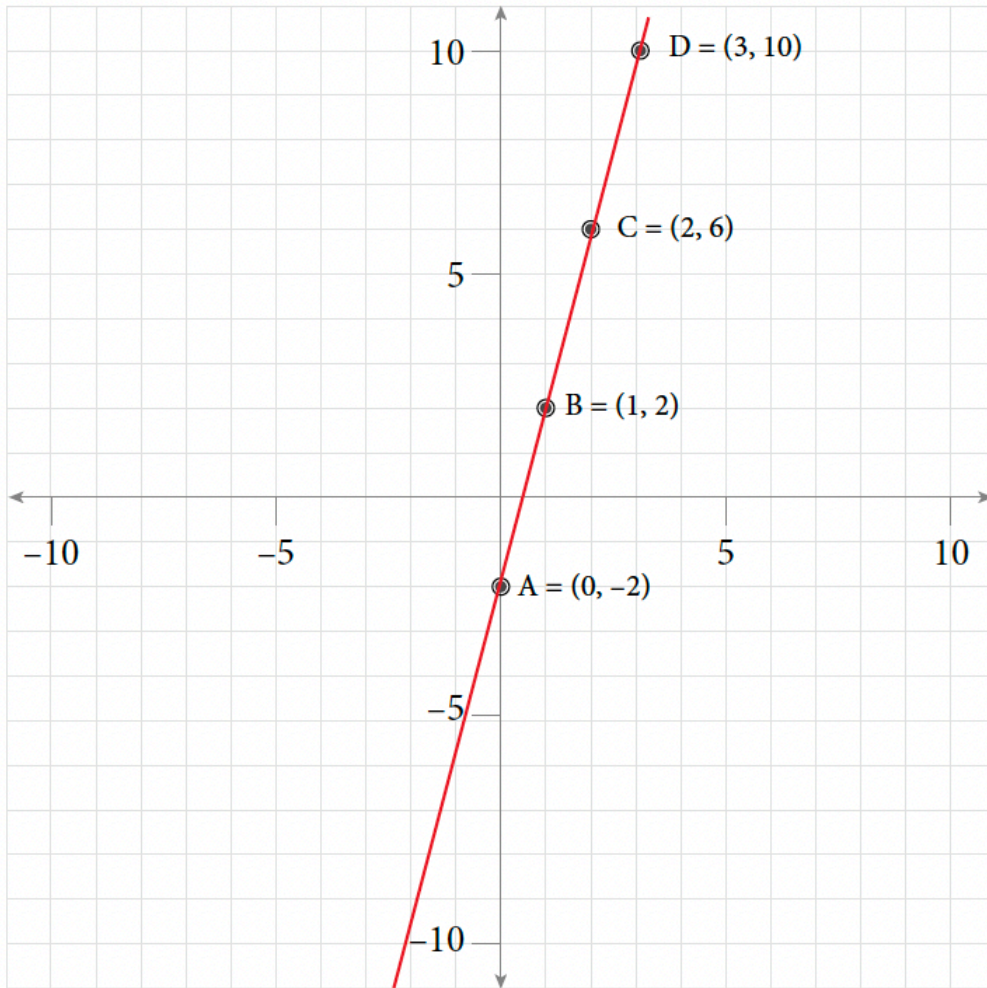
Table of Values for Exercise 7

x	y	(x, y)
0	4	$(0, 4)$
1	3	$(1, 3)$
2	2	$(2, 2)$
3	1	$(3, 1)$

9.
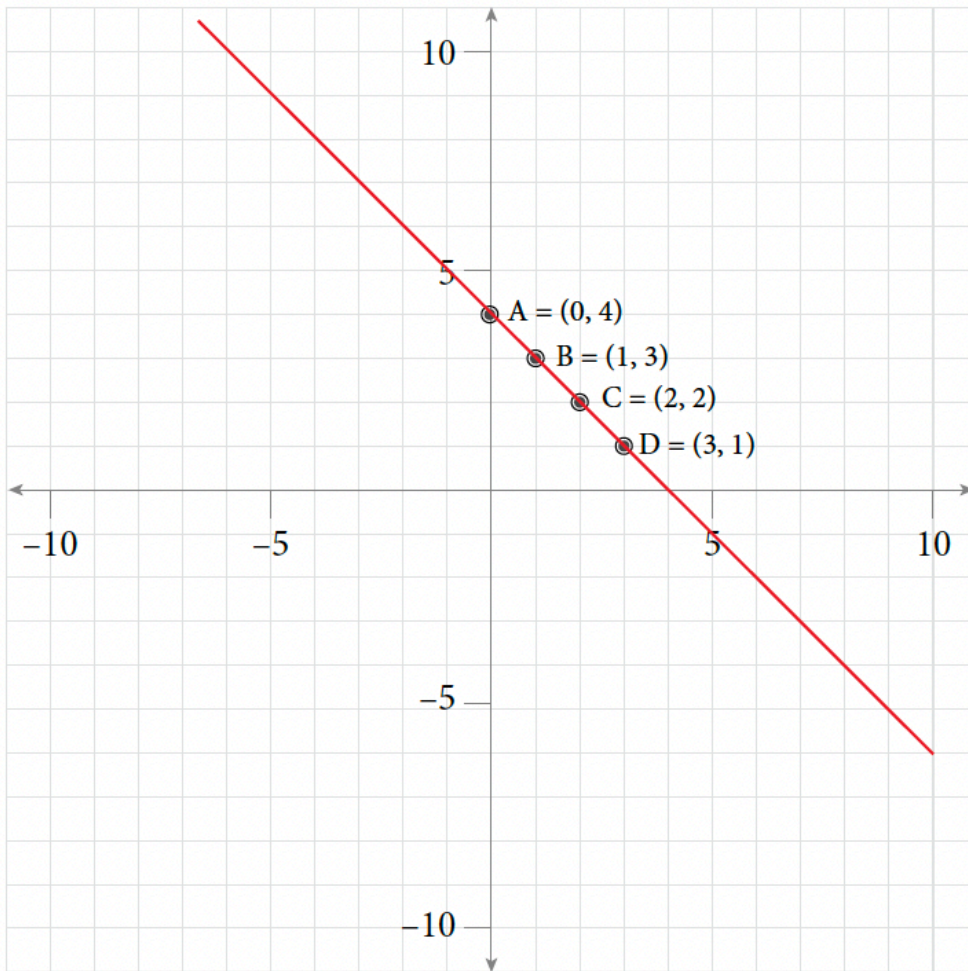
Table of Values for Exercise 9

x	y	(x, y)
0	2	$(0, 2)$
2	3	$(2, 3)$
4	4	$(4, 4)$
6	5	$(6, 5)$
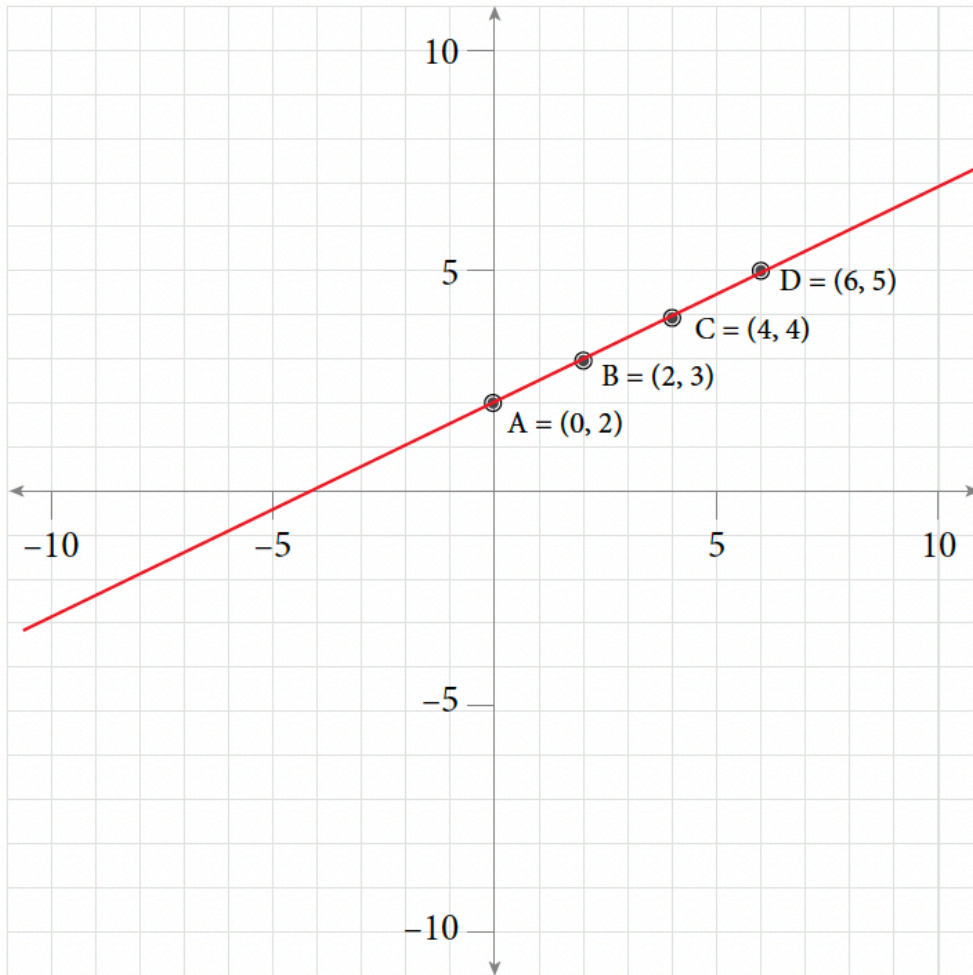
11.

Table of Values for Exercise 11

x	y	(x, y)
0	-3	$(0, -3)$
4	0	$(4, 0)$
8	3	$(8, 3)$
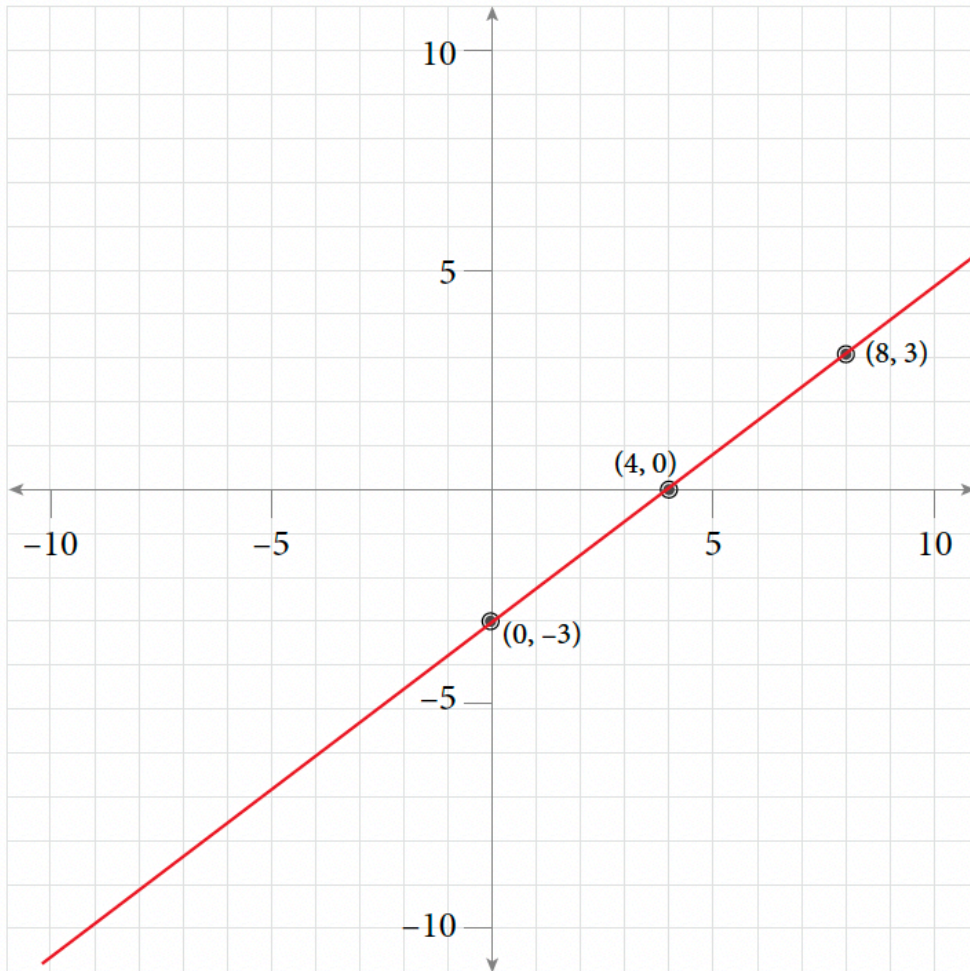
13.

Table of Values for Exercise 13

x	y	(x, y)
0	-3	$(0, -3)$
6	0	$(6, 0)$
2	-2	$(2, -2)$
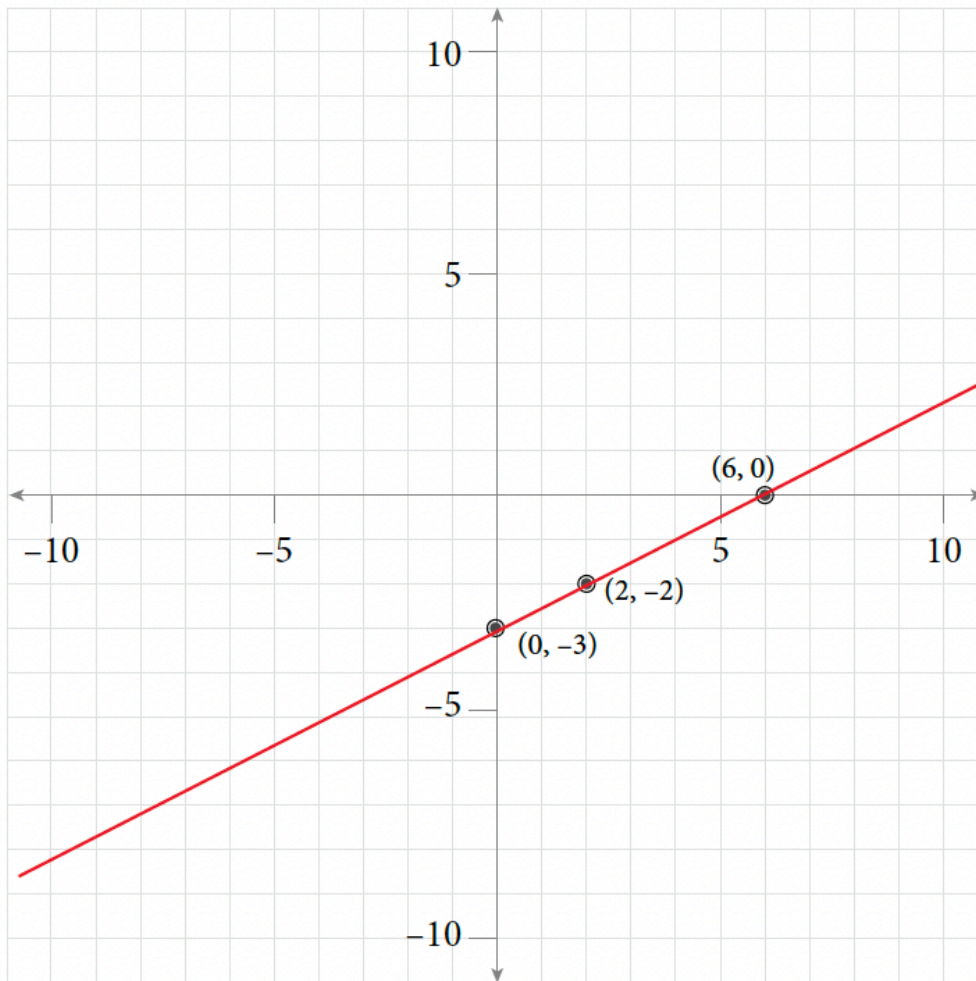
15.

Table of Values for Exercise 15

x	y	(x, y)
0	0	$(0, 0)$
1	4	$(1, 4)$
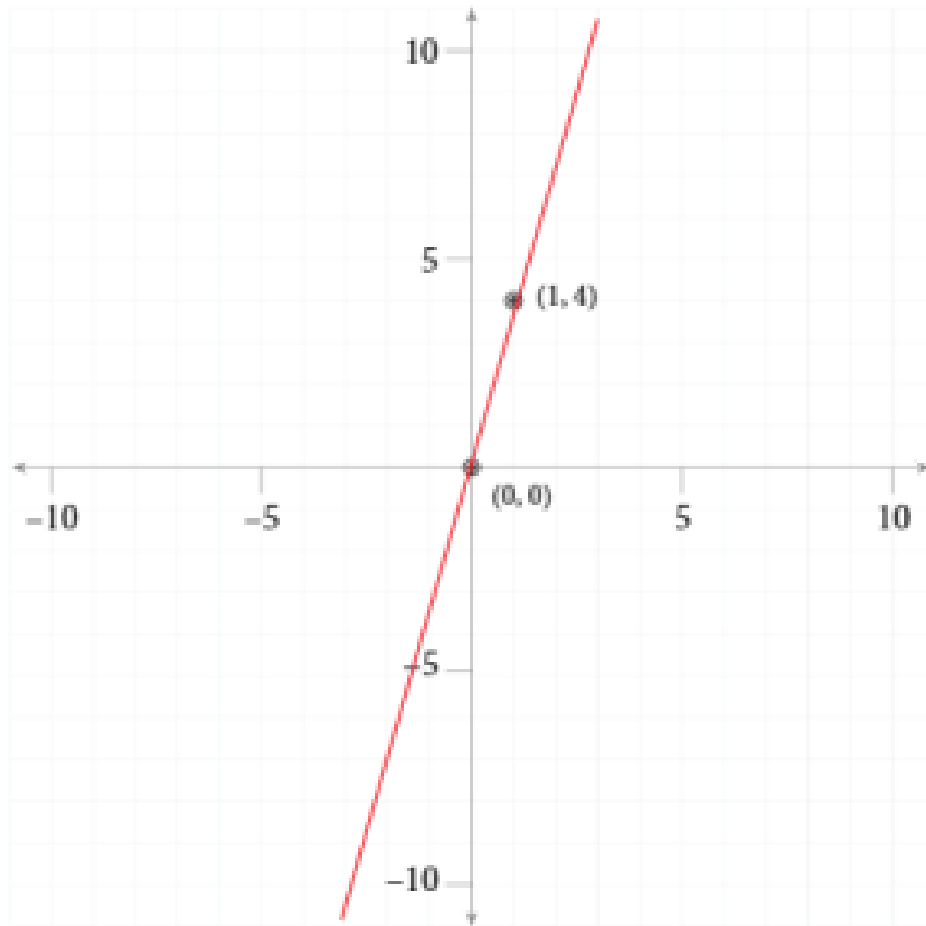
17. $m = 4; b = 6$
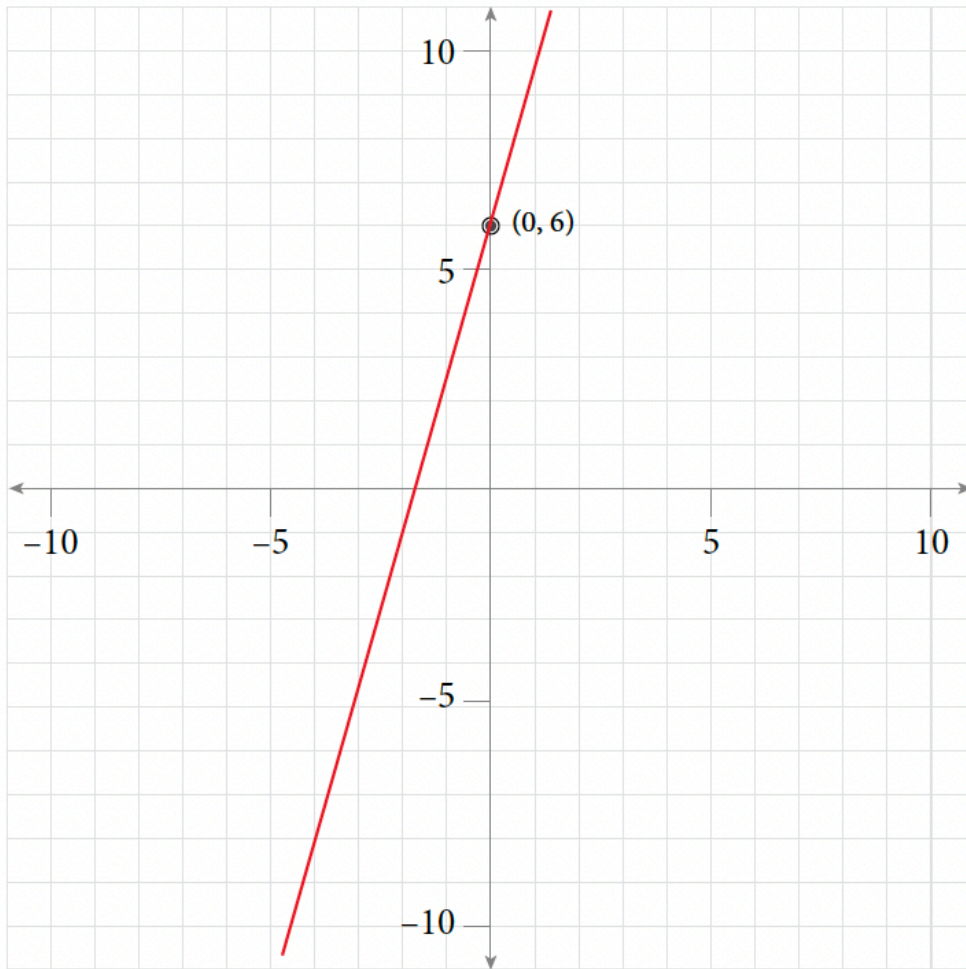
19. $m = -\dfrac{3}{2}; b = 6$
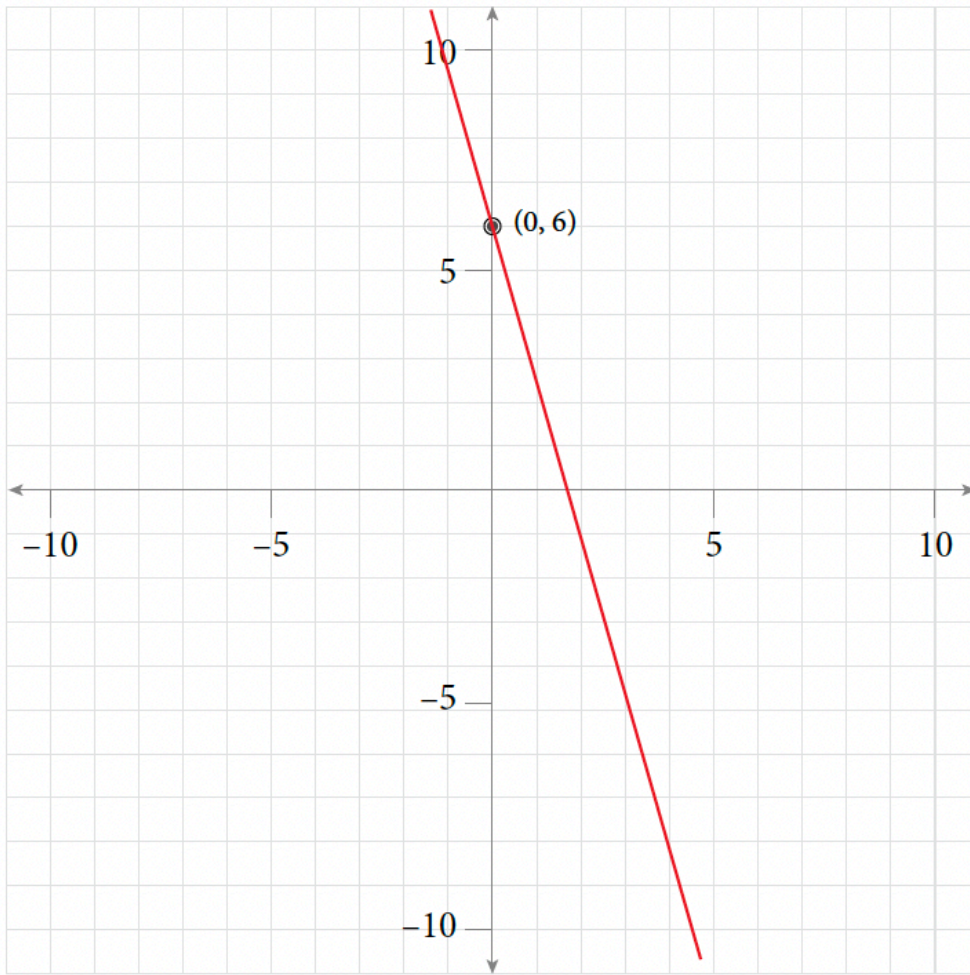
21. $m = -\dfrac{3}{4}; b = -1$
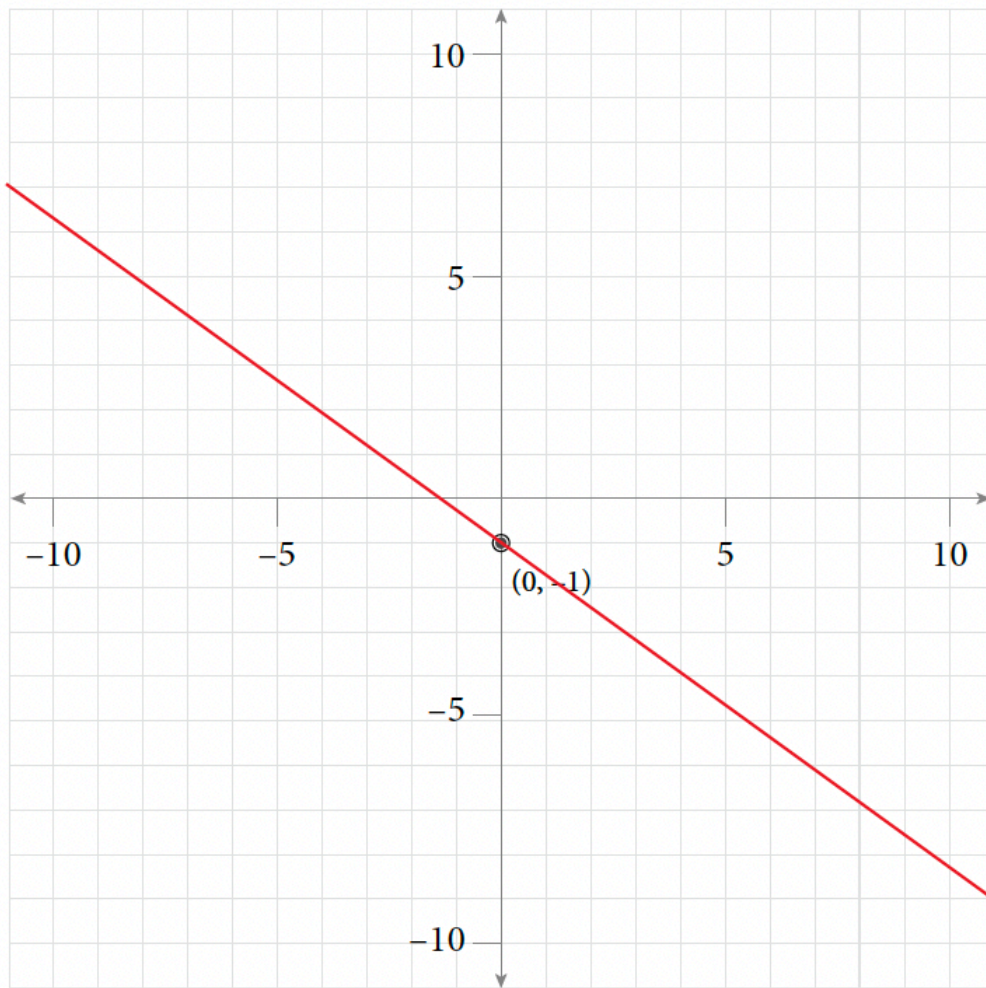
23. $y = \dfrac{3}{4}x - \dfrac{1}{4}$

25. $y = -\dfrac{4}{3}x + \dfrac{8}{3}$

27. $y = 3x - 5$

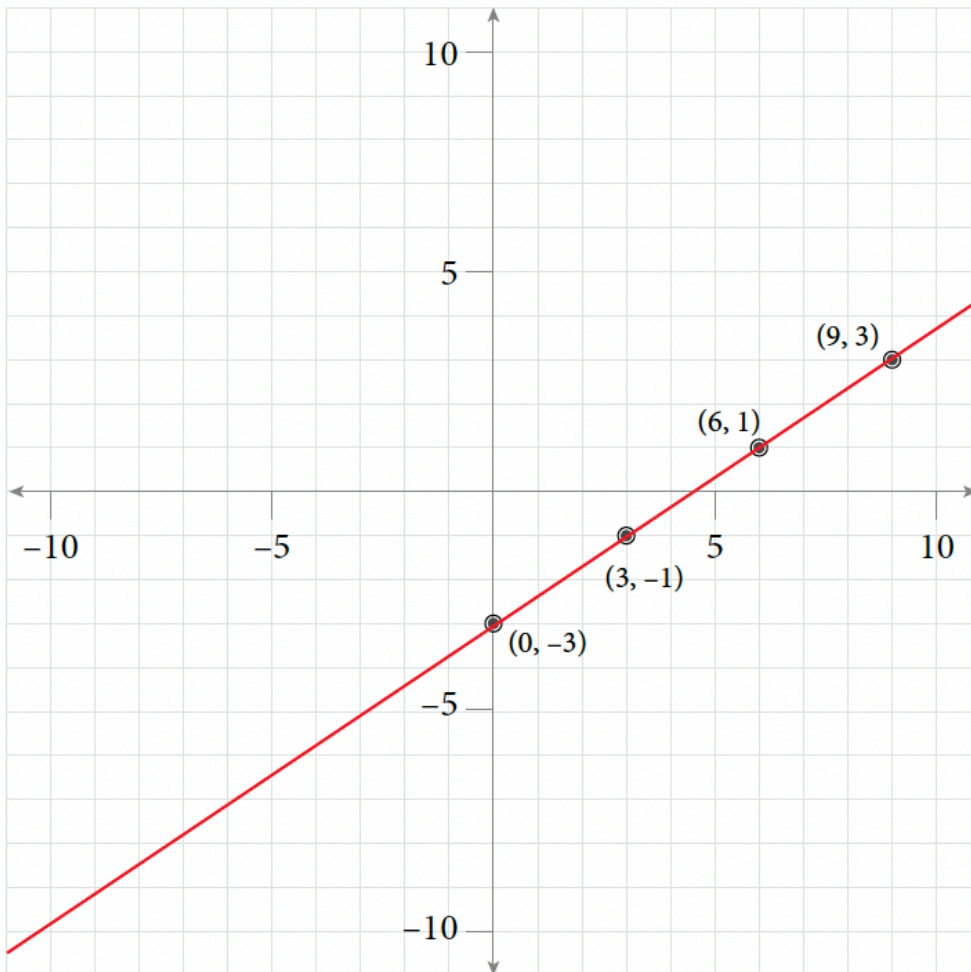29. $y = \dfrac{3}{4}x + \dfrac{9}{2}$

31. $y = -2x + 1$

Self-Test Exercises 8

1. $(-3, -1)$; Area = 40 square units

2.   a. $2x - 3y = 6$

    b. $x = \dfrac{25}{8}$

3.   a. $y = \dfrac{2}{3}x = 2$

    b. $y = -\dfrac{3}{4}x + \dfrac{5}{4}$
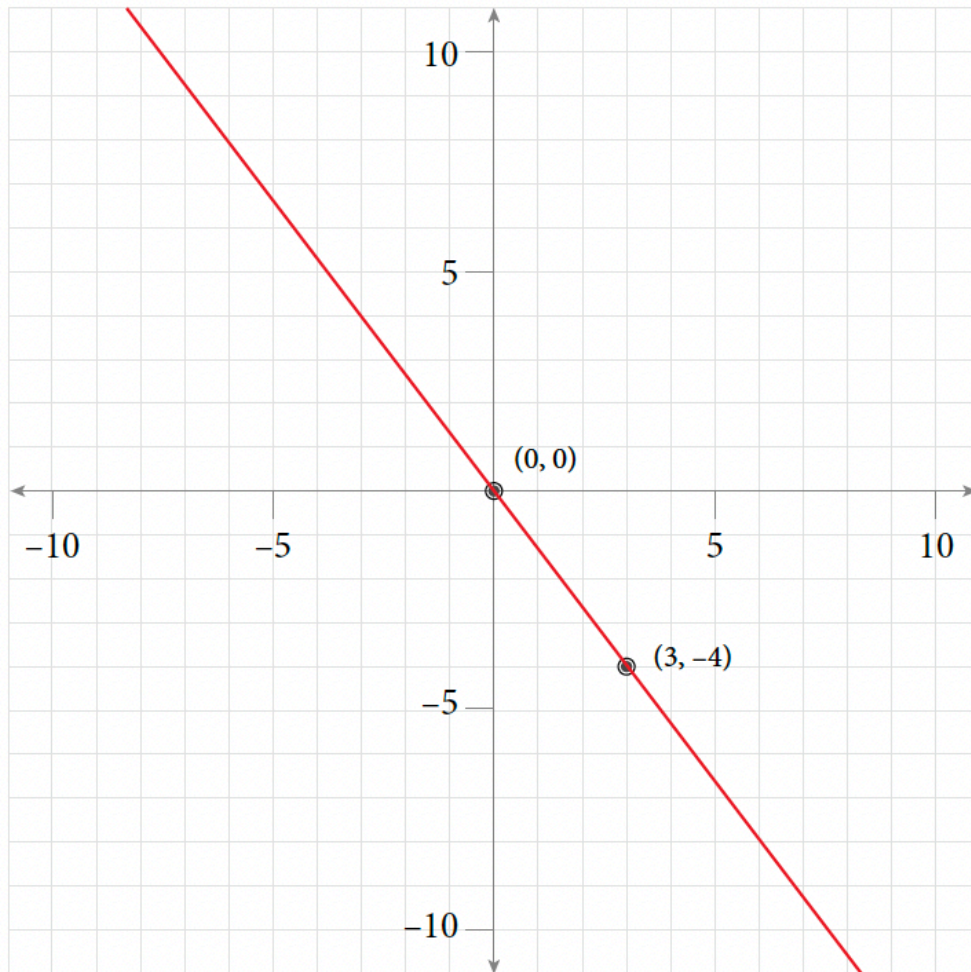
4.

Table of Values for Self-Test Exercise 4

x	y	(x, y)
0	-3	$(0, -3)$
3	-1	$(3, -1)$
6	1	$(6, 1)$
9	3	$(9, 3)$

5.

Table of Values for Self-Test Exercise 5

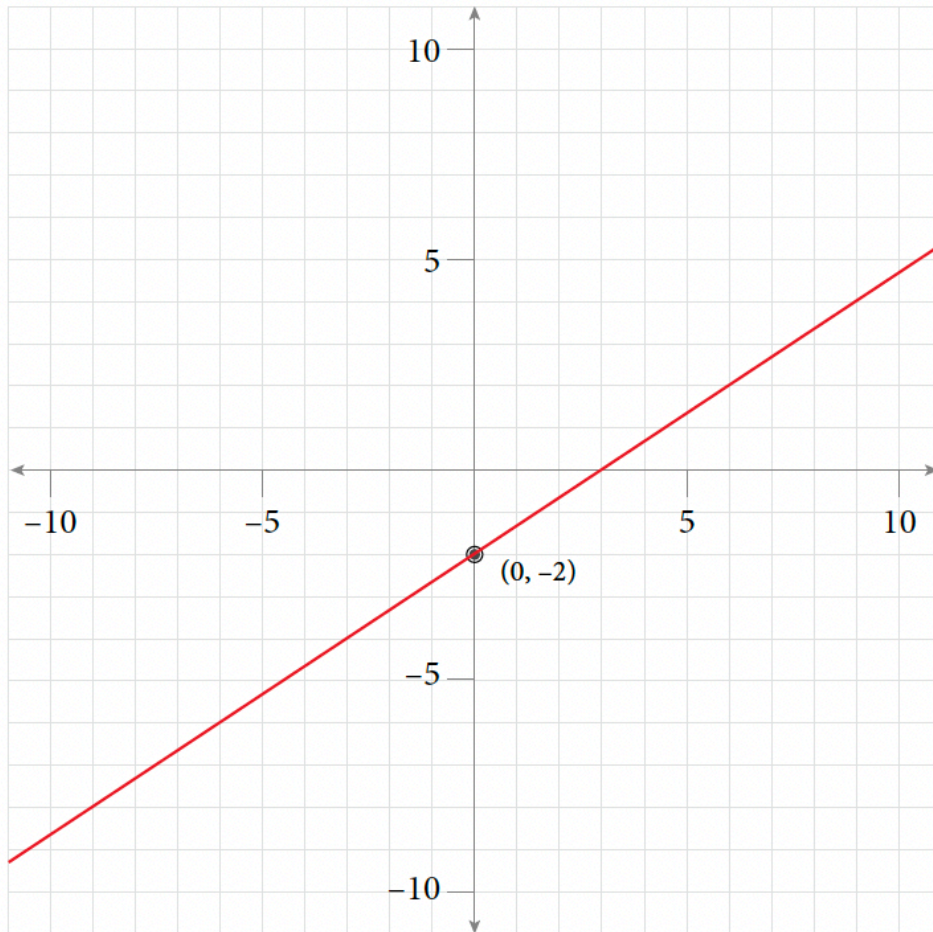| x	y	(x, y)
0	0	$(0, 0)$
3	-4	$(3, -4)$
6	1	$(6, 1)$

6. a)
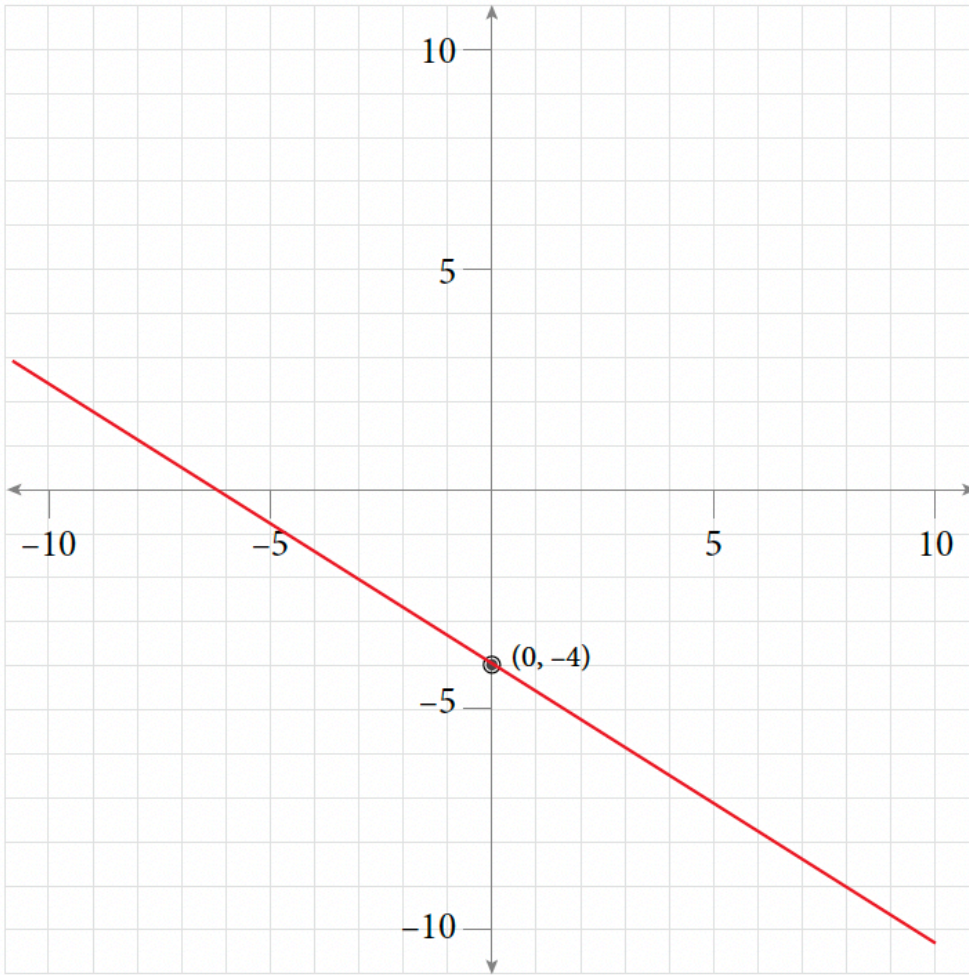
b)

7. $4x + 5y = 9$

8. $3x - 5y = 15$

9. $3x - 2y = -12$

10. $\dfrac{4}{3}x - y = 0$