Medical Mathematics

MEDICAL MATHEMATICS

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Windsor, ON

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Learning Objectives

Learning objectives will be included at the beginning of the chapter, within a green shaded box.

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When completing practice questions you should focus on reading each question carefully and reducing any distractions around you. Check your work after completing questions and try to determine where your mistakes are coming from if you get the wrong answer. Answers to sample exercises and practice questions have steps in solving shown to help you identify where you are making mistakes. You can compare your work to the work shown in the answers. Ask yourself the following questions to help you to understand why you are not getting the right answer:

- Did I understand what the question was asking?
- Did I set up the equation correctly?
- Did I make a mistake when using the calculator or doing mental math?
- Is there a difference between my answer and the provided answer because of the method I used when rounding?

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Welcome to Math Foundations for Health Care, and in particular, to MTH-63

In this course, we will cover the following topics.

- Unit 1: The Real Numbers and Introduction to Algebra
- Unit 2: Measurement and Rounding Rules
- Unit 3: Solving Linear Equations, Graphs of Linear Equations, and Applications of Linear Functions
- Unit 4: Systems of Linear Equations
- Unit 5: Medical Measurment
- Unit 6: Math for Medical Administration

Through our coverage of these topics, we will have the main goal of developing resilience, critical thinking skills, and self-directed learning skills. In addition to these transferable skills, we will aim to develop your attention to detail, determination, time management skills, as well as your mental mathematics skills.

In health care, making a small calculation error can be life-threatening, and therefore, we must develop our intuition with numbers and mathematics throughout the PHS math program. To do this, we will often work without a calculator to improve our mathematical abilities.

Mathematics can be challenging, and students must keep an open mind and remain positive. Remember that it takes a lot of practice to develop these foundational skills, so if you do not succeed right away, stay patient, ask for help, and keep working hard!

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Collaborators

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The following staff and students were involved in the creation of this project:

- Mike LePine
- Katrina Tomanelli
- Irene Stewart

PART I

UNIT 1: THE REAL NUMBERS AND INTRODUCTION TO ALGEBRA

1.1 INTRODUCTION TO WHOLE NUMBERS

Learning Objectives

By the end of this section, you will be able to:

- Use place value with whole numbers
- Identify multiples and apply divisibility tests
- Find prime factorizations and least common multiples

As we begin our study of elementary algebra, we need to refresh some of our skills and vocabulary. This chapter will focus on whole numbers, integers, fractions, decimals, and real numbers. We will also begin our use of algebraic notation and vocabulary.

Use Place Value with Whole Numbers

The most basic numbers used in algebra are the numbers we use to count objects in our world: 1, 2, 3, 4, and so on. These are called the **counting numbers**. Counting numbers are also called *natural numbers*. If we add zero to the counting numbers, we get the set of whole numbers.

Counting Numbers: 1, 2, 3, …

Whole Numbers: 0, 1, 2, 3, …

The notation "…" is called ellipsis and means "and so on," or that the pattern continues endlessly.

Our number system is called a place value system, because the value of a digit depends on its position in a number. Figure 1.1 shows the place values. The place values are separated into groups of three, which are called periods. The periods are *ones, thousands, millions, billions, trillions*, and so on. In a written number, commas separate the periods.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

Figure 1.1

Example 1

In the number 63,407,218, find the place value of each digit:

a. 7
b. 0
c. 1
d. 6
e. 3

Solution

Place the number in the place value chart:

Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

Figure 1.2

a. The 7 is in the thousands place.
b. The 0 is in the ten-thousands place.
c. The 1 is in the tens place.
d. The 6 is in the ten-millions place.
e. The 3 is in the millions place.

Try It

1) For the number 27,493,615, find the place value of each digit:

a. 2
b. 1
c. 4
d. 7
e. 5

Solution

a. ten millions
b. tens
c. hundred thousands
d. millions
e. ones

2) For the number 519,711,641,328, find the place value of each digit:

a. 9
b. 4
c. 2
d. 6
e. 7

Solution

a. billions
b. ten thousands
c. tens
d. hundred thousands
e. hundred millions

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period, followed by the name of the period, without the *s* at the end. Start at the left, where the periods have the largest value. The ones period is not named. The commas separate the periods, so wherever there is a comma in the number, put a comma between the words (see below). The number 74,218,369 is written as seventy-four million, two hundred eighteen thousand, three hundred sixty-nine.

How to

Name a Whole Number in Words

1. Start at the left and name the number in each period, followed by the period name.
2. Put commas in the number to separate the periods.
3. Do not name the ones period

Example 2

Name the number 8,165,432,098,710 using words.

Solution

***Step 1:* Name the number in each period, followed by the period name.**

***Step 2:* Put the commas in to separate the periods.**

So, 8,165,432,098,710 is named as eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

Try It

3) Name the number 9,258,137,904,061 using words.

Solution
Nine trillion, two hundred fifty-eight billion, one hundred thirty-seven million, nine hundred four thousand, sixty-one.

4) Name the number 17,864,325,619,004 using words.

Solution
Seventeen trillion, eight hundred sixty-four billion, three hundred twenty-five million, six hundred nineteen thousand four.

We are now going to reverse the process by writing the digits from the name of the number. To write the number in digits, we first look for the clue words that indicate the periods. It is helpful to draw three blanks for the needed periods and then fill in the blanks with the numbers, separating the periods with commas.

How to

Write a Whole Number Using Digits

1. Identify the words that indicate periods. (Remember, the ones period is never named.)
2. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
3. Name the number in each period and place the digits in the correct place value position.

Example 3

Write *nine billion, two hundred forty-six million, seventy-three thousand, one hundred eighty-nine* as a whole number using digits.

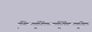
Solution

Step 1: **Identify the words that indicate periods.**

Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period.

Step 2: **Separate the periods by commas.**

Then write the digits in each period.



The number is 9,246,073,189.

Try It

5) Write the number two billion, four hundred sixty-six million, seven hundred fourteen thousand, fifty-one as a whole number using digits.

Solution

2,466,714,051

6) Write the number eleven billion, nine hundred twenty-one million, eight hundred thirty thousand, one hundred six as a whole number using digits.

Solution

11,921,830,106

In 2013, the U.S. Census Bureau estimated the population of the state of New York as 19,651,127. We could say the population of New York was approximately 20 million. In many cases, you don't need the exact value; an approximate number is good enough.

The process of approximating a number is called rounding. Numbers are rounded to a specific place value, depending on how much accuracy is needed. Saying that the population of New York is approximately 20 million means that we rounded to the millions place.

Example 4

Round 23,658 to the nearest hundred.

Solution

Step 1: **Locate the given place value with an arrow. All digits to the left do not change.**

Locate the hundreds place 23,658.

$$\text{Hundredths place. } 23,\underset{\uparrow}{6}58$$

Step 2: **Underline the digit to the right of the given place value.**

Underline the 5, which is to the right of the hundreds place.

$$\text{Hundredths place. } 23,6\underline{5}8$$

Step 3: **Is this digit greater than or equal to 5?**

Yes – add 1 to the digit in the given place value.

Add 1 to the 6 in the hundred place, since 5 is greater than or equal to 5.

$$\text{Add 1. } 23,658$$

Step 4: **Replace all digits to the right of the given place with zeros.**

Replace all digits to the right of the hundreds place with zeros.

Replace with zeros. 23,700

Try It

7) Round to the nearest hundred: 17,852.

Solution

17,900

8) Round to the nearest hundred: 468,751.

Solution

468,800

How to

Round Whole Numbers.

1. Locate the given place value and mark it with an arrow. All digits to the left of the arrow do not change.
2. Underline the digit to the right of the given place value.
3. Is this digit greater than or equal to 5?
    - Yes–add 1 to the digit in the given place value.
    - No–do *not* change the digit in the given place value.

4. Replace all digits to the right of the given place value with zeros.

Example 5

Round 103,978 to the nearest:

a. hundred
b. thousand
c. ten thousand

Solution

a.

***Step 1:* Locate the hundreds place in 103,978.**

$$\text{Hundreds place: } 103,\boxed{9}78$$

***Step 2:* Underline the digit to the right of the hundreds place.**

$$\text{Hundreds place: } 103,\boxed{9}\underline{7}8$$

***Step 3*: Since 7 is greater than or equal to 5, add 1 to the 9. Replace all digits to the right of the hundreds place with zeros.**

$$\text{Hundreds place: } 103,\boxed{\underbrace{9}\ \begin{array}{l}\text{Add 1}\\ 9+1=10\\ \text{replace 9 with 0}\\ \text{and carry the 1}\end{array}}\underbrace{78}_{\text{replace with 0s}} = 104,000$$

So, 104,000 is 103,978 rounded to the nearest hundred.

---

b.

***Step 1:* Locate the thousands place and underline the digit to the right of the thousands place.**

$$\text{Thousands place: } 10\boxed{3},\underline{9}78$$

***Step 2:* Since 9 is greater than or equal to 5, add 1 to the 3. Replace all digits to the right of the hundreds place with zeros.**

$$\text{Thousands place: } 10\boxed{\underbrace{3}\ \begin{array}{l}\text{Add 1}\\ 3+1=4\\ \text{replace 3 with 4}\end{array}}\underbrace{978}_{\text{replace with 0s}} = 104,000$$

So, 104,000 is 103,978 rounded to the nearest thousand.

---

c.

Step 1: **Locate the ten thousands place and underline the digit to the right of the ten thousands place.**

$$\text{Ten thousands place: } 1\boxed{0}3,978$$

Step 2: **Since 3 is less than 5, we leave the 0 as is, and then replace the digits to the right with zeros.**

$$100,000$$

So, 100,000 is 103,978 rounded to the nearest ten thousand.

Try It

9) Round 206,981 to the nearest:

a. hundred
b. thousand
c. ten thousand.

Solution

a. 207,000
b. 207,000
c. 210,000

10) Round 784,951 to the nearest:

a. hundred
b. thousand
c. ten thousand.

Solution

a. 785,000
b. 785,000
c. 780,000

Identify Multiples and Apply Divisibility Tests

The numbers 2, 4, 6, 8, 10, and 12 are called multiples of 2. A multiple of 2 can be written as the product of a counting number and 2.

2	$2 \cdot 1$
4	$2 \cdot 2$
6	$2 \cdot 3$
8	$2 \cdot 4$
10	$2 \cdot 5$
12	$2 \cdot 6$

Similarly, a multiple of 3 would be the product of a counting number and 3.

3	$3 \cdot 1$
6	$3 \cdot 2$
9	$3 \cdot 3$
12	$3 \cdot 4$
15	$3 \cdot 5$
18	$3 \cdot 6$

We could find the multiples of any number by continuing this process.

Table. 1.1 shows the multiples of 2 through 9 for the first 12 counting numbers.

Table 1.1

Counting Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Multiples of 2	2	4	6	8	10	12	14	16	18	20	22	24
Multiples of 3	3	6	9	12	15	18	21	24	27	30	33	36
Multiples of 4	4	8	12	16	20	24	28	32	36	40	44	48
Multiples of 5	5	10	15	20	25	30	35	40	45	50	55	60
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Multiples of 7	7	14	21	28	35	42	49	56	63	70	77	84
Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96
Multiples of 9	9	18	27	36	45	54	63	72	81	90	99	108
Multiples of 10	10	20	30	40	50	60	70	80	90	100	110	120

Multiple of a Number

A number is a **multiple** of n if it is the product of a counting number and n.

Another way to say that 15 is a multiple of 3 is to say that 15 is divisible by 3. That means that when we divide 3 into 15, we get a counting number. In fact, $15 \div 3$ is 5, so 15 is 15×3.

Divisible by a Number

If a number m is a multiple of n, then m is **divisible** by n.

Look at the multiples of 5 in Table 1.1. They all end in 5 or 0. Numbers with the last digit of 5 or 0 are divisible by 5. Looking for other patterns in Table 1.1 that show multiples of the numbers 2 through 9, we can discover the following divisibility tests:

Divisibility Tests

A number is divisible by:

- 2 if the last digit is 0, 2, 4, 6, or 8.
- 3 if the sum of the digits is divisible by 3.
- 4 if the last two digits are divisible by 4.
- 5 if the last digit is 5 or 0.
- 6 if it is divisible by both 2 and 3.
- 8 if it is divisible by 2 and 4.
- 9 if the sum of the digits is divisible by 9.
- 10 if it ends with 0.

Example 6

Is 5,625 divisible by 2? By 3? By 5? By 6? By 10?

Solution

Step 1: **Is 5,625 divisible by 2?**

No.

Step 2: **Does it end in 0,2,4,6, or 8?**

5,625 is not divisible by 2.

Step 3: **Is 5,625 divisible by 3?**

Yes. 5,625 is divisible by 3.

Step 4: **What is the sum of the digits?**

$$5+6+2+5=18$$

Step 5: **Is the sum divisible by 3?**

Yes, 18 is divisible by 3.

Step 6: **Is 5,625 divisible by 5 or 10?**

5,625 is divisible by 5 but not by 10.

Step 7: **What is the last digit?**

It is 5.

Step 8: **Is 5,625 divisible by 6?**

No, it is not.

Step 9: **Is it divisible by both 2 and 3?**

No, 5,625 is not divisible by 2, but it is divisible by 3.

Try It

11) Determine whether 4,962 is divisible by 2, by 3, by 5, by 6, and by 10.

Solution

By 2, 3, and 6.

12) Determine whether 3,765 is divisible by 2, by 3, by 5, by 6, and by 10.

Solution
By 3 and 5.

Find Prime Factorizations and Least Common Multiples

In mathematics, there are often several ways to talk about the same ideas. So far, we've seen that if m is a multiple of n, we can say that m is divisible by n. For example, since 72 is a multiple of 8, we say 72 is divisible by 8. Since 72 is a multiple of 9, we say 72 is divisible by 9. We can express this still another way.

Since $8 \times 9 = 72$, we say that 8 and 9 are **factors** of 72. When we write $72 = 8 \times 9$, we say we have factored 72.

$$\underbrace{8 \times 9}_{factors} = \underbrace{72}_{product}$$

Other ways to factor 72 are 1·72, 2·36, 3·24, 4·18, and 6·12. Seventy-two has many factors: 1, 2, 3, 4, 6, 8, 9, 12, 18, 36, and 72.

Factors

If $a \cdot b = m$ then a and b are factors of m.

Some numbers, like 72, have many factors. Other numbers have only two factors.

A **prime number** is a counting number greater than 1, whose only factors are 1 and itself.

A **composite number** is a counting number that is not prime. A composite number has factors other than 1 and itself.

The **counting numbers** from 2 to 19 are listed in the below table, with their factors. Make sure to agree with the "prime" or "composite" label for each!

The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19. Notice that the only even prime number is 2.

A composite number can be written as a unique product of primes. This is called the prime factorization of the number. Finding the **prime factorization** of a composite number will be useful later in this course.

Prime Factorization

The prime factorization of a number is the product of prime numbers that equals the number. These prime numbers are called the prime factors.

To find the prime factorization of a composite number, find any two factors of the number and use them to create two branches. If a factor is prime, that branch is complete. Circle that prime!

If the factor is not prime, find two factors of the number and continue the process. Once all the branches have circled primes at the end, the factorization is complete. The composite number can now be written as a product of prime numbers.

Example 7

Factor 48.

Solution

4 and 8 are not prime. Break them each into two factors.

2 and 3 are prime, so circle them.

Step 1: **Find two factors whose product is the given number. Use these numbers to create two branches.**

$$48 = 2 \times 24 \qquad \overbrace{2 \qquad 24}^{48}$$

Step 2: **If a factor is prime, that branch is complete. Circle the prime.**

2 is prime. Circle the prime.

$$\overbrace{\boxed{2} \qquad 24}^{48}$$

***Step 3*: If a factor is not prime, write it as the product of two factors and continue the process.**

24 is not prime break it into 2 more factors.4 and 6 are not prime. Break them each into two factors.

$$\overbrace{\boxed{2} \qquad \overbrace{4 \qquad 6}^{24}}^{48}$$

2 and 3 are prime, so circle them.

$$\overbrace{\boxed{2} \quad \overbrace{\overbrace{\boxed{2} \; \boxed{2}}^{4} \qquad \overbrace{\boxed{2} \; \boxed{3}}^{6}}^{24}}^{48}$$

***Step 4*: Write the composite number as the product of all the circled primes.**

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

We say 2·2·2·2·3 is the prime factorization of 48. We generally write the primes in ascending order. Be sure to multiply the factors to verify your answer!

If we first factored 48 in a different way, for example as 6·8, the result would still be the same. Finish the prime factorization and verify this for yourself.

Try It

13) Find the prime factorization of 80.

Solution

2·2·2·2·5

14) Find the prime factorization of 60.

Solution

2·2·3·5

How to

Find the Prime Factorization of a Composite Number.

1. Find two factors whose product is the given number, and use these numbers to create two branches.
2. If a factor is prime, that branch is complete. Circle the prime, like a bud on the tree.
3. If a factor is not prime, write it as the product of two factors and continue the process.
4. Write the composite number as the product of all the circled primes.

1.

Example 8

Find the prime factorization of 252.

Solution

Step 1: **Find two factors whose product is 252. 12 and 21 are not prime.**

Break 12 and 21 into two more factors. Continue until all primes are factored.

$$252\begin{cases}12\begin{cases}6\begin{cases}3\\2\end{cases}\\2\end{cases}\\21\begin{cases}3\\7\end{cases}\end{cases}$$

Step 2: **Write 252 as the product of all the circled primes.**

$$252=2\cdot2\cdot3\cdot3\cdot7$$

Try It

15) Find the prime factorization of 126.

Solution

2·3·3·7

16) Find the prime factorization of 294.

Solution

2·3·7·7

One of the reasons we look at multiples and primes is to use these techniques to find the least common multiple of two numbers. This will be useful when we add and subtract fractions with different denominators. Two methods are used most often to find the least common multiple and we will look at both of them.

The first method is the Listing Multiples Method. To find the least common multiple of 12 and 18, we list the first few multiples of 12 and 18:

12: 12, 24, *36*, 48, 60, 72, 84, 96, 108...

18: 18, *36*, 54, 72, 90, 108...

Common Multiples: *36*, 72, 108...

Least Common Multiple: 36

Notice that some numbers appear in both lists. They are the *common multiples* of 12 and 18.

We see that the first few common multiples of 12 and 18 are 36, 72, and 108. Since 36 is the smallest of the common multiples, we call it the **least common multiple**. We often use the abbreviation LCM.

Least Common Multiple

The least common multiple (LCM) of two numbers is the smallest number which is a multiple of both numbers.

The procedure box lists the steps to take to find the LCM using the prime factors method we used above for 12 and 18.

How to

Find the Least Common Multiple by Listing Multiples.

1. List several multiples of each number.
2. Look for the smallest number that appears on both lists.
3. This number is the LCM.

Example 9

Find the least common multiple of 15 and 20 by listing multiples.

Solution

Step 1: **Make lists of the first few multiples of 15 and of 20, and use them to find the least common multiple.**

15: 15, 30, 45, 60, 75, 90, 105, 120
20: 20, 40, 60, 80, 100, 120, 140, 160

Step 2: **Look for the smallest number that appears in both lists.**

The first number to appear on both lists is 60, so 60 is the least common multiple of 15 and 20.

Notice that 120 is in both lists, too. It is a common multiple, but it is not the *least* common multiple.

Try It

17) Find the least common multiple by listing multiples: 9 and 12.

Solution

36

18) Find the least common multiple by listing multiples: 18 and 24.

Solution

72

Our second method to find the least common multiple of two numbers is to use The Prime Factors Method. Let's find the LCM of 12 and 18 again, this time using their prime factors.

Example 10

Find the Least Common Multiple (LCM) of 12 and 18 using the prime factors method.

Solution

Step 1: Write each number as a product of primes.

$$\overbrace{\boxed{3} \quad \overbrace{\boxed{2} \quad \boxed{3}}^{6}}^{18}$$

$$\overbrace{\boxed{3} \quad \overbrace{\boxed{2} \quad \boxed{2}}^{4}}^{12}$$

***Step 2:* List the primes of each number. Match primes vertically when possible.**

List the primes of 12.

$$12 = 2 \times 2 \times 3$$

List the primes of 18. Line up with the primes of 12 when possible. If not create a new column.

$$18 = 2 \times 3 \times 3$$

Step 3: **Bring down the number from each column.**

$$\begin{aligned} 12 &= 2 \times 2 \times 3 \\ 18 &= 2 \times \quad\quad 3 \times 3 \\ LCM &= 2 \times 2 \times 3 \times 3 \end{aligned}$$

Step 4: **Multiply the factors.**

$$LCM = 36$$

Notice that the prime factors of 12 (2·2·3) and the prime factors of 18 (2·3·3) are included in the LCM (2·2·3·3). So 36 is the least common multiple of 12 and 18.

By matching up the common primes, each common prime factor is used only once. This way you are sure that 36 is the *least* common multiple.

How to

Find the Least Common Multiple Using the Prime Factors Method.

1. Write each number as a product of primes.
2. List the primes of each number. Match primes vertically when possible.
3. Bring down the columns.
4. Multiply the factors.

Try It

19) Find the LCM using the prime factors method: 9 and 12.

Solution

36

20) Find the LCM using the prime factors method: 18 and 24.

Solution

72

Example 11

Find the Least Common Multiple (LCM) of 24 and 36 using the prime factors method.

Solution

Step 1: **Find the primes of 24 and 36.**

Match primes vertically when possible.

Step 2: **Bring down all columns.**

$$\begin{aligned} 24 &= 2 \times 2 \times 2 \times 3 \\ 36 &= 2 \times 2 \qquad \times 3 \times 3 \end{aligned}$$

Step 3: **Multiply the factors.**

$$\begin{aligned} LCM &= 2 \times 2 \times 2 \times 3 \times 3 \\ LCM &= 72 \end{aligned}$$

The LCM of 24 and 36 is 72.

Try It

21) Find the LCM using the prime factors method: 21 and 28.

Solution

84

22) Find the LCM using the prime factors method: 24 and 32.

Solution

96

Key Concepts

- **Place Value**
- **Name a Whole Number in Words**
  1. Start at the left and name the number in each period, followed by the period name.
  2. Put commas in the number to separate the periods.
  3. Do not name the ones period.
- **Write a Whole Number Using Digits**
  1. Identify the words that indicate periods. (Remember the ones period is never named.)
  2. Draw 3 blanks to indicate the number of places needed in each period. Separate the periods by commas.
  3. Name the number in each period and place the digits in the correct place value position.
- **Round Whole Numbers**
  1. Locate the given place value and mark it with an arrow. All digits to the left of the arrow do not change.
  2. Underline the digit to the right of the given place value.
  3. Is this digit greater than or equal to 5?
     - Yes—add 1 to the digit in the given place value.
     - No—do *not* change the digit in the given place value.

4. Replace all digits to the right of the given place value with zeros.

- **Divisibility Tests:** A number is divisible by:
    - 2 if the last digit is 0, 2, 4, 6, or 8.
    - 3 if the sum of the digits is divisible by 3.
    - 5 if the last digit is 5 or 0.
    - 6 if it is divisible by both 2 and 3.
    - 10 if it ends with 0.
- **Find the Prime Factorization of a Composite Number**
    1. Find two factors whose product is the given number, and use these numbers to create two branches.
    2. If a factor is prime, that branch is complete. Circle the prime, like a bud on the tree.
    3. If a factor is not prime, write it as the product of two factors and continue the process.
    4. Write the composite number as the product of all the circled primes.
- **Find the Least Common Multiple by Listing Multiples**
    1. List several multiples of each number.
    2. Look for the smallest number that appears on both lists.
    3. This number is the LCM.
- **Find the Least Common Multiple Using the Prime Factors Method**
    1. Write each number as a product of primes.
    2. List the primes of each number. Match primes vertically when possible.
    3. Bring down the columns.
    4. Multiply the factors.

Glossary

composite number

A composite number is a counting number that is not prime. A composite number has factors other than 1 and itself.

counting numbers

The counting numbers are the numbers 1, 2, 3, …

divisible by a number

If a number m is a multiple of n, then m is divisible by n. (If 6 is a multiple of 3, then 6 is divisible by 3.)

factors

If $a \times b = m$, then (a and b) are factors of m. Since $3 \times 4 = 12$, then 3 and 4 are factors of 12.

least common multiple

The least common multiple of two numbers is the smallest number that is a multiple of both numbers.

multiple of a number

A number is a multiple of n if it is the product of a counting number and n.

number line

A number line is used to visualize numbers. The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left.

origin

The origin is the point labelled 0 on a number line.

prime factorization

The prime factorization of a number is the product of prime numbers that equals the number.

prime number

A prime number is a counting number greater than 1, whose only factors are 1 and itself.

whole numbers

The whole numbers are the numbers 0, 1, 2, 3, ….

Exercises: Place Value With Whole Numbers

Instructions: For questions 1-8, find the place value of each digit in the given numbers.

1) $51{,}493$

a) 1

b) 4

c) 9

d) 5

e) 3

2) $87{,}210$

a) 2

b) 8

c) 0

d) 7

e) 1

3) $164{,}285$

a) 5

b) 6

c) 1

- d) 8
- e) 2

4) $395{,}076$

- a) 5
- b) 3
- c) 7
- d) 0
- e) 9

5) $93{,}285{,}170$

- a) 9

b) 8

c) 7

d) 5

e) 3

6) $36{,}084{,}215$

a) 8

b) 6

c) 5

d) 4

e) 3

7) 7,284,915,860,132

a) 7

b) 4

c) 5

d) 3

e) 0

8) 2,850,361,159,433

a) 9

b) 8

c) 6

d) 4

e) 2

Odd Answers

1a) thousands
1b) hundreds
1c) tens
1d) ten thousands
1e) ones

3a) ones
3b) ten thousands
3c) hundred thousands
3d) tens
3e) hundreds

5a) ten millions
5b) ten thousands
5c) tens
5d) thousands
5e) millions

7a) trillions
7b) billions
7c) millions
7d) tens
7e) thousands

Exercises: Name Numbers Using Words

Instructions: For questions 9-16, name each number using words.

9) $1{,}078$

10) $5{,}902$

11) $364{,}510$

12) $146{,}023$

13) $5{,}846{,}103$

14) $1{,}458{,}398$

15) $37{,}889{,}005$

16) $62{,}008{,}465$

Odd Answers

9) one thousand, seventy-eight

11) three hundred sixty-four thousand, five hundred ten

13) five million, eight hundred forty-six thousand, one hundred three

15) thirty-seven million, eight hundred eighty-nine thousand, five

Exercises: Whole Numbers Using Digits

Instructions: For questions 17-24, write each number as a whole number using digits.

17) four hundred twelve

18) two hundred fifty-three

19) thirty-five thousand, nine hundred seventy-five

20) sixty-one thousand, four hundred fifteen

21) eleven million, forty-four thousand, one hundred sixty-seven

22) eighteen million, one hundred two thousand, seven hundred eighty-three

23) three billion, two hundred twenty-six million, five hundred twelve thousand, seventeen

24) eleven billion, four hundred seventy-one million, thirty-six thousand, one hundred six

---

Odd Answers

17) 412

19) $35{,}975$

21) $11{,}044{,}167$

23) $3{,}226{,}512{,}017$

Exercises: Round to Indicated Place Value

Instructions: For questions 25-32, round to the indicated place value.

25) Round to the nearest ten.

- **a)** 386
- **b)** $2,931$

26) Round to the nearest ten.

- **a)** 792
- **b)** $5,647$

27) Round to the nearest hundred.

- **a)** $13,748$
- **b)** $391,794$

28) Round to the nearest hundred.

- **a)** $28,166$
- **b)** $481,628$

29) Round to the nearest ten.

- **a)** $1,492$
- **b)** $1,497$

30) Round to the nearest ten.

- **a)** $2,791$
- **b)** $2,795$

31) Round to the nearest hundred.

- **a)** $63,994$
- **b)** $63,940$

32) Round to the nearest hundred.

a) $49{,}584$
b) $49{,}548$

Odd Answers

25a) 390
25b) $2{,}930$

27a) $13{,}700$
27b) $391{,}800$

29a) $1{,}490$
29b) $1{,}500$

31a) $64{,}000$
31b) $63{,}900$

Exercises: Round Numbers to Nearest Hundred, Thousand, and Ten Thousand

Instructions: For questions 33-36, round each number to the nearest:

a) hundred
b) thousand
c) ten thousand

33) $392{,}546$

34) $619{,}348$

35) $2{,}586{,}991$

36) $4,287,965$

Odd Answers

33a) $392,500$

33b) $393,000$

33c) $390,000$

35a) $2,587,000$

35b) $2,587,000$

35c) $2,590,000$

Exercises: Identify Multiples and Factors

Instructions: For questions 37-48, use the divisibility tests to determine whether each number is divisible by 2**,** 3**,** 5**,** 6**, and** 10**.**

37) 84

38) $9,696$

39) 75

40) 78

41) 900

42) 800

43) 986

44) 942

45) 350

46) 550

47) $22{,}335$

48) $39{,}075$

Odd Answers

37) divisible by 2, 3, and 6

39) divisible by 3 and 5

41) divisible by 2, 3, 5, 6, and 10

43) divisible by 2

45) divisible by 2, 5, and 10

47) divisible by 3 **and** 5

Exercises: Find Prime Factorizations

Instructions: For questions 49-59, find the prime factorization.

49) 86

50) 78

51) 132

52) 243

53) 693

54) 455

55) 432

56) 400

57) $2{,}160$

58) 627

59) $2{,}560$

Odd Answers

49) $2, 43$

51) $2, 2, 3, 11$

53) $3, 3, 7, 11$

55) $2, 2, 2, 2, 3, 3, 3$

57) $2, 2, 2, 2, 3, 3, 3, 5$

59) $2, 2, 2, 2, 2, 2, 2, 2, 2, 5$

Exercises: Find Least Common Multiples Using Multiples Method

Instructions: For questions 60-64, find the least common multiple of each pair of numbers using the multiples method.

60) $44, 55$ $8, 12$

61) $4, 3$

62) $12, 16$

63) $30, 40$

64) $20, 30$

Odd Answers

61) 12

63) 120

Exercises: Find Least Common Multiples using Prime Factors Method

Instructions: For questions 65-70, find the least common multiple of each pair of numbers using the prime factors method.

65) $8, 12$

66) $12, 16$

67) $28, 40$

68) $84, 90$

69) $55, 88$

70) $60, 72$

Odd Answers

65) 24

67) 280

69) 440

Exercises: Everyday Math

Instructions: For questions 71-78, answer the everyday math word problems.

71) Writing a Check Jorge bought a car for $\$24,493$. He paid for the car with a check. Write the purchase price in words.

72) Writing a Check. Marissa's kitchen remodeling cost $\$18,549$. She wrote a check to the contractor. Write the amount paid in words.

73) Buying a Car: Jorge bought a car for $\$24,493$. Round the price to the nearest

- **a) ten**
- **b) hundred**
- **c) thousand**
- **d) ten-thousand**

74) Remodeling a Kitchen.Marissa's kitchen remodeling cost $\$18,549$. Round the cost to the nearest

- **a) ten**
- **b) hundred**
- **c) thousand**
- **d) ten-thousand**

75) Population.The population of China was $1,339,724,852$ on November 1, 2010. Round the population to the nearest

- **a) billion**
- **b) hundred-million**
- **c) million**

76) Astronomy.The average distance between Earth and the sun is $149,597,888$ kilometers. Round the distance to the nearest

- **a) hundred-million**
- **b) ten-million**
- **c) million**

77) Grocery Shopping.Hot dogs are sold in packages of 10, but hot dog buns

come in packs of eight. What is the smallest number that makes the hot dogs and buns come out even?

78) Grocery Shopping.Paper plates are sold in packages of 12 and party cups come in packs of eight. What is the smallest number that makes the plates and cups come out even?

Odd Answers

71) twenty-four thousand, four hundred ninety-three dollars

73a) $\$24,490$

73b) $\$24,500$

73c) $\$24,000$

73d) $\$20,000$

75a) $1,000,000,000$

75b) $1,300,000,000$

75c) $1,340,000,000$

77) 40

Exercises: Writing Exercises

Instructions: For questions 79-82, answer the given writing exercises.

79) Give an everyday example where it helps to round numbers.

80) If a number is divisible by 2 and by 3 why is it also divisible by 6?

81) What is the difference between prime numbers and composite numbers?

82) Explain in your own words how to find the prime factorization of a composite number, using any method you prefer.

Odd Answers

79) Answers may vary.

81) Answers may vary.

1.2 INTRODUCTION TO THE LANGUAGE OF ALGEBRA

Learning Objectives

By the end of this section, you will be able to:

- Use variables and algebraic symbols
- Simplify expressions using the order of operations
- Evaluate an expression
- Identify and combine like terms
- Translate an English phrase to an algebraic expression

Use Variables and Algebraic Symbols

Suppose this year Greg is 20 years old and Alex is 23. You know that Alex is 3 years older than Greg. When Greg was 12, Alex was 15. When Greg is 35, Alex will be 38. No matter what Greg's age is, Alex's age will always be 3 years more, right? In the language of algebra, we say that Greg's age and Alex's age are **variables** and the 3 is a **constant**. The ages change ("vary") but the 3 years between them always stays the same ("constant"). Since Greg's age and Alex's age will always differ by 3 years, 3 is the *constant*.

In algebra, we use letters of the alphabet to represent variables. So if we call Greg's age g, then we could use $g+3$ to represent Alex's age.

Greg's age	Alex's age
12	15
20	23
35	38
g	$g+3$

Variable

A variable is a letter that represents a number whose value may change.

Constant

A constant is a number whose value always stays the same.

To write algebraically, we need some operation symbols as well as numbers and variables. There are several types of symbols we will be using.

There are four basic arithmetic operations: addition, subtraction, multiplication, and division. We'll list the symbols used to indicate these operations below table. You'll probably recognize some of them.

Operation	Notation	Say:	The result is…
Addition	$a+b$	a plus b	the sum of a and b
Subtraction	$a-b$	a minus b	the difference of a and b
Multiplication	$a\times b$	a times b	the product of a and b
Division	$a\div b$	a divided by b	the quotient of a and b, a is called the dividend, and b is called the divisor

We perform these operations on two numbers. When translating from symbolic form to English, or from English to symbolic form, pay attention to the words "of" and "and."

- The *difference of* 9 and 2 means subtract 9 and 2, in other words, 9 minus 2, which we write symbolically as $9 - 2$.
- The *product of* 4 and 8 means multiply 4 and 8, in other words 4 times 8, which we write symbolically as 4×8.

In algebra, the cross symbol, $\times$, is not used to show multiplication because that symbol may cause confusion. Does $3xy$ mean $3 \times y$ ('three times y') or ($3 \times x \times y$) ('three times x times y') To make it clear, use · or parentheses for multiplication.

When two quantities have the same value, we say they are equal and connect them with an equal sign.

Equality Symbol

$a = b$ is read "a is equal to b"

The symbol "=" is called the equal sign.

On the number line, the numbers get larger as they go from left to right. The number line can be used to explain the symbols "<" and ">".

Inequality Symbols

$a < b$ is read "a is less than b".

This means a is to the left of b on the number line.

$a > b$ is read a is greater than b.

This means a is to the right of b on the number line.

The expressions $a < b$ or $a > b$ can be read from left to right or right to left, though in English we usually read from left to right (Figure 1.2). In general, $a < b$ is equivalent to $a > b$. For example $7 < 11$ is equivalent to $11 > 7$. And $a > b$ is equivalent to $a < b$. For example $17 > 4$ is equivalent to $4 < 17$.

Figure 1.2

Inequality Symbols	Words
$a \neq b$	a is *not equal to* b
$a < b$	a is *less than* b
$a \leq b$	a is *less than or equal to* b
$a > b$	a is *greater than* b
$a \geq b$	a is *greater than or equal to* b

Example 1

Translate from algebra into English:

a. $17 < 26$
b. $8 \neq 17$
c. $12 > (27 \div 3)$
d. $y + 7 \leq 19$

Solution

a. $17 < 26$

17 is less than 26

b. $8 \neq 17$

8 is not equal to 17

c. $12 > (27 \div 3)$

12 is greater than 27 divided by 3

d. $y + 7 \leq 19$

y plus 7 is less than or equal to 19

Try It

1) Translate from algebra into English:

a. $14 \leq 27$
b. $19 - 2 \neq 8$
c. $12 > 4 \div 2$
d. $x - 7 < 1$

Solution

a. 14 is less than or equal to 27
b. 19 minus 2 is not equal to 8
c. 12 is greater than 4 divided by 2
d. x minus 7 is less than 1

2) Translate from algebra into English:

a. $19 \geq 15$
b. $7 = 12 - 5$
c. $15 \div 3 < 8$
d. $y + 3 > 6$

Solution

a. 19 is greater than or equal to 15
b. 7 is equal to 12 minus 5
c. 15 divided by 3 is less than 8
d. y plus 3 is greater than 6

Grouping symbols in algebra are much like the commas, colons, and other punctuation marks in English. They help to make clear which expressions are to be kept together and separate from other expressions. We will introduce three types now.

Grouping Symbols

Parentheses ( )
Brackets [ ]
Braces { }

Here are some examples of expressions that include grouping symbols. We will simplify expressions like these later in this section.

$$8(14 - 8)$$

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb. In algebra, we have *expressions* and *equations*.

Expression

An expression is a number, a variable, or a combination of numbers and variables using operation symbols.

An **expression** is like an English phrase. Here are some examples of expressions:

Expression	Words	English Phrase
$3+5$	3 plus 5	the sum of three and five
$n-1$	n minus one	the difference of n and one
6×7	6 times 7	the product of six and seven
$\frac{x}{y}$	x divided by y	the quotient of x and y

Notice that the English phrases do not form a complete sentence because the phrase does not have a verb.

An **equation** is two expressions linked with an equal sign. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb.

Equation

An equation is two expressions connected by an equal sign.

Here are some examples of equations.

Equation	English Sentence
$3+5=8$	The sum of three and five is equal to eight.
$n-1=14$	n minus one equals fourteen.
$6\times 7=42$	The product of six and seven is equal to forty-two.
$x=53$	x is equal to fifty-three.
$y+9=2y-3$	y plus nine is equal to two y minus three.

Example 2

1) Determine if each is an expression or an equation:

a. $2(x+3)=10$
b. $4(y-1)+1$
c. $x\div 25$
d. $y+8=40$

Solution

a. $2(x+3)=10$. This is an equation—two expressions are connected with an equal sign.
b. $4(y-1)+1$. This is an expression—no equal sign.

c. $x \div 25$. This is an expression—no equal sign.
d. $y + 8 = 40$. This is an equation—two expressions are connected with an equal sign.

Try It

3) Determine if each is an expression or an equation:

a. $3\left(x - 7\right) = 27$
b. $5\left(4y - 2\right) - 7$

Solution

a. equation
b. expression

4) Determine if each is an expression or an equation:

a. $y^3 \div 14$
b. $4x - 6 = 22$

Solution

a. expression
b. equation

Suppose we need to multiply 2 nine times. We could write this as $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$. This is tedious and it can be hard to keep track of all those 2s, so we use exponents. We write $2 \times 2 \times 2$ as 2^3 and $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ as 2^9. In expressions such as 2^3 the 2 is called the *base* and the 3 is called the *exponent*. The exponent tells us how many times we need to multiply the base.

$$\text{base} \leftarrow 2^{3 \rightarrow \text{exponent}}$$

means multiply 2 by itself, three times, as in $2 \times 2 \times 2$

We read 2^3 as "two to the third power" or "two cubed."

We say 2^3 is in *exponential notation* and $2 \times 2 \times 2$ is in *expanded notation*.

Exponential Notation

a^n means multiply a by itself, n times.

$base \leftarrow a^{n \rightarrow}\ exponent$

[illegible]

The expression a^n is read a to the nth power.

While we read a^n as "a to the nth power," we usually read:

- a^2 "a squared"
- a^3 "a cubed"

We'll see later why a^2 and a^3 have special names.

The table below shows how we read some expressions with exponents.

Expression	In Words
7^2	7 to the second power or 7 squared
5^3	5 to the third power or 5 cubed
9^4	9 to the fourth power
12^5	12 to the fifth power

Example 3

Simplify: 3^4

Solution

Step 1: **Expand the expression.**

$$3^4 = 3 \times 3 \times 3 \times 3$$

Step 2: **Multiply left to right.**

$$9 \times 3 \times 3$$

Step 3: **Multiply.**

$$27 \times 3$$

Step 4: **Multiply.**

$$81$$

Try It

5) Simplify

a. 5^3

b. 1^7

Solution

a. 125
b. 1

6) Simplify:

a. 7^2

b. 0^5

Solution

a. 49
b. 0

Simplify Expressions Using the Order of Operations

To **simplify an expression** means to do all the math possible. For example, to simplify $4 \times 2 + 1$ we'd first multiply 4×2 to get 8 and then add the 1 to get 9. A good habit to develop is to work down the page, writing each step of the process below the previous step. The example just described would look like this:

$$\begin{array}{r} 4 \times 2 + 1 \\ 8 + 1 \\ = 9 \end{array}$$

By not using an equal sign when you simplify an expression, you may avoid confusing expressions with equation.

Simplify an Expression

To simplify an expression, do all operations in the expression.

We've introduced most of the symbols and notation used in algebra, but now we need to clarify the order of operations. Otherwise, expressions may have different meanings, and they may result in different values. For example, consider the expression:

$$4 + 3 \times 7$$

If you simplify this expression, what do you get?

Some students say 49,	
	$4 + 3 \times 7$
since $4 + 3$ gives $\mathbf{7}$.	7×7
7×7 is 49	$\mathbf{49}$
Others say 25,	
	$4 + 3 \times 7$
Since 3×7 is 21.	$4 + 21$
And $21 + 4$ makes 25.	$\mathbf{25}$

Imagine the confusion in our banking system if every problem had several different correct answers!

The same expression should give the same result. So mathematicians early on established some guidelines that are called the Order of Operations.

How to

Perform the Order of Operations.

1. Parentheses and Other Grouping Symbols
   - Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.
2. Exponents
   - Simplify all expressions with exponents.
3. Multiplication and Division
   - Perform all multiplication and division in order from left to right. These operations have equal priority.
4. Addition and Subtraction
   - Perform all addition and subtraction in order from left to right. These operations have equal priority.

Doing the Manipulative Mathematics activity "Game of 24" gives you practice using the order of operations.

Students often ask, "How will I remember the order?" Here is a way to help you remember: Take the first letter of each keyword and substitute the silly phrase: "Please Excuse My Dear Aunt Sally."

Keyword	Mnemonic
Parentheses	Please
Exponents	Excuse
Multiplication Division	My Dear
Addition Subtraction	Aunt Sally

It's good that "**M**y **D**ear" goes together, as this reminds us that **m**ultiplication and **d**ivision have equal priority. We do not always do multiplication before division or always do division before multiplication. We do them in order from left to right.

Similarly, "**A**unt **S**ally" goes together and so reminds us that **a**ddition and **s**ubtraction also have equal priority and we do them in order from left to right.

Let's try an example.

Example 4

Simplify:

a. $4+3\times 7$

b. $(4+3)\times 7$

Solution

a.

Step 1: **Are there any parentheses?**

No.

Step 2: **Are there any exponents?**

No.

Step 3: **Is there any multiplication or division?**

Yes.

Step 4: **Multiply first.**

$$4+3\times 7$$

Step 5: **Add.**

$$4+21=25$$

---

b.

Step 1: **Are there any parentheses?**

Yes.

$$(4+3)\times 7$$

Step 2: **Simplify inside the parentheses.**

$$(7)\times 7$$

Step 3: **Are there any exponents?**

No.

Step 4: **Is there any multiplication or division?**

Yes.

Step 5: **Multiply.**

$$49$$

Try It

7) Simplify:

a. $12-5\times 2$
b. $(12-5)\times 2$

Solution

a. 2
b. 14

8) Simplify:

a. $8 + 3 \times 9$
b. $(8 + 3) \times 9$

Solution

a. 35
b. 99

Example 5

Simplify: $18 \div 6 + 4(5 - 2)$

Solution

***Step 1:* Parentheses?**

Yes, subtract first.

$$18 \div 6 + 4(5 - 2)$$
$$18 \div 6 + 4(3)$$

***Step 2:* Exponents?**

No.

***Step 3:* Multiplication or division?**

Yes.

***Step 4:* Divide first because we multiply and divide left to right.**

$$18 \div 6 + 4(3)$$

***Step 5:* Any other multiplication or division?**

Yes.

***Step 6:* Multiply.**

$$3 + 4(3)$$

***Step 7:* Any other multiplication or division?**

No.

***Step 8:* Any addition or subtraction?**

Yes, add.

$$3 + 12 = 15$$

Try It

9) Simplify: $30 \div 5 + 10(3 - 2)$

Solution

16

10) Simplify: $70 \div 10 + 4\,(6 - 2)$

Solution

23

When there are multiple grouping symbols, we simplify the innermost parentheses first and work outward.

Example 6

Simplify: $5 + 2^3 + 3[6 - 3(4 - 2)]$

Solution

Step 1: **Are there any parentheses (or other grouping symbol)?**

Yes.

Step 2: **Focus on the parentheses that are inside the brackets.**

$$5 + 2^3 + 3[6 - 3(4 - 2)]$$

Step 3: **Subtract.**

$$5 + 2^3 + 3[6 - 3(2)]$$

Step 4: **Continue inside the brackets and multiply.**

$$5 + 2^3 + 3[6 - 6]$$

Step 5: **Continue inside the brackets and subtract.**

The expression inside the brackets requires no further simplification.

$$5 + 2^3 + 3[0]$$

Step 6: **Are there any exponents?**

Yes.

Step 7: **Simplify exponents.**

$$5 + (8) + 3[0]$$

Step 8: **Is there any multiplication or division?**

Yes, multiply.

$$5 + 8 + (3 \times 0 = 0)$$

Step 9: **Is there any addition or subtraction?**

Yes, add.

$$5 + 8 + 0 = 13$$

Try It

11) Simplify: $9 + 5^3 - [4\,(9 + 3)]$

Solution

86

12) Simplify: $7^2 - 2\,[4\,(5 + 1)]$

Solution

1

Evaluate an Expression

In the last few examples, we simplified expressions using the order of operations. Now we'll evaluate some expressions—again following the order of operations. To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

To evaluate an expression means to find the value of the expression when the variable is replaced by a given number.

To evaluate an expression, substitute that number for the variable in the expression and then simplify the expression.

Example 7

Evaluate $7x - 4$, when

a. $x = 5$ and,
b. $x = 1$

Solution

a.

Step 1: **Substitute 5 for x in the expression.**

Simplify the expression.

$$7 \times 5 - 4$$

Step 2: **Multiply.**

$$35 - 4$$

Step 3: **Subtract.**

$$31$$

---

b.

Step 1: **Substitute 1 for x in the expression.**

Simplify the expression.

$$7 \times 1 - 4$$

Step 2: **Multiply.**

$$7 - 4$$

***Step 3:* Subtract.**

$$3$$

Try It

13) Evaluate $8x - 3$, when

a. $x = 2$
b. $x = 1$

Solution

a. 13
b. 5

14) Evaluate $4y - 4$, when

a. $y = 3$
b. $y = 5$

Solution

a. 8
b. 16

Example 8

Evaluate $x = 4,$ when

a. x^2

b. 3^x

Solution

a.

Step 1: **Replace x with 4.**

$$4^2$$

Step 2: **Use definition of exponent.**

$$4 \times 4$$

Step 3: **Simplify.**

$$16$$

---

b.

Step 1: **Replace x with 4.**

$$3^4$$

Step 2: **Use definition of exponent.**

$$3 \times 3 \times 3 \times 3$$

Step 3: **Simplify.**

$$81$$

Try It

15) Evaluate

a. x^2
b. 4^x

when $x = 3$

Solution

a. 9
b. 64

16) Evaluate

a. x^3
b. 2^x

when $x = 6$

Solution

a. 216

b. 64

Example 9

Evaluate $2x^2 + 3x + 8$ when $x = 4$

Solution

Step 1: **Substitute $x = 4$.**

$$2(4)^2 + 3(4) + 8$$

Step 2: **Follow the order of operations.**

$$2(16) + 12 + 8$$
$$32 + 12 + 8 = 52$$

Try It

17) Evaluate $3x^2 + 4x + 1$ when $x = 3$

Solution

40

18) Evaluate $6x^2 - 4x - 7$ when $x = 2$

Solution

$$9$$

Identify and Combine Like Terms

Term

Algebraic expressions are made up of terms. A term is a constant or the product of a constant and one or more variables.

Examples of terms are 7, y, $5x^2$, $9a$, b^5

The constant that multiplies the variable is called the **coefficient**.

Coefficient

The coefficient of a term is the constant that multiplies the variable in a term.

Think of the coefficient as the number in front of the variable. The coefficient of the term $3x$ is 3. When we write x, the coefficient is 1, since $x = 1x$.

Example 10

Identify the coefficient of each term:

a. $14y$
b. $15x^2$
c. a

Solution

1. The **coefficient** of $14y$ is 14.
2. The **coefficient** of $15x^2$ is 15.
3. The **coefficient** of a is 1 since $a = 1a$

Try It

19) Identify the coefficient of each term:

a. $17x$
b. $41b^2$

c. z

Solution

a. 17
b. 41
c. 1

20) Identify the coefficient of each term:

a. $9p$
b. $13a^3$
c. y^3

Solution

a. 9
b. 13
c. 1

Some terms share common traits. Look at the following 6 terms. Which ones seem to have traits in common?

$$5x \quad 7 \quad n^2 \quad 4 \quad 3x \quad 9n^2$$

The 7 and the 4 are both **constant** terms.

The $5x$ and the $3x$ are both terms with x.

The n^2 and the $9n^2$ are both terms with n^2

When two terms are constants or have the same variable and exponent, we say they are **like terms**.

- 7 and 4 are like terms.
- $5x$ and $3x$ are like terms.
- x^2 and $9x^2$ are like terms.

Like Terms

Terms that are either constants or have the same variables raised to the same powers are called like terms.

Example 11

Identify the like terms: y^3, $7x^2$, 14, 23, $4y^3$, $9x$, $5x^2$

Solution

y^3 and $4y^3$ are like terms because both have y^3 the variable and the exponent match.

$7x^2$ and $5x^2$ are like terms because both have x^2 the variable and the exponent match.

14 and 23 are like terms because both are constants.

There is no other term like $9x$

Try It

21) Identify the like terms: 9, $2x^3$, y^2, $8x^3$, 15, $9y$, $11y^2$

Solution

9 and 15, y^2 and $11y^2$, $2x^3$ and $8x^3$

22) Identify the like terms: $4x^3$, $8x^2$, 19, $3x^2$, 24, $6x^3$

Solution

19 and 24, $8x^2$ and $3x^2$, $4x^3$ and $6x^3$

Adding or subtracting terms forms an expression. In the expression $2x^2+3x+8$ when $x=4$ from Example 1.2.9, the three terms are $2x^2$, $3x$, and 8.

Example 12

Identify the terms in each expression.

a. $9x^2+7x+12$
b. $8x+3y$

Solution

a. The terms of $9x^2+7x+12$ are $9x^2$, $7x$, and 12.
b. The terms of $8x+3y$ are $8x$ and $3y$

Try It

23) Identify the terms in the expression $4x^2+5x+17$

Solution

$4x^2$, $5x$, 17

24) Identify the terms in the expression $5x+2y$

Solution

$5x$, $2y$

If there are like terms in an expression, you can simplify the expression by combining the like terms. What do you think $4x + 7x + x$ would simplify to? If you thought 12, you would be right!

$$4x + 7x + x$$

$$x + x + x + x \quad + x + x + x + x + x + x + x \quad + x$$

$$12x$$

Add the coefficients and keep the same variable. It doesn't matter what x is—if you have 4 of something and add 7 more of the same thing and then add 1 more, the result is 12 of them. For example, 4 oranges plus 7 oranges plus 1 orange is 12 oranges. We will discuss the mathematical properties behind this later.

Simplify: $4x + 7x + x$.

Add the coefficients. $12x$

Example 13

How To Combine Like Terms:

Simplify: $2x^2 + 3x + 7 + x^2 + 4x + 5$

Solution

***Step 1*: Identify the like terms**

$$2x^2 + 3x + 7 + x^2 + 4x + 5$$

$$\boxed{2x^2} + \boxed{3x} + 7 + \boxed{x^2} + \boxed{4x} + 5$$

***Step 2:* Rearrange the expression so the like terms are together.**

$$\boxed{2x^2} + \boxed{x^2} + \boxed{3x} + \boxed{4x} + 7 + 5$$

***Step 3*: Combine like terms.**

$$3x^2 + 7x + 12$$

Try It

25) Simplify: $3x^2 + 7x + 9 + 7x^2 + 9x + 8$

Solution

$10x^2 + 16x + 17$

26) Simplify: $4y^2 + 5y + 2 + 8y^2 + 4y + 5$

Solution

$12y^2 + 9y + 7$

How to

Combine Like Terms.

1. Identify like terms.
2. Rearrange the expression so like terms are together.
3. Add or subtract the coefficients and keep the same variable for each group of like terms.

Translate an English Phrase to an Algebraic

Expression

In the last section, we listed many operation symbols that are used in algebra, then we translated expressions and equations into English phrases and sentences. Now we'll reverse the process. We'll translate English phrases into algebraic expressions. The symbols and variables we've talked about will help us do that. Below table summarizes them.

Operation	Phrase	Expression
Addition	a plus b the sum of a and b a increased by b b more than a the total of a and b b added to a	$a+b$

Operation	Phrase	Expression
Subtraction	a minus b	$a - b$
	the difference of a and b	
	a decreased by b	
	b less than a	
	b subtracted from a	
Multiplication	a times b	$a \times b$, ab, $a(b)$, $(a)(b)$, $2a$
	the product of a and b	
	twice a	

Operation	Phrase	Expression
Division	a divided by b the quotient of a and b the ratio of a and b b divided into a	$a \div b$, $\frac{a}{b}$, $b\overline{)a}$

Look closely at these phrases using the four operations:

the sum *of* a *and* b

the difference *of* a *and* b

the product *of* a *and* b

the quotient *of* a *and* b

Each phrase tells us to operate on two numbers. Look for the words *of, and* to find the numbers.

Example 14

Translate each English phrase into an algebraic expression:

a. the difference of $17x$ and 5

b. the quotient of $10x^2$ and 7

Solution

a.

Step 1: **The key word is "difference", which tells us the operation is subtraction.**

The *difference* of $17x$ and 5.

Step 2**: Look for the words *of* and *and* find the numbers to subtract.**

The difference *of* $17x$ and 5.

Step 3**: Find the difference of $17x$ and 5**

$$17x - 5$$

---

b.

Step 1: **The key word is "quotient," which tells us the operation is division.**

The *quotient* of $10x^2$ and 7.

Step 2: **Divide $10x^2$ by 7**

$$10x^2 \div 7$$

This can also be written $10x^2/7$ or $\frac{10x^2}{7}$.

Try It

27) Translate the English phrase into an algebraic expression:

a. the difference of $14x^2$ and 13

b. the quotient of $12x$ and 2

Solution

a. $14x^2 - 13$
b. $12x \div 2$

28) Translate the English phrase into an algebraic expression:

a. the sum of $17y^2$ and 19
b. the product of 7 and y

Solution

a. $17y^2 + 19$
b. $7y$

How old will you be in eight years? What age is eight more years than your age now? Did you add 8 to your present age? Eight "more than" means 8 added to your present age. How old were you seven years ago? This is 7 years less than your age now. You subtract 7 from your present age. Seven "less than" means 7 subtracted from your present age.

Example 15

Translate the English phrase into an algebraic expression:

a. Seventeen more than y

b. Nine less than $9x^2$

Solution

a.

Step 1:* The key words are *more than.

They tell us the operation is addition. *More than* means "added to."

***Step 2:* Seventeen more than y.**

***Step 3:* Seventeen added to y.**

$$y + 17$$

---

b.

Step 1:* The key words are *less than.

They tell us to subtract. *Less than* means "subtracted from."

***Step 2:* Nine less than $9x^2$**

Step 3: **Nine subtracted from $9x^2$**

$$9x^2 - 9$$

Try It

29) Translate the English phrase into an algebraic expression:

a. Eleven more than x
b. Fourteen less than $11a$

Solution

a. $x + 11$
b. $11a - 14$

30) Translate the English phrase into an algebraic expression:

a. 13 more than z
b. 18 less than $8x$

Solution

a. $z + 13$
b. $8x - 18$

Example 16

Translate the English phrase into an algebraic expression:

a. five times the sum of m and n

b. the sum of five times m and n

Solution

a.

***Step 1:* There are two operation words—*times* tells us to multiply and *sum* tells us to add.**

***Step 2:* Because we are multiplying 5 times the sum we need parentheses around the sum of m and n, $(m + n)$.**

This forces us to determine the sum first. (Remember the order of operations.)

***Step 3:* Five times the sum of m and n.**

$$5(m + n)$$

---

b.

***Step 1:* To take a sum, we look for the words "of" and "and" to see what is being added.**

Here we are taking the sum *of* five times m and n.

***Step 2:* The sum of five times m and n.**

$$5m + n$$

Try It

31) Translate the English phrase into an algebraic expression:

a. four times the sum of p and q

b. the sum of four times p and q

Solution

a. $4(p + q)$
b. $4p + q$

32) Translate the English phrase into an algebraic expression:

a. the difference of two times x and 8

b. two times the difference of x and 8

Solution

a. $2x - 8$
b. $2(x - 8)$

Later in this course, we'll apply our skills in algebra to solving applications. The first step will be to translate an English phrase to an algebraic expression. We'll see how to do this in the next two examples.

Example 17

The length of a rectangle is 6 less than the width. Let w represent the width of the rectangle.

Write an expression for the length of the rectangle.

Solution

Step 1: **Write a phrase about the length of the rectangle.**

6 less than the width

Step 2: **Substitute w for "the width."**

6 less than w

Step 3: **Rewrite "less than" as "subtracted from."**

6 subtracted from w

Step 4: **Translate the phrase into algebra.**

$$w - 6$$

Try It

33) The length of a rectangle is 7 less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Solution

$w - 7$

34) The width of a rectangle is 6 less than the length. Let l represent the length of the rectangle. Write an expression for the width of the rectangle.

Solution

$l - 6$

Example 18

June has dimes and quarters in her purse. The number of dimes is three less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Solution

Step 1: **Write a phrase about the number of dimes.**

three less than four times the number of quarters

Step 2: **Substitute q for the number of quarters.**

3 less than 4 times q

Step 3: **Translate "4 times q".**

3 less than $4q$

Step 4: **Translate the phase into algebra.**

$$4q-3$$

Try It

35) Geoffrey has dimes and quarters in his pocket. The number of dimes is eight less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Solution
$4q - 8$

36) Lauren has dimes and nickels in her purse. The number of dimes is three more than seven times the number of nickels. Let n represent the number of nickels. Write an expression for the number of dimes.

Solution
$7n + 3$

Key Concepts

- **Notation**

Notation	The result is…
$a+b$	the sum of a and b
$a-b$	the difference of a and b
ab, $a \times b$, $(a)(b)$, $(a)b$, $a(b)$	the product of a and b
a/b, $a \div b$, $\frac{a}{b}$, $b\overline{)a}$	the quotient of a and b

- **Inequality**

$a<b$ is read "a is less than b"	a is to the left of b on the number line
$a<b$ is read "a is greater than b"	a is to the right of b on the number line

- **Inequality Symbols** **Words**

$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a \leq b$	a is less than or equal to b
$a > b$	a is greater than b
$a \geq b$	a is greater than or equal to b

- **Grouping Symbols**
  - Parentheses ( )
  - Brackets [ ]
  - Braces { }
- **Exponential Notation**
  - a^n means multiply a by itself, n times. The expression a^n is read a to the n^{th} power.

- **Order of Operations:** When simplifying mathematical expressions perform the operations in the following order:
  1. Parentheses and other Grouping Symbols: Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.
  2. Exponents: Simplify all expressions with exponents.
  3. Multiplication and Division: Perform all multiplication and division in order from left to right. These operations have equal priority.
  4. Addition and Subtraction: Perform all addition and subtraction in order from left to right. These operations have equal priority.
- **Combine Like Terms**
  1. Identify like terms.
  2. Rearrange the expression so like terms are together.
  3. Add or subtract the coefficients and keep the same variable for each group of like terms

Glossary

coefficient
: The coefficient of a term is the constant that multiplies the variable in a term.

constant
: A constant is a number whose value always stays the same.

equality symbol
: The symbol "=" is called the equal sign. We read $a = b$ as "a is equal to b."

equation
: An equation is two expressions connected by an equal sign.

evaluate an expression
: To evaluate an expression means to find the value of the expression when the variable is replaced by a given number.

expression
: An expression is a number, a variable, or a combination of numbers and variables using operation symbols.

like terms
: Terms that are either constants or have the same variables raised to the same powers are called like terms.

simplify an expression
: To simplify an expression, do all operations in the expression.

term
: A term is a constant or the product of a constant and one or more variables.

variable
: A variable is a letter that represents a number whose value may change.

Exercises: Use Variables and Algebraic Symbols

Instructions: For questions 1-14, translate from algebra to English.

1) $16-9$

2) $3\cdot 9$

3) $28\div 4$

4) $x+11$

5) $(2)(7)$

6) $(4)(8)$

7) $14<21$

8) $17<35$

9) $36\geq 19$

10) $6n=36$

11) $y - 1 > 6$

12) $y - 4 > 8$

13) $2 \leq 18 \div 6$

14) $a \neq 1 \cdot 12$

Odd Answers

1) 16 **minus** 9**, the difference of sixteen and nine**

3) 28 **divided by** 4**, the quotient of twenty-eight and four**

5) 2 **times** 7**, the product of two and seven**

7) **fourteen is less than twenty-one**

9) **thirty-six is greater than or equal to nineteen**

11) y **minus** 1 **is greater than** 6**, the difference of** y **and one is greater than six**

13) 2 **is less than or equal to** 18 **divided by** 6**; two is less than or equal to the quotient of eighteen and six**

Exercises: Expression or Equation

Instructions: For questions 15-20, determine if each is an expression or an equation.

15) $9 \cdot 6 = 54$

16) $7 \cdot 9 = 63$

17) $5 \cdot 4 + 3$

18) $x + 7$

19) $x + 9$

20) $y - 5 = 25$

Odd Answers

15) equation

17) expression

19) expression

Exercises: Simplify Expressions Using the Order of Operations

Instructions: For questions 21-24, simplify each expression.

21) 5^3

22) 8^3

23) 2^8

24) 10^5

Odd Answers

21) 125

23) 256

Exercises: Simplify Using Order of Operations

Instructions: For questions 25-46, simplify using the order of operations.

25) $3+8\cdot 5$

26) $2+6\cdot 3$

27) $(3+8)\cdot 5$

28) $(2+6)\cdot 3$

29) $2^3-12\div(9-5)$

30) $3^2-18\div(11-5)$

31) $3\cdot 8+5\cdot 2$

32) $4\cdot 7+3\cdot 5$

33) $2+8(6+1)$

34) $4+6(3+6)$

35) $4\cdot 12/8$

36) $2\cdot 36/6$

37) $(6+10)\div(2+2)$

38) $(9+12)\div(3+4)$

39) $20\div 4+6\cdot 5$

40) $33\div 3+8\cdot 2$

41) 3^2+7^2

42) $(3+7)^2$

43) $3(1+9\cdot 6)-4^2$

44) $5(2+8\cdot 4)-7^2$

45) $2\left[1+3(10-2)\right]$

46) $5\left[2+4(3-2)\right]$

Odd Answers

25) 43

27) 55

29) 5

31) 34

33) 58

35) 6

37) 4

39) 35

41) 58

43) 149

45) 50

Exercises: Evaluate an Expression

Instructions: For questions 47-60, evaluate the expressions.

47) $7x + 8$ **when** $x = 2$

48) $8x - 6$ **when** $x = 7$

49) x^2 **when** $x = 12$

50) x^3 **when** $x = 5$

51) x^5 when $x=2$

52) 4^x when $x=2$

53) x^2+3x-7 when $x=4$

54) $6x+3y-9$ when $x=6, y=9$

55) $(x-y)^2$ when $x=10, y=7$

56) $(x+y)^2$ when $x=6, y=9$

57) a^2+b^2 when $a=3, b=8$

58) r^2-s^2 when $r=12, s=5$

59) $2l+2w$ when $l=15, w=12$

60) $2l+2w$ when $l=18, w=14$

Odd Answers

47) 22

49) 144

51) 32

53) 21

55) 9

57) 73

59) 54

Exercises: Identifying the Coefficient

Instructions: For questions 61-64, identify the coefficient of each term.

61) $8a$

62) $13m$

63) $5r^2$

64) $6x^3$

Odd Answers

61) 8

63) 5

Exercises: Identify Like Terms

Instructions: For questions 65-68, identify the like terms.

65) $x^3, 8x, 14, 8y, 5, 8x^3$

66) $6z, 3w^2, 1, 6z^2, 4z, w^2$

67) $9a, a^2, 16, 16b^2, 4, 9b^2$

68) $3, 25r^2, 10s, 10r, 4r^2, 3s$

Odd Answers

65) x^3 and $8x^3$, 14 and 5

67) 16 and 4, $16b^2$ and $9b^2$

Exercises: Identify Terms in an Expression

Instructions: For questions 69-72, identify the terms in each expression

69) $15x^2 + 6x + 2$

70) $11x^2 + 8x + 5$

71) $10y^3 + y + 2$

72) $9y^3 + y + 5$

Odd Answers

69) $15x^2, 6x, 2$

71) $10y^3, y, 2$

Exercises: Simplify by Combining Like Terms

Instructions: For questions 73-82, simplify the expressions by combining like terms.

73) $10x + 3x$

74) $15x + 4x$

75) $4c + 2c + c$

76) $6y + 4y + y$

77) $7u + 2 + 3u + 1$

78) $8d + 6 + 2d + 5$

79) $10a + 7 + 5a - 2 + 7a - 4$

80) $7c + 4 + 6c - 3 + 9c - 1$

81) $3x^2 + 12x + 11 + 14x^2 + 8x + 5$

82) $5b^2 + 9b + 10 + 2b^2 + 3b - 4$

Odd Answers

73) $13x$

75) $7c$

77) $10u + 3$

79) $22a+1$

81) $17x^2+20x+16$

Exercises: Translate an English Phrase to an Algebraic Expression

Instructions: For questions 83-94, translate the phrases into algebraic expressions.

83) the difference of 14 **and** 9

84) the difference of 19 **and** 8

85) the product of 9 **and** 7

86) the product of 8 **and** 7

87) the quotient of 36 **and** 9

88) the quotient of 42 and 7

89) the sum of $8x$ and $3x$

90) the sum of $13x$ and $3x$

91) the quotient of y and 3

92) the quotient of y and 8

93) eight times the difference of y and nine

94) seven times the difference of y and one

Odd Answers

83) $14 - 9$

85) $9 \cdot 7$

87) $36 \div 9$

89) $8x + 3x$

91) $\frac{y}{3}$

93) $8(y-9)$

Exercises: Word Problems

Instructions: For questions 95-98, write an expression for the given word problems.

95) Eric has rock and classical CDs in his car. The number of rock CDs is 3 more than the number of classical CDs. Let c represent the number of classical CDs. Write an expression for the number of rock CDs.

96) The number of girls in a second-grade class is 4 less than the number of boys. Let b represent the number of boys. Write an expression for the number of girls.

97) Greg has nickels and pennies in his pocket. The number of pennies is seven less than twice the number of nickels. Let n represent the number of nickels. Write an expression for the number of pennies.

98) Jeannette has $\$5$ and $\$10$ bills in her wallet. The number of fives is three more than six times the number of tens. Let t represent the number of tens. Write an expression for the number of fives.

Odd Answers

95) $c+3$

97) $2n-7$

Exercises: Everyday Math

Instructions: For questions 99-100, answer the given everyday math word problems.

99) Car insurance: Justin's car insurance has a $\$750$ deductible per incident. This means that he pays $\$750$ and his insurance company will pay all costs beyond $\$750$. If Justin files a claim for $\$2,100$:

- **a) how much will he pay?**
- **b) how much will his insurance company pay?**

100) Home insurance: Armando's home insurance has a $\$2,500$ deductible per incident. This means that he pays $\$2,500$ and the insurance company will pay all costs beyond $\$2,500$. If Armando files a claim for $\$19,400$:

- **a) how much will he pay?**
- **b) how much will the insurance company pay?**

Odd Answers

99a) $\$750$

99b) $\$1,350$

Exercises: Writing Exercises

Instructions: For questions 101-104, answer the given writing exercises.

101) Explain the difference between an expression and an equation.

102) Why is it important to use the order of operations to simplify an expression?

103) Explain how you identify the like terms in the expression $8a^2+4a+9-a^2-1$.

104) Explain the difference between the phrases "4 times the sum of x and y" and "the sum of 4 times x and y."

Odd Answers

101) Answers may vary

103) Answers may vary

1.3 INTEGERS

Learning Objectives

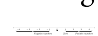
By the end of this section, you will be able to:

- Use negatives and opposites
- Simplify: expressions with absolute value
- Add integers
- Subtract integers
- Multiply integers
- Divide integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate English phrases to algebraic expressions
- Use integers in applications

Use Negatives and Opposites

Our work so far has only included counting numbers and the whole numbers. But if you have ever experienced a temperature below zero or accidentally overdrawn your checking account, you are already familiar with negative numbers. Negative numbers are numbers less than 0. The negative numbers are to the left of zero on the number line.

The number line shows the location of positive and negative numbers.



The arrows on the ends of the number line indicate that the numbers keep going forever. There is no biggest positive number, and there is no smallest negative number.

Is zero a positive or a negative number? Numbers larger than zero are positive, and numbers smaller than zero are negative. Zero is neither positive nor negative.

Consider how numbers are ordered on the number line. Going from left to right, the numbers increase in value. Going from right to left, the numbers decrease in value.

The numbers on a number line increase in value going from left to right and decrease in value going from right to left.

Remember that we use the notation:

$a < b$ (read "a is less than b") when a is to the left of b on the number line.

$a > b$ (read "a is greater than b") when a is to the right of b on the number line.

Now we need to extend the number line which shows the whole numbers to include negative numbers, too. The numbers marked by points in Figure 1.3.1 are called the integers. The integers are the numbers {...}−3,−2,−1,0,1,2,3{...}

All the marked numbers are called *integers*.

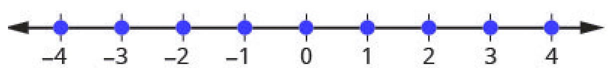


Figure 1.3.1

Example 1

Order each of the following pairs of numbers, using < or >:

a. $14\_\_\_6$
b. $-1\_\_\_9$
c. $-1\_\_\_-4$

d. $2\_\_ -20$

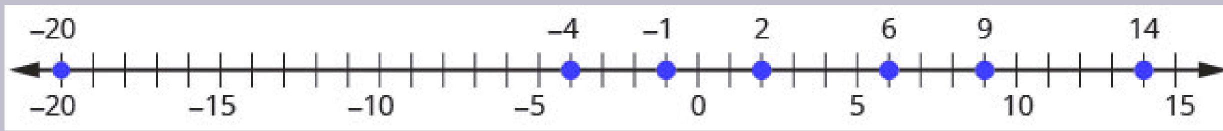


Figure 1.3.2

Solution

It may be helpful to refer to the number line shown.

a. 14 is to the right of 6 on the number line.

$14\_\_6 \qquad 14 > 6$

b. −1 is to the left of 9 on the number line.

$-1\_\_9 \qquad -1 < 9$

c. −1 is to the right of −4 on the number line.

$-1\_\_ -4 \qquad -1 > -4$

d. 2 is to the right of −20 on the number line.

$2\_\_ -20 \qquad 2 > -20$

Try It

1) Order each of the following pairs of numbers, using < or >:

a. $15\_\_7$

b. $-2\_\_5$

c. $-3\_\_ -7$

d. $5\_\_ -17$

Solution

a. >
b. <
c. >
d. >

2) Order each of the following pairs of numbers, using < or >:

a. $8\_\_\_13$
b. $3\_\_\_-4$
c. $-5\_\_\_-2$
d. $9\_\_\_-21$

Solution

a. <
b. >
c. <
d. >

You may have noticed that, on the number line, the negative numbers are a mirror image of the positive numbers, with zero in the middle. Because the numbers 2 and −2 are the same distance from zero, they are called **opposites**. The opposite of 2 is −2 and the opposite of −2 is 2.

Opposite

The opposite of a number is the number that is the same distance from zero on the number line but on the opposite side of zero.

Figure 1.3.3 illustrates the definition.

The opposite of 3 is −3.

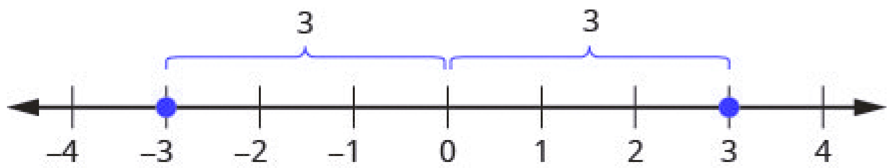


Figure 1.3.3

Sometimes in algebra, the same symbol has different meanings. Just like some words in English, the specific meaning becomes clear by looking at how it is used. You have seen the symbol "–" used in three different ways.

$10-4$	Between two numbers, it indicates the operation of subtraction. We read $10-4$ as " 10 minus 4."
-8	In front of a number, it indicates a negative number. We read –8 as "negative eight."
$-x$	In front of a variable, it indicates the opposite. We read $-x$ as "the opposite of x."
$-(-2)$	Here there are two "–" signs. The one is the parentheses tells us the number is negative 2. and the one outside the parentheses tells us to take the opposite of –2. We read $-(-2)$ as "the opposite of negative two."

Opposite Notation

$-a$ means the opposite of the number a.

The notation $-a$ is read as "the opposite of a."

Example 2

Find:

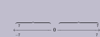
a. the opposite of 7

b. the opposite of -10

c. the opposite of -6

Solution

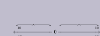
a. -7 is the same distance from 0 as 7, but on the opposite side of 0



The opposite of 7 is -7.

---

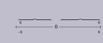
b. 10 is the same distance from 0 as -10, but on the opposite side of 0.



The opposite of -10 is 10.

---

c. $-(-6)$



The opposite of $-(-6)$ is -6.

Try It

3) Find:

a. the opposite of 4

b. the opposite of -3

c. $-(-1)$

Solution

a. -4

b. 3

c. 1

4) Find:

a. the opposite of 8

b. the opposite of -5

c. $-(-5)$

Solution

a. -8

b. 5

c. 5

Our work with opposites gives us a way to define the integers. The whole numbers and their opposites are called the **integers**. The integers are the numbers $-3,-2,-1,0,1,2,3$

Integers

The whole numbers and their opposites are called the integers.

The integers are the numbers $-3,-2,-1,0,1,2,3$

When evaluating the opposite of a variable, we must be very careful. Without knowing whether the variable represents a positive or negative number, we don't know whether x is positive or negative. We can see this in the below figure.

Example 3

Evaluate

a. $-x$, when $x = 8$
b. $-x$, when $x = -8$.

Solution
a.

To evaluate when $x = 8$ means to substitute 8 for x.

Step 1: **Substitute 8 for x.**

$$-(8)$$

Step 2: **Write the opposite of 8.**

$$-8$$

---

b.

To evaluate when $x = -8$ means to substitute -8 for x.

Step 1: **Substitute** -8 **for** x**.**

$$-(-8)$$

Step 2: **Write the opposite of** -8**.**

$$8$$

Try It

5) Evaluate $-n$ when

a. $n = 4$
b. $n = -4$

Solution

a. -4

b. 4

6) Evaluate $-m$ when

a. $m = 11$
b. $m = -11$

Solution

a. -11
b. 11

Simplify: Expressions with Absolute Value

We saw that numbers such as 2 and –2 are opposites because they are the same distance from 0 on the number line. They are both two units from 0. The distance between 0 and any number on the number line is called the **absolute value** of that number.

Absolute Value

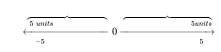
The absolute value of a number is its distance from 0 on the number line.

The absolute value of a number n is written as $|n|$.

For example,

- -5 is 5 units away from 0, so $|-5| = 5$.
- 5 is 5 units away from 0, so $|5| = 5$.

The figure below illustrates this idea. The integers 5 and −5 are 5 units away from 0.



The absolute value of a number is never negative (because distance cannot be negative). The only number with absolute value equal to zero is the number zero itself, because the distance from 0 to 0 on the number line is zero units.

Property of Absolute Value

$|n| \geq 0$ for all numbers

Absolute values are always greater than or equal to zero!

Mathematicians say it more precisely, "absolute values are always non-negative." Non-negative means greater than or equal to zero.

Example 4

7) Simplify:

a. $|3|$

b. $|-44|$

c. $|0|$

Solution

The absolute value of a number is the distance between the number and zero. Distance is never negative, so the absolute value is never negative.

a. $|3| = 3$

---

b. $|-44| = 44$

---

c. $|0| = 0$

Try It

7) Simplify:

a. $|4|$
b. $|-28|$
c. $|0|$

Solution

a. 4
b. 28
c. 0

8) Simplify:

a. $|-13|$
b. $|47|$

c. $|0|$

Solution

a. 13
b. 47
c. 0

In the next example, we'll order expressions with absolute values. Remember, positive numbers are always greater than negative numbers!

Example 5

Fill in $<$, $>$ or $=$ for each of the following pairs of numbers:

1. $|-5|\_\_\_ -|-5|$
2. $8\_\_\_ -|-8|$
3. $-9\_\_\_ -|-9|$
4. $-(-16)\_\_\_|-16|$

Solution

a.

Step 1: **Simplify.**

$$-5\_\_\_ - -5$$

Step 2: **Order.**

$$5 > -5$$

$$|5| > -|-5|$$

---

b.

Step 1: **Simplify.**

$$8\_\_\_ - -8$$

Step 2: **Order.**

$$8 > -8$$

$$|8| > -|-8|$$

---

c.

Step 1: **Simplify.**

$$-9\_\_\_ - -9$$

Step 2: **Order.**

$$-9 = -9$$
$$|9| = -|-9|$$

---

d.

Step 1: **Simplify.**

$$- - 16\_\_\_| - 16|$$

Step 2: **Order.**

$$16 = 16$$

$$-(-16) = -|-16|$$

Try It

9) Fill in $<$, $>$, or $=$ for each of the following pairs of numbers:

a. $|-9|\_\_\_ -|-9|$
b. $|2|\_\_\_ -|-2|$
c. $|-8|\_\_\_|-8|$
d. $-(-9)\_\_\_ -|-9|$

Solution

a. >
b. >
c. =
d. >

10) Fill in $<$, $>$, or $=$ for each of the following pairs of numbers:

a. $|7|\_\_\_ -|-7|$
b. $-(-10)\_\_\_ -|-10|$
c. $|-4|\_\_\_ -|-4|$
d. $|-1|\_\_\_|-1|$

Solution

a. >
b. >
c. >
d. =

We now add **absolute value** bars to our list of grouping symbols. When we use the order of operations, first we simplify inside the absolute value bars as much as possible, then we take the absolute value of the resulting number.

Grouping Symbols

Parentheses ( )	Braces { }
Brackets [ ]	Absolute value ||

In the next example, we simplify the expressions inside **absolute value** bars first, just like we do with parentheses.

Example 6

Simplify: $24 - |19 - 3(6 - 2)|$

Solution

Step 1: **Work inside parentheses first: subtract 2 from 6.**

$$24 - |19 - 3(4)|$$

Step 2: **Multiply 3(4).**

$$24 - |19 - 12|$$

Step 3: **Subtract inside the absolute value bars.**

$$24 - |7|$$

Step 4: **Take the absolute value.**

$$24 - 7$$

Step 5: **Subtract**

$$17$$

Try It

11) Simplify: $19 - |11 - 4(3 - 1)|$

Solution

$$16$$

12) Simplify: $9 - |8 - 4(7 - 5)|$

Solution

$$9$$

Example 7

Evaluate:

a. $\left|x\right|$ when $x = -35$

b. $|-y|$ when $y = -20$

c. $-|u|$ when $u = 12$

d. $-|p|$ when $p = -14$

Solution

a.

Step 1: **Substitute** -35**for** x.

$$|-35|$$

Step 2**: Take the absolute value.**

$$35$$

---

b.

Step 1: **Substitute** -20 **for** y.

\left|-
(-20)\ri
ght|

Step 2**: Simplify.**

$$|20|$$

Step 3**: Take the absolute value.**

$$20$$

---

c.

Step 1: **Substitute** 12 **for** u.

$$-|12|$$

Step 2**: Take the absolute value.**

$$-12$$

---

d.

***Step 1:* Substitute -14 for p.**

$$-|-14|$$

***Step 2*: Take the absolute value**

$$-14$$

Try It

13) Evaluate:

a. $|x|$ when $x = -17$
b. $|-y|$ when $y = 39$
c. $-|m|$ when $m = 22$
d. $-|p|$ when $p = -11$.

Solution

a. 17
b. 39
c. -22
d. -11

14) Evaluate:

a. $\left|y\right|$ when $y=-23$
b. $\left|-y\right|$ when $y=-21$
c. $-\left|n\right|$ when $n=37$
d. $-\left|q\right|$ when $q=-49$

Solution

a. 23
b. 21
c. -37
d. -49

Add Integers

Most students are comfortable with the addition and subtraction facts for positive numbers. However doing addition or subtraction with both positive and negative numbers may be more challenging.

We will use two colour counters to model the addition and subtraction of negatives so that you can visualize the procedures instead of memorizing the rules.

We let one colour (blue) represent positivity. The other colour (red) will represent the negatives. If we have one positive counter and one negative counter, the value of the pair is zero. They form a neutral pair. The value of this neutral pair is zero.

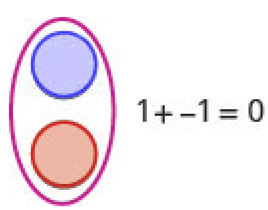


Figure 1.3.4

We will use the counters to show how to add the four addition facts using the numbers 5,−5 and 3,−3.

$5+3$ $-5+(-3)$ $-5+3$ $5+(-3)$

To add $5 + 3$, we realize that $5 + 3$ means the sum of 5 and 3.

We start with 5 positives.

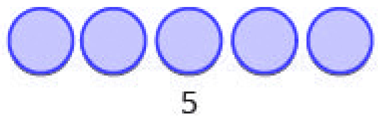


Figure 1.3.5

And then we add 3 positives.

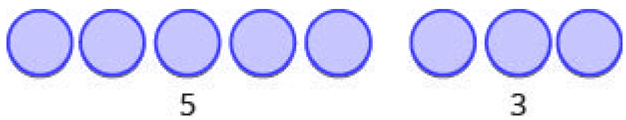


Figure 1.3.6

We now have 8 positives. The sum of 5 and 3 is 8.

Figure 1.3.7

Now we will add $-5 + (-3)$. Watch for similarities to the last example $5 + 3 = 8$.

To add $-5 + (-3)$, we realize this means the sum of −5 and −3.

We start with 5 negatives.

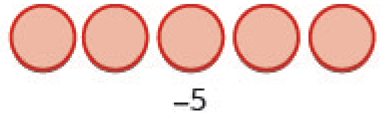


Figure 1.3.8

And then we add 3 negatives.

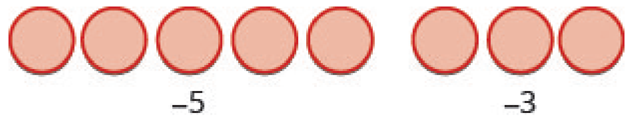


Figure 1.3.9

We now have 8 negatives. The sum of −5 and −3 is −8.

Figure 1.3.10

In what ways were these first two examples similar?

- The first example adds 5 positives and 3 positives—both positives.
- The second example adds 5 negatives and 3 negatives—both negatives.

In each case we got 8—either 8 positives or 8 negatives.

When the signs were the same, the counters were all the same colour, and so we added them.

Figure 1.3.11

Example 8

Add:

a. $1 + 4$
b. $-1 + (-4)$

Solution

a.

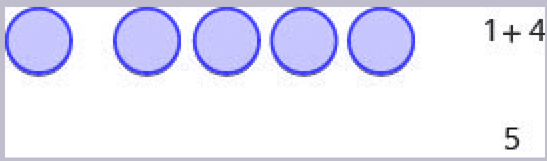


Figure 1.3.12

1 positive plus 4 positives is 5 positives.

---

b.

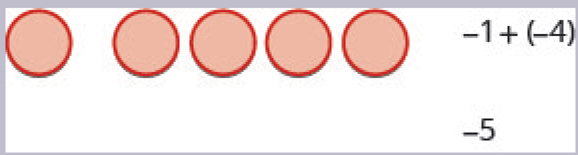


Figure 1.3.13

1 negative plus 4 negatives is 5 negatives.

Try It

15) Add:

a. $2+4$
b. $-2+(-4)$

Solution

a. 6
b. -6

16) Add:

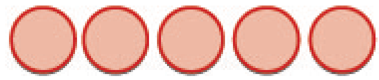
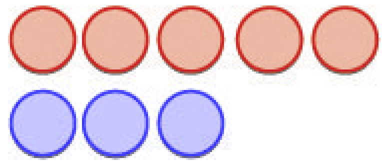
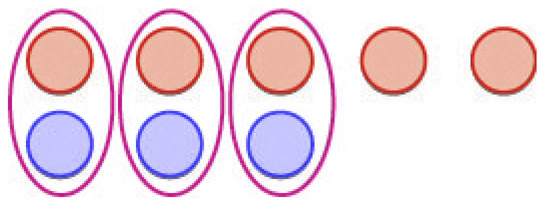
a. $2+5$
b. $-2+(-5)$

Solution

a. 7
b. -7

So what happens when the signs are different? Let's add $-5+3$. We realize this means the sum of -5 and 3. When the counters were the same colour, we put them in a row. When the counters are a different colour, we line them up under each other.

$-5+3$ means the sum of -5 and 3.

We start with 5 negatives.	 Figure 1.3.14
And then we add 3 positives.	 Figure 1.3.15
We remove any neutral pairs.	 Figure 1.3.16
We have 2 negatives left.	 Figure 1.3.17
The sum of -5 and 3 is -2.	$-5+3=-2$

Notice that there were more negatives than positives, so the result was negative.

Let's now add the last combination, $5+(-3)$.

$5 + (-3)$ means the sum of 5 and -3.

We start with 5 positives.

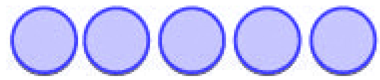


Figure 1.3.18

And then we add 3 negatives.

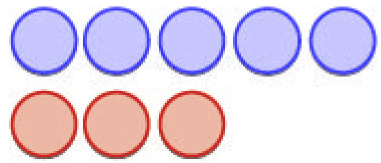


Figure 1.3.19

We remove any neutral pairs.

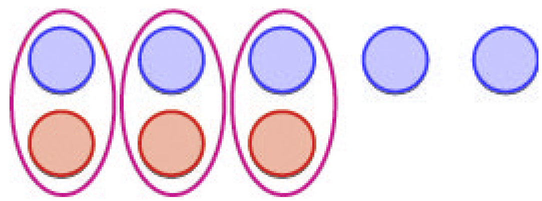


Figure 1.3.20

We have 2 positives left.

Figure 1.3.21

The sum of 5 and -3 is 2.

$$5 + (-3) = 2$$

When we use counters to model addition of positive and negative integers, it is easy to see whether there are more positive or more negative counters. So we know whether the sum will be positive or negative.

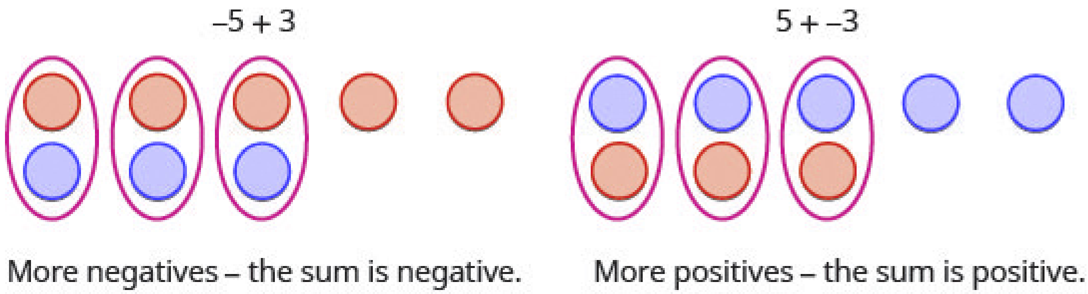


Figure 1.3.22

Example 9

Add:

a. $-1+5$
b. $1+(-5)$

Solution

a.

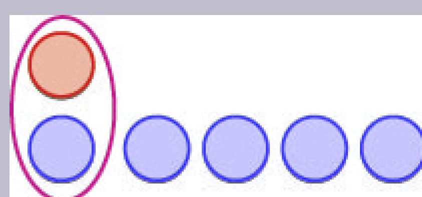


Figure 1.3.23

There are more positives, so the sum is positive.

$$-1+5=4$$

---

b.

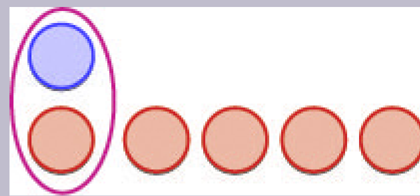


Figure 1.3.24

There are more negatives, so the sum is negative.

$$1+(-5)=-4$$

Try It

17) Add:

a. $-2+4$
b. $2+(-4)$

Solution

a. 2
b. -2

18) Add:

a. $-2+5$
b. $2+(-5)$

Solution

a. 3

b. -3

Now that we have added small positive and negative integers with a model, we can visualize the model in our minds to simplify problems with any numbers.

When you need to add numbers such as $37+(-53)$, you really don't want to have to count out 37 blue counters and 53 red counters. With the model in your mind, can you visualize what you would do to solve the problem?

Picture 37 blue counters with 53 red counters lined up underneath. Since there would be more red (negative) counters than blue (positive) counters, the sum would be *negative*. How many more red counters would there be? Because $53-37=16$, there are 16 more red counters.

Therefore, the sum of $37+(-53)$ is −16.

$$37+(-53)=-16$$

Let's try another one. We'll add $-74+(-27)$. Again, imagine 74 red counters and 27 more red counters, so we'd have 101 red counters. This means the sum is −101.

$$-74+(-27)=-101$$

Let's look again at the results of adding the different combinations of 5,−5 and 3,−3.

Addition of Positive and Negative Integers

$5+3$	$-5+(-3)$
8	-8
both positive, sum positive	both negative, sum negative

When the signs are the same, the counters would be all the same colour, so add them.

$-5+3$	$5+(-3)$
-2	2
different signs, more negatives, sum negative	different signs, more positives, sum positive

When the signs are different, some of the counters would make neutral pairs, so subtract to see how many are left.

Visualize the model as you simplify the expressions in the following examples.

Example 10

Simplify

a. $19+(-47)$
b. $-14+(-36)$

Solution
a. Since the signs are different, we subtract 19 from 47. The answer will be negative because there are more negatives than positives.

***Step 1:* Subtract**

$$19+(-47)$$
$$-28$$

---

b. Since the signs are the same, we add. The answer will be negative because there are only negatives.

***Step 1:* Add**

$$-14+(-36)$$
$$-50$$

Try It

19) Simplify:

a. $-31+(-19)$
b. $15+(-32)$

Solution

a. -50
b. -17

20) Simplify:

a. $-42+(-28)$
b. $25+(-61)$

Solution

a. -70
b. -36

The techniques used up to now extend to more complicated problems, like the ones we've seen before. Remember to follow the order of operations!

Example 11

Simplify: $-5+3(-2+7)$

Solution

Step 1: **Simplify inside the parentheses.**

$$-5+3(5)$$

Step 2: **Multiply.**

$$-5+15$$

Step 3: **Add left to right.**

$$10$$

Try It

21) Simplify: $-2+5(-4+7)$

Solution

13

22) Simplify: $-4+2(-3+5)$

Solution

0

Subtract Integers

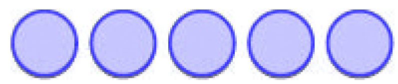
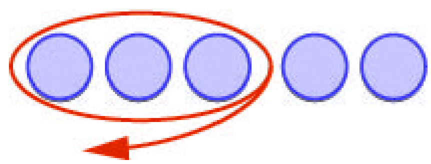
We will continue to use counters to model subtraction. Remember, the blue counters represent positive numbers and the red counters represent negative numbers.

Perhaps when you were younger, you read "5−3" as "5 take away 3". When you use counters, you can think of subtraction the same way!

We will model the four subtraction facts using the numbers 5 and 3.

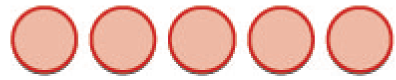
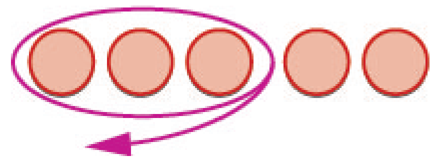
$$5-3 \qquad -5-(-3) \qquad -5-3 \qquad 5-(-3)$$

To subtract 5-3, we restate the problem as "5 takes away 3".

We start with 5 positives.	 Figure 1.3.25
We 'take away' 3 positives.	 Figure 1.3.26
We have 2 positives left.	The difference of 5 and 3 is 2.

Now we will subtract $-5-(-3)$. Watch for similarities to the last example $5-3=2$.

To subtract $-5-(-3)$, we restate this as "−5 take away −3".

We start with 5 negatives.	 Figure 1.3.27
We ‘take away’ 3 negatives.	 Figure 1.3.28
We have 2 negatives left.	The difference of -5 and -3 is -2.

Notice that these two examples are much alike: The first example, we subtract 3 positives from 5 positives and end up with 2 positives.

In the second example, we subtract 3 negatives from 5 negatives and end up with 2 negatives.

Each example used counters of only one colour, and the “take away” model of subtraction was easy to apply.

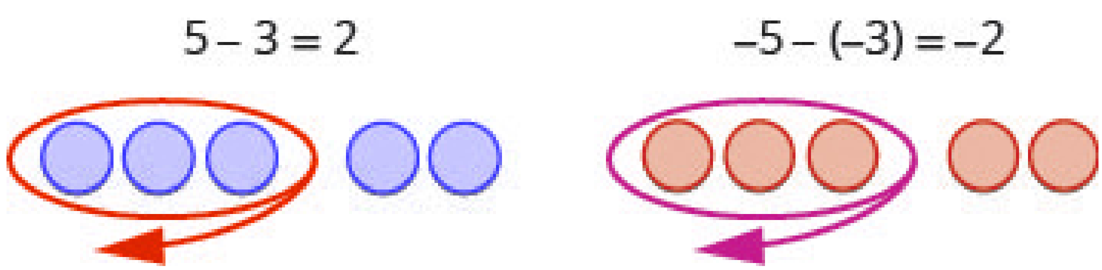


Figure 1.3.29

Example 12

Subtract:

a. $7-5$
b. $-7-(-5)$

Solution

a.

Step 1: **Take 5 positive from 7 positives and get 2 positives.**

$$7-5=2$$

---

b.

Step 1: **Take 5 negatives from 7 negatives and get 2 negatives.**

$$-7-(-5)=-2$$

Try It

23) Subtract:

a. $6-4$
b. $-6-(-4)$

Solution

a. 2
b. -2

24) Subtract:

a. $7 - 4$
b. $-7 - (-4)$

Solution

a. 3
b. -3

What happens when we have to subtract one positive and one negative number? We'll need to use both white and red counters as well as some neutral pairs. Adding a neutral pair does not change the value. It is like changing quarters to nickels—the value is the same, but it looks different.

- To subtract $-5 - 3$, we restate it as –5 take away 3.

We start with 5 negatives. We need to take away 3 positives, but we do not have any positives to take away.

Remember, a neutral pair has value zero. If we add 0 to 5 its value is still 5. We add neutral pairs to the 5 negatives until we get 3 positives to take away.

We start with 5 negatives.

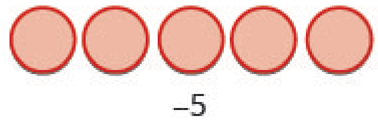


Figure 1.3.30

We now add the neutrals needed to get 3 positives.

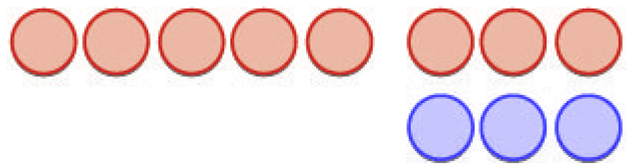


Figure 1.3.31

We remove the 3 positives.

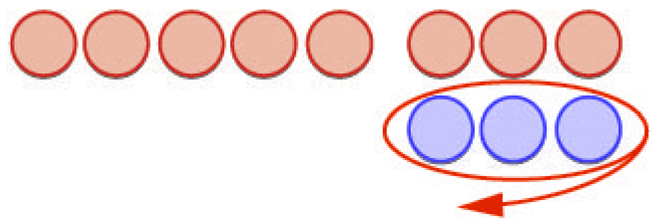


Figure 1.3.32

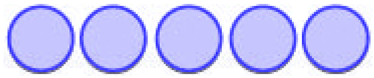
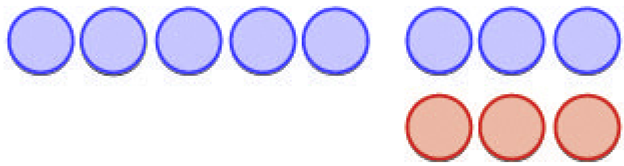
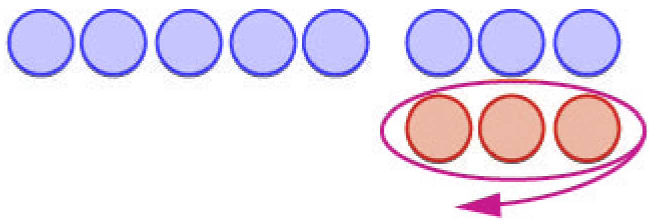
We are left with 8 negatives.

Figure 1.3.33

The difference of -5 and 3 is -8.

$$-5 - 3 = -8$$

And now, the fourth case, $5 - (-3)$. We start with 5 positives. We need to take away 3 negatives, but there are no negatives to take away. So we add neutral pairs until we have 3 negatives to take away.

We start with 5 positives.	 Figure 1.3.34
We now add the needed neutrals pairs.	 Figure 1.3.35
We remove the 3 negatives.	 Figure 1.3.36
We are left with 8 positives.	 Figure 1.3.37
The difference of 5 and −3 is 8.	$5-(-3)=8$

Example 13

Subtract:

a. $-3-1$
b. $3-(-1)$.

Solution

a.

Step 1: **Take 1 positive from the one added neutral pair.**

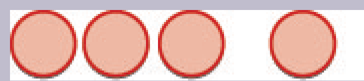


Figure 1.3.38

$-3-1$

1.3.39

$$-4$$

---

b.

Step 1: **Take 1 negative from the one added neutral pair.**

1.3.40

$3-(-1)$

Figure 1.3.41

4

Try It

25) Subtract:

a. $-6-4$
b. $6-(-4)$

Solution

a. -10
b. 10

26) Subtract:

a. $-7-4$
b. $7-(-4)$

Solution

a. -11
b. 11

Have you noticed that *subtraction of signed numbers can be done by adding the opposite*? In example 1.3.13 (above), $-3-1$ is the same as $-3+(-1)$ and $3-(-1)$ is the same as $3+1$. You will often see this idea, the subtraction property, written as follows:

Subtraction Property

$$a - b = a + (-b)$$

Subtracting a number is the same as adding its opposite.

Look at these two examples.

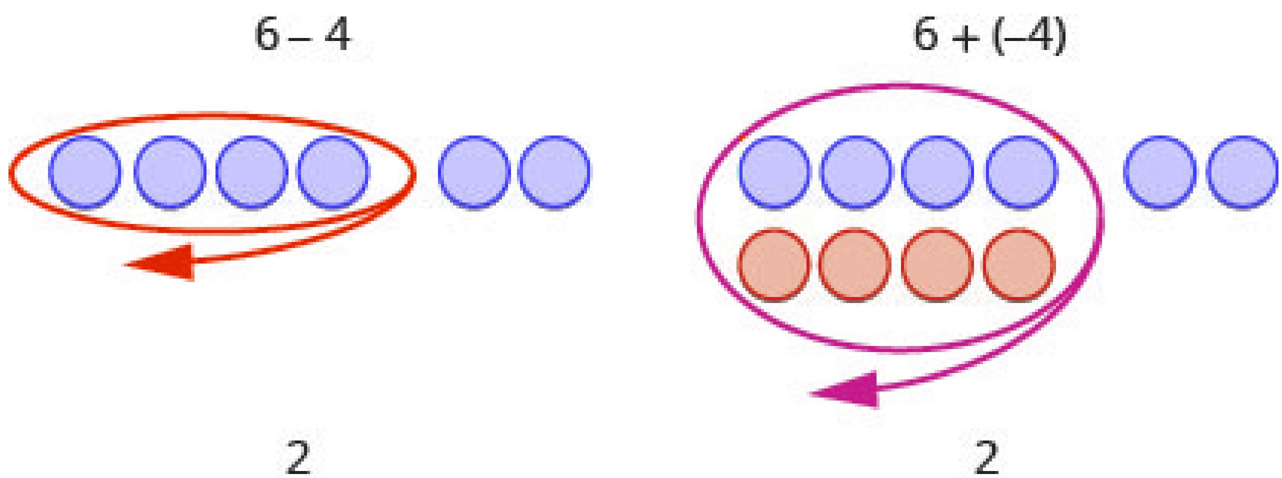


Figure 1.3.42

$6 - 4$ gives the same answer as $6 + (-4)$.

Of course, when you have a subtraction problem that has only positive numbers, like $6 - 4$, you just do the subtraction. You already knew how to subtract $6 - 4$ long ago. But *knowing* that $6 - 4$ gives the same answer as $6 + (-4)$ helps when you are subtracting negative numbers. Make sure that you understand how $6 - 4$ and $6 + (-4)$ give the same results!

Example 14

Simplify:

a. $13 - 8$ and $13 + (-8)$
b. $-17 - 9$ and $-17 + (-9)$

Solution

a.

Step 1: **Subtract.**

$$13 - 8 = 5$$
$$13 + (-8) = 5$$

---

b.

Step 1: **Subtract.**

$$-17 - 9 = -26$$
$$-17 + (-9) = -26$$

Try It

27) Simplify:

a. $21 - 13$ and $21 + (-13)$
b. $-11 - 7$ and $-11 + (-7)$.

Solution

a. 8

b. -18

28) Simplify:

a. $15 - 7$ and $15 + (-7)$

b. $-14 - 8$ and $-14 + (-8)$.

Solution

a. 8

b. -22

Look at what happens when we subtract a negative.

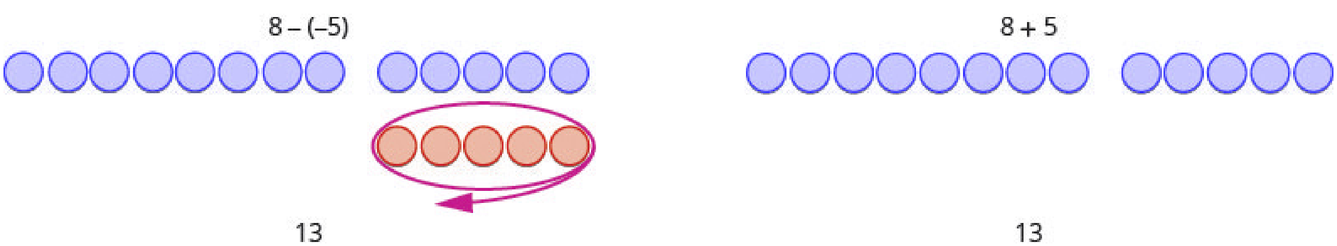


Figure 1.3.43

$8 - (-5)$ gives he same answer as $8 + 5$

Subtracting a negative number is like adding a positive!

You will often see this written as $a - b = a + b$.

Does that work for other numbers, too? Let's do the following example and see.

Example 15

Simplify:

a. $9-(-15)$ and $9+15$
b. $-7-(-4)$ and $-7+4$

Solution

a.

***Step 1:* Subtract.**

$$9-(-15)=24$$
$$9+15=24$$

---

b.

***Step 1:* Subtract.**

$$-7-(-4)=-3$$
$$-7+4=-3$$

Try It

29) Simplify

a. $6-(-13)$ and $6+13$
b. $-5-(-1)$ and $-5+1$

Solution

a. 19
b. -4

30) Simplify:

a. $4-(-19)$ and $4+19$
b. $-4-(-7)$ and $-4+7$

Solution

a. 23
b. 3

Let's look again at the results of subtracting the different combinations of 5,−5 and 3,−3.

Subtraction of Integers

$5-3$ 2 5 positives take 3 positives 2 positives	$-5-(-3)$ -2 5 negatives take away 3 negatives 2 negatives

When there would be enough counters of the colour to take away, subtract.

$-5-3$ -8 5 negatives, want to take away 3 positives need neutral pairs	$5-(-3)$ 8 5 positives, want to take away 3 negatives need neutral pairs

When there would be not enough counters of the colour to take away, add.

What happens when there are more than three integers? We just use the order of operations as usual.

Example 16

Simplify: $7-(-4-3)-9$

Solution

***Step 1:* Simplify inside the parentheses first.**

$$7-(-7)-9$$

***Step 2:* Subtract left to right**

$$14-9$$

***Step 3:* Subtract**

$$5$$

Try It

31) Simplify: $8-(-3-1)-9$

Solution

$$3$$

32) Simplify: $12-(-9-6)-14$

Solution

$$13$$

Multiply Integers

Since multiplication is mathematical shorthand for repeated addition, our model can easily be applied to show multiplication of integers. Let's look at this concrete model to see what patterns we notice. We will use the same examples that we used for addition and subtraction. Here, we will use the model just to help us discover the pattern.

We remember that $a\times b$ means add a, b times. Here, we are using the model just to help us discover the pattern.

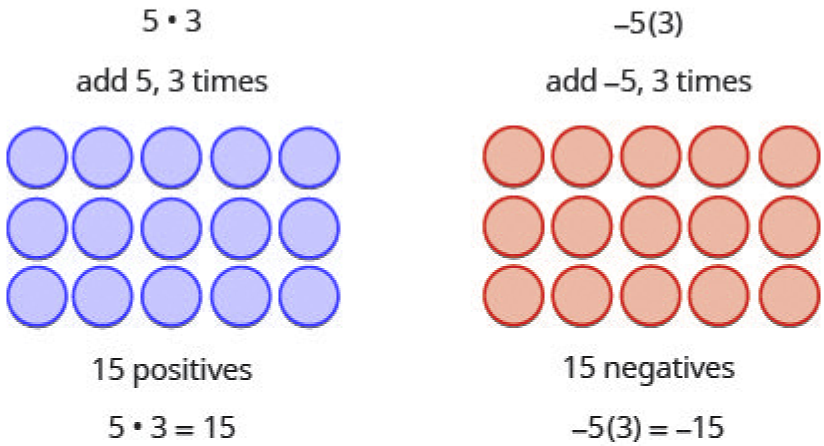


Figure 1.3.44

The next two examples are more interesting.

What does it mean to multiply 5 by −3? It means subtract 5, 3 times. Looking at subtraction as "taking away," it means to take away 5, 3 times. But there is nothing to take away, so we start by adding neutral pairs on the workspace. Then we take away 5 three times.

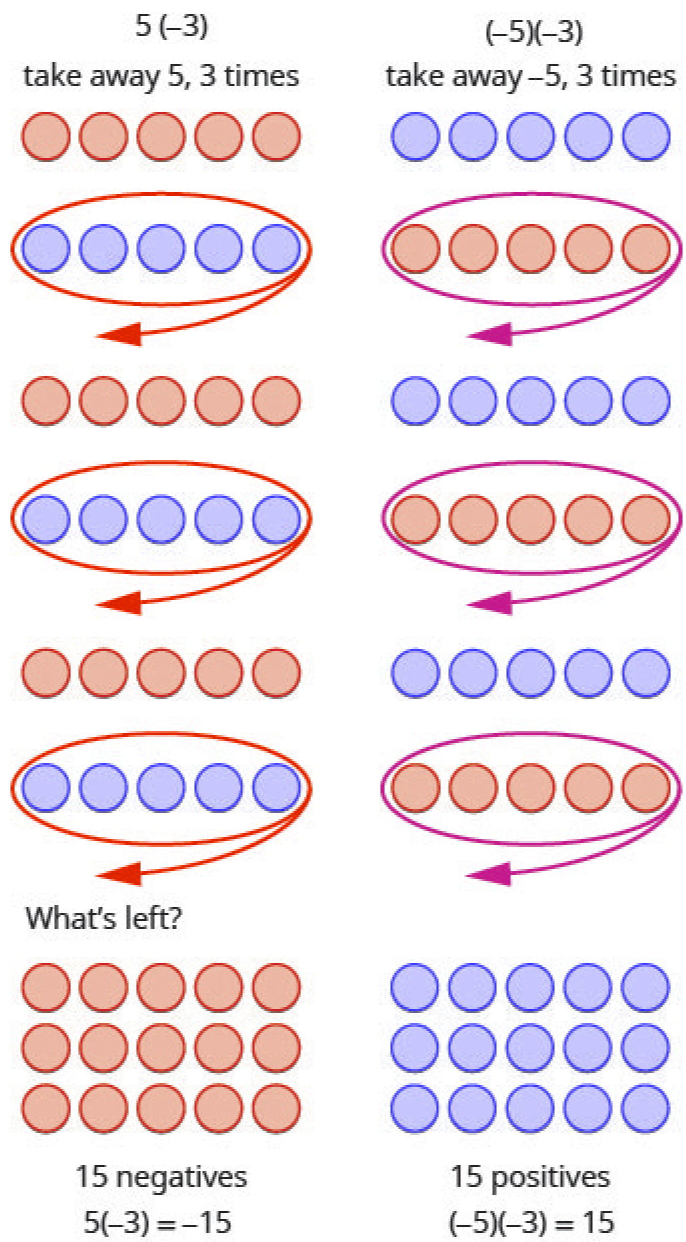


Figure 1.3.45

In summary:

$5 \times 3 = 15$	$-5 \times 3 = -15$
$5 \times -3 = -15$	$-5 \times -3 = 15$

Notice that for multiplication of two signed numbers, when the:

- signs are the *same*, the product is *positive*.

- signs are *different*, the product is *negative*.

We'll put this all together in the chart below.

Multiplication of Signed Numbers

For multiplication of two signed numbers:

Same signs	**Product**	**Example**
Two positives	Positive	$7 \times 4 = 28$
Two negatives	Positive	$-8 \times -6 = 48$

If the signs are the same, the result is positive.

Different signs	**Product**	**Example**
Positive and negative	Negative	$7 \times -9 = -63$
Negative and positive	Negative	$-5 \times 10 = -50$

If the signs are different, the result is negative.

Example 17

a. -9×3
b. $-2(-5)$
c. $4(-8)$
d. 7×6

Solution

a.

Step 1: **Multiply, noting that the signs are different so the product is negative.**

$$-9 \times 3 = -27$$

---

b.

***Step 1:* Multiply, noting that the signs are the same so the product is positive.**

$$-2(-5) = 10$$

---

c.

***Step 1:* Multiply, with different signs.**

$$4(-8) = -32$$

---

d.

***Step 1:* Multiply, with same signs.**

$$7 \times 6 = 42$$

Try It

33) Multiply:

a. -6×8
b. $-4(-7)$
c. $9(-7)$
d. 5×12

Solution

a. -48
b. 28
c. -63
d. 60

34) Multiply:

a. -8×7
b. $-6(-9)$
c. $7(-4)$
d. 3×13

Solution

a. -56
b. 54
c. -28
d. 39

When we multiply a number by 1, the result is the same number. What happens when we multiply a number by -1. Let's multiply a positive number and then a negative number by -1 to see what we get.

Each time we multiply a number by -1, we get its opposite!

Multiplication by -1

$-1a = -a$

Multiplying a number by -1 gives its opposite.

Example 18

Multiply:

a. -1×7
b. $-1(-11)$

Solution

a.

Step 1: **Multiply, noting that the signs are different so the product is negative.**

$$-1 \times 7 = -7$$

-7 is the opposite of 7.

---

b.

Step 1: **Multiply, noting that the signs are the same so the product is positive.**

$$-1(-11) = 11$$

11 is the opposite of -11.

Try It

35) Multiply:

a. -1×9
b. $-1(-17)$

Solution

a. -9
b. 17

36) Multiply:

a. -1×8
b. $-1(-16)$

Solution

a. -8
b. 16

Divide Integers

What about division? The division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $5 \times 3 = 15$ In words, this expression says that 15 can be divided into three groups of five each because adding five three times gives 15. Look at some examples of multiplying integers, to figure out the rules for dividing integers.

Division follows the same rules as multiplication!

For the division of two signed numbers, when the:

- signs are the *same*, and the quotient is *positive*.
- signs are *different*, the quotient is *negative*.

And remember that we can always check the answer to a division problem by multiplying.

Multiplication and Division of Signed Numbers

For multiplication and division of two signed numbers:

- If the signs are the same, the result is positive.
- If the signs are different, the result is negative.

Same signs	**Result**
Two positives	Positive
Two negatives	Positive
If the signs are the same, the result is positive.	

Different signs	**Result**
Positive and negative	Negative
Negative and positive	Negative
If the signs are different, the result is negative.	

Example 19

Divide:

a. $(-27) \div 3$
b. $-100 \div (-4)$

Solution

a.

Step 1: **Divide. With different signs, the quotient is negative.**

$$-27 \div 3 = -9$$

---

b.

Step 1: **Divide. With signs that are the same, the quotient is positive.**

$$-100 \div (-4) = 25$$

Try It

37) Divide:

a. $-42 \div 6$
b. $-117 \div -3$

Solution

a. -7
b. 39

38) Divide:

a. $-63 \div 7$

b. $-115 \div -5$

Solution

a. -9

b. 23

Simplify Expressions with Integers

What happens when there are more than two numbers in an expression? The order of operations still applies when negatives are included. Remember My Dear Aunt Sally?

Let's try some examples. We'll simplify expressions that use all four operations with integers—addition, subtraction, multiplication, and division. Remember to follow the order of operations.

Example 20

Simplify: $7(-2)+4(-7)-6$.

Solution

Step 1: **Multiply first.**

$$-14+(-28)-6$$

Step 2: **Add.**

$$-42-6$$

Step 3: **Subtract.**

$$-48$$

Try It

39) Simplify: $8(-3)+5(-7)-4$

Solution

-63

40) Simplify: $9(-3)+7(-8)-1$

Solution

-84

Example 21

Simplify:

a. $(-2)^4$
b. -2^4

Solution

a.

Step 1: **Write in expanded from.**

$$(-2)(-2)(-2)(-2)$$

Step 2: **Multiply.**

$$4(-2)(-2)$$

Step 3: **Multiply.**

$$-8(-2)$$

***Step 4:* Multiply.**

$$16$$

---

b.

***Step 1:* Write in expanded form. We are asked to find the opposite of 2^4.**

$$-(2\times2\times2\times2)$$

***Step 2:* Multiply.**

$$-(4\times2\times2)$$

***Step 3:* Multiply.**

$$-(8\times2)=-16$$

Notice the difference in parts a and b. In part a, the exponent means to raise what is in the parentheses, the (−2) to the 4^{th} power. In part b, the exponent means to raise just the 2 to the 4^{th} power and then take the opposite.

Try It

41) Simplify:

a. $(-3)^4$

b. -3^4

Solution

a. 81
b. -81

42) Simplify:

a. $(-7)^2$
b. -7^2

Solution

a. 49
b. -49

The next example reminds us to simplify inside parentheses first.

Example 22

Simplify: $12 - 3(9 - 12)$

Solution

Step 1: **Subtract in parentheses first.**

$$12 - 3(-3)$$

Step 2: **Multiply.**

$$12 - (-9)$$

Step 3: **Subtract.**

$$21$$

Try It

43) Simplify: $17 - 4(8 - 11)$

Solution

$$29$$

44) Simplify: $16 - 6(7 - 13)$

Solution

$$52$$

Example 23

Simplify: $8(-9) \div (-2)^3$

Solution

Step 1: **Exponents first.**

$$8(-9) \div (-8)$$

Step 2: **Multiply**

$$-72 \div -8$$

***Step 3:* Divide.**

$$9$$

Try It

45) Simplify: $12(-9) \div (-3)^3$

Solution

$$4$$

46) Simplify: $18(-4) \div (-2)^3$

Solution

$$9$$

Example 24

Simplify: $-30 \div +2 + (-3)(-7)$

Solution

Step 1: **Multiply and divide left to right, so divide first.**

$$-15 + (-3)(-7)$$

Step 2: **Add.**

$$-15 + 21 = 6$$

Try It

47) Simplify $-27 \div 3 + (-5)(-6)$

Solution

$$21$$

48) Simplify $-32 \div 4 + (-2)(-7)$

Solution

$$6$$

Evaluate Variable Expressions with Integers

Remember that to evaluate an expression means to substitute a number for the variable in the expression. Now we can use negative numbers as well as positive numbers.

Example 25

When $n = -5$, evaluate:

a. $n + 1$
b. $-n + 1$

Solution
a.

Step 1: **Substitute -5 for n.**

$$-5 + 1$$

Step 2: **Simplify.**

$$-4$$

---

b.

Step 1: **Substitute -5 for n.**

$$-(-5) + 1$$

Step 2: **Simplify.**

5+1

Step 3: **Add.**

$$6$$

Try It

49) When $n = -8$, evaluate

a. $n + 2$
b. $-n + 2$

Solution

a. -6
b. 10

50) When $y = -9$, evaluate

a. $y + 8$
b. $-y + 8$.

Solution

a. -1
b. 17

Example 26

Evaluate $(x+y)^2$ when $x=-18$ and $y=24$.

Solution

***Step 1:* Substitute -18 for x and 24 for y.**

$$(-18+24)^2$$

***Step 2:* Add inside parentheses.**

$$(6)^2$$

***Step 3:* Simplify.**

$$36$$

Try It

51) Evaluate $(x+y)^2$ when $x=-15$ and $y=29$

Solution
196

52) Evaluate $(x+y)^3$ when $x=-8$ and $y=10$

Solution

$$8$$

Example 27

Evaluate $20 - z$ when

a. $z = 12$
b. $z = -12$

Solution

a.

Step 1: **Substitute 12 for z.**

$$20 - 12$$

Step 2: **Subtract.**

$$8$$

---

b.

Step 1: **Substitute -12 for z.**

$$20 - (-12)$$

Step 2: **Subtract.**

$$32$$

Try It

53) Evaluate: $17 - k$ when

a. $k = 19$
b. $k = -19$

Solution

a. -2
b. 36

54) Evaluate: $-5 - b$ when

a. $b = 14$
b. $b = -14$

Solution

a. -19
b. 9

Example 28

Evaluate: $2x^2 + 3x + 8$ when $x = 4$.

Solution

Substitute 4 for x. Use parentheses to show multiplication.

***Step 1:* Substitute.**

$$2(4)^2 + 3(4) + 8$$

***Step 2:* Evaluate exponents.**

$$2(16) + 3(4) + 8$$

***Step 3:* Multiply.**

$$32 + 12 + 8$$

***Step 4:* Add.**

$$52$$

Try It

55) Evaluate: $3x^2 - 2x + 6$ when $x = -3$.

Solution

39

56) Evaluate: $4x^2 - x - 5$ when $x = -2$.

Solution

13

Translate English Phrases to Algebraic Expressions with Integers

Our earlier work translating English to algebra also applies to phrases that include both positive and negative numbers.

Example 29

Translate and simplify: the sum of 8 and -12, increased by 3.

Solution

Step 1: Translate.

$$[8 + (-12)] + 3$$

***Step 2:* Simplify. Be careful not to confuse the brackets with an absolute value sign.**

$$(-4) + 3$$

***Step 3:* Add.**

$$-1$$

Try It

57) Translate and simplify the sum of 9 and -16, increased by 4.

Solution

$(9-16)+4=-3$

58) Translate and simplify the sum of -8 and -12, increased by 7.

Solution

$(-8-12)+7=-13$

When we first introduced the operation symbols, we saw that the expression may be read in several ways. They are listed below.

- $a-b$
- a minus b
- the difference of a and b

- b subtracted from a
- b less than a

Be careful to get a and b in the right order!

Example 30

Translate and then simplify

a. The difference of 13 and -21
b. Subtract 24 from -19

Solution

a.

***Step 1:* Translate.**

$$13-(-21)$$

***Step 2:* Simplify.**

$$34$$

b.

Step 1: **Translate. Remember, "subtract b from a" means $a - b$.**

$$-19 - 24$$

Step 2: **Simplify.**

$$-43$$

Try It

59) Translate and simplify

a. the difference of 14 and -23
b. subtract 21 from -17

Solution

a. $14 - (-23) = 37$
b. $-17 - 21 = -38$

60) Translate and simplify

a. the difference of 11 and -19

b. subtract 18 from -11.

Solution

a. $11-(-19)=30$
b. $-11-18=-29$

Once again, our prior work translating English to algebra transfers to phrases that include both multiplying and dividing integers. Remember that the keyword for multiplication is "product" and for division is "quotient."

Example 31

Translate to an algebraic expression and simplify if possible: the product of -2 and 14.

Solution
Step 1: **Translate.**

$$(-2)(14)$$

Step 2: **Simplify.**

$$-28$$

Try It

61) Translate to an algebraic expression and simplify if possible: the product of -5 and 12.

Solution

$-5(12) = -60$

62) Translate to an algebraic expression and simplify if possible: the product of 8 and -13.

Solution

$8(-13) = -104$

Example 32

Translate to an algebraic expression and simplify if possible: the quotient of -56 and -7.

Solution

Step 1: **Translate.**

$$-56 \div -7$$

Step 2: **Simplify.**

$$8$$

Try It

63) Translate to an algebraic expression and simplify if possible: the quotient of -63 and -9.

Solution

$-63 \div -9 = 7$

64) Translate to an algebraic expression and simplify if possible: the quotient of -72 and -9.

Solution

$-72 \div -9 = 8$

Use Integers in Applications

We'll outline a plan to solve applications. It's hard to find something if we don't know what we're looking for or what to call it! So when we solve an application, we first need to determine what the problem is asking us to find. Then we'll write a phrase that gives the information to find it. We'll translate the phrase into an expression and then simplify the expression to get the answer. Finally, we summarize the answer in a sentence to make sure it makes sense.

How to Apply a Strategy to Solve Applications with Integers

Example 33

The temperature in Urbana, Illinois one morning was 11 degrees. By mid-afternoon, the temperature had dropped to -9 degrees. What was the difference of the morning and afternoon temperatures?

Solution

Step 1: **Read the problem. Make sure all the words and ideas are understood.**

Step 2: **Identify what we are asked to find.**

The difference of the morning and afternoon temperatures.

Step 3: **Write a phrase that gives the information to find it.**

The *difference of* 11 and -9.

Step 4: **Translate the phrase to an expression.**

$$11-(-9)$$

Step 5: **Simplify the expression.**

$$20$$

Step 6: **Write a complete sentence that answers the question.**

The difference in temperatures was 20 degrees.

Try It

65) The temperature in Anchorage, Alaska one morning was 15 degrees. By mid-afternoon the temperature had dropped to 30 degrees below zero. What was the difference in the morning and afternoon temperatures?

Solution

The difference in temperatures was 45 degrees.

66) The temperature in Denver was -6 degrees at lunchtime. By sunset the temperature had dropped to -15 degrees. What was the difference in the lunchtime and sunset temperatures?

Solution

The difference in temperatures was 9 degrees.

How to

Apply a Strategy to Solve Applications with Integers.

1. Read the problem. Make sure all the words and ideas are understood

2. Identify what we are asked to find.
3. Write a phrase that gives the information to find it.
4. Translate the phrase into an expression.
5. Simplify the expression.
6. Answer the question with a complete sentence.

Example 34

The Mustangs football team received three penalties in the third quarter. Each penalty gave them a loss of fifteen yards. What is the number of yards lost?

Solution

Step 1: **Read the problem. Make sure all the words and ideas are understood.**

Step 2**: Identify what we are asked to find.**

The number of yards lost.

Step 3**: Write a phrase that gives the information to find it.**

Three times a 15-yard penalty.

Step 4**: Translate the phrase to an expression.**

$$3(-15)$$

Step 5**: Simplify the expression.**

$$-45$$

Step 6**: Answer the question with a complete sentence.**

The team lost 45 yards.

Try It

67) The Bears played poorly and had seven penalties in the game. Each penalty resulted in a loss of 15 yards. What is the number of yards lost due to penalties?

Solution

The Bears lost 105 yards.

68) Bill uses the ATM on campus because it is convenient. However, each time he uses it he is charged a $2 fee. Last month he used the ATM eight times. How much was his total fee for using the ATM?

Solution

A $16 fee was deducted from his checking account.

Key Concepts

- **Multiplication and Division of Two Signed Numbers**
  - Same signs—Product is positive
  - Different signs—Product is negative
- **Strategy for Applications**
  - Identify what you are asked to find.
  - Write a phrase that gives the information to find it.
  - Translate the phrase to an expression.
  - Simplify the expression.

- Answer the question with a complete sentence

- **Addition of Positive and Negative Integers**

$5+3=8$ both positive, sum positive	$-5+(-3)=-8$ both negative, sum negative
$-5+3=-2$ different signs, more negatives sum negative	$5+(-3)=2$ different signs, more positives sum positive

- **Property of Absolute Value**: $|n| \geq 0$ for all numbers. Absolute values are always greater than or equal to zero!
- **Subtraction of Integers**

$5-3=2$ 5 positives take away 3 positives 2 positive	$-5-(-3)=-2$ 5 negatives, take away 3 negatives 2 negatives
$-5-3=-8$ 5 negatives, want to subtract 3 positives need neutral pairs	$5-(-3)=2$ 5 positives, want to subtract 3 negatives need neutral pairs

- **Subtraction Property:** Subtracting a number is the same as adding its opposite.
- **Multiplication and Division of Two Signed Numbers**
  - Same signs—Product is positive
  - Different signs—Product is negative
- **Strategy for Applications**
  1. Identify what you are asked to find.
  2. Write a phrase that gives the information to find it.
  3. Translate the phrase to an expression.
  4. Simplify the expression.
  5. Answer the question with a complete sentence

Glossary

absolute value

The absolute value of a number is its distance from 0 on the number line. The absolute value of a number n is written as $|n|$.

integers

The whole numbers and their opposites are called the integers: ...−3, −2, −1, 0, 1, 2, 3...

opposite

The opposite of a number is the number that is the same distance from zero on the number line but on the opposite side of zero: $-a$ means the opposite of the number a. The notation $-a$ is read as "the opposite of a.

Exercises: Multiply Integers

Instructions: For questions 1-8, multiply.

1) $-4 \cdot 8$

2) $-3 \cdot 9$

3) $9(-7)$

4) $13(-5)$

5) $-1 \cdot 6$

6) $-1 \cdot 3$

7) $-1(-14)$

8) $-1(-19)$

Odd Answers

1) -32

3) -63

5) -6

7) 14

Exercises: Divide Integers

Instructions: For questions 9-14, divide.

9) $-24 \div 6$

10) $35 \div (-7)$

11) $-52 \div (-4)$

12) $-84 \div (-6)$

13) $-180 \div 15$

14) $-192 \div 12$

Odd Answers

9) -4

11) 13

13) -12

Exercises: Simplify Expressions with Integers

Instructions: For questions 15-32, simplify each expression.

15) $5(-6)+7(-2)-3$

16) $8(-4)+5(-4)-6$

17) $(-2)^6$

18) $(-3)^5$

19) -4^2

20) -6^2

21) $-3(-5)(6)$

22) $-4(-6)(3)$

23) $(8-11)(9-12)$

24) $(6-11)(8-13)$

25) $26-3(2-7)$

26) $23-2(4-6)$

27) $65 \div (-5) + (-28) \div (-7)$

28) $52 \div (-4) + (-32) \div (-8)$

29) $9 - 2\left[3 - 8(-2)\right]$

30) $11 - 3\left[7 - 4(-2)\right]$

31) $(-3)^2 - 24 \div (8 - 2)$

32) $(-4)^2 - 32 \div (12 - 4)$

Odd Answers

15) -47

17) 64

19) -16

21) 90

23) 9

25) 41

27) -9

29) -29

31) 5

Exercises: Evaluate Variable Expressions with Integers

Instructions: For questions 33-50, evaluate each expression.

33) $y+(-14)$ **when**

- **a)** $y=-33$
- **b)** $y=30$

34) $x+(-21)$ **when**

- **a)** $x=-27$
- **b)** $x=44$

35a) $a+3$ **when** $a=-7$

35b) $-a+3$ **when** $a=-7$

36a) $d+(-9)$ **when** $d=-8$

36b) $-d+(-9)$ **when** $d=-8$

37) $m+n$ **when** $m=-15, n=7$

38) $p+q$ **when** $p=-9, q=17$

39) $r+s$ **when** $r=-9, s=-7$

40) $t+u$ **when** $t=-6, u=-5$

41) $(x+y)^2$ **when** $x=-3, y=14$

42) $(y+z)^2$ **when** $y=-3, z=15$

43) $-2x+17$ **when**

- **a)** $x=8$
- **b)** $x=-8$

44) $-5y+14$ **when**

- **a)** $y=9$
- **b)** $y=-9$

45) $10 - 3m$ **when**

a) $m = 5$

b) $m = -5$

46) $18 - 4n$ **when**

a) $n = 3$

b) $n = -3$

47) $2w^2 - 3w + 7$ **when** $w = -2$

48) $3u^2 - 4u + 5$ **when** $u = -3$

49) $9a - 2b - 8$ **when** $a = -6, b = -3$

50) $7m - 4n - 2$ **when** $m = -4, n = -9$

Odd Answers

33a) -47

33b) 16

35a) -4

35b) 10

37) -8

39) -16

41) 121

43a) 1

43b) 33

45a) -5

45b) 25

47) 21

49) -56

Exercises: Translate English Phrases to Algebraic Expressions

Instructions: For questions 51-64, translate to an algebraic expression and simplify if possible.

51) the sum of 3 **and** -15**, increased by** 7

52) the sum of -8 **and** -9**, increased by** 23

53) the difference of 10 **and** -18

54) subtract 11 **from** -25

55) the difference of -5 **and** -30

56) subtract -6 **from** -13

57) the product of -3 and 15

58) the product of -4 and 16

59) the quotient of -60 and -20

60) the quotient of -40 and -20

61) the quotient of -6 and the sum of a and b

62) the quotient of -7 and the sum of m and n

63) the product of -10 and the difference of p and q

64) the product of -13 and the difference of c and d

Odd Answers

51) $(3+(-15))+7$

$(3+(-15))+7=-5$

53) $10-(-18)$

$10-(-18)=28$

55) $-5-(-30)$

$-5-(-30)=25$

57) $-3 \cdot 15$

$-3 \cdot 15 = -45$

59) $-60 \div (-20)$

$-60 \div (-20) = 3$

61) $\dfrac{-6}{a+b}$

63) $-10(p-q)$

Exercises: Use Integers in Applications

Instructions: For questions 65-72, solve the given word problems.

65) Temperature: On January 15, the high temperature in Anaheim, California, was 84°. That same day, the high temperature in Embarrass, Minnesota was -12°. What was the difference between the temperature in Anaheim and the temperature in Embarrass?

66) Temperature: On January 21, the high temperature in Palm Springs, California, was 89°, and the high temperature in Whitefield, New Hampshire was -31°. What was the difference between the temperature in Palm Springs and the temperature in Whitefield

67) Football: At the first down, the Chargers had the ball on their 25 yard

line. On the next three downs, they lost 6 yards, gained 10 yards, and lost 8 yards. What was the yard line at the end of the fourth down?

68) Football: At the first down, the Steelers had the ball on their 30 yard line. On the next three downs, they gained 9 yards, lost 14 yards, and lost 2 yards. What was the yard line at the end of the fourth down?

69) Checking Account: Mayra has $\$124$ in her checking account. She writes a check for $\$152$. What is the new balance in her checking account?

70) Checking Account: Selina has $\$165$ in her checking account. She writes a check for $\$207$. What is the new balance in her checking account?

71) Checking Account: Diontre has a balance of $-\$38$ in his checking account. He deposits $\$225$ to the account. What is the new balance?

72) Checking Account: Reymonte has a balance of $-\$49$ in his checking account. He deposits $\$281$ to the account. What is the new balance?

Odd Answers

65) 96°

67) 21

69) $-\$28$

71) $\$187$

Exercises: Everyday Math

Instructions: For questions 73-74, solve the given everyday math word problems.

73) Stock market: Javier owns 300 shares of stock in one company. On Tuesday, the stock price dropped $\$12$ per share. What was the total effect on Javier's portfolio?

74) Weight loss: In the first week of a diet program, eight women lost an average of 3 pounds each. What was the total weight change for the eight women?

Odd Answers

73) $-\$3600$

Exercises: Writing Exercises

Instructions: For questions 75-78, answer the given writing exercises.

75) In your own words, state the rules for multiplying integers.

76) In your own words, state the rules for dividing integers.

77) Why is $-2^4 \neq (-2)^4$?

78) Why is $-4^3 = (-4)^3$?

Odd Answers

75) Answers may vary

77) Answers may vary

1.4 FRACTIONS

Learning Objectives

By the end of this section, you will be able to:

- Find equivalent fractions
- Simplify fractions
- Multiply fractions
- Divide fractions
- Simplify expressions written with a fraction bar
- Translate phrases to expressions with fractions
- Add or subtract fractions with a common denominator
- Add or subtract fractions with different denominators
- Use the order of operations to simplify complex fractions
- Evaluate variable expressions with fractions

Find Equivalent Fractions

Fractions are a way to represent parts of a whole. The fraction $\frac{1}{3}$ means that one whole has been divided into 3 equal parts and each part is one of the three equal parts. See (Figure 1.4.1). The fraction $\frac{2}{3}$

represents two of three equal parts. In the fraction $\dfrac{2}{3}$ the 2 is called the **numerator** and the 3 is called the **denominator**.

The circle on the left has been divided into 3 equal parts. Each part is $\dfrac{1}{3}$ of the 3 equal parts. In the circle on the right, $\dfrac{2}{3}$ of the circle is shaded (2 of the 3 equal parts).

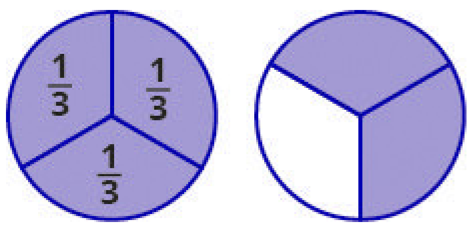


Figure 1.4.1. The circle on the left has been divided into 3 equal parts. Each part is 1/3 of the 3 equal parts. In the circle on the right, 2/3 of the circle is shaded (2 of the 3 equal parts).

Fraction

A fraction is written $\dfrac{a}{b}$ where $b \neq 0$ and

- a is the *numerator* and b is the *denominator*

A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

If a whole pie has been cut into 6 pieces and we eat all 6 pieces, we ate $\dfrac{6}{6}$ pieces, or, in other words, one whole pie.

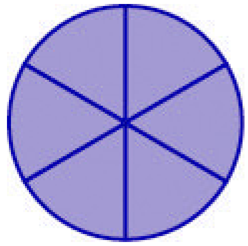


Figure 1.4.2

So $\frac{6}{6} = 1$. This leads us to the property of one that tells us that any number, except zero, divided by itself is 1.

Property of One

$\frac{a}{a} = 1 \quad (a \neq 0)$. Any number, except zero, divided by itself is one.

If a pie was cut into 6 pieces and we ate all 6, we ate $\frac{6}{6}$ pieces, or, in other words, one whole pie. If the pie was cut into 8 pieces and we ate all 8, we ate $\frac{8}{8}$ pieces, or one whole pie. We ate the same amount—one whole pie.

The fractions $\frac{6}{6}$ and $\frac{8}{8}$ have the same value, 1, and so they are called equivalent fractions.

Equivalent fractions are fractions that have the same value.

Let's think of pizzas this time. Figure 1.4.2 shows two images: a single pizza on the left, cut into two equal pieces, and a second pizza of the same size, cut into eight pieces on the right. This is a way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. In other words, they are equivalent fractions.

Since the same amount of each pizza is shaded, we see that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. They are equivalent fractions.

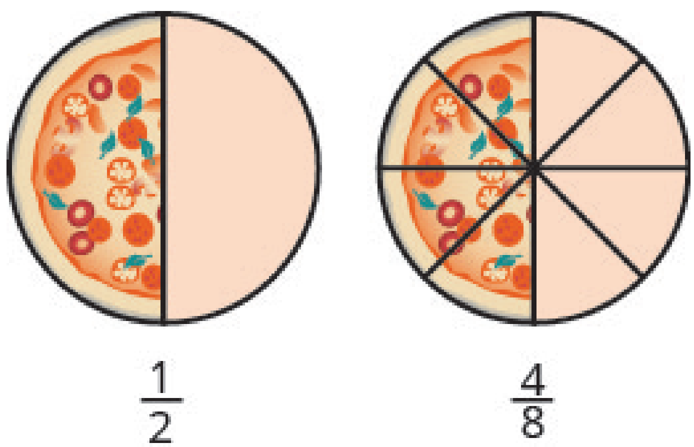


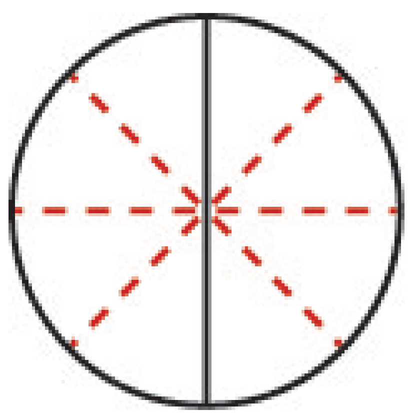
Figure 1.4.3. Since the same amount is of each pizza is shaded, we see that [latex]\frac{1}{2}[/latex] is equivalent to [latex]\frac{4}{8}[/latex]. They are equivalent fractions.

Equivalent Fractions

Equivalent fractions are fractions that have the same value.

How can we use mathematics to change $\frac{1}{2}$ into $\frac{4}{8}$? How could we take a pizza that is cut into 2 pieces and cut it into 8 pieces? We could cut each of the 2 larger pieces into 4 smaller pieces! The whole pizza would then be cut into 8 pieces instead of just 2. Mathematically, what we've described could be written like this as $\frac{1\times 4}{2\times 4}=\frac{4}{8}$. See Figure. 1.4.3.

Cutting each half of the pizza into 4 pieces, gives us pizza cut into 8 pieces: $\frac{1\times 4}{2\times 4}=\frac{4}{8}$.



1.4.4. Cutting each half of the pizza into 4 pieces, gives us pizza cut into 8 pieces.

This model leads to the following property:

Equivalent Fractions Property

If a, b, c are numbers where $b \neq 0, c \neq 0$, then

$\frac{a}{b} = \frac{ac}{bc}$ and $\frac{ac}{bc} = \frac{a}{b}$

If we had cut the pizza differently, we could get

$$\frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4} \quad \text{so} \quad \frac{1}{2} = \frac{2}{4}$$

$$\frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \quad \text{so} \quad \frac{1}{2} = \frac{3}{6}$$

$$\frac{1 \cdot 10}{2 \cdot 10} = \frac{10}{20} \quad \text{so} \quad \frac{1}{2} = \frac{10}{20}$$

So, we say $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \text{and} \frac{10}{20}$ are equivalent fractions.

Example 1

Find three fractions equivalent to $\frac{2}{5}$

Solution

To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number. We can choose any number, except for zero. Let's multiply them by 2, 3, and then 5.

$$\frac{2 \times 2}{5 \times 2} = \frac{4}{10} \qquad \frac{2 \times 3}{5 \times 3} = \frac{6}{15} \qquad \frac{2 \times 5}{5 \times 5} = \frac{10}{25}$$

So, $\frac{4}{10}, \frac{6}{15}, \text{and} \frac{10}{25}$ are equivalent to $\frac{2}{5}$.

Try It

1) Find three fractions equivalent to $\frac{3}{5}$

Solution

$\frac{6}{10}, \frac{9}{15}, \frac{12}{20}$; answers may vary

2) Find three fractions equivalent to $\frac{4}{5}$

Solution

$\frac{8}{10}, \frac{12}{15}, \frac{16}{20}$; answers may vary

Simplify Fractions

A fraction is considered *simplified* if there are no common factors, other than 1, in its numerator and denominator.

For example,

- $\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.
- $\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

Simplify Fraction

A fraction is considered simplified if there are no common factors in its numerator and denominator.

The phrase *reduces a fraction* means to simplify the fraction. We simplify, or reduce, a fraction by removing the common factors of the numerator and denominator. A fraction is not simplified until all common factors have been removed. If an expression has fractions, it is not completely simplified until the fractions are simplified.

In Example 1.4.2, we used the equivalent fractions property to find equivalent fractions. Now we'll use the equivalent fractions property in reverse to simplify fractions. We can rewrite the property to show both forms together.

Equivalent Fractions Property

If a, b, c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{ac}{bc}$ and $\frac{ac}{bc} = \frac{a}{b}$

Example 2

Simplify: $-\frac{32}{56}$.

Solution

Step 1: **Rewrite the numerator and denominator showing the common factors.**

$$-\frac{4 \times 8}{7 \times 8}$$

Step 2: **Simplify using the equivalent fractions property.**

$$-\frac{4}{7}$$

Notice that the fraction $-\frac{4}{7}$ is simplified because there are no more common factors.

Try It

3) Simplify: $-\frac{42}{54}$

Solution

$$-\frac{7}{9}$$

4) Simplify: $-\frac{45}{81}$

Solution

$$-\frac{5}{9}$$

Sometimes it may not be easy to find common factors of the numerator and denominator. When this happens, a good idea is to factor the numerator and the denominator into prime numbers. Then divide out the common factors using the equivalent fractions property.

Example 3

Simplify: $-\frac{210}{385}$

Solution

Step 1: **Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers first.**

Rewrite 210 and 385 as the product of the primes.

$$\begin{aligned} &= -\frac{210}{385} \\ &= -\frac{2 \times 3 \times 5 \times 7}{5 \times 7 \times 11} \end{aligned}$$

Step 2: **Simplify using the equivalent fractions property by dividing out common factors.**

Mark the common factors 5 and 7. Divide out the common factors.

$$\begin{aligned} &= -\frac{2 \times 3 \times \cancel{5} \times \cancel{7}}{\cancel{5} \times \cancel{7} \times 11} \\ &= -\frac{2 \times 3}{11} \end{aligned}$$

Step 3: **Multiply the remaining factors, if necessary.**

$$-\frac{6}{11}$$

Try It

5) Simplify: $-\frac{69}{120}$

Solution

$$-\frac{23}{40}$$

6) Simplify: $-\frac{120}{192}$

Solution

$$-\frac{5}{8}$$

We now summarize the steps you should follow to simplify fractions.

How to

Simplify a Fraction.

1. Rewrite the numerator and denominator to show the common factors.
   If needed, factor the numerator and denominator into prime numbers first.
2. Simplify using the equivalent fractions property by dividing out common factors.
3. Multiply any remaining factors, if needed.

Example 4

Simplify: $\dfrac{5x}{5y}$.

Solution

Step 1: Rewrite showing the common factors, then divide out the common factors.

$$\frac{\cancel{5} \times x}{\cancel{5} \times y}$$

***Step 2:* Simplify.**

$$\frac{x}{y}$$

Try It

7) Simplify: $\frac{7x}{7y}$

Solution

$$\frac{x}{y}$$

8)Simplify: $\frac{3a}{3b}$

Solution

$$\frac{a}{b}$$

Multiply Fractions

Many people find multiplying and dividing fractions easier than adding and subtracting fractions. So we will start with fraction multiplication.

We'll use a model to show you how to multiply two fractions and to help you remember the procedure. Let's start with $\frac{3}{4}$.

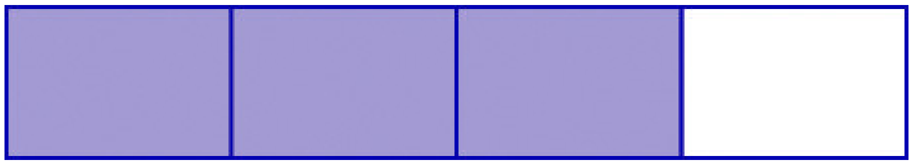


Figure 1.4.5

Now we'll take $\frac{1}{2}$ of $\frac{3}{4}$.

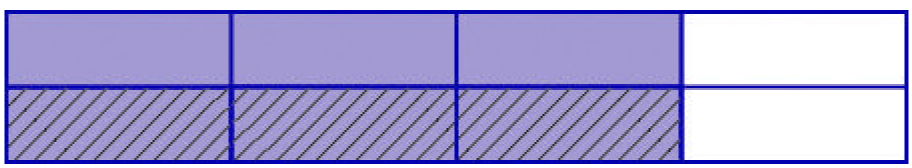


Figure 1.4.6

Notice that now, the whole is divided into 8 equal parts. So $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

To multiply fractions, we multiply the numerators and multiply the denominators.

Fraction Multiplication

If a, b, c and d are numbers where $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

To multiply fractions, multiply the numerators and multiply the denominators.

When multiplying fractions, the properties of positive and negative numbers still apply, of course. It is a good idea to determine the sign of the product as the first step. In Example 1.4.5, we will multiply negative and positive, so the product will be negative.

Example 5

Multiply: $-\frac{11}{12} \times \frac{5}{7}$

Solution

The first step is to find the sign of the product. Since the signs are different, the product is negative.

Step 1: **Determine the sign of the product; multiply.**

$$-\frac{11 \times 5}{12 \times 7}$$

Step 2: **Are there any common factors in the numerator and the denominator? No.**

$$-\frac{55}{84}$$

Try It

9) Multiply: $-\frac{10}{28} \times \frac{8}{15}$

Solution

$$-\frac{4}{21}$$

10) Multiply: $-\frac{9}{20} \times \frac{5}{12}$

Solution

$$-\frac{3}{16}$$

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a, can be written as $\frac{a}{1}$. So, for example, $3 = \frac{3}{1}$.

Example 6

Multiply: $-\frac{12}{5}(-20x)$

Solution

Determine the sign of the product. The signs are the same, so the product is positive.

Step 1: **Write $20x$ as a fraction.**

$$\frac{12}{5}\left(\frac{20x}{1}\right)$$

Step 2: **Multiply.**

Step 3: **Rewrite 20 to show the common factor 5 and divide it out.**

$$\frac{12 \times 4 \times \cancel{5}x}{\cancel{5} \times 1}$$

Step 4: **Simplify.**

$$48x$$

Try It

11) Multiply: $\frac{11}{3}(-9a)$

Solution
$-33a$

12) Multiply: $\frac{13}{7}(-14b)$

Solution
$-26b$

Divide Fractions

Now that we know how to multiply fractions, we are almost ready to divide. Before we can do that, we need some vocabulary.

The **reciprocal** of a fraction is found by inverting the fraction and placing the numerator in the denominator and the denominator in the numerator. The reciprocal of $\frac{2}{3}$ is $\frac{3}{20}$.

Notice that $\frac{2}{3} \cdot \frac{3}{2} = 1$. A number and its reciprocal multiplied will equal 1.

To get a product of positive 1 when multiplying two numbers, the numbers must have the same sign. So reciprocals must have the same sign.

The reciprocal of $-\dfrac{10}{7}$ is $-\dfrac{7}{10}$, since $-\frac{10}{7}\left(-\frac{7}{10}\right)=1$.

Reciprocal

The reciprocal of $\dfrac{a}{b}$ is $\dfrac{b}{a}$.

A number and its reciprocal multiply to one $\frac{a}{b}\times\frac{b}{a}=1$

To divide fractions, we multiply the first fraction by the reciprocal of the second.

Fraction Division

If a, b, c and d are numbers where $b \neq 0, c \neq 0$ and $d \neq 0$, then

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$$

To divide fractions, we multiply the first fraction by the reciprocal of the second.

We need to say $b \neq 0, c \neq 0$ and $d \neq 0$ to be sure we don't divide by zero!

Example 7

Divide: $-\frac{2}{3} \div \frac{n}{5}$.

Solution

***Step 1:* To divide, multiply the first fraction by the reciprocal of the second.**

$$-\frac{2}{3} \times \frac{5}{n}$$

***Step 2:* Multiply.**

$$-\frac{10}{3n}$$

Try It

13) Divide: $-\frac{3}{5} \div \frac{p}{7}$

Solution

$$-\frac{21}{5p}$$

14) Divide: $-\frac{5}{8} \div \frac{q}{3}$

Solution

$$-\frac{15}{8q}$$

Example 8

Find the quotient: $-\frac{7}{18}\div\left(-\frac{14}{27}\right)$.

Solution

***Step 1:* To divide, multiply the first fraction by the reciprocal of the second.**

$$-\frac{7}{18}\times-\frac{27}{14}$$

***Step 2:* Determine the sign of the product, and then multiply.**

$$\frac{7\times\ 27}{18\times\ 14}$$

***Step 3:* Rewrite showing common factors.**

$$\frac{\cancel{7}\times\cancel{9}\times 3}{\cancel{9}\times 2\times\cancel{7}\times 2}$$

***Step 4:* Remove common factors.**

$$\frac{3}{2\ \times\ 2}$$

***Step 5*: Simplify.**

$$\frac{3}{4}$$

Try It

15) Find the quotient: $-\frac{7}{27}\div\left(-\frac{35}{36}\right)$

Solution

$$\frac{4}{15}$$

16) Find the quotient: $-\frac{5}{14} \div \left(-\frac{15}{28}\right)$

Solution

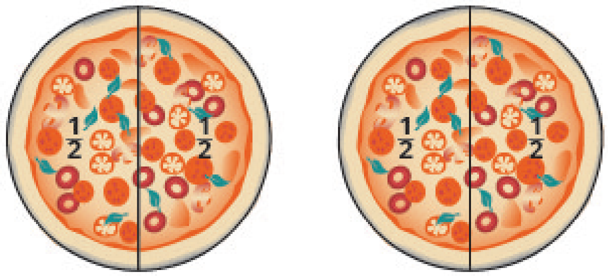
$$\frac{2}{3}$$

There are several ways to remember which steps to take to multiply or divide fractions. One way is to repeat the call outs to yourself. If you do this each time you do an exercise, you will have the steps memorized.

- "To multiply fractions, multiply the numerators and multiply the denominators."
- "To divide fractions, multiply the first fraction by the reciprocal of the second."

Another way is to keep two examples in mind:

One fourth of two pizzas is one half of a pizza.



$$2 \cdot \frac{1}{4}$$

$$\frac{2}{1} \cdot \frac{1}{4}$$

$$\frac{2}{4}$$

$$\frac{1}{2}$$

There are eight quarters in $2.00.

$$2 \div \frac{1}{4}$$

$$\frac{2}{1} \div \frac{1}{4}$$

$$\frac{2}{1} \cdot \frac{4}{1}$$

$$8$$

Figure 1.4.7

The numerators or denominators of some fractions contain fractions themselves. A fraction in which the numerator or the denominator is a fraction is called a **complex fraction**.

Complex Fraction

A complex fraction is a fraction in which the numerator or the denominator contains a fraction.

Some examples of complex fractions are:

$$\frac{\frac{6}{7}}{3} \quad \frac{\frac{3}{4}}{\frac{5}{8}} \quad \frac{\frac{x}{2}}{\frac{5}{6}}$$

To simplify a complex fraction, we remember that the fraction bar means division. For example, the complex fraction $\dfrac{\frac{3}{4}}{\frac{5}{8}}$ means $\frac{3}{4} \div \frac{5}{8}$.

Example 9

Simplify: $\dfrac{\frac{3}{4}}{\frac{5}{8}}$

Solution

Step 1: **Rewrite as division.**

$$\frac{3}{4} \div \frac{5}{8}$$

Step 2: **Multiply the first fraction by the reciprocal of the second.**

$$\frac{3}{4} \times \frac{8}{5}$$

Step 3: **Multiply.**

$$\frac{3 \times 8}{4 \times 5}$$

Step 4: **Look for common factors.**

$$\frac{3 \times \cancel{4} \times 2}{\cancel{4} \times 5}$$

Step 5: **Divide out common factors and simplify.**

$$\frac{6}{5}$$

Try It

17) Simplify: $\dfrac{\frac{2}{3}}{\frac{5}{6}}$

Solution

$$\frac{4}{5}$$

18) Simplify: $\dfrac{\frac{3}{7}}{\frac{6}{11}}$

Solution

$$\frac{11}{14}$$

Example 10

Simplify: $\dfrac{\frac{x}{2}}{\frac{xy}{6}}$

Solution

Step 1: **Rewrite as division.**

$$\frac{x}{2} \div \frac{xy}{6}$$

Step 2: **Multiply the first fraction by the reciprocal of the second.**

$$\frac{x}{2} \times \frac{6}{xy}$$

Step 3: **Multiply.**

$$\frac{x \times 6}{2 \times xy}$$

Step 4: **Look for common factors.**

$$\frac{\cancel{x} \times 3 \times \cancel{2}}{\cancel{2} \times \cancel{x} \times y}$$

Step 5: **Divide out common factors and simplify.**

$$\frac{3}{y}$$

Try It

19) Simplify: $\dfrac{\frac{a}{8}}{\frac{ab}{6}}$

Solution

$$\frac{3}{4b}$$

20) Simplify: $\dfrac{\frac{p}{2}}{\frac{pq}{8}}$

Solution

$$\frac{4}{q}$$

Simplify Expressions with a Fraction Bar

The line that separates the numerator from the denominator in a fraction is called a fraction bar. A fraction bar acts as a grouping symbol. The order of operations then tells us to simplify the numerator and then the denominator. Then we divide.

To simplify the expression $\frac{5-3}{7+1}$, we first simplify the numerator and the denominator separately. Then we divide.

$$= \frac{5-3}{7+1}$$
$$= \frac{2}{8}$$
$$= \frac{1}{4}$$

How to

Simplify an Expression with a Fraction Bar.

1. Simplify the expression in the numerator.
2. Simplify the expression in the denominator.
3. Simplify the fraction.

Example 11

Simplify: $\frac{4-2(3)}{2^2+2}$

Solution

Step 1: **Use the order of operations to simplify the numerator and the denominator.**

$$\frac{4-6}{4+2}$$

Step 2: **Simplify the numerator and the denominator.**

$$\frac{-2}{6}$$

Step 3: **Simplify. A negative divided by a positive is negative.**

$$-\frac{1}{3}$$

Try It

21) Simplify: $\frac{6-3(5)}{3^2+3}$

Solution

$$-\frac{3}{4}$$

22) Simplify: $\frac{4-4(6)}{3^2+3}$

Solution

$$-\frac{5}{3}$$

Where does the negative sign go in a fraction? Usually, the negative sign is in front of the fraction, but you will sometimes see a fraction with a negative numerator, or sometimes with a negative denominator. Remember that fractions represent division. When the numerator and denominator have different signs, the quotient is negative.

$$\begin{array}{ll} \frac{-1}{3} = -\frac{1}{3} & \frac{\text{negative}}{\text{positive}} = \text{negative} \\ \frac{1}{-3} = -\frac{1}{3} & \frac{\text{positive}}{\text{negative}} = \text{negative} \end{array}$$

Placement of Negative Sign in a Fraction

For any positive numbers a and b,

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Example 12

Simplify: $\frac{4(-3) + 6(-2)}{-3(2) - 2}$

Solution

Step 1: **Multiply.**

$$\frac{-12 + (-12)}{-6 - 2}$$

Step 2: **Simplify.**

$$\frac{-24}{-8}$$

Step 3: **Divide.**

$$3$$

Try It

23) Simplify: $\frac{8(-2)+4(-3)}{-5(2)+3}$

Solution

4

24) Simplify: $\frac{7(-1)+9(-3)}{-5(3)-2}$

Solution

2

Translate Phrases to Expressions with Fractions

Now that we have done some work with fractions, we are ready to translate phrases that would result in expressions with fractions.

The English words quotient and ratio are often used to describe fractions. Remember that "quotient" means division. The quotient of a and b is the result we get from dividing a by b, or $\frac{a}{b}$.

Example 13

Translate the English phrase into an algebraic expression: the quotient of the difference of m and n, and p.

Solution

We are looking for the *quotient of* the difference of m and n, and p. This means we want to divide the difference of m and n by p.

$$\frac{m-n}{p}$$

Try It

25) Translate the English phrase into an algebraic expression: the quotient of the difference of a and b, and cd.

Solution

$$\frac{a-b}{cd}$$

26) Translate the English phrase into an algebraic expression: the quotient of the sum of p and q, and r

Solution

$$\frac{p+q}{r}$$

Add or Subtract Fractions with a Common Denominator

When we multiplied fractions, we just multiplied the numerators and multiplied the denominators right straight across. To add or subtract fractions, they must have a common denominator.

Fraction Addition and Subtraction

If $a, b,$ and c are numbers where $c \neq 0$, then

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c} \text{ and } \frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$$

To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.

Example 14

Find the sum: $\frac{x}{3}+\frac{2}{3}$.

Solution

Step 1: **Add the numerators and place the sum over the common denominator.**

$$\frac{x+2}{3}$$

Try It

27) Find the sum: $\frac{x}{4}+\frac{3}{4}$

Solution

$$\frac{x+3}{4}$$

28) Find the sum: $\frac{y}{8}+\frac{5}{8}$

Solution

$$\frac{y+5}{8}$$

Example 15

Find the difference: $-\frac{23}{24} - \frac{13}{24}$.

Solution

Step 1: **Subtract the numerators and place the difference over the common denominator.**

$$\frac{-23-13}{24}$$

Step 2: **Simplify.**

$$\frac{-36}{24}$$

Step 3: **Simplify. Remember, $-\frac{a}{b} = \frac{-a}{b}$.**

$$-\frac{3}{2}$$

Try It

29) Find the difference: $-\frac{19}{28} - \frac{7}{28}$

Solution

$$-\frac{13}{14}$$

30) Find the difference: $-\frac{27}{32} - \frac{1}{32}$

Solution

$$-\frac{7}{8}$$

Example 16

Simplify: $-\frac{10}{x} - \frac{4}{x}$

Solution

Step 1: **Subtract the numerators and place the difference over the common denominator.**

$$\frac{-14}{x}$$

Step 2: **Rewrite with the sign in front of the fraction.**

$$-\frac{14}{x}$$

Try It

31) Find the difference: $-\frac{9}{x} - \frac{7}{x}$

Solution

$$-\frac{16}{x}$$

32) Find the difference: $-\frac{17}{a}-\frac{5}{a}$

Solution

$$-\frac{22}{a}$$

Now we will do an example that has both addition and subtraction.

Example 17

Simplify: $\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$

Solution

Step 1: **Add and subtract fractions—do they have a common denominator? Yes.**

$$\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}.$$

Step 2: **Add and subtract the numerators and place the difference over the common denominator.**

$$\frac{3+(-5)-1}{8}$$

Step 3: **Simplify left to right.**

$$\frac{-2-1}{8}$$

Step 4: **Simplify**.

$$-\frac{3}{8}$$

Try It

33) Simplify: $\frac{-2}{9}+\left(-\frac{4}{9}\right)-\frac{7}{9}$

Solution

$$-\frac{13}{9}$$

34) Simplify: $\frac{5}{9}+\left(-\frac{4}{9}\right)-\frac{7}{9}$

Solution

$$-\frac{2}{3}$$

Add or Subtract Fractions with Different Denominators

As we have seen, to add or subtract fractions, their denominators must be the same. The **least common denominator** (LCD) of two fractions is the smallest number that can be used as a common denominator of the fractions. The LCD of the two fractions is the least common multiple (LCM) of their denominators.

Least Common Denominator

The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.

After we find the least common denominator of two fractions, we convert the fractions to equivalent fractions with the LCD. Putting these steps together allows us to add and subtract fractions because their denominators will be the same!

Example 18

Add: $\frac{7}{12} + \frac{5}{18}$

Solution

Step 1: **Do they have a common denominator?**

$$\left.\begin{array}{l} 12 = 2 \times 2 \times 3 \\ 18 = 2 \times 3 \times 3 \end{array}\right\} \text{LCD} = 2 \times 2 \times 3 \times 3 = 36$$

No – rewrite each fraction with the LCD (least common denominator).

$$\begin{aligned} &= \frac{7}{12} + \frac{5}{18} \\ &= \frac{7 \times 3}{12 \times 3} + \frac{5 \times 2}{18 \times 2} \\ &= \frac{21}{36} + \frac{10}{36} \text{ Do not simplify LCD fractions.} \end{aligned}$$

Step 2: **Add or subtract the fractions.**

Add.

$$\frac{31}{36}$$

Step 3: **Simplify, if possible.**

Because 31 is a prime number, it has no factors in common with 36. The answer is simplified.

Try It

35) Add: $\frac{7}{12} + \frac{11}{15}$

Solution

$$\frac{79}{60}$$

36) Add: $\frac{13}{15}+\frac{17}{20}$

Solution

$$\frac{103}{60}$$

How to

Add or Subtract Fractions.

1. Do they have a common denominator?
   - Yes—go to step 2.
   - No—rewrite each fraction with the LCD (least common denominator). Find the LCD. Change each fraction into an equivalent fraction with the LCD as its denominator.
2. Add or subtract the fractions.
3. Simplify, if possible.

When finding the equivalent fractions needed to create the common denominators, there is a quick way to find the number we need to multiply both the numerator and denominator. This method works if we find the LCD by factoring into primes.

Look at the factors of the LCD and then at each column above those factors. The "missing" factors of each denominator are the numbers we need.

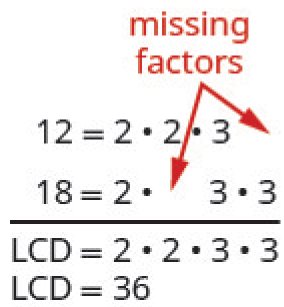


Figure 1.4.8

In the above figure the LCD, 36, has two factors of 2 and two factors of 3.

The numerator 12 has two factors of 2 but only one of 3—so it is "missing" one 3—we multiply the numerator and denominator by 3.

The numerator 18 is missing one factor of 2—so we multiply the numerator and denominator by 2.

We will apply this method as we subtract the fractions in Example 1.4.19

Example 19

Subtract: $\frac{7}{15} - \frac{19}{24}$

Solution

Step 1: **Find the LCD.**

Do the fractions have a common denominator? No, so we need to find the LCD.

Notice, 15 is "missing" three factors of 2 and 24 is "missing" the 5 from the factors of the LCD. So we multiply 8 in the first fraction and 5 in the second fraction to get the LCD.

$$\left.\begin{array}{lll} 15 & = & 3 \times 5 \\ 24 & = & 2 \times 2 \times 2 \times 3 \end{array}\right\} LCD = 2 \times 2 \times 2 \times 3 \times 5 = 120$$

$$= \frac{7}{15} - \frac{19}{24}$$

$$= \frac{7 \times 8}{15 \times 8} - \frac{19 \times 5}{24 \times 5}$$

Step 2: **Rewrite as equivalent fractions with the LCD.**

$$\frac{56}{120} - \frac{95}{120}$$

Step 3: **Subtract.**

$$-\frac{39}{120}$$

Step 4: **Check to see if the answer can be simplified.**

$$-\frac{13 \times 3}{40 \times 3}$$

Step 5: **Simplify.**

Both 39 and 120 have a factor of 3.

$$-\frac{13}{40}$$

Do not simplify the equivalent fractions! If you do, you'll get back to the original fractions and lose the common denominator!

Try It

37) Subtract: $\frac{13}{24} - \frac{17}{32}$

Solution

$$\frac{1}{96}$$

38) Subtract: $\frac{21}{32} - \frac{9}{28}$

Solution

$$\frac{75}{224}$$

In the next example, one of the fractions has a variable in its numerator. Notice that we do the same steps as when both numerators are numbers.

Example 20

Add: $\frac{3}{5} + \frac{x}{8}$

Solution

The fractions have different denominators.

Step 1: **Find the LCD.**

$$\left.\begin{array}{rcr} 5 & = & 5 \\ 8 & = & 2 \times 2 \times 2 \end{array}\right\} LCD = 2 \times 2 \times 2 \times 5 = 40$$

Step 2: **Rewrite as equivalent fractions with the LCD.**

$$\frac{3 \times 8}{5 \times 8} + \frac{x \times 5}{8 \times 5}$$

Step 3: **Simplify.**

$$\frac{24}{40} + \frac{5x}{40}$$

Step 4: **Add.**

$$\frac{24 + 5x}{40}$$

Remember, we can only add like terms: 24 and $5x$ are not like terms.

Try It

39) Add: $\frac{y}{6} + \frac{7}{9}$

Solution

$\frac{9y+42}{54}$

40) Add: $\frac{x}{6}+\frac{7}{15}$

Solution

$\frac{15x+42}{90}$

We now have all four operations for fractions. The table below summarizes fraction operations.

Fraction Multiplication	**Fraction Division**
$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$
Multiply the numerators and multiply the denominators	Multiply the first fraction by the reciprocal of the second.
Fraction Addition	**Fraction Subtraction**
$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$	$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$
Add the numerators and place the sum over the common denominator.	Subtract the numerators and place the difference over the common denominator.
To multiply or divide fractions, an LCD is NOT needed. To add or subtract fractions, an LCD is needed.	

Example 21

Simplify:

a. $\frac{5x}{6}-\frac{3}{10}$
b. $\frac{5x}{6}\times\frac{3}{10}$

Solution

First ask, “What is the operation?” Once we identify the operation that will determine whether we need a common denominator. Remember, we need a common denominator to add or subtract, but not to multiply or divide.

a. What is the operation? The operation is subtraction.

Step 1: **Do the fractions have a common denominator? No.**

$$\frac{5x}{6} - \frac{3}{10}$$

Step 2: **Rewrite each fraction as an equivalent fraction with the LCD.**

$$\frac{5x \times 5}{6 \times 5} - \frac{3 \times 3}{10 \times 3}$$

Step 3: **Subtract the numerators and place the difference over the common denominators.**

$$\frac{25x}{30} - \frac{9}{30}$$

Step 4: **Simplify, if possible There are no common factors. The fraction is simplified.**

$$\frac{25x - 9}{30}$$

---

b. What is the operation? Multiplication.

Step 1: **To multiply fractions, multiply the numerators and multiply the denominators.**

$$\frac{5x \times 3}{6 \times 10}$$

Step 2: **Rewrite, showing common factors**.

$$\frac{\cancel{5}x \times \cancel{3}}{2 \times \cancel{3} \times 2 \times \cancel{5}}$$

Step 3: **Remove common factors.**
Step 4: **Simplify.**

$$\frac{x}{4}$$

Notice we needed an LCD to add $\frac{5x}{6} - \frac{3}{10}$, but not to multiply $\frac{5x}{6} \times \frac{3}{10}$.

Try It

41) Simplify:

a. $\frac{3a}{4} - \frac{8}{9}$

b. $\frac{3a}{4} \times \frac{8}{9}$

Solution

a. $\frac{27a - 32}{36}$

b. $\frac{2a}{3}$

42) Simplify:

a. $\frac{4k}{5} - \frac{1}{6}$

b. $\frac{4k}{5} \times \frac{1}{6}$

Solution

a. $\frac{24k - 5}{30}$

b. $\frac{2k}{15}$

Use the Order of Operations to Simplify Complex Fractions

We have seen that a complex fraction is a fraction in which the numerator or denominator contains a fraction.

The fraction bar indicates division. We simplified the complex fraction $\dfrac{\frac{3}{4}}{\frac{5}{8}}$ by dividing $\dfrac{3}{4}$ by $\dfrac{5}{8}$

.

Now we'll look at complex fractions where the numerator or denominator contains an expression that can be simplified. So we first must completely simplify the numerator and denominator separately using the order of operations. Then we divide the numerator by the denominator.

Example 22

Simplify: $\dfrac{\left(\frac{1}{2}\right)^2}{4+3^2}$.

Solution

Step 1: Simplify the numerator.

*Remember, $\left(\dfrac{1}{2}\right)^2$ means $\dfrac{1}{2}\cdot\dfrac{1}{2}$

$$=\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$$
$$=\frac{\frac{1}{4}}{4+3^2}$$

***Step 2:* Simplify the denominator.**

$$=\frac{\frac{1}{4}}{4+9}$$
$$=\frac{\frac{1}{4}}{13}$$

***Step 3:* Divide the numerator by the denominator. Simplify if possible.**

*Remember, $13=\dfrac{13}{1}$

$$= \frac{1}{4} \div 13$$
$$= \frac{1}{4} \times \frac{1}{13}$$
$$= \frac{1}{52}$$

Try It

43) Simplify: $\dfrac{\left(\frac{1}{3}\right)^2}{2^3+2}$

Solution

$$\frac{1}{90}$$

44) Simplify: $\dfrac{1+4^2}{\left(\frac{1}{4}\right)^2}$

Solution

$$272$$

How to

Simplify Complex Fractions

1. Simplify the numerator.
2. Simplify the denominator.
3. Divide the numerator by the denominator. Simplify if possible.

Example 23

Simplify: $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$

Solution

It may help to put parentheses around the numerator and the denominator.

Step 1: **Simplify the numerator (LCD = 6) and simplify the denominator (LCD = 12).**

$$\frac{\left(\frac{3}{6}+\frac{4}{6}\right)}{\left(\frac{9}{12}-\frac{2}{12}\right)}$$

Step 2: **Simplify.**

$$\frac{\left(\frac{7}{6}\right)}{\left(\frac{7}{12}\right)}$$

Step 3: **Divide the numerator by the denominator.**

$$\frac{7}{6} \times \frac{12}{7}$$

Step 4: **Simplify.**

Step 5: **Divide out common factors.**

$$\frac{7 \times 6 \times 2}{6 \times 7}$$

Step 6: **Simplify.**

$$2$$

Try It

45) Simplify: $\frac{\frac{1}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{3}}$

Solution

2

46) Simplify: $\frac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$

Solution

$$\frac{2}{7}$$

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but now we can evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the value of the variable into the expression and then simplify.

Example 24

Evaluate $x + \frac{1}{3}$ when

a. $x = -\frac{1}{3}$

b. $x = -\frac{3}{4}$

Solution

a. To evaluate $x + \frac{1}{3}$ when $x = -\frac{1}{3}$, substitute $-\frac{1}{3}$ for x in the expression.

***Step 1:* Substitute $-\frac{1}{3}$ for x.**

$$-\frac{1}{3} + \frac{1}{3}$$

***Step 2:* Simplify.**

$$0$$

---

b. To evaluate $x + \frac{1}{3}$ when $x = -\frac{3}{4}$, we substitute $-\frac{3}{4}$ for x in the expression.

Step 1: **Substitute** $-\frac{3}{4}$ **for** x**.**

$$-\frac{3}{4}+\frac{1}{3}$$

Step 2: **Rewrite as equivalent fractions with the LCD,** 12**.**

$$-\frac{3\times 3}{4\times 3}+\frac{1\times 4}{3\times 4}$$

Step 3:**Simplify.**

$$-\frac{9}{12}+\frac{4}{12}$$

Step 4:**Add.**

$$-\frac{5}{12}$$

Try It

47) Evaluate $x+\frac{3}{4}$ when

a. $x=-\frac{7}{4}$

b. $x=-\frac{5}{4}$

Solution

a. -1

b. $-\frac{1}{2}$

48) Evaluate $y+\frac{1}{2}$ when

a. $y=\frac{2}{3}$

b. $y=-\frac{3}{4}$

Solution

a. $\frac{7}{6}$

b. $-\frac{1}{4}$

Example 25

Evaluate $-\frac{5}{6}-y$ when $y=-\frac{2}{3}$

Solution

Step 1: **Substitute** $-\frac{2}{3}$ **for** y.

$$-\frac{5}{6}-\left(-\frac{2}{3}\right)$$

Step 2: **Rewrite as equivalent fractions with the LCD,** 6**.**

$$-\frac{5}{6}-\left(-\frac{4}{6}\right)$$

Step 3: **Subtract.**

$$-\frac{5-(-4)}{6}$$

Step 4: **Simplify.**

$$-\frac{1}{6}$$

Try It

49) Evaluate $-\frac{1}{2}-y$ when $y=-\frac{1}{4}$

Solution

$$-\frac{1}{4}$$

50) Evaluate $-\frac{3}{8}-y$ when $x=-\frac{5}{2}$

Solution

$$\frac{17}{8}$$

Example 26

Evaluate $2x^2y$ when $x = \frac{1}{4}$ and $y = -\frac{2}{3}$

Solution

Substitute the values into the expression.

Step 1: **Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.**

$$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$$

Step 2: **Simplify exponents first.**

$$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$$

Step 3: **Multiply. Divide out the common factors. Notice we write 16 as $2 \cdot 2 \cdot 4$ to make it easy to remove common factors.**

$$-\frac{\cancel{2} \times 1 \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 4 \times 3}$$

Step 4: **Simplify.**

$$-\frac{1}{12}$$

Try It

51) Evaluate $3ab^2$ when $a = -\frac{2}{3}$ and $b = -\frac{1}{2}$

Solution

$$-\frac{1}{2}$$

52) Evaluate $4c^3d$ when $c = -\frac{1}{2}$ and $d = -\frac{4}{3}$

Solution

$$\frac{2}{3}$$

The next example will have only variables, no constants.

Example 27

Evaluate $\frac{p+q}{r}$ when $p = -4, q = -2, \text{and } r = 8$

Solution

To evaluate $\frac{p+q}{r}$ when $p = -4, q = -2, \text{and } r = 8$, we substitute the values into the expression.

Step 1: **Substitute -4 for p, -2 for q, and 8 for r.**

$$\frac{-4+(-2)}{8}$$

Step 2: **Add in the numerator first.**

$$\frac{-6}{8}$$

Step 3: **Simplify.**

$$-\frac{3}{4}$$

Try It

53) Evaluate $\frac{a+b}{c}$ when $a=-8, b=-7, \text{and} c=6$

Solution

$$-\frac{5}{2}$$

54) Evaluate $\frac{x+y}{z}$ when $x=9, y=-18, \text{and} z=-6$

Solution

$$\frac{3}{2}$$

Key Concepts

- **Equivalent Fractions Property:** If a, b, c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{a \times c}{b \times c}$ and $\frac{a \times c}{b \times c} = \frac{a}{b}$.
- **Fraction Division:** If a, b, c and d are numbers where $b \neq 0, c \neq 0$, and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$. To divide fractions, multiply the first fraction by the reciprocal of the second.
- **Fraction Multiplication:** If a, b, c and d are numbers where $b \neq 0$, and $d \neq 0$, then $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$. To multiply fractions, multiply the numerators and multiply the denominators.
- **Placement of Negative Sign in a Fraction:** For any positive numbers a and b, $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.
- **Property of One:** $\frac{a}{a} = 1$; Any number, except zero, divided by itself is one.
- **Simplify a Fraction**
  1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers first.
  2. Simplify using the equivalent fractions property by dividing out common factors.
  3. Multiply any remaining factors.
- **Simplify an Expression with a Fraction Bar**
  1. Simplify the expression in the numerator. Simplify the expression in the denominator.
  2. Simplify the fraction.
- **Fraction Addition and Subtraction:** If $a, b,$ and c are numbers where $c \neq 0$, then $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ and $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$.

To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.

- **Strategy for Adding or Subtracting Fractions**

  1. Do they have a common denominator?
     Yes—go to step 2.
     No—Rewrite each fraction with the LCD (Least Common Denominator). Find the LCD. Change each fraction into an equivalent fraction with the LCD as its denominator.
  2. Add or subtract the fractions.
  3. Simplify, if possible. To multiply or divide fractions, an LCD IS NOT needed. To add or subtract fractions, an LCD IS needed.

- **Simplify Complex Fractions**

  1. Simplify the numerator.
  2. Simplify the denominator.
  3. Divide the numerator by the denominator. Simplify if possible

Glossary

complex fraction
A complex fraction is a fraction in which the numerator or the denominator contains a fraction.

denominator
The denominator is the value on the bottom part of the fraction that indicates the number of equal parts into which the whole has been divided.

equivalent fractions
Equivalent fractions are fractions that have the same value.

fraction

A fraction is written $\frac{a}{b}$, where $b \neq 0$ a is the numerator and b is the denominator. A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

numerator

The numerator is the value on the top part of the fraction that indicates how many parts of the whole are included.

reciprocal

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$. A number and its reciprocal multiply to one: $\frac{a}{b} \cdot \frac{b}{a} = 1$.

simplified fraction

A fraction is considered simplified if there are no common factors in its numerator and denominator.

least common denominator

The least common denominator (LCD) of two fractions is the Least common multiple (LCM) of their denominators.

Exercises: Find Equivalent Fractions

Instructions: For questions 1-4, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

1) $$\frac{3}{8}$$

2) $$\frac{5}{8}$$

3) $$\frac{5}{9}$$

4) $$\frac{1}{8}$$

---

1) $\frac{6}{16}, \frac{9}{24}, \frac{12}{32}$ **(Answers may vary)**

3) $\frac{10}{18}, \frac{15}{27}, \frac{20}{36}$ **(Answers may vary)**

Exercises: Simplify Fractions

Instructions: For questions 5-14, simplify.

5) $-\frac{40}{88}$

6) $-\frac{63}{99}$

7) $-\frac{108}{63}$

8) $-\frac{104}{48}$

9) $\frac{120}{252}$

10) $\frac{182}{294}$

11) $-\frac{3x}{12y}$

12) $-\frac{4x}{32y}$

13) $\frac{14x^2}{21y}$

14) $\dfrac{24a}{32b^2}$

Odd Answers

5) $-\dfrac{5}{11}$

7) $-\dfrac{12}{7}$

9) $\dfrac{10}{21}$

11) $-\dfrac{x}{4y}$

13) $\dfrac{2x^2}{3y}$

Exercises: Multiply Fractions

Instructions: For questions 15-30, multiply.

15) $\dfrac{3}{4} \cdot \dfrac{9}{10}$

16) $\dfrac{4}{5} \cdot \dfrac{2}{7}$

17) $-\frac{2}{3}\left(-\frac{3}{8}\right)$

18) $-\frac{3}{4}\left(-\frac{4}{9}\right)$

19) $-\frac{5}{9}\cdot\frac{3}{10}$

20) $-\frac{3}{8}\cdot\frac{4}{15}$

21) $\left(-\frac{14}{15}\right)\left(\frac{9}{20}\right)$

22) $\left(-\frac{9}{10}\right)\left(\frac{25}{33}\right)$

23) $\left(-\frac{63}{84}\right)\left(-\frac{44}{90}\right)$

24) $\left(-\frac{63}{60}\right)\left(-\frac{40}{88}\right)$

25) $4\cdot\frac{5}{11}$

26) $5\cdot\frac{8}{3}$

27) $\frac{3}{7}\cdot 21n$

28) $\frac{5}{6}\cdot 30m$

29) $(-8)\left(\frac{17}{4}\right)$

30) $(-1)\left(-\frac{6}{7}\right)$

Odd Answers

15) $\frac{27}{40}$

17) $\frac{1}{4}$

19) $-\dfrac{1}{6}$

21) $-\dfrac{21}{50}$

23) $\dfrac{11}{30}$

25) $\dfrac{20}{11}$

27) $9n$

29) -34

Exercises: Divide Fractions

Instructions: For questions 31-44, divide.

31) $\dfrac{3}{4} \div \dfrac{2}{3}$

32) $\dfrac{4}{5} \div \dfrac{3}{4}$

33) $-\frac{7}{9} \div \left(-\frac{7}{9}\right)$

34) $-\frac{5}{6} \div \left(-\frac{5}{6}\right)$

35) $\dfrac{3}{4} \div \dfrac{x}{11}$

36) $\frac{2}{5} \div \frac{y}{9}$

37) $\frac{5}{18} \div \left(-\frac{15}{24}\right)$

38) $\frac{7}{18} \div \left(-\frac{14}{27}\right)$

39) $\frac{8u}{15} \div \frac{12v}{25}$

40) $\frac{12r}{25} \div \frac{18s}{35}$

41) $-5 \div \frac{1}{2}$

42) $-3 \div \frac{1}{4}$

43) $\frac{3}{4} \div (-12)$

44) $-15 \div \left(-\frac{5}{3}\right)$

Odd Answers

31) $\frac{9}{8}$

33) 1

35) $\frac{33}{4x}$

37) $-\dfrac{4}{9}$

39) $\dfrac{10u}{9v}$

41) -10

43) $-\dfrac{1}{16}$

Exercises: Simplify by Dividing

Instructions: For questions 45-50, simplify.

45) $\dfrac{-\frac{8}{21}}{\frac{12}{35}}$

46) $\dfrac{-\frac{9}{16}}{\frac{33}{40}}$

47) $\dfrac{-\frac{4}{5}}{2}$

48) $\dfrac{5}{\frac{3}{10}}$

49) $\dfrac{\frac{m}{3}}{\frac{n}{2}}$

50) $\dfrac{-\frac{3}{8}}{-\frac{y}{12}}$

Odd Answers

45) $-\dfrac{10}{9}$

47) $-\dfrac{2}{5}$

49) $\dfrac{2m}{3n}$

Exercises: Simplify Expressions Written with a Fraction Bar

Instructions: For questions 51-70, simplify.

51) $\dfrac{22+3}{10}$

52) $\dfrac{19-4}{6}$

53) $\dfrac{48}{24 - 15}$

54) $\dfrac{46}{4 + 4}$

55) $\dfrac{-6 + 6}{8 + 4}$

56) $\dfrac{-6 + 3}{17 - 8}$

57) $\dfrac{4 \cdot 3}{6 \cdot 6}$

58) $\dfrac{6 \cdot 6}{9 \cdot 2}$

59) $\dfrac{4^2 - 1}{25}$

60) $\dfrac{7^2 + 1}{60}$

61) $\dfrac{8 \cdot 3 + 2 \cdot 9}{14 + 3}$

62) $\dfrac{9 \cdot 6 - 4 \cdot 7}{22 + 3}$

63) $\dfrac{5 \cdot 6 - 3 \cdot 4}{4 \cdot 5 - 2 \cdot 3}$

64) $\dfrac{8 \cdot 9 - 7 \cdot 6}{5 \cdot 6 - 9 \cdot 2}$

65) $\dfrac{5^2 - 3^2}{3 - 5}$

66) $\dfrac{6^2 - 4^2}{4 - 6}$

67) $\dfrac{7 \cdot 4 - 2(8 - 5)}{9 \cdot 3 - 3 \cdot 5}$

68) $\dfrac{9 \cdot 7 - 3(12 - 8)}{8 \cdot 7 - 6 \cdot 6}$

69) $\dfrac{9(8 - 2) - 3(15 - 7)}{6(7 - 1) - 3(17 - 9)}$

70) $\dfrac{8(9 - 2) - 4(14 - 9)}{7(8 - 3) - 3(16 - 9)}$

Odd Answers

51) $\dfrac{5}{2}$

53) $\dfrac{16}{3}$

55) 0

57) $\dfrac{1}{3}$

59) $\dfrac{3}{5}$

61) $2\dfrac{8}{17}$

63) $\dfrac{9}{7}$

65) -8

67) $\frac{11}{6}$

69) $\frac{5}{2}$

Exercises: Translate Phrases to Expressions with Fractions

Instructions: For questions 71-74, translate each English phrase into an algebraic expression.

71) **the quotient of** r **and the sum of** s **and** 10

72) **the quotient of** A **and the difference of** 3 **and** B

73) **the quotient of the difference of** x **and** y**, and** -3

74) **the quotient of the sum of** m **and** n**, and** $4q$

Odd Answers

71) $\frac{r}{s+10}$

73) $\frac{x-y}{-3}$

Exercises: Everyday Math

Instructions: For questions 75-78, answer the given everyday math word problems.

75) Baking:A recipe for chocolate chip cookies calls for $\frac{3}{4}$ **cup brown sugar.**

Imelda wants to double the recipe.

- **a) How much brown sugar will Imelda need? Show your calculation.**
- **b) Measuring cups usually come in sets of** $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \text{ and } 1$ **cup. Draw a diagram to show two different ways that Imelda could measure the brown sugar needed to double the cookie recipe.**

76) Baking: Nina is making 4 pans of fudge to serve after a music recital. For each pan, she needs $\frac{2}{3}$ cup of condensed milk.

a) How much condensed milk will Nina need? Show your calculation.

b) Measuring cups usually come in sets of $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \text{ and } 1$ cup. Draw a diagram to show two different ways that Nina could measure the condensed milk needed for 4 pans of fudge.

77) Portions: Don purchased a bulk package of candy that weighs 5 pounds. He wants to sell the candy in little bags that hold $\frac{1}{4}$ pound. How many little bags of candy can he fill from the bulk package?

78) Portions:Kristen has $\frac{3}{4}$ yards of ribbon that she wants to cut into 6

equal parts to make hair ribbons for her daughter's 6 dolls. How long will each doll's hair ribbon be?

Odd Answers

75a) $1\frac{1}{2}$ **cups**

75b) Answers will vary

77) 20 **bags**

Exercises: Writing Exercises

Instructions: For questions 79-82, answer the given writing exercises.

79) Rafael wanted to order half a medium pizza at a restaurant. The waiter told him that a medium pizza could be cut into 6 **or** 8 **slices. Would he prefer** 3 **out of** 6 **slices or** 4 **out of** 8 **slices? Rafael replied**

that since he wasn't very hungry, he would prefer 3 out of 6 slices.

Explain what is wrong with Rafael's reasoning.

80) Give an example from everyday life that demonstrates how $\frac{1}{2} \cdot \frac{2}{3}$ is $\frac{1}{3}$.

81) Explain how you find the reciprocal of a fraction.

82) Explain how you find the reciprocal of a negative number.

Odd answers

79) Answers may vary

81) Answers may vary

Exercises: Add Fractions with a Common Denominator

Instructions: For questions 83-92, add.

83) $\frac{6}{13} + \frac{5}{13}$

84) $\frac{4}{15} + \frac{7}{15}$

85) $\frac{x}{4} + \frac{3}{4}$

86) $\frac{8}{q} + \frac{6}{q}$

87) $-\frac{3}{16} + \left(-\frac{7}{16}\right)$

88) $-\frac{5}{16} + \left(-\frac{9}{16}\right)$

89) $-\frac{8}{17} + \frac{15}{17}$

90) $-\frac{9}{19} + \frac{17}{19}$

91) $\frac{6}{13} + \left(-\frac{10}{13}\right) + \left(-\frac{12}{13}\right)$

92) $\frac{5}{12} + \left(-\frac{7}{12}\right) + \left(-\frac{11}{12}\right)$

Odd Answers

83) $\frac{11}{13}$

85) $\frac{x+3}{4}$

87) $-\frac{5}{8}$

89) $\frac{7}{17}$

91) $-\frac{16}{13}$

Exercises: Subtract Fractions with a Common Denominator

Instructions: For questions 93-106, subtract.

93) $\frac{11}{15} - \frac{7}{15}$

94) $\frac{9}{13} - \frac{4}{13}$

95) $\frac{11}{12} - \frac{5}{12}$

96) $\frac{7}{12} - \frac{5}{12}$

97) $\frac{19}{21} - \frac{4}{21}$

98) $\frac{17}{21} - \frac{8}{21}$

99) $\frac{5y}{8} - \frac{7}{8}$

100) $\frac{11z}{13} - \frac{8}{13}$

101) $-\frac{23}{u} - \frac{15}{u}$

102) $-\frac{29}{v} - \frac{26}{v}$

103) $-\frac{3}{5} - \left(-\frac{4}{5}\right)$

104) $-\frac{3}{7} - \left(-\frac{5}{7}\right)$

105) $-\frac{7}{9} - \left(-\frac{5}{9}\right)$

106) $-\frac{8}{11} - \left(-\frac{5}{11}\right)$

Odd Answers

93) $\frac{4}{15}$

95) $\dfrac{1}{2}$

97) $\dfrac{5}{7}$

99) $\dfrac{5y-7}{8}$

101) $-\dfrac{38}{u}$

103) $\dfrac{1}{5}$

105) $-\dfrac{2}{9}$

Exercises: Mixed Practice

Instructions: For questions 107-114, simplify.

107) $-\frac{5}{18}\cdot\frac{9}{10}$

108) $-\frac{3}{14} \cdot \frac{7}{12}$

109) $\frac{n}{5} - \frac{4}{5}$

110) $\frac{6}{11} - \frac{s}{11}$

111) $-\frac{7}{24} + \frac{2}{24}$

112) $-\frac{5}{18} + \frac{1}{18}$

113) $\frac{8}{15} \div \frac{12}{5}$

114) $\frac{7}{12} \div \frac{9}{28}$

Odd Answers

107) $-\frac{1}{4}$

109) $\frac{n-4}{5}$

111) $-\frac{5}{24}$

113) $\frac{2}{9}$

Exercises: Add or Subtract Fractions with Different Denominators

Instructions: For questions 115-138, add or subtract.

115) $\frac{1}{2}+\frac{1}{7}$

116) $\frac{1}{3}+\frac{1}{8}$

117) $\frac{1}{3}-\left(-\frac{1}{9}\right)$

118) $\frac{1}{4}-\left(-\frac{1}{8}\right)$

119) $\frac{7}{12}+\frac{5}{8}$

120) $\frac{5}{12}+\frac{3}{8}$

121) $\frac{7}{12}-\frac{9}{16}$

122) $\frac{7}{16}-\frac{5}{12}$

123) $\frac{2}{3}-\frac{3}{8}$

124) $\frac{5}{6}-\frac{3}{4}$

125) $-\frac{11}{30}+\frac{27}{40}$

126) $-\frac{9}{20}+\frac{17}{30}$

127) $-\frac{13}{30}+\frac{25}{42}$

128) $-\frac{23}{30}+\frac{5}{48}$

129) $-\frac{39}{56}-\frac{22}{35}$

130) $-\frac{33}{49}-\frac{18}{35}$

131) $-\frac{2}{3}-\left(-\frac{3}{4}\right)$

132) $-\frac{3}{4}-\left(-\frac{4}{5}\right)$

133) $1+\dfrac{7}{8}$

134) $1-\dfrac{3}{10}$

135) $\dfrac{x}{3}+\dfrac{1}{4}$

136) $\dfrac{y}{2}+\dfrac{2}{3}$

137) $\dfrac{y}{4}-\dfrac{3}{5}$

138) $\dfrac{x}{5}-\dfrac{1}{4}$

Odd Answers

115) $\dfrac{9}{14}$

117) $\dfrac{4}{9}$

119) $\dfrac{29}{24}$

121) $\dfrac{1}{48}$

123) $\frac{7}{24}$

125) $\frac{37}{120}$

127) $\frac{17}{105}$

129) $-\frac{53}{40}$

131) $\frac{1}{12}$

133) $\frac{15}{8}$

135) $\frac{4x+3}{12}$

137) $\frac{5y-12}{20}$

Exercises: Mixed Practice

Instructions: For questions 139-152, simplify.

139a) $\frac{2}{3}+\frac{1}{6}$

139b) $\frac{2}{3}\div\frac{1}{6}$

140a) $-\frac{2}{5}-\frac{1}{8}$

140b) $-\frac{2}{5}\cdot\frac{1}{8}$

141a) $\frac{5n}{6}\div\frac{8}{15}$

141b) $\frac{5n}{6}-\frac{8}{15}$

142a) $\frac{3a}{8}\div\frac{7}{12}$

142b) $\frac{3a}{8}-\frac{7}{12}$

143) $-\frac{3}{8}\div\left(-\frac{3}{10}\right)$

144) $-\frac{5}{12}\div\left(-\frac{5}{9}\right)$

145) $-\frac{3}{8}+\frac{5}{12}$

146) $-\frac{1}{8}+\frac{7}{12}$

147) $\frac{5}{6}-\frac{1}{9}$

148) $\frac{5}{9}-\frac{1}{6}$

149) $-\frac{7}{15}-\frac{y}{4}$

150) $-\frac{3}{8}-\frac{x}{11}$

151) $\frac{11}{12a}\cdot\frac{9a}{16}$

152) $\frac{10y}{13}\cdot\frac{8}{15y}$

Odd Answers

139a) $\frac{5}{6}$

139b) 4

141a) $\frac{25n}{16}$

141b) $\frac{25n-16}{30}$

143) $\frac{5}{4}$

145) $\frac{1}{24}$

147) $\frac{13}{18}$

149) $\frac{-28-15y}{60}$

151) $\frac{33}{64}$

Exercises: Use the Order of Operations to Simplify Complex

Fractions

Instructions: For questions 153-174, simplify.

153) $\dfrac{2^3+4^2}{\left(\frac{2}{3}\right)^2}$

154) $\dfrac{3^3-3^2}{\left(\frac{3}{4}\right)^2}$

155) $\dfrac{\left(\frac{3}{5}\right)^2}{\left(\frac{3}{7}\right)^2}$

156) $\dfrac{\left(\frac{3}{4}\right)^2}{\left(\frac{5}{8}\right)^2}$

157) $\dfrac{2}{\frac{1}{3}+\frac{1}{5}}$

158) $\dfrac{5}{\frac{1}{4}+\frac{1}{3}}$

159) $\dfrac{\frac{7}{8}-\frac{2}{3}}{\frac{1}{2}+\frac{3}{8}}$

160) $\dfrac{\frac{3}{4}-\frac{3}{5}}{\frac{1}{4}+\frac{2}{5}}$

161) $\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{12}$

162) $\frac{1}{3}+\frac{2}{5}\cdot\frac{3}{4}$

163) $1-\frac{3}{5}\div\frac{1}{10}$

164) $1-\frac{5}{6}\div\frac{1}{12}$

165) $\frac{2}{3}+\frac{1}{6}+\frac{3}{4}$

166) $\frac{2}{3}+\frac{1}{4}+\frac{3}{5}$

167) $\frac{3}{8}-\frac{1}{6}+\frac{3}{4}$

168) $\frac{2}{5}+\frac{5}{8}-\frac{3}{4}$

169) $12\left(\frac{9}{20}-\frac{4}{15}\right)$

170) $8\left(\frac{15}{16}-\frac{5}{6}\right)$

171) $\dfrac{\frac{5}{8}+\frac{1}{6}}{\frac{19}{24}}$

172) $\dfrac{\frac{1}{6}+\frac{3}{10}}{\frac{14}{30}}$

173) $\left(\frac{5}{9}+\frac{1}{6}\right)\div\left(\frac{2}{3}-\frac{1}{2}\right)$

174) $\left(\frac{3}{4}+\frac{1}{6}\right)\div\left(\frac{5}{8}-\frac{1}{3}\right)$

Odd Answers

153) 54

155) $\dfrac{49}{25}$

157) $\dfrac{15}{4}$

159) $\frac{5}{21}$

161) $\frac{7}{9}$

163) -5

165) $\frac{19}{12}$

167) $\frac{23}{24}$

169) $\frac{11}{5}$

171) 1

173) $\frac{13}{3}$

Exercises: Evaluate Variable Expressions with Fractions

Instructions: For questions 175-184, evaluate.

175) $x+\left(-\frac{5}{6}\right)$ **when**

- **a)** $x=\frac{1}{3}$
- **b)** $x=-\frac{1}{6}$

176) $x+\left(-\frac{11}{12}\right)$ **when**

- **a)** $x=\frac{11}{12}$
- **b)** $x=\frac{3}{4}$

177) $x-\frac{2}{5}$ **when**

- **a)** $x=\frac{3}{5}$
- **b)** $x=-\frac{3}{5}$

178) $x-\frac{1}{3}$ **when**

- **a)** $x=\frac{2}{3}$
- **b)** $x=-\frac{2}{3}$

179) $\frac{7}{10}-w$ **when**

- **a)** $w=\frac{1}{2}$
- **b)** $w=-\frac{1}{2}$

180) $\frac{5}{12} - w$ **when**

a) $w = \frac{1}{4}$

b) $w = -\frac{1}{4}$

181) $2x^2y^3$ **when** $x = -\frac{2}{3}$ **and** $y = -\frac{1}{2}$

182) $8u^2v^3$ **when** $u = -\frac{3}{4}$ **and** $v = -\frac{1}{2}$

183) $\frac{a+b}{a-b}$ **when** $a = -3, b = 8$

184) $\frac{r-s}{r+s}$ **when** $r = 10, s = -5$

Odd Answers

175a) $-\frac{1}{2}$

175b) -1

177a) $\frac{1}{5}$

177b) -1

179a) $\frac{1}{5}$

179b) $\frac{6}{5}$

181) $-\frac{1}{9}$

183) $-\frac{5}{11}$

Exercises: Everyday Math

Instructions: For questions 185-186, answer the given everyday math word problems.

185) Decorating: Laronda is making covers for the throw pillows on her sofa. For each pillow cover, she needs $\frac{1}{2}$ yard of print fabric and $\frac{3}{8}$ yard of solid fabric. What is the total amount of fabric Laronda needs for each pillow cover?

186) Baking: Vanessa is baking chocolate chip cookies and oatmeal cookies. She

needs $\frac{1}{2}$ cup of sugar for the chocolate chip cookies and $\frac{1}{4}$ of sugar for the oatmeal cookies. How much sugar does she need altogether?

Odd Answer

185) $\frac{7}{8}$ yard

Exercises: Writing Exercises

Instructions: For questions 187-188, answer the given writing exercises.

187) Why do you need a common denominator to add or subtract fractions? Explain.

188) How do you find the LCD of 2 fractions?

Odd Answer

187) Answers may vary

1.5 DECIMALS

Learning Objectives

By the end of this section, you will be able to:

- Name and write decimals
- Round decimals
- Add and subtract decimals
- Multiply and divide decimals
- Convert decimals, fractions, and percents

Name and Write Decimals

Decimals are another way of writing fractions whose denominators are powers of 10.

$0.1 = \frac{1}{10}$	0.1 is "one tenth"
$0.01 = \frac{1}{100}$	0.01 is "one hundredth"
$0.001 = \frac{1}{1000}$	0.001 is "one thousandth"
$0.0001 = \frac{1}{10000}$	0.0001 is "one ten-thousandth"

Notice that "ten thousand" is a number larger than one, but "one ten-thousand**th**" is a number smaller than one. The "th" at the end of the name tells you that the number is smaller than one.

When we name a whole number, the name corresponds to the place value based on the powers of ten. We read 10,000 as "ten thousand" and 10,000,000 as "ten million." Likewise, the names of the decimal places correspond to their fraction values. (Figure 1.5.1) shows the names of the place values to the left and right of the *decimal* point.

Place value of decimal numbers are shown to the left and right of the decimal point.

Place Value												
Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths
						•					

Figure 1.5.1

Example 1

Name the decimal 4.3.

Solution

Step 1: **Name the number to the left of the decimal point.**

4 is to the left of the decimal point.

4.3
four ________

Step 2: **Write 'and' for the decimal point.**

four and ______

Step 3: **Name the 'number' part to the right of the decimal point as if it were a whole number.**

3 is to the right of the decimal point.

$$\text{four and three } \_\_\_\_\_\_\_\_$$

Step 4: **Name the decimal place.**

Four and three tenths.

Try It

1) Name the decimal: 6.7

Solution

Six and seven tenths.

2) Name the decimal: 5.8

Solution

Five and eight tenths.

We summarize the steps needed to name a decimal below.

How to

Name a Decimal.

1. Name the number to the left of the decimal point.
2. Write "and" for the decimal point.
3. Name the "number" part to the right of the decimal point as if it were a whole number.

4. Name the decimal place of the last digit.

Example 2

Name the decimal: -15.571

Solution

Step 1: **Name the number to the left of the decimal point.**

$$\text{negative fifteen } \_\_\_\_\_\_$$

Step 2: **Write "and" for the decimal point.**

$$\text{negative fifteen and } \_\_\_\_\_\_$$

Step 3: **Name the number to the right of the decimal point.**

$$\text{negative fifteen and five hundred seventy one } \_\_\_\_\_\_$$

Step 4: **The 1 is in the thousandths place.**

Negative fifteen and five hundred seventy-one thousandths.

Try It

3) Name the decimal: -13.461

Solution

Negative thirteen and four hundred sixty-one thousandths.

4) Name the decimal: -2.053.

Solution
Negative two and fifty-three thousandths.

When we write a check we write both the numerals and the name of the number. Let's see how to write the decimal from the name.

Example 3

Write "fourteen and twenty-four thousandths" as a decimal.

Solution
***Step 1:* Look for the word 'and'; it locates the decimal point. Place a decimal point under the word 'and'.**

Translate the words before 'and' into the whole number and place to the left of the decimal point.

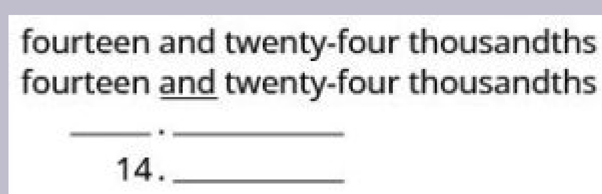


Figure 1.5.2

***Step 2:* Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.**

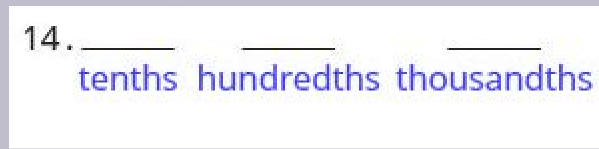


Figure 1.5.3

Step 3: **Translate the words after 'and' into the number to the right of the decimal point.**

Write the number in the spaces — putting the final digit in the last place.

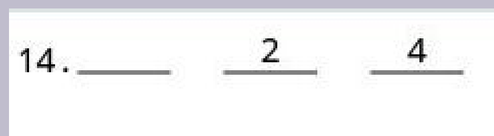


Figure 1.5.4

Step 4: **Fill in zeros for empty place holders as needed.**

Zeros are needed in the tenths place.

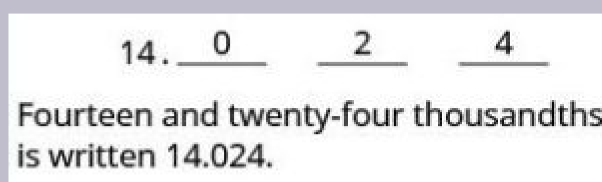


Figure 1.5.5

Try It

5) Write as a decimal: thirteen and sixty-eight thousandths.

Solution

13.068

6) Write as a decimal: five and ninety-four thousandths.

Solution

5.094

We summarize the steps to writing a decimal.

How to

Write a decimal.

1. Look for the word "and"—it locates the decimal point.
   - Place a decimal point under the word "and." Translate the words before "and" into the whole number and place it to the left of the decimal point.
   - If there is no "and," write a "0" with a decimal point to its right.
2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
3. Translate the words after "and" into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
4. Fill in zeros for placeholders as needed.

Round Decimals

Rounding decimals is very much like rounding whole numbers. We will round decimals with a method based on the one we used to round whole numbers.

Example 4

Round 18.379 to the nearest hundredth.

Solution

Step 1: **Locate the given place value and mark it with an arrow.**

Figure 1.5.6

Step 2: **Underline the digit to the right of the given place value.**

Figure 1.5.7

Step 3: **Is this digit greater than or equal to 5?**

Yes: Add 1 to the digit in the given place value.

No: Do not change the digit in the given place value.

Figure 1.5.8

Step 4: **Rewrite the number, removing all digits to the right of the rounding digit.**

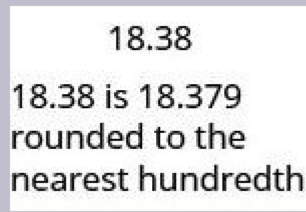


Figure 1.5.9

Try It

7) Round to the nearest hundredth: 1.047

Solution

1.05

8) Round to the nearest hundredth: 9.173

Solution

9.17

We summarize the steps for rounding a decimal here.

How to

Round Decimals.

1. Locate the given place value and mark it with an arrow.
2. Underline the digit to the right of the place value.
3. Is this digit greater than or equal to 5?
    - Yes—add 1 to the digit in the given place value.
    - No—do *not* change the digit in the given place value.
4. Rewrite the number, deleting all digits to the right of the rounding digit.

Example 5

Round 18.379 to the nearest

a. tenth
b. whole number.

Solution
Round 18.379

a. to the nearest tenth

Step 1: **Locate the tenths place with an arrow.**

Figure 1.5.10

Step 2: **Underline the digit to the right of the given place value.**

Figure 1.5.11

Step 3: **Because 7 is greater than or equal to 5, add 1 to the 3.**

Figure 1.5.12

Step 4: **Rewrite the number, deleting all digits to the right of the rounding digit.**

$$18.4$$

Step 5: **Notice that the deleted digits were NOT replaced with zeros.**

So, 18.379 rounded to the nearest tenth is 18.4.

---

b. to the nearest whole number

Step 1: **Locate the ones place with an arrow.**

Figure 1.5.13

Step 2: **Underline the digit to the right of the given place value.**

Figure 1.5.14

Step 3: **Since 3 is not greater than or equal to 5, do not add 1 to the 8.**

Figure 1.5.15

Step 4: **Rewrite the number, deleting all digits to the right of the rounding digit.**

So, 18.379 rounded to the nearest whole number is 18.

Try It

9) Round 6.582 to the nearest

a. hundredth
b. tenth
c. whole number.

Solution
a. 6.58
b. 6.6
c. 7

10) Round 15.2175 to the nearest

a. thousandth
b. hundredth
c. tenth.

Solution
a. 15.218
b. 15.22
c. 15.2

Add and Subtract Decimals

To add or subtract decimals, we line up the decimal points. By lining up the decimal points this way, we can add or subtract the corresponding place values. We then add or subtract the numbers as if they were whole numbers and then place the decimal point in the sum.

How to

Add or Subtract Decimals.

1. Write the numbers so the decimal points line up vertically.
2. Use zeros as placeholders, as needed.
3. Add or subtract the numbers as if they were whole numbers. Then place the decimal point in the answer under the decimal points in the given numbers.

Example 6

Add: $23.5 + 41.38$

Solution

Step 1: **Write the numbers so the decimal points line up vertically.**

$$\begin{array}{r} 23.50 \\ \underline{+41.38} \end{array}$$

Step 2: **Put 0 as a placeholder after the 5 in 23.5. Remember,** $\frac{5}{10} = \frac{50}{100}$ **so,** $0.5 = 0.50$**.**

$$\begin{array}{r} 23.50 \\ \underline{+41.38} \end{array}$$

Step 3: **Add the numbers as if they were whole numbers. Then place the decimal point in the sum.**

$$\begin{array}{r} 23.50 \\ \underline{+41.38} \\ 64.88 \end{array}$$

Try It

11) Add: $4.8 + 11.69$

Solution

16.49

12) Add: $5.123 + 18.47$

Solution

23.593

Example 7

Subtract: $20 - 14.65$

Solution

Step 1: **Write the numbers so the decimal points line up vertically.**

$$\begin{array}{r} 20. \\ \underline{-14.65} \end{array}$$

Step 2: **Put in zeros to the right as placeholders.**

Remember, 20 is a whole number, so place the decimal point after the 0.

$$\begin{array}{r} 20.00 \\ \underline{-14.65} \end{array}$$

Step 3: **Subtract and place the decimal point in the answer.**

$$\begin{array}{r} 20.00 \\ \underline{-14.65} \\ 5.35 \end{array}$$

Try It

13) Subtract $10 - 9.58$.

Solution

0.42

14) Subtract: $50 - 37.42$.

Solution

12.58

Multiply and Divide Decimals

Multiplying decimals is very much like multiplying whole numbers—we just have to determine where to place the decimal point. The procedure for multiplying decimals will make sense if we first convert them to fractions and then multiply.

So let's see what we would get as the product of decimals by converting them to fractions first. We will do two examples side-by-side. Look for a pattern!

	$(0.\underbrace{3}_{1\ place})\ (0.\underbrace{7}_{1\ place})$	$(0.\underbrace{2}_{1\ place})\ (0.\underbrace{46}_{2\ places})$
Convert to fractions.	$\frac{3}{10} \cdot \frac{7}{10}$	$\frac{2}{10} \cdot \frac{46}{100}$
Multiply.	$\frac{21}{100}$	$\frac{92}{1000}$
Convert to decimals.	$(0.\underbrace{21}_{2\ places})$	$(0.\underbrace{096}_{3\ places})$

Notice, in the first example, we multiplied two numbers that each had one digit after the decimal point and the product had two decimal places. In the second example, we multiplied a number with one decimal place by a number with two decimal places and the product had three decimal places.

We multiply the numbers just as we do whole numbers, temporarily ignoring the decimal point. We then count the number of decimal points in the factors and that sum tells us the number of decimal places in the product.

The rules for multiplying positive and negative numbers apply to decimals, too, of course!

When *multiplying* two numbers,

- if their signs are the *same* the product is *positive*.
- if their signs are *different* the product is *negative*.

When we multiply signed decimals, first we determine the sign of the product and then multiply as if the numbers were both positive. Finally, we write the product with the appropriate sign.

How to

Multiply Decimals

1. Determine the sign of the product.
2. Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
3. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors.
4. Write the product with the appropriate sign.

Example 8

Multiply: $(-3.9)(4.075)$

Solution

Step 1: **The signs are different. The product will be negative.**

Step 2: **Write in vertical format, lining up the numbers on the right.**

$$\begin{array}{r} 4.075 \\ \underline{\times \quad 3.9} \end{array}$$

Step 3: **Multiply.**

$$\begin{array}{r} 4.075 \\ \underline{\times \quad 3.9} \\ 158925 \end{array}$$

Step 4: **Add the number of decimal places in the factors** $(1+3)$**.**

$$(-3.\underbrace{9}_{1\ place})\ (4.\underbrace{075}_{3\ places})$$

Step 5: **Place the decimal point 4 places from the right.**

$$\begin{array}{r} 4.075 \\ \times\ \ 3.9 \\ \hline 15.\underbrace{8925}_{\text{4 places}} \end{array}$$

Step 6: **The signs are different, so the product is negative.**

$$(-3.9)(4.075) = -15.8925$$

Try It

15) Multiply: $-4.5(6.107)$

Solution

-27.4815

16) Multiply: $-10.79(8.12)$

Solution

-87.6148

In many of your other classes, especially in the sciences, you will multiply decimals by powers of 10 (10, 100, 1000, etc.). If you multiply a few products on paper, you may notice a pattern relating the number of zeros in the power of 10 to number of decimal places we move the decimal point to the right to get the product.

How to

Multiply a Decimal by a Power of Ten

1. Move the decimal point to the right in the same number of places as the number of zeros in the power of 10.
2. Add zeros at the end of the number as needed.

Example 9

Multiply 5.63

a. by 10
b. by 100
c. by 1,000

Solution
By looking at the number of zeros in the multiple of ten, we see the number of places we need to move the decimal to the right.

a.

Step 1: **There is 1 zero in 10, so move the decimal point 1 place to the right.**

$$5.63(10)$$

Fig. 1.5.16

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b.

Step 1: **There are 2 zeros in 100, so move the decimal point 2 places to the right.**

$$5.63(100)$$

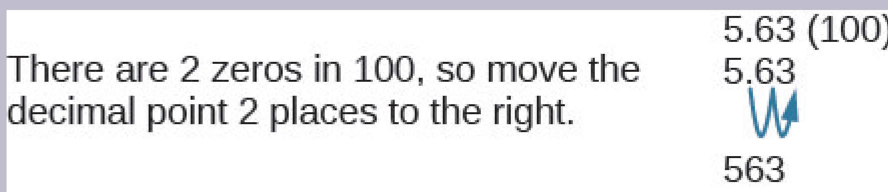

Figure 1.5.17

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c.

Step 1: **There are 3 zeros in 1,000, so move the decimal point 3 places to the right.**

$$5.63(1000)$$

Figure 1.5.18

Step 2: **A zero must be added at the end.**

$$5,630$$

Try It

17) Multiply 2.58

a. by 10
b. by 100
c. by 1,000

Solution
a. 25.8
b. 258
c. 2,580

18) Multiply 14.2
a. by 10
b. by 100
c. by 1,000

Solution
a. 142
b. 1,420
c. 14,200

Just as with multiplication, dividing decimals is like dividing whole numbers. We just have to figure out where the decimal point must be placed.

To divide decimals, determine what power of 10 to multiply the denominator by to make it a whole number. Then multiply the numerator by that same power of 10. Because of the equivalent fractions property, we haven't changed the value of the fraction! The effect is to move the decimal points in the numerator and denominator the same number of places to the right. For example:

$$\frac{0.8}{0.4}$$

$$\frac{0.8(10)}{0.4(10)}$$

$$\frac{8}{4}$$

We use the rules for dividing positive and negative numbers with decimals, too. When dividing signed decimals, first determine the sign of the quotient and then divide as if the numbers were both positive. Finally, write the quotient with the appropriate sign.

We review the notation and vocabulary for division:

Figure 1.5.19

We'll write the steps to take when dividing decimals, for easy reference.

HOW TO

Divide Decimals.

1. Determine the sign of the quotient.
2. Make the divisor a whole number by "moving" the decimal point all the way to the right. "Move" the decimal point in the dividend to the same number of places—adding zeros as needed.
3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
4. Write the quotient with the appropriate sign.

Example 10

Divide: $-25.65 \div (-0.06)$

Solution

Remember, you can "move" the decimals in the divisor and dividend because of the Equivalent Fractions Property.

***Step 1:* The signs are the same.**

The quotient is positive.

***Step 2:* Make the divisor a whole number by "moving" the decimal point all the way to the right.**

***Step 3:* "Move" the decimal point in the dividend the same number of places.**

$$0.06\overline{)25.65}$$

Figure 1.5.20

***Step 4:* Divide.**

Place the decimal point in the quotient above the decimal point in the dividend.

$$\begin{array}{r} 427.5 \\ 006.\overline{)2565.0} \\ \underline{-24} \\ 16 \\ \underline{-12} \\ 45 \\ \underline{-42} \\ 30 \\ \underline{30} \end{array}$$

Figure 1.5.21

***Step 5:* Write the quotient with the appropriate sign.**

$$-25.65 \div (-0.06) = 427.5$$

Try It

19) Divide: $-23.492 \div (-0.04)$

Solution

587.3

20) Divide: $-4.11 \div (-0.12)$

Solution

34.25

A common application of dividing whole numbers into decimals is when we want to find the price of one item that is sold as part of a multi-pack. For example, suppose a case of 24 water bottles costs $3.99. To find the price of one water bottle, we would divide $3.99 by 24. We show this division in Example 1.5.13. In calculations with money, we will round the answer to the nearest cent (hundredth).

Example 11

Divide: $\$3.99 \div 24$.

Solution

Step 1: **Place the decimal point in the quotient above the decimal point in the dividend.**

Step 2: **Divide as usual.**

When do we stop? Since this division involves money, we round it to the nearest cent (hundredth.) To do this, we must carry the division to the thousandths place.

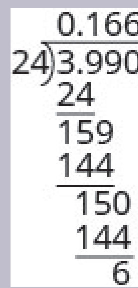


Figure 1.5.22

Step 3: **Round to the nearest cent.**

$$0.166 \approx 0.17$$
$$\$3.99 \div 24 \approx \$0.17$$

Try It

21) Divide: $\$6.99 \div 36$

Solution

0.19

22) Divide: $\$4.99 \div 12$

Solution

0.42

Convert Decimals, Fractions, and Percents

We convert decimals into fractions by identifying the place value of the last (farthest right) digit. In the decimal 0.03, the 3 is in the hundredth place, so 100 is the denominator of the fraction equivalent to 0.03.

$$0.03 = \frac{3}{100}$$

Notice, that when the number to the left of the decimal is zero, we get a fraction whose numerator is less than its denominator. Fractions like this are called proper fractions.

The steps to take to convert a decimal to a fraction are summarized in the procedure box.

How to

Convert a Decimal to a Proper Fraction.

1. Determine the place value of the final digit.
2. Write the fraction.
    - numerator—the “numbers” to the right of the decimal point
    - denominator—the place value corresponding to the final digit

Example 12

Write 0.374 as a fraction.

Solution

Step 1: **Determine the place value of the final digit.**

Figure 1.5.23

Step 2: **Write the fraction for 0.374:**

- The numerator is 374.
- The denominator is 1,000.

$$\frac{374}{1000}$$

Step 3: **Simplify the fraction.**

$$\frac{2 \times 187}{2 \times 500}$$

Step 4: **Divide out the common factors.**

$$\frac{187}{500}$$

So, $0.374 = \frac{187}{500}$.

Did you notice that the number of zeros in the denominator of $\frac{374}{1,000}$ is the same as the number of decimal places in 0.374?

Try It

23) Write 0.234 as a fraction.

Solution

$$\frac{117}{500}$$

24)Write 0.024 as a fraction.

Solution

$$\frac{3}{125}$$

We've learned to convert decimals to fractions. Now we will do the reverse—convert fractions to decimals. Remember that the fraction bar means division. So $\frac{4}{5}$ can be written $4 \div 5$ or $5\overline{)4}$. This leads to the following method for converting a fraction to a decimal.

How to

Convert a Fraction to a Decimal.

To convert a fraction to a decimal, divide the numerator of the fraction by the denominator of the fraction.

Example 13

Write $-\dfrac{5}{8}$ as a decimal.

Solution

Step 1: **Since a fraction bar means division, we begin by writing** $-\dfrac{5}{8}$ **as** $-8\overline{)5}$

Step 2: **Now divide.**

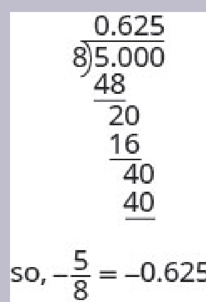


Figure 1.5.24

Try It

25) Write $\dfrac{-7}{8}$ as a decimal.

Solution

-0.875

26) Write $\frac{-3}{8}$ as a decimal.

Solution

-0.375

When we divide, we will not always get a zero remainder. Sometimes the quotient ends up with a decimal that repeats. A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly. A bar is placed over the repeating block of digits to indicate it repeats.

Repeating Decimal

A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly.

A bar is placed over the repeating block of digits to indicate it repeats.

Example 14

Write $\frac{43}{22}$ as a decimal.

Solution

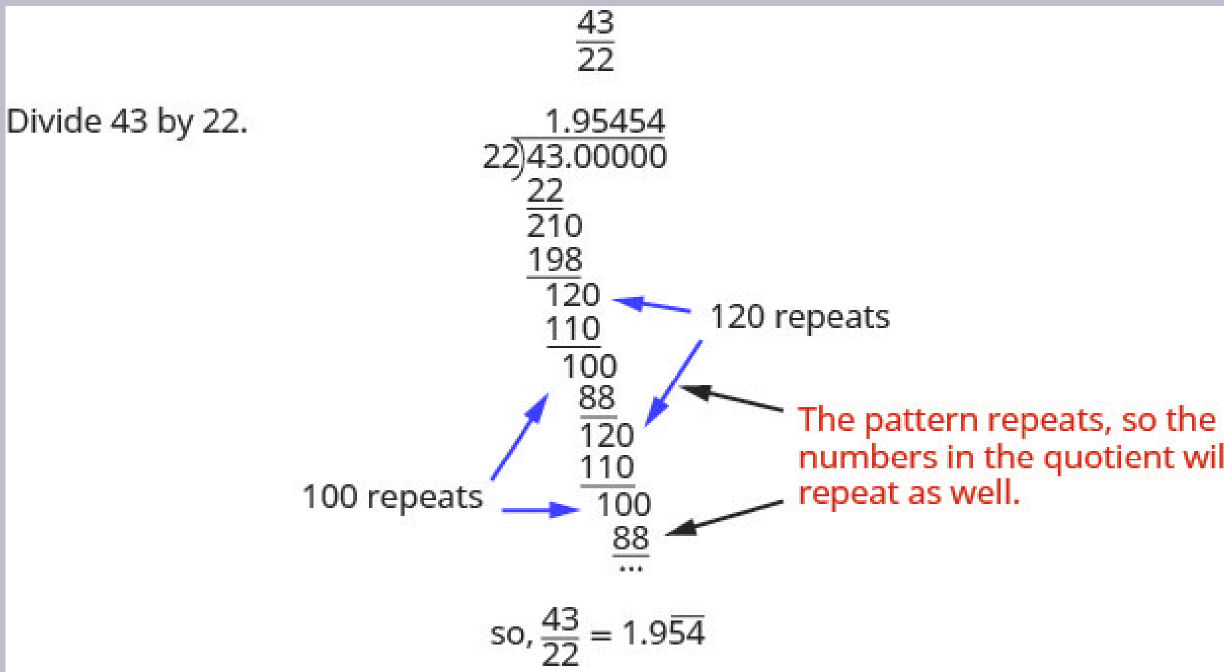


Figure 1.5.25

Try It

27) Write $\dfrac{27}{11}$ as a decimal.

Solution

$2.\overline{45}$

28) Write $\dfrac{51}{22}$ as a decimal.

Solution

$2.3\overline{18}$

Sometimes we may have to simplify expressions with fractions and decimals together.

Example 15

Simplify: $\frac{7}{8} + 6.4$

Solution

First we must change one number so both numbers are in the same form. We can change the fraction to a decimal, or change the decimal to a fraction. Usually it is easier to change the fraction to a decimal.

Step 1: **Change** $\frac{7}{8}$ **to a decimal.**

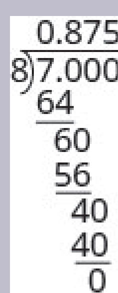


Figure 1.5.26

Step 2: **Add.**

$$0.875 + 6.4 = 7.275$$

So, $\frac{7}{8} + 6.4 = 7.275$

Try It

29) Simplify: $\frac{3}{8} + 4.9$

Solution

5.275

30) Simplify: $5.7 + \frac{13}{20}$

Solution

6.35

A **percent** is a ratio whose denominator is 100. Percent means per hundred. We use the percent symbol, %, to show percent.

Percent

A percent is a ratio whose denominator is 100.

Since a percent is a ratio, it can easily be expressed as a fraction. Percent means per 100, so the denominator of the fraction is 100. We then change the fraction to a decimal by dividing the numerator by the denominator.

	6%	78%	135%
Write as a ratio with a denominator of 100.	$\frac{6}{100}$	$\frac{78}{100}$	$\frac{135}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.06	0.78	1.35

Do you see the pattern? *To convert a percent number to a decimal number, we move the decimal point two places to the left.*

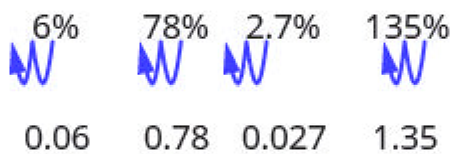


Figure 1.5.27

Example 16

Convert each percent to a decimal:

a. 62%

b. 135%

c. 5.7%

Solution

a.

Step 1: **Move the decimal point two places to the left.**

Figure 1.5.28

$$0.62$$

---

b.

Step 1: **Move the decimal point two places to the left.**

Figure 1.5.29

$$1.35$$

---

c.

Step 1: **Move the decimal point two places to the left.**

Figure 1.5.30

$$0.057$$

Try It

31) Convert each percent to a decimal:

a. 9%

b. 87%

c. 3.9%

Solution

a. 0.09

b. 0.87

c. 0.039

32) Convert each percent to a decimal:

a. 3%

b. 91%

c. 8.3%

Solution

a. 0.03

b. 0.91

c. 0.083

Converting a decimal to a percent makes sense if we remember the definition of percent and keep place value in mind.

To convert a decimal to a percent, remember that percent means per hundred. If we change the decimal to a fraction whose denominator is 100, it is easy to change that fraction to a percent.

	0.83	1.05	0.075
Write as a fraction.	$\frac{83}{100}$	$\frac{5}{100}$	$\frac{75}{1000}$
The denominator is 100.		$\frac{105}{100}$	$\frac{7.5}{100}$
Write the ratio as a percent.	83%	105%	7.5%

Recognize the pattern? *To convert a decimal to a percent, we move the decimal point two places to the right and then add the percent sign*.

Figure 1.5.31

Example 17

Convert each decimal to a percent:

a. 0.51

b. 1.25

c. 0.093

Solution

a.

Step 1: **Move the decimal point two places to the right.**

Figure 1.5.32

$$51\%$$

---

b.

Step 1: **Move the decimal point two places to the right.**

Figure 1.5.33

$$125\%$$

---

c.

Step 1: **Move the decimal point two places to the right.**

Figure 1.5.34

$$9.3\%$$

Try It

33) Convert each decimal to a percent:

a. 0.17

b. 1.75

c. 0.0825

Solution

a. 17%

b. 175%

c. 8.25%

34) Convert each decimal to a percent:

a. 0.41

b. 2.25

c. 0.0925

Solution

a. 41%

b. 225%

c. 9.25%

Key Concepts

- **Name a Decimal**
  1. Name the number to the left of the decimal point.
  2. Write "and" for the decimal point.
  3. Name the "number" part to the right of the decimal point as if it were a whole number.
  4. Name the decimal place of the last digit.
- **Write a Decimal**
  1. Look for the word 'and'—it locates the decimal point. Place a decimal point under the word 'and.' Translate the words before 'and' into the whole number and place it to the left of the decimal point. If there is no "and," write a "0" with a decimal point to its right.
  2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
  3. Translate the words after 'and' into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.

4. Fill in zeros for placeholders as needed.

- **Round a Decimal**
  1. Locate the given place value and mark it with an arrow.
  2. Underline the digit to the right of the place value.
  3. Is this digit greater than or equal to 5? Yes—add 1 to the digit in the given place value. No—do *not* change the digit in the given place value.
  4. Rewrite the number, deleting all digits to the right of the rounding digit.
- **Add or Subtract Decimals**
  1. Write the numbers so the decimal points line up vertically.
  2. Use zeros as place holders, as needed.
  3. Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.
- **Multiply Decimals**
  1. Determine the sign of the product.
  2. Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
  3. Place the decimal point. The number of decimal places in the product is the sum of the decimal places in the factors.
  4. Write the product with the appropriate sign.
- **Multiply a Decimal by a Power of Ten**
  1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
  2. Add zeros at the end of the number as needed.
- **Divide Decimals**
  1. Determine the sign of the quotient.
  2. Make the divisor a whole number by "moving" the decimal point all the way to the right. "Move" the decimal point in the dividend the same number of places – adding zeros as needed.
  3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
  4. Write the quotient with the appropriate sign.

- **Convert a Decimal to a Proper Fraction**
  1. Determine the place value of the final digit.
  2. Write the fraction: numerator—the 'numbers' to the right of the decimal point; denominator—the place value corresponding to the final digit.
- **Convert a Fraction to a Decimal** Divide the numerator of the fraction by the denominator

Glossary

decimal
: A decimal is another way of writing a fraction whose denominator is a power of ten.

percent
: A percent is a ratio whose denominator is 100.

repeating decimal
: A repeating decimal is a decimal in which the last digit or group of digits repeats endlessly.

Exercises: Name and Write Decimals

Instructions: For questions 1–8, write as a decimal.

1) Twenty-nine and eighty-one hundredths

2) **Sixty-one and seventy-four hundredths**

3) **Seven tenths**

4) **Six tenths**

5) **Twenty-nine thousandths**

6) **Thirty-five thousandths**

7) **Negative eleven and nine ten-thousandths**

8) **Negative fifty-nine and two ten-thousandths**

Odd Answers

1) 29.81

3) 0.7

5) 0.029

7) -11.0009

Exercises: Name and Write Decimals

Instructions: For questions 9–16, name each decimal.

9) 5.5

10) 14.02

11) 8.71

12) 2.64

13) **0.002**

14) **0.479**

15) -17.9

16) -31.4

Odd Answers

9) **five and five tenths**

11) **eight and seventy-one hundredths**

13) **two thousandths**

15) **negative seventeen and nine tenths**

Exercises: Round Decimals

Instructions: For questions 17–20, round each number to the nearest tenth.

17) 0.67

18) 0.49

19) 2.84

20) 4.63

Odd Answers

17) 0.7

19) 2.8

Exercises: Round Decimals

Instructions: For questions 21–26, round each number to the nearest hundredth.

21) 0.845

22) 0.761

23) 0.299

24) 0.697

25) 4.098

26) 7.096

Odd Answers

21) 0.85

23) 0.30

25) 4.10

Exercises: Round Decimals

Instructions: For questions 27–30, round each number to the nearest

a) hundredth
b) tenth
c) whole number

27) 5.781

28) 1.6381

29) 63.479

30) 84.281

Odd Answers

27a) 5.78

27b) 5.8

27c) 6

29a) 63.48

29b) 63.5

29c) 63

Exercises: Add and Subtract Decimals

Instructions: For questions 31–48, add or subtract.

31) $16.92 + 7.56$

32) $248.25 - 91.29$

33) $21.76 - 30.99$

34) $38.6 + 13.67$

35) $-16.53 - 24.38$

36) $-19.47 - 32.58$

37) $-38.69 + 31.47$

38) $29.83 + 19.76$

39) $72.5 - 100$

40) $86.2 - 100$

41) $15 + 0.73$

42) $27 + 0.87$

43) $91.95 - (-10.462)$

44) $94.69 - (-12.678)$

45) $55.01 - 3.7$

46) $59.08 - 4.6$

47) $2.51 - 7.4$

48) $3.84 - 6.1$

Odd Answers

31) 24.48

33) -9.23

35) -40.91

37) -7.22

39) -27.5

41) 15.73

43) 102.412

45) 51.31

47) -4.89

Exercises: Multiply Decimals

Instructions: For questions 49–62, multiply.

49) $(0.24)(0.6)$

50) $(0.81)(0.3)$

51) $(5.9)(7.12)$

52) $(2.3)(9.41)$

53) $(-4.3)(2.71)$

54) $(-8.5)(1.69)$

55) $(-5.18)(-65.23)$

56) $(-9.16)(-68.34)$

57) $(0.06)(21.75)$

58) $(0.08)(52.45)$

59) $(9.24)(10)$

60) $(6.531)(10)$

61) $(55.2)(1000)$

62) $(99.4)(1000)$

Odd Answers

49) 0.144

51) 42.008

53) -11.653

55) 337.8914

57) 1.305

59) 92.4

61) $55{,}200$

Exercises: Divide Decimals

Instructions: For questions 63–76, divide.

63) $4.75 \div 25$

64) $12.04 \div 43$

65) $\$117.25 \div 48$

66) $\$109.24 \div 36$

67) $0.6 \div 0.2$

58) $0.8 \div 0.4$

69) $1.44 \div (-0.3)$

70) $1.25 \div (-0.5)$

71) $-1.75 \div (-0.05)$

72) $-1.15 \div (-0.05)$

73) $5.2 \div 2.5$

74) $6.5 \div 3.25$

75) $11 \div 0.55$

76) $14 \div 0.35$

Odd Answers

63) 0.19

65) $\$2.44$

67) 3

69) -4.8

71) 35

73) 2.08

75) 20

Exercises: Convert Decimals and Fractions

Instructions: For questions 77–98, write each decimal as a fraction.

77) 0.04

78) 0.19

79) 0.52

80) 0.78

81) 1.25

82) 1.35

83) 0.375

84) 0.464

85) 0.095

86) 0.085

Instructions: For questions 87-98, convert each fraction to a decimal.

87) $\frac{17}{20}$

88) $\frac{13}{20}$

89) $\frac{11}{4}$

90) $\frac{17}{4}$

91) $-\frac{310}{25}$

92) $-\frac{284}{25}$

93) $\frac{15}{11}$

94) $\frac{18}{11}$

95) $\frac{15}{111}$

96) $\frac{25}{111}$

97) $2.4+\frac{5}{8}$

98) $3.9+\frac{9}{20}$

Odd Answers

77) $\frac{1}{25}$

79) $\frac{13}{25}$

81) $\frac{5}{4}$

83) $\frac{3}{8}$

85) $\frac{19}{200}$

87) 0.85

89) 2.75

91) -12.4

93) $1.\overline{36}$

95) $0.\overline{135}$

97) 3.025

Exercises: Convert Decimals and Percents

Instructions: For questions 99–108, convert each percent to a decimal.

99) 1%

100) 2%

101) 63%

102) 71%

103) 150%

104) 250%

105) 21.4%

106) 39.3%

107) 7.8%

108) 6.4%

Instructions: For questions 109–118, convert each decimal to a percent.

109) 0.01

110) 0.03

111) 1.35

112) 1.56

113) 3

114) 4

115) 0.0875

116) 0.0625

117) 2.254

118) 2.317

Odd Answers

99) 0.01

101) 0.63

103) 1.5

105) 0.214

107) 0.078

109) 1%

111) 135%

113) 300%

115) 8.75%

117) 225.4%

Exercises: Everyday Math

Instructions: For questions 119–124, answer the given everyday math word problems.

119) Salary Increase: Danny got a raise and now makes $\$58{,}965.95$ **a year. Round this number to the nearest**

a) dollar

b) thousand dollars
c) ten thousand dollars

120) New Car Purchase: Selena's new car cost $\$23,795.95$. Round this number to the nearest

a) dollar
b) thousand dollars
c) ten thousand dollars

121) Sales Tax: Hyo Jin lives in San Diego. She bought a refrigerator for $\$1,624.99$ and when the clerk calculated the sales tax it came out to exactly $\$142.186625$. Round the sales tax to the nearest

a) penny
b) dollar

122) Sales Tax: Jennifer bought a $\$1,038.99$ dining room set for her home in Cincinnati. She calculated the sales tax to be exactly $\$67.53435$. Round the sales tax to the nearest

a) penny
b) dollar

123) Paycheck. Annie has two jobs. She gets paid $\$14.04$ per hour for tutoring at City College and $\$8.75$ per

hour at a coffee shop. Last week she tutored for 8 hours and worked at the coffee shop for 15 hours.

a) How much did she earn?
b) If she had worked all 23 hours as a tutor instead of working both jobs, how much more would she have earned?

124) Paycheck. Jake has two jobs. He gets paid $\$7.95$ per hour at the college cafeteria and $\$20.25$ at the art gallery. Last week he worked 12 hours at the cafeteria and 5 hours at the art gallery.

a) How much did he earn?
b) If he had worked all 17 hours at the art gallery instead of working both jobs, how much more would he have earned?

Odd Answers

119a) $\$58,966$

119b) $\$59,000$

119c) $\$60,000$

121a) $\$142.19$

121b) $\$142$

123a) $\$243.57$

123b) $\$79.35$

Exercises: Writing Exercises

Instructions: For questions 125–128, answer the given writing exercises.

125) How does knowing about US money help you learn about decimals?

126) Explain how you write "three and nine hundredths" as a decimal.

127) Without solving the problem "44 is 80% of what number" think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

128) When the Szetos sold their home, the selling price was 500% of what they had paid for the house 30 years ago. Explain what 500% means in this context.

Odd Answers

125) Answers may vary

127) Answers may vary

1.6 THE REAL NUMBERS

Learning Objectives

By the end of this section, you will be able to:

- Simplify expressions with square roots
- Identify integers, rational numbers, irrational numbers, and real numbers
- Locate fractions on the number line
- Locate decimals on the number line

Simplify Expressions with Square Roots

Remember that when a number n is multiplied by itself, we write n^2 and read it "n squared." The result is called the square of n. For example,

8^2 read \text {8 squared}

64 is called the square of 8

Similarly, 121 is the square of 11, because 11^2 is 121.

Square of a Number

If $n^2 = m$, then m is the square of n.

Complete the following table to show the squares of the counting numbers 1 through 15.

Number	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square	n^2								64			121				

The numbers in the second row are called perfect square numbers. It will be helpful to learn to recognize the perfect square numbers.

The squares of the counting numbers are positive numbers. What about the squares of negative numbers? We know that when the signs of two numbers are the same, their product is positive. So the square of any negative number is also positive.

$$(-3)^2 = 9 \qquad (-8)^2 = 64 \qquad (-11)^2 = 121 \qquad (-15)^2 = 225$$

Did you notice that these squares are the same as the squares of the positive numbers?

Sometimes we will need to look at the relationship between numbers and their squares in reverse. Because $10^2 = 100$, we say 100 is the square of 10. We also say that 10 is a square root of 100. A number whose square is m is called a **square root** of m.

Square Root of a Number

If $n^2 = m$, then n is a square root of m.

Notice $(-10)^2 = 100$ also, so -10 is also a square root of 100. Therefore, both 10 and -10 are square roots of 100.

So, every positive number has two square roots—one positive and one negative. What if we only wanted the positive square root of a positive number? The **radical sign**, $\sqrt{m}$, denotes the positive square root. The positive square root is called the principal square root. When we use the radical sign that always means we want the principal square root.

We also use the radical sign for the square root of zero. Because $0^2 = 0$, $\sqrt{0} = 0$. Notice that zero has only one square root.

Square Root Notation

$\sqrt{m}$ is read "the square root of m"

$$\textit{radical sign} \rightarrow \sqrt{m} \leftarrow \textit{radicand}$$

If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.

The square root of m, $\sqrt{m}$, is the positive number whose square is m.

Since 10 is the principal square root of 100, we write $\sqrt{100} = 10$. You may want to complete the following table to help you recognize square roots.

[table id=9 /]

Try It

Simplify:

1) $\sqrt{25}$

2) $\sqrt{121}$

Solution

1) Since $5^2 = 25$

$\sqrt{25} = 5$

2)Since $11^2 = 121$

$\sqrt{121} = 11$

Try It

Simplify:

3) $\sqrt{36}$

4) $\sqrt{169}$

Solution

3) 6

4) 13

Try It

Simplify:

5) $\sqrt{16}$

6) $\sqrt{196}$

Solution

5) 4

6) 14

We know that every positive number has two square roots and the radical sign indicates the positive one. We write $\sqrt{100} = 10$. If we want to find the negative square root of a number, we place a negative in front of the radical sign. For example, $-\sqrt{100} = -10$. We read $-\sqrt{100}$ as "the opposite of the square root of 10."

Try It

Simplify:

7) $-\sqrt{9}$

8) $-\sqrt{144}$

Solution

7) The negative is in front of the radical sign. $-\sqrt{9} = -3$

8) The negative is in front of the radical sign. $-\sqrt{144} = -12$

Try It

Simplify:

9) $-\sqrt{4}$

10) $-\sqrt{225}$

Solution

9) -2

10) -15

Try It

Simplify:

11) $-\sqrt{81}$

12) $-\sqrt{100}$

Solution

11) -9

12) -10

Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

We have already described numbers as *counting numbers, whole numbers,* and *integers*. What is the difference between these types of numbers?

Counting numbers	$1, 2, 3, 4, \ldots$
Whole numbers	$0, 1, 2, 3, 4, \ldots$
Integers	$\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$

What type of numbers would we get if we started with all the integers and then included all the fractions? The numbers we would have form the set of rational numbers. A rational number is a number that can be written as a ratio of two integers.

Rational Number

A rational number is a number of the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$.

A rational number can be written as the ratio of two integers.

All signed fractions, such as $\frac{4}{5}, -\frac{7}{8}, \frac{13}{4}, -\frac{20}{3}$ are rational numbers. Each numerator and each denominator is an integer.

Are integers rational numbers? To decide if an integer is a rational number, we try to write it as a ratio of two integers. Each integer can be written as a ratio of integers in many ways. For example, 3 is equivalent to $\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \frac{12}{4}, \frac{15}{5} \ldots$

An easy way to write an integer as a ratio of integers is to write it as a fraction with denominator one.

$$3 = \frac{3}{1} \qquad -8 = -\frac{8}{1} \qquad 0 = \frac{0}{1}$$

Since any integer can be written as the ratio of two integers, all integers are rational numbers! Remember that the counting numbers and the whole numbers are also integers, and so they, too, are rational.

What about decimals? Are they rational? Let's look at a few to see if we can write each of them as the ratio of two integers.

We've already seen that integers are rational numbers. The integer -8 could be written as the decimal -8.0. So, clearly, some decimals are rational.

Think about the decimal 7.3. Can we write it as a ratio of two integers? Because 7.3 means $7\frac{3}{10}$, we can write it as an improper fraction, $\frac{73}{10}$. So 7.3 is the ratio of the integers 73 and 10. It is a rational number.

In general, any decimal that ends after a number of digits (such as 7.3 or -1.2684) is a rational number. We can use the place value of the last digit as the denominator when writing the decimal as a fraction.

Example 1

Write as the ratio of two integers:

a. −27
b. 7.31

Solution

a.

Step 1: **Write it as a fraction with denominator 1.**

$$-27 = \frac{-27}{1}$$

---

b.

Step 1: **Write it as a mixed number.**

Remember, 7 is the whole number and the decimal part, 0.31, indicates hundredths.

$$7\frac{31}{100}$$

Step 2: Convert to an improper fraction.

$$\frac{731}{100}$$

So we see that -27 and 7.31 are both rational numbers, since they can be written as the ratio of two integers.

Try It

Write as the ratio of two integers:

13) -24

14) 3.57

Solution

13) $\frac{-24}{1}$

14) $\frac{357}{100}$

Try It

Write as the ratio of two integers:

15) -19

16) 8.41

Solution

15) $\frac{-19}{1}$

16) $\frac{841}{100}$

Let's look at the decimal form of the numbers we know are rational.

We have seen that *every integer is a rational number*, since $a = \frac{a}{1}$ for any integer, a. We can also change any integer to a decimal by adding a decimal point and a zero.

Integer	-2	-1	0	1	2	3
Decimal form	-2.0	-1.0	0.0	1.0	2.0	3.0

These decimal numbers stop.

We have also seen that *every fraction is a rational number*. Look at the decimal form of the fractions we considered above.

Ratio of integers	$\frac{4}{5}$	$-\frac{7}{8}$	$\frac{13}{4}$	$-\frac{20}{3}$
The decimal form	0.8	-0.875	3.25	$-6.666\ldots$ $-6.\overline{6}$

These decimals either stop or repeat.

What do these examples tell us?

Every rational number can be written both as a ratio of integers, $\frac{p}{q}$, where p and q are integers and $q \neq 0$, *and as a decimal that either stops or repeats.*

Here are the numbers we looked at above expressed as a ratio of integers and as a decimal:

[table id=10 /]

Rational Number

A rational number is a number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Its decimal form stops or repeats.

Are there any decimals that do not stop or repeat? Yes!

The number π (the Greek letter pi, pronounced "pie"), which is very important in describing circles, has a decimal form that does not stop or repeat.

$$\pi = 3.141592654...$$

We can even create a decimal pattern that does not stop or repeat, such as

$$2.01001000100001\ldots$$

Numbers whose decimal form does not stop or repeat cannot be written as a fraction of integers. We call these numbers **irrational**.

Irrational Number

An irrational number is a number that cannot be written as the ratio of two integers.

Its decimal form does not stop and does not repeat.

Let's summarize a method we can use to determine whether a number is rational or irrational.

Rational or Irrational?

If the decimal form of a number

- *repeats or stops*, the number is *rational*.
- *does not repeat and does not stop*, the number is *irrational*.

Example 2

Given the numbers $0.58\overline{3}$, 0.47, $3.605551275\ldots$ list the

a. rational numbers
b. irrational numbers

Solution

a.

Step 1: **Look for decimals that repeat or stop.**

The 3 repeats in $0.58\overline{3}$.

The decimal 0.47 stops after the 7.

So $0.58\overline{3}$ and 0.47 are rational.

b.

***Step 1:* Look for decimals that repeat or stop.**

$3.605551275\ldots$ has no repeating block of digits and it does not stop.

So $3.605551275\ldots$ is irrational.

Try It

For the given numbers list the:

17) rational numbers
18) irrational numbers

0.29, $0.81\overline{6}$, $2.515115111\ldots$

Solution

17) $0.29, 0.81\overline{6}$
18) $2.515115111\ldots$

Try It

For the given numbers list the:

19) rational numbers
20) irrational numbers

$2.6\overline{3}$, 0.125, $0.418302\ldots$

Solution

19) $2.6\overline{3}$, 0.125

20) $0.418302\ldots$

Try It

For each number given, identify whether it is rational or irrational:

21) $\sqrt{36}$

22) $\sqrt{44}$

Solution

21) Recognize that 36 is a perfect square, since $6^2 = 36$. So $\sqrt{36} = 6$, therefore $\sqrt{36}$ is rational.

22) Remember that $6^2 = 36$ and $7^2 = 49$, so 44 is not a perfect square. Therefore, the decimal form of $\sqrt{44}$ will never repeat and never stop, so $\sqrt{44}$ is irrational.

Try It

For each number given, identify whether it is rational or irrational:

23) $\sqrt{81}$

24) $\sqrt{17}$

Solution

23) rational

24) irrational

Try It

For each number given, identify whether it is rational or irrational:

25) $\sqrt{116}$

26) $\sqrt{121}$

Solution

25) irrational

26) rational

We have seen that all counting numbers are whole numbers, all whole numbers are integers, and all integers are rational numbers. The irrational numbers are numbers whose decimal form does not stop and does not repeat. When we put together the rational numbers and the irrational numbers, we get the set of **real numbers**.

Real Number

A real number is a number that is either rational or irrational.

All the numbers we use in elementary algebra are real numbers. Figure 1.6.1 illustrates how the number sets we've discussed in this section fit together.

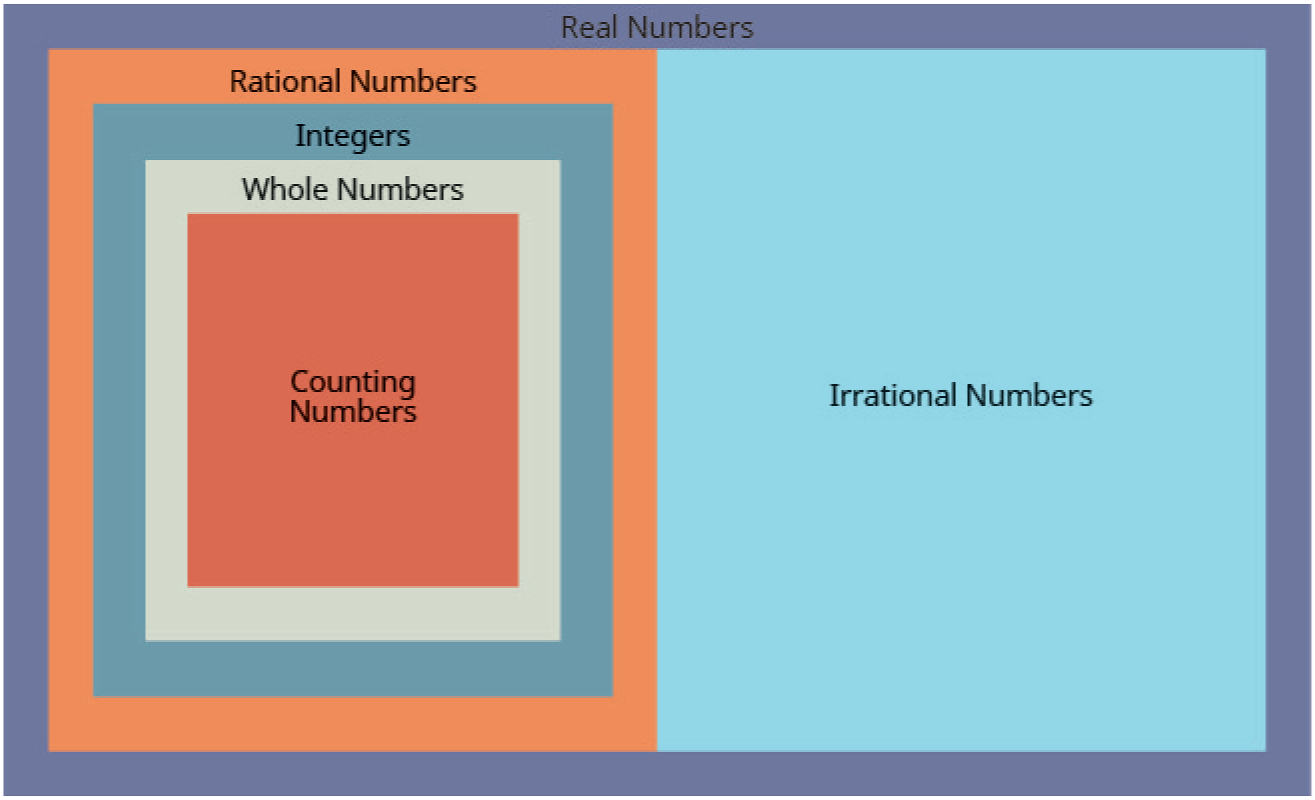


Figure 1.6.1. This chart shows the number sets that make up the set of real numbers. Does the term "real numbers" seem strange to you? Are there any numbers that are not "real," and, if so, what could they be?

This chart shows the number sets that make up the set of real numbers. Does the term "real numbers" seem strange to you? Are there any numbers that are not "real," and, if so, what could they be?

Can we simplify $\sqrt{-25}$? Is there a number whose square is -25?

$$(\quad)^2 = -25?$$

None of the numbers that we have dealt with so far has a square that is -25. Why? Any positive number squared is positive. Any negative number squared is positive. So we say there is no real number equal to $\sqrt{-25}$.

The square root of a negative number is not a real number.

Try It

For each number given, identify whether it is a real number or not a real number:

27) $\sqrt{-169}$

28) $-\sqrt{64}$

Solution

27) There is no real number whose square is -169. Therefore, $\sqrt{-169}$ is not a real number.

28) Since the negative is in front of the radical, $-\sqrt{64}$ is -8. Since -8 is a real number, $-\sqrt{64}$ is a real number.

Try It

For each number given, identify whether it is a real number or not a real number:

29) $\sqrt{-196}$

30) $-\sqrt{81}$

Solution

29) not a real number

30) real number

Try It

For each number given, identify whether it is a real number or not a real number:

31) $-\sqrt{49}$

32) $\sqrt{-121}$

Solution

31) real number

32) not a real number

Example 3

Given the numbers $-7, \frac{14}{5}, 8, \sqrt{5}, 5.9, -\sqrt{64}$, list the:

a. whole numbers
b. integers
c. rational numbers
d. irrational numbers
e. real numbers

Solution

a. Remember, the whole numbers are 0, 1, 2, 3, … and 8 is the only whole number given.

---

b. The integers are the whole numbers, their opposites, and 0. So the whole number 8 is an integer, and -7 is the opposite of a whole number so it is an integer, too. Also, notice that 64 is the square of 8 so $-\sqrt{64}=-8$. So the integers are $-7, 8, -\sqrt{64}$.

---

c. Since all integers are rational, then $-7, 8, -\sqrt{64}$ are rational. Rational numbers also include fractions and decimals that repeat or stop, so $\frac{14}{5}$ and 5.9 are rational. So the list of rational numbers is $-7, \frac{14}{5}, 8, 5.9, -\sqrt{64}$.

---

d. Remember that 5 is not a perfect square, so $\sqrt{5}$ is irrational.

---

e. All the numbers listed are real numbers.

Try It

For the given numbers, list the:

33) whole numbers
34) integers
35) rational numbers
36) irrational numbers
37) real numbers

$-3, -\sqrt{2}, 0.\overline{3}, \frac{9}{5}, 4, \sqrt{49}$.

Solution

33) $4, \sqrt{49}$

34) $-3, 4, \sqrt{49}$

35) $-3, 0.\overline{3}, \frac{9}{5}, 4, \sqrt{49}$

36) $-\sqrt{2}$

37) $-3, -\sqrt{2}, 0.\overline{3}, \frac{9}{5}, 4, \sqrt{49}$

Try It

For the given numbers, list the:

38) whole numbers
39) integers
40) rational numbers
41) irrational numbers
42) real numbers

$-\sqrt{25}, -\frac{3}{8}, -1, 6, \sqrt{121}, 2.041975\ldots$

Solution

38) $6, \sqrt{121}$

39) $-\sqrt{25}, -1, 6, \sqrt{121}$

40) $-\sqrt{25}, -\frac{3}{8}, -1, 6, \sqrt{121}$

41) $2.041975\ldots$

42) $-\sqrt{25}, -\frac{3}{8}, -1, 6, \sqrt{121}, 2.041975\ldots$

Locate Fractions on the Number Line

The last time we looked at the number line, it only had positive and negative integers on it. We now want to include fractions and decimals on it.

Let's start with fractions and locate $\frac{1}{5}, -\frac{4}{5}, 3, \frac{7}{4}, -\frac{9}{2}, -5, \text{ and } \frac{8}{3}$ on the number line.

We'll start with the whole numbers 3 and -5, because they are the easiest to plot. See Figure 1.6.2

The proper fractions listed are $\frac{1}{5}$ and $-\frac{4}{5}$. We know the proper fraction $\frac{1}{5}$ has value less than one and so would be located between 0 and 1. The denominator is 5, so we divide the unit from 0 to 1 into 5 equal parts $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$. We plot $\frac{1}{5}$. See Figure 1.6.2.

Similarly, $-\frac{4}{5}$ is between 0 and -1. After dividing the unit into 5 equal parts we plot $-\frac{4}{5}$. See Figure 1.6.2

Finally, look at the improper fractions $\frac{7}{4}, -\frac{9}{2}, \frac{8}{3}$. These are fractions in which the numerator is greater than the denominator. Locating these points may be easier if you change each of them to a mixed number. See Figure. 1.6.2.

$$\frac{7}{4} = 1\frac{3}{4} \qquad -\frac{9}{2} = -4\frac{1}{2} \qquad \frac{8}{3} = 2\frac{2}{3}$$

Figure 1.6.2 shows the number line with all the points plotted.

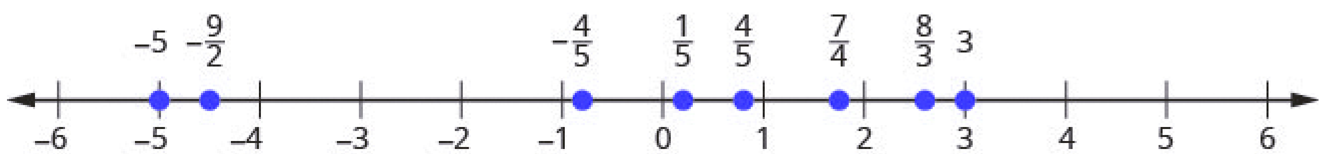


Figure 1.6.2

Example 4

Locate and label the following on a number line: $4, \frac{3}{4}, -\frac{1}{4}, -3, \frac{6}{5}, -\frac{5}{2}, \text{ and } \frac{7}{3}$.

Solution

Locate and plot the integers, $4, -3$

***Step 1:* Locate the proper fraction $\frac{3}{4}$ first.**

The fraction $\frac{3}{4}$ is between 0 and 1.

***Step 2:* Divide the distance between 0 and 1 into four equal parts then, we plot $\frac{3}{4}$.**

Similarly plot $-\frac{1}{4}$.

***Step 3:* Now locate the improper fractions $\frac{6}{5}, -\frac{5}{2}, \frac{7}{3}$.**

$$\frac{6}{5} = 1\frac{1}{5}$$
$$-\frac{5}{2} = -2\frac{1}{2}$$
$$\frac{7}{3} = 2\frac{1}{3}$$

It is easier to plot them if we convert them to mixed numbers and then plot them as described above:

Figure 1.6.3

Try It

43) Locate and label the following on a number line: $-1, \frac{1}{3}, \frac{6}{5}, -\frac{7}{4}, \frac{9}{2}, 5, -\frac{8}{3}$.

Solution

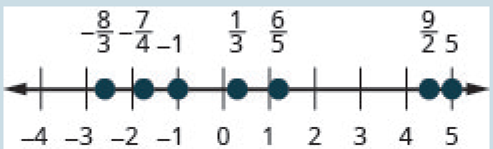


Figure 1.6.4

Try It

44) Locate and label the following on a number line: $-2, \frac{2}{3}, \frac{7}{5}, -\frac{7}{4}, \frac{7}{2}, 3, -\frac{7}{3}$.

Solution

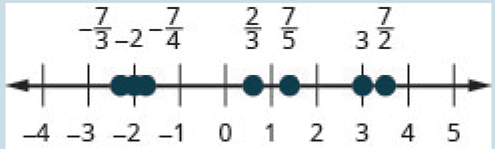


Figure 1.6.5

In Example 1.6.5, we'll use the inequality symbols to order fractions. In previous chapters we used the number line to order numbers.

- $a < b$ "a is less than b" when a is to the left of b on the number line
- $a > b$ "a is greater than b" when a is to the right of b on the number line

As we move from left to right on a number line, the values increase.

Example 5

Order each of the following pairs of numbers, using $<$ or $>$. It may be helpful to refer Figure 1.6.6.

a. $-\frac{2}{3}\_\_\_-1$
b. $-3\frac{1}{2}\_\_\_-3$
c. $-\frac{3}{4}\_\_\_-\frac{1}{4}$
d. $-2\_\_\_-\frac{8}{3}$

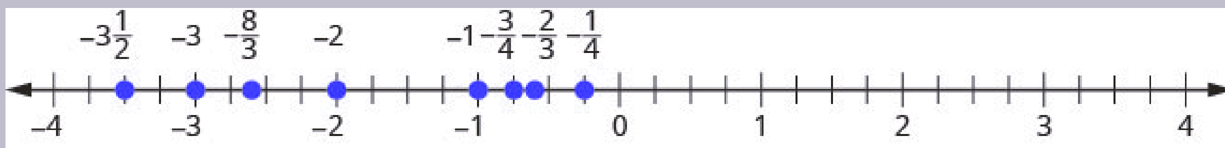


Figure 1.6.6

Solution

a.

Step 1: $-\frac{2}{3}$ **is to the right of** -1 **on the number line.**

$$-\frac{2}{3}\_\_\_-1 = -\frac{2}{3} > -1$$

b.

Step 1: $-3\frac{1}{2}$ **is to the right of** -3 **on the number line.**

$$-3\frac{1}{2}\_\_\_-3 = -3\frac{1}{2} < -3$$

c.

Step 1: $-\frac{3}{4}$ **is to the right of** $-\frac{1}{4}$ **on the number line.**

$$-\frac{3}{4}\_\_\_-\frac{1}{4} = -\frac{3}{4} < -\frac{1}{4}$$

d.

Step 1: **-2 is to the right of $-\dfrac{8}{3}$ on the number line.**

$$-2 \_\_\_ -\frac{8}{3} = -2 > -\frac{8}{3}$$

Try It

Order each of the following pairs of numbers, using < or >:

45) $-\frac{1}{3}\_\_\_-1$

46) $-1\frac{1}{2}\_\_\_-2$

47) $-\frac{2}{3}\_\_\_-\frac{1}{3}$

48) $-3\_\_\_-\frac{7}{3}$

Solution

45) $>$

46) $>$

47) $>$

48) $<$

Try It

Order each of the following pairs of numbers, using < or >:

49) $-1\_\_-\frac{2}{3}$

50) $-2\frac{1}{4}\_\_-2$

51) $-\frac{3}{5}\_\_-\frac{4}{5}$

52) $-4\_\_-\frac{10}{3}$

Solution

49) $<$

50) $<$

51) $>$

52) $<$

Locate Decimals on the Number Line

Since decimals are forms of fractions, locating decimals on the number line is similar to locating fractions on the number line.

Example 6

Locate 0.4 on the number line.

Solution

Step 1: A proper fraction has value less than one.

The decimal number 0.4 is equivalent to $\frac{4}{10}$, a proper fraction, so 0.4 is located between 0 and 1.

Step 2: **On a number line, divide the interval between 0 and 1 into 10 equal parts.**

Step 3: **Now label the parts 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0.**

We write 0 as 0.0 and 1 and 1.0, so that the numbers are consistently in tenths.

Step 4: **Finally, mark 0.4 on the number line.** See Figure 1.6.7.

Figure 1.6.7

Try It

53) Locate on the number line: 0.6.

Solution

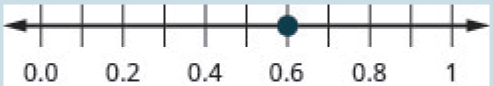


Figure 1.6.8

Try It

54) Locate on the number line: 0.9.

Solution

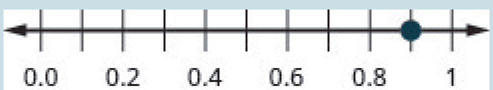


Figure 1.6.9

Example 7

Locate -0.74 on the number line.

Solution

Step 1: **The decimal** -0.74 **is equivalent to** $-\frac{74}{100}$**, so it is located between** 0 **and** -1.

Step 2: **On a number line, mark off and label the hundredths in the interval between** 0 **and** -1. See Figure 1.6.10.

Figure 1.6.10

Try It

55) Locate on the number line: -0.6.

Solution

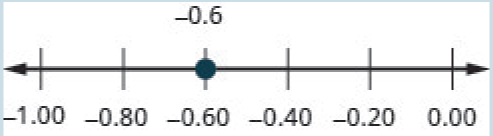


Figure 1.6.11

Try It

56) Locate on the number line: -0.7.

Solution

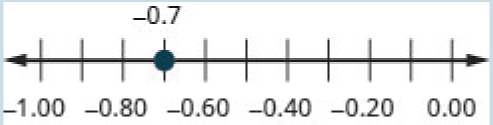


Figure 1.6.12

Which is larger, 0.04 or 0.40? If you think of this as money, you know that $\$0.40$ (forty cents) is greater than $\$0.04$ (four cents). So, $0.40 > 0.04$.

Again, we can use the number line to order numbers.

- $a < b$ "a is less than b" when a is to the left of b on the number line

- $a > b$ "a is greater than b" when a is to the right of b on the number line

Where are 0.04 and 0.40 located on the number line? See Figure 1.6.13.

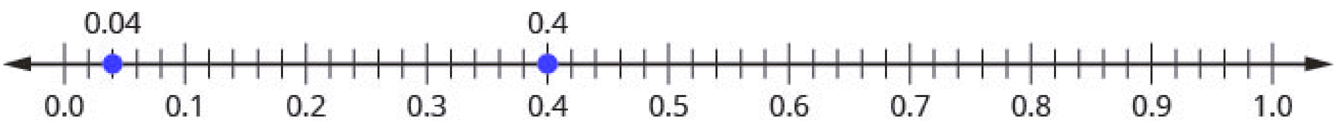


Figure 1.6.13

We see that 0.40 is to the right of 0.04 on the number line. This is another way to demonstrate that $0.40 > 0.04$.

How does 0.31 compare to 0.308? This doesn't translate into money to make it easy to compare. But if we convert 0.31 and 0.308 into fractions, we can tell which is larger.

[table id=11 /]

Because $310 > 308$, we know that $\frac{310}{1000} > \frac{308}{1000}$. Therefore, $0.31 > 0.308$.

Notice what we did in converting 0.31 to a fraction—we started with the fraction $\frac{31}{100}$ and ended with the equivalent fraction $\frac{310}{1000}$. Converting $\frac{310}{1000}$ back to a decimal gives 0.310. So 0.31 is equivalent to 0.310. Writing zeros at the end of a decimal does not change its value!

$$\frac{31}{100} = \frac{310}{1000} \quad \text{and} \quad 0.31 = 0.310$$

We say 0.31 and 0.310 are **equivalent decimals**.

Equivalent Decimals

Two decimals are equivalent if they convert to equivalent fractions.

We use equivalent decimals when we order decimals.

The steps we take to order decimals are summarized here.

How to

Order Decimals

Step 1: Write the numbers one under the other, lining up the decimal points.
Step 2: Check to see if both numbers have the same number of digits. If not, write zeros at the end of the one with fewer digits to make them match.
Step 3: Compare the numbers as if they were whole numbers.
Step 4: Order the numbers using the appropriate inequality sign.

Example 8

Order $0.64\_\_0.6$ using $<$ or $>$.

Solution

***Step 1:* Write the numbers one under the other, lining up the decimal points.**

$$\begin{array}{c}0.64\\0.6\end{array}$$

***Step 2:* Add a zero to 0.6 to make it a decimal with 2 decimal places.**

Now they are both hundredths.

$$0.64$$
$$0.60$$

Step 3: **64 is greater than 60.**

$$64 > 60$$

Step 4: **64 hundredths is greater than 60 hundredths.**

$$0.64 > 0.60$$

Try It

57) Order each of the following pairs of numbers, using $< \text{ or } > : 0.42\_\_\_0.4$.

Solution

$$>$$

Try It

58) Order each of the following pairs of numbers, using $< \text{ or } > : 0.18\_\_\_0.1$.

Solution

$$>$$

Example 9

Order $0.83\_\_0.803$ using $<$ or $>$.

Solution

Step 1: **Write the numbers one under the other, lining up the decimals.**

Step 2: **They do not have the same number of digits.**

$$\begin{array}{l} 0.83 \\ 0.803 \end{array}$$

Step 3: **Write one zero at the end of 0.83.**

$$\begin{array}{l} 0.830 \\ 0.803 \end{array}$$

Step 4: **Since $830 > 803$, 830 thousandths is greater than 803 thousandths.**

$$0.830 > 0.803$$

$$0.83 > 0.803$$

Try It

59) Order the following pair of numbers, using $< \text{ or } > :0.76\_\_0.706$.

Solution

$>$

Try It

60) Order the following pair of numbers, using $< \text{ or } >: 0.305\_\_0.35$.

Solution

$<$

When we order negative decimals, it is important to remember how to order negative integers. Recall that larger numbers are to the right on the number line. For example, because -2 lies to the right of -3 on the number line, we know that $-2 > -3$. Similarly, smaller numbers lie to the left on the number line. For example, because -9 lies to the left of -6 on the number line, we know that $-9 < -6$. See Figure 1.6.14.

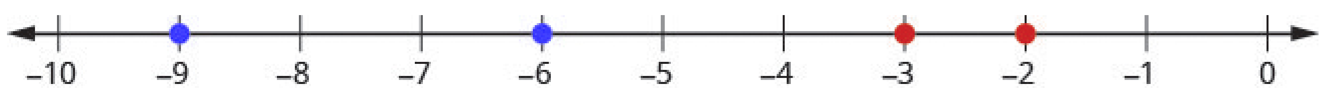


Figure 1.6.14

If we zoomed in on the interval between 0 and -1, as shown in Figure 1.6.6, we would see in the same way that $-0.2 > -0.3$ and $-0.9 < -0.6$.

Example 10

Use $<$ or $>$ to order $-0.1\_\_ -0.8$.

Solution

Step 1: **Write the numbers one under the other, lining up the decimal points.**

They have the same number of digits. $\begin{array}{r} -0.1 \\ -0.8 \end{array}$

Since $-1 > -8$, -1 tenth is greater than -8 tenths. $-0.1 > -0.8$

Try It

61) Order the following pair of numbers, using < or >: $-0.3\_\_ -0.5$.

Solution

$>$

Try It

62) Order the following pair of numbers, using < or >: $-0.6\_\_-0.7$.

Solution

$$>$$

Key Concepts

Square Root Notation

$\sqrt{m}$ is read 'the square root of m.' If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.

Order Decimals

1. Write the numbers one under the other, lining up the decimal points.
2. Check to see if both numbers have the same number of digits. If not, write zeros at the end of the one with fewer digits to make them match.
3. Compare the numbers as if they were whole numbers.
4. Order the numbers using the appropriate inequality sign.

Glossary

equivalent decimals
Two decimals are equivalent if they convert to equivalent fractions.

irrational number
An irrational number is a number that cannot be written as the ratio of two integers. Its decimal form does not stop and does not repeat.

rational numberA rational number is a number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. A rational number can be written as the ratio of two integers. Its decimal form stops or repeats.

radical sign
A radical sign is the symbol $\sqrt{m}$ that denotes the positive square root.

real number
A real number is a number that is either rational or irrational.

square and square root
If $n^2 = m$, then m is the square of n and n is a square root of m.

Exercises: Simplify Expressions with Square Roots

Instructions: For questions 1-12, simplify.

1) $\sqrt{36}$

2) $\sqrt{4}$

3) $\sqrt{64}$

4) $\sqrt{169}$

5) $\sqrt{9}$

6) $\sqrt{16}$

7) $\sqrt{100}$

8) $\sqrt{144}$

9) $-\sqrt{4}$

10) $-\sqrt{100}$

11) $-\sqrt{1}$

12) $-\sqrt{121}$

---

1) 6

3) 8

5) 3

7) 10

9) -2

11) -1

Exercises: Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

Instructions: For questions 13-16, write as the ratio of two integers.

13a) 5

13b) 3.19

14a) 8

14b) 1.61

15a) -12

15b) 9.279

16a) -16

16b) 4.399

Instructions: For questions 17-20, list the

- **a) rational numbers**
- **b) irrational numbers**

17) $0.75, 0.22\overline{3}, 1.39174...$

18) $0.36, 0.94729..., 2.52\overline{8}$

19) $0.4\overline{5}, 1.919293..., 3.59$

20) $0.1\overline{3}, 0.42982..., 1.875$

Instructions: For questions 21-24, identify whether each number is rational or irrational.

21a) $\sqrt{25}$

21b) $\sqrt{30}$

22a) $\sqrt{44}$

22b) $\sqrt{49}$

23a) $\sqrt{164}$

23b) $\sqrt{169}$

24a) $\sqrt{225}$

24b) $\sqrt{216}$

Instructions: For questions 25-28, identify whether each number is a real number or not a real number.

25a) $-\sqrt{81}$

25b) $\sqrt{-121}$

26a) $-\sqrt{64}$

26b) $\sqrt{-9}$

27a) $\sqrt{-36}$

27b) $-\sqrt{144}$

28a) $\sqrt{-49}$

28b) $-\sqrt{144}$

Instructions: For each set of numbers in questions 29-32, list the

- **a) whole numbers**
- **b) integers**
- **c) rational numbers**
- **d) irrational numbers**
- **e) real numbers**

29) $-8, 0, 1.95286..., \frac{12}{5}, \sqrt{36}, 9$

30) $-9, -3\frac{4}{9}, -\sqrt{9}, 0.40\overline{9}, \frac{11}{6}, 7$

31) $-\sqrt{100}, -7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$

32) $-6, -\frac{5}{2}, 0, 0.\overline{714285}, 2\frac{1}{5}, \sqrt{14}$

Odd Answers

13a) $\frac{5}{1}$

13b) $\frac{319}{100}$

15a) $\frac{-12}{1}$

15b) $\frac{9297}{1000}$

17a) $0.75, 0.22\overline{3}$

17b) $1.39174...$

19a) $0.4\overline{5}, 3.59$

19b) $1.919293...$

21a) **rational**

21b) **irrational**

23a) **irrational**

23b) **rational**

25a) **real number**

25b) **not a real number**

27a) **not a real number**

27b) **real number**

29a) $0, \sqrt{36}, 9$

29b) $-8, 0, \sqrt{36}, 9$

29c) $-8, 0, \frac{12}{5}, \sqrt{36}, 9$

29d) $1.95286...$

29e) $-8, 0, 1.95286..., \frac{12}{5}, \sqrt{36}, 9$

31a) **none**

31b) $-\sqrt{100}, -7, -1$

31c) $-\sqrt{100}, -7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$

31d) none

31e) $-\sqrt{100}, -7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$

Exercises: Locate Fractions on the Number Line

Instructions: For questions 33-40, locate the numbers on a number line.

33) $\frac{3}{4}, \frac{8}{5}, \frac{10}{3}$

34) $\frac{1}{4}, \frac{9}{5}, \frac{11}{3}$

35) $\frac{3}{10}, \frac{7}{2}, \frac{11}{6}, 4$

36) $\frac{7}{10}, \frac{5}{2}, \frac{13}{8}, 3$

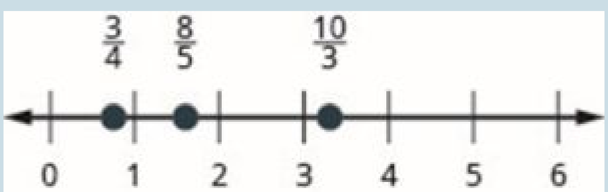
37) $\frac{2}{5}, -\frac{2}{5}$

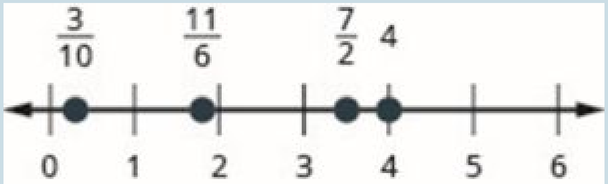
38) $\frac{3}{4}, -\frac{3}{4}$

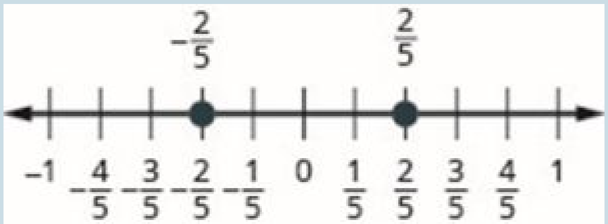
39) $\frac{3}{4}, -\frac{3}{4}, 1\frac{2}{3}, -1\frac{2}{3}, \frac{5}{2}, -\frac{5}{2}$

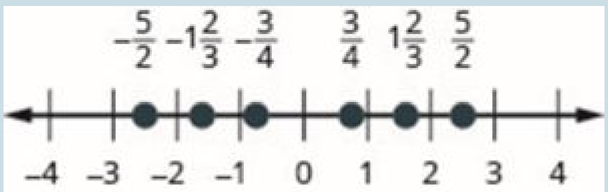
40) $\frac{2}{5}, -\frac{2}{5}, 1\frac{3}{4}, -1\frac{3}{4}, \frac{8}{3}, -\frac{8}{3}$

Odd Answers

33) 

35) 

37) 

39) 

Exercises: Order Pairs of Numbers

Instructions: For questions 41-48, order each of the pairs of numbers, using < or >.

41) $-1 \_\_ -\frac{1}{4}$

42) $-1 \_\_ -\frac{1}{3}$

43) $-2\frac{1}{2} \_\_ -3$

44) $-1\frac{3}{4} \_\_ -2$

45) $-\frac{5}{12} \_\_ -\frac{7}{12}$

46) $-\frac{9}{10} \_\_ -\frac{3}{10}$

47) $-3 \_\_ -\frac{13}{5}$

48) $-4 \_\_ -\frac{23}{6}$

Odd Answers

41) $<$

43) $>$

45) $>$

47) $<$

Exercises: Locate Decimals on the Number Line

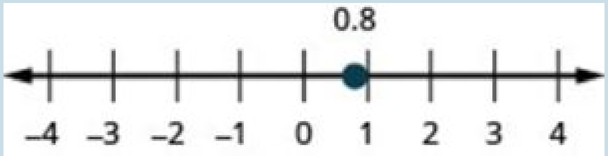
Instructions: For questions 49-52, locate the number on the number line.

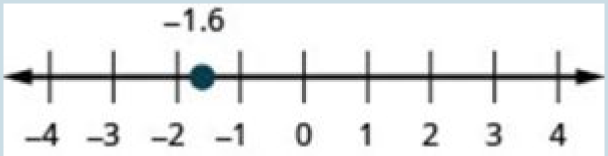
49) 0.8

50) -0.9

51) -1.6

52) 3.1

49) 

51) 

Exercises: Order Pairs of Numbers

Instructions: For questions 53-60, order each of the pairs of numbers, using $<$ or $>$.

53) $0.37 \_\_ 0.63$

54) $0.86 \_\_ 0.69$

55) $0.91 \_\_ 0.901$

56) $0.415 \_\_ 0.41$

57) $-0.5 \_\_ -0.3$

58) $-0.1 \_\_ -0.4$

59) $-0.62 \_\_ -0.619$

60) $-7.31 \_\_ -7.3$

Odd Answers

53) $<$

55) $>$

57) $<$

59) $<$

Exercises: Everyday Math

Instructions: For questions 61-62, answer the given everyday math word problems.

61) Field Trip: All the 5th graders at Lincoln Elementary School will go on a field trip to the science museum. Counting all the children, teachers, and chaperones, there will be 147 people. Each bus holds 44 people.

- **a) How many buses will be needed?**
- **b) Why must the answer be a whole number?**
- **c) Why shouldn't you round the answer the usual way, by choosing the whole number closest to the exact answer?**

62) Child Care: Serena wants to open a licensed childcare center. Her state requires there be no more than 12 children for each teacher. She would like her child care center to serve 40 children.

- **a) How many teachers will be needed?**
- **b) Why must the answer be a whole number?**
- **c) Why shouldn't you round the answer the usual way, by choosing the whole number closest to the exact answer?**

Odd Answers

a) 4 buses

b) Answers may vary

c) Answers may vary

Exercises: Writing Exercises

Instructions: For questions 63-64, answer the given writing exercises.

63) In your own words, explain the difference between a rational number and an irrational number.

64) Explain how the sets of numbers (counting, whole, integer, rational, irrationals, reals) are related to each other.

Odd Answers

63) Answers may vary

1.7 PROPERTIES OF REAL NUMBERS

Learning Objectives

By the end of this section, you will be able to:

- Use the commutative and associative properties
- Use the identity and inverse properties of addition and multiplication
- Use the properties of zero
- Simplify expressions using the distributive property

Use the Commutative and Associative Properties

Think about adding two numbers, say 5 and 3. The order we add them doesn't affect the result, does it?

$5+3$	$3+5$
8	8
$5+3$	$3+5$

The results are the same.

As we can see, the order in which we add does not matter!

What about multiplying 5 and 3?

5×3	3×5
15	15

$$5 \times 3 = 3 \times 5$$

Again, the results are the same!

The order in which we multiply does not matter!

These examples illustrate the *commutative property*. When adding or multiplying, changing the *order* gives the same result.

Commutative Property

of Addition	if a, b are real numbers, then $$a + b = b + a$$
of Multiplication	if a, b are real numbers, then $$a \times b = b \times a$$

When adding or multiplying, changing the order gives the same result.

The commutative property has to do with order. If you change the order of the numbers when adding or multiplying, the result is the same.

What about subtraction? Does order matter when we subtract numbers? Does $7-3$ give the same result as $3-7$?

$7-3$	$3-7$
4	-4

$$4 \neq -4$$

$$7-3 \neq -3-7$$

The results are not the same.

Since changing the order of the subtraction did not give the same result, we know that *subtraction is not commutative*.

Let's see what happens when we divide two numbers. Is division commutative?

$12 \div 4$	$14 \div 12$
$\frac{12}{4}$	$\frac{4}{12}$
3	$\frac{1}{3}$

$$3 \neq \frac{1}{3}$$

$$12 \div 4 \neq 4 \div 12$$

The results are not the same. Since changing the order of the division did not give the same result, *division is not commutative*.

The commutative properties only apply to addition and multiplication!

- Addition and Multiplication *are* commutative.
- Subtraction and Division *are not* commutative.

If you were asked to simplify this expression, how would you do it and what would your answer be?

$$7+8+2$$

Some people would think $7+8$ is 15 and then $15+2$ is 17. Others might start with $8+2$ makes 10 and then $7+10$ makes 17.

Either way gives the same result. Remember, we use parentheses as grouping symbols to indicate which operation should be done first.

$$\begin{array}{ll} \text{Add } 7+8 & (7+8)+2 \\ \text{Add} & 15+2 \\ & 17 \end{array}$$

$$\begin{array}{ll} \text{Add } 8+2 & 7+(8+2) \\ \text{Add} & 7+10 \\ & 17 \end{array}$$

$$(7+8)+2=7+(8+2)$$

When adding three numbers, changing the grouping of the numbers gives the same result.

This is true for multiplication, too.

$$\begin{array}{ll} \text{Multiply } 5\cdot\frac{1}{3} & \left(5\times\frac{1}{3}\right)\times 3 \\ \text{Multiply.} & \frac{5}{3}\times 3 \\ & 5 \end{array}$$

$$\begin{array}{ll} \text{Multiply } \frac{1}{3}\cdot 3 & 5\times\left(\frac{1}{3}\times 3\right) \\ \text{Multiply.} & 5\times 1 \\ & 5 \end{array}$$

$$(5\times\frac{1}{3})\times 3=5\times(\frac{1}{3}\times 3)$$

When multiplying three numbers, changing the grouping of the numbers gives the same result.

You probably know this, but the terminology may be new to you. These examples illustrate the associative property.

Associative Property

of Addition	If a, b, c are real numbers, then $(a+b)+c=a+(b+c)$
of Multiplication	If a, b, c are real numbers, then $(a\times b)\times c=a\times(b\times c)$

When adding or multiplying, changing the *grouping* gives the same result.

Let's think again about multiplying $5 \times \frac{1}{3} \times 3$. We got the same result both ways, but which way was easier? Multiplying $\frac{1}{3}$ and 3 first, as shown above on the right side, eliminates the fraction in the first step.

Using the associative property can make the math easier!

The associative property has to do with grouping. If we change how the numbers are grouped, the result will be the same. Notice it is the same three numbers in the same order—the only difference is the grouping.

We saw that subtraction and division were not commutative. They are not associative either.

When simplifying an expression, it is always a good idea to plan what the steps will be. In order to combine like terms in the next example, we will use the commutative property of addition to write the like terms together.

Example 1

Simplify: $18p + 6q + 15p + 5q$

Solution

Step 1: **Use the commutative property of addition to re-order so that like terms are together.**

Step 2: **Add like terms.**

$$33p + 11q$$

Try It

1) Simplify: $23r + 14s + 9r + 15s$

Solution

$32r + 29s$

2) Simplify: $37m + 21n + 4m - 15n$

Solution

$41m + 6n$

When we have to simplify algebraic expressions, we can often make the work easier by applying the commutative or associative property first, instead of automatically following the order of operations. When adding or subtracting fractions, combine those with a common denominator first.

Example 2

Simplify: $(\frac{5}{13}+\frac{3}{4})+\frac{1}{4}$

Solution

Step 1: **Notice that the last 2 terms have a common denominator, so change the grouping.**

$$\frac{5}{13}+(\frac{3}{4}+\frac{1}{4})$$

Step 2: **Add in parentheses first.**

$$\frac{5}{13}+(\frac{4}{4})$$

Step 3: **Simplify the fraction.**

$$\frac{5}{13}+(1)$$

Step 4: **Add.**

$$1\frac{5}{13}$$

Step 5: **Convert to an improper fraction.**

$$\frac{18}{13}$$

Try It

3) Simplify: $(\frac{7}{15}+\frac{5}{8})+\frac{3}{8}$

Solution

$$1\frac{7}{15}$$

4) Simplify: $(\frac{2}{9}+\frac{7}{12})+\frac{5}{12}$

Solution

$$1\frac{2}{9}$$

Example 3

Use the associative property to simplify $6(3x)$.

Solution

Step 1: **Change the grouping.**

$$(6\times 3)x$$

Step 2: **Multiply in the parentheses.**

$$18x$$

Notice that we can multiply 6×3 but we could not multiply $3x$ without having a value for x.

Try It

5) Use the associative property to simplify $8(4x)$

Solution
$32x$

6) Use the associative property to simplify $-9(7y)$

Solution
$-63y$

Use the Identity and Inverse Properties of Addition and Multiplication

What happens when we add 0 to any number? Adding 0 doesn't change the value. For this reason, we call 0 the **additive identity**.

For example,

$13+0$	$-14+0$	$0+(-8)$
13	-14	-8

These examples illustrate the *Identity Property of Addition* that states that for any real number, a, $a+0=a$ and $0+a=a$.

What happens when we multiply any number by one? Multiplying by 1 doesn't change the value. So we call 1 the **multiplicative identity**.

For example,

43×1	-27×1	$1\times\frac{3}{5}$
43	-27	$\frac{3}{5}$

These examples illustrate the *Identity Property of Multiplication* that states that for any real number, a, $a\cdot 1=a$ and $1\cdot a=a$.

We summarize the Identity Properties below.

Identity Property

of addition for any real number a:

0 is the additive identity $a + 0 = a$ $0 + a = a$

of multiplication for any real number a:

1 is the additive identity $a \times 1 = a$ $1 \times a = a$

What number added to 5 gives the additive identity, 0?

$5 + \_\_ = 0$ We know that $5 + (-5) = 0$

What number added to -6 gives the additive identity, 0?

$-6 + \_\_ = 0$ We know $(-6) + 6 = 0$

Notice that in each case, the missing number was the opposite of the number!

We call $-a$ the **additive inverse** of a. *The opposite of a number is its additive inverse.* A number and its opposite add to zero, which is the additive identity. This leads to the *Inverse Property of Addition* that states for any real number, a, $a + (-a) = 0$. Remember, a number and its opposite add to zero.

What number multiplied by $\frac{2}{3}$ gives the multiplicative identity, 1? In other words, $\frac{2}{3}$ times what results in 1?

$$\frac{2}{3}\cdot \_\_\_ = 1 \quad \text{We know } \frac{2}{3}\cdot\frac{3}{2} = 1$$

What number multiplied by 2 gives the multiplicative identity, 1? In other words 2 times what results in 1?

$$2\cdot \_\_\_ = 1 \quad \text{We know } 2\cdot\frac{1}{2} = 1$$

Notice that in each case, the missing number was the reciprocal of the number!

We call $\frac{1}{a}$ the **multiplicative inverse** of a. *The reciprocal of a number is its multiplicative inverse.* A number and its reciprocal multiply to one, which is the multiplicative identity. This leads to the *Inverse Property of Multiplication* that states that for any real number, a, $a \neq 0$, $a \times \frac{1}{a} = 1$.

We'll formally state the inverse properties here:

Inverse Property

of addition	For any real number a, $-a$ is the additive inverse of a. A number and its opposite add to zero.	$a+(-a)=0$
of multiplication	For any real number a, $\frac{1}{a}$ is the multiplicative inverse of a. A number and its reciprocal multiply to one.	$a\times\frac{1}{a}=1$

Example 4

Find the additive inverse of

a. $\frac{5}{8}$

b. 0.6

c. -8

d. $\frac{4}{3}$

Solution

To find the additive inverse, we find the opposite.

a. The additive inverse of $\frac{5}{8}$ is the opposite of $\frac{5}{8}$ The additive inverse of $\frac{5}{8}$ is $-\frac{5}{8}$

---

b. The additive inverse of 0.6 is the opposite of 0.6. The additive inverse of 0.6 is -0.6

.

---

c. The additive inverse of -8 is the opposite of -8. We write the opposite of -8 as

$-(-8)$, and then simplify it to 8. Therefore, the additive inverse of -8 is 8.

---

d. The additive inverse of $-\frac{4}{3}$ is the opposite of $-\frac{4}{3}$. We write this as $-(-\frac{4}{3})$ and then simplify to $\frac{4}{3}$. Thus, the additive inverse of $-\frac{4}{3}$ is $\frac{4}{3}$.

Try It

7) Find the additive inverse of:

a. $\frac{7}{9}$

b. 1.2

c. -14

d. $-\frac{9}{4}$

Solution

a. $-\frac{7}{9}$

b. -1.2

c. 14

d. $\frac{9}{4}$

8) Find the additive inverse of:

a. $\frac{7}{13}$

b. 8.4

c. -46

d. $-\frac{5}{2}$

Solution

a. $-\frac{7}{13}$

b. -8.4

c. 46

d. $\frac{5}{2}$

Example 5

Find the multiplicative inverse of

a. 9

b. $-\frac{1}{9}$

c. 0.9

Solution

To find the multiplicative inverse, we find the reciprocal.

a. The multiplicative inverse of 9 is the reciprocal of 9, which is $\frac{1}{9}$. Therefore, the multiplicative inverse of 9 is $\frac{1}{9}$.

---

b. The multiplicative inverse of $-\frac{1}{9}$ is the reciprocal of $-\frac{1}{9}$ which is -9. Thus, the multiplicative inverse of $-\frac{1}{9}$ is -9.

---

c.

Step 1: **To find the multiplicative inverse of 0.9, we first convert 0.9 to a fraction**

$$\frac{9}{10}$$

Step 2: **Then we find the reciprocal of the fraction.**

The reciprocal of $\frac{9}{10}$ is $\frac{10}{9}$.

So the multiplicative inverse of 0.9 is $\frac{10}{9}$.

Try It

9) Find the multiplicative inverse of

a. 4

b. $-\frac{1}{7}$

c. 0.3

Solution

a. $\frac{1}{4}$

b. -7

c. $\frac{10}{3}$

10) Find the multiplicative inverse of

a. 18

b. $-\frac{4}{5}$

c. 0.6

Solution

a. $\frac{1}{18}$

b. $-\frac{5}{4}$

c. $\frac{5}{3}$

Use the Properties of Zero

The identity property of addition says that when we add 0 to any number, the result is that same number. What happens when we multiply a number by 0? Multiplying by 0 makes the product equal zero.

Multiplication by Zero

For any real number a,

$0\times\ a=0\qquad a\times0=0$

The product of any real number and 0 is 0.

What about division involving zero? What is $0\div3$? Think about a real example: If there are no cookies in the cookie jar and 3 people are to share them, how many cookies does each person get? There are no cookies to share, so each person gets 0 cookies. So, $0\div3=0$

We can check division with the related multiplication fact.
$12\div6=2$ because $2\times6=12$

So we know $0 \div 3 = 0$ because $0 \times 3 = 0$.

Division of Zero

For any real number, a, except 0,

$\frac{0}{a} = 0$ and $0 \div a = 0$.

Zero is divided by any real number except zero is zero.

Now think about dividing *by* zero. What is the result of dividing 4 by 0? Think about the related multiplication fact: $4 \div 0 = ?$ means $? \times 0 = 4$ Is there a number that multiplied by 0 gives 4? Since any real number multiplied by 0 gives 0, there is no real number that can be multiplied by 0 to obtain 4.

We conclude that there is no answer to $4 \div 0$ and so we say that division by 0 is undefined.

Division by Zero

For any real number a, except 0,

$\frac{a}{0}$ and $a \div 0$ are undefined.

Division by zero is undefined.

We summarize the properties of zero below.

Properties of Zero

Multiplication by Zero: For any real number a,

$a \times 0 = 0 \qquad 0 \times a = 0$	The product of any number and 0 is 0.

Division by Zero: For any real number a, where $a \neq 0$

$\frac{0}{a} = 0$		Zero divided by any real number except itself is zero.
$\frac{a}{0}$	is undefined	Division by zero is undefined.

Example 6

Simplify:

a. -8×0

b. $\frac{0}{-2}$

c. $\frac{-32}{0}$

Solution

a.

Step 1: **The product of any real number and 0 is 0.**

$$-8 \times 0$$
$$0$$

b.

Step 1: **The product of any real number and** 0 **is** 0**.**

$$\frac{0}{-2}$$
$$0$$

c.

Step 1: **Division by** 0 **is undefined.**

$$\frac{-32}{0}$$
$$\text{Undefined}$$

Try It

11) Simplify:
a. $-14 \cdot 0$

b. $\frac{0}{-6}$
c. $\frac{-2}{0}$

Solution

a. 0

b. 0

c. undefined

12) Simplify:
a. $0(-17)$
b. $\frac{0}{-10}$
c. $\frac{-5}{0}$

Solution

a. 0

b. 0

c. undefined

We will now practice using the properties of identities, inverses, and zero to simplify expressions.

Example 7

Simplify:

a. $\dfrac{0}{n+5}$, where $n \neq -5$

b. $\dfrac{10-3p}{0}$, where $10-3p \neq 0$

Solution

a.

Step 1: **Zero divided by any real number except itself is 0.**

$$\frac{0}{n+5}$$
$$0$$

---

b.

Step 1: **Division by 0 is undefined.**

$$\frac{10-3p}{0}$$
$$\text{Undefined}$$

Example 8

Simplify: $-84n + (-73n) + 84n$

Solution

***Step 1:* Notice that the first and third terms are opposites; use the commutative property of addition to re-order the terms.**

$$-84n + 84n + (-73n)$$

***Step 2:* Add left to right.**

$$0 + (-73)$$

***Step 3:* Add.**

$$-73n$$

Try It

13) Simplify: $-27a + (-48a) + 27a$

Solution

$-48a$

14) Simplify: $39x + (-92x) + (-39x)$

Solution

$-92x$

Now we will see how recognizing reciprocals is helpful. Before multiplying left to right, look for reciprocals—their product is 1.

Example 9

Simplify: $\frac{7}{15} \times \frac{8}{23} \times \frac{15}{7}$

Solution

Step 1: **Notice that the first and third terms are reciprocals, so use the commutative property of multiplication to re-order the factors.**

$$\frac{7}{15} \times \frac{15}{7} \times \frac{8}{23}$$

Step 2: **Multiply left to right.**

$$\frac{8}{23}$$

Try It

15) Simplify: $\frac{9}{16} \times \frac{5}{49} \times \frac{16}{9}$

Solution

$$\frac{5}{49}$$

16) Simplify: $\frac{6}{17} \times \frac{11}{25} \times \frac{17}{6}$

Solution

$$\frac{11}{25}$$

Try It

17) Simplify:

a. $\frac{0}{m+7}$, where $m \neq -7$

b. $\frac{18-6c}{0}$, where $18-6c \neq 0$

Solution

a. 0

b. undefined

18) Simplify:

a. $\frac{0}{d-4}$, where $d \neq 4$

b. $\frac{15-4q}{0}$, $15-4q \neq 0$

Solution

a. 0

b. undefined

Example 10

Simplify: $\frac{3}{4} \times \frac{4}{3}(6x + 12)$

Solution

***Step 1:* There is nothing to do in the parentheses, so multiply the two fractions first—notice, they are reciprocals.**

$$1(6x + 12)$$

***Step 2:* Simplify by recognizing the multiplicative identity.**

$$(6x + 12)$$

Try It

19) Simplify: $\frac{2}{5} \times \frac{5}{2}(20y + 50)$

Solution

$20y + 50$

20) Simplify: $\frac{3}{8} \times \frac{8}{3}(12z + 16)$

Solution

$12z + 16$

Simplify Expressions Using the Distributive Property

Suppose that three friends are going to the movies. They each need $\$9.25$—that's 9 dollars and 1 quarter—to pay for their tickets. How much money do they need all together?

You can think about the dollars separately from the quarters. They need 3 times $\$9$ so $\$27$, and 3 times 1 quarter, so 75 cents. In total, they need $\$27.75$. If you think about doing the math in this way, you are using the *distributive property*.

Distributive Property

a(b+c)
=ab+a
c

If a, b, c are real numbers, then

$$(b+c)a = ba + ca$$

Also,

$$a(b-c) = ab - ac$$

$$(b-c)a = ba - ca$$

Back to our friends at the movies, we could find the total amount of money they need like this:

$$\begin{array}{l} 3(9.25) \\ 3(9+0.25) \\ 3(9)+3(0.25) \\ 27+0.75 \\ 27.75 \end{array}$$

In algebra, we use the distributive property to remove parentheses as we simplify expressions.

For example, if we are asked to simplify the expression $3(x+4)$ the order of operations says to work in the parentheses first. But we cannot add x and 4, since they are not like terms. So we use the distributive property, as shown in Example 1.7.11.

Some students find it helpful to draw arrows to remind them how to use the distributive property. Then the first step in Example 1.7.11 would look like this:

Figure 1.7.1

Example 11

Simplify: $3(x+4)$

Solution

Step 1: **Distribute**

$$3\times x+3\times 4$$

Step 2: **Multiply.**

$$3x+12$$

Try It

21) Simplify: $4(x+2)$

Solution

$4x+8$

22) Simplify: $6(x+7)$

Solution

$6x+42$

Example 12

Simplify: $8(\frac{3}{8}x + \frac{1}{4})$

Solution

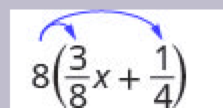


Figure 1.7.2

Step 1: **Distribute.**

$$8 \cdot \frac{3}{8}x + 8 \cdot \frac{1}{4}$$

Step 2: **Multiply.**

$$3x + 2$$

Try It

23) Simplify: $6(\frac{5}{6}y + \frac{1}{2})$

Solution

$5y + 3$

24) Simplify: $12(\frac{1}{3}n + \frac{3}{4})$

Solution

$4n + 9$

Using the distributive property as shown in Example 1.7.13 will be very useful when we solve money applications in later chapters.

Example 13

Simplify: $100(0.3 + 0.25q)$

Solution

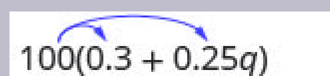


Figure 1.7.3

Step 1: **Distribute.**

$$100\,(0.3) + 100\,(0.25q)$$

Step 2: **Multiply.**

$$30 + 25q$$

Try It

25) Simplify: $100(0.7 + 0.15p)$

Solution

$70 + 15p$

26) Simplify: $100(0.04 + 0.35d)$

Solution

$4 + 35d$

When we distribute a negative number, we need to be extra careful to get the signs correct!

Example 14

Simplify: $-2(4y+1)$

Solution

Figure 1.7.4

Step 1: **Distribute.**

$$-2\cdot 4y+(-2)\cdot 1$$

Step 2: **Multiply.**

$$-8y-2$$

Try It

27) Simplify: $-3(6m+5)$

Solution

$-18m-15$

28) Simplify: $-6(8n+11)$

Solution

$-48n-66$

Example 15

Simplify: $-11(4-3a)$

Solution

***Step 1:* Distribute.**

Figure 1.7.5

***Step 2:* Multiply.**

$$\begin{array}{l} -11 \cdot 4 - (-11) \cdot 13a \\ -44 - (-33a) \end{array}$$

***Step 3:* Simplify.**

$$-44+33a$$

Notice that you could also write the result as $33a-44$. Do you know why?

Try It

29) Simplify: $-5(2-3a)$

Solution

$-10+15a$

30) Simplify: $-7(8-15y)$

Solution

$-56+105y$

Example 1.7.16 will show how to use the distributive property to find the opposite of an expression.

Example 16

Simplify: $-(y+5)$

Solution

***Step 1:* Multiplying by -1 results in the opposite.**

$$-1(y+5)$$

***Step 2:* Distribute.**

$$-1\times y+-1\times 5$$

***Step 3:* Simplify.**

$$\begin{array}{l} -y+(-5) \\ -y-5 \end{array}$$

Try It

31) Simplify: $-(z-11)$

Solution

$-z+11$

32) Simplify: $-(x-4)$

Solution

$-x+4$

There will be times when we'll need to use the distributive property as part of the order of operations. Start by looking at the parentheses. If the expression inside the parentheses cannot be simplified, the next step would be multiply using the distributive property, which removes the parentheses. The next two examples will illustrate this.

Example 17

Simplify: $8-2(x+3)$

Be sure to follow the order of operations. Multiplication comes before subtraction, so we will distribute the 2 first and then subtract.

Solution

Step 1: **Distribute.**

$$8-2\times x-2\times 3$$

Step 2: **Multiply.**

$$8-2x-6$$

Step 3: **Combine like terms.**

$$-2x+2$$

Try It

33) Simplify: $9-3(x+2)$

Solution

$3-3x$

34) Simplify: $7x-5(x+4)$

Solution

$2x-20$

Example 18

Simplify: $4(x-8)-(x+3)$

Solution

Step 1: **Distribute.**

$$4x-32-x-3$$

Step 2: **Combine like terms.**

$$3x-35$$

Try It

35) Simplify: $6(x-9)-(x+12)$

Solution

$5x-66$

36) Simplify: $8(x-1)-(x+5)$

Solution

$7x - 13$

Key Concepts

- **Commutative Property of**
  - *Addition:* If a, b, are real numbers, then $a + b = b + a$.
  - *Multiplication:* If a, b, are real numbers, then $a \cdot b = b \cdot a$. When adding or multiplying, changing the *order* gives the same result.
- **Associative Property of**
  - *Addition:* If a, b, c are real numbers, then $(a + b) + c = a + (b + c)$
  - *Multiplication:* If a, b, c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

    When adding or multiplying, changing the *grouping* gives the same result.

- **Distributive Property:** If a, b, c are real numbers, then
  - $a(b+c) = ab + ac$
  - $(b+c)a = ba + ca$
  - $a(b-c) = ab - ac$
  - $(b-c)a = ba - ca$
- **Identity Property**
  - *of Addition:* For any real number $a + 0 = a$, $0 + a = a$

    0 is the *additive identity*
  - *of Multiplication:* For any real number $a \cdot 1 = a$, $1 \cdot a = a$

    1 is the *multiplicative identity*
- **Inverse Property**
  - *of Addition:* For any real number a, $a + (-a) = 0$. A number and its *opposite* add to zero. $-a$ is the *additive inverse* of a.
  - *of Multiplication:* For any real number a, $a \neq 0$ $\frac{1}{a} = 1$. A number and its *reciprocal* multiply to one. $\frac{1}{a}$ is the *multiplicative inverse* of a.

- **Properties of Zero**
  - For any real number a,

    $a \cdot 0 = 0$, $0 \cdot a = 0$ – The product of any real number and 0 is 0.
  - $\frac{0}{a} = 0$ for $a \neq 0$ – Zero divided by any real number except zero is zero.
  - $\frac{a}{0}$ is undefined – Division by zero is undefined.

Glossary

additive identity

The additive identity is the number 0; adding 0 to any number does not change its value.

additive inverse

The opposite of a number is its additive inverse. A number and it additive inverse add to 0.

multiplicative identity

The multiplicative identity is the number 1; multiplying 1 by any number does not change the value of the number.

multiplicative inverse

The reciprocal of a number is its multiplicative inverse. A number and its multiplicative inverse multiply to one.

Exercises: Use the Commutative and Associative Properties

Instructions: For questions 1–4, use the associative property to simplify.

1) $3(4x)$

2) $4(7m)$

3) $(y+12)+28$

4) $(n+17)+33$

Odd Answers

1) $12x$

3) $y+40$

Exercises: Use the Commutative and Associative Properties

Instructions: For questions 5–26, simplify.

5) $\frac{1}{2}+\frac{7}{8}+\left(-\frac{1}{2}\right)$

6) $\frac{2}{5}+\frac{5}{12}+\left(-\frac{2}{5}\right)$

7) $\frac{3}{20}\times\frac{49}{11}\times\frac{20}{3}$

8) $\frac{13}{18}\times\frac{25}{7}\times\frac{18}{13}$

9) $-24\times7\times\frac{3}{8}$

10) $-36\times11\times\frac{4}{9}$

11) $\left(\frac{5}{6}+\frac{8}{15}\right)+\frac{7}{15}$

12) $\left(\frac{11}{12}+\frac{4}{9}\right)+\frac{5}{9}$

13) $17(0.25)(4)$

14) $36(0.2)(5)$

15) $[2.48(12)]\,(0.5)$

16) $[9.731(4)]\,(0.75)$

17) $7(4a)$

18) $9(8w)$

19) $-15(5m)$

20) $-23\,(2n)$

21) $12\left(\frac{5}{6}p\right)$

22) $20\left(\frac{3}{5}q\right)$

23) $43m+(-12n)+(-16m)+(-9n)$

24) $-22p+17q+(-35p)+(-27q)$

25) $\frac{3}{8}g + \frac{1}{12}h + \frac{7}{8}g + \frac{5}{12}h$

26) $\frac{5}{6}a + \frac{3}{10}b + \frac{1}{6}a + \frac{9}{10}b$

27) $6.8p + 9.14q + (-4.37p) + (-0.88q)$

28) $9.6m + 7.22n + (-2.19m) + (-0.65n)$

Odd Answers

5) $\frac{7}{8}$

7) $\frac{49}{11}$

9) -63

11) $1\frac{5}{6}$

13) 17

15) 14.88

17) $28a$

19) $-75m$

21) $10p$

23) $27m + (-21n)$

25) $\frac{5}{4}g + \frac{1}{2}h$

27) $2.43p + 8.26q$

Exercises: Use the Identity and Inverse Properties of Addition and Multiplication

Instructions: For questions 29–32, find the additive inverse of each number.

29a) $\frac{2}{5}$

29b) 4.3

29c) -8

29d) $-\frac{10}{3}$

30a) $\frac{5}{9}$

30b) 2.1

30c) -3

30d) $-\frac{9}{5}$

31a) $-\frac{7}{6}$

31b) -0.075

31c) 23

31d) $\frac{1}{4}$

32a) $-\frac{8}{3}$

32b) -0.019

32c) 52

32d) $\frac{5}{6}$

Odd Answers

29a) $-\frac{2}{5}$

29b) -4.3

29c) 8

29d) $\frac{10}{3}$

31a) $\dfrac{7}{6}$

31b) 0.075

31c) -23

31d) $-\dfrac{1}{4}$

Exercises: Use the Identity and Inverse Properties of Addition and Multiplication

Instructions: For questions 33–36, find the multiplicative inverse of each number.

33a) 6

33b) $-\dfrac{3}{4}$

33c) 0.7

34a) 12

34b) $-\frac{9}{2}$

34c) 0.13

35a) $\frac{11}{12}$

35b) -1.1

35c) -4

36a) $\frac{17}{20}$

36b) -1.5

36c) -3

Odd Answers

33a) $\frac{1}{6}$

33b) $-\frac{4}{3}$

33c) $\frac{10}{7}$

35a) $\frac{12}{11}$

35b) $-\frac{10}{11}$

35c) $-\frac{1}{4}$

Exercises: Use the Properties of Zero

Instructions: For questions 37–44, simplify.

37) $\frac{0}{6}$

38) $\frac{3}{0}$

39) $0 \div \frac{11}{12}$

40) $\dfrac{6}{0}$

41) $\dfrac{0}{3}$

42) $0 \times \frac{8}{15}$

43) $(-3.14)\,(0)$

44) $\dfrac{\frac{1}{10}}{0}$

Odd Answers

37) 0

39) 0

41) 0

43) 0

Exercises: Mixed Practice

Instructions: For questions 45–58, simplify.

45) $19a + 44 - 19a$

46) $27c + 16 - 27c$

47) $10(0.1d)$

48) $100(0.01p)$

49) $\frac{0}{u - 4.99}$, **where** $u \neq 4.99$

50) $\frac{0}{v - 65.1}$, **where** $v \neq 65.1$

51) $0 \div \left(x - \frac{1}{2}\right)$, **where** $x \neq \frac{1}{2}$

52) $0 \div \left(y - \frac{1}{6}\right)$, **where** $x \neq \frac{1}{6}$

53) $\frac{32 - 5a}{0}$, **where** $32 - 5a \neq 0$

54) $\frac{28 - 9b}{0}$, **where** $28 - 9b \neq 0$

55) $\left(\frac{3}{4} + \frac{9}{10}m\right) \div 0$ **where** $\frac{3}{4} + \frac{9}{10}m \neq 0$

56) $\left(\frac{5}{16}n - \frac{3}{7}\right) \div 0$ **where** $\frac{5}{16}n - \frac{3}{7} \neq 0$

57) $15 \times \frac{3}{5}(4d + 10)$

58) $18 \times \frac{5}{6}(15h + 24)$

Odd Answers

45) 44

47) d

49) 0

51) 0

53) undefined

55) undefined

57) $36d + 90$

Exercises: Simplify Expressions Using the Distributive Property

Instructions: For questions 59–94, simplify using the distributive property.

59) $8(4y + 9)$

60) $9(3w + 7)$

61) $6(c-13)$

62) $7(y-13)$

63) $\frac{1}{4}(3q+12)$

64) $\frac{1}{5}(4m+20)$

65) $9(\frac{5}{9}y-\frac{1}{3})$

66) $10\left(\frac{3}{10}x-\frac{2}{5}\right)$

67) $12\left(\frac{1}{4}+\frac{2}{3}r\right)$

68) $12\left(\frac{1}{6}+\frac{3}{4}s\right)$

69) $r(s-18)$

70) $u(v-10)$

71) $(y+4)p$

72) $(a+7)x$

73) $-7(4p+1)$

74) $-9(9a+4)$

75) $-3(x-6)$

76) $-4(q-7)$

77) $-(3x-7)$

78) $-(5p-4)$

79) $16-3(y+8)$

80) $18-4(x+2)$

81) $4-11(3c-2)$

82) $9-6(7n-5)$

83) $22-(a+3)$

84) $8-(r-7)$

85) $(5m-3)-(m+7)$

86) $(4y-1)-(y-2)$

87) $5(2n+9)+12(n-3)$

88) $9(5u+8)+2(u-6)$

89) $9(8x-3)-(-2)$

90) $4(6x-1)-(-8)$

91) $14(c-1)-8(c-6)$

92) $11(n-7)-5(n-1)$

93) $6(7y+8)-(30y-15)$

94) $7(3n+9)-(4n-13)$

Odd Answers

59) $32y+72$

61) $6c-78$

63) $\frac{3}{4}q+3$

65) $5y-3$

67) $3+8r$

69) $rs-18r$

71) $yp+4p$

73) $-28p-7$

75) $-3x+18$

77) $-3x+7$

79) $-3y-8$

81) $-33c+26$

83) $-a+19$

85) $4m-10$

87) $22n+9$

89) $72x-25$

91) $6c + 34$

93) $12y + 63$

Exercises: Everyday Math

Instructions: For questions 95–98, answer the given everyday math word problems.

95) Insurance co-payment: Carrie had to have 5 fillings done. Each filling cost $\$80$. Her dental insurance required her to pay 20% of the cost as a copay. Calculate Carrie's copay:

a) First, by multiplying 0.20 by 80 to find her copay for each filling and then multiplying your answer by 5 to find her total copay for 5 fillings.

b) Next, by multiplying $[5(0.20)](80)$

c) Which of the properties of real numbers says that your answers to parts (a), where you multiplied $5[(0.20)(80)]$ **and (b), where you multiplied** $[5(0.20)](80)$**, should be equal?**

96) Cooking time. Helen bought a 24-pound turkey for her family's Thanksgiving dinner and wants to know what time to put the turkey in to the

oven. She wants to allow 20 minutes per pound cooking time. Calculate the length of time needed to roast the turkey:

a) First, by multiplying 24×20 to find the total number of minutes and then multiplying the answer by $\frac{1}{60}$ to convert minutes into hours.

b) Next, by multiplying $24\left(20 \times \frac{1}{60}\right)$.

c) Which of the properties of real numbers says that your answers to parts (a), where you multiplied $(24 \times 20)\frac{1}{60}$, and (b), where you multiplied $24\left(20 \times \frac{1}{60}\right)$, should be equal?

97) Buying by the case. Trader Joe's grocery stores sold a bottle of wine they called "Two Buck Chuck" for $\$1.99$. They sold a case of 12 bottles for $\$23.88$. To find the cost of 12 bottles at $\$1.99$, notice that 1.99 is $2 - 0.01$.

a) Multiply $12(1.99)$ by using the distributive property to multiply $12(2 - 0.01)$.

b) Was it a bargain to buy "Two Buck Chuck" by the case?

98) Multi-pack purchase. Adele's shampoo sells for $\$3.99$ per bottle at the grocery store. At the warehouse store, the same shampoo is sold as a 3 pack for $\$10.49$. To find the cost of 3 bottles at $\$3.99$, notice that 3.99 is $4 - 0.01$.

a) Multiply $3(3.99)$ by using the distributive property to multiply $3(4 - 0.01)$.

b. How much would Adele save by buying 3 bottles at the warehouse store instead of at the grocery store?

Odd Answers

95a) $\$80$

95b) $\$80$

95c) Answers will vary

97a) $\$23.88$

97b) no, the price is the same

Exercises: Writing Exercises

Instructions: For questions 99–102, answer the given writing exercises.

99) In your own words, state the commutative property of addition.

100) What is the difference between the additive inverse and the multiplicative inverse of a number?

101) Simplify $8\left(x-\frac{1}{4}\right)$ **using the distributive property and explain each step.**

102) Explain how you can multiply $4(\$5.97)$ **without paper or calculator by thinking of** $\$5.97$ **as** $6-0.03$ **and then using the distributive property.**

Odd Answers

99) Answers may vary

101) Answers may vary

1.8 PERCENTAGES

Learning Outcomes

In this section you will be able to complete the following:

- Convert Decimals to Percentages
- Calculations with Rates, Bases, and Proportions

For this section you will need the following:

Symbols Used

- $\times 100$ = percentage conversion factor
- Base = the whole quantity
- dec = decimal number to be converted to percentage
- Portion = a part of the whole quantity
- Rate = the relationship between the Portion and the Base

Formulas Used

- Formula 1.8a – **Percentage**

$$\% = \text{dec} \times 100$$

- Formula 1.8b – **Rate, Portion, Base**

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$$

Introduction

Your class just wrote its first math quiz. You got 13 out of 19 questions correct, or $\frac{13}{19}$.

In speaking with your friends Sandhu and Illija, who are in other classes, you find out that they also wrote math quizzes; however, theirs were different. Sandhu scored 16 out of 23, or $\frac{16}{23}$ while Illija got 11 out of 16, or $\frac{11}{16}$.

Who achieved the highest grade? Who had the lowest? The answers are not readily apparent, because fractions are difficult to compare.

Now express your grades in percentages rather than fractions. You scored 68%, Sandhu scored 70%, and Illija scored 69%. Notice you can easily answer the questions now. The advantage of percentages is that they facilitate comparison and comprehension.

Converting Decimals to Percentages

A **percentage** is a part of a whole expressed in hundredths. In other words, it is a value out of 100. For example, 93% means 93 out of 100, or $\frac{93}{100}$.

Formula 1.8a: Percentage

$$\% = \text{dec} \times 100$$

$\%$ is Percentage: This is the decimal expressed as a percentage. It must always be written with the percent ($\%$) symbol immediately following the number.

$\times 100$ is Conversion Factor: A percentage is always expressed in hundredths.

dec is Decimal Number: This is the decimal number needing to be converted into a percentage.

How to

Convert a Decimal to a Percentage

Assume you want to convert the decimal number 0.0875 into a percentage. This number represents the dec variable in the formula. Substitute into Formula 1.8a:

$$\begin{aligned}\% &= \text{dec} \times 100 \\ \% &= 0.0875 \times 100 \\ \% &= 8.75\%\end{aligned}$$

Key Concepts

You can also solve this formula for the decimal number. To convert any percentage back into its decimal form, you need to perform a mathematical opposite. Since a percentage is a result of multiplying by 100, the mathematical opposite is achieved by dividing by 100. Therefore, to convert 81% back into decimal form:

$$\frac{81\%}{100} = 0.81$$

Things To Watch Out For

Your Texas Instruments BAII Plus calculator has a $\%$ key that can be used to convert any percentage number into its decimal format. For example, if you press 81 and then $\%$, your calculator displays 0.81.

While this function works well when dealing with a single percentage, it causes problems when your math problem involves multiple percentages. For example, try keying $4\% + 3\% =$ into the calculator using the $\%$ key. This should be the same as $0.04 + 0.03 = 0.07$. Notice, however, that your calculator has 0.0412 on the display.

Why is this? As a business calculator, you BAII Plus is programmed to take portions of a whole. When you key 3% into the calculator, it takes 3% of the first number keyed in, which was 4%. As a formula, the calculator sees $4\% + (3\% \times 4\%)$. This works out to $0.04 + 0.0012 = 0.0412$.

To prevent this from happening, your best course of action is not to use the $\%$ key on your calculator. It is best to key all percentages as decimal numbers whenever possible, thus eliminating any chance that the $\%$ key takes a portion of your whole. Throughout this textbook, all percentages are converted to decimals before calculations take place.

When working with percentages, you can use some tricks for remembering the formula and moving the decimal point.

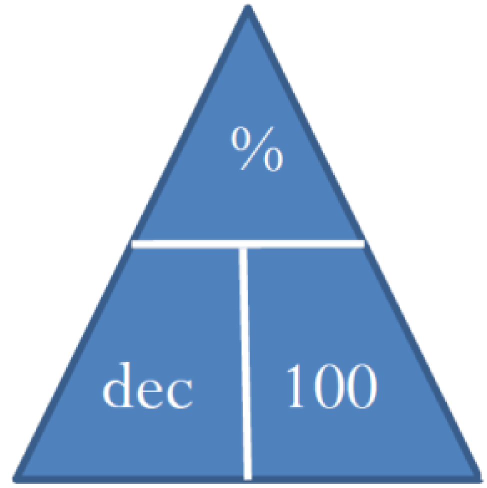


Figure 1.8.1

Remembering the Formula

When an equation involves only multiplication of all terms on one side with an isolated solution on the other side, use a mnemonic called the triangle technique. In this technique, draw a triangle with a horizontal line through its middle. Above the line goes the solution, and below the line, separated by vertical lines, goes each of the terms involved in the multiplication. The figure to the right shows how **Formula 1.8a** $\% = \text{dec} \times 100$ would be drawn using the triangle technique.

To use this triangle, identify the unknown variable, which you then calculate from the other variables in the triangle:

- Anything on the same line gets multiplied together. If solving for $\%$, then the other variables are on the same line and multiplied as $\text{dec} \times 100$.
- Any pair of items with one above the other is treated like a fraction and divided. If solving for dec, then the other variables are above/below each other and are divided as $\frac{\%}{100}$.

Moving the Decimal

Another easy way to work with percentages is to remember that multiplying or dividing by 100 moves the decimal over two places.

- If you are multiplying by 100, the decimal position moves two positions to the right.

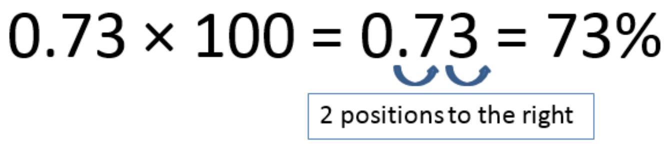


Figure 1.8.2

- If you are dividing by 100, the decimal position moves two positions to the left.

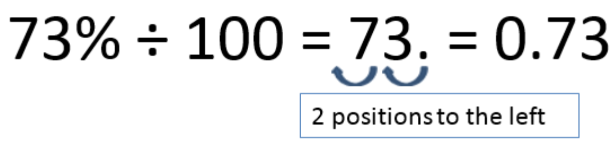


Figure 1.8.3

Example 1

Convert (a) and (b) into percentages. Convert (c) back into decimal format.

a. $\dfrac{3}{8}$

b. 1.3187

c. 12.399%

Solution

***Step 1:* What are we looking for?**

For questions (a) and (b), you need to convert these into percentage format. For question (c), you need to convert it back to decimal format.

Step 2: **What do we already know?**

a. This is a fraction to be converted into a decimal, or **dec**.
b. This is **dec**.
c. This is **%**.

Step 3: **Make substitutions using the information known above.**

a. Convert the fraction into a decimal to have **dec**.

$$\text{dec} = \frac{3}{8}$$
$$\text{dec} = 0.375$$

Then apply Formula 1.8a to get the percentage:

$$\% = \text{dec} \times 100$$
$$\% = 0.375 \times 100$$
$$\% = 37.5\%$$

b. As this term is already in decimal format, apply Formula 1.8a to get the percentage.

$$\% = \text{dec} \times 100$$
$$\% = 1.3187 \times 100$$
$$\% = 131.87\%$$

c. This term is already in percentage format. Using the triangle technique, calculate the decimal number through

$$\text{dec} = \frac{\%}{100}$$
$$\text{dec} = \frac{12.399\%}{100}$$
$$\text{dec} = 0.12399$$

Step 4: **Provide the information in a worded statement.**

In percentage format, the first two numbers are 37.5% and 131.87%. In decimal format, the last number is 0.12399.

Calculations with Rates, Bases, and Proportions

In your personal life and career, you will often need to either calculate or compare various quantities involving fractions. For example, if your income is $\$3,000$ per month and you can't spend more than 30% on housing, what is your maximum housing dollar amount? Or perhaps your manager tells you that this year's sales of $\$1,487,003$ are 102% of last year's sales. What were your sales last year?

Formula 1.8b: Rate, Portion, Base:

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$$

$\text{Portion is The Part Of The Quantity:}$ The portion represents the part of the whole. Compare it against the base to assess the rate.

$\text{Rate is The Relationship:}$ The rate is the decimal form expressing the relationship between the portion and the base. Convert it to a percentage if needed by applying **Formula 1.8a** $\% = \text{dec} \times 100$. This variable can take on any value, whether positive or negative.

$\text{Base is The Entire Quantity:}$ The base is the entire amount or quantity that is of concern. It represents a whole, standard, or benchmark that you assess the portion against.

How to

Calculate the Percentage of a Part to the Whole

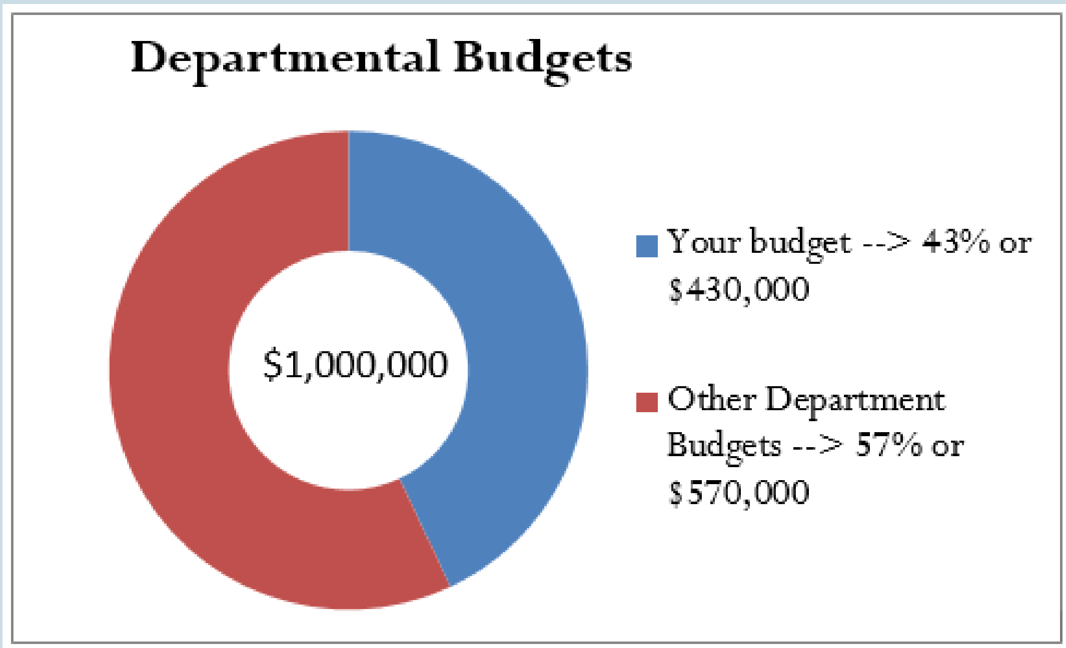


Figure 1.8.4

Assume that your company has set a budget of $\$1,000,000$. This is the entire amount of the budget and represents your *base*. Your department gets $\$430,000$ of the budget—this is your department's part of the whole and represents the *portion*. You want to know the relationship between your budget and the company's budget. In other words, you are looking for the *rate*.

Apply Formula 1.8b:

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$$
$$\text{Rate} = \frac{\$430,000}{\$1,000,000}$$
$$\text{Rate} = 0.43$$

Your budget is 0.43, or 43%, of the company's budget.

Key Concepts

There are three parts to this formula. Mistakes commonly occur through incorrect assignment of a quantity to its associated variable. The table below provides some tips and clue words to help you make the correct assignment.

Table 1.8.1

Variable	Key Words	Example
Base	of	If your department can spend 43% of the company's total budget ***of $1,000,000***, what is your maximum departmental spending?
Portion	is, are	If your department can spend 43% of the company's total budget of $1,000,000, ***what is your maximum departmental spending***?
Rate	%, percent, rate	If your department can spend ***43%*** of the company's total budget of $1,000,000, what is your maximum departmental spending?

Things To Watch Out For

In resolving the rate, you must express all numbers in the same units—you cannot have apples and oranges in the same sequence of calculations. In the above example, both the company's budget and the departmental budget are in the units of dollars. Alternatively, you would not be able to calculate the rate if you had a base expressed in kilometres and a portion expressed in metres. Before you perform the rate calculation, express both in kilometres or both in metres.

Formula 1.8b

$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$ is another formula you can use the triangle technique for. You do not need to memorize multiple versions of the formula for each of the variables. The triangle appears to the right.

Be very careful when performing operations involving rates, particularly in summing or averaging rates.

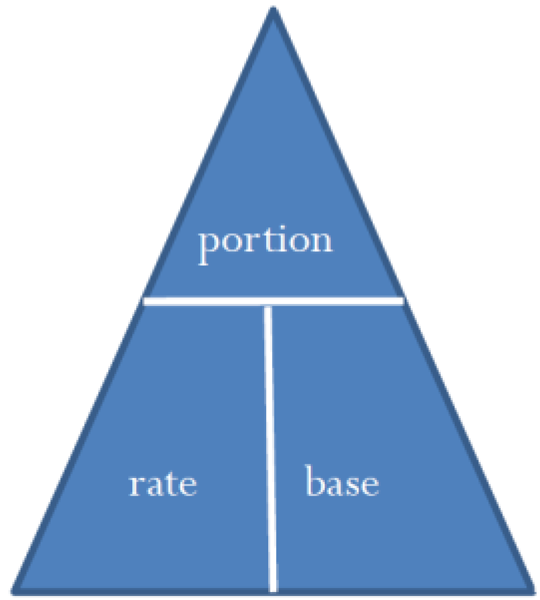


Figure 3.1.5

Summing Rates

Summing rates requires each rate to be a part of the same whole or base. If Bob has 5% of the kilometres travelled and Sheila has 6% of the oranges, these are not part of the same whole and cannot be added. If you did, what does the 11% represent? The result has no interpretation. However, if there are 100 oranges of which Bob has 5% and Sheila has 6%, the rates can be added and you can say that in total they have 11% of the oranges.

Averaging Rates

Simple averaging of rates requires each rate to be a measure of the same variable with the same base. If 36% of your customers are female and 54% have high income, the average of 45% is meaningless since each rate measures a different variable. Recall that earlier in this chapter you achieved 68% on your test and Sandhu scored 70%. However, your test involved 19 questions and Sandhu's involved 23 questions. These rates also cannot be simply averaged to 69% on the reasoning that $\frac{(68\% + 70\%)}{2} = 69\%$, since the bases are not the same. When two variables measure the same characteristic but have different bases (such as the math quizzes), you must use a weighted-average technique.

When can you average rates? Hypothetically, assume Sandhu achieved his 70% by writing the same test with 19 questions. Since both rates measure the same variable and have the same base, the simple average of 69% is now calculable.

Try It

Consider the following situations and select the best answer without performing any calculations.

1) If the rate is 0.25%, in comparison to the base the portion is:

a. a little bit smaller than the base.
b. a lot smaller than the base.
c. a little bit bigger than the base.
d. a lot bigger than the base.

Solution
b. (0.25% is 0.0025, resulting in a very small portion)

Try It

Consider the following situations and select the best answer without performing any calculations.

2) If the portion is $\$44,931$ and the base is $\$30,000$, the rate is:

a. smaller than 100%.
b. equal to 100%.
c. larger than 100% but less than 200%.
d. larger than 200%.

Solution

c. (the portion is larger than the base, but not twice as large)

Try It

Consider the following situations and select the best answer without performing any calculations.

3) If the rate is 75% and the portion is $\$50,000$, the base is:

a. smaller than $\$50,000$.
b. larger than $\$50,000$.
c. the same as the portion and equal to $\$50,000$.

Solution

b. (the portion represents 75% of the base, meaning the base must be larger)

Example 2

Solve for the unknown in the following three scenarios.

a. If your total income is $\$3,000$ per month and you can't spend more than 30% on housing, what is the maximum amount of your total income that can be spent on housing?
b. Your manager tells you that 2014 sales are 102% of 2013 sales. The sales for 2014 are $\$1,487,003$. What were the sales in 2013?

c. In Calgary, total commercial real estate sales in the first quarter of 2008 were $\$1.28\text{ billion}$. The industrial, commercial, and institutional (ICI) land sector in Calgary had sales of $\$409.6\text{ million}$. What percentage of commercial real estate sales is accounted for by the ICI land sector?

Solution

***Step 1:* What are we looking for?**

a. You are looking for the maximum amount of your income that can be spent on housing.
b. You need to figure out the sales for 2013.
c. You must determine the percentage of commercial real estate sales accounted for by the ICI land sector in Calgary.

***Step 2:* What do we already know?**

a. Look for key words in the question: "what ***is*** the maximum amount" and "***of*** your total income." The total income is the base, and the maximum amount is the portion.

$$\begin{aligned}\text{Base} &= \$3,000\\ \text{Rate} &= 30\%\\ \text{Portion} &= \text{maximum amount}\end{aligned}$$

b. Look for key words in the question: "sales for 2014 ***are*** $1,487,003" and "***of*** 2013 sales." The 2014 sales is the portion, and the 2013 sales is the base.

$$\begin{aligned}\text{Portion} &= \$1,487,003\\ \text{Rate} &= 102\%\\ \text{Base} &= 2013\text{ sales}\end{aligned}$$

c. Look for key words in the question: "***of*** commercial real estate sales" and "***are*** accounted for by the ICI land sector." The commercial real estate sales are the base, and the ICI land sector sales are the portion.

$$\begin{aligned}\text{Base} &= \$1.28\text{ billion}\\ \text{Portion} &= \$409.6\text{ million}\\ \text{Rate} &= \text{percentage}\end{aligned}$$

***Step 3:* Make substitutions using the information known above.**

a. Apply Formula 1.8b, but rearrange using the triangle technique to have:

$$\begin{aligned}\text{Portion} &= \text{Rate} \times \text{Base}\\ \text{Portion} &= 30\% \times \$3,000\\ \text{Portion} &= 0.3 \times \$3,000\\ \text{Portion} &= \$900\end{aligned}$$

b. Apply Formula 1.8b, but rearrange using the triangle technique to have:

$$\begin{aligned}\text{Base} &= \frac{\text{Portion}}{\text{Rate}}\\ \text{Base} &= \frac{\$1,487,003}{102\%}\\ \text{Base} &= \frac{\$1,487,003}{1.02}\\ \text{Base} &= \$1,457,846.08\end{aligned}$$

c. Apply Formula 1.8b:

$$\begin{aligned}\text{Rate} &= \frac{\text{Portion}}{\text{Base}}\\ \text{Rate} &= \frac{\$409.6\text{ million}}{\$1.28\text{ billion}}\\ \text{Rate} &= \frac{\$409,600,000}{\$1,280,000,000}\\ \text{Rate} &= 0.32\\ \text{Rate} &= 32\%\end{aligned}$$

Step 4: **Provide the information in a worded statement.**

a. The maximum you can spend on housing is $\$900$ per month.
b. 2013 sales were $\$1,457,846.08$.
c. The ICI land sector accounted for 32% of commercial real estate sales in Calgary for the first quarter of 2008.

Exercises: Mechanics

1) Convert the following decimals into percentages.

a) 0.4638
b) 3.1579
c) 0.000138
d) 0.015

2) Convert the following fractions into percentages.

a) $\dfrac{3}{8}$

b) $\dfrac{17}{32}$

c) $\dfrac{42}{12}$

d) $2\dfrac{4}{5}$

3) Convert the following fractions into percentages. Round to four decimals or express in repeating decimal format as needed.

a) $\dfrac{46}{12}$

b) $\dfrac{2}{9}$

c) $\dfrac{3}{11}$

d) $\dfrac{48}{93}$

4) Convert the following percentages into decimal form.

a) 15.3%

b) 0.03%

c) 153.987%

d) 14.0005%

5) What percentage of $\$40,000$ **is** $\$27,000$**?**

6) What is $\dfrac{1}{2}\%$ **of** $\$500,000$**?**

7) $\$0.15$ **is** $4,900\%$ **of what number?**

Odd Answers

1a) 46.38%

1b) 315.79%

1c) 0.0138%

1d) 1.5%

3a) $383.\overline{3}\%$

3b) $22.\overline{2}\%$

3c) $27.\overline{27}\%$

3d) 51.6129%

5) 67.5%

7) $\$0.003061$

Exercises: Applications

8) In 2009, medical experts predicted that one out of two Manitobans would contract some form of the H1N1 virus. If the population of Manitoba in 2009 was $1,217,200$**, how many Manitobans were predicted to become ill?**

9) In February 2009, $14,676$ **mortgages were in arrears in Canada, which represented** 0.38% **of all mortgages. How many total mortgages were in the Canadian market at that time?**

10) During Michael Jordan's NBA career (1984–2003), he averaged a free throw

completion percentage of 83.5% in regular season play. If Jordan threw $8,772$ free throws in his career, how many completed free throws did he make?

11) In August 2004, Google Inc. offered its stocks to the public at $\$85$ per share. In October 2007, the share price had climbed to $\$700.04$. Express the 2007 share price as a percentage of the 2004 price.

12) If the new minimum wage of $\$8.75$ per hour is 102.9412% of the old minimum wage, what was the old minimum wage?

13) If total advertising expenditures on television advertising declined 4.1% to $\$141.7$ billion in the current year, how much was spent on television advertising in the previous year? Round your answer to one decimal.

14) A machine can produce $2,500$ products per hour. If 37 of those products were defective, what is the defect rate per hour for the machine?

Odd Answers

9) $3,862,105$

11) 823.5765%

13) $\$147.8$ billion

Exercises: Challenge, Critical Thinking, & Other Applications

15) In 2011, Manitoba progressive income tax rates were 10.8% on the first $\$31,000$, 12.75% on the next $\$36,000$, and 17.4% on any additional income. If your gross taxable earnings for the year were $\$85,000$, what percentage of your earnings did you pay in taxes?

16) In 2011, the maximum amount that you could have contributed to your RRSP

(Registered Retirement Savings Plan) was the lesser of $\$22,450$ or 18% of your earned income from the previous year. How much income do you need to claim a $\$22,450$ contribution in 2011?

17) 17. Maria, a sales representative for a large consumer goods company, is paid 3% of the total profits earned by her company. Her company averages 10% profit on sales. If Maria's total income for the year was $\$60,000$, what total sales did her company realize?

18) A house was purchased six years ago for $\$214,000$. Today it lists at a price that is 159.8131% of the original purchase price. In dollars, how much has the price of the house increased over the six years?

19) An investor buys $1,000$ shares of WestJet Airlines at $\$10.30$ per share. A few months later, the investor sells the shares when their value hits 120% of the original share price. What is the price of a WestJet share when the investor sells these shares? How much money did the investor make?

20) A Honda Insight has fuel economy of 3.2 litres consumed per 100 kilometres driven. It has a fuel tank capacity of 40 litres. A Toyota Prius is rated at 4.2 L per 100 km driven. It has a fuel tank capacity of 45 L. What percentage is the total distance drivable (rounded to the nearest kilometre before calculating) of a Honda Insight compared to that of a Toyota Prius?

Odd Anwers

15) 13.0235%

17) $\$20,000,000$

19) $\text{Share Price} = \$12.36$; $\text{Money made} = \$2,060$

1.9 PERCENT CHANGE

Learning Objectives

By the end of this section, you will be able to:

- Make calculations with the Percent Change Formula
- Make calculations with the Rate of Change Over Time Formula

For this section you will need the following:

Symbols Used

- $\%C =$ percent change
- $V_{\text{i}} =$ original value
- $V_{\text{f}} =$ updated value
- $RoC =$ rate of change
- $n =$ total number of time periods

Formulas Introduced

- Formula 1.9a – **Percent Change**

$$\%C = \frac{V_{\text{f}} - V_{\text{i}}}{V_{\text{i}}} \times 100$$

- Formula 1.9b – **Rate of Change Over Time**

$$RoC = \left(\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)^{\frac{1}{n}}\right) \times 100$$

Introduction

On your way to work, you notice that the price of gasoline is about 10% higher than it was last month. At the office, reports indicate that input costs are down 5.4% and sales are up 3.6% over last year. Your boss asks you to analyze the year-over-year change in industry sales and submit a report. During your coffee break, you look through the day's e-flyers in your inbox. Home Depot is advertising that all garden furniture is 40% off this week; Safeway's ad says that Tuesday is 10% off day; and a *Globe and Mail* story informs you that the cost of living has risen by 3% since last year. You then log in to your financial services account, where you are happy to find that the mutual funds in your RRSP are up 6.7% from last year. What are you going to do with all this information?

Understanding how data changes from one period to the next is a critical business skill. It allows for quick assessment as to whether matters are improving or getting worse. It also allows for easy comparison of changes in different types of data over time. In this section, the concept of *percent change* is explored, which allows for the calculation of change between two points in time. Then a rate of change over time is introduced, which allows you to determine the change per period when multiple points in time are involved in the calculation.

Make calculations with the Percent Change Formula

It can be difficult to understand a change when it is expressed in absolute terms. Can you tell at a glance how good a deal it is to buy a $\$359$ futon for $215.40? It can also be difficult to understand a change when it is expressed as a percentage of its end result. Are you getting a good deal if that $215.40 futon is 60% of the regular price? What most people do find easier to understand is a change expressed as a percentage of its starting amount. Are you getting a good deal if that $\$359$ futon is on sale at 40% off? A **percent change** expresses in percentage form how much any quantity changes from a starting period to an ending period.

Formula 1.9a: Percent Change

$$\%C = \frac{V_{\text{f}} - V_{\text{i}}}{V_{\text{i}}} \times 100$$

$\%C$ is Percent Change: The change in the quantity is always expressed in percent format.

V_f is Final Quantity Value: This is the value that the quantity has become, or the number that you want to compare against a starting point.

V_i is Initial Quantity Value: This is the value that the quantity used to be, or the number that you want all others to be compared against. Notice how the formula is structured:

1. First calculate "$V_f - V_i$," the change in the quantity. This is the numerator.
2. Divide the change by "V_i" to express the quantity change first as a fraction, then convert it into its decimal format by performing the division.

$\times 100$ is Percent Conversion: Recall from Section 1.8 that you convert a decimal to a percentage by multiplying it by 100. As the language suggests, percent changes are always percentages; therefore, you must include this component in the formula.

—

To calculate the percent change in a variable, you need to know the starting number and the ending number. Once you know these, Formula 1.9a represents how to express the change as a percentage. Remember two critical concepts about percent change:

Do Not Include the Original Quantity in the Change

Percent change represents the changes in the quantities, not the values of the quantities themselves. The original quantity does not count toward the percent change. Therefore, if any quantity doubles, its percent change is 100%, not 200%. For example, if the old quantity was 25 and the new quantity is 50, note that the quantity has doubled. However, 25 of the final 50 comes from the original amount and therefore does not count toward the change. We subtract it out of the numerator through calculating $50 - 25 = 25$. Therefore, the change as a percentage is:

$$\frac{50-25}{25} \times 100 = 100\%.$$

The same applies to a tripling of quantity. If our new quantity is 75 (triple the old quantity of 25), then:

$$\frac{75-25}{25} \times 100 = 200\%.$$

The original value of 25 is once again subtracted out of the numerator. The original 100% is always removed from the formula.

Negative Changes

A negative change must be expressed with a negative sign or equivalent wording. For example, if the old quantity was 20 and the new quantity is 15, this is a decrease of 5, or a change of $15 - 20 = -5$.

The percent change is

$$\frac{15-20}{20} \times 100 = -25\%.$$

Be careful in expressing a negative percent change. There are two correct ways to do this properly:

1. "The change is $-$ 25%."
2. "It has decreased by 25%."

Note in the second statement that the word "decreased" replaces the negative sign. To avoid confusion, do not combine the negative sign with the word "decreased" — recall that two negatives make a positive, so "It has decreased by -25%" would actually mean the quantity has *increased* by 25.

How to

Solve Any Question About Percent Change

Step 1: Notice that there are three variables in the formula. Identify the two known variables and the one unknown variable.

Step 2: Solve for the unknown variable using **Formula 1.9a** $\%C = \frac{V_f - V_i}{V_i} \times 100$.

Assume last month your sales were $\$10,000$, and they have risen to $\$15,000$ this month. You want to express the percent change in sales.

The known variables are $V_i = \$10,000$ and $V_f = \$15,000$. The unknown variable is percent change, or $\%C$.

Using **Formula 1.9a** $\%C = \frac{V_f - V_i}{V_i} \times 100$:

$$\%C = \frac{\$15,000 - \$10,000}{\$10,000} \times 100 = 0.5 \times 100 = 50\%$$

Observe that the change in sales is $\$5,000$ month-over-month. Relative to sales of $\$10,000$ last month, this month's sales have risen by 50%.

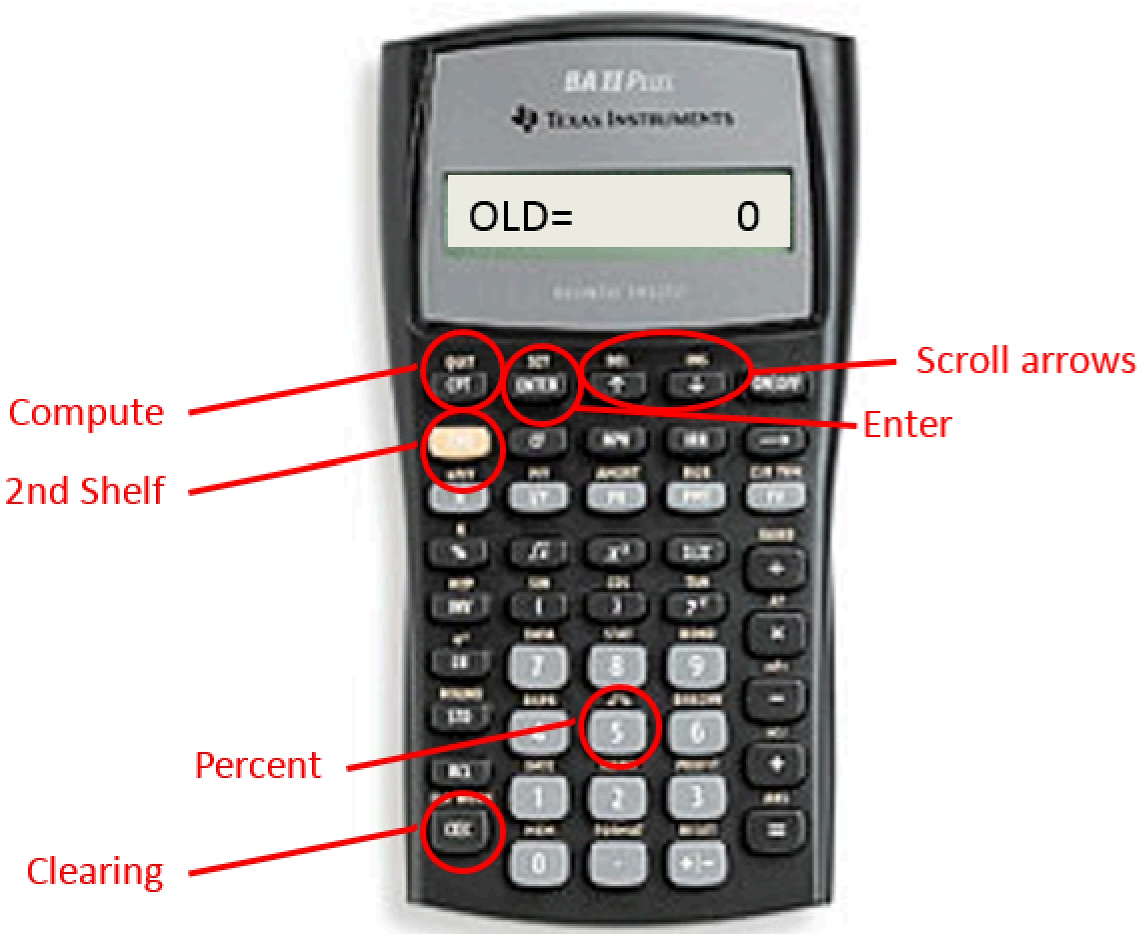


Figure 1.9.1

Key Concepts

To access the percent change function on your BAII Plus calculator, press $\text{2nd}\Delta\%$ located above the number 5 on your keypad. Always clear the memory of any previous question by pressing **2nd CLR Work** once the function is open. Use the ↑ **(up)** and ↓ **(down)** arrows to scroll through the four lines of this function. To solve for an unknown variable, key in three of the four variables and then press **Enter**. With the unknown variable on your display, press **CPT**. Each variable in the calculator is as follows:

- **OLD** = The old or original quantity; the number that represents the starting point
- **NEW** = The new or current quantity; the number to compare against the starting point
- **%CH** = The percent change, or $\%C$ in its percent format without the $\%$ sign
- **#PD** = Number of consecutive periods for change. By default, it is set to 1. For now, do not touch this variable. Later in this section, when we cover rate of change over time, this variable will be explained.

Things To Watch Out For

Watch out for two common difficulties involving percent changes.

1. **Rates versus Percent Changes**. Sometimes you may be confused about whether questions involve rates (Section 3.1) or percent changes. Recall that a *rate* expresses the relationship between a portion and a base. Look for some key identifiers, such as "is/are" (the portion) and "of" (the base). For percent change, key identifiers are "by" or "than." For example, "x has increased *by* $y\%$" and "x is $y\%$ more *than* last year" both represent a percent change.

2. **Mathematical Operations**. You may imagine that you are supposed to add or subtract percent changes, but you cannot do this. Remember that percentages come from fractions. According to the rules of algebra, you can add or subtract fractions only if they share the same base (denominator). For example, if an investment increases in value in the first year by 10% and then declines in the second year by 6%, this is not an overall increase of $10\% - 6\% = 4\%$. Why? If you originally had $\$100$, an increase of 10% (which is $\$100 \times 10\% = \$10$) results in $\$110$ at the end of the first year. You must calculate the 6% decline in the second year using the $\$110$ balance, not the original $\$100$. This is a decline of $\$110 \times (-6\%) = -\$6.60$, resulting in a final balance of $\$103.40$. Overall, the percent change is 3.4%.

A percent change extends the rate, portion, and base calculations introduced in Section 1.8. The primary difference lies in the portion. Instead of the portion being a part of a whole, the portion represents the change of the whole. Putting the two formulas side by side, you can calculate the rate using either approach.

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}} = \frac{V_{\text{f}} - V_{\text{i}}}{V_{\text{i}}}$$

Try It

1) It has been five years since Juan went shopping for a new car. On his first visit to a car lot, he had sticker shock when he realized that new car prices had risen by about 20%. What does this situation involve?

a. Percent Change
b. Rate, portion, Base

Solution

a. (the question involves how car prices have changed; note the keyword "by")

Try It

2) Manuel had his home custom built in 2006 for $\$300,000$. In 2014 he had it professionally appraised at $\$440,000$. He wants to figure out how much his home has appreciated. How would he do so?

a. The 2006 price is V_{f} and the 2014 price is V_{i}.

b. The 2006 price is V_{i} and the 2014 price is the V_{f}.

Solution

b. (the 2006 price is what the house used to be worth, which is V_{i}; the 2014 price represents the new value of the home, or V_{f}).

Example 1

In 1982, the average price of a new car sold in Canada was $\$10,668$. By 2009, the average price of a new car had increased to $\$25,683$. By what percentage has the price of a new car changed over these years?

Solution

***Step 1:* What are you looking for?**

You are trying to find the percent change in the price of the new car, or $\%C$.

Step 2: **What do you already know?**

You know the old and new prices for the cars: $V_= \$10,668$ and $V_{\text{f}} = \$25,683$. You also know that you can apply **Formula 3.2a** $\%C = \frac{V_{\text{f}} - V_{\text{i}}}{V_{\text{i}}} \times 100$ to find $\%C$.

Step 3: **Make substitutions using the information known above.**

$$\%C = \frac{V_{\text{f}} - V_{\text{i}}}{V_{\text{i}}} \times 100$$
$$\%C = \frac{\$25,683 - \$10,668}{\$10,668} \times 100$$
$$\%C = \frac{\$15,015}{\$10,668} \times 100$$
$$\%C = 140.748\%$$

Step 4: **Provide the information in a worded statement.**

From 1982 to 2009, average new car prices in Canada have increased by 140.748%.

Example 2

When you purchase a retail item, the tax increases the price of the product. If you buy a $\$799.00$ Bowflex Classic Home Gym machine in Ontario, it is subject to 13% HST. What amount do you pay for the Bowflex including taxes?

Solution

Step 1: **What are you looking for?**

You are looking for the price of the Bowflex after increasing it by the sales tax. This is the "Final" price for the Bowflex.

Step 2: **What do you already know?**

You know the original price of the machine and how much to increase it by: $V_{\text{i}} = \$799.00$ and $\%C = 13\%$. You also know that, given these values, you can apply **Formula 1.9a** $\%C = \frac{V_{\text{f}} - V_{\text{i}}}{V_{\text{i}}} \times 100$ to find the value of V_{f}.

Step 3: **Make substitutions using the information known above.**

$$\begin{aligned}\%C &= \frac{V_{\text{f}} - V_{\text{i}}}{V_{\text{i}}} \times 100 \\ 13\% &= \frac{V_{\text{f}} - \$799}{\$799} \times 100 \\ 0.13 &= \frac{V_{\text{f}} - \$799}{\$799} \\ \$103.87 &= V_{\text{f}} - \$799 \\ \$902.87 &= V_{\text{f}}\end{aligned}$$

Step 4: **Provide the information in a worded statement.**

The price of a Bowflex, after increasing the price by the taxes of 13%, is $\$902.87$.

Example 3

Consumers often object to price changes in many daily products, even though inflation and other cost changes may justify these increases. To reduce the resistance to a price increase, many manufacturers adjust both prices and product sizes at the same time. For example, the regular selling price for a bottle of shampoo was $\$5.99$ for 240 mL. To account for cost changes, the manufacturer decided to change the price to $\$5.79$, but also reduce the bottle size to 220 mL. What was the percent change in the price per millilitre?

Solution

Step 1: **What are you looking for?**

You need to find the percent change in the price per millilitre, or $\%C$.

Step 2: **What do you already know?**

You know the old price and bottle size, as well as the planned price and bottle size:

$$\begin{aligned}\text{Old price} &= \$5.99 \qquad \text{Old size} = 240mL \\ \text{New price} &= \$5.79 \qquad \text{New size} = 220mL\end{aligned}$$

You can convert the price and size to a price per millilitre by taking the price and dividing by the size, and then find the percent change per millilitre by applying **Formula 1.9a** $\%C = \frac{V_{\text{f}} - V_{\text{i}}}{V_{\text{i}}} \times 100$.

Step 3: **Make substitutions using the information known above.**

First, find price per millilitre for Old and New prices / sizes:

$$\frac{\text{Old price}}{\text{Old size}} = \frac{\$5.99}{240mL} = \$0.024958/mL$$

$$\frac{\text{New price}}{\text{New size}} = \frac{\$5.79}{220mL} = \$0.026318/mL$$

Next, substitute into Formula 1.9a:

$$\%C = \frac{V_f - V_i}{V_i} \times 100$$
$$\%C = \frac{\$0.026318 - \$0.024958}{\$0.024958} \times 100$$
$$\%C = \frac{\$0.001359}{\$0.024958} \times 100$$
$$\%C = 5.4485\%$$

Step 4: **Provide the information in a worded statement.**

The price per mL has increased by 5.4485%. Note that to the consumer, it would appear as if the price has been lowered from $\$5.99$ to $\$5.79$, as most consumers would not notice the change in the bottle size.

Make calculations with the Rate of Change Over Time Formula

The percent change measures the change in a variable from start to end overall. It is based on the assumption that only a single change occurs. What happens when the ending number results from multiple changes and you want to know the typical value of each change? For example, the population of the Toronto census metropolitan area (CMA) has grown from $4,263,759$ in 1996 to $5,113,149$ in 2006. What annual percentage growth in population does this reflect? Notice that we are not interested in calculating the change in population over the 10 years; instead, we want to determine the percentage change in *each* of the 10 years. The **rate of change over time** measures the percent change in a variable per time period.

$\boxed{3.2b}$ Rate of Change Over Time

$$RoC = \left(\left(\frac{V_f}{V_i}\right)^{\frac{1}{n}} - 1\right) \times 100$$

n is Total Number of Periods: The total number of periods reflects the number of periods of change that have occurred between the Old and New quantities.

RoC is Rate of Change per time period: This is a percentage that expresses how the quantity is changing per time period. It recognizes that any change in one period affects the change in the next period.

V_f is Final Quantity Value: What the quantity has become.

V_i is Initial Quantity Value: What the quantity used to be.

$\times 100$ is Percent Conversion: Rates of change over time are always expressed as percentages.

Calculating the rate of change over time is not as simple as dividing the percent change by the number of time periods involved, because you must consider the change for each time period relative to a different starting quantity. For example, in the Toronto census example, the percent change from 1996 to 1997 is based on the original population size of $4,263,759$. However, the percent change from 1997 to 1998 is based on the new population figure for 1997. Thus, even if the same number of people were added to the city in both years, the percent change in the second year is smaller because the population base became larger after the first year. As a result, when you need the percent change per time period, you must use Formula 3.2b.

How to

Work With Rate of Change Over Time

When you work with any rate of change over time, follow these steps:

Step 1: Identify the three known variables and the one unknown variable.

Step 2: Solve for the unknown variable using **Formula 1.9b** $RoC = \left(\left(\frac{V_f}{V_i}\right)^{\frac{1}{n}}\right) \times 100$.

Key Concepts

On your calculator, calculate the rate of change over time using the percent change ($\%C$) function. Previously, we had ignored the **#PD** variable in the function and it was always assigned a value of 1. In rate of change, this variable is the same as n in our equation. Therefore, if our question involved 10 changes, such as the annual population change of the Toronto CMA from 1996 to 2006, then this variable is set to 10.

Paths To Success

You may find it difficult to choose which formula to use: percent change or rate of change over time. To distinguish between the two, consider the following:

- If you are looking for the **overall** rate of change from beginning to end, you need to calculate the percent change.
- If you are looking for the rate of change **per interval**, you need to calculate the rate of change over time.

Ultimately, the percent change formula is a simplified version of the rate of change over time formula where $n = 1$. Thus you can solve any percent change question using **Formula 1.9b** $RoC = \left(\left(\frac{V_\text{f}}{V_\text{i}}\right)^{\frac{1}{n}}\right) \times 100$ instead of **Formula 1.9a** $\%C = \frac{V_\text{f} - V_\text{i}}{V_\text{i}} \times 100$.

Try It

For each of the following, distinguish whether you should solve the question by the percent change formula or the rate of change over time formula.

3) When Peewee started five-pin bowling with the Youth Bowling Canada (YBC) in 1997, his average was 53. In 2011, he finished his last year of the YBC with an average of 248. How did his average change from 1997 to 2011?

Solution
Percent change (looking for an overall change)

Try It

4) A stock was priced at $\$4.34$ per share in 2006 and reached $\$7.15$ per share in 2012. What annual return did a shareholder realize?

Solution
Rate of change over time (looking for change per year)

Try It

5) In 2004, total sales reached $\$1.2 \text{ million}$. By 2010, sales had climbed to $\$4.25 \text{ million}$. What is the growth in sales per year?

Solution

Rate of change over time (looking for change per year)

Example 4

Using the Toronto CMA information, where the population grew from $4,263,759$ in 1996 to $5,113,149$ in 2006, calculate the annual percent growth in the population.

Solution

Step 1: **What are you looking for?**

We need to calculate the percent change per year, which is the rate of change over time, or RoC.

Step 2: **What do you already know?**

We know the starting and ending numbers for the population along with the time frame involved.

$$
\begin{aligned}
V_\text{i} &= 4,263,759 \\
V_\text{f} &= 5,113,149 \\
n &= 2006 - 1996 = 10 \text{ years}
\end{aligned}
$$

Step 3: **Make substitutions using the information known above.**

$$
\begin{aligned}
RoC &= \left(\left(\frac{V_\text{f}}{V_\text{i}}\right)^{\frac{1}{n}} - 1\right) \times 100 \\
RoC &= \left(\left(\frac{5,113,149}{4,263,759}\right)^{\frac{1}{10}} - 1\right) \times 100 \\
RoC &= \left(1.199211^{\frac{1}{10}} - 1\right) \times 100 \\
RoC &= (1.018332 - 1) \times 100 \\
RoC &= 0.018332 \times 100 \\
RoC &= 1.8332\%
\end{aligned}
$$

Step 4: **Provide the information in a worded statement.**

Over the 10 year span from 1996 to 2006, the CMA of Toronto grew by an average of 1.8332% per year.

Example 5

Kendra collects hockey cards. In her collection, she has a rookie year Wayne Gretzky card in mint condition. The book value of the card varies depending on demand for the card and its condition. If the estimated book value of the card fell by $84 in the first year and then rose by $\$113$ in the second year, determine the following:

a. What is the percent change in each year if the card is valued at $\$1,003.33$ at the end of the first year?
b. Over the course of the two years, what was the overall percent change in the value of the card?
c. What was the rate of change per year?

Solution

Step 1: **What are you looking for?**

We need to provide four answers to the questions and find the percent change in Year 1, or $\%C_1$, then the percent change in Year 2, or $\%C_2$. Using these first two solutions, we calculate both the overall percent change across both years, or $\%C_{\text{overall}}$, and the rate of change per year, or RoC.

Step 2: **What do you already know?**

We know the price of the card at the end of the first year as well as how it has changed each year.

$$\begin{aligned} \text{Price at end of first year} &= \$1,003.33 \\ \text{Price change in first year} &= -\$84 \\ \text{Price change in second year} &= \$113 \end{aligned}$$

Furthermore, we know that we can find the percent change using **Formula 1.9a** $\%C = \frac{V_{\text{f}} - V_{\text{i}}}{V_{\text{i}}} \times 100$ to answer questions a and b, and use **Formula 1.9b** $RoC = \left(\left(\frac{V_f}{V_i}\right)^{\frac{1}{n}}\right) \times 100$ to find the rate of change over time for question c.

Step 3: **Make substitutions using the information known above.**

First, calculate the price at the beginning of the first year:

$$\begin{aligned} V_{f1} &= V_{i1} + \text{Change}_1 \\ \$1,003.33 &= V_{i1} - \$84.00 \\ \$1,087.33 &= V_{i1} \end{aligned}$$

For the percent change in $\text{Year } 1$, apply **Formula 1.9a** $\%C = \frac{V_f - V_i}{V_i} \times 100$:

$$\begin{aligned} \%C_1 &= \frac{V_{f1} - V_{i1}}{V_{i1}} \times 100 \\ \%C_1 &= \frac{\$1,003.33 - \$1,087.33}{\$1,087.33} \times 100 \\ \%C_1 &= -7.7253\% \end{aligned}$$

Calculate the price at the end of the second year:

$$\begin{aligned} V_{f2} &= V_{i2} + \text{Change}_2 \\ V_{f2} &= \$1,003.33 + \$113.00 \\ V_{f2} &= \$1,116.33 \end{aligned}$$

For the percent change in $\text{Year } 2$, apply **Formula 1.9a** $\%C = \frac{V_f - V_i}{V_i} \times 100$:

$$\begin{aligned} \%C_2 &= \frac{V_{f2} - V_{i2}}{V_{i2}} \times 100 \\ \%C_2 &= \frac{\$1,116.33 - \$1,003.33}{\$1,003.33} \times 100 \\ \%C_2 &= 11.2625\% \end{aligned}$$

For the overall percent change, take the old price at the beginning of the first year and compare it to the new price at the end of the second year. Apply **Formula 1.9a** $\%C = \frac{V_f - V_i}{V_i} \times 100$:

$$\begin{aligned} \%C_{\text{overall}} &= \frac{V_{f2} - V_{i1}}{V_{i1}} \times 100 \\ \%C_{\text{overall}} &= \frac{\$1,116.33 - \$1,087.33}{\$1,087.33} \times 100 \\ \%C_{\text{overall}} &= 2.6671\% \end{aligned}$$

Calculate the rate of change over the two years using the same two prices, but apply **Formula 1.9b** $RoC = \left(\left(\frac{V_f}{V_i}\right)^{\frac{1}{n}}\right) \times 100$.

$$\begin{aligned} RoC &= \left(\left(\frac{V_{f2}}{V_{i1}}\right)^{\frac{1}{n}} - 1\right) \times 100 \\ RoC &= \left(\left(\frac{\$1,116.33}{\$1,087.33}\right)^{\frac{1}{2}} - 1\right) \times 100 \\ RoC &= 1.3248\% \end{aligned}$$

Step 4: **Provide the information in a worded statement.**

The value of the hockey card dropped 7.7253% in the first year and rose 11.2625% in the second year. Overall, the card rose by 2.6671% across both years, which represents a growth of 1.3248% in each year.

Exercises: Mechanics

For questions 1–3, solve for the unknown (?) using Formula 1.9a $\%C = \frac{V_f - V_i}{V_i} \times 100$ (percent change).

Table 1.9.1

	Old	New	Δ%
1)	$109.95	$115.45	?
2)	?	$622.03	13.25%
3)	5.94%	?	−10%

4) If $\$9.99$ is changed to $\$10.49$, what is the percent change?

5) $\$19.99$ lowered by 10% is what dollar amount?

6) What amount when increased by 40% is $\$3,500$?

7) If $10,000$ grows to $20,000$ over a period of 10 years, what is the annual rate of change?

Odd Answers

1) 5.0023%

3) 5.346%

5) $\$17.99$

7) 7.1773%

Exercises: Applications

8) How much, including taxes of 12%, would you pay for an item with a retail price of $\$194.95$?

9) From September 8, 2007 to November 7, 2007, the Canadian dollar experienced a rapid

appreciation against the US dollar, going from $\$0.9482$ to $\$1.1024$. What was the percent increase in the Canadian dollar?

10) From 1996 to 2006, the "big three" automakers in North America (General Motors, Ford, and Chrysler) saw their market share drop from 71.5% to 52.7%. What is the overall change and the rate of change per year?

11) The average price of homes in Calgary fell by $\$10,000$ to $\$357,000$ from June 2009 to July 2009. The June 2009 price was 49% higher than the June 2005 price.

- **a) What was the percent change from June 2009 to July 2009?**
- **b) What was the average price of a home in June 2005?**
- **c) What was the annual rate of change from June 2005 to June 2009?**

12) On October 28, 2006, Saskatchewan lowered its provincial sales tax (PST) from 7% to 5%. What percent reduction does this represent?

13) A local Superstore sold $21,983$ cases of its President's Choice cola at $\$2.50$ per case. In the following year, it sold $19,877$ cases at $\$2.75$ per case.

- **a) What is the percent change in price year-over-year?**
- **b) What is the percent change in quantity year-over-year?**
- **c) What is the percent change in total revenue year-over-year? (Hint: revenue = price × quantity)**

14) A bottle of liquid laundry detergent priced at $\$16.99$ for a 52-load bottle has been changed to $\$16.49$ for a 48-load bottle. By what percentage has the price per load changed?

Odd Answers

9) 16.2624%

11a) -2.7248%

11b) $\$246,308.73$

11c) 10.4833%

13a) 10%

13b) -9.5801%

13c) -0.5381%

Exercises: Challenge, Critical Thinking, & Other Applications

15) At a boardroom meeting, the sales manager is happy to announce that sales have risen from $\$850,000$ to $\$1,750,000$ at a rate of 4.931998% per year. How many years did it take for the sales to reach $\$1,750,000$?

16) The Nova Scotia Pension Agency needs to determine the annual cost of living adjustment (COLA) for the pension payments made to its members. To do this, it averages the consumer price index (CPI) for both the previous fiscal year and the current fiscal year. It then calculates the percent change between the two years to arrive at the COLA. If CPI information is as follows, determine the COLA that the pensioners will receive.

Table 1.9.2

Previous Fiscal Year	Current Fiscal Year						
Nov.	109.2	May	112.1	Nov.	111.9	May	114.6
Dec.	109.4	June	111.9	Dec.	112.0	June	115.4
Jan.	109.4	July	112.0	Jan.	111.8	July	115.8
Feb.	110.2	Aug.	111.7	Feb.	112.2	Aug.	115.6
Mar.	111.1	Sep.	111.9	Mar.	112.6	Sep.	115.7
Apr.	111.6	Oct.	111.6	Apr.	113.5	Oct.	114.5

17) During The Bay's warehouse clearance days, it has reduced merchandise by 60%. As a bonus, today is Scratch 'n' Save day, where you can receive up to an additional 25% off the reduced price. If you scratched the maximum of 25% off, how many

dollars would you save off an item that is regularly priced at $\$275.97$? What percent savings does this represent?

18) Federal Canadian tax rates for 2010 and 2011 are listed below. For example, you pay no tax on income within the first bracket, 15% on income within the next bracket, and so on. If you earned $\$130,000$ in each year, by what percentage did your federal tax rate change? In dollars, what was the difference?

Table 1.9.3

2010 Tax Brackets	Taxed at	2011 Tax Brackets
$0–$10,382	0%	$0–$10,527
$10,383–$40,970	15%	$10,528–$41,544
$40,971–$81,941	22%	$41,545–$83,088
$81,942–$127,021	26%	$83,089–$128,800
$127,022+	29%	$128,801+

19) Melina is evaluating two colour laser printers for her small business. A Brother model is capable of printing 21 colour pages per minute and operates 162.5% faster than a similar Hewlett-Packard model. She needs to print $15,000$ pages for a promotion. How much less time (stated as a percentage) will it take on the Brother model?

20) A chocolate bar has been priced at $\$1.25$ for a 52 gram bar. Due to vending machine restrictions, the manufacturer needs to keep the price the same. To adjust for rising costs, it lowers the weight of the bar to 48 grams.

a) By what percentage has the price per gram changed?
b) If this plan is implemented over two periods, what rate of change occurs in each period?

Odd Answers

15) 15 years

17) $\text{Amount saved} = \$193.18$; $\%C = 70.0004\%$

19) 62.5% less time

PART II

UNIT 2: MEASUREMENT AND ROUNDING RULES

2.1 SYSTEMS OF MEASUREMENT

Learning Objectives

By the end of this section, you will be able to:

- Perform metric-to-metric unit conversions.
- Make Unit Conversions in the U.S. System
- Perform unit conversions (from any system) using dimensional analysis.

Perform metric-to-metric unit conversions.

The Metric System

Metric system (SI – international system of units): the most widely used system of measurement in the world. It is based on the basic units of meter, kilogram, second, etc.

In the metric system, units are related by powers of 10. The roots words of their names reflect this relation. For example, the basic unit for measuring length is a meter. One kilometer is 1,000 meters; the prefix *kilo* means *thousand*. One centimeter is $\frac{1}{100}$ of a meter, just like one cent is $\frac{1}{100}$ of one dollar.

SI common units:

Table 2.1.1

Quantity	Unit	Unit Symbol
Length	meter	m
Mass (or weight)	gram	kg
Volume	litre	L
Time	second	s
Temperature	degree (Celsius)	°C

Metric prefixes (SI prefixes): large and small numbers are made by adding SI prefixes, which is based on multiples of 10.

Metric conversion table:

Table 2.1.2

Prefix	Symbol (abbreviation)	Power of 10	Multiple value	Example
giga	G	10^9	1,000,000,000	1 Gm = 1,000,000,000 m
mega	M	10^6	1,000,000	1 Mm = 1,000,000 m
kilo-	k	10^3	1,000	1 km = 1,000 m
hecto-	h	10^2	100	1 hm = 100 m
deka-	da	10^1	10	1 dam = 10 m
meter/gram/ litre		1 (100)		
deci-	d	10^{-1}	0.1	1 m = 10 dm
centi-	c	10^{-2}	0.01	1 m = 100 cm
milli-	m	10^{-3}	0.001	1 m = 1,000 mm
micro	µ or mc	10^{-6}	0.000 001	1 m = 1,000,000 µm
nano	n	10^{-9}	0.000 000 001	1 m = 1,000,000,000 nm
pico	p	10^{-12}	0.000 000 000 001	1 m = 1,000,000,000,000pm

A good way to remember the order of the metric prefixes is by using a mnemonic device such as “Great

Mighty King Henry died by drinking chocolate malted milk not poison". Notice that the first letter of each word reminds you of the metric prefix, and the word "by" represents the base units. Feel free to use this particular mnemonic device, or come up with your own!

Metric prefix for length, weight and volume:

Table 2.1.3

Prefix	Length (m – meter)	Weight (g – gram)	Liquid volume (L – litre)
giga (G)	Gm (Gigameter)	Gg (Gigagram)	GL (Gigalitre)
mega (M)	Mm (Megameter)	Mg (Megagram)	ML (Megalitre)
kilo (k)	km (Kilometer)	kg (Kilogram)	kL (Kilolitre)
hecto (h)	hm (hectometer)	hg (hectogram)	hL (hectolitre)
deka (da)	dam (dekameter)	dag (dekagram)	daL (dekalitre)
meter/gram/litre	m (meter)	g (gram)	L (litre)
deci (d)	dm (decimeter)	dg (decigram)	dL (decilitre)
centi (c)	cm (centimeter)	cg (centigram)	cL (centilitre)
milli (m)	mm (millimeter)	mg (milligram)	mL (millilitre)
micro (µ or mc)	µm or mcm (micrometer)	µg or mcg (microgram)	µL or mcL (microlitre)
nano (n)	nm (nanometer)	ng (nanogram)	nL (nanolitre)
pico (p)	pm (picometer)	pg (picogram)	pL (picolitre)

The more commonly used equivalencies of measurements in the metric system are shown in Table 2.1.4. The common abbreviations for each measurement are given in parentheses. Please note, that you will need to be able to convert the units outside of this table as well.

Metric System of Measurement

Table 2.1.4

Length	Mass	Capacity
1 kilometer (km) = 1,000 m	1 kilogram (kg) = 1,000 g	1 kiloliter (kL) = 1,000 L
1 hectometer (hm) = 100 m	1 hectogram (hg) = 100 g	1 hectoliter (hL) = 100 L
1 dekameter (dam) = 10 m	1 dekagram (dag) = 10 g	1 dekaliter (daL) = 10 L
1 meter (m) = 1 m	1 gram (g) = 1 g	1 liter (L) = 1 L
1 decimeter (dm) = 0.1 m	1 decigram (dg) = 0.1 g	1 deciliter (dL) = 0.1 L
1 centimeter (cm) = 0.01 m	1 centigram (cg) = 0.01 g	1 centiliter (cL) = 0.01 L
1 millimeter (mm) = 0.001 m	1 milligram (mg) = 0.001 g	1 milliliter (mL) = 0.001 L
1 meter = 100 centimeters	1 gram = 100 centigrams	1 liter = 100 centiliters
1 meter = 1,000 millimeters	1 gram = 1,000 milligrams	1 liter = 1,000 milliliters

Perform metric-to-metric unit conversions using the decimal point method.

How to

Performing Metric to Metric Conversions

One of the most convenient things about the metric system is that we can use its decimal nature to convert from one unit to the other simply by moving the decimal point to the left or to the right.

Steps for metric conversion:

- Identify the number of places to move the decimal point.

– Convert a ***smaller*** unit ***to*** a ***larger*** unit: move the decimal point to the ***left***.
– Convert a ***larger*** unit ***to*** a ***smaller*** unit: move the decimal point to the ***right***.

Example 1

326 mm = (?) m

Solution
Step 1: **Identify mm (millimeters) and m (meters) on the conversion table.**
Step 2: **Count places from mm to m:**

3 places left

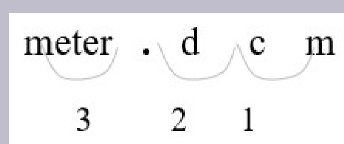


Figure 2.1.1

Step 3: **Move 3 decimal places.**

Convert a smaller unit (mm) to a larger (m) unit: move the decimal point to the left.

$$(1m = 1000mm)$$

Step 4: **Move the decimal point three places to the left.**

$$326.mm = 0.326m$$

Example 2

4.675 hg = (?) g

Solution

Step 1: **Identify hg (hectograms) and g (grams) on the conversion table.**

Step 2: **Count places from hg to g:**

2 places right

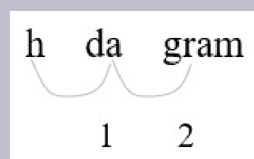


Figure 2.1.2

Step 3: **Move 2 decimal places.**

$$(1hg = 100g)$$

Step 4: **Convert a larger unit (hg) to a smaller (g) unit.**

Move the decimal point to the right.

Step 5: **Move the decimal point two places to the right.**

$$4.765hg = 476.5g$$

Try It

1) Convert 0.2744kg to micrograms.

Solution

274,400,000 mcg (or 274,400,000 μg)

2) Convert 12,940,000 nL to decilitres.

Solution
0.1294 dL

Being able to convert units by shifting the decimal point left or right is convenient and does work for a lot of our metric-to-metric conversions. However, as we will see later in the section, converting more complex units may be confusing if we are using the decimal point method. Thus, it is important to have an understanding of the technique of Dimensional Analysis (or the Unit Factor Method).

Perform metric-to-metric unit conversions using dimensional analysis.

How to

Convert units using the dimensional analysis or (the Unit Factor Method)

Step 1: **Write the original term as a fraction (over 1).**

Example: 10g can be written as $\frac{10g}{1}$

Step 2: **Write the conversion formula as a fraction, \frac{ 1}{( )} or \frac{ ( )}{1}**

Example: 1m = 100 cm can be written as $\frac{1m}{(100cm)}$ or $\frac{(100cm)}{1m}$

Step 3: **Put the desired or unknown unit on the top.**

\frac{ \frac{
Step 4: **Multiply the original term by 1}{(or ((Cancel out the same units)**
)})}{1}

Example 3

1200 g = (?) kg

Solution

Step 1: **Write the original term (the left side) as a fraction.**

$$1200g = \frac{1200g}{1}$$

Step 2: **Write the conversion formula as a fraction.**

"kg" is the desired unit.

$$1kg = 1000g : \frac{1kg}{(1000g)}$$

Step 3: **Multiply.**

The units "g" cancel out.

$$\begin{aligned} 1200g &= \frac{1200\cancel{g}}{1} \times \frac{1kg}{(1000\cancel{g})} \\ &= \frac{1200kg}{1000} \\ &= 1.2kg \end{aligned}$$

Example 4

30 cm = (?) mm

Solution

Step 1: **Write the original term (the left side) as a fraction.**

$$30cm = \frac{30cm}{1}$$

Step 2: **Write the conversion formula as a fraction.**

"mm" is the desired unit.

$$1cm = 10mm : \frac{(10mm)}{1cm}$$

Step 3: **Multiply.**

The units "cm" cancel out.

$$\begin{aligned} 30cm &= \frac{30\cancel{cm}}{(1mm)} \times \frac{(10mm)}{1\cancel{cm}} \\ &= \frac{(30)(10)mm}{1} \\ &= 300mm \end{aligned}$$

Try It

Use dimensional analysis to convert the following units:

3) Convert 28.4 dag to g.

Solution

284 g

4) Convert 0.00485kL to dL.

Solution

48.5 dL

Example 5

Have you ever run a 5K or 10K race? The length of those races are measured in kilometers. The metric system is commonly used in the United States when talking about the length of a race.

Nick ran a 10K race. How many meters did he run?

Solution

We will convert kilometers to meters using the identity property of multiplication.

Step 1: **Multiply the measurement to be converted by 1.**

$$10 \text{ kilometers } \times 1$$

Step 2: **Write 1 as a fraction relating kilometers and meters.**

$$10 \text{ kilometers} \cdot \frac{1,000 \text{ meters}}{1 \text{ kilometers}}$$

Step 3: **Simplify.**

$$\frac{10 \cancel{\text{kilometers}} \cdot 1,000\, m}{1 \cancel{\text{kilometer}}}$$

Step 4: **Multiply.**

$10,000$ meters. Nick ran $10,000$ meters.

Try It

5) Sandy completed her first 5K race! How many meters did she run?

Solution

5,000 meters

6) Herman bought a rug 2.5 meters in length. How many centimeters is the length?

Solution

250 centimeters

Example 6

Eleanor's newborn baby weighed 3,200 grams. How many kilograms did the baby weigh?

Solution

We will convert grams into kilograms.

Step 1: **Multiply the measurement to be converted by 1.**

$$3,200 \text{ grams } \times \text{ } 1$$

Step 2: **Write 1 as a function relating kilograms and grams.**

$$3,200 \text{ grams} \cdot \frac{1kg}{1,000 \text{ grams}}$$

Step 3: **Simplify.**

$$3,200 \text{ } \cancel{\text{grams}} \cdot \frac{1kg}{1,000 \text{ } \cancel{\text{grams}}}$$

Step 4: **Multiply.**

$$\frac{3,200 \text{ kilograms}}{1,000}$$

Step 5: **Divide.**

3.2 kilograms. The baby weighed 3.2 kilograms.

Try It

7) Kari's newborn baby weighed 2,800 grams. How many kilograms did the baby weigh?

Solution

2.8 kilograms

8) Anderson received a package that was marked 4,500 grams. How many kilograms did this package weigh?

Solution

4.5 kilograms

Example 7

Samadia took 800mg of Ibuprofen for her inflammation. How many grams of Ibuprofen did she take?

Solution

We will convert milligrams to grams using the identity property of multiplication.

Step 1: **Multiply the measurement to be converted by 1.**

$$800 \text{ milligrams} \times 1$$

Step 2: **Write 1 as a fraction relating kilometres and metres.**

$$800 \text{ milligrams} \times \frac{1 \text{ gram}}{1000 \text{ milligrams}}$$

Step 3: **Simplify.**

$$800 \text{ milligrams} \times \frac{1 \text{ gram}}{1000 \text{ milligrams}}$$

Step 4: **Multiply.**

$$0.8 \text{ grams}$$

Samadia took 0.8 grams of Ibuprofen.

Example 8

Dena's recipe for lentil soup calls for 150 milliliters of olive oil. Dena wants to triple the recipe. How many liters of olive oil will she need?

Solution

We will find the amount of olive oil in millileters then convert to liters.

Step 1: **Translate to algebra.**

$$3 \times 150$$

Step 2: **Multiply.**

$$450mL$$

Step 3: **Convert to liters.**

$$450mL \times \frac{0.001L}{1mL}$$

Step 4: **Simplify.**

$$0.45L$$

Dena needs 0.45 liters of olive oil.

Try It

9) Klaudia took 0.125 grams of Ibuprofen for his headache. How many milligrams of the medication did she take?

Solution
125 milligrams

10) A recipe for Alfredo sauce calls for 250 milliliters of milk. Renata is making pasta with Alfredo sauce for a big party and needs to multiply the recipe amounts by 8. How many liters of milk will she need?

Solution
2 liters

11) To make one pan of baklava, Dorothea needs 400 grams of filo pastry. If Dorothea plans to make 6 pans of baklava, how many kilograms of filo pastry will she need?

Solution
2.4 kilograms

Example 9

The volume of blood coursing throughout an adult human body is about 5 litres. Convert it to millilitres.

Solution
We will convert litres to millilitres. In the Metric System of Measurement table, we see that 1 litre = 1,000 millilitres.

Step 1: **Multiply by 1, writing 1 as a fraction relating litres to millilitres.**

$$5L \times \frac{1000mL}{1L}$$

Step 2: **Simplify.**

$$5\cancel{L} \times \frac{1000mL}{1\cancel{L}} = 5 \times 1000mL$$

Step 3: **Multiply.**

$$5000mL$$

As we saw before, when we are converting metric to metric units, you may see a pattern. Since the system is based on multiples of ten, the calculations involve multiplying by multiples of ten. We have learned how to simplify these calculations by just moving the decimal.

Remember that to multiply by 10, 100, or 1,000, we move the decimal to the right one, two, or three places, respectively. To multiply by 0.1, 0.01, or 0.001, we move the decimal to the left one, two, or three places, respectively.

We can apply this pattern when we make measurement conversions in the metric system. In Figure 2.1.1 , we changed 3,200 grams to kilograms by multiplying by $\frac{1}{1000}$ (or 0.001). This is the same as moving the decimal three places to the left.

Figure 2.1.3

Example 10

Convert:

a. 350 L to kiloliters
b. 4.1 L to milliliters.

Solution

a. We will convert liters to kiloliters. In Table 2.1.4, we see that 1 kiloliter= 1,000 liters.

Step 1: **Multiply by 1, writing 1 as a fraction relating liters to kiloliters.**

$$350L \cdot \frac{1kL}{1,000L}$$

Step 2: **Simplify.**

$$350 \cancel{L} \cdot \frac{1kL}{1,000 \cancel{L}}$$

Step 3: **Move the decimal 3 units to the left.**

$$0.35kL$$

---

b. We will convert liters to milliliters. From Table 2.1.4 we see that 1 liter=1,000 milliliters.

Step 1: **Multiply by 1, writing 1 as a fraction relating liters to milliliters.**

$$4.1L \cdot \frac{1,000mL}{1L}$$

Step 2: **Simplify.**

$$4.1 \cancel{L} \cdot \frac{1,000mL}{1,000 \cancel{L}}$$

Step 3: **Move the decimal 3 units to the right.**

$$4\overset{\rightarrow}{.}1_1 0_2 0_3 = 4,100.\ mL$$

Try It

12) Convert:

a. 725 L to kiloliters
b. 6.3 L to milliliters

Solution
a 7,250 kiloliters
b 6,300 milliliters

13) Convert:

a. 350 hL to liters
b. 4.1 L to centiliters

Solution
a 35,000 liters
b 410 centiliters

As we see, even when doing dimensional analysis, we can use the pattern of multiplying by powers of ten and shift our decimal point to the left or right accordingly to find our answers and make our calculations more simple. However, what might we do if we needed to convert from milligrams per decilitre to grams per litre. When we use these types of units, it can make it more difficult to simply move the decimal point to the left and right. In the following example, we see how dimensional analysis can help us stay organized and convert these types of units.

Example 11

100 m/s = (?) km/h

Solution
Step 1: **Write the original term (the left side) as a fraction.**

$$100m/s = \frac{100m}{1s}$$

Step 2: **Write the conversion formulas required as fractions.**

"km/h" is the desired unit

$$1000m = 1km \text{ and } 1h = 3600s$$

$$\frac{1km}{1000m} \text{ and } \frac{3600s}{1h}$$

Step 3: **Multiply.**

The units "m" and "s" cancel out.

$$\begin{aligned} 100m/s &= \frac{100\cancel{m}}{1\cancel{s}} \times \frac{1km}{1000\cancel{m}} \times \frac{3600\cancel{s}}{1h} \\ &= \frac{100 \times 3600km}{1 \times 1000h} \\ &= 360km/h \end{aligned}$$

Try It

14) Convert 0.000005kg/L to micrograms per decilitre.

Solution

500 mcg/dL or 500 mu g/dL.

Adding and subtracting SI measurements:

Example 12

Combine after converting to the same unit.

a. $\begin{array}{r} 3m \\ \underline{-2000mm} \end{array}$

b. $\begin{array}{r} 25kg \\ \underline{+4g} \end{array}$

Solution

a.

Step 1: **Convert to the same unit.**

$$1m = 1,000mm$$

Step 2: **Subtract.**

$$\begin{array}{r} 3000mm \\ \underline{-2000mm} \\ 1000mm \end{array}$$

---

b.

Step 1: **Convert to the same unit.**

$$1kg = 1000g$$

Step 2: **Add.**

$$\begin{array}{r} 25000g \\ +4g \\ \hline 25004g \end{array}$$

The Relationship between mL, g, and cm^3

How are mL, g, and cm^3 related?

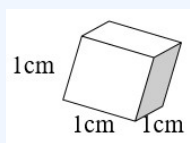


Figure 2.1.4

- A cube takes up 1 cm^3 of space (1 cm × 1 cm × 1 cm = 1cm^3).
- A cube holds 1 mL of water and has a mass of 1 gram at 4°C.
- 1 cm^3 = 1 mL = 1 g

Example 13

Convert.

a. $16cm^3$ = (?) g
b. 9 L = (?) cm^3
c. 35 cm^3 = (?) cL
d. 450 kg = (?) L

Solution
a. $16cm^3$ = (?) g

***Step 1:* Covert cm^3 to g.**

$$\begin{aligned} 1\ cm^3 &= 1g \\ 16\ cm^3 &= 16g \end{aligned}$$

---

b. 9 L = (?) cm^3

***Step 1:* Convert L to mL.**

$$\begin{aligned} 1\ L &= 1,000\ mL \\ 9\ L &= 9,000\ mL \end{aligned}$$

***Step 2:* Convert mL to cm^3**

$$\begin{aligned} 1\ mL &= 1\ cm^3 \\ &= 9000\ cm^3 \end{aligned}$$

---

c. 35 cm^3 = (?) cL

***Step 1:* Convert cm^3 to mL.**

$$\begin{aligned} 1\ cm^3 &= 1\ mL \\ 35\ cm^3 &= 35\ mL \end{aligned}$$

***Step 2:* Move 1 decimal place left.**

$$= 3.5cL$$

---

d. 450 kg = (?) L

***Step 1:* Convert kg to g.**

$$\begin{aligned} 1\ kg &= 1,000\ g \\ 450\ kg &= 450,000\ g \end{aligned}$$

***Step 2:* Convert g to mL.**

$$\begin{aligned} 1\ g &= 1\ mL \\ &= 450,000\ mL \end{aligned}$$

Step 3: **Covert mL to L.**

$$\begin{aligned} 1\ L &= 1,000\ mL \\ &= 450\ L \end{aligned}$$

Example 14

A swimming pool measures 10 m by 8 m by 2 m. How many kilolitres of water will it hold?

Solution

Step 1: **Find the volume in m^3.**

$$160\ m^3 = (\ ?\ )\ kL$$

$$V = w \times l \times h = (8m)\ (10m)\ (2m) = 160\ m^3$$

Step 2: **Convert to cm^3**

$1m = 100cm$, $3 \times 2 = 6$, move 6 places right for volume.

$$160m^3 = 160,000,000\ cm^3$$

Step 3: **Convert to mL**

$$\begin{aligned} 1\ mL &= 1\ cm^3 \\ 160,000,000\ cm^3 &= 160,000,000\ mL \end{aligned}$$

Step 4: **Convert to kL.**

$$\begin{aligned} 160,000,000\ mL &= 160\ kL \\ 1\ kL &= 1,000,000\ mL \\ 160\ m^3 &= 160\ kL \end{aligned}$$

The swimming pool will hold 160 kL of water.

Use Mixed Units of Measurement in the Metric System

Performing arithmetic operations on measurements with mixed units of measures in the metric system requires care. Make sure to add or subtract like units.

Example 15

Ryland is 1.6 meters tall. His younger brother is 85 centimeters tall. How much taller is Ryland than his younger brother?

Solution

We can convert both measurements to either centimeters or meters. Since meters is the larger unit, we will subtract the lengths in meters. We convert 85 centimeters to meters by moving the decimal 2 places to the left.

***Step 1:* Write the 85 centimeters as meters.**

85cm is 0.85m.

***Step 2:* Subtract.**

$$\begin{array}{r} 1.6m \\ -0.85m \\ \hline 0.75m \end{array}$$

Ryland is 0.75 m taller than his brother.

Try It

15) Mariella is 1.58 meters tall. Her daughter is 75 centimeters tall. How much taller is Mariella than her daughter? Write the answer in centimeters.

Solution

83 centimeters

16) The fence around Hank's yard is 2 meters high. Hank is 96 centimeters tall. How much shorter than the fence is Hank? Write the answer in meters.

Solution
1.04 meters

Make Unit Conversions in the U.S. System

There are two systems of measurement commonly used around the world. Most countries use the metric system. The U.S. uses a different system of measurement, usually called the U.S. system. We will look at the U.S. system now.

The U.S. system of measurement uses units of inch, foot, yard, and mile to measure length and pound and ton to measure weight. For capacity, the units used are cups, pints, quarts, and gallons. Both the U.S. system and the metric system measure time in seconds, minutes, and hours.

The equivalencies of measurements are following, and also show, in parentheses, the common abbreviations for each measurement.

Table 2.1.5

Length	1 foot (ft.) = 12 inches (in.)	Volume	3 teaspoons (t) = 1 tablespoon (T)
	1 yard (yd.) = 3 feet (ft.)		16 tablespoons(T) = 1 cup (C)
	1 mile (mi.) = 5,280 feet (ft.)		1 cup (C) = 8 fluid ounces (fl. oz.)
			1 pint (pt.) = 2 cups (C)
			1 quart (qt) = 2 pints (pt.)
			1 gallon (gal) = 4 quarts (qt.)
Weight	1 pound (lb.) = 16 ounces (oz.)	Time	1 minute = 60 seconds (sec)
	1 ton = 2205 pounds (lb.)		1 hour (hr) = 60 minutes (min)
			1 day = 24 hours (hr)
			1 week (wk) = 7 days
			1 year (yr) = 365 days

In many real-life applications, we need to convert between units of measurement, such as feet and yards, minutes and seconds, quarts and gallons, etc. We will use the identity property of multiplication to do these conversions. We'll restate the identity property of multiplication here for easy reference.

Identity Property of Multiplication

For any real number a: $a \cdot 1 = a \qquad 1 \cdot a = a$

1 is the multiplicative identity

As we saw earlier in the section, dimensional analysis can be used to convert units. In the U.S. system, since it is not a decimal system, it is best that we always use dimensional analysis to convert our units. Here, we elaborate on that concept.

To use the identity property of multiplication, we write 1 in a form that will help us convert the units. For example, suppose we want to change inches to feet. We know that 1 foot is equal to 12 inches, so we will write 1 as the fraction $\frac{1 foot}{12 inch}$ When we multiply by this fraction we do not change the value but just change the units.

But $\frac{1 foot}{12 inch}$ also equals 1. How do we decide whether to multiply by $\frac{1 foot}{12 inch}$ or $\frac{1 foot}{12 inch}$? We choose the fraction that will make the units we want to convert *from* divide out. Treat the unit words like factors and "divide out" common units like we do common factors. If we want to convert 66 inches to feet, which multiplication will eliminate the inches?

$$66\ inches \cdot \frac{1\ foot}{12\ inches} \quad \text{or} \quad 66\ inches \cdot \frac{12\ inches}{1\ foot}$$

The first form works since $66\ \cancel{inches} \cdot \frac{1\ foot}{12\ \cancel{inches}}$

The inches divide out and leave only feet. The second form does not have any units that will divide out and so will not help us.

Example 16

MaryAnne is 66 inches tall. Convert her height into feet.

Solution

***Step 1:* Multiply the measurement to be converted by 1; write 1 as a fraction relating the units given and the units needed.**

Multiply 66 inches by 1, writing 1 as a fraction relating inches and feet. We need inches in the denominator so that the inches will divide out!

$$66 \text{ inches} \times 1 = 66 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}}$$

***Step 2:* Multiply.**

Think of 66 inches as $\frac{66 \text{ inches}}{1}$

$$\frac{66 \text{ inches} \times 1 \text{ foot}}{12 \text{ inches}}$$

***Step 3:* Simplify the fraction.**

Notice: inches divide out.

$$66 \cancel{\text{inches}} \times \frac{1 \text{ foot}}{12 \cancel{\text{inches}}} = \frac{66 \text{ inches}}{12}$$

***Step 4:* Simplify.**

Divide 66 by 12.

$$5.5 feet$$

Try It

17) Lexie is 30 inches tall. Convert her height to feet.

Solution

2.5 feet

18) Rene bought a hose that is 18 yards long. Convert the length to feet.

Solution

54 feet

How To

Make Unit Conversions.

1. Multiply the measurement to be converted by 1; write 1 as a fraction relating the units given and the units needed.
2. Multiply.
3. Simplify the fraction.
4. Simplify.

When we use the identity property of multiplication to convert units, we need to make sure the units we want to change from will divide out. Usually this means we want the conversion fraction to have those units in the denominator.

Example 17

Eli's six month son is 102.4 ounces. Convert his weight to pounds.

Solution

To convert ounces into pounds we will multiply by conversion factors of 1.

Step 1: **Write 1 as $\frac{1 \text{ pound}}{16 \text{ ounces}}$.**

$$102.4 \text{ ounces} \times \frac{1 \text{ pound}}{16 \text{ ounces}}$$

Step 2: **Divide out the common units.**

$$102.4 \cancel{\text{ ounces}} \times \frac{1 \text{ pound}}{16 \cancel{\text{ ounces}}}$$

Step 3: **Simplify the fraction.**

$$\frac{102.4 \text{ ounces}}{16 \text{ ounces}}$$

Step 4: **Simplify.**

$$6.4 \text{ pounds}$$

Eli's six months son weighs 6.4 pounds.

Example 18

Ndula, an elephant at the San Diego Safari Park, weighs almost 3.2 tons. Convert her weight to pounds.

Solution
We will convert 3.2 tons into pounds. We will use the identity property of multiplication, writing 1 as the fraction:

$$\frac{2000 \text{ pounds}}{1 \text{ ton}}$$

Step 1: **Multiply the measurement to be converted, by 1.**

$$3.2 \text{ tons} \times 1$$

Step 2: **Write 1 as a fraction relating tons and pounds.**

$$3.2 \text{ tons} \times \frac{2,000 \text{ pounds}}{1 \text{ ton}}$$

Step 3: **Simplify.**

$$\frac{3.2 \cancel{\text{tons}} \times 2,000 \text{ pounds}}{1 \cancel{\text{ton}}}$$

Step 4: **Multiply.**

$$6,400 \text{ pounds}$$

Try It

19) Arnold's SUV weighs about 4.3 tons. Convert the weight to pounds.

Solution

8,600 pounds

20) The Carnival *Destiny* cruise ship weighs 51,000 tons. Convert the weight to pounds.

Solution
102,000,000 pounds

21) One-year-old girl weighs 11 pounds. Convert her weight to ounces.

Solution
176 ounces.

As was the case with the metric system, sometimes, to convert from one unit to another, we may need to use several other units in between, so we will need to multiply several fractions.

Example 19

Juliet is going with her family to their summer home. She will be away from her boyfriend for 9 weeks. Convert the time to minutes.

Solution
To convert weeks into minutes we will convert weeks into days, days into hours, and then hours into minutes. To do this we will multiply by conversion factors of 1.

Step 1: **Write 1 as** $\frac{7 \text{ days}}{1 \text{ week}}$**,** $\frac{24 \text{ hours}}{1 \text{ day}}$**, and** $\frac{60 \text{ minutes}}{1 \text{ hour}}$**.**

$$\frac{9wk}{1} \times \frac{7days}{1wk} \times \frac{24hr}{1day} \times \frac{60min}{1hr}$$

Step 2: **Divide out the common units.**

$$\frac{9\,\cancel{wk}}{1} \cdot \frac{7}{\cancel{days}} 1\,\cancel{wk} \cdot \frac{24\,\cancel{hr}}{1\,\cancel{day}} \cdot \frac{60\,min}{1} \cancel{hr}$$

Step 3: **Multiply.**

$$\frac{9 \times 7 \times 24 \times 60 \text{ min}}{1 \times 1 \times 1 \times 1} = 90,720 \text{ minutes}$$

Step 4: **Multiply.**

Juliet and her boyfriend will be apart for 90,720 minutes (although it may seem like an eternity!).

Try It

22) The distance between the earth and the moon is about 250,000 miles. Convert this length to yards.

Solution
440,000,000 yards

23) The astronauts of Expedition 28 on the International Space Station spend 15 weeks in space. Convert the time to minutes.

Solution
151,200 minutes

Example 20

How many ounces are in 1 gallon?

Solution
We will convert gallons to ounces by multiplying by several conversion factors. Refer to Table 2.1.5.

Step 1: **Multiply the measurement to be converted by 1.**

$$\frac{1\text{ gallon}}{1} \times \frac{4\text{ quarts}}{1\text{ gallon}} \times \frac{2\text{ pints}}{1\text{ quart}} \times \frac{2\text{ cups}}{1\text{ pint}} \times \frac{8\text{ ounces}}{1\text{ cup}}$$

Step 2: **Use conversion factors to get to the right unit.**

Simplify.

$$\frac{1\,\cancel{\text{gallon}}}{1} \times \frac{4\,\cancel{\text{quarts}}}{1\,\cancel{\text{gallon}}} \times \frac{2\,\cancel{\text{pints}}}{1\,\cancel{\text{quarts}}} \times \frac{2\,\cancel{\text{cups}}}{1\,\cancel{\text{pint}}} \times \frac{8\text{ ounces}}{1\,\cancel{\text{cup}}}$$

Step 3: **Multiply.**

$$\frac{1 \times 4 \times 2 \times 2 \times 8\text{ ounces}}{1 \times 1 \times 1 \times 1 \times 1}$$

Step 4: **Simplify.**

128 ounces

There are 128 ounces in a gallon.

Try It

24) How many cups are in 1 gallon?

Solution

16 cups

25) How many teaspoons are in 1 cup?

Solution

48 teaspoons

Use Mixed Units of Measurement in the U.S. System

We often use mixed units of measurement in everyday situations. Suppose Joe is 5 feet 10 inches tall, stays at work for 7 hours and 45 minutes, and then eats a 1-pound 2-ounce steak for dinner—all these measurements have mixed units.

Performing arithmetic operations on measurements with mixed units of measures requires care. Be sure to add or subtract like units!

Example 21

Seymour bought three steaks for a barbecue. Their weights were 14 ounces, 1 pound 2 ounces, and 1 pound 6 ounces. How many total pounds of steak did he buy?

Solution

We will add the weights of the steaks to find the total weight of the steaks.

Step 1: **Add the ounces. Then add the pounds.**

$$\begin{aligned} &+ 14 \text{ ounces} \\ 1 \text{ pound} &+ 2 \text{ ounces} \\ 1 \text{ pound} &+ 6 \text{ ounces} \\ = 2 \text{ pounds} &+ 22 \text{ ounces} \end{aligned}$$

Step 2: **Convert 22 ounces to pounds and ounces.**

2 pounds 1 pound, 6 ounces.

Step 3: **Add the pounds.**

3 pounds, 6 ounces.

Seymour bought 3 pounds 6 ounces of steak.

Try It

26) Laura gave birth to triplets weighing 3 pounds 3 ounces, 3 pounds 3 ounces, and 2 pounds 9 ounces. What was the total birth weight of the three babies?

Solution

9 lbs. 15 oz

27) Stan cut two pieces of crown moulding for his family room that were 8 feet 7 inches and 12 feet 11 inches. What was the total length of the moulding?

Solution

21 ft. 6 in.

Example 22

Anthony bought four planks of wood that were each 6 feet 4 inches long. What is the total length of the wood he purchased?

Solution

We will multiply the length of one plank to find the total length.

Step 1: **Multiply the inches and then the feet.**

$$\begin{array}{rr} & 6\ feet\ \ 4\ inches \\ \times & 4 \\ \hline & 24\ feet\ \ 16\ inches \end{array}$$

Step 2: **Convert the 16 inches to feet.**

$$\begin{aligned} \text{16 inches} - \text{12 inches} &= \text{1 foot} + \text{4 inches} \\ \text{1 foot} + \text{4 inches} + \text{24 feet} &= \text{25 feet} + \text{4 inches} \end{aligned}$$

Step 3: **Add the feet.**

Anthony bought 25 feet and 4 inches of wood.

Try It

28) Henri wants to triple his spaghetti sauce recipe that uses 1 pound 8 ounces of ground turkey. How many pounds of ground turkey will he need?

Solution

4 lbs. 8 oz.

29) Joellen wants to double a solution of 5 gallons 3 quarts. How many gallons of solution will she have in all?

Solution

11 gallons 2 qt.

Perform unit conversions (from any system) using dimensional analysis.

Many measurements in the United States are made in metric units. The soda may come in 2-liter bottles, calcium may come in 500-mg capsules, and people may run a 5K race. To work easily in both systems, we need to be able to convert between the two systems.

The table below shows some of the most common conversions.

Conversion Factors Between U.S. and Metric Systems

Table 2.1.6

Length	Mass	Capacity
1 in. = 2.54 cm	1 lb. = 0.45 kg	1 qt. = 0.95 L
1 ft. = 0.305 m	1 oz. = 28 g	1 fl. oz. = 30 mL
1 yd. = 0.914 m	1 kg = 2.2 lb.	1 L = 1.06 qt.
1 mi. = 1.61 km		
1 m = 3.28 ft.		

Figure 2.1.2 shows how inches and centimeters are related on a ruler.

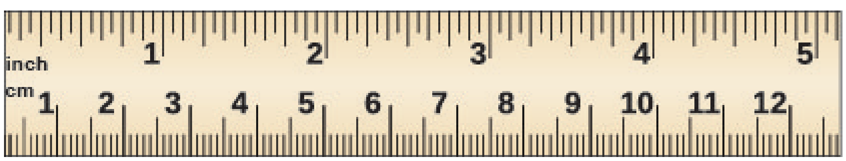


Figure 2.1.5: This ruler shows inches and centimeters.

Figure 2.1.3 shows the ounce and milliliter markings on a measuring cup.

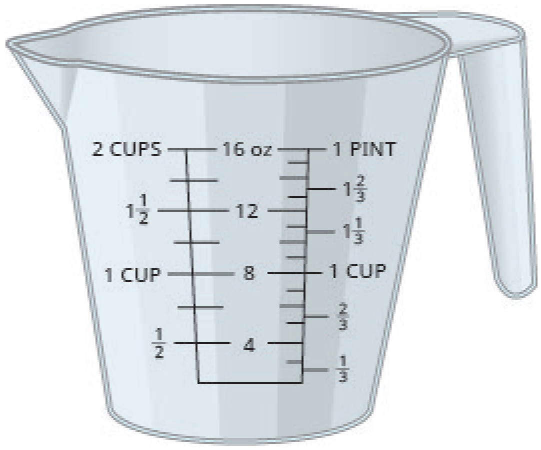


Figure 2.1.6: This measuring cup shows ounces and milliliters.

Figure 2.1.4 shows how pounds and kilograms marked on a bathroom scale.

Figure 2.1.7: This scale shows pounds and kilograms.

We make conversions between the systems just as we do within the systems—by multiplying by unit conversion factors.

Example 23

The plastic bag used for transfusion holds 500 mL of packed red cells. How many ounces are in the bag? Round to the nearest tenth of an ounce.

Solution

Step 1: **Multiply by a unit conversion factor relating mL and ounces.**

$$500 \text{ millilitres} \cdot \frac{1 \text{ ounce}}{30 \text{ millilitres}}$$

Step 2: **Simplify.**

$$\frac{500 \text{ ounces}}{30}$$

Step 3: **Divide.**

$$16.7 \text{ ounces}$$

The plastic bag has 16.7 ounces of packed red cells.

Try It

30) Adam donated 450 ml of blood. How many ounces is that?

Solution

15 ounces.

31) How many quarts of soda are in a 2-L bottle?

Solution

2.12 quarts

32)How many liters are in 4 quarts of milk?

Solution

3.8 liters

Example 24

Soleil was on a road trip and saw a sign that said the next rest stop was in 100 kilometers. How many miles until the next rest stop?

Solution

Step 1: **Multiply by a unit conversion factor relating km and mi.**

$$100 \text{ kilometers} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometer}}$$

Step 2: **Simplify.**

$$\frac{100 \text{ miles}}{1.61}$$

Step 3: **Divide.**

$$62 \text{ miles}$$

Soleil will travel 62 miles.

Example 25

A human brain weights about 3 pounds. How many kilograms is that? Round to the nearest tenth of a kilogram.

Solution

Step 1: **Multiply by a unit conversion factor relating km and mi.**

$$3 \text{ pounds} \times \frac{1 \text{ kilogram}}{2.2 \text{ pounds}}$$

Step 2: **Simplify.**

$$\frac{3 \text{ kilograms}}{2.2}$$

Step 3: **Divide.**

$$1.4 \text{ kilograms}$$

A human brain weights around 1.4 kilograms.

Try It

33) A human liver normally weights approximately 1.5 kilograms. Convert it to pounds.

Solution

3.3 pounds

34) The height of Mount Kilimanjaro is 5,895 meters. Convert the height to feet.

Solution

19,327.9 feet

35) The flight distance from New York City to London is 5,586 kilometers. Convert the distance to miles.

Solution

8,993.46 km

Convert between Fahrenheit and Celsius Temperatures

Have you ever been in a foreign country and heard the weather forecast? If the forecast is for 79°F what does that mean?

The U.S. and metric systems use different scales to measure temperature. The U.S. system uses degrees Fahrenheit, written °F The metric system uses degrees Celsius, written °C. Figure 2.1.5 shows the relationship between the two systems. The diagram shows normal body temperature, along with the freezing and boiling temperatures of water in degrees Fahrenheit and degrees Celsius.

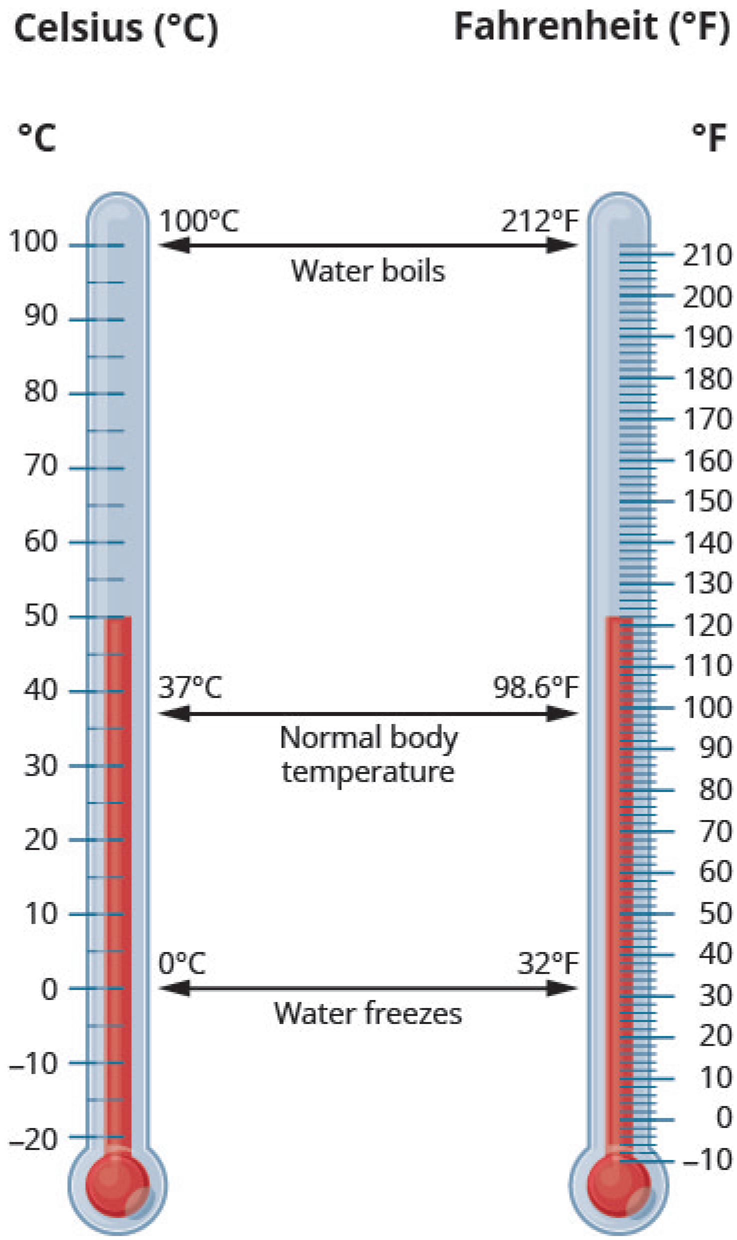


Figure 2.1.8

Temperature Conversion

To convert from Fahrenheit temperature, F, to Celsius temperature, C, use the formula:

$C = \frac{5}{9}(F - 32)$

To convert from Celsius temperature, C, to Fahrenheit temperature, F, use the formula:

$F = \frac{9}{5}C + 32$

Example 26

Convert 50° Fahrenheit into degrees Celsius.

Solution

We will substitute 50°F into the formula to find C.

Step 1: **Substitute 50 for F.**

$$C = \frac{5}{9}(50 - 32)$$

Step 2: **Simplify in parentheses.**

$$C = \frac{5}{9}(18)$$

Step 3: **Multiply.**

$$C = 10$$

So we found that 50°F is equivalent to 10°C.

Example 27

Before mixing, the Pfizer-BioNTech COVID-19 vaccine may be stored in an ultra-cold freezer between -112°F and -76°F. Convert the temperatures into degrees Celsius.

Solution

We will substitute a) -112°F and b) -76°F into the formula to find C.

a.

Step 1: **Substitute -112 for F.**

$$C = \frac{5}{9}(-112 - 32)$$

Step 2: **Simplify in parentheses.**

$$C = \frac{5}{9}(-144)$$

Step 3: **Multiply.**

$$C = -80$$

So we found that -112°F is equivalent to -80°C

---

b.

Step 1: **Substitute -76 for F.**

$$C = \frac{5}{9}(-76 - 32)$$

Step 2: **Simplify.**

$$C = \frac{5}{9}(-108)$$
$$C = -60$$

So we found that -76°F is equivalent to – 60°C.

Try It

36) Convert the Fahrenheit temperature to degrees Celsius: 59° Fahrenheit.

Solution

15°C

37) Convert the Fahrenheit temperature to degrees Celsius: 41° Fahrenheit.

Solution

5°C

Example 28

While visiting Paris, Woody saw the temperature was 20° Celsius. Convert the temperature into degrees Fahrenheit.

Solution

We will substitute 20°C into the formula to find F.

Step 1: **Substitute 20 for C.**

$$F = \frac{9}{5}(20)\,32$$

Step 2: **Multiply.**

$$F = 3632$$

Step 3: **Add.**

$$F = 68$$

So we found that 20°C is equivalent to 68°F.

Example 29

Once mixed, the Pfizer-BioNTech COVID-19 vaccine can be left at room temperature 2°C to 25°C. Convert the temperatures into degrees Fahrenheit.

Solution

We will substitute a) 2°C and b) 25°C into the formula to find F.

a.

Step 1: **Substitute 2 for C.**

$$F = \frac{9}{5} \times 232$$

Step 2: **Simplify.**

$$F = 35.6$$

So we found that 2°C is equivalent to 35.6°F.

---

b.

Step 1: **Substitute 25 for C.**

$$F = \frac{9}{5} \times 2532$$

Step 2: **Simplify.**

$$F = 77$$

So we found that 25°C is equivalent to 77°F.

Try It

38) Convert the Celsius temperature to degrees Fahrenheit: the temperature in Helsinki, Finland, was 15° Celsius.

Solution

59°F

39) Convert the Celsius temperature to degrees Fahrenheit: the temperature in Sydney, Australia, was 10° Celsius.

Solution

50°F

Key Concepts

- **Metric System of Measurement**

Metric System of Measurement

Length	Mass	Capacity
1 kilometer (km) = 1,000 m	1 kilogram (kg) = 1,000 g	1 kiloliter (kL) = 1,000 L
1 hectometer (hm) = 100 m	1 hectogram (hg) = 100 g	1 hectoliter (hL) = 100 L
1 dekameter (dam) = 10 m	1 dekagram (dag) = 10 g	1 dekaliter (daL) = 10 L
1 meter (m) = 1 m	1 gram (g) = 1 g	1 liter (L) = 1 L
1 decimeter (dm) = 0.1 m	1 decigram (dg) = 0.1 g	1 deciliter (dL) = 0.1 L
1 centimeter (cm) = 0.01 m	1 centigram (cg) = 0.01 g	1 centiliter (cL) = 0.01 L
1 millimeter (mm) = 0.001 m	1 milligram (mg) = 0.001 g	1 milliliter (mL) = 0.001 L
1 meter = 100 centimeters	1 gram = 100 centigrams	1 liter = 100 centiliters
1 meter = 1,000 millimeters	1 gram = 1,000 milligrams	1 liter = 1,000 milliliters

- **Temperature Conversion**
  - To convert from Fahrenheit temperature, F, to Celsius temperature, C, use the formula $C = \frac{5}{9}(F - 32)$
  - To convert from Celsius temperature, C, to Fahrenheit temperature, F, use the formula $F = \frac{9}{5}C + 32$

Exercises: Make Unit Conversions in the Metric System

Instructions: For questions 1–14, convert the units.

1) Ghalib ran 5 kilometres. Convert the length to meters.

2) Kitaka hiked 8 kilometres. Convert the length to meters.

3) Estrella is 1.55 meters tall. Convert her height to centimetres.

4) The width of the wading pool is 2.45 meters. Convert the width to centimetres.

5) Mount Whitney is $3{,}072$ meters tall. Convert the height to kilometres.

6) The depth of the Mariana Trench is $10{,}911$ meters. Convert the depth to kilometres.

7) June's multivitamin contains $1{,}500$ milligrams of calcium. Convert this to grams.

8) A typical ruby-throated hummingbird weights 3 grams. Convert this to milligrams.

9) One stick of butter contains 91.6 grams of fat. Convert this to milligrams.

10) One serving of gourmet ice cream has 25 grams of fat. Convert this to milligrams.

11) The maximum mass of an airmail letter is 2 kilograms. Convert this to grams.

12) Dimitri's daughter weighed 3.8 kilograms at birth. Convert this to grams.

13) A bottle of wine contained 750 millilitres. Convert this to litres.

14) A bottle of medicine contained 300 millilitres. Convert this to litres.

Odd Answers

1) $5{,}000$ **meters**

3) 155 **centimetres**

5) 3.072 **kilometres**

7) 1.5 **grams**

9) $91{,}600$ **milligrams**

11) $2{,}000$ **grams**

13) 0.75 **litres**

Exercises: Use Mixed Units of Measurement in the Metric System

Instructions: For questions 15–24, solve.

15) Matthias is 1.8 metres tall. His son is 89 centimetres tall. How much taller is Matthias than his son?

16) Stavros is 1.6 metres tall. His sister is 95 centimetres tall. How much taller is Stavros than his sister?

17) A typical dove weighs 345 grams. A typical duck weighs 1.2 kilograms. What is the difference, in grams, of the weights of a duck and a dove?

18) Concetta had a 2-kilogram bag of flour. She used 180 grams of flour to make biscotti. How many kilograms of flour are left in the bag?

19) Harry mailed 5 packages that weighed 420 grams each. What was the total weight of the packages in kilograms?

20) One glass of orange juice provides 560 milligrams of potassium. Linda drinks one glass of orange juice every morning. How many grams of potassium does Linda get from her orange juice in 30 days?

21) Jonas drinks 200 millilitres of water 8 times a day. How many litres of water does Jonas drink in a day?

22) Complete:

- **a)** $38\ \text{cm}^3 = (?)\ \text{g}$
- **b)** $5\ \text{L} = (?)\ \text{cm}^3$
- **c)** $18\ \text{L}$ **of water has a volume of** $\_\_\ \text{cm}^3$.
- **d)** **A water tank measures** 45 **cm by** 35 **cm by** 25 **cm. How many kilolitres of water will it hold?**

23) One serving of whole grain sandwich bread provides 6 grams of protein. How many milligrams of protein are provided by 7 servings of whole grain sandwich bread?

Combine:

- **a)** $7\ \text{m} - 3000\ \text{mm} = (?)\ \text{mm}$
- **b)** $63\ \text{kg} + 6\ \text{g} = (?)\ \text{g}$

Odd Answers

15) 91 **centimetres**

17) 855 **grams**

19) 2.1 **kilograms**

21) 1.6 **litres**

23a) $4{,}000$ **mm**

23b) $63{,}006$ **g**

Exercises: Make Unit Conversions in the U.S. System

Instructions: For questions 25–50, convert the units.

25) A park bench is 6 **feet long. Convert the length to inches.**

26) A floor tile is 2 **feet wide. Convert the width to inches.**

27) A ribbon is 18 **inches long. Convert the length to feet.**

28) Carson is 45 **inches tall. Convert his height to feet.**

29) A football field is 160 **feet wide. Convert the width to yards.**

30) On a baseball diamond, the distance from home plate to first base is 30 **yards. Convert the distance to feet.**

31) Ulises lives 1.5 **miles from school. Convert the distance to feet.**

32) Denver, Colorado, is $5,183$ feet above sea level. Convert the height to miles.

33) A killer whale weighs 4.6 tons. Convert the weight to pounds.

34) Blue whales can weigh as much as 150 tons. Convert the weight to pounds.

35) An empty bus weighs $35,000$ pounds. Convert the weight to tons.

36) At take-off, an airplane weighs $220,000$ pounds. Convert the weight to tons.

37) Rocco waited $1\frac{1}{2}$ hours for his appointment. Convert the time to seconds.

38) Misty's surgery lasted $2\frac{1}{4}$ hours. Convert the time to seconds.

39) How many teaspoons are in a pint?

40) How many tablespoons are in a gallon?

41) JJ's cat, Posy, weighs 14 pounds. Convert her weight to ounces.

42) April's dog, Beans, weighs 8 pounds. Convert his weight to ounces.

43) Crista will serve 20 cups of juice at her son's party. Convert the volume to gallons.

44) Lance needs 50 cups of water for the runners in a race. Convert the volume to gallons.

45) Jon is 6 feet 4 inches tall. Convert his height to inches.

46) Faye is 4 feet 10 inches tall. Convert her height to inches.

47) The voyage of the *Mayflower* took 2 months and 5 days. Convert the time to days.

48) Lynn's cruise lasted 6 days and 18 hours. Convert the time to hours.

49) Baby Preston weighed 7 pounds 3 ounces at birth. Convert his weight to ounces.

50) Baby Audrey weighted 6 pounds 15 ounces at birth. Convert her weight to ounces.

Odd Answers

25) 72 inches

27) 1.5 **feet**

29) $53\frac{1}{3}$ **yards**

31) $7{,}920$ **feet**

33) $9{,}200$ **pounds**

35) $17\frac{1}{2}$ **tons**

37) $5{,}400$ **s**

39) 96 **teaspoons**

41) 224 **ounces**

43) $1\frac{1}{4}$ **gallons**

45) 76 **in.**

47) 65 **days**

49) 115 **ounces**

Exercises: Use Mixed Units of Measurement in the U.S. System

Instructions: For questions 51–58, solve.

51) Eli caught three fish. The weights of the fish were 2 pounds 4 ounces, 1 pound 11 ounces, and 4 pounds 14 ounces. What was the total weight of the three fish?

52) Judy bought 1 pound 6 ounces of almonds, 2 pounds 3 ounces of walnuts, and 8 ounces of cashews. How many pounds of nuts did Judy buy?

53) One day Anya kept track of the number of minutes she spent driving. She recorded $45, 10, 8, 65, 20,$ and 35. How many hours did Anya spend driving?

54) Last year Eric went on 6 business trips. The number of days of each was $5, 2, 8, 12, 6,$ and 3. How many weeks did Eric spend on business trips last year?

55) Renee attached a 6 feet 6 inch extension cord to her computer's 3 feet 8 inch power cord. What was the total length of the cords?

56) Fawzi's SUV is 6 feet 4 inches tall. If he puts a 2 feet 10 inch box on top of his SUV, what is the total height of the SUV and the box?

57) Leilani wants to make 8 placemats. For each placemat she needs 18 inches of fabric. How many yards of fabric will she need for the 8 placemats?

58) Mireille needs to cut 24 inches of ribbon for each of the 12 girls in her dance class. How many yards of ribbon will she need altogether?

Odd Answers

51) 8 lbs. 13 oz.

53) 3.05 hours

55) 10 ft. 2 in.

57) 4 yards

Exercises: Convert Between the U.S. and the Metric Systems of Measurement

Instructions: For questions 59–70, make the unit conversions. Round to the nearest tenth.

59) Bill is 75 inches tall. Convert his height to centimetres.

60) Frankie is 42 inches tall. Convert his height to centimetres.

61) Marcus passed a football 24 yards. Convert the pass length to metres

62) Connie bought 9 yards of fabric to make drapes. Convert the fabric length to metres.

63) Each American throws out an average of $1,650$ pounds of garbage per year. Convert this weight to kilograms.

64) An average American will throw away $90,000$ pounds of trash over his or her lifetime. Convert this weight to kilograms.

65) A 5K run is 5 kilometres long. Convert this length to miles.

66) Kathryn is 1.6 metres tall. Convert her height to feet.

67) Dawn's suitcase weighed 20 kilograms. Convert the weight to pounds.

68) Jackson's backpack weighed 15 kilograms. Convert the weight to pounds.

69) Ozzie put 14 gallons of gas in his truck. Convert the volume to litres.

70) Bernard bought 8 gallons of paint. Convert the volume to litres.

Odd Answers

59) 190.5 **centimetres**

61) 21.9 **meters**

63) 742.5 **kilograms**

65) 3.1 **miles**

67) 44 **pounds**

69) 30.4 **litres**

Exercises: Convert between Fahrenheit and Celsius Temperatures

Instructions: For questions 71–78, convert the Fahrenheit temperatures to degrees Celsius. Round to the nearest tenth.

71) 86° **Fahrenheit**

72) 77° **Fahrenheit**

73) 104° **Fahrenheit**

74) 14° **Fahrenheit**

75) 72° **Fahrenheit**

76) 4° **Fahrenheit**

77) 0° **Fahrenheit**

78) 120° **Fahrenheit**

Odd Answers

71) $30^{\circ}\mathrm{C}$

73) $40^{\circ}\mathrm{C}$

75) $22.2^{\circ}\mathrm{C}$

77) $-17.8^{\circ}\mathrm{C}$

Exercises: Convert between Fahrenheit and Celsius Temperatures

Instructions: For questions 79–86, convert the Celsius temperatures to degrees Fahrenheit. Round to the nearest tenth.

79) 5° **Celsius**

80) 25° **Celsius**

81) -10° **Celsius**

82) -15° **Celsius**

83) 22° **Celsius**

84) 8° **Celsius**

85) 43° **Celsius**

86) 16° **Celsius**

Odd Answers

79) 41°F

81) 14°F

83) 71.6°F

85) 109.4°F

Exercises: Everyday Math

Instructions: For questions 87–88, answer the given everyday math word problem.

87) Nutrition: Julian drinks one can of soda every day. Each can of soda contains 40 grams of sugar. How many kilograms of sugar does Julian get from soda in 1 year?

88) Reflectors: The reflectors in each lane-marking stripe on a highway are spaced 16 yards apart. How many reflectors are needed for a one mile long lane-marking stripe?

Odd Answers

87) 14.6 **kilograms**

Exercises: Writing Exercises

Instructions: For questions 89–90, answer the given writing exercises.

89) **Some people think that 65° to 75° Fahrenheit is the ideal temperature range.**

- **a) What is your ideal temperature range? Why do you think so?**
- **b) Convert your ideal temperatures from Fahrenheit to Celsius.**

90) **Read the prompts and answer accordingly.**

- **a) Did you grow up using the U.S. or the metric system of measurement?**
- **b) Describe two examples in your life when you had to convert between the two systems of measurement.**

Odd Answers

89) **Answers may vary.**

2.2 ACCURACY, PRECISION, AND ROUNDING RULES

Learning Objectives

By the end of this section, you will be able to:

- Identify exact and inexact numbers.
- Significant Figures

Identify Exact and Inexact Numbers

When considering measurement, we must consider the nature of the numbers that we are using in our calculations.

A number is exact if it is known with complete certainty. An exact number is a number that you can get by counting. Exact numbers have infinitely many significant figures and decimal places. For example, there are exactly 100 centimeters in one meter and exactly 4 quarts in a gallon.

A number is inexact if it has **uncertainty** associated with it. This uncertainty can arise due to measurement or rounding. Some examples would be a person's height or weight.

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the centre of a bull's-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In Figure 2.2.1 you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the centre of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 2.2.2 the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.

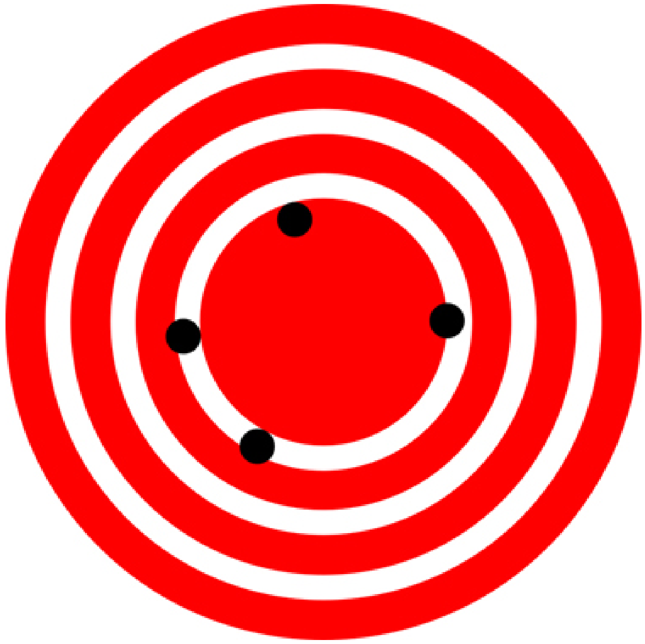


Figure 2.2.1**.** A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil).

Figure 2.2.2. In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil).

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A, is often denoted as δA ("delta A"), so the measurement result would be recorded as $A \pm \delta A$. In our paper example, the length of the paper could be expressed as $11 in. \pm 0.2$.

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object being measured,
4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Making Connections: Real World Connections – Fever or Chills?

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check his or her temperature with a thermometer. What if the uncertainty of the thermometer were 3.0 °C? If the child's temperature reading was 37.0 °C (which is normal body temperature), the "true" temperature could be anywhere from a hypothermic 34.0 °C to a dangerously high 40.0 °C. A thermometer with an uncertainty of 3.0 °C would be useless.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with $uncertainty$, δA, the **percent uncertainty** (%unc) is defined to be:

$$\% \text{ unc} = \frac{\delta A}{A} \times 100\%$$

Example 1

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: **4.8 lb**
- Week 2 weight: **5.3 lb**
- Week 3 weight: **4.9 lb**
- Week 4 weight: **5.4 lb**

You determine that the weight of the 5-lb bag has an uncertainty of ±0.4 lb. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

$$\% \text{ unc} = \frac{\delta \text{A}}{\text{A}} \times 100\%$$

Solution

Plug the known values into the equation:

$$\begin{aligned} \% \text{ unc} &= \frac{0.4 \text{ lb}}{5 \text{ lb}} \times 100\% \\ \% \text{ unc} &= 8\% \end{aligned}$$

Discussion

We can conclude that the weight of the apple bag is 5 lb ±8%. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainties in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method says that *the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation*. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2% and 1%, respectively, then the area of the floor is 12.0 m and has an uncertainty of 3%. (Expressed as an area this is 0.36 m^2, which we round to 0.4 m^2 since the area of the floor is given to a tenth of a square meter.)

Try It

1) A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of ±0.05 s. Runners on the track coach's team regularly clock 100-m sprints of 11.49 s to 15.01 s. At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

Solution

No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

Significant Figures

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest

0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7 cm. You could not express this value as 36.71 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6 cm and 36.7 cm, and he or she must estimate the value of the last digit. Using the method of significant figures, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value 36.7 cm has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Significant Figures: Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers*.

Try It

2) Determine the number of significant figures in the following measurements:

a) 0.0009
b) 15,450.0
c) 6×10^3

d) 87.990
e) 30.42

Solution
a) 1
b) 6
c) 1
d) 5
e) 4

As you have probably realized by now, the biggest issue in determining the number of significant figures in a value is the zero. Is the zero significant or not? One way to unambiguously determine whether a zero is significant or not is to write a number in scientific notation. Scientific notation will include zeros in the coefficient of the number *only if they are significant*. Thus, the number 8.666×10^6 has four significant figures. However, the number 8.6660×10^6 has five significant figures. That last zero is significant; if it were not, it would not be written in the coefficient. So when in doubt about expressing the number of significant figures in a quantity, use scientific notation and include the number of zeros that are truly significant. We will learn more about scientific notation in a later section in the text.

Significant Figures

If you use a calculator to evaluate the expression $\frac{337}{217}$, you will get the following:

$$\frac{337}{217} = 1.55299539171...$$

and so on for many more digits. Although this answer is correct, it is somewhat presumptuous. You start with two values that each have three digits, and the answer has *twelve* digits? That does not make much sense from a strict numerical point of view.

Consider using a ruler to measure the width of an object, as shown in Figure 2.2.3 “Expressing Width”. The object is definitely more than 1 cm long, so we know that the first digit in our measurement is 1. We see by counting the tick marks on the ruler that the object is at least three ticks after the 1. If each tick represents 0.1 cm, then we know the object is at least 1.3 cm wide. But our ruler does not have any more ticks between the 0.3 and the 0.4 marks, so we can’t know exactly how much the next decimal place is. But with a practised eye we can estimate it. Let us estimate it as about six-tenths of the way between the third and fourth tick marks, which estimates our hundredths place as 6, so we identify a measurement of 1.36 cm for the width of the object.

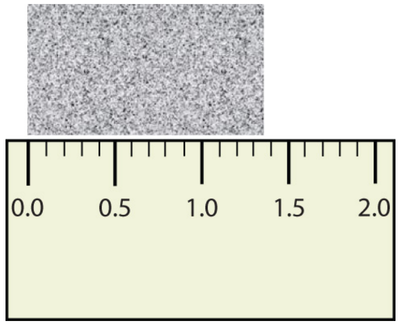


Figure 2.2.3. Expressing Width

What is the proper way to express the width of this object?

Does it make any sense to try to report a thousandths place for the measurement? No, it doesn't; we are not exactly sure of the hundredths place (after all, it was an estimate only), so it would be fruitless to estimate a thousandths place. Our best measurement, then, stops at the hundredths place, and we report 1.36 cm as proper measurement.

This concept of reporting the proper number of digits in a measurement or a calculation is called **significant figures**. Significant figures (sometimes called significant digits) represent the limits of what values of a measurement or a calculation we are sure of. The convention for a measurement is that the quantity reported should be all known values and the first estimated value. The conventions for calculations are discussed as follows.

Example 2

Use each diagram to report a measurement to the proper number of significant figures.

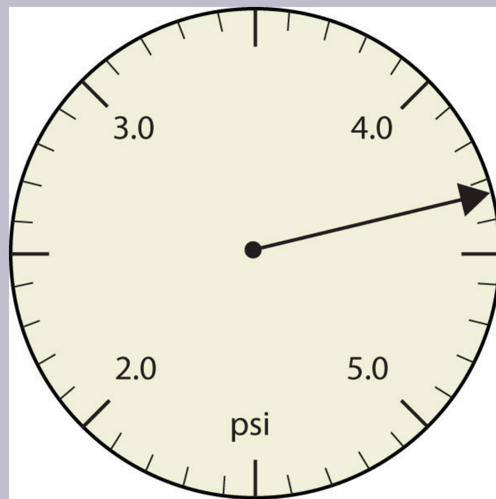
a. 

Figure 2.2.4. Pressure gauge in units of pounds per square inch

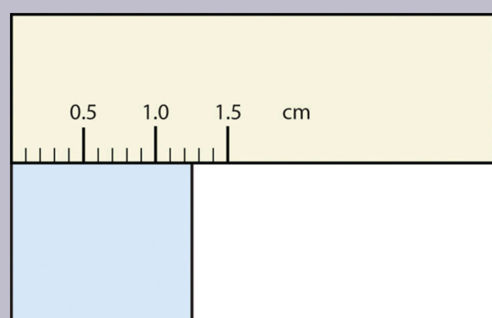
b. 

Figure 2.2.5. A measuring ruler

Solution

a. The arrow is between 4.0 and 5.0, so the measurement is at least 4.0. The arrow is between the third and fourth small tick marks, so it's at least 0.3. We will have to estimate the last place. It looks like about one-third of the way across the space, so let us estimate the hundredths place as 3. Combining the digits, we have a measurement of 4.33 psi (psi stands for "pounds per square inch" and is a unit of pressure, like air in a tire). We say that the measurement is reported to three significant figures.
b. The rectangle is at least 1.0 cm wide but certainly not 2.0 cm wide, so the first significant digit is 1. The rectangle's width is past the second tick mark but not the third; if each tick mark represents 0.1, then the rectangle is at least 0.2 in the next significant digit. We have to

estimate the next place because there are no markings to guide us. It appears to be about halfway between 0.2 and 0.3, so we will estimate the next place to be a 5. Thus, the measured width of the rectangle is 1.25 cm. Again, the measurement is reported to three significant figures

Try It

3) What would be the reported width of this rectangle?

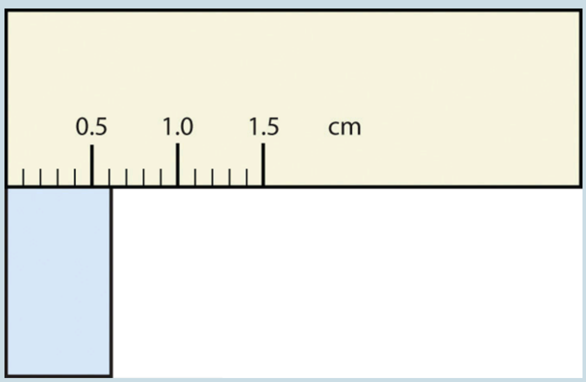


Figure 2.2.6. A measuring ruler

Solution

0.61cm

In many cases, you will be given a measurement. How can you tell by looking at what digits are significant? For example, the reported population of the United States is 306,000,000. Does that mean that it is *exactly* three hundred six million or is some estimation occurring?

The following conventions dictate which numbers in a reported measurement are significant and which are not significant:

1. Any nonzero digit is significant.
2. Any zeros between nonzero digits (i.e., embedded zeros) are significant.
3. Zeros at the end of a number:
    - without a decimal point (i.e., trailing zeros) are not significant; they serve only to put the significant digits in the correct positions.
    - with a decimal point are significant.
4. Zeros at the beginning of a decimal number (i.e., leading zeros) are not significant; again, they serve only to put the significant digits in the correct positions.

So, by these rules, the population figure of the United States has only three significant figures: the 3, the 6, and the zero between them. The remaining six zeros simply put the 306 in the millions position.

Example 3

Give the number of significant figures in each measurement.

a. 36.7 m
b. 0.006606 s
c. 2,002 kg
d. 306,490,000 people

Solution

a. By rule 1, all nonzero digits are significant, so this measurement has three significant figures.
b. By rule 4, the first three zeros are not significant, but by rule 2 the zero between the sixes is; therefore, this number has four significant figures.

c. By rule 2, the two zeros between the twos are significant, so this measurement has four significant figures.
d. The four trailing zeros in the number are not significant, but the other five numbers are, so this number has five significant figures.

Try It

Give the number of significant figures in each measurement.

4) 0.000601 m

Solution

3 significant figures

5) 65.080 kg

Solution

5 significant figures

Significant Figures in Calculations

How are significant figures handled in calculations? It depends on what type of calculation is being performed. If the calculation is an addition or a subtraction, the rule is as follows: limit the reported answer to the rightmost column that all numbers have significant figures in common. For example, if you were to add 1.2 and 4.71, we note that the first number stops its significant figures in the tenths column, while the second number stops its significant figures in the hundredths column. We therefore limit our answer to the tenths column.

1.2
4.41
5.61
↑ limit final answer to the tenths column: 5.6

Figure 2.2.7.

We drop the last digit—the 1—because it is not significant to the final answer.

The dropping of positions in sums and differences brings up the topic of rounding. Although there are several conventions, in this text we will adopt the following rule: the final answer should be rounded up if the first dropped digit is 5 or greater and rounded down if the first dropped digit is less than 5.

77.2
10.46
87.66
↑ limit final answer to the tenths column and round up: 87.7

Figure 2.2.8.

Example 4

Express the final answer to the proper number of significant figures.

a. $101.2 + 18.702 = ?$
b. $202.88 - 1.013 = ?$

Solution

a. If we use a calculator to add these two numbers, we would get 119.902. However, most calculators do not understand significant figures, and we need to limit the final answer to the tenths place. Thus, we drop the 02 and report a final answer of 119.9 (rounding down).
b. A calculator would answer 201.867. However, we have to limit our final answer to the hundredths place. Because the first number being dropped is 7, which is greater than 7, we

round up and report a final answer of 201.87.

Try It

6) Express the answer for $3.445 + 90.83 - 72.4$ to the proper number of significant figures.

Solution

$$21.9$$

If the operations being performed are multiplication or division, the rule is as follows: limit the answer to the number of significant figures that the data value with the *least* number of significant figures has. So if we are dividing 23 by 448, which have two and three significant figures each, we should limit the final reported answer to two significant figures (the lesser of two and three significant figures):

$$\frac{23}{448} = 0.051339286... = 0.051$$

The same rounding rules apply in multiplication and division as they do in addition and subtraction.

Example 5

Express the final answer to the proper number of significant figures.

a. $76.4 \times 180.4 = ?$
b. $934.9 \div 0.00455 = ?$

Solution

a. The first number has three significant figures, while the second number has four significant figures. Therefore, we limit our final answer to three significant figures:

$$76.4 \times 180.4 = 13,782.56 = 13,800$$

.

---

b. The first number has four significant figures, while the second number has three significant figures. Therefore we limit our final answer to three significant figures:

$$934.9 \div 0.00455 = 205,472.5275\ldots = 205,000$$

.

Try It

Express the final answer to the proper number of significant figures.

7) $22.4 \times 8.314 =$?

Solution

186

8) $1.381 \div 6.02 =$?

Solution

0.229

Summary

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

1. For multiplication and division: *The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation*. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area has if the radius has only two—say, $r = 1.2m$. Then,

$$A = \pi r^2 = 3.1415927 \times (1.2\ \mathrm{m})^2 = 4.5238934\ \mathrm{m}^2$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated quantity to two significant figures or

$$A = \pi r^2 = 3.1415927 \times (1.2\ \mathrm{m})^2 = 4.5\ \mathrm{m}^2$$

even though π is good to at least eight digits.

2. For addition and subtraction: *The answer can contain no more decimal places than the least precise measurement*. Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$\begin{array}{l} 7.56kg \\ -6.052kg \\ \underline{+\ 13.7kg} \\ 15.208kg = 15.2kg \end{array}$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

Rounding in Practical Situations

It is important to note that the rounding rules above are the theoretical rules used in chemical and physical laboratory papers and results. In various health care professions, the precision of the measurement tool being used will often dictate how you would round your medication dosages. In this sense, it is important to understand these theoretical rules and be aware that in practical situations, you may be taught best practices to coincide with the measurement tools available to you.

Key Concepts:

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- Significant figures in a quantity indicate the number of known values plus one place that is estimated.
- The following conventions dictate which numbers in a reported measurement are significant and which are not significant:

    1. Any nonzero digit is significant.
    2. Any zeros between nonzero digits (i.e., embedded zeros) are significant.
    3. Zeros at the end of a number without a decimal point (i.e., trailing zeros) are not significant; they serve only to put the significant digits in the correct positions. However, zeros at the end of any number with a decimal point are significant.
    4. Zeros at the beginning of a decimal number (i.e., leading zeros) are not significant; again, they serve only to put the significant digits in the correct positions.

- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.
- When performing mixed operations, round according to the rules for addition/subtraction and multiplication/division while using the order of operations.

Glossary

accuracy
: the degree to which a measure value agrees with the correct value for that measurement

method of adding percents
: the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

percent uncertainty
: the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

precision
: the degree to which repeated measurements agree with each other

significant figures
: express the precision of a measuring tool used to measure a value

uncertainty
: a quantitative measure of how much your measured values deviate from a standard or expected value

Exercises: Significant Figures and Proper Units

Instructions: For questions 1–18, express your answer to problems in this section to the correct number of significant figures and proper units.

1. Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

Solution

2 kg

---

2. A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

---

3. A car speedometer has a 5.0% uncertainty.

a. What is the range of possible speeds when it reads 90 km/h?

b. Convert this range to miles per hour.

Solution

$(1\text{ km} = 0.6214\text{ mi})$

a. 85.5 to 94.5 km/h

b. 53.1 to 58.7 mi/h

---

4. An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?

---

5. Suppose that a person has an average heart rate of 72.0 beats/min.

a. How many beats does he or she have in 2.0 y?

b. In 2.00 y?

c. In 2.000 y?

Solution

a. 7.6×10^7 beats

b. 7.57×10^7 beats

c. 7.57×10^7 beats

---

6. A can contains 375 mL of soda. How much is left after 308 mL is removed?

---

7. State how many significant figures are proper in the results of the following calculations:

a. $(106.7)(98.2)/(46.210)(1.01)$

b. $(18.7)^2$

c. $(1.60 \times 10^{-19})(3712)$

Solution

a. 3

b. 3

c. 3

8.

a. How many significant figures are in the numbers 99 and 100?

b. If the uncertainty in each number is 1, what is the percent uncertainty in each?

c. Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

9.

a. If your speedometer has an uncertainty of 2.0 km/h at a speed of 90 km/h, what is the percent uncertainty?

b. If it has the same percent uncertainty when it reads 60 km/h, what is the range of speeds you could be going?

Solution

a. 2.2%

b. 59 to 61 km/h

10.

a. A person's blood pressure is measured to be 120 ± 2 mm Hg. What is its percent uncertainty?

b. Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80 mm Hg?

---

11. A person measures his or her heart rate by counting the number of beats in 30 s. If 40 ± 1 beats are counted in 30.0 ± 0.5 s, what is the heart rate and its uncertainty in beats per minute?

Solution

80\
pm3\
text{ b
eats/
min}

---

12. What is the area of a circle 3.102 cm in diameter?

---

13. If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?

Solution

2.8 h

---

14. A marathon runner completes a 42.188-km course in 2 h, 30 min, and 12

s. There is an uncertainty of 25 m in the distance travelled and an uncertainty of 1 s in the elapsed time.

a. Calculate the percent uncertainty in the distance.
b. Calculate the uncertainty in the elapsed time.
c. What is the average speed in meters per second?
d. What is the uncertainty in the average speed?

---

15. The sides of a small rectangular box are measured to be 1.80 ± 0.01 cm, 2.05 ± 0.02 cm, and 3.1 ± 0.1 cm long. Calculate its volume and uncertainty in cubic centimetres.

Solution

$11.4 \pm 0.5 \text{ cm}^3$

---

16. When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where 1 lbm = 0.4539 kg.

a. If there is an uncertainty of 0.0001 kg in the pound-mass unit, what is its percent uncertainty?
b. Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

---

17. The length and width of a rectangular room are measured to be 3.955 ± 0.005 m and 3.050 ± 0.005 m. Calculate the area of the room and its uncertainty in square meters.

Solution

$12.06 \pm 0.04\ \text{m}^2$

---

18. A car engine moves a piston with a circular cross section of 7.500 ± 0.002 cm diameter a distance of 3.250 ± 0.001 cm to compress the gas in the cylinder.

a. By what amount is the gas decreased in volume in cubic centimetres?
b. Find the uncertainty in this volume.

Exercises: Significant Figures

Instructions: For questions 19–34, express each measurement to the correct number of significant figures.

19. Express each measurement to the correct number of significant figures.

a.

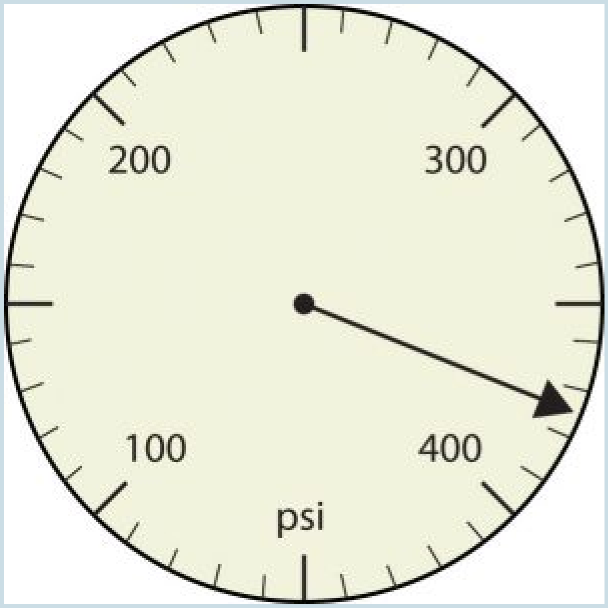


Figure 2P.2.1. Pressure gauge in units of pounds per square inch

b.

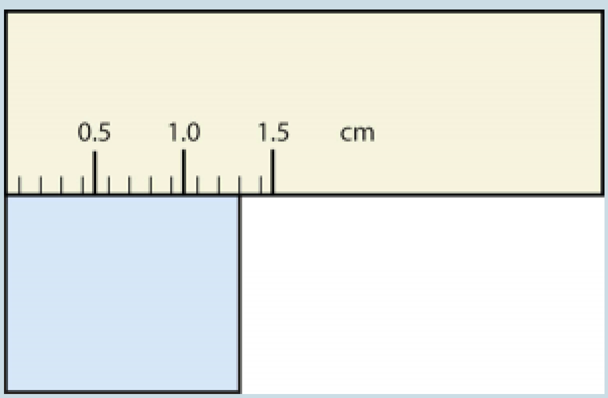


Figure 2P.2.2. A measuring ruler

Solution

a. 375 psi

b. 1.30 cm

---

20. Express each measurement to the correct number of significant figures.

a.

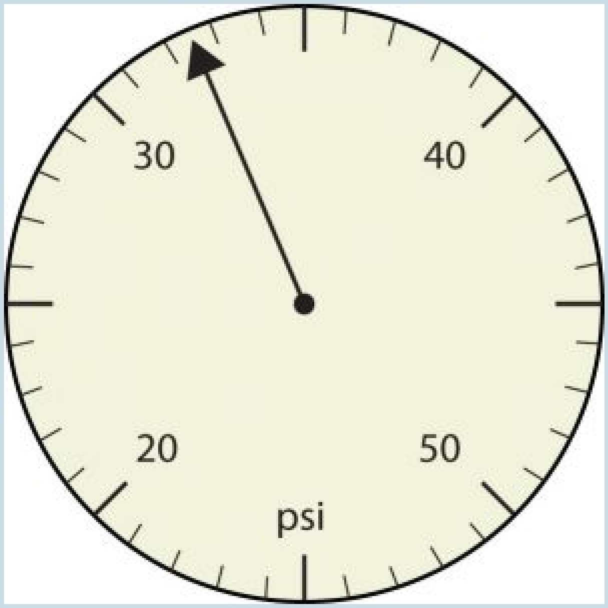


Figure 2P.2.3. Pressure gauge in units of pounds per square inch

b.

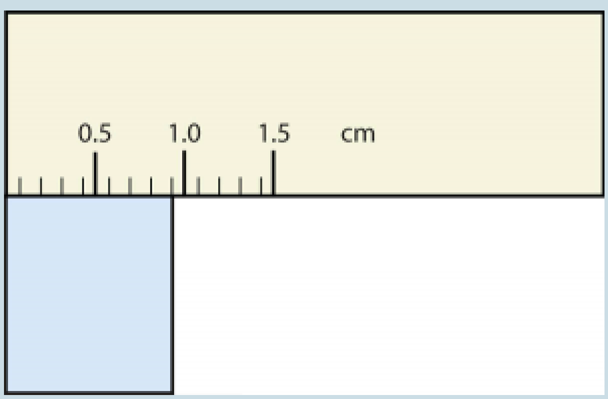


Figure 2P.2.4. A measuring ruler

---

21. How many significant figures do these numbers have?

a. 23

b. 23.0

c. 0.00023

d. 0.0002302

Solution

a. Two

b. Three

c. Two

d. Four

---

22. How many significant figures do these numbers have?

a. 5.44×10^{8}

b. 1.008×10^{-5}

c. 43.09

d. 0.0000001381

---

23. How many significant figures do these numbers have?

a. $765{,}890$

b. $765{,}890.0$

c. 1.2000×10^{5}

d. 0.0005060

Solution

a. Five

b. Seven

c. Five

d. Four

---

24. How many significant figures do these numbers have?

a. 0.009

b. 0.0000009

c. $65{,}444$

d. $65{,}040$

---

25. Compute and express each answer with the proper number of significant figures, rounding as necessary.

a. $56.0 + 3.44 = ?$

b. $0.00665 + 1.004 = ?$

c. $45.99 - 32.8 = ?$

d. $45.99 - 32.8 + 75.02 = ?$

Solution

a. 59.4

b. 1.011

c. 13.2

d. 88.2

---

26. Compute and express each answer with the proper number of significant figures, rounding as necessary.

a. $1.005 + 17.88 = ?$

b. $56,700 - 324 = ?$

c. $405,007 - 123.3 = ?$

d. $55.5 + 66.66 - 77.777 = ?$

---

27. Compute and express each answer with the proper number of significant figures, rounding as necessary.

a. $56.7 \times 66.99 = ?$

b. $1.000/77 = ?$

c. $1.000/77.0 = ?$

d. $6.022 \times 1.89 = ?$

Solution

a. 3.80×10^3
b. 0.013
c. 0.0130
d. 11.4

---

28. Compute and express each answer with the proper number of significant figures, rounding as necessary.

- **a.** $0.000440 \times 17.22 = ?$
- **b.** $203{,}000/0.044 = ?$
- **c.** $67 \times 85.0 \times 0.0028 = ?$
- **d.** $999{,}999/3{,}310 = ?$

---

29. Write the number $87{,}449$ in scientific notation with four significant figures.

Solution

8.745×10^4

---

30. Write the number $306{,}000{,}000$ in scientific notation to the proper number of significant figures.

---

31. Write the number 0.000066600 in scientific notation with five significant figures.

Solution

6.6600×10^{-5}

---

32. Write the number 0.0000558 in scientific notation with two significant figures.

---

33. Perform each calculation and limit each answer to three significant figures.

- **a.** $67{,}883 \times 0.004321 = ?$
- **b.** $(9.67 \times 10^3) \times 0.0055087 = ?$

Solution

a. 293

b. 53.3

---

34. Perform each calculation and limit each answer to four significant figures.

- **a.** $18{,}900 \times 76.33/0.00336 = ?$
- **b.** $0.77604/76{,}003 \times 8.888 = ?$

2.3 SCIENTIFIC NOTATION

Learning Objectives

By the end of this section, you will be able to:

- Review the Laws of Exponents
- Convert from decimal notation to scientific notation
- Convert scientific notation to decimal form
- Multiply and divide using scientific notation

Try It

Before you get started, take this readiness quiz:

1) What is the place value of the 6 in the number $64,891$?

2) Name the decimal: 0.0012.

3) Subtract: $5-(-3)$.

Review the Laws of Exponents

If a and b are real numbers, and m and n are integers, then:

[table id=17 /]

Scientific Notation

Sometimes, in real-life scenarios, we may need to deal with numbers that are very large, or numbers that are very small. Thus, when dealing with numbers like $1,280,000,000$ or 0.00000000274, it may be beneficial to represent these numbers in a different way. We will use scientific notation to help us with this.

Scientific Notation and Significant Figures

Before we work on converting between forms of numbers, let's consider the concept of significant figures and scientific notation. We can determine the number of significant figures in a number in scientific notation in the same way we would a number in decimal notation. Let's remind ourselves of the rules for significant figures that were covered in the previous section:

The following conventions dictate which numbers in a reported measurement are significant and which are not significant:

1. Any nonzero digit is significant.
2. Any zeros between nonzero digits (i.e., embedded zeros) are significant.
3. Zeros at the end of a number without a decimal point (i.e., trailing zeros) are not significant; they serve only to put the significant digits in the correct positions. However, zeros at the end of any number with a decimal point are significant.
4. Zeros at the beginning of a decimal number (i.e., leading zeros) are not significant; again, they serve only to put the significant digits in the correct positions.

Of course, in scientific notation, we no longer have to worry about leading zeros. This is one of the benefits of using scientific notation, as the digits present are always significant.

Example 1

Determine the number of significant figures in the following numbers:

a. 1.41×10^{3}

b. 1.034×10^{-5}

c. 4.0000×10^{-2}

Solution

a. All non-zero digits are significant so there are 3 significant figures.

b. All confined zeros are significant, as well as all non-zero digits, so there are 4 significant figures.

c. All zeros at the end of the number after a decimal point are significant, so there are 5 significant figures.

Try It

4) Determine the number of significant figures in the following numbers:

a. 1.9×10^{3}

b. 4.00×10^{-5}

c. 5.001×10^{-2}

Solution

a. 2 significant figures

b. 3 significant figures

c. 4 significant figures

Convert from Decimal Notation to Scientific Notation

Remember working with place value for whole numbers and decimals? Our number system is based on powers of 10. We use tens, hundreds, thousands, and so on. Our decimal numbers are also based on powers of tens—tenths, hundredths, thousandths, and so on. Consider the numbers 4,000 and 0.004. We know that 4,000 means 4×1000 and 0.004 means $4 \times \frac{1}{1,000}$

If we write the 1000 as a power of ten in exponential form, we can rewrite these numbers in this way:

4000	0.004
4×1000	$4 \times \frac{1}{1000}$
4×10^3	$4 \times \frac{1}{10^3} = 4 \times 10^{-3}$

When a number is written as a product of two numbers, where the first factor is a number greater than or equal to one but less than 10, and the second factor is a power of 10 written in exponential form, it is said to be in **scientific notation**.

Scientific Notation

A number is in scientific notation if it is in the form $M \times 10^n$ where $1 \leq M < 10$.

It is customary in scientific notation to use as the $\times$ multiplication sign, even though we avoid using this sign elsewhere in algebra.

If we look at what happened to the decimal point, we can see a method to easily convert from decimal notation to scientific notation.

$$4 \times 10^3 = \text{Decimal moved 3 to the right.}\ 4.0_1 0_2 0_3 = 4000.$$

$$4 \times 10^{-3} = \text{Decimal moved 3 to the left.}\ 0_3 0_2 0_1 4. = 0.004$$

In both cases, the decimal was moved 3 places to get the first factor between 1 and 10.

The power of 10 is positive when the number is larger than 1: $4,000 = 4 \times 10^3$

The power of 10 is negative when the number is between 0 and 1: $0.004 = 4 \times 10^{-3}$

Example 2

Write in scientific notation: 37,000.

Solution

Step 1: **Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.**

Remember, there is a decimal at the end of 37,000.

Move the decimal after the 3. 3.700 is between 1 and 10.

Step 2: **Count the number of decimal places, n, that the decimal point was moved.**

The decimal place was moved 4 places to the left.

$$\text{Decimal moved 4 to the left. } 3_4 7_3 0_2 0_1 0. = 3.7$$

Step 3: **Write the number as a product with a power of 10.**

If the original number is:

Greater than 1, the power of 10 will be 10^a.

Between 0 and 1, the power of 10 will be 10^{-a}.

37,000 is greater than 1 so the power of 10 will have exponent 4.

$$3.7 \times 10^4$$

Step 4: **Check to see if your answer makes sense.**

10^4 is 10,000 and 10,000 times 3.7 will be 37,000.

$$3.7 \times 10^4 = 37,000$$

Try It

5) Write in scientific notation: $96,000$.

Solution

$9.6 \times \ 10^4$

6) Write in scientific notation: $48,300$.

Solution

$4.83 \times \ 10^4$

7) Write in scientific notation: $54,\tilde{0}00$

Solution

$5.40 \times \ 10^4$

Example 3

Write in scientific notation: 0.0052

Solution

The original number, 0.0052, is between 0 and 1 so we will have a negative power of 10.

Step 1: **Move the decimal point to get 5.2, a number between 1 and 10.**

$$0.0{\uparrow}0{\uparrow}5{\uparrow}2 \ = \ 5.2$$

Step 2: **Count the number of decimal places the point was moved.**

$$0.0_1 0_2 5_3 2 \ = \ \text{The decimal was moved } 3 \text{ places.}$$

Step 3: **Write as a product with a power of 10.**

$$5.2 \times \ 10^{-3}$$

Step 4: **Check.**

$$\begin{aligned} 5.2 \times \ 10^{-3} \\ 5.2 \times \ \frac{1}{10^3} \\ 5.2 \times \ \frac{1}{1000} \\ 5.2 \times 0.001 \ = \ 0.0052 \\ 0.0052 = 5.2 \times \ 10^{-3} \end{aligned}$$

Try It

8) Write in scientific notation: 0.0078

Solution

7.8×10^{-3}

9)Write in scientific notation: 0.0129

Solution

1.29×10^{-2}

How to

Convert from decimal notation to scientific notation

1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
2. Count the number of decimal places, n, that the decimal point was moved.
3. Write the number as a product with a power of 10.
   If the original number is:
   - greater than 1, the power of 10 will be 10^n.
   - between 0 and 1, the power of 10 will be 10^{-n}.
4. Check.

Preserving Significant Zeros Using Scientific Notation

As we saw in the previous section, when considering a number with trailing zeros, it is not easy to determine whether or not they are significant. For example, the number 10,000 could have one significant figure only, but is is possible that some of the zeros are significant as well. One way to communicate a significant zero, when there is no decimal present, is to use a tilde to indicate its significance. For instance, $10,0\tilde{0}0$ indicates that the third zero is significant, which makes the zeros in between significant as well. In this sense, this number $10,0\tilde{0}0$ has 4 significant figures. If we wanted to put this number into scientific notation, we would need to include those significant zeros and it would be represented by 1.000×10^4.

Try It

10) Convert the following numbers to scientific notation. Be sure to preserve the number of significant figures.

a. 0.002840

b. 129.00

c. $1800\tilde{0}$

Solution

a. 2.840×10^{-3}

b. 1.2900×10^2

c. 1.8000×10^4

Convert Scientific Notation to Decimal Form

How can we convert from scientific notation to decimal form? Let's look at two numbers written in scientific notation and see.

$$\begin{array}{rl} 9.12 \times & 10^4 \\ 9.12 \times & 10^{-4} \\ 9.12 \times & 10,000 \\ 9.12 \times & 0.0001 \\ 91,200 \times & 0.000912 \end{array}$$

If we look at the location of the decimal point, we can see an easy method to convert a number from scientific notation to decimal form.

$$9.12 \times 10^4 = \text{Move the decimal 4 to the right. } 9.1_1 2_2 0_3 0_4 = 91,200.$$

$$9.12 \times 10^{-4} = \text{Move the decimal 4 to the left. } = 0_4 0_3 0_2 0_1 9.12 = 0.000912$$

In both cases, the decimal point moved 4 places. When the exponent was positive, the decimal moved to the right. When the exponent was negative, the decimal point moved to the left.

Example 4

Convert to decimal form: 6.2×10^3

Solution

Step 1: **Determine the exponent, n, on the factor 10.**

The exponent is 3.

$$6.2 \times 10^3$$

Step 2: **Move the decimal n spaces, adding zeros if needed.**

If the exponent is positive, move the decimal point n places to the right.

If the exponent is negative, move the decimal point $|n|$ places to the left.

$$\text{Exponent is positive so we move the decimal 3 spaces to the right, adding 2 zeros as placeholders.}$$
$$6.2_1 0_2 0_3 = 6,200$$

Step 3: **Check to see if your answer makes sense.**

10^3 is 1000 and 1000 times 6.2 is $6,200$.

Try It

11) Convert to decimal form: 1.3×10^3

Solution

$1,300$

12) Convert to decimal form: 9.25×10^4

Solution

$92,500$

13) Convert to decimal form: 3.900×10^5

Solution

$390,\tilde{0}00$

Example 5

Convert to decimal form: 8.9×10^{-2}

Solution

Step 1: **Determine the exponent, n, on the factor 10.**

The exponent is -2.

Step 2: **Since the exponent is negative, move the decimal point 2 places to the left.**

$$8.9 \times 10^{-2} = 0_2 0_1 8 \overset{\leftarrow}{.} 9$$

Step 3: **Add zeros as needed for placeholders.**

$$8.9 \times 10^{-2} = 0.089$$

Try It

14) Convert to decimal form: 1.2×10^{-4}

Solution

0.00012

15) Convert to decimal form: 7.5×10^{-2}

Solution

0.075

How to

Convert scientific notation to decimal form.

The steps are summarized below.

To convert scientific notation to decimal form:

1. Determine the exponent, n, on the factor 10.
2. Move the decimal n places, adding zeros if needed.
    - If the exponent is positive, move the decimal point n places to the right.

- If the exponent is negative, move the decimal point $|n|$ places to the left.

3. Check.

Multiply and Divide Using Scientific Notation

Astronomers use very large numbers to describe distances in the universe and the ages of stars and planets. Chemists use very small numbers to describe the size of an atom or the charge on an electron. When scientists perform calculations with very large or very small numbers, they use scientific notation. Scientific notation provides a way for the calculations to be done without writing a lot of zeros. We will see how the Properties of Exponents are used to multiply and divide numbers in scientific notation.

Example 6

Multiply. Write answers in decimal form: $(4 \times 10^5)(2 \times 10^{-7})$

Solution

Step 1: **Use the Commutative Property to rearrange the factors.**

$$4 \times 2 \times 10^5 \times 10^{-7}$$

Step 2: **Multiply.**

$$8 \times 10^{-2}$$

Step 3: **Change to decimal form by moving the decimal two places left.**

$$0.08$$

Try It

16) Multiply $(3 \times 10^{6})(2 \times 10^{-8})$. Write answers in decimal form.

Solution

0.06

17) Multiply $(3 \times 10^{-2})(3 \times 10^{-1})$. Write answers in decimal form.

Solution

0.009

Example 7

Divide. Write answers in decimal form: $\frac{9 \times 10^{3}}{3 \times 10^{-2}}$

Solution

Step 1: **Separate the factors, rewriting as the product of two fractions.**

$$\frac{9 \times 10^{3}}{3 \times 10^{-2}}$$

Step 2: **Divide.**

$$3 \times \ 10^{5}$$

Step 3: **Change to decimal form by moving the decimal five places right.**

$$300,000$$

Try It

18) Divide $\frac{8 \times 10^4}{2 \times 10^{-1}}$ Write answers in decimal form.

Solution

$400,000$

19) Divide $\frac{8 \times 10^2}{4 \times 10^{-2}}$. Write answers in decimal form.

Solution

$20,000$

Access these online resources for additional instruction and practice with integer exponents and scientific notation:

- Negative Exponents
- Scientific Notation
- Scientific Notation 2

Key Concepts

- **To convert a decimal to scientific notation:**
    1. Move the decimal point so that the first factor is greater than or equal to 1 but less

than 10.

2. Count the number of decimal places, n, that the decimal point was moved.
3. Write the number as a product with a power of 10. If the original number is:
    - greater than 1, the power of 10 will be 10^n
    - between 0 and 1, the power of 10 will be 10^{-n}
4. Check.

- **To convert scientific notation to decimal form:**
    1. Determine the exponent, n, on the factor 10.
    2. Move the decimal n places, adding zeros if needed.
        - If the exponent is positive, move the decimal point n places to the right.
        - If the exponent is negative, move the decimal point $|n|$ places to the left.
    3. Check.
- Use a tilde to indicate a significant zero when necessary.

Glossary

scientific notation

A number is expressed in scientific notation when it is of the form $M \times 10^n$ where $1 \leq M < 10$ and n is an integer.

Exercises: Convert from Decimal Notation to Scientific Notation

Instructions: For questions 1–7, write each number in scientific notation.

1. $340{,}000$

Solution

3.4×10^5

---

2. $8{,}750{,}000$

---

3. $1{,}290{,}000$

Solution

1.29×10^{6}

---

4. 0.026

---

5. 0.041

Solution

4.1×10^{-2}

---

6. 0.00000871

---

7. 0.00000103

Solution

1.03×10^{-6}

Exercises: Convert Scientific Notation to Decimal Form

Instructions: For questions 8–15, convert each number to decimal form.

8. 5.2×10^{2}

---

9. 8.3×10^2

Solution

830

---

10. 7.5×10^6

---

11. 1.6×10^{10}

Solution

16,000,000,000

---

12. 2.5×10^{-2}

---

13. 3.8×10^{-2}

Solution

0.038

---

14. 4.13×10^{-5}

---

15. 1.93×10^{-5}

Solution

0.0000193

Exercises: Multiply and Divide Using Scientific Notation

Instructions: For questions 16–19, multiply. Write your answer in decimal form.

16. $\left(3 \times 10^{-5}\right)\left(3 \times 10^{9}\right)$

---

17. $\left(2 \times 10^{2}\right)\left(1 \times 10^{-4}\right)$

Solution

0.02

---

18. $\left(7.1 \times 10^{-2}\right)\left(2.4 \times 10^{-4}\right)$

---

19. $\left(3.5 \times 10^{-4}\right)\left(1.6 \times 10^{-2}\right)$

Solution

5.6×10^{-6}

Exercises: Multiply and Divide Using Scientific Notation

Instructions: For questions 20–23, divide. Write your answer in decimal form.

20. $\frac{7 \times 10^{-3}}{1 \times 10^{-7}}$

---

21. $\frac{5 \times 10^{-2}}{1 \times 10^{-10}}$

Solution

$500{,}000{,}000$

---

22. $\frac{6 \times 10^{4}}{3 \times 10^{-2}}$

---

23. $\frac{8 \times 10^{6}}{4 \times 10^{-1}}$

Solution

$20{,}000{,}000$

Exercises: Everyday Math

Instructions: For questions 24–35, answer the given everyday math word problem.

24. The population of the United States on July 4, 2010 was almost $310{,}000{,}000$. Write the number in scientific notation.

---

25. The population of the world on July 4, 2010 was more than $6{,}850{,}000{,}000$. Write the number in scientific notation

Solution

6.85×10^9.

---

26. The average width of a human hair is 0.0018 centimetres. Write the number in scientific notation.

---

27. The probability of winning the 2010 Megamillions lottery was about 0.0000000057. Write the number in scientific notation.

Solution

5.7×10^{-9}

---

28. In 2010, the number of Facebook users each day who changed their status to 'engaged' was 2×10^4. Convert this number to decimal form.

---

29. At the start of 2012, the US federal budget had a deficit of more than $\$1.5 \times 10^{13}$. Convert this number to decimal form.

Solution

$15{,}000{,}000{,}000{,}000$

---

30. The concentration of carbon dioxide in the atmosphere is 3.9×10^{-4}. Convert this number to decimal form.

---

31. The width of a proton is 1×10^{-5} of the width of an atom. Convert this number to decimal form.

Solution

0.00001

---

32. Health care costs. The Centres for Medicare and Medicaid projects that consumers will spend more than $\$4$ trillion on health care by 2017.

a. Write 4 trillion in decimal notation.

b. Write 4 trillion in scientific notation.

---

33. Coin production. In 1942, the U.S. Mint produced $154{,}500{,}000$ nickels. Write $154{,}500{,}000$ in scientific notation.

Solution

1.545×10^{8}

---

34. Distance. The distance between Earth and one of the brightest stars in the night star is 33.7 light years. One light year is about $6{,}000{,}000{,}000{,}000$ (6 trillion), miles.

a. Write the number of miles in one light year in scientific notation.
b. Use scientific notation to find the distance between Earth and the star in miles. Write the answer in scientific notation.

---

35. Debt. At the end of fiscal year 2015 the gross United States federal government debt was estimated to be approximately $\$18,600,000,000,000$ ($\$18.6$ trillion), according to the Federal Budget. The population of the United States was approximately $300,000,000$ people at the end of fiscal year 2015.

a. Write the debt in scientific notation.
b. Write the population in scientific notation.
c. Find the amount of debt per person by using scientific notation to divide the debt by the population. Write the answer in scientific notation.

Solution

a. 1.86×10^{13}

b. 3×10^{8}

c. 6.2×10^{4}

Exercises: Writing Exercises

Instructions: For question 36, answer the given writing exercise.

36. When you convert a number from decimal notation to scientific notation, how do you know if the exponent will be positive or negative?

Solution

Answers will vary

2.4 MORE UNIT CONVERSIONS AND ROUNDING RULES

Learning Objectives

By the end of this section, you will be able to:

- Perform unit conversions while respecting the appropriate rounding rules for accuracy and precision.
- Perform operations with scientific notation while respecting the appropriate rounding rules for accuracy and precision.

Perform unit conversions while respecting the appropriate rounding rules for accuracy and precision.

Consider a simple example: how many feet are there in 4 yards? Most people will almost automatically answer that there are 12 feet in 4 yards. How did you make this determination? Well, if there are 3 feet in 1 yard and there are 4 yards, then there are 4 × 3 = 12 feet in 4 yards.

This is correct, of course, but it is informal. Let us formalize it in a way that can be applied more generally. We know that 1 yard (yd) equals 3 feet (ft):

$$1 \text{ yd} = 3 \text{ ft}$$

In math, this expression is called an *equality*. The rules of algebra say that you can change (i.e., multiply or divide or add or subtract) the equality (as long as you don't divide by zero) and the new expression will still be an equality. For example, if we divide both sides by 2, we get

$$\frac{1}{2}yd = \frac{3}{2}ft$$

We see that one-half of a yard equals 3/2, or one and a half, feet—something we also know to be true, so the above equation is still an equality. Going back to the original equality, suppose we divide both sides of the equation by 1 yard (number *and* unit):

$$\frac{1yd}{1yd} = \frac{3ft}{1yd}$$

The expression is still an equality, by the rules of algebra. The left fraction equals 1. It has the same quantity in the numerator and the denominator, so it must equal 1. The quantities in the numerator and denominator cancel, both the number *and* the unit:

$$\frac{\cancel{1}\,\cancel{yd}}{\cancel{1}\,\cancel{yd}} = \frac{3ft}{1yd}$$

When everything cancels in a fraction, the fraction reduces to 1:

$$1 = \frac{3ft}{1yd}$$

We have an expression, 3 ft1 yd, that equals 1. This is a strange way to write 1, but it makes sense: 3 ft equal 1 yd, so the quantities in the numerator and denominator are the same quantity, just expressed with different units. The expression 3 ft 1 yd is called a *conversion factor*, and it is used to formally change the unit of a quantity into another unit. (The process of converting units in such a formal fashion is sometimes called *dimensional analysis* or the *factor label method*.)

To see how this happens, let us start with the original quantity:

4 yd

Now let us multiply this quantity by 1. When you multiply anything by 1, you don't change the value of the quantity. Rather than multiplying by just 1, let us write 1 as 3 ft1 yd:

$$4yd \times \frac{3ft}{1yd}$$

The 4 yd term can be thought of as $\frac{4yd}{1}$; that is, it can be thought of as a fraction with 1 in the denominator. We are essentially multiplying fractions. If the same thing appears in the numerator and denominator of a fraction, they cancel. In this case, what cancels is the unit *yard*:

$$4\,\cancel{yd} \times \frac{3ft}{1\,\cancel{yd}}$$

$$\frac{4 \times 3ft}{1} = \frac{12ft}{1} = 12ft$$

That is all that we can cancel. Now, multiply and divide all the numbers to get the final answer:

Again, we get an answer of 12 ft, just as we did originally. But in this case, we used a more formal procedure that is applicable to a variety of problems.

How many millimeters are in 14.66 m? To answer this, we need to construct a conversion factor between millimeters and meters and apply it correctly to the original quantity. We start with the definition of a millimeter, which is

$$1mm = \frac{1}{1,000m}$$

The 1/1,000 is what the prefix *milli-* means. Most people are more comfortable working without fractions, so we will rewrite this equation by bringing the 1,000 into the numerator of the other side of the equation:

1,000 mm = 1 m

Now we construct a conversion factor by dividing one quantity into both sides. But now a question arises: which quantity do we divide by? It turns out that we have two choices, and the two choices will give us different conversion factors, both of which equal 1:

$$\frac{1000mm}{1000mm} = \frac{1m}{1000mm} \ or \ \frac{1000mm}{1m} = \frac{1m}{1m}$$

$$1 = \frac{1m}{1000mm} \ or \ \frac{1000mm}{1m} = 1$$

Which conversion factor do we use? The answer is based on *what unit you want to get rid of in your initial quantity*. The original unit of our quantity is meters, which we want to convert to millimeters. Because the original unit is assumed to be in the numerator, to get rid of it, we want the meter unit in the *denominator*; then they will cancel. Therefore, we will use the second conversion factor. Cancelling units and performing the mathematics, we get

$$14.66 \cancel{m} \times \frac{1000mm}{1 \cancel{m}} = 14660mm$$

Note how m cancels, leaving mm, which is the unit of interest.

The ability to construct and apply proper conversion factors is a very powerful mathematical technique in chemistry. You need to master this technique if you are going to be successful in this and future courses.

Example 1

a. Convert 35.9 kL to liters.
b. Convert 555 nm to meters.

Solution

a.

Step 1: **We will use the fact that** $1kL = 1,000L$**.**

Of the two conversion factors that can be defined, the one that will work is $\frac{1,000L}{1kL}$.

Step 2: **Applying this conversion factor, we get:**

$$35.9 \cancel{kL} \times \frac{1000L}{1 \cancel{kL}} = 35900L$$

---

b.

***Step 1:* We will use the fact that $1nm = \frac{1}{1,000,000,000m}$, which we will rewrite as 1,000,000,000 nm = 1 m, or 10 nm = 1 m.**

***Step 2:* Of the two possible conversion factors, the appropriate one has the nm unit in the denominator.**

$$\frac{1m}{10^9 nm}$$

***Step 3:* Applying this conversion factor, we get:**

$$555nm \times \frac{1m}{10^9 nm} = 0.000000555m$$

***Step 4:* In the final step, we expressed the answer in scientific notation.**

$$= 5.55 \times 10^{-7} m$$

Try It

1) Convert $67.08\mu L$ to liters. Give your answer in scientific notation.
2) Convert $56.8m$ to kilometers. Give your answer in scientific notation.

Solution

1. 6.708×10^{-5} L
2. 5.68×10^{-2} km

What if we have a derived unit that is the product of more than one unit, such as m^2? Suppose we want to convert square meters to square centimeters? The key is to remember that m^2 means m × m, which means we have *two* meter units in our derived unit. That means we have to include *two* conversion factors, one for each unit. For example, to convert 17.6 m^2 to square centimeters, we perform the conversion as follows:

$$17.6m^2 = 17.6(\cancel{m} \times \cancel{m}) \times \frac{100cm}{1\cancel{m}} \times \frac{100cm}{1\cancel{m}} = 176000cm \times cm = 1.76 \times 10^5 cm^2$$

Example 2

How many cubic centimeters are in 0.883 m^3?

Solution
With an exponent of 3, we have three length units, so by extension we need to use three conversion factors between meters and centimeters. Thus, we have

$$0.883\,\cancel{m^3} \times \frac{100\,cm}{1\,\cancel{m}} \times \frac{100\,cm}{1\,\cancel{m}} \times \frac{100\,cm}{1\,\cancel{m}} = 883000\,cm^3 = 8.83 \times 10^5\,cm^3$$

You should demonstrate to yourself that the three meter units do indeed cancel.

Try It

3) How many cubic millimeters are present in 0.0923 m^3? Give your answer in scientific notation.

Solution
$9.23 \times 10^7\ mm^3$

Suppose the unit you want to convert is in the denominator of a derived unit; what then? Then, in the conversion factor, the unit you want to remove must be in the *numerator*. This will cancel with the original unit in the denominator and introduce a new unit in the denominator. The following example illustrates this situation.

Example 3

Convert 88.4 m/min to meters/second.

Solution

***Step 1:* We want to change the unit in the denominator from minutes to seconds.**

Because there are 60 seconds in 1 minute (60 s = 1 min), we construct a conversion factor so that the unit we want to remove, minutes, is in the numerator: $1min/60$s.

***Step 2:* Apply and perform the math.**

$$\frac{88.4m}{min} \times \frac{1min}{60s} = 1.47m/s$$

Notice how the 88.4 automatically goes in the numerator. That's because any number can be thought of as being in the numerator of a fraction divided by 1.

Figure 2.4.1. How Fast Is Fast? A common garden snail moves at a rate of about 0.2 m/min, which is about 0.003 m/s, which is 3 mm/s!
Source: "Grapevine snail"by Jürgen Schoneris licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license.

Try It

4) Convert 0.203 m/min to meters/second. Give your answer in scientific notation.

Solution
0.00338 m/s or 3.38×10^{-3} m/s

Sometimes there will be a need to convert from one unit with one numerical prefix to another unit with a different numerical prefix. How do we handle those conversions? Well, you could memorize the conversion factors that interrelate all numerical prefixes. Or you can go the easier route: first convert the quantity to the base unit, the unit with no numerical prefix, using the definition of the original prefix. Then convert the quantity in the base unit to the desired unit using the definition of the second prefix. You can do the conversion in two separate steps or as one long algebraic step. For example, to convert 2.77 kg to milligrams:

$$2.77\,\cancel{kg} \times \frac{1000g}{1\,\cancel{kg}} = 2770g \;\; (convert\ to\ the\ base\ unit\ of\ grams)$$

$$2770\,\cancel{g} \times \frac{1000mg}{1\,\cancel{g}} = 2770000mg = 2.77 \times 10^{6}\ mg \;\; (convert\ to\ the\ desired\ unit)$$

Alternatively, it can be done in a single multistep process:

$$2.77\,\cancel{kg} \times \frac{1000\,\cancel{g}}{1\,\cancel{kg}} \times \frac{1000mg}{1\,\cancel{g}} = 2770000m = 2.77 \times 10^{6} mg$$

You get the same answer either way.

Example 4

How many nanoseconds are in 368.09 μs?

Solution
You can either do this as a one-step conversion from microseconds to nanoseconds or convert to

the base unit first and then to the final desired unit. We will use the second method here, showing the two steps in a single line. Using the definitions of the prefixes *micro-* and *nano-*,

$$368.09\,\cancel{\mu s} \times \frac{1\,\cancel{s}}{10^6\,\cancel{\mu s}} \times \frac{10^9\,ns}{1\,\cancel{s}} = 368090\,ns = 3.6809 \times 10^5\,ns$$

Try It

5) How many milliliters are in 607.8 kL? Give your answer in scientific notation.

Solution
6.078×10^8 mL

When considering the significant figures of a final numerical answer in a conversion, there is one important case where a number does not impact the number of significant figures in a final answer—the so-called exact number. An exact number is a number from a defined relationship, not a measured one. For example, the prefix *kilo-* means 1,000—*exactly* 1,000, no more or no less. Thus, in constructing the conversion factor

$$\frac{1000g}{1kg}$$

neither the 1,000 nor the 1 enter into our consideration of significant figures. The numbers in the numerator and denominator are defined exactly by what the prefix *kilo-* means. Another way of thinking about it is that these numbers can be thought of as having an infinite number of significant figures, such as

$$\frac{1000.0000000000...g}{1.0000000000...kg}$$

The other numbers in the calculation will determine the number of significant figures in the final answer.

Example 5

A rectangular plot in a garden has the dimensions 36.7 cm by 128.8 cm. What is the area of the garden plot in square meters? Express your answer in the proper number of significant figures.

Solution

Area is defined as the product of the two dimensions, which we then have to convert to square meters and express our final answer to the correct number of significant figures, which in this case will be three.

$$36.7\,\cancel{cm} \times 128.8\,\cancel{cm} \times \frac{1\,m}{100\,\cancel{cm}} \times \frac{1\,m}{100\,\cancel{cm}} = 0.472696\,m^2 = 0.473\,m^2$$

The 1 and 100 in the conversion factors do not affect the determination of significant figures because they are exact numbers, defined by the centi- prefix.

Try It

6) What is the volume of a block in cubic meters whose dimensions are 2.1 cm × 34.0 cm × 118 cm?

Solution

0.0084 m^3

In the examples above, the answers provided are all consistent with the rounding rules from the previous section, but we did not emphasize what choices were being made while reporting our answers. In the next sections, we will go through a series of examples to combine what we've learned over the last few sections. Throughout this section (and for the rest of the course), be sure to always pay attention to the instructions so that you are always rounding appropriately. Sometimes, you will be required to follow the rounding rules for accuracy and precision, and other times, you will be asked to round to a specific place value. It is always important to pay attention to detail and to answer questions with care.

Rounding Rules When Converting Within the Same System of Measurement

When considering metric-to-metric or U.S. System-to-U.S. System conversions, we must first realize that our conversion equations within each system are exact. For example, $100cm = 1m$ means that there are exactly 100 centimeters in 1 meter and $1ft = 12in$ means that there are exactly 12 inches in 1 foot. Since these conversion equations are exact, this means that the numbers involved have infinitely many significant figures and infinitely many decimal places. As such, when we are performing metric-to-metric or U.S. System-to-U.S. System conversions, we will use the measurement that we are converting to determine the appropriate number of significant figures to include in our answer. Please note that metric-to-metric conversion equations must be recreated by memory, whereas units from the U.S. system will always be provided in our course.

Example 6

Convert 31.5 meters per minute to meters per second. Give your answer with the appropriate number of significant figures.

Solution

Step 1: **Write down any relevant unit conversion equations.**

$$1min = 60s$$

Step 2: **Convert the units using dimensional analysis.**

$$\begin{aligned} &= \frac{31.5m}{1min} \times \frac{1min}{60s} \\ &= \frac{31.5m}{60s} \\ &= 0.525\ m/s \end{aligned}$$

Step 3: **Be sure that the answer is given to the proper number of significant figures.**

Since the operations we performed were all multiplications and divisions, we need to round to the least number of significant figures. The unit conversion equation used, $1min = 60s$, is exact so there are infinitely many significant figures in those values. In this case, we need to look at the measurement given. Since $31.5m/min$ has 3 significant figures, our answer should have 3 significant figures as well. So our answer of $0.525m/s$ is appropriate.

Example 7

Convert $18\tilde{0} mg/dL$ to g/L. Give your answer with the appropriate number of significant figures.

Solution

***Step 1:* Write down any relevant unit conversion equations.**

$1000mg = 1g$ and $10dL = 1L$

For the purpose of the work, we will make note that there is a tilde over the zero and so $18\tilde{0}$ has 3 significant figures.

***Step 2:* Convert the units using dimensional analysis.**

$$= \frac{180mg}{1dL} \times \frac{10dL}{1L} \times \frac{1g}{1000mg}$$
$$= \frac{180 \ \times 10g}{1 \ times \ 1000L}$$
$$= 1.8 \ g/L$$

***Step 3:* Be sure that the answer is given to the proper number of significant figures.**

The operations we used in this question were multiplications and divisions, and so we need to round to the least number of significant figures in our values. Our calculator gives us $1.8g/L$, but we need to take into account the fact that our measurement had 3 significant figures. Thus, our answer in this case should have 3 significant figures.

Our answer in this case is that $18\tilde{0} mg/dL = 1.80g/L$.

Example 8

Convert 32.8 ounces to cups. Give your answer with the appropriate number of significant figures.

Solutions

The following unit conversion equations may be useful for this example: $1cup = 8oz$.

Step 1: **Write down any relevant unit conversion equations.**

Note, that the U.S. system conversion equation is given.

Step 2: **Convert the units using dimensional analysis.**

$$\begin{aligned} &= \frac{32.8oz}{1} \times \frac{1cup}{8oz} \\ &= \frac{32.8\ cup}{1 \cdot 8} \\ &= 4.1\ cups \end{aligned}$$

Step 3: **Be sure that the answer is given to the proper number of significant figures.**

The operations we used in this question were multiplications and divisions, and so we need to round to the least number of significant figures in our values. Our calculator does not take into account significant figures. Since our unit conversion equation is exact (i.e. there are exactly 8 ounces in 1 cup), those numbers have infinitely many significant figures. Thus, we need to look to our measurement to decide how many significant figures our answer needs. 32.8 ounces has 3 significant figures, so our answer must also have 3 significant figures.

Therefore, $32.8oz = 4.10cups$.

Example 9

Convert 161.8 miles per hour to feet per second. Give your answer with the appropriate number of significant figures.

Solution

The following unit conversion equations may be useful for this example: $1mi = 5280ft$.

Step 1: **Write down any relevant unit conversion equations.**

Note, that the U.S. system conversion equation is given. To convert from hours to seconds, we need to know: $1h = 3600s$.

Step 2: **Convert the units using dimensional analysis.**

$$\begin{aligned} &= \frac{161.8mi}{1h} \times \frac{5280ft}{1mi} \times \frac{1h}{3600s} \\ &= \frac{161.8 \cdot 5280\ ft}{1 \cdot 3600s} \\ &= 237.30\overline{6}ft/s \end{aligned}$$

Step 3: **Be sure that the answer is given to the proper number of significant figures.**

The operations we used in this question were multiplications and divisions, and so we need to round to the least number of significant figures in our values. Our calculator does not take into account significant figures. Since our unit conversion equation is exact, the numbers involved in the conversion equations have infinitely many significant figures. Thus, we need to look to our measurement to decide how many significant figures our answer needs. 161.8 mi/h has 4 significant figures, so our answer must also have 4 significant figures.

Therefore, $161.8 mi/h = 237.3 ft/s$.

As we have seen in the examples above, when we are converting metric-to-metric or U.S. System-to-U.S. System units, we round according to the measurement that we are converting.

Rounding Rules When Converting Between Two Different Systems of Measurement

When considering the unit conversion equations that establish relationships between two units of different measurement systems, it is important to note the number of significant figures present in the equations. This is due to the fact that when we convert from the metric system to the U.S. system of measurement, the relationships between the units are not exact, but are rather approximations. For example, if we wanted to convert between grams (metric) and pounds (U.S. system), we may be given one of the following unit conversion equations.

Unit Conversion Equations for Converting Between Grams and Pounds

$1.00 lb = 454 g$

$1.000 lb = 453.6 g$

$1.0000 lb = 453.59 g$

$1.00000 lb = 453.592 g$

The equations above show that the precision of the unit conversion equation depends on the precision of the measurement tool that was used. Therefore, these numbers in these unit conversion equations are approximate and will be taken into account while performing our unit conversions.

Example 10

Convert 8.4 miles to kilometers. Give your answer with the appropriate number of significant figures.

Solution

The following unit conversion equations may be useful for this example: $1.0000mi = 1.6093km$.

Step 1: **Write down any relevant unit conversion equations.**

Note, that the U.S. system to metric conversion equation is provided.

Step 2: **Convert the units using dimensional analysis.**

$$\begin{aligned} &= \frac{8.4mi}{1} \times \frac{1.6093km}{1.0000mi} \\ &= \frac{8.4 \cdot 1.6093km}{1} \\ &= 13.51812km \end{aligned}$$

Step 3: **Be sure that the answer is given to the proper number of significant figures.**

All the operations in our conversion above were multiplications and divisions, so we need to round to the least number of significant figures. Don't forget, our calculator does not take into account significant figures. Since our unit conversion is 1.0000 mi = 1.6093 km, each of those numbers has 5 significant figures, whereas our measurement of 8.4mi has only 2 significant figures. Thus, we need to round our final answer to 2 significant figures.

Therefore, $8.4mi = 14km$.

Example 11

Convert 20.18 kilogram per liter to pound per gallon. Give your answer with the appropriate number of significant figures.

Solution

The following unit conversion equations may be useful for this example: $1.00kg = 2.20lbs$ and $1.00L = 0.264gal(U.S)$.

Step 1: **Write down any relevant unit conversion equations.**

Note, that the U.S. system to metric conversion equations are provided.

Step 2: **Convert the units using dimensional analysis.**

$$\begin{aligned} &= \frac{20.18kg}{1L} \times \frac{2.20lbs}{1.00kg} \frac{1.00L}{0.264gal} \\ &= \frac{20.18 \times\ 2.20 \times\ 1.00lbs}{1 \times\ 1.00 \times\ 0.264gal} \\ &= 168.1\overline{6}lbs/gal \end{aligned}$$

Step 3: **Be sure that the answer is given to the proper number of significant figures.**

All the operations in our conversion above were multiplications and divisions, so we need to round to the least number of significant figures. Don't forget, our calculator does not take into account significant figures. Since our unit conversions are $1.00kg = 2.20lbs$ and $1.00L = 0.264U.S.gal$, each of those numbers have 3 significant figures, whereas our measurement of 20.18 kg/L has 4 significant figures. Thus, we need to round our final answer to 3 significant figures.

Therefore, $20.18kg/L = 168lbs/gal$.

Perform operations with scientific notation while respecting the appropriate rounding rules for accuracy and precision.

Now that we know how to use scientific notation, we can combine this with our knowledge of our rounding rules to round appropriately. In the following examples, we will perform the indicated operations and round according to the instructions provided.

Example 12

Perform the indicated operations. Give your answer with the appropriate number of significant figures.

a. $3.92 \times 10^4 + 2.3 \times 10^4$

b. $1.9 \times 10^{-2} - 1.49 \times 10^{-1}$

c. $(5.8 \times 10^5) \times (2.84 \times 10^{-7})$

d. $\frac{(3.8 \times 10^2)(2.93 \times 10^3)}{(8 \times 10^{-3})(8.935 \times 10^3)}$

Solution

a.

Step 1: **Since the powers of ten are the same, we can simply add the values.**

$$3.92 \times 10^4 + 2.3 \times 10^4 = 6.22 \times 10^4$$

Step 2: **As we were adding, we need to round to the correct precision. This means, we need to round according to the 2.3×10^4 which has 2 significant figures.**

Thus, the answer is: 6.2×10^4

---

b.

Step 1: **First, we need to write the numbers with the same power of ten.**

$$\begin{aligned} &= 1.9 \times 10^{-2} - 1.49 \times 10^{-1} \\ &= 0.19 \times 10^{-1} - 1.49 \times 10^{-1} \\ &= -1.3 \times 10^{-1} \end{aligned}$$

Step 2: **Our answer already has the correct precision, so no further rounding is necessary.**

---

c.

Step 1: **To multiply scientific notation, multiply the coefficients and add the exponents on the tens.**

$$\begin{aligned} &= (5.8 \times 10^5) \times (2.84 \times 10^{-7})10^{-1} \\ &= (5.8 \cdot 2.84) \times 10^{5+(-7)} \\ &= 16.472 \times 10^{-2} \end{aligned}$$

Step 2: **Before we round, we need to write this number in correct scientific notation.**

$$= 1.6472 \times 10^{-1}$$

Step 3: **Now, since we multiplied to get this result, we need to round to the least number of significant figures, which in this case is 2.**

$$= 1.6 \times 10^{-1}$$

---

d.

Step 1: **To answer this question, let's first perform the multiplications in the numerator and denominator, and then we will do the division.**

$$\frac{(3.8 \times 10^{2})(2.93 \times 10^{3})}{(8 \times 10^{-3})(8.935 \times 10^{3})}$$

$$= \frac{3.8 \times 2.93 \times 10^{2+3}}{8 \times 8.935 \times 10^{-3+3}}$$

$$= \frac{11.134 \times 10^{5}}{71.48 \times 10^{0}}$$

Step 2: **Now, we can perform the division of the coefficients and subtract the exponents on the tens.**

$$= 0.15576385 \times 10^{5}$$

Step 3: **Finally, let's put this into correct scientific notation before we round appropriately.**

$$= 1.5576385 \times 10^{4}$$

Step 4: **Since the 8 in the initial problem only has 1 significant figure, our answer should only have 1 significant figure.**

$$= 2 \times 10^{4}$$

Important Note

Instructions on rounding expectations will always be given on evaluations. Be sure to read instructions carefully to know if you are expected to use the rounding rules covered in this section, or if there are any other expectations. If questions are vague and do not specify rounding instructions, do not round your answer.

Key Concepts

- Units can be converted to other units using the proper conversion factors.
- Conversion factors are constructed from equalities that relate two different units.
- Conversions can be a single step or multistep.
- Unit conversion is a powerful mathematical technique in chemistry, physics, and mathematics, that must be mastered.
- Exact numbers do not affect the determination of significant figures.

Exercises: Significant Figures

Instructions: For questions 1–22, ensure that your answers have the appropriate number of significant figures according to our rules for precision and accuracy.

1. Write the two conversion factors that exist between the two given units.

a. millilitres and litres
b. microseconds and seconds
c. kilometres and meters

Solution

a. $1{,}000 \text{ mL}/1 \text{ L}$ and $1 \text{ L}/1{,}000 \text{ mL}$

b. $1{,}000{,}000 \ \mu\text{s}/1 \text{ s}$ and $1 \text{ s}/1{,}000{,}000 \ \mu\text{s}$

c. $1{,}000 \text{ m}/1 \text{ km}$ and $1 \text{ km}/1{,}000 \text{ m}$

---

2. Write the two conversion factors that exist between the two given units.

a. kilograms and grams

b. milliseconds and seconds

c. centimetres and meters

---

3. Perform the following conversions.

a. 5.4 **km to meters**

b. 0.665 **m to millimetres**

c. 0.665 **m to kilometres**

Solution

a. $5{,}400$ m

b. 665 mm

c. 6.65×10^{-4} km

---

4. Perform the following conversions.

a. 90.6 **mL to litres**

b. 0.00066 **mL to litres**

c. 750 **L to kilolitres**

---

5. Perform the following conversions.

a. 17.8 **μg to grams**

b. 7.22×10^2 **kg to grams**

c. 0.00118 **g to nanograms**

Solution

a. 1.78×10^{-5} g

b. 7.22×10^5 g

c. 1.18×10^6 ng

---

6. Perform the following conversions.

a. 833 **ns to seconds**

b. 5.809 **s to milliseconds**

c. 2.77×10^6 **s to megaseconds**

---

7. Perform the following conversions.

a. $9.44\ \mathrm{m}^2$ **to square centimetres**

b. $3.44 \times 10^8\ \mathrm{mm}^3$ **to cubic meters**

Solution

a. $94{,}400\ \mathrm{cm}^2$

b. $0.344\ \mathrm{m}^3$

---

8. Perform the following conversions.

a. $0.00444\ \mathrm{cm}^3$ **to cubic meters**

b. $8.11 \times 10^2\ \mathrm{m}^2$ **to square nanometres**

---

9. Why would it be inappropriate to convert square centimetres to cubic meters?

Solution

One is a unit of area, and the other is a unit of volume.

---

10. Why would it be inappropriate to convert from cubic meters to cubic seconds?

---

11. Perform the following conversions.

- **a.** 45.0 **m/min to meters/second**
- **b.** 0.000444 **m/s to micrometers/second**
- **c.** 60.0 **km/h to kilometres/second**

Solution

a. 0.750 m/s

b. 444 μm/s

c. 1.67×10^{-2} km/s

---

12. Perform the following conversions.

- **a.** 3.4×10^{2} **cm/s to centimetres/minute**
- **b.** 26.6 **mm/s to millimetres/hour**
- **c.** 13.7 **kg/L to kilograms/millilitres**

---

13. Perform the following conversions.

- **a.** 0.674 **kL to millilitres**
- **b.** 2.81×10^{12} **mm to kilometres**
- **c.** 94.5 **kg to milligrams**

Solution

a. $674{,}000$ mL

b. 2.81×10^{6} km

c. 9.45×10^{7} mg

14. Perform the following conversions.

- **a.** 6.79×10^{-6} **kg to micrograms**
- **b.** 1.22 **mL to kilolitres**
- **c.** 9.508×10^{-9} **ks to milliseconds**

15. Perform the following conversions.

- **a.** 6.77×10^{14} **ms to kiloseconds**
- **b.** $34{,}550{,}000$ **cm to kilometres**

Solution

a. 6.77×10^{8} ks

b. 345.5 km

16. Perform the following conversions.

- **a.** 4.701×10^{15} **mL to kilolitres**
- **b.** 8.022×10^{-11} **ks to microseconds**

17. Perform the following conversions. Note that you will have to convert units in both the numerator and the denominator.

- **a.** 88 **ft/s to miles/hour (Hint: use** $5{,}280 \text{ ft} = 1 \text{ mi.}$**)**
- **b.** 0.00667 **km/h to meters/second**

Solution

a. 6.0×10^{1} mi/h

b. 0.00185 m/s

18. Perform the following conversions. Note that you will have to convert units in both the numerator and the denominator.

a. 3.88×10^2 mm/s to kilometres/hour
b. 1.004 kg/L to grams/millilitre

---

19. What is the area in square millimetres of a rectangle whose sides are $2.44 \text{ cm} \times 6.077 \text{ cm}$? Express the answer to the proper number of significant figures.

Solution

$1.48 \times 10^3 \text{ mm}^2$

---

20. What is the volume in cubic centimetres of a cube with sides of 0.774 m? Express the answer to the proper number of significant figures.

---

21. The formula for the area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$. What is the area of a triangle in square centimetres if its base is 1.007 m and its height is 0.665 m? Express the answer to the proper number of significant figures.

Solution

$3.35 \times 10^3 \text{ cm}^2$

---

22. The formula for the area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$. What is the area of a triangle in square meters if its base is 166 mm and its height is 930.0 mm? Express the answer to the proper number of significant figures.

PART III

UNIT 3: SOLVING LINEAR EQUATIONS, GRAPHS OF LINEAR EQUATIONS, AND APPLICATIONS OF LINEAR FUNCTIONS

3.1 SOLVE EQUATIONS USING THE SUBTRACTION AND ADDITION PROPERTIES OF EQUALITY

Learning Objectives

By the end of this section, you will be able to:

- Verify a solution of an equation
- Solve equations using the Subtraction and Addition Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

Try It

Before you get started, take this readiness quiz:

1) Evaluate $x + 4$ when $x = -3$.

2) Evaluate $15 - y$ when $y = -5$.

3) Simplify $4(4n+1)-15n$.

4) Translate into algebra "5 is less than x".

Verify a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. The purpose of solving an equation is to find the value or values of the variable that make each side of the equation the same – so that we end up with a true statement. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle!

Solution of an equation

A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

Exercises

To determine whether a number is a solution to an equation.

1. Substitute the number for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting equation is true (the left side is equal to the right side)
    - If it is true, the number is a solution.
    - If it is not true, the number is not a solution.

Example 1

Determine whether $x = \frac{3}{2}$ is a solution of $4x - 2 = 2x + 1$.

Solution

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

Step 1: **Substitute $\frac{3}{2}$ for x.**

$$4\left(\frac{3}{2}\right) - 2 \stackrel{?}{=} 2\left(\frac{3}{2}\right) + 1$$

Step 2: **Multiply.**

$$6 - 2 \stackrel{?}{=} 3 + 1$$

Step 3: **Subtract.**

$$4 = 4 \checkmark$$

Since $x = \frac{3}{2}$ results in a true equation (4 is in fact equal to 4), $\frac{3}{2}$ is a solution to the equation $4x - 2 = 2x + 1$

Try It

5) Is $y = \frac{4}{3}$ a solution of $9y + 2 = 6y + 3?$

Solution
no

6) Is $y = \frac{7}{5}$ a solution of $5y + 3 = 10y - 4$?

Solution
yes

Solve Equations Using the Subtraction and Addition Properties of Equality

We are going to use a model to clarify the process of solving an equation. An envelope represents the variable – since its contents are unknown – and each counter represents one. We will set out one envelope and some counters in our workspace, as shown in Figure 3.1.1. Both sides of the workspace have the same number of counters, but some counters are "hidden" in the envelope. Can you tell how many counters are in the envelope?

The illustration shows a model of an equation with one variable. On the left side of the workspace is an unknown (envelope) and three counters, while on the right side of the workspace are eight counters.

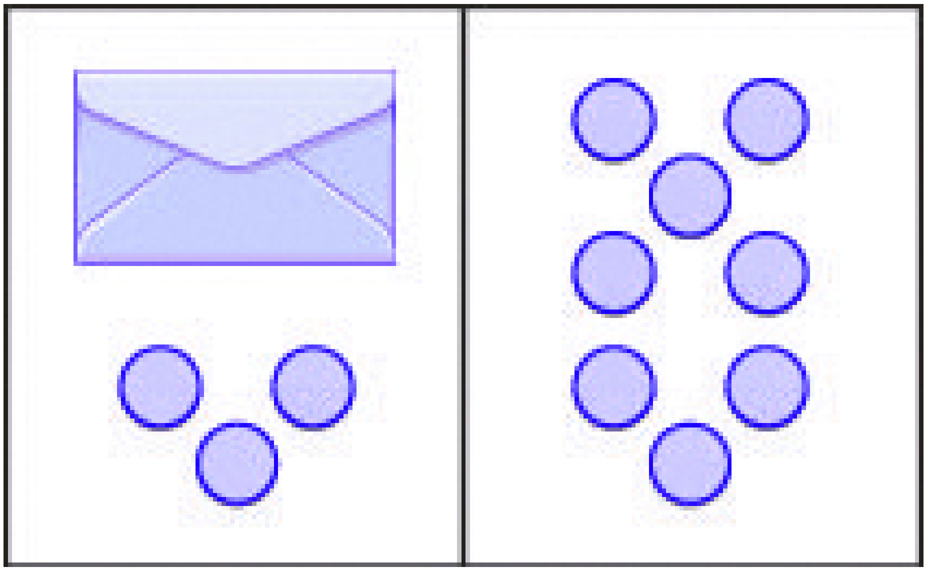


Figure 3.1.1 The illustration shows a model of an equation with one variable. On the left side of the workspace is an unknown (envelope) and three counters, while on the right side of the workspace are eight counters.

What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope?

Perhaps you are thinking: "I need to remove the 3 counters at the bottom left to get the envelope by itself. The 3 counters on the left can be matched with 3 on the right so I can take them away from both sides. That leaves five on the right—so there must be 5 counters in the envelope." See Figure 3.1.2. for an illustration of this process.

The illustration shows a model for solving an equation with one variable. On both sides of the workspace remove three counters, leaving only the unknown (envelope) and five counters on the right side. The unknown is equal to five counters.

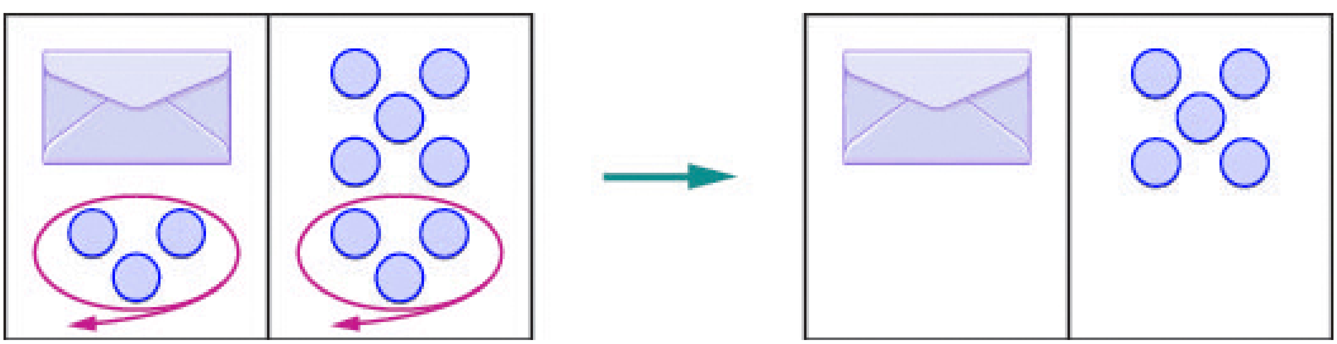
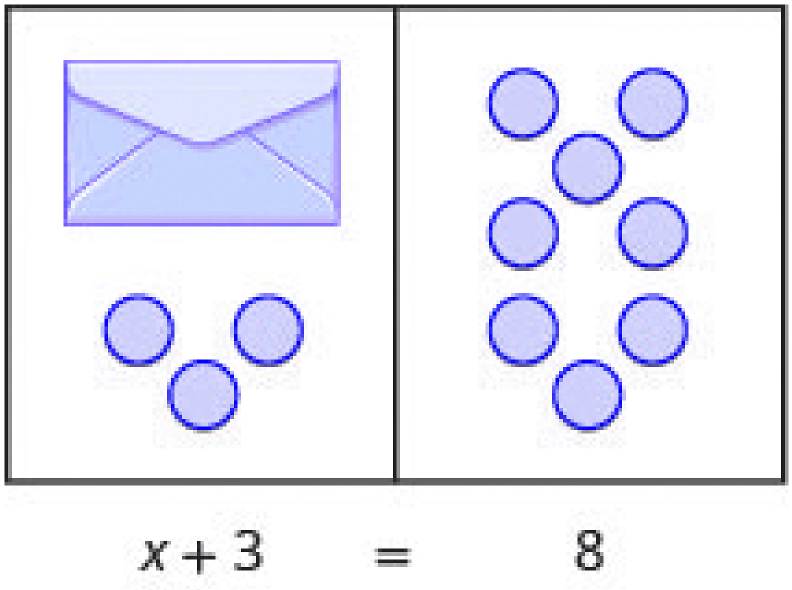


Figure 3.1.2-The illustration shows a model for solving an equation with one variable. On both sides of the workspace remove three counters, leaving only the unknown (envelope) and five counters on the right side. The unknown is equal to five counters.

What algebraic equation would match this situation? In Figure 3.1.3 each side of the workspace represents an expression and the centre line takes the place of the equal sign. We will call the contents of the envelope x

.

The illustration shows a model for the equation $x+3=8$.



3.1.3 – The illustration shows a model for the equation �+3=8.

Let’s write algebraically the steps we took to discover how many counters were in the envelope:

	$x+3=8$
First, we took away three from each side.	$x+3-3=8-3$
Then we were left with five.	$x=5$
Check:	Five in the envelope plus three more does equal eight!
	$5+3=8$

Our model has given us an idea of what we need to do to solve one kind of equation. The goal is to isolate the variable by itself on one side of the equation. To solve equations such as these mathematically, we use the *Subtraction Property of Equality*.

Subtraction Property of Equality

For any numbers a, b, and c,

if $a = b$,
then $a - c = b - c$

When you subtract the same quantity from both sides of an equation, you still have equality.

Let's see how to use this property to solve an equation. Remember, the goal is to isolate the variable on one side of the equation. And we check our solutions by substituting the value into the equation to make sure we have a true statement.

Example 2

Solve: $y + 37 = -13$.

Solution

To get y by itself, we will undo the addition of 37 by using the Subtraction Property of Equality.

Step 1: **Subtract 37 from each side to 'undo' the addition.**

$$y + 37 - 37 = -13 - 37$$

Step 2: **Simplify.**

$$y = -50$$

***Step 3:* Check:**

$$y + 37 = -13$$

***Step 4:* Substitute** $y = -50$

$$\begin{aligned} -50 + 37 &= -13 \\ -13 &\stackrel{?}{=} -13 \checkmark \end{aligned}$$

Since $y = -50$ makes $y + 37 = -13$ a true statement, we have the solution to this equation.

Try It

7) Solve: $x + 19 = -27$.

Solution

$x = -46$

8) Solve: $x + 16 = -34$.

Solution

$x = -50$

What happens when an equation has a number subtracted from the variable, as in the equation $x - 5 = 8$? We use another property of equations to solve equations where a number is subtracted from the variable. We want to isolate the variable, so to 'undo' the subtraction we will add the number to both sides. We use the Addition Property of Equality.

Addition Property of Equality

For any numbers a, b, and c,

if $a = b$,

then $a + c = b + c$

When you add the same quantity to both sides of an equation, you still have equality.

In Example 3.1.2, 37 was added to the y and so we subtracted 37 to 'undo' the addition. In 3.1.3, we will need to 'undo' subtraction by using the Addition Property of Equality.

Example 3

Solve: $a - 28 = -37$.

Solution

Step 1: **Add 28 to each side to 'undo' the subtraction.**

$$a - 28 + 28 = -37 + 28$$

Step 2: **Simplify.**

$$a = -9$$

Step 3: **Check:**

$$a - 28 = -37$$

Step 4: **Substitute** $a = -9$

$$-9 - 28 = 37$$
$$-37 \stackrel{?}{=} -37\checkmark$$

The solution to $a - 28 = -37$ is $a = -9$.

Try It

9) Solve: $n - 61 = -75$.

Solution

$n = -14$

10) Solve: $p - 41 = -73$.

Solution

$p = -32$

Example 4

Solve: $x - \frac{5}{8} = \frac{3}{4}$

Solution

Step 1: **Use the Addition Property of Equality.**

$$x - \frac{5}{8} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8}$$

Step 2: **Find the LCD to add the fractions on the right.**

$$\begin{array}{ll} & x-\frac{5}{8}+\frac{5}{8}=\frac{6}{8}+\frac{5}{8} \\ \text{Simplify} & x=\frac{11}{8} \end{array}$$

***Step 3:* Check:**

$$x-\frac{5}{8}=\frac{3}{4}$$

***Step 4:* Substitute $x=\frac{11}{8}$.**

$$\begin{array}{ll} & \frac{11}{8}-\frac{5}{8}\stackrel{?}{=}\frac{3}{4} \\ \text{Subtract.} & \frac{6}{8}\stackrel{?}{=}\frac{3}{4} \\ \text{Simplify.} & \frac{3}{4}=\frac{3}{4}\checkmark \end{array}$$

The solution to $x-\frac{5}{8}=\frac{3}{4}$ is $x=\frac{11}{8}$

Try It

11) Solve: $p-\frac{2}{3}=\frac{5}{6}$

Solution

$p=\frac{3}{2}$

12) Solve: $q-\frac{1}{2}=\frac{5}{6}$

Solution

$q=\frac{4}{3}$

The next example will be an equation with decimals.

Example 5

Solve: $n - 0.63 = -4.2$.

Solution

Step 1: **Use the Addition Property of Equality.**

$$\begin{array}{ll} & n - 0.63 + 0.63 = -4.2 + 0.63 \\ \text{Add.} & n = -3.57 \end{array}$$

Step 2: **Check:**

$$n = -3.57$$

Step 3: **Let** $n = -3.57$**.**

$$\begin{array}{r} -3.57 - 0.63 \stackrel{?}{=} -4.2 \\ -4.2 - 4.2\checkmark \end{array}$$

Try It

13) Solve: $b - 0.47 = -2.1$.

Solution

$b = -1.63$

14) Solve: $c - 0.93 = -4.6$.

Solution

$c = -3.67$

Solve Equations That Require Simplification

In the previous examples, we were able to isolate the variable with just one operation. Most of the equations we encounter in algebra will take more steps to solve. Usually, we will need to simplify one or both sides of an equation before using the *Subtraction or Addition Properties of Equality*.

You should always simplify as much as possible before you try to isolate the variable. Remember that simplifying an expression means doing all the operations in the expression. Simplify one side of the equation at a time. Note that simplification is different from the process used to solve an equation in which we apply an operation to both sides.

Example 6

Solve: $9x-5-8x-6=7$.

Solution

***Step 1:* Simplify the expressions on each side as much as possible.**

Rearrange the terms, using the Commutative Property of Addition.

$$\begin{array}{ll} \text{Combine like terms.} & 9x-5-8x-6=7 \\ & 9x-8x-5-6=7 \\ & x-11=7 \end{array}$$

Notice that each side is now simplified as much as possible.

***Step 2:* Isolate the variable.**

Now isolate x.

$$\text{Undo subtraction by adding 11 to both sides.} \quad x-11+11=7+11$$

***Step 3:* Simplify the expressions on both sides of the equation.**

$$x=18$$

***Step 4:* Check the solution.**

Substitute $x=18$

$$\begin{aligned} 9x-5-8x-6&=7 \\ 9(18)-5-8(8)-6&\stackrel{?}{=}7 \\ 162-5-144-6&\stackrel{?}{=}7 \\ 157-144&\stackrel{?}{=}7 \\ 13-6&\stackrel{?}{=}7 \\ 7&=7\checkmark \end{aligned}$$

The solution to $9x-5-8x-6=7$ is $x=18$

Try It

15) Solve: $8y - 4 - 7y - 7 = 4$.

Solution
$y = 15$

16) Solve: $6z + 5 - 5z - 4 = 3$.

Solution
$z = 2$

Example 7

Solve: $5(n - 4) - 4n = -8$.

Solution
We simplify both sides of the equation as much as possible before we try to isolate the variable.

Step 1: **Distribute on the left.**

$$5n - 20 - 4n = -8$$

Step 2: **Use the Commutative Property to rearrange terms.**

$$\begin{array}{ll} & 5n - 4n - 20 = -8 \\ \text{Combine like terms.} & n - 20 = -8 \end{array}$$

Step 3: **Each side is as simplified as possible. Next, isolate n.**

Step 4: **Undo subtraction by using the Addition Property of Equality.**

$$\begin{array}{ll} & n - 20 + 20 = -8 + 20 \\ \text{Add.} & n = 12 \end{array}$$

Step 5: **Check.**

Substitute $n = 12$.

$$\begin{aligned} 5(n-4)-4n &= -8 \\ 5(12-4)-4(12) &\stackrel{?}{=} -8 \\ 5(8)-48 &\stackrel{?}{=} -8 \\ 40-48 &\stackrel{?}{=} -8 \\ -8 &= -8 \checkmark \end{aligned}$$

The solution to $5(n-4)-4n=-8$ is $n = 12$.

Try It

17) Solve: $5(p-3)-4p=-10$.

Solution

$p = 5$

18) Solve: $4(q+2)-3q=-8$.

Solution

$q = -16$

Example 8

Solve: $3(2y-1)-5y=2(y+1)-2(y+3)$.

Solution

We simplify both sides of the equation before we isolate the variable.

Step 1: **Distribute on both sides.**

$$6y-3-5y=2y+2-2y-6$$

***Step 2:* Use the Commutative Property of Addition.**

$$\begin{array}{ll} & 6y-5y-3=2y-2y+2-6 \\ \text{Combine like terms.} & y-3=-4 \end{array}$$

***Step 3:* Each side is as simplified as possible. Next, isolate y.**

***Step 4:* Undo subtraction by using the Addition Property of Equality.**

$$\begin{array}{ll} & y-3+3=-4+3 \\ \text{Add.} & y=-1 \end{array}$$

***Step 5:* Check.**

Let $y=-1$.

$$\begin{aligned} 3(2y-1)-5y&=2(y+1)-2(y+3) \\ 3(2(12-1)-1)-5(-1)&\stackrel{?}{=}2(-1+1)-2(-1+3) \\ 3(-2-1)+5&\stackrel{?}{=}2(0)-2(2) \\ 3(-3)+5&\stackrel{?}{=}-4 \\ -9+5&=-4 \\ -4&=-4\checkmark \end{aligned}$$

The solution to $3(2y-1)-5y=2(y+1)-2(y+3$ is $y=-1$.

Try It

19) Solve: $4(2h-3)-7h=6(h-2)-6(h-1)$

Solution

$h=6$

20) Solve: $2(5x+2)-9x=3(x-2)-3(x-4)$

Solution

$x=2$

Translate to an Equation and Solve

To solve applications algebraically, we will begin by translating English sentences into equations. Our first step is to look for the word (or words) that would translate to the equals sign. The below table shows us some of the words that are commonly used.

Equals =	is is equal to is the same as the result is gives was will be

The steps we use to translate a sentence into an equation are listed below.

How to

Translate an English sentence to an algebraic equation.

1. Locate the "equals" word(s). Translate to an equals sign (=).
2. Translate the words to the left of the "equals" word(s) into an algebraic expression.
3. Translate the words to the right of the "equals" word(s) into an algebraic expression.

Example 9

Translate and solve: Eleven more than x is equal to 54.

Solution

Step 1: **Translate.**

$$\underbrace{Eleven\ more\ than\ x}_{x+11}\ \underbrace{is\ equal\ to}_{=}\ \underbrace{54}_{54}$$

Step 2: **Subtract 11 from both sides.**

$$x+11-11=54-11$$

Step 3: **Simplify.**

$$x=43$$

Step 4: **Check**

$$\begin{aligned}43+11 &\stackrel{?}{=} 54\\ 54 &= 54\end{aligned}$$

Try It

21) Translate and solve: Ten more than x is equal to 41.

Solution

$x+10=41$; $x=31$

22) Translate and solve: Twelve less than x is equal to 51.

Solution

$y-12=51$; $y=63$

Example 10

Translate and solve: The difference of $12t$ and $11t$ is -14.

Solution

Step 1: Translate.

$$\underbrace{The\ difference\ of\ 12t\ and\ 11t}_{12t-11t}\ \underbrace{is}_{=}\ \underbrace{-14}_{-14}$$

Step 2: **Simplify.**

$$t = -14$$

Step 3: **Check:**

$$\begin{aligned} 12(-14) - 11(-14) &\stackrel{?}{=} -14 \\ (-168 + 154) &\stackrel{?}{=} -14 \\ -14 &= -14 \end{aligned}$$

Try It

23) Translate and solve: The difference of $4x$ and $3x$ is 14.

Solution

$$\begin{aligned} 4x - 3x &= 14 \\ x &= 14 \end{aligned}$$

24) Translate and solve: The difference of $7a$ and $6a$ is -8.

Solution

$$\begin{aligned} 7a - 6a &= -8 \\ a &= -8 \end{aligned}$$

Translate and Solve Applications

Most of the time a question that requires an algebraic solution comes out of a real-life question. To begin with, that question is asked in English (or the language of the person asking) and not in math symbols. Because of this, it is an important skill to be able to translate an everyday situation into algebraic language.

We will start by restating the problem in just one sentence, assigning a variable, and then translating the sentence into an equation to solve. When assigning a variable, choose a letter that reminds you of what you are looking for. For example, you might use q for the number of quarters if you were solving a problem about coins.

Example 11

The MacIntyre family recycled newspapers for two months. The two months of newspapers weighed a total of 57 pounds. The second month, the newspapers weighed 28 pounds. How much did the newspapers weigh the first month?

Solution

***Step 1:* Read the problem.**

Make sure all the words and ideas are understood.

The problem is about the weight of newspapers.

***Step 2:* Identify what we are asked to find.**

What are we asked to find?

"How much did the newspapers wight the 2nd month?"

***Step 3:* Name what we are looking for.**

Choose a variable to represent that quantity.

Let w = weight of the newspapers the 1st month

Step 4: **Translate into an equation.**

It may be helpful to restate the problem in one sentence with the important information.

Restate the problem.

Weight of newspapers the 1st month plus the weight of the newspapers the 2nd month equals 57 pounds.

We know the weight of the newspapers the second month is 28 pounds.

Weight from 1st month plus 2nd month equals 57 pounds.

Translate into an equation, using the variable w.

$$w + 28 = 57$$

Step 5: **Solve the equation using good algebra techniques.**

$$\begin{aligned} w + 28 - 28 &= 57 - 28 \\ w &= 29 \end{aligned}$$

Step 6: **Check the answer in the problem and make sure it makes sense.**

Does 1st month's weight plus 2nd month's weight equal 57 pounds?

$$\begin{aligned} 29 + 28 &\stackrel{?}{=} 57 \\ 57 &= 57\checkmark \end{aligned}$$

Step 7: **Answer the question with a complete sentence.**

Write a sentence to answer "How much did the newspapers weigh the 2nd month?"

The 2nd month the newspapers weighed 29 pounds.

Translate into an algebraic equation and solve:

How to

Solve an application.

1. Read the problem. Make sure all the words and ideas are understood.
2. Identify what we are looking for.
3. Name what we are looking for. Choose a variable to represent that quantity.
4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.
5. Solve the equation using good algebra techniques.
6. Check the answer in the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

Example 12

Randell paid $\$28,675$ for his new car. This was $\$875$ less than the sticker price. What was the sticker price of the car?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

"What was the sticker price of the car?"

Step 3: **Name what we are looking for.**

Choose a variable to represent that quantity.

Let s = the sticker price of the car.

Step 4: **Translate into an equation. Restate the problem in one sentence.**

$\$28,675$ is $\$875$ less than the sticker price

$\$28,675$ is $\$875$ less than s

Step 5: **Solve the equation.**

$$\begin{aligned} 28675 &= s - 875 \\ 28675 + 875 &= s - 875 + 875 \\ 29550 &= s \end{aligned}$$

Step 6: **Check the answer**.

Is $\$875$ less than $\$29,550$? Equal to $\$28,675$?

$$\begin{aligned} 29,550 - 875 &\stackrel{?}{=} 28,675 \\ 28,675 &= 28,675 \end{aligned}$$

Step 7: **Answer the question with a complete sentence.**

The sticker price of the car was $\$29,550$.

Try It

25) Translate into an algebraic equation and solve:

The Pappas family has two cats, Zeus and Athena. Together, they weigh 23 pounds. Zeus weighs 16 pounds. How much does Athena weigh?

Solution

7 pounds

26) Translate into an algebraic equation and solve:

Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

Solution

42 books

27) Translate into an algebraic equation and solve:

Eddie paid $19,875 for his new car. This was $1,025 less than the sticker price. What was the sticker price of the car?

Solution

$20,900

28) Translate into an algebraic equation and solve:

The admission price for the movies during the day is $7.75. This is $3.25 less the price at night. How much does the movie cost at night?

Solution

$11.00

Key Concepts

- **To Determine Whether a Number is a Solution to an Equation**
    1. **Substitute the number in for the variable in the equation.**
    2. **Simplify the expressions on both sides of the equation.**
    3. **Determine whether the resulting statement is true.**
        - If it is true, the number is a solution.
        - If it is not true, the number is not a solution.
- **Addition Property of Equality**

- For any numbers a, b, and c, if $a = b$, then $a + c = b + c$.

- **Subtraction Property of Equality**
    - For any numbers
    - a, b, and c, if $a = b$, then $a - c = b - c$.

- **To Translate a Sentence to an Equation**
    1. Locate the "equals" word(s). Translate to an equal sign ($=$).
    2. Translate the words to the left of the "equals" word(s) into an algebraic expression.
    3. Translate the words to the right of the "equals" word(s) into an algebraic expression.

- **To Solve an Application**
    1. Read the problem. Make sure all the words and ideas are understood.
    2. Identify what we are looking for.
    3. Name what we are looking for. Choose a variable to represent that quantity.
    4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.
    5. Solve the equation using good algebra techniques.
    6. Check the answer in the problem and make sure it makes sense.
    7. Answer the question with a complete sentence.

Glossary

solution of an equation
solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

Exercises: Verify a Solution of an Equation

Instructions: For questions 1-4, determine whether the given value is a solution to the equation.

1. Is $y = \frac{5}{3}$ a solution of $6y + 10 = 12y$?

Solution

Yes

---

2. Is $x = \frac{9}{4}$ a solution of $4x + 9 = 8x$?

---

3. Is $u = -\frac{1}{2}$ a solution of $8u - 1 = 6u$?

Solution

No

---

4. Is $v = -\frac{1}{3}$ a solution of $9v - 2 = 3v$?

Exercises: Solve Equations using the Subtraction and Addition Properties of Equality

Instructions: For questions 5-28, solve each equation using the Subtraction and Addition Properties of Equality.

5. $x + 24 = 35$

Solution

$x = 11$

---

6. $x + 17 = 22$

---

7. $y + 45 = -66$

Solution

$y = -111$

---

8. $y + 39 = -83$

---

9. $b + \frac{1}{4} = \frac{3}{4}$

Solution

$b = \frac{1}{2}$

---

10. $a + \frac{2}{5} = \frac{4}{5}$

---

11. $p + 2.4 = -9.3$

Solution

$p = -11.7$

---

12. $m + 7.9 = 11.6$

---

13. $a - 45 = 76$

Solution

$a = 121$

---

14. $a - 30 = 57$

---

15. $m - 18 = -200$

Solution

$m = -182$

---

16. $m - 12 = -12$

---

17. $x - \frac{1}{3} = 2$

Solution

$x = \frac{7}{3}$

---

18. $x - \frac{1}{5} = 4$

---

19. $y - 3.8 = 10$

Solution

$y = 13.8$

---

20. $y - 7.2 = 5$

---

21. $x - 165 = -420$

Solution

$x = -255$

---

22. $z - 101 = -314$

---

23. $z + 0.52 = -8.5$

Solution

$z = -9.02$

---

24. $x + 0.93 = -4.1$

---

25. $q + \frac{3}{4} = \frac{1}{2}$

Solution

$q = -\frac{1}{4}$

---

26. $p + \frac{1}{3} = \frac{5}{6}$

---

27. $p - \frac{2}{5} = \frac{2}{3}$

Solution

$p = \frac{16}{15}$

---

28. $y - \frac{3}{4} = \frac{3}{5}$

Exercises: Solve Equations that Require Simplification

Instructions: For questions 29-50, solve each equation.

29. $c + 31 - 10 = 46$

Solution

$c = 25$

---

30. $m + 16 - 28 = 5$

---

31. $9x + 5 - 8x + 14 = 20$

Solution

$x = 1$

---

32. $6x + 8 - 5x + 16 = 32$

---

33. $-6x - 11 + 7x - 5 = -16$

Solution

$x = 0$

---

34. $-8n - 17 + 9n - 4 = -41$

---

35. $5(y-6)-4y=-6$

Solution

$y=24$

---

36. $9(y-2)-8y=-16$

---

37. $8(u+1.5)-7u=4.9$

Solution

$u=-7.1$

---

38. $5(w+2.2)-4w=9.3$

---

39. $6a-5(a-2)+9=-11$

Solution

$a=-30$

---

40. $8c-7(c-3)+4=-16$

---

41. $6(y-2)-5y=4(y+3)-4(y-1)$

Solution

$y=28$

---

42. $9(x-1)-8x=-3(x+5)+3(x-5)$

---

43. $3(5n-1)-14n+9=10(n-4)-6n-4(n+1)$

Solution

$n=-50$

---

44. $2(8m+3)-15m-4=9(m+6)-2(m-1)-7m$

---

45. $-(j+2)+2j-1=5$

Solution

$j=8$

---

46. $-(k+7)+2k+8=7$

---

47. $-\left(\frac{1}{4}a-\frac{3}{4}\right)+\frac{5}{4}a=-2$

Solution

$a=-\frac{11}{4}$

---

48. $-\left(\frac{2}{3}d-\frac{1}{3}\right)+\frac{5}{3}d=-4$

---

49. $8(4x+5)-5(6x)-x=53-6(x+1)+3(2x+2)$

Solution

$x = 13$

---

50. $6(9y-1)-10(5y)-3y=22-4(2y-12)+8(y-6)$

Exercises: Translate to an Equation and Solve

Instructions: For questions 51-62, translate to an equation and then solve it.

51. Nine more than x is equal to 52.

Solution

$$\begin{aligned} x+9 &= 52 \\ x &= 43 \end{aligned}$$

---

52. The sum of x and -15 is 23.

---

53. Ten less than m is -14.

Solution

$$\begin{aligned} m-10 &= -14 \\ m &= -4 \end{aligned}$$

---

54. Three less than y is -19.

---

55. The sum of y and -30 is 40.

Solution

$$\begin{aligned} y+(-30) &= 40 \\ y &= 70 \end{aligned}$$

---

56. Twelve more than p is equal to 67.

---

57. The difference of $9x$ and $8x$ is 107.

Solution

$$\begin{aligned} 9x-8x &= 107 \\ x &= 107 \end{aligned}$$

---

58. The difference of $5c$ and $4c$ is 602.

---

59. The difference of n and $\frac{1}{6}$ is $\frac{1}{2}$.

Solution

$$\begin{aligned} n - \frac{1}{6} &= \frac{1}{2} \\ n &= \frac{1}{3} \end{aligned}$$

---

60. The difference of f and $\frac{1}{3}$ is $\frac{1}{12}$.

---

61. The sum of $-4n$ and $5n$ is -82.

Solution

$$\begin{aligned} -4n + 5n &= -82 \\ n &= -82 \end{aligned}$$

---

62. The sum of $-9m$ and $10m$ is -95.

Exercises: Translate and Solve Applications

Instructions: For questions 63-72, translate into an equation and solve.

63. Distance. Avril rode her bike a total of 18 miles, from home to the library and then to the beach. The distance from Avril's house to the library is 7 miles. What is the distance from the library to the beach?

Solution

11 miles

---

64. Reading. Jeff read a total of 54 pages in his History and Sociology textbooks. He read 41 pages in his History textbook. How many pages did he read in his Sociology textbook?

---

65. Age. Eva's daughter is 15 years younger than her son. Eva's son is 22 years old. How old is her daughter?

Solution

7 years old

---

66. Age. Pablo's father is 3 years older than his mother. Pablo's mother is 42 years old. How old is his father?

---

67. Groceries. For a family birthday dinner, Celeste bought a turkey that weighed 5 pounds less than the one she bought for Thanksgiving. The birthday turkey weighed 16 pounds. How much did the Thanksgiving turkey weigh?

Solution

21 pounds

---

68. Weight. Allie weighs 8 pounds less than her twin sister Lorrie. Allie weighs 124 pounds. How much does Lorrie weigh?

---

69. Health. Connor's temperature was 0.7 degrees higher this morning than it had been last night. His temperature this morning was 101.2 degrees. What was his temperature last night?

Solution

100.5 degrees

---

70. Health. The nurse reported that Tricia's daughter had gained 4.2 pounds since her last checkup and now weighs 31.6 pounds. How much did Tricia's daughter weigh at her last checkup?

---

71. Salary. Ron's paycheck this week was $\$17.43$ less than his paycheck last week. His paycheck this week was $\$103.76$. How much was Ron's paycheck last week?

Solution

$\$121.19$

---

72. Textbooks. Melissa's math book cost $\$22.85$ less than her art book cost. Her math book cost $\$93.75$. How much did her art book cost?

Exercises: Everyday Math

Instructions: For questions 73-74, answer the given everyday math word problems.

73. Construction. Miguel wants to drill a hole for a $\dfrac{5}{8}$ inch screw. The hole should be

$\frac{1}{12}$ inch smaller than the screw. Let d equal the size of the hole he should drill. Solve the equation $d-\frac{1}{12}=\frac{5}{8}$ to see what size the hole should be.

Solution

$d=\frac{17}{24}$ inch

---

74. Baking. Kelsey needs $\frac{2}{3}$ cup of sugar for the cookie recipe she wants to make. She only has $\frac{3}{8}$ cup of sugar and will borrow the rest from her neighbor. Let s equal the amount of sugar she will borrow. Solve the equation $\frac{3}{8}+s=\frac{2}{3}$ to find the amount of sugar she should ask to borrow.

Exercises: Writing Exercises

Instructions: For questions 75-76, answer the given writing exercises.

75. Is -8 a solution to the equation $3x=16-5x$? How do you know?

Solution

No. Justifications will vary.

---

76. What is the first step in your solution to the equation $10x + 2 = 4x + 26$?

3.2 SOLVE EQUATIONS USING THE DIVISION AND MULTIPLICATION PROPERTIES OF EQUALITY

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using the Division and Multiplication Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

Try It

Before you get started, take this readiness quiz:

1) Simplify: $-7 \cdot \frac{1}{-7}$.

2) Evaluate $9x + 2$ when $x = -3$.

Solve Equations Using the Division and

Multiplication Properties of Equality

You may have noticed that all of the equations we have solved so far have been of the form $x + a = b$ or $x - a = b$. We were able to isolate the variable by adding or subtracting the constant term on the side of the equation with the variable. Now we will see how to solve equations that have a variable multiplied by a constant and so will require the division to isolate the variable.

Let's look at our puzzle again with the envelopes and counters in Figure 3.2.1.

The illustration shows a model of an equation with one variable multiplied by a constant. On the left side of the workspace are two instances of the unknown (envelope), while on the right side of the workspace are six counters.

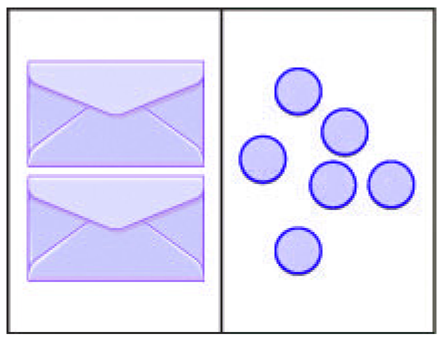


Figure 3.2.1 – The illustration shows a model of an equation with one variable multiplied by a constant. On the left side of the workspace are two instances of the unknown (envelope), while on the right side of the workspace are six counters.

In the illustration, two identical envelopes contain the same number of counters. Remember, the left side of the workspace must equal the right side, but the counters on the left side are "hidden" in the envelopes. So how many counters are in each envelope?

How do we determine the number? We have to separate the counters on the right side into two groups of the same size to correspond with the two envelopes on the left side. The 6 counters divided into 2 equal groups gives 3 counters in each group (since $6 \div 2 = 3$).

What equation models the situation shown in Figure 3.2.2? There are two envelopes, and each contains x counters. Together, the two envelopes must contain a total of 6 counters.

The illustration shows a model of the equation $2x = 6$.

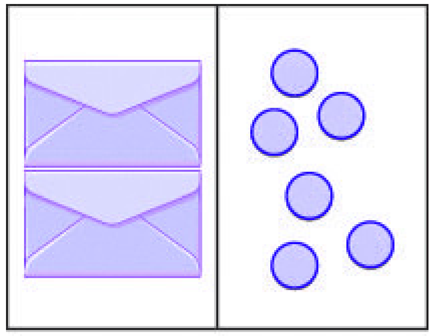


Figure 3.2.2 – The illustration shows a model of the equation [latex]2�=6[/latex].

	$2x = 6$
If we divide both sides of the equation by 2, as we did with the envelopes and counters,	$\frac{2x}{2} = \frac{6}{2}$
we get:	$x = 3$

We found that each envelope contains 3 counters. Does this check? We know $2 \cdot 3 = 6$, so it works! Three counters in each of two envelopes do equal six!

This example leads to the Division Property of Equality.

The Division Property of Equality

For any numbers a, b, and c, if $a = b$, and $c \neq 0$,

If $a = b$,
then $\frac{a}{c} = \frac{b}{c}$
When you divide both sides of an equation by any non-zero number, you still have equality.

The goal in solving an equation is to 'undo' the operation on the variable. In the next example, the variable is multiplied by 5, so we will divide both sides by 5 to 'undo' the multiplication.

Example 1

Solve: $5x = -27$.

Solution

***Step 1:* To isolate x, "undo" the multiplication by 5.**

$$5x = -27$$

***Step 2:* Divide to 'undo' the multiplication.**

$$\frac{5x}{5} = -\frac{27}{5}$$

$$\text{Simplify.} \qquad x = -\frac{27}{5}$$

***Step 3:* Check:**

$$5x = -27$$

***Step 5:* Substitute $-\frac{27}{5}$ for x.**

$$5\left(-\frac{27}{5}\right) \stackrel{?}{=} -27$$

$$-27 = -27 \checkmark$$

Since this is a true statement, $x = -\frac{27}{5}$ is the solution to $5x = -27$.

Try It

3) Solve: $3y = -41$.

Solution

$y = \frac{-41}{3}$

4) Solve: $4z = -55$.

Solution

$z = -\frac{55}{4}$

Consider the equation $\frac{x}{4} = 3$. We want to know what number divided by 4 gives 3. So to "undo" the division, we will need to multiply by 4. The Multiplication Property of Equality will allow us to do this. This property says that if we start with two equal quantities and multiply both by the same number, the results are equal.

The Multiplication Property of Equality

For any numbers a, b, and c,

If $a = b$,

Then $ac = bc$

If you multiply both sides of an equation by the same number, you still have equality.

Example 2

Solve: $\frac{y}{-7} = -14$.

Solution

Here y is divided by -7. We must multiply by -7 to isolate y.

Step 1: **Multiply both sides by -7.**

$$\frac{y}{-7} \times -7 = -14 \times -7$$

$$\text{Simplify.} \qquad y = 98$$

Step 2: **Check:** $\frac{y}{-7} = -14$

Step 3: **Substitute $y = 98$.**

$$\frac{98}{-7} \stackrel{?}{=} -14$$

$$-14 = -14 \checkmark$$

Try It

5) Solve: $\frac{a}{-7} = -42$.

Solution

$a = 294$

6) Solve: $\frac{b}{-6} = -24$.

Solution

$b = 144$

Example 3

Solve: $-n = 9$.

Solution

Step 1: **Remember $-n$ is equivalent to $-1n$.**

$$-1n = 9$$

Step 2: **Divide both sides by -1.**

$$\begin{array}{ll} & \frac{-1n}{-1} = \frac{9}{-1} \\ \text{Divide} & n = -9 \end{array}$$

Notice that there are two other ways to solve $-n = 9$. We can also solve this equation by multiplying both sides by -1 and also by taking the opposite of both sides.

Step 3: **Check:**

$$-n = 9$$

Step 4: **Substitute $n = -9$.**

$$\begin{array}{ll} & -(-9) \stackrel{?}{=} 9 \\ \text{Simplify} & 9 = 9\checkmark \end{array}$$

Try It

7) Solve: $-k = 8$.

Solution

$k = -8$

8) Solve: $-g = 3$.

Solution

$g = -3$

Example 4

Solve: $\frac{3}{4}x = 12$.

Solution

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{3}{4}$.

Step 1: **Multiply by the reciprocal of $\frac{3}{4}$.**

	$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot 12$
Reciprocals multiply to 1	$1x = \frac{4}{3} \cdot \frac{12}{1}$
Mulitply.	$x = 16$

Notice that we could have divided both sides of the equation $\frac{3}{4}x = 12$ by $\frac{3}{4}$ to isolate x. While this would work, most people would find multiplying by the reciprocal easier.

Step 2: **Check:**

Step 5: **Substitute $x = 16$.**

$$\frac{3}{4}x = 12$$
$$\frac{3}{4} \cdot 16 \stackrel{?}{=} 12$$
$$12 = 12 \checkmark$$

Try It

9) Solve: $\frac{2}{5}n = 14$.

Solution

$n = 35$

10) Solve: $\frac{5}{6}y = 15$.

Solution

$y = 18$

In the next example, all the variable terms are on the right side of the equation. As always, our goal in solving the equation is to isolate the variable.

Example 5

Solve: $\frac{8}{15} = -\frac{4}{5}x$.

Solution

Step 1: **Multiply by the reciprocal of $-\dfrac{4}{5}$.**

$$\left(-\frac{5}{4}\right)\left(\frac{8}{15}\right)=\left(-\frac{5}{4}\right)\left(-\frac{4}{5}x\right)$$

Reciprocals multiply to 1 $\quad -\dfrac{5\cdot 4\cdot 2}{4\cdot 3\cdot 5}=1x$

Mulitply. $\quad -\dfrac{2}{3}=x$

Step 2: **Let $x=-\frac{2}{3}$.**

$$\frac{8}{15}=-\frac{4}{5}x$$
$$\frac{8}{15}=-\frac{4}{5}\left(-\frac{2}{3}\right)$$
$$\frac{8}{15}=\frac{8}{15}\checkmark$$

Try It

11) Solve: $\frac{9}{25}=-\frac{4}{5}z$.

Solution

$z=-\dfrac{9}{20}$

12) Solve: $\frac{5}{6}=-\frac{8}{3}r$

Solution

$r=-\dfrac{5}{16}$

Solve Equations That Require Simplification

Many equations start out more complicated than the ones we have been working with.

With these more complicated equations, the first step is to simplify both sides of the equation as much as possible. This usually involves combining like terms or using the distributive property.

Example 6

Solve: $14-23=12y-4y-5y$.

Solution

Begin by simplifying each side of the equation.

Step 1: **Simplify each side.**

$$-9=3y$$

Step 2: **Divide both sides by 3 to isolate y.**

$$\begin{aligned} &\frac{-9}{3}=\frac{3y}{3} \\ \text{Divide.}\quad &-3=y \end{aligned}$$

Step 3: **Check:**

$$14-23\stackrel{?}{=}12y-4y-5y$$

Step 4: **Substitute $y=-3$.**

$$\begin{aligned} 14-23 &\stackrel{?}{=} 12(-3)-4(-3)-5(-3) \\ 14-23 &\stackrel{?}{=} -36+12+15 \\ -9 &= -9\checkmark \end{aligned}$$

Try It

13) Solve: $18-27=15c-9c-3c$.

Solution

$c=-3$

14) Solve: $18-22=12x-x-4x$.

Solution

$x = -\frac{4}{7}$

Example 7

Solve: $-4(a-3)-7=25$.

Solution

Here we will simplify each side of the equation by using the distributive property first.

Step 1: **Distribute.**

$$\begin{array}{lr} & -4a+12-7=25 \\ \text{Simplify.} & -4a+5=25 \\ \text{Simplify.} & -4a=20 \end{array}$$

Step 2: **Divide both sides by -4 to isolate a.**

$$\begin{array}{lc} & \left(\frac{-4a}{-4}\right)=\left(\frac{20}{-4}\right) \\ \text{Divide.} & a=-5 \end{array}$$

Step 3: **Check:**

$$-4(a-3)-7=25$$

Step 4: **Substitute $a=-5$.**

$$\begin{aligned} -4(-5-3)-7 &\stackrel{?}{=} 25 \\ -4(-8)-7 &\stackrel{?}{=} 25 \\ 32-7 &\stackrel{?}{=} 25 \\ 25 &= 25\checkmark \end{aligned}$$

Try It

15) Solve: $-4(q-2)-8=24$.

Solution

$q=-6$

16) Solve: $-6(r-2)-12=30$.

Solution

$r=-5$

Now we have covered all four properties of equality—subtraction, addition, division, and multiplication. We'll list them all together here for easy reference.

Properties of Equality

Subtraction Property of Equality	**Addition Property of Equality**
for any real numbers a, b, and c, If $a = b$, then $a - c = b - c$.	for any real numbers a, b, and c, If $a = b$, then $a + c = b + c$.
Division Property of Equality	**Multiplication Property of Equality**
for any real numbers a, b, and c, and $c \neq 0$ If $a = b$, then $\frac{a}{c} = \frac{b}{c}$.	for any real numbers a, b, and c, If $a = b$, then $a \times c = b \times c$.

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Translate to an Equation and Solve

In the next few examples, we will translate sentences into equations and then solve the equations. You might want to review the translation table in the previous chapter.

Example 8

Translate and solve: The number 143 is the product of -11 and y.

Solution

Begin by translating the sentence into an equation.

Step 1: **Translate.**

$$\underbrace{\text{The number } 143}_{143}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{the product of } -11 \text{ and } y.}_{-11y}$$

Step 2: **Divide by -11.**

$$\frac{143}{-11} = \frac{-11y}{-11}$$

Step 3: **Simplify.**

$$-13 = y$$

Step 4: **Check:**

$$143 = -11y$$
$$143 \stackrel{?}{=} -11(-13)$$
$$143 = 143\checkmark$$

Try It

17) Translate and solve: The number 132 is the product of -12 and y.

Solution

$132=-12y$; $y=-11$

18) Translate and solve: The number 117 is the product of -13 and z.

Solution

$117=-13z$; $z=-9$

Example 9

Translate and solve: n divided by 8 is -32.

Solution

Step 1: **Begin by translating the sentence into an equation.**

$$\underbrace{n\ divided\ by\ 8}_{\frac{n}{8}}\ \underbrace{is}_{=}\ \underbrace{-32.}_{-32}$$

Step 2: **Multiple both sides by 8.**

$$8 \cdot \frac{n}{8} = 8(-32)$$

Simplify. $n = -256$

Step 3: **Check:**

Is n divided by 8 equal to -32?

Step 4: **Let $n = -256$.**

$$\frac{-256}{8} \stackrel{?}{=} -32$$
$$-32 = -32\checkmark$$

Try It

19) Translate and solve: n divided by 7 is equal to -21.

Solution

$\frac{n}{7} = -21$; $n = -147$

20) Translate and solve: n divided by 8 is equal to -56.

Solution

$\frac{n}{8} = -56$; $n = -448$

Example 10

Translate and solve: The quotient of y and -4 is 68.

Solution

Begin by translating the sentence into an equation.

Step 1: **Translate.**

$$\underbrace{\text{The quotient of } y \text{ and } -4}_{\frac{y}{-4}} \underbrace{\text{is}}_{=} \underbrace{68.}_{68}$$

Step 2: **Multiply both sides by -4.**

$$-4\left(\frac{y}{-4}\right) = -4(68)$$

Simplify. $$y = -272$$

Step 3: **Check:**

Is the quotient of y and -4 equal to 68?

Step 4: **Let $y = -272$.**

$$\frac{-272}{-4} \stackrel{?}{=} 68$$

$$68 = 68 \checkmark$$

Try It

21) Translate and solve: The quotient of q and -8 is 72.

Solution

$\frac{q}{-8} = 72$; $q = -576$

22) Translate and solve: The quotient of p and -9 is 81.

Solution

$\frac{p}{-9} = 81$; $p = -729$

Example 11

Translate and solve: Three-fourths of p is 18.

Solution

Begin by translating the sentence into an equation. Remember, "of" translates into multiplication.

Step 1: **Translate.**

$$\underbrace{Three - fourths\ of\ p}_{\frac{3}{4}p}\ \underbrace{is}_{=}\ \underbrace{18.}_{18}$$

Step 2: **Multiply both sides by $\frac{4}{3}$.**

$$\frac{4}{3} \cdot \frac{3}{4}p = \frac{4}{3} \cdot 18$$

Simplify. $p = 24$

Step 3: **Check:**

Is three-fourths of p equal to 18?

Step 5: **Let $p = 24$.**

Is three-fourths of 24 equal to 18?

$$\frac{3}{4} \times 24 \stackrel{?}{=} 18$$

$$18 = 18 \checkmark$$

Try It

23) Translate and solve: Two-fifths of f is 16.

Solution

$\frac{2}{5}f = 16$; $f = 40$

24) Translate and solve: Three-fourths of f is 21.

Solution

$\frac{3}{4}f = 21$; $f = 28$

Example 12

Translate and solve: The sum of three-eighths and x is one-half.

Solution

Begin by translating the sentence into an equation.

Step 1: **Translate.**

$$\underbrace{The\ sum\ of\ three-eighths\ and\ x}_{\frac{3}{8}+x}\ \underbrace{is}_{=}\ \underbrace{\frac{1}{2}}_{\frac{1}{2}}$$

Step 2: **Subtract $\frac{3}{8}$ from each side.**

$$\frac{3}{8} - \frac{3}{8} + x = \frac{1}{2} - \frac{3}{8}$$

Simplify and rewrite fractions with common denominators. $x = \frac{4}{8} - \frac{3}{8}$

Simplify. $x = \frac{1}{8}$

Step 3: **Check:**

Is the sum of three-eighths and x equal to one-half?

***Step 4:* Let** $x = \frac{1}{8}$

Is the sum of three-eighths and one-eighth equal to one-half?

$$\begin{array}{ll} & \frac{3}{8} + \frac{1}{8} \stackrel{?}{=} \frac{1}{2} \\ \text{Simplify.} & \frac{4}{8} \stackrel{?}{=} \frac{1}{2} \\ \text{Simplify.} & \frac{1}{2} = \frac{1}{2} \checkmark \end{array}$$

Try It

25) Translate and solve: The sum of five-eighths and x is one-fourth.

Solution

$\frac{5}{8} + x = \frac{1}{4}$; $\quad x = -\frac{3}{8}$

26) Translate and solve: The sum of three-fourths and x is five-sixths.

Solution

$\frac{3}{4} + x = \frac{5}{6}$; $\quad x = \frac{1}{12}$

Translate and Solve Applications

To solve applications using the Division and Multiplication Properties of Equality, we will follow the same steps we used in the last section. We will restate the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve.

Example 13

Denae bought 6 pounds of grapes for $\$10.74$. What was the cost of one pound of grapes?

Solution

***Step 1:* What are you asked to find?**

The cost of 1 pound of grapes

***Step 2:* Assign a variable.**

Let c = the cost of one pound.

***Step 3:* Write a sentence that gives the information to find it.**

The cost of 6 pounds is $\$10.74$.

***Step 4:* Translate into an equation.**

$$6c = 10.74$$

***Step 5:* Solve.**

$$\begin{aligned} \frac{4}{3}(12,000) &= \frac{4}{3} \times \frac{3}{4}p \\ 16,000 &= p \end{aligned}$$

The grapes cost $\$1.79$ per pound.

***Step 6:* Check:**

If one pound costs $\$1.79$, do 6 pounds cost $\$10.74$?

$$6(1.79) \stackrel{?}{=} 10.74$$
$$10.74 = 10.74\checkmark$$

Try It

27) Translate and solve:

Arianna bought a 24-pack of water bottles for $\$9.36$. What was the cost of one water bottle?

Solution
$\$0.39$

28) Translate and solve:

At JB's Bowling Alley, 6 people can play on one lane for $\$34.98$. What is the cost for each person?

Solution
$\$5.83$

Example 14

Andreas bought a used car for $\$12,000$. Because the car was 4-years old, its price was $\frac{3}{4}$ of the original price, when the car was new. What was the original price of the car?

Solution

Step 1: **What are you asked to find?**

The original price of the car

Step 2: **Assign a variable.**

p = the original price.

Step 3: **Write a sentence that gives the information to find it.**

$\$12,000$ is $\frac{3}{4}$ of the original price.

Step 4: **Translate into an equation.**

$$12,000 = \frac{3}{4}p$$

Step 5: **Solve.**

$$\begin{aligned} \frac{4}{3} \times (12,000) &= \frac{4}{3} \times \frac{3}{4}p \\ 16,000 &= p \end{aligned}$$

The original cost of the car was $\$16,000$.

Step 6: **Check:**

Is $\frac{3}{4}$ of $\$16,000$ equal to $\$12,000$?

$$\begin{aligned} \frac{3}{4} \times (16,000) &\stackrel{?}{=} 12,000 \\ 12,000 &= 12,000 \checkmark \end{aligned}$$

Try It

29) Translate and solve:

The annual property tax on the Mehta's house is $\$1,800$, calculated as $\frac{15}{1,000}$ of the assessed value of the house. What is the assessed value of the Mehta's house?

Solution

$\$120,000$

30) Translate and solve:

Stella planted 14 flats of flowers in $\frac{2}{3}$ of her garden. How many flats of flowers would she need to fill the whole garden?

Solution

21 flats

Key Concepts

- **The Division Property of Equality**—For any numbers a, b, and c, and

$c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

When you divide both sides of an equation by any non-zero number, you still have equality.

- **The Multiplication Property of Equality**—For any numbers a, b, and c, if

  $a = b$, then $ac = bc$.

  If you multiply both sides of an equation by the same number, you still have equality.

Exercises: Solve Equations Using the Division and Multiplication Properties of Equality

Instructions: For questions 1-36, solve each equation using the Division and Multiplication Properties of Equality and check the solution.

1. $8x = 56$

Solution

$x = 7$

---

2. $7p = 63$

---

3. $-5c = 55$

Solution

$c = -11$

4. $-9x = -27$

5. $-809 = 15y$

Solution

$y = -\frac{809}{15}$

6. $-731 = 19y$

7. $-37p = -541$

Solution

$p = -\frac{541}{37}$

8. $-19m = -586$

9. $0.25z = 3.25$

Solution

$z = 13$

10. $0.75a = 11.25$

11. $-13x = 0$

Solution

$x = 0$

---

12. $24x = 0$

---

13. $\frac{x}{4} = 35$

Solution

$x = 140$

---

14. $\frac{z}{2} = 54$

---

15. $-20 = \frac{q}{-5}$

Solution

$q = 100$

---

16. $\frac{c}{-3} = -12$

---

17. $\frac{y}{9} = -16$

Solution

$y = -144$

---

18. $\frac{q}{6} = -38$

---

19. $\frac{m}{-12} = 45$

Solution

$m = -540$

---

20. $-24 = \frac{p}{-20}$

---

21. $-y = 6$

Solution

$y = -6$

---

22. $-u = 15$

---

23. $-v = -72$

Solution

$v = 72$

---

24. $-x = -39$

---

25. $\frac{2}{3}y = 48$

Solution

$y = 72$

---

26. $\frac{3}{5}r = 75$

---

27. $-\frac{5}{8}w = 40$

Solution

$w = -64$

---

28. $24 = -\frac{3}{4}x$

---

29. $-\frac{2}{5} = \frac{1}{10}a$

Solution

$a = -4$

---

30. $-\frac{1}{3}q = -\frac{5}{6}$

---

31. $-\frac{7}{10}x = -\frac{14}{3}$

Solution

$x = \frac{20}{3}$

---

32. $\frac{3}{8}y = -\frac{1}{4}$

---

33. $\frac{7}{12} = -\frac{3}{4}p$

Solution

$$p = -\frac{7}{9}$$

---

34. $\frac{11}{18} = -\frac{5}{6}q$

---

35. $-\frac{5}{18} = -\frac{10}{9}u$

Solution

$$u = \frac{1}{4}$$

---

36. $-\frac{7}{20} = -\frac{7}{4}v$

Exercises: Solve Equations that Require Simplification

Instructions: For questions 37-46, solve each equation requiring simplification

37. $100 - 16 = 4p - 10p - p$

Solution

$p = -12$

---

38. $-18 - 7 = 5t - 9t - 6t$

---

39. $\frac{7}{8}n - \frac{3}{4}n = 9 + 2$

Solution

$n = 88$

---

40. $\frac{5}{12}q + \frac{1}{2}q = 25 - 3$

---

41. $0.25d + 0.10d = 6 - 0.75$

Solution

$d = 15$

---

42. $0.05p - 0.01p = 2 + 0.24$

---

43. $-10(q - 4) - 57 = 93$

Solution

$q = -11$

---

44. $-12(d - 5) - 29 = 43$

---

45. $-10(x+4)-19=85$

Solution

$x=-\frac{72}{5}$

---

46. $-15(z+9)-11=75$

Exercises: Mixed Practice

Instructions: For questions 47-65, solve each equation.

47. $\frac{9}{10}x=90$

Solution

$x=100$

---

48. $\frac{5}{12}y=60$

---

49. $y+46=55$

Solution

$y = 9$

---

50. $x + 33 = 41$

---

51. $\frac{w}{-2} = 99$

Solution

$w = -198$

---

52. $\frac{s}{-3} = -60$

---

53. $27 = 6a$

Solution

$a = \frac{9}{2}$

---

54. $-a = 7$

---

55. $-x = 2$

Solution

$x = -2$

---

56. $z - 16 = -59$

---

57. $m-41=-14$

Solution

$m=27$

---

58. $0.04r=52.60$

---

59. $63.90=0.03p$

Solution

$p=2130$

---

60. $-15x=-120$

---

61. $84=-12z$

Solution

$y=-7$

---

62. $19.36=x-0.2x$

---

63. $c-0.3c=35.70$

Solution

$c=51$

64. $-y = -9$

65. $-x = -8$

Solution

$x = 8$

Exercises: Translate to an Equation and Solve

Instructions: For questions 66-85, translate to an equation and then solve

66. 187 **is the product of** -17 **and** m**.**

67. 133 **is the product of** -19 **and** n**.**

Solution

$$\begin{aligned} 133 &= -19n \\ n &= -7 \end{aligned}$$

68. -184 **is the product of** 23 **and** p.

---

69. -152 **is the product of** 8 **and** q.

Solution

$$\begin{aligned} -152 &= 8q \\ q &= -19 \end{aligned}$$

---

70. u **divided by** 7 **is equal to** -49.

---

71. r **divided by 12 is equal to** -48.

Solution

$$\begin{aligned} \frac{r}{12} &= -48 \\ r &= -576 \end{aligned}$$

---

72. h **divided by** -13 **is equal to** -65.

---

73. j **divided by** -20 **is equal to** -80.

Solution

$$\begin{aligned}\frac{j}{-20} &= -80\\ j &= 1,600\end{aligned}$$

---

74. The quotient c **and** -19 **is** 38.

---

75. The quotient of b **and** -6 **is** 18.

Solution

$$\begin{aligned}\frac{b}{-6} &= 18\\ b &= -108\end{aligned}$$

---

76. The quotient of h **and** 26 **is** -52.

---

77. The quotient k **and** 22 **is** -66.

Solution

$\frac{k}{22} = -66; k = -1,452$

---

78. Five-sixths of y is 15.

---

79. Three-tenths of x is 15.

Solution

$$\begin{aligned} \frac{3}{10}x &= 15 \\ x &= 50 \end{aligned}$$

---

80. Four-thirds of w is 36.

---

81. Five-halves of v is 50.

Solution

$$\begin{aligned} \frac{5}{2}v &= 50 \\ v &= 20 \end{aligned}$$

---

82. The sum of nine-tenths and g is two-thirds.

83. The sum of two-fifths and f is one-half.

Solution

$$\begin{aligned}\frac{2}{5}+f &= \frac{1}{2}\\ f &= \frac{1}{10}\end{aligned}$$

84. The difference of p and one-sixth is two-thirds.

85. The difference of q and one-eighth is three-fourths.

Solution

$$\begin{aligned}q-\frac{1}{8} &= \frac{3}{4}\\ q &= \frac{7}{8}\end{aligned}$$

Exercises: Translate and Solve Applications

Instructions: For questions 86-93, translate into an equation and solve.

86. Kindergarten. Connie's kindergarten class has 24 children. She wants them to get into 4 equal groups. How many children will she put in each group?

---

87. Balloons. Ramona bought 18 balloons for a party. She wants to make 3 equal bunches. How many balloons did she use in each bunch?

Solution

6 balloons

---

88. Tickets. Mollie paid $\$36.25$ for 5 movie tickets. What was the price of each ticket?

---

89. Shopping. Serena paid $\$12.96$ for a pack of 12 pairs of sport socks. What was the price of pair of sport socks?

Solution

$\$1.08$

---

90. Sewing. Nancy used 14 yards of fabric to make flags for one-third of the drill team. How much fabric, would Nancy need to make flags for the whole team?

---

91. MPG. John's SUV gets 18 miles per gallon (mpg). This is half as many mpg as his wife's hybrid car. How many miles per gallon does the hybrid car get?

Solution

36 mpg

---

92. Height. Aiden is 27 inches tall. He is $\frac{3}{8}$ as tall as his father. How tall is his father?

---

93. Real estate. Bea earned $\$11,700$ commission for selling a house, calculated as $\frac{6}{100}$ of the selling price. What was the selling price of the house?

Solution

$\$195,000$

Exercises: Everyday Math

Instructions: For questions 94-95, solve the given everyday math word problems.

94. Commission. Every week Perry gets paid $\$150$ plus 12% of his total sales amount. Solve the equation $840=150+0.12(a-1250)$ for a, to find the total amount Perry must sell in order to be paid $\$840$ one week.

---

95. Stamps. Travis bought $\$9.45$ worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 5 less than the number of 49-cent stamps. Solve the equation $0.49s+0.21(s-5)=9.45$ for s, to find the number of 49-cent stamps Travis bought.

Solution

$15\,49$-cent stamps

Exercises: Writing Exercises

Instructions: For questions 96-97, answer the given writing exercises.

96. Frida started to solve the equation $-3x=36$ by adding 3 to both sides. Explain why Frida's method will not solve the equation.

---

97. Emiliano thinks $x=40$ is the solution to the equation $\frac{1}{2}x=80$. Explain why he is wrong.

Solution

Answers will vary.

3.3 SOLVE EQUATIONS WITH VARIABLES AND CONSTANTS ON BOTH SIDES

Learning Objectives

By the end of this section, you will be able to:

- Solve an equation with constants on both sides
- Solve an equation with variables on both sides
- Solve an equation with variables and constants on both sides

Try It

Before you get started, take this readiness quiz:

1) Simplify: $4y-9+9$.

Solve Equations with Constants on Both Sides

In all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we will learn to solve equations in which the variable terms, or constant terms, or both are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the "variable side", and the other side of the equation to be the "constant side." Then, we will use the Subtraction and Addition Properties of Equality to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that began with variables and constants on both sides into the form $ax = b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

Example 1

Solve: $7x + 8 = -13$.

Solution

In this equation, the variable is found only on the left side. It makes sense to call the left side the "variable" side. Therefore, the right side will be the "constant" side. We will write the labels above the equation to help us remember what goes where.

$$\overset{variable}{7x + 8 =} \overset{constant}{-13}$$

Since the left side is the "x", or variable side, the 8 is out of place. We must "undo" adding 8 by subtracting 8, and to keep the equality we must subtract 8 from both sides.

Step 1: **Use the Subtraction Property of Equality.**

$$\begin{array}{lrcl} & 7x + 8 - 8 &=& -13 - 8 \\ \text{Simplify} & 7x &=& -21 \end{array}$$

Now all the variables are on the left and the constant on the right.
The equation looks like those you learned to solve earlier.

Step 2: **Use the Division Property of Equality.**

$$\begin{array}{ll} & \frac{7x}{7} = \frac{-21}{7} \\ \text{Simplify} & x = -3 \end{array}$$

***Step 3:* Check:**

$$7x + 8 = -13$$

***Step 4:* Let $x = -3$.**

$$\begin{aligned} 7(-3) + 8 &\stackrel{?}{=} -13 \\ -21 + 8 &\stackrel{?}{=} -13 \\ -13 &= -13 \checkmark \end{aligned}$$

Try It

2) Solve: $3x + 4 = -8$.

Solution

$x = -4$

3) Solve: $5a + 3 = -37$.

Solution

$a = -8$

Example 2

Solve: $8y - 9 = 31$.

Solution

Notice, the variable is only on the left side of the equation, so we will call this side the "variable" side, and the right side will be the "constant" side. Since the left side is the "variable" side, the

9 is out of place. It is subtracted from the $8y$, so to "undo" subtraction, add 9 to both sides. Remember, whatever you do to the left, you must do to the right.

$$\underset{\text{variable}}{8y - 9} = \underset{\text{constant}}{31}$$

Step 1: **Add 9 to both sides.**

$$8y - 9 + 9 = 31 + 9$$

Simplify $$8y = 40$$

The variables are now on one side and the constants on the other.
We continue from here as we did earlier.

Step 2: **Divide both sides by 8.**

$$\frac{8y}{8} = \frac{40}{8}$$

Simplify $$y = 5$$

Step 3: **Check:**

$$8y - 9 = 31$$

Step 4: **Let $y = 5$.**

$$8 \cdot 5 - 9 \stackrel{?}{=} 31$$
$$40 - 9 \stackrel{?}{=} 31$$
$$31 = 31 \checkmark$$

Try It

4) Solve: $5y - 9 = 16$.

Solution

$y = 5$

5) Solve: $3m - 8 = 19$.

Solution
$m = 9$

Solve Equations with Variables on Both Sides

What if there are variables on both sides of the equation? For equations like this, begin as we did above—choose a "variable" side and a "constant" side, and then use the subtraction and addition properties of equality to collect all variables on one side and all constants on the other side.

Example 3

Solve: $9x = 8x - 6$.

Solution
Here the variable is on both sides, but the constants only appear on the right side, so let's make the right side the "constant" side. Then the left side will be the "variable" side.

$$\overset{variable}{9x} = \overset{constant}{8x - 6}$$

Step 1: **We don't want any x's on the right, so subtract the $8x$ from both sides.**

$$\begin{array}{lr} & 9x - 8x = 8x - 8x - 6 \\ \text{Simplify} & x = 6 \end{array}$$

We succeeded in getting the variables on one side and the constants on the other, and have obtained the solution.

Step 3: **Check:**

$$9x = 8x - 6$$

***Step 4:* Let** $x = -6$**.**

$$\begin{aligned} 9x(-6) &\stackrel{?}{=} 8x(-6) - 6 \\ -54 &\stackrel{?}{=} -48 - 6 \\ -54 &= -54\checkmark \end{aligned}$$

Try It

6) Solve: $6n = 5n - 10$.

Solution

$n = -10$

7) Solve: $-6c = -7c - 1$.

Solution

$c = -1$

Example 4

Solve: $5y - 9 = 8y$.

Solution

The only constant is on the left and the y's are on both sides. Let's leave the constant on the left and get the variables to the right.

$$\underbrace{5y - 9}_{\text{constant}} = \underbrace{8y}_{\text{variable}}$$

Step 1: **Subtract $5y$ from both sides.**

$$\begin{array}{lr} & 5y - 5y - 9 = 8y - 5y \\ \text{Simplify} & -9 = 3y \end{array}$$

Step 2: **We have the y's on the right and the constants on the left. Divide both sides by 3.**

$$\begin{array}{lr} & \frac{-9}{3} = \frac{3y}{3} \\ \text{Simplify} & -3 = y \end{array}$$

Step 3: **Check:**

$$5y - 9 = 8y$$

Step 4: **Let $y = -3$.**

$$\begin{aligned} 5(-3) - 9 &\stackrel{?}{=} 8(-3) \\ -15 - 9 &\stackrel{?}{=} -24 \\ -24 &= -24 \checkmark \end{aligned}$$

Try It

8) Solve: $3p - 14 = 5p$.

Solution

$p = -7$

9) Solve: $8m + 9 = 5m$.

Solution

$m = -3$

Example 5

Solve: $12x = -x + 26$.

Solution

The only constant is on the right, so let the left side be the "variable" side.

$$\underset{\text{variable}}{12x} = \underset{\text{constant}}{-x + 26}$$

***Step 1:* Remove the $-x$ from the right side by adding x to both sides.**

$$\begin{array}{ll} & 12x + x = -x + x + 26 \\ \text{Simplify} & 13x = 26 \end{array}$$

***Step 2:* All the x's are on the left and the constants are on the right. Divide both sides by 13.**

$$\begin{array}{ll} & \frac{13x}{13} = \frac{26}{13} \\ \text{Simplify} & x = 2 \end{array}$$

Try It

10) Solve: $12j = -4j + 32$.

Solution

$j = 2$

11) Solve: $8h = -4h + 12$.

Solution

$h = 1$

Solve an Equation with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

Example 6

Solve: $7x + 5 = 6x + 2$.

Solution

***Step 1:* Choose which side will be the "variable" side – the other side will be the "constant" side.**

The variable terms are $7x$ and $6x$.

Since 7 is greater than 6, we will make the left side the "x" side.

The right side will be the "constant" side.

$$\overset{variable}{7x + 5} = \overset{constant}{6x + 2}$$

***Step 2:* Collect the variable terms to the "variable" side of the equation, using the addition or subtraction property of equality.**

With the right side as the "constant" side, the $6x$ is out of place, so subtract $6x$ from both sides.

$$7x - 6x + 5 = 6 - 6x + 2$$

Combine like terms. $$x + 5 = 2$$

Now, the variable is only on the left side!

Step 3: **Collect all the constants to the other side of the equation, using the addition or subtraction property of equality.**

The right side is the "constant" side, so the 5 is out of place.

Subtract from both sides.	$x+5-5=2-5$
Simplify.	$x=-3$

Step 4: **Make the coefficient of the variable equal 1, using the multiplication or division property of equality.**

The coefficient of x is one.

The equation is solved.

Step 5: **Check.**

Let x=-3.	$7x+6=6x+2$
Simplify.	$(-3)+5=6(-3)+2$
Add.	$-21+5=-18+2$
	$-16=-16\checkmark$

Try It

12) Solve: $12x+8=6x+2$.

Solution

$x=-1$

13) Solve: $9y+4=7y+12$.

Solution

$y=4$

We'll list the steps below so you can easily refer to them. But we'll call this the 'Beginning Strategy' because we'll be adding some steps later in this chapter.

How to

Beginning Strategy for Solving Equations with Variables and Constants on Both Sides of the Equation.

1. Choose which side will be the "variable" side—the other side will be the "constant" side.
2. Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
5. Check the solution by substituting it into the original equation.

In Step 1, a helpful approach is to make the "variable" side the side that has the variable with the larger coefficient. This usually makes the arithmetic easier.

Example 7

Solve: $8n-4=-2n+6$.

Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

***Step 1:* Since $8>-2$, make the left side the "variable" side.**

$$\underbrace{8n-4}_{}^{\textit{variable}} = \overbrace{-2n+6}^{\textit{constant}}$$

Step 2: **We don't want variable terms on the right side—add $2n$ to both sides to leave only constants on the right.**

$$\begin{array}{ll} & 8n+2n-4=-2n+2n+6 \\ \text{Combine like terms.} & 10n-4=6 \end{array}$$

Step 3: **We don't want any constants on the left side, so add 4 to both sides.**

$$\begin{array}{ll} & 10n-4+4=6+4 \\ \text{Simplify} & 10n=10 \end{array}$$

Step 4: **The variable term is on the left and the constant term is on the right. To get the coefficient of n to be one, divide both sides by 10.**

$$\begin{array}{ll} & \frac{10n}{10}=\frac{10}{10} \\ \text{Simplify} & n=1 \end{array}$$

Step 5: **Check:**

$$8n-4=-2n+6$$

Step 6: **Let $n=1$.**

$$\begin{array}{c} 8\cdot 1-4\stackrel{?}{=}2\cdot 1+6 \\ 8-4\stackrel{?}{=}-2+6 \\ 4=4\checkmark \end{array}$$

Try It

14) Solve: $8q-5=-4q+7$.

Solution
$q=1$

15) Solve: $7n-3=n+3$.

Solution
$n=1$

Example 8

Solve: $7a-3=13a+7$.

Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since $13>7$, make the right side the "variable" side and the left side the "constant" side.

$$\underset{\text{constant}}{7a-3}=\underset{\text{variable}}{13a+7}$$

Step 1: **Subtract $7a$ from both sides to remove the variable term from the left.**

$$7a-7a-3=13a-7a+7$$

Combine like terms. $$-3=6a+7$$

Step 2: **Subtract 7 from both sides to remove the constant from the right.**

$$-3-7=6a+7-7$$

Simplify. $$-10=6a$$

Step 3: **Divide both sides by 6 to make 1 the coefficient of a.**

$$\frac{-10}{6}=\frac{6a}{6}$$

Simplify. $$-\frac{5}{3}=a$$

Step 4: **Check:**

$$7a-3=13a+7$$

Step 5: **Let $a=-\frac{5}{3}$.**

$$8\left(-\frac{5}{3}\right)-3\stackrel{?}{=}13\left(-\frac{5}{3}\right)+7$$
$$-\frac{35}{3}-\frac{9}{3}\stackrel{?}{=}-\frac{65}{3}+\frac{21}{3}$$
$$-\frac{54}{3}=-\frac{54}{3}\checkmark$$

Try It

16) Solve: $2a-2=6a+18$.

Solution

$a=-5$

17) Solve: $4k-1=7k+17$.

Solution

$k=-6$

In the last example, we could have made the left side the "variable" side, but it would have led to a negative coefficient on the variable term. (Try it!) While we could work with the negative, there is less chance of errors when working with the positives. The strategy outlined above helps avoid the negatives!

To solve an equation with fractions, we just follow the steps of our strategy to get the solution!

Example 9

Solve: $\frac{5}{4}x+6=\frac{1}{4}x-2$.

Solution

Since $\frac{5}{4}>\frac{1}{4}$, make the left side the "variable" side and the right side the "constant" side.

$$\underbrace{\frac{5}{4}x+6}_{\text{variable}}=\underbrace{\frac{1}{4}x-2}_{\text{constant}}$$

***Step 1:* Subtract $\frac{1}{4}x$ from both sides.**

$$\begin{array}{ll} & \frac{5}{4}x - \frac{1}{4}x + 6 = \frac{1}{4}x - \frac{1}{4}x - 2 \\ \text{Combine like terms.} & x + 6 = -2 \end{array}$$

Step 2: **Subtract 6 from both sides.**

$$x + 6 - 6 = -2 - 6$$

Step 3: **Simplify.**

$$x = -8$$

Step 4: **Check: Let $x = -8$**

$$\begin{aligned} \frac{5}{4}x + 6 &= \frac{1}{4}x - 2 \\ \frac{5}{4}(-8) + 6 &\stackrel{?}{=} \frac{1}{4}(-8) - 2 \\ -10 + 6 &\stackrel{?}{=} -2 - 2 \\ -4 &= -4 \checkmark \end{aligned}$$

Try It

18) Solve: $\frac{7}{8}x - 12 = -\frac{1}{8}x - 2$.

Solution

$x = 10$

19) Solve: $\frac{7}{6}y + 11 = \frac{1}{6}y + 8$.

Solution

$y = -3$

We will use the same strategy to find the solution for an equation with decimals.

Example 10

Solve: $7.8x + 4 = 5.4x - 8$.

Solution

Since $7.8 > 5.4$, make the left side the "variable" side and the right side the "constant" side.

$$\underset{\text{variable side}}{7.8x + 4} = \underset{\text{constant side}}{5.4x - 8}$$

Step 1: **Subtract $5.4x$ from both sides.**

$$7.8x - 5.4x + 4 = 5.4x - 5.4x - 8$$

Combine like terms. $2.4x + 4 = -8$

Step 2: **Subtract 4 from both sides.**

$$2.4x + 4 - 4 = -8 - 4$$

Simplify. $2.4x = -12$

Step 3: **Use the Division Property of Equality.**

$$\frac{2.4x}{2.4} = \frac{-12}{2.4}$$

Simplify. $x = -5$

Step 4: **Check:**

$$7.8x + 4 = 5.4x - 5$$

Step 5: **Let $x = -5$.**

$$7.8(-5) + 4 = 5.4(-5) - 8$$
$$-39 + 4 \stackrel{?}{=} -27 - 8$$
$$-35 = -35 \checkmark$$

Try It

20) Solve: $2.8x + 12 = -1.4x - 9$.

Solution

$x = -5$

21) Solve: $3.6y + 8 = 1.2y - 4$.

Solution

$y = -5$

Key Concepts

Beginning Strategy for Solving an Equation with Variables and Constants on Both Sides of the Equation

1. Choose which side will be the "variable" side—the other side will be the "constant" side.
2. Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
5. Check the solution by substituting it into the original equation.

Exercises: Solve Equations with Constants on Both Sides

Instructions: For questions 1-11, solve the following equations with constants on both sides.

1. $12x - 8 = 64$

Solution

$x = 6$

---

2. $14w + 5 = 117$

---

3. $15y + 7 = 97$

Solution

$y = 6$

---

4. $2a + 8 = -28$

---

5. $3m + 9 = -15$

Solution

$m = -8$

---

6. $-62 = 8n - 6$

7. $-77 = 9b - 5$

Solution

$b = -8$

8. $35 = -13y + 9$

9. $60 = -21x - 24$

Solution

$x = -4$

10. $-12p - 9 = 9$

11. $-14q - 2 = 16$

Solution

$q = -\frac{9}{7}$

Exercises: Solve Equations with Variables on Both Sides

Instructions: For questions 12-23, solve the following equations with variables on both sides.

12. $19z = 18z - 7$

---

13. $21k = 20k - 11$

Solution

$k = -11$

---

14. $9x + 36 = 15x$

---

15. $8x + 27 = 11x$

Solution

$x = 9$

---

16. $c = -3c - 20$

---

17. $b = -4b - 15$

Solution

$b = -3$

18. $9q = 44 - 2q$

19. $5z = 39 - 8z$

Solution

$z = 3$

20. $6y + \frac{1}{2} = 5y$

21. $4x + \frac{3}{4} = 3x$

Solution

$x = -\frac{3}{4}$

22. $-18a - 8 = -22a$

23. $-11r - 8 = -7r$

Solution

$r = -2$

Exercises: Solve Equations with Variables and Constants on Both Sides

Instructions: For questions 24-51, solve the equations with variables and constants on both sides.

24. $8x - 15 = 7x + 3$

---

25. $6x - 17 = 5x + 2$

Solution

$x = 19$

---

26. $26 + 13d = 14d + 11$

---

27. $21 + 18f = 19f + 14$

Solution

$f = 7$

---

28. $2p - 1 = 4p - 33$

---

29. $12q - 5 = 9q - 20$

Solution

$q = -5$

---

30. $4a + 5 = -a - 40$

---

31. $8c + 7 = -3c - 37$

Solution

$c = -4$

---

32. $5y - 30 = -5y + 30$

---

33. $7x - 17 = -8x + 13$

Solution

$x = 2$

---

34. $7s + 12 = 5 + 4s$

---

35. $9p + 14 = 6 + 4p$

Solution

$p = -\frac{8}{5}$

---

36. $2z - 6 = 23 - z$

---

37. $3y - 4 = 12 - y$

Solution

$y = 4$

---

38. $\frac{5}{3}c - 3 = \frac{2}{3}c - 16$

---

39. $\frac{7}{4}m - 7 = \frac{3}{4}m - 13$

Solution

$m = -6$

---

40. $8 - \frac{2}{5}q = \frac{3}{5}q + 6$

---

41. $11 - \frac{1}{5}a = \frac{4}{5}a + 4$

Solution

$a = 7$

---

42. $\frac{4}{3}n + 9 = \frac{1}{3}n - 9$

---

43. $\frac{5}{4}a + 15 = \frac{3}{4}a - 5$

Solution

$a = -40$

---

44. $\frac{1}{4}y + 7 = \frac{3}{4}y - 3$

---

45. $\frac{3}{5}p + 2 = \frac{4}{5}p - 1$

Solution

$p = 15$

---

46. $14n + 8.25 = 9n + 19.60$

---

47. $13z + 6.45 = 8z + 23.75$

Solution

$z = 3.46$

---

48. $2.4w - 100 = 0.8w + 28$

---

49. $2.7w - 80 = 1.2w + 10$

Solution

$w = 60$

---

50. $5.6r + 13.1 = 3.5r + 57.2$

---

51. $6.6x - 18.9 = 3.4x + 54.7$

Solution

$x = 23$

Exercises: Everyday Math

Instructions: For questions 52-53, answer the given everyday math word problems.

52. Concert tickets. At a school concert the total value of tickets sold was $\$1506$. Student tickets sold for $\$6$ and adult tickets sold for 9. The number of adult tickets sold was 5 less than 3 times the number of student tickets. Find the number of student tickets sold, s, by solving the equation $6s + 27s - 45 = 1506$.

---

53. Making a fence. Jovani has 150 feet of fencing to make a rectangular garden in his backyard. He wants the length to be 15 feet more than the width. Find the width, w, by solving the equation $150 = 2w + 30 + 2w$.

Solution

30 feet

Exercises: Writing Exercises

Instructions: For questions 54-57, answer the given writing exercises.

54. Solve the equation $\frac{6}{5}y - 8 = \frac{1}{5}y + 7$ explaining all the steps of your solution as in the examples in this section.

---

55. Solve the equation $10x + 14 = -2x + 38$ explaining all the steps of your solution as in the examples in this section.

Solution

$x = 2$ (Justifications will vary.)

---

56. When solving an equation with variables on both sides, why is it usually better to choose the side with the larger coefficient of x to be the "variable" side?

---

57. Is $x = -2$ a solution to the equation $5 - 2x = -4x + 1$? How do you know?

Solution

Yes. Justifications will vary.

3.4 USE A GENERAL STRATEGY TO SOLVE LINEAR EQUATIONS

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using a general strategy
- Classify equations

Try It

Before you get started, take this readiness quiz:

1) Simplify: $-(a-4)$
2) Multiply: $\frac{3}{2}(12x+20)$.
3) Simplify: $5-2(n+1)$.
4) Multiply: $3(7y+9)$.
5) Multiply: $(2.5)(6.4)$.

Solve Equations Using a General Strategy

Until now we have dealt with solving one specific form of a linear equation. It is time now to lay out one overall strategy that can be used to solve any linear equation. Some equations we solve will not require all these steps to solve, but many will.

Beginning by simplifying each side of the equation makes the remaining steps easier.

How to

The general strategy for solving linear equations.

1. Simplify each side of the equation as much as possible.
   Use the Distributive Property to remove any parentheses.
   Combine like terms.
2. Collect all the variable terms on one side of the equation.
   Use the Addition or Subtraction Property of Equality.
3. Collect all the constant terms on the other side of the equation.
   Use the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable term to equal to 1.
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. Check the solution.
   Substitute the solution into the original equation to make sure the result is a true statement.

Example 1

Solve: $-6(x+3)=24$.

Solution

***Step 1:* Simplify each side of the equation as much as possible.**

Use the Distributive Property.

$$\begin{aligned}-6\left(x+3\right)&=24\\-6x-18&=24\end{aligned}$$

Notice that each side of the equation is simplified as much as possible.

***Step 2:* Collect all variable terms on one side of the equation.**

Nothing to to – all x 's are on the left side.

***Step 3:* Collect constant terms on the other side of the equation.**

To get constants only on the right, add 18 to each side.

$$\begin{aligned}&-6x-18+18=24+18\\\text{Simplify.}\quad&-6x=42\end{aligned}$$

***Step 4:* Make the coefficient of the variable term to equal 1.**

$$\begin{aligned}\text{Divide each side by -6.}\quad&\frac{-6x}{-6}=\frac{42}{-6}\\\text{Simplify.}\quad&x=-7\end{aligned}$$

***Step 5:* Check the solution.**

Let $x=-7$

$$\begin{aligned}-6\left(x+3\right)&=24\\-6\left(-7+3\right)&\stackrel{?}{=}24\\\text{Simplify.}-6\left(-4\right)&\stackrel{?}{=}24\\\text{Multiply.}\ 24&=24\end{aligned}$$

Try It

6) Solve: $5(x+3)=35$.

Solution

$x=4$

7) Solve: $6(y-4)=-18$.

Solution

$y=1$

Example 2

Solve: $-(y+9)=8$.

Solution

Step 1: **Simplify each side of the equation as much as possible by distributing.**

$$-y-9=8$$

Step 2: **The only y term is on the left side, so all variable terms are on the left side of the equation.**

Step 3: **Add 9 to both sides to get all constant terms on the right side of the equation.**

$$\begin{array}{ll} & -y-9+9=8+9 \\ \text{Simplify.} & -y=17 \end{array}$$

Step 4: **Rewrite** $-y$ **as** $-1y$**.**

$$-1y=17$$

Step 5: **Make the coefficient of the variable term to equal to 1 by dividing both sides by -1.**

$$\frac{-1y}{-1} = \frac{17}{-1}$$

Simplify. $$y = -17$$

Step 6: **Check:**

$$-(y+9) = 8$$

Step 7: **Let $y = -17$.**

$$-(-17+9) \stackrel{?}{=} 8$$
$$-(-8) \stackrel{?}{=} 8$$
$$8 = 8 \checkmark$$

Try It

8) Solve: $-(y+8) = -2$.

Solution

$y = -6$

9) Solve: $-(z+4) = -12$

Solution

$z = 8$

Example 3

Solve: $5(a-3)+5 = -10$.

Solution

Step 1: **Simplify each side of the equation as much as possible.**

$$\begin{array}{ll} \text{Distribute.} & 5a - 15 + 5 = -10 \\ \text{Combine like terms.} & 5a - 10 = -10 \end{array}$$

Step 2: **The only a term is on the left side, so all variable terms are on one side of the equation.**

Step 3: **Add 10 to both sides to get all constant terms on the other side of the equation.**

$$\begin{array}{ll} & 5a - 10 + 10 = -10 + 10 \\ \text{Simplify.} & 5a = 0 \end{array}$$

Step 4: **Make the coefficient of the variable term to equal to *1* by dividing both sides by 5.**

$$\begin{array}{ll} & \frac{5a}{5} = \frac{0}{5} \\ \text{Simplify.} & a = 0 \end{array}$$

Step 5: **Check:**

$$5(a-3) + 5 = -10$$

Step 6: **Let $a = 0$.**

$$\begin{aligned} 5(0-3) + 5 &\stackrel{?}{=} -10 \\ 5(-3) + 5 &\stackrel{?}{=} -10 \\ -15 + 5 &\stackrel{?}{=} -10 \\ -10 &= -10 \checkmark \end{aligned}$$

Try It

10) Solve: $2(m-4)+3=-1$.

Solution

$m = 2$

11) Solve: $7(n-3)-8=-15$.

Solution

$n = 2$

Example 4

Solve: $\frac{2}{3}(6m-3)=8-m$.

Solution

***Step 1:* Distribute.**

$$4m-2=8-m$$

***Step 2:* Add m to get the variables only to the left.**

$$\begin{array}{ll} & 4m+m-2=8-m+m \\ \text{Simplify.} & 5m-2=8 \end{array}$$

***Step 3:* Add 2 to get constants only on the right.**

$$\begin{array}{ll} & 5m-2+2=8+2 \\ \text{Simplify.} & 5m=10 \end{array}$$

***Step 4:* Divide by 5.**

$$\begin{array}{ll} & \frac{5m}{5}=\frac{10}{5} \\ \text{Simplify.} & m=2 \end{array}$$

***Step 5:* Check:**

$$\frac{2}{3}(6m-3)=8-m$$

***Step 6:* Let $m=2$.**

$$\begin{array}{r} \frac{2}{3}(6\cdot 2\cdot -3)\stackrel{?}{=}8-2 \\ \frac{2}{3}(12-3)\stackrel{?}{=}6 \\ \frac{2}{3}(9)\stackrel{?}{=}6 \\ 6=6\checkmark \end{array}$$

Try It

12) Solve: $\frac{1}{3}(6u+3)=7-u$.

Solution

$u = 2$

13) Solve: $\frac{2}{3}(9x - 12) = 8 + 2x$.

Solution
$x = 4$

Example 5

Solve: $8 - 2(3y + 5) = 0$.

Solution
Step 1: **Simplify—use the Distributive Property.**

$$\begin{array}{lr} & 8 - 6y - 10 = 0 \\ \text{Combine like terms.} & -6y - 2 = 0 \end{array}$$

Step 2: **Add 2 to both sides to collect constants on the right.**

$$\begin{array}{lr} & -6y - 2 + 2 = 0 + 2 \\ \text{Simplify.} & -6y = 2 \end{array}$$

Step 3: **Divide both sides by -6.**

$$\begin{array}{lr} & \frac{-6y}{-6} = \frac{2}{-6} \\ \text{Simplify.} & y = -\frac{1}{3} \end{array}$$

Step 4: **Check:**

Let $y = -\frac{1}{3}$

$$\begin{aligned} 8 - 2(3y + 5) &= 0 \\ 8 - 2\left[3\left(-\frac{1}{3}\right) + 5\right] &= 0 \\ 8 - 2(-1 + 5) &\stackrel{?}{=} 0 \\ 8 - 2(4) &\stackrel{?}{=} 0 \\ 8 - 8 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

Try It

14) Solve: $12-3(4j+3)=-17$

Solution

$$j=\frac{5}{3}$$

15) Solve: $-6-8(k-2)=-10$.

Solution

$$k=\frac{5}{2}$$

Example 6

Solve: $4(x-1)-2=5(2x+3)+6$.

Solution

Step 1: **Distribute.**

$$\begin{array}{ll} & 4x-4-2=10x+15+6 \\ \text{Combine like terms.} & 4x-6=10x+21 \end{array}$$

Step 2: **Subtract** $4x$ **to get the variables only on the right side since** $10>4$**.**

$$\begin{array}{ll} & 4x-4x-6=10-4x+21 \\ \text{Simplify.} & -6=6x+21 \end{array}$$

Step 3: **Subtract 21 to get the constants on left.**

$$\begin{array}{ll} & 6-21=6x+21-21 \\ \text{Simplify.} & -27=6x \end{array}$$

Step 4: **Divide by 6.**

Simplify. $$\frac{-27}{6} = \frac{6x}{6}$$ $$-\frac{9}{2} = x$$

Step 5: **Check:**

$$4(x-1)-2=5(2x+3)+6$$

Step 6: **Let $x = -\frac{9}{2}$.**

$$\begin{aligned} 4\left(-\frac{9}{2}\right)-2 &\stackrel{?}{=} 5\left[2\left(-\frac{9}{2}\right)+3\right]+6 \\ 4\left(-\frac{11}{2}\right)-2 &\stackrel{?}{=} 5(-9+3)+6 \\ -22-2 &\stackrel{?}{=} 5(-6)+6 \\ -24 &\stackrel{?}{=} -30+6 \\ -24 &= -24\checkmark \end{aligned}$$

Try It

16) Solve: $6(p-3)-7=5(4p+3)-12$.

Solution

$p=-2$

17) Solve: $8(q+1)-5=3(2q-4)-1$.

Solution

$q=-8$

Example 7

Solve: $10[3-8(2s-5)]=15(40-5s)$.

Solution

Step 1: **Simplify from the innermost parentheses first.**

$$10[3-16s+40]=15(40-5s)$$

***Step 2:* Combine like terms in the brackets.**

$$10[43-16s]=15(40-5s)$$

***Step 3:* Distribute.**

$$430-160s=600-75s$$

***Step 4:* Add $160s$ to get the S's to the right.**

$$\begin{array}{ll} & 430-160+160s=600-85s+160s \\ \text{Simplify.} & 430=600+85s \end{array}$$

***Step 5:* Subtract 600 to get the constants to the left.**

$$\begin{array}{ll} & 430-600=85s+600-600 \\ \text{Simplify.} & -170=85s \end{array}$$

***Step 6:* Divide.**

$$\begin{array}{ll} & \frac{-170}{85}=\frac{85s}{85} \\ \text{Simplify.} & -2=s \end{array}$$

***Step 7:* Check:**

$$10[3-8(2s-5)]=15(40-5s)$$

***Step 8:* Substitute $s=-2$.**

$$\begin{aligned} 10\left[3-8\left(2(-2)-5\right)\right] &\stackrel{?}{=} 15\left(40-5\left(-2\right)\right) \\ 10\left[3-8\left(-4-5\right)\right] &\stackrel{?}{=} 15\left(40+10\right) \\ 10\left[3-8\left(-9\right)\right] &\stackrel{?}{=} 15\left(50\right) \\ 10\left[3+72\right] &\stackrel{?}{=} 750 \\ 10\left[75\right] &\stackrel{?}{=} 750 \\ 750 &= 750\checkmark \end{aligned}$$

Try It

18) Solve: $6[4-2(7y-1)]=8(13-8y)$.

Solution

$y=-\frac{17}{5}$

19) Solve: $12[1-5(4z-1)]=3(24+11z)$.

Solution

$z=0$

Example 8

Solve: $0.36(100n+5)=0.6(30n+15)$.

Solution

Step 1: **Distribute.**

$$36n+1.8=18n+9$$

Step 2: **Subtract $18n$ to get the variables to the left.**

$$\begin{array}{ll} & 36n-18n+1.8=18n-18n+9 \\ \text{Simplify.} & 18n+1.8=9 \end{array}$$

Step 3: **Subtract 1.8 to get the constants to the right.**

$$\begin{array}{ll} & 18n+1.8-1.8=9-1.8 \\ \text{Simplify.} & 18n=7.2 \end{array}$$

Step 4: **Divide.**

$$\begin{array}{ll} & \frac{18n}{18}=\frac{7.2}{18} \\ \text{Simplify.} & n=0.4 \end{array}$$

Step 5: **Check:**

$$0.35(100n+5)=0.6(30n+15)$$

Step 6: **Let $n=0.4$.**

$$\begin{array}{c} 0.36\left(100\left(0.4\right)+5\right)\stackrel{?}{=}0.6\left(30\left(0.4\right)+15\right) \\ 0.36\left(40+5\right)\stackrel{?}{=}0.6\left(12+15\right) \\ 0.36\left(45\right)\stackrel{?}{=}0.6\left(27\right) \\ 16.2=16.2\checkmark \end{array}$$

Try It

20) Solve: $0.55(100n + 8) = 0.6(85n + 14)$.

Solution

$n = 1$

21) Solve: $0.15(40m - 120) = 0.5(60m + 12)$.

Solution

$m = -1$

Classify Equations

Consider the equation we solved at the start of the last section, $7x + 8 = -13$. The solution we found was $x = -3$. This means the equation $7x + 8 = -13$ is true when we replace the variable, x, with the value -3. We showed this when we checked the solution $x = -3$ and evaluated $7x + 8 = -13$ for $x = -3$.

$$\begin{aligned} 7(-3) + 8 &\stackrel{?}{=} -13 \\ -21 + 8 &\stackrel{?}{=} -13 \\ -13 &= -13 \checkmark \end{aligned}$$

If we evaluate $7x + 8$ for a different value of x, the left side will not be -13.

The equation $7x + 8 = -13$ is true when we replace the variable, x, with the value -3, but not true when we replace x with any other value. Whether or not the equation $7x + 8 = -13$ is true depends on the value of the variable. Equations like this are called conditional equations.

All the equations we have solved so far are **conditional equations**.

Conditional equation

An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

Now let's consider the equation $2y + 6 = 2(y + 3)$. Do you recognize that the left side and the right side are equivalent?

Let's see what happens when we solve for y.

	$2y + 6 = 2(y + 3)$
Distribute	$-2y + 6 = 2y + 6$
Subtract $2y$ to get the y's to one side.	$2y - 2y + 6 = 2 - 2y + 6$
Simplify-the y's are gone!	$6 = 6$

But $6 = 6$ is true.

This means that the equation $2y + 6 = 2(y + 3)$ is true for any value of y. We say the solution to the equation is all of the real numbers. An equation that is true for any value of a variable like this is called an **identity**.

Identity

An equation that is true for any value of the variable is called an identity.

The solution of an identity is all real numbers.

What happens when we solve the equation $5z = 5z - 1$?

	$5z = 5z - 1$
Subtract $5z$ to get the constant alone on the right.	$5z - 5z = 5z - 5z - 1$
Simplify-the z's are gone!	$0 \neq -1$

But $0 \neq -1$

Solving the equation $5z = 5z - 1$ led to the false statement $0 = -1$. The equation $5z = 5z - 1$ will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a **contradiction**.

Contradiction

An equation that is false for all values of the variable is called a contradiction.

A contradiction has no solution.

Example 9

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$$6(2n-1)+3=2n-8+5(2n+1)$$

Solution
***Step 1:* Distribute.**

$$12n-6+3=2n-8+10n+5$$

Combine like terms. $$12n-3=12n-3$$

***Step 2:* Subtract $12n$ to get the n's to one side.**

$$12n-12n-3=12n-12n-3$$

Simplify. $$-3=-3$$

This is a true statement.

The equation is an identity.
The solution is all real numbers.

Try It

22) Classify the equation as a conditional equation, an identity, or a contradiction, and then state the solution:

$$4+9(3x-7)=-42x-13+23(3x-2)$$

Solution
identity; all real numbers

23) Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$$8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$$

Solution
identity; all real numbers

Example 10

Classify as a conditional equation, an identity, or a contradiction. Then state the solution.

$10+4(p-5)=0$

Solution
Step 1: **Distribute.**

$$\begin{array}{lr} & 10+4p-20=0 \\ \text{Combine like terms.} & 4p-10=0 \end{array}$$

Step 2: **Add 10 to both sides.**

$$\begin{array}{lr} & 4p-10+10=0+10 \\ \text{Simplify.} & 4p=10 \end{array}$$

Step 3: **Divide.**

$$\frac{4p}{4} = \frac{10}{4}$$

Simplify. $$p = \frac{5}{2}$$

The equation is true when $p = \frac{5}{2}$.

This is a conditional equation.

The solution is $p = \frac{5}{2}$.

Try It

24) Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11(q+3) - 5 = 19$

Solution

conditional equation; $q = -\frac{9}{11}$

25) Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6 + 14(k-8) = 95$

Solution

conditional equation; $k = \frac{201}{14}$

Example 11

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$5m + 3(9 + 3m) = 2(7m - 11)$

Solution
Step 1: **Distribute.**

$$\begin{array}{lr} & 5m + 27 + 9m = 14m - 22 \\ \text{Combine like terms.} & 14m + 27 = 14m - 22 \end{array}$$

Step 2: **Subtract $14m$ from both sides.**

$$\begin{array}{lc} & 14m + 27 - 14m = 14m - 22 - 14m \\ \text{Simplify.} & 27 \neq -22 \end{array}$$

But $27 \neq -22.$

The equation is a contradiction.
It has no solution.

Try It

26) Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$$10m + 4(3 - 2m) = 2(m - 6)$$

Solution
contradiction; no solution

27) Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$$4(7d+18)=13(3d-2)-11d$$

Solution

contradiction; no solution

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution

Key Concepts

- **General Strategy for Solving Linear Equations**
    1. Simplify each side of the equation as much as possible.
       Use the Distributive Property to remove any parentheses.
       Combine like terms.
    2. Collect all the variable terms on one side of the equation.
       Use the Addition or Subtraction Property of Equality.
    3. Collect all the constant terms on the other side of the equation.
       Use the Addition or Subtraction Property of Equality.
    4. Make the coefficient of the variable term to equal to 1.
       Use the Multiplication or Division Property of Equality.
       State the solution to the equation.
    5. Check the solution.

Substitute the solution into the original equation.

Glossary

conditional equation
An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

contradiction
An equation that is false for all values of the variable is called a contradiction. A contradiction has no solution.

identity
An equation that is true for any value of the variable is called an identity. The solution of an identity is all real numbers.

Exercises: Solve Equations Using the General Strategy for Solving Linear Equations

Instructions: For questions 1-59, solve each linear equation.

1. $21(y-5)=-42$

Solution
$y=3$

---

2. $-9(2n+1)=36$

---

3. $-16(3n+4)=32$

Solution

$n=-2$

---

4. $8(22+11r)=0$

---

5. $5(8+6p)=0$

Solution

$p=-\frac{4}{3}$

---

6. $-(w-12)=30$

---

7. $-(t-19)=28$

Solution

$t=-9$

---

8. $9(6a+8)+9=81$

---

9. $8(9b-4)-12=100$

Solution

$b = 2$

---

10. $32 + 3(z + 4) = 41$

---

11. $21 + 2(m - 4) = 25$

Solution

$m = 6$

---

12. $51 + 5(4 - q) = 56$

---

13. $-6 + 6(5 - k) = 15$

Solution

$k = \frac{3}{2}$

---

14. $2(9s - 6) - 62 = 16$

---

15. $8(6t - 5) - 35 = -27$

Solution

$t = 1$

---

16. $3(10-2x)+54=0$

---

17. $-2(11-7x)+54=4$

Solution

$x=-2$

---

18. $\frac{2}{3}(9c-3)=22$

---

19. $\frac{3}{5}(10x-5)=27$

Solution

$x=5$

---

20. $\frac{1}{5}(15c+10)=c+7$

---

21. $\frac{1}{4}(20d+12)=d+7$

Solution

$d=1$

---

22. $18-(9r+7)=-16$

---

23. $15-(3r+8)=28$

Solution

$r = -7$

---

24. $5 - (n - 1) = 19$

---

25. $-3 - (m - 1) = 13$

Solution

$m = -15$

---

26. $11 - 4(y - 8) = 43$

---

27. $18 - 2(y - 3) = 32$

Solution

$y = -4$

---

28. $24 - 8(3v + 6) = 0$

---

29. $35 - 5(2w + 8) = -10$

Solution

$w = \frac{1}{2}$

---

30. $4(a - 12) = 3(a + 5)$

---

31. $-2(a-6)=4(a-3)$

Solution

$a=4$

---

32. $2(5-u)=-3(2u+6)$

---

33. $5(8-r)=-2(2r-16)$

Solution

$r=8$

---

34. $3(4n-1)-2=8n+3$

---

35. $9(2m-3)-8=4m+7$

Solution

$m=3$

---

36. $12+2(5-3y)=-9(y-1)-2$

---

37. $-15+4(2-5y)=-7(y-4)+4$

Solution

$y=-3$

---

38. $8(x-4)-7x=14$

---

39. $5(x-4)-4x=14$

Solution

$x=34$

---

40. $5+6(3s-5)=-3+2(8s-1)$

---

41. $-12+8(x-5)=-4+3(5x-2)$

Solution

$x=-6$

---

42. $4(u-1)-8=6(3u-2)-7$

---

43. $7(2n-5)=8(4n-1)-9$

Solution

$n=-1$

---

44. $4(p-4)-(p+7)=5(p-3)$

---

45. $3(a-2)-(a+6)=4(a-1)$

Solution

$a = -4$

---

46. $-(9y + 5) - (3y - 7) = 16 - (4y - 2)$

---

47. $-(7m + 4) - (2m - 5) = 14 - (5m - 3)$

Solution

$m = -4$

---

48. $4\,[5 - 8(4c - 3)] = 12(1 - 13c) - 8$

---

49. $5\,[9 - 2(6d - 1)] = 11(4 - 10d) - 139$

Solution

$d = -3$

---

50. $3\,[-9 + 8(4h - 3)] = 2(5 - 12h) - 19$

---

51. $3\,[-14 + 2(15k - 6)] = 8(3 - 5k) - 24$

Solution

$k = \dfrac{3}{5}$

---

52. $5\,[2(m + 4) + 8(m - 7)] = 2\,[3(5 + m) - (21 - 3m)]$

53. $10\left[5(n+1)+4(n-1)\right]=11\left[7(5+n)-(25-3n)\right]$

Solution

$n=-5$

54. $5(1.2u-4.8)=-12$

55. $4(2.5v-0.6)=7.6$

Solution

$v=1$

56. $0.25(q-6)=0.1(q+18)$

57. $0.2(p-6)=0.4(p+14)$

Solution

$p=-34$

58. $0.2(30n+50)=28$

59. $0.5(16m+34)=-15$

Solution

$m=-4$

Exercises: Classify Equations

Instructions: For questions 60-79, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

60. $23z+19=3(5z-9)+8z+46$

---

61. $15y+32=2(10y-7)-5y+46$

Solution

identity; all real numbers

---

62. $5(b-9)+4(3b+9)=6(4b-5)-7b+21$

---

63. $9(a-4)+3(2a+5)=7(3a-4)-6a+7$

Solution

identity; all real numbers

---

64. $18(5j-1)+29=47$

---

65. $24(3d-4)+100=52$

Solution

conditional equation; $d = \frac{2}{3}$

---

66. $22(3m-4)=8(2m+9)$

---

67. $30(2n-1)=5(10n+8)$

Solution

conditional equation; $n = 7$

---

68. $7v+42=11(3v+8)-2(13v-1)$

---

69. $18u-51=9(4u+5)-6(3u-10)$

Solution

contradiction; no solution

---

70. $3(6q-9)+7(q+4)=5(6q+8)-5(q+1)$

---

71. $5(p+4)+8(2p-1)=9(3p-5)-6(p-2)$

Solution

contradiction; no solution

---

72. $12(6h-1)=8(8h+5)-4$

73. $9(4k-7)=11(3k+1)+4$

Solution

conditional equation; $k=26$

74. $45(3y-2)=9(15y-6)$

75. $60(2x-1)=15(8x+5)$

Solution

contradiction; no solution

76. $16(6n+15)=48(2n+5)$

77. $36(4m+5)=12(12m+15)$

Solution

identity; all real numbers

78. $9(14d+9)+4d=13(10d+6)+3$

79. $11(8c+5)-8c=2(40c+25)+5$

Solution

identity; all real numbers

Exercises: Everyday Math

Instructions: For questions 80-81, answer the given everyday math word problems.

80. Fencing. Micah has 44 feet of fencing to make a dog run in his yard. He wants the length to be 2.5 feet more than the width. Find the length, L, by solving the equation $2L + 2(L - 2.5) = 44$.

---

81. Coins. Rhonda has $\$1.90$ in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, n, by solving the equation $0.05n + 0.10(2n - 1) = 1.90$.

Solution

8 nickels

Exercises: Writing Exercises

Instructions: For questions 82-85, answer the given writing exercises.

82. Using your own words, list the steps in the general strategy for solving linear equations.

---

83. Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

Solution

Answers will vary.

---

84. What is the first step you take when solving the equation $3-7(y-4)=38$? Why is this your first step?

---

85. Solve the equation $\frac{1}{4}(8x+20)=3x-4$ explaining all the steps of your solution as in the examples in this section.

Solution

Answers will vary.

3.5 SOLVE EQUATIONS WITH FRACTIONS OR DECIMALS

Learning Objectives

By the end of this section, you will be able to:

- Solve equations with fraction coefficients
- Solve equations with decimal coefficients

Before you get started, take this readiness quiz:

Try It

1) Multiply: $8 \times \frac{3}{8}$

2) Find the LCD of $\frac{5}{6}$ and $\frac{1}{4}$

3) Multiply 4.78 by 100

Solve Equations with Fraction Coefficients

Let's use the general strategy for solving linear equations introduced earlier to solve the equation, $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

	$\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$
To isolate the x term, subtract $\frac{1}{2}$ from both sides.	$\frac{1}{8}x + \frac{1}{2} - \frac{1}{2} = \frac{1}{4} - \frac{1}{2}$
Simplify the left side.	$\frac{1}{8}x = \frac{1}{4} - \frac{1}{2}$
Change the constants to equivalent fractions with the LCD.	$\frac{1}{8}x = \frac{1}{4} - \frac{2}{4}$
Subtract.	$\frac{1}{8}x = -\frac{1}{4}$
Multiply both sides by the reciprocal of $\frac{1}{8}$	$\frac{8}{1} \cdot \frac{1}{8}x = \frac{8}{1}\left(-\frac{1}{4}\right)$
Simplify.	$x = -2$

This method worked fine, but many students did not feel very confident when they saw all those fractions. So, we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the *Multiplication Property of Equality* and multiply both sides of an equation by the least common denominator of all the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but without fractions. This process is called "clearing" the equation of fractions.

Let's solve a similar equation, but this time use the method that eliminates the fractions.

Example 1

Solve: $\frac{1}{6}y - \frac{1}{3} = \frac{5}{6}$

Solution

Step 1: **Find the least common denominator of *all* the fractions in the equation.**

What is the LCD of $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{5}{6}$?

$$\frac{1}{6}y - \frac{1}{3} = \frac{5}{6} \quad LCD = 6$$

Step 2: **Multiply both sides of the equation by that LCD. This clears the fractions.**

Multiply both sides of the equation by the LCD 6.	$6\left(\frac{1}{6}y - \frac{1}{3}\right) = 6\left(\frac{5}{6}\right)$
Use the Distributive Property.	$6 \cdot \frac{1}{6}y - 6 \cdot \frac{1}{3} = 6 \cdot \frac{5}{6}$
Simplify - and notice, no more fractions!	$y - 2 = 5$

Step 3: **Sole using the General Strategy for Solving Linear Equations.**

To isolate the "y" term, add 2.	$y - 2 + 2 = 5 + 2$
Simplify.	$y = 7$

Try It

4) Solve: $\frac{1}{4}x + \frac{1}{2} = \frac{5}{8}$

Solution

$x = \frac{1}{2}$

5) Solve: $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$

Solution

$x = -2$

Notice in Example 1, that once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve! We then used the *General Strategy for Solving Linear Equations*.

How to

Strategy to solve equations with fraction coefficients.

1. Find the least common denominator of *all* the fractions in the equation.
2. Multiply both sides of the equation by that LCD. This clears the fractions.
3. Solve using the General Strategy for Solving Linear Equations.

Example 2

Solve: $6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$

Solution

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Step 1: **Find the LCD of all fractions in the equation.**

$$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$$

Step 2: **The LCD is 20.**

Step 3: **Multiply both sides of the equation by 20.**

$$20(6)=20\cdot\left(\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v\right)$$

Step 4: **Distribute.**

$$\begin{array}{ll} & 20(6)=20\cdot\frac{1}{2}v+20\cdot\frac{2}{5}v-20\cdot\frac{3}{4}v \\ \text{Simplify – notice, no more fractions.} & 120=10v+8v-15v \end{array}$$

Step 5: **Combine like terms.**

$$120=3v$$

Step 6: **Divide by 3.**

$$\begin{array}{ll} & \frac{120}{3}=\frac{3v}{3} \\ \text{Simplify.} & 40=v \end{array}$$

Step 7: **Check:**

$$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$$

Step 8: **Let $v=40$.**

$$\begin{array}{l} 6\stackrel{?}{=}\frac{1}{2}(40)+\frac{2}{5}(40)-\frac{3}{4}(40) \\ 6\stackrel{?}{=}20+16-30 \\ 6=6\checkmark \end{array}$$

Try It

6) Solve: $7=\frac{1}{2}x+\frac{3}{4}x-\frac{2}{3}x$

Solution

$x=12$

7) Solve: $-1=\frac{1}{2}u+\frac{1}{4}u-\frac{2}{3}u$

Solution

$u=-12$

In the next example, we again have variables on both sides of the equation.

Example 3

Solve: $a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$

Solution

Step 1: **Find the LCD of all fractions in the equation.**

The LCD is 8.

Step 2: **Multiply both sides by the LCD.**

$$8\left(a+\frac{3}{4}\right)=8\left(\frac{3}{8}a-\frac{1}{2}\right)$$

Step 3: **Distribute.**

```
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Step 4: **Subtract $3a$ from both sides.**

$$8a{-3a}+6=3a{-3a}-4$$

Simplify. $$5a+6=-4$$

Step 5: **Subtract 6 from both sides.**

$$\begin{aligned} 5a+6-6 &= -4-6 \\ \text{Simplify.} \quad 5a &= -10 \end{aligned}$$

Step 6: **Divide by 5.**

$$\begin{aligned} \frac{5a}{5} &= \frac{-10}{5} \\ \text{Simplify.} \quad a &= -2 \end{aligned}$$

Step 7: **Check:**

$$a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$$

Step 8: **Let $a=-2$.**

$$\begin{aligned} -2+\frac{3}{4} &\stackrel{?}{=} \frac{3}{8}(-2)-\frac{1}{2} \\ -\frac{8}{4}+\frac{3}{4} &\stackrel{?}{=} -\frac{16}{8}-\frac{4}{8} \\ -\frac{5}{4} &= -\frac{10}{8} \\ -\frac{5}{4} &= -\frac{5}{4}\checkmark \end{aligned}$$

Try It

8) Solve: $x+\frac{1}{3}=\frac{1}{6}x-\frac{1}{2}$

Solution

$x=-1$

9) Solve: $c+\frac{3}{4}=\frac{1}{2}c-\frac{1}{4}$

Solution

$c=-2$

In the next example, we start by using the *Distributive Property*. This step clears the fractions right away.

Example 4

Solve: $-5 = \frac{1}{4}(8x + 4)$

Solution

Step 1: **Distribute.**

$$\begin{array}{ll} & -5 = \frac{1}{4} \cdot 8x + \frac{1}{4} \cdot 4 \\ \text{Simplify.} & -5 = 2x + 1 \end{array}$$

Now there are no fractions.

Step 2: **Subtract** 1 **from both sides.**

$$\begin{array}{ll} & -5 - 1 = 2 + 1 - 1 \\ \text{Simplify.} & -6 = 2x \end{array}$$

Step 3: **Divide by** 2**.**

$$\begin{array}{ll} & \frac{-6}{2} = \frac{2x}{2} \\ \text{Simplify.} & -3 = x \end{array}$$

Step 4: **Check:**

$$-5 = \frac{1}{4}(8x + 4)$$

Step 5: **Let** $x = -3$**.**

$$\begin{array}{l} -5 \stackrel{?}{=} \frac{1}{2}(4(-3) + 2) \\ -5 \stackrel{?}{=} \frac{1}{2}(-12 + 2) \\ -5 \stackrel{?}{=} \frac{1}{2}(-10) \\ -5 = -5 \checkmark \end{array}$$

Try It

10) Solve: $-11 = \frac{1}{2}(6p + 2)$

Solution

$p = -4$

11) Solve: $8 = \frac{1}{3}(9q + 6)$

Solution

$q = 2$

In the next example, even after distributing, we still have fractions to clear.

Example 5

Solve: $\frac{1}{2}(y - 5) = \frac{1}{4}(y - 1)$

Solution

Step 1: **Distribute.**

$$\frac{1}{2} \cdot y - \frac{1}{2} \cdot 5 = \frac{1}{4} \cdot y - \frac{1}{4} \cdot 1$$

Simplify. $$\frac{1}{2}y - \frac{5}{2} = \frac{1}{4}y - \frac{1}{4}$$

Step 2: **Multiply by the LCD, 4.**

$$4\left(\frac{1}{2}y - \frac{5}{2}\right) = 4\left(\frac{1}{4}y - \frac{1}{4}\right)$$

Distribute. $$4 \cdot \frac{1}{2}y - 4 \cdot \frac{5}{2} = 4 \cdot \frac{1}{4}y - 4 \cdot \frac{1}{4}$$

Simplify. $$2y - 10 = y - 1$$

Step 3: **Collect the variables to the left.**

$$2y - y - 10 = y - y - 1$$

Simplify. $$y - 10 = -1$$

Step 4: **Collect the constants to the right.**

$$\begin{array}{ll} & y - 10 + 10 = -1 + 10 \\ \text{Simplify.} & y = 9 \end{array}$$

Step 5: **Check:**

$$\frac{1}{2}(y - 5) = \frac{1}{4}(y - 1)$$

Step 6: **Let** $y = 9$**.**

$$\frac{1}{2}(9 - 5) \stackrel{?}{=} \frac{1}{4}(9 - 1)$$

Finish the check on your own.

Try It

12) Solve: $\frac{1}{5}(n + 3) = \frac{1}{4}(n + 2)$

Solution

$n = 2$

13) Solve: $\frac{1}{2}(m - 3) = \frac{1}{4}(m - 7)$

Solution

$m = -1$

Example 6

Solve: $\frac{5x - 3}{4} = \frac{x}{2}$

Solution

Step 1: **Multiply by the LCD,** 4**.**

$$4\left(\frac{5x-3}{4}\right)=4\left(\frac{x}{2}\right)$$

Simplify. $$5x-3=2x$$

Step 2: **Collect the variables to the right.**

$$5x-5x-3=2x-5x$$

Simplify. $$-3=-3x$$

Step 3: **Divide.**

$$\frac{-3}{-3}=\frac{-3x}{-3}$$

Simplify. $$1=x$$

Step 4: **Check:**

$$\frac{5x-3}{4}=\frac{x}{2}$$

Step 5: **Let** $x=1$**.**

$$\frac{5(1)-3}{4}\overset{?}{=}\frac{1}{2}$$
$$\frac{2}{4}\overset{?}{=}\frac{1}{2}$$
$$\frac{1}{2}=\frac{1}{2}\checkmark$$

Try It

14) Solve: $\frac{4y-7}{3}=\frac{y}{6}$

Solution

$y=2$

15) Solve: $\frac{-2z-5}{4}=\frac{z}{8}$

Solution

$z=-2$

Example 7

Solve: $\frac{a}{6}+2=\frac{a}{4}+3$

Solution

Step 1: **Multiply by the LCD,** 12.

$$12\left(\frac{a}{6}+2\right)=12\left(\frac{a}{4}+3\right)$$

Step 2: **Distribute.**

$$12\cdot\frac{a}{6}+12\cdot 2=12\cdot\frac{a}{4}+12\cdot 3$$

$$\text{Simplify.}\quad 2a+24=3a+36$$

Step 3: **Collect the variables to the right.**

$$2a-2a+24=3a-2a+36$$

$$\text{Simplify.}\quad 24=a+36$$

Step 4: **Collect the constants to the left.**

$$24-36=a+36-36$$

$$\text{Simplify.}\quad a=-12$$

Step 5: **Check:**

$$\frac{a}{6}+2=\frac{a}{4}+3$$

Step 6: **Let** $a=-12$.

$$\frac{-12}{6}+2\stackrel{?}{=}\frac{-12}{4}+3$$

$$-2+2\stackrel{?}{=}-3+3$$

$$0=0\checkmark$$

Try It

16) Solve: $\frac{b}{10}+2=\frac{b}{4}+5$

Solution

$b = -20$

17) Solve: $\frac{c}{6} + 3 = \frac{c}{3} + 4$

Solution

$c = -6$

Example 8

Solve: $\frac{4q+3}{2} + 6 = \frac{3q+5}{4}$

Solution

Step 1: **Multiply by the LCD, 4.**

$$4\left(\frac{4q+3}{2} + 6\right) = 4\left(\frac{3q+5}{4}\right)$$

Step 2: **Distribute.**

$$4\left(\frac{4q+3}{2}\right) + 4 \cdot 6 = 4 \cdot \left(\frac{3q+5}{4}\right)$$

Simplify.

$$2(4q+3) + 24 = 3q + 5$$
$$8q + 6 + 24 = 3q + 5$$
$$8q + 30 = 3q + 5$$

Step 3: **Collect the variables to the left.**

$$8q - 3q + 30 = 3q - 3q + 5$$

Simplify.

$$5q + 30 = 5$$

Step 4: **Collect the constants to the right.**

$$5q + 30 - 30 = 5 - 30$$

Simplify.

$$5q = -25$$

Step 5: **Divide by 5.**

$$\frac{5q}{5} = \frac{-25}{5}$$

Simplify.

$$q = -5$$

Step 6: **Check:**

$$\frac{4q+3}{2}+6=\frac{3q+5}{4}$$

Step 7: **Let $q=-5$.**

$$\frac{4(-5)+3}{2}+6\stackrel{?}{=}\frac{3(-5)+5}{4}$$

Finish the check on your own.

Try It

18) Solve: $\frac{3r+5}{6}+1=\frac{4r+3}{3}$

Solution

$r=1$

19) Solve: $\frac{2s+3}{2}+1=\frac{3s+2}{4}$

Solution

$s=-8$

Solve Equations with Decimal Coefficients

Some equations have decimals in them. This kind of equation will occur when we solve problems dealing with money or percentages. But decimals can also be expressed as fractions. For example, $0.3=\frac{3}{10}$ and $0.17=\frac{17}{100}$. So, with an equation with decimals, we can use the same method we used to clear fractions—multiply both sides of the equation by the least common denominator.

Example 9

Solve: $0.06x + 0.02 = 0.25x - 1.5$

Solution

Look at the decimals and think of the equivalent fractions.

$0.06 = \frac{6}{100}$ 0.02=\frac{2}{100} $0.25 = \frac{25}{100}$ $1.5 = 1\frac{5}{10}$

Notice, that the LCD is 100.

By multiplying by the LCD, we will clear the decimals from the equation.

$$0.06x + 0.02 = 0.25x - 1.5$$

Step 1: **Multiply both sides by 100.**

$$100(0.06x + 0.02) = 100(0.25x - 1.5)$$

Step 2: **Distribute.**

$$100(0.06x) + 100(0.02) = 100(0.25x) - 100(1.5)$$

Step 3: **Multiply, and now we have no more decimals.**

$$6x + 2 = 25x - 150$$

Step 4: **Collect the variables to the right.**

$$6x - 6x + 2 = 25x - 6x - 150$$

Simplify. $2 = 19x - 150$

Step 5: **Collect the constants to the left.**

$$2 + 150 = 19x - 150 + 150$$

Simplify. $152 = 19x$

Step 6: **Divide by 19.**

$$\frac{152}{19} = \frac{19x}{19}$$

Simplify. $8 = x$

Step 7: **Check: Let $x = 8$.**

$$0.06(8) + 0.02 \stackrel{?}{=} 0.25(8) - 1.5$$
$$0.48 + 0.02 \stackrel{?}{=} 2.00 - 1.5$$
$$0.50 = 0.50 \checkmark$$

Try It

20) Solve: $0.14h + 0.12 = 0.35h - 2.4$

Solution

$h = 12$

21) Solve: $0.65k - 0.1 = 0.4k - 0.35$

Solution

$k = -1$

The next example uses an equation that is typical of the money applications in the next chapter. Notice that we distribute the decimal before we clear all the decimals.

Example 10

Solve: $0.25x + 0.05(x + 3) = 2.85$

Solution

Step 1: **Distribute first.**

$$0.25x + 0.05x + 0.15 = 2.85$$

Step 2: **Combine like terms.**

$$0.30x + 0.15 = 2.85$$

Step 3: **To clear decimals, multiply by 100.**

$$100(0.30x + 0.15) = 100(2.8)$$

Step 4: **Distribute.**

$$30x + 15 = 285$$

Step 5: **Subtract 15 from both sides.**

$$30x + 15 - 15 = 285 - 15$$
Simplify. $$30x = 270$$

Step 6: **Divide by 30.**

$$\frac{30x}{30} = \frac{270}{30}$$
Simplify. $$x = 9$$

Step 7: **Check it yourself by substituting $x = 9$ into the original equation.**

Try It

22) Solve: $0.25n + 0.05(n + 5) = 2.95$

Solution

$n = 9$

23) Solve: $0.10d + 0.05(d - 5) = 2.15$

Solution

$d = 16$

Key Concepts

- **Strategy to Solve an Equation with Fraction Coefficients**

1. Find the least common denominator of all the fractions in the equation.
2. Multiply both sides of the equation by that LCD. This clears the fractions.
3. Solve using the General Strategy for Solving Linear Equations.

Exercises: Solve Equations with Fraction Coefficients

Instructions: For questions 1-35, solve each equation with fraction coefficients.

1. $\frac{3}{4}x - \frac{1}{2} = \frac{1}{4}$

Solution

$x = 1$

---

2. $\frac{5}{6}y - \frac{2}{3} = -\frac{3}{2}$

---

3. $\frac{5}{6}y - \frac{1}{3} = -\frac{7}{6}$

Solution

$y = -1$

---

4. $\frac{1}{2}a + \frac{3}{8} = \frac{3}{4}$

---

5. $\frac{5}{8}b + \frac{1}{2} = -\frac{3}{4}$

Solution

$b = -2$

---

6. $2 = \frac{1}{3}x - \frac{1}{2}x + \frac{2}{3}x$

---

7. $2 = \frac{3}{5}x - \frac{1}{3}x + \frac{2}{5}x$

Solution

$x = 3$

---

8. $\frac{1}{4}m - \frac{4}{5}m + \frac{1}{2}m = -1$

---

9. $\frac{5}{6}n - \frac{1}{4}n - \frac{1}{2}n = -2$

Solution

$n = -24$

---

10. $x + \frac{1}{2} = \frac{2}{3}x - \frac{1}{2}$

---

11. $x + \frac{3}{4} = \frac{1}{2}x - \frac{5}{4}$

Solution

$x = -4$

---

12. $\frac{1}{3}w + \frac{5}{4} = w - \frac{1}{4}$

---

13. $\frac{3}{2}z + \frac{1}{3} = z - \frac{2}{3}$

Solution

$z = -2$

---

14. $\frac{1}{2}x - \frac{1}{4} = \frac{1}{12}x + \frac{1}{6}$

---

15. $\frac{1}{2}a - \frac{1}{4} = \frac{1}{6}a + \frac{1}{12}$

Solution

$a = 1$

---

16. $\frac{1}{3}b + \frac{1}{5} = \frac{2}{5}b - \frac{3}{5}$

---

17. $\frac{1}{3}x + \frac{2}{5} = \frac{1}{5}x - \frac{2}{5}$

Solution

$x = -6$

---

18. $1 = \frac{1}{6}(12x - 6)$

---

19. $1 = \frac{1}{5}(15x - 10)$

Solution

$x = 1$

---

20. $\frac{1}{4}(p - 7) = \frac{1}{3}(p + 5)$

21. $\frac{1}{5}(q+3)=\frac{1}{2}(q-3)$

Solution

$q=7$

22. $\frac{1}{2}(x+4)=\frac{3}{4}$

23. $\frac{1}{3}(x+5)=\frac{5}{6}$

Solution

$x=-\frac{5}{2}$

24. $\frac{5q-8}{5}=\frac{2q}{10}$

25. $\frac{4m+2}{6}=\frac{m}{3}$

Solution

$m=-1$

26. $\frac{4n+8}{4}=\frac{n}{3}$

27. $\frac{3p+6}{3}=\frac{p}{2}$

Solution

$p=-4$

28. $\frac{u}{3}-4=\frac{u}{2}-3$

29. $\frac{v}{10}+1=\frac{v}{4}-2$

Solution

$v=20$

30. $\frac{c}{15}+1=\frac{c}{10}-1$

31. $\frac{d}{6}+3=\frac{d}{8}+2$

Solution

$d=-24$

32. $\frac{3x+4}{2}+1=\frac{5x+10}{8}$

33. $\frac{10y-2}{3}+3=\frac{10y+1}{9}$

Solution

$y=-1$

34. $\frac{7u-1}{4}-1=\frac{4u+8}{5}$

35. $\frac{3v-6}{2}+5=\frac{11v-4}{5}$

Solution

$v = 4$

Exercises: Solve Equations with Decimal Coefficients

Instructions: For questions 36-51, solve each equation with decimal coefficients.

36. $0.6y + 3 = 9$

---

37. $0.4y - 4 = 2$

Solution

$y = 15$

---

38. $3.6j - 2 = 5.2$

---

39. $2.1k + 3 = 7.2$

Solution

$k = 2$

---

40. $0.4x + 0.6 = 0.5x - 1.2$

---

41. $0.7x + 0.4 = 0.6x + 2.4$

Solution

$x = 20$

---

42. $0.23x + 1.47 = 0.37x - 1.05$

---

43. $0.48x + 1.56 = 0.58x - 0.64$

Solution

$x = 22$

---

44. $0.9x - 1.25 = 0.75x + 1.75$

---

45. $1.2x - 0.91 = 0.8x + 2.29$

Solution

$x = 8$

---

46. $0.05n + 0.10(n + 8) = 2.15$

---

47. $0.05n + 0.10(n + 7) = 3.55$

Solution

$n = 19$

---

48. $0.10d + 0.25(d + 5) = 4.05$

---

49. $0.10d + 0.25(d + 7) = 5.25$

Solution

$d = 10$

---

50. $0.05(q - 5) + 0.25q = 3.05$

---

51. $0.05(q - 8) + 0.25q = 4.10$

Solution

$q = 15$

Exercises: Everyday Math

Instructions: For questions 52-53, answer the given everyday math word problems.

52. Coins. Taylor has $\$2.00$ in dimes and pennies. The number of pennies is 2 more than the number of dimes. Solve the equation $0.10d + 0.01(d + 2) = 2$ for d, the number of dimes.

---

53. Stamps. Paula bought $\$22.82$ worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 8 less than the number of 49-cent stamps. Solve the equation $0.49s + 0.21(s - 8) = 22.82$ for

s, to find the number of 49-cent stamps Paula bought.

Solution

$s = 35$

Exercises: Writing Exercises

Instructions: For questions 54-57, answer the given writing exercises.

54. Explain how you find the least common denominator of $\frac{3}{8}$, $\frac{1}{6}$, and $\frac{2}{3}$.

---

55. If an equation has several fractions, how does multiplying both sides by the LCD make it easier to solve?

Solution

Answers will vary.

---

56. If an equation has fractions only on one side, why do you have to multiply both sides of the equation by the LCD?

---

57. In the equation $0.35x + 2.1 = 3.85$ what is the LCD? How do you know?

Solution

100. Justifications will vary.

3.6 SOLVE A FORMULA FOR A SPECIFIC VARIABLE

Learning Objectives

By the end of this section, you will be able to:

- Use the Distance, Rate, and Time formula
- Solve a formula for a specific variable

Try It

Before you get started, take this readiness quiz:

1) Solve: $15t = 120$
2) Solve: $6x + 24 = 96$

Use the Distance, Rate, and Time Formula

One formula you will use often in algebra and everyday life is the distance formula travelled by an object moving at a constant rate. Rate is an equivalent word for "speed." The basic idea of rate may already be familiar to you. Do you know what distance you travel if you drive at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car's cruise control while driving on the highway.) If you said 120 miles, you already know how to use this formula!

For an object moving at a uniform (constant) rate, the distance travelled, the elapsed time, and the rate are related by the formula:

$$\begin{array}{l} d = rt, \quad when \\ d = distance \\ r = rate \\ t = time \end{array}$$

We will use the *Strategy for Solving Applications* that we used earlier in this chapter. When our problem requires a formula, we change Step 4: In place of writing a sentence, we write the appropriate formula. We write the revised steps here for reference.

How to

Solve an application (with a formula).

1. Read the problem. Make sure all the words and ideas are understood.
2. Identify what we are looking for.
3. Name what we are looking for. Choose a variable to represent that quantity.
4. Translate into an equation. Write the appropriate formula for the situation. Substitute in the given information.
5. Solve the equation using good algebra techniques.
6. Check the answer to the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

You may want to create a mini-chart to summarize the information in the problem. See the chart in this first example.

Example 1

Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. What distance has he travelled?

Solution

Step 1: **Read the problem.**

Step 2**: Identify what you are looking for.**

distance travelled

Step 3**: Name. Choose a variable to represent it.**

Let d = distance.

Step 4: **Translate: Write the appropriate formula.**

$$\begin{aligned} &d = rt \\ &d = ? \\ &r = 12\ mph \\ &t = 3\frac{1}{2}\ hours \end{aligned}$$

Step 5**: Substitute in the given information.**

$$d = 12 \cdot 3\frac{1}{2}$$

Step 6**: Solve the equation.**

$$d = 42\ miles$$

Step 7: **Check**

Does 42 miles make sense?

Jamal rides:

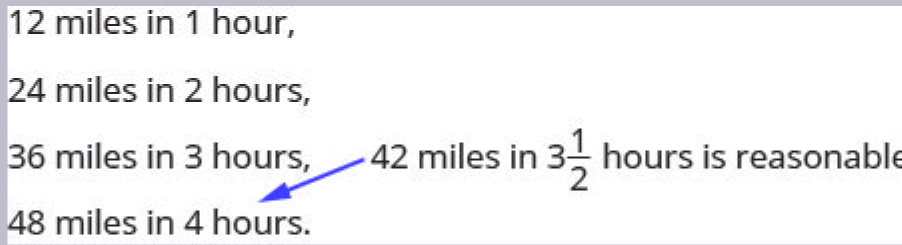


Figure 3.6.1

***Step 8*: Answer the question with a complete sentence.**

Jamal rode 42 miles.

Try It

3) Lindsay drove for $5\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Solution

330 miles

4) Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

Solution

7 miles

Example 2

Rey is planning to drive from his house in San Diego to visit his grandmother in Sacramento, a distance of 520 miles. If he can drive at a steady rate of 65 miles per hour, how many hours will the trip take?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what you are looking for.**

How many hours (time)

Step 3: **Name.**

Choose a variable to represent it.

Let t = time.

$$\begin{aligned} d &= 520\ miles \\ r &= 65\ mph \\ t &= ?\ hours \end{aligned}$$

Step 4: **Translate.**

Write the appropriate formula. Substitute in the given information.

$$\begin{aligned} d &= rt \\ 520 &= 65t \\ t &= 8 \end{aligned}$$

Step 5: **Solve the equation.**

$$t = 8$$

Step 6: **Check.**

Substitute the numbers into the formula and make sure the result is a true statement.

$$\begin{aligned} d &= rt \\ 520 &\stackrel{?}{=} 65 \times 8 \\ 520 &= 520\checkmark \end{aligned}$$

Step 7: **Answer the question with a complete sentence.**

Rey's trip will take 8 hours.

Try It

5) Lee wants to drive from Phoenix to his brother's apartment in San Francisco, a distance of 770 miles. If he drives at a steady rate of 70 miles per hour, how many hours will the trip take?

Solution

11 hours

6) Yesenia is 168 miles from Chicago. If she needs to be in Chicago in 3 hours, at what rate does she need to drive?

Solution

56 mph

Solve a Formula for a Specific Variable

You are probably familiar with some geometry formulas. A formula is a mathematical description of the relationship between variables. Formulas are also used in the sciences, such as chemistry, physics, and biology. In medicine, they are used for calculations for dispensing medicine or determining body mass index.

Spreadsheet programs rely on formulas to make calculations. It is important to be familiar with formulas and be able to manipulate them easily.

In Example 3.6.1 and Example 3.6.2, we used the formula $d = rt$. This formula gives the value of d, distance, when you substitute in the values of r and t, the rate and time. But in Example 3.6.2 we had to find the value of t. We substituted in values of d and r and then used algebra to solve for t. If you had to do this often, you might wonder why there is not a formula that gives the value of t when you substitute in the values of d and r. We can make a formula like this by solving the formula $d = rt$ for t.

To solve a formula for a specific variable means to isolate that variable on one side of the equals sign with a coefficient of 1. All other variables and constants are on the other side of the equals sign. To see how to solve a formula for a specific variable, we will start with the distance, rate and time formula.

Example 3

Solve the formula $d = rt$ for t:

a. when $d = 520$ and $r = 65$
b. in general

Solution
We will write the solutions side-by-side to demonstrate that solving a formula in general uses the same steps as when we have numbers to substitute.

a. when $d = 520$ and $r = 65$		b. in general	
Step 1: **Write the formula.**	$d = rt$	***Step 1:*** **Write the formula.**	$d = rt$
Step 2: **Substitute.**	$520 = 65t$		
Step 3: **Divide, to isolate** t.	$\frac{520}{65} = \frac{65t}{65}$	***Step 2:*** **Divide, to isolate** t.	$\frac{d}{r} = \frac{rt}{r}$
Step 4: **Simplify.**	$8 = t$	***Step 3:*** **Simplify.**	$\frac{d}{r} = t$

We say the formula $t = \frac{d}{r}$ is solved for t.

Try It

7) Solve the formula $d = rt$ for r

a. when $d = 180$ and $t = 4$

b. in general

Solution

a. $r = 45$

b. $r = \frac{d}{t}$

8) Solve the formula $d = rt$ for r

a. when $d = 780$ and $t = 12$

b. in general

Solution

a. $r = 65$

b. $r = \frac{d}{t}$

Example 4

Solve the formula $A = \frac{1}{2}bh$ for h:

a. when $A = 90$ and $b = 15$

b. in general

Solution

a. when $A = 90$ and $b = 15$		b. in general	
Step 1: **Write the formula.**	$A = \frac{1}{2}bh$	***Step 1:*** **Write the formula.**	$A = \frac{1}{2}bh$
Step 2: **Substitute.**	$90 = \frac{1}{2} \cdot 15 \cdot h$		
Step 3: **Clear the fractions.**	$2 \cdot 90 = 2 \cdot \frac{1}{2} 15h$	***Step 2:*** **Clear the fractions.**	$2 \cdot A = 2 \cdot \frac{1}{2} bh$
Step 4: **Simplify.**	$180 = 15h$	***Step 3:*** **Simplify.**	$2A = bh$
Step 5: **Solve for** h.	$12 = h$	***Step 4:*** **Solve for** h.	$\frac{2A}{b} = h$

We can now find the height of a triangle, if we know the area and the base, by using the formula $h = \frac{2A}{b}$

Try It

9) Use the formula $A = \frac{1}{2}bh$ to solve for h

a. when $A = 170$ and $b = 17$
b. in general

Solution
a. $h = 20$
b. $h = \frac{2A}{b}$

10) Use the formula $A = \frac{1}{2}bh$ to solve for b

a. when $A = 62$ and $h = 31$
b. in general

Solution
a. $b = 4$
b. $b = \frac{2A}{h}$

The formula $I = Prt$ is used to calculate simple interest, I, for a principal, P, invested at rate, r, for t years.

Example 5

Solve the formula $I = Prt$ to find the principal, P:

a. when $I = \$5,600$, $r = 4\%$, $t = 7\ years$

b. in general

Solution

a. $\$5,600, r = 4\%, t = 7years$		b. in general	
Step 1: **Write the formula.**	$I = Prt$	***Step 1:*** **Write the formula.**	$I = Prt$
Step 2: **Substitute.**	$5600 = P(0.04)(7)$		
Step 3: **Simplify.**	$5600 = P(0.28)$	***Step 2:*** **Simplify.**	$I = P(rt)$
Step 4: **Divide, to isolate** P.	$\frac{560}{0.28} = \frac{P(0.28)}{0.28}$	***Step 3:*** **Divide, to isolate** P.	$\frac{I}{rt} = \frac{P(rt)}{rt}$
Step 5: **Simplify.**	$20,000 = P$	***Step 4:*** **Simplify.**	$\frac{I}{rt} = P$
The principal is	$\$20,000$		$P = \frac{I}{rt}$

Try It

11) Use the formula $I = Prt$ to find the principal, P:

a. when $I = \$2,160$, $r = 6\%$, $t = 3years$

b. in general

Solution

a. $\$12,000$

b. $P = \frac{I}{rt}$

12) Use the formula $I = Prt$ to find the principal, P:

a. when $I = \$5,400$, $r = 12\%$, $t = 5\ years$

b. in general

Solution

a. $\$9,000$

b. $P = \frac{I}{rt}$

Later in this class, and future algebra classes, you'll encounter equations that relate two variables, usually x and y. You might be given an equation that is solved for y and need to solve it for x

, or vice versa. In the following example, we're given an equation with both x and y on the same side and we'll solve it for y.

Example 6

Solve the formula $3x + 2y = 18$ for y:

a. when $x = 4$
b. in general

Solution

a. when $x = 4$		b. in general	
Step 1: **Substitute.**	$3(4) + 2y = 18$		
Step 2: **Subtract to isolate the y-term.**	$12 - 12 + 2y = 18 - 12$	***Step 1:*** **Subtract to isolate the y-term.**	$3x - 3x + 2y = 18 - 3x$
Step 3: **Divide.**	$\frac{2y}{2} = \frac{6}{2}$	***Step 2:*** **Divide.**	$\frac{2y}{2} = \frac{18}{2} - \frac{3x}{2}$
Step 4: **Simplify.**	$y = 3$	***Step 3:*** **Simplify.**	$y = -\frac{3x}{2} + 9$

Try It

13) Solve the formula $3x + 4y = 10$ for y:

a. when $x = \frac{14}{3}$

b. in general

Solution

a. $y = 1$

b. $y = \frac{10 - 3x}{4}$

14) Solve the formula $5x + 2y = 18$ for y:

a. when $x = 4$

b. in general

Solution

a. $y = -1$

b. $y = \frac{18 - 5x}{2}$

Now we will solve a formula in general without using numbers as a guide.

Example 7

Solve the formula $P=a+b+c$ for a

Solution

Step 1: **We will isolate a on one side of the equation.**

$$P=a+b+c$$

Step 2: **Both b and c are added to a, so we subtract them from both sides of the equation.**

$$P-b-c=a+b+c-b-c$$

Step 3: **Simplify.**

$$\begin{aligned}P-b-c&=a\\a&=P-b-c\end{aligned}$$

Try It

15) Solve the formula $P = a + b + c$ for b

Solution

$b = P - a - c$

16) Solve the formula $P = a + b + c$ for c

Solution

$c = P - a - b$

Example 8

Solve the formula $6x + 5y = 13$ for y

Solution

Step 1: **Subtract $6x$ from both sides to isolate the term with y**

$$6x - 6x + 5y = 13 - 6x$$

Simplify. $$5y = 13 - 6x$$

Step 2: **Divide by 5 to make the coefficient 1.**

$$\frac{5y}{5} = \frac{13 - 6x}{5}$$

Simplify. $$y = \frac{13 - 6x}{5}$$

The fraction is simplified. We cannot divide $13 - 6x$ by 5.

Try It

17) Solve the formula $4x + 7y = 9$ for y.

Solution

$y = \frac{9 - 4x}{7}$

18) Solve the formula $5x + 8y = 1$ for y.

Solution

$y = \frac{1 - 5x}{8}$

In the next example, we will solve this formula for the height.

Example 9

Solve the formula $V=\frac{1}{3}\pi r^2 h$ for h.

Solution

Step 1: **Write the formula.**

$$V=\frac{1}{3}\pi r^2 h$$

Step 2: **Remove the fraction on the right.**

$$3\cdot V=3\cdot\frac{1}{3}\pi r^2 h$$

Simplify. $$3V=\pi r^2 h$$

Step 4: **Divide both sides by** πr^2

$$\frac{3V}{\pi r^2}=h$$

We could now use this formula to find the height of a right circular cone when we know the volume and the radius of the base, by using the formula $h=\frac{3V}{\pi r^2}$.

Try It

19) Use the formula $A = \frac{1}{2}bh$ to solve for b.

Solution

$b = \frac{2A}{h}$

20) Use the formula $A = \frac{1}{2}bh$ to solve for h.

Solution

$h = \frac{2A}{b}$

In the sciences, we often need to change temperature from Fahrenheit to Celsius or vice versa. If you travel in a foreign country, you may want to change the Celsius temperature to the more familiar Fahrenheit temperature.

Example 10

Solve the formula $C = \frac{5}{9}(F - 32)$ for F.

Solution

***Step 1:* Write the formula.**

$$C = \frac{5}{9}(F - 32)$$

Step 2: **Remove the fraction on the right.**

$$\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$$

$$\text{Simplify.} \quad \frac{9}{5}C = (F - 32)$$

Step 4: **Add 32 to both sides.**

$$\frac{9}{5}C + 32 = F$$

We can now use the formula $F = \frac{9}{5}C + 32$ to find the Fahrenheit temperature when we know the Celsius temperature.

Try It

21) Solve the formula $F = \frac{9}{5}C + 32$ for C.

Solution

$C = \frac{5}{9}(F - 32)$

22) Solve the formula $A = \frac{1}{2}h(b + B)$ for b.

Solution

$b = \frac{2A - Bh}{h}$

The next example uses the formula for the surface area of a right cylinder.

Example 11

Solve the formula $S=2\pi r^2+2\pi rh$ for h.

Solution

Step 1: **Write the formula.**

$$S=2\pi r^2+2\pi rh$$

Step 2: **Isolate the** h **term by subtracting** $2\pi r^2$ **from each side.**

$$S-2\pi r^2=2\pi r^2-2\pi r^2+2\pi rh$$

$$\text{Simplify.}\quad S-2\pi r^2=2\pi rh$$

Step 3: **Solve for** h **by dividing both sides by** $2\pi r$

$$\frac{S-2\pi r^2}{2\pi r}=\frac{2\pi rh}{2\pi r}$$

$$\text{Simplify.}\quad \frac{S-2\pi r^2}{2\pi r}=h$$

Try It

23) Solve the formula $A=P+Prt$ for t.

Solution

$t = \frac{A - P}{Pr}$

24) Solve the formula $A = P + Prt$ for r.

Solution

$r = \frac{A - P}{Pt}$

Key Concepts

- **To Solve an Application (with a formula)**
  1. Read the problem. Make sure all the words and ideas are understood.
  2. Identify what we are looking for.
  3. Name what we are looking for. Choose a variable to represent that quantity.
  4. Translate into an equation. Write the appropriate formula for the situation. Substitute in the given information.
  5. Solve the equation using good algebra techniques.
  6. Check the answer in the problem and make sure it makes sense.
  7. Answer the question with a complete sentence.
- **Distance, Rate and Time**
  For an object moving at a uniform (constant) rate, the distance travelled, the elapsed time, and the rate are related by the formula: $d = rt$ where $d = distance$, $r = rate$, $t = time$.
- **To solve a formula for a specific variable** means to get that variable by itself with a coefficient of 1 on one side of the equation and all other variables and constants on the other side.

Exercises: Use the Distance, Rate, and Time Formula

Instructions: For questions 1-11, solve.

1. Socorro drove for $4\frac{5}{6}$ hours at 60 miles per hour. How much distance did she travel?

Solution

290 miles

---

2. Yuki walked for $1\frac{3}{4}$ hours at 4 miles per hour. How far did she walk?

---

3. Francie rode her bike for $2\frac{1}{2}$ hours at 12 miles per hour. How far did she ride?

Solution

30 miles

---

4. Connor wants to drive from Tucson to the Grand Canyon, a distance of 338 miles. If he drives at a steady rate of 52 miles per hour, how many hours will the trip take?

---

5. Megan is taking the bus from New York City to Montreal. The distance is 380 miles and the bus travels at a steady rate of 76 miles per hour. How long will the bus ride be?

Solution

5 hours

---

6. Aurelia is driving from Miami to Orlando at a rate of 65 miles per hour. The distance is 235 miles. To the nearest tenth of an hour, how long will the trip take?

---

7. Kareem wants to ride his bike from St. Louis to Champaign, Illinois. The distance is 180 miles. If he rides at a steady rate of 16 miles per hour, how many hours will the trip take?

Solution

11.25 hours

---

8. Javier is driving to Bangor, 240 miles away. If he needs to be in Bangor in 4 hours, at what rate does he need to drive?

---

9. Alejandra is driving to Cincinnati, 450 miles away. If she wants to be there in 6 hours, at what rate does she need to drive?

Solution

75 mph

---

10. Aisha took the train from Spokane to Seattle. The distance is 280 miles and the trip took 3.5 hours. What was the speed of the train?

---

11. Philip got a ride with a friend from Denver to Las Vegas, a distance of 750 miles. If the trip took 10 hours, how fast was the friend driving?

Solution

75 mph

Exercises: Solve a Formula for a Specific Variable

Instructions: For questions 12-19, use the formula $d = rt$.

12. Solve for t

a. when $d = 350$ **and** $r = 70$
b. in general

---

13. Solve for t

a. when $d = 240$ **and** $r = 60$
b. in general

Solution

a. $t = 4$

b. $t = \frac{d}{r}$

---

14. Solve for t

a. when $d = 510$ **and** $r = 60$
b. in general

---

15. Solve for t

a. when $d = 175$ **and** $r = 50$
b. in general

Solution

a. $t = 3.5$

b. $t = \frac{d}{r}$

---

16. Solve for r

a. when $d = 204$ **and** $t = 3$
b. in general

---

17. Solve for r

a. when $d = 420$ **and** $t = 6$
b. in general

Solution

a. $r = 70$

b. $r = \frac{d}{t}$

---

18. Solve for r

a. when $d = 160$ **and** $t = 2.5$
b. in general

---

19. Solve for r

a. when $d = 180$ **and** $t = 4.5$
b. in general

Solution
a. $r = 40$
b. $r = \frac{d}{t}$

Exercises: Use the Area of a Triangle Formula (Area, Base, and Height)

Instructions: For questions 20-23, use the formula $A = \frac{1}{2}bh$.

20. Solve for b

a. when $A = 126$ **and** $h = 18$

b. in general

---

21. Solve for h

a. when $A = 176$ **and** $b = 22$

b. in general

Solution

a. $h = 16$

b. $h = \frac{2A}{b}$

---

22. Solve for h

a. when $A = 375$ **and** $b = 25$

b. in general

---

23. Solve for b

a. when $A = 65$ **and** $h = 13$

b. in general

Solution

a. $b = 10$

b. $b = \frac{2A}{h}$

Exercises: Use the Interest, Principal, Rate, and Time Formula

Instructions: For questions 24-27, use the formula $I = Prt$

24. Solve for the principal, P for

a. $I = \$5,480, r = 4\%, t = 7 \text{ years}$

b. in general

---

25. Solve for the principal, P for

a. $I = \$3,950, r = 6\%, t = 5 \text{ years}$

b. in general

Solution

a. $P = \$13,166.67$

b. $P = \frac{I}{rt}$

---

26. Solve for the time, t **for**

a. $I = \$2,376, P = \$9,000, r = 4.4\%$
b. in general

---

27. Solve for the time, t **for**

a. $I = \$624, P = \$6,000, r = 5.2\%$
b. in general

Solution

a. $t = 2$ years

b. $t = \frac{I}{Pr}$

Exercises: Solve a Formula for a Specific Variable

Instructions: For questions 28-49, solve.

28. Solve the formula $2x + 3y = 12$ **for** y

a. when $x = 3$

b. in general

---

29. Solve the formula $5x + 2y = 10$ for y

a. when $x = 4$

b. in general

Solution

a. $y = -5$

b. $y = \frac{10 - 5x}{2}$

---

30. Solve the formula $3x - y = 7$ for y

a. when $x = -2$

b. in general

---

31. Solve the formula $4x + y = 5$ for y

a. when $x = -3$

b. in general

Solution

a. $y = 17$

b. $y = 5 - 4x$

---

32. Solve $a+b=90$ for b.

---

33. Solve $a+b=90$ for a.

Solution

$a=90-b$

---

34. Solve $180=a+b+c$ for a.

---

35. Solve $180=a+b+c$ for c.

Solution

$c=180-a-b$

---

36. Solve the formula $8x+y=15$ for y.

---

37. Solve the formula $9x + y = 13$ for y.

Solution

$y = 13 - 9x$

---

38. Solve the formula $-4x + y = -6$ for y.

---

39. Solve the formula $-5x + y = -1$ for y.

Solution

$y = -1 + 5x$

---

40. Solve the formula $4x + 3y = 7$ for y.

---

41. Solve the formula $3x + 2y = 11$ for y.

Solution

$y = \frac{11 - 3x}{2}$

---

42. Solve the formula $x - y = -4$ **for** y**.**

---

43. Solve the formula $x - y = -3$ **for** y**.**

Solution

$y = 3 + x$

---

44. Solve the formula $P = 2L + 2W$ **for** L**.**

---

45. Solve the formula $P = 2L + 2W$ **for** W**.**

Solution

$W = \frac{P - 2L}{2}$

---

46. Solve the formula $C = \pi d$ **for** d**.**

---

47. Solve the formula $C = \pi d$ **for** π**.**

Solution

$\pi = \frac{C}{d}$

---

48. Solve the formula $V = LWH$ for L.

---

49. Solve the formula $V = LWH$ for H.

Solution

$H = \frac{V}{LW}$

Exercises: Everyday Math

Instructions: For questions 50-51, answer the given everyday math word problems.

50. Converting temperature. While on a tour in Greece, Tatyana saw that the temperature was 40° Celsius. Solve for F in the formula $C = \frac{5}{9}(F - 32)$ to find the Fahrenheit temperature.

---

51. Converting temperature. Yon was visiting the United States and he saw that the temperature in Seattle one day was 50° Fahrenheit. Solve for C in the formula $F=\frac{9}{5}C+32$ to find the Celsius temperature.

Solution

10° C

Exercises: Writing Exercises

Instructions: For questions 52-53, answer the given writing exercises.

52. Solve the equation $2x+3y=6$ for y

a. when $x=-3$
b. in general
c. Which solution is easier for you, a. or b.? Why?

---

53. Solve the equation $5x-2y=10$ for x

a. when $y=10$
b. in general
c. Which solution is easier for you, a. or b.? Why?

Solution

a. 6

b. $x=\frac{10+2y}{5}$

c. Answers will vary.

3.7 USE A PROBLEM-SOLVING STRATEGY AND APPLICATIONS

Learning Objectives

By the end of this section, you will be able to:

- Approach word problems with a positive attitude
- Use a problem-solving strategy for word problems
- Solve number problems
- Translate and solve basic percent equations
- Solve percent applications
- Find the percent increase and percent decrease
- Solve simple interest applications
- Solve applications with discount or mark-up

Try It

Before you get started, take this readiness quiz:

1) Translate "6 less than twice x" into an algebraic expression.

2) Solve: $\frac{2}{3}x = 24$

3) Solve: $3x + 8 = 14$

4) Convert 4.5% to a decimal.

5) Convert 0.6 to a percent.

6) Round 0.875 to the nearest hundredth.

7) Multiply $(4.5)(2.38)$

8) Solve $3.5 = 0.7n$

9) Subtract $50 - 37.45$

Approach Word Problems with a Positive Attitude

"If you think you can… or think you can't… you're right."—Henry Ford

The world is full of word problems! Will my income qualify me to rent that apartment? How much punch do I need to make for the party? What size diamond can I afford to buy my girlfriend? Should I fly or drive to my family reunion?

How much money do I need to fill the car with gas? How much tip should I leave at a restaurant? How many socks should I pack for vacation? What size turkey do I need to buy for Thanksgiving dinner, and then what time do I need to put it in the oven? If my sister and I buy our mother a present, how much does each of us pay?

Now that we can solve equations, we are ready to apply our new skills to word problems. Do you know anyone who has had negative experiences in the past with word problems? Have you ever had thoughts like the student below?

Negative thoughts can be barriers to success.

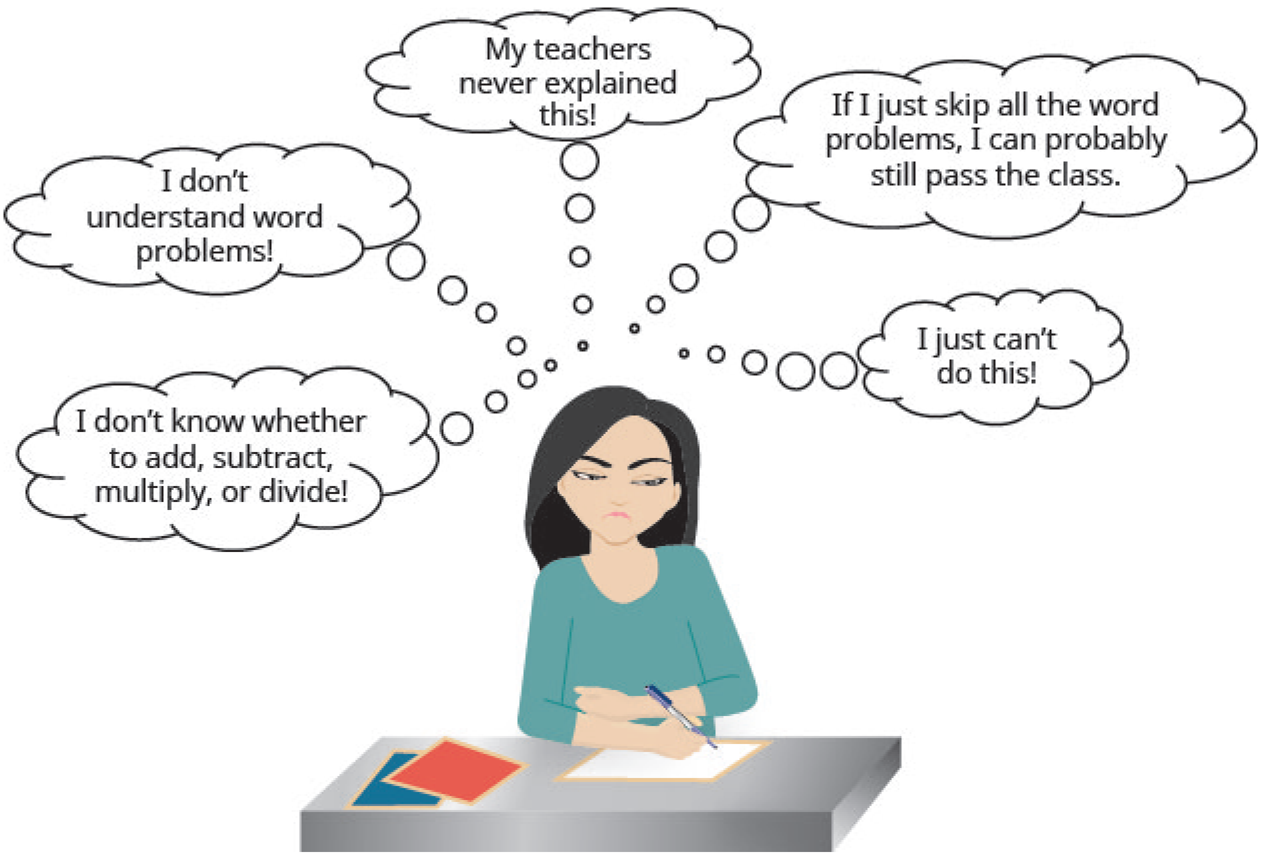


Figure 3.7.1 Negative thoughts can be barriers to success.

When we feel we have no control, and continue repeating negative thoughts, we set up barriers to success. We need to calm our fears and change our negative feelings.

Start with a fresh slate and begin to think positive thoughts. If we take control and believe we can be successful, we will be able to master word problems! Read the positive thoughts in Figure 3.7.2 and say them out loud.

Thinking positive thoughts is the first step towards success.

Figure 3.7.2 Thinking positive thoughts is a first step towards success.

Think of something, outside of school, that you can do now but couldn't do 3 years ago. Is it driving a car? Snowboarding? Cooking a gourmet meal? Speaking a new language? Your past experiences with word problems happened when you were younger—now you're older and ready to succeed!

Use a Problem-Solving Strategy for Word Problems

We have reviewed translating English phrases into algebraic expressions, using some basic mathematical vocabulary and symbols. We have also translated English sentences into algebraic equations and solved some word problems. The word problems applied math to everyday situations. We restated the situation in one sentence, assigned a variable, and then wrote an equation to solve the problem. This method works as long as the situation is familiar and the math is not too complicated.

Now, we'll expand our strategy so we can use it to successfully solve any word problem. We'll list the strategy here, and then we'll use it to solve some problems. We summarize below an effective strategy for problem-solving.

How to

Use a Problem-Solving Strategy to Solve Word Problems.

1. Read the problem. Make sure all the words and ideas are understood.
2. Identify what we are looking for.
3. Name what we are looking for. Choose a variable to represent that quantity.
4. Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.
5. Solve the equation using good algebra techniques.
6. Check the answer to the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

Example 1

Pilar bought a purse on sale for $\$18$, which is one-half of the original price. What was the original price of the purse?

Solution

Step 1: **Read the problem. Read the problem two or more times if necessary. Look up any unfamiliar words in a dictionary or on the internet.**

In this problem, is it clear what is being discussed? Is every word familiar?

Step 2: **Identify what you are looking for.**

Did you ever go into your bedroom to get something and then forget what you were looking for? It's hard to find something if you are not sure what it is! Read the problem again and look for words that tell you what you are looking for!

In this problem, the words "what was the original price of the purse" tell us what we need to find.

Step 3: **Name what we are looking for.**

Choose a variable to represent that quantity. We can use any letter for the variable, but choose one that makes it easy to remember what it represents.

Let p = the original price of the purse.

Step 4: **Translate into an equation.**

It may be helpful to restate the problem in one sentence with all the important information. Translate the English sentence into an algebraic equation.

Reread the problem carefully to see how the given information is related. Often, there is one sentence that gives this information, or it may help to write one sentence with all the important information. Look for clue words to help translate the sentence into algebra. Translate the sentence into an equation.

Restate the problem in one sentence with all the important information.

$$\underbrace{18}\ \underbrace{\text{is}}\ \underbrace{\text{one-half the original price.}}$$

Translate into an equation.

$$18 \ = \ \frac{1}{2} \ \times \ p$$

Step 5: Solve the equation using good algebraic techniques.

Even if you know the solution right away, using good algebraic techniques here will better prepare you to solve problems that do not have obvious answers.

Solve the equation:

$$\begin{array}{ll} & 18 = \frac{1}{2}p \\ \text{Multiply both sides by 2.} & 2 \times 18 = 2 \times \frac{1}{2}p \\ \text{Simplify} & 36 = p \end{array}$$

Step 6: **Check the answer in the problem to make sure it makes sense.**

We solved the equation and found that $p = 36$, which means "the original price" was $\$36$.

Does $\$36$ make sense in the problem? Yes, because 18 is one-half of 36, and the purse was on sale at half the original price.

***Step 7:* Answer the question with a complete sentence.** The problem asked "What was the original price of the purse?"

The answer to the question is: "The original price of the purse was $\$36$."

If this were a homework exercise, our work might look like this:

Pilar bought a purse on sale for $\$18$, which is one-half the original price. What was the original price of the purse?

Let $p =$ the original price.

18 is one-half the original price.

$$\begin{array}{lr} & 18 = \frac{1}{2}p \\ \text{Multiply both sides by 2.} & 2 \times 18 = 2 \times \frac{1}{2}p \\ \text{Simplify} & 36 = p \end{array}$$

***Step 8:* Check. Is $\$36$ a reasonable price for a purse?**

Yes.

Is 18 one half of 36?

$$\begin{array}{l} 18 \stackrel{?}{=} \frac{1}{2} \times 36 \\ 18 = 18 \checkmark \end{array}$$

The original price of the purse was $\$36$.

Try It

10) Joaquin bought a bookcase on sale for $\$120$, which was two-thirds of the original price. What was the original price of the bookcase?

Solution

$\$180$

11) Two-fifths of the songs in Mariel's playlist are country. If there are 16 country songs, what is the total number of songs in the playlist?

Solution

40

Let's try this approach with another example.

Example 2

Ginny and her classmates formed a study group. The number of girls in the study group was three more than twice the number of boys. There were 11 girls in the study group. How many boys were in the study group?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

How many boys were in the study group?

Step 3: **Name. Choose a variable to represent the number of boys.**

Let $n =$ the number of boys.

Step 4: **Translate. Restate the problem in one sentence with all the important information.**

$$\underbrace{\text{The number of girls (11)}} \; \underbrace{\text{was}} \; \underbrace{\text{three more than twice the number of boys.}}$$

Translate into an equation.

$$11 = 2b + 3$$

Step 5: **Solve the equation.**

	$11 = 2b + 3$
Subtract 3 from each side	$11 - 3 = 2b + 3 - 3$
Simplify	$8 = 2b$
Divide each side by 2	$\frac{8}{2} = \frac{2b}{2}$
Simplify	$4 = b$

Step 6: **Check. First, is our answer reasonable?**

Yes, having 4 boys in a study group seems OK. The problem says the number of girls was 3 more than twice the number of boys. If there are four boys, does that make eleven girls? Twice 4 boys is 8. Three more than 8 is 11.

Step 7: **Answer the question.**

There were 4 boys in the study group.

Try It

12) Guillermo bought textbooks and notebooks at the bookstore. The number of textbooks was 3 more than twice the number of notebooks. He bought 7 textbooks. How many notebooks did he buy?

Solution

2

13) Gerry worked Sudoku puzzles and crossword puzzles this week. The number of Sudoku puzzles he completed is eight more than twice the number of crossword puzzles. He completed 22 Sudoku puzzles. How many crossword puzzles did he do?

Solution

7

Solve Number Problems

Now that we have a problem-solving strategy, we will use it on several different types of word problems. The first type we will work on is "number problems." Number problems give some clues about one or more numbers. We use these clues to write an equation. Number problems don't usually arise on an everyday basis, but they provide a good introduction to practicing the problem-solving strategy outlined above.

Example 3

The difference between a number and six is 13. Find the number.

Solution

Step 1: **Read the problem. Are all the words familiar?**

Step 2: **Identify what we are looking for.**

The number.

Step 3: **Name. Choose a variable to represent the number.**

Let n = the number.

Step 4: **Translate. Remember to look for clue words like "difference… of… and…"**

Restate the problem in one sentence.

$$\underbrace{\text{The difference of the number and 6}}\ \underbrace{\text{is}}\ \underbrace{13.}$$

Translate into an equation.

$$n-6=13$$

Step 5: **Solve the equation.**

Simplify.

$$\begin{aligned} n-6&=13 \\ n&=19 \end{aligned}$$

Step 6: **Check.**

The difference of 19 and 6 is 13. It checks!

Step 7: **Answer the question.**

The number is 19.

Try It

14) The difference between a number and eight is 17. Find the number.

Solution

$$25$$

15) The difference between a number and eleven is -7. Find the number.

Solution

$$4$$

Example 4

The sum of twice a number and seven is 15. Find the number.

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

The number.

Step 3: **Name. Choose a variable to represent the number.**

Let n = the number.

Step 4: **Translate.**

Restate the problem in one sentence.

$$\underbrace{\text{The sum of twice a number and 7}}\ \underbrace{\text{is}}\ \underbrace{\text{13.}}$$

Translate into an equation.

$$2n \;+\; 7 \;=\; 15$$

Step 5: **Solve the equation.**

Subtract 7 from each side and simplify.	$2n + 7 = 15$
Divide each side by 2 and simplify.	$2n = 8$
	$n = 4$

Step 6: **Check.**

Is the sum of twice 4 and 7 equal to 15?

Step 7: **Answer the question.**

The number is 4.

Did you notice that we left out some of the steps as we solved this equation? If you're not yet ready to leave out these steps, write down as many as you need.

Try It

16) The sum of four times a number and two is 14. Find the number.

Solution

3

17) The sum of three times a number and seven is 25. Find the number.

Solution

6

Some number word problems ask us to find two or more numbers. It may be tempting to name them all with different variables, but so far we have only solved equations with one variable. To avoid using more than one variable, we will define the numbers in terms of the same variable. Be sure to read the problem carefully to discover how all the numbers relate to each other.

Example 5

One number is five more than another. The sum of the numbers is 21. Find the numbers.

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

We are looking for two numbers.

Step 3: **Name. We have two numbers to name and need a name for each.**

Choose a variable to represent the first number.

$$n = 1^{st}\ number$$

What do we know about the second number?

$$n + 5 = 2^{nd}\ number$$

Step 4: **Translate. Restate the problem as one sentence with all the important information.**

The sum of the 1^{st} number and the 2^{nd} number is 21.

Translate into an equation.

$$\underbrace{1^{st}\text{ number}} + \underbrace{2^{nd}\text{ number}} \underbrace{=} \underbrace{21}$$

Substitute the variable expressions.

$$n + n + 5 = 21$$

Step 5: **Solve the equation.**

	$n + n + 5 = 21$	
Combine like terms.	$2n + 5 = 21$	
Subtract 5 from both sides and simplify.	$2n = 16$	
Divide by 2 and simplify.	$n = 8$	1^{st} number
Find the second number, too	$n + 5$	2^{nd} number
	$8 + 5 = 13$	

Step 6: **Check.**

Do these numbers check in the problem?

Is one number 5 more than the other?	$13 \stackrel{?}{=} 8 + 5$
Is thirteen 5 more than 8? Yes.	$13 = 13\checkmark$
Is the sum of the two numbers 21?	$8 + 13 \stackrel{?}{=} 21$
	$21 = 21\checkmark$

Step 7: **Answer the question.**

The numbers are 8 and 13.

Try It

18) One number is six more than another. The sum of the numbers is twenty-four. Find the numbers.

Solution

9, 15

19) The sum of two numbers is fifty-eight. One number is four more than the other. Find the numbers.

Solution

$$27, 31$$

Example 6

The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

We are looking for two numbers.

Step 3: **Name.**

Choose a variable.

$$n = 1^{st}\ number$$

One number is 4 less than the other.

$$n - 4 = 2^{nd}\ number$$

Step 4: **Translate.**

The sum of the 2 numbers is negative 14.

Write as one sentence.

$$\underbrace{1^{st} \text{ number}} + \underbrace{2^{nd} \text{ number}} \underbrace{\text{is}} \underbrace{\text{negative } 14}$$

Translate into an equation.

$$n + n - 4 = -14$$

Step 5: **Solve the equation.**

	$n + n - 4 = -14$	
Combine like terms.	$2n - 4 = -14$	
Add 4 to each side and simplify.	$2n = -10$	
	$n = -5$	1^{st} number
Simplify.	$n - 4$	2^{nd} number
	$-5 - 4 = -9$	

Step 6: **Check.**

Is -9 four less than -5?	$-5 - 4 \stackrel{?}{=} -9$
	$-9 = -9 \checkmark$
Is their sum -14?	$-5 + (-9) \stackrel{?}{=} -14$
	$-14 = -14 \checkmark$

Step 7: **Answer the question.**

The numbers are -5 and -9.

Try It

20) The sum of two numbers is negative twenty-three. One number is seven less than the other. Find the numbers.

Solution

-15, -8

21) The sum of two numbers is -18. One number is 40 more than the other. Find the numbers.

Solution

-29, 11

Example 7

One number is ten more than twice another. Their sum is one. Find the numbers.

Solution

***Step 1:* Read the problem.**

***Step 2:* Identify what you are looking for.**

We are looking for two numbers.

***Step 3:* Name.**

Choose a variable.

$$x = 1^{st}\ number$$

One number is 10 more than twice another.

$$2x + 10 = 2^{nd}\ number$$

***Step 4:* Translate.**

Their sum is one.

Restate as one sentence.

The sum of the two numbers is 1.

Translate into an equation.

$$x + 2x + 10 = 1$$

***Step 5:* Solve the equation.**

	$x + 2x + 10 = 1$
Combine like terms.	$3x + 10 = 1$
Subtract 10 from each side.	$3x = -9$
Divide each side by 3.	$x = -3$ $\quad 1^{st}$ number
	$2x + 10$ $\quad 2^{nd}$ number
	$2(-3) + 10 = 4$

***Step 6:* Check.**

Is ten more than twice −3 equal to 4?	$2(-3) + 10 \stackrel{?}{=} 4$
	$-6 + 10 \stackrel{?}{=} 4$
	$4 = 4 ✓$
Is their sum 1?	$-3 + 4 \stackrel{?}{=} 1$
	$1 = 1$

***Step 7:* Answer the question.**

The numbers are -3 and 4.

Try It

22) One number is eight more than twice another. Their sum is negative four. Find the numbers.

Solution

$-4, 0$

23) One number is three more than three times another. Their sum is -5. Find the numbers.

Solution

$-3, -2$

Some number problems involve consecutive integers. *Consecutive integers* are integers that immediately follow each other. Examples of consecutive integers are:

$$1, 2, 3, 4$$

$$-10, -9, -8, -7$$

$$150, 151, 152, 153$$

Notice that each number is one more than the number preceding it. So if we define the first integer as n, the next consecutive integer is $n+1$. The one after that is one more than $n+1$, so it is $n+1+1$, which is $n+2$.

n	1^{st} integer
$n+1$	2nd consecutive integer
$n+2$	3rd consecutive integer… etc.

Example 8

The sum of two consecutive integers is 47. Find the numbers.

Solution

Step 1: **Read the problem.**

Step 2: **Identify what you are looking for.**

Two consecutive integers.

Step 3: **Name each number.**

$$\begin{aligned} n &= 1^{st}\ integer \\ n+1 &= 2^{nd}\ next\ consecutive\ integer \end{aligned}$$

Step 4: **Translate.**

Restate as one sentence.

The sum of the integers is 47.

Translate into an equation.

$$n+n+1=47$$

***Step 5:* Solve the equation.**

$$\begin{array}{lrl} & n+n+1=47 & \\ \text{Combine like terms.} & 2n+1=47 & \\ \text{Subtract 1 from each side.} & 2n=46 & \\ \text{Divide each side by 2.} & n=23 & 1^{st}\ integer \\ & n+1 & 2^{nd}\ integer \\ & 23+1=24 & \end{array}$$

***Step 6:* Check.**

$$\begin{aligned} 23+24 &\stackrel{?}{=} 47 \\ 47 &= 47\checkmark \end{aligned}$$

***Step 7:* Answer the question.**

The two consecutive integers are 23 and 24.

Try It

24) The sum of two consecutive integers is 95. Find the numbers.

Solution

47, 48

25) The sum of two consecutive integers is -31. Find the numbers.

Solution

-16, -15

Example 9

Find three consecutive integers whose sum is -42.

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

three consecutive integers

Step 3: **Name each of the three numbers.**

$$\begin{aligned} n &= 1^{st}\ integer \\ n+1 &= 2^{nd}\ consecutive\ integer \\ n+2 &= 3^{rd}\ consecutive\ integer \end{aligned}$$

Step 4: **Translate.**

Restate as one sentence.

The sum of the three integers is -42.

Translate into an equation.

$$n + n + 1 + n + 2 = -42$$

Step 5: **Solve the equation.**

	$n + n + 1 + n + 2 = -42$	
Combine like terms.	$3n + 3 = -42$	
Subtract 3 from each side.	$3n = -45$	
Divide each side by 3.	$n = -15$	1^{st} *integer*
	$n + 1$	
	$-15 + 1 = -14$	2^{nd} *integer*
	$-15 + 2 = -13$	3^{rd} *integer*

Step 6: **Check.**

$$\begin{aligned} -13 + (-14) + (-15) &\stackrel{?}{=} -42 \\ -42 &= -42 \checkmark \end{aligned}$$

Step 7: **Answer the question.**

The three consecutive integers are -13, -14, and -15.

Try It

26) Find three consecutive integers whose sum is -96.

Solution
$-33, -32, -31$

27) Find three consecutive integers whose sum is -36.

Solution
$-13, -12, -11$

Now that we have worked with consecutive integers, we will expand our work to include consecutive even integers and consecutive odd integers. *Consecutive even integers* are even integers that immediately follow one another. Examples of consecutive even integers are:

$$18, 20, 22$$
$$64, 66, 68$$
$$-12, -10, -8$$

Notice each integer is 2 more than the number preceding it. If we call the first one n, then the next one is $n+2$. The next one would be $n+2+2$ or $n+4$.

n	1st even integer
$n+2$	2nd a consecutive even integer
$n+4$	3rd consecutive even integer… etc.

Consecutive odd integers are odd integers that immediately follow one another. Consider the consecutive odd integers 77, 79, and 81.

$$77, 79, 81$$

$$n, n+2, n+4$$

n	1st odd integer
$n+2$	2nd consecutive odd integer
$n+4$	3rd consecutive odd integer… etc.

Does it seem strange to add 2 (an even number) to get from one odd integer to the next? Do you get an odd number or an even number when we add 2 to 3? to 11? to 47?

Whether the problem asks for consecutive even numbers or odd numbers, you don't have to do anything different. The pattern is still the same—to get from one odd or one even integer to the next, add 2.

Example 10

Find three consecutive even integers whose sum is 84.

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

three consecutive even integers

Step 3: **Name the integers.**

Let $\begin{aligned} n &= 1^{st} \textit{ integer} \\ n+2 &= 2^{nd} \textit{ consecutive even integer} \\ n+4 &= 3^{rd} \textit{ consecutive even integer} \end{aligned}$

Step 4: **Translate.**

Restate as one sentence.

The sum of the three even integers is 84.

Translate into an equation.

$$n + n + 2 + n + 4 = 84$$

Step 5: **Solve the equation.**

$$\begin{array}{llr} & n + n + 2 + n + 4 = 84 & \\ \text{Combine like terms.} & 3n + 6 = 84 & \\ \text{Subtract 6 from each side.} & 3n + 6 - 6 = 84 - 6 & \\ \text{Divide each side by 3.} & 3n = 78 & \\ & n = 26 & 1^{st} \text{ integer} \\ & n + 2 & 2^{nd} \text{ integer} \\ & 26 + 2 = 28 & \\ & n + 4 & 3^{rd} \text{ integer} \\ & 26 + 4 = 30 & \end{array}$$

Step 6: **Check.**

$$\begin{aligned} 26 + 28 + 30 &\stackrel{?}{=} 84 \\ 84 &= 84\checkmark \end{aligned}$$

Step 7: **Answer the question.**

The three consecutive integers are 26, 28, and 30.

Try It

28) Find three consecutive even integers whose sum is 102.

Solution

32, 34, 36

29) Find three consecutive even integers whose sum is -24.

Solution

-10, -8, -6

Example 11

A married couple together earns $\$110,000$ a year. The wife earns $\$16,000$ less than twice what her husband earns. What does the husband earn?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

How much does the husband earn?

Step 3: **Name.**

Choose a variable to represent the amount the husband earns.

Let h $=$ the amount the husband earns.

The wife earns $\$16,000$ less than twice that.

$2h-16,000$ the amount the wife earns.

Together the husband and wife earn *$110,000*.

Step 4: **Translate.**

Restate the problem in one sentence with all the important information.

$$\underbrace{\text{The amount the husband earns}} + \underbrace{\text{the amount the wife earns}} \underbrace{\text{is}} \underbrace{\$110,000}$$

Translate into an equation.

$$h+2h-16,000=110,000$$

Step 5: **Solve the equation.**

$$\begin{array}{lrl} & h+2h-16,000=110,000 & \\ \text{Combine like terms.} & 3h-16,000=110,000 & \\ \text{Add 16,000 to both sides and simplify.} & 3h=126,000 & \\ \text{Divide each side by 3.} & h=42,000 & \\ & \$42,000 & \textit{the amount the husband earns} \\ & 2h-16 & \textit{the amount the wife earns} \\ & 2(42,000)-16,000 & \\ & 84,000-16,000=68,000 & \end{array}$$

Step 6: **Check.**

If the wife earns $\$68,000$ and the husband earns $\$42,000$ is the total $\$110,000$? Yes!

Step 7: **Answer the question.**

The husband earns $\$42,000$ a year.

Try It

30) According to the National Automobile Dealers Association, the average cost of a car in 2014 was $\$28,500$. This was $\$1,500$ less than 6 times the cost in 1975. What was the average cost of a car in 1975?

Solution

$\$5,000$

31) U.S. Census data shows that the median price of a new home in the United States in November 2014 was $\$280,900$. This was $\$10,700$ more than 14 times the price in November 1964. What was the median price of a new home in November 1964?

Solution

$\$19,300$

Translate and Solve Basic Percent Equations

We will solve percent equations using the methods we used to solve equations with fractions or decimals. Without the tools of algebra, the best method available to solve percent problems was by setting them up as proportions. Now as an algebra student, you can just translate English sentences into algebraic equations and then solve the equations.

We can use any letter you like as a variable, but it is a good idea to choose a letter that will remind us of what you are looking for. We must be sure to change the given percent to a decimal when we put it in the equation.

Example 12

Translate and solve: What number is 35% of 90?

Solution

$$\underbrace{\text{What number}}\ \underbrace{\text{is}}\ \underbrace{35\%}\ \underbrace{\text{of}}\ \underbrace{90?}$$

Step 1: **Translate into algebra. Let $n =$ be the number.**

Remember "of" means multiply, and "is" means equals.

$$n = 0.25 \cdot 90$$

Step 2: **Multiply.**

$$n = 31.5$$

31.5 is 35% of 90.

Try It

32) Translate and solve:

What number is 45% of 80?

Solution

36

33) Translate and solve:

What number is 55% of 60?

Solution

33

We must be very careful when we translate the words in the next example. The unknown quantity will not be isolated at first like it was in Example 3.7.12. We will again use direct translation to write the equation.

Example 13

Translate and solve: 6.5% of what number is $\$1.17$?

Solution

$$\underbrace{6.5\%}\ \underbrace{\text{of}}\ \underbrace{\text{what number}}\ \underbrace{\text{is}}\ \underbrace{\$1.17?}$$

***Step 1:* Translate. Let $n =$ be the number.**

$$0.065 \cdot n = 1.17$$

***Step 2:* Solve.**

$$\begin{array}{ll} & 0.065 \cdot n = 1.17 \\ \text{Multiply.} & 0.065n = 1.17 \\ \text{Divide both sides by 0.065 and simplify.} & n = 18 \end{array}$$

6.5% of $\$18$ is $\$1.17$.

Try It

34) Translate and solve: 7.5% of what number is $\$1.95$?

Solution

$\$26$

35) Translate and solve: 8.5% of what number is $\$3.06$?

Solution

$\$36$

In the next example, we are looking for the percent.

Example 14

Translate and solve: 144 is what percent of 96?

Solution

$$\underbrace{144}\ \underbrace{\text{is}}\ \underbrace{\text{what percent}}\ \underbrace{\text{of}}\ \underbrace{96?}$$

Step 1: **Translate into algebra. Let $p =$ the percent.**

$$144 = p \cdot 96$$

Step 2: **Solve.**

	$144 = p \cdot 96$
Multiply.	$144 = 96p$
Divide both sides by 96 and simplify.	$1.5 = p$
Convert to percent.	$150\% = p$

144 is 150% of 96.

Note that we are asked to find percent, so we must have our final result in percent form.

Try It

36) Translate and solve:

110 is what percent of 88?

Solution

125%

37) Translate and solve:

126 is what percent of 72?

Solution

175%

Solve Percent Applications

Many applications of percent—such as tips, sales tax, discounts, and **interest**—occur in our daily lives. To solve these applications we'll translate to a basic percent equation, just like those we solved in previous examples. Once we translate the sentence into a percent equation, we know how to solve it.

We will restate the problem-solving strategy we used earlier for easy reference.

How To

Use a Problem-Solving Strategy to Solve an Application.

1. Read the problem. Make sure all the words and ideas are understood.
2. Identify what we are looking for.
3. Name what we are looking for. Choose a variable to represent that quantity.
4. Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.
5. Solve the equation using good algebra techniques.
6. Check the answer to the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

Now that we have the strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications will involve everyday situations, you can rely on your own experience.

Example 15

Dezohn and his girlfriend enjoyed a nice dinner at a restaurant and his bill was $\$68.50$. He wants to leave an 18% tip. If the tip will be 18% of the total bill, how much tip should he leave?

Solution

***Step 1:* Read the problem.**

***Step 2:* Identify what we are looking for.**

the amount of tip should Dezohn leave

***Step 3:* Name what we are looking for.**

Choose a variable to represent it.

Let t $=$ amount of tip.

***Step 4:* Translate into an equation.**

Write a sentence that gives the information to find it.

The tip is 18% of the total bill.

Translate the sentence into an equation.

$$\underbrace{\text{The tip}}\ \underbrace{\text{is}}\ \underbrace{18\%}\ \underbrace{\text{of}}\ \underbrace{\$68.50}$$

***Step 5:* Solve the equation.**

$$\begin{array}{ll} & t = 0.18 \cdot 68.50 \\ \text{Multiply.} & t = 12.33 \end{array}$$

***Step 6:* Check. Does this make sense?**

Yes, 20% of $\$70$ is $\$14$.

***Step 7:* Answer the question with a complete sentence.**

Dezohn should leave a tip of $\$12.33$.

Notice that we used t to represent the unknown tip.

Try It

38) Cierra and her sister enjoyed dinner in a restaurant and the bill was $\$81.50$. If she wants to leave 18% of the total bill as her tip, how much should she leave?

Solution
$\$14.67$

39) Kimngoc had lunch at her favourite restaurant. She wants to leave 15% of the total bill as her tip. If her bill was $\$14.40$, how much will she leave for the tip?

Solution
$\$2.16$

Example 16

The label on Masao's breakfast cereal said that one serving of cereal provides 85 milligrams (mg) of potassium, which is 2% of the recommended daily amount. What is the total recommended daily amount of potassium?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

the total amount of potassium that is recommended

Step 3: **Name what we are looking for**.

Choose a variable to represent it.

Let $a =$ total amount of potassium.

Step 4: **Translate.**

Write a sentence that gives the information to find it.

$$\underbrace{85mg}\ \underbrace{\text{is}}\ \underbrace{2\%}\ \underbrace{\text{of the}}\ \underbrace{\text{total amount}}$$

Translate into an equation.

$$85 = 0.02 \cdot a$$

Step 5: **Solve the equation.**

$$\begin{aligned} 85 &= 0.02 \cdot a \\ 4,250 &= a \end{aligned}$$

Step 6: **Check. Does this make sense?**

Yes, 2% is a small percent and 85 is a small part of $4,250$.

Step 7: **Answer the question with a complete sentence.**

The amount of potassium that is recommended is $4,250mg$.

Try It

40) One serving of wheat square cereal has seven grams of fibre, which is 28% of the recommended daily amount. What is the total recommended daily amount of fibre?

Solution

$25g$

41) One serving of rice cereal has $190mg$ of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Solution

$2,375mg$

Example 17

Mitzi received some gourmet brownies as a gift. The wrapper said each brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

the percent of the total calories from fat

Step 3: **Name what we are looking for.**

Choose a variable to represent it.

Let $p =$ percent of fat.

Step 4: **Translate. Write a sentence that gives the information to find it.**

$$\underbrace{\text{What percent}}\ \underbrace{\text{of}}\ \underbrace{480}\ \underbrace{\text{is}}\ \underbrace{240?}$$

Translate into an equation.

$$p \cdot 480 = 240$$

Step 5: **Solve the equation.**

$$\begin{array}{ll} & 480p = 240 \\ \text{Divide by 480} & p = 0.5 \\ \text{Put in percent form} & p = 50\% \end{array}$$

Step 6: **Check. Does this make sense?**

Yes, 240 is half of 480, so 50% makes sense.

Step 7: **Answer the question with a complete sentence.**

Of the total calories in each brownie, 50% is fat.

Try It

42) Solve. Round to the nearest whole percent.

Veronica is planning to make muffins from a mix. The package says each muffin will be 230 calories and 60 calories will be from fat. What percent of the total calories is from fat?

Solution

26%

43) Solve. Round to the nearest whole percent.

The mix Ricardo plans to use to make brownies says that each brownie will be 190 calories, and 76 calories are from fat. What percent of the total calories are from fat?

Solution

40%

Find the Percent Increase and Percent Decrease

People in the media often talk about how much an amount has increased or decreased over a certain period of time. They usually express this increase or decrease as a percentage.

To find the percent increase, first, we find the amount of increase, the difference between the new amount and the original amount. Then we find what percent the amount of the increase is of the original amount.

How To

Find the Percent Increase.

1. Find the amount of increase.
   new amount − original amount = increase
2. Find the percent increase.
   The increase is what percent of the original amount?

Example 18

In 2011, the California governor proposed raising community college fees from $\$26$ a unit to $\$36$ a unit. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

the percent increase

Step 3: **Name what we are looking for.**

Choose a variable to represent it.

Let $p =$ the percent.

Step 4: **Translate**.

Write a sentence that gives the information to find it.

new amount − original amount = increase

First find the amount of increase.

$$36 - 26 = 10$$

Find the percent.

The increase is what percent of the original amount?

$$\underbrace{10}\ \underbrace{\text{is}}\ \underbrace{\text{what percent}}\ \underbrace{\text{of}}\ \underbrace{26?}$$

Translate into an equation.

$$10 = p \cdot 26$$

Step 5: **Solve the equation.**

$$\begin{array}{lr} & 10 = 26p \\ \text{Divide by 26.} & 0.3846 = p \\ \text{Change to percent form; round to the nearest tenth.} & 38.5\% = p \end{array}$$

Step 6: **Check. Does this make sense?**

Yes, 38.5% is close to $\frac{1}{3}$ and 10 is close to $\frac{1}{3}$ of 26.

Step 7: **Answer the question with a complete sentence.**

The new fees represent a 38.5% increase over the old fees.

Notice that we rounded the division to the nearest thousandth to round the percent to the nearest tenth.

Try It

44) Find the percent increase. (Round to the nearest tenth of a percent.)

In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents.

Solution

8.8%

45) Find the percent increase.

In 1995, the standard bus fare in Chicago was $\$1.50$. In 2008, the standard bus fare was $\$2.25$.

Solution

50%

Finding the percent decrease is very similar to finding the percent increase, but now the amount of decrease is the difference between the original amount and the new amount. Then we find what percent the amount of the decrease is of the original amount.

How To

Find the Percent Decrease.

1. Find the amount of decrease.
   original amount − new amount = decrease
2. Find the percent decrease.
   Decrease is what percent of the original amount?

Example 19

The average price of a gallon of gas in one city in June 2014 was $\$3.71$. The average price in that city in July was $\$3.64$. Find the percent decrease.

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

the percent decrease

Step 3: **Name what we are looking for.**

Choose a variable to represent that quantity.

Let p = the percent decrease.

Step 4: **Translate.**

Write a sentence that gives the information to find it.

First, find the amount of decrease.

$$3.71 - 3.64 = 0.07$$

Find the percent.

Decrease is what percent of the original amount?

$$\underbrace{0.07}\ \underbrace{\text{is}}\ \underbrace{\text{what percent}}\ \underbrace{\text{of}}\ \underbrace{3.71?}$$

Translate into an equation.

$$0.07 = p \cdot 3.71$$

Step 5: **Solve the equation.**

$$\begin{array}{ll} & 0.07 = 3.71p \\ \text{Divide by } 3.71. & 0.019 = p \\ \text{Change to percent form; round to the nearest tenth.} & 1.9\% = p \end{array}$$

Step 6: **Check. Does this make sense?**

Yes, if the original price was $\$4$, a 2% decrease would be 8 cents.

Step 7: **Answer the question with a complete sentence.**

The price of gas decreased 1.9%.

Try It

46) Find the percent decrease. (Round to the nearest tenth of a percent.)

The population of North Dakota was about $672,000$ in 2010. The population is projected to be about $630,000$ in 2020.

Solution

6.3%

47) Find the percent decrease.

Last year, Sheila's salary was $\$42,000$. Because of furlough days, this year, her salary was $\$37,800$.

Solution

10%

Solve Simple Interest Applications

Do you know that banks pay you to keep your money? The money a customer puts in the bank is called the principal, P, and the money the bank pays the customer is called the **interest**. The interest is computed as a certain percent of the principal; called the **rate of interest**, r. We usually express the rate of interest as a percent per year, and we calculate it by using the decimal equivalent of the percent. The variable t, (for *time*) represents the number of years the money is in the account.

To find the interest we use the **simple interest** formula, $I = Prt$.

Simple Interest

If an amount of money, P, called the principal, is invested for a period of t years at an annual interest rate r, the amount of interest, I, earned is

$$I = Prt \quad \text{where} \quad \begin{array}{l} I = interest \\ P = principal \\ r = rate \\ t = time \end{array}$$

Interest earned according to this formula is called simple interest.

Interest may also be calculated another way, called compound interest. This type of interest will be covered in later math classes.

The formula we use to calculate simple interest is $I = Prt$. To use the formula, we substitute the values the problem gives us for the variables, and then solve for the unknown variable. It may be helpful to organize the information in a chart.

Example 20

Nathaly deposited $\$12,500$ in her bank account where it will earn 4% interest. How much interest will Nathaly earn in 5 years?

$I = ?$

$P = \$12,500$

$r = 4\%$

$t = 5\ years$

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

the amount of interest earned

Step 3: **Name what we are looking for.**

Choose a variable to represent that quantity.

Let $I =$ the amount of interest.

Step 4: **Translate into an equation.**

Write the formula	$I = Prt$
Substitute in the given information	$I = (12,500)(0.04)(5)$

Step 5: **Solve the equation.**

$I = 2,500$

Step 6: **Check**: **Does this make sense?**

Is $\$2,500$ a reasonable interest on $\$12,500$? Yes.

Step 7: **Answer the question with a complete sentence.**

The interest is $\$2,500$.

Try It

48) Areli invested a principal of $\$950$ in her bank account with interest rate 3%. How much interest did she earn in 5 years?

Solution

$\$142.50$

49) Susana invested a principal of $\$36,000$ in her bank account with interest rate 6.5%. How much interest did she earn in 3 years?

Solution

$\$7,020$

There may be times when we know the amount of interest earned on a given principal over a certain length of time, but we don't know the rate. To find the rate, we use the simple interest formula, substitute the given values for the principal and time, and then solve for the rate.

Example 21

Loren loaned his brother $\$3,000$ to help him buy a car. In 4 years his brother paid him back the $\$3,000$ plus $\$660$ in interest. What was the rate of interest?

$I = \$660$

$P = \$3,000$

$r =$

$t = 4\ years$

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

the rate of interest

Step 3: **Name what we are looking for.**

Choose a variable to represent that quantity.

Let $r =$ the rate of interest.

Step 4: **Translate into an equation.**

Write the formula	$I = Prt$
Substitute in the given information	$660 = (3000)r(4)$

Step 5: **Solve the equation.**

	$660 = (3,000)r(4)$
Multiply.	$660 = (12,000)r$
Divide.	$0.055 = r$
Change to percent form.	$5.5\% = r$

Step 6: **Check**: **Does this make sense?**

$$I = Prt$$
$$660 \stackrel{?}{=} (3000)(0.55)(4)$$
$$660 = 660\checkmark$$

Step 7: **Answer the question with a complete sentence.**

The rate of interest was 5.5%.

Notice that in this example, Loren's brother paid Loren interest, just like a bank would have paid interest if Loren invested his money there.

Try It

50) Jim loaned his sister $\$5,000$ to help her buy a house. In 3 years, she paid him the $\$5,000$, plus $\$900$ interest. What was the rate of interest?

Solution

6%

51) Hang borrowed $\$7,500$ from her parents to pay her tuition. In 5 years, she paid them $\$1,500$ interest in addition to the $\$7,500$ she borrowed. What was the rate of interest?

Solution

4%

Example 22

Eduardo noticed that his new car loan papers stated that with a 7.5% interest rate, he would pay $\$6,596.25$ in interest over 5 years. How much did he borrow to pay for his car?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

the amount borrowed (the principal)

Step 3**: Name what we are looking for.**

Choose a variable to represent that quantity.

Let P = principal borrowed.

Step 4: **Translate into an equation.**

$$\begin{array}{ll} \text{Write the formula} & I = Prt \\ \text{Substitute in the given information} & 6,596.25 = P(0.075)(5) \end{array}$$

Step 5: **Solve the equation.**

$$\begin{array}{lr} & 6,596 = P(0.075)(5) \\ \text{Multiply.} & 6,596.25 = 0.375P \\ \text{Divide.} & 17,590 = P \end{array}$$

Step 6: **Check**: **Does this make sense?**

$$\begin{array}{r} I = Prt \\ 6.596.25 \stackrel{?}{=} (14,590)(0.075)(5) \\ 6,596.25 = 6,596.25\checkmark \end{array}$$

Step 7: **Answer the question with a complete sentence.**

The principal was $\$17,590$.

Try It

52) Sean's new car loan statement said he would pay $\$4,866.25$ in interest from an interest rate of 8.5% over 5 years. How much did he borrow to buy his new car?

Solution
$\$11,450$

53) In 5 years, Gloria's bank account earned $\$2,400$ interest at 5%. How much had she deposited in the account?

Solution
$\$9,600$

Solve Applications with Discount or Mark-up

Applications of discount are very common in retail settings. When you buy an item on sale, the original price has been discounted by some dollar amount. The **discount rate**, usually given as a percent, is used to determine the amount of the discount. To determine the **amount of discount**, we multiply the discount rate by the original price.

We summarize the discount model in the box below.

Discount

amount of discount = discount rate × original price

sale price = original price − amount of discount

Keep in mind that the sale price should always be less than the original price.

Example 23

Elise bought a dress that was discounted 35% off of the original price of $\$140$.

What was a. the amount of discount and b. the sale price of the dress?

Solution

a. Original Price = $\$140$ Discount rate = 35% Discount = ?

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

the amount of discount

Step 3: **Name what we are looking for.**

Choose a variable to represent that quantity.

Let $d =$ the amount of discount.

Step 4: **Translate into an equation.**

Write a sentence that gives the information to find it.

The discount is 35% of $140.

Translate into an equation.

$$d = 0.35(140)$$

Step 5: **Solve the equation.**

$$\begin{aligned} d &= 0.35(140) \\ d &= 49 \end{aligned}$$

***Step 6*: Check: Does this make sense?**

Is a $\$49$ discount reasonable for a $\$140$ dress? Yes.

***Step 7:* Write a complete sentence to answer the question.**

The amount of discount was $\$49$.

---

b. Read the problem again.

***Step 1:* Identify what we are looking for.**

the sale price of the dress

***Step 2:* Name what we are looking for.**

Choose a variable to represent that quantity.

Let s $=$ the sale price.

***Step 3:* Translate into an equation.**

Write a sentence that gives the information to find it.

$$\underbrace{\text{The sale price}}\ \underbrace{\text{is}}\ \underbrace{\text{the \$140}}\ \underbrace{\text{minus}}\ \underbrace{\text{the \$49 discount}}$$

Translate into an equation.

$$s = 140 - 49$$

***Step 4:* Solve the equation.**

$$\begin{aligned} s &= 140 - 49 \\ s &= 91 \end{aligned}$$

***Step 5:* Check. Does this make sense?**

Is the sale price less than the original price?

Yes, $\$91$ is less than $\$140$.

***Step 6:* Answer the question with a complete sentence.**

The sale price of the dress was $\$91$.

Try It

54) Find
a. the amount of discount and
b. the sale price:

Sergio bought a belt that was discounted 40% from an original price of $\$29$.

Solution
a. $\$11.60$
b. $\$17.40$

55) Find
a. the amount of discount and
b. the sale price:

Oscar bought a barbecue that was discounted 65% from an original price of $\$395$.

Solution
a. $\$256.75$
b. $\$138.25$

There may be times when we know the original price and the sale price, and we want to know the discount rate. To find the discount rate, first, we will find the amount of discount and then use it to compute the rate as a percent of the original price. Example 3.7.24 will show this case.

Example 24

Jeannette bought a swimsuit at a sale price of $\$13.95$. The original price of the swimsuit was $\$31$. Find the:

a. amount of discount and
b. discount rate.

Solution

a.

Original price = $\$31$

Discount = ?

Sale price = $\$13.95$

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

the amount of discount

Step 3: **Name what we are looking for.**

Choose a variable to represent that quantity.

Let $d =$ the amount of discount.

Step 4: **Translate into an equation.**

Write a sentence that gives the information to find it.

The discount is the difference between the original price and the sale price.

Translate into an equation.

$$d = 31 - 13.95$$

Step 5: **Solve the equation.**

$$d = 31 - 13.95$$
$$d = 17.05$$

Step 6: **Check**: **Does this make sense?**

Is 17.05 less than 31? Yes.

***Step 7:* Answer the question with a complete sentence.**

The amount of discount was $\$17.05$.

---

b. Read the problem again.

1. When we translate this into an equation, we obtain 17.05 equals r times 31. We are told to solve the equation 17.05 equals $31r$. We divide by 31 to obtain 0.55 equals r. We put this in percent form to obtain r equals 55. We are told to check: does this make sense? Is 7.05 equal to 55 of $> \ 1$? Below this, we have 17.05 equals with a question mark over it 0.55 times 31. Below this, we have 17.05 equals 17.05 with a checkmark next to it. Then we are told to answer the question with a complete sentence: The rate of discount was 55%.

***Step 1:* Identify what we are looking for.**

the discount rate

***Step 2:* Name what we are looking for.**

Choose a variable to represent it.

Let $r =$ the discount rate.

***Step 3:* Translate into an equation.**

Write a sentence that gives the information to find it.

$$\underbrace{\text{The discount of \$17.05}}\ \underbrace{\text{is}}\ \underbrace{\text{what percent}}\ \underbrace{\text{of}}\ \underbrace{\text{\$31?}}$$

Translate into an equation.

$$17.05 = r \cdot 31$$

Step 4: **Solve the equation.**

$$\begin{array}{lr} & 17.05 = 31r \\ \text{Divide both sides by 31.} & 0.55 = r \\ \text{Change to percent form} & r = 55\% \end{array}$$

Step 5: **Check. Does this make sense?**

Is $\$17.05$ equal to 55% of $\$31$?

$$\begin{array}{l} 17.05 = 0.55 \times (31) \\ 17.05 = 17.05 \checkmark \end{array}$$

Step 6: **Answer the question with a complete sentence.**

The rate of discount was 55%.

Try It

56) Find
a. the amount of discount and
b. the discount rate.

Lena bought a kitchen table at the sale price of $\$375.20$. The original price of the table was $\$560$.

Solution
a. $\$184.80$
b. 33%

57) Find
a. the amount of discount and
b. the discount rate.

Nick bought a multi-room air conditioner at a sale price of $\$340$. The original price of the air conditioner was $\$400$.

Solution

a. $\$60$
b. 15%

Applications of **mark-up** are very common in retail settings. The price a retailer pays for an item is called the original cost. The retailer then adds a mark-up to the original cost to get the list price, the price he sells the item for. The mark-up is usually calculated as a percent of the original cost. To determine the amount of mark-up, multiply the mark-up rate by the original cost.

We summarize the mark-up model in the box below.

Mark-Up

amount of mark-up = mark-up rate × original cost
list price = original cost + amount of mark up

Keep in mind that the **list price** should always be more than the **original cost**.

Example 25

Adam's art gallery bought a photograph at the original cost $\$250$. Adam marked the price up 40%. Find the:

a. amount of mark-up and
b. the list price of the photograph.

Solution

a.

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

the amount of mark-up

Step 3: **Name what we are looking for.**

Choose a variable to represent it.

Let $m =$ the amount of markup.

Step 4: **Translate into an equation.**

Write a sentence that gives the information to find it.

$$\underbrace{\text{The mark-up}}\ \underbrace{\text{is}}\ \underbrace{40\%}\ \underbrace{\text{of the \$250 original cost}}$$

Translate into an equation.

$$m = 0.40 \cdot 250$$

Step 5: **Solve the equation.**

$$m = 0.40 \cdot 250)$$
$$m = 100$$

Step 6: **Check. Does this make sense?**

Yes, 40% is less than one-half and 100 is less than half of 250.

Step 7: **Answer the question with a complete sentence.**

The mark-up on the photograph was $\$100$.

---

b.

Step 1: **Read the problem again.**

Step 2: **Identify what we are looking for.**

the list price

Step 3: **Name what we are looking for.**

Choose a variable to represent it.

Let $p =$ the list price.

Step 4: **Translate into an equation.**

Write a sentence that gives the information to find it.

$$\underbrace{\text{The list price}}\ \underbrace{\text{is}}\ \underbrace{\text{original cost}}\ \underbrace{\text{plus}}\ \underbrace{\text{the mark-up}}$$

Translate into an equation.

$$p = 250 + 100$$

Step 5: **Solve the equation.**

$$\begin{aligned} p &= 250 + 100 \\ p &= 350 \end{aligned}$$

Step 6: **Check. Does this make sense?**

Is the list price more than the net price?
Is $\$350$ more than $\$250$? Yes

Step 7: **Answer the question with a complete sentence.**

The list price of the photograph was $\$350$.

Try It

58) Find
a. the amount of mark-up and
b. the list price.

Jim's music store bought a guitar at the original cost $\$1,200$. Jim marked the price up 50%.

Solution
a. $\$600$
b. $\$1,800$

59) Find
a. the amount of mark-up and
b. the list price.

The Auto Resale Store bought Pablo's Toyota for $\$8,500$. They marked the price up 35%.

Solution

a. $\$2,975$

b. $\$11,475$

Glossary

amount of discount
: The amount of discount is the amount resulting when a discount rate is multiplied by the original price of an item.

discount rate
: The discount rate is the percent used to determine the amount of a discount, common in retail settings.

interest
: Interest is the money that a bank pays its customers for keeping their money in the bank.

list price
: The list price is the price a retailer sells an item for.

mark-up
: A mark-up is a percentage of the original cost used to increase the price of an item.

original cost
: The original cost in a retail setting, is the price that a retailer pays for an item.

principal
: The principal is the original amount of money invested or borrowed for a period of time at a specific interest rate.

rate of interest
: The rate of interest is a percent of the principal, usually expressed as a percent per year.

simple interest
: Simple interest is the interest earned according to the formula $I = Prt$.

Exercises: Use the Approach Word Problems with a Positive Attitude

Instructions: For questions 1-2, prepare the lists described.

1. List five positive thoughts you can say to yourself that will help you approach word problems with a positive attitude. You may want to copy them on a sheet of paper and put them in the front of your notebook, where you can read them often.
Solution
Answers will vary

---

2. List five negative thoughts that you have said to yourself in the past that will hinder your progress on word problems. You may want to write each one on a small piece of paper and rip it up to symbolically destroy the negative thoughts.

Exercises: Use a Problem-Solving Strategy for Word Problems

Instructions: For questions 3-14, solve using the problem-solving strategy for word problems. Remember to write a complete sentence to answer each question

3. Two-thirds of the children in the fourth-grade class are girls. If there are 20 girls, what is the total number of children in the class?
Solution

30 children

---

4. Three-fifths of the members of the school choir are women. If there are 24 women, what is the total number of choir members?

---

5. Zachary has 25 country music CDs, which is one-fifth of his CD collection. How many CDs does Zachary have?

Solution

125 CDs

---

6. One-fourth of the candies in a bag of M&M's are red. If there are 23 red candies, how many candies are in the bag?

---

7. There are 16 girls in a school club. The number of girls is four more than twice the number of boys. Find the number of boys.

Solution

6 boys

---

8. There are 18 Cub Scouts in Pack 645. The number of scouts is three more than five times the number of adult leaders. Find the number of adult leaders.

---

9. Huong is organizing paperback and hardback books for her club's used book sale. The number of paperbacks is 12 less than three times the number of hardbacks. Huong had 162 paperbacks. How many hardback books were there?

Solution

58 books

---

10. Jeff is lining up children's and adult bicycles at the bike shop where he works. The number of children's bicycles is nine less than three times the number of adult bicycles. There are 42 adult bicycles. How many children's bicycles are there?

---

11. Philip pays $\$1,620$ in rent every month. This amount is $\$120$ more than twice what his brother Paul pays for rent. How much does Paul pay for rent?

Solution

$\$750$

---

12. Marc just bought an SUV for $\$54,000$. This is $\$7,400$ less than twice what his wife paid for her car last year. How much did his wife pay for her car?

---

13. Laurie has $\$46,000$ invested in stocks and bonds. The amount invested in stocks is $\$8,000$ less than three times the amount invested in bonds. How much does Laurie have invested in bonds?

Solution

$\$9,500$

---

14. Erica earned a total of $\$50,450$ last year from her two jobs. The amount she earned from her job at the store was $\$1,250$ more than three times the amount she earned from

her job at the college. How much did she earn from her job at the college?

Exercises: Solve Number Problems

Instructions: For questions 15-56, solve each number word problem.

15. The sum of a number and eight is 12. Find the number.

Solution

4

---

16. The sum of a number and nine is 17. Find the number.

---

17. The difference of a number and 12 is three. Find the number.

Solution

15

---

18. The difference between a number and eight is four. Find the number.

---

19. The sum of three times a number and eight is 23. Find the number.

Solution

5

---

20. The sum of twice a number and six is 14. Find the number.

---

21. The difference between twice a number and seven is 17. Find the number.

Solution

12

---

22. The difference of four times a number and seven is 21. Find the number.

---

23. Three times the sum of a number and nine is 12. Find the number.

Solution

-5

---

24. Six times the sum of a number and eight is 30. Find the number.

25. One number is six more than the other. Their sum is 42. Find the numbers.

Solution

18, 24

26. One number is five more than the other. Their sum is 33. Find the numbers.

27. The sum of two numbers is 20. One number is four less than the other. Find the numbers.

Solution

$8, 12$

28. The sum of two numbers is 27. One number is seven less than the other. Find the numbers.

29. The sum of two numbers is -45. One number is nine more than the other. Find the numbers.

Solution

$-18, -27$

30. The sum of two numbers is -61. One number is 35 more than the other. Find the numbers.

---

31. The sum of two numbers is -316. One number is 94 less than the other. Find the numbers.

Solution

$-111, -205$

---

32. The sum of two numbers is -284. One number is 62 less than the other. Find the numbers.

---

33. One number is 14 less than another. If their sum is increased by seven, the result is 85. Find the numbers.

Solution

$32, 46$

---

34. One number is 11 less than another. If their sum is increased by eight, the result is 71. Find the numbers.

---

35. One number is five more than another. If their sum is increased by nine, the result is

60. Find the numbers.

Solution
$23, 28$

---

36. One number is eight more than another. If their sum is increased by 17, the result is 95. Find the numbers.

---

37. One number is one more than twice another. Their sum is -5. Find the numbers.

Solution
$-2, -3$

---

38. One number is six more than five times another. Their sum is six. Find the numbers.

---

39. The sum of two numbers is 14. One number is two less than three times the other. Find the numbers.

Solution
$4, 10$

---

40. The sum of two numbers is zero. One number is nine less than twice the other. Find the numbers.

---

41. The sum of two consecutive integers is 77. Find the integers.

Solution

$38, 39$

---

42. The sum of two consecutive integers is 89. Find the integers.

---

43. The sum of two consecutive integers is -23. Find the integers.

Solution

$-11, -12$

---

44. The sum of two consecutive integers is -37. Find the integers.

---

45. The sum of three consecutive integers is 78. Find the integers.

Solution

$25, 26, 27$

---

46. The sum of three consecutive integers is 60. Find the integers.

---

47. Find three consecutive integers whose sum is -36.

Solution

$-11, -12, -13$

---

48. Find three consecutive integers whose sum is -3.

---

49. Find three consecutive even integers whose sum is 258.

Solution

$84, 86, 88$

---

50. Find three consecutive even integers whose sum is 222.

---

51. Find three consecutive odd integers whose sum is 171.

Solution

$55, 57, 59$

---

52. Find three consecutive odd integers whose sum is 291.

---

53. Find three consecutive even integers whose sum is -36.

Solution

$-10, -12, -14$

---

54. Find three consecutive even integers whose sum is -84.

55. Find three consecutive odd integers whose sum is -213.

Solution

$-69, -71, -73$

56. Find three consecutive odd integers whose sum is -267.

Exercises: Translate and Solve Basic Percent Equations

Instructions: For questions 57-80, translate and solve.

57. What number is 45% of 120?

Solution

$$54$$

58. What number is 65% of 100?

59. What number is 24% of 112?

Solution

26.88

60. What number is 36% of 124?

61. 250% of 65 is what number?

Solution

162.5

62. 150% of 90 is what number?

63. 800% of 2250 is what number?

Solution

$18{,}000$

64. 600% of 1740 is what number?

65. 28 is 25% of what number?

Solution

112

66. 36 **is** 25% **of what number?**

---

67. 81 **is** 75% **of what number?**

Solution

108

---

68. 93 **is** 75% **of what number?**

---

69. 8.2% **of what number is** $\$2.87$**?**

Solution

$\$35$

---

70. 6.4% **of what number is** $\$2.88$**?**

---

71. 11.5% **of what number is** $\$108.10$**?**

Solution

$\$940$

---

72. 12.3% **of what number is** $\$92.25$**?**

---

73. What percent of 260 is 78?

Solution

30%

---

74. What percent of 215 is 86?

---

75. What percent of 1500 is 540?

Solution

36%

---

76. What percent of 1800 is 846?

---

77. 30 is what percent of 20?

Solution

150%

---

78. 50 is what percent of 40?

---

79. 840 is what percent of 480?

Solution

175%

---

80. 790 **is what percent of** 395**?**

Exercises: Solve Percent Applications

Instructions: For questions 81-98, solve.

81. Geneva treated her parents to dinner at their favorite restaurant. The bill was $\$74.25$**. Geneva wants to leave** 16% **of the total bill as a tip. How much should the tip be?**

Solution

$\$11.88$

---

82. When Hiro and his co-workers had lunch at a restaurant near their work, the bill was $\$90.50$**. They want to leave** 18% **of the total bill as a tip. How much should the tip be?**

---

83. Trong has 12% **of each paycheck automatically deposited to his savings account. His last paycheck was** $\$2165$**. How much money was deposited to Trong's savings account?**

Solution

$\$259.80$

---

84. Cherise deposits 8% of each paycheck into her retirement account. Her last paycheck was $\$1,485$. How much did Cherise deposit into her retirement account?

---

85. One serving of oatmeal has eight grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?

Solution

24.2 g

---

86. One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?

---

87. A bacon cheeseburger at a popular fast food restaurant contains 2070 milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?

Solution

2407 mg

---

88. A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?

---

89. After 3 months on a diet, Lisa had lost 12% of her original weight. She lost 21

pounds. What was Lisa's original weight?

Solution

175 lb.

---

90. Tricia got a 6% raise on her weekly salary. The raise was $\$30$ per week. What was her original salary?

---

91. Yuki bought a dress on sale for $\$72$. The sale price was 60% of the original price. What was the original price of the dress?

Solution

$\$120$

---

92. Kim bought a pair of shoes on sale for $\$40.50$. The sale price was 45% of the original price. What was the original price of the shoes?

---

93. Tim left a $\$9$ tip for a $\$50$ restaurant bill. What percent tip did he leave?

Solution

18%

---

94. Rashid left a $\$15$ tip for a $\$75$ restaurant bill. What percent tip did he leave?

---

95. The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?

Solution

45%

---

96. The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?

---

97. Emma gets paid $\$3{,}000$ per month. She pays $\$750$ a month for rent. What percent of her monthly pay goes to rent?

Solution

25%

---

98. Dimple gets paid $3{,}200$ per month. She pays $\$960$ a month for rent. What percent of her monthly pay goes to rent?

Exercises: Find Percent Increase and Percent Decrease

Instructions: For questions 99-110, solve.

99. Tamanika got a raise in her hourly pay, from $\$15.50$ to $\$17.36$. Find the percent increase.

Solution

12%

---

100. Ayodele got a raise in her hourly pay, from $\$24.50$ to $\$25.48$. Find the percent increase.

---

101. Annual student fees at the University of California rose from about $\$4,000$ in 2000 to about $12,000$ in 2010. Find the percent increase.

Solution

200%

---

102. The price of a share of one stock rose from $\$12.50$ to $\$50$. Find the percent increase.

---

103. According to *Time* magazine annual global seafood consumption rose from 22 pounds per person in the 1960s to 38 pounds per person in 2011. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution

72.7%

---

104. In one month, the median home price in the Northeast rose from $\$225,400$ to $\$241,500$. Find the percent increase. (Round to the nearest tenth of a percent.)

---

105. A grocery store reduced the price of a loaf of bread from $\$2.80$ to $\$2.73$. Find the percent decrease.

Solution

2.5%

---

106. The price of a share of one stock fell from $\$8.75$ to $\$8.54$. Find the percent decrease.

---

107. Hernando's salary was $\$49,500$ last year. This year his salary was cut to $\$44,055$. Find the percent decrease.

Solution

11%

---

108. In 10 years, the population of Detroit fell from 950,000 to about 712,500. Find the percent decrease.

---

109. In 1 month, the median home price in the West fell from $\$203{,}400$ to $\$192{,}300$. Find the percent decrease. (Round to the nearest tenth of a percent.)

Solution

5.5%

---

110. Sales of video games and consoles fell from $\$1{,}150$ million to $\$1{,}030$ million in 1 year. Find the percent decrease. (Round to the nearest tenth of a percent.)

Exercises: Solve Simple Interest Applications

Instructions: For questions 111-122, solve.

111. Casey deposited $\$1,450$ in a bank account with interest rate 4%. How much interest was earned in two years?

Solution

$\$116$

---

112. Terrence deposited $\$5,720$ in a bank account with interest rate 6%. How much interest was earned in 4 years?

---

113. Robin deposited $\$31,000$ in a bank account with interest rate 5.2%. How much interest was earned in 3 years?

Solution

$\$4,836$

---

114. Carleen deposited $\$16,400$ in a bank account with interest rate 3.9%. How much interest was earned in 8 years?

---

115. Hilaria borrowed $\$8,000$ from her grandfather to pay for college. Five years later, she paid him back the $\$8,000$, plus $\$1,200$ interest. What was the rate of interest?

Solution

3%

---

116. Kenneth loaned his niece $\$1{,}200$ to buy a computer. Two years later, she paid him back the $\$1{,}200$, plus $\$96$ interest. What was the rate of interest?

---

117. Lebron loaned his daughter $\$20{,}000$ to help her buy a condominium. When she sold the condominium four years later, she paid him the $\$20{,}000$, plus $\$3{,}000$ interest. What was the rate of interest?

Solution

3.75%

---

118. Pablo borrowed $\$50{,}000$ to start a business. Three years later, he repaid the $\$50{,}000$, plus $\$9{,}375$ interest. What was the rate of interest?

---

119. In 10 years, a bank account that paid 5.25% earned $\$18{,}375$ interest. What was the principal of the account?

Solution

$\$35{,}000$

---

120. In 25 years, a bond that paid 4.75% earned $\$2{,}375$ interest. What was the principal of the bond?

---

121. Joshua's computer loan statement said he would pay $\$1{,}244.34$in interest for a 3-year loan at 12.4%. How much did Joshua borrow to buy the computer?

Solution

$\$3,345$

---

122. Margaret's car loan statement said she would pay $\$7,683.20$ in interest for a 5-year loan at 9.8%. How much did Margaret borrow to buy the car?

Exercises: Solve Applications with Discount or Mark-up

Instructions: For questions 123-126, find the sale price.

123. Perla bought a cell phone that was on sale for $\$50$ off. The original price of the cell phone was $\$189$.

Solution

$\$139$

---

124. Sophie saw a dress she liked on sale for $\$15$ off. The original price of the dress was $\$96$.

---

125. Rick wants to buy a tool set with original price $\$165$. Next week the tool set will be on sale for $\$40$ off.

Solution

$\$125$

---

126. Angelo's store is having a sale on televisions. One television, with original price $\$859$, is selling for $\$125$ off.

Exercises: Find Discount and Sales Price Amounts

Instructions: For questions 127-132, find

a. the amount of discount
b. the sale price.

127. Janelle bought a beach chair on sale at 60% off. The original price was $\$44.95$.

Solution

a. $\$26.97$

b. $\$17.98$

---

128. Errol bought a skateboard helmet on sale at 40% off. The original price was $\$49.95$.

---

129. Kathy wants to buy a camera that lists for $\$389$. The camera is on sale with a 33% discount.

Solution

a. $\$128.37$

b. $\$260.63$

---

130. Colleen bought a suit that was discounted 25% from an original price of $\$245$.

---

131. Erys bought a treadmill on sale at 35% off. The original price was $\$949.95$ (round to the nearest cent.)

Solution

a. $\$332.48$

b. $\$617.47$

---

132. Jay bought a guitar on sale at 45% off. The original price was $\$514.75$ (round to the nearest cent.)

Exercises: Find Discount and Discount Rate Amounts

Instructions: For questions 133-138, find

a. the amount of discount

b. the discount rate (Round to the nearest tenth of a percent if needed)

133. Larry and Donna bought a sofa at the sale price of $\$1,344$. The original price of the sofa was $\$1,920$.

Solution

a. $\$576$

b. 30%

---

134. Hiroshi bought a lawnmower at the sale price of $\$240$. The original price of the lawnmower is $\$300$.

---

135. Patty bought a baby stroller on sale for $\$301.75$. The original price of the stroller was $\$355$.

Solution

a. $\$53.25$

b. 15%

---

136. Bill found a book he wanted on sale for $\$20.80$. The original price of the book was $\$32$.

---

137. Nikki bought a patio set on sale for $\$480$. The original price was $\$850$. To the nearest tenth of a percent, what was the rate of discount?

Solution

a. $\$370$

b. 43.5%

---

138. Stella bought a dinette set on sale for $\$725$. The original price was $\$1,299$. To the nearest tenth of a percent, what was the rate of discount?

Exercises: Find Mark-up and List Price Amounts

Instructions: For questions 139-144, find

a. the amount of the mark-up
b. the list price.

139. Daria bought a bracelet at original cost $\$16$ to sell in her handicraft store. She marked the price up 45%.

Solution

a. $\$7.20$

b. $\$23.20$

---

140. Regina bought a handmade quilt at original cost $\$120$ to sell in her quilt store. She marked the price up 55%.

---

141. Tom paid $\$0.60$ a pound for tomatoes to sell at his produce store. He added a 33% mark-up.

Solution

a. $\$0.20$

b. $\$0.80$

---

142. Flora paid her supplier $\$0.74$ a stem for roses to sell at her flower shop. She added an 85% mark-up.

143. Alan bought a used bicycle for $\$115$. After re-conditioning it, he added 225% mark-up and then advertised it for sale.

Solution

a. $\$258.75$

b. $\$373.75$

144. Michael bought a classic car for $\$8,500$. He restored it, then added 150% mark-up before advertising it for sale.

Exercises: Everyday Math

Instructions: For questions 149-152, answer the given everyday math word problems.

145. Sale Price. Patty paid $\$35$ for a purse on sale for $\$10$ off the original price. What was the original price of the purse?

Solution

$\$45$

146. Sale Price. Travis bought a pair of boots on sale for $\$25$ off the original price. He paid $\$60$ for the boots. What was the original price of the boots?

147. Buying in Bulk. Minh spent $\$6.25$ on five sticker books to give his nephews. Find the cost of each sticker book.

Solution

$\$1.25$

---

148. Buying in Bulk. Alicia bought a package of eight peaches for $\$3.20$. Find the cost of each peach.

---

149. Price before Sales Tax. Tom paid $1{,}166.40$ for a new refrigerator, including $\$86.40$ tax. What was the price of the refrigerator?

Solution

$\$1080$

---

150. Price before Sales Tax. Kenji paid $2{,}279$ for a new living room set, including $\$129$ tax. What was the price of the living room set?

---

151. Leaving a Tip. At the campus coffee cart, a medium coffee costs $\$1.65$. MaryAnne brings $\$2.00$ with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?

Solution

21.2%

---

152. Splitting a Bill. Four friends went out to lunch and the bill came to $\$53.75$. They decided to add enough tip to make a total of $\$64$, so that they could easily split the bill evenly among themselves. What percent tip did they leave?

Exercises: Writing Exercises

Instructions: For questions 153-160, answer the given writing exercises.

153. What has been your past experience solving word problems?

Solution

Answers will vary.

---

154. When you start to solve a word problem, how do you decide what to let the variable represent?

---

155. What are consecutive odd integers? Name three consecutive odd integers between 50 and 60.

Solution

Consecutive odd integers are odd numbers that immediately follow each other. An example of three consecutive odd integers between 50 and 60 would be 51, 53, and 55.

---

156. What are consecutive even integers? Name three consecutive even integers between -50 and -40.

---

157. Without solving the problem "44 is 80% of what number" think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

Solution

The number should be greater than 44. Since 80% equals 0.8 in decimal form, 0.8 is less than one, and we must multiply the number by 0.8 to get 44, the number must be greater than 44.

---

158. Without solving the problem "What is 20% of 300?" think about what the solution might be. Should it be a number that is greater than 300 or less than 300? Explain your reasoning.

---

159. After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.

Solution

He meant that he should have packed half the shorts and twice the shirts.

---

160. Because of road construction in one city, commuters were advised to plan that their Monday morning commute would take 150% of their usual commuting time. Explain what this means.

3.8 SOLVE MIXTURE AND UNIFORM MOTION APPLICATIONS

Learning Objectives

By the end of this section, you will be able to:

- Solve coin word problems
- Solve ticket and stamp word problems
- Solve mixture word problems
- Solve uniform motion applications

Try It

Before you get started, take this readiness quiz:

1) Multiply: $14(0.25)$.

2) Solve: $(0.25x) + 0.10(x + 4) = 2.5$

3) The number of dimes is three more than the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

4) Find the distance travelled by a car going 70 miles per hour for 3 hours.

5) Solve $x+1.2(x-10)=98$.

6) Convert 90 minutes to hours.

Solve Coin Word Problems

In **mixture problems**, we will have two or more items with different values to combine. The mixture model is used by grocers and bartenders to make sure they set fair prices for the products they sell. Many other professionals, like chemists, investment bankers, and landscapers also use the mixture model. Doing the Manipulative Mathematics activity *Coin Lab* will help you develop a better understanding of mixture word problems.

We will start by looking at an application everyone is familiar with—money!

Imagine that we take a handful of coins from a pocket or purse and place them on a desk. How would we determine the value of that pile of coins? If we can form a step-by-step plan for finding the total value of the coins, it will help us as we begin solving coin word problems.

So what would we do? To get some order to the mess of coins, we could separate the coins into piles according to their value. Quarters would go with quarters, dimes with dimes, nickels with nickels, and so on. To get the total value of all the coins, we would add the total value of each pile.

Figure 3.8.1.

How would we determine the value of each pile? Think about the dime pile—how much is it worth? If we count the number of dimes, we'll know how many we have—the *number* of dimes.

But this does not tell us the *value* of all the dimes. Say we counted 17 dimes, how much are they worth? Each dime is worth $0.10—that is the *value* of one dime. To find the total value of the pile of 17 dimes, multiply 17 by $0.10 to get $1.70. This is the total value of all 17 dimes. This method leads to the following model.

How To

Total Value of Coins

For the same type of coin, the total value of several coins is found by using the model

$$number \times value = total$$

Where

number is the number of coins
value is the value of each coin
total value is the total value of all the coins

The number of dimes times the value of each dime equals the total value of the dimes.

$$Number \times Value = Total\ Value$$

We could continue this process for each type of coin, and then we would know the total value of each type of coin. To get the total value of *all* the coins, add the total value of each type of coin.

Let's look at a specific case. Suppose there are 14 quarters, 17 dimes, 21 nickels, and 39 pennies.

Type	Number ·	Value($)	= Total Value ($)
Quarters	14	0.25	3.50
Dimes	17	0.10	1.70
Nickels	21	0.05	1.05
Pennies	39	0.01	0.39
			6.64

The total value of all the coins is $\$6.64$.

Notice how the chart helps organize all the information! Let's see how we use this method to solve a coin word problem.

Example 1

Adalberto has $\$2.25$ in dimes and nickels in his pocket. He has nine more nickels than dimes. How many of each type of coin does he have?

Solution

***Step 1:* Read the problem. Make sure all the words and ideas are understood.**

- **Determine** the types of coins involved.
  Think about the strategy we used to find the value of the handful of coins. The first thing we need is to notice what types of coins are involved. Adalberto has dimes and nickels.
- **Create a table** to organize the information. See chart below.
  - Label the columns "type," "number," "value," "total value."
  - List the types of coins.
  - Write in the value of each type of coin.
  - Write in the total value of all the coins.

Type	Number ·	Value ($)	= Total Value ($)
Dimes		0.10	
Nickels		0.05	
			2.25

We can work this problem all in cents or in dollars. Here we will do it in dollars and put in the dollar sign ($) in the table as a reminder.
The value of a dime is $\$0.10$ and the value of a nickel is $\$0.05$. The total value of all the coins is $\$2.25$. The table below shows this information.

Step 2: **Identify what we are looking for.**

- We are asked to find the number of dimes and nickels Adalberto has.

Step 3: **Name what we are looking for. Choose a variable to represent that quantity.**

- Use variable expressions to represent the number of each type of coin and write them in the table.
- Multiply the number times the value to get the total value of each type of coin.

Next we counted the number of each type of coin. In this problem we cannot count each type of coin—that is what you are looking for—but we have a clue. There are nine more nickels than dimes. The number of nickels is nine more than the number of dimes.

Let $d =$ number of dimes.

$d + 9 =$ number of nickles

Fill in the “number” column in the table to help get everything organized.

Type	Number ·	Value ($)	= Total Value ($)
Dimes	d	0.10	
Nickels	$d+9$	0.05	
			2.25

Now we have all the information we need from the problem!

We multiply the number times the value to get the total value of each type of coin. While we do not know the actual number, we do have an expression to represent it.

And so now multiply $number \times value = total\ value$ See how this is done in the table below.

Type	Number ·	Value ($)	= Total Value ($)
Dimes	d	0.10	$0.10d$
Nickels	$d+9$	0.05	$0.05(d+9)$
			2.25

Notice that we made the heading of the table show the model.

Step 4: **Translate into an equation. It may be helpful to restate the problem in one sentence. Translate the English sentence into an algebraic equation.**

Write the equation by adding the total values of all the types of coins.

	Value of dimes	+	Value of nickels	=	Total value of coins
Translate to an equation	$0.10d$	+	$0.05(d+9)$	=	2.25

Step 5: **Solve the equation using good algebra techniques.**

Now solve this equation	$0.10d + 0.05(d+9) = 2.25$
Distribute	$0.10d + 0.05d + 0.45 = 2.25$
Combine like terms	$0.15d + 0.45 = 2.25$
Subtract 0.45 from each side	$0.15d = 2.25$
Divide	$d = 12$
So there are 12 dimes	
The number of nickels is $(d+9)$	$d + 9$
	$12 + 9 = 21$

Step 6: **Check the answer in the problem and make sure it makes sense.**

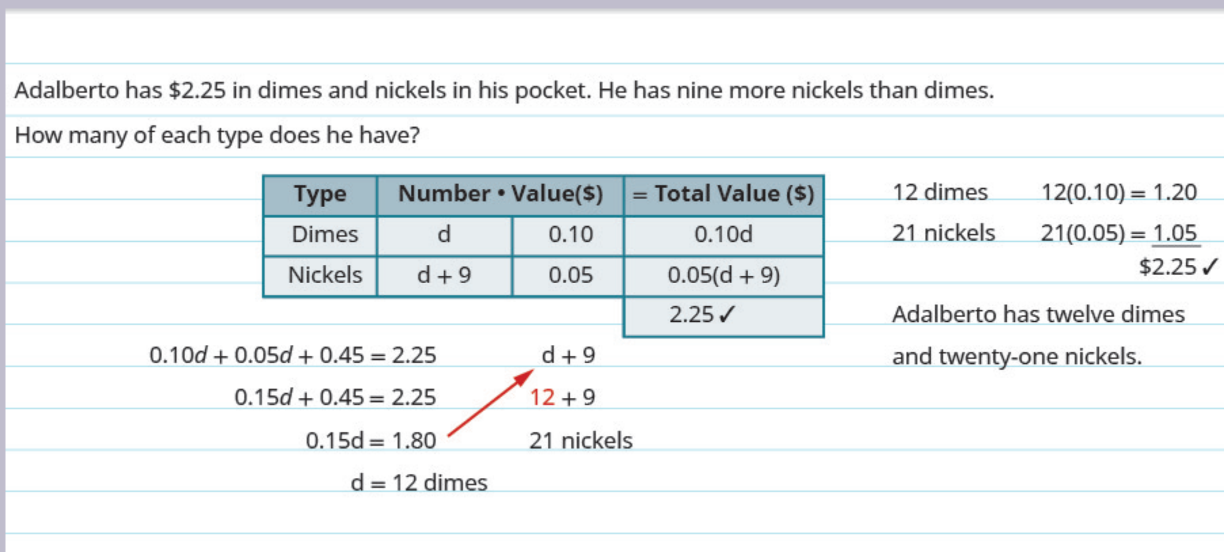
Does this check?

12 dimes	$12(0.10) = 1.20$
21 nickels	$21(0.05) = 2.25\surd$

Step 7: **Answer the question with a complete sentence.**

- Adalberto has twelve dimes and twenty-one nickels.

  If this were a homework exercise, our work might look like the following.



Adalberto has $2.25 in dimes and nickels in his pocket. He has nine more nickels than dimes. How many of each type does he have?

Type	Number • Value($)		= Total Value ($)
Dimes	d	0.10	0.10d
Nickels	d + 9	0.05	0.05(d + 9)
			2.25 ✓

$0.10d + 0.05d + 0.45 = 2.25$

$0.15d + 0.45 = 2.25$

$0.15d = 1.80$

d = 12 dimes

d + 9

12 + 9

21 nickels

12 dimes 12(0.10) = 1.20

21 nickels 21(0.05) = 1.05

$2.25 ✓

Adalberto has twelve dimes and twenty-one nickels.

Figure 3.8.2

Try It

7) Michaela has $\$2.05$ in dimes and nickels in her change purse. She has seven more dimes than nickels. How many coins of each type does she have?

Solution

9 nickels, 16 dimes

8) Liliana has $\$2.10$ in nickels and quarters in her backpack. She has 12 more nickels than quarters. How many coins of each type does she have?

Solution

17 nickels, 5 quarters

Example 2

Maria has $\$2.43$ in quarters and pennies in her wallet. She has twice as many pennies as quarters. How many coins of each type does she have?

Solution

***Step 1:* Read the problem.**

Determine the types of coins involved.
We know that Maria has quarters and pennies.
Create a table to organize the information.

- Label the columns "type," "number," "value," "total value."
- List the types of coins.
- Write in the value of each type of coin.
- Write in the total value of all the coins.

Type	Number ·	Value ($)	= Total Value ($)
Quarters		0.25	
Pennies		0.01	
			2.43

Step 2: **Identify what you are looking for.**

- We are looking for the number of quarters and pennies.

Step 3: **Name. Represent the number of quarters and pennies using variables.**

- We know Maria has twice as many pennies as quarters. The number of pennies is defined in terms of quarters.
- Let q represent the number of quarters.
- Then the number of pennies is $2q$.

Type	Number ·	Value ($)	= Total Value ($)
Quarters	q	0.25	
Pennies	$2q$	0.01	
			2.43

Multiply the 'number' and the 'value' to get the 'total value' of each type of coin.

Type	Number ·	Value ($)	= Total Value ($)
Quarters	q	0.25	$0.25q$
Pennies	$2q$	0.01	$0.01(2q)$
			2.43

Step 4: **Translate. Write the equation by adding the 'total value' of all the types of coins.**

Step 5: **Solve the equation.**

	$0.25q + 0.01(2q) = 2.43$
Multiply	$0.25q + 0.02q = 2.43$
Combine like terms	$0.27q = 2.43$
Divide by 0.27	$q = 9$
The number of pennies is 2q	$2q$
	$2 \cdot 9 = 18$ pennies

Step 6: **Check the answer in the problem.**

Maria has 9 quarters and 18 pennies. Does this make $\$2.43$?

9 quarters	$9(0.25) = 2.25$
18 pennies	$18(0.01) = 0.18$
Total	$\$2.43$✓

Step 7: **Answer the question.**

Maria has nine quarters and eighteen pennies.

Try It

9) Sumanta has $\$4.20$ in nickels and dimes in her piggy bank. She has twice as many nickels as dimes. How many coins of each type does she have?

Solution

42 nickels, 21 dimes.

10) Alison has three times as many dimes as quarters in her purse. She has $\$9.35$ altogether. How many coins of each type does she have?

Solution

51 dimes, 17 quarters.

In the next example, we'll show only the completed table—remember the steps we take to fill in the table.

How To

Solve Coin Word Problems.

Step 1: **Read the problem. Make sure all the words and ideas are understood.**

- Determine the types of coins involved.
- Create a table to organize the information.
- Label the columns "type," "number," "value," and "total value."
- List the types of coins.
- Write in the value of each type of coin.
- Write in the total value of all the coins.

Type	Number •	Value($) =	Total Value($)

Figure 3.8.3

Step 2: **Identify what we are looking for.**

Step 3: **Name what we are looking for. Choose a variable to represent that quantity.**

Use variable expressions to represent the number of each type of coin and write them in the table.

Multiply the number times the value to get the total value of each type of coin.

Step 4: **Translate into an equation.**

It may be helpful to restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.

Write the equation by adding the total values of all the types of coins.

Step 5: **Solve the equation using good algebra techniques.**

Step 6: **Check the answer in the problem and make sure it makes sense.**

Step 7: **Answer the question with a complete sentence.**

Example 3

Danny has $\$2.14$ worth of pennies and nickels in his piggy bank. The number of nickels is two more than ten times the number of pennies. How many nickels and how many pennies does Danny have?

Solution

Step 1: **Read the problem.**

Determine the types of coins involved.

pennies and nickels

Create a table. Write in the value of each type of coin.

Pennies are worth $\$0.01$.

Nickels are worth $\$0.05$.

Step 2: **Identify what we are looking for.**

the number of pennies and nickels

Step 3: **Name. Represent the number of each type of coin using variables.**

The number of nickels is defined in terms of the number of pennies, so start with pennies.

Let p = number of pennies

The number of nickels is two more than ten times the number of pennies.

Let $10p + 2$ = number of nickels

Multiply the number and the value to get the total value of each type of coin.

Type	Number ·	Value ($)	= Total Value ($)
Pennies	p	0.01	$0.01p$
Nickels	$10p + 2$	0.05	$0.05(10p + 2)$
			2.14

Step 4: **Translate. Write the equation by adding the total value of all the types of coins.**

$$0.01p + 0.05(10p + 2) = 2.14$$

Step 5: **Solve the equation.**

$$\begin{aligned} 0.01p + 0.50p + 0.10 &= 2.14 \\ 0.51p + 0.10 &= 2.14 \\ 0.51p &= 2.04 \\ p &= 4 \text{ pennies} \end{aligned}$$

How many nickels?

$$\begin{aligned} &10p + 2 \\ &10(4) + 2 \\ &42 \text{ nickles} \end{aligned}$$

Step 7: **Answer the question.**

Danny has four pennies and 42 nickels.

Try It

11) Jesse has $\$6.55$ worth of quarters and nickels in his pocket. The number of nickels is five more than two times the number of quarters. How many nickels and how many quarters does Jesse have?

Solution

41 nickels, 18 quarters

12) Elane has $\$7.00$ total in dimes and nickels in her coin jar. The number of dimes that Elane has is seven less than three times the number of nickels. How many of each coin does Elane have?

Solution

22 nickels, 59 dimes

Solve Ticket and Stamp Word Problems

Problems involving tickets or stamps are very much like coin problems. Each type of ticket and stamp has a value, just like each type of coin does. So to solve these problems, we will follow the same steps we used to solve coin problems.

Example 4

At a school concert, the total value of tickets sold was $\$1,506$. Student tickets sold for $\$6$ each and adult tickets sold for $\$9$ each. The number of adult tickets sold was five less than three times the number of student tickets sold. How many student tickets and how many adult tickets were sold?

Solution

***Step 1:* Read the problem.**

- Determine the types of tickets involved. There are student tickets and adult tickets.
- Create a table to organize the information.

Type	Number ·	Value ($)	= Total Value ($)
Student		6	
Adult		9	
			1506

***Step 2:* Identify what we are looking for.**

- We are looking for the number of student and adult tickets.

***Step 3:* Name. Represent the number of each type of ticket using variables.**

We know the number of adult tickets sold was five less than three times the number of student tickets sold.

- Let s be the number of student tickets.
- Then $3s-5$ is the number of adult tickets

Multiply the number times the value to get the total value of each type of ticket.

Type	Number ·	Value ($)	= Total Value ($)
Student	s	6	$6s$
Adult	$3s-5$	9	$9(3s-5)$
			1506

Step 4: **Translate. Write the equation by adding the total values of each type of ticket.**

$$6s+9(3s-5)=1506$$

Step 5: **Solve the equation.**

$$\begin{aligned} 6s+27s-45 &= 1506 \\ 33s-45 &= 1506 \\ s &= 47 \\ &= 3s-5 \\ &= 3(47)-5 \end{aligned}$$

136 adult tickets.

Step 6: **Check the answer.**

There were 47 student tickets at $\$6$ each and 136 adult tickets at $\$9$ each. Is the total value $\$1,506$? We find the total value of each type of ticket by multiplying the number of tickets times its value then add to get the total value of all the tickets sold.

$$\begin{aligned} 47 \times 6 &= 282 \\ 136 \times 9 &= 1,224 \\ &= 1,506 \checkmark \end{aligned}$$

Step 7: Answer the question. They sold 47 student tickets and 136 adult tickets.

Try It

13) On The first day of a water polo tournament the total value of tickets sold was $\$17,610$. One-day passes sold for $\$20$ and tournament passes sold for $\$30$. The number of tournament passes sold was 37 more than the number of day passes sold. How many day passes and how many tournament passes were sold?

Solution

330 day passes, 367 tournament passes

14) At the movie theatre, the total value of tickets sold was $\$2,612.50$. Adult tickets sold for $\$10$ each and senior/child tickets sold for $\$7.50$ each. The number of senior/child tickets sold was 25 less than twice the number of adult tickets sold. How many senior/child tickets and how many adult tickets were sold?

Solution

112 adult tickets, 199 senior/child tickets

We have learned how to find the total number of tickets when the number of one type of ticket is based on the number of the other type. Next, we'll look at an example where we know the total number of tickets and have to figure out how the two types of tickets relate.

Suppose Bianca sold a total of 100 tickets. Each ticket was either an adult ticket or a child ticket. If she sold 20 child tickets, how many adult tickets did she sell?

- *Did you say '80'? How did you figure that out? Did you subtract 20 from 100?*

If she sold 45 child tickets, how many adult tickets did she sell?

- *Did you say '*55*'? How did you find it? By subtracting* 45 *from* 100*?*

What if she sold 75 child tickets? How many adult tickets did she sell?

- *The number of adult tickets must be* $100 - 75$*. She sold* 25 *adult tickets.*

Now, suppose Bianca sold x child tickets. Then how many adult tickets did she sell? To find out, we would follow the same logic we used above. In each case, we subtracted the number of child tickets from 100 to get the number of adult tickets. We now do the same with x.

We have summarized this below.

Child tickets	Adult tickets
20	80
45	55
75	25
x	$100 - x$

We can apply these techniques to other examples

Example 5

Galen sold 810 tickets for his church's carnival for a total of $\$2,820$. Children's tickets cost $\$3$ each and adult tickets cost $\$5$ each. How many children's tickets and how many adult tickets did he sell?

Solution

Step 1: **Read the problem.**

- Determine the types of tickets involved. There are children tickets and adult tickets.
- Create a table to organize the information.

Type	Number ·	Value ($)	= Total Value ($)
Children		3	
Adult		5	
			2820

Step 2: **Identify what we are looking for.**

- We are looking for the number of children and adult tickets.

Step 3: **Name. Represent the number of each type of ticket using variables.**

- We know the total number of tickets sold was 810. This means the number of children's tickets plus the number of adult tickets must add up to 810.

- Let c be the number of children tickets.
- Then $810 - c$ is the number of adult tickets.
- Multiply the number times the value to get the total value of each type of ticket.

Type	Number ·	Value ($)	= Total Value ($)
Children	c	3	$3c$
Adult	$810 - c$	5	$5(810 - c)$
			2820

Step 4: **Translate.**

Write the equation by adding the total values of each type of ticket.

Step 5: **Solve the equation.**

$$\begin{aligned} 3c + 5(810 - c) &= 2,820 \\ 3c + 5(810 - c) &= 2,820 \\ -2c &= -1,230 \\ c &= 615 \text{ children tickets} \end{aligned}$$

How many adults?

$$\begin{aligned} &810 - c \\ &810 - 615 \end{aligned}$$

195 adult tickets

Step 6: **Checkthe answer. There were 615 children's tickets at $\$3$ each and 195 adult tickets at $\$5$ each. Is the total value $\$2,820$?**

$$\begin{aligned} 615 \times 3 &= 1,845 \\ 195 \times 5 &= 975 \\ &= 2,820 \checkmark \end{aligned}$$

Step 7: **Answer the question. Galen sold 615 children's tickets and 195 adult tickets.**

Try It

15) During her shift at the museum ticket booth, Leah sold 115 tickets for a total of $\$1,163$. Adult tickets cost $\$12$ and student tickets cost $\$5$. How many adult tickets and how many student tickets did Leah sell?

Solution

84 adult tickets, 31 student tickets

16) A whale-watching ship had 40 paying passengers on board. The total collected from tickets was $\$1,196$. Full-fare passengers paid $\$32$ each and reduced-fare passengers paid $\$26$ each. How many full-fare passengers and how many reduced-fare passengers were on the ship?

Solution

26 full-fare, 14 reduced fare

Now, we'll do one where we fill in the table all at once.

Example 6

Monica paid $8.36 for stamps. The number of 41-cent stamps was four more than twice the number of two-cent stamps. How many 41-cent stamps and how many two-cent stamps did Monica buy?

Solution

The types of stamps are 41 -cent stamps and two-cent stamps. Their names also give the value!

"The number of 41 -cent stamps was four more than twice the number of two-cent stamps."

Let $x =$ number of 2 – cent stamps.

$2x + 4 =$ number of 41 – cent stamps

Type	Number ·	Value ($)	= Total Value ($)
41 cent stamps	$2x + 4$	0.41	$0.41(2x + 4)$
2 cent stamps	x	0.02	$0.02x$
			8.36

Step 1: **Write the equation from the total values.**

$$0.41(2x + 4) + 0.02x = 8.36$$

Step 2: **Solve the equation.**

$$\begin{aligned} 0.82x + 1.64 + 0.02x &= 8.36 \\ 0.84x + 1.64 &= 8.36 \\ 0.84x &= 6.72 \\ x &= 8 \end{aligned}$$

Monica bought eight two-cent stamps.

$2x + 4$ for $x = 8$

Step 3: **Find the number of 41-cent stamps she bought by evaluating.**

$$\begin{aligned} &2x + 4 \\ &2(8) + 4 \\ &20 \end{aligned}$$

Step 4: **Check.**

$$\begin{aligned} 8(0.02) + 20(0.41) &\stackrel{?}{=} 8.36 \\ 0.16 + 8.20 &\stackrel{?}{=} 8.36 \\ 8.36 &= 8.36\checkmark \end{aligned}$$

Monica bought eight two-cent stamps and 20 41-cent stamps.

Try It

17) Eric paid $\$13.36$ for stamps. The number of 41 -cent stamps was eight more than twice the number of two-cent stamps. How many 41 -cent stamps and how many two-cent stamps did Eric buy?

Solution

32 at $\$0.41$, 12 at $\$0.02$

18) Kailee paid $\$12.66$ for stamps. The number of 41 -cent stamps was four less than three times the number of 20-cent stamps. How many 41 -cent stamps and how many 20 -cent stamps did Kailee buy?

Solution

26 at $\$0.41$, 10 at $\$0.20$

Solve Mixture Word Problems

Now we'll solve some more general applications of the mixture model. Grocers and bartenders use the mixture model to set a fair price for a product made from mixing two or more ingredients. Financial planners use the mixture model when they invest money in a variety of accounts and want to find the overall interest rate. Landscape designers use the mixture model when they have an assortment of plants and a fixed budget, and event coordinators do the same when choosing appetizers and entrees for a banquet.

Our first mixture word problem will be making trail mix from raisins and nuts.

Example 7

Henning is mixing raisins and nuts to make 10 pounds of trail mix. Raisins cost $\$2$ a pound and nuts cost $\$6$ a pound. If Henning wants his cost for the trail mix to be $\$5.20$ a pound, how many pounds of raisins and how many pounds of nuts should he use?

Solution

As before, we fill in a chart to organize our information.

The 10 pounds of trail mix will come from mixing raisins and nuts.

$$\begin{aligned} x &= \text{number of pounds of raisins} \\ 10 - x &= \text{number of pounds of nuts} \end{aligned}$$

We enter the price per pound for each item.

We multiply the number times the value to get the total value.

Type	Number of pounds ·	Price per pound ($)	= Total Value ($)
Raisins	x	2	$2x$
Nuts	$10 - x$	6	$6(10 - x)$
Trail mix	10	5.20	$10(5.20)$

Notice that the last line in the table gives the information for the total amount of the mixture.

We know the value of the raisins plus the value of the nuts will be the value of the trail mix.

Step 1: **Write the equation from the total values.**

$$2x + 6(10 + x) = 10(5.20)$$

Step 2: **Solve the equation.**

$$\begin{aligned} 2x + 60 - 6x &= 52 \\ -4x &= -8 \\ x &= 2 \text{ pounds of raisins} \end{aligned}$$

Step 3: **Find the number of pounds of nuts.**

$$\begin{aligned} &10 - x \\ &10 - 2 \\ &8 \text{ pounds of nuts} \end{aligned}$$

Step 4: **Check.**

$$\begin{aligned} 2(\$2) + 8(\$6) &\stackrel{?}{=} 10(\$5.20) \\ \$4 + 48 &\stackrel{?}{=} \$52 \\ \$52 &= \$52 \checkmark \end{aligned}$$

Henning mixed two pounds of raisins with eight pounds of nuts.

Try It

19) Orlando is mixing nuts and cereal squares to make a party mix. Nuts sell for $\$7$ a pound and cereal squares sell for $\$4$ a pound. Orlando wants to make 30 pounds of party mix at a cost of $\$6.50$ a pound, how many pounds of nuts and how many pounds of cereal squares should he use?

Solution

5 pounds cereal squares, 25 pounds nuts

20) Becca wants to mix fruit juice and soda to make a punch. She can buy fruit juice for $\$3$ a

gallon and soda for $\$4$ a gallon. If she wants to make 28 gallons of punch at a cost of $\$3.25$ a gallon, how many gallons of fruit juice and how many gallons of soda should she buy?

Solution

21 gallons of fruit punch, 7 gallons of soda

We can also use the mixture model to solve investment problems using simple interest. We have used the simple interest formula, $I = Prt$, where t represented the number of years. When we just need to find the interest for one year, $t = 1$, so then $I = Pr$.

Example 8

Stacey has $\$20,000$ to invest in two different bank accounts. One account pays interest at 3 per year and the other account pays interest at 5 per year. How much should she invest in each account if she wants to earn 4.5 interest per year on the total amount?

Solution

We will fill in a chart to organize our information. We will use the simple interest formula to find the interest earned in the different accounts.

The interest on the mixed investment will come from adding the interest from the account earning 3 and the interest from the account earning 5 to get the total interest on the $\$20,000$.

$$\begin{aligned} \text{Let } x &= \text{amount invested at } 3\% \\ 20,000 - x &= \text{amount invested at } 5\% \end{aligned}$$

The amount invested is the *principal* for each account.

We enter the interest rate for each account.

We multiply the amount invested times the rate to get the interest.

Type	Amount Invested $\cdot$	Rate	$=$ Interest
3%	x	0.03	$0.03x$
5%	$20,000-x$	0.05	$0.05(20,000-x)$
4.5%	$20,000$	0.045	$0.045(20,000)$

Notice that the total amount invested, $20,000$, is the sum of the amount invested at 3 and the amount invested at 5. The total interest, $0.045(20,000)$ is the sum of the interest earned in the 3 account and the interest earned in the 5% account.

As with the other mixture applications, the last column in the table gives us the equation to solve.

Step 1: **Write the equation from the interest earned.**

$$0.03x + 0.05(20,000 - x) = 0.045(20,000)$$

Step 2: **Solve the equation.**

$$\begin{aligned} 0.03x + 0.05(20,000 - x) &= 0.045(20,000) \\ 0.03x + 1000 - 0.05x &= 900 \\ -0.02x + 1000 &= 900 \\ -0.02x &- 100 \\ x &= 5,000 \text{ amount invested at } 3\% \end{aligned}$$

Step 3: **Find the amount invested at 5%.**

$$\begin{aligned} &20,000 - x \\ &20,000 - 5,000 \\ &15,000 = \text{amount invested at } 5\% \end{aligned}$$

Step 4: **Check.**

$$\begin{aligned} 0.03x + 0.05(15,000 + x) &\stackrel{?}{=} 0.045(20,000) \\ 150 + 750 &\stackrel{?}{=} -900 \\ 900 &= 900\checkmark \end{aligned}$$

Stacey should invest $5,000 in the account that earns 3% and $15,000 in the account that earns 5%.

Try It

21) Remy has $\$14,000$ to invest in two mutual funds. One fund pays interest at 4 per year and the other fund pays interest at 7 per year. How much should she invest in each fund if she wants to earn 6.1 interest on the total amount?

Solution

$\$4,200$ at 4, $\$9,800$ at 7

22) Marco has $\$8,000$ to save for his daughter's college education. He wants to divide it between

one account that pays 3.2 interest per year and another account that pays 8 interest per year. How much should he invest in each account if he wants the interest on the total investment to be 6.5?

Solution

$\$2,500$ at 3.2, $\$5,500$ at 8

Solve Uniform Motion Applications

When planning a road trip, it often helps to know how long it will take to reach the destination or how far to travel each day. We would use the distance, rate, and time formula, $D = rt$, which we have already seen.

In this section, we will use this formula in situations that require a little more algebra to solve than the ones we saw earlier. Generally, we will be looking at comparing two scenarios, such as two vehicles travelling at different rates or in opposite directions. When the speed of each vehicle is constant, we call applications like this *uniform motion problems*.

Our problem-solving strategies will still apply here, but we will add to the first step. The first step will include drawing a diagram that shows what is happening in the example. Drawing the diagram helps us understand what is happening so that we can write an appropriate equation. Then we will make a table to organize the information, like we did for the money applications.

The steps are listed here for easy reference:

How To

Use a Problem-Solving Strategy in Distance, Rate, and Time Applications.

Step 1: **Read the problem. Make sure all the words and ideas are understood.**

- Draw a diagram to illustrate what is happening.
- Create a table to organize the information.
- Label the column's rate, time, and distance.
- List the two scenarios.
- Write in the information you know.

	Rate •	Time =	Distance

Figure 3.8.4

Step 2: **Identify what we are looking for.**

Step 3: **Name what we are looking for. Choose a variable to represent that quantity.**

- Complete the chart.
- Use variable expressions to represent that quantity in each row.
- Multiply the rate times the time to get the distance.

Step 4: **Translate into an equation.**

- Restate the problem in one sentence with all the important information.
- Then, translate the sentence into an equation.

Step 5: **Solve the equation using good algebra techniques.**

Step 6: **Check the answer in the problem and make sure it makes sense.**

Step 7: **Answer the question with a complete sentence.**

Example 9

An express train and a local train leave Pittsburgh to travel to Washington, D.C. The express train can make the trip in 4 hours and the local train takes 5 hours for the trip. The speed of the express train is 12 miles per hour faster than the speed of the local train. Find the speed of both trains.

Solution

Step 1: **Read the problem. Make sure all the words and ideas are understood.**

- Draw a diagram to illustrate what it happening. Shown below is a sketch of what is happening in the example.

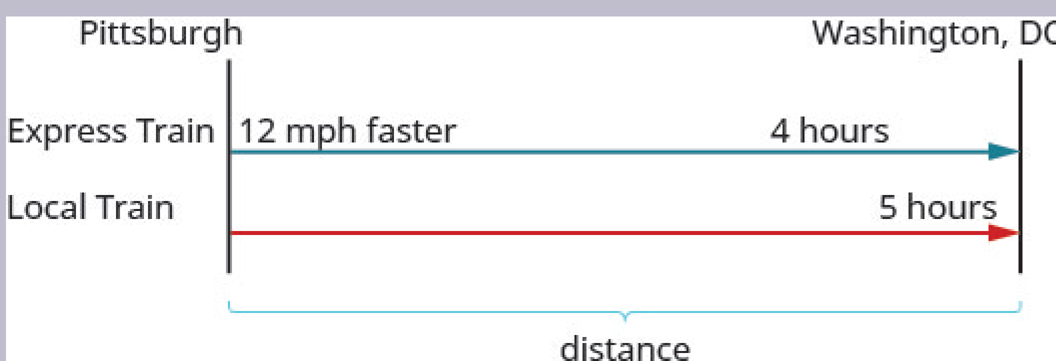


Figure 3.8.5

	Rate (mph) ·	Time (hrs)	= Distance (miles)
Express		4	
Local		5	

- Create a table to organize the information.
- Label the columns "Rate," "Time," and "Distance."
- List the two scenarios.
- Write in the information you know.

Step 2: **Identify what we are looking for.**

- We are asked to find the speed of both trains.
- Notice that the distance formula uses the word "rate," but it is more common to use "speed" when we talk about vehicles in everyday English.

Step 3: **Name what we are looking for. Choose a variable to represent that quantity.**

- Complete the chart
- Use variable expressions to represent that quantity in each row.
- We are looking for the speed of the trains. Let's let r represent the speed of the local train. Since the speed of the express train is 12 mph faster, we represent that as $r+12$.

$r =$ speed of the local train

$r+12 =$ speed of the express train

Fill in the speeds on the chart.

	Rate (mph) ·	Time (hrs)	= Distance (miles)
Express	$r+12$	4	
Local	r	5	

Multiply the rate times the time to get the distance.

	Rate (mph) ·	Time (hrs)	= Distance (miles)
Express	$r+12$	4	$4(r+12)$
Local	r	5	$5r$

Step 4: **Translate into an equation.**

- Restate the problem in one sentence with all the important information.
- Then, translate the sentence into an equation.
- The equation to model this situation will come from the relation between the distances. Look at the diagram we drew above. How is the distance travelled by the express train related to the distance travelled by the local train?
- Since both trains leave from Pittsburgh and travel to Washington, D.C. they travel the same distance. So we write:

$$\text{Translate to an equation} \quad \underbrace{\text{distance travelled by express train}}_{4(r+12)} = \underbrace{\text{distance travelled by local train}}_{5r}$$

Step 5: **Solve the equation using good algebra techniques.**

Now solve this equation.

$$\begin{aligned} 4(r+12) &= 5r \\ 4r+48 &= 5r \\ 48 &= r \end{aligned}$$

So the speed of the local train is 48 mph.

Find the speed of the express train.

$$\begin{aligned} &{\color{red}r}+12 \\ &{\color{red}48}+12 \\ &60 \end{aligned}$$

The speed of the express train is 60 mph.

Step 6: **Check the answer in the problem and make sure it makes sense.**

$$\begin{array}{ll} \text{express train} & 60\ mph(4\ hours) = 240\ miles \\ \text{local train} & 48\ mph(5\ hour) = 240\ miles \end{array}$$

Step 7: **Answer the question with a complete sentence.**

- The speed of the local train is 48 mph and the speed of the express train is 60 mph.

Try It

23) Wayne and Dennis like to ride the bike path from Riverside Park to the beach. Dennis's speed is seven miles per hour faster than Wayne's speed, so it takes Wayne 2 hours to ride to the beach while it takes Dennis 1.5 hours for the ride. Find the speed of both bikers.

Solution

Wayne 21 mph, Dennis 28 mph

24) Jeromy can drive from his house in Cleveland to his college in Chicago in 4.5 hours. It takes his mother 6 hours to make the same drive. Jeromy drives 20 miles per hour faster than his mother. Find Jeromy's speed and his mother's speed.

Solution

Jeromy 80 mph, mother 60 mph

The diagram and the chart helped us write the equation we solved. Let's see how this works in another case.

Example 10

Christopher and his parents live 115 miles apart. They met at a restaurant between their homes to celebrate his mother's birthday. Christopher drove 1.5 hours while his parents drove 1 hour to get to the restaurant. Christopher's average speed was 10 miles per hour faster than his parents' average speed. What were the average speeds of Christopher and his parents as they drove to the restaurant?

Solution

Step 1: **Read the problem. Make sure all the words and ideas are understood.**

- Draw a diagram to illustrate what is happening. Below is a sketch of what is happening in the example.

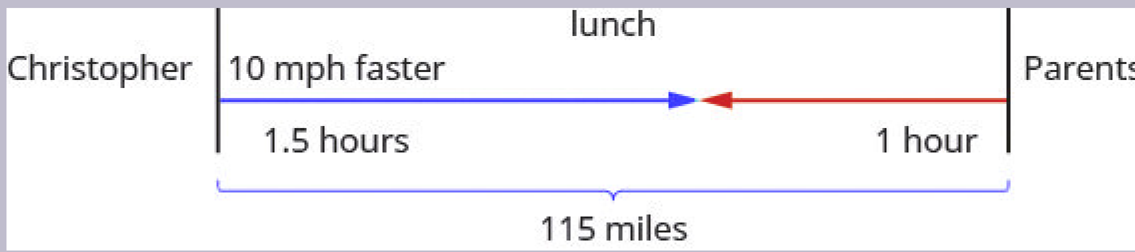


Figure 3.8.6

- Create a table to organize the information.
- Label the columns' rate, time, and distance.
- List the two scenarios.
- Write in the information you know.

	Rate (mph) ·	Time (hrs)	= Distance (miles)
Christopher		1.5	
Parents		1	
			115

Step 2: **Identify what we are looking for.**

- We are asked to find the average speeds of Christopher and his parents.

Step 3: **Name what we are looking for. Choose a variable to represent that quantity.**

- Complete the chart.
- Use variable expressions to represent that quantity in each row.
- We are looking for their average speeds. Let's let r represent the average speed of the parents. Since the Christopher's speed is 10 mph faster, we represent that as $r + 10$.

Fill in the speeds on the chart.

	Rate (mph) ·	Time (hrs)	= Distance (miles)
Christopher	$r+10$	1.5	$1.5(r+10)$
Parents	r	1	r
			115

Multiply the rate times the time to get the distance.

Step 4: **Translate into an equation.**

- Restate the problem in one sentence with all the important information.
- Then, translate the sentence into an equation.
- Again, we need to identify a relationship between the distances to write an equation. Look at the diagram we created above and notice the relationship between the distance Christopher travelled and the distance his parents travelled.

The distance Christopher travelled plus the distance his parents travelled must add up to 115 miles. So we write:

$$\begin{array}{lcccc} & \underbrace{\text{distance travelled by Christopher}} & + & \underbrace{\text{distance travelled by his parents}} & = & 115 \\ \text{Translate to an equation} & 1.5(r+10) & + & r & = & 115 \end{array}$$

Step 5: **Solve the equation using good algebra techniques.**

Now solve this equation.

$$\begin{aligned} 1.5(r+10)+r &= 115 \\ 1.5r+15+r &= 115 \\ 2.5r+15 &= 115 \\ 2.5r &= 100 \\ r &= 40 \end{aligned}$$

So the parents' speed was 40 mph.

Christopher's speed is $r+10$.

$$\begin{aligned} r+10 \\ 40+10=50 \end{aligned}$$

Step 6: **Check the answer in the problem and make sure it makes sense.**

$$\begin{array}{lr} \text{Christopher drove} & 50\ mph(1.5\ hours) = 75\ miles \\ \text{His parents drove} & 40\ mph(1\ hour) = 40\ miles \\ & = 115\ miles \end{array}$$

Step 7: **Answer the question with a complete sentence.**

Christopher's speed was 50 mph.

His parents' speed was 40 mph.

Try It

25) Carina is driving from her home in Anaheim to Berkeley on the same day her brother is driving from Berkeley to Anaheim, so they decide to meet for lunch along the way in Buttonwillow. The distance from Anaheim to Berkeley is 410 miles. It takes Carina 3 hours to get to Buttonwillow, while her brother drives 4 hours to get there. The average speed Carina's brother drove was 15 miles per hour faster than Carina's average speed. Find Carina's and her brother's average speeds.

Solution

Carina 50 mph, brother 65 mph

26) Ashley goes to college in Minneapolis, 234 miles from her home in Sioux Falls. She wants her parents to bring her more winter clothes, so they decide to meet at a restaurant on the road between Minneapolis and Sioux Falls. Ashley and her parents both drove 2 hours to the

restaurant. Ashley's average speed was seven miles per hour faster than her parents' average speed. Find Ashley's and her parents' average speed.

Solution

Parents 55 mph, Ashley 62 mph

As you read the next example, think about the relationship between the distances travelled. Which of the previous two examples is more similar to this situation?

Example 11

Two truck drivers leave a rest area on the interstate at the same time. One truck travels east and the other one travels west. The truck travelling west travels at 70 mph and the truck travelling east has an average speed of 60 mph. How long will they travel before they are 325 miles apart?

Solution

Step 1: **Read the problem. Make sure all the words and ideas are understood.**

- Draw a diagram to illustrate what is happening.

Figure 3.8.7

- Create a table to organize the information.

	Rate (mph) ·	Time (hrs)	= Distance (miles)
West		70	
East		60	
			325

Step 2: **Identify what we are looking for.**

- We are asked to find the amount of time the trucks will travel until they are 325 miles apart.

Step 3: **Name what we are looking for. Choose a variable to represent that quantity.**

- We are looking for the time travelled. Both trucks will travel the same amount of time. Let's call the time t. Since their speeds are different, they will travel different distances.
- Complete the chart.

	Rate (mph) ·	Time (hrs)	= Distance (miles)
West	70	t	$70t$
East	60	t	$60t$
			325

Step 4: **Translate into an equation.**

- We need to find a relation between the distances to write an equation. Looking at the diagram, what is the relationship between the distance each of the trucks will travel?
- The distance travelled by truck going west plus the distance travelled by the truck going east must add up to 325 miles. So we write:

$$\underbrace{\text{distance travelled by westbound truck}}_{70t} + \underbrace{\text{distance travelled by eastbound truck}}_{60t} = 325$$

Step 5: **Solve the equation using good algebra techniques.**

$$\begin{aligned} 70t + 60t &= 325 \\ 130t &= 325 \\ t &= 2.5 \end{aligned}$$

So it will take the trucks 2.5 hours to be 325 miles apart.

Step 6: **Check the answer in the problem and make sure it makes sense.**

$$\begin{array}{ll} \text{Truck going West} & 70\ mph(2.5\ hours) = 175\ miles \\ \text{Truck going East} & 60\ mph(2.5\ hour) = 150\ miles \\ & = 325\ miles \end{array}$$

Step 7: **Answer the question with a complete sentence.**

It will take the trucks 2.5 hours to be 325 miles apart.

Try It

27) Pierre and Monique leave their home in Portland at the same time. Pierre drives north on the turnpike at a speed of 75 miles per hour while Monique drives south at a speed of 68 miles per hour. How long will it take them to be 429 miles apart?

Solution

3 hours

28) Thanh and Nhat leave their office in Sacramento at the same time. Thanh drives north on I-5 at a speed of 72 miles per hour. Nhat drives south on I-5 at a speed of 76 miles per hour. How long will it take them to be 330 miles apart?

Solution

2.2 hours

Matching Units in Problems

It is important to make sure the units match when we use the distance rate and time formula. For instance, if the rate is in miles per hour, then the time must be in hours.

Example 12

When Katie Mae walks to school, it takes her 30 minutes. If she rides her bike, it takes her 15 minutes. Her speed is three miles per hour faster when she rides her bike than when she walks. What are her walking speed and her speed riding her bike?

Solution

First, we draw a diagram that represents the situation to help us see what is happening.

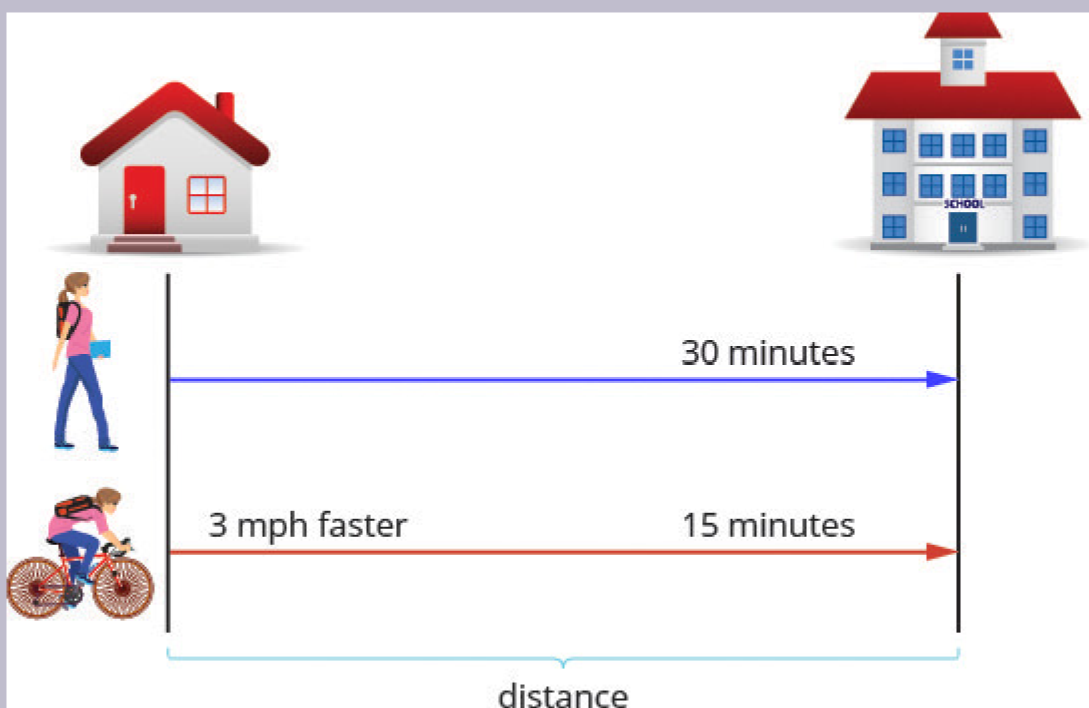


Figure 3.8.8

We are asked to find her speed walking and riding her bike. Let's call her walking speed r.

Since her biking speed is three miles per hour faster, we will call that speed $r+3$. We write the speeds in the chart.

The speed is in miles per hour, so we need to express the times in hours, too, in order for the units to be the same. Remember, one hour is 60 minutes. So:

30 minutes is $\frac{30}{60}$ or $\frac{1}{2}$ hour

15 minutes is $\frac{15}{60}$ or $\frac{1}{4}$ hour

Next, we multiply the rate times time to fill in the distance column.

	Rate (mph) ·	Time (hrs)	= Distance (miles)
Walk	r	$\frac{1}{2}$	$\frac{1}{2}r$
Bike	$r+3$	$\frac{1}{4}$	$\frac{1}{4}(r+3)$

The equation will come from the fact that the distance from Katie Mae's home to her school is the same whether she is walking or riding her bike.

So we say:

$$\underbrace{\text{distance walked}} = \underbrace{\text{distance covered by bike}}$$

Step 1: **Translate into an equation.**

$$\frac{1}{2}r = \frac{1}{4}(r + 3)$$

Step 2: **Solve this equation.**

$$\frac{1}{2}r = \frac{1}{4}(r + 3)$$

Step 3: **Clear the fractions by multiplying by the LCD of all the fractions in the equation.**

$$8 \times \frac{1}{2}r = 8 \times \frac{1}{4}(r + 3)$$

Step 4: **Simplify.**

$$\begin{aligned} 4r &= 2(r + 3) \\ 4r &= 2r + 6 \\ 2r &= 6 \\ r &= 3\ mph \end{aligned}$$

3 mph is Katie Mae's walking speed

$r + 3$ biking speed

$3 + 3$

6 mph (Katie Mae's biking speed)

Step 5: **Let's check if this works.**

$$\begin{array}{ll} \text{Walk} & 3\ mph(0.5\ hours) = 1.5\ miles \\ \text{Bike} & 6\ mph(0.25\ hour) = 1.5\ miles \end{array}$$

Yes, either way, Katie Mae travels 1.5 miles to school.

Katie Mae's walking speed is 3 mph.

Her speed riding her bike is 6 mph.

Try It

29) Suzy takes 50 minutes to hike uphill from the parking lot to the lookout tower. It takes her 30 minutes to hike back down to the parking lot. Her speed going downhill is 1.2 miles per hour faster than her speed going uphill. Find Suzy's uphill and downhill speeds.

Solution

Uphill 1.8 mph, downhill 3 mph

30) Llewyn takes 45 minutes to drive his boat upstream from the dock to his favorite fishing spot. It takes him 30 minutes to drive the boat back downstream to the dock. The boat's speed going downstream is four miles per hour faster than its speed going upstream. Find the boat's upstream and downstream speeds.

Solution

Upstream 8 mph, downstream 12 mph

In the distance, rate, and time formula, time represents the actual amount of elapsed time (in hours, minutes, etc.). If a problem gives us starting and ending times as clock times, we must find the elapsed time in order to use the formula.

Example 13

Hamilton loves to travel to Las Vegas, 255 miles from his home in Orange County. On his last trip, he left his house at 2:00 pm. The first part of his trip was on congested city freeways. At 4:00 pm, the traffic cleared and he was able to drive through the desert at a speed 1.75 times faster than when he drove in the congested area. He arrived in Las Vegas at 6:30 pm. How fast was he driving during each part of his trip?

Solution

A diagram will help us model this trip.

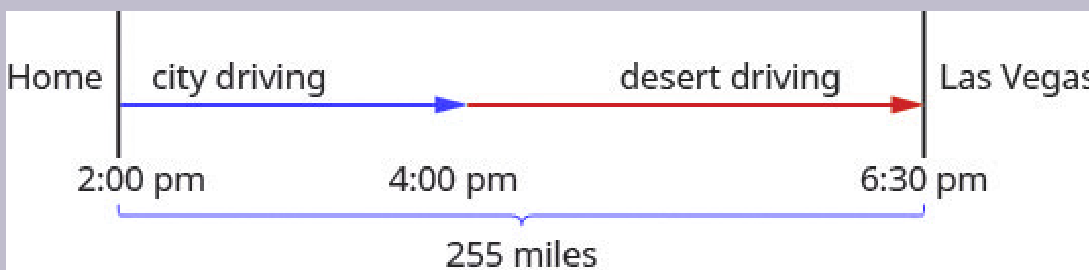


Figure 3.8.9

Next, we create a table to organize the information.

We know the total distance is 255 miles. We are looking for the rate of speed for each part of the trip. The rate in the desert is 1.75 times the rate in the city. If we let $r\ =$ the rate in the city, then the rate in the desert is $1.75r$.

The times here are given as clock times. Hamilton started from home at 2:00 pm and entered the desert at 4:30 pm. So he spent two hours driving the congested freeways in the city. Then he drove faster from 4:00 pm until 6:30 pm in the desert. So he drove 2.5 hours in the desert.

Now, we multiply the rates by the times.

	Rate (mph) ·	Time (hrs)	= Distance (miles)
City	r	2	$2r$
Desert	$1.75r$	2.5	$2.5(1.5r)$
			225

By looking at the diagram below, we can see that the sum of the distance driven in the city and the distance driven in the desert is 255 miles.

$$\underbrace{\text{distance driven in the city}} + \underbrace{\text{distance driven in desert}} = 255$$

Step 1: **Translate into an equation.**

$$2r + 2.5(1.75r) = 255$$

Step 2: **Solve this equation.**

$$\begin{aligned} 2r + 2.5(1.75r) &= 255 \\ 2r + 4.375r &= 255 \\ 6.375r &= 255 \\ r &= 40\ mph\ city \end{aligned}$$

$$\begin{array}{l} 1.75r \text{ desert speed} \\ 1.75(40) \\ 70\ mph \end{array}$$

Step 3: **Check.**

$$\begin{aligned} \text{City } 40\ mph\ (2 \text{ hours}) &= 80\ miles \\ \text{Desert } 70\ mph\ (2.5 \text{ hours}) &= \frac{175\ miles}{255\ miles} \end{aligned}$$

Hamilton drove 40 mph in the city and 70 mph in the desert.

Try It

31) Cruz is training to compete in a triathlon. He left his house at 6:00 and ran until 7:30. Then he

rode his bike until 9:45. He covered a total distance of 51 miles. His speed when biking was 1.6 times his speed when running. Find Cruz's biking and running speeds.

Solution

Biking 16 mph, running 10 mph

32) Phuong left home on his bicycle at 10:00. He rode on the flat street until 11:15, then rode uphill until 11:45. He rode a total of 31 miles. His speed riding uphill was 0.6 times his speed on the flat street. Find his speed biking uphill and on the flat street.

Solution

Uphill 12 mph, flat street 20 mph

Key Concepts

- **Total Value of Coins** For the same type of coin, the total value of a number of coins is found by using the model.
  - number × value=total value where *number* is the number of coins and *value* is the value of each coin; *total value* is the total value of all the coins
- **Problem-Solving Strategy—Coin Word Problems**
  1. **Read the problem. Make all the words and ideas are understood. Determine the types of coins involved.**
     - Create a table to organize the information.
     - Label the columns type, number, value, total value.

- List the types of coins.
- Write in the value of each type of coin.
- Write in the total value of all the coins.

2. **Identify what we are looking for.**
3. **Name what we are looking for. Choose a variable to represent that quantity.**
   Use variable expressions to represent the number of each type of coin and write them in the table.
   Multiply the number times the value to get the total value of each type of coin.
4. **Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.**
   Write the equation by adding the total values of all the types of coins.
5. **Solve the equation using good algebra techniques.**
6. **Check the answer in the problem and make sure it makes sense.**
7. **Answer the question with a complete sentence.**

- **Distance, Rate, and Time**
  - $D = rt$ where $D =$ distance, $r =$ rate, $t =$ time
- **Problem-Solving Strategy—Distance, Rate, and Time Applications**

1. **Read the problem. Make sure all the words and ideas are understood.**
   Draw a diagram to illustrate what it happening.
   Create a table to organize the information: Label the columns rate, time, distance. List the two scenarios. Write in the information you know.
2. **Identify what we are looking for.**
3. **Name what we are looking for. Choose a variable to represent that quantity.**
   Complete the chart.
   Use variable expressions to represent that quantity in each row.
   Multiply the rate times the time to get the distance.
4. **Translate into an equation.**
   Restate the problem in one sentence with all the important information.
   Then, translate the sentence into an equation.
5. **Solve the equation using good algebra techniques.**
6. **Check the answer in the problem and make sure it makes sense.**

7. **Answer the question with a complete sentence.**

Glossary

Mixture problems
Mixture problems combine two or more items with different values together.

Exercises: Solve Coin Word Problems

Instructions: For questions 1-18, solve each coin word problem.

1. Jaime has $\$2.60$ in dimes and nickels. The number of dimes is 14 more than the number of nickels. How many of each coin does he have?

Solution

8 nickels, 22 dimes

---

2. Lee has $\$1.75$ in dimes and nickels. The number of nickels is 11 more than the number of dimes. How many of each coin does he have?

---

3. Ngo has a collection of dimes and quarters with a total value of $\$3.50$. The number of dimes is seven more than the number of quarters. How many of each coin does he have?
Solution

15 dimes, 8 quarters

---

4. Connor has a collection of dimes and quarters with a total value of $\$6.30$. The number of dimes is 14 more than the number of quarters. How many of each coin does he have?

---

5. A cash box of $\$1$ and $\$5$ bills is worth $\$45$. The number of $\$1$ bills is three more than the number of $\$5$ bills. How many of each bill does it contain?

Solution

10 at $\$1$, 7 at $\$5$

---

6. Joe's wallet contains $\$1$ and $\$5$ bills worth $\$47$. The number of $\$1$ bills is five more than the number of $\$5$ bills. How many of each bill does he have?

---

7. Rachelle has $\$6.30$ in nickels and quarters in her coin purse. The number of nickels is twice the number of quarters. How many coins of each type does she have?
Solution

18 quarters, 36 nickels

---

8. Deloise has $\$1.20$ in pennies and nickels in a jar on her desk. The number of pennies is three times the number of nickels. How many coins of each type does she have?

---

9. Harrison has $\$9.30$ in his coin collection, all in pennies and dimes. The number of dimes is three times the number of pennies. How many coins of each type does he have?

Solution

30 pennies, 90 dimes

---

10. Ivan has $\$8.75$ in nickels and quarters in his desk drawer. The number of nickels is twice the number of quarters. How many coins of each type does he have?

---

11. In a cash drawer there is $\$125$ in $\$5$ and $\$10$ bills. The number of $\$10$ bills is twice the number of $\$5$ bills. How many of each are in the drawer?

Solution

10 at $\$10$, 5 at $\$5$

---

12. John has $\$175$ in $\$5$ and $\$10$ bills in his drawer. The number of $\$5$ bills is

three times the number of $\$10$ bills. How many of each are in the drawer?

---

13. Carolyn has $\$2.55$ in her purse in nickels and dimes. The number of nickels is nine less than three times the number of dimes. Find the number of each type of coin.

Solution

12 dimes and 27 nickels

---

14. Julio has $\$2.75$ in his pocket in nickels and dimes. The number of dimes is 10 less than twice the number of nickels. Find the number of each type of coin.

---

15. Chi has $\$11.30$ in dimes and quarters. The number of dimes is three more than three times the number of quarters. How many of each are there?

Solution

63 dimes, 20 quarters

---

16. Tyler has $\$9.70$ in dimes and quarters. The number of quarters is eight more than four times the number of dimes. How many of each coin does he have?

---

17. Mukul has $\$3.75$ in quarters, dimes and nickels in his pocket. He has five more dimes than quarters and nine more nickels than quarters. How many of each coin are in his pocket?

Solution

16 nickels, 12 dimes, 7 quarters

---

18. Vina has $\$4.70$ in quarters, dimes and nickels in her purse. She has eight more dimes than quarters and six more nickels than quarters. How many of each coin are in her purse?

Exercises: Solve Ticket and Stamp Word Problems

Instructions: For questions 19-32, solve each ticket or stamp word problem.

19. The school play sold $\$550$ in tickets one night. The number of $\$8$ adult tickets was 10 less than twice the number of $\$5$ child tickets. How many of each ticket were sold?

Solution

30 child tickets, 50 adult tickets

---

20. If the number of $\$8$ child tickets is seventeen less than three times the number of $\$12$ adult tickets and the theater took in $\$584$, how many of each ticket were sold?

---

21. The movie theater took in $\$1,220$ one Monday night. The number of $\$7$ child tickets was ten more than twice the number of $\$9$ adult tickets. How many of each

were sold?
Solution
110 child tickets, 50 adult tickets

---

22. The ball game sold $\$1{,}340$ in tickets one Saturday. The number of $\$12$ adult tickets was 15 more than twice the number of $\$5$ child tickets. How many of each were sold?

---

23. The ice rink sold 95 tickets for the afternoon skating session, for a total of $\$828$. General admission tickets cost $\$10$ each and youth tickets cost $\$8$ each. How many general admission tickets and how many youth tickets were sold?
Solution
34 general, 61 youth

---

24. For the 7:30 show time, 140 movie tickets were sold. Receipts from the $\$13$ adult tickets and the $\$10$ senior tickets totalled $1{,}664$. How many adult tickets and how many senior tickets were sold?

---

25. The box office sold 360 tickets to a concert at the college. The total receipts were $\$4{,}170$. General admission tickets cost $\$15$ and student tickets cost $\$10$. How many of each kind of ticket was sold?
Solution

114 general, 246 student

---

26. Last Saturday, the museum box office sold 281 tickets for a total of $\$3,954$. Adult tickets cost $\$15$ and student tickets cost $\$12$. How many of each kind of ticket was sold?

---

27. Julie went to the post office and bought both $\$0.41$ stamps and $\$0.26$ postcards. She spent $\$51.40$. The number of stamps was 20 more than twice the number of postcards. How many of each did she buy?

Solution

40 postcards, 100 stamps

---

28. Jason went to the post office and bought both $\$0.41$ stamps and $\$0.26$ postcards and spent $\$10.28$. The number of stamps was four more than twice the number of postcards. How many of each did he buy?

---

29. Maria spent $\$12.50$ at the post office. She bought three times as many $\$0.41$ stamps as $\$0.02$ stamps. How many of each did she buy?

Solution

30 at $\$0.41$, 10 at $\$0.02$

---

30. Hector spent $\$33.20$ at the post office. He bought four times as many $\$0.41$ stamps as $\$0.02$ stamps. How many of each did he buy?

---

31. Hilda has $\$210$ worth of $\$10$ and $\$12$ stock shares. The numbers of $\$10$ shares is five more than twice the number of $\$12$ shares. How many of each does she have?

Solution

15 $\$10$ shares, 5 $\$12$ shares

---

32. Mario invested $\$475$ in $\$45$ and $\$25$ stock shares. The number of $\$25$ shares was five less than three times the number of $\$45$ shares. How many of each type of share did he buy?

Exercises: Solve Mixture Word Problems

Instructions: For questions 33-44, solve each mixture word problem.

33. Lauren in making 15 liters of mimosas for a brunch banquet. Orange juice costs her $\$1.50$ per liter and champagne costs her $\$12$ per liter. How many liters of orange juice and how many liters of champagne should she use for the mimosas to cost Lauren $\$5$ per liter?

Solution

5 liters champagne, 10 liters orange juice

---

34. Macario is making 12 pounds of nut mixture with macadamia nuts and almonds. Macadamia nuts cost $\$9$ per pound and almonds cost $\$5.25$ per pound. How many pounds of macadamia nuts and how many pounds of almonds should Macario use for the mixture to cost $\$6.50$ per pound to make?

---

35. Kaapo is mixing Kona beans and Maui beans to make 25 pounds of coffee blend. Kona beans cost Kaapo $\$15$ per pound and Maui beans cost $\$24$ per pound. How many pounds of each coffee bean should Kaapo use for his blend to cost him $\$17.70$ per pound?

Solution

7.5 lbs Maui beans, 17.5 Kona beans

---

36. Estelle is making 30 pounds of fruit salad from strawberries and blueberries. Strawberries cost $\$1.80$ per pound and blueberries cost $\$4.50$ per pound. If Estelle wants the fruit salad to cost her $\$2.52$ per pound, how many pounds of each berry should she use?

---

37. Carmen wants to tile the floor of his house. He will need 1000 square feet of tile. He will do most of the floor with a tile that costs $\$1.50$ per square foot, but also wants to

use an accent tile that costs $\$9.00$ per square foot. How many square feet of each tile should he plan to use if he wants the overall cost to be $\$3$ per square foot?

Solution

800 at $\$1.50$, 200 at $\$9.00$

---

38. Riley is planning to plant a lawn in his yard. He will need nine pounds of grass seed. He wants to mix Bermuda seed that costs $\$4.80$ per pound with Fescue seed that costs $\$3.50$ per pound. How much of each seed should he buy so that the overall cost will be $\$4.02$ per pound?

---

39. Vartan was paid $\$25,000$ for a cell phone app that he wrote and wants to invest it to save for his son's education. He wants to put some of the money into a bond that pays 4% annual interest and the rest into stocks that pay 9% annual interest. If he wants to earn 7.4% annual interest on the total amount, how much money should he invest in each account?

Solution

$\$8,000$ at 4%, $\$17,000$ at 9%

---

40. Vern sold his 1964 Ford Mustang for $\$55,000$ and wants to invest the money to earn him 5.8% interest per year. He will put some of the money into Fund A that earns 3% per year and the rest in Fund B that earns 10% per year. How much should he invest into each fund if he wants to earn 5.8% interest per year on the total amount?

---

41. Stephanie inherited $\$40,000$. She wants to put some of the money in a certificate of

deposit that pays 2.1% interest per year and the rest in a mutual fund account that pays 6.5% per year. How much should she invest in each account if she wants to earn 5.4% interest per year on the total amount?

Solution

$\$10{,}000$ in CD, $\$30{,}000$ in mutual fund

---

42. Avery and Caden have saved $\$27{,}000$ towards a down payment on a house. They want to keep some of the money in a bank account that pays 2.4% annual interest and the rest in a stock fund that pays 7.2% annual interest. How much should they put into each account so that they earn 6% interest per year?

---

43. Dominic pays 7% interest on his $\$15{,}000$ college loan and 12% interest on his $\$11{,}000$ car loan. What average interest rate does he pay on the total $\$26{,}000$ he owes? (Round your answer to the nearest tenth of a percent.)

Solution

9.1%

---

44. Liam borrowed a total of $\$35{,}000$ to pay for college. He pays his parents 3% interest on the $\$8{,}000$ he borrowed from them and pays the bank 6.8% on the rest. What average interest rate does he pay on the total $\$35{,}000$? (Round your answer to the nearest tenth of a percent.)

Exercises: Solve Uniform Motion Applications

Instructions: For questions 45-66, solve.

45. Lilah is moving from Portland to Seattle. It takes her three hours to go by train. Mason leaves the train station in Portland and drives to the train station in Seattle with all Lilah's boxes in his car. It takes him 2.4 hours to get to Seattle, driving at 15 miles per hour faster than the speed of the train. Find Mason's speed and the speed of the train.

Solution

Mason 75 mph, train 60 mph

---

46. Kathy and Cheryl are walking in a fundraiser. Kathy completes the course in $4.8 hours$ and Cheryl completes the course in 8 hours. Kathy walks two miles per hour faster than Cheryl. Find Kathy's speed and Cheryl's speed.

---

47. Two buses go from Sacramento for San Diego. The express bus makes the trip in 6.8 hours and the local bus takes 10.2 hours for the trip. The speed of the express bus is 25 mph faster than the speed of the local bus. Find the speed of both buses.

Solution

express bus 75 mph, local 50 mph

---

48. A commercial jet and a private airplane fly from Denver to Phoenix. It takes the commercial jet 1.1 hours for the flight, and it takes the private airplane 1.8 hours. The speed of the commercial jet is 210 miles per hour faster than the speed of the private airplane. Find the speed of both airplanes.

---

49. Saul drove his truck 3 hours from Dallas towards Kansas City and stopped at a truck stop to get dinner. At the truck stop he met Erwin, who had driven 4 hours from Kansas City towards Dallas. The distance between Dallas and Kansas City is 542 miles, and Erwin's speed was eight miles per hour slower than Saul's speed. Find the speed of the two truckers.

Solution

Saul 82 mph, Erwin 74 mph

---

50. Charlie and Violet met for lunch at a restaurant between Memphis and New Orleans. Charlie had left Memphis and drove 4.8 hours towards New Orleans. Violet had left New Orleans and drove 2 hours towards Memphis, at a speed 10 miles per hour faster than Charlie's speed. The distance between Memphis and New Orleans is 394 miles. Find the speed of the two drivers.

---

51. Sisters Helen and Anne live 332 miles apart. For Thanksgiving, they met at their other sister's house partway between their homes. Helen drove 3.2 hours and Anne drove 2.8 hours. Helen's average speed was four miles per hour faster than Anne's. Find Helen's average speed and Anne's average speed.

Solution

Helen 67 mph, Anne 53 mph

---

52. Ethan and Leo start riding their bikes at the opposite ends of a 65-mile bike path. After Ethan has ridden 1.5 hours and Leo has ridden 2 hours, they meet on the path. Ethan's speed is six miles per hour faster than Leo's speed. Find the speed of the two bikers.

---

53. Elvira and Aletheia live 3.1 miles apart on the same street. They are in a study group that meets at a coffee shop between their houses. It took Elvira half an hour and Aletheia two-thirds of an hour to walk to the coffee shop. Aletheia's speed is 0.6 miles per hour slower than Elvira's speed. Find both women's walking speeds.

Solution

Aletheia 2.4 mph, Elvira 3 mph

---

54. DaMarcus and Fabian live 23 miles apart and play soccer at a park between their homes. DaMarcus rode his bike for three-quarters of an hour and Fabian rode his bike for half an hour to get to the park. Fabian's speed was six miles per hour faster than DaMarcus' speed. Find the speed of both soccer players.

---

55. Cindy and Richard leave their dorm in Charleston at the same time. Cindy rides her bicycle north at a speed of 18 miles per hour. Richard rides his bicycle south at a speed of 14 miles per hour. How long will it take them to be 96 miles apart?

Solution

3 hours

---

56. Matt and Chris leave their uncle's house in Phoenix at the same time. Matt drives west on I-60 at a speed of 76 miles per hour. Chris drives east on I-60 at a speed of 82 miles per hour. How many hours will it take them to be 632 miles apart?

---

57. Two buses leave Billings at the same time. The Seattle bus heads west on I-90 at a speed of 73 miles per hour while the Chicago bus heads east at a speed of 79 miles an hour. How many hours will it take them to be 532 miles apart?

Solution

3.5 hours

---

58. Two boats leave the same dock in Cairo at the same time. One heads north on the Mississippi River while the other heads south. The northbound boat travels four miles per hour. The southbound boat goes eight miles per hour. How long will it take them to be 54 miles apart?

---

59. Lorena walks the path around the park in 30 minutes. If she jogs, it takes her 20 minutes. Her jogging speed is 1.5 miles per hour faster than her walking speed. Find Lorena's walking speed and jogging speed.
Solution

walking 3 mph, jogging 4.5 mph

---

60. Julian rides his bike uphill for 45 minutes, then turns around and rides back downhill. It takes him 15 minutes to get back to where he started. His uphill speed is 3.2 miles per hour slower than his downhill speed. Find Julian's uphill and downhill speed.

---

61. Cassius drives his boat upstream for 45 minutes. It takes him 30 minutes to return downstream. His speed going upstream is three miles per hour slower than his

speed going downstream. Find his upstream and downstream speeds.
Solution

upstream 6 mph, downstream 9 mph

---

62. It takes Darline 20 minutes to drive to work in light traffic. To come home, when there is heavy traffic, it takes her 36 minutes. Her speed in light traffic is 24 miles per hour faster than her speed in heavy traffic. Find her speed in light traffic and in heavy traffic.

---

63. At 1:30, Marlon left his house to go to the beach, a distance of 7.6 miles. He rode his skateboard until 2:15, then walked the rest of the way. He arrived at the beach at 3:00. Marlon's speed on his skateboard is 2.5 times his walking speed. Find his speed when skateboarding and when walking.
Solution

skateboarding 7.2 mph, walking 2.9 mph

---

64. Aaron left at 9:15 to drive to his mountain cabin 108 miles away. He drove on the freeway until 10:45, and then he drove on the mountain road. He arrived at 11:05. His speed on the freeway was three times his speed on the mountain road. Find Aaron's speed on the freeway and on the mountain road.

---

65. Marisol left Los Angeles at 2:30 to drive to Santa Barbara, a distance of 95 miles.

The traffic was heavy until 3:20. She drove the rest of the way in very light traffic and arrived at 4:20. Her speed in heavy traffic was 40 miles per hour slower than her speed in light traffic. Find her speed in heavy traffic and in light traffic.

Solution

heavy traffic 30 mph, light traffic 70 mph

---

66. Lizette is training for a marathon. At 7:00, she left her house and ran until 8:15, then she walked until 11:15. She covered a total distance of 19 miles. Her running speed was five miles per hour faster than her walking speed. Find her running and walking speeds.

Exercises: Everyday Math

Instructions: For questions 67-70, answer the given everyday math word problems.

67. As the treasurer of her daughter's Girl Scout troop, Laney collected money for some girls and adults to go to a 3-day camp. Each girl paid $\$75$ and each adult paid $\$30$. The total amount of money collected for camp was $\$765$. If the number of girls is three times the number of adults, how many girls and how many adults paid for camp?

Solution

9 girls, 3 adults

---

68. Laurie was completing the treasurer's report for her son's Boy Scout troop at the end of the school year. She didn't remember how many boys had paid the $\$15$ full-year registration fee and how many had paid the $\$10$ partial-year fee. She knew that the number of boys who paid for a full-year was ten more than the number who paid for a partial-year. If $\$250$ was collected for all the registrations, how many boys had paid the full-year fee and how many had paid the partial-year fee?

---

69. John left his house in Irvine at 8:35 am to drive to a meeting in Los Angeles, 45 miles away. He arrived at the meeting at 9:50. At 3:30 pm, he left the meeting and drove home. He arrived home at 5:18.

a. What was his average speed on the drive from Irvine to Los Angeles?
b. What was his average speed on the drive from Los Angeles to Irvine?
c. What was the total time he spent driving to and from this meeting?
d. John drove a total of 90 miles roundtrip. Find his average speed. (Round to the nearest tenth.)

Solution

a. 36 mph

b. 25 mph

c. 3.05 hours

d. 29.5 mph

---

70. Sarah wants to arrive at her friend's wedding at 3:00. The distance from Sarah's house to the wedding is 95 miles. Based on usual traffic patterns, Sarah predicts she

can drive the first 15 miles at 60 miles per hour, the next 10 miles at 30 miles per hour, and the remainder of the drive at 70 miles per hour.

a. How long will it take Sarah to drive the first 15 miles?

b. How long will it take Sarah to drive the next 10 miles?

c. How long will it take Sarah to drive the rest of the trip?
d. What time should Sarah leave her house?

Exercises: Writing Exercises

Instructions: For questions 71-76, answer the given writing exercises.

71. Suppose you have six quarters, nine dimes, and four pennies. Explain how you find the total value of all the coins.
Solution
Answers will vary.

---

72. Do you find it helpful to use a table when solving coin problems? Why or why not?

---

73. In the table used to solve coin problems, one column is labeled "number" and another column is labeled "value." What is the difference between the "number" and the "value?"
Solution

Answers will vary.

---

74. What similarities and differences did you see between solving the coin problems and the ticket and stamp problems?

---

75. When solving a uniform motion problem, how does drawing a diagram of the situation help you?
Solution
Answers will vary.

---

76. When solving a uniform motion problem, how does creating a table help you?

3.9 GRAPH LINEAR EQUATIONS IN TWO VARIABLES

Learning Objectives

By the end of this section, you will be able to:

- Plot points in a rectangular coordinate system
- Verify solutions to an equation in two variables
- Complete a table of solutions to a linear equation
- Find solutions to a linear equation in two variables
- Recognize the relationship between the solutions of an equation and its graph.
- Graph a linear equation by plotting points.
- Graph vertical and horizontal lines.
- Identify the x-intercept and y-intercept on a graph
- Find the x-intercept and y-intercept from an equation of a line
- Graph a line using the intercepts

Try It

Before you get started, take this readiness quiz:

1) Evaluate $x+3$ when $x=-1$.
2) Evaluate $2x-5y$ when $x=3$ and $y=-2$.
3) Solve for y: $40-4y=20$.
4) Evaluate $3x+2$ when $x=-1$.
5) Solve $3x+2y=12$ for y in general.
6) Solve: $3\cdot 0+4y=-2$.

Plot Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a **rectangular coordinate system**. The rectangular coordinate system is also called the xy-plane or the 'coordinate plane'.

The horizontal number line is called the x*-axis*. The vertical number line is called the y*-axis.*

The x-axis and the y-axis together form the rectangular coordinate system. These axes divide a plane into four regions, called **quadrant**s. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise. See Figure 3.9.1.

'Quadrant' has the root 'quad,' which means 'four.'

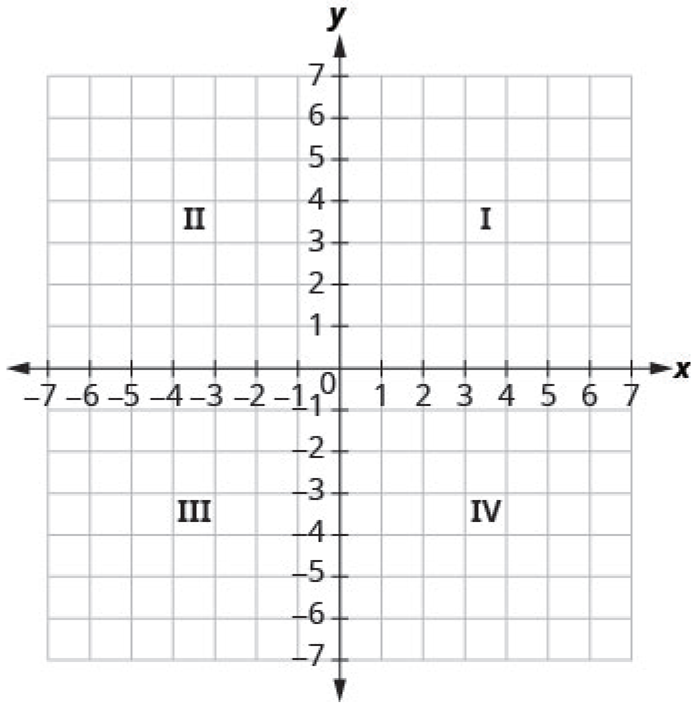


Figure 3.9.1-'Quadrant' has the root 'quad,' which means 'four.'

In the rectangular coordinate system, every point is represented by an **ordered pair**. The first number in the ordered pair is the **[latex]x[/latex]-coordinate** of the point, and the second number is the **[latex]y[/latex]-coordinate** of the point.

Ordered Pair

An ordered pair, (x, y), gives the coordinates of a point in a rectangular coordinate system.

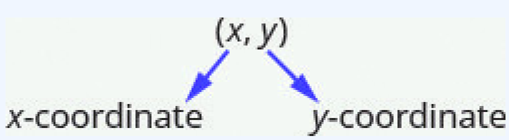


Figure 3.9.2

The first number is the x-coordinate.

The second number is the y-coordinate.

The phrase 'ordered pair' means the order is important. What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the **origin**.

The Origin

The point $(0, 0)$ is called the origin. It is the point where the x-axis and y-axis intersect.

We use the coordinates to locate a point on the xy-plane. Let's plot the point $(1, 3)$ as an example. First, locate 1 on the x-axis and lightly sketch a **vertical line** through $x = 1$. Then, locate 3 on the y-axis and sketch a **horizontal line** through $y = 3$. Now, find the point where these two lines meet—that is the point with coordinates $(1, 3)$.

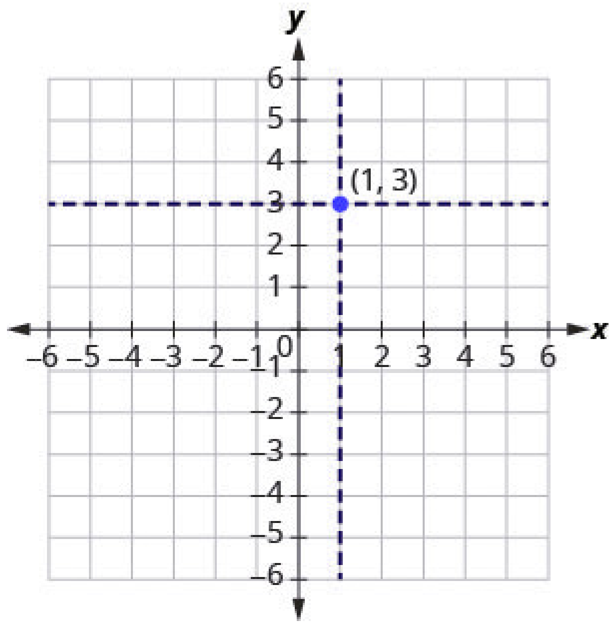


Figure 3.9.3

Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

Example 1

Plot each point in the rectangular coordinate system and identify the quadrant in which the point is located:

a. $(-5, 4)$
b. $(-3, -4)$
c. $(2, -3)$
d. $(-2, 3)$
e. $(3, \frac{5}{2})$

Solution

The first number of the coordinate pair is the x-coordinate, and the second number is the y-coordinate.

a. Since $x = -5$, the point is to the left of the y-axis. Also, since $y = 4$, the point is above the x-axis. The point $(-5, 4)$ is in Quadrant II.

b. Since $x = -3$, the point is to the left of the y-axis. Also, since $y = -4$, the point is below the x-axis. The point $(-3, -4)$ is in Quadrant III.

c. Since $x = 2$, the point is to the right of the y-axis. Since $y = -3$, the point is below the x-axis. The point $(2, -3)$ is in Quadrant lV.

d. Since $x = -2$, the point is to the left of the y-axis. Since $y = 3$, the point is above the x-axis. The point $(-2, 3)$ is in Quadrant II.

e. Since $x = 3$, the point is to the right of the y-axis. Since $y = \frac{5}{2}$, the point is above the

x-axis. (It may be helpful to write $\frac{5}{2}$) as a mixed number or decimal.) The point $(3, \frac{5}{2})$ is in Quadrant I.

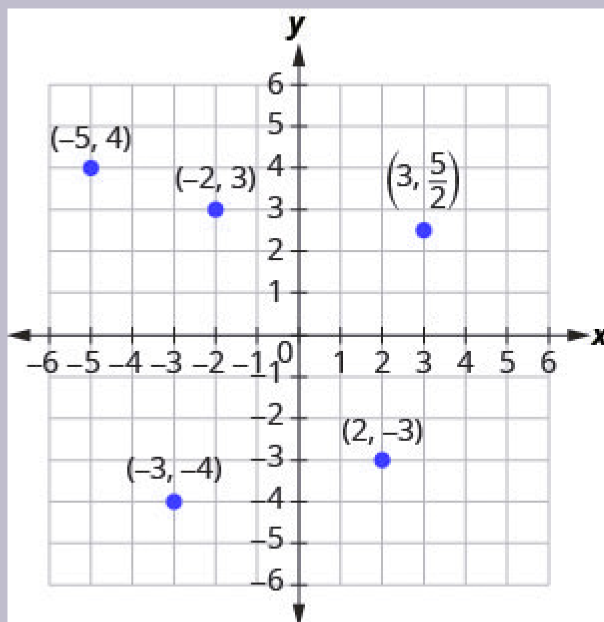


Figure 3.9.4

Try It

7) Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

a. $(-2, 1)$

b. $(-3, -1)$

c. $(4, -4)$

d. $(-4, 4)$

e. $(-4, \frac{3}{2})$

Solution

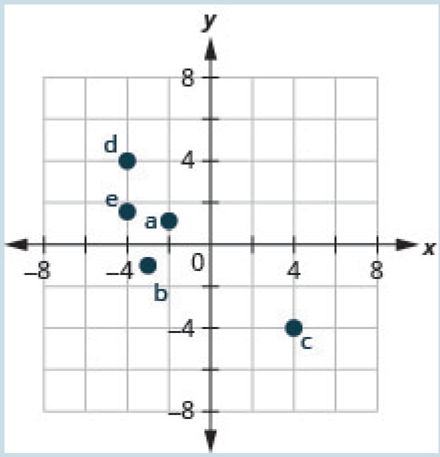


Figure 3.9.5

8) Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

a. $(-4, 1)$

b. $(-2, 3)$

c. $(2, -5)$

d. $(-2, 5)$

e. $(-3, \frac{5}{2})$

Solution

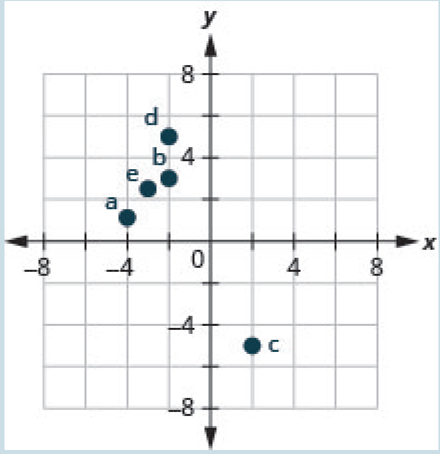


Figure 3.9.6

How do the signs affect the location of the points? You may have noticed some patterns as you graphed the points in the previous example.

For the point in Figure 3.9.4 in Quadrant IV, what do you notice about the signs of the coordinates? What about the signs of the coordinates of points in the third quadrant? The second quadrant? The first quadrant?

Can you tell just by looking at the coordinates in which quadrant the point $(-2, 5)$ is located? In which quadrant is $(2, -5)$ located?

Quadrants

We can summarize sign patterns of the quadrants in this way.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$

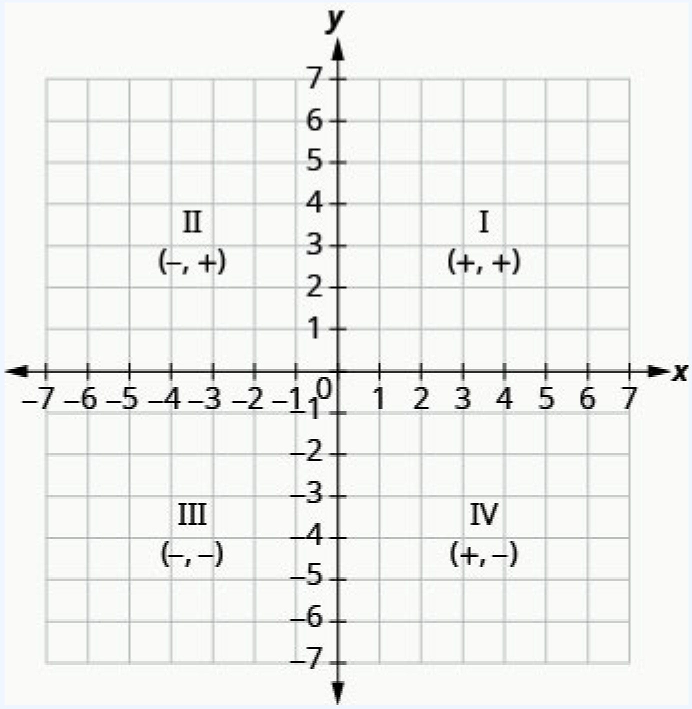


Figure 3.9.7

What if one coordinate is zero as shown in Figure 3.9.8? Where is the point $(0, 4)$ located? Where is the point $(-2, 0)$ located?

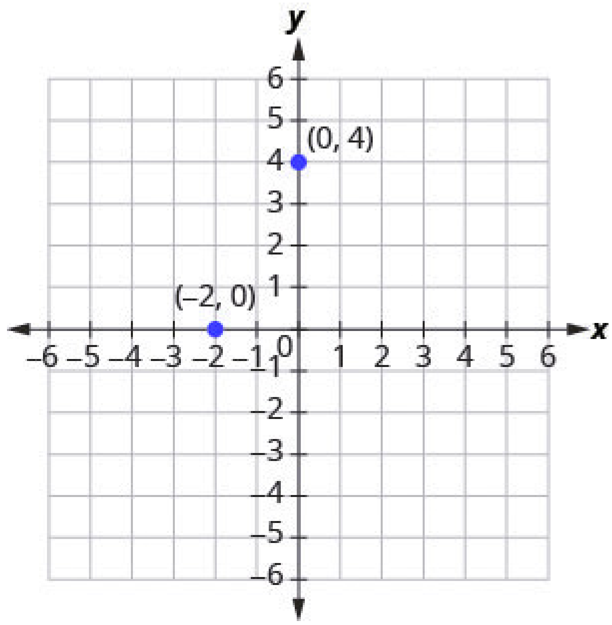


Figure 3.9.8

The point $(0, 4)$ is on the y-axis and the point $(-2, 0)$ is on the x-axis.

Points on the Axes

Points with a y-coordinate equal to 0 are on the x-axis, and have coordinates $(a, 0)$.

Points with an x-coordinate equal to 0 are on the y-axis, and have coordinates $(0, b)$.

Example 2

Plot each point:

a. $(0, 5)$

b. $(4, 0)$

c. $(-3, 0)$

d. $(0, 0)$

e. $(0, -1)$

Solution

a. Since $x = 0$, the point whose coordinates are $(0, 5)$ is on the y-axis.

b. Since $y = 0$, the point whose coordinates are $(4, 0)$ is on the x-axis.

c. Since $y = 0$, the point whose coordinates are $(-3, 0)$ is on the x-axis.

d. Since $x = 0$ and $y = 0$, the point whose coordinates are $(0, 0)$ is the origin.

e. Since $x = 0$, the point whose coordinates are $(0, -1)$ is on the y-axis.

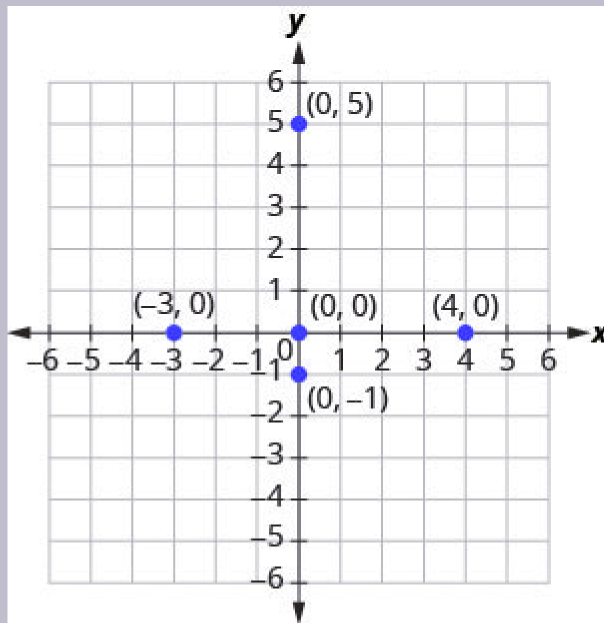


Figure 3.9.9

Try It

9) Plot each point:

a. $(4, 0)$

b. $(-2, 0)$

c. $(0, 0)$

d. $(0, 2)$

e. $(0, -3)$

Solution

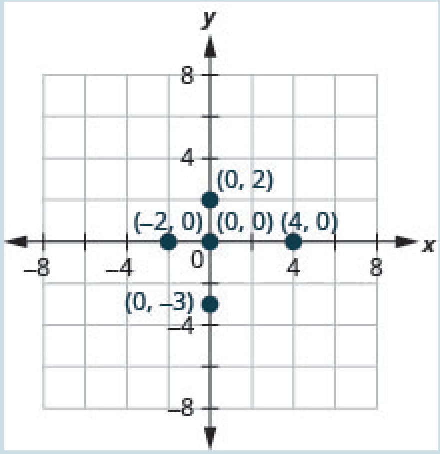


Figure 3.9.10

10) Plot each point:

a. $(-5, 0)$

b. $(3, 0)$

c. $(0, 0)$

d. $(0, -1)$

e. $(0, 4)$

Solution

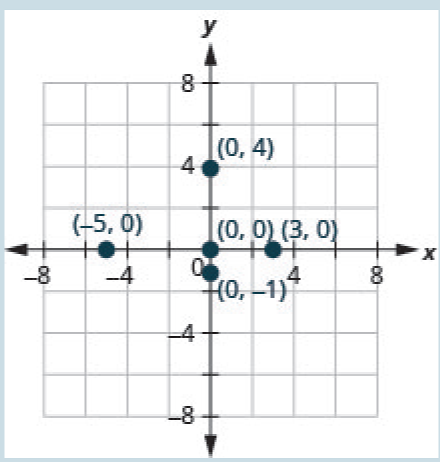


Figure 3.9.11

In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the x-coordinate of a point on a graph, read the number on the x-axis directly above or below the point. To identify the y-coordinate of a point, read the number on the y-axis directly to the left or right of the point. Remember, when you write the ordered pair use the correct order, (x, y).

Example 3

Name the ordered pair of each point shown in the rectangular coordinate system.

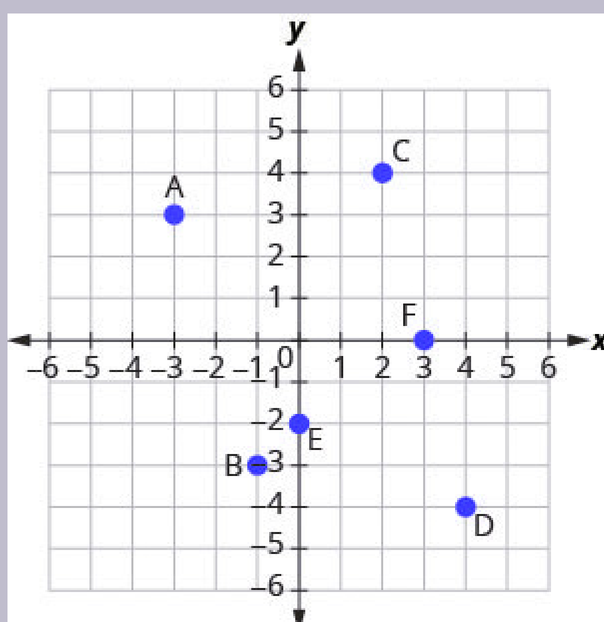


Figure 3.9.12

Solution

Point A is above -3 on the x-axis, so the x-coordinate of the point is -3.

- The point is to the left of 3 on the y-axis, so the y-coordinate of the point is 3.
- The coordinates of the point are $(-3, 3)$.

Point B is below -1 on the x-axis, so the x-coordinate of the point is -1.

- The point is to the left of -3 on the y-axis, so the y-coordinate of the point is -3.
- The coordinates of the point are $(-1, -3)$.

Point C is above 2 on the x-axis, so the x-coordinate of the point is 2.

- The point is to the right of 4 on the y-axis, so the y-coordinate of the point is 4.
- The coordinates of the point are $(2, 4)$.

Point D is below 4 on the x-axis, so the x-coordinate of the point is 4.

- The point is to the right of -4 on the y-axis, so the y-coordinate of the point is -4.
- The coordinates of the point are $(4, -4)$.

Point E is on the y-axis at $y = -2$. The coordinates of point E are $(0, -2)$.

Point F is on the x-axis at $x = 3$. The coordinates of point F are $(3, 0)$.

Try It

11) Name the ordered pair of each point shown in the rectangular coordinate system.

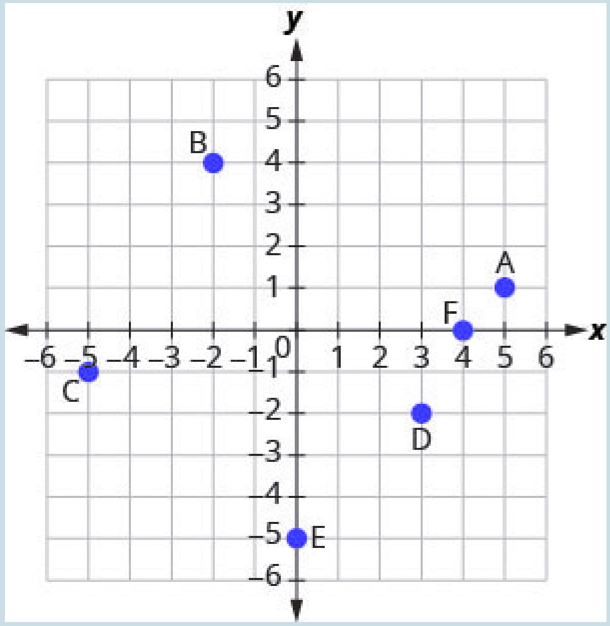


Figure 3.9.13

Solution

a: $(5, 1)$

b: $(-2, 4)$

c: $(-5, -1)$

d: $(3, -2)$

e: $(0, -5)$

f: $(4, 0)$

12) Name the ordered pair of each point shown in the rectangular coordinate system.

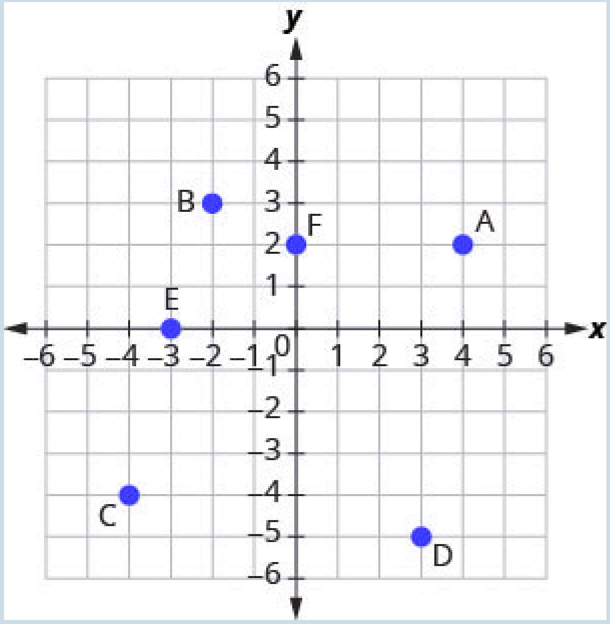


Figure 3.9.14

Solution

a: $(4, 2)$

b: $(-2, 3)$

c: $(-4, -4)$

d: $(3, -5)$

e: $(-3, 0)$

f: $(0, 2)$

Verify Solutions to an Equation in Two Variables

Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. The process of solving an equation ended with a statement like $x = 4$. (Then, you check the solution by substituting back into the equation.)

Here's an example of an equation in one variable, and its one solution.

$$\begin{aligned} 3x + 5 &= 17 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

However, equations can have more than one variable. Equations with two variables may be of the form $Ax + By = C$. Equations of this form are called **linear equations** in two variables.

Linear Equation

An equation of the form $Ax + By = C$, where A and B are not both zero, is called a linear equation *in two variables.*

Notice the word *line* in *linear*. Here is an example of a linear equation in two variables, x and y.

$$\begin{array}{c} Ax + By = C \\ x + 4y = 8 \\ A=1,\ B=4,\ C=8 \end{array}$$

The equation $y = -3x + 5$ is also a linear equation. But it does not appear to be in the form $Ax + By = C$. We can use the Addition Property of Equality and rewrite it in $Ax + By = C$ form.

	$y = -3x + 5$
Step 1: Add to both sides.	$y + 3x = -3x + 5 + 3x$
Step 2: Simplify.	$y + 3x = 5$
Step 3: Use the Commutative Property to put it in Ax+By=C form.	$3x + y = 5$

By rewriting $y = -3x + 5$ as $3x + y = 5$, we can easily see that it is a linear equation in two variables because it is of the form $Ax + By = C$. When an equation is in the form $Ax + By = C$, we say it is in *standard form*.

Standard Form of Linear Equation

A linear equation is in standard form when it is written $Ax + By = C$.

Most people prefer to have A, B, and C be integers and $A \geq 0$ when writing a linear equation in standard form, although it is not strictly necessary.

Linear equations have infinitely many solutions. For every number that is substituted for x there is a corresponding y value. This pair of values is a *solution* to the linear equation and is represented by the ordered pair (x, y). When we substitute these values of x and y into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

Solution of a Linear Equation in Two Variables

An ordered pair (x, y) is a *solution* of the linear equation $Ax + By = C$, if the equation is a true statement when the x and y values of the ordered pair are substituted into the equation.

Example 4

Determine which ordered pairs are solutions to the equation $x + 4y = 8$.

a. $(0, 2)$

b. $(2, -4)$

c. $(-4, 3)$

Solution

Substitute the x– and y-values from each ordered pair into the equation and determine if the result is a true statement.

(a)	(b)	(c)
$(0,2)$ $x=0, y=2$ $x+4y=8$ $0+4\cdot 2\stackrel{?}{=}8$ $0+8\stackrel{?}{=}8$ $8=8\checkmark$	$(2,-4)$ $x=2, y=-4$ $x+4y=8$ $2+4\cdot -4\stackrel{?}{=}8$ $2+(-16)\stackrel{?}{=}8$ $-14\neq 8$	$(-4,3)$ $x=-4, y=3$ $x+4y=8$ $-4+4\cdot 3\stackrel{?}{=}8$ $-4+12\stackrel{?}{=}8$ $8=8\checkmark$
$(0,2)$ is a solution	$(2,-4)$ is not a solution	$(-4,3)$ is a solution

Try It

13) Which of the following ordered pairs are solutions to $2x+3y=6$?

a. $(3,0)$

b. $(2,0)$

c. $(6,-2)$

Solution

a, c

14) Which of the following ordered pairs are solutions to the equation $4x-y=8$?

a. $(0,8)$

b. $(2,0)$

c. $(1,-4)$

Solution

b, c

Example 5

Which of the following ordered pairs are solutions to the equation $y = 5x - 1$?

a. $(0, -1)$

b. $(1, 4)$

c. $(-2, -7)$

Solution

Substitute the x and y values from each ordered pair into the equation and determine if it results in a true statement.

(a)	(b)	(c)
$(0, -1)$ $x = 0, y = -1$ $y = 5x - 1$ $-1 \stackrel{?}{=} 5(0) - 1$ $-1 \stackrel{?}{=} 0 - 1$ $-1 = -1 \checkmark$	$(1, 4)$ $x = 1, y = 4$ $y = 5x - 1$ $4 \stackrel{?}{=} 5(1) - 1$ $4 \stackrel{?}{=} 5 - 1$ $4 = 4 \checkmark$	$(-2, -7)$ $x = -2, y = -7$ $y = 5x - 1$ $-7 \stackrel{?}{=} 5(-2) - 1$ $-7 \stackrel{?}{=} -10 - 1$ $7 \neq -11$
$(0, -1)$ is a solution.	$(1, 4)$ is a solution.	$(-2, -7)$ is not a solution.

Try It

15) Which of the following ordered pairs are solutions to the equation $y = 4x - 3$?

a. $(0, 3)$

b. $(1, 1)$

c. $(-1, -1)$

Solution

b

16) Which of the following ordered pairs are solutions to the equation $y = -2x + 6$?

a. $(0, 6)$

b. $(1, 4)$

c. $(-2, -2)$

Solution

a, b

Complete a Table of Solutions to a Linear Equation in Two Variables

In the examples above, we substituted the x and y values of a given ordered pair to determine whether or not it was a solution to a linear equation. But how do you find the ordered pairs if they are not given? It's easier than you might think—you can just pick a value for x and then solve the equation for y. Or, pick a value for y and then solve for x.

We'll start by looking at the solutions to the equation $y = 5x - 1$ that we found in Example 3.9.5. We can summarize this information in a table of solutions, as shown in the below table.

$y = 5x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	4	$(1, 4)$

To find a third solution, we'll let $x = 2$ and solve for y.

	$y = 5x - 1$
Substitute $x = 2$.	$y = 5(2) - 1$
Multiply.	$y = 10 - 1$
Simplify.	$y = 9$

The ordered pair $(2, 9)$ is a solution to $y = 5x - 1$. We will add it to the below table.

$y = 5x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	4	$(1, 4)$
2	9	$(2, 9)$

We can find more solutions to the equation by substituting in any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are infinitely many solutions to this equation.

Example 6

Complete the below table to find three solutions to the equation $y = 4x - 2$.

$y = 4x - 2$		
x	y	(x, y)
0		
-1		
2		

Solution

Substitute $x = 0$, $x = -1$, and $x = 2$ into $y = 4x - 2$

$x = 0$	$x = -1$	$x = 2$
$y = 4x - 2$	$y = 4x - 2$	$y = 4x - 2$
$y = 4 \cdot 0 - 2$	$y = 4(-1) - 2$	$y = 4 \cdot 2 - 2$
$y = -2$	$y = -4 - 2$	$y = 8 - 2$
$y = -2$	$y = -6$	$y = 6$
$(0, -2)$	$(-1, -6)$	$(2, 6)$

The results are summarized in the below table.

$y = 4x - 2$		
x	y	(x, y)
0	-2	$(0, -2)$
-1	-6	$(-1, -6)$
2	6	$(2, 6)$

Try It

17) Complete the table to find three solutions to this equation: $y = 3x - 1$.

$y = 3x - 1$		
x	y	(x, y)
0		
-1		
2		

Solution

$y = 3x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
-1	-4	$(-1, -4)$
2	5	$(2, 5)$

18) Complete the table to find three solutions to this equation: $y = 6x - 1$.

$y = 6x - 1$		
x	y	(x, y)
0		
-1		
2		

Solution

$y = 6x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
-1	-7	$(-1, -7)$
2	11	$(2, 11)$

Example 7

Complete the below table to find three solutions to the equation $5x - 4y = 20$.

$5x - 4y = 20$		
x	y	(x, y)
0		
	0	
	5	

Solution

Substitute the given value into the equation $5x - 4y = 20$ and solve for the other variable. Then, fill in the values in the table.

$$\begin{array}{r} x = 0 \\ 5x - 4y = 20 - 2 \\ 5 \cdot 0 = 20 \\ 0 - 4y = 20 \\ -4y = 20 \\ y = -5 \\ (0, -5) \end{array} \qquad \begin{array}{r} y = 0 \\ 5x - 4y = 20 \\ 5x - 4 \cdot 0 = 20 \\ 5x - 0 = 20 \\ 5x = 20 \\ x = 4 \\ (4, 0) \end{array} \qquad \begin{array}{r} y = 5 \\ 5x - 4y = 20 \\ 5x - 4 \cdot 5 = 20 \\ 5x - 20 = 20 \\ 5x = 40 \\ x = 8 \\ (8, 5) \end{array}$$

The results are summarized in the below table.

$5x - 4y = 20$		
x	y	(x, y)
0	-5	$(0, -5)$
4	0	$(4, 0)$
8	5	$(8, 5)$

Try It

19) Complete the table to find three solutions to this equation: $2x - 5y = 20$.

$2x - 5y = 20$		
x	y	(x, y)
0		
	0	
-5		

Solution

$2x - 5y = 20$		
x	y	(x, y)
0	-4	$(0, -4)$
10	0	$(10, 0)$
-5	-6	$(-5, -6)$

20) Complete the table to find three solutions to this equation: $3x - 4y = 12$.

$3x - 4y = 12$		
x	y	(x, y)
0		
	0	
-4		

Solution

$3x - 4y = 12$		
x	y	(x, y)
0	-3	$(0, -3)$
4	0	$(4, 0)$
-4	-6	$(-4, -6)$

Find Solutions to a Linear Equation in Two Variables

To find a solution to a linear equation, you really can pick *any* number you want to substitute into the equation for x or y. But since you'll need to use that number to solve for the other variable it's a good idea to choose a number that's easy to work with.

When the equation is in y-form, with the y by itself on one side of the equation, it is usually easier to choose values of x and then solve for y.

Example 8

Find three solutions to the equation $y = -3x + 2$.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in y-form, it will be easier to substitute in values of x.

Let's pick $x = 0$, $x = 1$, and $x = -1$.

$x = 0$ $y = -3x + 2$	$x = 1$ $y = -3x + 2$	$x = -1$ $y = -3x + 2$

Step 1: **Substitute the value into the equation.**

$y = -3 \times 0 + 2$	$y = -3 \times 1 + 2$	$y = -3 \times (-1) + 2$

Step 2: **Simplify.**

$y = 0 + 2$	$y = -3 + 2$	$y = 3 + 2$

Step 3: **Simplify.**

$y = 2$	$y = -1$	$y = 5$

Step 4: **Write the ordered pair.**

$(0, 2)$	$(1, -1)$	$(-1, 5)$

Step 5: **Check.**

$y = -3x + 2$ $2 \stackrel{?}{=} -3(0) + 2$ $2 = 2 \checkmark$	$y = -3x + 2$ $-1 \stackrel{?}{=} -3(1) + 2$ $-1 = -1 \checkmark$	$y = -3x + 2$ $5 \stackrel{?}{=} -3(-1) + 2$ $5 = 5 \checkmark$

So, $(0, 2)$, $(1, -1)$ and $(-1, 5)$ are all solutions to $y = -3x + 2$. We show them in the below table.

$y = -3x + 2$		
x	y	(x, y)
0	2	$(0, 2)$
1	-1	$(1, -1)$
-1	5	$(-1, 5)$

Try It

21) Find three solutions to this equation: $y=-2x+3$.

Solution

Answers will vary.

22) Find three solutions to this equation: $y=-4x+1$.

Solution

Answers will vary.

We have seen how using zero as one value of x makes finding the value of y easy. When an equation is in standard form, with both the x and y on the same side of the equation, it is usually easier to first find one solution when $x=0$ find a second solution when $y=0$, and then find a third solution.

Example 9

Find three solutions to the equation $3x+2y=6$.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in standard form, let's pick first $x = 0$, then $y = 0$, and then find a third point.

$x = 0$	$y = 0$	$x = 1$

Step 1: **Substitute the value into the equation.**

$3x + 2y = 6$	$3x + 2y = 6$	$3x + 2y = 6$

Step 2. **Simplify.**

$3(0) + 2y = 6$	$3x + 2(0) = 6$	$3(1) + 2y = 6$

Step 3: **Solve.**

$\begin{aligned} 0 + 2y &= 6 \\ 2y &= 6 \\ y &= 3 \end{aligned}$	$\begin{aligned} 3x + 0 &= 6 \\ 3x &= 6 \\ x &= 2 \end{aligned}$	$\begin{aligned} 3 + 2y &= 6 \\ 2y &= 3 \\ y &= \frac{3}{2} \end{aligned}$

Step 4: **Write the ordered pair.**

$(0, 3)$	$(2, 0)$	$(1, \frac{3}{2})$

Step 5: **Check.**

$\begin{aligned} 3x + 2y &= 6 \\ 3(0) + 2(3) &\stackrel{?}{=} 6 \\ 0 + 6 &= 6 \\ 6 &= 6 ✓ \end{aligned}$	$\begin{aligned} 3x + 2y &= 6 \\ 3(2) + 2(0) &\stackrel{?}{=} 6 \\ 6 + 0 &= 6 \\ 6 &= 6 ✓ \end{aligned}$	$\begin{aligned} 3x + 2y &= 6 \\ 3(1) + 2(\frac{3}{2}) &\stackrel{?}{=} 6 \\ 3 + 3 &\stackrel{?}{=} 6 \\ 6 &= 6 ✓ \end{aligned}$

So $(0, 3)$, $(2, 0)$, and $(1, \frac{3}{2})$ are all solutions to the equation $3x + 2y = 6$. We can list these three solutions in the below table.

$3x + 2y = 6$		
x	y	(x, y)
0	3	$(0, 3)$
2	0	$(2, 0)$
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

Try It

23) Find three solutions to the equation $2x + 3y = 6$.

Solution

Answers will vary.

24) Find three solutions to the equation $4x + 2y = 8$.

Solution

Answers will vary.

Recognize the Relationship Between the Solutions

of an Equation and its Graph

In the previous section, we found several solutions to the equation $3x + 2y = 6$. They are listed in the table below. So, the ordered pairs $(0, 3)$, $(2, 0)$, and $(1, \frac{3}{2})$ are some solutions to the equation $3x + 2y = 6$. We can plot these solutions in the rectangular coordinate system as shown in the below table.

$3x + 2y = 6$		
x	y	(x, y)
0	3	$(0, 3)$
2	0	$(2, 0)$
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

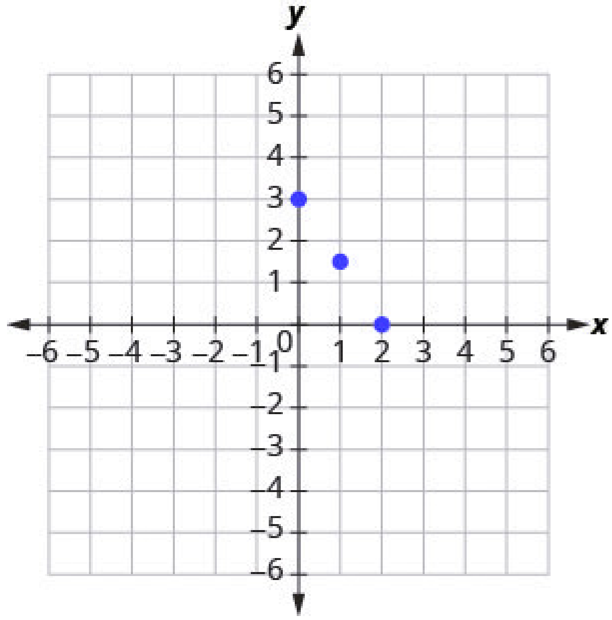


Figure 3.9.15

Notice how the points line up perfectly? We connect the points with a line to get the graph of the equation $3x + 2y = 6$. See Figure 3.9.16. Notice the arrows on the ends of each side of the line. These arrows indicate the line continues.

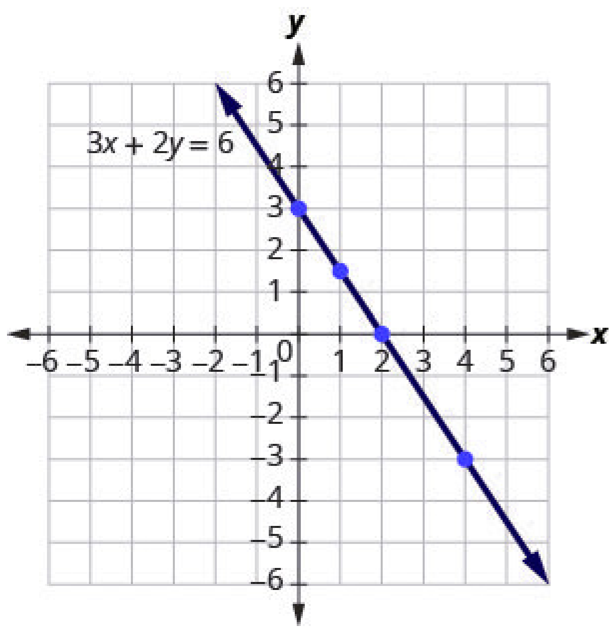


Figure 3.9.16

Every point on the line is a solution of the equation. Also, every solution of this equation is a point on this line. Points *not* on the line are not solutions.

Notice that the point whose coordinates are $(-2, 6)$ is on the line shown in Figure 3.9.17. If you substitute $x = -2$ and $y = 6$ into the equation, you find that it is a solution to the equation.

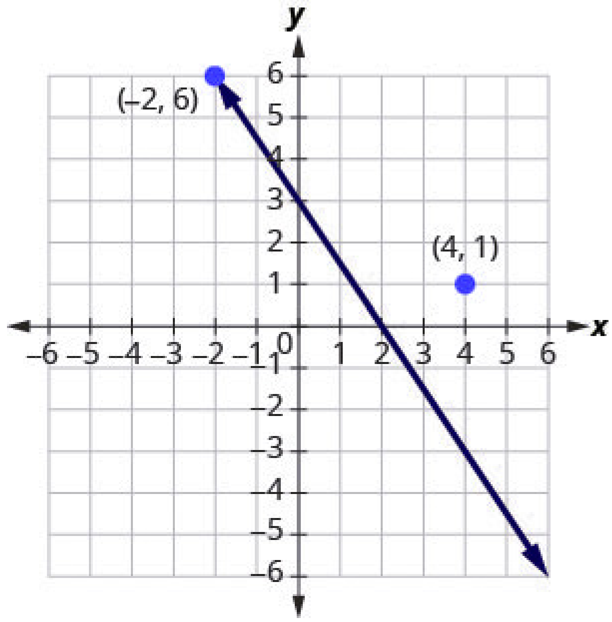


Figure 3.9.17

$$\begin{aligned} &\text{Test}(-2,6)\\ 3x+2y &= 6\\ 3(-2)+2(6) &= 6\\ -6+12 &= 6\\ 6 &= 6\checkmark \end{aligned}$$

So the point $(-2, 6)$ is a solution to the equation $3x + 2y = 6$. (The phrase "the point whose coordinates are $(-2, 6)$" is often shortened to "the point $(-2, 6)$.")

$$\begin{aligned} &\text{What about}(4,1)\\ 3x+2y &= 6\\ 3(4)+2(1) &= 6\\ 12+2 &\stackrel{?}{=} 6\\ 14 &\neq 6\checkmark \end{aligned}$$

So $(4, 1)$ is not a solution to the equation $3x + 2y = 6$. Therefore, the point $(4, 1)$ is not on the line. See Figure 3.9.17. This is an example of the saying, "A picture is worth a thousand words." The line shows you *all* the solutions to the equation. Every point on the line is a solution of the equation. And, every solution of this equation is on this line. This line is called the *graph* of the equation $3x + 2y = 6$.

Graph of a Linear Equation

The **graph of a linear equation** $Ax + By = C$ is a line.

- Every point on the line is a solution of the equation.
- Every solution of this equation is a point on this line.

Example 10

The graph of $y = 2x - 3$ is shown.

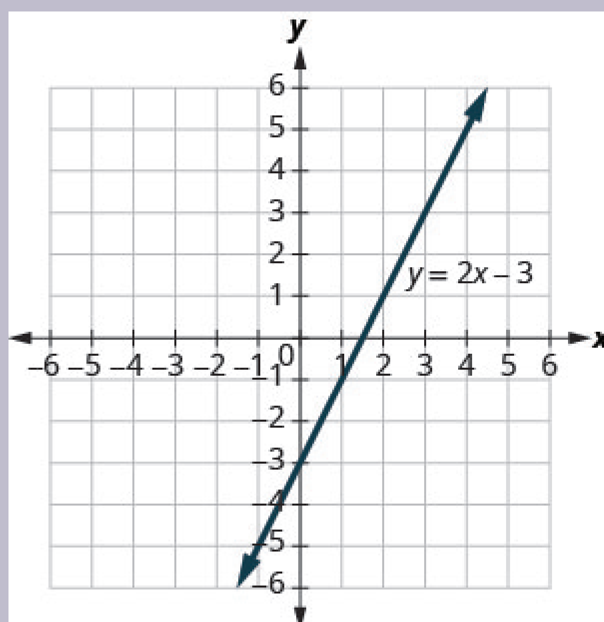


Figure 3.9.18

For each ordered pair, decide:

a. Is the ordered pair a solution to the equation?
b. Is the point on the line?

a. $(0, -3)$
b. $(3, 3)$
c. $(2, -3)$
d. $(-1, -5)$

Solution

Substitute the x – and y – values into the equation to check if the ordered pair is a solution to the equation.

a.

A:	B:	C:	D:
$(0, -3)$ $y = 2x - 3$ $-3 \stackrel{?}{=} 2(0) - 3$ $-3 = -3 \checkmark$	$(3, 3)$ $y = 2x - 3$ $3 \stackrel{?}{=} 2(3) - 3$ $3 = 3 \checkmark$	$(2, -3)$ $y = 2x - 3$ $-3 \stackrel{?}{=} 2(2) - 3$ $-3 \neq 1$	$(-1, -5)$ $y = 2x - 3$ $-5 \stackrel{?}{=} 2(-1) - 3$ $-5 = -5 \checkmark$
$(0, -3)$ is a solution.	$(3, 3)$ is a solution	$(2, -3)$ is not a solution	$(-1, -5)$ is a solution

---

b. Plot the points A $(0, 3)$, B $(3, 3)$, C $(2, -3)$, and D $(-1, -5)$.

The points $(0, 3)$, $(3, 3)$, and $(-1, -5)$ are on the line $y = 2x - 3$, and the point $(2, -3)$ is not on the line.

The points that are solutions to $y = 2x - 3$ are on the line, but the point that is not a solution is not on the line.

Try It

25) Use the graph of $y = 3x - 1$ to decide whether each ordered pair is:

- a solution to the equation.
- on the line.

a. $(0, -1)$

b. $(2, 5)$

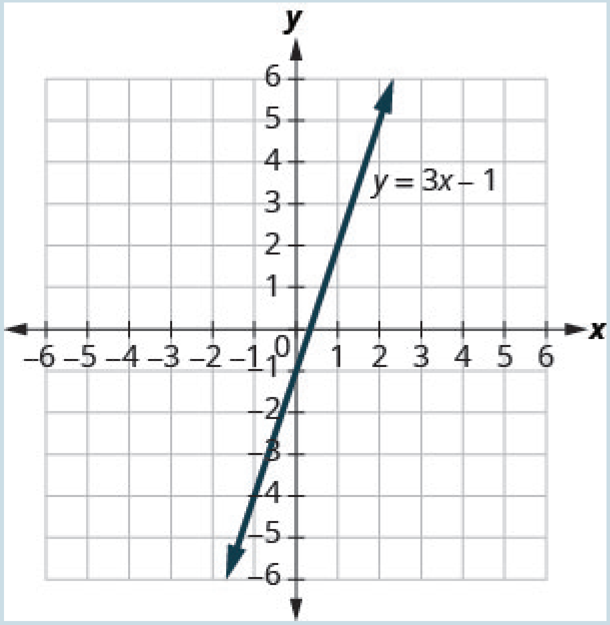


Figure 3.9.19

Solution

a. yes, yes

b. yes, yes

26) Use graph of $y = 3x - 1$ to decide whether each ordered pair is:

- a solution to the equation
- on the line

a. $(3, -1)$

b. $(-1, -4)$

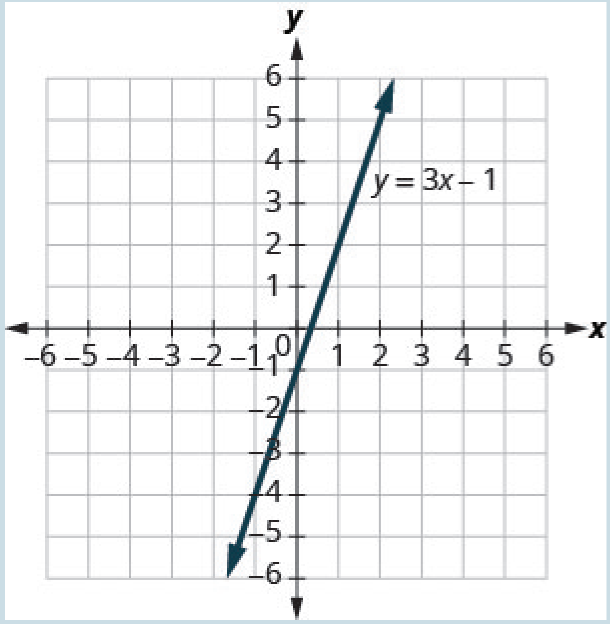


Figure 3.9.20

Solution

a. no, no

b. yes, yes

Graph a Linear Equation by Plotting Points

Several methods can be used to graph a linear equation. The method we used to graph $3x + 2y = 6$ is called plotting points, or the Point–point-plotting method.

Example 11

Graph the equation $y = 2x + 1$ by plotting points.

Solution

Step 1: **Find three points whose coordinates are solutions to the equation.**

You can choose any values for x or y.

In this case, since y is isolated on the left side of the equation, it is easier to choose values for x.

$$y = 2x + 1$$

$$\begin{aligned} x &= 0 \\ y &= 2x + 1 \\ y &= 2 \cdot 0 + 1 \\ y &= 0 + 1 \\ y &= 1 \end{aligned}$$

$$\begin{aligned} x &= 1 \\ y &= 2x + 1 \\ y &= 2 \cdot 1 + 1 \\ y &= 2 + 1 \\ y &= 3 \end{aligned}$$

$$\begin{aligned} x &= -2 \\ y &= 2x + 1 \\ y &= 2 \cdot -2 + 1 \\ y &= -4 + 1 \\ y &= -3 \end{aligned}$$

Organize the solutions in a table.

Put the three solutions in a table.

y=2x+1		
x	**y**	**(x,y)**
0	1	(0,1)
1	3	(1,3)
-2	-3	(-2.-3)

Step 2: **Plot the points in a rectangular coordinate system.**

Plot: (0,1), (1, 3), (-2,-3).

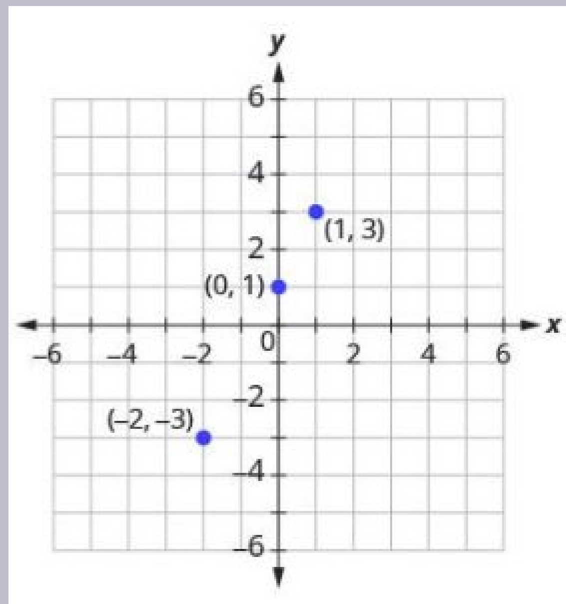


Figure 3.9.21

Check that the points line up. If they do not, carefully check your work!

Do the points line up? Yes, the points line up.

Step 3: **Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.**

This line is the graph of $y = 2x + 1$.

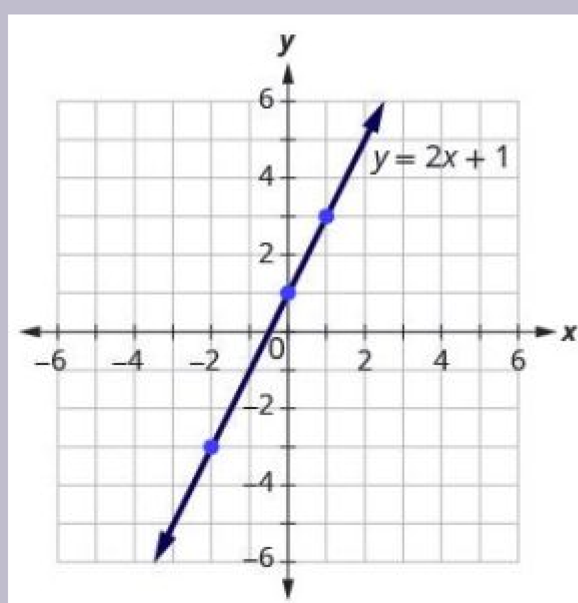


Figure 3.9.22

Try It

27) Graph the equation by plotting points: $y = 2x - 3$.

Solution

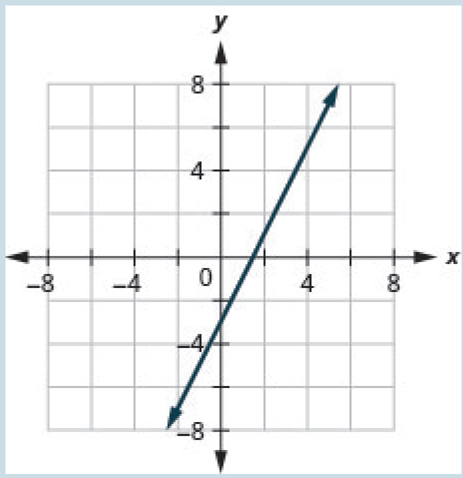


Figure 3.9.23

28) Graph the equation by plotting points: $y = -2x + 4$.

Solution

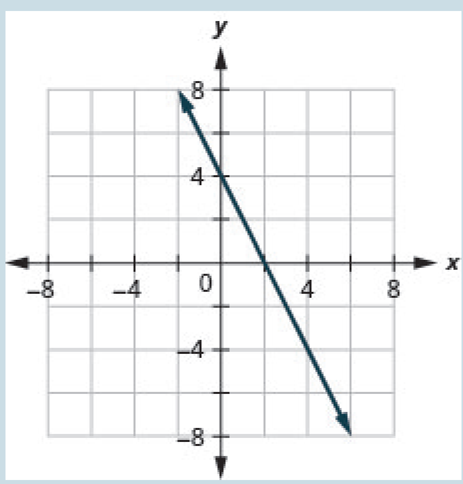


Figure 3.9.24

The steps to take when graphing a linear equation by plotting points are summarized below.

How to

Graph a linear equation by plotting points.

1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.
3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

It is true that it only takes two points to determine a line, but it is a good habit to use three points. If you only plot two points and one of them is incorrect, you can still draw a line but it will not represent the solutions to the equation. It will be the wrong line.

If you use three points, and one is incorrect, the points will not line up. This tells you something is wrong and you need to check your work. Look at the difference between part (a) and part (b) in Figure 3.9.25.

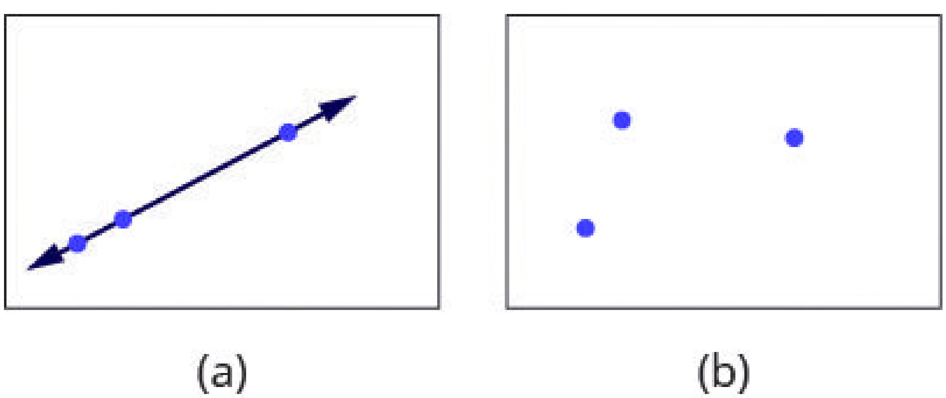


Figure 3.9.25

Let's do another example. This time, we'll show the last two steps all on one grid.

Example 12

Graph the equation $y = -3x$.

Solution

Step 1: **Find three points that are solutions to the equation.**

Here, again, it's easier to choose values for x. Do you see why?

$x = 0$	$x = 1$	$x = -2$
$y = -3x$	$y = -3$	$y = -3$
$y = -3 \cdot 0$	$y = -3 \cdot 1$	$y = -3(-2)$
$y = 0$	$y = -3$	$y = 6$

Step 2: **List the points in the following table.**

$y = -3x$		
x	y	(x, y)
0	0	$(0, 0)$
1	-3	$(1, -3)$
-2	6	$(-2, 6)$

Step 3: **Plot the points, check that they line up, and draw the line.**

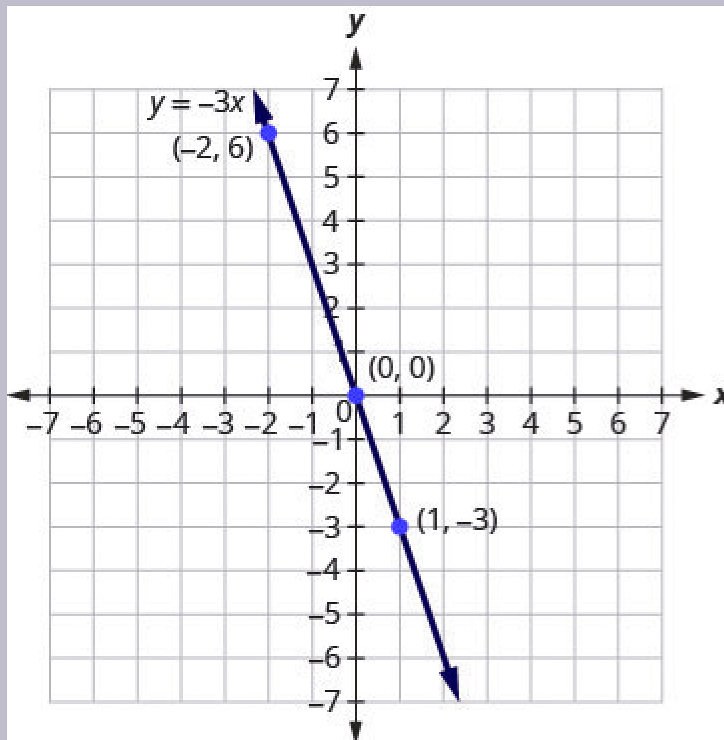


Figure 3.9.26

Try It

29) Graph the equation by plotting points: $y = -4x$.

Solution

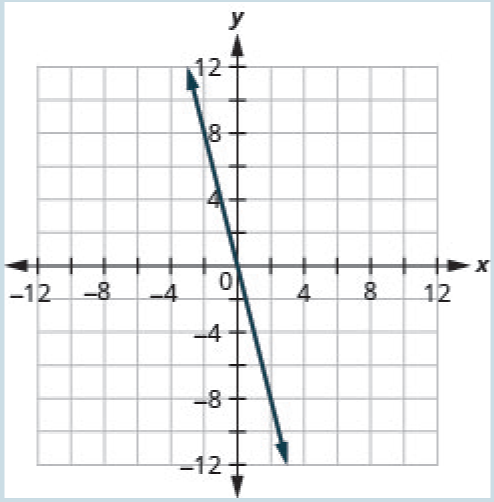


Figure 3.9.27

30) Graph the equation by plotting points: $y = x$.

Solution

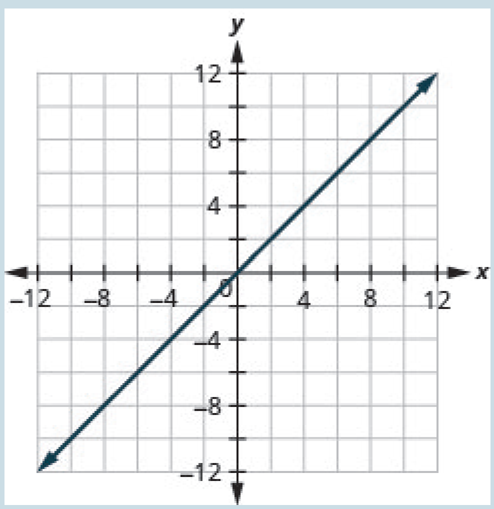


Figure 3.9.28

When an equation includes a fraction as the coefficient of x, we can still substitute any numbers for

x. But the math is easier if we make 'good' choices for the values of x. This way we will avoid fraction answers, which are hard to graph precisely.

Example 13

Graph the equation $y = \frac{1}{2}x + 3$

Solution

Step 1: **Find three points that are solutions to the equation.**

Since this equation has the fraction $\frac{1}{2}$ as a coefficient of x, we will choose values of x carefully. We will use zero as one choice and multiples of 2 for the other choices. Why are multiples of 2 a good choice for values of x?

$$\begin{array}{lll} x = 0 & x = 2 & x = 4 \\ y = \frac{1}{2}x + 3 & y = \frac{1}{2}x + 3 & y = \frac{1}{2}x + 3 \\ y = \frac{1}{2}(0) + 3 & y = \frac{1}{2}(2) + 3 & y = \frac{1}{2}(4) + 3 \\ y = 0 + 3 & y = 1 + 3 & y = 2 + 3 \\ y = 3 & y = 4 & y = 5 \end{array}$$

The points are shown in the below table.

$y=\frac{1}{2}x+3$		
x	y	(x,y)
0	3	$(0,3)$
2	4	$(2,4)$
4	5	$(4,5)$

Step 2: **Plot the points, check that they line up, and draw the line.**

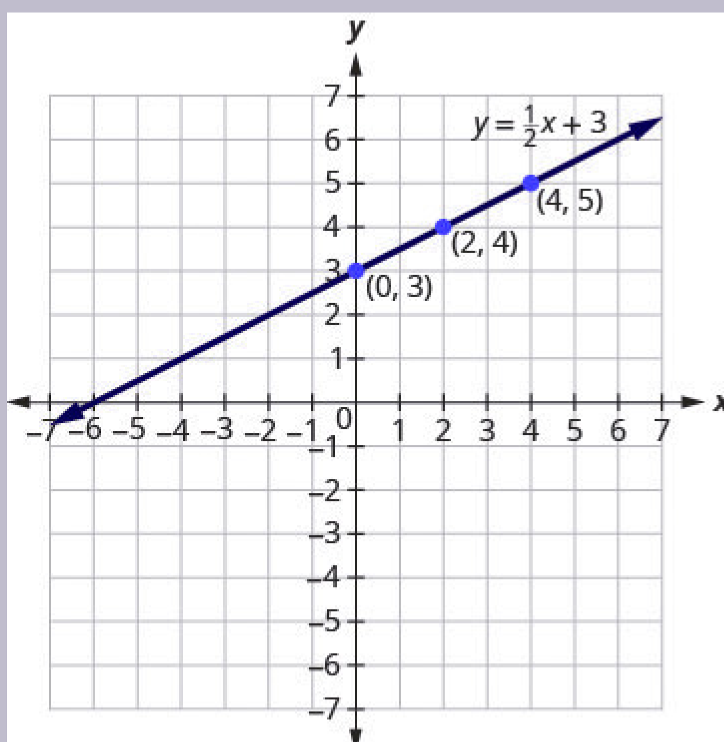


Figure 3.9.29

Try It

31) Graph the equation $y = \frac{1}{3}x - 1$

Solution

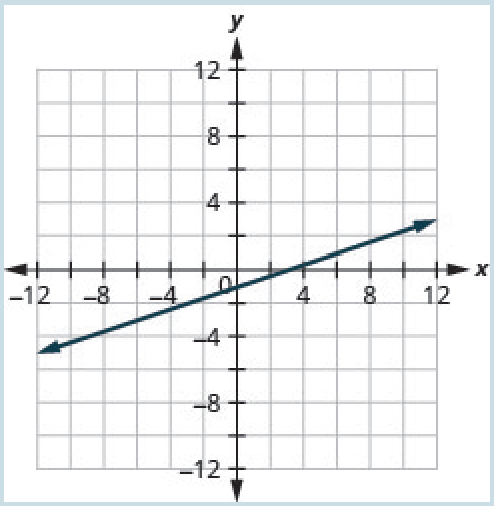


Figure 3.9.30

32) Graph the equation $y = \frac{1}{4}x + 2$.

Solution

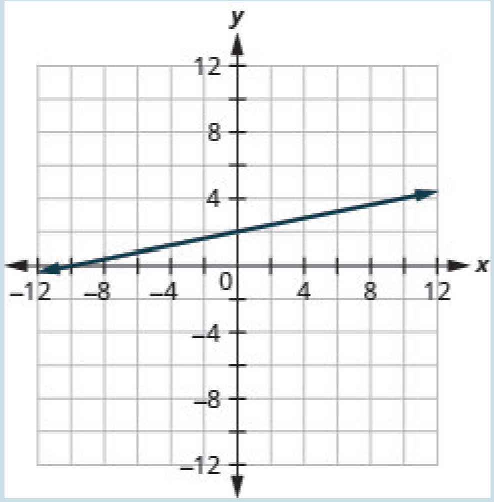


Figure 3.9.31

So far, all the equations we graphed had y given in terms of x. Now we'll graph an equation with x and y on the same side. Let's see what happens in the equation $2x + y = 3$. If $y = 0$ what is the value of x?

$$\begin{aligned} y &= 0 \\ 2x + y &= 3 \\ 2x + 0 &= 3 \\ 2x &= 3 \\ x &= \frac{3}{2} \\ &\left(\frac{3}{2}, 0\right) \end{aligned}$$

This point has a fraction for the x – coordinate and, while we could graph this point, it is hard to be precise graphing fractions. Remember in the example $y = \frac{1}{2}x + 3$, we carefully chose values for x so as not to graph fractions at all. If we solve the equation $2x + y = 3$ for y, it will be easier to find three solutions to the equation.

$$\begin{aligned} 2x + y &= 3 \\ y &= -2x + 3 \end{aligned}$$

The solutions for $x = 0$, $x = 1$, and $x = -1$ are shown in the below table. The graph is shown in the Figure 3.9.32.

$2x + y = 3$		
x	y	(x, y)
0	3	$(0, 3)$
1	1	$(1, 1)$
-1	5	$(-1, 5)$

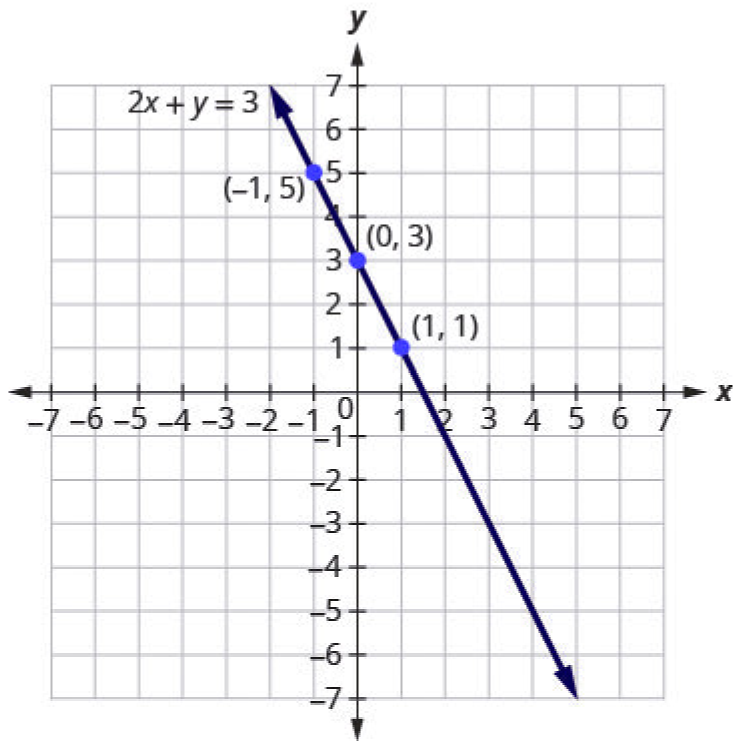


Figure 3.9.32

Can you locate the point $(\frac{3}{2}, 0)$, which we found by letting $y = 0$, on the line?

Example 14

Graph the equation $3x + y = -1$.

Solution

Step 1: **Find three points that are solutions to the equation.**

$$3x + y = -1$$

Step 2: **First, solve the equation for** y**.**

$$y = -3x - 1$$

Step 3: **We'll let** x **be** 0, 1**, and** -1 **to find 3 points.**

The ordered pairs are shown in the below table. Plot the points, check that they line up, and draw the line. See Figure 3.9.33.

$3x + y = -1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	-4	$(1, -4)$
-1	2	$(-1, 2)$

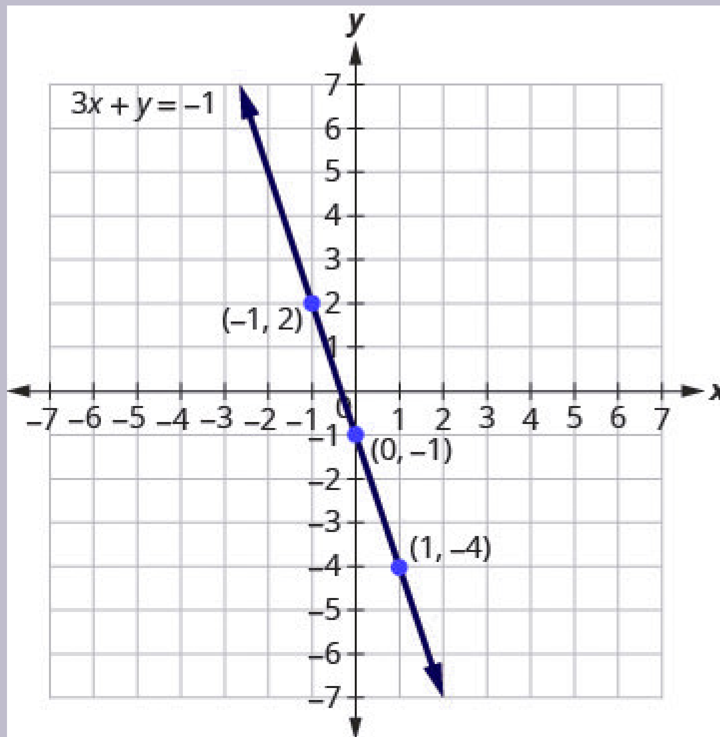


Figure 3.9.33

Try It

33) Graph the equation $2x + y = 2$.

Solution

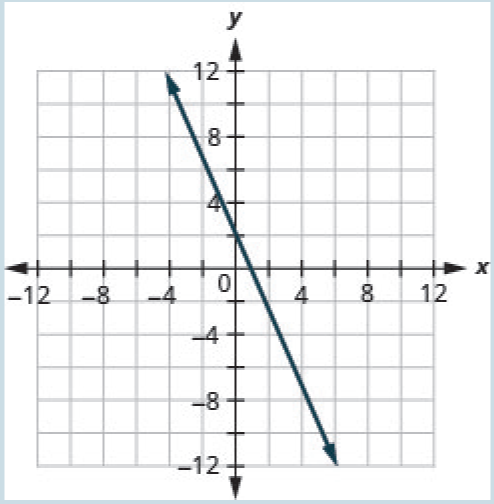


Figure 3.9.34

34) Graph the equation $4x + y = -3$.

Solution

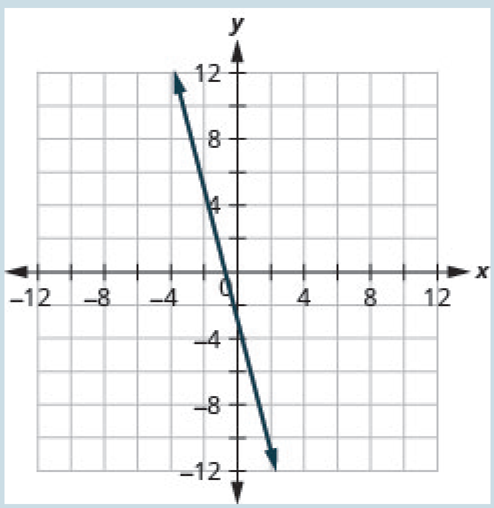


Figure 3.9.35

If you can choose any three points to graph a line, how will you know if your graph matches the one shown in the answers in the book? If the points where the graphs cross the x – and y -axis are the same, the graphs match!

The equation in Example 3.9.14, was written in standard form, with both x and y on the same side. We solved that equation for y in just one step. But for other equations in standard form, it is not that easy to solve for y, so we will leave them in standard form. We can still find a first point to plot by letting $x = 0$ and solving for y. We can plot a second point by letting $y = 0$ and then solving for x. Then we will plot a third point by using some other value for x or y.

Example 15

Graph the equation $2x - 3y = 6$

Solution

Step 1: **Find three points that are solutions to the equation.**

$$2x - 3y = 6$$

Step 2: **First, let $x = 0$.**

$$2(0) - 3y = 6$$

Step 3: **Solve for y.**

$$-3y = 6$$
$$y = -2$$

Step 4: **Now let $y = 0$.**

$$2x - 3(0) = 6$$

Step 5: **Solve for x.**

$$2x = 6$$
$$x = 3$$

Step 6: **We need a third point. Remember, we can choose any value for x or y. We'll let $x = 6$.**

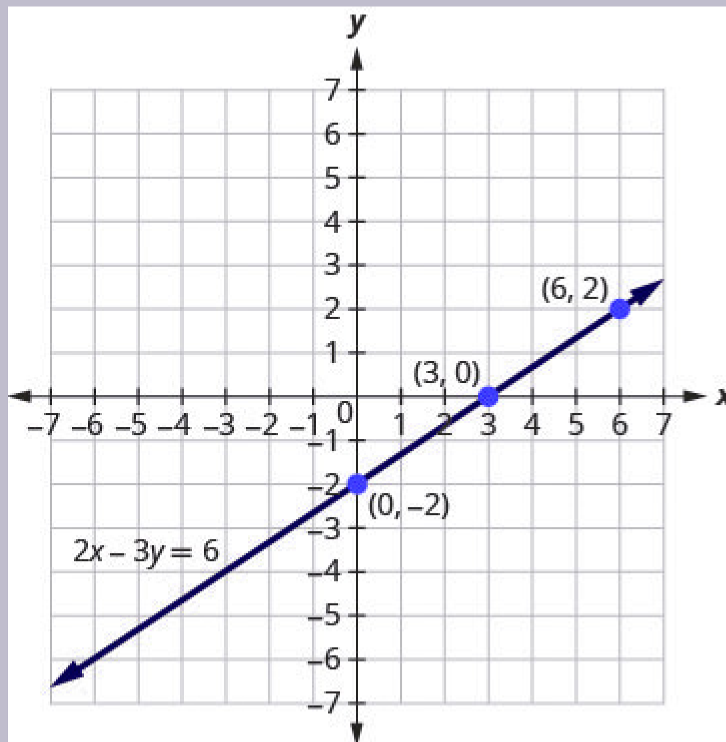
$$2(6) - 3y = 6$$

Step 7: **Solve for y.**

$$12 - 3y = 6$$
$$-3y = -6$$
$$y = 2$$

We list the ordered pairs in the below table. Plot the points, check that they line up, and draw the line. See Figure 3.9.36

$2x - 3y = 6$		
x	y	(x, y)
0	-2	$(0, -2)$
3	0	$(3, 0)$
6	2	$(6, 2)$



3.9.36

Try It

35) Graph the equation $4x + 2y = 8$.

Solution

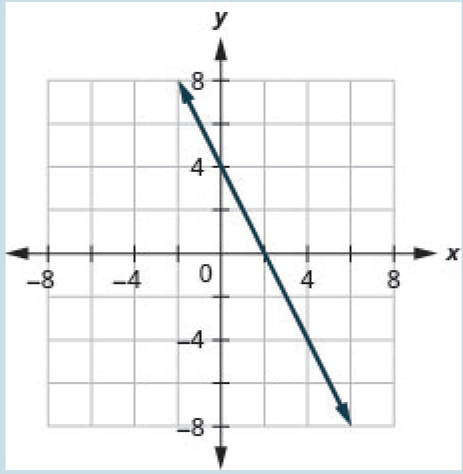


Figure 3.9.37

36) Graph the equation $2x - 4y = 8$.

Solution

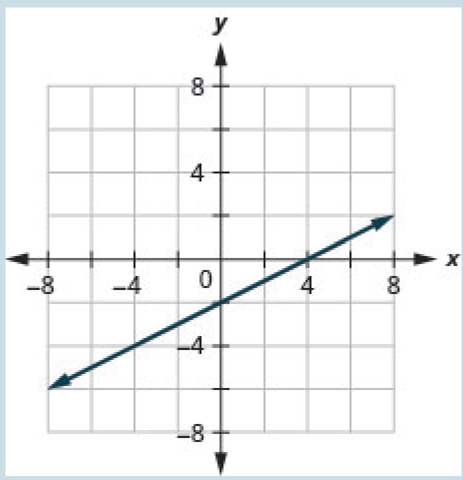


Figure 3.9.38

Graph Vertical and Horizontal Lines

Can we graph an equation with only one variable? Just x and no y, or just y without an x? How will we make a table of values to get the points to plot?

Let's consider the equation $x = -3$. This equation has only one variable, x. The equation says that x is *always* equal to -3, so its value does not depend on y. No matter what y is, the value of x is always -3.

So to make a table of values, write -3 in for all the x values. Then choose any values for y. Since x does not depend on y, you can choose any numbers you like. But to fit the points on our coordinate graph, we'll use 1, 2, and 3 for the y-coordinates. See the table below.

$x = -3$		
x	y	(x, y)
-3	1	$(-3, 1)$
-3	2	$(-3, 2)$
-3	3	$(-3, 3)$

Plot the points from the above table and connect them with a straight line. Notice in Figure 3.9.39 that we have graphed a vertical line.

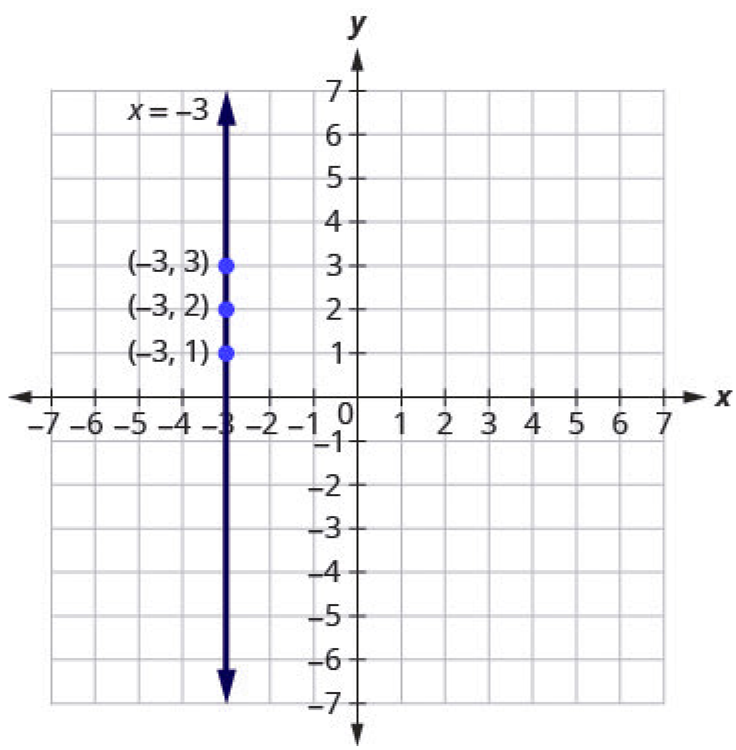


Figure 3.9.39

Vertical Line

A vertical line is the graph of an equation of the form $x = a$.

The line passes through the x-axis at $(a, 0)$.

Example 16

Graph the equation $x = 2$.

Solution

The equation has only one variable, x, and x is always equal to 2. We create the table below where x is always 2 and then put in any values for y. The graph is a vertical line passing through the x-axis at 2. See Figure 3.9.40.

$x = 2$		
x	y	(x, y)
2	1	$(2, 1)$
2	2	$(2, 2)$
2	3	$(2, 3)$

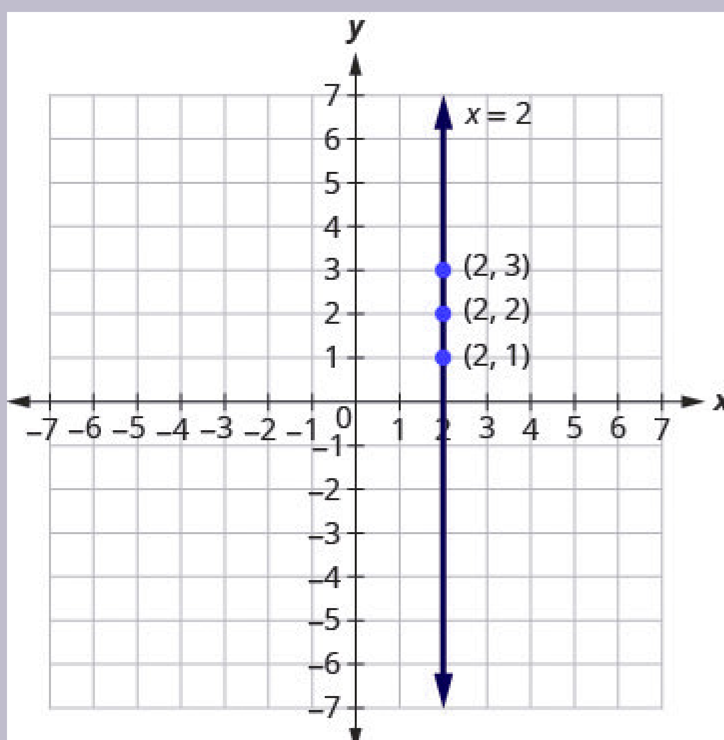


Figure 3.9.40

Try It

37) Graph the equation $x = 5$.

Solution

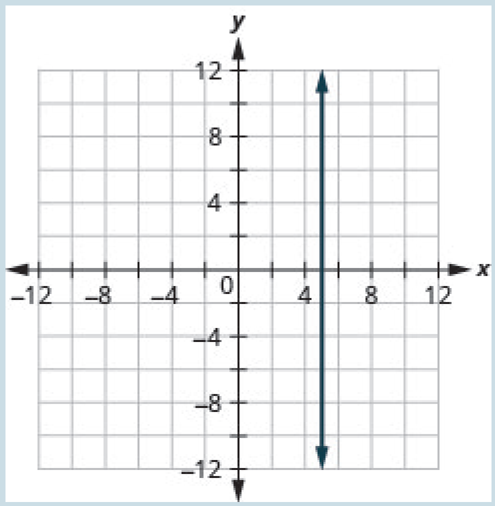


Figure 3.9.41

38) Graph the equation $x = -2$.

Solution

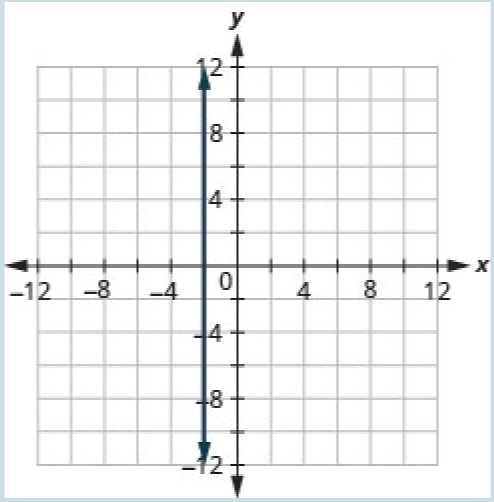


Figure 3.9.42

What if the equation has y but no x? Let's graph the equation $y = 4$. This time the y-value is a constant, so in this equation, y does not depend on x. Fill in 4 for all the y's in the below table and then choose any values for x. We'll use 0, 2, and 4 for the x-coordinates.

$y = 4$		
x	y	(x, y)
0	4	$(0, 4)$
2	4	$(2, 4)$
4	4	$(4, 4)$

The graph is a horizontal line passing through the y-axis at 4. See Figure 3.9.43.

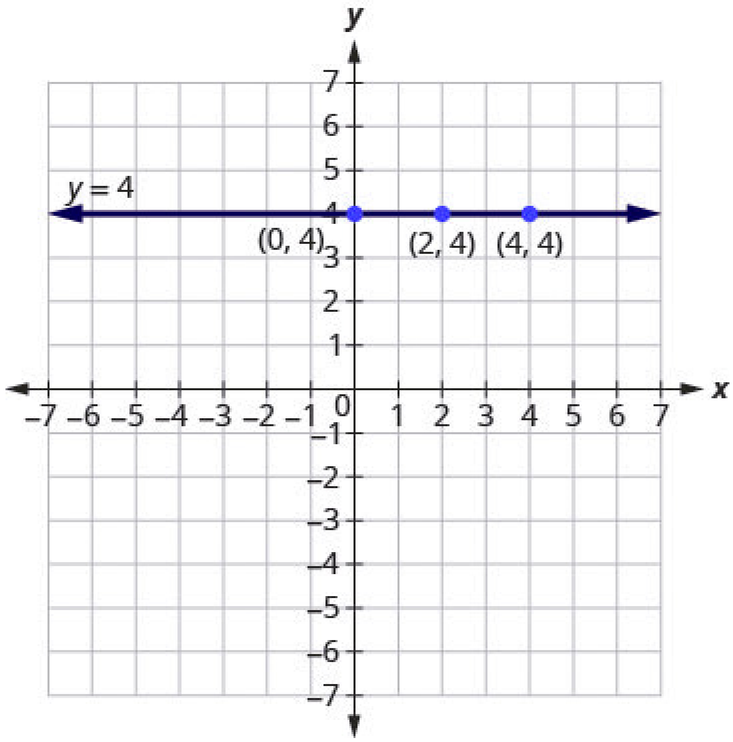


Figure 3.9.43

Horizontal Line

A horizontal line is the graph of an equation of the form $y = b$.

The line passes through the y-axis at $(0, b)$.

Example 17

Graph the equation $y = -1$

Solution

The equation $y = -1$ has only one variable, y. The value of y is constant. All the ordered pairs in the below table have the same y-coordinate. The graph is a horizontal line passing through the y-axis at -1, as shown in Figure 3.9.44.

$y = -1$		
x	y	(x, y)
0	-1	$(0, -1)$
3	-1	$(3, -1)$
-3	-1	$(-3, -1)$

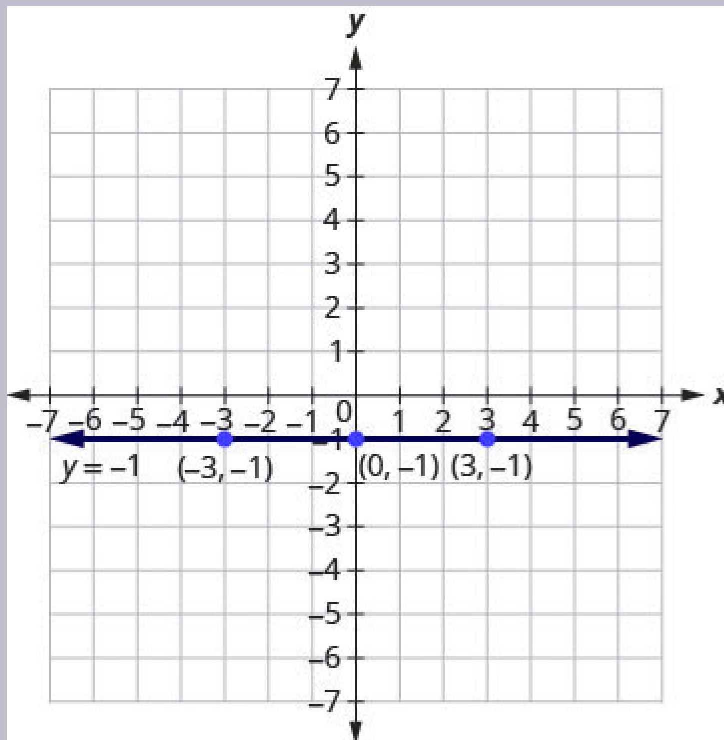


Figure 3.9.44

Try It

39) Graph the equation $y = -4$.

Solution

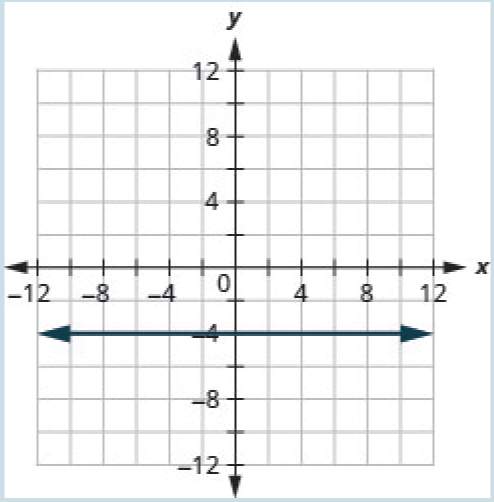


Figure 3.9.45

40) Graph the equation $y = 3$.

Solution

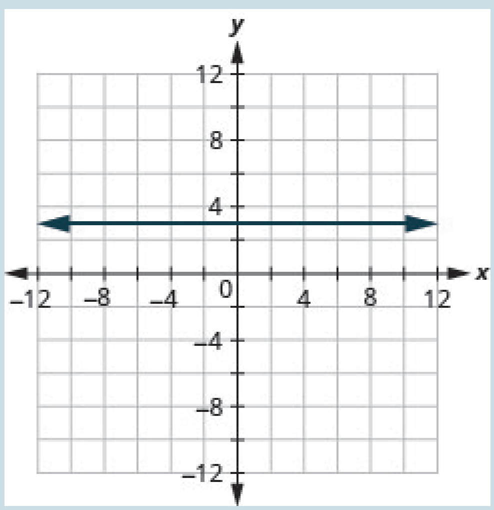


Figure 3.9.46

The equations for vertical and horizontal lines look very similar to equations like $y = 4x$. What is the difference between the equations $y = 4x$ and $y = 4$?

The equation $y = 4x$ has both x and y. The value of y depends on the value of x. The y-coordinate changes according to the value of x. The equation $y = 4$ has only one variable. The value of y is constant. The y-coordinate is always 4. It does not depend on the value of x. See the table below.

$y = 4x$				$y = 4$		
x	y	(x, y)		x	y	(x, y)
0	0	$(0, 0)$		0	4	$(0, 4)$
1	4	$(1, 4)$		1	4	$(1, 4)$
2	8	$(2, 8)$		2	4	$(2, 4)$

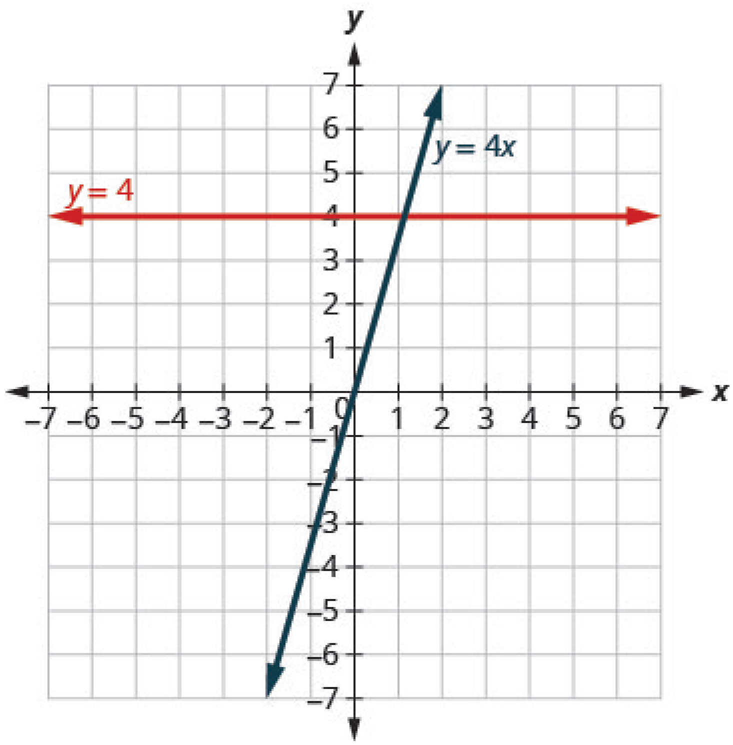


Figure 3.9.47

Notice, in Figure 3.9.47, the equation $y = 4x$ gives a slanted line, while $y = 4$ gives a horizontal line.

Example 18

Graph $y = -3x$ and $y = -3$ in the same rectangular coordinate system.

Solution

Notice that the first equation has the variable x, while the second does not. See the below table. The two graphs are shown in Figure 3.9.48.

$y=-3x$			$y=-3$		
x	y	(x,y)	x	y	(x,y)
0	0	$(0,0)$	0	-3	$(0,-3)$
1	-3	$(1,-3)$	1	-3	$(1,-3)$
2	-6	$(2,-6)$	2	-3	$(2,-3)$

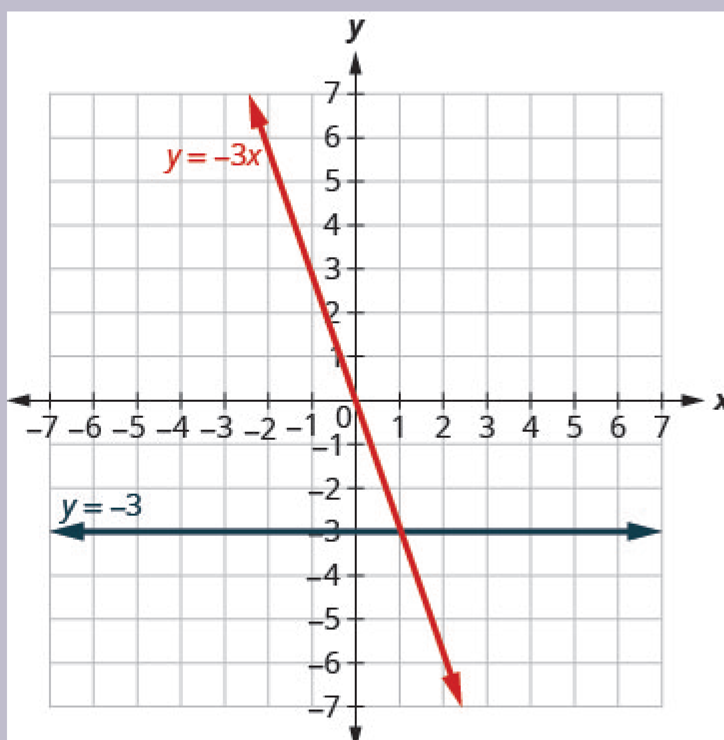


Figure 3.9.48

Try It

41) Graph $y = -4x$ and $y = -4$ in the same rectangular coordinate system.

Solution

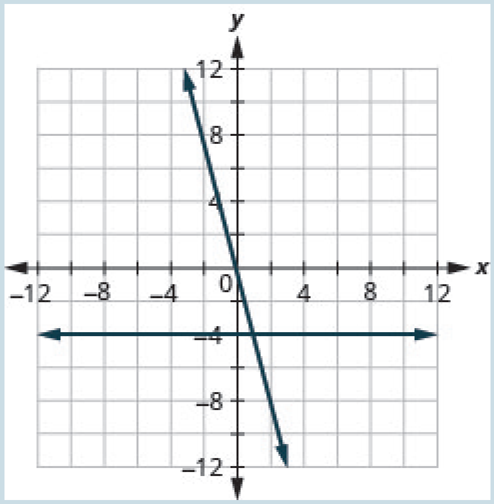


Figure 3.9.49

42) Graph $y = 3$ and $y = 3x$ in the same rectangular coordinate system.

Solution

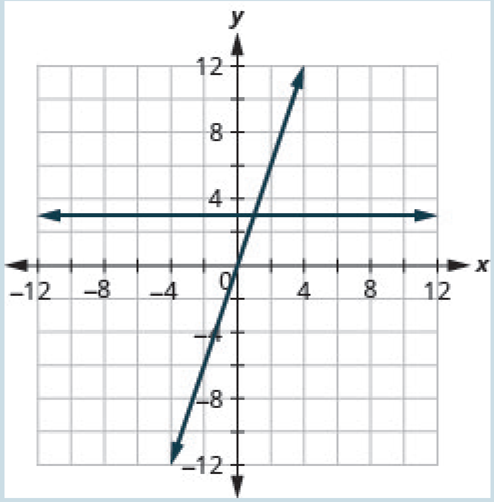


Figure 3.9.50

Identify the x-Intercept and y-Intercept from an Equation and its Graph

Every linear equation can be represented by a unique line that shows all the solutions of the equation. We have seen that when graphing a line by plotting points, you can use any three solutions to graph. This means that two people graphing the line might use different sets of three points.

At first glance, their two lines might not appear to be the same, since they would have different points labeled. But if all the work was done correctly, the lines should be the same. One way to recognize that they are indeed the same line is to look at where the line crosses the x-axis and the y-axis. These points are called the **intercepts of the line**.

Intercepts of a Line

The points where a line crosses the x-axis and the y-axis are called the intercepts of a line.

Let's look at the graphs of the lines in Figure 3.9.51.

Examples of graphs crossing the x-negative axis.

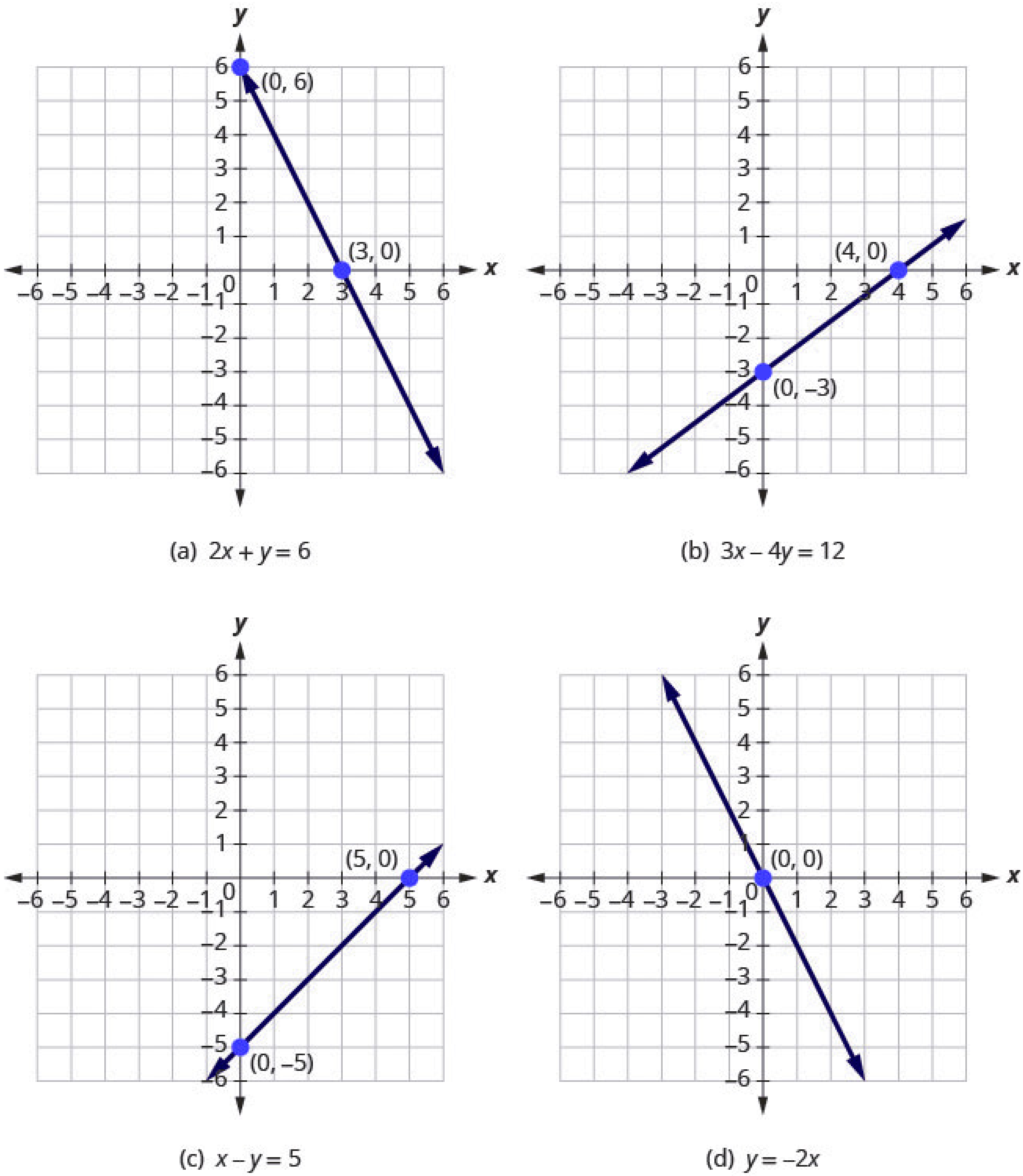


Figure 3.9.51

First, notice where each of these lines crosses the x negative axis. See the table below.

Figure	The line crosses the x-axis at:	Ordered pair of this point
Figure (a)	3	$(3, 0)$
Figure (b)	4	$(4, 0)$
Figure (c)	5	$(5, 0)$
Figure (d)	0	$(0, 0)$

Do you see a pattern?

For each row, the y-coordinate of the point where the line crosses the x-axis is zero. The point where the line crosses the x-axis has the form $(a, 0)$ and is called the **[latex]x[/latex]-intercept** of a line. The x-intercept occurs when y is zero.

Now, let's look at the points where these lines cross the y-axis. See the table below.

Figure	**The line crosses the y-axis at:**	**Ordered pair for this point**
Figure (a)	6	$(0, 6)$
Figure (b)	-3	$(0, -3)$
Figure (c)	-5	$(0, 5)$
Figure (d)	0	$(0, 0)$

What is the pattern here?

In each row, the x -coordinate of the point where the line crosses the y -axis is zero. The point where the line crosses the y -axis has the form $(0, b)$ and is called the y –intercept of the line. The y -intercept occurs when x is zero.

The x -intercept is the point $(a, 0)$ where the line crosses the x -axis.

The y -intercept is the point $(0, b)$ where the line crosses the y -axis.

	x	y
The x -intercept occurs when y is zero.	a	0
The y -intercept occurs when x is zero	0	b

Example 19

Find the x -intercept and y -intercept on each graph.

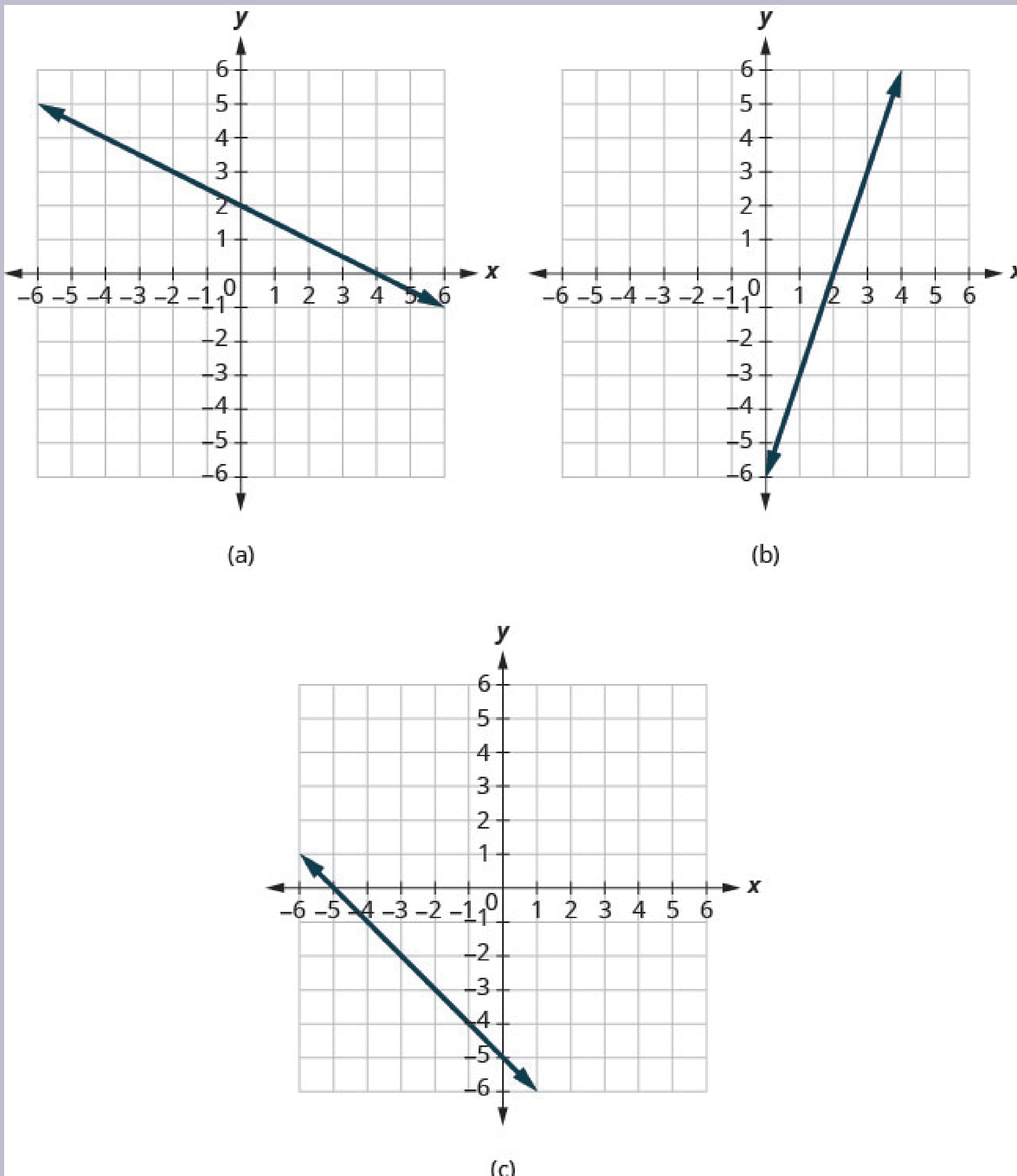


Figure 3.9.52

Solution

a. The graph crosses the x-axis at the point $(4, 0)$. The x-intercept is $(4, 0)$.

The graph crosses the y-axis at the point $(0, 2)$. The y-intercept is $(0, 2)$.

b. The graph crosses the x-axis at the point $(2, 0)$. The x-intercept is $(2, 0)$.

The graph crosses the y-axis at the point $(0, -6)$. The y-intercept is $(0, -6)$.

c. The graph crosses the x-axis at the point $(-5, 0)$. The x-intercept is $(-5, 0)$.

The graph crosses the y-axis at the point $(0, -5)$. The y-intercept is $(0, -5)$.

Try It

43) Find the x-intercept and y-intercept on the graph.

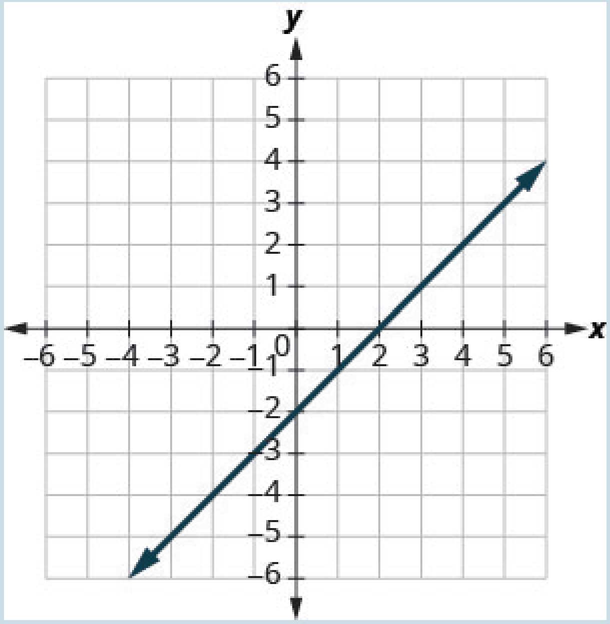


Figure 3.9.53

Solution

x-intercept: $(2, 0)$; y-intercept: $(0, -2)$

44) Find the x-intercept and y-intercept on the graph.

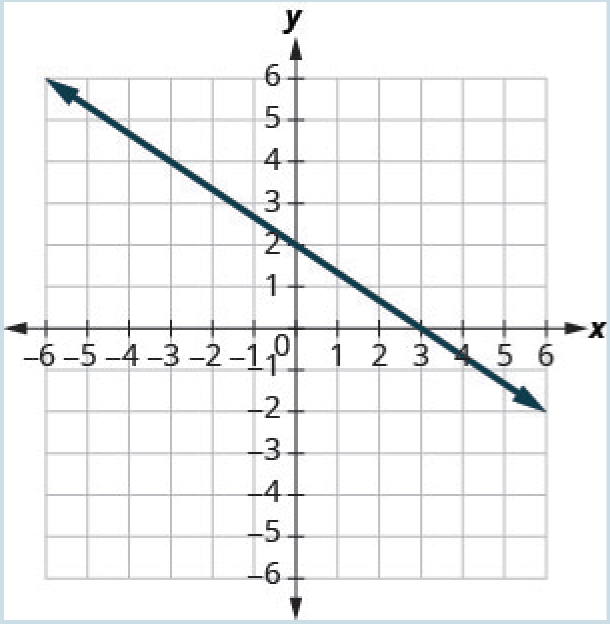


Figure 3.9.54

Solution

x-intercept: $(3, 0)$, y-intercept: $(0, 2)$

Intercepts from an Equation of a Line

Recognizing that the x-intercept occurs when y is zero and that the y-intercept occurs when x is zero, gives us a method to find the intercepts of a line from its equation. To find the x-intercept, let $y = 0$ and solve for x. To find the y-intercept, let $x = 0$ and solve for y.

Find the x-intercept and y-intercept from the *Equation of a Line*

Use the equation of the line. To find:

- the x-intercept of the line, let $y = 0$ and solve for x.
- the y-intercept of the line, let $x = 0$ and solve for y.

Example 20

Find the intercepts of $2x + y = 6$.

Solution

We will let $y = 0$ to find the x-intercept, and let $x = 0$ to find the y-intercept. We will fill in the table, which reminds us of what we need to find.

$2x+y=6$		
x	y	
	0	x-intercept
0		y-intercept

To find the x-intercept, let $y=0$.

Step 1: **Let $y=0$.**

$$\begin{aligned}2x+y&=6\\2x+0&=6\end{aligned}$$

Step 2: **Simplify.**

$$\begin{aligned}2x&=6\\x&=3\end{aligned}$$

Step 3: **The x-intercept is.**

$$(3,0)$$

Step 4: **To find the y-intercept, let $x=0$.**

Step 5: **Let $x=0$.**

$$\begin{aligned}2x+y&=6\\2\times0+y&=6\end{aligned}$$

Step 6: **Simplify.**

$$\begin{aligned}0+y&=6\\y&=6\end{aligned}$$

Step 7: **The y-intercept is.**

$$(0, 6)$$

The intercepts are the points $(3, 0)$ and $(0, 6)$ as shown in the below table.

$2x + y = 6$	
x	y
3	0
0	6

Try It

45) Find the intercepts of $3x + y = 12$.

Solution

x-intercept: $(4, 0)$, y-intercept: $(0, 12)$

46) Find the intercepts of $x + 4y = 8$.

Solution

x-intercept: $(8, 0)$, y-intercept: $(0, 2)$

Example 21

Find the intercepts of $4x-3y=12$.

Solution

***Step 1:* To find the x-intercept, let $y=0$.**

$$\begin{aligned} 4x - 3y &= 12 \\ 4x - 3 \times (0) &= 12 \end{aligned}$$

***Step 3:* Simplify.**

$$\begin{aligned} 4x - 0 &= 12 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$

***Step 4:* The x-intercept is:**

$$(3,0)$$

***Step 5:* To find the y-intercept, let $x=0$.**

***Step 6:* Let $x=0$.**

$$\begin{aligned} 4x - 3y &= 12 \\ 4 \times 0 - 3y &= 12 \end{aligned}$$

***Step 7:* Simplify.**

$$\begin{aligned} 0 - 3y &= 12 \\ -3y &= 12 \\ y &= -4 \end{aligned}$$

***Step 8:* The y-intercept is:**

$$(0,-4)$$

The intercepts are the points $(3,0)$ and $(0,-4)$ as shown in the following table.

$4x - 3y = 12$	
x	y
3	0
0	-4

Try It

47) Find the intercepts of $3x-4y = 12$.

Solution

x-intercept: $(4, 0)$, y-intercept: $(0, -3)$

48) Find the intercepts of $2x-4y = 8$.

Solution

x-intercept: $(4, 0)$, y-intercept: $(0, -2)$

Graph a Line Using the Intercepts

To graph a linear equation by plotting points, you need to find three points whose coordinates are solutions to the equation. You can use the x-intercept and y-intercept as two of your three points. Find the intercepts, and then find a third point to ensure accuracy. Make sure the points line up—then draw the line. This method is often the quickest way to graph a line.

How to

Graph a linear equation using the intercepts.

The steps to graph a linear equation using the intercepts are summarized below.

1. Find the x-intercept and y-intercept of the line.
    - Let $y = 0$ and solve for x.
    - Let $x = 0$ and solve for y.
2. Find a third solution to the equation.
3. Plot the three points and check that they line up.
4. Draw the line.

Example 22

Graph $-x + 2y = 6$ using the intercepts.

Solution

***Step 1:* Find the x – and y-intercepts of the line.**

Let $y = 0$ and solve for x.

Let $x = 0$ and solve for y.

Find the x-intercept.

$$\begin{aligned} \text{Let } y &= 0 \\ -x + 2y &= 6 \\ -x + 2(0) &= 6 \\ -x &= 6 \\ x &= 6 \end{aligned}$$

The x-intercept is (-6,0).

Find the y-intercept.

$$\begin{aligned} \text{Let } x &= 0 \\ -x + 2y &= 6 \\ -0 + 2y &= 6 \\ 2y &= 6 \\ y &= 3 \end{aligned}$$

The y-intercept is (0,3).

***Step 2:* Find another solution to the equation.**

We'll use $x = 2$.

$$\begin{aligned} \text{Let } x &= 2 \\ -x + 2y &= 6 \\ -2 + 2y &= 6 \\ 2y &= 8 \\ y &= 4 \end{aligned}$$

A third point is (2,4).

***Step 3:* Plot the three points.**

Check that the points line up.

x	y	(x, y)
-6	0	(-6,0)
0	3	(0,3)
2	4	(2,4)

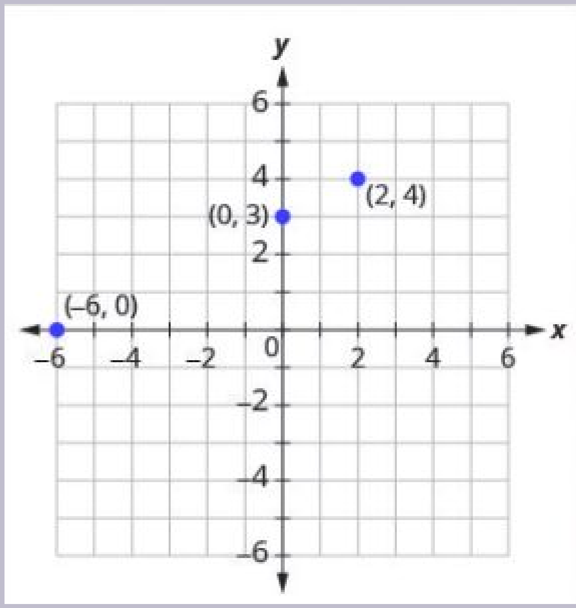


Figure 3.9.55

Step 4: **Draw the line.**

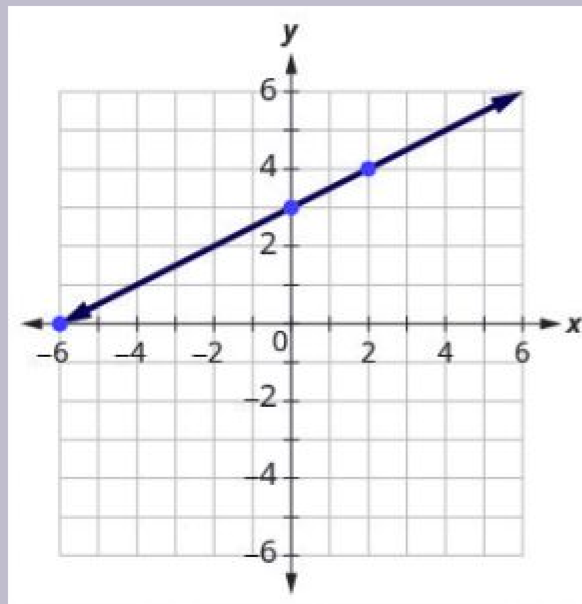


Figure 3.9.56

See the graph.

Try It

49) Graph $x - 2y = 4$ using the intercepts.

Solution

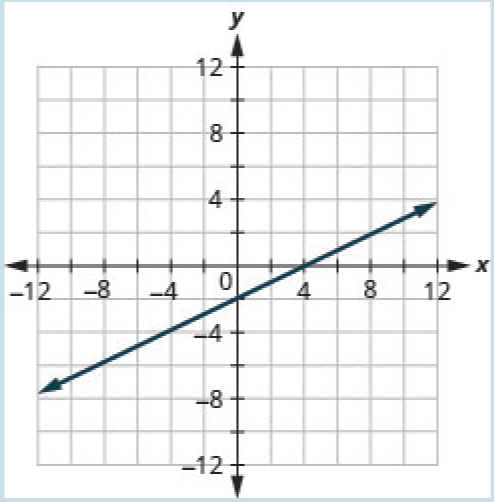


Figure 3.9.57

50) Graph $-x+3y=6$ using the intercepts.

Solution

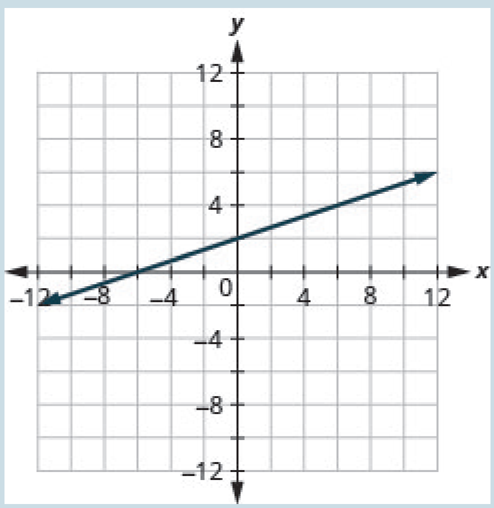


Figure 3.9.58

Example 23

Graph $4x-3y=12$ using the intercepts.

Solution

$$\begin{array}{lll} x\text{-intercept, let } y=0 & y\text{-intercept, let } x=0 & \text{third point, let } y=4 \\ 4x-3y=12 & 4x-3y=12 & 4x-3y=12 \\ 4x-3(0)=12 & 4(0)-3y=12 & 4x-3(4)=12 \\ 4x=12 & -3y=12 & 4x-12=12 \\ x=3 & y=-4 & 4x=24 \\ & & x=6 \end{array}$$

Find the intercepts and a third point. We list the points in the table below and show the graph below.

$4x-3y=12$		
x	y	(x,y)
3	0	$(3,0)$
0	-4	$(0,-4)$
6	4	$(6,4)$

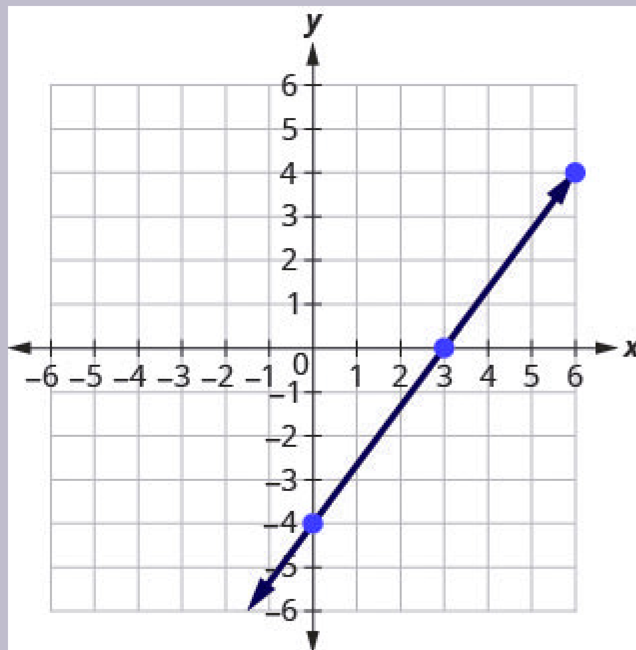


Figure 3.9.59

Try It

51) Graph $5x-2y=10$ using the intercepts.

Solution

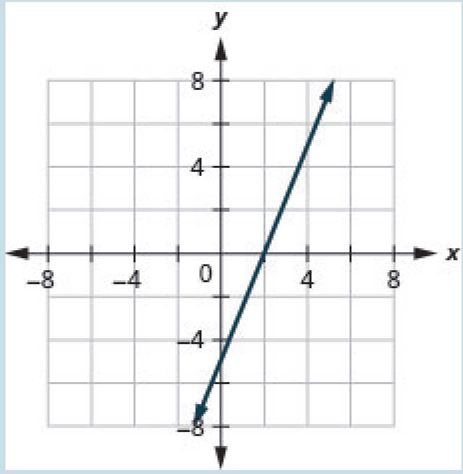


Figure 3.9.60

52) Graph $3x-4y=12$ using the intercepts.

Solution

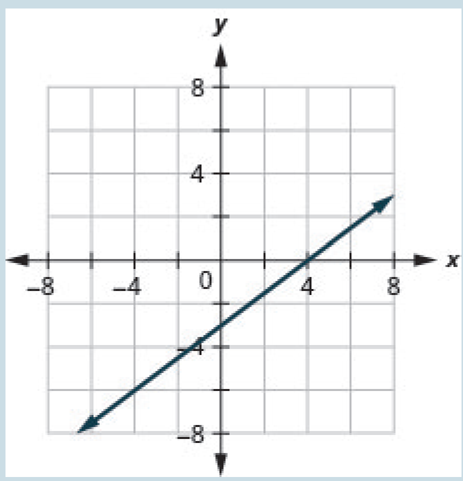


Figure 3.9.61

Example 24

Graph $y = 5x$ using the intercepts.

Solution

Step 1: **Find the x – and y -intercepts of the line.**

x-intercept	y-intercept
Let $y = 0$	Let $x = 0$
$y = 5x$	$y = 5x$
$0 = 5x$	$y = 5 \cdot 0$
$0 = x$	$y = 0$
$(0,0)$	$(0,0)$

This line has only one intercept. It is the point $(0, 0)$.

Step 2: **To ensure accuracy we need to plot three points. Since the x -intercept and y -intercept are the same point, we need *two* more points to graph the line.**

Let $x = 1$	Let $x = -1$
$y = 5x$	$y = 5x$
$y = 5 \cdot (1)$	$y = 5(-1)$
$y = 5$	$y = -5$

See table below.

$y = 5x$		
x	y	(x, y)
0	0	$(0, 0)$
1	5	$(1, 5)$
-1	-5	$(-1, -5)$

Step 3: **Plot the three points, check that they line up, and draw the line.**

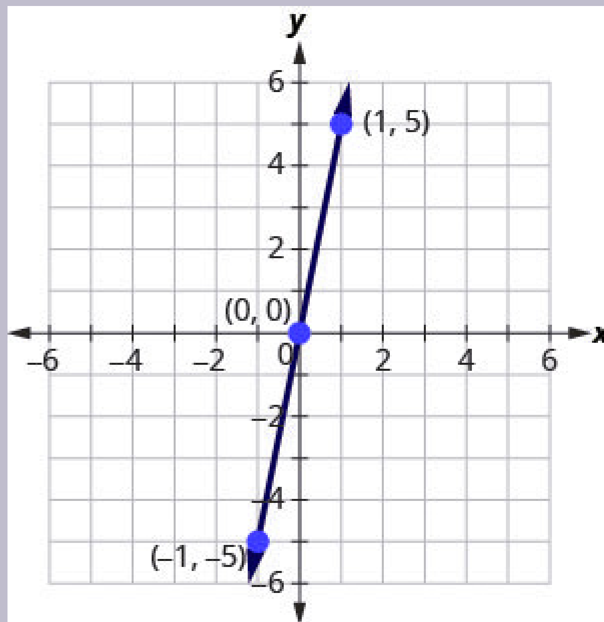


Figure 3.9.362

Try It

53) Graph $y = 4x$ using the intercepts.

Solution

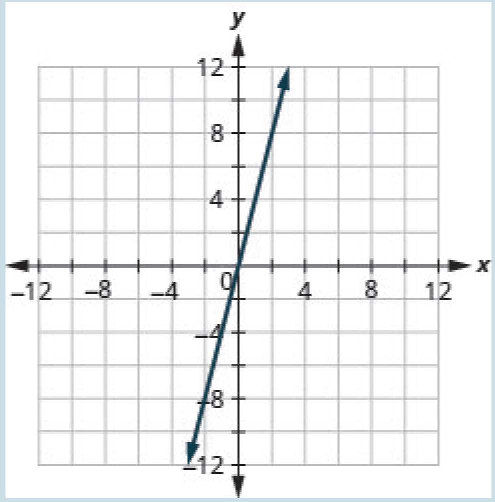


Figure 3.9.63

54) Graph $y = -x$ the intercepts.

Solution

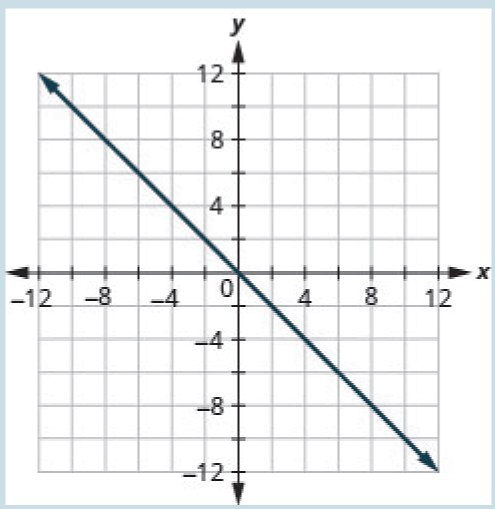


Figure 3.9.64

Key Concepts

- **Sign Patterns of the Quadrants**

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$

- **Points on the Axes**
    - On the x-axis, $y = 0$. Points with a y-coordinate equal to 0 are on the x-axis, and have coordinates $(a, 0)$.
    - On the y-axis, $x = 0$. Points with an x-coordinate equal to 0 are on the y-axis, and have coordinates $(0, b)$.
- **Solution of a Linear Equation**
    - An ordered pair (x, y) is a solution of the linear equation $Ax + By = C$, if the equation is a true statement when the x and y values of the ordered pair are substituted

into the equation.

- **Graph a Linear Equation by Plotting Points**
    1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
    2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!
    3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.
- **Find the x-intercept and y-Intercept from the Equation of a Line**
    - Use the equation of the line to find the x-intercept of the line, let $y = 0$ and solve for x.
    - Use the equation of the line to find the y-intercept of the line, let $x = 0$ and solve for y.
- **Graph a Linear Equation using the Intercepts**
    1. Find the x-intercept and y-intercept of the line.

       Let $y = 0$ and solve for x.

       Let $x = 0$ and solve for y.

2. Find a third solution to the equation.
3. Plot the three points and then check that they line up.
4. Draw the line.

- **Strategy for Choosing the Most Convenient Method to Graph a Line:**
  - Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.

    $x = a$ is a vertical line passing through the x-axis at a.

    $y = b$ is a horizontal line passing through the y-axis at b.

  - If y is isolated on one side of the equation, graph by plotting points.
  - Choose any three values for x and then solve for the corresponding y-values.
  - If the equation is of the form $Ax + By = C$, find the intercepts. Find the x-intercept and y-intercept, then a third point.

Glossary

linear equation

A linear equation is of the form $Ax + By = C$, where A and B are not both zero, is called a linear equation in two variables.

ordered pair

An ordered pair (x, y) gives the coordinates of a point in a rectangular coordinate system.

origin

The point $(0, 0)$ is called the origin. It is the point where the x-axis and y-axis intersect.

quadrant

The x-axis and the y-axis divide a plane into four regions, called quadrants.

rectangular coordinate system

A grid system is used in algebra to show a relationship between two variables; also called the xy-plane or the 'coordinate plane'.

***x*-coordinate**

The first number in an ordered pair (x, y).

y-coordinate

The second number in an ordered pair (x, y).

graph of a linear equation

The graph of a linear equation $Ax + By = C$ is a straight line. Every point on the line is a solution of the equation. Every solution of this equation is a point on this line.

horizontal line

A horizontal line is the graph of an equation of the form $y = b$. The line passes through the

y-axis at $(0, b)$.

vertical line

A vertical line is the graph of an equation of the form $x = a$. The line passes through the x-axis at $(a, 0)$.

intercepts of a line

The points where a line crosses the x-axis and the y-axis are called the intercepts of the line.

x-intercept

The point $(a, 0)$ where the line crosses the x-axis; the x–intercept occurs when y is zero.

y-intercept

The point $(0, b)$ where the line crosses the y-axis; the y-intercept occurs when x is zero.

Exercises: Plot Points in a Rectangular Coordinate System

Instructions: For questions 1-8, plot each point in a rectangular coordinate system and identify the quadrant in which the point is located.

1.

a. $(-4, 2)$
b. $(-1, -2)$
c. $(3, -5)$
d. $(-3, 5)$
e. $\left(\frac{5}{3}, 2\right)$

Solution

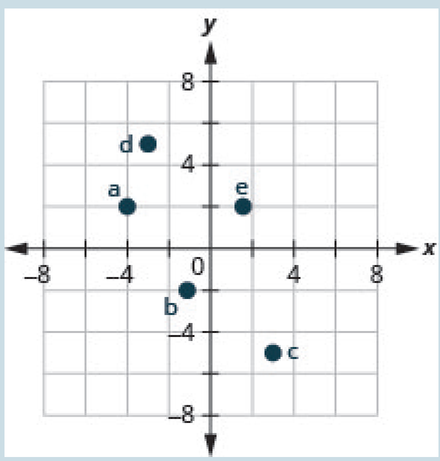


Figure P3.9.1

2.

a. $(-2, -3)$
b. $(3, -3)$
c. $(-4, 1)$

d. $(4, -1)$

e. $\left(\frac{3}{2}, 1\right)$

3.

a. $(3, -1)$

b. $(-3, 1)$

c. $(-2, 2)$

d. $(-4, -3)$

e. $\left(1, \frac{14}{5}\right)$

Solution

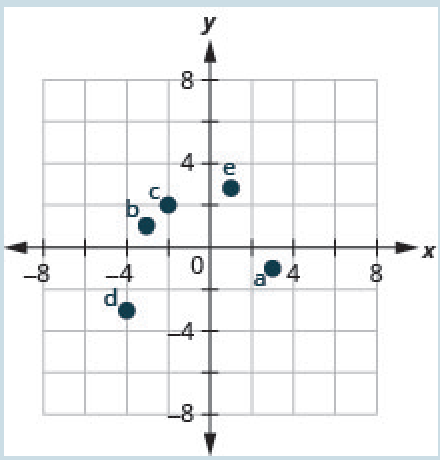


Figure P3.9.2

4.

a. $(-1, 1)$

b. $(-2, -1)$

c. $(2, 1)$

d. $(1, -4)$

e. $\left(3, \frac{7}{2}\right)$

5.

a. $(-2, 0)$
b. $(-3, 0)$
c. $(0, 0)$
d. $(0, 4)$
e. $(0, 2)$

Solution

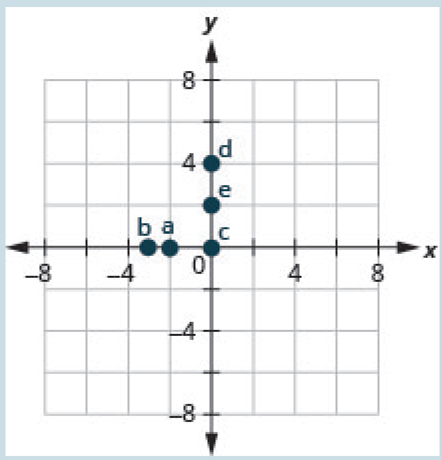


Figure 3P.9.3

6.

a. $(0, 1)$
b. $(0, -4)$
c. $(-1, 0)$
d. $(0, 0)$
e. $(5, 0)$

7.

- **a.** $(0, 0)$
- **b.** $(0, -3)$
- **c.** $(-4, 0)$
- **d.** $(1, 0)$
- **e.** $(0, -2)$

Solution

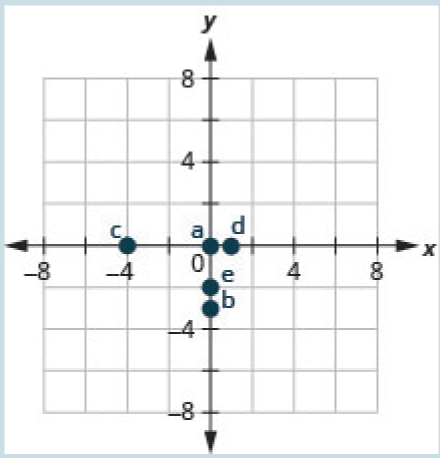


Figure 3P.9.4

8.

- **a.** $(-3, 0)$
- **b.** $(0, 5)$
- **c.** $(0, -2)$
- **d.** $(2, 0)$
- **e.** $(0, 0)$

Exercises: Name Ordered Pairs in a Rectangular Coordinate System

Instructions: For questions 9-12, name the ordered pair of each point shown in the rectangular coordinate system.

9.

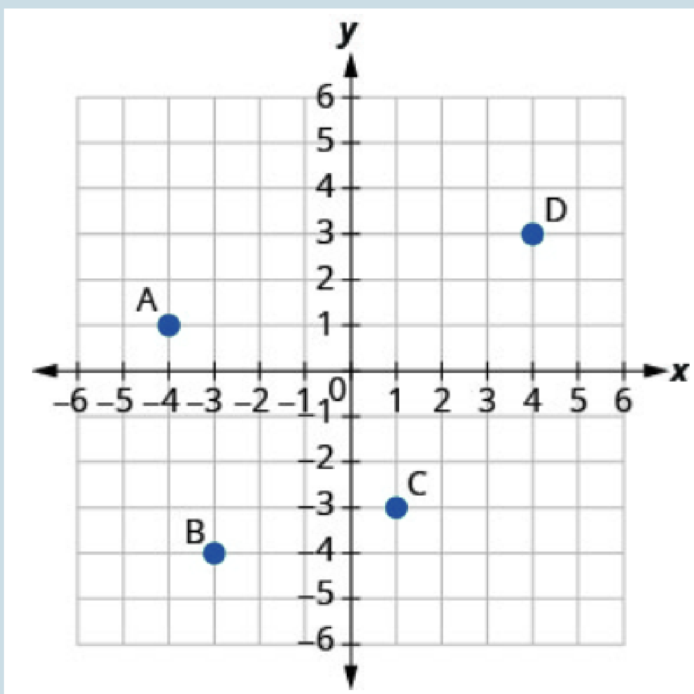


Figure 3P.9.5

Solution

A: $(-4, 1)$

B: $(-3, -4)$

C: $(1, -3)$

D: $(4, 3)$

10.

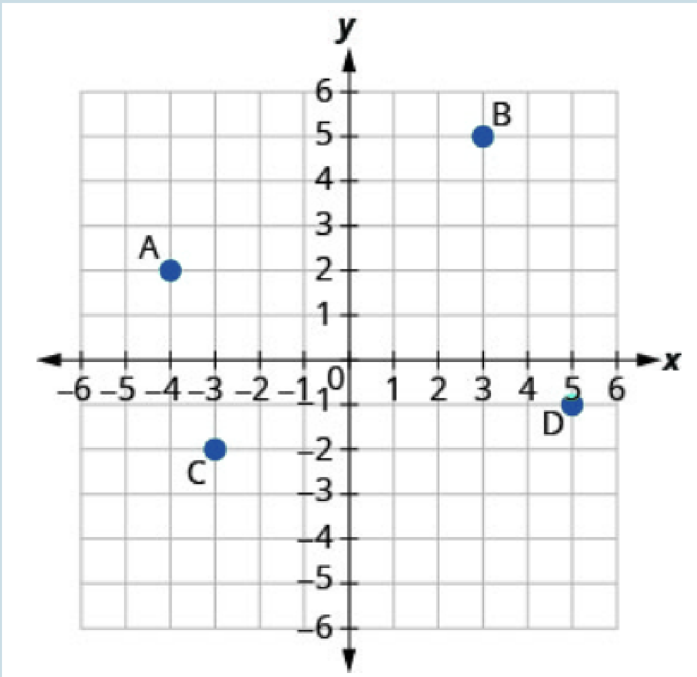


Figure 3P.9.6

11.

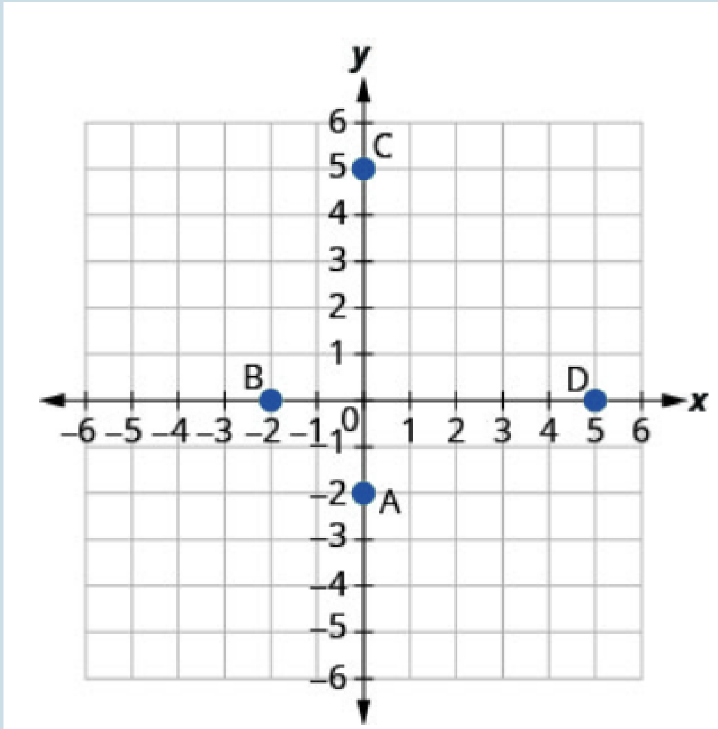


Figure 3P.9.7

Solution

A: $(0, -2)$

B: $(-2, 0)$

C: $(0, 5)$

D: $(5, 0)$

12.

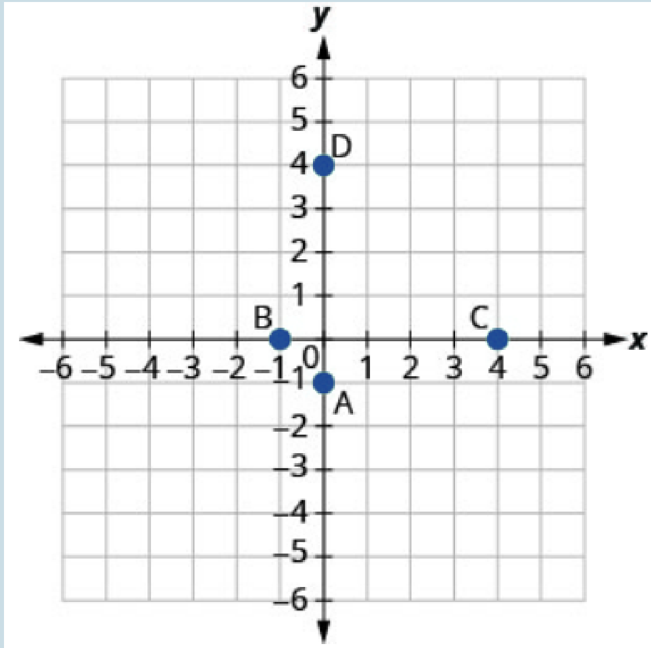


Figure 3P.9.8

Exercises: Verify Solutions to an Equation in Two Variables

Instructions: For questions 13-20, which ordered pairs are solutions to the given equations?

13. $2x + y = 6$

a. $(1, 4)$

b. $(3, 0)$

c. $(2, 3)$

Solution

a, b

14. $x + 3y = 9$

a. $(0, 3)$

b. $(6, 1)$

c. $(-3, -3)$

---

15. $4x - 2y = 8$

a. $(3, 2)$

b. $(1, 4)$

c. $(0, -4)$

Solution

a, c

---

16. $3x - 2y = 12$

a. $(4, 0)$

b. $(2, -3)$

c. $(1, 6)$

---

17. $y = 4x + 3$

a. $(4, 3)$

b. $(-1, -1)$

c. $\left(\frac{1}{2}, 5\right)$

Solution

b, c

---

18. $y = 2x - 5$

- **a.** $(0, -5)$
- **b.** $(2, 1)$
- **c.** $\left(\frac{1}{2}, -4\right)$

---

19. $y = \frac{1}{2}x - 1$

- **a.** $(2, 0)$
- **b.** $(-6, -4)$
- **c.** $(-4, -1)$

Solution

a, b

---

20. $y = \frac{1}{3}x + 1$

- **a.** $(-3, 0)$
- **b.** $(9, 4)$
- **c.** $(-6, -1)$

Exercises: Complete a Table of Solutions to a Linear Equation

Instructions: For questions 21-32, complete the table to find solutions to each linear equation.

21. $y = 2x - 4$

x	y	(x, y)
0		
2		
-1		

Solution

x	y	(x, y)
0	-4	$(0, -4)$
2	0	$(2, 0)$
-1	-6	$(-1, -6)$

---

22. $y = 3x - 1$

x	y	(x, y)
0		
2		
-1		

---

23. $y = -x + 5$

x	y	(x, y)
0		
3		
-2		

Solution

x	y	(x,y)
0	5	$(0,5)$
3	2	$(3,2)$
-2	7	$(-2,7)$

---

24. $y=-x+2$

x	y	(x,y)
0		
3		
-2		

---

25. $y = \frac{1}{3}x + 1$

x	y	(x, y)
0		
3		
6		

Solution

x	y	(x, y)
0	1	$(0, 1)$
3	2	$(3, 2)$
6	3	$(6, 3)$

---

26. $y = \frac{1}{2}x + 4$

x	y	(x, y)
0		
2		
4		

27. $y = -\frac{3}{2}x - 2$

x	y	(x, y)
0		
2		
-2		

Solution

x	y	(x, y)
0	-2	$(0, -2)$
2	-5	$(2, -5)$
-2	1	$(-2, 1)$

28. $y = -\frac{2}{3}x - 1$

x	y	(x, y)
0		
3		
-3		

29. $x + 3y = 6$

x	y	(x, y)
0		
3		
	0	

Solution

x	y	(x, y)
0	2	$(0, 2)$
3	1	$(3, 1)$
6	0	$(6, 0)$

---

30. $x + 2y = 8$

x	y	(x, y)
0		
4		
	0	

---

31. $2x - 5y = 10$

x	y	(x, y)
0		
10		
	0	

Solution

x	y	(x, y)
0	-2	$(0, -2)$
10	2	$(10, 2)$
5	0	$(5, 0)$

---

32. $3x - 4y = 12$

x	y	(x, y)
0		
8		
	0	

Exercises: Find Solutions to a Linear Equation

Instructions: For questions 33-48, find three solutions to each linear equation.

33. $y = 5x - 8$

Solution

Answers will vary.

---

34. $y = 3x - 9$

---

35. $y = -4x + 5$

Solution

Answers will vary.

---

36. $y = -2x + 7$

---

37. $x + y = 8$

Solution

Answers will vary.

---

38. $x + y = 6$

---

39. $x + y = -2$

Solution

Answers will vary.

---

40. $x + y = -1$

---

41. $3x + y = 5$

Solution

Answers will vary.

---

42. $2x + y = 3$

---

43. $4x - y = 8$

Solution

Answers will vary.

---

44. $5x - y = 10$

---

45. $2x + 4y = 8$

Solution

Answers will vary.

---

46. $3x + 2y = 6$

---

47. $5x - 2y = 10$

Solution

Answers will vary.

---

48. $4x - 3y = 12$

Exercises: Recognize the Relationship Between the Solutions of an Equation and its Graph

Instructions: For questions 49-52, for each ordered pair, decide:

a. Is the ordered pair a solution to the equation?
b. Is the point on the line?

49. $y = x + 2$

a. $(0, 2)$
b. $(1, 2)$
c. $(-1, 1)$
d. $(-3, -1)$

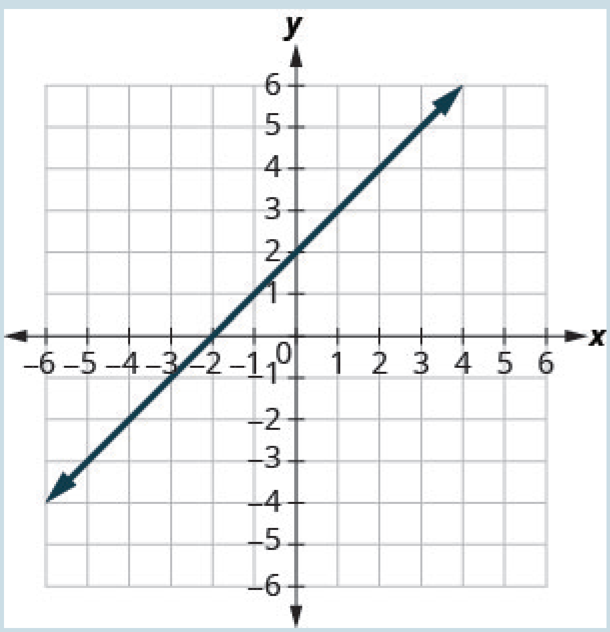


Figure 3P.9.9

Solution

a. yes; yes

b. no; no
c. yes; yes
d. yes; yes

---

50. $y = x - 4$

a. $(0, -4)$
b. $(3, -1)$
c. $(2, 2)$
d. $(1, -5)$

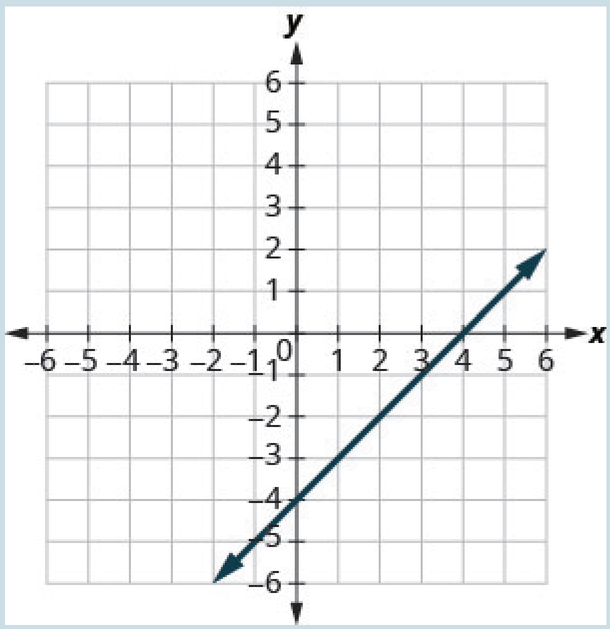


Figure 3P.9.10

---

51. $y = \frac{1}{2}x - 3$

a. $(0, -3)$
b. $(2, -2)$
c. $(-2, -4)$
d. $(4, 1)$

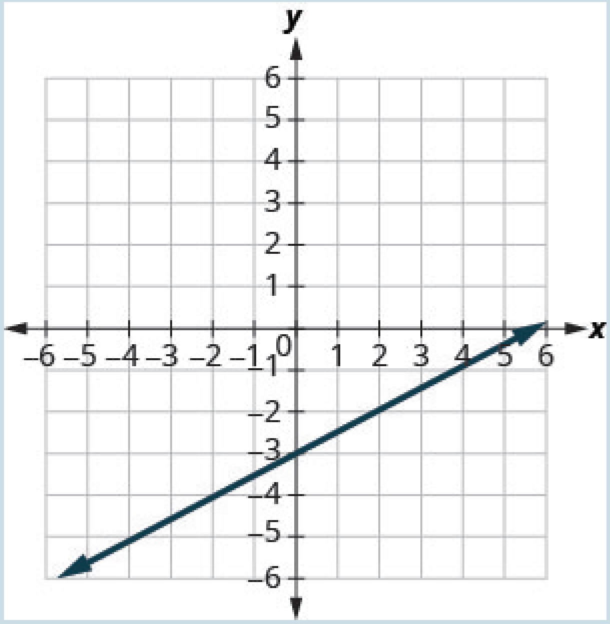


Figure 3P.9.11

Solution

a. yes; yes

b. yes; yes

c. yes; yes

d. no; no

---

52. $y = \frac{1}{3}x + 2$

a. $(0, 2)$

b. $(3, 3)$

c. $(-3, 2)$

d. $(-6, 0)$

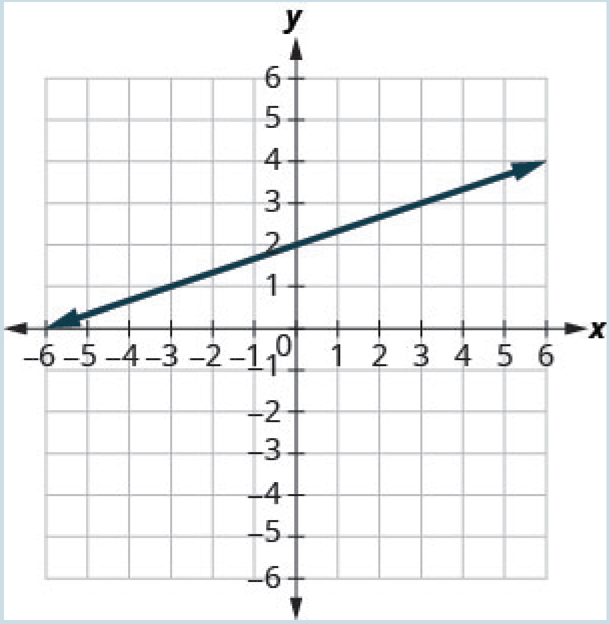


Figure 3P.9.12

Exercises: Graph a Linear Equation by Plotting Points

Instructions: For questions 53-96, graph by plotting points.

53. $y = 3x - 1$

Solution

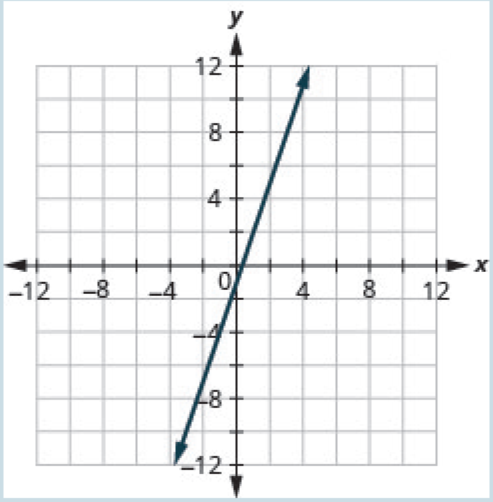


Figure 3P.9.13

---

54. $y = 2x + 3$

---

55. $y = -\frac{9}{4}x + 3$

Solution

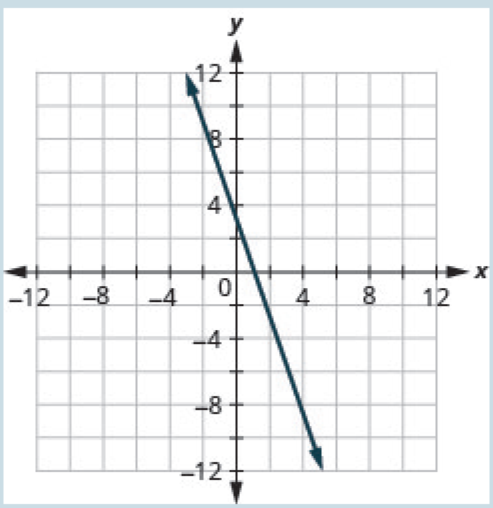


Figure 3P.9.14

---

56. $y = -3x + 1$

---

57. $y = x + 2$

Solution

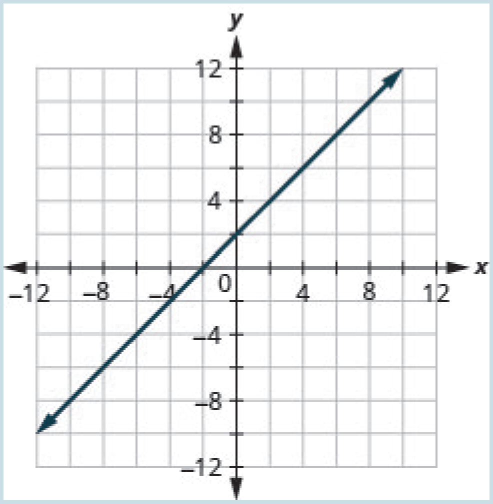


Figure 3P.9.15

---

58. $y = x - 3$

---

59. $y = -x - 3$

Solution

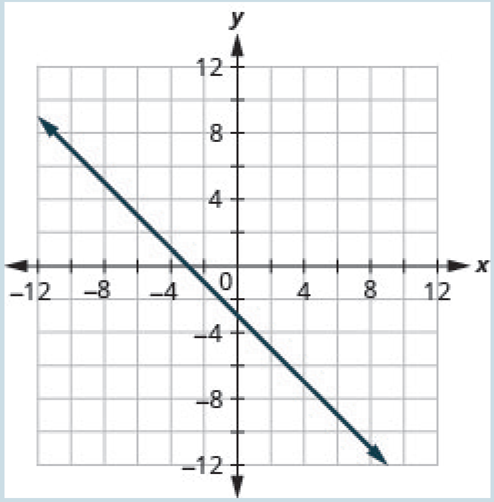


Figure 3P.9.16

---

60. $y = -x - 2$

---

61. $y = 2x$

Solution

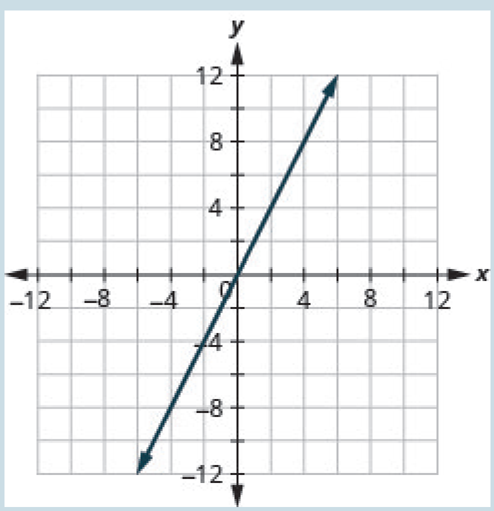


Figure 3P.9.17

---

62. $y = 3x$

---

63. $y = -4x$

Solution

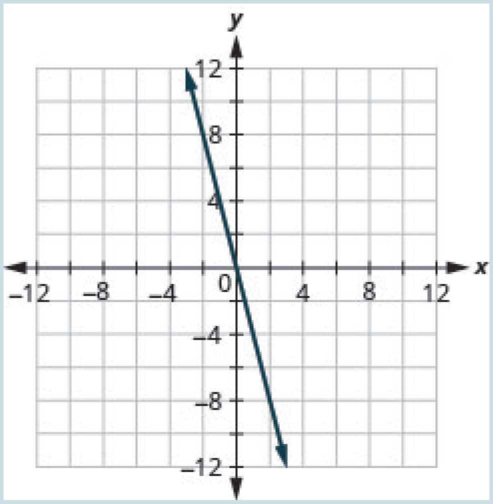


Figure 3P.9.18

---

64. $y = -2x$

---

65. $y = \frac{1}{2}x + 2$

Solution

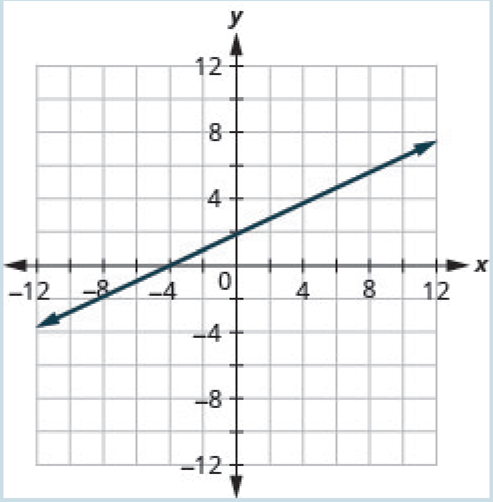


Figure 3P.9.19

---

66. $y = \frac{1}{3}x - 1$

---

67. $y = \frac{4}{3}x - 5$

Solution

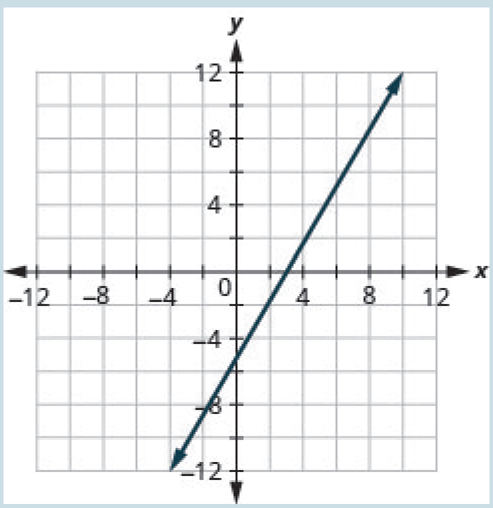


Figure 3P.9.20

---

68. $y = \frac{3}{2}x - 3$

---

69. $y = -\frac{2}{5}x + 1$

Solution

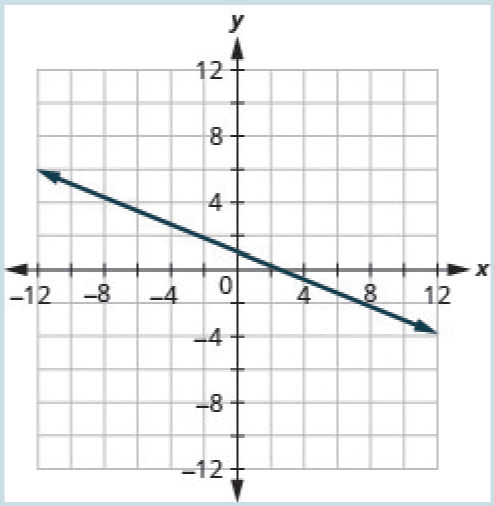


Figure 3P.9.21

---

70. $y = -\frac{4}{5}x - 1$

---

71. $y = -\frac{3}{2}x + 2$

Solution

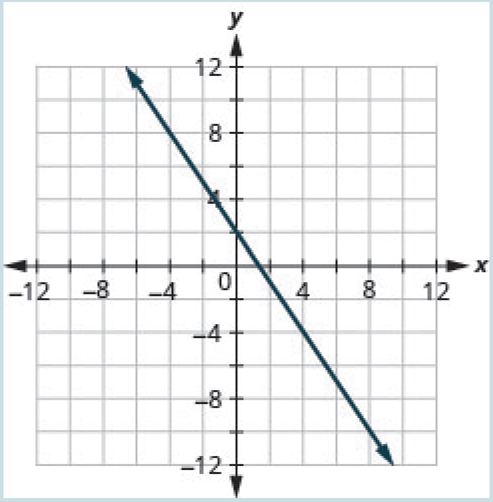


Figure 3P.9.22

---

72. $y = -\frac{5}{3}x + 4$

---

73. $x + y = 6$

Solution

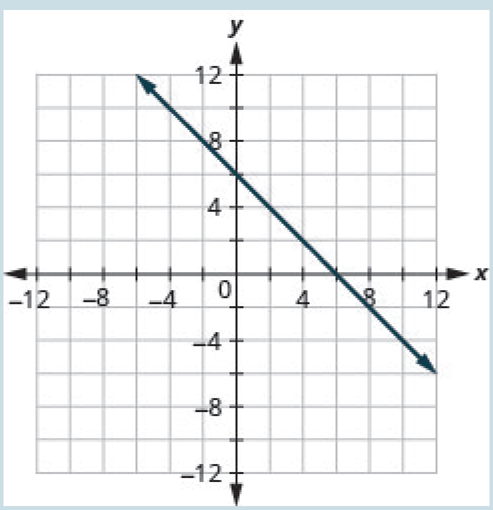


Figure 3P.9.23

---

74. $x + y = 4$

---

75. $x + y = -3$

Solution

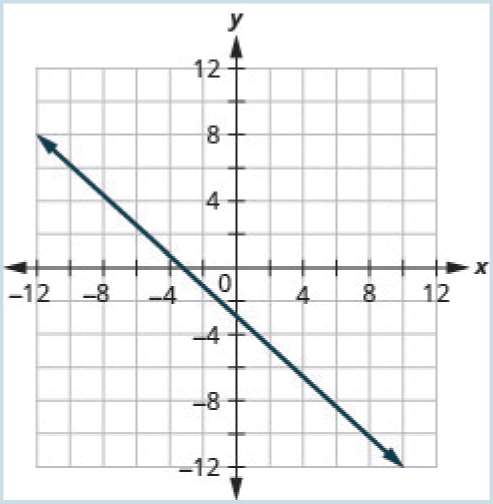


Figure 3P.9.24

---

76. $x + y = -2$

---

77. $x - y = 2$

Solution

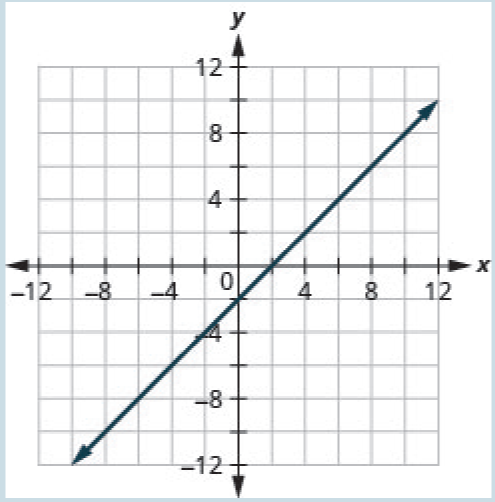


Figure 3P.9.25

---

78. $x - y = 1$

---

79. $x - y = -1$

Solution

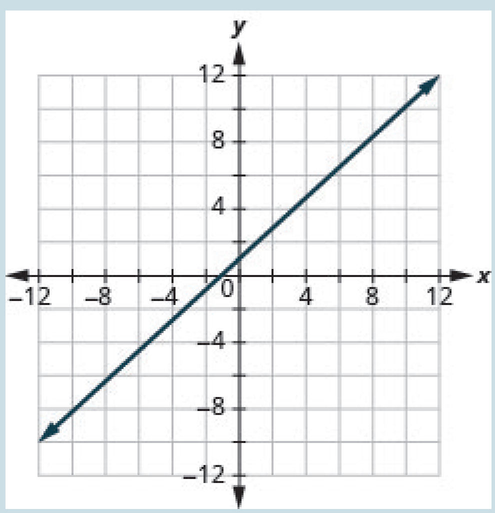


Figure 3P.9.26

---

80. $x - y = -3$

---

81. $3x + y = 7$

Solution

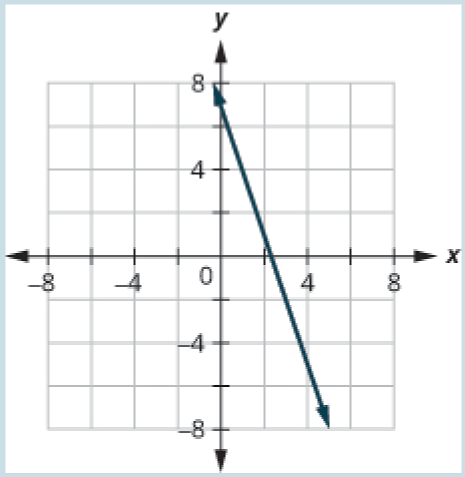


Figure 3P.9.27

---

82. $5x + y = 6$

---

83. $2x + y = -3$

Solution

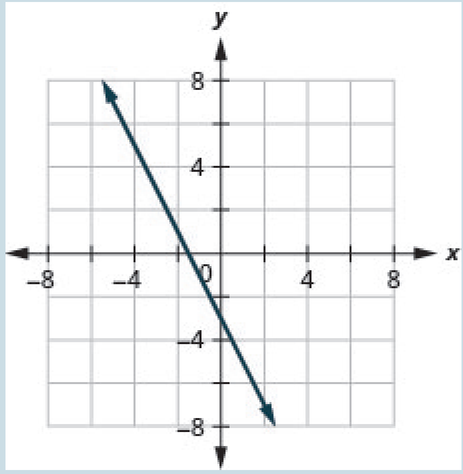


Figure 3P.9.28

84. $4x + y = -5$

85. $\frac{1}{3}x + y = 2$

Solution

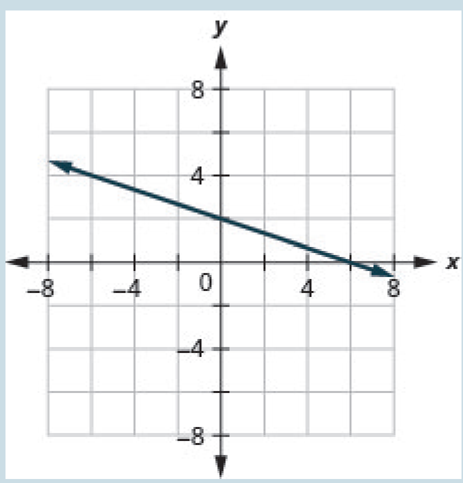


Figure 3P.9.29

86. $\frac{1}{2}x + y = 3$

---

87. $\frac{2}{5}x - y = 4$

Solution

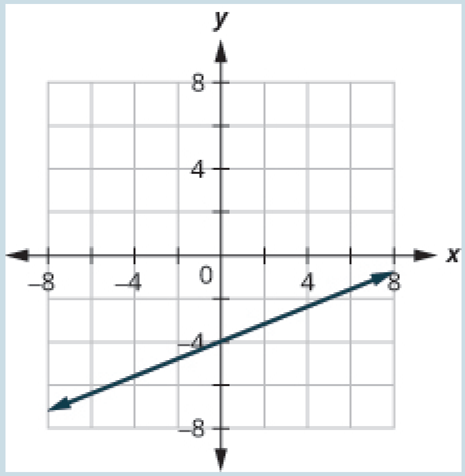


Figure 3P.9.30

---

88. $\frac{3}{4}x - y = 6$

---

89. $2x + 3y = 12$

Solution

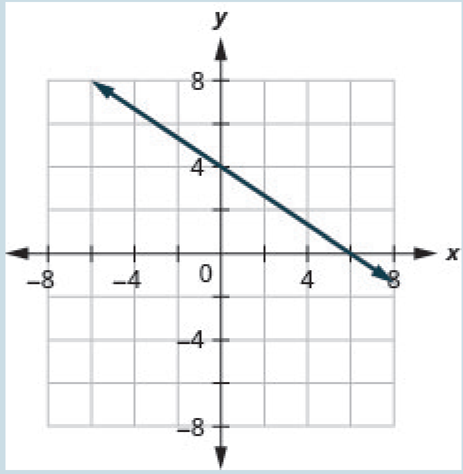


Figure 3P.9.31

---

90. $4x + 2y = 12$

---

91. $3x - 4y = 12$

Solution

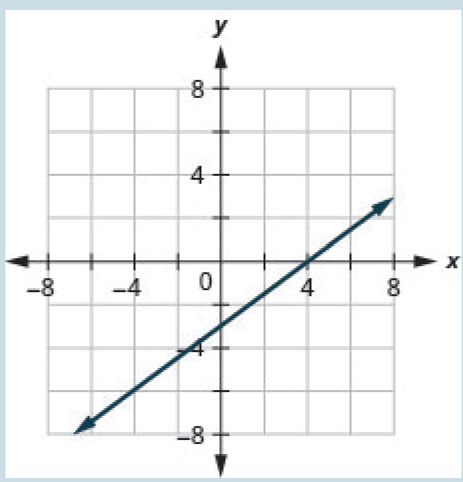


Figure 3P.9.32

---

92. $2x - 5y = 10$

---

93. $x - 6y = 3$

Solution

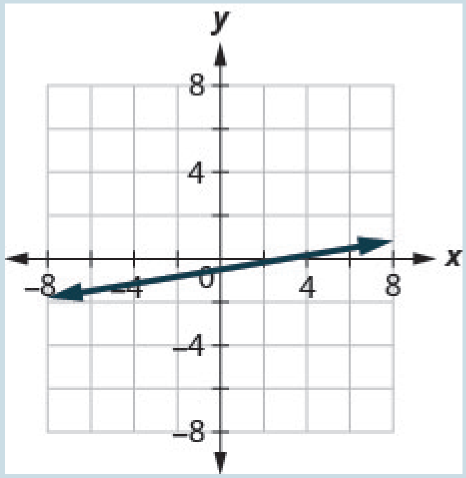


Figure 3P.9.33

---

94. $x - 4y = 2$

---

95. $5x + 2y = 4$

Solution

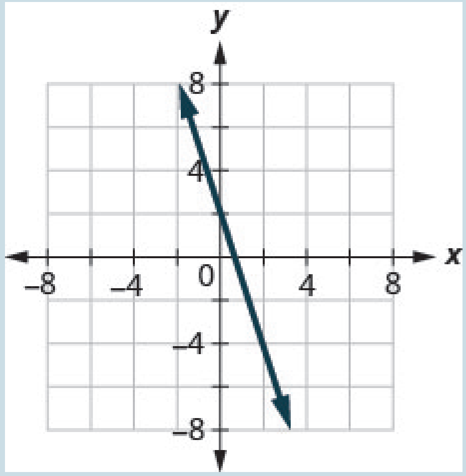


Figure 3P.9.34

---

96. $3x + 5y = 5$

Exercises: Graph Vertical and Horizontal Lines

Instructions: For questions 97-108, graph each equation.

97. $x = 4$

Solution

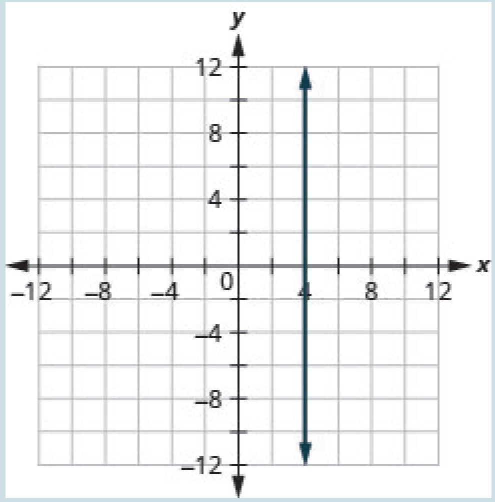


Figure 3P.9.35

---

98. $x = 3$

---

99. $x = -2$

Solution

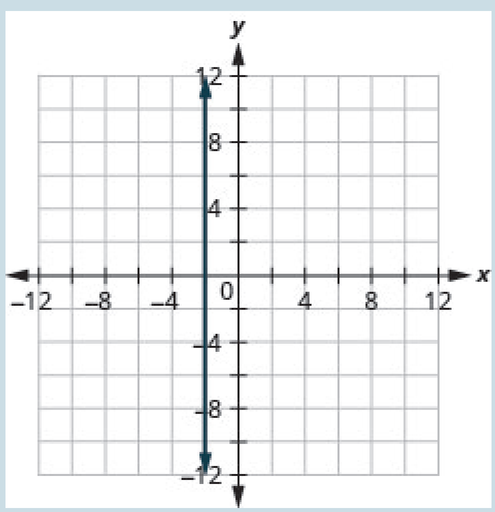


Figure 3P.9.36

---

100. $x = -5$

---

101. $y = 3$

Solution

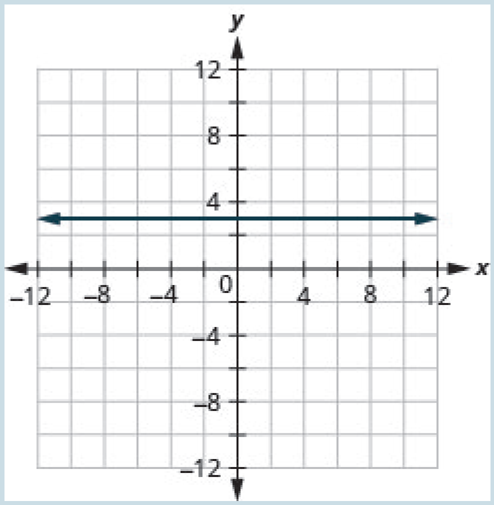


Figure 3P.9.37

---

102. $y = 1$

---

103. $y = -5$

Solution

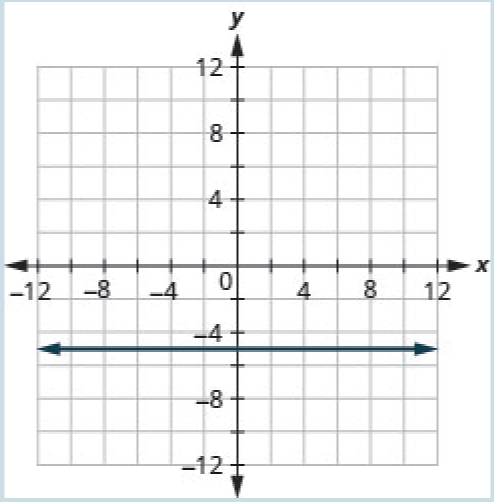


Figure 3P.9.38

---

104. $y = -2$

---

105. $x = \frac{7}{3}$

Solution

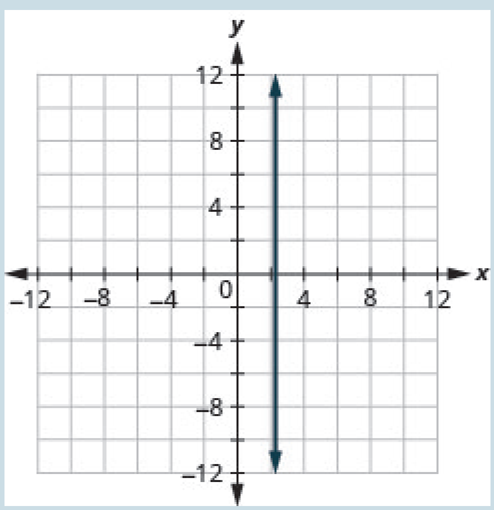


Figure 3P.9.39

---

106. $x = \frac{5}{4}$

---

107. $y = -\frac{15}{4}$

Solution

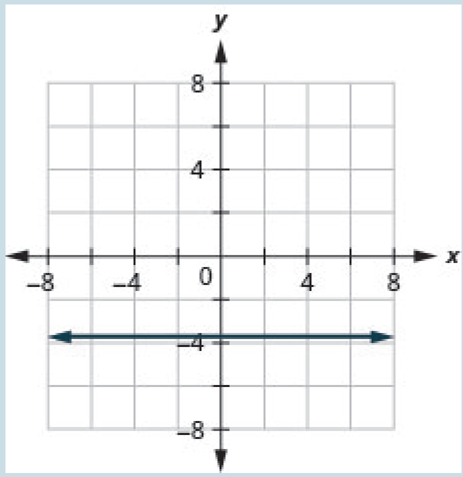


Figure 3P.9.40

---

108. $y = -\frac{5}{3}$

Exercises: Graph a Pair of Equations in the Same Rectangular Coordinate System

Instructions: For questions 109-112, graph each pair of equations in the same rectangular coordinate system.

109. $y = 2x$ **and** $y = 2$

Solution

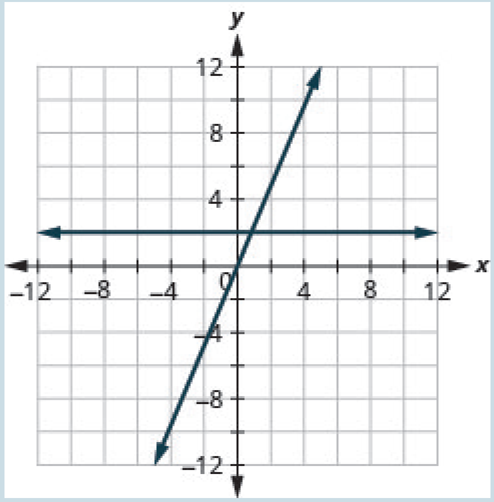


Figure 3P.9.41

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110. $y = 5x$ **and** $y = 5$

---

111. $y = -\frac{1}{2}x$ **and** $y = -\frac{1}{2}$

Solution

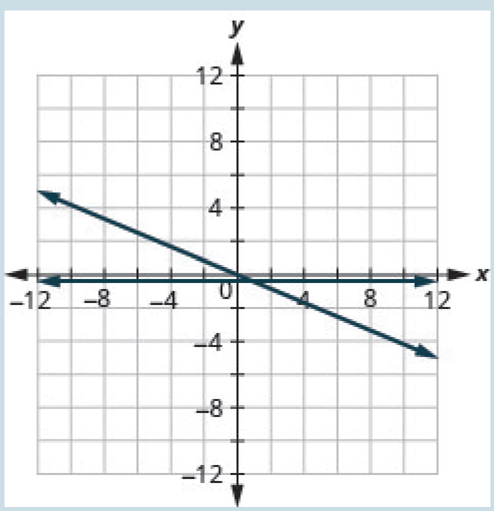

Figure 3P.9.42

---

112. $y = -\frac{1}{3}x$ **and** $y = -\frac{1}{3}$

Exercises: Mixed Practice

Instructions: For questions 113-128, graph each equation.

113. $y = 4x$

Solution

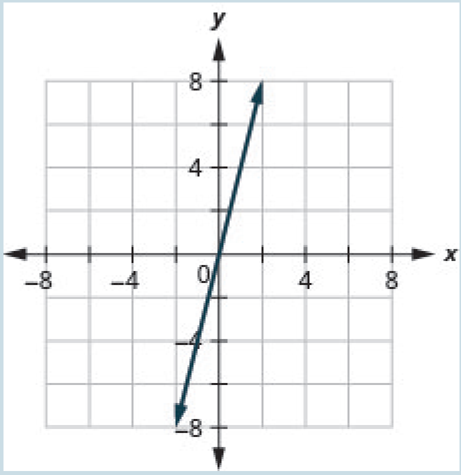

Figure 3P.9.43

---

114. $y = 2x$

---

115. $y=-\frac{1}{2}x+3$

Solution

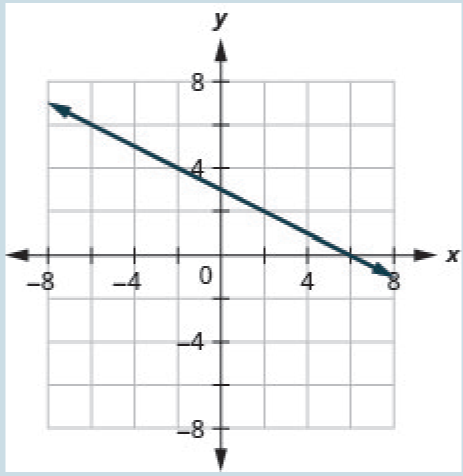

Figure 3P.9.44

---

116. $y=\frac{1}{4}x-2$

---

117. $y=-x$

Solution

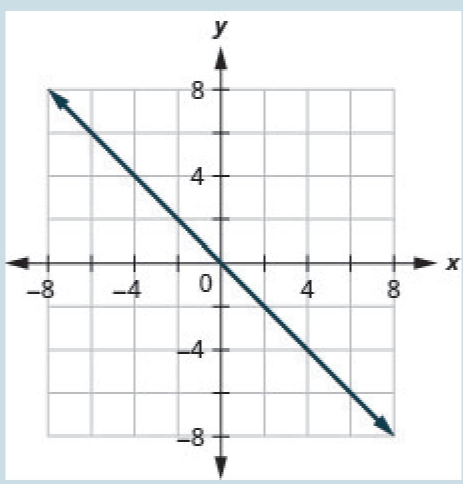

Figure 3P.9.45

---

118. $y = x$

---

119. $x - y = 3$

Solution

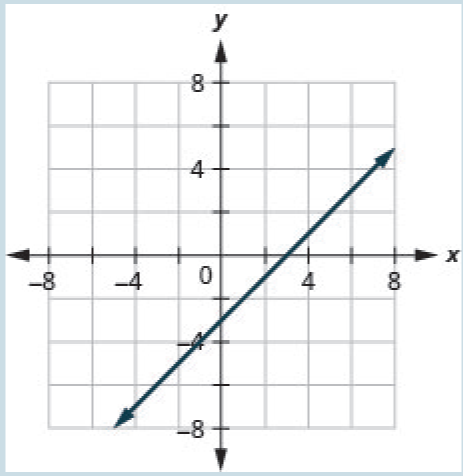

Figure 3P.9.46

---

120. $x + y = -5$

---

121. $4x + y = 2$

Solution

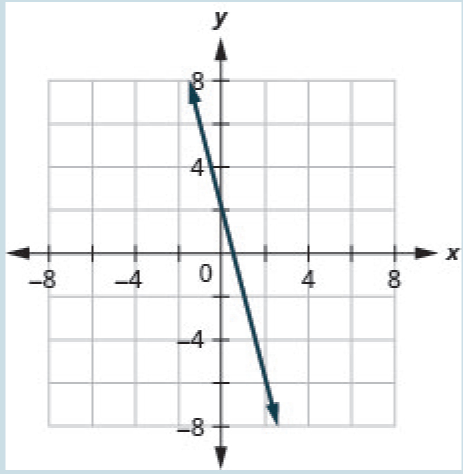

Figure 3P.9.47

---

122. $2x + y = 6$

---

123. $y = -1$

Solution

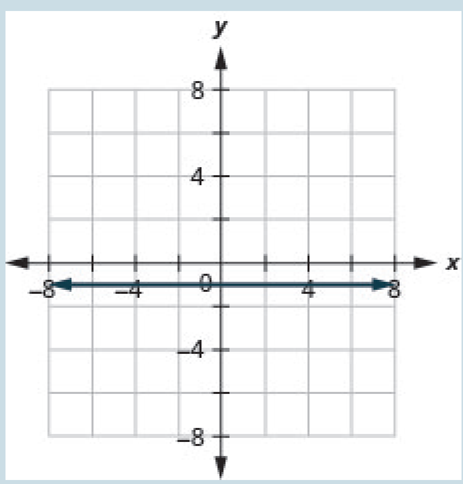

Figure 3P.9.48

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124. $y = 5$

125. $2x + 6y = 12$

Solution

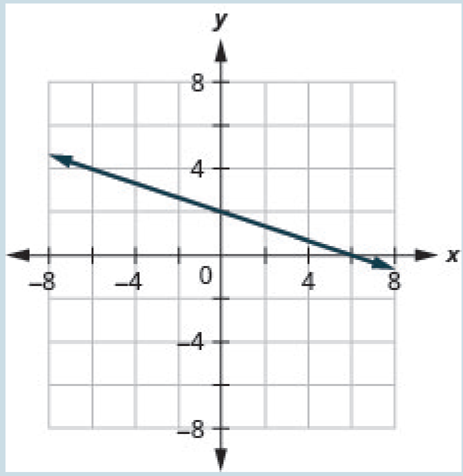

Figure 3P.9.49

126. $5x + 2y = 10$

127. $x = 3$

Solution

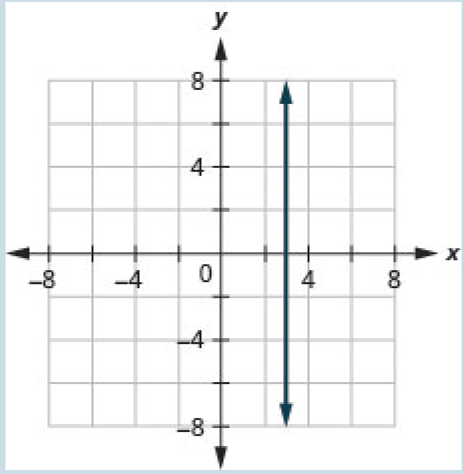

Figure 3P.9.50

---

128. $x = -4$

Exercises: Identify the Formula does not parse x and Formula does not parse y-Intercepts on a Graph

Instructions: For questions 129-140, find the x and y-intercepts on each graph.

129.

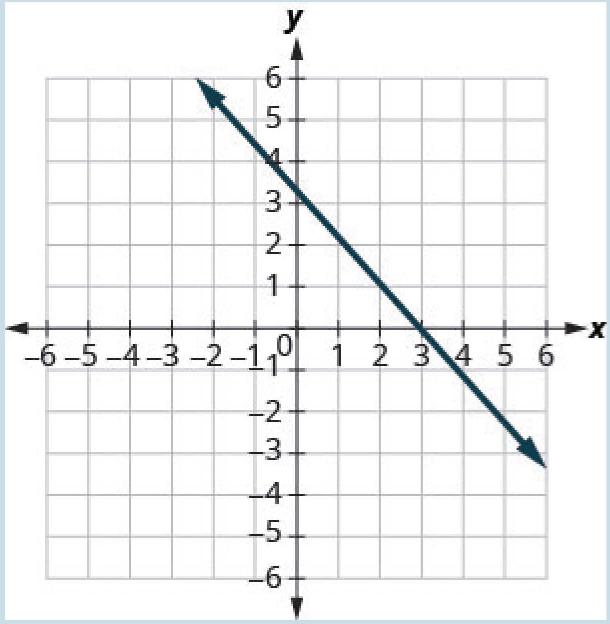

Figure 3P.9.51

Solution

$(3, 0), (0, 3)$

130.

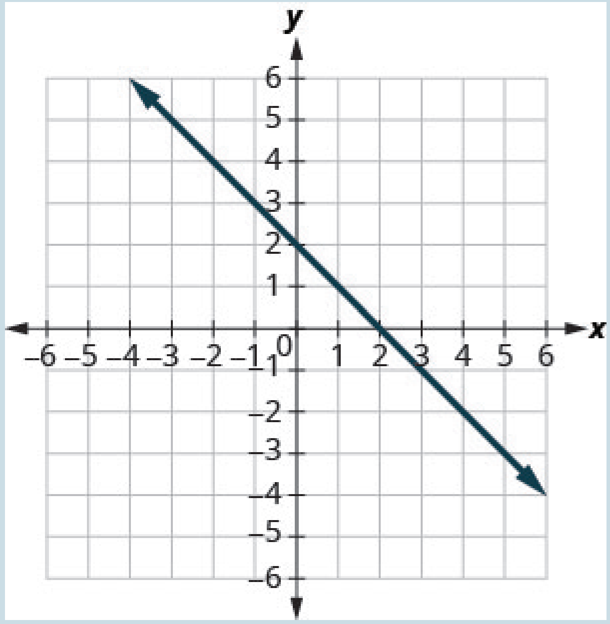

Figure 3P.9.52

131.

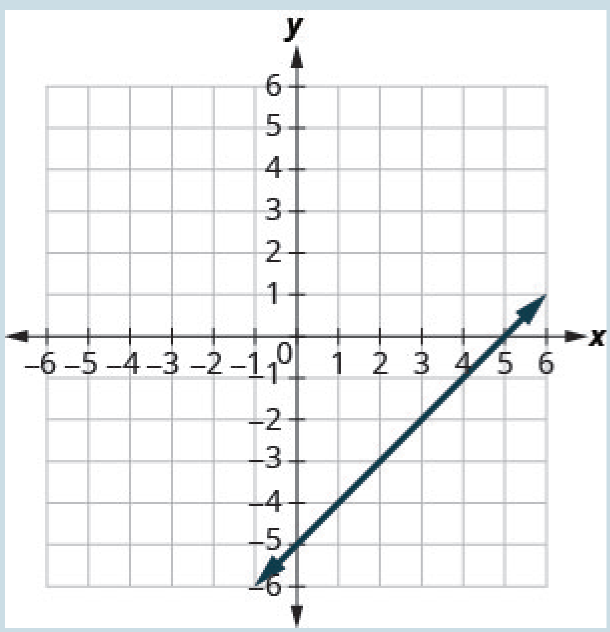

Figure 3P.9.53

Solution

$(5, 0), (0, -5)$

132.

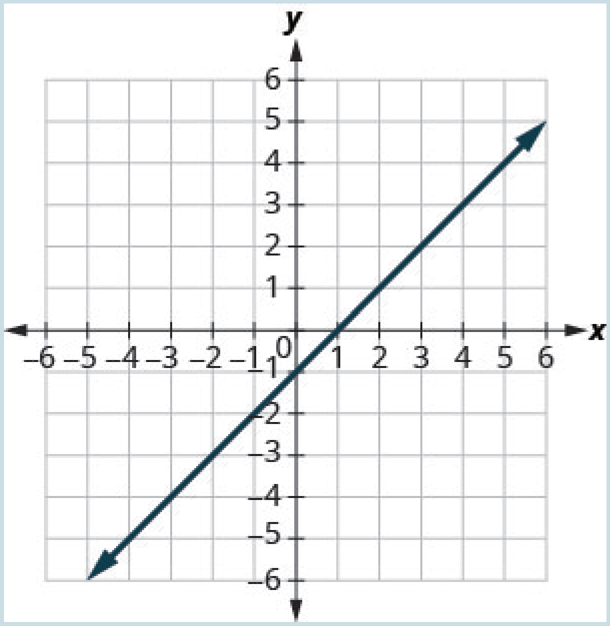

Figure 3P.9.54

133.

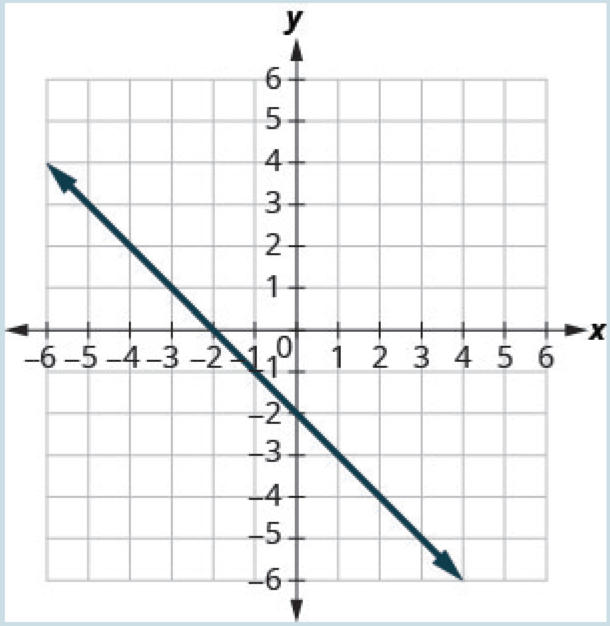

Figure 3P.9.55

Solution

$(-2,0)\,,(0,-2)$

134.

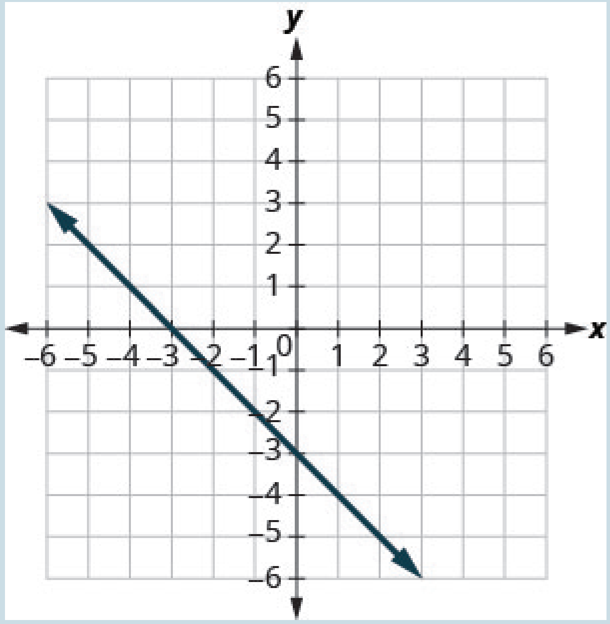

Figure 3P.9.56

135.

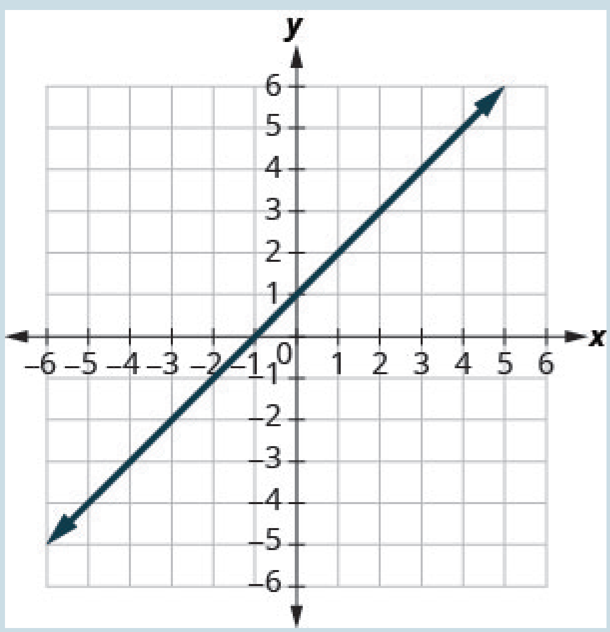

Figure 3P.9.57

Solution

$(-1, 0), (0, 1)$

136.

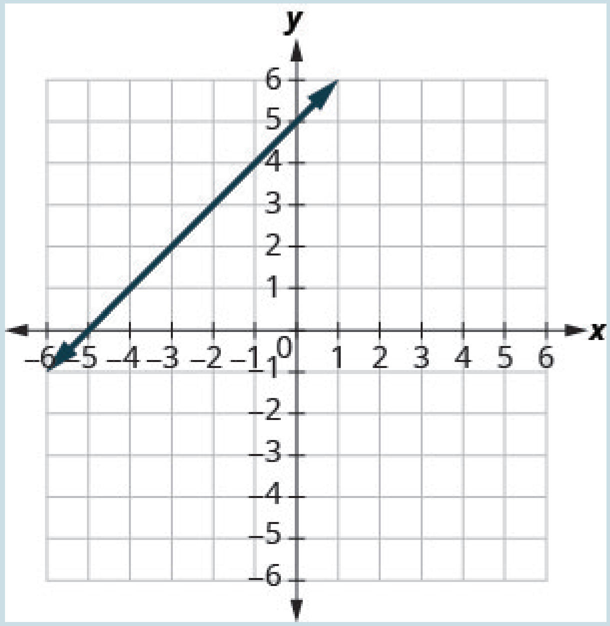

Figure 3P.9.58

137.

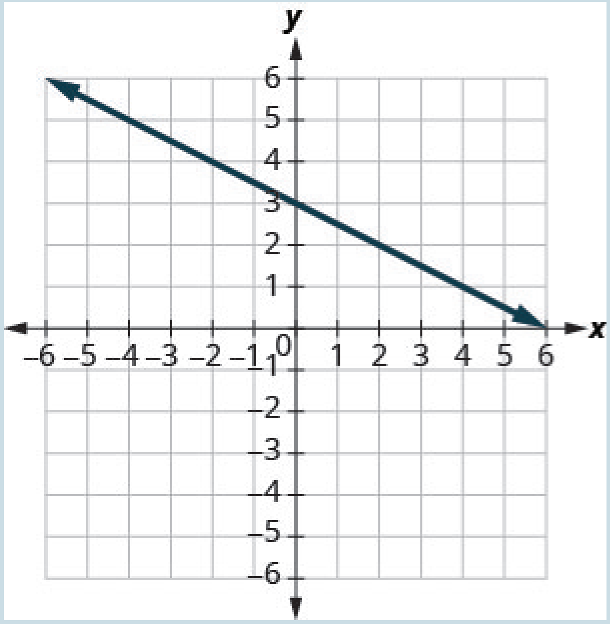

Figure 3P.9.59

Solution

$(6, 0), (0, 3)$

138.

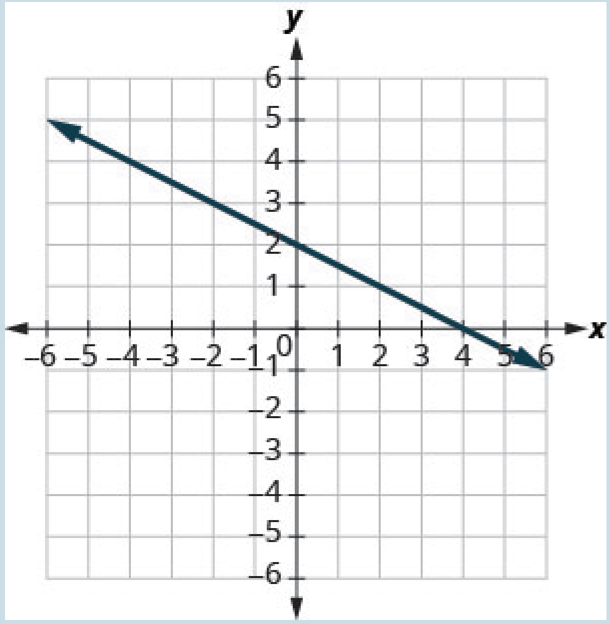

Figure 3P.9.60

139.

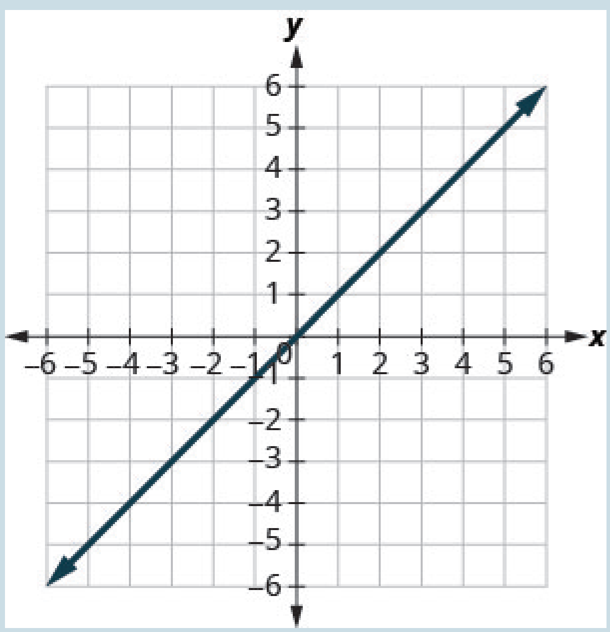

Figure 3P.9.61

Solution

$(0, 0)$

140.

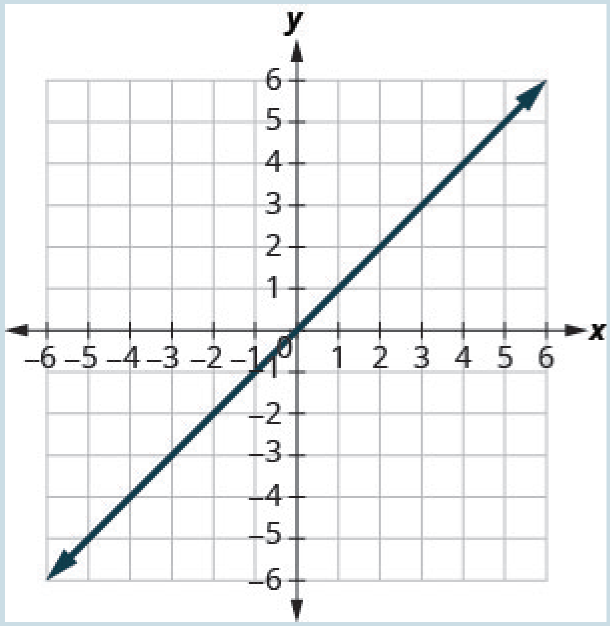

Figure 3P.9.62

Exercises: Find the Formula does not parse x and Formula does not parse y-Intercepts from an Equation of a Line

Instructions: For questions 141-168, find the intercepts for each equation.

141. $x + y = 4$

Solution

$(4, 0), (0, 4)$

---

142. $x + y = 3$

---

143. $x + y = -2$

Solution

$(-2, 0), (0, -2)$

---

144. $x + y = -5$

---

145. $x - y = 5$

Solution

$(5, 0), (0, -5)$

---

146. $x - y = 1$

---

147. $x - y = -3$

Solution

$(-3, 0), (0, 3)$

---

148. $x - y = -4$

---

149. $x + 2y = 8$

Solution

$(8, 0), (0, 4)$

---

150. $x + 2y = 10$

---

151. $3x + y = 6$

Solution

$(2, 0), (0, 6)$

---

152. $3x + y = 9$

---

153. $x - 3y = 12$

Solution

$(12, 0), (0, -4)$

---

154. $x - 2y = 8$

---

155. $4x - y = 8$

Solution

$(2, 0), (0, -8)$

---

156. $5x - y = 5$

157. $2x + 5y = 10$

Solution

$(5, 0), (0, 2)$

158. $2x + 3y = 6$

159. $3x - 2y = 12$

Solution

$(4, 0), (0, -6)$

160. $3x - 5y = 30$

161. $y = \frac{1}{3}x + 1$

Solution

$(-3, 0), (0, 1)$

162. $y = \frac{1}{4}x - 1$

163. $y = \frac{1}{5}x + 2$

Solution

$(-10, 0), (0, 2)$

164. $y = \frac{1}{3}x + 4$

---

165. $y = 3x$

Solution

$(0, 0)$

---

166. $y = -2x$

---

167. $y = -4x$

Solution

$(0, 0)$

---

168. $y = 5x$

Exercises: Graph a Line Using the Intercepts

Instructions: For questions 169-194, graph using the intercepts.

169. $-x + 5y = 10$

Solution

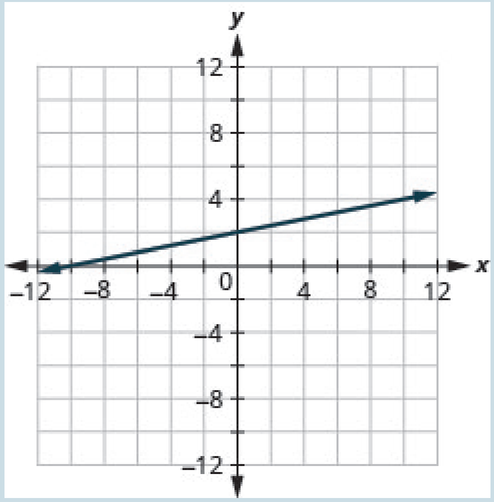

Figure 3P.9.63

---

170. $-x + 4y = 8$

---

171. $x + 2y = 4$

Solution

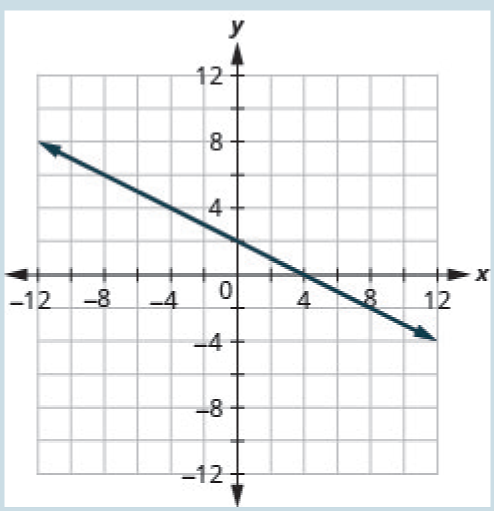

Figure 3P.9.64

172. $x + 2y = 6$

---

173. $x + y = 2$

Solution

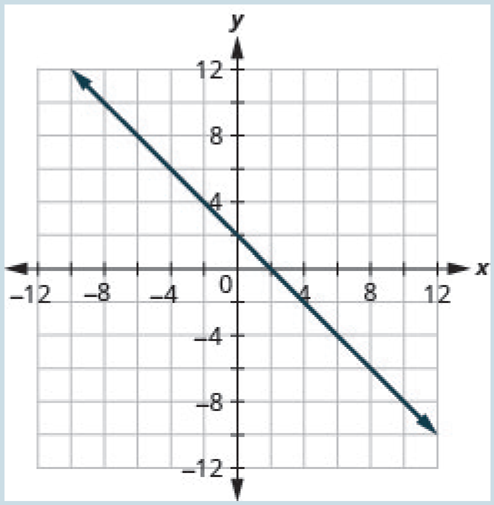

Figure 3P.9.65

---

174. $x + y = 5$

---

175. $x + y = -3$

Solution

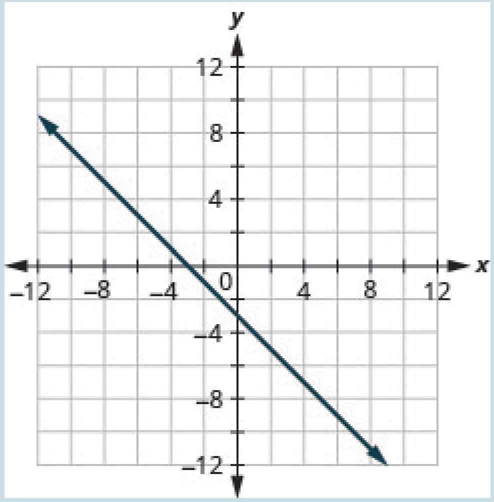

Figure 3P.9.66

---

176. $x + y = -1$

---

177. $x{-}y = 1$

Solution

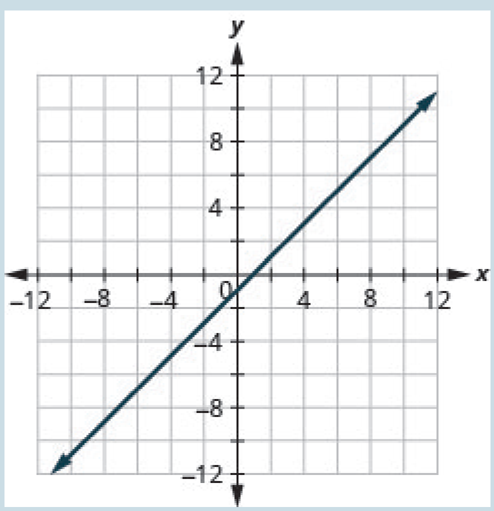

Figure 3P.9.67

---

178. $x - y = 2$

---

179. $x - y = -4$

Solution

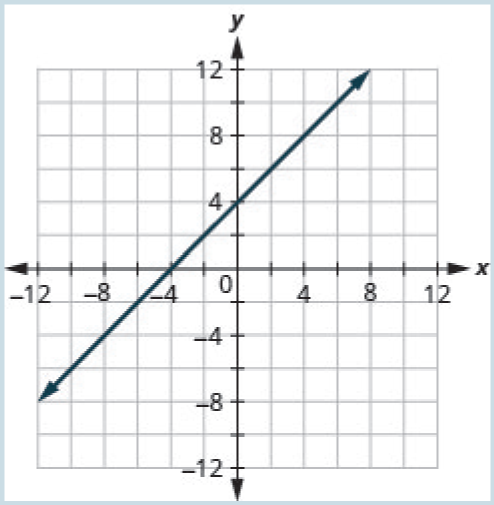

Figure 3P.9.68

---

180. $x - y = -3$

---

181. $4x + y = 4$

Solution

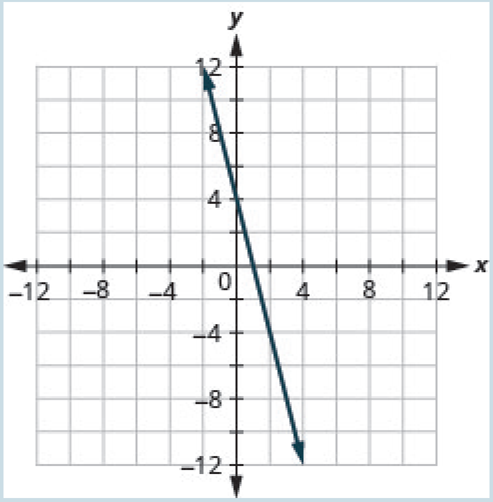

Figure 3P.9.69

---

182. $3x + y = 3$

---

183. $2x + 4y = 12$

Solution

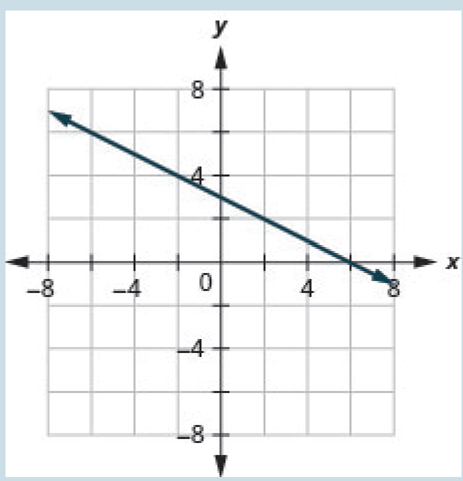

Figure 3P.9.70

---

184. $3x + 2y = 12$

185. $3x - 2y = 6$

Solution

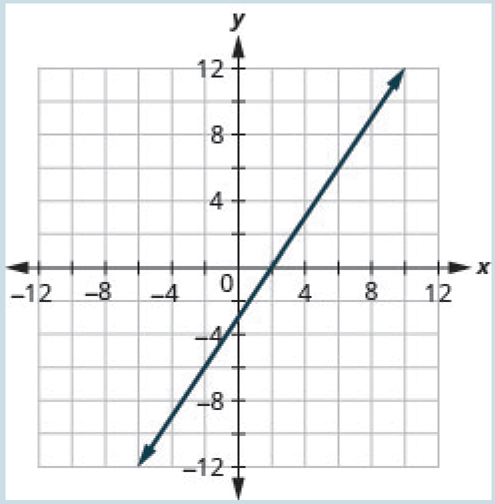

Figure 3P.9.71

186. $5x - 2y = 10$

187. $2x - 5y = -20$

Solution

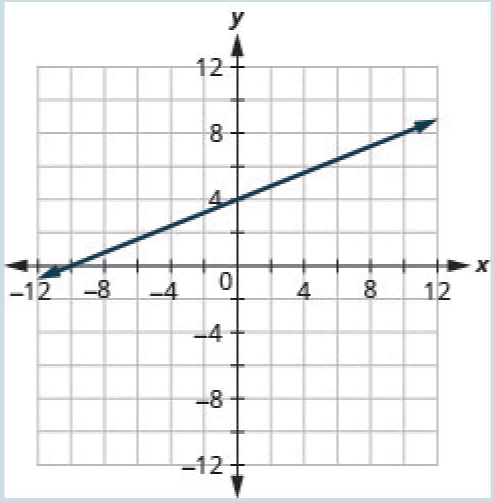

Figure 3P.9.72

---

188. $3x - 4y = -12$

---

189. $3x - y = -6$

Solution

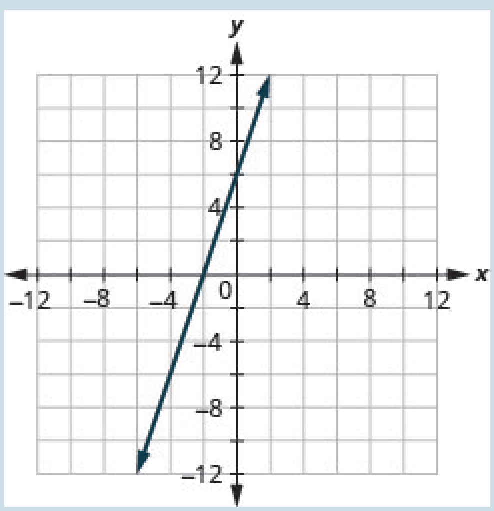

Figure 3P.9.73

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190. $2x - y = -8$

---

191. $y = -2x$

Solution

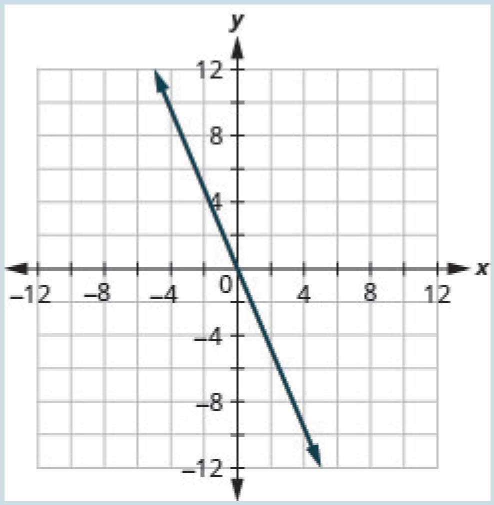

Figure 3P.9.74

---

192. $y = -4x$

---

193. $y = x$

Solution

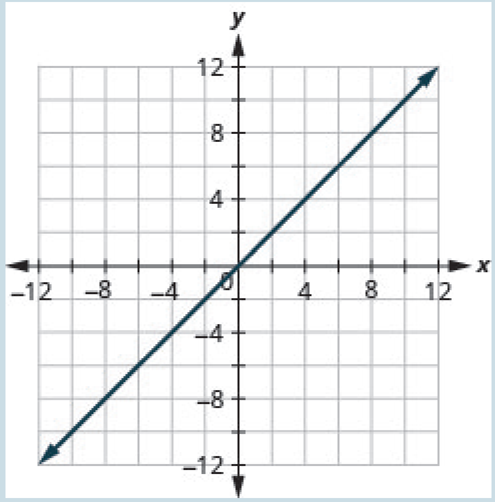

Figure 3P.9.75

---

194. $y = 3x$

Exercises: Everyday Math

Instructions: For questions 195-200, answer the given everyday math word problems.

195. Weight of a baby. Mackenzie recorded her baby's weight every two months. The baby's age, in months, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

a. Plot the points on a coordinate plane.

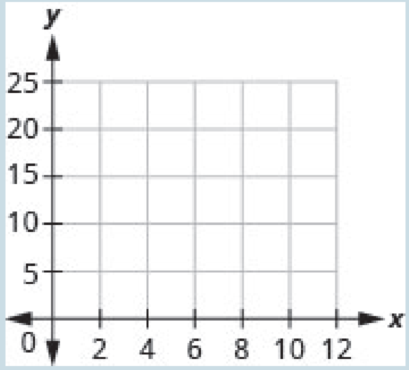

Figure 3P.9.76

b. Why is only Quadrant I needed?

Age x	Weight y	(x, y)
0	7	$(0, 7)$
2	11	$(2, 11)$
4	15	$(4, 15)$
6	16	$(6, 16)$
8	19	$(8, 19)$
10	20	$(10, 20)$
12	21	$(12, 21)$

Solution

a.

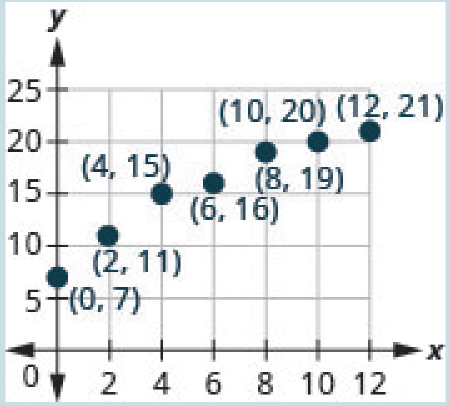

Figure 3P.9.77

b. Age and weight are only positive.

---

196. Weight of a child. Latresha recorded her son's height and weight every year. His height, in inches, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

a. Plot the points on a coordinate plane.

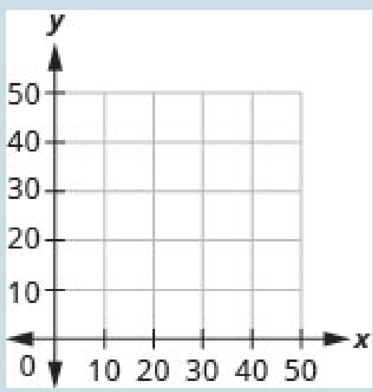

Figure 3P.9.78

b. Why is only Quadrant I needed?

Height x	Weight y	(x, y)
28	22	$(28, 22)$
31	27	$(31, 27)$
33	33	$(33, 33)$
37	35	$(37, 35)$
40	41	$(40, 41)$
42	45	$(42, 45)$

---

197. Motor home cost. The Robinsons rented a motor home for one week to go on vacation. It cost them $\$594$ plus $\$0.32$ per mile to rent the motor home, so the linear equation $y = 594 + 0.32x$ gives the cost, y, for driving x miles. Calculate the rental cost for driving $400, 800,$ and 1200 miles, and then graph the line.

Solution

$\$722, \$850, \$978$

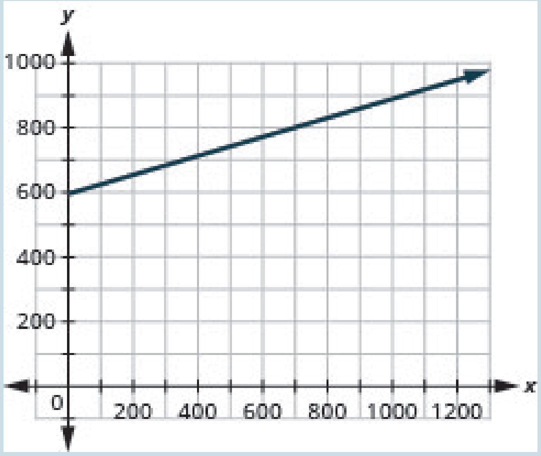

Figure 3P.9.79

---

198. Weekly earnings. At the art gallery where he works, Salvador gets paid $\$200$ per week plus 15% of the sales he makes, so the equation $y = 200 + 0.15x$ gives the amount, y, he earns for selling x dollars of artwork. Calculate the amount Salvador earns for selling $\$900$, $\$1600$, and $\$2000$, and then graph the line.

---

199. Road trip. Damien is driving from Chicago to Denver, a distance of 1000 miles. The x-axis on the graph below shows the time in hours since Damien left Chicago. The y-axis represents the distance he has left to drive.

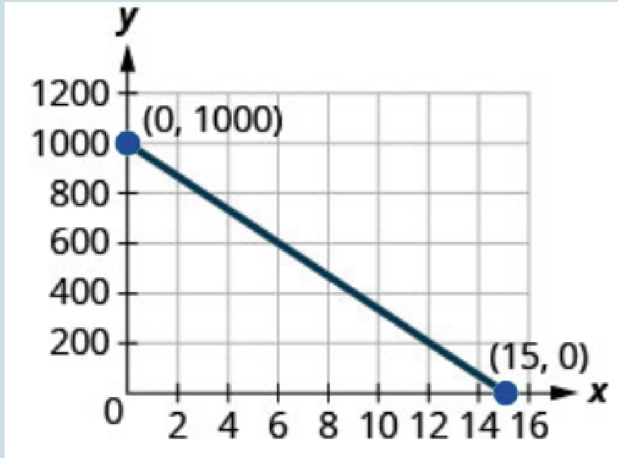

Figure 3P.9.80

a. Find the x and y-intercepts.

b. Explain what the x and y-intercepts mean for Damien.

Solution

a. $(0, 1000), (15, 0)$

b. At $(0, 1000)$, he has been gone 0 hours and has 1000 miles left. At $(15, 0)$, he has been gone 15 hours and has 0 miles left to go.

---

200. Road trip. Ozzie filled up the gas tank of his truck and headed out on a road trip. The x-axis on the graph below shows the number of miles Ozzie drove since filling up.

The y-axis represents the number of gallons of gas in the truck's gas tank.

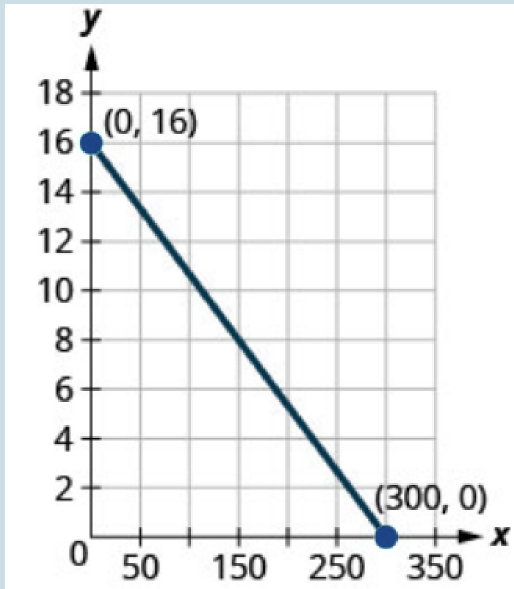

Figure 3P.9.81

a. Find the x and y-intercepts.

b. Explain what the x and y-intercepts mean for Ozzie.

Exercises: Writing Exercises

Instructions: For questions 201-210, answer the given writing exercises.

201. Explain in words how you plot the point $(4, -2)$ in a rectangular coordinate system.

Solution

Answers will vary.

---

202. How do you determine if an ordered pair is a solution to a given equation?

---

203. Is the point $(-3, 0)$ on the x-axis or y-axis? How do you know?

Solution

Answers will vary.

---

204. Is the point $(0, 8)$ on the x-axis or y-axis? How do you know?

---

205. Explain how you would choose three x-values to make a table to graph the line $y = \frac{1}{5}x - 2$.

Solution

Answers will vary.

---

206. What is the difference between the equations of a vertical and a horizontal line?

---

207. How do you find the x-intercept of the graph of $3x - 2y = 6$?

Solution

Answers will vary.

---

208. Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $4x + y = -4$? Why?

---

209. Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $y = \frac{2}{3}x - 2$? Why?

Solution

Answers will vary.

---

210. Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $y = 6$? Why?

3.10 SLOPE OF A LINE

Learning Objectives

By the end of this section, you will be able to:

- Use $m = \frac{rise}{run}$ to find the slope of a line from its graph
- Find the slope of horizontal and vertical lines
- Use the slope formula to find the slope of a line between two points
- Graph a line given a point and the slope
- Solve slope applications
- Recognize the relation between the graph and the slope–intercept form of an equation of a line
- Identify the slope and y-intercept form of an equation of a line
- Graph a line using its slope and intercept
- Choose the most convenient method to graph a line
- Graph and interpret applications of slope–intercept
- Use slopes to identify parallel lines
- Use slopes to identify perpendicular lines

Try It

Before you get started, take this readiness quiz:

1) Simplify: $\frac{1-4}{8-2}$

2) Divide: $\frac{0}{4}, \frac{4}{0}$

3) Simplify: $\frac{15}{-3}, \frac{-15}{3}, \frac{-15}{-3}$

4) Add: $\frac{x}{4} + \frac{1}{4}$

5) Find the reciprocal of $\frac{3}{7}$

6) Solve $2x - 3y = 12$

When you graph linear equations, you may notice that some lines tilt up as they go from left to right and some lines tilt down. Some lines are very steep and some lines are flatter. What determines whether a line tilts up or down or if it is steep or flat?

In mathematics, the 'tilt' of a line is called the *slope* of the line. The concept of slope has many applications in the real world. The pitch of a roof, the grade of a highway, and a ramp for a wheelchair are some examples where you see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

In this section, we will explore the concept of slope.

Use $m = \frac{rise}{run}$ to Find the Slope of a Line from its Graph

Now, we'll look at some graphs on the xy-coordinate plane and see how to find their slopes. The method we will use here will be similar to how one would use **geoboard**s.

To find the slope, we must count out the **rise** and the **run**. But where do we start?

We locate two points on the line whose coordinates are integers. We then start with the point on the left and sketch a right triangle, so we can count the rise and run.

How to

Find the slope of a line from its graph using $m = \frac{rise}{run}$

1. Locate two points on the line whose coordinates are integers.
2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
3. Count the rise and the run on the legs of the triangle.
4. Take the ratio of rise to run to find the slope, $m = \frac{rise}{run}$

Example 1

How to Use $m = \frac{rise}{run}$ to Find the **Slope of a Line** from its Graph

Find the slope of the line shown.

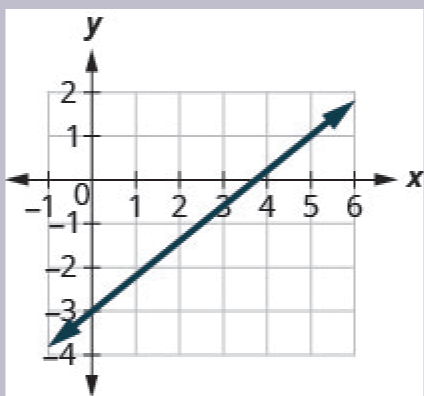

Figure 3.10.1

Solution

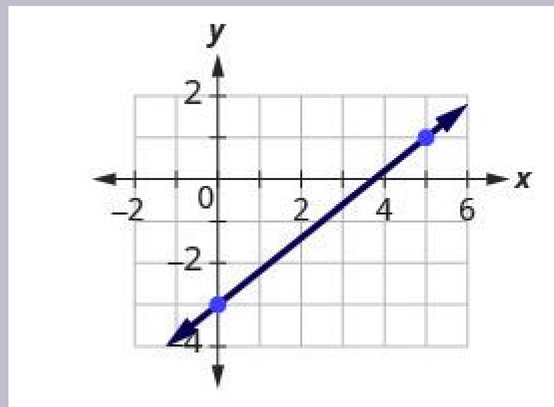

Figure 3.10.2

Step 1: **Locate two points whose coordinates are integers.**

Mark $(0-,3)(5,1)$

Step 2: **Starting with the point on the left, sketch a right triangle, going from the first point to the second point.**

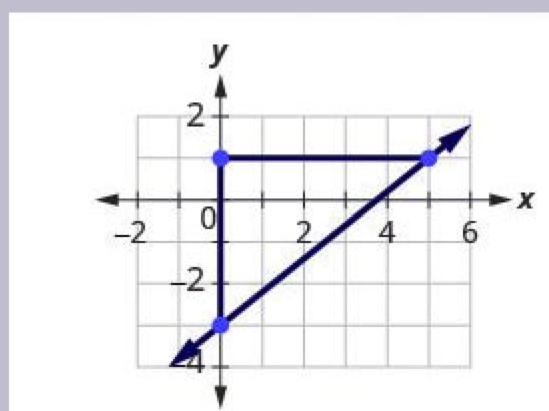

Figure 3.10.3

Step 3: **Count the rise and the run on the legs of the triangle.**

Count the rise.

Count the run.

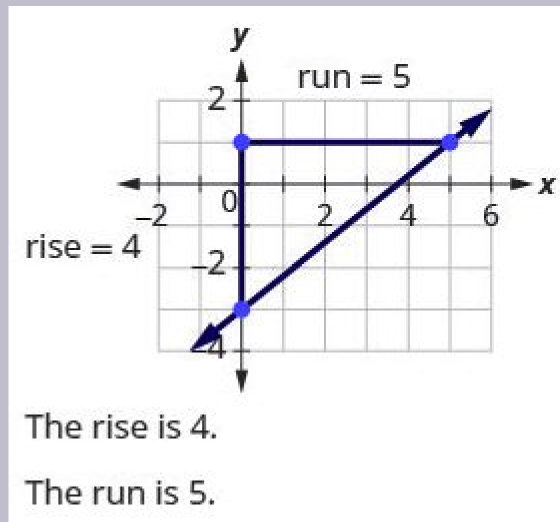

Figure 3.10.4

Step 4: **Take the ratio of rise to run to find the slope.**

The slope of the line is $\frac{4}{5}$.

Try It

7) Find the slope of the line shown.

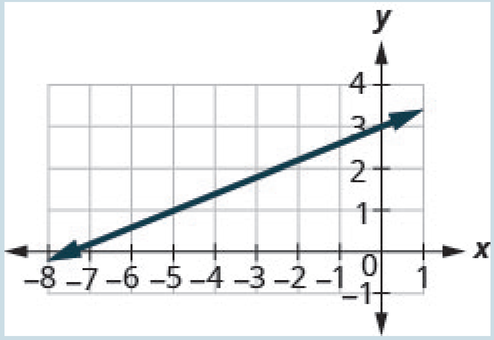

Figure 3.10.5

Solution

$$\frac{2}{5}$$

8) Find the slope of the line shown.

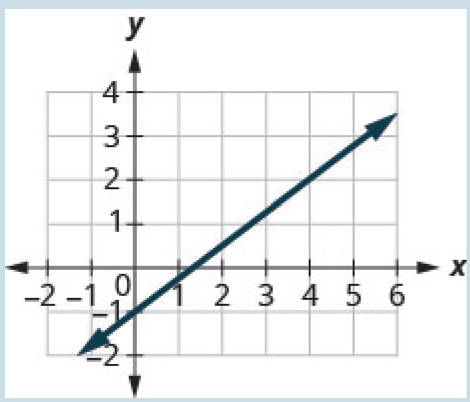

Figure 3.10.6

Solution

$$\frac{3}{4}$$

Example 2

Find the slope of the line shown.

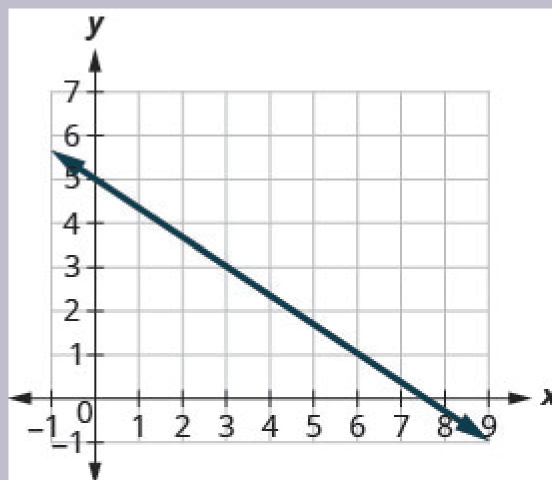

Figure 3.10.7

Solution

m=\fr
ac{rise
}{run}

***Step 1*. Locate two points on the graph whose coordinates are integers.**

$$(0, 5) \text{ and } (3, 3)$$

***Step 2:* Which point is on the left?**

$$(0, 5)$$

***Step 3:* Starting at $(0, 5)$, sketch a right triangle to $(3, 3)$.**

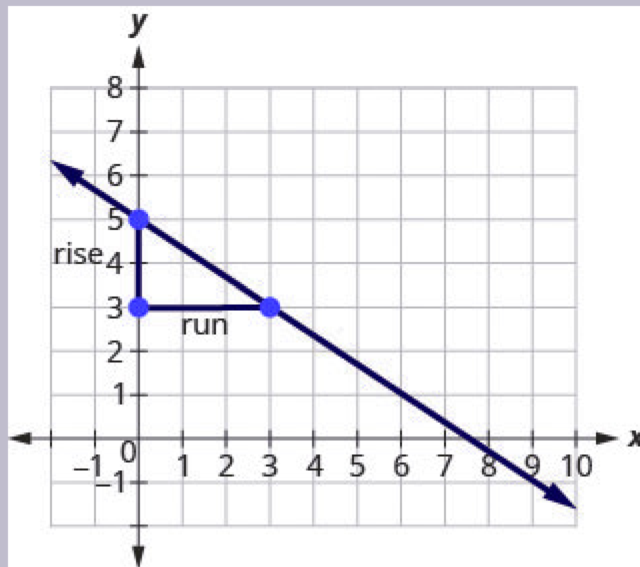

Figure 3.10.8

***Step 4:* Count the rise—it is negative.**

The rise is -2.

***Step 5:* Count the run.**

The run is 3.

Step 6: Use the slope formula.

$$m = \frac{rise}{run}$$

***Step 7:* Substitute the values of the rise and run.**

$$m = \frac{-2}{3}$$

Simplify. $m = -\frac{2}{3}$

The slope of the line is $-\dfrac{2}{3}$.

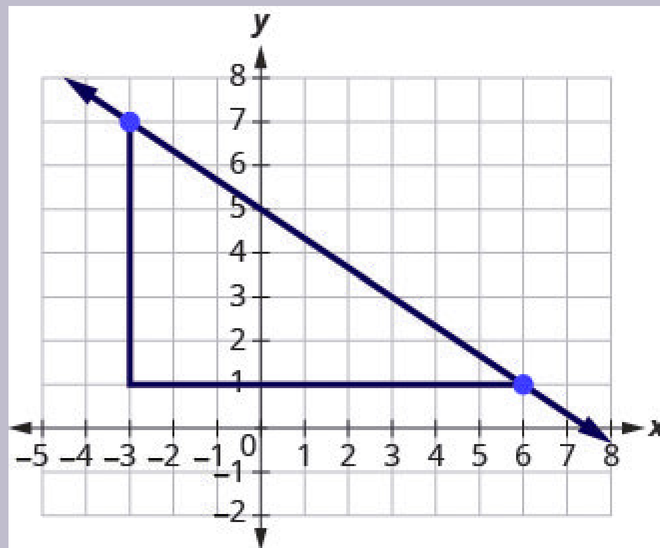

Figure 3.10.9

So y increases by 3 units as x decreases by 2 units. What if we used the points left $(-3, 7)$ and $(6, 1)$ to find the slope of the line? The rise would be -6 and the run would be 9. Then $m = \frac{-6}{9}$, and that simplifies to $m = -\frac{2}{3}$. Remember, it does not matter which points you use—the slope of the line is always the same.

Try It

9) Find the slope of the line shown.

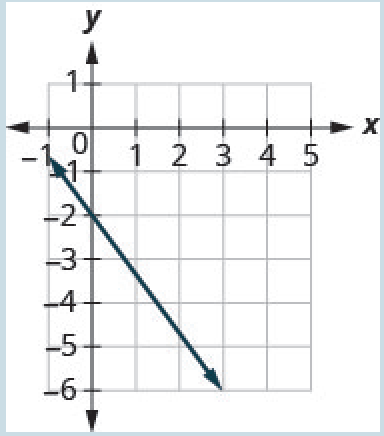

Figure 3.10.10

Solution

$$-\frac{4}{3}$$

10) Find the slope of the line shown.

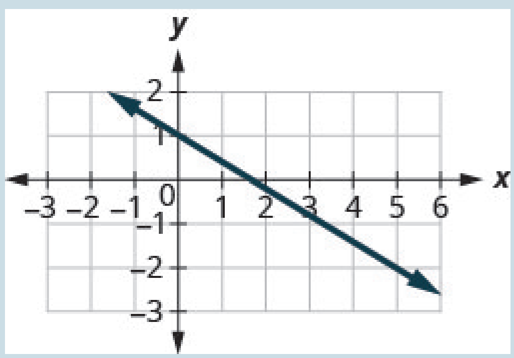

Figure 3.10.11

Solution

$$-\frac{3}{5}$$

In the last two examples, the lines had y-intercepts with integer values, so it was convenient to use the y-intercept as one of the points to find the slope. In the next example, the y-intercept is a fraction. Instead of using that point, we'll look for two other points whose coordinates are integers. This will make the slope calculations easier.

Example 3

Find the slope of the line shown.

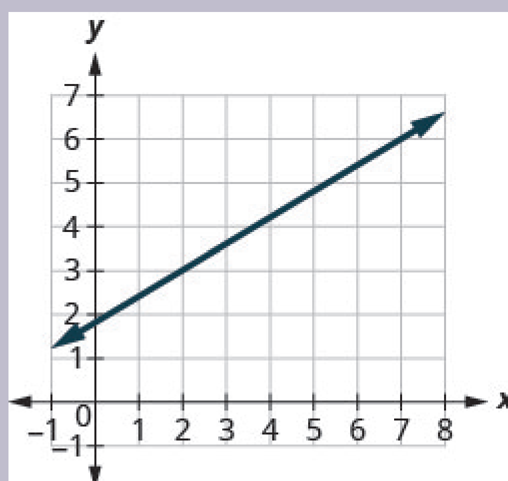

Figure 3.10.12

Solution

Step 1: **Locate two points on the graph whose coordinates are integers.**

$$(2, 3) \text{ and } (7, 6)$$

Step 2: **Which point is on the left?**

$$(2, 3)$$

Step 3: **Starting at $(2, 3)$, sketch a right triangle to $(7, 6)$.**

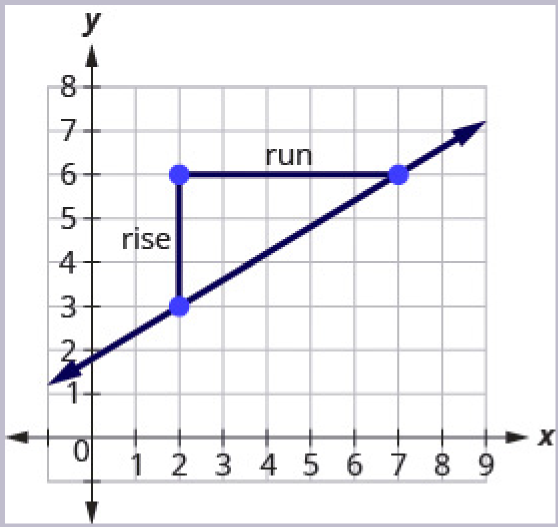

Figure 3.10.13

Step 4: **Count the rise.**

The rise is 3.

Step 5: **Count the run.**

The run is 5.

Step 6: **Use the slope formula.**

$$m = \frac{rise}{run}$$

Step 7: **Substitute the values of the rise and run.**

The slope of the line is $\frac{3}{5}$

This means that y increases 5 units as x increases 3 units. If you used a geoboard to introduce the concept of slope, you would always start with the point on the left and count the rise and the run to get to the point on the right. That way the run was always positive and the rise determined whether the slope was positive or negative. What would happen if we started with the point on the right? Let's use the points $(2, 3)$ and $(7, 6)$ again, but now we'll start at $(7, 6)$.

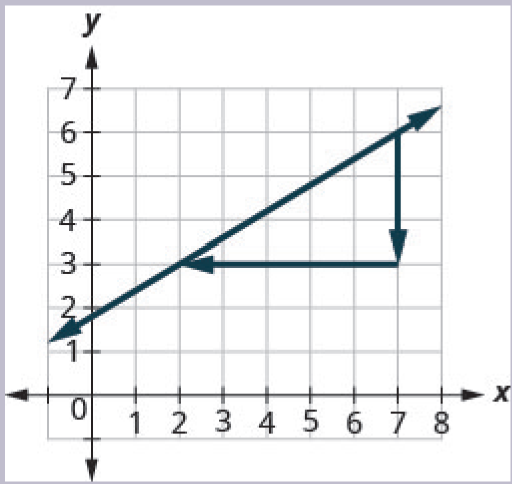

Figure 3.10.14

Step 1: **Count the rise.**

The rise is -3.

Step 2: **Count the run. It goes from right to left, so it is negative.**

The run is -5.

Step 3: **Use the slope formula.**

$$m = \frac{rise}{run}$$

Step 4: **Substitute the values of the rise and run.**

$$m = \frac{-3}{-5}$$

Step 5: **The slope of the line is** $\frac{3}{5}$**.**

It does not matter where you start—the slope of the line is always the same.

Try It

11) Find the slope of the line shown.

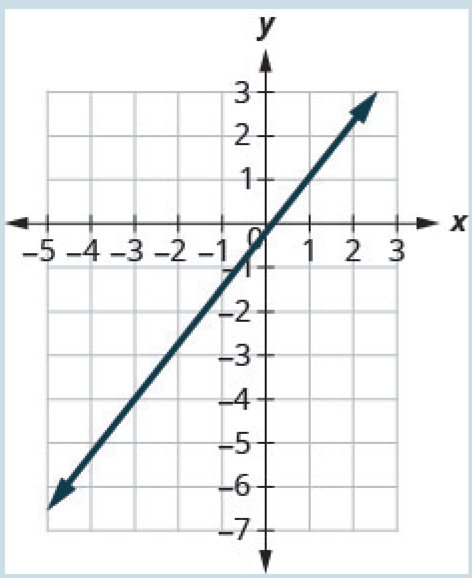

Figure 3.10.15

Solution

$$\frac{5}{4}$$

12) Find the slope of the line shown.

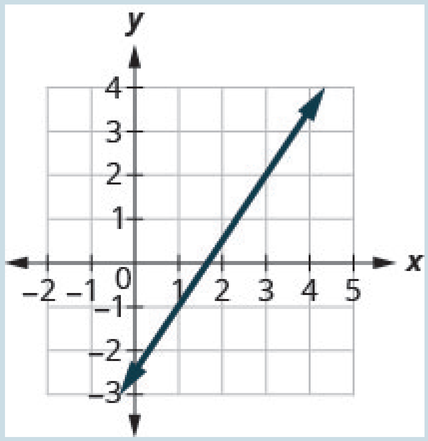

Figure 3.10.16

Solution

$$\frac{3}{2}$$

Find the Slope of Horizontal and Vertical Lines

Do you remember what was special about horizontal and vertical lines? Their equations had just one variable.

Horizontal line $y = b$	Vertical line $x = a$
y-coordinates are the same.	x-coordinates are the same.

So how do we find the slope of the horizontal line $y = 4$? One approach would be to graph the horizontal line, find two points on it, and count the rise and the run. Let's see what happens when we do this.

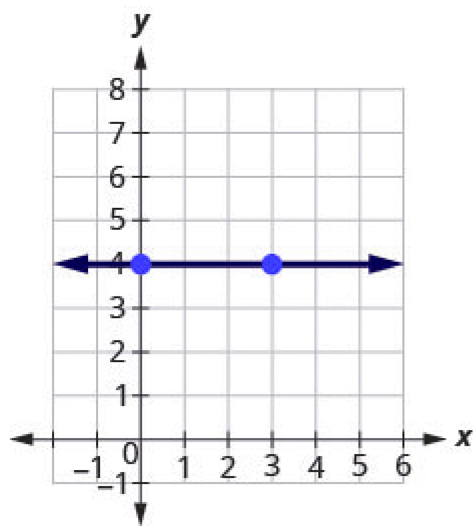

Figure 3.10.17

Step 1: **What is the rise?**

The rise is 0.

Step 2: **Count the run.**

The run is 3.

Step 3: **What is the slope?**

$$m = \frac{rise}{run}$$
$$m = \frac{0}{3}$$
$$m = 0$$

The slope of the horizontal line $y = 4$ is 0.

All horizontal lines have slope 0. When the y-coordinates are the same, the rise is 0.

Slope of a Horizontal Line

The slope of a horizontal line, $y = b$, is 0.

The floor of your room is horizontal. Its slope is 0. If you carefully placed a ball on the floor, it would not roll away.

Now, we'll consider a vertical line, the line.

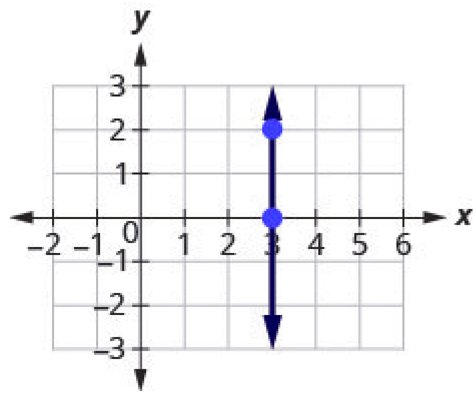

Figure 3.10.18

Step 1: **What is the rise?**

The rise is 2.

Step 2: **Count the run.**

The run is 0.

Step 3: **What is the slope?**

$$m = \frac{rise}{run}$$
$$m = \frac{2}{0}$$

But we can't divide by 0. Division by 0 is not defined. So we say that the slope of the vertical line $x = 3$ is undefined.

The slope of any vertical line is undefined. When the x-coordinates of a line are all the same, the run is 0.

The slope of a Vertical Line

The slope of a vertical line, $x = a$, is undefined.

Example 4

Find the slope of each line:

a. $x = 8$
b. $y = -5$

Solution

a. $x = 8$

This is a vertical line.
Its slope is undefined.

---

b. $y = -5$

This is a horizontal line.

It has slope 0.

Try It

13) Find the slope of the line: $x = -4$.

Solution

undefined

14) Find the slope of the line: $y = 7$.

Solution

$$0$$

Quick Guide to the Slopes of Lines

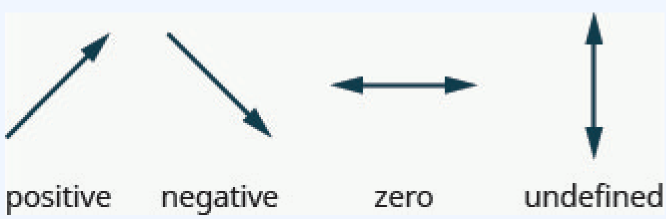

Figure 3.10.19

Remember, we 'read' a line from left to right, just like we read written words in English.

Use the Slope Formula to find the Slope of a Line Between Two Points

Sometimes we'll need to find the slope of a line between two points when we don't have a graph to count out the rise and the run. We could plot the points on grid paper, and then count out the rise and the run, but as we'll see, there is a way to find the slope without graphing. Before we get to it, we need to introduce some algebraic notation.

We have seen that an ordered pair (x, y) gives the coordinates of a point. But when we work with slopes, we use two points. How can the same symbol (x, y) be used to represent two different points? Mathematicians use subscripts to distinguish the points.

(x_1, y_1)	read ' x sub 1, y sub 1 '
(x_2, y_2)	read ' x sub 2, y sub 2 '

The use of subscripts in math is very much like the use of last name initials in elementary school. Maybe you remember Laura C. and Laura M. in your third grade class?

We will use (x_1, y_1) to identify the first point and (x_2, y_2) to identify the second point.

If we had more than two points, we could use (x_3, y_3), (x_4, y_4), and so on.

Let's see how the rise and run are related to the coordinates of the two points by taking another look at the slope of the line between the points $(2, 3)$ and $(7, 6)$.

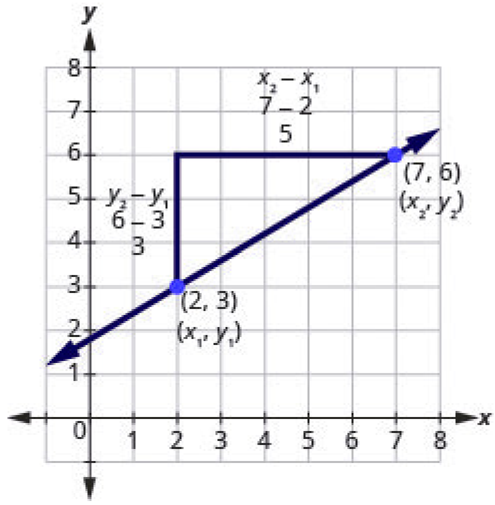

Figure 3.10.20

Since we have two points, we will use subscript notation,

$$\begin{pmatrix} x_1 & y_1 \\ 2 & 3 \end{pmatrix} \quad \begin{pmatrix} x_2 & y_2 \\ 7 & 6 \end{pmatrix}$$

On the graph, we counted the rise of 3 and the run of 5.

Notice that the rise of 3 can be found by subtracting the y-coordinates 6 and 3.

$$3 = 6 - 3$$

And the run of 5 can be found by subtracting the x-coordinates 7 and 2.

$$5 = 7 - 2$$

We know $m = \frac{rise}{run}$. So $m = \frac{3}{5}$.

We rewrite the rise and run by putting in the coordinates $m = \frac{6-3}{7-2}$.

But 6 is y_2, the y-coordinate of the second point and 3 is y_1 the y-coordinate of the first point.

So we can rewrite the slope using subscript notation. $m = \frac{y_2 - y_1}{7 - 2}$

Also, 7 is x_2, the x-coordinate of the second point and 2 is x_1, the x-coordinate of the first point.

So, again, we rewrite the slope using subscript notation. $m = \frac{y_2 - y_1}{x_2 - x_1}$

We've shown that $m = \frac{y_2 - y_1}{x_2 - x_1}$ is really another version of $m = \frac{rise}{run}$. We can use this formula to find the slope of a line when we have two points on the line.

Slope Formula

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

This is the **slope formula**.

The slope is:

y of the second point minus y of the first point

over

x of the second points minus x of the first point

Example 5

Use the slope formula to find the slope of the line between the points $(1, 2)$ and $(4, 5)$.

Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 1: **We'll call $(1, 2)$ point #1 and $(4, 5)$ point #2.**

$$\begin{pmatrix} x_1 & y_1 \\ 1 & 2 \end{pmatrix} \qquad \begin{pmatrix} x_2 & y_2 \\ 4 & 5 \end{pmatrix}$$

Step 2: **Use the slope formula.**

$$m = \frac{rise}{run}$$

Step 3: **Substitute the values.**

y of the second point minus y of the first point.

$$m = \frac{5 - 2}{x_2 - x_1}$$

x of the second point minus x of the first point.

$$m = \frac{5 - 2}{4 - 1}$$

Step 4: **Simplify the numerator and the denominator.**

$$\begin{array}{ll} & m = \frac{3}{3} \\ \text{Simplify.} & m = 1 \end{array}$$

Let's confirm this by counting out the slope on a graph using $m = \frac{rise}{run}$.

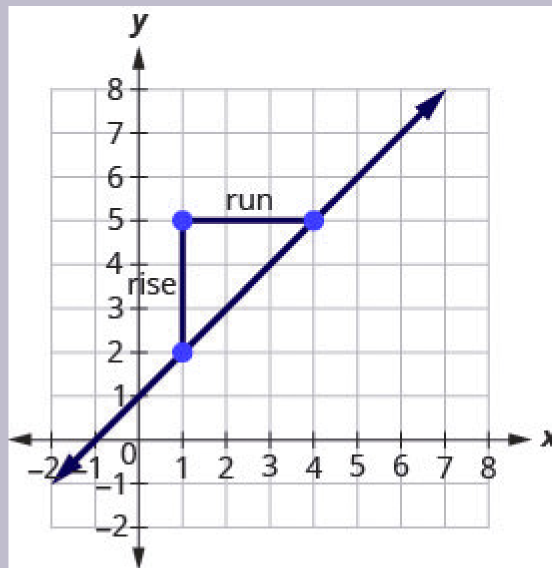

Figure 3.10.21

It doesn't matter which point you call point #1 and which one you call point #2. The slope will be the same. Try the calculation yourself.

Try It

15) Use the slope formula to find the slope of the line through the points: $(8, 5)$ and $(6, 3)$.

Solution

$$1$$

16) Use the slope formula to find the slope of the line through the points: $(1, 5)$ and $(5, 9)$.

Solution

1

Example 6

Use the slope formula to find the slope of the line through the points $(-2,-3)$ and $(-7,4)$.

Solution

***Step 1.* We'll call $(-2,-3)$ point #1 and $(-7,4)$ point #2.**

$$\begin{pmatrix} x_1 & y_1 \\ -2 & -3 \end{pmatrix} \qquad \begin{pmatrix} x_2 & y_2 \\ -7 & 4 \end{pmatrix}$$

***Step 2.* Use the slope formula.**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

***Step 3.* Substitute the values.**

y of the second point minus y of the first point.

$$m = \frac{4-(-3)}{x_2 - x_1}$$

x of the second point minus x of the first point.

$$m = \frac{4-(-3)}{-7-(-2)}$$

***Step 4.* Simplify.**

$$m = \frac{7}{-5}$$
$$m = -\frac{7}{5}$$

Let's verify this slope on the graph shown.

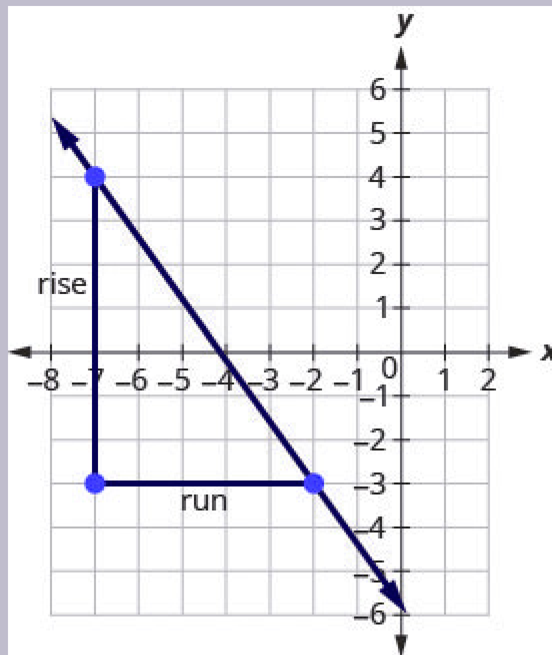

Figure 3.10.22

$$m = \frac{rise}{run}$$
$$m = \frac{-7}{5}$$
$$m = -\frac{7}{5}$$

Try It

17) Use the slope formula to find the slope of the line through the points: $(-3, 4)$ and $(2, -1)$

Solution

$$-1$$

18) Use the slope formula to find the slope of the line through the pair of points: $(-2, 6)$ and $(-3, -4)$

.

Solution

10

Graph a Line Given a Point and the Slope

Up to now, in this chapter, we have graphed lines by plotting points, by using intercepts, and by recognizing horizontal and vertical lines.

One other method we can use to graph lines is called the *point-slope method*. We will use this method when we know one point and the slope of the line. We will start by plotting the point and then use the definition of slope to draw the graph of the line.

How to

Graph a line given a point and the slope.

1. Plot the given point.
2. Use the slope formula $m = \frac{rise}{run}$ to identify the rise and the run.
3. Starting at the given point, count out the rise and run to mark the second point.
4. Connect the points with a line.

Example 7

Graph the line passing through the point $(1, -1)$ whose slope is $m = \frac{3}{4}$.

Solution

***Step 1:* Plot the given point.**

Plot $(1, -1)$.

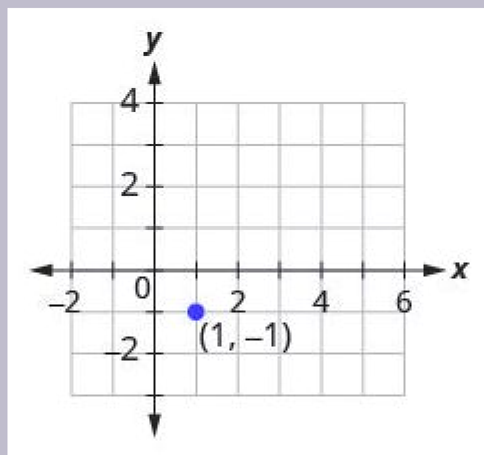

Figure 3.10.23

***Step 2:* Use the slope formula $m = \frac{rise}{run}$ to identify the rise and the run.**

Identify the rise and the run.

$$m = \frac{3}{4}$$

$$\frac{rise}{run} = \frac{3}{4}$$

$$\text{rise} = 3$$
$$\text{run} = 4$$

***Step 3:* Starting at the given point, count out the rise and run to mark the second point**

Start at $(1, -1)$ and count the rise and the run. Up 3 units, right 4 units.

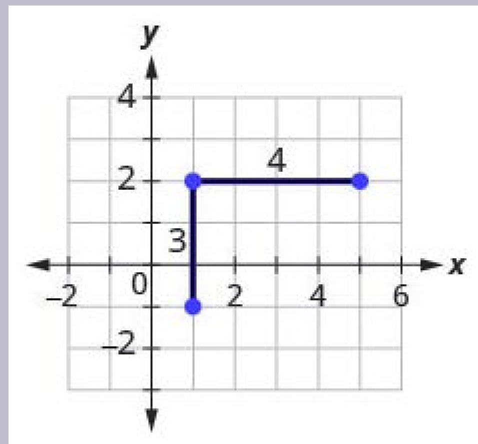

Figure 3.10.24

Step 4: **Connect the points with a line.**

Connect the two points with a line.

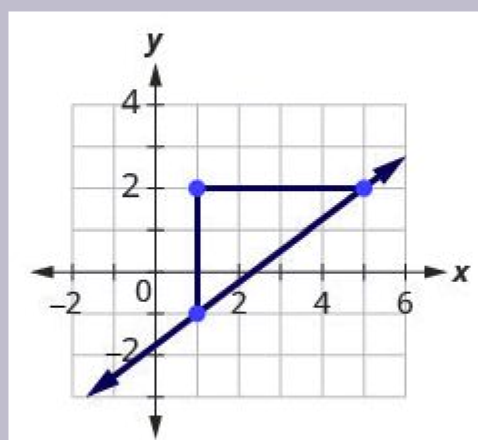

Figure 3.10.25

Try It

19) Graph the line passing through the point $(2, -2)$ with the slope $m = \frac{4}{3}$.

Solution

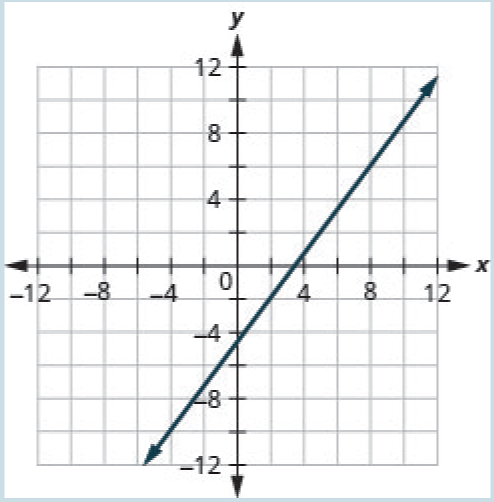

Figure 3.10.26

20) Graph the line passing through the point $(-2, 3)$ with the slope $m = \frac{1}{4}$.

Solution

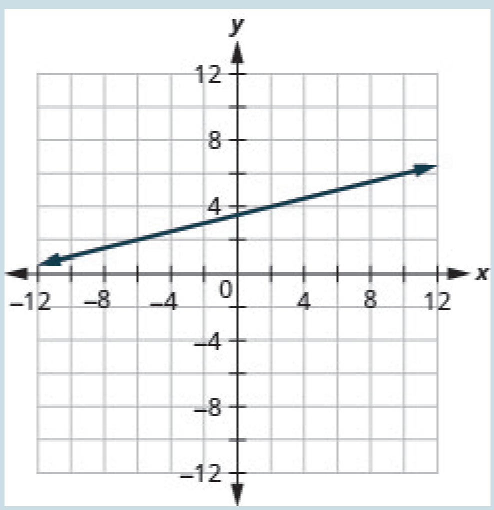

Figure 3.10.27

Example 8

Graph the line with y-intercept 2 whose slope is $m = -\frac{2}{3}$.

Solution

Step 1: **Plot the given point, the y-intercept, $(0, 2)$.**

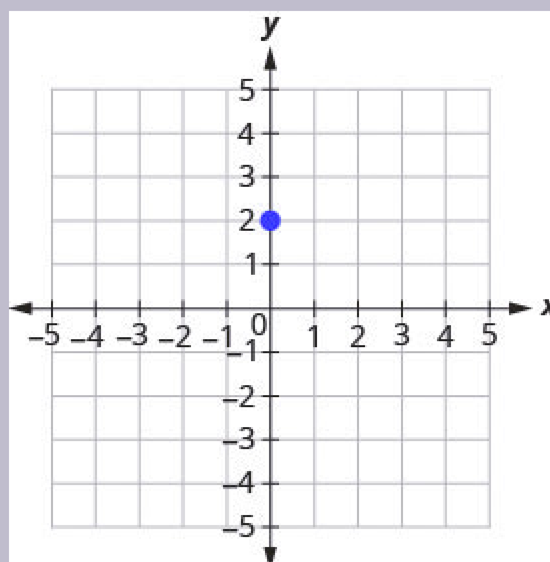

Figure 3.10.28

$$m = -\frac{2}{3}$$

Step 2: **Identify the rise and the run.**

$$\left(\frac{rise}{run}\right) = \frac{-2}{3}$$
$$rise = -2$$
$$run = 3$$

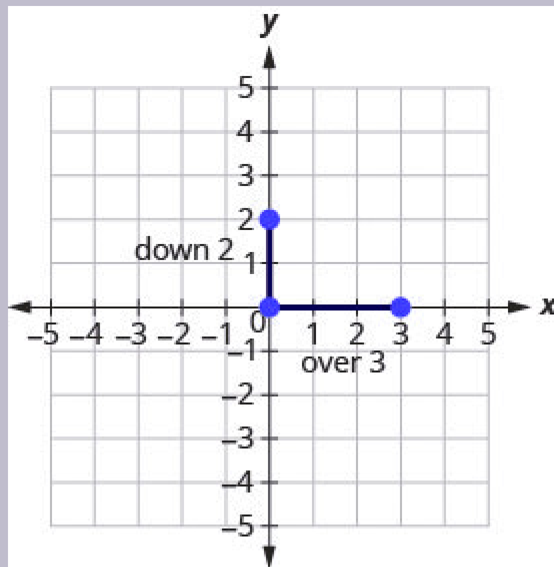

Figure 3.10.29

Step 3: **Count the rise and the run. Mark the second point.**

Step 4: **Connect the two points with a line.**

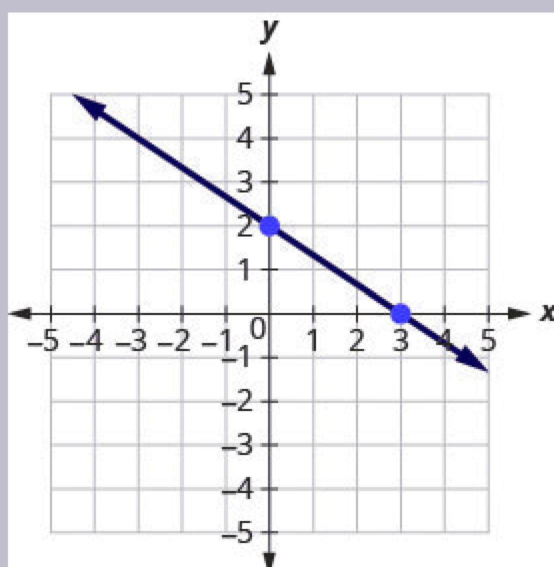

Figure 3.10.30

You can check your work by finding a third point. Since the slope is $m = -\frac{2}{3}$, it can be written as $m = \frac{2}{-3}$. Go back to $(0, 2)$ and count out the rise, 2, and the run, -3

Try It

21) Graph the line with the y-intercept 4 and slope $m = -\frac{5}{2}$.

Solution

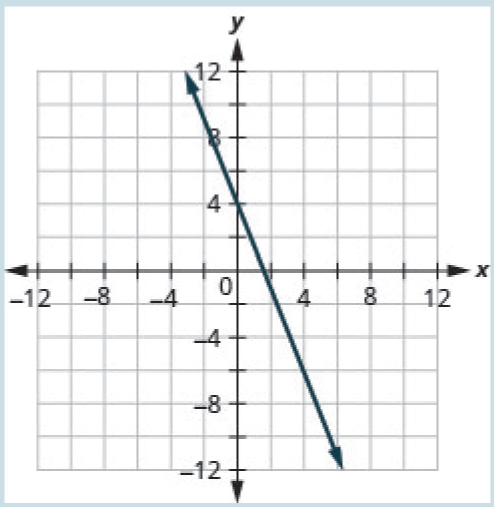

Figure 3.10.31

22) Graph the line with the x-intercept -3 and slope $m = -\frac{3}{4}$.

Solution

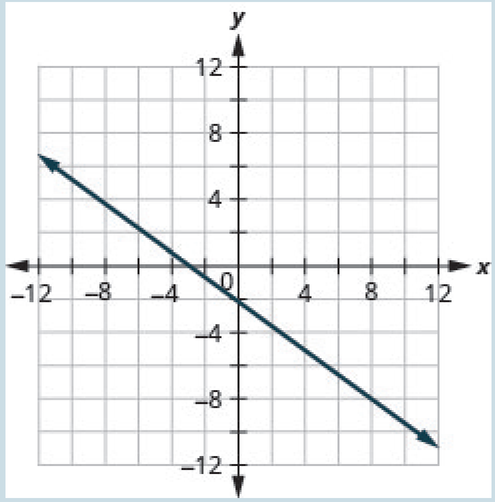

Figure 3.10.32

Example 9

Graph the line passing through the point $(-1, -3)$ whose slope is $m = 4$.

Solution

Step 1: **Plot the given point.**

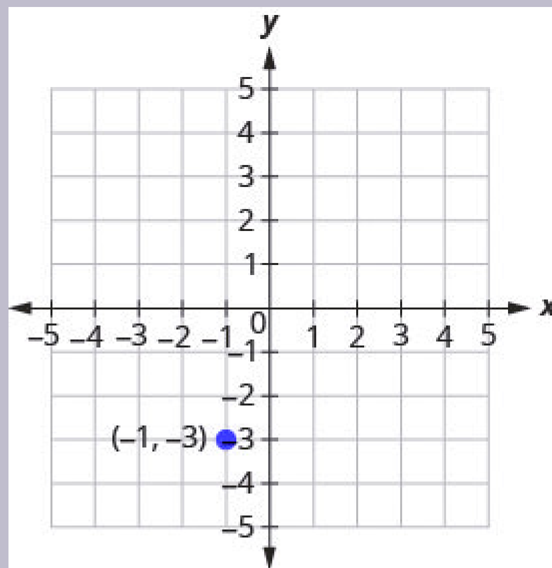

Figure 3.10.33

Step 2: **Identify the rise and the run.**

$$m = 4$$

Step 3: **Write** 4 **as a fraction.**

$$\frac{rise}{run} = \frac{4}{1}$$
$$rise = 4$$
$$run = 1$$

Step 4: **Count the rise and run and mark the second point.**

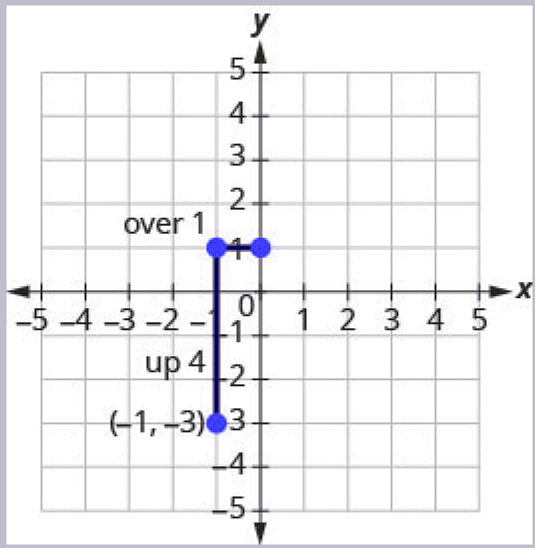

Figure 3.10.34

Step 5: Connect the two points with a line.

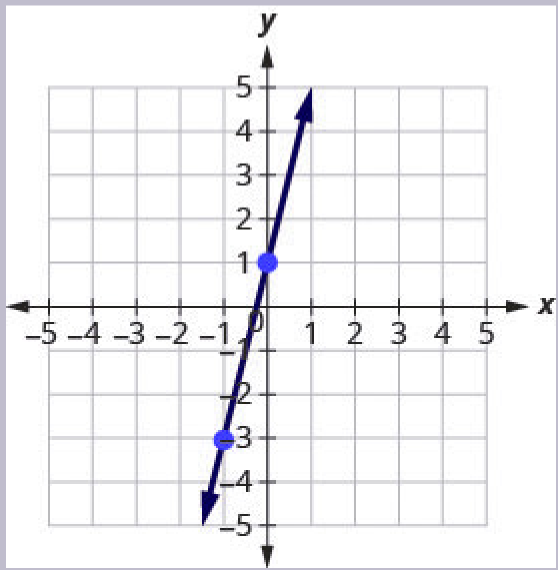

Figure 3.10.35

You can check your work by finding a third point. Since the slope is $m = 4$, it can be written as $m = \frac{-4}{-1}$. Go back to $(-1, -3)$ and count out the rise, -4, and the run, -1.

Try It

23) Graph the line with the point $(-2, 1)$ and slope $m = 3$.

Solution

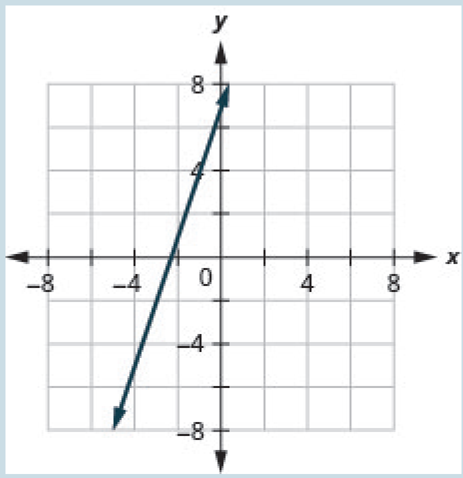

Figure 3.10.36

24) Graph the line with the point $(4, -2)$ and slope $m = -2$.

Solution

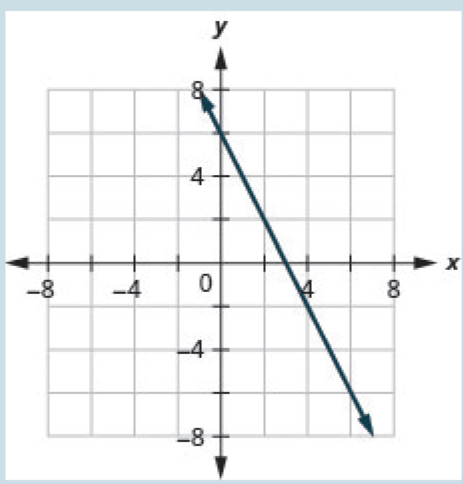

Figure 3.10.37

Solve Slope Applications

At the beginning of this section, we said there are many applications of slope in the real world. Let's look at a few now.

Example 10

The 'pitch' of a building's roof is the slope of the roof. Knowing the pitch is important in climates where there is heavy snowfall. If the roof is too flat, the weight of the snow may cause it to collapse. What is the slope of the roof shown?

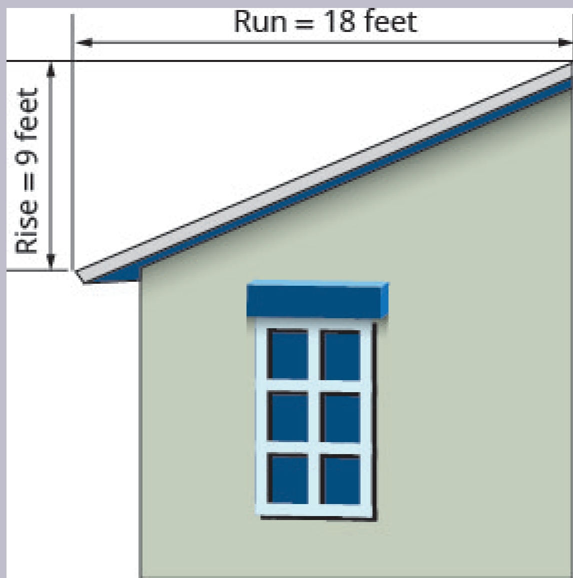

Figure 3.10.38

Solution

Step 1: **Use the slope formula.**

$$m = \frac{rise}{run}$$

Step 2: **Substitute the values for rise and run.**

$$m = \frac{9}{18}$$

Step 3: **Simplify.**

$$m = \frac{1}{2}$$

Step 4: **The slope of the roof is** $\frac{1}{2}$.

The roof rises 1 foot for every 2 feet of horizontal run.

Try It

25) Use Example 3.10.10, substituting the $rise = 14$ and $run = 24$.

Solution

$\frac{7}{12}$

26) Use Example 3.10.10, substituting $rise = 15$ and $run = 36$.

Solution

$\frac{5}{12}$

Example 11

Have you ever thought about the sewage pipes going from your house to the street? They must slope down $\frac{1}{4}$ inch per foot in order to drain properly. What is the required slope?

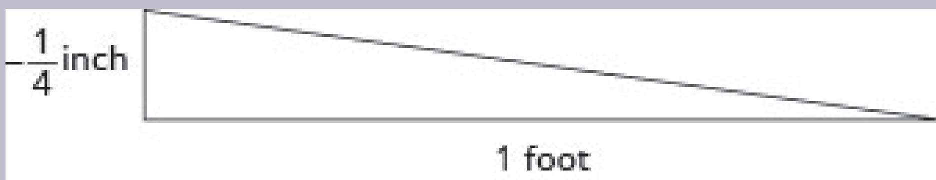

Figure 3.10.39

Solution

***Step 1:* Use the slope formula**

$$m = \frac{rise}{run}$$

$$m = \frac{-\frac{1}{4}inch}{1\ foot}$$

$$m = \frac{-\frac{1}{4}inch}{12\ inches}$$

Simplify. $$m = -\frac{1}{48}$$

The slope of the pipe is $-\frac{1}{48}$

The pipe drops 1 inch for every 48 inches of horizontal run.

Try It

27) Find the slope of a pipe that slopes down $\frac{1}{3}$ inch per foot.

Solution

$-\frac{1}{36}$

28) Find the slope of a pipe that slopes down $\frac{3}{4}$ inch per yard.

Solution

$-\frac{1}{48}$

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

We have graphed linear equations by plotting points, using intercepts, recognizing horizontal and vertical lines, and using the point-slope method. Once we see how an equation in slope–intercept form and its graph are related, we'll have one more method we can use to graph lines.

In Graph Linear Equations in Two Variables, we graphed the line of the equation $y = \frac{1}{2}x + 3$ by plotting points. See Figure 3.10.40. Let's find the slope of this line.

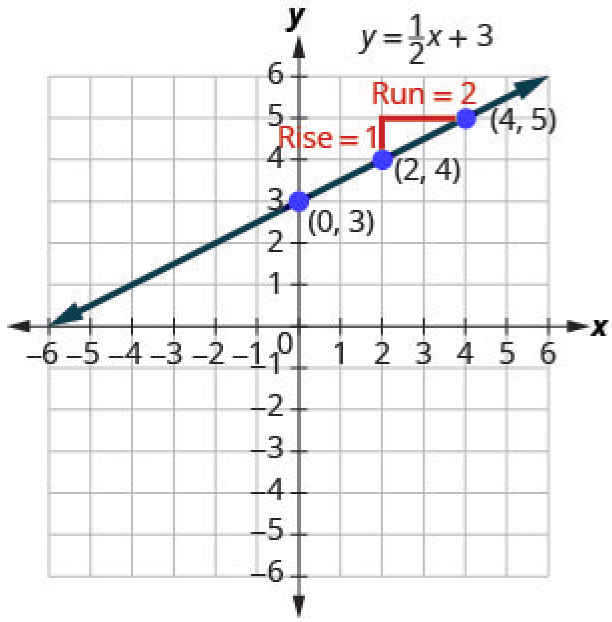

Figure 3.10.40

The red lines show us the rise is 1 and the run is 2. Substituting into the slope formula:

$$m = \frac{rise}{run}$$
$$m = \frac{1}{5}$$

What is the y-intercept of the line? The y-intercept is where the line crosses the y-axis, so y-intercept is $(0, 3)$. The equation of this line is:

$$y = \frac{1}{2}x + 3$$

Notice, that the line has:

Slope $m = \frac{1}{2}$

When a linear equation is solved for y, the coefficient of the x term is the slope and the constant term is the y-coordinate of the y-intercept. We say that the equation $y = \frac{1}{2}x + 3$ is in slope–intercept form.

Slope $m = \frac{1}{2}$

$$y = \frac{1}{2}x + 3$$
$$y = mx + b$$

Slope-Intercept Form of an Equation of a Line

The slope-intercept form of an equation of a line with slope m and y-intercept, $(0, b)$ is,

$$y = mx + b$$

Sometimes the slope-intercept form is called the "y-form."

Example 12

Use the graph to find the slope and y-intercept of the line, $y = 2x + 1$.

Compare these values to the equation $y = mx + b$.

Solution

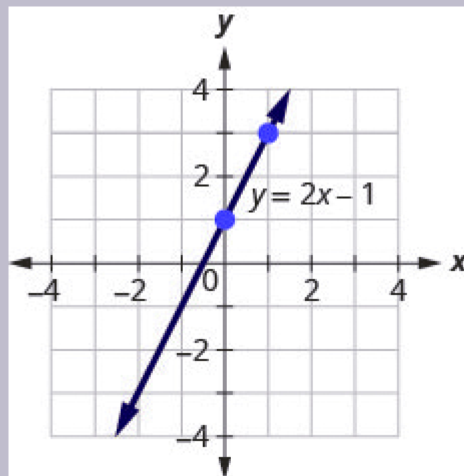

Figure 3.10.41

To find the slope of the line, we need to choose two points on the line. We'll use the points $(0, 1)$ and $(1, 3)$.

Step 1: **Find the rise and run.**

$$m = \frac{rise}{run}$$
$$m = \frac{2}{1}$$
$$m = 2$$

Step 2: **Find the y-intercept of the line.**

The y-intercept is the point $(0, 1)$.

Step 3: **We found the slope $m = 2$ and the y-intercept $(0, 1)$.**

$$y = 2x + 1$$
$$y = mx + b$$

The slope is the same as the coefficient of x and the y-coordinate of the y-intercept is the same as the constant term.

Try It

29) Use the graph to find the slope and y-intercept of the line $y = \frac{2}{3}x - 1$. Compare these values to the equation $y = mx + b$.

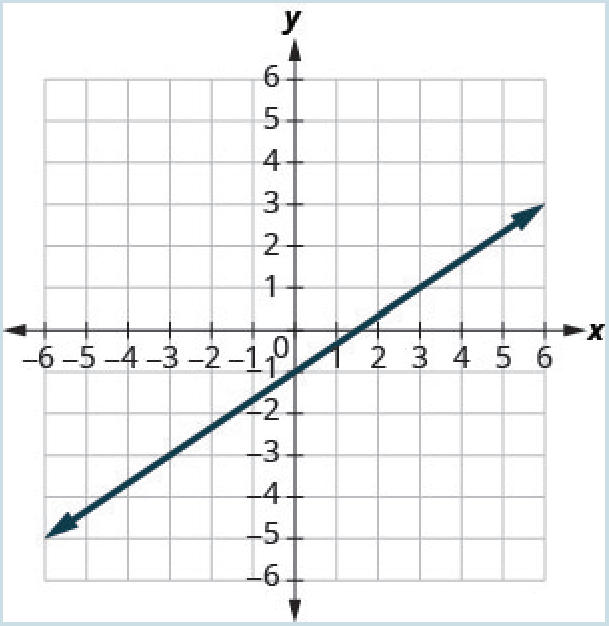

Figure 3.10.42

Solution

slope $m = \frac{2}{3}$ and y-intercept $(0, -1)$

30) Use the graph to find the slope and y-intercept of the line $y = \frac{1}{2}x + 3$. Compare these values to the equation $y = mx + b$.

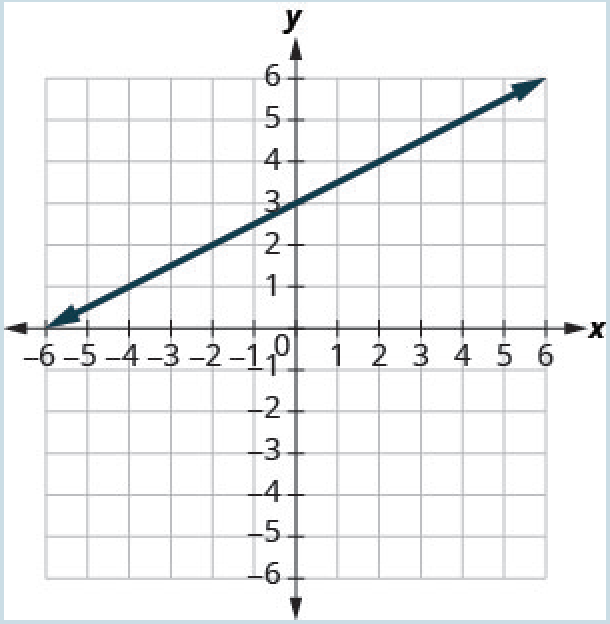

Figure 3.10.43

Solution

slope $m = \frac{1}{2}$ and y-intercept $(0, 3)$

Identify the Slope and y-Intercept From an Equation of a Line

In Understand Slope of a Line, we graphed a line using the slope and a point. When we are given an equation in slope–intercept form, we can use the y-intercept as the point, and then count out the slope from there.

Let's practice finding the values of the slope and y-intercept from the equation of a line.

Example 13

Identify the slope and y-intercept of the line with equation $y = -3x + 5$.

Solution

We compare our equation to the slope–intercept form of the equation.

$$y = mx + b$$

Step 1: **Write the equation of the line.**

$$y = -3x + 5$$

Step 2: **Identify the slope.**

$$m = -3$$

Step 3: **Identify the y-intercept.**

y-intercept is $(0, 5)$.

Try It

31) Identify the slope and y-intercept of the line $y = \frac{2}{5}x - 1$

Solution

$\frac{2}{5}$; $(0, -1)$

32) Identify the slope and y-intercept of the line $y = -\frac{4}{3}x + 1$

Solution

$-\frac{4}{3}$; $(0, 1)$

When an equation of a line is not given in slope–intercept form, our first step will be to solve the equation for y.

Example 14

Identify the slope and y-intercept of the line with equation $x + 2y = 6$.

Solution

This equation is not in slope-intercept form. To compare it to the slope-intercept form we must first solve the equation for y.

Step 1: **Solve for y.**

$$x + 2y = 6$$

Step 2: **Subtract x from each side.**

$$2y = -x + 6$$

Step 3: **Divide both sides by 2.**

$$\frac{2y}{2} = \frac{-x + 6}{2}$$

$$\text{Simplify.} \quad \frac{2y}{2} = \frac{-x}{2} + \frac{6}{2}$$

Remember: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$$y = -\frac{1}{2}x + 3$$

Step 6: **Write the slope-intercept form of the equation of the line.**

$$y = mx + b$$

Step 7: **Write the equation of the line.**

$$y = -\frac{1}{2}x + 3$$

Step 8: **Identify the slope.**

$$m = -\frac{1}{2}$$

Step 9: **Identify the y-intercept.**

y-intercept = $(0, 3)$

Try It

33) Identify the slope and y-intercept of the line $x + 4y = 8$.

Solution

$-\frac{1}{4}$; $(0, 2)$

34) Identify the slope and y-intercept of the line $3x + 2y = 12$.

Solution

$-\frac{3}{2}$; $(0, 6)$

Graph a Line Using its Slope and Intercept

Now that we know how to find the slope and y-intercept of a line from its equation, we can graph the line by plotting the y-intercept and then using the slope to find another point.

How to

Graph a line using its slope and y-intercept.

1. Find the slope-intercept form of the equation of the line.
2. Identify the slope and y-intercept.
3. Plot the y-intercept.
4. Use the slope formula $m = \frac{rise}{run}$ to identify the **rise** and the **run**.
5. Starting at the y-intercept, count out the rise and run to mark the second point.
6. Connect the points with a line.

Example 15

Graph the line of the equation $y = 4x - 2$ using its slope and y-intercept.

Solution

***Step 1:* Find the slope-intercept form of the equation.**

This equation is in slope-intercept form

$$y = 4x - 2$$

***Step 2:* Identify the slope and y-intercept.**

Use $y = mx + b$

Find the slope.

Find the y-intercept.

$$\begin{aligned} y &= mx + b \\ y &= 4x + (-2) \\ m &= 4 \\ b &= -2, (0, -2) \end{aligned}$$

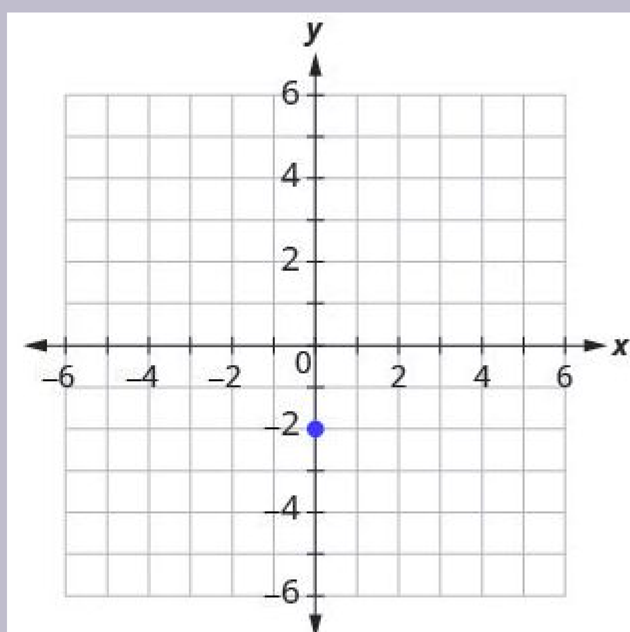

Figure 3.10.44

***Step 4:* Use the slope formula $m = \frac{rise}{run}$ to identify the rise and the run.**

Identify the rise and the run.

$$m = 4$$
$$\frac{rise}{run} = \frac{4}{1}$$
$$rise = 4$$
$$run = 1$$

***Step 5:* Starting at the y-intercept, count out the rise and run to mark the second point.**

Start at (0,-2) and count the rise and run.

Up 4, right 1.

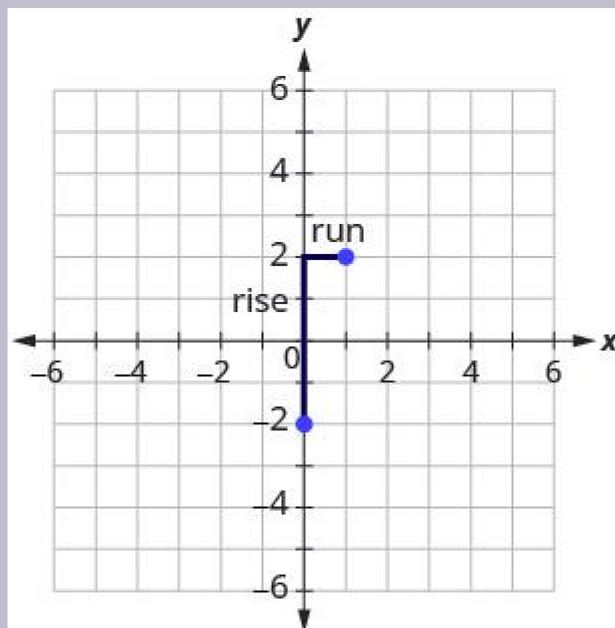

Figure 3.10.45

***Step 6:* Connect the points with a line.**

Connect the two points with a line.

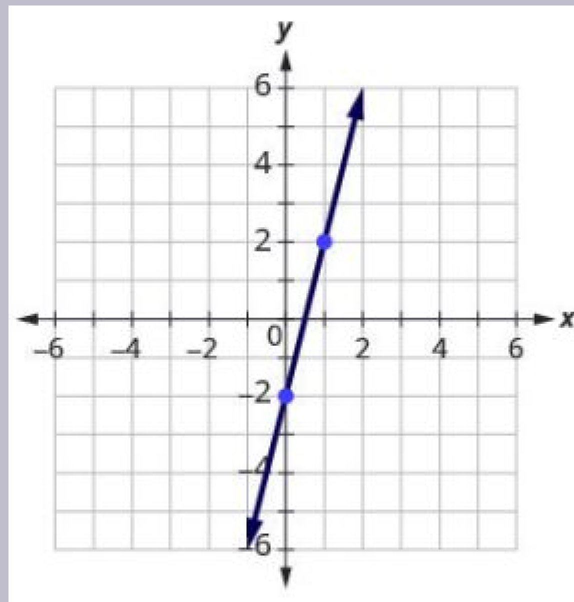

Figure 3.10.46

Try It

35) Graph the line of the equation $y = 4x + 1$ using its slope and y-intercept.

Solution

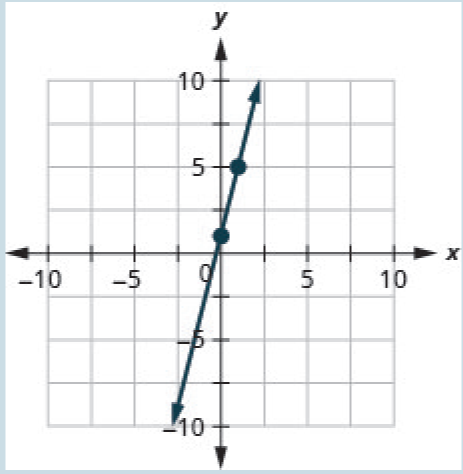

Figure 3.10.47

36) Graph the line of the equation $y = 2x - 3$ using its slope and y-intercept.

Solution

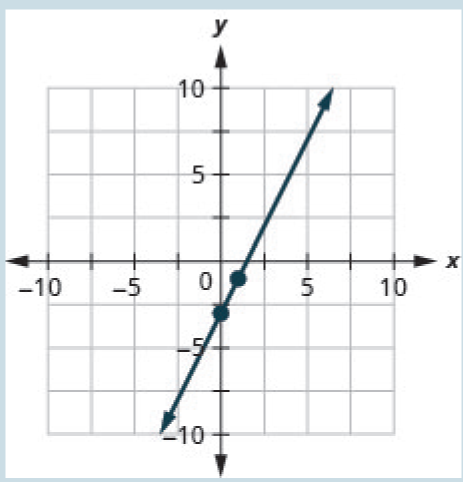

Figure 3.10.48

Example 16

Graph the line of the equation $y=-x+4$ using its slope and y-intercept.

Solution

$$y=mx+b$$

Step 1: **The equation is in slope–intercept form.**

$$y=-x+4$$

Step 2: **Identify the slope and y-intercept.**

$$m=-1$$

y-intercept is $(0,4)$

Step 3: **Plot the y-intercept.**

See graph below.

Step 4: **Identify the rise and the run.**

$$m=\frac{-1}{1}$$

Step 5: **Count out the rise and run to mark the second point.**

$rise=-1$, $run=1$

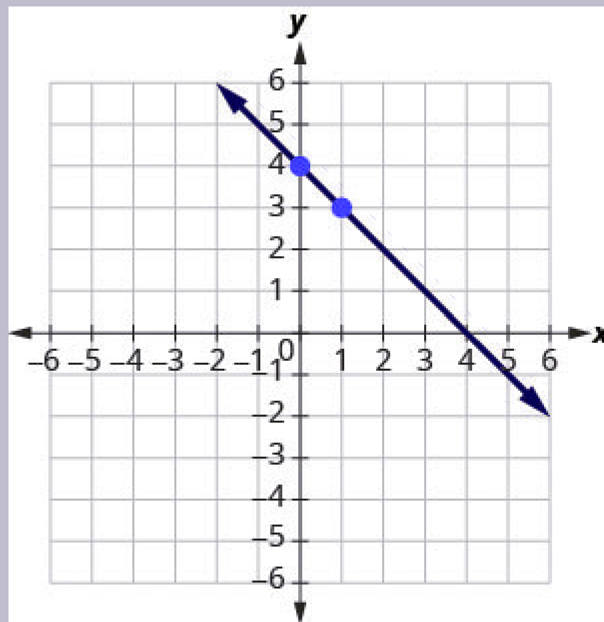

Figure 3.10.49

Step 6: **Draw the line.**

Step 7: **To check your work, you can find another point on the line and make sure it is a solution to the equation.**

In the graph, we see the line goes through $(4, 0)$.

Step 8: **Check.**

$$\begin{aligned} y &= -x + 4 \\ 0 &\stackrel{?}{=} -4 + 4 \\ 0 &= 0 \checkmark \end{aligned}$$

Try It

37) Graph the line of the equation $y = -x - 3$ using its slope and y-intercept.

Solution

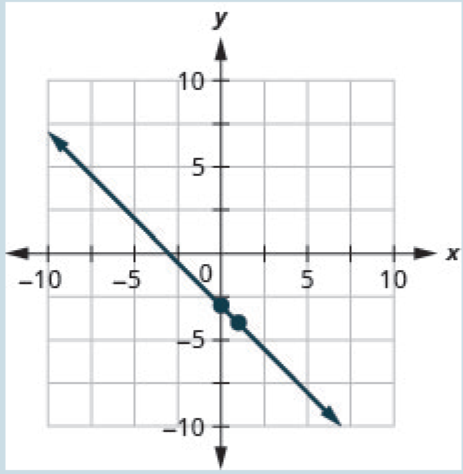

Figure 3.10.50

38) Graph the line of the equation $y = -x - 1$ using its slope and y-intercept.

Solution

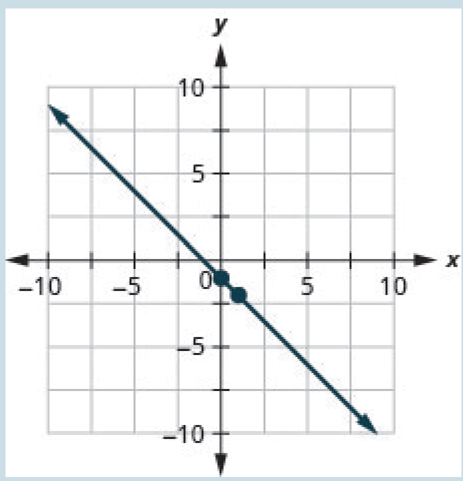

Figure 3.10.51

Example 17

Graph the line of the equation $y = -\frac{2}{3}x - 3$ using its slope and y-intercept.

Solution

$$y = mx + b$$

Step 1: **The equation is in slope-intercept form.**

$m = -\frac{2}{3}$; y-intercept is $(0, -3)$

Step 2: **Identify the slope and y-intercept.**

See graph below.

Step 3: **Plot the y-intercept.**

Step 4: **Identify the rise and the run.**

Step 5: **Count out the rise and run to mark the second point.**

Step 6: **Draw the line.**

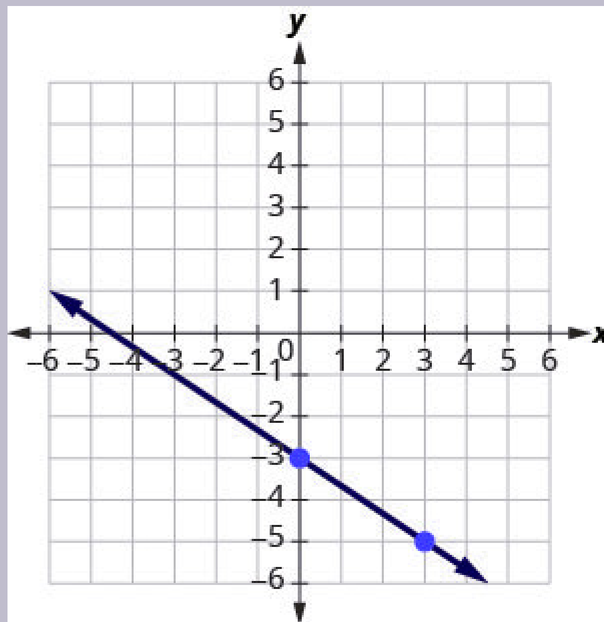

Figure 3.10.52

Try It

39) Graph the line of the equation $y=-\frac{5}{2}x+1$ using its slope and y-intercept.

Solution

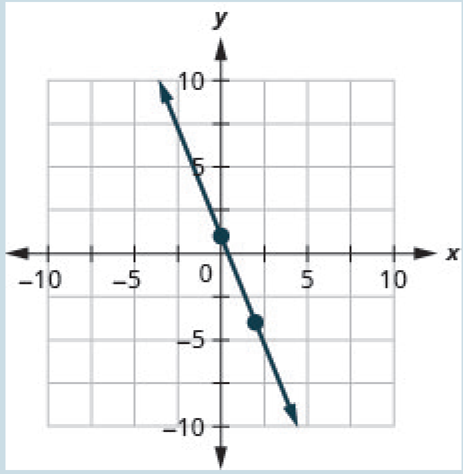

Figure 3.10.53

40) Graph the line of the equation $y=-\frac{3}{4}x-2$ using its slope and y-intercept.

Solution

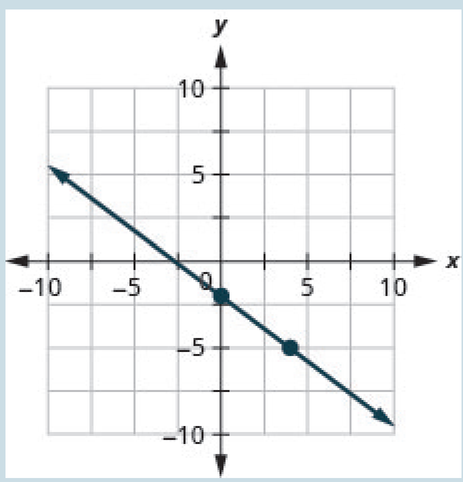

Figure 3.10.54

Example 18

Graph the line of the equation $4x - 3y = 12$ using its slope and y-intercept.

Solution

$$4x - 3y = 12$$

Step 1: **Find the slope-intercept form of the equation.**

$$\begin{aligned} 4x - 3y &= 12 \\ -3y &= -4x + 12 \\ -\frac{3y}{3} &= \frac{-4x+12}{-3} \end{aligned}$$

Step 2: **The equation is now in slope-intercept form.**

$$y = \frac{4}{3}x - 4$$

Step 3: **Identify the slope and y-intercept.**

$$m = \frac{4}{3}$$

y-intercept is $(0, -4)$

See graph below.

Step 4: **Plot the y-intercept.**

Step 5: **Identify the rise and the run; count out the rise and run to mark the second point.**

Step 6: **Draw the line.**

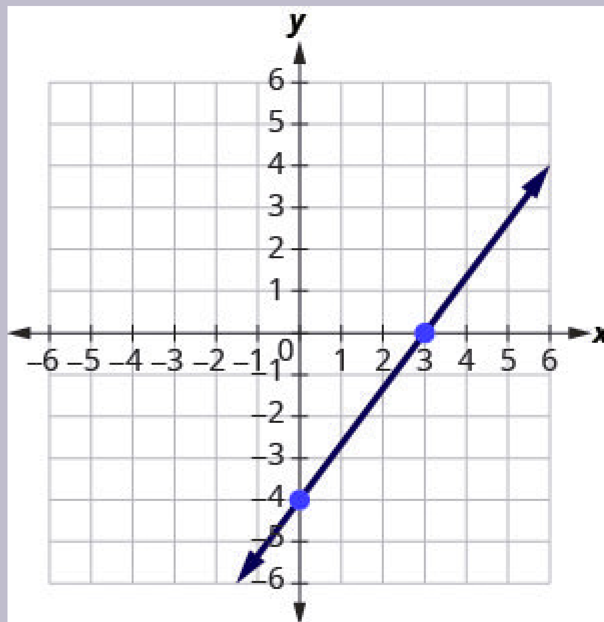

Figure 3.10.55

Try It

41) Graph the line of the equation $2x - y = 6$ using its slope and y-intercept.

Solution

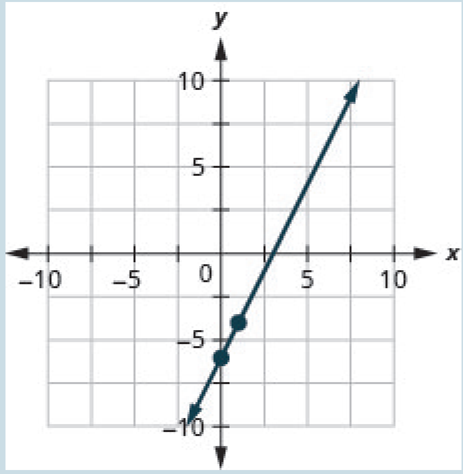

Figure 3.10.56

42) Graph the line of the equation $3x - 2y = 8$ using its slope and y-intercept.

Solution

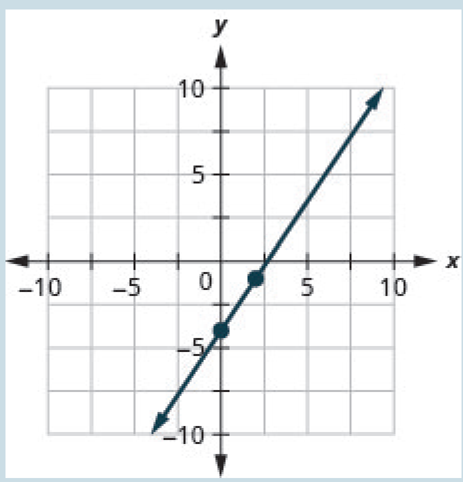

Figure 3.10.57

We have used a grid with x and y both going from about -10 to 10 for all the equations we've graphed so far. Not all linear equations can be graphed on this small grid. Often, especially in applications with real-world data, we'll need to extend the axes to bigger positive or smaller negative numbers.

Example 19

Graph the line of the equation $y = 0.2x + 45$ using its slope and y-intercept.

Solution

We'll use a grid with the axes going from about -80 to 80.

$$y = mx + b$$

***Step 1:* The equation is in slope-intercept form.**

$$y = 0.2x + 45$$

***Step 2:* Identify the slope and y-intercept.**

$$m = 0.2$$

The y-intercept is $(0, 45)$.

***Step 3:* Plot the y-intercept.**

See graph below.

***Step 4:* Count out the rise and run to mark the second point.**

The slope is $m = 0.2$; in fraction form this means $m = \frac{2}{10}$. Given the scale of our graph, it would be easier to use the equivalent fraction $m = \frac{10}{50}$.

***Step 5:* Draw the line.**

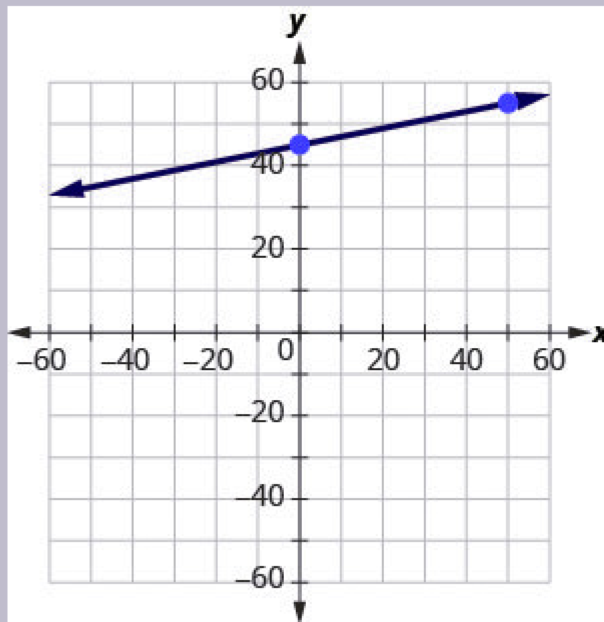

Figure 3.10.58

Try It

43) Graph the line of the equation $y = 0.5x + 25$ using its slope and y-intercept.

Solution

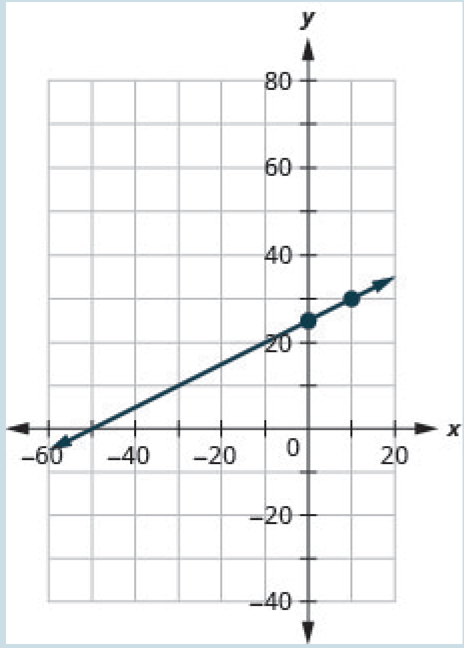

Figure 3.10.59

44) Graph the line of the equation $y = 0.1x - 30$ using its slope and y-intercept.

Solution

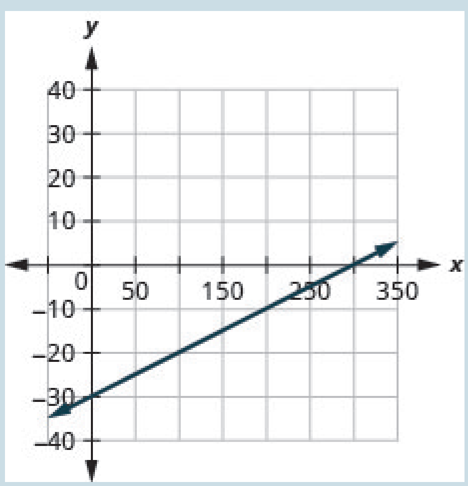

Figure 3.10.60

Now that we have graphed lines by using the slope and y-intercept, let's summarize all the methods we have used to graph lines. See Figure 3.10.61.

Methods to Graph Lines			
Point Plotting	**Slope–Intercept**	**Intercepts**	**Recognize Vertical and Horizontal Lines**
x | y (table with three blank rows)	$y = mx + b$	x | y; 0 | ; | 0; |	
Find three points. Plot the points, make sure they line up, then draw the line.	Find the slope and y-intercept. Start at the y-intercept, then count the slope to get a second point.	Find the intercepts and a third point. Plot the points, make sure they line up, then draw the line.	The equation has only one variable. $x = a$ vertical $y = b$ horizontal

Figure 3.10.61

Choose the Most Convenient Method to Graph a Line

Now that we have seen several methods we can use to graph lines, how do we know which method to use for a given equation?

While we could plot points, use the slope-intercept form, or find the intercepts for *any* equation, if we recognize the most convenient way to graph a certain type of equation, our work will be easier. Generally, plotting points is not the most efficient way to graph a line. We saw better methods in previous sections and earlier in this section. Let's look for some patterns to help determine the most convenient method to graph a line.

Here are six equations we graphed in this chapter, and the method we used to graph each of them.

	Equation	Method
#1	$x = 2$	Vertical Line
#2	$y = 4$	Horizontal Line
#3	$-x + 2y = 6$	Intercepts
#4	$4x - 3y = 12$	Intercepts
#5	$y = 4x - 2$	Slope-intercept
#6	$y = -x + 4$	Slope-intercept

Equations #1 and #2 each have just one variable. Remember, in equations of this form the value of that one variable is constant; it does not depend on the value of the other variable. Equations of this form have graphs that are vertical or horizontal lines.

In equations #3 and #4, both x and y are on the same side of the equation. These two equations are of the form $Ax + By = C$. We substituted $y = 0$ to find the x-intercept and $x = 0$ to find the y-intercept, and then found a third point by choosing another value for x or y.

Equations #5 and #6 are written in slope-intercept form. After identifying the slope and y-intercept from the equation we used them to graph the line.

This leads to the following strategy.

How to

Strategy for Choosing the Most Convenient Method to Graph a Line

Consider the form of the equation.

- **If it only has one variable, it is a vertical or horizontal line.**
  - $x = a$ is a vertical line passing through the x-axis at a.
  - $y = b$ is a horizontal line passing through the y-axis at b.
- **If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.**
  - Identify the slope and y-intercept and then graph.
- **If the equation is of the form $Ax + By = C$, find the intercepts**.
  - Find the x– and y-intercepts, a third point, and then graph.

Example 20

Determine the most convenient method to graph each line.

a. $y = -6$

b. $5x - 3y = 15$

c. $x = 7$

d. $y = \frac{2}{5}x - 1$

Solution

a. $y = -6$

This equation has only one variable, y. Its graph is a horizontal line crossing the y-axis at -6.

---

b. $5x - 3y = 15$

This equation is of the form $Ax + By = C$. The easiest way to graph it will be to find the intercepts and one more point.

---

c. $x = 7$

There is only one variable, x. The graph is a vertical line crossing the x-axis at 7.

---

d. $y = \frac{2}{5}x - 1$

Since this equation is in $y = mx + b$ form, it will be easiest to graph this line by using the slope and y-intercept.

Try It

45) Determine the most convenient method to graph each line:

a. $3x+2y=12$

b. $y=4$

c. $y=\frac{1}{5}x-4$

d. $x=-7$

Solution

a. intercepts

b. horizontal line

c. slope–intercept

d. vertical line

46) Determine the most convenient method to graph each line:

a. $x=6$

b. $y=-\frac{3}{4}x+1$

c. $y=-8$

d. $4x-3y=-1$

Solution

a. vertical line

b. slope–intercept

c. horizontal line

d. intercepts

Graph and Interpret Applications of Slope–Intercept

Many real-world applications are modeled by linear equations. We will take a look at a few applications here so you can see how equations written in slope–intercept form relate to real-world situations.

Usually, when a linear equation models a real-world situation, different letters are used for the variables, instead of x and y. The variable names remind us of what quantities are being measured.

Example 21

The equation $F = \frac{9}{5}C + 32$ is used to convert temperatures, C, on the Celsius scale to temperatures, F, on the Fahrenheit scale.

a. Find the Fahrenheit temperature for a Celsius temperature of 0.

b. Find the Fahrenheit temperature for a Celsius temperature of 20.

c. Interpret the slope and F-intercept of the equation.

d. Graph the equation.

Solution

a.

Step 1: **Find the Fahrenheit temperature for a Celsius temperature of 0.**

Step 2: **Find F when $C = 0$.**

$$F = \frac{9}{5}C + 32$$
$$F = \frac{9}{5} \times 0 + 32$$
$$F = 32$$

b.

Step 1: **Find the Fahrenheit temperature for a Celsius temperature of** 20**.**

Step 2: **Find** F **when** $C = 20$**.**

$$\begin{aligned} F &= \frac{9}{5}C + 32 \\ F &= \frac{9}{5}(20) + 32 \\ F &= 36 + 32 \\ F &= 68 \end{aligned}$$

c.

Interpret the slope and F-intercept of the equation. Even though this equation uses F and C, it is still in slope–intercept form.

$$\begin{aligned} y &= mx + b \\ F &= mC + b \\ F &= \frac{9}{5}C + 32 \end{aligned}$$

The slope, $\frac{9}{5}$, means that the temperature Fahrenheit (F) increases 9 degrees when the temperature Celsius (C) increases 5 degrees. The F-intercept means that when the temperature is (0) on the Celsius scale, it is 32° on the Fahrenheit scale.

d.

Graph the equation. We'll need to use a larger scale than our usual. Start at the F-intercept $(0, 32)$ then count out the rise of 9 and the run of 5 to get a second point. See Figure 3.10.62

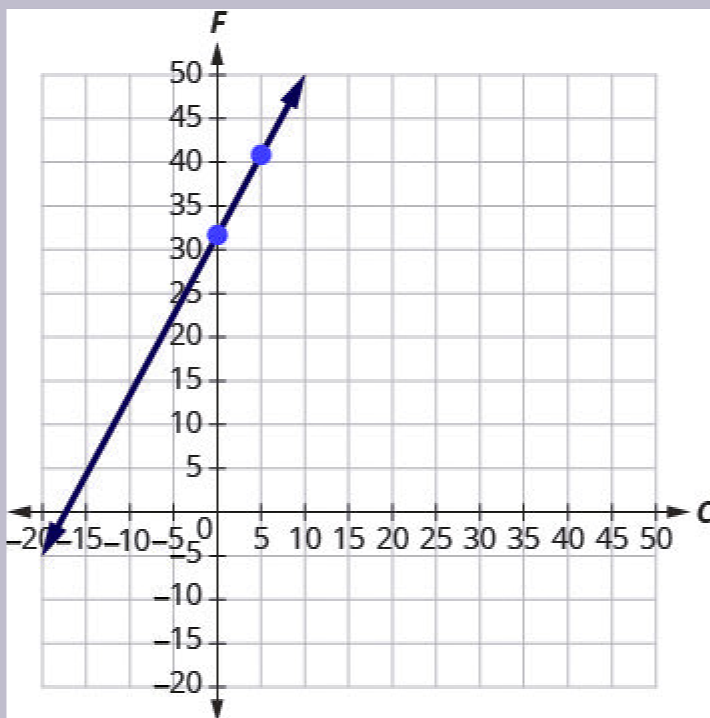

Figure 3.10.62

Try It

47) The equation $h = 2s + 50$ is used to estimate a woman's height in inches, h, based on her shoe size, s.

a. Estimate the height of a child who wears woman's shoe size 0.

b. Estimate the height of a woman with shoe size 8.

c. Interpret the slope and h-intercept of the equation.

d. Graph the equation.

Solution

a. 50 inches

b. 66 inches

c. The slope, 2, means that the height, h, increases by 2 inches when the shoe size,

s, increases by 1. The h-intercept means that when the shoe size is 0, the height is 50 inches.

d.

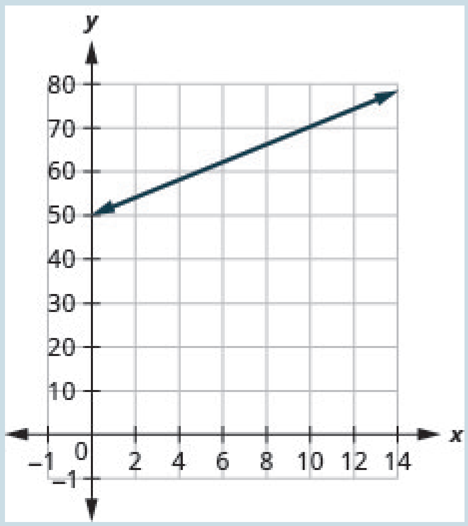

Figure 3.10.63

48) The equation $T = \frac{1}{4}n + 40$ is used to estimate the temperature in degrees Fahrenheit, T, based on the number of cricket chirps, n, in one minute.

a. Estimate the temperature when there are no chirps.
b. Estimate the temperature when the number of chirps in one minute is 100.

c. Interpret the slope and T-intercept of the equation.

d. Graph the equation.

Solution

a. 40 degrees

b. 65 degrees

c. The slope, $\frac{1}{4}$, means that the temperature Fahrenheit (F) increases 1 degree when the number of chirps, n, increases by 4. The T-intercept means that when the number of chirps is 0, the temperature is $40°$

d.

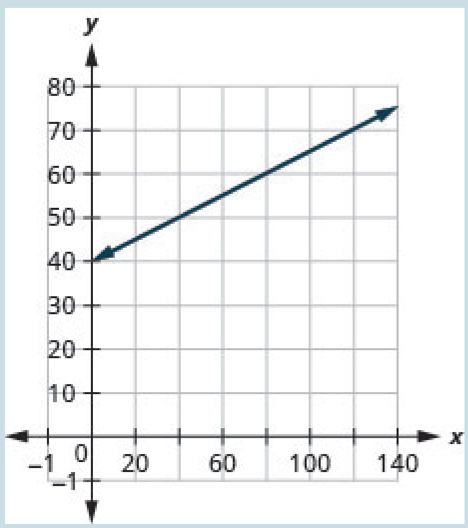

Figure 3.10.64

The cost of running some types of business has two components—a *fixed cost* and a *variable cost*. The fixed cost is always the same regardless of how many units are produced. This is the cost of rent, insurance, equipment, advertising, and other items that must be paid regularly. The variable cost depends on the number of units produced. It is for the material and labour needed to produce each item.

Example 22

Stella has a home business selling gourmet pizzas. The equation $C = 4p + 25$ models the relation between her weekly cost, C, in dollars and the number of pizzas, p, that she sells.

a. Find Stella's cost for a week when she sells no pizzas.

b. Find the cost for a week when she sells 15 pizzas.

c. Interpret the slope and C-intercept of the equation.

d. Graph the equation.

Solution

a.

Step 1: **Find Stella's cost for a week when she sells no pizzas.**

$$C = 4p + 25$$

Step 2: **Find C when $p = 0$.**

$$\begin{array}{ll} & C = 4p + 25 \\ & C = 4(0) + 25 \\ \text{Simplify.} & C = 25 \end{array}$$

Stella's fixed cost is $\$25$ when she sells no pizzas.

---

b.

Step 1: **Find the cost for a week when she sells 15 pizzas.**

$$C = 4p + 25$$

Step 2: **Find C when $p = 15$.**

$$\begin{array}{ll} & C = 4p + 25 \\ & C = 4(15) + 25 \\ \text{Simplify.} & C = 60 + 25 \\ & C = 85 \end{array}$$

Stella's costs are $\$85$ when she sells 15 pizzas.

---

c.

Step 1: **Interpret the slope and C-intercept of the equation.**

$$y = mx + b$$
$$C = 4p + 25$$

The slope, 4, means that the cost increases by $\$4$ for each pizza Stella sells. The C-intercept means that even when Stella sells no pizzas, her costs for the week are $\$25$.

---

d.

Step 1: **Graph the equation.**

We'll need to use a larger scale than our usual. Start at the C-intercept $(0, 25)$ then count out the rise of 4 and the run of 1 to get a second point

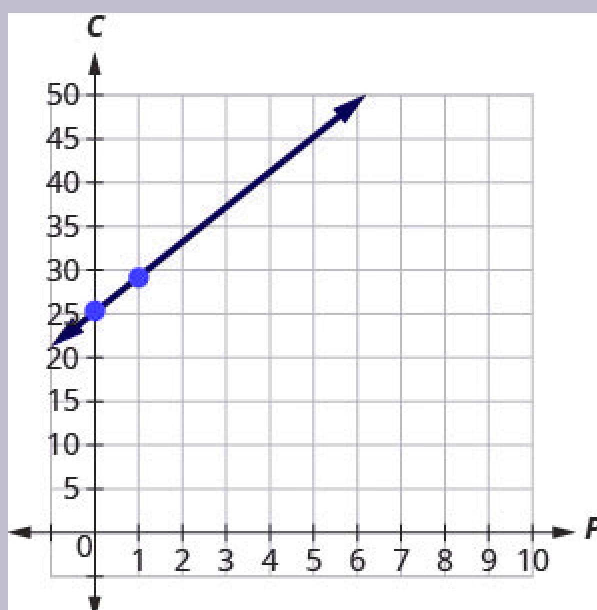

Figure 3.10.65

Try It

49) Sam drives a delivery van. The equation $C = 0.5m + 60$ models the relation between his weekly cost, C, in dollars and the number of miles, m, that he drives.

a. Find Sam's cost for a week when he drives 0 miles.

b. Find the cost for a week when he drives 250 miles.

c. Interpret the slope and C-intercept of the equation.

d. Graph the equation.

Solution

a. $\$60$

b. $\$185$

c. The slope, 0.5, means that the weekly cost, C, increases by $\$0.50$ when the number of miles driven, n, increases by 1. The C-intercept means that when the number of miles driven is 0, the weekly cost is $\$60$.

d.

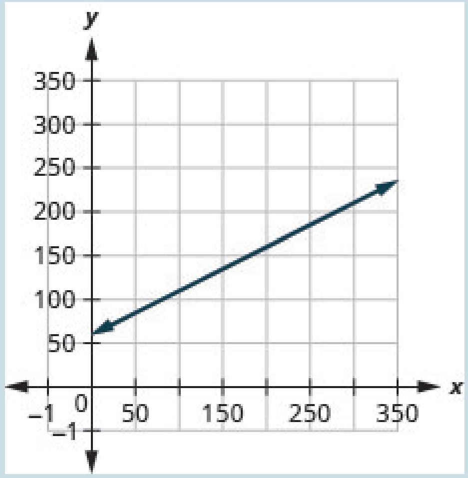

Figure 3.10.66

50) Loreen has a calligraphy business. The equation $C = 1.8n + 35$ models the relation between her weekly cost, C, in dollars and the number of wedding invitations, n, that she writes.

a. Find Loreen's cost for a week when she writes no invitations.

b. Find the cost for a week when she writes 75 invitations.

c. Interpret the slope and C-intercept of the equation.

d. Graph the equation.

Solution

a. $\$35$

b. $\$170$

c. The slope, 1.8, means that the weekly cost, C, increases by $\$1.80$ when the number of invitations, n, increases by 1.80.

The C-intercept means that when the number of invitations is 0, the weekly cost is $\$35$.

d.

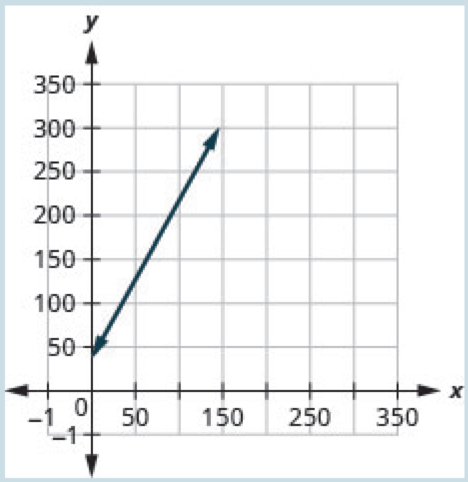

Figure 3.10.67

Use Slopes to Identify Parallel Lines

The slope of a line indicates how steep the line is and whether it rises or falls as we read it from left to right. Two lines that have the same slope are called **parallel lines**. Parallel lines never intersect.

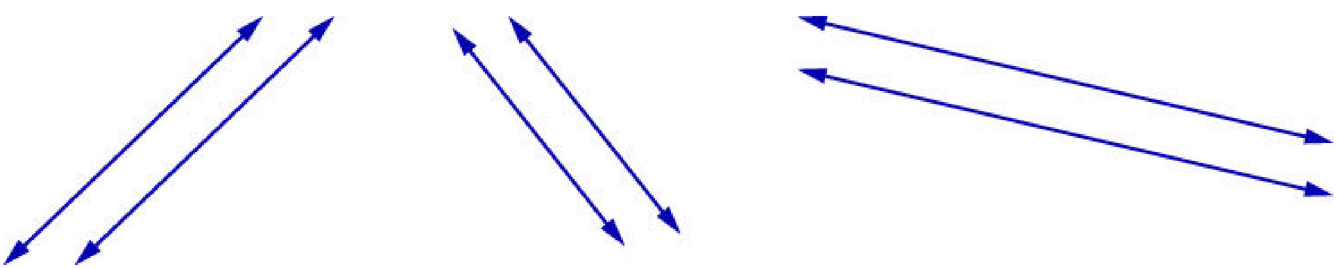

Figure 3.10.68

We say this more formally in terms of the rectangular coordinate system. Two lines that have the same slope and different y-intercepts are called parallel lines. See Figure 3.10.69.

Verify that both lines have the same slope, $m = \frac{2}{5}$, and different y-intercepts.

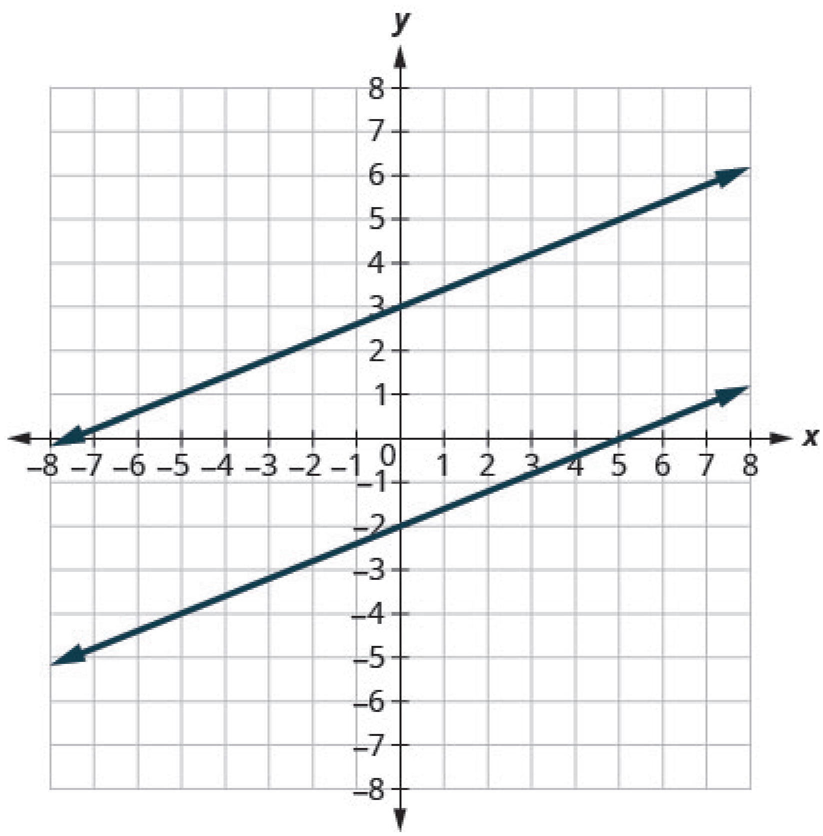

Figure 3.10.69

What about vertical lines? The slope of a vertical line is undefined, so vertical lines don't fit in the definition above. We say that vertical lines that have different x-intercepts are parallel. See Figure 3.10.70.

Vertical lines with different x-intercepts are parallel.

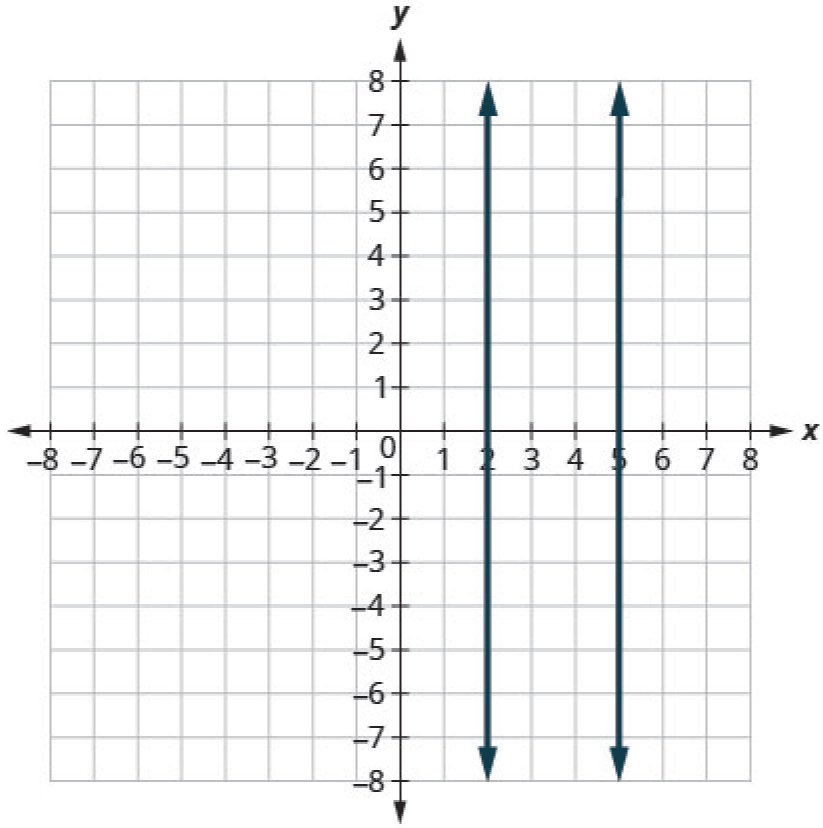

Figure 3.10.70

Parallel Lines

Parallel lines are lines in the same plane that do not intersect.

- Parallel lines have the same slope and different y-intercepts.
- If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
- Parallel vertical lines have different x-intercepts.

Let's graph the equations $y = -2x + 3$ and $2x + y = -1$ on the same grid. The first equation is already in slope-intercept form: $y = -2x + 3$. We solve the second equation for y:

$$2x + y = -1$$

$$y = -2x - 1$$

Graph the lines.

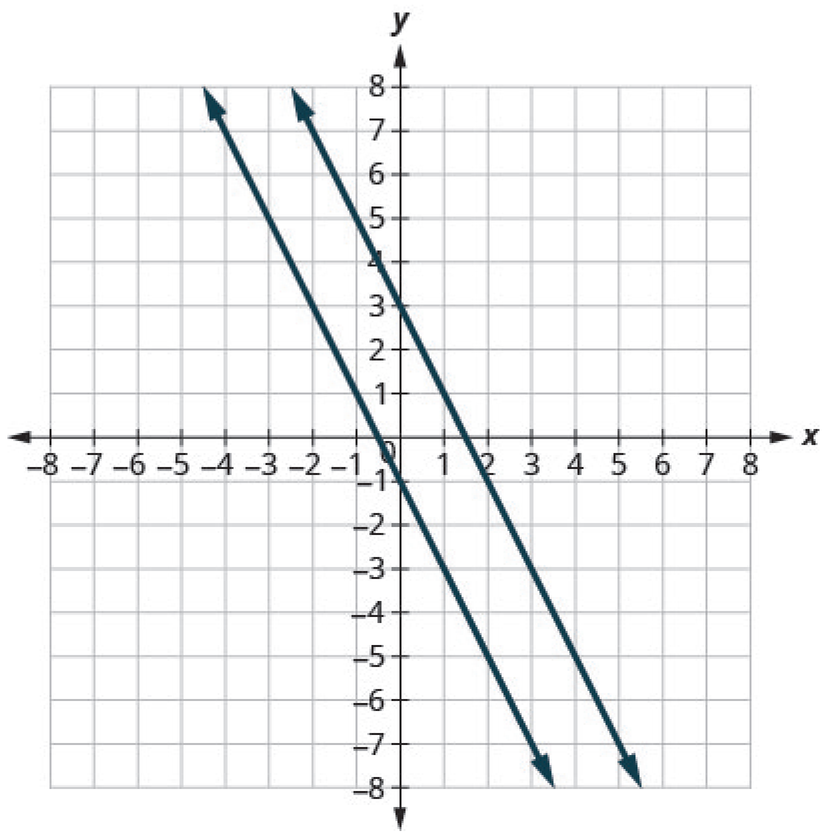

Figure 3.10.71

Notice the lines look parallel. What is the slope of each line? What is the y-intercept of each line?

$y = mx + b$	$y = mx + b$
$y = -2x + 3$	$y = -2x - 1$
$m = -2$	$m = -2$
$b = 3, (0, 3)$	$b = -1, (0, -1)$

The slopes of the lines are the same and the y-intercept of each line is different. So we know these lines are parallel.

Since parallel lines have the same slope and different y-intercepts, we can now just look at the slope-intercept form of the equations of lines and decide if the lines are parallel.

Example 23

Use slopes and y-intercepts to determine if the lines $3x - 2y = 6$ and $y = \frac{3}{2}x + 1$ are parallel.

Solution

Step 1: **Solve the first equation for y.**

$\begin{aligned} 3x - 2y &= 6 \\ -2y &= -3x + 6 \\ \frac{-2y}{-2} &= \frac{3x + 6}{-2} \\ y &= \frac{3}{2}x - 3 \end{aligned}$	$y = \frac{3}{2}x + 1$

Step 2: **The equation is now in slope-intercept form.**

$y = \frac{3}{2}x - 3$	

Step 3: **The equation of the second line is already in slope-intercept form.**

	$y = \frac{3}{2}x + 1$

Step 4: **Identify the slope and y-intercept of both lines.**

$y = mx + b$ $m = \frac{3}{2}$ $y = \frac{3}{2}x - 3$	$y = mx + b$ $m = \frac{3}{2}$ $y = \frac{3}{2}x + 1$ $y = \frac{3}{2}x + 1$
y-intercept is $(0, -3)$	y-intercept is $(0, 1)$

The lines have the same slope and different y-intercepts and so they are parallel. You may want to graph the lines to confirm whether they are parallel.

Try It

51) Use slopes and y-intercepts to determine if the lines $2x + 5y = 5$ and $y = -\frac{2}{5}x - 4$ are parallel.

Solution

parallel

52) Use slopes and y-intercepts to determine if the lines $4x-3y=6$ and $y=\frac{4}{3}x-1$ are parallel.

Solution

parallel

Example 24

Use slopes and y-intercepts to determine if the lines $y=-4$ and $y=3$ are parallel.

Solution

$y=-4$	$y=3$

Step 1: **Write each equation in slope-intercept form.**

Since there is no x term we write $0x$.

$y=0x-4$	$y=0x+3$

Step 2: **Identify the slope and y-intercept of both lines.**

$m = 0$	$m = 0$
y-intercept is $(0, 4)$	y-intercept is $(0, 3)$

The lines have the same slope and different y-intercepts so they are parallel.

There is another way you can look at this example. If you recognize right away from the equations that these are horizontal lines, you know their slopes are both 0. Since the horizontal lines cross the y-axis at $y = -4$ and at $y = 3$, we know the y-intercepts are $(0, -4)$ and $(0, 3)$. The lines have the same slope and different y-intercepts and so they are parallel.

Try It

53) Use slopes and y-intercepts to determine if the lines $y = 8$ and $y = -6$ are parallel.

Solution
parallel

54) Use slopes and y-intercepts to determine if the lines $y = 1$ and $y = -5$ are parallel.

Solution

parallel

Example 25

Use slopes and y-intercepts to determine if the lines $x = -2$ and $x = -5$ are parallel.

Solution

$$x = -2 \text{ and } x = -5$$

Since there is no y, the equations cannot be put in slope-intercept form. But we recognize them as equations of vertical lines. Their x-intercepts are -2 and -5. Since their x-intercepts are different, the vertical lines are parallel.

Try It

55) Use slopes and y-intercepts to determine if the lines $x = 1$ and $x = -5$ are parallel.

Solution
parallel

56) Use slopes and y-intercepts to determine if the lines $x = 8$ and $x = -6$ are parallel.

Solution
parallel

Example 26

Use slopes and y-intercepts to determine if the lines $y = 2x - 3$ and $-6x + 3y = -9$ are parallel. You may want to graph these lines, too, to see what they look like.

Solution

$y = 2x - 3$	$-6x + 3y = -9$

***Step 1:* The first equation is already in slope-intercept form.**

$y = 2x - 3$	

Step 2: **Solve the second equation for y.**

	$\begin{aligned} -6x + 3y &= -9 \\ 3y &= 6x - 9 \\ \frac{3y}{3} &= \frac{6x-9}{3} \\ y &= 2x - 3 \end{aligned}$

Step 3: **The second equation is now in slope-intercept form.**

$y = 2x - 3$	$y = 2x - 3$

Step 4: **Identify the slope and y-intercept of both lines.**

$y = 2x - 3$ $y = mx + b$ $m = 2$	$y = 2x - 3$ $y = mx + b$ $m = 2$
y-intercept is $(0, -3)$	y-intercept is $(0, -3)$

The lines have the same slope, but they also have the same y-intercepts. Their equations represent the same line. They are not parallel; they are the same line.

Try It

57) Use slopes and y-intercepts to determine if the lines $y=-\frac{1}{2}x+1$ and $x+2y=2$ are parallel.

Solution
not parallel; same line

58) Use slopes and y-intercepts to determine if the lines $y=\frac{3}{4}x-3$ and $3x-4y=12$ are parallel.

Solution
not parallel; same line

Use Slopes to Identify Perpendicular Lines

Let's look at the lines whose equations are $y=\frac{1}{4}x-1$ and $y=-4x+2$, shown in Figure 3.10.72.

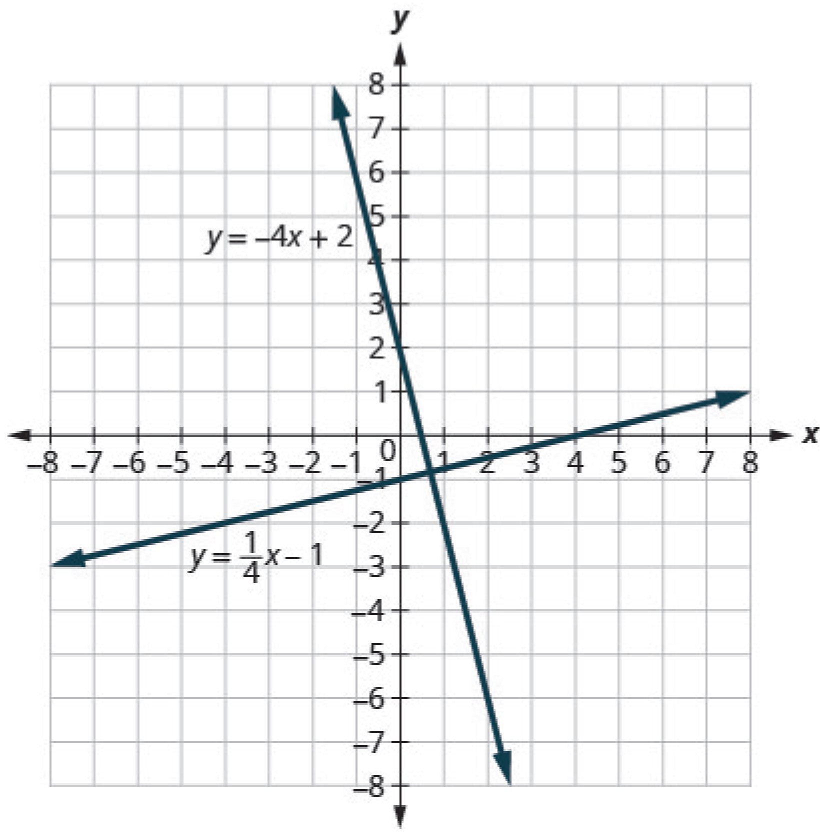

Figure 3.10.72

These lines lie in the same plane and intersect at right angles. We call these lines **perpendicular**.

What do you notice about the slopes of these two lines? As we read from left to right, the line $y = \frac{1}{4}x - 1$ rises, so its slope is positive. The line $y = -4x + 2$ drops from left to right, so it has a **negative slope**. Does it make sense to you that the slopes of two perpendicular lines will have opposite signs?

If we look at the slope of the first line, $m_1 = \frac{1}{4}$, and the slope of the second line, $m_2 = -4$, we can see that they are *negative reciprocals* of each other. If we multiply them, their product is -1.

$$m_1 \cdot m_2$$
$$\frac{1}{4}(-4)$$
$$-1$$

This is always true for perpendicular lines and leads us to this definition.

Perpendicular Lines

Perpendicular lines are lines in the same plane that form a right angle.

If m_1 and m_2 are the slopes of two perpendicular lines, then:

$m_1 \cdot m_2 = -1$ and $m_1 = -\frac{1}{m_2}$

Vertical lines and horizontal lines are always perpendicular to each other.

We were able to look at the slope-intercept form of linear equations and determine whether or not the lines were parallel. We can do the same thing for perpendicular lines.

We find the slope-intercept form of the equation and then see if the slopes are negative reciprocals. If the product of the slopes is -1, the lines are perpendicular. Perpendicular lines may have the same y-intercepts.

Example 27

Use slopes to determine if the lines, $y = -5x - 4$ and $x - 5y = 5$ are perpendicular.

Solution

***Step 1:* The first equation is already in slope-intercept form.**

$y = -5x - 4$	

***Step 2:* Solve the second equation for y.**

	$x - 5y = 5$ $-5y = x + 5$ $\frac{-5y}{-5} = \frac{x+5}{-5}$ $y = \frac{1}{-5}x - 1$

Step 3: **Identify the slope of each line.**

$y = -5x - 4$ $y = mx + b$ $m_1 = -5$	$y = \frac{1}{5}x - 4$ $y = mx + b$ $m_2 = \frac{1}{5}$

The slopes are negative reciprocals of each other, so the lines are perpendicular. We check by multiplying the slopes,

$$\begin{aligned} &m_1 \cdot m_2 \\ &= -5(\tfrac{1}{5}) \\ &= -1 \end{aligned}$$

Try It

59) Use slopes to determine if the lines $y = -3x + 2$ and $x - 3y = 4$ are perpendicular.

Solution

perpendicular

60) Use slopes to determine if the lines $y = 2x - 5$ and $x + 2y = -6$ are perpendicular.

Solution

perpendicular

Example 28

Use slopes to determine if the lines, $7x + 2y = 3$ and $2x + 7y = 5$ are perpendicular.

Solution

Step 1: **Solve the equations for y.**

$7x + 2y = 3$ $2y = -7x + 3$ $\frac{2y}{2} = \frac{-7x+3}{2}$ $y = \frac{-7}{2}x + \frac{3}{2}$	$2x + 7y = 5$ $7y = -2x + 5$ $\frac{7y}{7} = \frac{-2x+5}{7}$ $y = \frac{-2}{7}x + \frac{5}{7}$

Step 2: **Identify the slope of each line.**

$y = mx + b$ $m_1 = -\frac{7}{2}$	$y = mx + b$ $m_2 = -\frac{2}{7}$

The slopes are reciprocals of each other, but they have the same sign. Since they are not negative reciprocals, the lines are not perpendicular.

Try It

61) Use slopes to determine if the lines $5x + 4y = 1$ and $4x + 5y = 3$ are perpendicular.

Solution

not perpendicular

62) Use slopes to determine if the lines $2x - 9y = 3$ and $9x - 2y = 1$ are perpendicular.

Solution

not perpendicular

Key Concepts

- **Find the Slope of a Line from its Graph using** $m = \frac{rise}{run}$

1. Locate two points on the line whose coordinates are integers.
2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
3. Count the rise and the run on the legs of the triangle.
4. Take the ratio of rise to run to find the slope.

- **Graph a Line Given a Point and the Slope**

1. Plot the given point.
2. Use the slope formula $m = \frac{rise}{run}$ to identify the rise and the run.
3. Starting at the given point, count out the rise and run to mark the second point.
4. Connect the points with a line.

- **Slope of a Horizontal Line**
    - The slope of a horizontal line, $y = b$, is 0.

- **Slope of a vertical line**
    - The slope of a vertical line, $x = a$, is undefined

Graph a Line Using its Slope and *y*-Intercept

1. The slope-intercept form of an equation of a line with slope m and y-intercept, $(0, b)$ is, $y = mx + b$.
2. Find the slope-intercept form of the equation of the line.

3. Identify the slope and y-intercept.
4. Plot the y-intercept.
5. Use the slope formula $m = \frac{rise}{run}$ to identify the rise and the run.
6. Starting at the y-intercept, count out the rise and run to mark the second point.
7. Connect the points with a line.

- **Strategy for Choosing the Most Convenient Method to Graph a Line:** Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.

    $x = a$ is a vertical line passing through the x-axis at a.

    $y = b$ is a horizontal line passing through the y-axis at b.
  - If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.

    Identify the slope and y-intercept and then graph.
  - If the equation is of the form $Ax + By = C$, find the intercepts.

Find the x-intercept and y-intercept, a third point, and then graph.

- **Parallel lines** are lines in the same plane that do not intersect.
    - Parallel lines have the same slope and different y-intercepts.
        - If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
    - Parallel vertical lines have different x-intercepts.
- **Perpendicular lines** are lines in the same plane that form a right angle.
    - If m_1 and m_2 are the slopes of two perpendicular lines, then $m_1 \cdot m_2 = -1$ and $m_1 = -\frac{1}{m_2}$
    - Vertical lines and horizontal lines are always perpendicular to each other.

Glossary

geoboard
A geoboard is a board with a grid of pegs on it.

negative slope
A negative slope of a line goes down as you read from left to right.

positive slope
A positive slope of a line goes up as you read from left to right.

rise
The rise of a line is its vertical change.

run
The run of a line is its horizontal change.

slope formula
The slope of the line between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$

slope of a line
The slope of a line is $m = \frac{rise}{run}$. The rise measures the vertical change and the run measures the horizontal change.

parallel lines
Lines in the same plane that do not intersect.

perpendicular lines
Lines in the same plane that form a right angle.

slope-intercept form of an equation of a line
The slope–intercept form of an equation of a line with slope m and y-intercept, $(0, b)$ is, $y = mx + b$.

Exercises: Use $m = \frac{rise}{run}$ to find the Slope of a Line from its Graph

Instructions: For questions 1-16, find the slope of each line shown.

1.

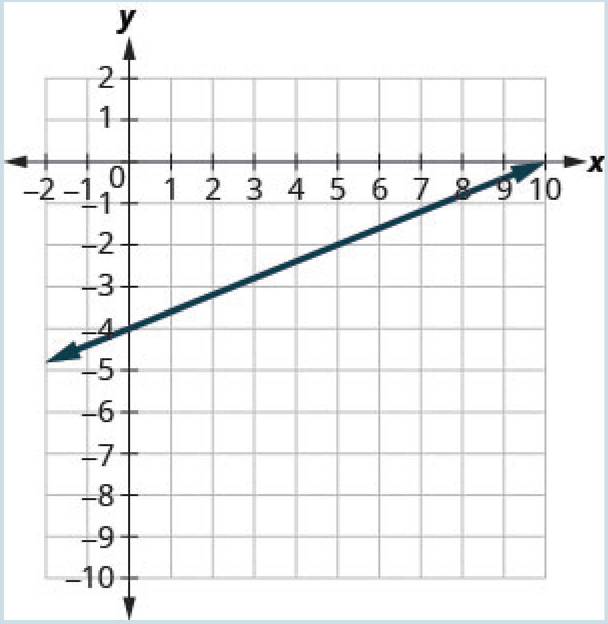

Figure 3P.10.1

Solution

$$\frac{2}{5}$$

2.

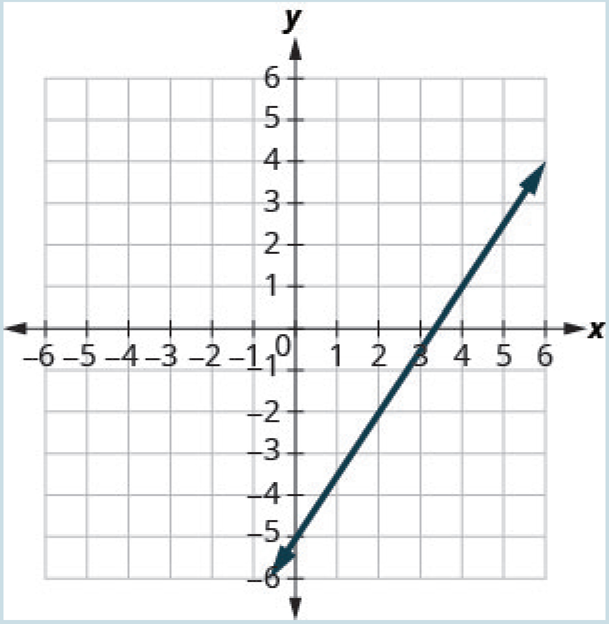

Figure 3P.10.2

3.

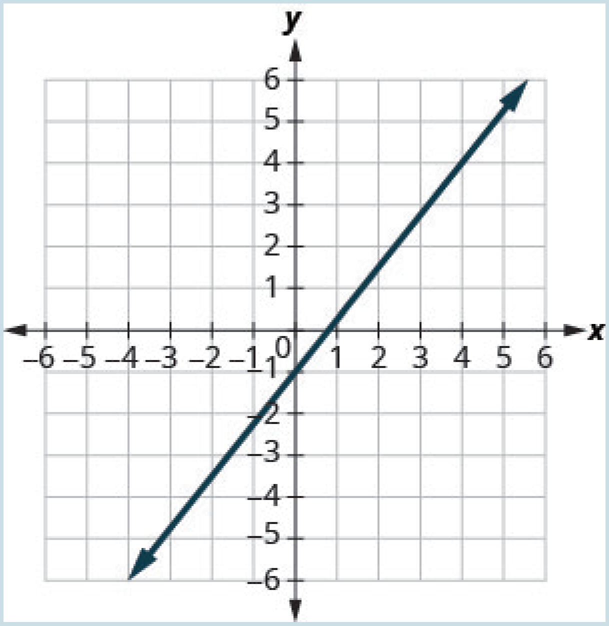

Figure 3P.10.3

Solution

$$\frac{5}{4}$$

4.

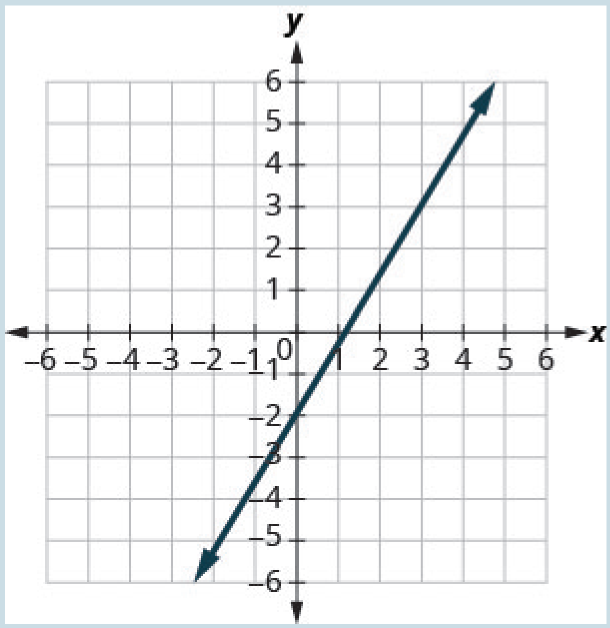

Figure 3P.10.4

5.

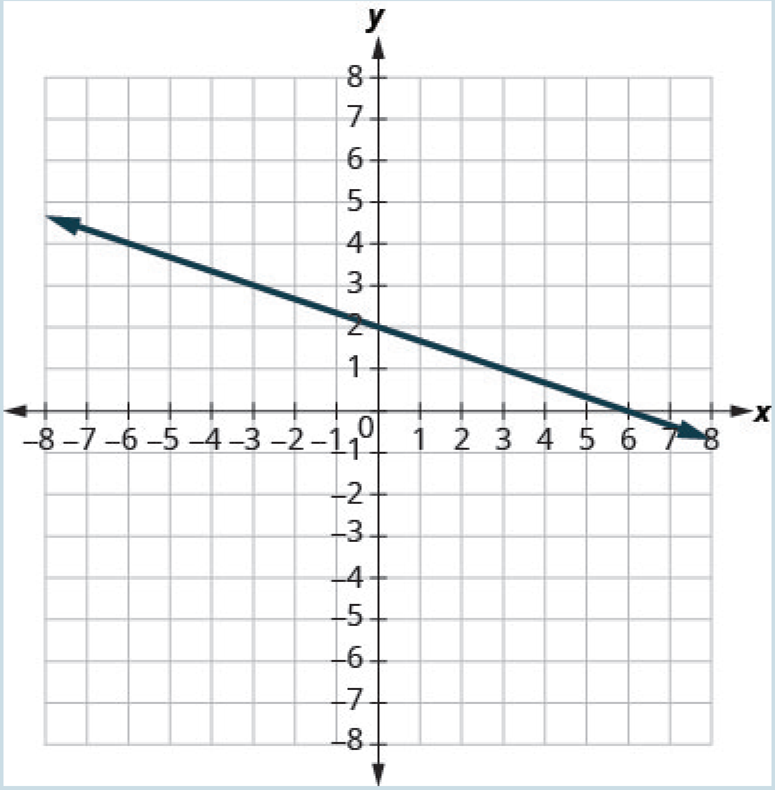

Figure 3P.10.5

Solution

$$-\frac{1}{3}$$

6.

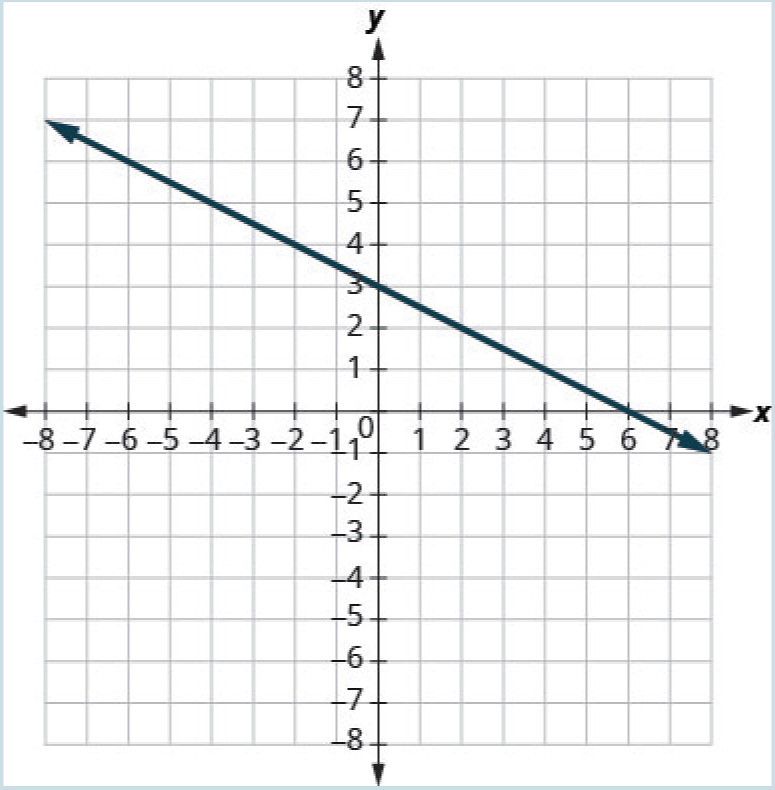

Figure 3P.10.6

7.

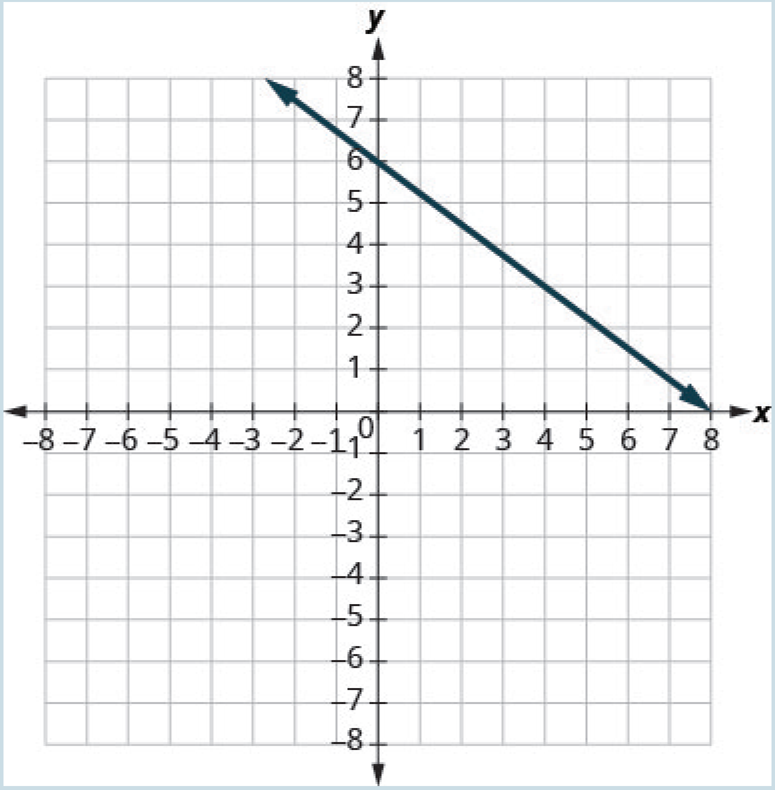

Figure 3P.10.7

Solution

$-\frac{3}{4}$

8.

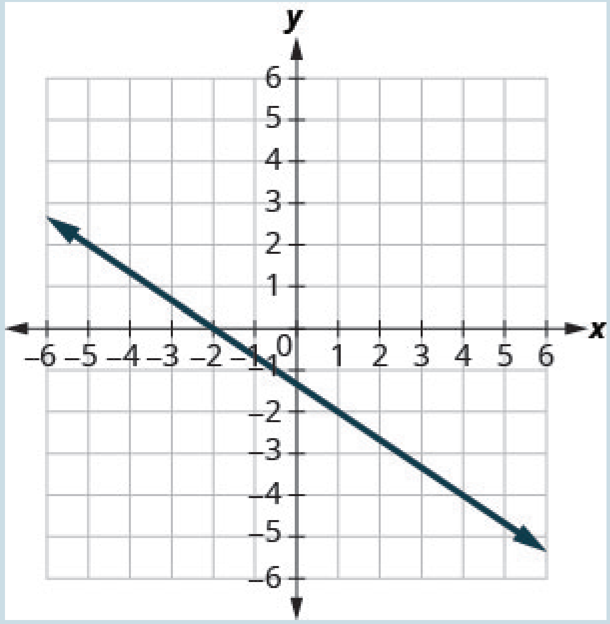

Figure 3P.10.8

9.

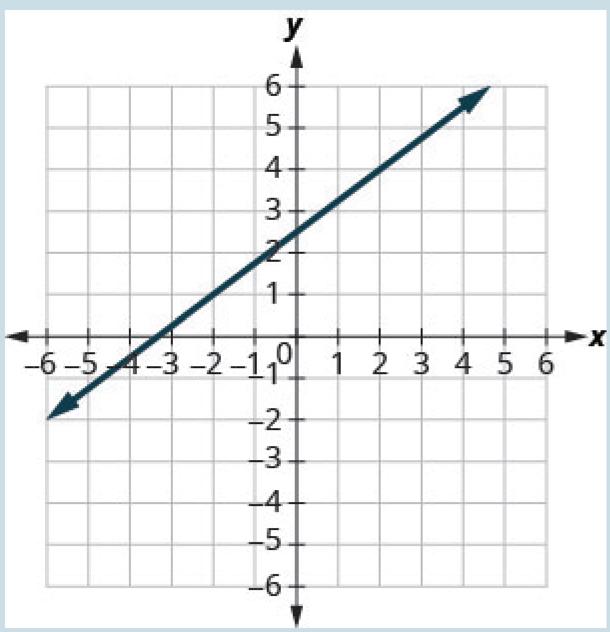

Figure 3P.10.9

Solution

$$\frac{3}{4}$$

10.

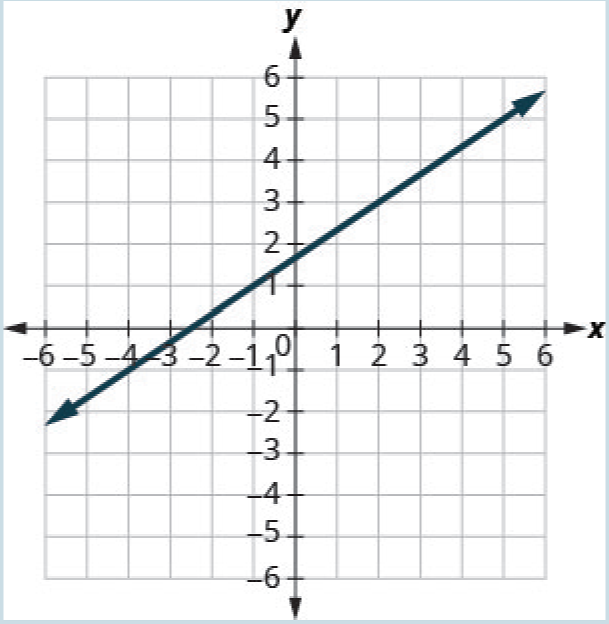

Figure 3P.10.10

11.

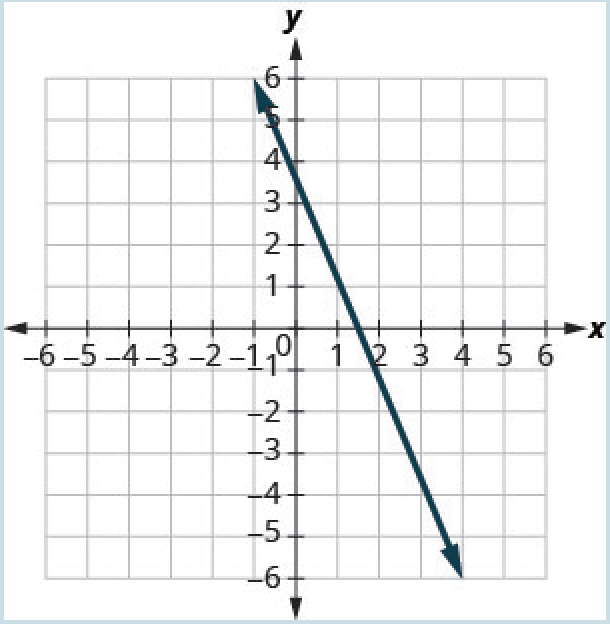

Figure 3P.10.11

Solution

$$-\frac{5}{2}$$

12.

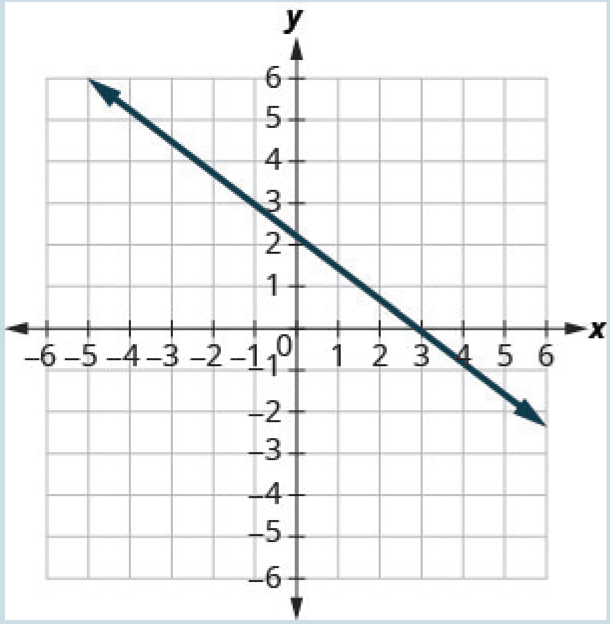

Figure 3P.10.12

13.

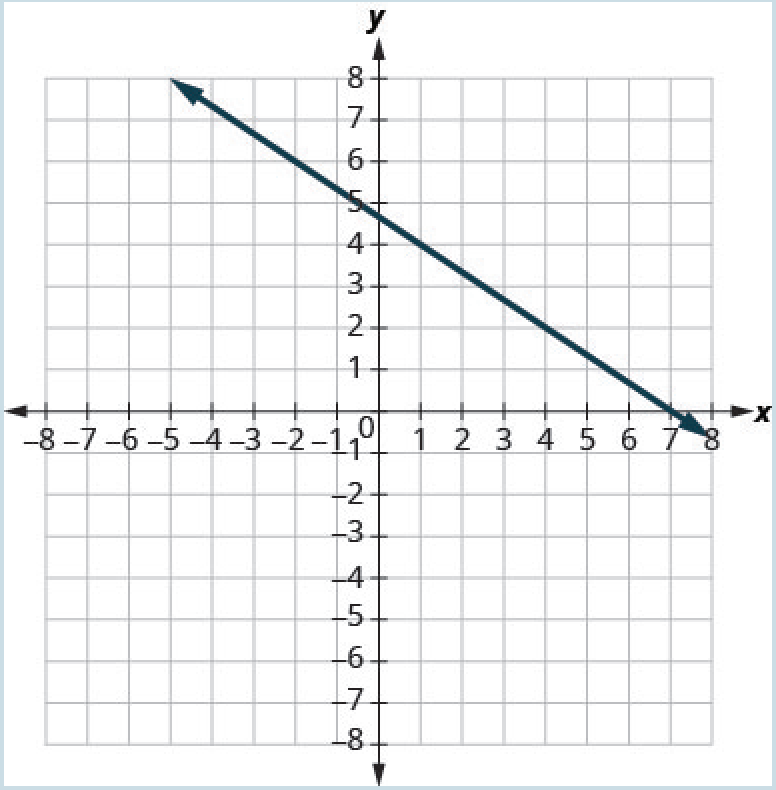

Figure 3P.10.13

Solution

$$-\frac{2}{3}$$

14.

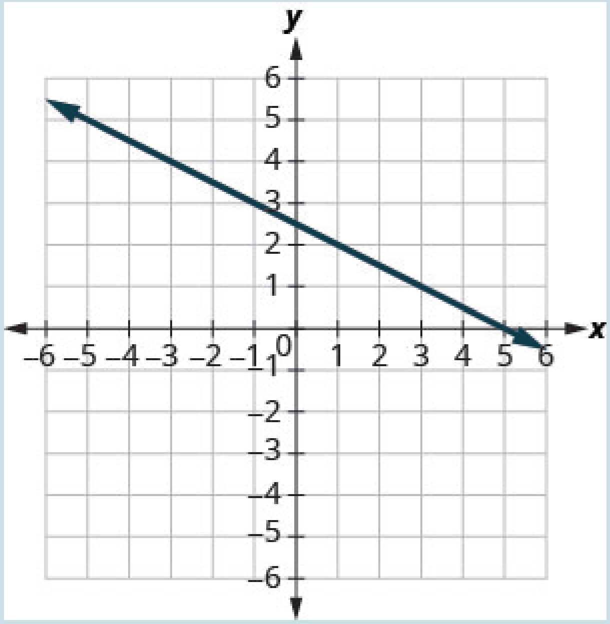

Figure 3P.10.14

15.

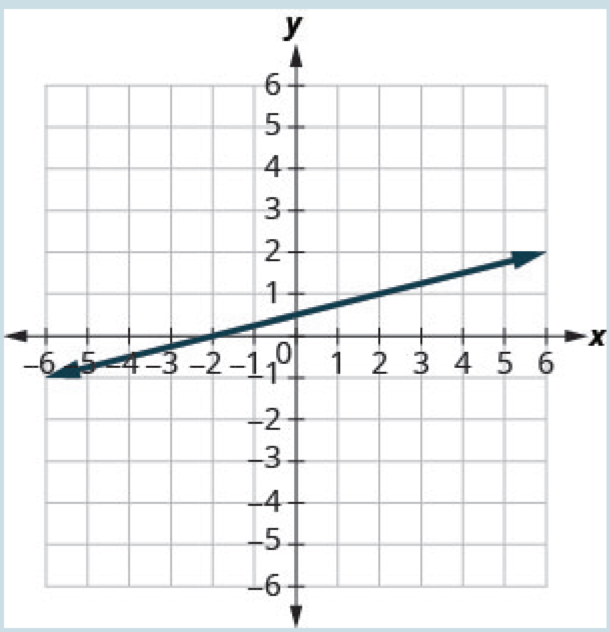

Figure 3P.10.15

Solution

$$\frac{1}{4}$$

16.

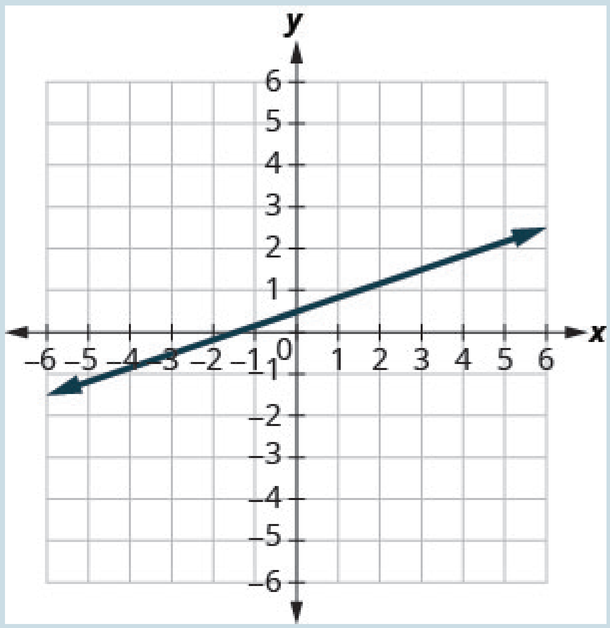

Figure 3P.10.16

Exercises: Find the Slope of Horizontal and Vertical Lines

Instructions: For questions 17-24, find the slope of each line.

17. $y = 3$

Solution

$$0$$

---

18. $y = 1$

---

19. $x = 4$

Solution

undefined

---

20. $x = 2$

---

21. $y = -2$

Solution

$$0$$

---

22. $y = -3$

---

23. $x = -5$

Solution

undefined

---

24. $x = -4$

Exercises: Use the Slope Formula to find the Slope of a Line between Two Points

Instructions: For questions 25-36, use the slope formula to find the slope of the line between each pair of points.

25. $(1, 4), (3, 9)$

Solution

$$\frac{5}{2}$$

---

26. $(2, 3), (5, 7)$

---

27. $(0, 3), (4, 6)$

Solution

$$\frac{3}{4}$$

---

28. $(0, 1), (5, 4)$

---

29. $(2, 5), (4, 0)$

Solution

$$-\frac{5}{2}$$

---

30. $(3, 6), (8, 0)$

---

31. $(-3, 3), (4, -5)$

Solution

$$-\frac{8}{7}$$

---

32. $(-2, 4), (3, -1)$

---

33. $(-1, -2), (2, 5)$

Solution

$$\frac{7}{3}$$

---

34. $(-2,-1),(6,5)$

---

35. $(4,-5),(1,-2)$

Solution

$$-1$$

---

36. $(3,-6),(2,-2)$

Exercises: Graph a Line Given a Point and the Slope

Instructions: For questions 37-52, graph each line with the given point and slope.

37. $(1,-2)$; $m=\frac{3}{4}$

Solution

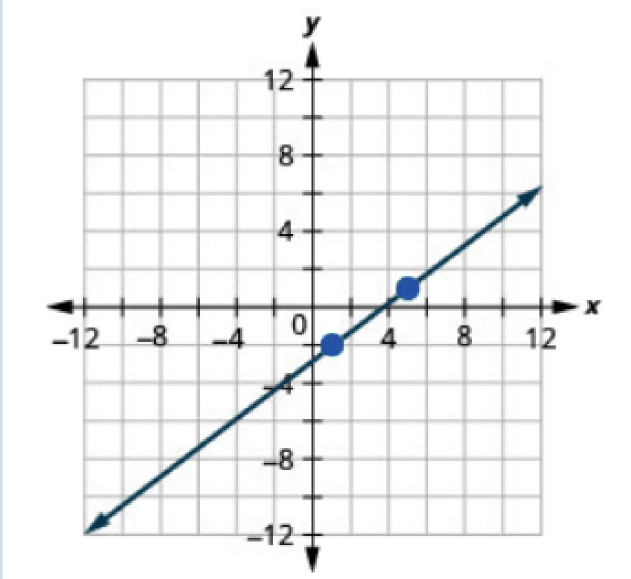

Figure 3P.10.17

---

38. $(1,-1)$**;** $m=\frac{2}{3}$

---

39. $(2,5)$**;** $m=-\frac{1}{3}$

Solution

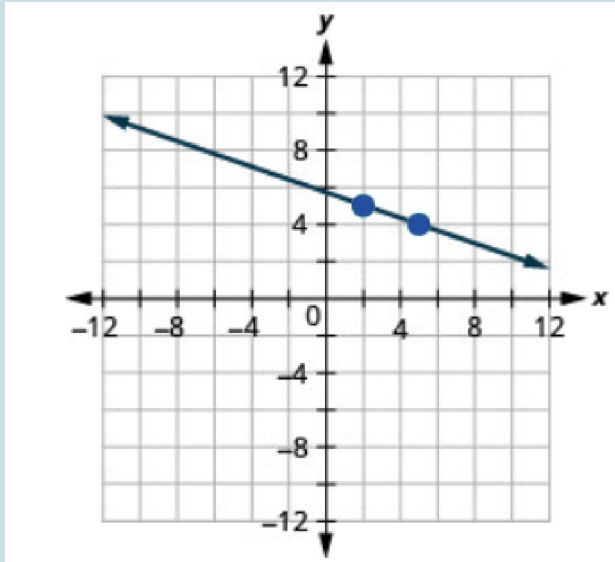

Figure 3P.10.18

---

40. $(1, 4)$**;** $m = -\frac{1}{2}$

---

41. $(-3, 4)$**;** $m = -\frac{3}{2}$

Solution

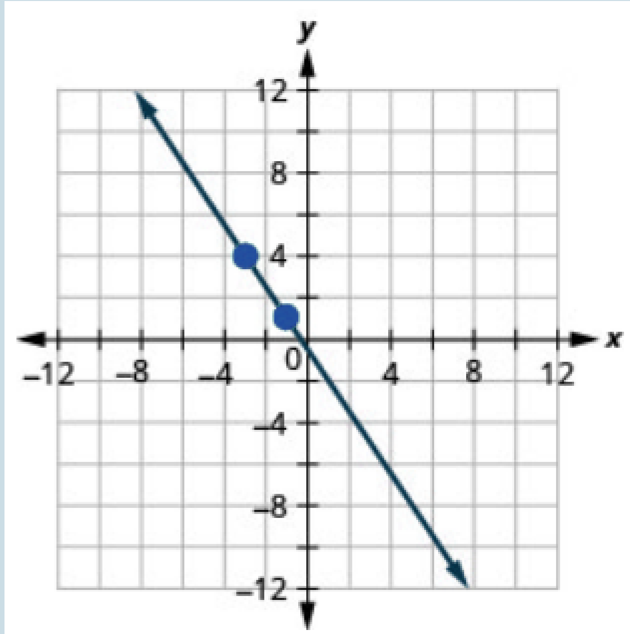

Figure 3P.10.19

---

42. $(-2, 5)$**;** $m = -\frac{5}{4}$

---

43. $(-1, -4)$**;** $m = \frac{4}{3}$

Solution

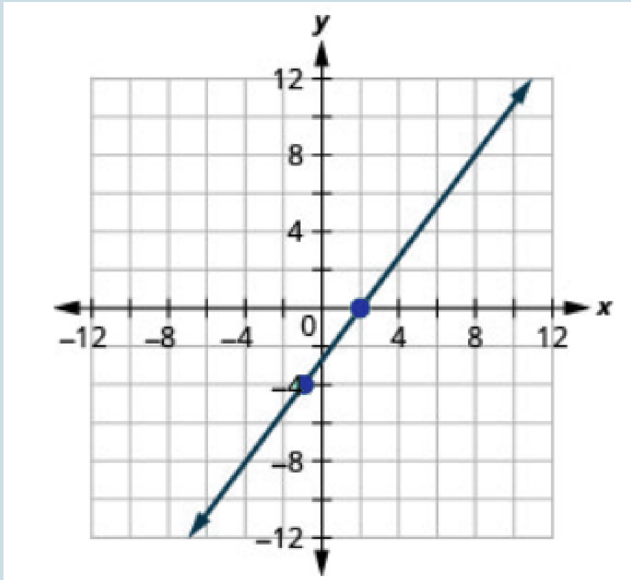

Figure 3P.10.20

---

44. $(-3,-5)$**;** $m=\frac{3}{2}$

---

45. y**-intercept 3;** $m=-\frac{2}{5}$

Solution

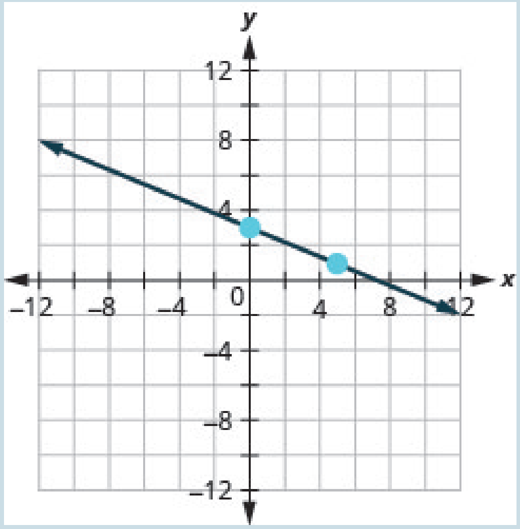

Figure 3P.10.21

---

46. y**-intercept 5;** $m = -\frac{4}{3}$

---

47. x**-intercept** -2**;** $m = \frac{3}{4}$

Solution

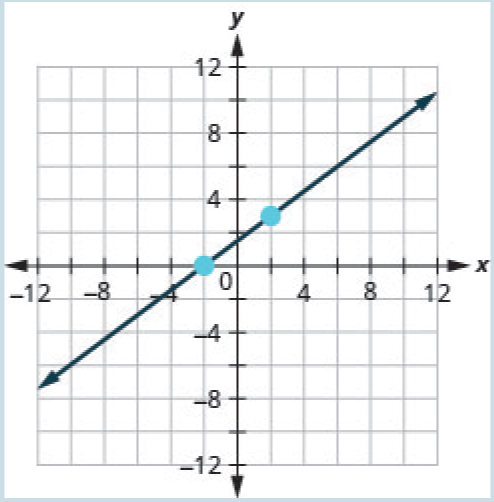

Figure 3P.10.22

---

48. x **-intercept** -1**;** $m = \frac{1}{5}$

---

49. $(-3, 3)$**;** $m = 2$

Solution

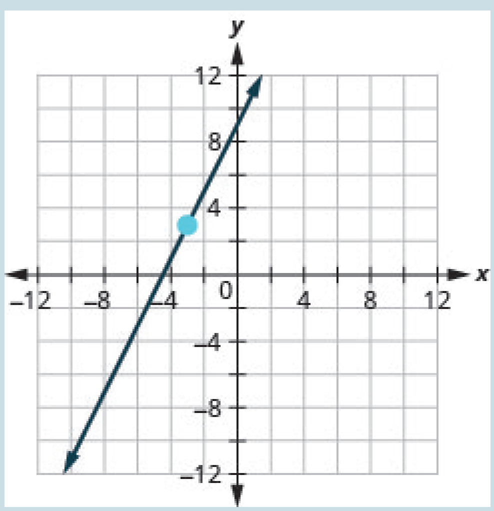

Figure 3P.10.23

---

50. $(-4, 2)$; $m = 4$

---

51. $(1, 5)$; $m = -3$

Solution

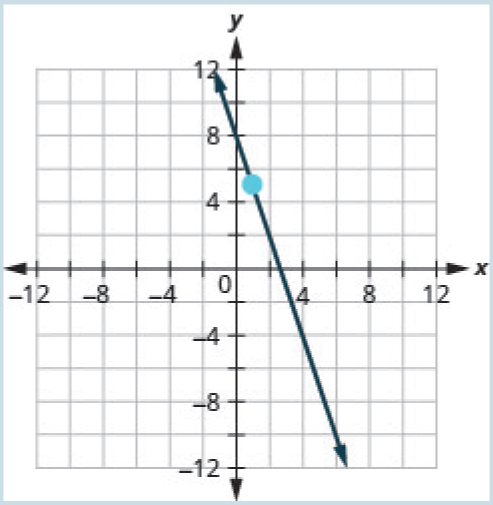

Figure 3P.10.24

---

52. $(2, 3)$; $m = -1$

Exercises: Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

Instructions: For questions 53-58, use the graph to find the slope and y-intercept of each line. Compare the values to the equation $y = mx + b$.

53. $y = 4x - 2$

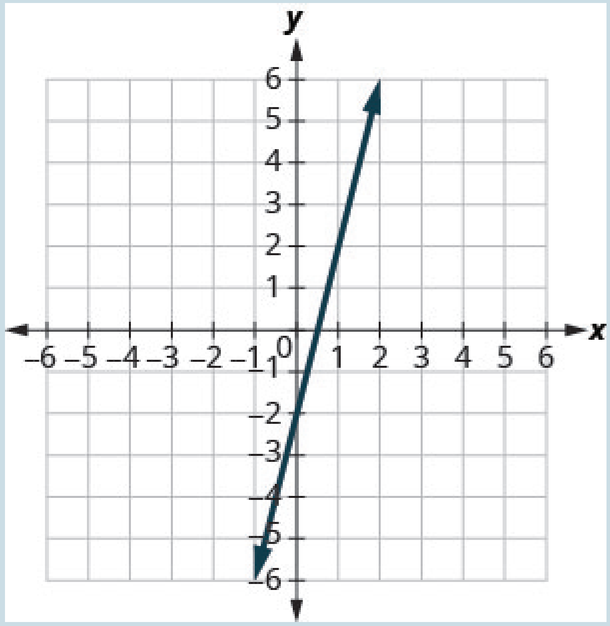

Figure 3P.10.25

Solution

slope $m = 4$ and y-intercept $(0, -2)$

---

54. $y = 3x - 5$

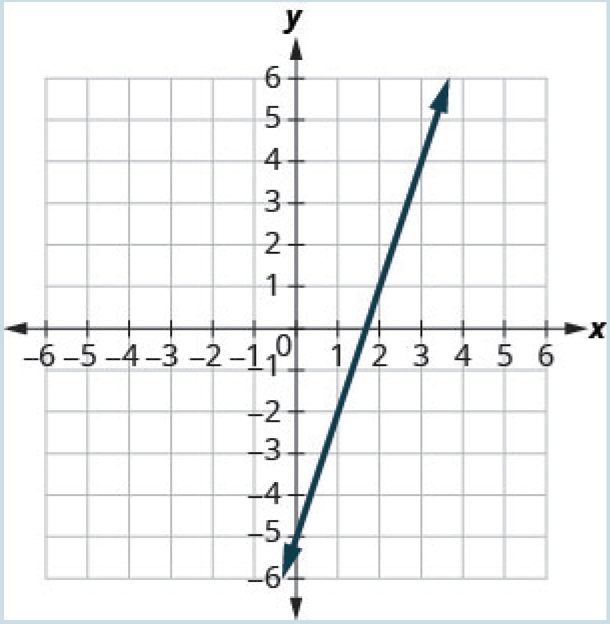

Figure 3P.10.26

---

55. $y = -3x + 1$

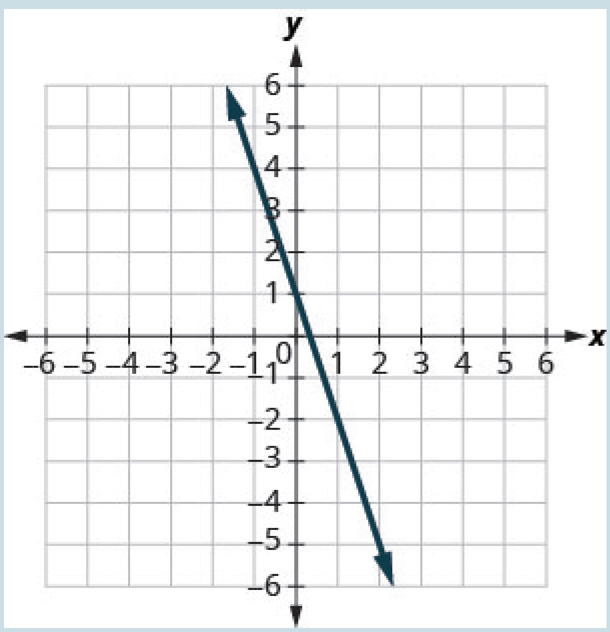

Figure 3P.10.27

Solution

slope $m = -3$ and y-intercept $(0, 1)$

---

56. $y = -x + 4$

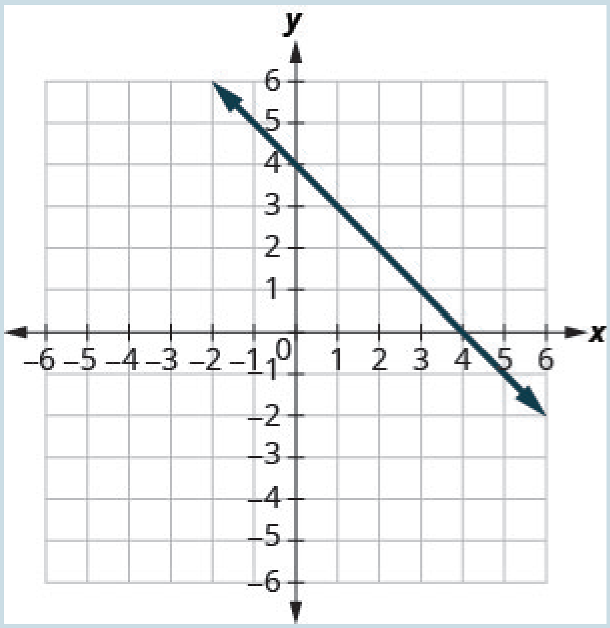

Figure 3P.10.28

---

57. $y = -\frac{2}{5}x + 3$

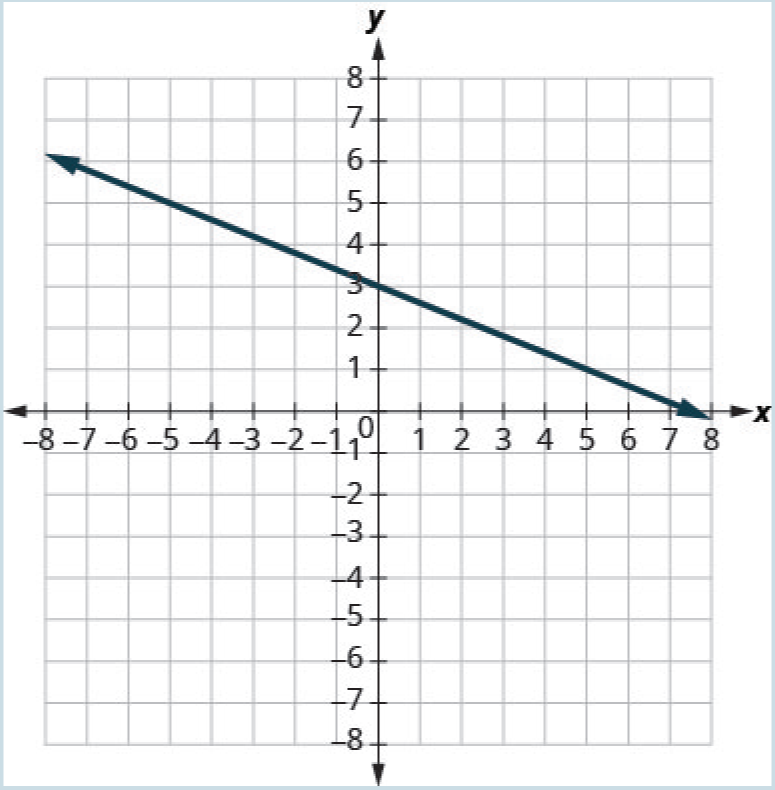

Figure 3P.10.29

Solution

slope $m = -\frac{2}{5}$ and y-intercept $(0, 3)$

---

58. $y = -\frac{4}{3}x + 1$

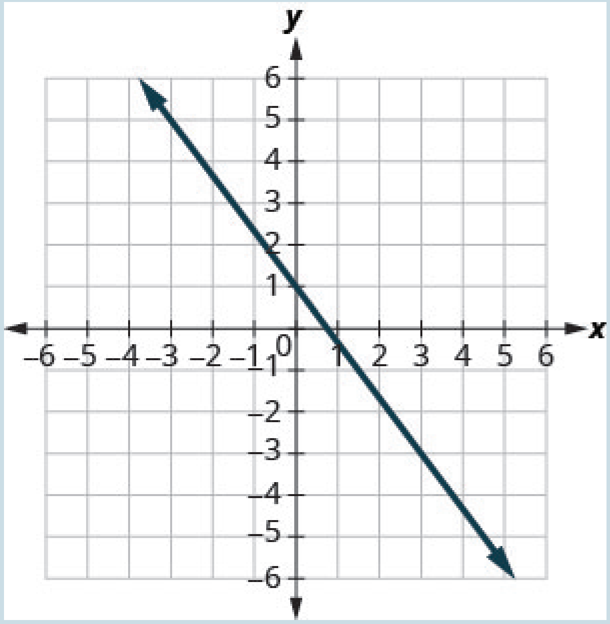

Figure 3P.10.30

Exercises: Identify the Slope and y-Intercept From an Equation of a Line

Instructions: For questions 59-68, identify the slope and y-intercept of each line.

59. $y=-9x+7$

Solution

$-9;(0,7)$

---

60. $y=-7x+3$

---

61. $y=4x-10$

Solution

$4;(0,-10)$

---

62. $y=6x-8$

---

63. $4x+y=8$

Solution

$-4;(0,8)$

---

64. $3x+y=5$

---

65. $8x+3y=12$

Solution

$-\frac{8}{3};(0,4)$

---

66. $6x+4y=12$

67. $7x - 3y = 9$

Solution

$\frac{7}{3}; (0, -3)$

68. $5x - 2y = 6$

Exercises: Graph a Line Using Its Slope and Intercept

Instructions: For questions 69-84, graph the line of each equation using its slope and y-intercept.

69. $y = x + 4$

Solution

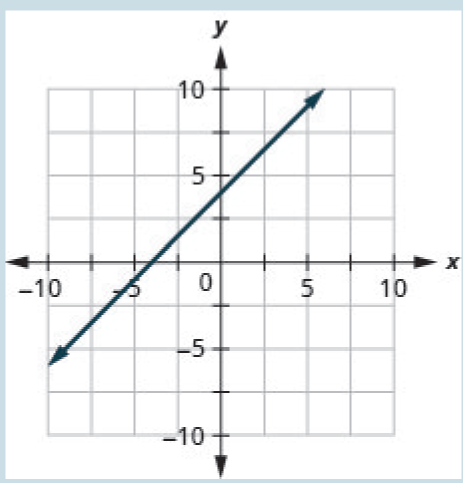

Figure 3P.10.31

70. $y = x + 3$

---

71. $y = 2x - 3$

Solution

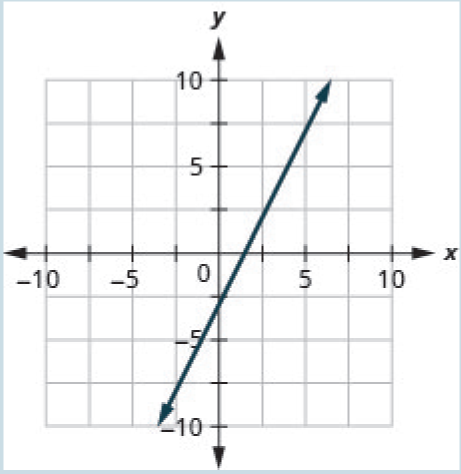

Figure 3P.10.32

---

72. $y = 3x - 1$

---

73. $y = -x + 3$

Solution

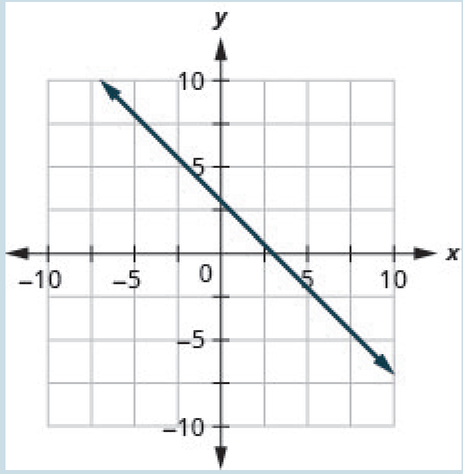

Figure 3P.10.33

---

74. $y = -x + 2$

---

75. $y = -x - 2$

Solution

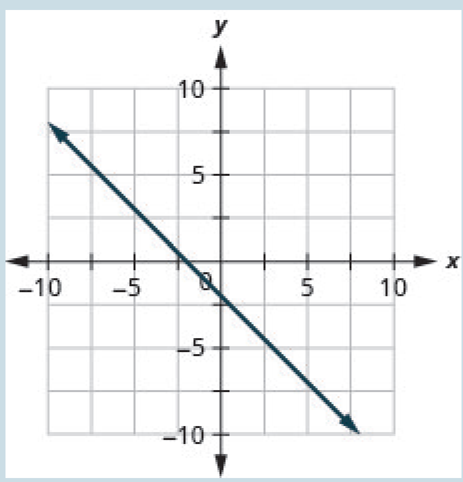

Figure 3P.10.34

---

76. $y = -x - 4$

---

77. $y=-\frac{2}{5}x-3$

Solution

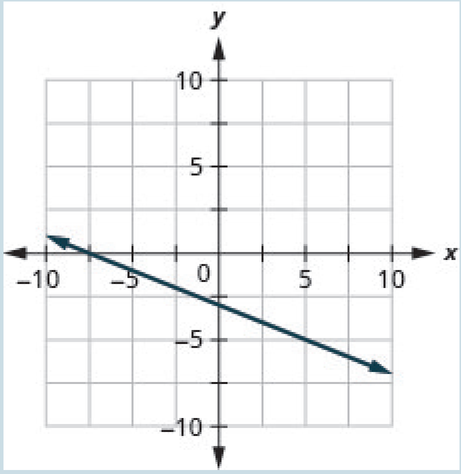

Figure 3P.10.35

---

78. $y=-\frac{3}{4}x-1$

---

79. $y=-\frac{2}{3}x+1$

Solution

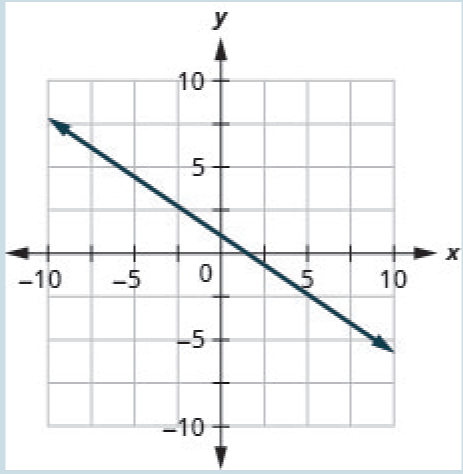

Figure 3P.10.36

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80. $y=-\frac{3}{5}x+2$

---

81. $4x-3y=6$

Solution

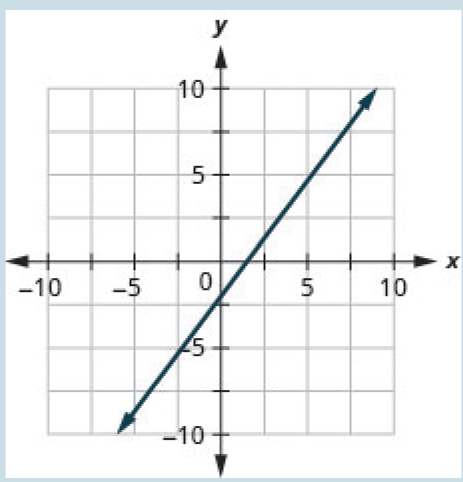

Figure 3P.10.37

---

82. $3x-4y=8$

83. $y = 0.3x + 25$

Solution

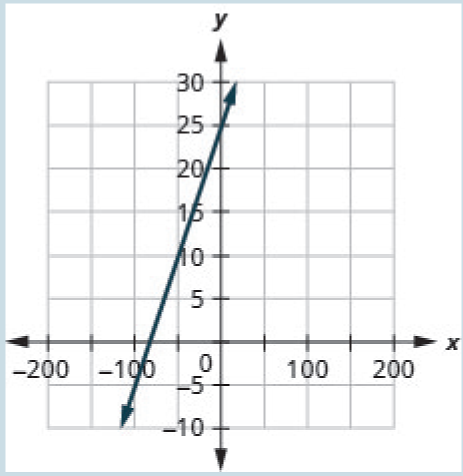

Figure 3P.10.38

84. $y = 0.1x + 15$

Exercises: Choose the Most Convenient Method to Graph a Line

Instructions: For questions 85-100, determine the most convenient method to graph each line.

85. $y = 4$

Solution

horizontal line

86. $x = 2$

---

87. $x = -3$

Solution

vertical line

---

88. $y = 5$

---

89. $y = -5x + 2$

Solution

slope–intercept

---

90. $y = -3x + 4$

---

91. $x - y = 1$

Solution

intercepts

---

92. $x - y = 5$

---

93. $y = \frac{4}{5}x - 3$

Solution

slope–intercept

94. $y = \frac{2}{3}x - 1$

95. $y = -1$

Solution

horizontal line

96. $y = -3$

97. $2x - 5y = -10$

Solution

intercepts

98. $3x - 2y = -12$

99. $y = -\frac{1}{3}x + 5$

Solution

slope–intercept

100. $y = -\frac{1}{4}x + 3$

Exercises: Graph and Interpret Applications of Slope–Intercept

Instructions: For questions 101-108, graph and interpret each application of slope-intercept.

101. The equation $P = 28 + 2.54w$ models the relation between the amount of Randy's monthly water bill payment, P, in dollars, and the number of units of water, w, used.

a. Find the payment for a month when Randy used 0 units of water.

b. Find the payment for a month when Randy used 15 units of water.

c. Interpret the slope and P-intercept of the equation.

d. Graph the equation.

Solution

a. $\$28$

b. $\$66.10$

c. The slope, 2.54, means that Randy's payment, P, increases by $\$2.54$ when the number of units of water he used, w, increases by 1. The P–intercept means that if the number units of water Randy used was 0, the payment would be $\$28$.

d.

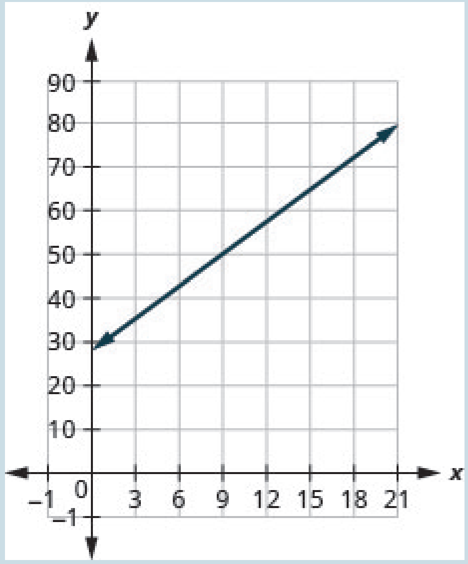

Figure 3P.10.39

---

102. The equation $P = 31 + 1.75w$ models the relation between the amount of Tuyet's monthly water bill payment, P, in dollars, and the number of units of water, w, used.

a. Find Tuyet's payment for a month when 0 units of water are used.

b. Find Tuyet's payment for a month when 12 units of water are used.

c. Interpret the slope and P-intercept of the equation.

d. Graph the equation.

---

103. Janelle is planning to rent a car while on vacation. The equation $C = 0.32m + 15$ models the

relation between the cost in dollars, C, per day and the number of miles, m, she drives in one day.

a. Find the cost if Janelle drives the car 0 miles one day.

b. Find the cost on a day when Janelle drives the car 400 miles.

c. Interpret the slope and C–intercept of the equation.

d. Graph the equation.

Solution

a. $\$15$

b. $\$143$

c. The slope, 0.32, means that the cost, C, increases by $\$0.32$ when the number of miles driven, m, increases by 1. The C-intercept means that if Janelle drives 0 miles one day, the cost would be $\$15$.

d.

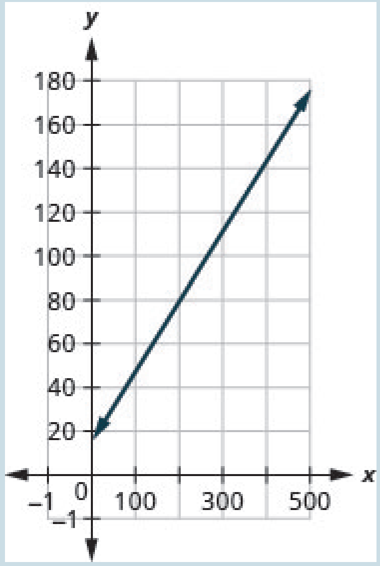

Figure 3P.10.40

---

104. Bruce drives his car for his job. The equation $R = 0.575m + 42$ models the relation between the amount in dollars, R, that he is reimbursed and the number of miles, m, he drives in one day.

a. Find the amount Bruce is reimbursed on a day when he drives 0 miles.

b. Find the amount Bruce is reimbursed on a day when he drives 220 miles.

c. Interpret the slope and R-intercept of the equation.

d. Graph the equation.

---

105. Patel's weekly salary includes a base pay plus commission on his sales. The equation

$S=750+0.09c$ models the relation between his weekly salary, S, in dollars and the amount of his sales, c, in dollars.

a. Find Patel's salary for a week when his sales were $\$0$.

b. Find Patel's salary for a week when his sales were $\$18,540$.

c. Interpret the slope and S-intercept of the equation.

d. Graph the equation.

Solution

a. $\$750$

b. $\$2418.60$

c. The slope, 0.09, means that Patel's salary, S, increases by $\$0.09$ for every $\$1$ increase in his sales. The S-intercept means that when his sales are $\$0$, his salary is $\$750$.

d.

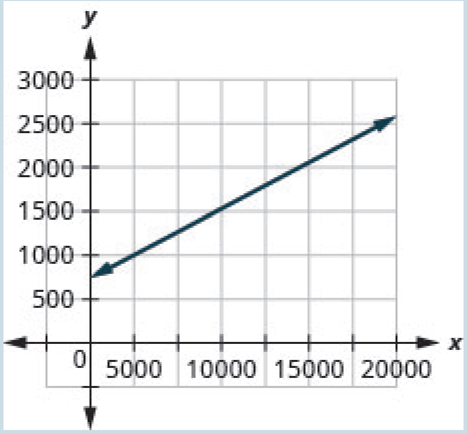

Figure 3P.10.41

---

106. Cherie works in retail and her weekly salary includes commission for the amount she sells. The equation $S = 400 + 0.15c$ models the relation between her weekly salary, S, in dollars and the amount of her sales, c, in dollars.

- **a. Find Cherie's salary for a week when her sales were $\$0$.**
- **b. Find Cherie's salary for a week when her sales were $\$3600$.**
- **c. Interpret the slope and S–intercept of the equation.**
- **d. Graph the equation.**

---

107. Margie is planning a dinner banquet. The equation $C = 750 + 42g$ models the relation between the cost in dollars, C of the banquet and the number of guests, g.

a. Find the cost if the number of guests is 50.

b. Find the cost if the number of guests is 100.

c. Interpret the slope and C–intercept of the equation.

d. Graph the equation.

Solution

a. $\$2850$

b. $\$4950$

c. The slope, 42, means that the cost, C, increases by $\$42$ for when the number of guests increases by 1. The C-intercept means that when the number of guests is 0, the cost would be $\$750$.

d.

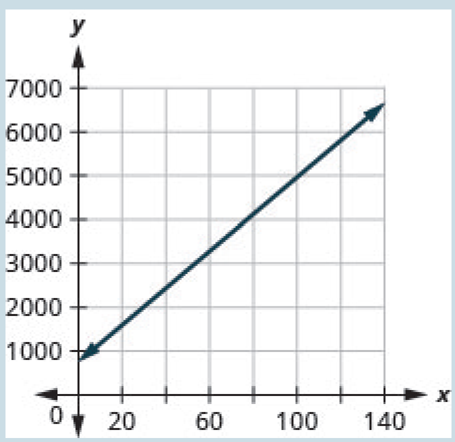

Figure 3P.10.42

108. Costa is planning a lunch banquet. The equation $C = 450 + 28g$ models the relation between the cost in dollars, C, of the banquet and the number of guests, g.

a. Find the cost if the number of guests is 40.

b. Find the cost if the number of guests is 80.

c. Interpret the slope and C-intercept of the equation.

d. Graph the equation.

Exercises: Use Slopes to Identify Parallel Lines

Instructions: For questions 109-134, use slopes and y-intercepts to determine if the lines are parallel.

109. $y = \frac{2}{3}x - 1; \; 2x - 3y = -2$

Solution

parallel

---

110. $y = \frac{3}{4}x - 3; \; 3x - 4y = -2$

---

111. $3x - 4y = -2; \;\; y = \frac{3}{4}x - 3$

Solution

parallel

---

112. $2x - 5y = -3; \;\; y = \frac{2}{5}x + 1$

---

113. $6x - 3y = 9; \;\; 2x - y = 3$

Solution

parallel

---

114. $2x - 4y = 6; \;\; x - 2y = 3$

---

115. $8x + 6y = 6; \;\; 12x + 9y = 12$

Solution

parallel

---

116. $4x + 2y = 6; \;\; 6x + 3y = 3$

---

117. $x = 7; \;\; x = -8$

Solution

parallel

---

118. $x = 5; \;\; x = -6$

119. $x=-3;\ \ x=-2$

Solution

parallel

120. $x=-4;\ \ x=-1$

121. $y=5;\ \ y=1$

Solution

parallel

122. $y=2;\ \ y=6$

123. $y=-1;\ \ y=2$

Solution

parallel

124. $y=-4;\ \ y=3$

125. $4x+4y=8;\ \ x+y=2$

Solution

parallel

126. $x - y = 2; \;\; 2x - 2y = 4$

---

127. $5x - 2y = 11; \;\; 5x - y = 7$

Solution

not parallel

---

128. $x - 3y = 6; \;\; 2x - 6y = 12$

---

129. $2x - 8y = 16; \;\; x - 2y = 4$

Solution

not parallel

---

130. $3x - 6y = 12; \;\; 6x - 3y = 3$

---

131. $x - 5y = 10; \;\; 5x - y = -10$

Solution

not parallel

---

132. $9x - 3y = 6; \;\; 3x - y = 2$

---

133. $9x - 5y = 4; \;\; 5x + 9y = -1$

Solution

not parallel

---

134. $7x-4y=8;\;\;4x+7y=14$

Exercises: Use Slopes to Identify Perpendicular Lines

Instructions: For questions 135-146, use slopes and y-intercepts to determine if the lines are perpendicular.

135. $x-4y=8;\;\;4x+y=2$

Solution

perpendicular

---

136. $3x-2y=8;\;\;2x+3y=6$

---

137. $2x+3y=5;\;\;3x-2y=7$

Solution

perpendicular

---

138. $2x+5y=3;\;\;5x-2y=6$

---

139. $3x - 4y = 8; \;\; 4x - 3y = 6$

Solution

not perpendicular

---

140. $3x - 2y = 1; \;\; 2x - 3y = 2$

---

141. $2x + 4y = 3; \;\; 6x + 3y = 2$

Solution

not perpendicular

---

142. $5x + 2y = 6; \;\; 2x + 5y = 8$

---

143. $2x - 6y = 4; \;\; 12x + 4y = 9$

Solution

perpendicular

---

144. $4x - 2y = 5; \;\; 3x + 6y = 8$

---

145. $8x - 2y = 7; \;\; 3x + 12y = 9$

Solution

perpendicular

---

146. $6x - 4y = 5; \;\; 8x + 12y = 3$

Exercises: Everyday Math

Instructions: For questions 147-154, answer the given everyday math word problems.

147. Slope of a roof. An easy way to determine the slope of a roof is to set one end of a 12 inch level on the roof surface and hold it level. Then take a tape measure or ruler and measure from the other end of the level down to the roof surface. This will give you the slope of the roof. Builders, sometimes, refer to this as pitch and state it as an "x 12 pitch" meaning $\frac{x}{12}$, where x is the measurement from the roof to the level—the rise. It is also sometimes stated as an "x-in-12 pitch".

- **a. What is the slope of the roof in this picture?**
- **b. What is the pitch in construction terms?**

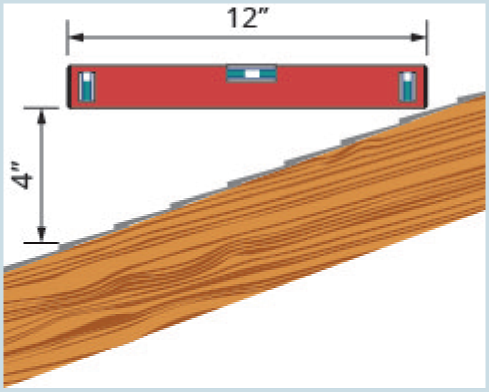

Figure 3P.10.43

Solution

a. $\dfrac{1}{3}$

b. $4\ 12\ \text{pitch}$ or 4-in-12 pitch

---

148. The slope of the roof shown here is measured with a 12" level and a ruler. What is the slope of this roof?

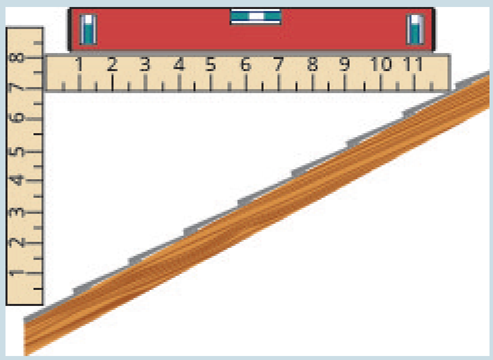

Figure 3P.10.44

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149. Road grade. A local road has a grade of 6%. The grade of a road is its slope expressed as a percent. Find the slope of the road as a fraction and then simplify. What rise and run would reflect this slope or grade?

Solution

$\dfrac{3}{50}$; $\text{rise} = 3$, $\text{run} = 50$

---

150. Highway grade. A local road rises 2 feet for every 50 feet of highway.

a. What is the slope of the highway?
b. The grade of a highway is its slope expressed as a percent. What is the grade of this highway?

---

151. Wheelchair ramp. The rules for wheelchair ramps require a maximum 1-inch rise for a 12-inch run.

a. How long must the ramp be to accommodate a 24-inch rise to the door?

b. Create a model of this ramp.

Solution

a. 288 inches (24 feet)

b. Models will vary.

---

152. Wheelchair ramp. A 1-inch rise for a 16-inch run makes it easier for the wheelchair rider to ascend a ramp.

a. How long must a ramp be to easily accommodate a 24-inch rise to the door?
b. Create a model of this ramp.

---

153. The equation $n = 4T - 160$ is used to estimate the number of cricket chirps, n, in one minute based on the temperature in degrees Fahrenheit, T.

a. Explain what the slope of the equation means.

b. Explain what the n–intercept of the equation means. Is this a realistic situation?

Solution

a. For every increase of one degree Fahrenheit, the number of chirps increases by four.

b. There would be -160 chirps when the Fahrenheit temperature is 0°. (Notice that this does not make sense; this model cannot be used for all possible temperatures.)

---

154. The equation $C = \frac{5}{9}F - 17.8$ can be used to convert temperatures F, on the Fahrenheit scale to temperatures, C, on the Celsius scale.

a. Explain what the slope of the equation means.

b. Explain what the C–intercept of the equation means.

Exercises: Writing Exercises

Instructions: For questions 155-158, answer the given writing exercises.

155. What does the sign of the slope tell you about a line?

Solution

When the slope is a positive number the line goes up from left to right. When the slope is a negative number the line goes down from left to right.

---

156. How does the graph of a line with slope $m = \frac{1}{2}$ differ from the graph of a line with slope $m = 2$?

---

157. Why is the slope of a vertical line "undefined"?

Solution

A vertical line has 0 run and since division by 0 is undefined the slope is undefined.

---

158. Explain in your own words how to decide which method to use to graph a line.

3.11 FIND THE EQUATION OF A LINE

Learning Objectives

By the end of this section, you will be able to:

- Find an equation of the line given the slope and y-intercept
- Find an equation of the line given the slope and a point
- Find an equation of the line given two points
- Find an equation of a line parallel to a given line
- Find an equation of a line perpendicular to a given line

Try It

Before you get started, take this readiness quiz:

1) Solve: $\frac{2}{3} = \frac{x}{5}$

2) Simplify: $-\frac{2}{5}(x - 15)$

How do online retailers know that 'you may also like' a particular item based on something you just ordered? How can economists know how a rise in the minimum wage will affect the unemployment rate? How do medical researchers create drugs to target cancer cells? How can traffic engineers predict the effect on your commuting time of an increase or decrease in gas prices? It's all mathematics.

You are at an exciting point in your mathematical journey as the mathematics you are studying has interesting applications in the real world.

The physical sciences, social sciences, and the business world are full of situations that can be modeled with linear equations relating two variables. Data is collected and graphed. If the data points appear to form a straight line, an equation of that line can be used to predict the value of one variable based on the value of the other variable.

To create a mathematical model of a linear relation between two variables, we must be able to find the equation of the line. In this section, we will look at several ways to write the equation of a line. The specific method we use will be determined by what information we are given.

Find an Equation of the Line Given the Slope and y-Intercept

We can easily determine the slope and intercept of a line if the equation was written in slope–intercept form, $y = mx + b$. Now, we will do the reverse—we will start with the slope and y-intercept and use them to find the equation of the line.

Example 1

Find an equation of a line with slope -7 and y-intercept $(0, -1)$.

Solution

Since we are given the slope and y-intercept of the line, we can substitute the needed values into the slope–intercept form, $y = mx + b$.

Step 1: **Name the slope.**

$$m = 7$$

Step 2: **Name the y-intercept.**

$$y\text{-intercept} = (0, -1)$$

Step 3: **Substitute the values into $y = mx + b$.**

$$\begin{aligned} y &= mx + b \\ y &= -7x + (-1) \\ y &= -7x - 1 \end{aligned}$$

Try It

3) Find an equation of a line with slope $\frac{2}{5}$ and y-intercept $(0, 4)$.

Solution

$y = \frac{2}{5}x + 4$

4) Find an equation of a line with slope -1 and y-intercept $(0, -3)$.

Solution

$y = -x - 3$

Sometimes, the slope and intercept need to be determined from the graph.

Example 2

Find the equation of the line shown.

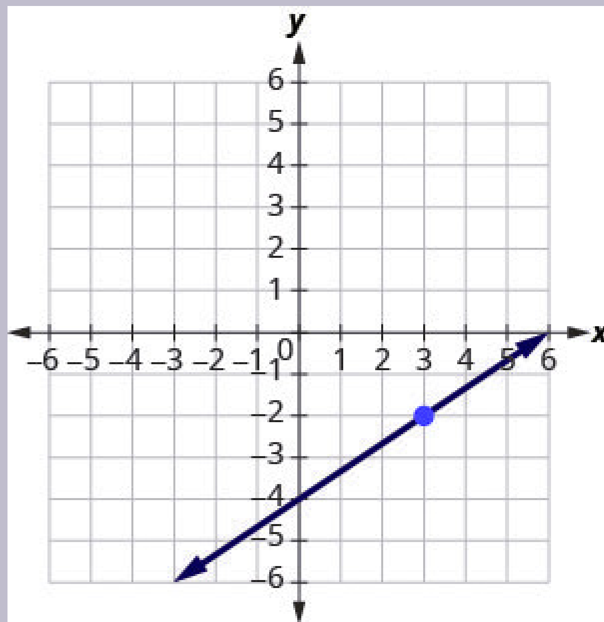


Figure 3.11.1

Solution

We need to find the slope and y-intercept of the line from the graph so we can substitute the needed values into the slope–intercept form, $y = mx + b$.

To find the slope, we choose two points on the graph.

The y-intercept is $(0, -4)$ and the graph passes through $(3, -2)$.

Step 1: **Find the slope by counting the rise and run.**

$$m = \frac{rise}{run}$$
$$m = \frac{2}{3}$$

Step 2: **Find the y-intercept.**

$$\text{y-intercept} = (0, -4)$$

Step 3: **Substitute the values into $y = mx + b$.**

$$y = mx + b$$
$$y = \frac{2}{3}x - 4$$

Try It

5) Find the equation of the line shown in the graph.

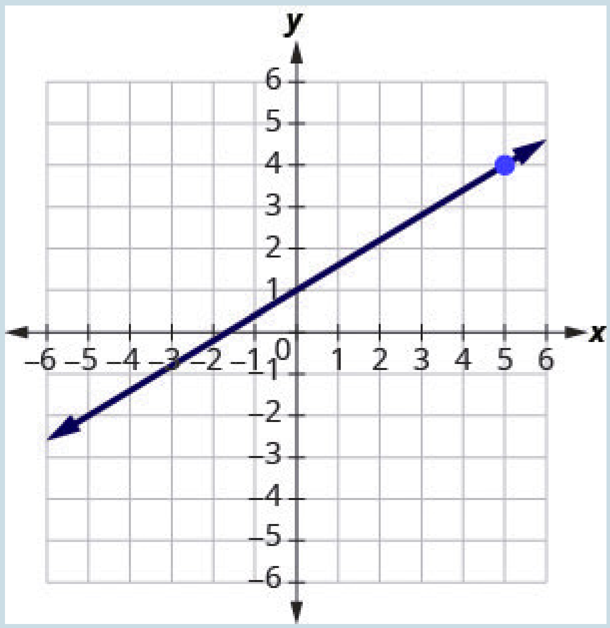


Figure 3.11.2

Solution

$y = \frac{3}{5}x + 1$

6) Find the equation of the line shown in the graph.

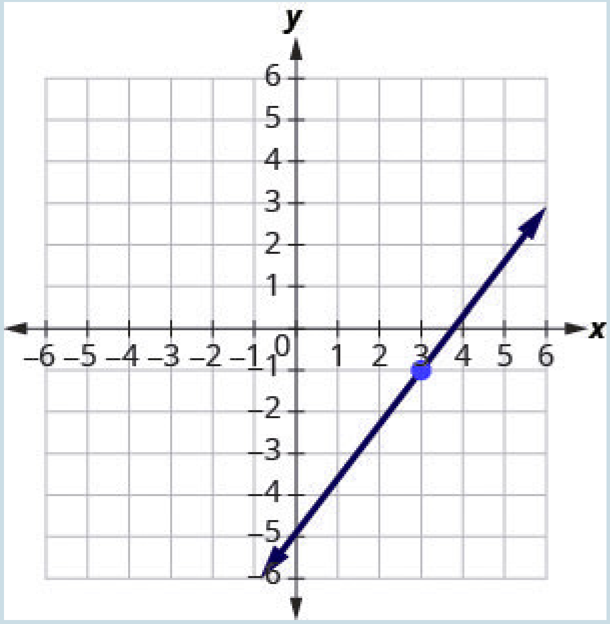


Figure 3.11.3

Solution

$y = \frac{4}{3}x - 5$

Find an Equation of the Line Given the Slope and a Point

Finding an equation of a line using the slope-intercept form of the equation works well when you are given the slope and y-intercept or when you read them off a graph. But what happens when you have another point instead of the y-intercept?

We are going to use the slope formula to derive another form of an equation of the line. Suppose we have a line that has slope m and that contains some specific point (x_1, y_1) and some other point, which we will just call (x, y). We can write the slope of this line and then change it to a different form.

$$m = \frac{y - y_1}{x - x_1}$$

Step 1: Multiply both sides of the equation by $(x - x_1)$.	$m(x - x_1) = \left(\frac{y - y_1}{x - x_1}\right)(x - x_1)$
Step 2: Simplify.	$m(x - x_1) = y - y_1$
Step 3: Rewrite the equation with the y terms on the left.	$(y - y_1) = m(x - x_1)$

This format is called the **point–slope form** of an equation of a line.

Point–slope Form of an Equation of a Line

The point-slope form of an equation of a line with slope m and containing the point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

We can use the point-slope form of an equation to find an equation of a line when we are given the slope and one point. Then we will rewrite the equation in slope-intercept form. Most applications of linear equations use the slope-intercept form.

How to

Find an equation of a line given the slope and a point.

1. Identify the slope.
2. Identify the point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope-intercept form.

Example 3

Find an equation of a line with slope $m = \frac{2}{5}$ that contains the point $(10, 3)$. Write the equation in slope-intercept form.

Solution

Step 1: **Identify the slope.**

The slope is given.

$$m = \frac{2}{5}$$

Step 2: **Identify the point.**

The point is given.

$$\begin{pmatrix} x_1 & y_1 \\ 10, & 3 \end{pmatrix}$$

Step 3: **Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.**

$$\begin{aligned} & y - y_1 = m(x - x_1) \\ & y - 3 = \frac{2}{5}(x - 10) \\ \text{Simplify.} \quad & y - 3 = \frac{2}{5}x - 4 \end{aligned}$$

Step 4: **Write the equation in slope-intercept form.**

$$y = \frac{2}{5}x - 1$$

Try It

7) Find an equation of a line with slope $m = \frac{5}{6}$ containing the point left $(6, 3)$.

Solution

$y = \frac{5}{6}x - 2$

8) Find an equation of a line with slope $m = \frac{2}{3}$ and containing the point $(9, 2)$.

Solution

$y = \frac{2}{3}x - 4$

Example 4

Find an equation of a line with slope $m = -\frac{1}{3}$ that contains the point $(6, -4)$. Write the equation in slope–intercept form.

Solution

Since we are given a point and the slope of the line, we can substitute the needed values into the point–slope form, $y - y_1$

Step 1: **Identify the slope.**

$$m = -\frac{1}{3}$$

Step 2: **Identify the point.**

$$\left(\overset{x_1}{6}, \overset{y_1}{-4}\right)$$

Step 3: **Substitute the values into** $y - y_1 = m(x - x_1)$

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = -\frac{1}{3}(x - 6)$$

Step 4: **Simplify.**

$$y - 4 = -\frac{1}{3}x - 6$$

Step 5: **Write in slope-intercept form.**

$$y = -\frac{1}{3}x - 2$$

Try It

9) Find an equation of a line with slope $m=-\frac{2}{5}$ and containing the point $(10,-5)$.

Solution

$y=-\frac{2}{5}x-1$

10) Find an equation of a line with slope $m=-\frac{3}{4}$, and containing the point $(4,-7)$.

Solution

$y=-\frac{3}{4}x-4$

Example 5

Find an equation of a horizontal line that contains the point $(-1,2)$. Write the equation in slope-intercept form.

Solution

Every horizontal line has slope 0. We can substitute the slope and points into the point-slope form, $y-y_1=m(x-x_1)$.

***Step 1:* Identify the slope.**

$$m=0$$

***Step 2:* Identify the point.**

$$\left(\overset{x_1}{-1},\ \overset{y_1}{2}\right)$$

***Step 3:* Substitute the values into $y-y_1=m(x-x_1)$.**

$$\begin{array}{ll} & y-y_1=m(x-x_1) \\ & y-2=0(x-(-1)) \\ \text{Simplify.} & y-2=(x+1) \\ & y-2=0 \\ & y=2 \end{array}$$

Step 4: **Write in slope-intercept form.**

It is in y-form, but could be written $y = 0x + 2$.

Did we end up with the form of a horizontal line, $y = a$?

Try It

11) Find an equation of a horizontal line containing the point $(-3, 8)$.

Solution
$y = 8$

12) Find an equation of a horizontal line containing the point $(-1, 4)$.

Solution
$y = 4$

Find an Equation of the Line Given Two Points

When real-world data is collected, a linear model can be created from two data points. In the next example, we'll see how to find an equation of a line when just two points are given.

We have two options so far for finding an equation of a line: slope-intercept or point-slope. Since we will know two points, it will make more sense to use the point-slope form.

But then we need the slope. Can we find the slope with just two points? Yes. Then, once we have the slope, we can use it and one of the given points to find the equation.

How to

Find an equation of a line given two points.

1. Find the slope using the given points.
2. Choose one point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope-intercept form.

Example 6

Find an equation of a line that contains the points $(5, 4)$ and $(3, 6)$. Write the equation in slope-intercept form.

Solution

Step 1: Find the slope using the given points.

Find the slope of the line through (5,4) and (3,6)

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{6 - 4}{3 - 5} \\ m &= \frac{2}{-2} \\ m &= -1 \end{aligned}$$

Step 2: Choose one point.

Choose either point.

$$\begin{pmatrix} x_1 & y_1 \\ 5, & 4 \end{pmatrix}$$

Step 3: Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= -1(x - 5) \\ y - 4 &= -1x + 5 \\ y &= -1x + 9 \end{aligned}$$

***Step 4:* Write the equation in slope-intercept form.**

$$y = -1x + 9$$

Use the point $(3, 6)$ and see that you get the same equation.

Try It

13) Find an equation of a line containing the points $(3, 1)$ and $(5, 6)$.

Solution

$y = \frac{5}{2}x - \frac{13}{2}$

14) Find an equation of a line containing the points $(1, 4)$ and $(6, 2)$.

Solution

$y = -\frac{2}{5}x + \frac{22}{5}$

Example 7

Find an equation of a line that contains the points $(-3, -1)$ and $(2, -2)$. Write the equation in slope–intercept form.

Solution

Since we have two points, we will find an equation of the line using the point–slope form. The first step will be to find the slope.

***Step 1:* Find the slope of the line through $(-3, -1)$ and $(2, -2)$.**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-2 - (-1)}{2 - (-3)}$$
$$m = \frac{-1}{5}$$
$$m = \frac{-1}{5}$$

Step 2: **Choose either point.**

$$\left(\overset{x_1}{2}, \overset{y_1}{-2}\right)$$

Step 3: **Substitute the values into** $y - y_1 = m(x - x_1)$**.**

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{1}{5}(x - 2)$$
$$y + 2 = -\frac{1}{5}x + \frac{2}{5}$$

Step 4: **Write in slope-intercept form.**

$$y = -\frac{1}{5}x - \frac{8}{5}$$

Try It

15) Find an equation of a line containing the points $(-2, -4)$ and $(1, -3)$.

Solution

$y = \frac{1}{3}x - \frac{10}{3}$

16) Find an equation of a line containing the points $(-4, -3)$ and $(1, -5)$.

Solution

$y = -\frac{2}{5}x - \frac{23}{5}$

Example 8

Find an equation of a line that contains the points $(-2, 4)$ and $(-2, -3)$. Write the equation in slope–intercept form.

Solution

Again, the first step will be to find the slope.

Find the slope of the line through $(-2, 4)$ and $(-2, -3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-3 - 4}{-2 - (-2)}$$
$$m = \frac{-7}{0}$$

The slope is undefined.

This tells us it is a vertical line. Both of our points have an x-coordinate of -2. So our equation of the line is $x = -2$. Since there is no y, we cannot write it in slope–intercept form.

You may want to sketch a graph using the two given points. Does the graph agree with our conclusion that this is a vertical line?

Try It

17) Find an equation of a line containing the points $(5, 1)$ and $(5, -4)$.

Solution

$x = 5$

18) Find an equation of a line containing the points $(-4, 4)$ and $(-4, 3)$.

Solution

$x = -4$

We have seen that we can use either the slope–intercept form or the point–slope form to find an equation of a line. Which form we use will depend on the information we are given. This is summarized in the table below.

To Write an Equation of a Line		
If given:	**Use:**	**Form:**
Slope and y-intercept	slope–intercept	$y = mx + b$
Slope and a point	point–slope	$y - y_1 = m(x - x_1)$
Two points	point–slope	$y - y_1 = m(x - x_1)$

Find an Equation of a Line Parallel to a Given Line

Suppose we need to find an equation of a line that passes through a specific point and is parallel to a given line. We can use the fact that parallel lines have the same slope. So we will have a point and the slope—just what we need to use the point–slope equation.

First, let's look at this graphically.

The graph shows the graph of $y = 2x - 3$. We want to graph a line parallel to this line and pass through the point $(-2, 1)$.

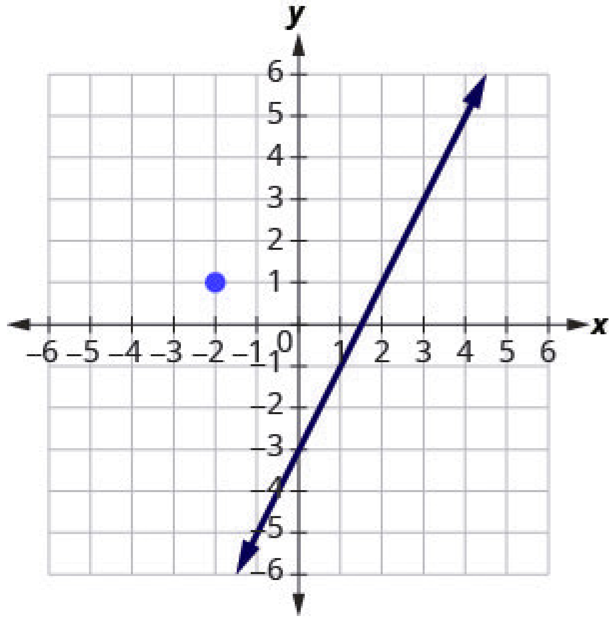


Figure 3.11.4

We know that parallel lines have the same slope. So the second line will have the same slope as $y = 2x - 3$. That slope is $m_{\|} = 2$. We'll use the notation m to represent the slope of a line parallel to a line with slope m. (Notice that the subscript || looks like two parallel lines.)

The second line will pass through $(-2, 1)$ and have $m = 2$. To graph the line, we start at $(-2, 1)$ and count out the rise and run. With $m = 2$ (or $m = \frac{2}{1}$), we count out the rise 2 and the run 1. We draw the line.

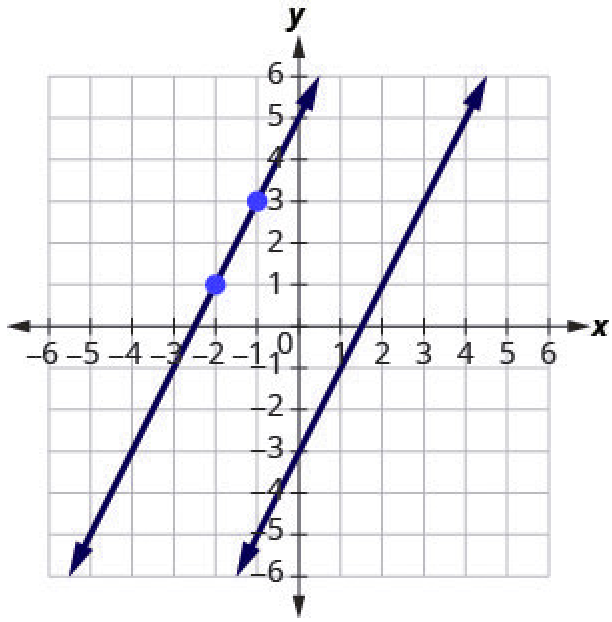


Figure 3.11.5

Do the lines appear parallel? Does the second line pass through $(-2, 1)$?

Now, let's see how to do this algebraically.

We can use either the slope–intercept form or the point–slope form to find an equation of a line. Here we know one point and can find the slope. So we will use the point–slope form.

Example 9

Find an equation of a line parallel to $y = 2x - 3$ that contains the point $(-2, 1)$. Write the equation in slope–intercept form.

Solution

***Step 1:* Find the slope of the given line.**

The line is in slope-intercept form, $y = 2x - 3$.

$$m = 2$$

***Step 2:* Find the slope of the parallel line.**

Parallel lines have the same slope.

$$m_2 = 2$$

Step 3: **Identify the point.**

The given point is, (-2,1).

$$\begin{pmatrix} x_1 & y_1 \\ -2, & 1 \end{pmatrix}$$

Step 4: **Substitute the values into the point-slope form,** $y - y_1 = m(x - x_1)$**.**

Simplify.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= 2(x - (-2)) \\ y - 1 &= 2(x + 2) \\ y - 1 &= 2x + 4 \end{aligned}$$

Step 5: **Write the equation in slope-intercept form.**

$$y = 2x + 5$$

Does this equation make sense? What is the y-intercept of the line? What is the slope?

Try It

19) Find an equation of a line parallel to the line $y = 3x + 1$ that contains the point $(4, 2)$. Write the equation in slope–intercept form.

Solution

$y = 3x - 10$

20) Find an equation of a line parallel to the line $y = \frac{1}{2}x - 3$ that contains the point $(6, 4)$.

Solution

$y = \frac{1}{2}x + 1$

HOW TO

Find an equation of a line parallel to a given line.

1. Find the slope of the given line.
2. Find the slope of the parallel line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.
5. Write the equation in slope–intercept form.

Find an Equation of a Line Perpendicular to a Given Line

Now, let's consider perpendicular lines. Suppose we need to find a line passing through a specific point that is perpendicular to a given line. We can use the fact that perpendicular lines have slopes that are negative reciprocals. We will again use the point–slope equation, like we did with parallel lines.

The graph shows the graph of $y = 2x - 3$. Now, we want to graph a line perpendicular to this line and passing through $(-2, 1)$.

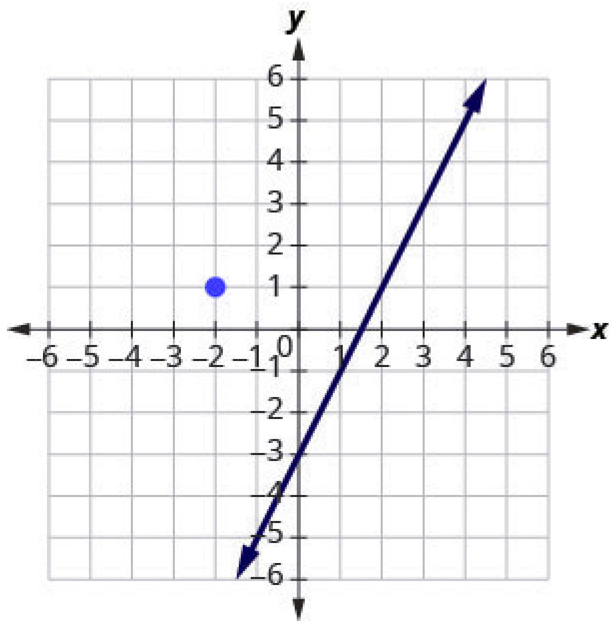


Figure 3.11.6

We know that perpendicular lines have slopes that are negative reciprocals. We'll use the notation $m_{\perp}$ to represent the slope of a line perpendicular to a line with slope m. (Notice that the subscript $\perp$ looks like the right angles made by two perpendicular lines.)

$y = 2x - 3$ perpendicular line

$m = 2$ $m - \perp = \frac{1}{2}$

We now know the perpendicular line will pass through $(-2, 1)$ with $m_{\perp} = -\frac{1}{2}$

To graph the line, we will start at $(-2, 1)$ and count out the rise -1 and the run 2. Then we draw the line.

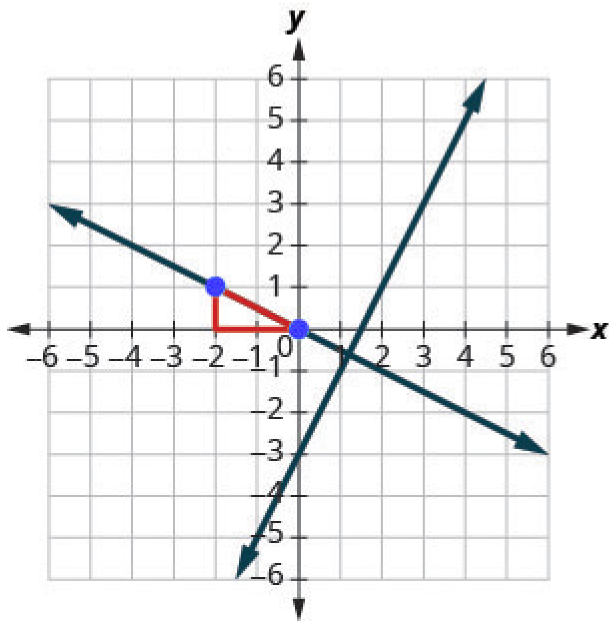


Figure 3.11.7

Do the lines appear perpendicular? Does the second line pass through $(-2, 1)$?

Now, let's see how to do this algebraically. We can use either the slope–intercept form or the point–slope form to find an equation of a line. In this example we know one point, and can find the slope, so we will use the point–slope form.

Example 10

Find an equation of a line perpendicular to $y = 2x - 3$ that contains the point $(-2, 1)$. Write the equation in slope–intercept form.

Solution

Step 1: **Find the slope of the given line.**

The line is in slope-intercept form, $y = 2x - 3$.

$$m = 2$$

Step 2: **Find the slope of the perpendicular line.**

The slopes of perpendicular lines are negative reciprocals.

$$m_{\perp} = -\frac{1}{2}$$

Step 3: **Identify the point.**

The given point is, (-2,1)

$$\begin{pmatrix} x_1 & y_1 \\ -2, & 1 \end{pmatrix}$$

Step 4: **Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.**

Simplify.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= -\frac{1}{2}(x - (-2)) \\ y - 1 &= -\frac{1}{2}(x + 2) \\ y - 1 &= -\frac{1}{2}x - 1 \end{aligned}$$

Step 5: **Write in slope-intercept form.**

$$y = -\frac{1}{2}x$$

Try It

21) Find an equation of a line perpendicular to the line $y = 3x + 1$ that contains the point $(4, 2)$. Write the equation in slope–intercept form.

Solution

$y = -\frac{1}{3}x + \frac{10}{3}$

22) Find an equation of a line perpendicular to the line $y = \frac{1}{2}x - 3$ that contains the point $(6, 4)$.

Solution

$y = -2x + 16$

HOW TO

Find an equation of a line perpendicular to a given line.

1. Find the slope of the given line.
2. Find the slope of the perpendicular line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.
5. Write the equation in slope–intercept form.

Example 11

Find an equation of a line perpendicular to $x = 5$ that contains the point $(3, -2)$. Write the equation in slope–intercept form.

Solution

Again, since we know one point, the point–slope option seems more promising than the slope–intercept option. We need the slope to use this form, and we know the new line will be perpendicular to $x = 5$. This line is vertical, so its perpendicular will be horizontal. This tells us the $m_\perp = 0$.

***Step 1:* Identify the point.**

$$(3, -2)$$

***Step 2:* Identify the slope of the perpendicular line.**

$$m_\perp = 0$$

***Step 3:* Substitute the values into $y - y_1 = m(x - x_1)$.**

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-2) &= 0(x - 3) \\ y + 2 &= 0 \\ y &= -2 \end{aligned}$$

***Step 4:* Simplify.**

$$y = -2$$

Sketch the graph of both lines. Do they appear to be perpendicular?

Try It

23) Find an equation of a line that is perpendicular to the line $x = 4$ that contains the point $(4, -5)$. Write the equation in slope–intercept form.

Solution

$y = -5$

24) Find an equation of a line that is perpendicular to the line $x = 2$ that contains the point $(2, -1)$. Write the equation in slope–intercept form.

Solution

$y = -1$

In Example 3.11.11, we used the point–slope form to find the equation. We could have looked at this differently.

We want to find a line that is perpendicular to $x = 5$ that contains the point $(3, -2)$. The graph shows us the line $x = 5$ and the point $(3, -2)$.

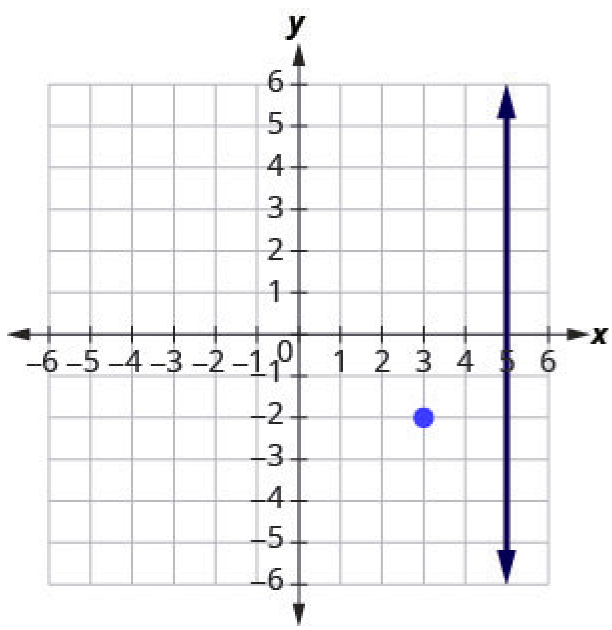


Figure 3.11.8

We know every line perpendicular to a vertical line is horizontal, so we will sketch the horizontal line through $(3, -2)$.

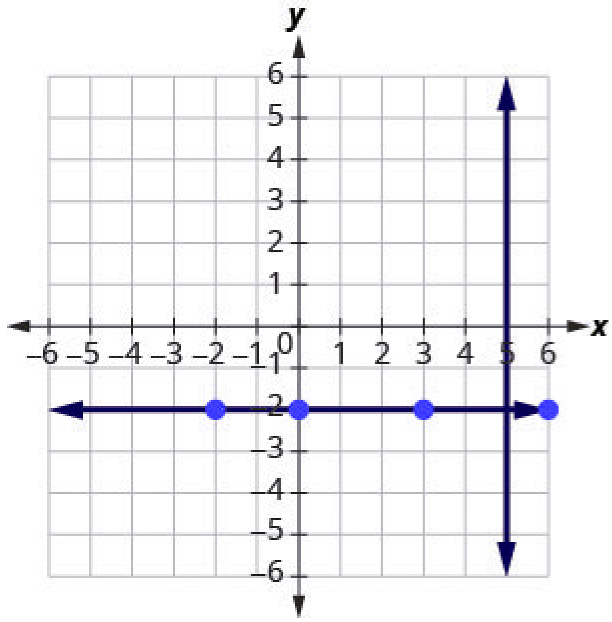


Figure 3.11.9

Do the lines appear perpendicular?

If we look at a few points on this horizontal line, we notice they all have y-coordinates of -2. So, the equation of the line perpendicular to the vertical line $x = 5$ is $y = -2$.

Example 12

Find an equation of a line that is perpendicular to $y = -4$ that contains the point $(-4, 2)$. Write the equation in slope–intercept form.

Solution

The line $y = -4$ is horizontal. Any line perpendicular to it must be vertical, in the form $x = a$. Since the perpendicular line is vertical and passes through $(-4, 2)$, every point on it has an x-coordinate of -4.

The equation of the perpendicular line is $x = -4$. You may want to sketch the lines. Do they appear perpendicular?

Try It

25) Find an equation of a line that is perpendicular to the line $y = 1$ that contains the point $(-5, 1)$. Write the equation in slope–intercept form.

Solution

$x = -5$

26) Find an equation of a line that is perpendicular to the line $y = -5$ that contains the point $(-4, -5)$.

Solution

$x = -4$

Access this online resource for additional instruction and practice with finding the equation of a line.

- Use the Point-Slope Form of an Equation of a Line

Key Concepts

- **To Find an Equation of a Line Given the Slope and a Point**
    1. Identify the slope.
    2. Identify the point.
    3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
    4. Write the equation in slope-intercept form.
- **To Find an Equation of a Line Given Two Points**
    1. Find the slope using the given points.
    2. Choose one point.
    3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
    4. Write the equation in slope-intercept form.
- **To Write and Equation of a Line**
    - If given slope and y-intercept, use slope–intercept form $y = mx + b$.
    - If given slope and a point, use point–slope form $y - y_1 = m(x - x_1)$.
    - If given two points, use point–slope form $y - y_1 = m(x - x_1)$.
- **To Find an Equation of a Line Parallel to a Given Line**
    1. Find the slope of the given line.
    2. Find the slope of the parallel line.
    3. Identify the point.
    4. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
    5. Write the equation in slope-intercept form.
- **To Find an Equation of a Line Perpendicular to a Given Line**
    - Find the slope of the given line.

- Find the slope of the perpendicular line.
- Identify the point.
- Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
- Write the equation in slope-intercept form.

Glossary

point–slope form

The point–slope form of an equation of a line with slope m and containing the point (x_1, y_1) is $y - y_1 = m(x - x_1)$.

Exercises: Find an Equation of the Line Given the Slope and y-Intercept

Instructions: For questions 1-15, find the equation of a line with given slope and y-intercept.

Write the equation in slope–intercept form.

1. slope 4 **and** y**-intercept** $(0, 1)$

Solution

$y = 4x + 1$

---

2. slope 6 **and** y**-intercept** $(0, -4)$

---

3. slope 8 **and** y**-intercept** $(0, -6)$

Solution

$y = 8x - 6$

---

4. slope -1 **and** y**-intercept** $(0, 3)$

---

5. slope -1 **and** y**-intercept** $(0, 7)$

Solution

$y = -x + 7$

---

6. slope -2 **and** y**-intercept** $(0, -3)$

---

7. slope -3 **and** y**-intercept** $(0, -1)$

Solution

$y = -3x - 1$

---

8. slope $\frac{3}{5}$ **and** y**-intercept** $(0, -1)$

---

9. slope $\frac{1}{5}$ **and** y**-intercept** $(0, -5)$

Solution

$y = \frac{1}{5}x - 5$

---

10. slope $-\frac{3}{4}$ **and** y **-intercept** $(0, -2)$

---

11. slope $-\frac{2}{3}$ **and** y **-intercept** $(0, -3)$

Solution

$y = -\frac{2}{3}x - 3$

---

12. slope 0 **and** y **-intercept** $(0, -1)$

---

13. slope 0 **and** y **-intercept** $(0, 2)$

Solution

$y = 2$

---

14. slope -3 **and** y **-intercept** $(0, 0)$

---

15. slope -4 **and** y**-intercept** $(0,0)$

Solution

$y=-4x$

Exercises: Find the Equation of a Line Shown on a Graph

Instructions: For questions 16-23, find the equation of the line shown in each graph. Write the equation in slope–intercept form.

16.

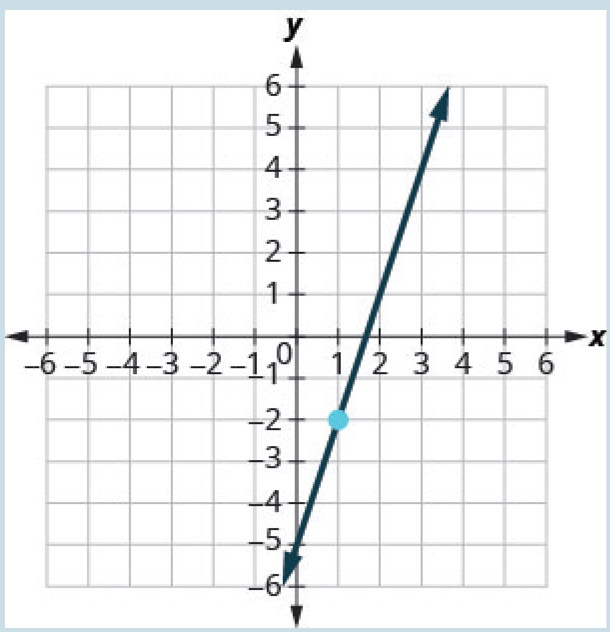


Figure 3P.11.1

17.

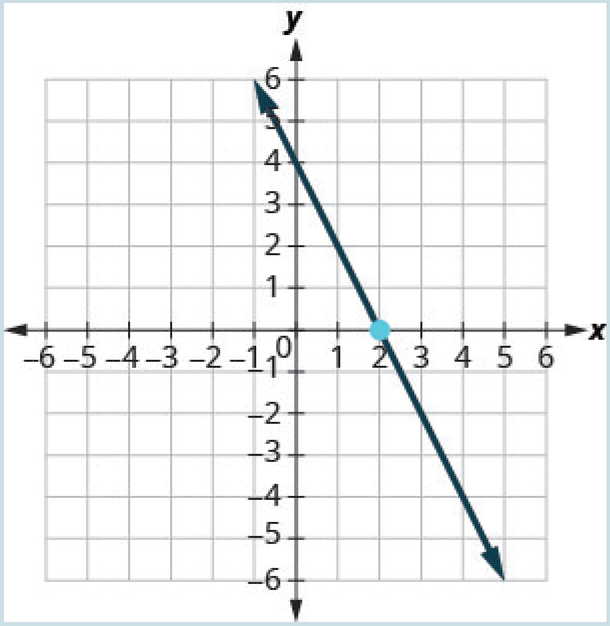


Figure 3P.11.2

Solution

$y = -2x + 4$

18.

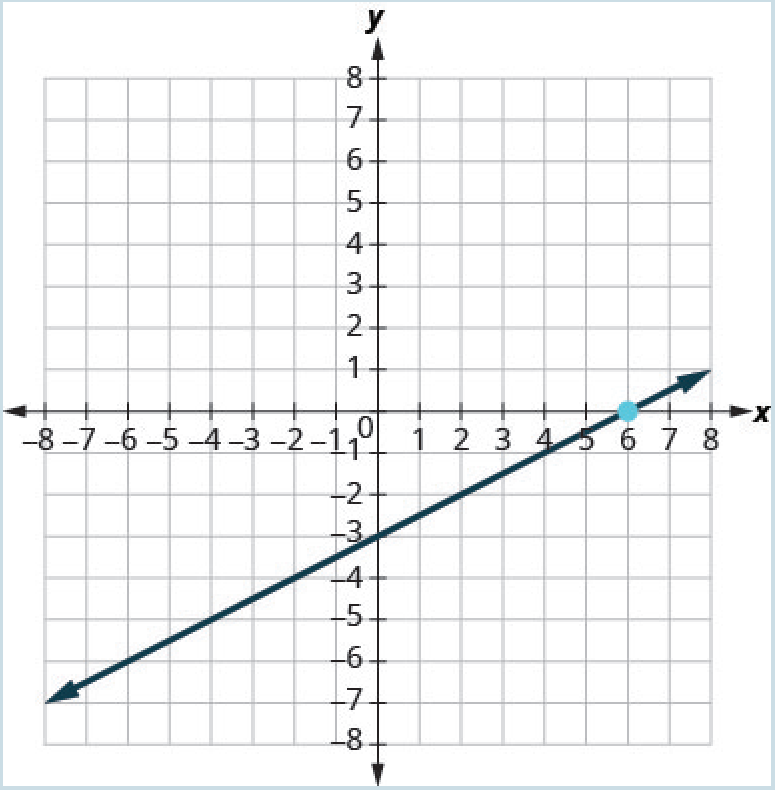


Figure 3P.11.3

19.

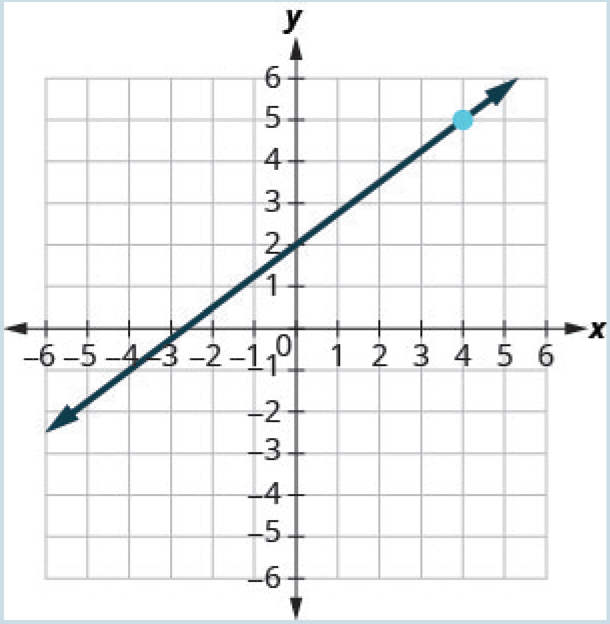


Figure 3P.11.4

Solution

$y = \frac{3}{4}x + 2$

20.

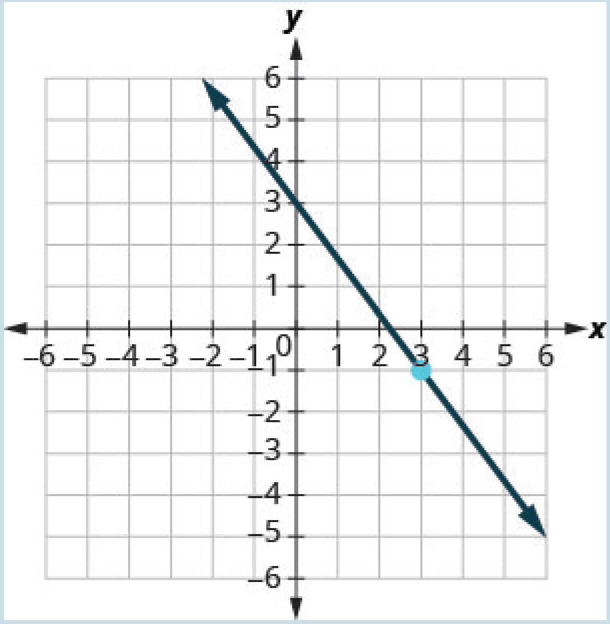


Figure 3P.11.5

21.

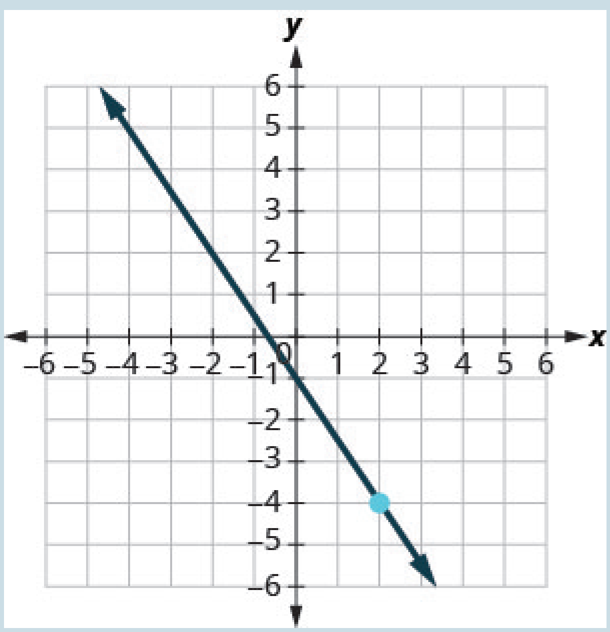


Figure 3P.11.6

Solution

$y=-\frac{3}{2}x-1$

22.

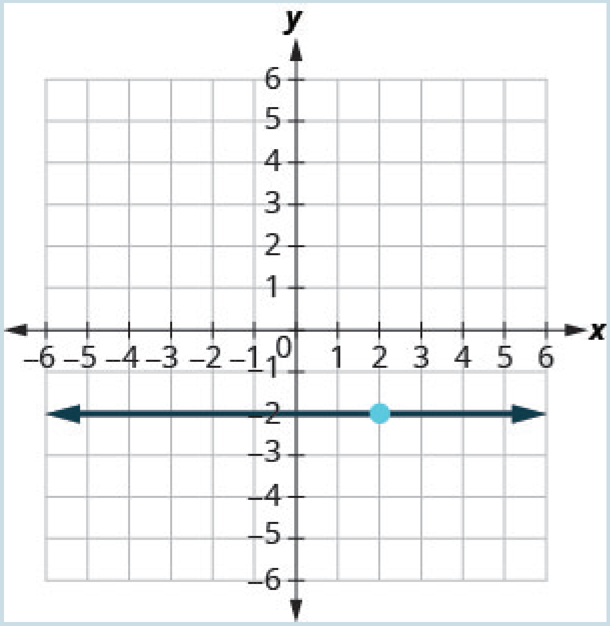


Figure 3P.11.7

23.

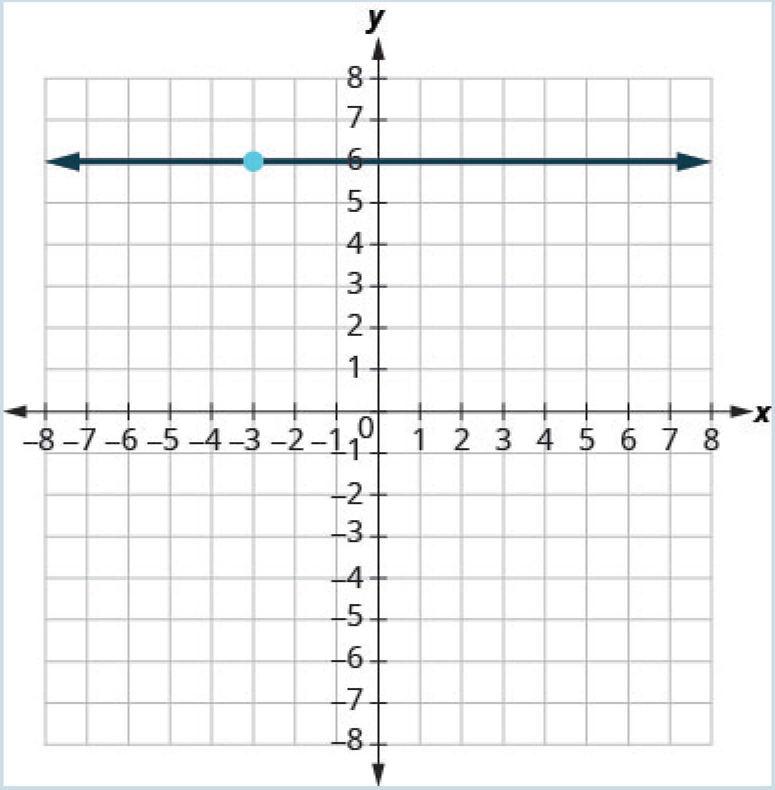


Figure 3P.11.8

Solution

$y = 6$

Exercises: Find an Equation of the Line Given the Slope and a Point

Instructions: For questions 24-41, find the equation of a line with given slope and containing the given point. Write the equation in slope–intercept form.

24. $m = \frac{5}{8}$**, point** $(8, 3)$

---

25. $m = \frac{3}{8}$**, point** $(8, 2)$

Solution

$y = \frac{3}{8}x - 1$

---

26. $m = \frac{1}{6}$**, point** $(6, 1)$

---

27. $m = \frac{5}{6}$**, point** $(6, 7)$

Solution

$y = \frac{5}{6}x + 2$

---

28. $m = -\frac{3}{4}$**, point** $(8, -5)$

---

29. $m = -\frac{3}{5}$**, point** $(10, -5)$

Solution

$y = -\frac{3}{5}x + 1$

---

30. $m = -\frac{1}{4}$**, point** $(-12, -6)$

---

31. $m=-\frac{1}{3}$, **point** $(-9,-8)$

Solution

$y=-\frac{1}{3}x-11$

---

32. Horizontal line containing $(-2,5)$

---

33. Horizontal line containing $(-1,4)$

Solution

$y=4$

---

34. Horizontal line containing $(-2,-3)$

---

35. Horizontal line containing $(-1,-7)$

Solution

$y=-7$

---

36. $m=-\frac{3}{2}$, **point** $(-4,-3)$

---

37. $m=-\frac{5}{2}$, **point** $(-8,-2)$

Solution

$y=-\frac{5}{2}x-22$

---

38. $m = -7$**, point** $(-1, -3)$

---

39. $m = -4$**, point** $(-2, -3)$

Solution

$y = -4x - 11$

---

40. Horizontal line containing $(2, -3)$

---

41. Horizontal line containing $(4, -8)$

Solution

$y = -8$

Exercises: Find an Equation of the Line Given Two Points

Instructions: For questions 42-67, find the equation of a line containing the given points. Write the equation in slope–intercept form.

42. $(2, 6)$ **and** $(5, 3)$

---

43. $(3, 1)$ **and** $(2, 5)$

Solution

$y = -4x + 13$

---

44. $(4, 3)$ **and** $(8, 1)$

---

45. $(2, 7)$ **and** $(3, 8)$

Solution

$y = x + 5$

---

46. $(-3, -4)$ **and** $(5 - 2)$

---

47. $(-5, -3)$ **and** $(4, -6)$

Solution

$y = -\frac{1}{3}x - \frac{14}{3}$

---

48. $(-1, 3)$ **and** $(-6, -7)$

---

49. $(-2, 8)$ **and** $(-4, -6)$

Solution

$y = 7x + 22$

---

50. $(6, -4)$ **and** $(-2, 5)$

---

51. $(3, -2)$ **and** $(-4, 4)$

Solution

$y = -\frac{6}{7}x + \frac{4}{7}$

---

52. $(0, 4)$ **and** $(2, -3)$

---

53. $(0, -2)$ **and** $(-5, -3)$

Solution

$y = \frac{1}{5}x - 2$

---

54. $(7, 2)$ **and** $(7, -2)$

---

55. $(4, 2)$ **and** $(4, -3)$

Solution

$x = 4$

---

56. $(-7, -1)$ **and** $(-7, -4)$

---

57. $(-2, 1)$ **and** $(-2, -4)$

Solution

$x = -2$

---

58. $(6, 1)$ **and** $(0, 1)$

---

59. $(6, 2)$ **and** $(-3, 2)$

Solution

$y = 2$

---

60. $(3, -4)$ **and** $(5, -4)$

---

61. $(-6, -3)$ **and** $(-1, -3)$

Solution

$y = -3$

---

62. $(4, 3)$ **and** $(8, 0)$

---

63. $(0, 0)$ **and** $(1, 4)$

Solution

$y = 4x$

---

64. $(-2, -3)$ **and** $(-5, -6)$

---

65. $(-3, 0)$ **and** $(-7, -2)$

Solution

$y=\frac{1}{2}x+\frac{3}{2}$

---

66. $(8,-1)$ **and** $(8,-5)$

---

67. $(3,5)$ **and** $(-7,5)$

Solution

$y=5$

Exercises: Find an Equation of a Line Parallel to a Given Line

Instructions: For questions 68-83, find an equation of a line parallel to the given line and contains the given point. Write the equation in slope–intercept form.

68. line $y=4x+2$**, point** $(1,2)$

---

69. line $y=3x+4$**, point** $(2,5)$

Solution

$y=3x-1$

---

70. line $y = -2x - 3$**, point** $(-1, 3)$

---

71. line $y = -3x - 1$**, point** $(2, -3)$

Solution

$y = -3x + 3$

---

72. line $3x - y = 4$**, point** $(3, 1)$

---

73. line $2x - y = 6$**, point** $(3, 0)$

Solution

$y = 2x - 6$

---

74. line $4x + 3y = 6$**, point** $(0, -3)$

---

75. line $2x + 3y = 6$**, point** $(0, 5)$

Solution

$y = -\frac{2}{3}x + 5$

---

76. line $x = -3$**, point** $(-2, -1)$

---

77. line $x = -4$**, point** $(-3, -5)$

Solution

$x = -3$

---

78. line $x - 2 = 0$**, point** $(1, -2)$

---

79. line $x - 6 = 0$**, point** $(4, -3)$

Solution

$x = 4$

---

80. line $y = 5$**, point** $(2, -2)$

---

81. line $y = 1$**, point** $(3, -4)$

Solution

$y = -4$

---

82. line $y + 2 = 0$**, point** $(3, -3)$

---

83. line $y + 7 = 0$**, point** $(1, -1)$

Solution

$y = -1$

Exercises: Find an Equation of a Line Perpendicular to a Given Line

Instructions: For questions 84-95, find an equation of a line perpendicular to the given line and contains the given point. Write the equation in slope–intercept form.

84. line $y = -2x + 3$, point $(2, 2)$

---

85. line $y = -x + 5$, point $(3, 3)$

Solution

$y = x$

---

86. line $y = \frac{3}{4}x - 2$, point $(-3, 4)$

---

87. line $y = \frac{2}{3}x - 4$, point $(2, -4)$

Solution

$y = -\frac{3}{2}x - 1$

---

88. line $2x - 3y = 8$, point $(4, -1)$

---

89. line $4x - 3y = 5$, point $(-3, 2)$

Solution

$y = -\frac{3}{4}x - \frac{1}{4}$

---

90. line $2x + 5y = 6$, point $(0, 0)$

---

91. line $4x + 5y = -3$, point $(0, 0)$

Solution

$y = \frac{5}{4}x$

---

92. line $y - 3 = 0$, point $(-2, -4)$

---

93. line $y - 6 = 0$, point $(-5, -3)$

Solution

$x = -5$

---

94. line y-axis, point $(3, 4)$

---

95. line y-axis, point $(2, 1)$

Solution

$y = 1$

Exercises: Mixed Practice

Instructions: For questions 96-113, find the equation of each line. Write the equation in slope–intercept form.

96. Containing the points $(4,3)$ and $(8,1)$

---

97. Containing the points $(2,7)$ and $(3,8)$

Solution

$y=x+5$

---

98. $m=\frac{1}{6}$**, containing point $(6,1)$**

---

99. $m=\frac{5}{6}$**, containing point $(6,7)$**

Solution

$y=\frac{5}{6}x+2$

---

100. Parallel to the line $4x+3y=6$, containing point $(0,-3)$

---

101. Parallel to the line $2x+3y=6$, containing point $(0,5)$

Solution

$y=-\frac{2}{3}x+5$

---

102. $m=-\frac{3}{4}$**, containing point** $(8,-5)$

---

103. $m=-\frac{3}{5}$**, containing point** $(10,-5)$

Solution

$y=-\frac{3}{5}x+1$

---

104. Perpendicular to the line $y-1=0$**, point** $(-2,6)$

---

105. Perpendicular to the line y**-axis, point** $(-6,2)$

Solution

$y=2$

---

106. Containing the points $(4,3)$ **and** $(8,1)$

---

107. Containing the points $(-2,0)$ **and** $(-3,-2)$

Solution

$y=2x+4$

---

108. Parallel to the line $x = -3$, containing point $(-2, -1)$

---

109. Parallel to the line $x = -4$, containing point $(-3, -5)$

Solution

$x = -3$

---

110. Containing the points $(-3, -4)$ and $(2, -5)$

---

111. Containing the points $(-5, -3)$ and $(4, -6)$

Solution

$y = -\frac{1}{3}x - \frac{14}{3}$

---

112. Perpendicular to the line $x - 2y = 5$, containing point $(-2, 2)$

---

113. Perpendicular to the line $4x + 3y = 1$, containing point $(0, 0)$

Solution

$y = \frac{3}{4}x$

Exercises: Everyday Math

Instructions: For questions 114-115, answer the given everyday math word problems.

114. Cholesterol. The age, x, and LDL cholesterol level, y, of two men are given by the points $(18, 68)$ and $(27, 122)$. Find a linear equation that models the relationship between age and LDL cholesterol level.

---

115. Fuel consumption. The city mpg, x, and highway mpg, y, of two cars are given by the points $(29, 40)$ and$(19, 28)$. Find a linear equation that models the relationship between city mpg and highway mpg.

Solution

$y = 1.2x + 5.2$

Exercises: Writing Exercises

Instructions: For questions 116-117, answer the given writing exercises.

116. Why are all horizontal lines parallel?

---

117. Explain in your own words why the slopes of two perpendicular lines must have opposite signs.

Solution

Answers will vary.

3.12 LINEAR FUNCTIONS AND APPLICATIONS OF LINEAR FUNCTIONS

Learning Objectives

In this section, you will:

- Represent a linear function.
- Write and interpret an equation for a linear function.
- Model real-world problems with linear functions.
- Build linear models from verbal descriptions.

Just as with the growth of a bamboo plant, many situations involve constant change over time. Consider, for example, the first commercial maglev train in the world, the Shanghai MagLev Train Figure 3.12.1. It carries passengers comfortably for a 30-kilometer trip from the airport to the subway station in only eight minutes[1] .

Figure 3.12.1 Shanghai MagLev Train by Jody McIntyre CC-BY-SA 2.0

Suppose a maglev train travels a long distance and maintains a constant speed of 83 meters per second for some time once it is 250 meters from the station. How can we analyze the train's distance from the station as a function of time? In this section, we will investigate a kind

1. http://www.chinahighlights.com/shanghai/transportation/maglev-train.htm

of function that is useful for this purpose, and use it to investigate real-world situations such as the train's distance from the station at a given point in time.

Representing a Linear Function

The function describing the train's motion is a **linear function**, which is defined as a function with a constant rate of change. This is a polynomial of degree 1. There are several ways to represent a linear function, including word form, function notation, tabular form, and graphical form. We will describe the train's motion as a function using each method.

Representing a Linear Function in Word Form

Let's begin by describing the linear function in words. For the train problem, we just considered, the following word sentence may be used to describe the function relationship.

- *The train's distance from the station is a function of the time during which the train moves at a constant speed plus its original distance from the station when it began moving at a constant speed.*

The speed is the rate of change. Recall that a rate of change is a measure of how quickly the dependent variable changes with respect to the independent variable. The rate of change for this example is constant, which means that it is the same for each input value. As the time (input) increases by 1 second, the corresponding distance (output) increases by 83 meters. The train began moving at this constant speed at a distance of 250 meters from the station.

Representing a Linear Function in Function Notation

Another approach to representing linear functions is by using function notation. One example of function notation is an equation written in the slope-intercept form of a line, where x is the input value, m is the rate of change, and b is the initial value of the dependent variable.

Equation form	$y = mx + b$
Function notation	$f(x) = mx + b$

In the example of the train, we might use the notation D where the total distance D is a function of the time t. The rate, m, is 83 meters per second. The initial value of the dependent variable b is the original distance from the station, 250 meters. We can write a generalized equation to represent the motion of the train.

$$D = 83t + 250.$$

Can the input in the previous example be any real number?

No. The input represents time so while nonnegative rational and irrational numbers are possible, negative real numbers are not possible for this example. The input consists of non-negative real numbers.

Ask yourself what numbers can be input to the function. In other words, what is the domain of the function? The domain is comprised of all real numbers because any number may be doubled, and then have one added to the product.

Linear Function

A linear function is a function whose graph is a line. Linear functions can be written in the slope-intercept form of a line

$$f(x) = mx + b$$

where b is the initial or starting value of the function (when input, $x = 0$), and m, is the constant rate of change, or slope of the function. The y-intercept is at $(0, b)$.

Example 1

Using a Linear Function to Find the Pressure on a Diver

The pressure, P in pounds per square inch *(PSI)* on the diver in Figure 3.12.4 depends upon her depth below the water surface, d in feet. This relationship may be modeled by the equation, $P(d) = 0.434d + 14.696$. Restate this function in words.

Figure 3.12.4 Photo by Adam Reeder CC-BY-NC 2.0

Solution

To restate the function in words, we need to describe each part of the equation. The pressure as a function of depth equals four hundred thirty-four thousandths times depth plus fourteen and six hundred ninety-six thousandths.

Analysis

The initial value, 14.696, is the pressure in PSI on the diver at a depth of 0 feet, which is the surface of the water. The rate of change, or slope, is 0.434 PSI per foot. This tells us that the pressure on the diver increases 0.434 PSI for each foot her depth increases.

Model Real-World Problems with Linear Functions

In the real world, problems are not always explicitly stated in terms of a function or represented with a graph. Fortunately, we can analyze the problem by first representing it as a linear function and then interpreting the components of the function. As long as we know, or can figure out, the initial value and the rate of change of a **linear function**, we can solve many different kinds of real-world problems.

How to

Given a linear function f and the initial value and rate of change, evaluate f.

1. Determine the initial value and the rate of change (slope).
2. Substitute the values into $f(x) = mx + b$.
3. Evaluate the function at $x = c$.

Example 2

Using a Linear Function to Calculate Salary Based on Commission

Working as an insurance salesperson, Ilya earns a base salary plus a commission on each new policy. Therefore, Ilya's weekly income depends on the number of new policies, n he sells during the week. Last week he sold 3 new policies, and earned $\$760$ for the week. The week before, he sold 5 new policies and earned $\$920$. Find an equation for I and interpret the meaning of the components of the equation.

Solution

The given information gives us two input-output pairs: $(3, \$760)$ and $(5, \$920)$ We start by finding the rate of change.

$$\begin{aligned} \mathrm{m} &= \frac{\$920 - \$760}{5 - 3} \\ &= \frac{\$160}{2\ policies} \\ &= \$80\ per\ policy \end{aligned}$$

Keeping track of units can help us interpret this quantity. Income increased by $\$160$ when the number of policies increased by 2, so the rate of change is $\$80$ per policy. Therefore, Ilya earns a commission of $\$80$ for each policy sold during the week.

We can then solve for the initial value.

$$\begin{aligned} \mathrm{I(n)} &= 80n + b \\ 760 &= 80(3) + b \text{ when n=3, I(3)=760} \\ 760 - 80(3) &= b \\ 520 &= b \end{aligned}$$

The value of b is the starting value for the function and represents Ilya's income when $n = 0$ or when no new policies are sold. We can interpret this as Ilya's base salary for the week, which does not depend upon the number of policies sold.

We can now write the final equation.

$$I = 80n + 520$$

Our final interpretation is that Ilya's base salary is $\$520$ per week and he earns an additional $\$80$ commission for each policy sold.

Example 3

Using Tabular Form to Write an Equation for a Linear Function

The below table relates the number of rats in a population to time, in weeks. Use the table to write a linear equation.

number of weeks, w	0	2	4	6
number of rats, $P(w)$	1000	1080	1160	1240

Solution

We can see from the table that the initial value for the number of rats is 1000, so $b = 1000$.

Rather than solving for m we can tell from looking at the table that the population increases by 80 for every 2 weeks that pass. This means that the rate of change is 80 rats per 2 weeks, which can be simplified to 40 rats per week.

$$P(w) = 40w + 1000$$

If we did not notice the rate of change from the table we could still solve for the slope using any two points from the table. For example, using $(2, 1080)$ and $(6, 1240)$

$$\begin{aligned} \text{m} &= \frac{1240 - 1080}{6 - 2} \\ &= \frac{160}{4} \\ &= 40 \end{aligned}$$

Is the initial value always provided in a table of values like Example 3.12.4?

No. Sometimes the initial value is provided in a table of values, but sometimes it is not. If you see an input of 0*, then the initial value would be the corresponding output. If the initial value is not provided because there is no value of input on the table equal to* 0*, find the slope, substitute one coordinate pair, and the slope into* $f(x) = mx + b$ *and solve for* b*.*

Try It

1) A new plant food was introduced to a young tree to test its effect on the height of the tree. The table shows the height of the tree, in feet, x months since the measurements began. Write a linear function, $H(x)$, where x is the number of months since the start of the experiment.

x	0	2	4	8	12
$H(x)$	12.5	13.5	14.5	16.5	18.5

Solution

$H(x) = 0.5x + 12.5$

Building Linear Models from Verbal Descriptions

When building linear models to solve problems involving quantities with a constant rate of change, we typically follow the same problem strategies that we would use for any type of function. Let's briefly review them:

How to

Building Linear Models from Verbal Descriptions

1. Identify changing quantities, and then define descriptive variables to represent those quantities. When appropriate, sketch a picture or define a coordinate system.
2. Carefully read the problem to identify important information. Look for information that provides values for the variables or values for parts of the functional model, such as slope and initial value.
3. Carefully read the problem to determine what we are trying to find, identify, solve, or interpret.
4. Identify a solution pathway from the provided information to what we are trying to find. Often this will involve checking and tracking units, building a table, or even finding a formula

for the function being used to model the problem.

5. When needed, write a formula for the function.
6. Solve or evaluate the function using the formula.
7. Reflect on whether your answer is reasonable for the given situation and whether it makes sense mathematically.
8. Clearly convey your result using appropriate units, and answer in full sentences when necessary.

Using a Given Intercept to Build a Model

Some real-world problems provide the y-intercept, which is the constant or initial value. Once the y-intercept is known, the x-intercept can be calculated. Suppose, for example, that Hannah plans to pay off a no-interest loan from her parents. Her loan balance is $\$1,000$. She plans to pay $\$250$ per month until her balance is $\$0$. The y-intercept is the initial amount of her debt, or $\$1,000$. The rate of change, or slope, is $-\$250$ per month. We can then use the slope-intercept form and the given information to develop a linear model.

$$\begin{aligned} \text{f(x)} &= mx + b \\ &= -250x + 1000 \end{aligned}$$

Now we can set the function equal to 0, and solve for x to find the x-intercept.

$$\begin{aligned} 0 &= -250x + 1000 \\ 1000 &= 250x \\ 4 &= x \\ x &= 4 \end{aligned}$$

The x-intercept is the number of months it takes her to reach a balance of $\$0$. The x-intercept is 4 months, so it will take Hannah four months to pay off her loan.

Using a Given Input and Output to Build a Model

Many real-world applications are not as direct as the ones we just considered. Instead, they require us to identify some aspect of a linear function. We might sometimes instead be asked to evaluate the linear model at a given input or set the equation of the linear model equal to a specified output.

How to

Given a word problem that includes two pairs of input and output values, use the linear function to solve a problem.

1. Identify the input and output values.
2. Convert the data to two coordinate pairs.
3. Find the slope.
4. Write the linear model.
5. Use the model to make a prediction by evaluating the function at a given x-value.
6. Use the model to identify an x-value that results in a given y-value.
7. Answer the question posed.

Example 4

Using a Linear Model to Investigate a Town's Population

A town's population has been growing linearly. In 2004, the population was 6,200. By 2009, the population had grown to 8,100. Assume this trend continues.

a. Predict the population in 2013.
b. Identify the year in which the population will reach 15,000.

Solution
The two changing quantities are the population size and time. While we could use the actual year value as the input quantity, doing so tends to lead to very cumbersome equations because the y-intercept would correspond to the year 0, more than 2000 years ago!

To make computation a little nicer, we will define our input as the number of years since 2004.

Input: t, years since 2004

Output: $P(t)$, the town's population

To predict the population in 2013 $t=9$, we would first need an equation for the population. Likewise, to find when the population would reach 15,000, we would need to solve for the input that would provide an output of 15,000. To write an equation, we need the initial value and the rate of change, or slope.

To determine the rate of change, we will use the change in output per change in input.

$$m=\frac{\text{change in output}}{\text{change in input}}$$

The problem gives us two input-output pairs. Converting them to match our defined variables, the year 2004 would correspond to $t = 0$ giving the point $(0, 6200)$. Notice that through our clever choice of variable definition, we have "given" ourselves the y-intercept of the function. The year 2009 would correspond to, $t = 5$ giving the point $(5, 8100)$.

The two coordinate pairs are $(0, 6200)$ and $(5, 8100)$. Recall that we encountered examples in which we were provided two points earlier in the chapter. We can use these values to calculate the slope.

$$\mathrm{m} = \frac{8100 - 6200}{5 - 0}$$

$$= \frac{1900}{5}$$

$$= 380$$

We already know the y-intercept of the line, so we can immediately write the equation:

$$P(t) = 380t + 6200$$

To predict the population in 2013, we evaluate our function at, $t = 9$.

$$\mathrm{P}(9) = 380(9) + 6,200$$

$$= 9,620$$

if the trend continues, our model predicts a population of 9,620 in 2013.

To find when the population will reach 15,000, we can set

$$\mathrm{P(t)} = 15000$$

$$150000 = 380t + 6200$$

$$8800 = 380t$$

$$\approx 23.158$$

Our model predicts the population will reach 15,000 in a little more than 23 years after 2004, or somewhere around the year 2027.

Try It

2) A company sells doughnuts. They incur a fixed cost of $25,000 for rent, insurance, and other expenses. It costs $0.25 to produce each doughnut.

a. Write a linear model to represent the cost C of the company as a function of x, the number of doughnuts produced.

b. Find and interpret the y-intercept.

Solution

a. $C(x) = 0.25x + 25,000$

b. The y-intercept is $(0, 25,000)$. If the company does not produce a single doughnut, they still incur a cost of 25,000.

3) A city's population has been growing linearly. In 2008, the population was 28,200. By 2012, the population was 36,800. Assume this trend continues.

a. Predict the population in 2014.
b. Identify the year in which the population will reach 54,000.

Solution

a. 41,100
b. 2020

Key Concepts

- Linear functions can be represented in words, function notation, tabular form, and graphical form.

- The equation for a linear function can be written if the slope m and initial value b are known.
- A linear function can be used to solve real-world problems given information in different forms.
- We can use the same problem strategies that we would use for any type of function.
- When modeling and solving a problem, identify the variables and look for key values, including the slope and y-intercept.
- Draw a diagram, where appropriate.
- Check for the reasonableness of the answer.
- Linear models may be built by identifying or calculating the slope and using the y-intercept.
  - The x-intercept may be found by setting, $y=0$, which is setting the expression $mx+b$ equal to 0.
  - The point of intersection of a system of linear equations is the point where the x– and y-values are the same.
  - A graph of the system may be used to identify the points where one line falls below (or above) the other line.

Glossary

decreasing linear function
a function with a negative slope: If $f(x) = mx + b$, $m < 0$.

increasing linear function
a function with a positive slope: If $f(x) = mx + b$, $m > 0$.

linear function
a function with a constant rate of change that is a polynomial of degree 1, and whose graph is a straight line

Exercises: Verbal Problems

Instructions: For questions 1-7, answer the given verbal word problems.

1. Terry is skiing down a steep hill. Terry's elevation, $E(t)$, in feet after t seconds is given by $E(t) = 3000 - 70t$. Write a complete sentence describing Terry's starting elevation and how it is changing over time.

Solution

Terry starts at an elevation of 3000 feet and descends 70 feet per second.

2. Jessica is walking home from a friend's house. After 2 minutes she is 1.4 miles from home. Twelve minutes after leaving, she is 0.9 miles from home. What is her rate in miles per hour?

---

3. A boat is 100 miles away from the marina, sailing directly toward it at 10 miles per hour. Write an equation for the distance of the boat from the marina after t hours.

Solution

$d(t) = 100 - 10t$

---

4. Explain how to find the output variable in a word problem that uses a linear function.

---

5. Explain how to find the input variable in a word problem that uses a linear function.

Solution

Determine the independent variable. This is the variable upon which the output depends.

---

6. Explain how to determine the slope in a word problem that uses a linear function.

---

7. Explain how to interpret the initial value in a word problem that uses a linear function.

Solution

To determine the initial value, find the output when the input is equal to zero.

Exercises: Algebraic Problems

Instructions: For questions 8-9, consider this scenario: A town's population has been decreasing at a constant rate. In 2010 the population was $5{,}900$. By 2012 the population had dropped to $4{,}700$. Assume this trend continues.

8. Identify the year in which the population will reach 0.

---

9. Predict the population in 2016.

Solution

$2{,}300$

Exercises: Algebraic Problems

Instructions: For questions 10-11, consider this scenario: A town's population has been increasing at a constant rate. In 2010 the population was $46{,}020$. By 2012 the population had increased to $52{,}070$. Assume this trend continues.

10. Identify the year in which the population will reach $75{,}000$.

---

11. Predict the population in 2016.
Solution
$64{,}170$

Exercises: Algebraic Problems

Instructions: For questions 12-17, consider this scenario: A town has an initial population of $75{,}000$. It grows at a constant rate of $2{,}500$ per year for 5 years.

12. Find a reasonable domain and range for the function P.

---

13. Find the linear function that models the town's population P as a function of the year, t, where t is the number of years since the model began.

Solution

$P(t) = 75,000 + 2500t$

---

14. If the function P is graphed, find and interpret the slope of the function.

---

15. If the function P is graphed, find and interpret the x- and y-intercepts.

Solution

$(-30, 0)$ Thirty years before the start of this model, the town had no citizens.

$(0, 75,000)$ Initially, the town had a population of $75,000$.

---

16. What is the population in the year 12 years from the onset of the model?

---

17. When will the population reach $100,000$?

Solution

Ten years after the model began

Exercises: Algebraic Problems

Instructions: For questions 18-23, consider this scenario: The weight of a newborn is 7.5 pounds. The baby gained one-half pound a month for its first year.

18. Find a reasonable domain and range for the function W.

---

19. Find the linear function that models the baby's weight W as a function of the age of the baby, in months, t.

Solution

$W(t) = 0.5t + 7.5$

---

20. If the function W is graphed, find and interpret the slope of the function.

---

21. If the function W is graphed, find and interpret the x– and y-intercepts.

Solution

$(-15, 0)$: The x -intercept is not a plausible set of data for this model because it means the baby weighed 0 pounds 15 months prior to birth. $(0, 7.5)$: The baby weighed 7.5 pounds at birth.

---

22. What is the output when the input is 6.2?

---

23. When did the baby weigh 10.4 pounds?

Solution

At age 5.8 months

Exercises: Algebraic Problems

Instructions: For questions 24-29, consider this scenario: The number of people afflicted with the common cold in the winter months steadily decreased by 205 each year from 2005 until 2010. In 2005, $12{,}025$ people were inflicted.

24. Find a reasonable domain and range for the function C.

---

25. Find the linear function that models the number of people inflicted with the common cold C as a function of the year, t.

Solution

$C(t) = 12{,}025 - 205t$

---

26. If the function C is graphed, find and interpret the slope of the function.

---

27. If the function C is graphed, find and interpret the x- and y-intercepts.

Solution

$(58.7, 0)$: In roughly 59 years, the number of people inflicted with the common cold would be 0.

$(0, 12{,}025)$: Initially there were $12{,}025$ people afflicted by the common cold.

---

28. In what year will the number of people be $9{,}700$?

---

29. When will the output reach 0?

Solution

2063

Exercises: Graphical Problems

Instructions: For questions 30-33, use the graph in Figure 3P.12.1, which shows the profit, y, in thousands of dollars, of a company in a given year, t, where t represents the number of years since 1980.

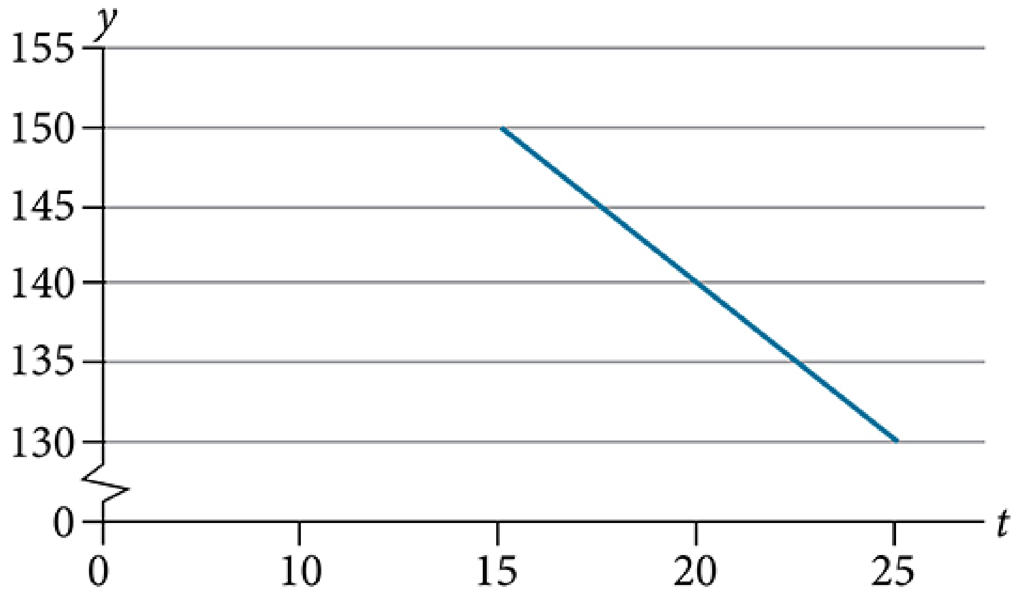


Figure 3P.12.1

30. Find and interpret the y-intercept.

---

31. Find the linear function y, where y depends on t, the number of years since 1980.

Solution

$y = -2t + 180$

32. Find and interpret the slope.

---

33. Find and interpret the x-intercept.

Solution
In 2070, the company's profit will be zero.

Exercises: Graphical Problems

Instructions: For questions 34-37, use the graph in Figure 3P.12.2, which shows the profit, y, in thousands of dollars, of a company in a given year, t, where t represents the number of years since 1980.

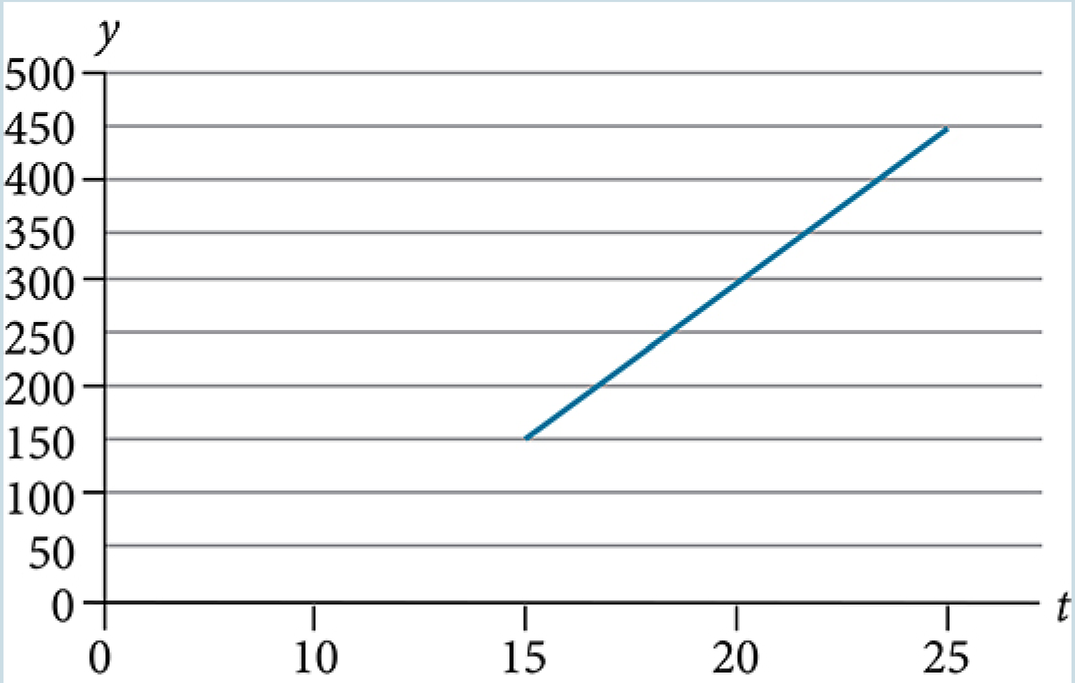


Figure 3P.12.2

34. Find the linear function y, where y depends on t, the number of years since 1980.

Solution

$y = 30t - 300$

---

35. Find and interpret the y-intercept.

Solution

$(0, -300)$ In 1980, the company lost $\$300,000$.

---

36. Find and interpret the x-intercept.

---

37. Find and interpret the slope.

Solution

$y = 30t - 300$ of form $y = mx + b, m = 30$. For each year after 1980, the company's profits increased $\$30,000$ per year

Exercises: Numeric Problems

Instructions: For questions 38-40, use the median home values in Mississippi and Hawaii (adjusted for inflation) shown in Table 3P.12.1. Assume that the house values are changing linearly.

Table 3P.12.1: Median Home Values of Mississipi and Hawaii in 1950 and 2000

Year	Mississippi	Hawaii
1950	$\$25,200$	$\$74,400$
2000	$\$71,400$	$\$272,700$

38. In which state have home values increased at a higher rate?

---

39. If these trends were to continue, what would be the median home value in Mississippi in 2010?

Solution

$\$80,640$

---

40. If we assume the linear trend existed before 1950 and continues after 2000, the two states' median house values will be (or were) equal in what year? (The answer might be absurd.)

Exercises: Numeric

Instructions: For questions 41-43, use the median home values in Indiana and Alabama (adjusted for inflation) shown in (Figure). Assume that the house values are changing linearly.

Table 3P.12.2: Median Home Values of Indiana and Alabama in 1950 and 2000

Year	Indiana	Alabama
1950	$37,700	$27,100
2000	$94,300	$85,100

41. In which state have home values increased at a higher rate?

Solution

Alabama

---

42. If these trends were to continue, what would be the median home value in Indiana in 2010?

---

43. If we assume the linear trend existed before 1950 and continues after 2000, the two states' median house values will be (or were) equal in what year? (The answer might be absurd.)

Solution

2328

Exercises: Real-World Applications

Instructions: For questions 44-66, solve the given real-world application problems.

44. A gym membership with two personal training sessions costs $\$125$, while gym membership with five personal training sessions costs $\$260$. What is cost per session?

---

45. At noon, a barista notices that she has $\$20$ in her tip jar. If she makes an average of $\$0.50$ from each customer, how much will she have in her tip jar if she serves n more customers during her shift?

Solution

$20 + 0.5n$

---

46. A phone company charges for service according to the formula: $C(n) = 24 + 0.1n$, where n is the number of minutes talked, and $C(n)$ is the monthly charge, in dollars. Find and interpret the rate of change and initial value.

---

47. A clothing business finds there is a linear relationship between the number of shirts, n, it can sell and the price, p, it can charge per shirt. In particular, historical data

shows that $1,000$ shirts can be sold at a price of $\$30$, while $3,000$ shirts can be sold at a price of $\$22$. Find a linear equation in the form $p(n) = mn + b$ that gives the price p they can charge for n shirts.

Solution

$p(n) = -0.004n + 34$

---

48. A city's population in the year 1960 was $287,500$. In 1989 the population was $275,900$. Compute the rate of growth of the population and make a statement about the population rate of change in people per year.

---

49. A farmer finds there is a linear relationship between the number of bean stalks, n, she plants and the yield, y, each plant produces. When she plants 30 stalks, each plant yields 30 oz of beans. When she plants 34 stalks, each plant produces 28 oz of beans. Find a linear relationships in the form $y = mn + b$ that gives the yield when n stalks are planted.

Solution

$y = -0.5n + 45$

---

50. Suppose that average annual income (in dollars) for the years 1990 through 1999 is

given by the linear function: $I(x) = 1054x + 23{,}286$, where x is the number of years after 1990.

Which of the following interprets the slope in the context of the problem?

a. As of 1990, average annual income was $\$23{,}286$.
b. In the ten-year period from 1990–1999, average annual income increased by a total of $\$1{,}054$.
c. Each year in the decade of the 1990s, average annual income increased by $\$1{,}054$.
d. Average annual income rose to a level of $\$23{,}286$ by the end of 1999.

---

51. A town's population has been growing linearly. In 2003, the population was $45{,}000$, and the population has been growing by $1{,}700$ people each year. Write an equation, $P(t)$, for the population t years after 2003.

Solution

$P(t) = 1700t + 45,000$

---

52. In 2004, a school population was 1001. By 2008 the population had grown to 1697. Assume the population is changing linearly.

a. How much did the population grow between the year 2004 and 2008?
b. How long did it take the population to grow from 1001 students to 1697 students?
c. What is the average population growth per year?
d. What was the population in the year 2000?
e. Find an equation for the population, P, of the school t years after

2000.
f. Using your equation, predict the population of the school in 2011.

---

53. When temperature is 0 degrees Celsius, the Fahrenheit temperature is 32. When the Celsius temperature is 100, the corresponding Fahrenheit temperature is 212. Express the Fahrenheit temperature as a linear function of C, the Celsius temperature, $F(C)$.

a. Find the rate of change of Fahrenheit temperature for each unit change temperature of Celsius.
b. Find and interpret $F(28)$.
c. Find and interpret $F(-40)$.

Solution

a. $\text{Rate of change} = \frac{\Delta F}{\Delta C} = \frac{212-32}{100-0} = 1.8 \text{ degrees F for one degree change in C}$

b. $F(28) = 1.8(28) + 32 = 82.4 \text{ degrees F is 28 degrees C}$

c. $F(-40) = 1.8(-40) + 32 = -40 \text{ degrees F is -40 degrees C}$

---

54. A phone company has a monthly cellular plan where a customer pays a flat monthly fee and then a certain amount of money per minute used on the phone. If a customer uses 410 minutes, the monthly cost will be $\$71.50$. If the customer uses 720 minutes, the monthly cost will be $\$118$.

a. Find a linear equation for the monthly cost of the cell plan as a function of x, the number of monthly minutes used.

b. Interpret the slope and y-intercept of the equation.

c. Use your equation to find the total monthly cost if 687 minutes are used.

---

55. In 2003, a town's population was 1431. By 2007 the population had grown to 2134. Assume the population is changing linearly.

a. How much did the population grow between the years 2003 and 2007?
b. How long did it take the population to grow from 1431 people to 2134 people?
c. What is the average population growth per year?
d. What was the population in the year 2000?

e. Find an equation for the population, P, of the town t years after 2000.

f. Using your equation, predict the population of the town in 2014.

Solution

a. $2134 - 1431 = 703$ people
b. $2007 - 2003 = 4$ years
c. Average rate of growth $= \frac{703}{4} = 175.75$ people per year. So, using $y = mx + b$, we have $y = 175.75x + 1431$.
d. The year 2000 corresponds to $t = -3$.
So, $y = 175.75(-3) + 1431 = 903.75$ or roughly 904 people in year 2000
e. If the year 2000 corresponds to $t = 0$, then we have ordered pair $(0, 903.75)$.
$y = 175.75x + 903.75$ corresponds to $P(t) = 175.75t + 903.75$.
f. The year 2014 corresponds to $t = 14$. Therefore, $P(14) = 175.75(14) + 903.75 = 3364.25$.
So, a population of 3364.

---

56. In 1991, the moose population in a park was measured to be $4{,}360$. By 1999, the population was measured again to be $5{,}880$. Assume the population continues to

change linearly.

a. Find a formula for the moose population, P since 1990.

b. What does your model predict the moose population to be in 2003?

---

57. A phone company has a monthly cellular data plan where a customer pays a flat monthly fee of $\$10$ and then a certain amount of money per megabyte (MB) of data used on the phone. If a customer uses 20 MB, the monthly cost will be $\$11.20$. If the customer uses 130 MB, the monthly cost will be $\$17.80$.

a. Find a linear equation for the monthly cost of the data plan as a function of x, the number of MB used.

b. Interpret the slope and y-intercept of the equation.

c. Use your equation to find the total monthly cost if 250 MB are used.

Solution

a.

$$\begin{aligned}
&\text{Ordered pairs are } (20, 11.20) \text{ and } (130, 17.80)\\
m &= \frac{17.80-11.20}{130-20} = 0.06 \text{ and } b \text{ is at the point } (0, 10)\\
y &= mx + b\\
y &= 0.06x + 10\\
\text{or } C(x) &= 0.06x + 10
\end{aligned}$$

b.

0.06 – For every MB, the client is charged 6 cents.

$(0, 10)$ – If no usage occurs, the client is charged $\$10$.

c.

$$\begin{aligned}
C(250) &= 0.06(250) + 10\\
&= \$25
\end{aligned}$$

---

58. The Federal Helium Reserve held about 16 billion cubic feet of helium in 2010 and is being depleted by about 2.1 billion cubic feet each year.

a. Give a linear equation for the remaining federal helium reserves, R, in terms of t, the number of years since 2010.

b. In 2015, what will the helium reserves be?
c. If the rate of depletion doesn't change, in what year will the Federal Helium Reserve be depleted?

---

59. In 2003, the owl population in a park was measured to be 340. By 2007, the population was measured again to be 285. The population changes linearly. Let the input be years since 1990.

a. Find a formula for the owl population, P. Let the input be years since 2003.

b. What does your model predict the owl population to be in 2012?

Solution

a.

Ordered pairs are $(0, 340)$ and $(4, 285)$

$$\begin{aligned} m &= \frac{285-340}{4-0} = -13.75 \text{ and } b \text{ is at the point } (0, 340) \\ y &= mx + b \\ y &= -13.75x + 340 \\ \text{or } P(t) &= -13.75t + 340 \end{aligned}$$

b.

The year 2012 corresponds to $t = 9$

$$\begin{aligned} P(9) &= -13.75(9) + 340 \\ &= 216.25 \text{ or } 216 \text{ owls} \end{aligned}$$

---

60. You are choosing between two different prepaid cell phone plans. The first plan charges a rate of 26 cents per minute. The second plan charges a monthly fee of

$\$19.95$ *plus* 11 cents per minute. How many minutes would you have to use in a month in order for the second plan to be preferable?

---

61. Suppose the world's oil reserves in 2014 are $1,820$ billion barrels. If, on average, the total reserves are decreasing by 25 billion barrels of oil each year:

a. Give a linear equation for the remaining oil reserves, R, in terms of t, the number of years since now.
b. Seven years from now, what will the oil reserves be?
c. If the rate at which the reserves are decreasing is constant, when will the world's oil reserves be depleted?

Solution

a.

The year 2014 corresponds to $t = 0$.
We have $m = -25$ and b at the point $(0, 1820)$

$$\begin{aligned} y &= mx + b \\ y &= -25x + 1820 \\ \text{or } R(t) &= -25t + 1820 \end{aligned}$$

b.

$$\begin{aligned} R(7) &= -25(7) + 1820 \\ &= 1645 \text{ billion cubic feet} \end{aligned}$$

c.

$$\begin{aligned} 0 &= -25t + 1820 \\ -1820 &= -25t \\ 72.8 &= t \Rightarrow 2014 + 72.8 = 2086.8. \text{ So, in the year 2086} \end{aligned}$$

---

62. When hired at a new job selling jewelry, you are given two pay options:

Option A: Base salary of $\$17,000$ a year with a commission of 12% of your sales

Option B: Base salary of $\$20,000$ a year with a commission of 5% of your sales

How much jewelry would you need to sell for option A to produce a larger income?

---

63. You are choosing between two different window washing companies. The first charges $\$5$ per window. The second charges a base fee of $\$40$ plus $\$3$ per window. How many windows would you need to have for the second company to be preferable?

Solution

Plan 1: $y = 5x$ where x is number of windows
Plan 2: $y = 3x + 40$ where x is number of windows

$$\begin{aligned} 3x + 40 &\leq 5x \\ 40 &\leq 2x \\ 20 &\leq x \end{aligned}$$

So, more than 20 windows

---

64. When hired at a new job selling electronics, you are given two pay options:

Option A: Base salary of $\$20,000$ a year with a commission of 12% of your sales

Option B: Base salary of $\$26,000$ a year with a commission of 3% of your sales

How many electronics would you need to sell for option A to produce a larger income?

---

65. When hired at a new job selling electronics, you are given two pay options:

Option A: Base salary of $\$14,000$ a year with a commission of 10% of your sales

Option B: Base salary of $\$19,000$ a year with a commission of 4% of your sales

How many electronics would you need to sell for option A to produce a larger income?

Solution

Option A: $y = 0.10x + 14,000$ where x is dollars of sales.
Option B: $y = 0.04x + 19,000$ where x is dollars of sales.

$$\begin{aligned} 0.10x + 14,000 &\geq 0.04x + 19,000 \\ 0.06x + 14,000 &\geq 19,000 \\ 0.06x &\geq 5,000 \\ x &\geq 83,333.33 \end{aligned}$$

So, more than \$83,333.33 in sales.

---

66. When hired at a new job selling electronics, you are given two pay options:

Option A: Base salary of $\$10,000$ a year with a commission of 9% of your sales

Option B: Base salary of $\$20,000$ a year with a commission of 4% of your sales

How much electronics would you need to sell for option A to produce a larger income?

PART IV

UNIT 4: SYSTEMS OF LINEAR EQUATIONS

4.1 SOLVE SYSTEMS OF EQUATIONS BY GRAPHING

Learning Objectives

By the end of this section, you will be able to:

- Determine whether an ordered pair is a solution of a system of equations
- Solve a system of linear equations by graphing
- Determine the number of solutions of linear system
- Solve applications of systems of equations by graphing

Try It

Before you get started, take this readiness quiz:

1) For the equation $y = \frac{2}{3}x - 4$

a. is $(6, 0)$ a solution? b. is $(-3, -2)$ a solution?

2) Find the slope and y-intercept of the line $3x - y = 12$.

3) Find the x- and y-intercepts of the line $2x - 3y = 12$.

Determine Whether an Ordered Pair is a Solution of a System of Equations

In Solving Linear Equations and Applications of Linear Functions we learned how to solve linear equations with one variable. Remember that the solution of an equation is a value of the variable that makes a true statement when substituted into the equation.

Now we will work with systems of linear equations, two or more linear equations grouped.

System of Linear Equations

When two or more linear equations are grouped, they form a **system of linear equations**.

We will focus our work here on systems of two linear equations in two unknowns. Later, you may solve larger systems of equations.

An example of a system of two linear equations is shown below. We use a brace to show the two equations are grouped to form a system of equations.

$$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$$

A linear equation in two variables, like $2x + y = 7$, has an infinite number of solutions. Its graph is a line. Remember, every point on the line is a solution to the equation and every solution to the equation is a point on the line.

To solve a system of two linear equations, we want to find the values of the variables that are solutions to both equations. In other words, we are looking for the ordered pairs (x, y) that make both equations true. These are called the solutions to a system of equations.

Solutions of a System of Equations

Solutions of a system of equations are the values of the variables that make all the equations true. A solution of a system of two linear equations is represented by an ordered pair (x, y).

To determine if an ordered pair is a solution to a system of two equations, we substitute the values of the variables into each equation. If the ordered pair makes both equations true, it is a solution to the system.

Let's consider the system below:

$$\begin{cases} 3x - y = 7 \\ x - 2y = 4 \end{cases}$$

Is the ordered pair $(2, -1)$ a solution?

We substitute $x = 2$ into both equations.

$$\begin{aligned} 3x - y &= 7 \\ 3(2) - (-1) &\stackrel{?}{=} 7 \\ 7 &= 7 \checkmark \end{aligned}$$

$$\begin{aligned} x - 2y &= 4 \\ 2 - 2(-1) &\stackrel{?}{=} 4 \\ 4 &= 4 \checkmark \end{aligned}$$

The ordered pair $(2, -1)$ made both equations true. Therefore $(2, -1)$ is a solution to this system.

Let's try another ordered pair. Is the ordered pair $(3, 2)$ a solution?

We substitute $x = 3$ into both equations.

$$\begin{aligned} 3x - y &= 7 \\ 3(3) - 2 &\stackrel{?}{=} 7 \\ 7 &= 7 \checkmark \end{aligned}$$

$$\begin{aligned} x - 2y &\stackrel{?}{=} 4 \\ 2 - 2(2) &= 4 \\ -2 &= 4 \; False \end{aligned}$$

The ordered pair $(3, 2)$ made one equation true, but it made the other equation false. Since it is not a solution to **both** equations, it is not a solution to this system.

Example 1

Determine whether the ordered pair is a solution to the system: $\begin{cases} x - y = -1 \\ 2x - y = -5 \end{cases}$

a. $(-2, -1)$ b. $(-4, -3)$

Solution

a.

$$\begin{cases} x - y = -1 \\ 2x - y = -5 \end{cases}$$

We substitute x = -2 and y = -1 into both equations.

$$\begin{aligned} x - y &= -1 \\ -2 - (-1) &\stackrel{?}{=} -1 \\ -1 &= -1 \checkmark \end{aligned}$$

$$\begin{aligned} 2x - y &= -5 \\ 2(-2) - (-1) &\stackrel{?}{=} -5 \\ 5 &\neq -5 \end{aligned}$$

$(-2, -1)$ does not make both equations true. $(-2, -1)$ is not a solution.

---

b.

We substitute $x = -4$ and $y = -3$ into both equations.

$$\begin{aligned} x - y &= -1 \\ -4 - (-3) &\stackrel{?}{=} -1 \\ -1 &= -1 \checkmark \end{aligned}$$

$$\begin{aligned} 2x - y &= -5 \\ 2(-4) - (-3) &\stackrel{?}{=} -5 \\ -5 &= -5 \checkmark \end{aligned}$$

$(-4, -3)$ does make both equations true. $(-4, -3)$ is a solution.

Try It

4) Determine whether the ordered pair is a solution to the system: $\begin{cases} 3x + y = 0 \\ x + 2y = -5 \end{cases}$

a. $(1,-3)$ b. $(0,0)$

Solution

a. yes b. no

5) Determine whether the ordered pair is a solution to the system: $\begin{cases} x-3y=-8 \\ -3x-y=4 \end{cases}$

a. $(2,-2)$ b. $(-2,2)$

Solution

a. no b. yes

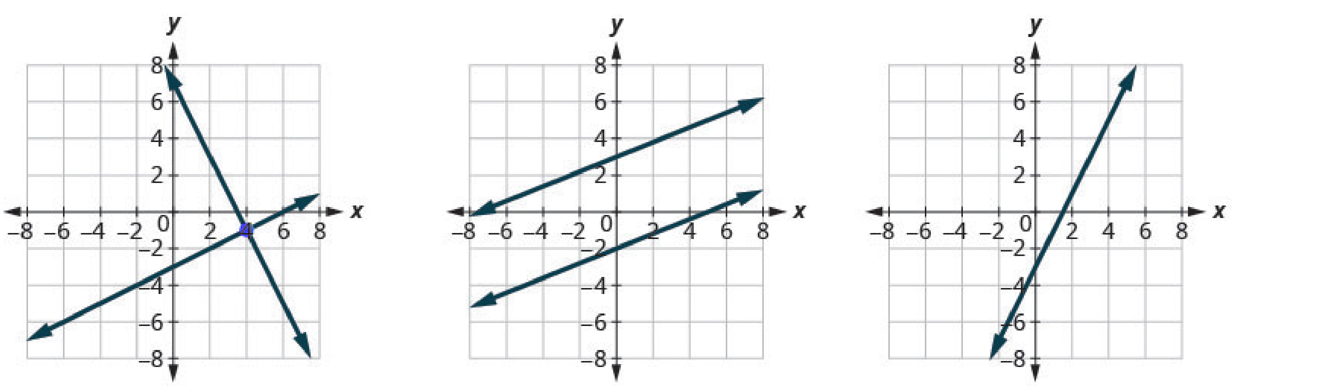
Solve a System of Linear Equations by Graphing

In this chapter, we will use three methods to solve a system of linear equations. The first method we'll use is graphing.

The graph of a linear equation is a line. Each point on the line is a solution to the equation. For a system of two equations, we will graph two lines. Then we can see all the points that are solutions to each equation. And, by finding what the lines have in common, we'll find the solution to the system.

Most linear equations in one variable have one solution, but we saw that some equations, called contradictions, have no solutions, and for other equations, called identities, all numbers are solutions.

Similarly, when we solve a system of two linear equations represented by a graph of two lines in the same plane, there are three possible cases, as shown in Figure 4.1.1:



The lines intersect. Intersecting lines have one point in common. There is one solution to this system.

The lines are parallel. Parallel lines have no points in common. There is no solution to this system.

Both equations give the same line. Because we have just one line, there are infinitely many solutions.

Figure 4.1.1

For the first example of solving a system of linear equations in this section and the next two sections, we will solve the same system of two linear equations. But we'll use a different method in each section. After seeing the third method, you'll decide which method was the most convenient way to solve this system.

Example 2

Solve the system by graphing: $\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$

Solution

Step 1: **Graph the first equation.**

To graph the first line, write the equation in slope-intercept form.

$$\begin{aligned} 2x + y &= 7 \\ y &= -2x + 7 \\ m &= -2 \\ b &= 7 \end{aligned}$$

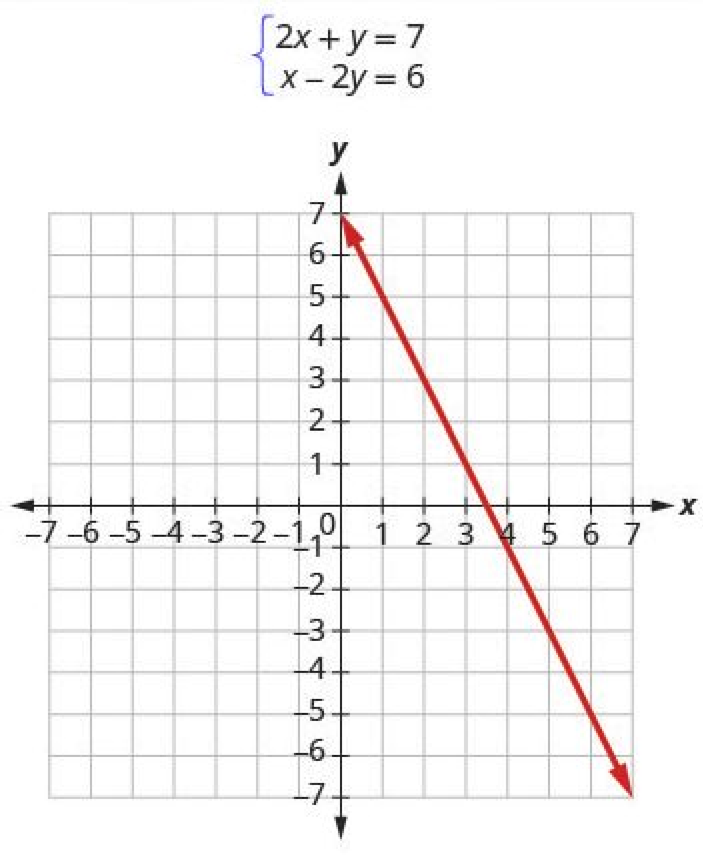


Figure 4.1.2

Step 2: **Graph the second equation on the same rectangular coordinate system.**

To graph the second line, use intercepts.

$x - 2y = 6$

$(0, -3)$ $(6, 0)$

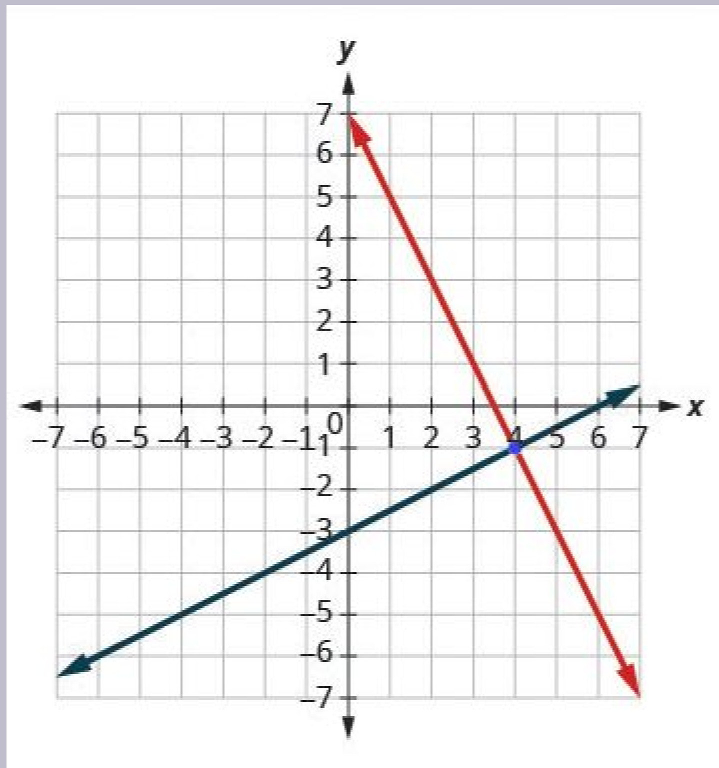


Figure 4.1.3

Step 3: **Determine whether the lines intersect, are parallel, or are the same line.**

Look at the graph of the lines.

The lines intersect.

Step 4: **Identify the solution to the system.**

If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system.

If the lines are parallel, the system has no solution. If the lines are the same, the system has an infinite number of solutions.

Since the lines intersect, find the point of intersection.

Check the point in both equations.

The lines intersect at $(4, -1)$

$$\begin{aligned} 2x + y &= 7 \\ 2(4) + (-1) &\stackrel{?}{=} 7 \\ 8 - 1 &\stackrel{?}{=} 7 \\ 7 &= 7\checkmark \end{aligned}$$

$$\begin{aligned} x - 2y &= 6 \\ 4 - 2(-1) &\stackrel{?}{=} 6 \\ 6 &= 6\checkmark \end{aligned}$$

The solution is $(4, -1)$.

Try It

6) Solve each system by graphing: $\begin{cases} x - 3y = -3 \\ x + y = 5 \end{cases}$

Solution
$(3, 2)$

7) Solve each system by graphing: $\begin{cases} -x + y = 1 \\ 3x + 2y = 12 \end{cases}$

Solution
$(2, 3)$

The steps to use to solve a system of linear equations by graphing are shown below.

How to

Solve a system of linear equations by graphing.

1. Graph the first equation.
2. Graph the second equation on the same rectangular coordinate system.
3. Determine whether the lines intersect, are parallel, or are the same line.
4. Identify the solution to the system.

- ***If the lines intersect***, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system.
- ***If the lines are parallel***, the system has no solution.
- ***If the lines are the same***, the system has an infinite number of solutions.

Example 3

Solve the system by graphing: $\begin{cases} y = 2x + 1 \\ y = 4x - 1 \end{cases}$

Solution

Both of the equations in this system are in slope-intercept form, so we will use their slopes and y-intercepts to graph them. $\begin{cases} y = 2x + 1 \\ y = 4x - 1 \end{cases}$

Step 1: **Find the slope and y-intercept of the first equation.**

$$\begin{aligned} y &= 2x + 1 \\ m &= 2 \\ &= 1 \end{aligned}$$

Step 2: **Find the slope and y-intercept of the second equation.**

$$\begin{aligned} y &= 4x - 1 \\ m &= 4 \\ &= -1 \end{aligned}$$

Step 3: **Graph the two lines.**

Step 4: **Determine the point of intersection.**

The lines intersect at $(1, 3)$.

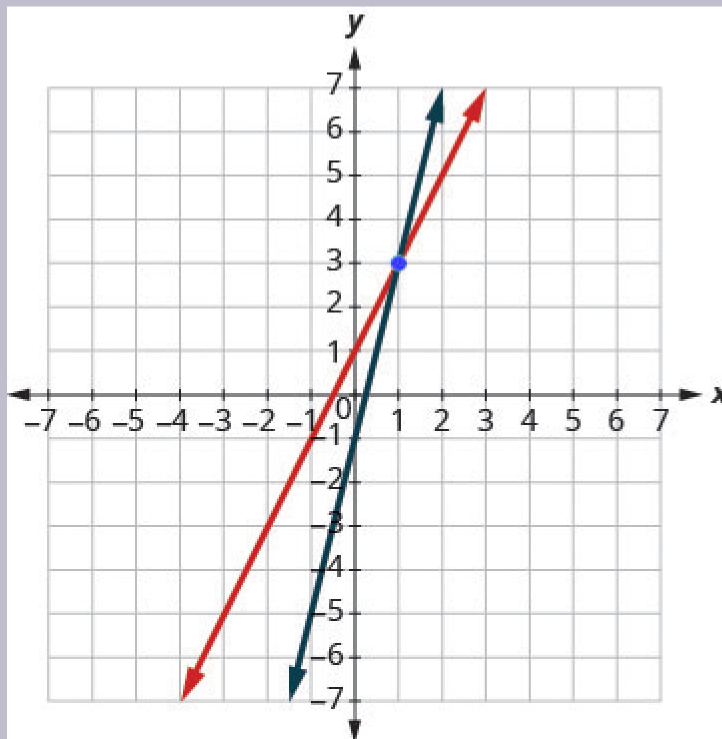


Figure 4.1.4

Step 5: **Check the solution in both equations.**

$$\begin{array}{rcl} y & = & 2x+1 \\ 3 & \stackrel{?}{=} & 2\cdot 1+1 \\ 3 & = & 3\checkmark \end{array} \qquad \begin{array}{rcl} y & = & 4x-1 \\ 3 & \stackrel{?}{=} & 4\cdot 1-1 \\ 3 & = & 3\checkmark \end{array}$$

Step 6: **Write the solution as point (x, y).**

The solution is $(1, 3)$.

Try It

8) Solve each system by graphing: $\begin{cases} y = 2x + 2 \\ y = -x - 4 \end{cases}$

Solution

$(-2,-2)$

9) Solve each system by graphing: $\begin{cases} y=3x+3 \\ y=-x+7 \end{cases}$

Solution

$(1,6)$

Both equations in Example 4.1.3 were given in slope–intercept form. This made it easy for us to quickly graph the lines. In the next example, we'll first re-write the equations into slope–intercept form.

Example 4

Solve the system by graphing: $\begin{cases} 3x+y=-1 \\ 2x+y=0 \end{cases}$

Solution

We'll solve both of these equations for y so that we can easily graph them using their slopes and y-intercepts. $\begin{cases} 3x+y=-1 \\ 2x+y=0 \end{cases}$

Step 1: **Solve the first equation for y.**

Step 2: **Find the slope and y-intercept.**

$$\begin{aligned} 3x+y &= -1 \\ y &= -3x-1 \end{aligned}$$

$$\begin{aligned} m &= -3 \\ b &= -1 \end{aligned}$$

$$\begin{aligned} 2x+y &= 0 \\ y &= -2x \end{aligned}$$

$$\begin{aligned} m &= -2 \\ b &= 0 \end{aligned}$$

Step 3: **Solve the second equation for y.**

Step 4: **Find the slope and y-intercept.**

Step 5: **Graph the lines.**

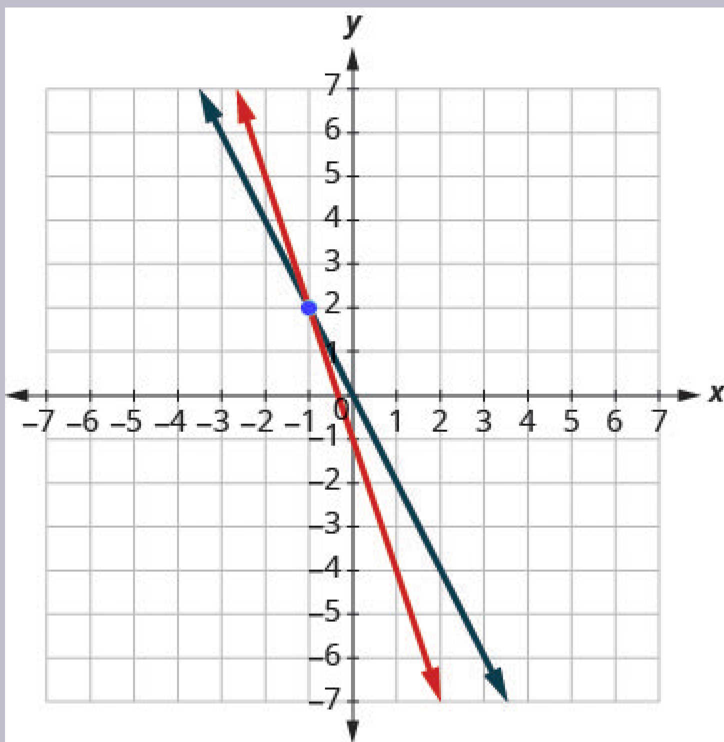


Figure 4.1.5

Step 6: **Determine the point of intersection.**

The lines intersect at $(-1, 2)$.

Step 7: **Check the solution in both equations.**

$$\begin{aligned} 3x+y &= -1 \\ 3(-1)+2 &\stackrel{?}{=} -1 \\ -1 &= -1\checkmark \end{aligned} \qquad \begin{aligned} 2x+y &= 0 \\ 2(-1)+2 &\stackrel{?}{=} 0 \\ 0 &= 0\checkmark \end{aligned}$$

The solution is $(-1, 2)$.

Try It

10) Solve each system by graphing: $\begin{cases} -x+y=1 \\ 2x+y=10 \end{cases}$

Solution

$(3, 4)$

11) Solve each system by graphing: $\begin{cases} 2x+y=6 \\ x+y=1 \end{cases}$

Solution

$(5, -4)$

Usually, when equations are given in standard form, the most convenient way to graph them is by using the intercepts. We'll do this in Example 5.

Example 5

Solve the system by graphing: $\begin{cases} x+y=2 \\ x-y=4 \end{cases}$

Solution

We will find the x– and y-intercepts of both equations and use them to graph the lines.

$$x + y = 2$$

Step 1: **To find the intercepts, let $x = 0$ and solve for y, then let $y = 0$ and solve for x.**

$$\begin{aligned} x + y &= 2 \\ 0 + y &= 2 \\ y &= 2 \end{aligned} \qquad \begin{aligned} x + y &= 2 \\ x + 0 &= 2 \\ x &= 2 \end{aligned}$$

x	y
0	2
2	0

Step 2: **To find the intercepts, let $x = 0$ then let $y = 0$.**

$$\begin{aligned} x - y &= 4 \\ 0 - y &= 4 \\ -y &= 4 \\ y &= -4 \end{aligned} \qquad \begin{aligned} x - y &= 4 \\ x - 0 &= 4 \\ x &= 4 \end{aligned}$$

x	y
0	-4
4	0

Step 3: **Graph the line.**

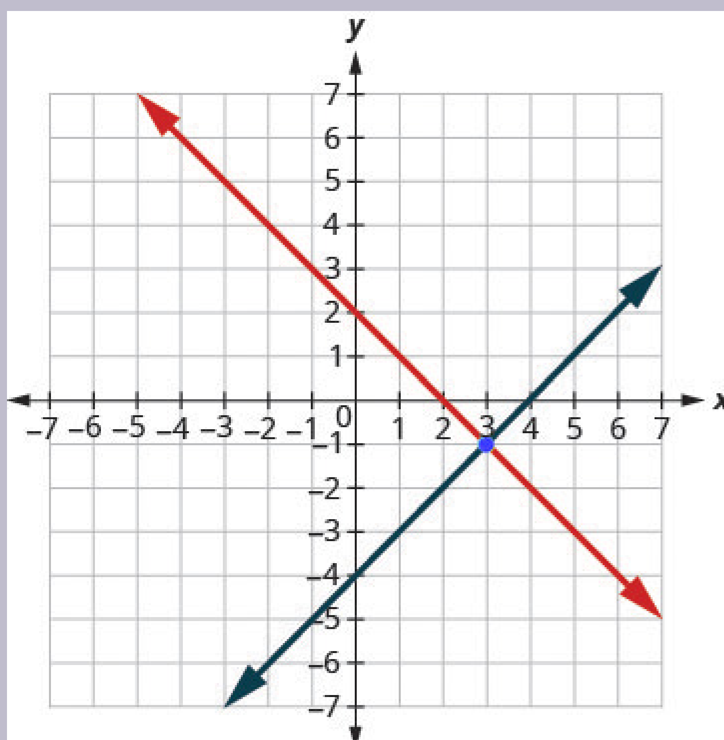


Figure 4.1.6

Step 4: **Determine the point of intersection.**

The lines intersect at $(3, -1)$.

Step 5: **Check the solution in both equations.**

$$\begin{aligned} x+y &= 2 \\ 3+(-1) &\stackrel{?}{=} 2 \\ 2 &= 2\checkmark \end{aligned} \qquad \begin{aligned} x-y &= 4 \\ 3-(-1) &\stackrel{?}{=} 4 \\ 4 &= 4\checkmark \end{aligned}$$

The solution is $(3, -1)$.

Try It

12) Solve each system by graphing: $\begin{cases} x+y=6 \\ x-y=2 \end{cases}$

Solution

$(4, 2)$

13) Solve each system by graphing: $\begin{cases} x+y=2 \\ x-y=-8 \end{cases}$

Solution

$(-3, 5)$

Do you remember how to graph a linear equation with just one variable? It will be either a vertical or a horizontal line.

Example 6

Solve the system by graphing: $\begin{cases} y=6 \\ 2x+3y=12 \end{cases}$

Solution

$$\begin{cases} y=6 \\ 2x+3y=12 \end{cases}$$

Step 1: **We know the first equation represents a horizontal line whose** y**-intercept is** 6.

$$y = 6$$

Step 2: **The second equation is most conveniently graphed using intercepts.**

$$2x + 3y = 12$$

Step 3: **To find the intercepts, let** $x = 0$ **and then** $y = 0$**.**

x	y
0	4
6	0

Step 4: **Graph the lines.**

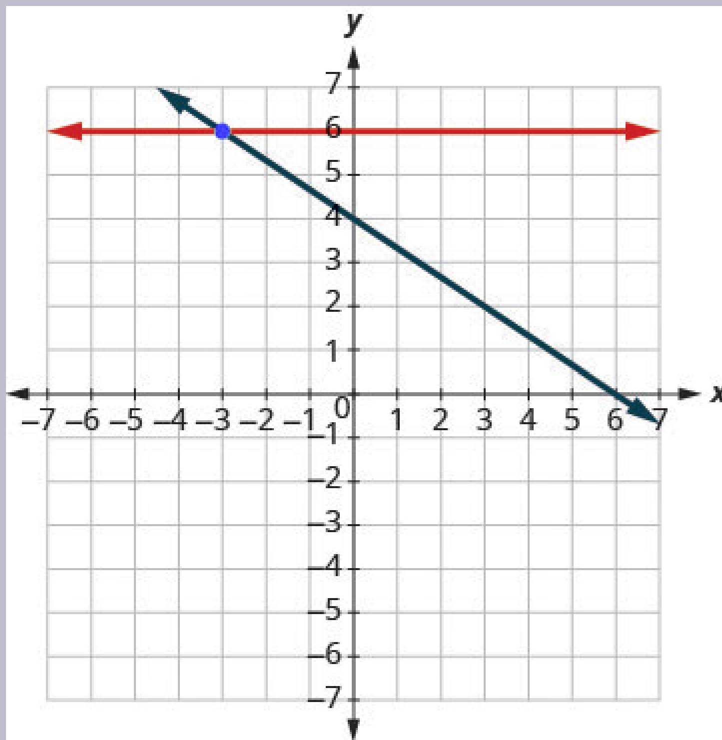


Figure 4.1.7

Step 5: **Determine the point of intersection.**

The lines intersect at $(-3, 6)$.

Step 6: **Check the solution to both equations.**

$$\begin{array}{rcl} y & = & 6 \\ 6 & \stackrel{?}{=} & 6\checkmark \\ 2 & = & 2 \end{array} \qquad \begin{array}{rcl} 2x + 3y & = & 12 \\ 2(-3) + 3(6) & \stackrel{?}{=} & 12 \\ -6 + 18 & \stackrel{?}{=} & 12 \\ 12 & = & 12\checkmark \end{array}$$

The solution is $(-3, 6)$.

Try It

14) Solve each system by graphing: $\begin{cases} y = -1 \\ x + 3y = 6 \end{cases}$

Solution
$(9, -1)$

15) Solve each system by graphing: $\begin{cases} x = 4 \\ 3x - 2y = 24 \end{cases}$

Solution
$(4, -6)$

In all the systems of linear equations so far, the lines intersected and the solution was one point. In the next two examples, we'll look at a system of equations that has no solution and at a system of equations that has an infinite number of solutions.

Example 7

Solve the system by graphing: $\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$

Solution

$$\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$$

Step 1: **To graph the first equation, we will use its slope and y-intercept.**

$$y = \frac{1}{2}x - 3$$
$$m = \frac{1}{2}$$
$$b = -3$$

Step 2: **To graph the second equation, we will use the intercepts.**

$$x - 2y = 4$$

x	y
0	−2
4	0

Step 3: **Graph the lines.**

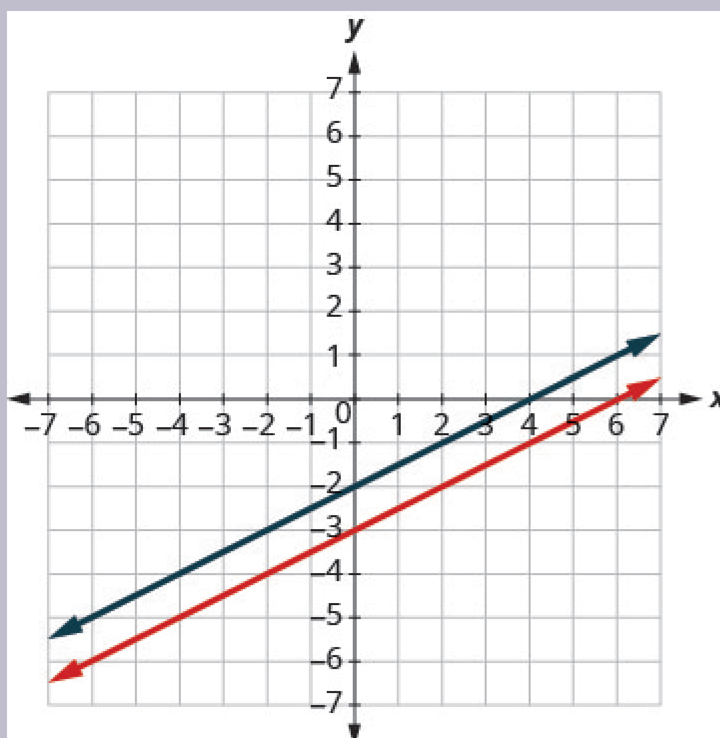


Figure 4.1.8

Step 4: **Determine the point of intersection.**

The lines are parallel.

Since no point is on both lines, there is no ordered pair that makes both equations true. There is no solution to this system.

Try It

16) Solve each system by graphing: $\begin{cases} y = -\frac{1}{4}x + 2 \\ x + 4y = -8 \end{cases}$

Solution

no solution

17) Solve each system by graphing: $\begin{cases} y = 3x - 1 \\ 6x - 2y = 6 \end{cases}$

Solution

no solution

Example 8

Solve the system by graphing: $\begin{cases} y = 2x - 3 \\ -6x + 3y = -9 \end{cases}$

Solution

$$\begin{cases} y = 2x - 3 \\ -6x + 3y = -9 \end{cases}$$

Step 1: **Find the slope and y-intercept of the first equation.**

$$y = 2x - 3$$
$$m = 2$$
$$b = -3$$

Step 2: **Find the intercepts of the second equation.**

$$-6x + 3y = -9$$

x	y
0	-3
$\frac{3}{2}$	0

Step 3: **Graph the lines.**

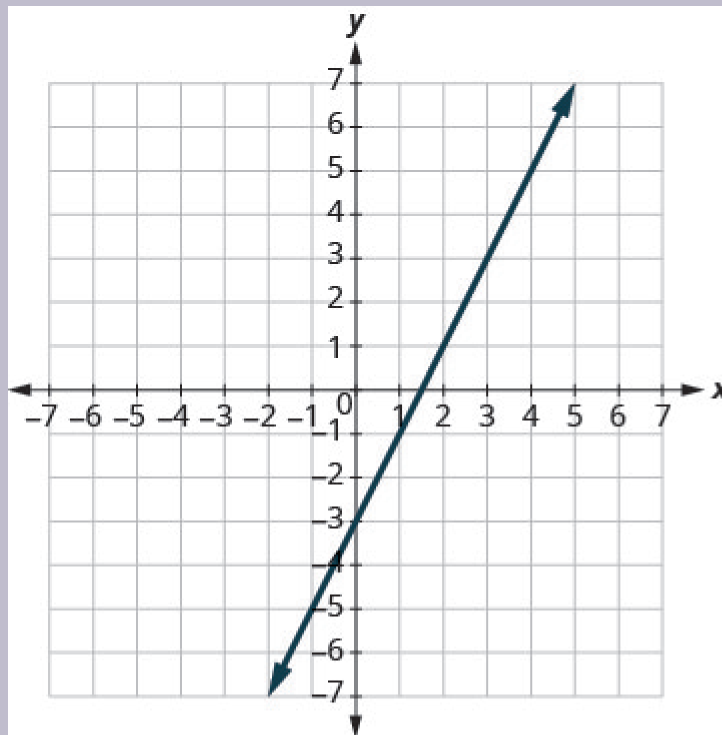


Figure 4.1.9

***Step 4:* Determine the point of intersection.**

The lines are the same!

Since every point on the line makes both equations true, there are infinitely many ordered pairs that make both equations true.

There are infinitely many solutions to this system.

Try It

18) Solve each system by graphing: $\begin{cases} y = -3x - 6 \\ 6x + 2y = -12 \end{cases}$

Solution

infinitely many solutions

19) Solve each system by graphing: $\begin{cases} y = \frac{1}{2}x - 4 \\ 2x - 4y = 16 \end{cases}$

Solution
infinitely many solutions

If you write the second equation in Example 4.1.8 in slope-intercept form, you may recognize that the equations have the same slope and same y-intercept.

When we graphed the second line in the last example, we drew it right over the first line. We say the two lines are coincident. **Coincident lines** have the same slope and same y-intercept.

Coincident Lines

Coincident lines have the same slope and same y-intercept.

Determine the Number of Solutions of a Linear System

There will be times when we will want to know how many solutions there will be to a system of linear equations, but we might not actually have to find the solution. It will be helpful to determine this without graphing.

We have seen that two lines in the same plane must either intersect or are parallel. The systems of equations in Example 4.1.2 through Example 4.1.6 all had two intersecting lines. Each system had one solution.

A system with parallel lines, like Example 4.1.7, has no solution. What happened in Example 4.1.8? The equations have coincident lines, and so the system had infinitely many solutions.

We'll organize these results in Figure 4.2.10 below:

Figure 4.1.10

Graph	**Number of solutions**
2 intersecting lines	1
Parallel lines	None
Same line	Infinitely many

Parallel lines have the same slope but different y-intercepts. So, if we write both equations in a system of linear equations in slope–intercept form, we can see how many solutions there will be without graphing! Look at the system we solved in Example 4.1.7.

$$\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$$

The first line is in slope–intercept form.

$$y = \frac{1}{2}x - 3$$

$$m = \frac{1}{2}, b = -3$$

If we solve the second equation for y, we get

$$\begin{aligned} x - 2y &= 4 \\ -2y &= -x + 4 \\ y &= \frac{1}{2}x - 2 \end{aligned}$$

$$m = \frac{1}{2}, b = -2$$

The two lines have the same slope but different y-intercepts. They are parallel lines.

Figure 4.1.11 shows how to determine the number of solutions of a linear system by looking at the slopes and intercepts.

Number of Solutions of a Linear System of Equations

Figure 4.1.11

Slopes	**Intercepts**	**Type of Lines**	**Number of Solutions**
Different		Intersecting	1 point
Same	Different	Parallel	No solution
Same	Same	Coincident	Infinitely many solutions

Let's take one more look at our equations in Example 4.1.7 which gave us parallel lines.

$$\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$$

When both lines were in slope-intercept form we had:

$$y = \frac{1}{2}x - 3 \qquad y = \frac{1}{2}x - 2$$

Do you recognize that it is impossible to have a single ordered pair (x, y) that is a solution to both of those equations?

We call a system of equations like this an **inconsistent system**. It has no solution.

A system of equations that has at least one solution is called a **consistent system**.

Consistent and Inconsistent Systems

A consistent system of equations is a system of equations with at least one solution.

An inconsistent system of equations is a system of equations with no solution.

We also categorize the equations in a system of equations by calling the equations *independent* or *dependent*. If two equations are **independent equations**, they each have their own set of solutions. Intersecting lines and parallel lines are independent.

If two equations are dependent, all the solutions of one equation are also solutions of the other equation. When we graph two **dependent equations**, we get coincident lines.

Independent and Dependent Equations

Two equations are independent if they have different solutions.

Two equations are dependent if all the solutions of one equation are also solutions of the other equation.

Let's sum this up by looking at the graphs of the three types of systems. See Figure 4.1.12 and Figure 4.2.13.

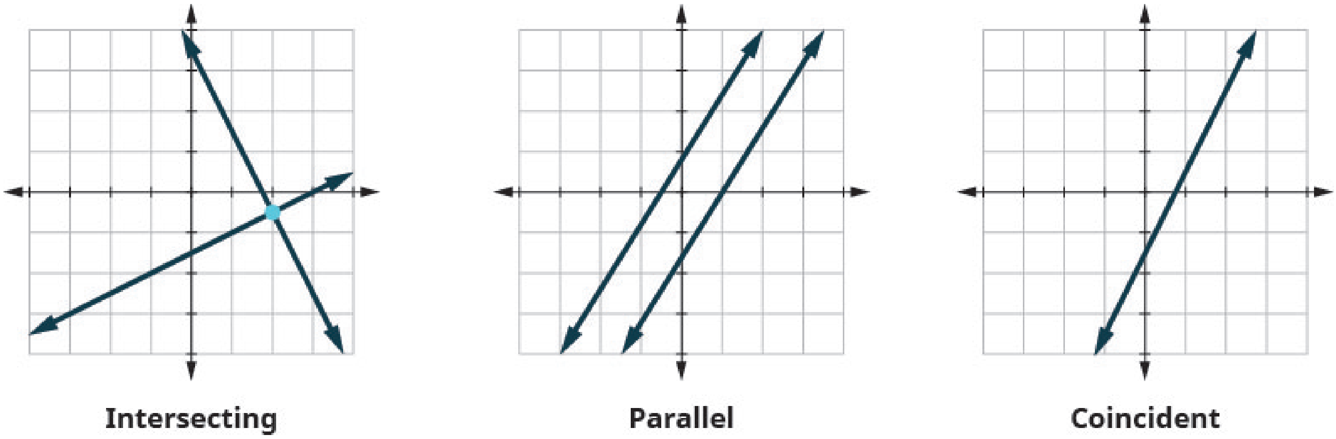


Figure 4.1.12

Figure 4.1.13

Lines	**Intersecting**	**Parallel**	**Coincident**
Number of solutions	1 point	No solution	Infinitely many
Consistent/ Inconsistent	Consistent	Inconsistent	Consistent
Dependent/ Independent	Independent	Independent	Dependent

Example 9

Without graphing, determine the number of solutions and then classify the system of equations:
$\begin{cases} y = 3x - 1 \\ 6x - 2y = 12 \end{cases}$

Solution

Step 1: **We will compare the slopes and intercepts of the two lines.**

$$\begin{cases} y = 3x - 1 \\ 6x - 2y = 12 \end{cases}$$

Step 2: **The first equation is already in slope-intercept form.**

$$y = 3x - 1$$

Step 3: **Write the second equation in slope-intercept form.**

$$\begin{aligned} 6x - 2y &= 12 \\ -2y &= -6x + 12 \\ \frac{-2y}{-2} &= \frac{-6x+12}{-2} \\ y &= 3x - 6 \end{aligned}$$

Step 4: **Find the slope and intercept of each line.**

$$\begin{aligned} y &= 3x - 1 \\ m &= 3 \\ b &= -1 \end{aligned} \qquad \begin{aligned} y &= 3x - 6 \\ m &= 3 \\ b &= -6 \end{aligned}$$

Since the slopes are the same and y-intercepts are different, the lines are parallel.

A system of equations whose graphs are parallel lines has no solution and is inconsistent and independent.

Try It

20) Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} y = -2x - 4 \\ 4x + 2y = 9 \end{cases}$$

Solution

no solution, inconsistent, independent

21) Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} y = \frac{1}{3}x - 5 \\ x - 3y = 6 \end{cases}$$

Solution

no solution, inconsistent, independent

Example 10

Without graphing, determine the number of solutions and then classify the system of equations:
$\begin{cases} 2x+y=-3 \\ x-5y=5 \end{cases}$

Solution

Step 1: **We will compare the slope and intercepts of the two lines.**

$$\begin{cases} 2x+y=-3 \\ x-5y=5 \end{cases}$$

Step 2: **Write both equations in slope-intercept form.**

$$\begin{array}{rcl} 2x+y & = & -3 \\ y & = & -2x-3 \\ x-5y=5 & = & 5 \\ -5y & = & -x+5 \\ \frac{-5y}{-5} & = & \frac{-x+5}{-5} \\ y & = & \frac{1}{5}x-1 \end{array}$$

Step 3: **Find the slope and intercept of each line.**

$$\begin{array}{rcl} y & = & -2x-3 \\ m & = & -2 \\ b & = & -3 \\ y & = & \frac{1}{5}x-1 \\ m & = & \frac{1}{5} \\ b & = & -1 \end{array}$$

Since the slopes are different, the lines intersect.

A system of equations whose graphs are intersect has 1 solution and is consistent and independent.

Try It

22) Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} 3x+2y=2 \\ 2x+y=1 \end{cases}$$

Solution
one solution, consistent, independent

23) Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} x + 4y = 12 \\ -x + y = 3 \end{cases}$$

Solution
one solution, consistent, independent

Example 11

Without graphing, determine the number of solutions and then classify the system of equations.
$\begin{cases} 3x - 2y = 4 \\ y = \frac{3}{2}x - 2 \end{cases}$

Solution

Step 1: **We will compare the slopes and intercepts of the two lines.**

$$\begin{cases} 3x - 2y = 4 \\ y = \frac{3}{2}x - 2 \end{cases}$$

Step 2: **Write the first equation in slope-intercept form.**

$$\begin{aligned} 3x - 2y &= 4 \\ -2y &= -3x + 4 \\ \frac{-2y}{-2} &= \frac{-3x+4}{-2} \\ y &= \frac{3}{2}x - 2 \end{aligned}$$

Step 3: **The second equation is already in slope-intercept form.**

$$y = \frac{3}{2}x - 2$$

Since the slopes are the same, they have the same slope and same y-intercept and so the lines are coincident.

A system of equations whose graphs are coincident lines has infinitely many solutions and is consistent and dependent.

Try It

24) Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} 4x - 5y = 20 \\ y = \frac{4}{5}x - 4 \end{cases}$$

Solution

infinitely many solutions, consistent, dependent

25) Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} -2x - 4y = 8 \\ y = -\frac{1}{2}x - 2 \end{cases}$$

Solution

infinitely many solutions, consistent, dependent

Solve Applications of Systems of Equations by Graphing

We will use the same problem-solving strategy we used in Math Models to set up and solve applications of systems of linear equations. We'll modify the strategy slightly here to make it appropriate for systems of equations.

How to

Use a problem-solving strategy for systems of linear equations.

1. Read the problem. Make sure all the words and ideas are understood.
2. Identify what we are looking for.
3. Name what we are looking for. Choose variables to represent those quantities.
4. Translate into a system of equations.
5. Solve the system of equations using good algebra techniques.
6. Check the answer in the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

Step 5 is where we will use the method introduced in this section. We will graph the equations and find the solution.

Example 12

Sondra is making 10 quarts of punch from fruit juice and club soda. The number of quarts of fruit juice is 4 times the number of quarts of club soda. How many quarts of fruit juice and how many quarts of club soda does Sondra need?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

We are looking for the number of quarts of fruit juice and the number of quarts of club soda that Sondra will need.

Step 3: **Name what we are looking for. Choose variables to represent those quantities.**

Let f $=$ number of quarts of fruit juice.

Let c $=$ number of quarts of club soda

Step 4: **Translate into a system of equations.**

$$\text{The } \underbrace{\text{number of quarts of fruit juice}}_{f} \underbrace{\text{and}}_{+} \text{ the } \underbrace{\text{number of quarts of club soda}}_{c} \underbrace{\text{is } 10}_{=10}$$

$$\text{The } \underbrace{\text{number of quarts of fruit juice}}_{f} \underbrace{\text{is}}_{=} \underbrace{\text{four times the number of quarts of club soda}}_{4c}$$

We now have the system. $\begin{cases} f + c = 10 \\ f = 4c \end{cases}$

Step 5: **Solve the system of equations using good algebra techniques.**

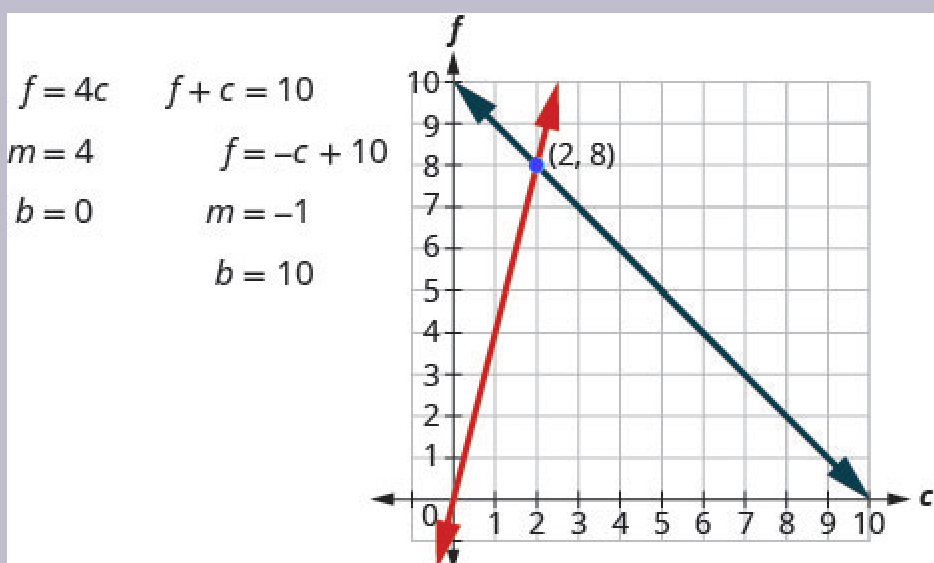


Figure 4.1.14

The point of intersection $(2, 8)$ is the solution. This means Sondra needs 2 quarts of club soda and 8 quarts of fruit juice.

Step 6: **Check the answer in the problem and make sure it makes sense.**

Does this make sense in the problem?

Yes, the number of quarts of fruit juice, 8 is 4 times the number of quarts of club soda, 2.

Yes, 10 quarts of punch is 8 quarts of fruit juice plus 2 quarts of club soda.

Step 7: **Answer the question with a complete sentence.**

Sondra needs 8 quarts of fruit juice and 2 quarts of soda.

Try It

26) Manny is making 12 quarts of orange juice from concentrate and water. The number of quarts of water is 3 times the number of quarts of concentrate. How many quarts of concentrate and how many quarts of water does Manny need?

Solution

Manny needs 3 quarts juice concentrate and 9 quarts water.

27) Alisha is making an 18 ounce coffee beverage that is made from brewed coffee and milk. The number of ounces of brewed coffee is 5 times greater than the number of ounces of milk. How many ounces of coffee and how many ounces of milk does Alisha need?

Solution

Alisha needs 15 ounces of coffee and 3 ounces of milk.

Access these online resources for additional instruction and practice with solving systems of equations by graphing.

- Instructional Video Solving Linear Systems by Graphing
- Instructional Video Solve by Graphing

Key Concepts

- **To solve a system of linear equations by graphing**
  1. Graph the first equation.
  2. Graph the second equation on the same rectangular coordinate system.
  3. Determine whether the lines intersect, are parallel, or are the same line.
  4. Identify the solution to the system.
     If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system.
     If the lines are parallel, the system has no solution.
     If the lines are the same, the system has an infinite number of solutions.
  5. Check the solution in both equations.

- Determine the number of solutions from the graph of a linear system

Graph	**Number of solutions**
2 intersecting lines	1
Parallel lines	None
Same line	Infinitely many

- Determine the number of solutions of a linear system by looking at the slopes and intercepts

Number of Solutions of a Linear System of Equations

Slopes	**Intercepts**	**Type of Lines**	**Number of Solutions**
Different		Intersecting	1 point
Same	Different	Parallel	No solution
Same	Same	Coincident	Infinitely many solutions

- Determine the number of solutions and how to classify a system of equations

Lines	**Intersecting**	**Parallel**	**Coincident**
Number of solutions	1 point	No colution	Infinitely many
Consistent/ Inconsistent	Consistent	Inconsistent	Consistent
Dependent/ Independent	Independent	Independent	Dependent

- **Problem Solving Strategy for Systems of Linear Equations**

  1. Read the problem. Make sure all the words and ideas are understood.
  2. Identify what we are looking for.
  3. Name what we are looking for. Choose variables to represent those quantities.
  4. Translate into a system of equations.
  5. Solve the system of equations using good algebra techniques.
  6. Check the answer in the problem and make sure it makes sense.
  7. Answer the question with a complete sentence.

Glossary

coincident lines

Coincident lines are lines that have the same slope and same y-intercept.

consistent system

A consistent system of equations is a system of equations with at least one solution.

dependent equations

Two equations are dependent if all the solutions of one equation are also solutions of the other equation.

inconsistent system

An inconsistent system of equations is a system of equations with no solution.

independent equations

Two equations are independent if they have different solutions.

solutions of a system of equations

Solutions of a system of equations are the values of the variables that make all the equations true. A solution of a system of two linear equations is represented by an ordered pair (x, y).

system of linear equations

When two or more linear equations are grouped together, they form a system of linear equations.

Exercises: Determine Whether an Ordered Pair is a Solution of a System of Equations

Instructions: For questions 1-8, determine if the following points are solutions to the given system of equations.

1. $\begin{cases} 2x-6y=0 \\ 3x-4y=5 \end{cases}$

a. $(3,1)$

b. $(-3,4)$

Solution

a. yes

b. no

---

2. $\begin{cases} 7x-4y=-1 \\ -3x-2y=1 \end{cases}$

a. $(1,2)$

b. $(1,-2)$

---

3. $\begin{cases} 2x+y=5 \\ x+y=1 \end{cases}$

a. $(4,-3)$

b. $(2,0)$

Solution

a. yes

b. no

---

4. $\begin{cases} -3x+y=8 \\ -x+2y=-9 \end{cases}$

a. $(-5,-7)$

b. $(-5,7)$

---

5. $\begin{cases} x+y=2 \\ y=\frac{3}{4}x \end{cases}$

a. $\left(\frac{8}{7},\frac{6}{7}\right)$

b. $\left(1,\frac{3}{4}\right)$

Solution

a. yes

b. no

---

6. $\begin{cases} x+y=1 \\ y=\frac{2}{5}x \end{cases}$

a. $\left(\frac{5}{7},\frac{2}{7}\right)$

b. $(5,2)$

---

7. $\begin{cases} x+5y=10 \\ y=\frac{3}{5}x+1 \end{cases}$

a. $(-10,4)$

b. $\left(\frac{5}{4},\frac{7}{4}\right)$

Solution

a. no

b. yes

---

8. $\begin{cases} x+3y=9 \\ y=\frac{2}{3}x-2 \end{cases}$

a. $(-6,5)$

b. $\left(5,\dfrac{4}{3}\right)$

Exercises: Solve a System of Linear Equations by Graphing

Instructions: For questions 9-50, solve the following systems of equations by graphing.

9. $\begin{cases} 3x+y=-3 \\ 2x+3y=5 \end{cases}$

Solution

$(-2,3)$

---

10. $\begin{cases} -x+y=2 \\ 2x+y=-4 \end{cases}$

---

11. $\begin{cases} -3x+y=-1 \\ 2x+y=4 \end{cases}$

Solution

$(1,2)$

---

12. $\begin{cases} -2x+3y=-3 \\ x+y=4 \end{cases}$

---

13. $\begin{cases} y = x + 2 \\ y = -2x + 2 \end{cases}$

Solution

$(0, 2)$

---

14. $\begin{cases} y = x - 2 \\ y = -3x + 2 \end{cases}$

---

15. $\begin{cases} y = \frac{3}{2}x + 1 \\ y = -\frac{1}{2}x + 5 \end{cases}$

Solution

$(2, 4)$

---

16. $\begin{cases} y = \frac{2}{3}x - 2 \\ y = -\frac{1}{3}x - 5 \end{cases}$

---

17. $\begin{cases} -x + y = -3 \\ 4x + 4y = 4 \end{cases}$

Solution

$(2, -1)$

---

18. $\begin{cases} x - y = 3 \\ 2x - y = 4 \end{cases}$

---

19. $\begin{cases} -3x + y = -1 \\ 2x + y = 4 \end{cases}$

Solution

$(1, 2)$

---

20. $\begin{cases} -3x + y = -2 \\ 4x - 2y = 6 \end{cases}$

---

21. $\begin{cases} x + y = 5 \\ 2x - y = 4 \end{cases}$

Solution

$(3, 2)$

---

22. $\begin{cases} x - y = 2 \\ 2x - y = 6 \end{cases}$

---

23. $\begin{cases} x + y = 2 \\ x - y = 0 \end{cases}$

Solution

$(1, 1)$

---

24. $\begin{cases} x + y = 6 \\ x - y = -8 \end{cases}$

---

25. $\begin{cases} x + y = -5 \\ x - y = 3 \end{cases}$

Solution

$(-1, -4)$

---

26. $\begin{cases} x + y = 4 \\ x - y = 0 \end{cases}$

---

27. $\begin{cases} x + y = -4 \\ -x + 2y = -2 \end{cases}$

Solution

$(-2,-2)$

---

28. $\begin{cases} -x+3y=3 \\ x+3y=3 \end{cases}$

---

29. $\begin{cases} -2x+3y=3 \\ x+3y=12 \end{cases}$

Solution

$(3,3)$

---

30. $\begin{cases} 2x-y=4 \\ 2x+3y=12 \end{cases}$

---

31. $\begin{cases} 2x+3y=6 \\ y=-2 \end{cases}$

Solution

$(6,-2)$

---

32. $\begin{cases} -2x+y=2 \\ y=4 \end{cases}$

---

33. $\begin{cases} x-3y=-3 \\ y=2 \end{cases}$

Solution

$(3,2)$

---

34. $\begin{cases} 2x-2y=8 \\ y=-3 \end{cases}$

---

35. $\begin{cases} 2x - y = -1 \\ x = 1 \end{cases}$

Solution

$(1, 3)$

---

36. $\begin{cases} x + 2y = 2 \\ x = -2 \end{cases}$

---

37. $\begin{cases} x - 3y = -6 \\ x = -3 \end{cases}$

Solution

$(-3, 1)$

---

38. $\begin{cases} x + y = 4 \\ x = 1 \end{cases}$

---

39. $\begin{cases} 4x - 3y = 8 \\ 8x - 6y = 14 \end{cases}$

Solution

no solution

---

40. $\begin{cases} x + 3y = 4 \\ -2x - 6y = 3 \end{cases}$

---

41. $\begin{cases} -2x + 4y = 4 \\ y = \frac{1}{2}x \end{cases}$

Solution

no solution

---

42. $\begin{cases} 3x + 5y = 10 \\ y = -\frac{3}{5}x + 1 \end{cases}$

---

43. $\begin{cases} x = -3y + 4 \\ 2x + 6y = 8 \end{cases}$

Solution

infinitely many solutions

---

44. $\begin{cases} 4x = 3y + 7 \\ 8x - 6y = 14 \end{cases}$

---

45. $\begin{cases} 2x + y = 6 \\ -8x - 4y = -24 \end{cases}$

Solution

infinitely many solutions

---

46. $\begin{cases} 5x + 2y = 7 \\ -10x - 4y = -14 \end{cases}$

---

47. $\begin{cases} x + 3y = -6 \\ 4y = -\frac{4}{3}x - 8 \end{cases}$

Solution

infinitely many solutions

---

48. $\begin{cases} -x + 2y = -6 \\ y = -\frac{1}{2}x - 1 \end{cases}$

---

49. $\begin{cases} -3x + 2y = -2 \\ y = -x + 4 \end{cases}$

Solution

$(2, 2)$

---

50. $\begin{cases} -x + 2y = -2 \\ y = -x - 1 \end{cases}$

Exercises: Determine the Number of Solutions of a Linear System

Instructions: For questions 51-62, determine the number of solutions and then classify the system of equations **without** graphing.

51. $\begin{cases} y = \frac{2}{3}x + 1 \\ -2x + 3y = 5 \end{cases}$

Solution

no solutions, inconsistent, independent

---

52. $\begin{cases} y = \frac{1}{3}x + 2 \\ x - 3y = 9 \end{cases}$

---

53. $\begin{cases} y = -2x + 1 \\ 4x + 2y = 8 \end{cases}$

Solution

no solutions, inconsistent, independent

---

54. $\begin{cases} y = 3x + 4 \\ 9x - 3y = 18 \end{cases}$

---

55. $\begin{cases} y = \frac{2}{3}x + 1 \\ 2x - 3y = 7 \end{cases}$

Solution

no solutions, inconsistent, independent

---

56. $\begin{cases} 3x + 4y = 12 \\ y = -3x - 1 \end{cases}$

---

57. $\begin{cases} 4x + 2y = 10 \\ 4x - 2y = -6 \end{cases}$

Solution

consistent, 1 solution

---

58. $\begin{cases} 5x + 3y = 4 \\ 2x - 3y = 5 \end{cases}$

---

59. $\begin{cases} y = -\frac{1}{2}x + 5 \\ x + 2y = 10 \end{cases}$

Solution

infinitely many solutions, consistent, dependent

---

60. $\begin{cases} y = x + 1 \\ -x + y = 1 \end{cases}$

---

61. $\begin{cases} y = 2x + 3 \\ 2x - y = -3 \end{cases}$

Solution

infinitely many solutions, consistent, dependent

---

62. $\begin{cases} 5x - 2y = 10 \\ y = \frac{5}{2}x - 5 \end{cases}$

Exercises: Solve Applications of Systems of Equations by Graphing

Instructions: For questions 63-66, solve.

63. Molly is making strawberry infused water. For each ounce of strawberry juice, she uses three times as many ounces of water. How many ounces of strawberry juice and how many ounces of water does she need to make 64 ounces of strawberry infused water?

Solution

Molly needs 16 ounces of strawberry juice and 48 ounces of water.

---

64. Jamal is making a snack mix that contains only pretzels and nuts. For every ounce of nuts, he will use 2 ounces of pretzels. How many ounces of pretzels and how many ounces of nuts does he need to make 45 ounces of snack mix?

---

65. Enrique is making a party mix that contains raisins and nuts. For each ounce of nuts, he uses twice the amount of raisins. How many ounces of nuts and how many ounces of raisins does he need to make 24 ounces of party mix?

Solution

Enrique needs 8 ounces of nuts and 16 ounces of raisins.

---

66. Owen is making lemonade from concentrate. The number of quarts of water he needs is 4 times the number of quarts of concentrate. How many quarts of water and how many quarts of concentrate does Owen need to make 100 quarts of lemonade?

Exercises: Everyday Math

Instructions: For questions 67-68, answer the given everyday math word problems.

67. Leo is planning his spring flower garden. He wants to plant tulip and daffodil bulbs. He will plant 6 times as many daffodil bulbs as tulip bulbs. If he wants to plant 350 bulbs, how many tulip bulbs and how many daffodil bulbs should he plant?

Solution

Leo should plant 50 tulips and 300 daffodils.

---

68. A marketing company surveys $1{,}200$ people. They surveyed twice as many females as males. How many males and females did they survey?

Exercises: Writing Exercises

Instructions: For questions 69-70, answer the given writing exercises.

69. In a system of linear equations, the two equations have the same slope. Describe the possible solutions to the system.

Solution

Given that it is only known that the slopes of both linear equations are the same, there are either no solutions (the graphs of the equations are parallel) or infinitely many.

---

70. In a system of linear equations, the two equations have the same intercepts. Describe the possible solutions to the system.

4.2 SOLVE SYSTEMS OF EQUATIONS BY SUBSTITUTION

Learning Objectives

By the end of this section, you will be able to:

- Solve a system of equations by substitution
- Solve applications of systems of equations by substitution

Try It

Before you get started, take this readiness quiz:

1) Simplify $-5(3-x)$.
2) Simplify $4-2(n+5)$.
3) Solve for y: $8y-8=32-2y$.
4) Solve for x: $3x-9y=-3$.

Solving systems of linear equations by graphing is a good way to visualize the types of solutions that may result.

However, there are many cases where solving a system by graphing is inconvenient or imprecise. If the graphs extend beyond the small grid with x and y both between -10 and 10, graphing the lines may be cumbersome. And if the solutions to the system are not integers, it can be hard to read their values precisely from a graph.

In this section, we will solve systems of linear equations by the substitution method.

Solve a System of Equations by Substitution

We will use the same system we used first for graphing.

$$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$$

We will first solve one of the equations for either x or y. We can choose either equation and solve for either variable—but we'll try to make a choice that will keep the work easy.

Then we substitute that expression into the other equation. The result is an equation with just one variable—and we know how to solve those!

After we find the value of one variable, we will substitute that value into one of the original equations and solve for the other variable. Finally, we check our solution and make sure it makes both equations true.

We'll fill in all these steps now in Example 1

Example 1

How to Solve a System of Equations by Substitution

Solve the system by substitution. $\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$

Solution

Step 1. Solve one of the equations for either variable.	We'll solve the first equation for *y*.	$2x + y = 7$ $y = 7 - 2x$

Figure 4.2.1

Step 2. Substitute the expression from Step 1 into the other equation.	We replace *y* in the second equation with the expression $7 - 2x$.	$x - 2y = 6$ $x - 2(7 - 2x) = 6$

Figure 4.2.2

Step 3. Solve the resulting equation.	Now we have an equation with just 1 variable. We know how to solve this!	$x - 2(7 - 2x) = 6$ $x - 14 + 4x = 6$ $5x = 20$ $x = 4$

Figure 4.2.3

Step 4. Substitute the solution in Step 3 into one of the original equations to find the other variable.	We'll use the first equation and replace *x* with 4.	$2x + y = 7$ $2(4) + y = 7$ $8 + y = 7$ $y = -1$

Figure 4.2.4

Step 5. Write the solution as an ordered pair.	The ordered pair is (x, y).	$(4, -1)$

Figure 4.2.5

Step 6. Check that the ordered pair is a solution to **both** original equations.	Substitute (4, –1) into both equations and make sure they are both true.	$2x + y = 7$ $2(4) + (-1) \stackrel{?}{=} 7$ $7 = 7$ ✓ $x - 2y = 6$ $4 - 2(-1) \stackrel{?}{=} 6$ $6 = 6$ ✓ Both equations are true. (4, –1) is the solution to the system.

Figure 4.2.6

Try It

5) Solve the system by substitution.

$$\begin{cases} -2x + y = -11 \\ x + 3y = 9 \end{cases}$$

Solution

$(6, 1)$

6) Solve the system by substitution.

$$\begin{cases} x + 3y = 10 \\ 4x + y = 18 \end{cases}$$

Solution

$(4, 2)$

How to

Solve a system of equations by substitution.

1. Solve one of the equations for either variable.
2. Substitute the expression from Step 1 into the other equation.
3. Solve the resulting equation.
4. Substitute the solution in Step 3 into one of the original equations to find the other variable.
5. Write the solution as an ordered pair.
6. Check that the ordered pair is a solution to **both** original equations.

If one of the equations in the system is given in slope–intercept form, Step 1 is already done! We'll see this in Example 2

Example 2

Solve the system by substitution.

$$\begin{cases} x + y = -1 \\ y = x + 5 \end{cases}$$

Solution

The second equation is already solved for y. We will substitute the expression in place of y in the first equation.

The second equation is already solved for y. We will substitute it into the first equation.

$$\begin{cases} x + y = -1 \\ y = x + 5 \end{cases}$$

Step 1: **Replace the y with $x + 5$**

$$\begin{aligned} y &= \boxed{x + 5} \\ x + y &= -1 \\ x + x + 5 &= -1 \end{aligned}$$

Step 2: **Solve the resulting equation for x**

$$\begin{aligned} x + x + 5 &= -1 \\ 2x + 5 &= -1 \\ 2x &= -6 \end{aligned}$$

Step 3: **Substitute $x = -3$ into $y = x + 5$ to find y**

$$\begin{aligned} x &= -3 \\ y &= x + 5 \\ y &= -3 + 5 \\ y &= 2 \end{aligned}$$

The ordered pair is $(-3, 2)$.

Step 4: **Check the ordered pair in both equations:**

$x + y = -1$ $-3 + 2 \stackrel{?}{=} -1$ $-1 = -1 \checkmark$	$y = x + 5$ $2 \stackrel{?}{=} -3 + 5$ $2 = 2 \checkmark$

The solution is $(-3, 2)$.

Try It

7) Solve the system by substitution.

$$\begin{cases} x+y=6 \\ y=3x-2 \end{cases}$$

Solution

$(2,4)$

8) Solve the system by substitution.

$$\begin{cases} 2x-y=1 \\ y=-3x-6 \end{cases}$$

Solution

$(-1,-3)$

If the equations are given in standard form, we'll need to start by solving for one of the variables. In this next example, we'll solve the first equation for y.

Example 3

Solve the system by substitution.

$$\begin{cases} 3x+y=5 \\ 2x+4y=-10 \end{cases}$$

Solution

We need to solve one equation for one variable. Then we will substitute that expression into the other equation.

Solve for y

$$3x + y = 5$$

Step 1: **Substitute into the other equation.**

Step 2: **Replace the y with $-3x + 5$.**

$$\begin{aligned} 3x + y &= 5 \\ y &= -3 + 5 \\ 2x + 4y &= -10 \\ 2x + 4(-3 + 5) &= -10 \end{aligned}$$

Step 3: **Solve the resulting equation for x.**

$$\begin{aligned} 2x - 12x + 20 &= -10 \\ -10x + 20 &= -10 \\ -10x &= -30 \\ x &= 3 \end{aligned}$$

Step 4: **Substitute $x = 3$ into $3x + y = 5$ to find y.**

$$\begin{aligned} x &= 3 \\ 3x + y &= 5 \\ 3(3) + y &= 5 \\ 9 + y &= 5 \\ y &= -4 \end{aligned}$$

The ordered pair is $(3, -4)$.

Step 5: **Check the ordered pair in both equations:**

$\begin{aligned} 3x + y &= 5 \\ -3 \times 3 + (-4) &\stackrel{?}{=} 5 \\ 9 - 4 &\stackrel{?}{=} 5 \\ 5 &= 5 \checkmark \end{aligned}$	$\begin{aligned} 2x + 4y &= -10 \\ 2 \times 3 + 4(-4) &= -10 \\ 6 - 16 &\stackrel{?}{=} -10 \\ -10 &= -10 \checkmark \end{aligned}$

The solution is $(3, -4)$.

Try It

9) Solve the system by substitution.

$$\begin{cases} 4x + y = 2 \\ 3x + 2y = -1 \end{cases}$$

Solution

$(1, -2)$

10) Solve the system by substitution.

$$\begin{cases} -x + y = 4 \\ 4x - y = 2 \end{cases}$$

Solution

$(2, 6)$

In Example 3 it was easiest to solve for y in the first equation because it had a coefficient of 1. In Example 4 it will be easier to solve for x.

Example 4

Solve the system by substitution.

$$\begin{cases} x - 2y = -2 \\ 3x + 2y = 34 \end{cases}$$

Solution

We will solve the first equation for x and then substitute the expression into the second equation.

$$x - 2y = -2$$

Step 1: **Solve for x.**

Step 2: **Substitute into the other equation.**

Step 3: **Replace the x with $2y - 2$.**

$$\begin{aligned} 3x + 2y &= 34 \\ 3(2y - 2) + 2y &= 34 \end{aligned}$$

Step 4: **Solve the resulting equation for y.**

$$\begin{aligned} 6y - 6 + 2y &= 34 \\ 8y - 6 &= 34 \\ 8y &= 40 \\ y &= 5 \end{aligned}$$

Step 5: **Substitute $y = 5$ into $x - 2y = -2$ to find x.**

$$\begin{aligned} x - 2y &= -2 \\ x - 2 \times 5 &= -2 \\ x - 10 &= -2 \\ x &= 8 \end{aligned}$$

The ordered pair is $(8, 5)$.

Step 6: **Check the ordered pair in both equations:**

$x - 2y = -2$ $8 - 2 \times 5 \stackrel{?}{=} -2$ $8 - 10 \stackrel{?}{=} -2$ $-2 = -2 \checkmark$	$3x + 2y = 34$ $3 \times 8 + 2 \times 5 \stackrel{?}{=} 34$ $24 + 10 \stackrel{?}{=} 34$ $34 = 34 \checkmark$

The solution is $(8, 5)$.

Try It

11) Solve the system by substitution.

$$\begin{cases} x - 5y = 13 \\ 4x - 3y = 1 \end{cases}$$

Solution

$(-2, -3)$

12) Solve the system by substitution.

$$\begin{cases} x - 6y = -6 \\ 2x - 4y = 4 \end{cases}$$

Solution

$(6, 2)$

When both equations are already solved for the same variable, it is easy to substitute!

Example 5

Solve the system by substitution.

$$\begin{cases} y = -2x + 5 \\ y = \frac{1}{2}x \end{cases}$$

Solution

Since both equations are solved for y, we can substitute one into the other.

Step 1: **Substitute $\frac{1}{2}x$ for y in the first equation.**

$$\begin{aligned} y &= \frac{1}{2}x \\ y &= -2x + 5 \end{aligned}$$

Step 2: **Replace the y with $\frac{1}{2}x$**

$$\frac{1}{2}x = -2x + 5$$

Step 3: **Solve the resulting equation. Start by clearing the fraction.**

$$2\left(\frac{1}{2}x\right) = 2(-2x + 5)$$

Step 4: **Solve for x.**

$$\begin{aligned} x &= -4x + 10 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

Step 5: **Substitute $x = 2$ into $y = \frac{1}{2}x$ to find y.**

$$\begin{aligned} x &= 2 \\ y &= \frac{1}{2}x \\ y &= \frac{1}{2} \times 2 \\ y &= 1 \end{aligned}$$

The ordered pair is $(2, 1)$.

Step 6: **Check the ordered pair in both equations**

$y = \frac{1}{2}x$ $1 \overset{?}{=} \frac{1}{2} \times 2$ $1 = 1 \checkmark$	$y = -2x + 5$ $1 \overset{?}{=} -2 \times 2 + 5$ $1 = 1 \checkmark$

The solution is $(2, 1)$.

Try It

13) Solve the system by substitution.

$\begin{cases} x + y = 6 \\ y = 3x - 2 \end{cases}$

Solution

$(2, 4)$

14) Solve the system by substitution.

$\begin{cases} 4x - y = 0 \\ 2x - 3y = 5 \end{cases}$

Solution

$(-\frac{1}{2}, -2)$

Be very careful with the signs in the next example.

Example 6

Solve the system by substitution.

$$\begin{cases} 4x + 2y = 4 \\ 6x - y = 8 \end{cases}$$

Solution

We need to solve one equation for one variable. We will solve the first equation for y.

$$4x + 2y = 4$$

Step 1: **Solve the first equation for** y**.**

$$2y = -4x + 4$$

Step 2: **Substitute** $-2x+2$ **for** y **in the second equation.**

$$\begin{aligned} y &= -2x+2 \\ 6x - y &= 8 \end{aligned}$$

Step 3: **Replace the** y **with** $-2x+2$**.**

$$6x - (-2x+2) = 8$$

Step 4: **Solve the equation for** x**.**

$$\begin{aligned} 6x + 2x - 2 &= 8 \\ 8x - 2 &= 8 \\ 8x &= 10 \\ x &= \frac{5}{4} \end{aligned}$$

Step 5: **Substitute** $x = \frac{5}{4}$ **into** $4x+2y=4$ **to find** y**.**

$$\begin{aligned} x &= \frac{5}{2} \\ 4x + 2y &= 4 \\ 4\left(\frac{5}{2}\right) + 2y &= 4 \\ 5 + 2y &= 4 \\ 2y &= -1 \\ y &= -\frac{1}{2} \end{aligned}$$

The ordered pair is $(\frac{5}{4}, -\frac{1}{2})$

Step 6: **Check the ordered pair in both equations.**

$4x + 2y = 4$ $4(\frac{5}{4}) + 2(-\frac{1}{2}) \stackrel{?}{=} 4$ $5 - 1 \stackrel{?}{=} 4$ $4 = 4 \checkmark$	$6x - y = 8$ $6\left(\frac{5}{4}\right) - \left(-\frac{1}{2}\right) \stackrel{?}{=} 8$ $\frac{15}{4} - \left(-\frac{1}{2}\right) \stackrel{?}{=} 8$ $\frac{16}{2} \stackrel{?}{=} 8$ $8 = 8 \checkmark$

The solution is $(\frac{5}{4}, -\frac{1}{2})$

Try It

15) Solve the system by substitution.

$$\begin{cases} 3x + y = 5 \\ 2x + 4y = 10 \end{cases}$$

Solution

$(-1, 8)$

16) Solve the system by substitution.

$$\begin{cases} 4x - y = 0 \\ 2x - 3y = 5 \end{cases}$$

Solution

$(-\frac{1}{2}, -2)$

In Example 7, it will take a little more work to solve one equation for x or y.

Example 7

Solve the system by substitution.

$$\begin{cases} 4x - 3y = 6 \\ 15y - 20x = -30 \end{cases}$$

Solution

We need to solve one equation for one variable. We will solve the first equation for x.

$$\begin{aligned} x &= \frac{5}{2} \\ 4x - 3y &= 6 \end{aligned}$$

Step 1: **Solve the first equation for x.**

$$4x = 3y + 6$$

Step 2: **Substitute $\frac{3}{4}y + \frac{3}{2}$ for x in the second equation.**

$$\begin{aligned} x &= \frac{3}{4}y + \frac{3}{2} \\ 15y - 20x &= -30 \end{aligned}$$

Step 3: **Replace the x with $\frac{3}{4}y + \frac{3}{2}$**

$$15y - 20\left(\frac{3}{4}y + \frac{3}{2}\right) = -30$$

Step 4: **Solve for y.**

$$\begin{aligned} 15y - 15y - 30 &= -30 \\ 0 - 30 &= -30 \\ 0 &= 0 \end{aligned}$$

Since $0 = 0$ is a true statement, the system is consistent. The equations are dependent. The graphs of these two equations would give the same line. The system has infinitely many solutions.

Try It

17) Solve the system by substitution.

$$\begin{cases} 2x - 3y = 12 \\ -12y + 8x = 48 \end{cases}$$

Solution

Infinitely many solutions.

18) Solve the system by substitution.

$$\begin{cases} 5x + 2y = 12 \\ -4y - 10x = -24 \end{cases}$$

Solution

Infinitely many solutions.

Look back at the equations in Example 7. Is there any way to recognize that they are the same line?

Let's see what happens in the next example.

Example 8

Solve the system by substitution.

$$\begin{cases} 5x - 2y = -10 \\ y = \frac{5}{2}x \end{cases}$$

Solution

The second equation is already solved for y, so we can substitute for y in the first equation.

***Step 1:* Substitute x for y in the first equation.**

$$y = \frac{5}{2}x$$
$$5x - 2y = -10$$

***Step 2:* Replace the y with $\frac{5}{2}x$.**

$$5x - 2\left(\frac{5}{2}x\right) = -10$$

***Step 3:* Solve for x.**

$$\begin{aligned} 5x - 5x &= -10 \\ 0 &\neq -10 \end{aligned}$$

Since $0 = -10$ is a false statement the equations are inconsistent. The graphs of the two equation would be parallel lines. The system has no solutions.

Try It

19) Solve the system by substitution.

$$\begin{cases} 3x + 2y = 9 \\ y = -\frac{3}{2}x + 1 \end{cases}$$

Solution

No solution.

20) Solve the system by substitution.

$$\begin{cases} 5x - 3y = 2 \\ y = \frac{5}{3}x - 4 \end{cases}$$

Solution

No solution.

Solve Applications of Systems of Equations by Substitution

We'll copy here the problem-solving strategy we used in the Solving Systems of Equations by Graphing section for solving systems of equations. Now that we know how to solve systems by substitution, that's what we'll do in Step 5:

How to use a problem-solving strategy for systems of linear equations.

How to

How to use a problem-solving strategy for systems of linear equations.

1. Read the problem. Make sure all the words and ideas are understood.
2. Identify what we are looking for.
3. Name what we are looking for. Choose variables to represent those quantities.
4. Translate into a system of equations.
5. Solve the system of equations using good algebra techniques.
6. Check the answer to the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

Some people find setting up word problems with two variables easier than setting them up with just one variable. Choosing the variable names is easier when all you need to do is write down two letters. Think about this in the next example—how would you have done it with just one variable?

Example 9

The sum of two numbers is zero. One number is nine less than the other. Find the numbers.

Solution
Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

We are looking for two numbers.

Step 3: **Name what we are looking for.**

Let n $=$ the first number

Let m $=$ the second number

Step 4: **Translate into a system of equations.**

The sum of two numbers is zero.

$$n + m = 0$$

One number is nine less than the other.

$$n = m - 9$$

The system is:

$$\begin{cases} n + m = 0 \\ n = m - 9 \end{cases}$$

Step 5: **Solve the system of equations.**

We will use substitution since the second equation is solved for n.

Substitute $m - 9$ for n in the first equation.

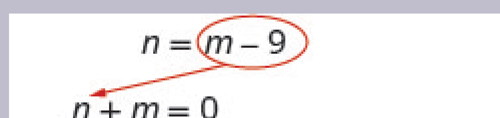


Figure 4.2.7

Solve for m.

$$\begin{aligned} {\color{red}m-9} + m &= 0 \\ 2m - 9 &= 0 \\ 2m &= 9 \end{aligned}$$

Substitute $m = \frac{9}{2}$ into the second equation and then solve for n.

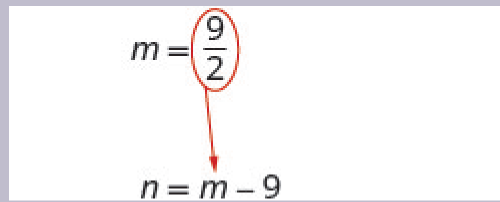


Figure 4.2.8

$$\begin{aligned} m &= \frac{9}{2} - 9 \\ m &= \frac{9}{2} - \frac{18}{2} \\ n &= -\frac{9}{2} \end{aligned}$$

Step 6: **Check the answer to the problem.**

Do these numbers make sense in the problem? We will leave this to you!

Step 7: **Answer the question.**

The numbers are $\frac{9}{2}$ and $-\frac{9}{2}$

Try It

21) The sum of two numbers is 10. One number is 4 less than the other. Find the numbers.

Solution

The numbers are 3 and 7.

22) The sum of two numbers is -6. One number is 10 less than the other. Find the numbers.

Solution

The numbers are 2 and -8.

In Example 10, we'll use the formula for the perimeter of a rectangle, $P = 2L + 2W$.

Example 10

The perimeter of a rectangle is 88. The length is five more than twice the width. Find the length and the width.

Solution

Step 1: **Read the problem.**

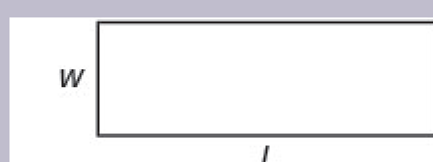


Figure 4.2.9

Step 2: **Identify what you are looking for.**

We are looking for the length and width.

Step 3: **Name what we are looking for.**

Let L = the length

Let W = the width

Step 4: **Translate into a system of equations.**

The perimeter of a rectangle is 88.

$$2L+2W=P$$

$$2L+2W=88$$

The length is five more than twice the width.

$$L=2W+5$$

The system is:

$$\begin{cases}2L+2W=88\\L=2W+5\end{cases}$$

Step 5: **Solve the system of equations.**

We will use substitution since the second equation is solved for L.

Substitute $2W+5$ for L in the first equation.

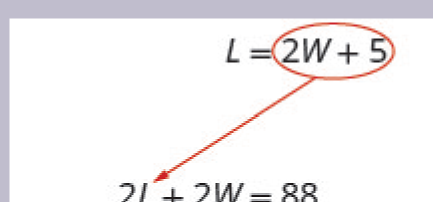


Figure 4.2.10

Solve for W.

$$\begin{aligned} 2(2W+5)+2W &= 88 \\ 4W+10+2W &= 88 \\ 6W+10 &= 88 \\ 6W &= 78 \end{aligned}$$

Substitute $W=13$ into the second equation and then solve for L.

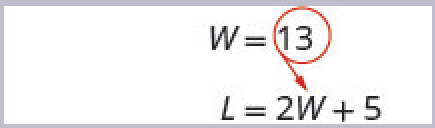


Figure 4.2.11

$$\begin{aligned} L &= 2\times 13+5 \\ L &= 31 \end{aligned}$$

***Step 6:* Check the answer in the problem.**

Does a rectangle with length 31 and width 13 have perimeter 88? Yes.

***Step 7:* Answer the equation.**

The length is 31 and the width is 13.

Try It

23) The perimeter of a rectangle is 40. The length is 4 more than the width. Find the length and width of the rectangle.

Solution

The length is 12 and the width is 8.

24) The perimeter of a rectangle is 58. The length is 5 more than three times the width. Find the length and width of the rectangle.

Solution

The length is 23 and the width is 6.

For Example 11, we need to remember that the sum of the measures of the angles of a triangle is 180 degrees and that a right triangle has one 90 degree angle.

Example 11

The measure of one of the small angles of a right triangle is ten more than three times the measure of the other small angle. Find the measures of both angles.

Solution

We will draw and label a figure.

Step 1: **Read the problem.**

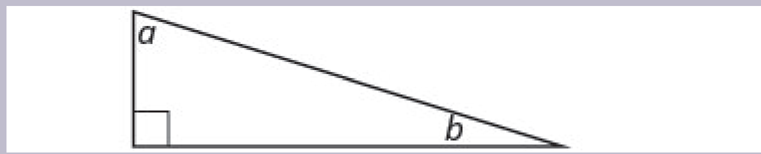


Figure 4.2.12

Step 2: **Identify what you are looking for.**

We are looking for the measures of the angles.

Step 3: **Name what we are looking for.**

Let a $=$ the measure of the 1st angle.

Let b $=$ the measure of the 2nd angle.

Step 4: **Translate into a system of equations.**

The measure of one of the small angles of a right triangle is ten more than three times the measure of the other small angle.

$$a = 3b + 10$$

The sum of the measures of the angles of a triangle is 180.

$$a + b + 90 = 180$$

The system is:

$$\begin{cases} a = 3b + 10 \\ a + b + 90 = 180 \end{cases}$$

Step 5: **Solve the system of equations.**

We will use substitution since the first equation is solved for a.

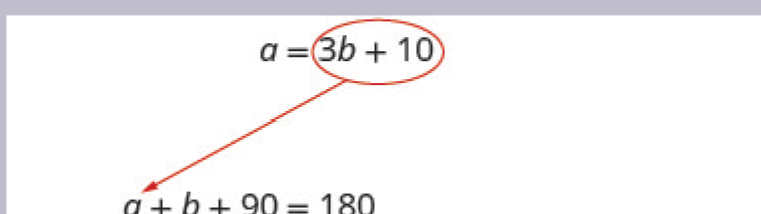


Figure 4.2.13

Substitute $3b+10$ for a in the second equation.

$$(3b+10)+b+90=180$$

Solve for b.

$$\begin{aligned} 4b+100 &= 180 \\ 4b &= 80 \end{aligned}$$

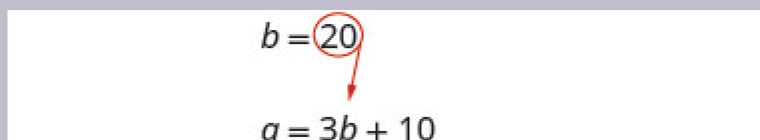


Figure 4.2.14

Substitute $b=20$ into the first equation and then solve for a.

$$\begin{aligned} a &= 3 \times 20 + 10 \\ a &= 70 \end{aligned}$$

Step 6: **Check the answer in the problem.**

We will leave this to you!

Step 7: **Answer the question.**

The measures of the small angles are 20 and 70.

Try It

25) The measure of one of the small angles of a right triangle is 2 more than 3 times the measure of the other small angle. Find the measure of both angles.

Solution

The measure of the angles are 22 degrees and 68 degrees.

26) The measure of one of the small angles of a right triangle is 18 less than twice the measure of the other small angle. Find the measure of both angles.

Solution

The measure of the angles are 36 degrees and 54 degrees.

Example 12

Heather has been offered two options for her salary as a trainer at the gym. Option A would pay her $\$25,000$ plus $\$15$ for each training session. Option B would pay her $\$10,000 + \$40$ for each training session. How many training sessions would make the salary options equal?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what you are looking for.**

We are looking for the number of training sessions that would make the pay equal.

Step 3: **Name what we are looking for.**

Let $s =$ Heather's salary.

Let $n =$ be the number of training sessions

Step 4: **Translate into a system of equations.**

Option A would pay her $\$25,000$ plus 15 for each training session.

$$s = 25,000 + 15n$$

Option B would pay her $\$10,000 + \$40$ for each training session

$$s = 10,000 + 40n$$

The system is:

$$\begin{cases} s & = 25,000 + 15n \\ s & = 10,000 + 40n \end{cases}$$

Step 5: **Solve the system of equations.**

We will use substitution.

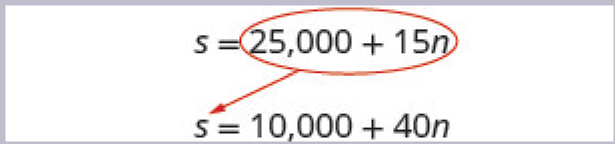


Figure 4.2.15

Substitute $25,000 + 15n$ for s in the second equation.

$$25,000 + 15n = 10,000 + 40n$$

Solve for n.

$$\begin{aligned} 25,000 &= 10,000 + 25n \\ 15,000 &= 25n \\ 600 &= n \end{aligned}$$

Step 6: **Check the answer.**

Are 600 training sessions a year reasonable?

Are the two options equal when $n = 600$?

***Step 7:* Answer the question.**

The salary options would be equal for 600 training sessions.

Try It

27) Geraldine has been offered positions by two insurance companies. The first company pays a salary of $\$12,000$ plus a commission of $\$100$ for each policy sold. The second pays a salary of $\$20,000$ plus a commission of $\$50$ for each policy sold. How many policies would need to be sold to make the total pay the same?

Solution

There would need to be 160 policies sold to make the total pay the same.

28) Kenneth currently sells suits for company A at a salary of $\$22,000$ plus a $\$10$ commission for each suit sold. Company B offers him a position with a salary of $\$28,000$ plus a $\$4$ commission for each suit sold. How many suits would Kenneth need to sell for the options to be equal?

Solution

Kenneth would need to sell $1,000$ suits.

Access these online resources for additional instruction and practice with solving systems of equations by substitution.

Instructional Video-Solve Linear Systems by Substitution

Instructional Video-Solve by Substitution

Key Concepts

- **Solve a system of equations by substitution**
    1. Solve one of the equations for either variable.
    2. Substitute the expression from Step 1 into the other equation.
    3. Solve the resulting equation.
    4. Substitute the solution in Step 3 into one of the original equations to find the other variable.
    5. Write the solution as an ordered pair.
    6. Check that the ordered pair is a solution to both original equations.

Exercises: Solve a System of Equations by Substitution

Instructions: For questions 1-36, solve the systems of equations by substitution.

1. $\begin{cases} 2x + y = -4 \\ 3x - 2y = -6 \end{cases}$

Solution

$(-2, 0)$

---

2. $\begin{cases} 2x + y = -2 \\ 3x - y = 7 \end{cases}$

---

3. $\begin{cases} x - 2y = -5 \\ 2x - 3y = -4 \end{cases}$

Solution

$(7, 6)$

---

4. $\begin{cases} x - 3y = -9 \\ 2x + 5y = 4 \end{cases}$

---

5. $\begin{cases} 5x - 2y = -6 \\ y = 3x + 3 \end{cases}$

Solution

$(0, 3)$

---

6. $\begin{cases} -2x + 2y = 6 \\ y = -3x + 1 \end{cases}$

---

7. $\begin{cases} 2x + 3y = 3 \\ y = -x + 3 \end{cases}$

Solution

$(6, -3)$

---

8. $\begin{cases} 2x + 5y = -14 \\ y = -2x + 2 \end{cases}$

---

9. $\begin{cases} 2x+5y=1 \\ y=\frac{1}{3}x-2 \end{cases}$

Solution

$(3,-1)$

---

10. $\begin{cases} 3x+4y=1 \\ y=-\frac{2}{5}x+2 \end{cases}$

---

11. $\begin{cases} 3x-2y=6 \\ y=\frac{2}{3}x+2 \end{cases}$

Solution

$(6,6)$

---

12. $\begin{cases} -3x-5y=3 \\ y=\frac{1}{2}x-5 \end{cases}$

---

13. $\begin{cases} 2x+y=10 \\ -x+y=-5 \end{cases}$

Solution

$(5,0)$

---

14. $\begin{cases} -2x+y=10 \\ -x+2y=16 \end{cases}$

---

15. $\begin{cases} 3x+y=1 \\ -4x+y=15 \end{cases}$

Solution

$(-2,7)$

16. $\begin{cases} x+y=0 \\ 2x+3y=-4 \end{cases}$

17. $\begin{cases} x+3y=1 \\ 3x+5y=-5 \end{cases}$

Solution

$(-5, 2)$

18. $\begin{cases} x+2y=-1 \\ 2x+3y=1 \end{cases}$

19. $\begin{cases} 2x+y=5 \\ x-2y=-15 \end{cases}$

Solution

$(-1, 7)$

20. $\begin{cases} 4x+y=10 \\ x-2y=-20 \end{cases}$

21. $\begin{cases} y=-2x-1 \\ y=-\frac{1}{3}x+4 \end{cases}$

Solution

$(-3, 5)$

22. $\begin{cases} y=x-6 \\ y=-\frac{3}{2}x+4 \end{cases}$

23. $\begin{cases} y = 2x - 8 \\ y = \frac{3}{5}x + 6 \end{cases}$

Solution

(10, 12)

---

24. $\begin{cases} y = -x - 1 \\ y = x + 7 \end{cases}$

---

25. $\begin{cases} 4x + 2y = 8 \\ 8x - y = 1 \end{cases}$

Solution

$\left(\frac{1}{2}, 3\right)$

---

26. $\begin{cases} -x - 12y = -1 \\ 2x - 8y = -6 \end{cases}$

---

27. $\begin{cases} 15x + 2y = 6 \\ -5x + 2y = -4 \end{cases}$

Solution

$\left(\frac{1}{2}, -\frac{3}{4}\right)$

---

28. $\begin{cases} 2x - 15y = 7 \\ 12x + 2y = -4 \end{cases}$

---

29. $\begin{cases} y = 3x \\ 6x - 2y = 0 \end{cases}$

Solution

Infinitely many solutions

---

30. $\begin{cases} x = 2y \\ 4x - 8y = 0 \end{cases}$

---

31. $\begin{cases} 2x + 16y = 8 \\ -x - 8y = -4 \end{cases}$

Solution

Infinitely many solutions

---

32. $\begin{cases} 15x + 4y = 6 \\ -30x - 8y = -12 \end{cases}$

---

33. $\begin{cases} y = -4x \\ 4x + y = 1 \end{cases}$

Solution

No solution

---

34. $\begin{cases} y = -\frac{1}{4}x \\ x + 4y = 8 \end{cases}$

---

35. $\begin{cases} y = \frac{7}{8}x + 4 \\ -7x + 8y = 6 \end{cases}$

Solution

No solution

---

36. $\begin{cases} y = -\frac{2}{3}x + 5 \\ 2x + 3y = 11 \end{cases}$

Exercises: Solve Applications of Systems of Equations by Substitution

Instructions: For questions 37-51, translate to a system of equations and solve.

37. The sum of two numbers is 15. One number is 3 less than the other. Find the numbers.

Solution

The numbers are 6 and 9.

---

38. The sum of two numbers is 30. One number is 4 less than the other. Find the numbers.

---

39. The sum of two numbers is -26. One number is 12 less than the other. Find the numbers.

Solution

The numbers are -7 and -19.

---

40. The perimeter of a rectangle is 50. The length is 5 more than the width. Find the length and width.

---

41. The perimeter of a rectangle is 60. The length is 10 more than the width. Find the length and width.

Solution

The length is 20 and the width is 10.

---

42. The perimeter of a rectangle is 58. The length is 5 more than three times the width. Find the length and width.

---

43. The perimeter of a rectangle is 84. The length is 10 more than three times the width. Find the length and width.

Solution

The length is 34 and the width is 8.

---

44. The measure of one of the small angles of a right triangle is 14 more than 3 times the measure of the other small angle. Find the measure of both angles.

---

45. The measure of one of the small angles of a right triangle is 26 more than 3 times the measure of the other small angle. Find the measure of both angles.

Solution

The measures are $16°$ and $74°$.

---

46. The measure of one of the small angles of a right triangle is 15 less than twice the measure of the other small angle. Find the measure of both angles.

---

47. The measure of one of the small angles of a right triangle is 45 less than twice the measure of the other small angle. Find the measure of both angles.

Solution

The measures are $45°$ and $45°$.

---

48. Maxim has been offered positions by two car dealers. The first company pays a salary of $\$10,000$ plus a commission of $\$1,000$ for each car sold. The second pays a salary of $\$20,000$ plus a commission of $\$500$ for each car sold. How many cars would need to be sold to make the total pay the same?

---

49. Jackie has been offered positions by two cable companies. The first company pays a salary of $14,000$ plus a commission of $\$100$ for each cable package sold. The second pays a salary of $\$20,000$ plus a commission of $\$25$ for each cable package sold. How many cable packages would need to be sold to make the total pay the same?

Solution

80 cable packages would need to be sold.

---

50. Amara currently sells televisions for company. A at a salary of $\$17,000$ plus a $\$100$ commission for each television she sells. Company B offers her a position with a salary of $\$29,000$ plus a $\$20$ commission for each television she sells. How televisions would Amara need to sell for the options to be equal?

---

51. Mitchell currently sells stoves for company. A at a salary of $\$12,000$ plus a $\$150$ commission for each stove he sells. Company B offers him a position with a salary of $\$24,000$ plus a $\$50$ commission for each stove he sells. How many stoves would Mitchell need to sell for the options to be equal?

Solution

Mitchell would need to sell 120 stoves.

Exercises: Everyday Math

Instructions: For questions 52-53, answer the given everyday math word problems.

52. When Gloria spent 15 minutes on the elliptical trainer and then did circuit training for 30 minutes, her fitness app says she burned 435 calories. When she spent 30 minutes on the elliptical trainer and 40 minutes circuit training she burned 690 calories. Solve the system $\begin{cases} 15e+30c=435 \\ 30e+40c=690 \end{cases}$ for e, the number of calories she burns for each minute on the elliptical trainer, and c, the number of calories she burns for each minute of circuit training.

---

53. Stephanie left Riverside, California, driving her motorhome north on Interstate 15 towards Salt Lake City at a speed of 56 miles per hour. Half an hour later, Tina left Riverside in her car on the same route as Stephanie, driving 70 miles per hour. Solve the system $\begin{cases} 56s=70t \\ s=t+\frac{1}{2} \end{cases}$.

a. for t to find out how long it will take Tina to catch up to Stephanie.

b. what is the value of s, the number of hours Stephanie will have driven before Tina catches up to her?

Solution

a. $t = 2$ hours

b. $s = 2\frac{1}{2}$ hours

Exercises: Writing Exercises

Instructions: For questions 54-55, answer the given writing exercises.

54. Solve the system of equations $\begin{cases} x+y=10 \\ x-y=6 \end{cases}$

a. by graphing
b. by substitution
c. Which method do you prefer? Why?

---

55. Solve the system of equations $\begin{cases} 3x+y=12 \\ x=y-8 \end{cases}$ by substitution and explain all your steps in words.

Solution

Answers will vary.

4.3 SOLVE SYSTEMS OF EQUATIONS BY ELIMINATION

Learning Objectives

By the end of this section, you will be able to:

- Solve a system of equations by elimination
- Solve applications of systems of equations by elimination
- Choose the most convenient method to solve a system of linear equations

Try It

Before you get started, take this readiness quiz.

1) Simplify $-5(6-3a)$.
2) Solve the equation $\frac{1}{3}x+\frac{5}{8}=\frac{31}{24}$.

We have solved systems of linear equations by graphing and by substitution. Graphing works well when the variable coefficients are small and the solution has integer values. Substitution works well when we can easily solve one equation for one of the variables and not have too many fractions in the resulting expression.

The third method of solving systems of linear equations is called the Elimination Method. When we solved a system by substitution, we started with two equations and two variables and reduced it to one equation with one variable. This is what we'll do with the elimination method, too, but we'll have a different way to get there.

Solve a System of Equations by Elimination

The Elimination Method is based on the Addition Property of Equality. The Addition Property of Equality says that when you add the same quantity to both sides of an equation, you still have equality. We will extend the Addition Property of Equality to say that when you add equal quantities to both sides of an equation, the results are equal.

For any expressions a, b, c, and d,

$$\begin{array}{lrcl} \text{if} & a & = & b \\ \text{and} & c & = & d \\ \text{then} & a+c & = & b+d \end{array}$$

To solve a system of equations by elimination, we start with both equations in standard form. Then we decide which variable will be easiest to eliminate. How do we decide? We want to have the coefficients of one variable be opposites so that we can add the equations together and eliminate that variable.

Notice how that works when we add these two equations together:

$$\begin{array}{rcl} 3x+y & = & 5 \\ 2x-y & = & 0 \\ \hline 5x & = & 5 \end{array}$$

The y's add to zero and we have one equation with one variable.

Let's try another one:

$$\begin{array}{rcl} x+4y & = & 2 \\ 2x+5y & = & -2 \end{array}$$

This time we don't see a variable that can be immediately eliminated if we add the equations.

But if we multiply the first equation by -2, we will make the coefficients of x opposites. We must multiply every term on both sides of the equation by -2.

$$\begin{gathered} -2(x+4y)=-2(2) \\ 2x+5y=-2 \\ -2x-8y=-4 \\ 2x+5y=-2 \end{gathered}$$

Now we see that the coefficients of the x terms are opposites, so x will be eliminated when we add these two equations.

Add the equations yourself—the result should be $-3y = -6$. And that looks easy to solve, doesn't it? Here is what it would look like.

$$\begin{cases} -2x - 8y = -4 \\ \underline{\quad 2x + 5y = -2} \end{cases}$$
$$-3y = -6$$

We'll do one more:

$$\begin{aligned} 4x - 3y &= 10 \\ 3x + 5y &= -7 \end{aligned}$$

It doesn't appear that we can get the coefficients of one variable to be opposites by multiplying one of the equations by a constant unless we use fractions. So instead, we'll have to multiply both equations by a constant.

We can make the coefficients of x be opposites if we multiply the first equation by 3 and the second by -4, so we get $12x$ and $-12x$.

$$\begin{aligned} (4x - 3y) &= 3(10) \\ -4(3x + 5y) &= -4(-7) \end{aligned}$$

This gives us these two new equations:

$$\begin{aligned} 12x - 9y &= 30 \\ -12x + 20y &= 28 \end{aligned}$$

When we add these equations,

$$\begin{aligned} 12x - 9y &= 30 \\ \underline{-12x + 20y} &\underline{= 28} \\ -29y &= 58 \end{aligned}$$

the x's are eliminated and we just have $-29y = 58$.

Once we get an equation with just one variable, we solve it. Then we substitute that value into one of the original equations to solve for the remaining variable. And, as always, we check our answer to make sure it is a solution to both of the original equations.

Now we'll see how to use elimination to solve the same system of equations we solved by graphing and by substitution.

Example 1

How to Solve a System of Equations by Elimination

Solve the system by elimination.

$$\begin{aligned} 2x + y &= 7 \\ x - 2y &= 6 \end{aligned}$$

Solution

Step 1. Write both equations in standard form. If any coefficients are fractions, clear them.	Both equations are in standard form, $Ax + By = C$. There are no fractions.	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$

Figure 4.3.1

Step 2. Make the coefficients of one variable opposites. Decide which variable you will eliminate. Multiply one or both equations so that the coefficients of that variable are opposites.	We can eliminate the y's by multiplying the first equation by 2. Multiply both sides of $2x + y = 7$ by 2.	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$ $\begin{cases} 2(2x + y) = 2(7) \\ x - 2y = 6 \end{cases}$

Figure 4.3.2

Step 3. Add the equations resulting from Step 2 to eliminate one variable.	We add the x's, y's, and constants.	$\begin{cases} 4x + 2y = 14 \\ x - 2y = 6 \end{cases}$ $\overline{5x \qquad = 20}$

Figure 4.3.3

Step 4. Solve for the remaining variable.	Solve for x.	$x = 4$

Figure 4.3.4

Step 5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.	Substitute $x = 4$ into the second equation, $x - 2y = 6$. Then solve for y.	$x - 2y = 6$ $4 - 2y = 6$ $-2y = 2$ $y = -1$

Figure 4.3.5

Step 6. Write the solution as an ordered pair.	Write it as (x, y).	$(4, -1)$

Figure 4.3.6

Step 7. Check that the ordered pair is a solution to **both** original equations.	Substitute $(4, -1)$ into $2x + y = 7$ and $x - 2y = 6$. Do they make both equations true? Yes!	$2x + y = 7$, $2(4) + (-1) \stackrel{?}{=} 7$, $7 = 7$ ✓ $x - 2y = 6$, $4 - 2(-1) \stackrel{?}{=} 6$, $6 = 6$ ✓ The solution is $(4, -1)$.

Figure 4.3.7

Try It

3) Solve the system by elimination.

$$\begin{aligned} 3x + y &= 5 \\ 2x - 3y &= 7 \end{aligned}$$

Solution

$(2, -1)$

4) Solve the system by elimination.

$$\begin{aligned} 4x + y &= -5 \\ -2x - 2y &= -2 \end{aligned}$$

Solution

$(-2, 3)$

HOW TO

How to solve a system of equations by elimination.

1. Write both equations in standard form. If any coefficients are fractions, clear them.
2. Make the coefficients of one variable opposite.
    - Decide which variable you will eliminate.
    - Multiply one or both equations so that the coefficients of that variable are opposites.
3. Add the equations resulting from Step 2 to eliminate one variable.
4. Solve for the remaining variable.
5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.
6. Write the solution as an ordered pair.
7. Check that the ordered pair is a solution to both original equations.

First, we'll do an example where we can eliminate one variable right away.

Example 2

Solve the system by elimination.

$$\begin{aligned} x+y&=10 \\ x-y&=12 \end{aligned}$$

Solution

$$\begin{cases} x+y=10 \\ x-y=12 \end{cases}$$

Both equations are in standard form.

The coefficients of y are already opposites.

Step 1: **Add the two equations to eliminate y.**

The resulting equation has only 1 variable, x.

$$\frac{\begin{cases} x+y &= 10 \\ x-y &= 12 \end{cases}}{2x=22}$$

Step 2: **Solve for x, the remaining variable.**

Step 3: **Substitute $x=11$ into one of the original equations.**

Figure 4.3.8

$$11+y=10$$

Step 4: **Solve for the other variable, y.**

$$y = -1$$

Step 5: **Write the solution as an ordered pair.**

The ordered pair is $(11, -1)$.

***Step 6*. Check that the ordered pair is a solution to both original equations.**

$$\begin{aligned} \text{Equation 1: } x + y &= 10 \\ 11 + (-1) &\stackrel{?}{=} 10 \\ 10 &= 10 \checkmark \\ \\ \text{Equation 2: } x - y &= 12 \\ 11 - (-1) &\stackrel{?}{=} 12 \\ 12 &= 12 \checkmark \end{aligned}$$

The solution is $(11, -1)$.

Try It

5) Solve the system by elimination.

$$\begin{aligned} 2x + y &= 5 \\ x - y &= 4 \end{aligned}$$

Solution

$(3, -1)$

6) Solve the system by elimination.

$$\begin{aligned} x + y &= 3 \\ -2x - y &= -1 \end{aligned}$$

Solution

$(-2, 5)$

In Example 3, we will be able to make the coefficients of one variable opposite by multiplying one equation by a constant.

Example 3

Solve the system by elimination.

$$\begin{array}{c} 3x - 2y = -2 \\ 5x - 6y = 10 \end{array}$$

Solution

$$\begin{cases} 3x - 2y = -2 \\ 5x - 6y = 10 \end{cases}$$

Both equations are in standard form.

None of the coefficients are opposites.

Step 1: **We can make the coefficients of y opposites by multiplying the first equation by -3.**

$$\begin{cases} -3(3x - 2y) = -3(-2) \\ 5x - 6y = 10 \end{cases}$$

Step 2: **Simplify.**

$$\begin{cases} -9x + 6y = 6 \\ 5x - 6y = 10 \end{cases}$$

Step 3: **Add the two equations to eliminate y.**

$$\frac{\begin{cases} -9x + 6y = 6 \\ 5x - 6y = 10 \end{cases}}{-4x = 16}$$

Step 4: **Solve for the remaining variable, x.**

Step 5: **Substitute $x = -4$ into one of the original equations.**

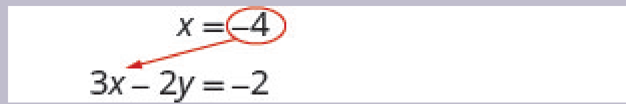


Figure 4.3.9

$$3(-4)-2y=-2$$

Step 6: **Solve for** y**.**

$$\begin{aligned}-12-2y&=-2\\-2y&=10\\y&=-5\end{aligned}$$

Step 7: **Write the solution as an ordered pair.**

The ordered pair is $(-4,-5)$.

Step 8: **Check that the ordered pair is a solution to both original equations.**

$$\begin{aligned}\text{Equation 1: }3x-2y&=-2\\3(-4)-2(-5)&\stackrel{?}{=}-2\\-12+10&\stackrel{?}{=}-2\\-2&=-2\ \checkmark\\[1em]\text{Equation 2: }5x-6y&=10\\3(-4)-6(-5)&\stackrel{?}{=}10\\-20+30&\stackrel{?}{=}10\\10&=10\checkmark\end{aligned}$$

The solution is $(-4,-5)$.

Try It

7) Solve the system by elimination.

$$\begin{aligned}4x-3y&=1\\5x-9y&=-4\end{aligned}$$

Solution

$(1,1)$

8) Solve the system by elimination.

$$\begin{array}{l} 3x + 2y = 2 \\ 6x + 5y = 8 \end{array}$$

Solution

$(-2, 4)$

Now we'll do an example where we need to multiply both equations by constants to make the coefficients of one variable opposites.

Example 4

Solve the system by elimination. $\begin{array}{l} 4x - 3y = 9 \\ 7x + 2y = -6 \end{array}$

Solution

In this example, we cannot multiply just one equation by any constant to get opposite coefficients. So we will strategically multiply both equations by a constant to get the opposites.

$$\begin{cases} 4x - 3y = 9 \\ 7x + 2y = -6 \end{cases}$$

Step 1: **Both equations are in standard form. To get opposite coefficients of y, we will multiply the first equation by 2 and the second equation by 3.**

$$\begin{cases} -3(3x - 2y) = -3(-2) \\ 5x - 6y = 10 \end{cases}$$

Step 2: **Simplify.**

$$\begin{cases} 8x - 6y = 18 \\ 21x + 6y = -18 \end{cases}$$

Step 3: **Add the two equations to eliminate y.**

$$\begin{array}{r}\begin{cases}8x-6y=18\\21x+6y=-18\end{cases}\\ \hline 39x=0\end{array}$$

Step 4: **Solve for x.**

Step 5: **Substitute $x=0$ into one of the original equations.**

Figure 4.3.10

$$7\times0+2y=-6$$

Step 6: **Solve for y.**

$$2y=-6$$

$$y=-3$$

Step 7: **Write the solution as an ordered pair.**

The ordered pair is $(0,-3)$.

Step 8: **Check that the ordered pair is a solution to both original equations.**

$$\begin{array}{rcl}4x-3y&=&9\\4(0)-3(-3)&\stackrel{?}{=}&9\\9&=&9\checkmark\end{array}\qquad\begin{array}{rcl}7x+2y&=&-6\\7(0)+2(-3)&\stackrel{?}{=}&-6\\-6&=&-6\checkmark\end{array}$$

The solution is $(0,-3)$.

What other constants could we have chosen to eliminate one of the variables? Would the solution be the same?

Try It

9) Solve the system by elimination.

$$\begin{array}{l} 3x - 4y = -9 \\ 5x + 3y = 14 \end{array}$$

Solution

$(1, 3)$

10) Solve the system by elimination.

$$\begin{array}{l} 7x + 8y = 4 \\ 3x - 5y = 27 \end{array}$$

Solution

$(4, -3)$

When the system of equations contains fractions, we will first clear the fractions by multiplying each equation by its LCD.

Example 5

Solve the system by elimination.

$$\begin{array}{l} x + \frac{1}{2}y = 6 \\ \frac{3}{2}x + \frac{2}{3}y = \frac{17}{2} \end{array}$$

Solution

In this example, both equations have fractions. Our first step will be to multiply each equation by its LCD to clear the fractions.

$$\begin{cases} x + \frac{1}{2}y = 6 \\ \frac{3}{2}x + \frac{2}{3}y = \frac{17}{2} \end{cases}$$

Step 1: **To clear the fractions, multiply each equation by its LCD.**

$$\begin{cases} 2(x+\frac{1}{2}y)=2(6) \\ 6(\frac{3}{2}x+\frac{2}{3}y)=6(\frac{17}{2}) \end{cases}$$

Step 2: **Simplify.**

$$\begin{cases} 2x+y=12 \\ 9x+4y=51 \end{cases}$$

Now we are ready to eliminate one of the variables. Notice that both equations are in standard form.

Step 3: **We can eliminate y by multiplying the top equation by -4.**

$$\begin{cases} -4(2x+y)=-4(12) \\ 9x+4y=51 \end{cases}$$

Step 4: **Simplify and add.**

Step 5: **Substitute $x=3$ into one of the original equations.**

$$\begin{cases} -8x-4y=-48 \\ 9x+4y=\ \ 51 \end{cases}$$
$$x = 3$$
$$x+\frac{1}{2}y=6$$

Figure 4.3.11

Step 6: **Solve for y.**

$$\begin{aligned} 3+\frac{1}{2}y&=6 \\ \frac{1}{2}y&=3 \\ y&=6 \end{aligned}$$

Step 7: **Write the solution as an ordered pair.**

The ordered pair is $(3,6)$.

Step 8: **Check that the ordered pair is a solution to both original equations.**

$$\begin{aligned} \text{Equation 1: } x+\frac{1}{2}y &= 6 \\ 3+\frac{1}{2}(6) &\stackrel{?}{=} 6 \\ 3+3 &= 6 \\ 6 &= 6 \\ \text{Equation 2: } \frac{3}{2}x+\frac{2}{3}y &= \frac{17}{2} \\ \frac{3}{2}(3)+\frac{2}{3}(6) &\stackrel{?}{=} \frac{17}{2} \\ \frac{9}{2}+4 &\stackrel{?}{=} \frac{17}{2} \\ \frac{9}{2}+\frac{8}{2} &\stackrel{?}{=} \frac{17}{2} \\ \frac{17}{2} &= \frac{17}{2} \end{aligned}$$

The solution is $(3, 6)$.

Try It

11) Solve the system by elimination.

$$\begin{aligned} \frac{1}{3}x-\frac{1}{2}y &= 1 \\ \frac{3}{4}x-y &= \frac{5}{2} \end{aligned}$$

Solution

$(6, 2)$

12) Solve the system by elimination.

$$\begin{aligned} x+\frac{3}{5}y &= -\frac{1}{5} \\ -\frac{1}{2}x-\frac{2}{3}y &= \frac{5}{6} \end{aligned}$$

Solution

$(1, -2)$

In the Solving Systems of Equations by Graphing we saw that not all systems of linear equations have a single ordered pair as a solution. When the two equations were the same line, there were infinitely many solutions. We called that a consistent system. When the two equations described parallel lines, there was no solution. We called that an inconsistent system.

Example 6

Solve the system by elimination. $\begin{array}{r} 3x + 4y = 12 \\ y = 3 - \frac{3}{4}x \end{array}$

Solution

$$\begin{cases} 23x + 4y = 12 \\ y = 3 - \frac{3}{4}x \end{cases}$$

Step 1: **Write the second equation in standard form.**

$$\begin{cases} 23x + 4y = 12 \\ y = 3 - \frac{3}{4}x \end{cases}$$

Step 2: **Clear the fractions by multiplying the second equation by** 4**.**

$$\begin{cases} 3x + 4y = 12 \\ 4\left(\frac{3}{4}x + y\right) = 4(3) \end{cases}$$

Step 3: **Simplify.**

$$\begin{cases} 3x + 4y = 12 \\ 3x + 4y = 12 \end{cases}$$

Step 4: **To eliminate a variable, we multiply the second equation by** -1**.**

Step 5: **Simplify and add.**

$$\begin{cases} 3x + 4y = 12 \\ \underline{-3x - 4y = 12} \end{cases} \\ 0 = 0$$

This is a true statement. The equations are consistent but dependent. Their graphs would be the same line. The system has infinitely many solutions.

After we cleared the fractions in the second equation, did you notice that the two equations were the same? That means we have coincidental lines.

Try It

13) Solve the system by elimination.

$$\begin{aligned} 5x - 3y &= 15 \\ y &= -5 + \frac{5}{3}x \end{aligned}$$

Solution

Infinitely many solutions.

14) Solve the system by elimination.

$$\begin{aligned} x + 2y &= 6 \\ y &= -\frac{1}{2}x + 3 \end{aligned}$$

Solution

Infinitely many solutions.

Example 7

Solve the system by elimination. $\begin{aligned} -6x + 15y &= 10 \\ 2x - 5y &= -5 \end{aligned}$

Solution

Step 1: **The equations are in standard form.**

$$\begin{cases} -6x + 15y = 10 \\ 2x - 5y = -5 \end{cases}$$

Step 2: **Multiply the second equation by** 3 **to eliminate a variable.**

$$\begin{cases} -6x + 15y = 10 \\ 3(2x - 5y) = 3(-5) \end{cases}$$

Step 3: **Simplify and add.**

$$\begin{array}{r} \left\{\begin{array}{l} -6x+15y=10 \\ \underline{6x-15y=-15} \end{array}\right. \\ 0\neq -5 \end{array}$$

This statement is false. The equations are inconsistent and so their graphs would be parallel lines.

The system does not have a solution.

Try It

15) Solve the system by elimination.

$$\begin{array}{r} -3x+2y=8 \\ 9x-6y=13 \end{array}$$

Solution

No solution.

16) Solve the system by elimination.

$$\begin{array}{r} 7x-3y=-2 \\ -14x+6y=8 \end{array}$$

Solution

No solution.

Solve Applications of Systems of Equations by Elimination

Some application problems translate directly into equations in standard form, so we will use the elimination method to solve them. As before, we use our Problem-Solving Strategy to help us stay focused and organized.

Example 8

The sum of two numbers is 39. Their difference is 9. Find the numbers.

Solution

***Step 1:* Read the problem.**

***Step 2:* Identify what we are looking for.**

We are looking for two numbers.

***Step 3:* Name what we are looking for.**

Choose a variable to represent that quantity.

Let $n =$ be the first number.

Let $m =$ be the second number.

***Step 4:* Translate into a system of equations.**

The system is:

The sum of two numbers is 39.

$$n + m = 39$$

Their difference is 9.

$$n - m = 9$$

$$n + m = 39$$
$$n - m = 9$$

***Step 5:* Solve the system of equations.**

To solve the system of equations, use elimination.

The equations are in standard form and the coefficients of m are opposites. Add.

Solve for n.

Substitute $n = 24$ into one of the original equations and solve for m.

$$\begin{cases} n + m = 39 \\ -n - m = 9 \end{cases}$$
$$\overline{\quad 2n = 48 \quad}$$

$$\begin{aligned} n &= 24 \\ n + m &= 39 \\ 24 + m &= 39 \\ m &= 15 \end{aligned}$$

Step 6: **Check the answer.**

Since $24 + 15 = 39$ and $24 - 15 = 9$, the answers check.

Step 7: **Answer the question.**

The numbers are 24 and 15.

Try It

17) The sum of two numbers is 42. Their difference is 8. Find the numbers.

Solution

The numbers are 25 and 17.

18) The sum of two numbers is -15. Their difference is -35. Find the numbers.

Solution

The numbers are -25 and 10.

Example 9

Joe stops at a burger restaurant every day on his way to work. Monday he had one order of medium fries and two small sodas, which had a total of 620 calories. Tuesday he had two orders of medium fries and one small soda, for a total of 820 calories. How many calories are there in one order of medium fries? How many calories in one small soda?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

We are looking for the number of calories in one order of medium fries and in one small soda.

Step 3: **Name what we are looking for.**

Let f $=$ the number of calories in 1 order of medium fries.

Let s $=$ the number of calories in 1 small soda.

Step 4: **Translate into a system of equations:**

Our system is:

one medium fries and two small sodas had a total of 620 calories

$$f + 2s = 620$$

two medium fries and one small soda had a total of 820 calories.

$$2f + s = 820$$

$$\begin{cases} f + 2s = 620 \\ 2f + s = 820 \end{cases}$$

Step 5: **Solve the system of equations.**

To solve the system of equations, use elimination. The equations are in standard form. To get opposite coefficients of f, multiply the top equation by -2. Simplify and add. Solve for s. Substitute $s = 140$ into one of the original equations and then solve for f.

$$\begin{cases} -2(f + 2s) = -2(620) \\ 2f + s = 820 \end{cases}$$

$$\begin{array}{r} \begin{cases} -2f - 4s = -1240 \\ 2f + s = 820 \end{cases} \\ \hline -3s = -420 \end{array}$$

$$\begin{aligned} s &= 140 \\ f + 2s &= 620 \\ f + 2 \times 140 &= 620 \\ f + 280 &= 620 \\ f &= 340 \end{aligned}$$

Step 6: **Check the answer.**

Verify that these numbers make sense in the problem and that they are solutions to both equations. We leave this to you!

Step 7: **Answer the question.**

The small soda has 140 calories and the fries have 340 calories.

Try It

19) Malik stops at the grocery store to buy a bag of diapers and 2 cans of formula. He spends a total of $\$37$. The next week he stops and buys 2 bags of diapers and 5 cans of formula for a total of $\$87$. How much does a bag of diapers cost? How much is one can of formula?

Solution

The bag of diapers costs $\$11$ and the can of formula costs $\$13$.

20) To get her daily intake of fruit for the day, Sasha eats a banana and 8 strawberries on Wednesday for a calorie count of 145. On the following Wednesday, she eats two bananas and 5 strawberries for a total of 235 calories for the fruit. How many calories are there in a banana? How many calories are in a strawberry?

Solution

There are 105 calories in a banana and 5 calories in a strawberry.

Choose the Most Convenient Method to Solve a System of Linear Equations

When you will have to solve a system of linear equations in a later math class, you will usually not be told which method to use. You will need to make that decision yourself. So you'll want to choose the method that is easiest to do and minimizes your chance of making mistakes.

Graphing	Substitution	Elimination
Use when you need a picture of the situation.	Use when one equation is already solved for one variable.	Use when the equations are in standard form.

Figure 4.1.1

Example 10

For each system of linear equations decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

a.

$$\begin{aligned} 3x + 8y &= 40 \\ 7x - 4y &= -32 \end{aligned}$$

b.

$$\begin{aligned} 5x + 6y &= 12 \\ y &= \frac{2}{3}x - 1 \end{aligned}$$

Solution

a.

$$\begin{aligned} 3x + 8y &= 40 \\ 7x - 4y &= -32 \end{aligned}$$

Since both equations are in standard form, using elimination will be most convenient.

b.

$$\begin{aligned} 5x + 6y &= 12 \\ y &= \frac{2}{3}x - 1 \end{aligned}$$

Since one equation is already solved for y, using substitution will be most convenient.

Try It

21) For each system of linear equations, decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

a.

$$\begin{aligned} 4x - 5y &= -32 \\ 3x + 2y &= -1 \end{aligned}$$

b.

$$\begin{aligned} x &= 2y - 1 \\ 3x - 5y &= -7 \end{aligned}$$

Solution

a. Since both equations are in standard form, using elimination will be most convenient.

b. Since one equation is already solved for x, using substitution will be most convenient.

22) For each system of linear equations, decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

a.

$$\begin{aligned} y &= 2x - 1 \\ 3x - 4y &= -6 \end{aligned}$$

b.

$$\begin{aligned} 6x - 2y &= 12 \\ 3x + 7y &= -13 \end{aligned}$$

Solution

a. Since one equation is already solved for y, using substitution will be most convenient

b. Since both equations are in standard form, using elimination will be most convenient.

Access these online resources for additional instruction and practice with solving systems of linear equations by elimination.

- Instructional Video-Solving Systems of Equations by Elimination
- Instructional Video-Solving by Elimination
- Instructional Video-Solving Systems by Elimination

Key Concepts

- **To Solve a System of Equations by Elimination**
  1. Write both equations in standard form. If any coefficients are fractions, clear them.
  2. Make the coefficients of one variable opposites.
     - Decide which variable you will eliminate.
     - Multiply one or both equations so that the coefficients of that variable are opposites.
  3. Add the equations resulting from Step 2 to eliminate one variable.
  4. Solve for the remaining variable.
  5. Substitute the solution from Step 4 into one of the original equations. Then solve for

the other variable.
6. Write the solution as an ordered pair.
7. Check that the ordered pair is a solution to both original equations.

Exercises: Solve a System of Equations by Elimination

Instructions: For questions 1-41, solve the systems of equations by elimination.

1. $\begin{cases} 5x+2y=2 \\ -3x-y=0 \end{cases}$

---

2. $\begin{cases} -3x+y=-9 \\ x-2y=-12 \end{cases}$

Solution

$(6,9)$

---

3. $\begin{cases} 6x-5y=-1 \\ 2x+y=13 \end{cases}$

---

4. $\begin{cases} 3x-y=-7 \\ 4x+2y=-6 \end{cases}$

Solution

$(-2,1)$

---

5. $\begin{cases} x+y=-1 \\ x-y=-5 \end{cases}$

---

6. $\begin{cases} x + y = -8 \\ x - y = -6 \end{cases}$

Solution

$(-7, -1)$

---

7. $\begin{cases} 3x - 2y = 1 \\ -x + 2y = 9 \end{cases}$

---

8. $\begin{cases} -7x + 6y = -10 \\ x - 6y = 22 \end{cases}$

Solution

$(-2, -4)$

---

9. $\begin{cases} 3x + 2y = -3 \\ -x - 2y = -19 \end{cases}$

---

10. $\begin{cases} 5x + 2y = 1 \\ -5x - 4y = -7 \end{cases}$

Solution

$(-1, 3)$

---

11. $\begin{cases} 6x + 4y = -4 \\ -6x - 5y = 8 \end{cases}$

---

12. $\begin{cases} 3x - 4y = -11 \\ x - 2y = -5 \end{cases}$

Solution

$(-1, 2)$

---

13. $\begin{cases} 5x - 7y = 29 \\ x + 3y = -3 \end{cases}$

---

14. $\begin{cases} 6x - 5y = -75 \\ -x - 2y = -13 \end{cases}$

Solution

$(-5, 9)$

---

15. $\begin{cases} -x + 4y = 8 \\ 3x + 5y = 10 \end{cases}$

---

16. $\begin{cases} 2x - 5y = 7 \\ 3x - y = 17 \end{cases}$

Solution

$(6, 1)$

---

17. $\begin{cases} 5x - 3y = -1 \\ 2x - y = 2 \end{cases}$

---

18. $\begin{cases} 7x + y = -4 \\ 13x + 3y = 4 \end{cases}$

Solution

$(-2, 10)$

---

19. $\begin{cases} -3x + 5y = -13 \\ 2x + y = -26 \end{cases}$

---

20. $\begin{cases} 3x - 5y = -9 \\ 5x + 2y = 16 \end{cases}$

Solution

$(2, 3)$

---

21. $\begin{cases} 4x - 3y = 3 \\ 2x + 5y = -31 \end{cases}$

---

22. $\begin{cases} 4x + 7y = 14 \\ -2x + 3y = 32 \end{cases}$

Solution

$(-7, 6)$

---

23. $\begin{cases} 5x + 2y = 21 \\ 7x - 4y = 9 \end{cases}$

---

24. $\begin{cases} 3x + 8y = -3 \\ 2x + 5y = -3 \end{cases}$

Solution

$(-9, 3)$

---

25. $\begin{cases} 11x + 9y = -5 \\ 7x + 5y = -1 \end{cases}$

---

26. $\begin{cases} 3x + 8y = 67 \\ 5x + 3y = 60 \end{cases}$

Solution

$(9, 5)$

---

27. $\begin{cases} 2x + 9y = -4 \\ 3x + 13y = -7 \end{cases}$

---

28. $\begin{cases} \frac{1}{3}x - y = -3 \\ x + \frac{5}{2}y = 2 \end{cases}$

Solution

$(-3, 2)$

---

29. $\begin{cases} x + \frac{1}{2}y = \frac{3}{2} \\ \frac{1}{5}x - \frac{1}{5}y = 3 \end{cases}$

---

30. $\begin{cases} x + \frac{1}{3}y = -1 \\ \frac{1}{2}x - \frac{1}{3}y = -2 \end{cases}$

Solution

$(-2, 3)$

---

31. $\begin{cases} \frac{1}{3}x - y = -3 \\ \frac{2}{3}x + \frac{5}{2}y = 3 \end{cases}$

---

32. $\begin{cases} 2x + y = 3 \\ 6x + 3y = 9 \end{cases}$

Solution

infinitely many solutions

---

33. $\begin{cases} x - 4y = -1 \\ -3x + 12y = 3 \end{cases}$

---

34. $\begin{cases} -3x - y = 8 \\ 6x + 2y = -16 \end{cases}$

Solution

infinitely many solutions

---

35. $\begin{cases} 4x + 3y = 2 \\ 20x + 15y = 10 \end{cases}$

---

36. $\begin{cases} 3x + 2y = 6 \\ -6x - 4y = -12 \end{cases}$

Solution

infinitely many solutions

---

37. $\begin{cases} 5x - 8y = 12 \\ 10x - 16y = 20 \end{cases}$

---

38. $\begin{cases} -11x + 12y = 60 \\ -22x + 24y = 90 \end{cases}$

Solution

inconsistent, no solution

---

39. $\begin{cases} 7x - 9y = 16 \\ -21x + 27y = -24 \end{cases}$

---

40. $\begin{cases} 5x - 3y = 15 \\ y = \frac{5}{3}x - 2 \end{cases}$

Solution

inconsistent, no solution

---

41. $\begin{cases} 2x + 4y = 7 \\ y = -\frac{1}{2}x - 4 \end{cases}$

Exercises: Solve Applications of Systems of Equations by Elimination

Instructions: For questions 42-49, translate to a system of equations and solve.

42. The sum of two numbers is 65. Their difference is 25. Find the numbers.

Solution

The numbers are 20 and 45.

---

43. The sum of two numbers is 37. Their difference is 9. Find the numbers.

---

44. The sum of two numbers is -27. Their difference is -59. Find the numbers.

Solution

The numbers are 16 and -43.

---

45. The sum of two numbers is -45. Their difference is -89. Find the numbers.

---

46. Andrea is buying some new shirts and sweaters. She is able to buy 3 shirts and 2 sweaters for $\$114$ or she is able to buy 2 shirts and 4 sweaters for $\$164$. How much does a shirt cost? How much does a sweater cost?

Solution

A shirt costs $\$16$ and a sweater costs $\$33$.

---

47. Peter is buying office supplies. He is able to buy 3 packages of paper and 4 staplers for $\$40$ or he is able to buy 5 packages of paper and 6 staplers for $\$62$. How much does a package of paper cost? How much does a stapler cost?

---

48. The total amount of sodium in 2 hot dogs and 3 cups of cottage cheese is 4720 mg. The total amount of sodium in 5 hot dogs and 2 cups of cottage cheese is 6300 mg. How much sodium is in a hot dog? How much sodium is in a cup of cottage cheese?

Solution

There are 860 mg in a hot dog. There are $1,000$ mg in a cup of cottage cheese.

---

49. The total number of calories in 2 hot dogs and 3 cups of cottage cheese is 960 calories. The total number of calories in 5 hot dogs and 2 cups of cottage cheese is 1190 calories. How many calories are in a hot dog? How many calories are in a cup of cottage cheese?

Exercises: Choose the Most Convenient Method to Solve a System of Linear Equations

Instructions: For questions 50-53, decide whether it would be more convenient to solve the system of equations by substitution or elimination.

50.

a. $\begin{cases} 8x - 15y = -32 \\ 6x + 3y = -5 \end{cases}$

b. $\begin{cases} x = 4y - 3 \\ 4x - 2y = -6 \end{cases}$

Solution

a. elimination

b. substitution

51.

a. $\begin{cases} y = 7x - 5 \\ 3x - 2y = 16 \end{cases}$

b. $\begin{cases} 12x - 5y = -42 \\ 3x + 7y = -15 \end{cases}$

52.

a. $\begin{cases} y = 4x + 9 \\ 5x - 2y = -21 \end{cases}$

b. $\begin{cases} 9x - 4y = 24 \\ 3x + 5y = -14 \end{cases}$

Solution

a. substitution

b. elimination

53.

a. $\begin{cases} 14x - 15y = -30 \\ 7x + 2y = 10 \end{cases}$

b. $\begin{cases} x = 9y - 11 \\ 2x - 7y = -27 \end{cases}$

Exercises: Everyday Math

Instructions: For questions 54-55, answer the given everyday math word problems.

54. Norris can row 3 miles upstream against the current in the same amount of time it takes him to row 5 miles downstream, with the current. Solve the system. $\begin{cases} r - c = 3 \\ r + c = 5 \end{cases}$

a. for r, his rowing speed in still water.

b. Then solve for c, the speed of the river current.

Solution

a. $r = 4$

b. $c = 1$

---

55. Josie wants to make 10 pounds of trail mix using nuts and raisins, and she wants the total cost of the trail mix to be $\$54$. Nuts cost $\$6$ per pound and raisins cost $\$3$ per pound. Solve the system $\begin{cases} n + r = 10 \\ 6n + 3r = 54 \end{cases}$ to find n, the number of pounds of nuts, and r, the number of pounds of raisins she should use.

Exercises: Writing Exercises

Instructions: For questions 56-57, answer the given writing exercises.

56. Solve the system $\begin{cases} x+y=10 \\ 5x+8y=56 \end{cases}$

- **a. by substitution**
- **b. by graphing**
- **c. Which method do you prefer? Why?**

Solution

a. (8, 2)

b.

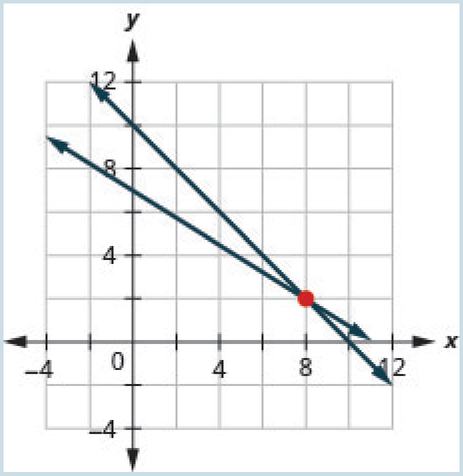


Figure 4P.3.1

c. Answers will vary.

---

57. Solve the system $\begin{cases} x+y=-12 \\ y=4-\frac{1}{2}x \end{cases}$

- **a. by substitution**
- **b. by graphing**
- **c. Which method do you prefer? Why?**

4.4 SOLVE APPLICATIONS WITH SYSTEMS OF EQUATIONS

Learning Objectives

By the end of this section, you will be able to:

- Translate to a system of equations
- Solve direct translation applications
- Solve geometry applications
- Solve uniform motion applications

Try It

Before you get started, take this readiness quiz:

1) The sum of twice a number and nine is 31. Find the number.

2) Twins Jon and Ron together earned $\$96,000$ last year. Ron earned $\$8,000$ more than three times what Jon earned. How much did each of the twins earn?

3) Alessio rides his bike $3\frac{1}{2}$ hours at a rate of 10 miles per hour. How far did he ride?

Previously in this chapter, we solved several applications with systems of linear equations. In this section, we'll look at some specific types of applications that relate to two quantities. We'll translate the words into linear equations, decide which is the most convenient method to use, and then solve them.

We will use our Problem-Solving Strategy for Systems of Linear Equations.

How to

Use a problem-solving strategy for systems of linear equations.

1. Read the problem. Make sure all the words and ideas are understood.
2. Identify what we are looking for.
3. Name what we are looking for. Choose variables to represent those quantities.
4. Translate into a system of equations.
5. Solve the system of equations using good algebra techniques.
6. Check the answer to the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

Translate to a System of Equations

Many of the problems we solved in earlier applications related to two quantities. Here are two of the examples from the chapter on **Math Models**.

- The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.
- A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

In that chapter, we translated each situation into one equation using only one variable. Sometimes it was a bit of a challenge figuring out how to name the two quantities, wasn't it?

Let's see how we can translate these two problems into a system of equations with two variables. We'll focus on Steps 1 through 4 of our Problem-Solving Strategy.

Example 1

How to Translate to a System of Equations

Translate to a system of equations:

The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.

Solution

Step 1. Read the problem. Make sure you understand all the words and ideas.	This is a number problem.	The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.

Figure 4.4.1

Step 2. Identify what you are looking for.	"Find the numbers."	We are looking for 2 numbers.

Figure 4.4.2

Step 3. Name what you are looking for. Choose variables to represent those quantities.	We will use two variables, m and n.	Let m = one number n = second number

Figure 4.4.3

Step 4. Translate into a system of equations.	We will write one equation for each sentence.	$\underbrace{\text{The sum of the numbers}}_{m+n}\ \underbrace{\text{is}}_{=}\ \underbrace{-14}_{-14}$ $\underbrace{\text{One number}}_{m}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{four less than the other}}_{n-4}$ The system is: $\begin{cases} m+n=-14 \\ m=n-4 \end{cases}$

Figure 4.4.4

Try It

4) Translate to a system of equations: The sum of two numbers is negative twenty-three. One number is 7 less than the other. Find the numbers.

Solution

$$\begin{cases} m+n=-23 \\ m=n-7 \end{cases}$$

5) Translate to a system of equations: The sum of two numbers is negative eighteen. One number is 40 more than the other. Find the numbers.

Solution

$$\begin{cases} m+n=-18 \\ m=n+40 \end{cases}$$

We'll do another example where we stop after we write the system of equations.

Example 2

Translate to a system of equations:

A married couple together earns $\$110,000$ a year. The wife earns $\$16,000$ less than twice what her husband earns. What does the husband earn?

Solution

Step 1: **We are looking for the amount that the husband and wife each earn.**

Let h $=$ the amount the husband earns.

Let w $=$ the amount the wife earns.

Step 2: **Translate.**

A married couple together earns $\$110,000$.

$$w+h=110,000$$

The wife earns $\$16,000$ less than twice what the husband earns.

$$w=2h-16,000$$

Step 3: **The system of equations is:**

$$\begin{cases} w+h=110,000 \\ w=2h-16,000 \end{cases}$$

Try It

6) Translate to a system of equations: A couple has a total household income of $84,000. The husband earns $18,000 less than twice what the wife earns. How much does the wife earn?

Solution

$\begin{cases} w+h=84,000 \\ h=2w-18,000 \end{cases}$

7) Translate to a system of equations: A senior employee makes $5 less than twice what a new employee makes per hour. Together they make $43 per hour. How much does each employee make per hour?

Solution

$\begin{cases} s=2n-5 \\ s+n=43 \end{cases}$

Solve Direct Translation Applications

We set up but did not solve, the systems of equations in Examples 1 and 2. Now we'll translate a situation to a system of equations and then solve it.

Example 3

Translate to a system of equations and then solve: Devon is 26 years older than his son Cooper. The sum of their ages is 50. Find their ages.

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

We are looking for the ages of Devon and Cooper.

Step 3: **Name what we are looking for.**

Let $d=$ Devon's age.

Let $c=$ Cooper's age

Step 4: **Translate into a system of equations.**

Devon is 26 years older than Cooper.

$$d = c + 26$$

The sum of their ages is 50.

$$d + c = 50$$

The system is:

$$\begin{cases} d = c + 26 \\ d + c = 50 \end{cases}$$

***Step 5:* Solve the system of equations.**

Solve by substitution.

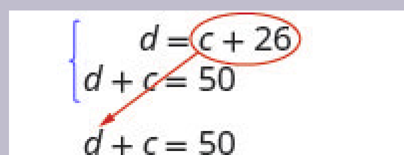


Figure 4.4.5

Substitute $c + 26$ into the second equation.

$$c + 26 + c = 50$$

Solve for c.

$$\begin{aligned} 2c + 26 &= 50 \\ 2c &= 24 \\ c &= 12 \\ \text{Substitute c = 12.} \quad d &= c + 26 \\ \text{Solve for d.} \quad d &= 12 + 26 \\ d &= 38 \end{aligned}$$

***Step 6:* Check the answer in the problem.**

Is Devon's age 26 more than Cooper's?

Yes, 38 is 26 more than 12.

Is the sum of their ages 50?

Yes, 38 plus 12 is 50.

***Step 7:* Answer the question.**

Devon is 38 and Cooper is 12 years old.

Try It

8) Translate to a system of equations and then solve: Ali is 12 years older than his youngest sister, Jameela. The sum of their ages is 40. Find their ages.

Solution

Ali is 26 and Jameela is 14.

9) Translate to a system of equations and then solve: Jake's dad is 6 more than 3 times Jake's age. The sum of their ages is 42. Find their ages.

Solution

Jake is 9 and his dad is 33.

Example 4

Translate to a system of equations and then solve:

When Jenna spent 10 minutes on the elliptical trainer and then did circuit training for 20 minutes, her fitness app said she burned 278 calories. When she spent 20 minutes on the elliptical trainer and 30 minutes on circuit training she burned 473 calories. How many calories does she burn for each minute on the elliptical trainer? How many calories does she burn for each minute of circuit training?

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

We are looking for the number of calories burned each minute on the elliptical trainer and each minute of circuit training.

Step 3: **Name what we are looking for.**

Let $e =$ be the number of calories burned per minute on the elliptical trainer.

Let $c =$ be the number of calories burned per minute while circuit training.

Step 4: **Translate into a system of equations.**

10 minutes on the elliptical and circuit training for 20 minutes, burned 278 calories

$$10e + 20c = 278$$

20 minutes on the elliptical and 30 minutes of circuit training burned 473 calories

$$20e + 30c = 473$$

The system is:

$$\begin{cases} 10e + 20c = 278 \\ 20e + 30c = 473 \end{cases}$$

Step 5: **Solve the system of equations.**

Multiply the first equation by -2 to get opposite coefficients of e.

$$\begin{cases} -2(10e + 20c) = -2(278) \\ 20e + 30c = 473 \end{cases}$$

Simplify and add the equations.

Solve for c.

$$\begin{array}{r} \begin{cases} -20e - 40c = -556 \\ 20e + 30c = 473 \end{cases} \\ \hline -10c = -83 \\ c = 8.3 \end{array}$$

Substitute $c = 8.3$ into one of the original equations to solve for e.

$$\begin{aligned} 10e + 20c &= 278 \\ 10e + 20(8.3) &= 278 \\ 10e + 166 &= 278 \\ 10e &= 112 \\ e &= 11.2 \end{aligned}$$

Step 6: **Check the answer to the problem.**

Check the math on your own.

$$\begin{cases} 10(11.2) + 20(8.3) \stackrel{?}{=} 278 \\ 20(11.2) + 30(8.3) \stackrel{?}{=} 473 \end{cases}$$

Step 7: **Answer the question.**

Jenna burns 8.3 calories per minute circuit training and 11.2 calories per minute while on the elliptical trainer.

Try It

10) *Translate to a system of equations and then solve:* Mark went to the gym and did 40 minutes of Bikram hot yoga and 10 minutes of jumping jacks. He burned 510 calories. The next time he went to the gym, he did 30 minutes of Bikram hot yoga and 20 minutes of jumping jacks burning 470 calories. How many calories were burned for each minute of yoga? How many calories were burned for each minute of jumping jacks?

Solution

Mark burned 11 calories for each minute of yoga and 7 calories for each minute of jumping jacks.

11) *Translate to a system of equations and then solve:* Erin spent 30 minutes on the rowing machine and 20 minutes lifting weights at the gym and burned 430 calories. During her next visit to the gym, she spent 50 minutes on the rowing machine and 10 minutes lifting weights and burned 600 calories. How many calories did she burn for each minute on the rowing machine? How many calories did she burn for each minute of weight lifting?

Solution

Erin burned 11 calories for each minute on the rowing machine and 5 calories for each minute of weight lifting.

Solve Geometry Applications

When we learned about **Math Models**, we solved geometry applications using properties of triangles and rectangles. Now we'll add to our list some properties of angles.

The measures of two **complementary angles** add to 90 degrees. The measures of two **supplementary angles** add to 180 degrees.

Complementary and Supplementary Angles

Two angles are **complementary** if the sum of the measures of their angles is 90 degrees.

Two angles are **supplementary** if the sum of the measures of their angles is 180 degrees.

If two angles are complementary, we say that *one angle is the **complement** of the other.*
If two angles are supplementary, we say that *one angle is the **supplement** of the other.*

Example 5

Translate to a system of equations and then solve: The difference of two complementary angles is 26 degrees. Find the measures of the angles.

Solution

***Step 1:* Read the problem.**

***Step 2:* Identify what we are looking for.**

We are looking for the measure of each angle.

***Step 3:* Name what we are looking for.**

Let x $=$ the measure of the first angle.

Let m $=$the measure of the second angle.

***Step 4:* Translate into a system of equations.**

The angles are complementary.

$$x+y=90$$

The difference of the two angles is *26* degrees.

$$x-y=26$$

The system is

$$\begin{cases} x+y=90 \\ x-y=26 \end{cases}$$

***Step 5:* Solve the system of equations by elimination.**

$$\begin{cases} x+y=90 \\ x-y=26 \end{cases}$$

$$2x = 116$$
$$x = 58$$

Substitute $x = 58$ into the first equation.

$$x + y = 90$$
$$58 + y = 90$$
$$y = 32$$

Step 6: **Check the answer in the problem.**

$$58 + 32 = 90\checkmark$$
$$58 - 32 = 26\checkmark$$

Step 7: **Answer the question.**

The angle measures are 58 degrees and 32 degrees.

Try It

12) Translate to a system of equations and then solve: The difference between two complementary angles is 20 degrees. Find the measures of the angles.

Solution

The angle measures are 55 degrees and 35 degrees.

13) Translate to a system of equations and then solve: The difference between two complementary angles is 80 degrees. Find the measures of the angles.

Solution

The angle measures are 5 degrees and 85 degrees.

Example 6

Translate to a system of equations and then solve:

Two angles are supplementary. The measure of the larger angle is twelve degrees less than five times the measure of the smaller angle. Find the measures of both angles.

Solution

Step 1: **Read the problem.**

Step 2: **Identify what we are looking for.**

We are looking for the measure of each angle.

Step 3: **Name what we are looking for.**

Let x $=$ the measure of the first angle.

Let y $=$ the measure of the second angle

Step 4: **Translate into a system of equations.**

The angles are supplementary.

$$x+y=180$$

The larger angle is twelve less than five times the smaller angle

$$y=5x-12$$

The system is:

$$\begin{cases} x+y=180 \\ y=5x-12 \end{cases}$$

Step 5: **Solve the system of equations substitution.**

Figure 4.4.6

Substitute $5x - 12$ for y in the first equation.

$$x + 5x - 12 = 180$$

Solve for x.

$$\begin{aligned} 6x - 12 &= 180 \\ 6x &= 192 \\ \text{Substitute in x=32.} \quad y &= 5x - 12 \\ \text{Solve for y.} \quad y &= 5 \times 32 - 12 \\ y &= 160 - 12 \\ y &= 148 \end{aligned}$$

Step 6: **Check the answer to the problem.**

$$\begin{aligned} 32 + 158 &= 180 \checkmark \\ 5 \times 32 - 12 &= 147 \checkmark \end{aligned}$$

Step 7: **Answer the question.**

The angle measures are 148 and 32.

Try It

14) *Translate to a system of equations and then solve:* Two angles are supplementary. The measure of the larger angle is 12 degrees more than three times the smaller angle. Find the measures of the angles.

Solution

The angle measures are 42 degrees and 138 degrees.

15) *Translate to a system of equations and then solve:* Two angles are supplementary. The measure of the larger angle is 18 less than twice the measure of the smaller angle. Find the measures of the angles.

Solution

The angle measures are 66 degrees and 114 degrees.

Example 7

Translate to a system of equations and then solve:

Randall has 125 feet of fencing to enclose the rectangular part of his backyard adjacent to his house. He will only need to fence around three sides because the fourth side will be the wall of the house. He wants the length of the fenced yard (parallel to the house wall) to be 5 feet more than four times as long as the width. Find the length and the width.

Solution

Step 1: **Read the problem.**

Step 2: **Identify what you are looking for.**

We are looking for the length and width.

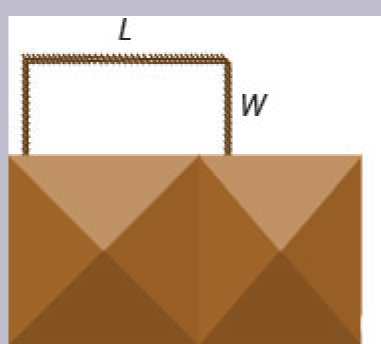


Figure 4.4.7

Step 3: **Name what we are looking for.**

Let L $=$ the length of the fenced yard.

Let W $=$the width of the fenced yard

Step 4: **Translate into a system of equations.**

One length and two widths equal 125.

$$L+2W=125$$

The length will be 5 feet more than four times the width.

$$L=4W+5$$

The system is:

$$\begin{cases} L+2W=125 \\ L=4W+5 \end{cases}$$

Step 5: **Solve the system of equations by substitution.**

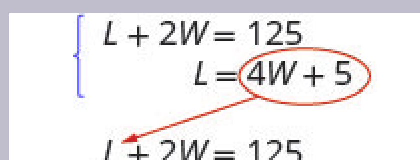


Figure 4.4.8

Substitute $L=4W+5$ into the first equation, then solve for W.

$$\begin{aligned} 4W+5+2W&=125 \\ 6W+5&=125 \\ 6W&=120 \end{aligned}$$

Substitute 20 for W in the second equation, then solve for L.

$$\begin{aligned} \text{Substitute W=20.} \quad L&=4W+5 \\ \text{Solve for L.} \quad L&=4\times 20+5 \\ L&=80+5 \\ L&=85 \end{aligned}$$

Step 6: **Check the answer in the problem.**

$$\begin{aligned} 20+28+20&=125\checkmark \\ 85&=4\times 20+5\checkmark \end{aligned}$$

Step 7: **Answer the equation.**

The length is 85 feet and the width is 20 feet.

Try It

16) *Translate to a system of equations and then solve:* Mario wants to put a rectangular fence around the pool in his backyard. Since one side is adjacent to the house, he will only need to fence three sides. There are two long sides and the one shorter side is parallel to the house. He needs 155 feet of fencing to enclose the pool. The length of the long side is 10 feet less than twice the width. Find the length and width of the pool area to be enclosed.

Solution

The length is 60 feet and the width is 35 feet.

17) *Translate to a system of equations and then solve:* Alexis wants to build a rectangular dog run in her yard adjacent to her neighbour's fence. She will use 136 feet of fencing to completely enclose the rectangular dog run. The length of the dog run along the neighbour's fence will be 16 feet less than twice the width. Find the length and width of the dog run.

Solution

The length is 60 feet and the width is 38 feet.

Solve Uniform Motion Applications

We used a table to organize the information in uniform motion problems when we introduced them earlier.

We'll continue using the table here. The basic equation was $D = rt$ where D is the distance travelled, r is the rate, and t is the time.

Our first example of a uniform motion application will be for a situation similar to some we have already seen, but now we can use two variables and two equations.

Example 8

Translate to a system of equations and then solve:

Joni left St. Louis on the interstate, driving west towards Denver at a speed of 65 miles per hour. Half an hour later, Kelly left St. Louis on the same route as Joni, driving 78 miles per hour. How long will it take Kelly to catch up to Joni?

Solution

A diagram is useful in helping us visualize the situation.

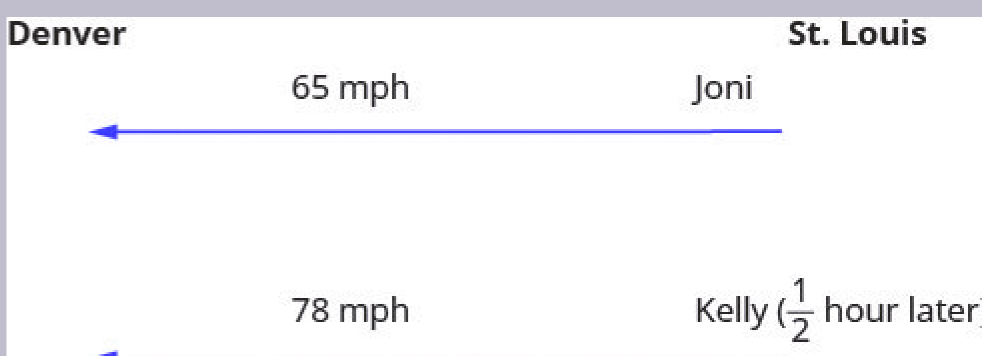


Figure 4.4.9

Step 1: **Identify and name what we are looking for.**

A chart will help us organize the data. We know the rates of both Joni and Kelly and so we enter them in the chart.

We are looking for the length of time Kelly, k, and Joni, j, will each drive. Since

$D = r \cdot t$ we can fill in the Distance column.

Type	Rate x Time = Distance		
Joni	65	j	65j
Kelly	78	k	78k

***Step 2:* Translate into a system of equations.**

To make the system of equations, we must recognize that Kelly and Joni will drive the same distance. So, $65j = 78k$.

Also, since Kelly left later, her time will be $\frac{1}{2}$ hour less than Joni's time.

So, $k = j - \frac{1}{2}$

Now we have the system.

$$\begin{cases} k = j - \frac{1}{2} \\ 65j = 78k \end{cases}$$

***Step 3:* Solve the system of equations by substitution.**

$$65j = 78k$$

Substitute $k = j - \frac{1}{2}$ into the second equation, then solve for j.

$$\begin{aligned} 65j &= 78(j - \tfrac{1}{2}) \\ 65j &= 78j - 39 \\ -13j &= -39 \\ j &= 3 \end{aligned}$$

To find Kelly's time, substitute $j = 3$ into the first equation, then solve for k.

$$\text{Solve for k. } \begin{aligned} k &= j - \tfrac{1}{2} \\ k &= 3 - \tfrac{1}{2} \\ k &= \tfrac{5}{2} \text{ or } k = 2\tfrac{1}{2} \end{aligned}$$

***Step 4:* Check the answer to the problem.**

Joni 3 hours (65 mph) = 195 miles.

Kelly $2\frac{1}{2}$ hours (78 mph) = 195 miles.

Yes, they will have travelled the same distance when they meet.

***Step 5:* Answer the question.**

Kelly will catch up to Joni in $2\frac{1}{2}$ hours. By then, Joni will have travelled 3 hours.

Try It

18) *Translate to a system of equations and then solve:* Mitchell left Detroit on the interstate driving south towards Orlando at a speed of 60 miles per hour. Clark left Detroit 1 hour later

travelling at a speed of 75 miles per hour, following the same route as Mitchell. How long will it take Clark to catch Mitchell?

Solution

It will take Clark 4 hours to catch Mitchell.

19) *Translate to a system of equations and then solve:* Charlie left his mother's house travelling at an average speed of 36 miles per hour. His sister Sally left 15 minutes ($\frac{1}{4}$ hour) later travelling the same route at an average speed of 42 miles per hour. How long before Sally catches up to Charlie?

Solution

It will take Sally $1\frac{1}{2}$ hours to catch up to Charlie.

Many real-world applications of uniform motion arise because of the effects of currents—of water or air—on the actual speed of a vehicle. Cross-country airplane flights in the United States generally take longer going west than going east because of the prevailing wind currents.

Let's take a look at a boat travelling on a river. Depending on which way the boat is going, the current of the water is either slowing it down or speeding it up.

Figures 4.4.1 and 4.4.2 show how a river current affects the speed at which a boat is travelling. We'll call the speed of the boat in still water b and the speed of the river current c.

In Figure 4.4.1 boat is going downstream, in the same direction as the river current. The current helps push the boat, so the boat's actual speed is faster than its speed in still water. The actual speed at which the boat is moving is $b + c$.

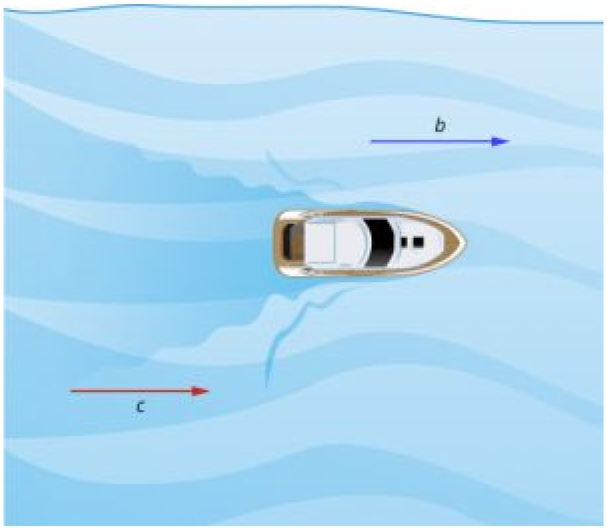


Figure 4.4.10

In Figure 4.4.2 the boat is going upstream, opposite to the river current. The current is going against the boat, so the boat's actual speed is slower than its speed in still water. The actual speed of the boat is $b - c$.

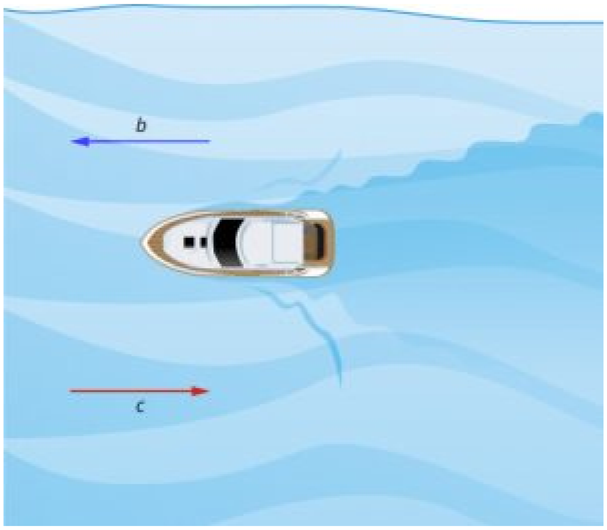


Figure 4.4.11

We'll put some numbers to this situation in Example 9

Example 9

Translate to a system of equations and then solve:

A river cruise ship sailed 60 miles downstream for 4 hours and then took 5 hours sailing upstream to return to the dock. Find the speed of the ship in still water and the speed of the river current.

Solution

Read the problem.

This is a uniform motion problem and a picture will help us visualize the situation.

Figure 4.4.12

Step 1: **Identify what we are looking for.**

We are looking for the speed of the ship in still water and the speed of the current.

Step 2: **Name what we are looking for.**

Let s $=$ the rate of the ship in still water.

Let c $=$ the rate of the current

A chart will help us organize the information. The ship goes downstream and then upstream. Going downstream, the current helps the ship; therefore, the ship's actual rate is $s + c$. Going upstream, the current slows the ship; therefore, the actual rate is $s - c$.

Type	Rate x Time = Distance		
Downstream	$s+c$	4	60
Upstream	$s-c$	5	60

Downstream it takes 4 hours. Upstream it takes 5 hours. Each way the distance is 60 miles.

***Step 3:* Translate into a system of equations.**

Since rate times time is distance, we can write the system of equations.

$$\begin{cases} 4(s+c)=60 \\ 5(s-c)=60 \end{cases}$$

***Step 4:* Solve the system of equations.**

Distribute to put both equations in standard form, then solve by elimination.

$$\begin{cases} 4s+4c=60 \\ 5s-5c=60 \end{cases}$$

Multiply the top equation by 5 and the bottom equation by 4. Add the equations, then solve for s.

$$\begin{array}{r} \begin{cases} 20s+20c=300 \\ 20s-20c=240 \end{cases} \\ \hline 40s=540 \end{array}$$

Substitute $s=13.5$ into one of the original equations.

$$\begin{array}{rr} \text{Substitute s=13.5.} & 4(s+c)=60 \\ & 4(13.5+c)=60 \\ \text{Solve for c.} & 54+4c=60 \\ & 4c=6 \\ & c=1.5 \end{array}$$

***Step 5:* Check the answer in the problem.**

The downstream rate would be $13.5+1.5=15\ mph$.

In 4 hours the ship would travel $15\cdot 4=60\ miles$.

The upstream rate would be $13.5-1.5=12\ mph$.

In 5 hours the ship would travel $12 \cdot 5 = 60\ miles$.

***Step 6:* Answer the question**.

The rate of the ship is 13.5 mph and the rate of the current is 1.5 mph.

Try It

20) *Translate to a system of equations and then solve* A Mississippi river boat cruise sailed 120 miles upstream for 12 hours and then took 10 hours to return to the dock. Find the speed of the riverboat in still water and the speed of the river current.

Solution

The rate of the boat is 11 mph and the rate of the current is 1 mph.

21) *Translate to a system of equations and then solve:* Jason paddled his canoe 24 miles upstream for 4 hours. It took him 3 hours to paddle back. Find the speed of the canoe in still water and the speed of the river current.

Solution

The speed of the canoe is 7 mph and the speed of the current is 1 mph.

Wind currents affect airplane speeds in the same way as water currents affect boat speeds. We'll see this in Example 10. A wind current in the same direction as the plane is flying is called a *tailwind*. A wind current blowing against the direction of the plane is called a ***headwind***.

Example 10

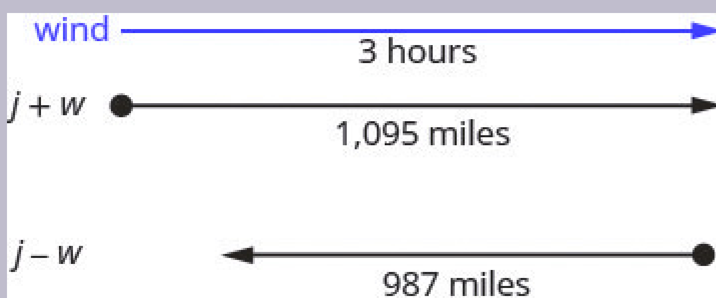
Translate to a system of equations and then solve:

A private jet can fly 1095 miles in three hours with a tailwind but only 987 miles in three hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Solution

Read the problem.

This is a uniform motion problem and a picture will help us visualize.



4.4.13

Step 1: **Identify what we are looking for.**

We are looking for the speed of the jet in still air and the speed of the wind.

Step 2: **Name what we are looking for.**

Let j $=$ the speed of the jet in still air.

Let w $=$ the speed of the wind

A chart will help us organize the information. The jet makes two trips-one in a tailwind and one in a headwind. In a tailwind, the wind helps the jet and so the rate is $j+w$. In a headwind, the wind slows the jet and so the rate is $j-w$.

Type	Rate x Time = Distance		
Tailwind	$j+w$	3	1095
Headwind	$j-w$	3	987

Each trip takes 3 hours. In a tailwind the jet flies 1095 miles. In a headwind the jet flies 987 miles.

Step 3: **Translate into a system of equations.**

Since rate times time is distance, we get the system of equations.

$$\begin{cases} 3(j+w)=1095 \\ 3(j-w)=987 \end{cases}$$

Step 4: **Solve the system of equations.**

Distribute, then solve by elimination.

$$\begin{array}{r} \begin{cases} 3j+3w=1095 \\ 3j-3w=987 \end{cases} \\ \hline 6j=2082 \end{array}$$

Add, and solve for j.

Substitute $j=347$ into one of the original equations, then solve for w.

$$\begin{array}{lr} \text{Substitute j=347.} & 3(j+w)=1095 \\ & 3(347+w)=1095 \\ \text{Solve for w.} & 1041+3w=1095 \\ & 3w=54 \\ & w=18 \end{array}$$

Step 5: **Check the answer in the problem.**

With the tailwind, the actual rate of the jet would be $347+18=365\ mph.$

In 3 hours the jet would travel $365\times3=1095\ miles.$

Going into the headwind, the jet's actual rate would be $347-18=329\ mph.$

In 3 hours the jet would travel $329 \times 3 = 987\ miles.$

Step 6: **Answer the question.**

The rate of the jet is 347 mph and the rate of the wind is $18\ mph.$

Try It

22) Translate to a system of equations and then solve: A small jet can fly $1,325$ miles in 5 hours with a tailwind but only 1025 miles in 5 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Solution

The speed of the jet is 235 mph and the speed of the wind is 30 mph.

23) Translate to a system of equations and then solve: A commercial jet can fly 1728 miles in 4 hours with a tailwind but only 1536 miles in 4 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Solution

The speed of the jet is 408 mph and the speed of the wind is 24 mph.

Glossary

complementary angles

Two angles are complementary if the sum of the measures of their angles is 90 degrees.

supplementary angles

Two angles are supplementary if the sum of the measures of their angles is 180 degrees.

Exercises: Translate to a System of Equations

Instructions: For questions 1-14, translate to a system of equations and solve the system.

1. The sum of two numbers is fifteen. One number is three less than the other. Find the numbers.

Solution

The numbers are 6 and 9.

---

2. The sum of two numbers is twenty-five. One number is five less than the other. Find the numbers.

---

3. The sum of two numbers is negative thirty. One number is five times the other. Find the numbers.

Solution

The numbers are -5 and -25.

---

4. The sum of two numbers is negative sixteen. One number is seven times the other. Find the numbers.

---

5. Twice a number plus three times a second number is twenty-two. Three times the first number plus four times the second is thirty-one. Find the numbers.

Solution

The numbers are 5 and 4.

---

6. Six times a number plus twice a second number is four. Twice the first number plus four times the second number is eighteen. Find the numbers.

---

7. Three times a number plus three times a second number is fifteen. Four times the first plus twice the second number is fourteen. Find the numbers.

Solution

The numbers are 2 and 3.

---

8. Twice a number plus three times a second number is a negative one. The first number plus four times the second number is two. Find the numbers.

9. A married couple together earn $\$75{,}000$. The husband earns $\$15{,}000$ more than five times what his wife earns. What does the wife earn?

Solution

$\$10{,}000$

10. During two years in college, a student earned $\$9{,}500$. The second year she earned $\$500$ more than twice the amount she earned the first year. How much did she earn the first year?

11. Daniela invested a total of $\$50{,}000$, some in a certificate of deposit (CD) and the remainder in bonds. The amount invested in bonds was $\$5000$ more than twice the amount she put into the CD. How much did she invest in each account?

Solution

She put $\$15{,}000$ into a CD and $\$35{,}000$ in bonds.

12. Jorge invested $\$28{,}000$ into two accounts. The amount he put in his money market account was $\$2{,}000$ less than twice what he put into a CD. How much did he invest in each account?

13. In her last two years in college, Marlene received $\$42{,}000$ in loans. The first year she received a loan that was $\$6{,}000$ less than three times the amount of the second year's loan. What was the amount of her loan for each year?

Solution

The amount of the first year's loan was $\$30{,}000$ and the amount of the second year's loan was $\$12{,}000$.

14. Jen and David owe $\$22,000$ in loans for their two cars. The amount of the loan for Jen's car is $\$2000$ less than twice the amount of the loan for David's car. How much is each car loan?

Exercises: Solve Direct Translation Applications

Instructions: For questions 15-24, translate to a system of equations and solve.

15. Alyssa is twelve years older than her sister, Bethany. The sum of their ages is forty-four. Find their ages.

Solution

Bethany is 16 years old and Alyssa is 28 years old.

16. Robert is 15 years older than his sister, Helen. The sum of their ages is sixty-three. Find their ages.

17. The age of Noelle's dad is six less than three times Noelle's age. The sum of their ages is seventy-four. Find their ages.

Solution

Noelle is 20 years old and her dad is 54 years old.

18. The age of Mark's dad is 4 less than twice Marks's age. The sum of their ages is ninety-five. Find their ages.

---

19. Two containers of gasoline hold a total of fifty gallons. The big container can hold ten gallons less than twice the small container. How many gallons does each container hold?
Solution

The small container holds 20 gallons and the large container holds 30 gallons.

---

20. June needs 48 gallons of punch for a party and has two different coolers to carry it in. The bigger cooler is five times as large as the smaller cooler. How many gallons can each cooler hold?

---

21. Shelly spent 10 minutes jogging and 20 minutes cycling and burned 300 calories. The next day, Shelly swapped times, doing 20 minutes of jogging and 10 minutes of cycling and burned the same number of calories. How many calories were burned for each minute of jogging and how many for each minute of cycling?
Solution

There were 10 calories burned jogging and 10 calories burned cycling.

---

22. Drew burned 1800 calories Friday playing one hour of basketball and canoeing for two hours. Saturday he spent two hours playing basketball and three hours canoeing and

burned 3200 calories. How many calories did he burn per hour when playing basketball?

---

23. Troy and Lisa were shopping for school supplies. Each purchased different quantities of the same notebook and thumb drive. Troy bought four notebooks and five thumb drives for $\$116$. Lisa bought two notebooks and three thumb dives for $\$68$. Find the cost of each notebook and each thumb drive.

Solution

Notebooks are $\$4$ and thumb drives are $\$20$.

---

24. Nancy bought seven pounds of oranges and three pounds of bananas for $\$17$. Her husband later bought three pounds of oranges and six pounds of bananas for $\$12$. What was the cost per pound of the oranges and the bananas?

Exercises: Solve Geometry Applications

Instructions: For questions 25-40, translate to a system of equations and solve.

25. The difference of two complementary angles is 30 degrees. Find the measures of the angles.

Solution

The measures are 60 degrees and 30 degrees.

---

26. The difference of two complementary angles is 68 degrees. Find the measures of the angles.

---

27. The difference of two supplementary angles is 70 degrees. Find the measures of the angles.

Solution

The measures are 125 degrees and 55 degrees.

---

28. The difference of two supplementary angles is 24 degrees. Find the measure of the angles.

---

29. The difference of two supplementary angles is 8 degrees. Find the measures of the angles.

Solution

94 degrees and 86 degrees

---

30. The difference of two supplementary angles is 88 degrees. Find the measures of the angles.

---

31. The difference of two complementary angles is 55 degrees. Find the measures of the angles.

Solution

72.5 degrees and 17.5 degrees

---

32. The difference of two complementary angles is 17 degrees. Find the measures of the angles.

---

33. Two angles are supplementary. The measure of the larger angle is four more than three times the measure of the smaller angle. Find the measures of both angles.

Solution

The measures are 44 degrees and 136 degrees.

---

34. Two angles are supplementary. The measure of the larger angle is five less than four times the measure of the smaller angle. Find the measures of both angles.

---

35. Two angles are complementary. The measure of the larger angle is twelve less than twice the measure of the smaller angle. Find the measures of both angles.

Solution

The measures are 34 degrees and 56 degrees.

---

36. Two angles are complementary. The measure of the larger angle is ten more than four times the measure of the smaller angle. Find the measures of both angles.

---

37. Wayne is hanging a string of lights 45 feet long around the three sides of his rectangular patio, which is adjacent to his house. The length of his patio, the side along the house, is five feet longer than twice its width. Find the length and width of the patio.

Solution

The width is 10 feet and the length is 25 feet.

---

38. Darrin is hanging 200 feet of Christmas garland on the three sides of fencing that enclose his rectangular front yard. The length, the side along the house, is five feet less than three times the width. Find the length and width of the fencing.

---

39. A frame around a rectangular family portrait has a perimeter of 60 inches. The length is fifteen less than twice the width. Find the length and width of the frame.

Solution

The width is 15 feet and the length is 15 feet.

---

40. The perimeter of a rectangular toddler play area is 100 feet. The length is ten more than three times the width. Find the length and width of the play area.

Exercises: Solve Uniform Motion Applications

Instructions: For questions 41-52, translate to a system of equations and solve.

41. Sarah left Minneapolis heading east on the interstate at a speed of 60 mph. Her sister followed her on the same route, leaving two hours later and driving at a rate of 70 mph. How long will it take for Sarah's sister to catch up to Sarah?

Solution

It took Sarah's sister 12 hours.

---

42. College roommates John and David were driving home to the same town for the holidays. John drove 55 mph, and David, who left an hour later, drove 60 mph. How long will it take for David to catch up to John?

---

43. At the end of spring break, Lucy left the beach and drove back towards home, driving at a rate of 40 mph. Lucy's friend left the beach for home 30 minutes (half an hour) later, and drove 50 mph. How long did it take Lucy's friend to catch up to Lucy?

Solution

It took Lucy's friend 2 hours.

---

44. Felecia left her home to visit her daughter driving 45 mph. Her husband waited for the dog sitter to arrive and left home twenty minutes ($\frac{1}{3}$ hour) later. He drove 55 mph to catch up to Felecia. How long before he reaches her?

---

45. The Jones family took a 12 mile canoe ride down the Indian River in two hours. After lunch, the return trip back up the river took three hours. Find the rate of the canoe in still water and the rate of the current.

Solution

The canoe rate is 5 mph and the current rate is 1 mph.

---

46. A motor boat travels 60 miles down a river in three hours but takes five hours to return upstream. Find the rate of the boat in still water and the rate of the current.

---

47. A motor boat travelled 18 miles down a river in two hours but going back upstream, it took 4.5 hours due to the current. Find the rate of the motor boat in still water and the rate of the current. (Round to the nearest hundredth.).

Solution

The boat rate is 6.5 mph and the current rate is 2.5 mph.

48. A river cruise boat sailed 80 miles down the Mississippi River for four hours. It took five hours to return. Find the rate of the cruise boat in still water and the rate of the current. (Round to the nearest hundredth.).

---

49. A small jet can fly $1,072$ miles in 4 hours with a tailwind but only 848 miles in 4 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Solution

The jet rate is 240 mph and the wind speed is 28 mph.

---

50. A small jet can fly $1,435$ miles in 5 hours with a tailwind but only $1,215$ miles in 5 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

51. A commercial jet can fly 868 miles in 2 hours with a tailwind but only 792 miles in 2 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Solution

The jet rate is 415 mph and the wind speed is 19 mph.

---

52. A commercial jet can fly $1{,}320$ miles in 3 hours with a tailwind but only $1{,}170$ miles in 3 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Exercises: Everyday Math

Instructions: For questions 53-54, answer the given everyday math word problems.

53. At a school concert, 425 tickets were sold. Student tickets cost $\$5$ each and adult tickets cost $\$8$ each. The total receipts for the concert were $\$2{,}851$.

Solve the system $\begin{cases} s+a=425 \\ 5s+8a=2{,}851 \end{cases}$ to find s, the number of student tickets and a, the number of adult tickets.

Solution

$s=183, a=242$

---

54. The first graders at one school went on a field trip to the zoo. The total number of children and adults who went on the field trip was 115. The number of adults was $\frac{1}{4}$ the number of children.

Solve the system $\begin{cases} c+a=115 \\ a=\frac{1}{4}c \end{cases}$ to find c, the number of children and a, the number of adults.

Exercises: Writing Exercises

Instructions: For questions 55-56, answer the given writing exercises.

55. Write an application problem similar to (Example 4.4.3) using the ages of two of your friends or family members. Then translate to a system of equations and solve it.

Solution

Answers will vary.

---

56. Write a uniform motion problem similar to (Example 4.4.8) that relates to where you live with your friends or family members. Then translate to a system of equations and solve it.

4.5 SOLVE MIXTURE APPLICATIONS WITH SYSTEMS OF EQUATIONS

Learning Objectives

By the end of this section, you will be able to:

- Solve mixture applications
- Solve interest applications

Try It

Before you get started, take this readiness quiz:

1) Multiply $4.025(1,562)$.
2) Write 8.2% as a decimal.
3) Earl's dinner bill came to $\$32.50$ and he wanted to leave an 18% tip. How much should the tip be?

Solve Mixture Applications

When we solved a mixture of applications with coins and tickets earlier, we started by creating a table so we could organize the information. For a coin example with nickels and dimes, the table looked like this:

Figure 4.5.1

Type	Number x Value ($) = Total Value ($)		
Nickels		0.05	
Dimes		0.10	

Using one variable meant that we had to relate the number of nickels and the number of dimes. We had to decide if we were going to let n be the number of nickels and then write the number of dimes in terms of n, or if we would let d be the number of dimes and write the number of nickels in terms of d.

Now that we know how to solve systems of equations with two variables, we'll just let n be the number of nickels and d be the number of dimes. We'll write one equation based on the total value column like we did before, and the other equation will come from the number column.

For the first example, we'll do a ticket problem where the ticket prices are in whole dollars, so we won't need to use decimals just yet.

Example 1

Translate to a system of equations and solve:

The box office at a movie theatre sold 147 tickets for the evening show, and receipts totalled $\$1,302$. How many $\$11$ adult and how many $\$8$ child tickets were sold?

Solution

***Step 1:* Read the problem.**

We will create a table to organize the information.

***Step 2:* Identify what we are looking for.**

We are looking for the number of adult tickets and the number of child tickets sold.

***Step 3:* Name what we are looking for.**

Let a $=$ the number of adult tickets.
Let c $=$ the number of child tickets

A table will help us organize the data. We have two types of tickets: adult and child.

Write a and c for the number of tickets.

Write the total number of tickets sold at the bottom of the Number column.

Altogether 147 were sold.

Write the value of each type of ticket in the Value column.

The value of each adult ticket is $\$11$. The value of each child ticket is $\$8$.

The number of times the value gives the total value, so the total value of adult tickets is $a \cdot 11 = 11a$, and the total value of child tickets is $c \cdot 8 = 8c$.

Type	Number $\times$	Value (\$) =	Total Value (\$)
Adult	a	11	$11a$
Child	c	8	$8c$
	147		1302

Altogether the total value of the tickets was $\$1,302$.

Fill in the Total Value column.

Step 4: **Translate into a system of equations.**

The Number column and the Total Value column give us the system of equations. We will use the elimination method to solve this system.

$$\begin{cases} a + c = 147 \\ 11a + 8c = 1302 \end{cases}$$

Multiply the first equation by -8.

$$\begin{cases} -8(a + c) = -8(147) \\ 11a + 8c = 1302 \end{cases}$$

Simplify and add, then solve for a.

$$\frac{\begin{cases} -8a + 8c = -1176 \\ 11a + 8c = 1302 \end{cases}}{3a = 126}$$

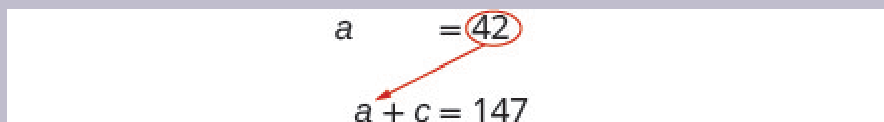


Figure 4.5.2

Substitute $a = 42$ into the first equation, then solve for c.

$$\begin{aligned} 42 + c &= 147 \\ c &= 105 \end{aligned}$$

Step 5: **Check the answer in the problem.**

42 adult tickets at $\$11$ per ticket makes $\$462$.

105 child tickets at $\$8$ per ticket makes $\$840$.

The total receipts are $\$1,302 \checkmark$.

Step 6: **Answer the question.**

The movie theatre sold 42 adult tickets and 105 child tickets.

Try It

4) *Translate to a system of equations and solve:* The ticket office at the zoo sold 553 tickets in one day. The receipts totalled $3,936. How many $9 adult tickets and how many $6 child tickets were sold?

Solution
There were 206 adult tickets sold and 347 children tickets sold.

5) *Translate to a system of equations and solve:* A science centre sold 1,363 tickets on a busy weekend. The receipts totalled $12,146. How many $12 adult tickets and how many $7 child tickets were sold?

Solution
There were 521 adult tickets sold and 842 children tickets sold.

In Example 2 we'll solve a coin problem. Now that we know how to work with systems of two variables, naming the variables in the 'number' column will be easy.

Example 2

Translate to a system of equations and solve:

Priam has a collection of nickels and quarters, with a total value of $\$7.30$. The number of nickels is six less than three times the number of quarters. How many nickels and how many quarters does he have?

Solution
***Step 1:* Read the problem.**

We will create a table to organize the information.

Step 2: **Identify what we are looking for.**

We are looking for the number of nickels and the number of quarters.

Step 3: **Name what we are looking for.**

Let $n =$ be the number of nickels.
Let $q =$ be the number of quarters

A table will help us organize the data. We have two types of coins, nickels and quarters.

Write n and q for the number of each type of coin.

Fill in the Value column with the value of each type of coin.

The value of each nickel is $\$0.05$.

The value of each quarter is $\$0.25$.

The number of times the value gives the total value, so, the total value of the nickels is $n(0.05) = 0.05n$ and the total value of quarters is $q(0.25) = 0.25q$. Altogether the total value of the coins is $\$7.30$.

Type	Number $\times$	Value ($) =	Total Value ($)
Nickels	n	0.05	$0.05n$
Quarters	q	0.25	$0.25q$
			7.30

Step 4: **Translate into a system of equations**.

The Total Value column gives one equation.

$$0.05n + 0.25q = 7.30$$

We also know the number of nickels is six less than three times the number of quarters. Translate to get the second equation.

$$n = 3q - 6$$

Now we have the system to solve.

$$\begin{cases} 0.05n + 0.25q = 7.30 \\ n = 3q - 6 \end{cases}$$

Step 5: **Solve the system of equations**

We will use the substitution method. Substitute $n = 3q - 6$ into the first equation. Simplify and solve for q.

$$\begin{aligned} 0.05n + 0.25q &= 7.30 \\ 0.05(3q-6) + 0.25q &= 7.3 \\ 0.15q - 0.3 + 0.25q &= 7.3 \\ 0.4q - 0.3 &= 7.3 \\ 0.4q &= 7.6 \\ q &= 19 \end{aligned}$$

To find the number of nickels, substitute $q = 19$ into the second equation.

$$\begin{aligned} n &= 3q - 6 \\ n &= 3 \times 19 - 6 \\ n &= 51 \end{aligned}$$

Step 6: **Check the answer to the problem.**

19 quarters at $\$0.25 = \$4.75$

51 nickels at $\$0.05 = \$2.55$

$$\begin{aligned} \text{Total} &= \$7.30 \\ 3.19 - 16 &= \$51 \checkmark \end{aligned}$$

Step 7: **Answer the question.**

Priam has 19 quarters and 51 nickels.

Try It

6) *Translate to a system of equations and solve:* Matilda has a handful of quarters and dimes, with a total value of $\$8.55$. The number of quarters is 3 more than twice the number of dimes. How many dimes and how many quarters does she have?

Solution

Matilda has 13 dimes and 29 quarters.

7) *Translate to a system of equations and solve:* Juan has a pocketful of nickels and dimes. The total value of the coins is $\$8.10$. The number of dimes is 9 less than twice the number of nickels. How many nickels and how many dimes does Juan have?

Solution

Juan has 36 nickels and 63 dimes.

Some mixture applications involve combining foods or drinks. Example situations might include combining raisins and nuts to make a trail mix or using two types of coffee beans to make a blend.

Example 3

Translate to a system of equations and solve:

Carson wants to make 20 pounds of trail mix using nuts and chocolate chips. His budget requires that the trail mix costs him $\$7.60$ per pound. Nuts cost $\$9.00$ per pound and chocolate chips cost $\$2.00$ per pound. How many pounds of nuts and how many pounds of chocolate chips should he use?

Solution

***Step 1:* Read the problem.**

We will create a table to organize the information.

***Step 2:* Identify what we are looking for.**

We are looking for the number of pounds of nuts and the number of pounds of chocolate chips.

***Step 3:* Name what we are looking for.**

Let $n =$ be the number of pounds of nuts.
Let $c =$ be the number of pounds of chips

Carson will mix nuts and chocolate chips to get trail mix. Write in n and c for the number of pounds of nuts and chocolate chips.

There will be 20 pounds of trail mix. Put the price per pound of each item in the Value column. Fill in the last column using Number $\times$ Value = Total Value

Type	Number of Pounds × Value ($) = Total Value ($)		
Nuts	n	9.00	$9n$
Chocolate Chips	c	2.00	$2c$
Trail Mix	20	7.60	$7.60(20) = 152$

***Step 4:* Translate into a system of equations. We get the equations from the Number and Total Value columns.**

$$\begin{cases} n + c = 20 \\ 9n + 2c = 152 \end{cases}$$

***Step 5:* Solve the system of equations. We will use elimination to solve the system.**

Multiply the first equation by -2 to eliminate c.

$$\begin{cases} -2(n + c) = -2(20) \\ 9n + 2c = 152 \end{cases}$$

Simplify and add. Solve for n.

$$\begin{array}{r} \begin{cases} -2n - 2c = -40 \\ 9n + 2c = 152 \end{cases} \\ \hline 7n = 112 \end{array}$$

$$n = 16$$

To find the number of pounds of chocolate chips, substitute $n = 16$ into the first equation, then solve for c.

$$\begin{aligned} n + c &= 20 \\ 16 + c &= 20 \\ c &= 4 \end{aligned}$$

***Step 6:* Check the answer to the problem.**

$$\begin{aligned} 16+4&=20\checkmark \\ 9.16+2.4&=152\checkmark \end{aligned}$$

***Step 7:* Answer the question.**

Carson should mix 16 pounds of nuts with 4 pounds of chocolate chips to create the trail mix.

Try It

8) *Translate to a system of equations and solve:* Greta wants to make 5 pounds of a nut mix using peanuts and cashews. Her budget requires the mixture to cost her $\$6$ per pound. Peanuts are $\$4$ per pound and cashews are $\$9$ per pound. How many pounds of peanuts and how many pounds of cashews should she use?

Solution

Greta should use 3 pounds of peanuts and 2 pounds of cashews.

9) *Translate to a system of equations and solve:* Sammy has most of the ingredients he needs to make a large batch of chili. The only items he lacks are beans and ground beef. He needs a total of 20 pounds combined of beans and ground beef and has a budget of $\$3$ per pound. The

price of beans is $\$1$ per pound and the price of ground beef is $\$5$ per pound. How many pounds of beans and how many pounds of ground beef should he purchase?

Solution

Sammy should purchase 10 pounds of beans and 10 pounds of ground beef.

Another application of mixture problems relates to concentrated cleaning supplies, other chemicals, and mixed drinks. The concentration is given as a percent. For example, a 20% concentrated household cleanser means that 20 of the total amount is cleanser, and the rest is water. To make 35 ounces of a 20 concentration, you mix 7 ounces (20 of 35) of the cleanser with 28 ounces of water.

For these kinds of mixture problems, we'll use percent instead of value for one of the columns in our table.

Example 4

Translate to a system of equations and solve: Sasheena is a lab assistant at her community college. She needs to make 200 millilitres of a 40 solution of sulphuric acid for a lab experiment. The lab has only 25 and 50 solutions in the storeroom. How much should she mix of the 25 and the 50 solutions to make the 40 solution?

Solution

***Step 1:* Read the problem.**

A figure may help us visualize the situation, then we will create a table to organize the information.

Sasheena must mix some of the 25% solution and some of the 50% solution to get 200 ml of the 40% solution.

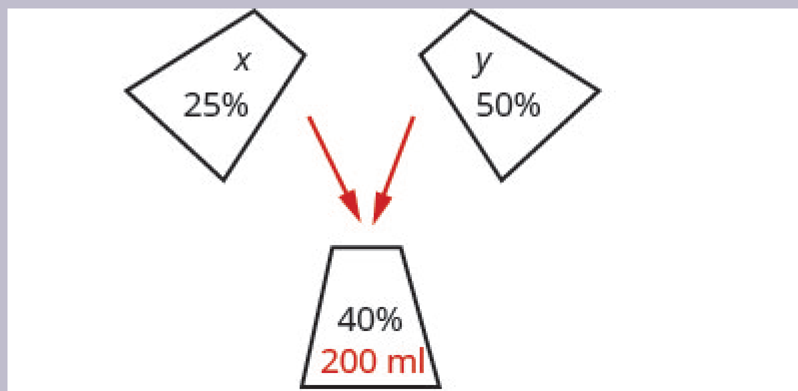


Figure 4.5.3

Step 2: **Identify what we are looking for.**

We are looking for how much of each solution she needs.

Step 3: **Name what we are looking for.**

Let $x =$ number of ml of 25% solution.
Let $y =$ number of ml of 50% solution.

A table will help us organize the data.

She will mix x ml of 25 with y ml of 50 to get 200 ml of 40 solution.

We write the percentages as decimals in the chart.

We multiply the number of units times the concentration to get the total amount of sulphuric acid in each solution.

Type	Number of Units	× Concentration % =	Amount
25%	x	0.25	$0.25x$
50%	y	0.50	$0.50y$
40%	200	0.40	$0.40(200)$

Step 4: **Translate into a system of equations. We get the equations from the Number column and the Amount column.**

Now we have the system.

$$\begin{cases} x + y = 200 \\ 0.25x + 0.50y = 0.40(200) \end{cases}$$

Step 5: **Solve the system of equations. We will solve the system by elimination. Multiply the first equation by -0.5 to eliminate y.**

$$\begin{cases} -0.5(x + y) = -0.5(200) \\ 0.25x + 0.50y = 0.40(200) \end{cases}$$

Simplify and add to solve for x.

$$\begin{array}{r} \begin{cases} -0.5x - 0.5y = -100 \\ 0.25x + 0.50y = 0.40(200) \end{cases} \\ \hline -0.25x = -20 \\ x = 80 \end{array}$$

To solve for y, substitute $x = 80$ into the first equation.

$$\begin{aligned} x + y &= 200 \\ 80 + y &= 200 \\ y &= 120 \end{aligned}$$

Step 6: **Check the answer to the problem.**

$$\begin{aligned} 80 + 120 = 120 \checkmark \\ 0.25(80) + 0.50(120) = 80 \checkmark \end{aligned}$$

Step 7: **Answer the question.**

Sasheena should mix 80 ml of the 25 solution with 120 ml of the 50 solution to get the 200 ml of the 40 solution.

Try It

10) *Translate to a system of equations and solve:* LeBron needs 150 millilitres of a 30 solution of sulphuric acid for a lab experiment but only has access to a 25 and a 50 solution. How much of the 25 and how much of the 50 solution should he mix to make the 30 solution?

Solution

LeBron needs 120 ml of the 25 solution and 30 ml of the 50 solution.

11) *Translate to a system of equations and solve:* Anatole needs to make 250 millilitres of a 25 solution of hydrochloric acid for a lab experiment. The lab only has a 10 solution and a 40 solution in the storeroom. How much of the 10 and how much of the 40 solutions should he mix to make the 25 solution?

Solution

Anatole should mix 125 ml of the 10 solution and 125 ml of the 40 solution.

Solve Interest Applications

The formula to model interest applications is $I = Prt$. Interest, I, is the product of the principal, P, the rate, r, and the time, t. In our work here, we will calculate the interest earned in one year, so t will be 1.

We modify the column titles in the mixture table to show the interest formula, as you'll see in Example 5.

Example 5

Translate to a system of equations and solve:

Adnan has $\$40,000$ to invest and hopes to earn 7.1 interest per year. He will put some of the money into a stock fund that earns 8 per year and the rest into bonds that earn 3 per year. How much money should he put into each fund?

Solution

Step 1: **Read the problem.**

A chart will help us organize the information.

Step 2: **Identify what we are looking for.**

We are looking for the amount to invest in each fund.

Step 3: **Name what we are looking for.**

Let s $=$ the amount invested in stocks.

Let b $=$ the amount invested in bonds.

Write the interest rate as a decimal for each fund. Multiply: Principal $\times$ Rate $\times$ Time to get the Interest.

Account	Principal $\times$	Rate $\times$	Time =	Interest
Stock Fund	s	0.08	1	$0.08s$
Bonds	b	0.03	1	$0.03b$
Total	$40,000$	0.071		$0.071(40,000)$

Step 4: **Translate into a system of equations.**

We get our system of equations from the Principal column and the Interest column.

$$\begin{cases} s + b = 40,000 \\ 0.08s + 0.03b = 0.071(40,000) \end{cases}$$

Step 5: **Solve the system of equations.**

Solve by elimination. Multiply the top equation by -0.03.

$$\begin{cases} -0.03(s + b) = -0.03(40,000) \\ 0.08s + 0.03b = 2,840 \end{cases}$$

$$s = 32,800$$

Simplify and add to solve for s.

$$\begin{cases} -0.03s - 0.03b = -1,200 \\ 0.08s + 0.03b = 2,840 \end{cases}$$
$$0.05s = 1,640$$

To find b, substitute $s = 32,800$ into the first equation.

$$\begin{aligned} s + b &= 40,000 \\ 32{,}800 + b &= 40,000 \\ b &= 7,200 \end{aligned}$$

Step 6: **Check the answer to the problem.**

We leave the check to you.

Step 7: **Answer the question.**

Adnan should invest $\$32,800$ in stock and $\$7,200$ in bonds.

Did you notice that the Principal column represents the total amount of money invested while the Interest column represents only the interest earned? Likewise, the first equation in our system, $s + b = 40,000$, represents the total amount of money invested and the second equation, $0.08s + 0.03b = 0.071(40,000)$, represents the interest earned.

Try It

12) *Translate to a system of equations and solve:* Leon had $\$50,000$ to invest and hopes to earn 6.2 interest per year. He will put some of the money into a stock fund that earns 7 per year and the rest in to a savings account that earns 2 per year. How much money should he put into each fund?

Solution

Leon should put $\$42,000$ in the stock fund and $\$8000$ in the savings account.

13) *Translate to a system of equations and solve:* Julius invested $\$7,000$ into two stock investments. One stock paid 11 interest and the other stock paid 13 interest. He earned 12.5 interest on the total investment. How much money did he put in each stock?

Solution

Julius invested $\$1,750$ at 11 and $\$5,250$ at 13.

Example 6

Translate to a system of equations and solve: Rosie owes $\$21,540$ on her two student loans. The interest rate on her bank loan is 10.5 and the interest rate on the federal loan is 5.9. The total amount of interest she paid last year was $\$1,669.68$. What was the principal for each loan?

Solution

Step 1: **Read the problem.**

A chart will help us organize the information.

Step 2: **Identify what we are looking for.**

We are looking for the principal of each loan.

Step 3: **Name what we are looking for.**

Let $b =$ be the principal for the bank loan.

Let $f =$ be the principal on the federal loan.

The total loans are $\$21,540$.

Record the interest rates as decimals in the chart.

Account	Principal	× Rate	× Time	= Interest
Bank	b	0.105	1	$0.105b$
Federal	f	0.059	1	$0.059f$
Total	$21,540$			$1,669.68$

Multiply using the formula $I = Prt$ to get the Interest.

Step 4: **Translate into a system of equations.**

The system of equations comes from the Principal column and the Interest column.

$$\begin{cases} b + f = 21,540 \\ 0.105b + 0.059f = 1,669.68 \end{cases}$$

Step 5: **Solve the system of equations.**

We will use substitution to solve. Solve the first equation for b.

$$\begin{aligned} b + f &= 21,540 \\ b &= -f + 21,540 \end{aligned}$$

Substitute $b = -f + 21,540$ into the second equation.

$$\begin{aligned} 0.105b + 0.59f &= 1,669.68 \\ 0.105(-f + 21,540) + 0.59f &= 1,669.68 \end{aligned}$$

Simplify and solve for f.

$$\begin{aligned} -0.105f + 2,261.70 + 0.059f &= 1,669.68 \\ -0.46f + 2,261.70 &= 1,669.68 \\ -0.046f &= -592.02 \\ f &= 12,870 \end{aligned}$$

To find *b*, substitute $f = 12,870$ into the first equation.

$$\begin{aligned} b + f &= 21,540 \\ b + 12,870 &= 21,540 \\ b &= 8,670 \end{aligned}$$

Step 6: **Check the answer to the problem.**

We leave the check to you.

***Step 7:* Answer the question.**

The principal of the bank loan is $\$8670$ and the principal for the federal loan is $\$12,870.$

Try It

14) *Translate to a system of equations and solve:* Laura owes $\$18,000$ on her student loans. The interest rate on a bank loan is 2.5 and the interest rate on a federal loan is 6.9. The total amount of interest she paid last year was $\$1,066$. What was the principal for each loan?

Solution
The principal amount for the bank loan was $\$4,000$. The principal amount for the federal loan was $\$14,000$.

15) *Translate to a system of equations and solve:* Jill's Sandwich Shoppe owes $\$65,200$ on two business loans, one at 4.5 interest and the other at 7.2 interest. The total amount of interest owed last year was $\$3,582$. What was the principal for each loan?

Solution
The principal amount for was $\$41,200$ at 4.5. The principal amount was, $\$24,000$ at 7.2.

Access these online resources for additional instruction and practice with solving application problems with systems of linear equations.

- Cost and Mixture Word Problems
- Mixture Problems

Key Concepts

Table for coin and mixture applications

Type	Number	• Value($)	= Total Value($)
Total			

Figure 4.5.4

Table for concentration applications

Type	Number of units	• Concentration %	= Amount
Total			

Figure 4.5.5

Table for interest applications

Account	Principal •	Rate •	Time	= Interest
			1	
			1	
Total				

Figure 4.5.6

Exercises: Solve Mixture Applications

Instructions: For questions 1-24, translate to a system of equations and solve.

1. Tickets to a Broadway show cost $\$35$ for adults and $\$15$ for children. The total receipts for 1650 tickets at one performance were $\$47{,}150$. How many adult and how many child tickets were sold?

Solution

There 1120 adult tickets and 530 child tickets sold.

---

2. Tickets for a show are $\$70$ for adults and $\$50$ for children. One evening performance had a total of 300 tickets sold and the receipts totalled $\$17{,}200$. How many adult and how many child tickets were sold?

---

3. Tickets for a train cost $\$10$ for children and $\$22$ for adults. Josie paid $\$1{,}200$ for a

total of 72 tickets. How many children's tickets and how many adult tickets did Josie buy?

Solution

Josie bought 40 adult tickets and 32 children tickets.

---

4. Tickets for a baseball game are $\$69$ for Main Level seats and $\$39$ for Terrace Level seats. A group of sixteen friends went to the game and spent a total of $\$804$ for the tickets. How many of Main Level and how many Terrace Level tickets did they buy?

---

5. Tickets for a dance recital cost $\$15$ for adults and $\$7$ for children. The dance company sold 253 tickets and the total receipts were $\$2,771$. How many adult tickets and how many child tickets were sold?

Solution

There were 125 adult tickets and 128 children tickets sold.

---

6. Tickets for the community fair cost $\$12$ for adults and $\$5$ dollars for children. On the first day of the fair, 312 tickets were sold for a total of $\$2,204$. How many adult tickets and how many child tickets were sold?

---

7. Brandon has a cup of quarters and dimes with a total value of $\$3.80$. The number of quarters is four less than twice the number of dimes. How many quarters and how many dimes does Brandon have?

Solution

Brandon has 12 quarters and 8 dimes.

---

8. Sherri saves nickels and dimes in a coin purse for her daughter. The total value of the coins in the purse is $\$0.95$. The number of nickels is two less than five times the number of dimes. How many nickels and how many dimes are in the coin purse?

---

9. Peter has been saving his loose change for several days. When he counted his quarters and dimes, he found they had a total value $\$13.10$. The number of quarters was fifteen more than three times the number of dimes. How many quarters and how many dimes did Peter have?

Solution

Peter had 11 dimes and 48 quarters.

---

10. Lucinda had a pocketful of dimes and quarters with a value of ? $\$6.20$. The number of dimes is eighteen more than three times the number of quarters. How many dimes and how many quarters does Lucinda have?

---

11. A cashier has 30 bills, all of which are $\$10$ or $\$20$ bills. The total value of the money is $\$460$. How many of each type of bill does the cashier have?

Solution

The cashier has fourteen $\$10$ bills and sixteen $\$20$ bills.

---

12. A cashier has 54 bills, all of which are $\$10$ or $\$20$ bills. The total value of the money is $\$910$. How many of each type of bill does the cashier have?

---

13. Marissa wants to blend candy selling for $\$1.80$ per pound with candy costing $\$1.20$ per pound to get a mixture that costs her $\$1.40$ per pound to make. She wants to make 90 pounds of the candy blend. How many pounds of each type of candy should she use?

Solution

Marissa should use 60 pounds of the $\$1.20/\text{lb}$ candy and 30 pounds of the $\$1.80/\text{lb}$ candy.

---

14. How many pounds of nuts selling for $\$6$ per pound and raisins selling for $\$3$ per pound should Kurt combine to obtain 120 pounds of trail mix that cost him $\$5$ per pound?

---

15. Hannah has to make twenty-five gallons of punch for a potluck. The punch is made of soda and fruit drink. The cost of the soda is $\$1.79$ per gallon and the cost of the fruit drink is $\$2.49$ per gallon. Hannah's budget requires that the punch cost $\$2.21$ per gallon. How many gallons of soda and how many gallons of fruit drink does she need?

Solution

Hannah needs 10 gallons of soda and 15 gallons of fruit drink.

---

16. Joseph would like to make 12 pounds of a coffee blend at a cost of $\$6.25$ per pound. He blends Ground Chicory at $\$4.40$ a pound with Jamaican Blue Mountain at $\$8.84$ per pound. How much of each type of coffee should he use?

---

17. Julia and her husband own a coffee shop. They experimented with mixing a City Roast Columbian coffee that cost $\$7.80$ per pound with French Roast Columbian coffee that cost $\$8.10$ per pound to make a 20 pound blend. Their blend should cost them $\$7.92$ per pound. How much of each type of coffee should they buy?

Solution

Julia and her husband should buy 12 pounds of City Roast Columbian coffee and 8 pounds of French Roast Columbian coffee.

---

18. Melody wants to sell bags of mixed candy at her lemonade stand. She will mix chocolate pieces that cost $\$4.89$ per bag with peanut butter pieces that cost $\$3.79$ per bag to get a total of twenty-five bags of mixed candy. Melody wants the bags of mixed candy to cost her $\$4.23$ a bag to make. How many bags of chocolate pieces and how many bags of peanut butter pieces should she use?

---

19. Jotham needs 70 liters of a 50% alcohol solution. He has a 30% and an 80% solution available. How many liters of the 30% and how many liters of the 80% solutions should he mix to make the 50% solution?

Solution

Jotham should mix 42 liters of the 30% solution and 28 liters of the 80% solution.

---

20. Joy is preparing 15 liters of a 25% saline solution. She only has 40% and 10% solution in her lab. How many liters of the 40% and how many liters of the 10% should she mix to make the 25% solution?

---

21. A scientist needs 65 liters of a 15% alcohol solution. She has available a 25% and a 12% solution. How many liters of the 25% and how many liters of the 12% solutions should she mix to make the 15% solution?

Solution

The scientist should mix 15 liters of the 25% solution and 50 liters of the 12% solution.

---

22. A scientist needs 120 liters of a 20% acid solution for an experiment. The lab has available a 25% and a 10% solution. How many liters of the 25% and how many liters of the 10% solutions should the scientist mix to make the 20% solution?

---

23. A 40% antifreeze solution is to be mixed with a 70% antifreeze solution to get 240 liters of a 50% solution. How many liters of the 40% and how many liters of the 70% solutions will be used?

Solution

160 liters of the 40% solution and 80 liters of the 70% solution will be used.

24. A 90% antifreeze solution is to be mixed with a 75% antifreeze solution to get 360 liters of a 85% solution. How many liters of the 90% and how many liters of the 75% solutions will be used?

Exercises: Solve Interest Applications

Instructions: For questions 25-32, translate to a system of equations and solve.

25. Hattie had $\$3,000$ to invest and wants to earn 10.6% interest per year. She will put some of the money into an account that earns 12% per year and the rest into an account that earns 10% per year. How much money should she put into each account?

Solution

Hattie should invest $\$900$ at 12% and $\$2,100$ at 10%.

26. Carol invested $\$2,560$ into two accounts. One account paid 8% interest and the other paid 6% interest. She earned 7.25% interest on the total investment. How much money did she put in each account?

27. Sam invested $\$48,000$, some at 6% interest and the rest at 10%. How much did he invest at each rate if he received $\$4,000$ in interest in one year?

Solution

Sam invested $\$28,000$ at 10% and $\$20,000$ at 6%.

---

28. Arnold invested $\$64,000$, some at 5.5% interest and the rest at 9%. How much did he invest at each rate if he received $\$4,500$ in interest in one year?

---

29. After four years in college, Josie owes $\$65,800$ in student loans. The interest rate on the federal loans is 4.5% and the rate on the private bank loans is 2%. The total interest she owed for one year was $\$2,878.50$. What is the amount of each loan?

Solution

The federal loan is $\$62,500$ and the bank loan is $\$3,300$.

---

30. Mark wants to invest $\$10,000$ to pay for his daughter's wedding next year. He will invest some of the money in a short term CD that pays 12% interest and the rest in a money market savings account that pays 5% interest. How much should he invest at each rate if he wants to earn $\$1,095$ in interest in one year?

---

31. A trust fund worth $\$25,000$ is invested in two different portfolios. This year, one portfolio is expected to earn 5.25% interest and the other is expected to earn 4%. Plans are for the total interest on the fund to be $\$1,150$ in one year. How much money should be invested at each rate?

Solution

$\$12,000$ should be invested at 5.25% and $\$13,000$ should be invested at 4%.

---

32. A business has two loans totalling $\$85,000$. One loan has a rate of 6% and the other has a rate of 4.5%. This year, the business expects to pay $\$4,650$ in interest on the two loans. How much is each loan?

Exercises: Everyday Math

Instructions: For questions 33-34, translate to a system of equations and solve.

33. Laurie was completing the treasurer's report for her son's Boy Scout troop at the end of the school year. She didn't remember how many boys had paid the $\$15$ full-year registration fee and how many had paid the $\$10$ partial-year fee. She knew that the number of boys who paid for a full-year was ten more than the number who paid for a partial-year. If $\$250$ was collected for all the registrations, how many boys had paid the full-year fee and how many had paid the partial-year fee?

Solution

14 boys paid the full-year fee. 4 boys paid the partial-year fee,

---

34. As the treasurer of her daughter's Girl Scout troop, Laney collected money for some girls and adults to go to a three-day camp. Each girl paid $\$75$ and each adult paid $\$30$. The total amount of money collected for camp was $\$765$. If the number of girls is three times the number of adults, how many girls and how many adults paid for camp?

Exercises: Writing Exercises

Instructions: For questions 35-36, answer the given writing exercises.

35. Take a handful of two types of coins, and write a problem similar to (Example 4.5.2) relating the total number of coins and their total value. Set up a system of equations to describe your situation and then solve it.

Solution

Answers will vary.

---

36. In (Example 4.5.6) we solved the system of equations

$$\begin{cases} b + f = 21{,}540 \\ 0.105b + 0.059f = 1669.68 \end{cases}$$

by substitution. Would you have used substitution or elimination to solve this system? Why?

PART V
UNIT 5: MEDICAL MEASUREMENT

5.1 COMMON UNITS IN NURSING

Learning Objectives

By the end of this chapter, learners will be able to:

- identify common units of measurement for amount, mass, and liquid volume in the metric system
- identify the correct abbreviations for common units of measure.

Identify Common Units of Measurement for Amount, Mass, and Liquid Volume in the Metric System

Types of Units

You have likely learned about the metric system (also known as The International System of Units) at some point in your past education, but perhaps some of the details are a little fuzzy. There are seven basic types of measure, which relate to quantities of time, length, mass, electric current, thermodynamic temperature, amount of substance, and luminous intensity (National Institute of Standards and Technology, 2020). However, we will focus on only four types, as they are the measurements most commonly used in nursing. The mole, grams, metres, and litres. Respectively, these units measure the amount of a substance, mass (weight), length, and volume (capacity).

Difference Between Volume and Capacity

You might think of litres as a way to measure the volume of a liquid, but it's not the most precise definition. Volume is a measure of how much space an object takes up, always measured in cubic units, such as cubic centimetres. Can you remember calculating the volume of objects in high school geometry? Multiplying the height, width, and length of an object would give the volume of the object. Capacity is the measurement of how much of a substance can be inside an object, which could be matter existing in any state. Although there are many units to measure capacity, litre is the most commonly used unit for measuring liquid matter. For use in nursing work, litre is commonly referred to as a measure of volume.

Identify the Correct Abbreviations for Common Units of Measure

When units of measure go up and down in size, they do so by a power of ten. A prefix is added to the base unit to indicate the size of the unit. For instance, a unit ten times larger than a gram is a decagram. You will not often see all of the possible units of measure being used in nursing work, so for this text, we will focus on the units you will use most often.

Table 5.1 Common Base Units

Metric Prefix	Symbol	Power of 10	Meaning	Multiply By
tera	T	10^{12}	one trillion	1,000,000,000,000
giga	G	10^{9}	one billion	1,000,000,000
mega	M	10^{6}	one million	1,000,000
kilo	k	10^{3}	one thousand	1,000
hecto	h	10^{2}	one hundred	100
deca	da	10^{1}	ten	10
deci	d	10^{-1}	one tenth	1/10
centi	c	10^{-2}	one hundredth	1/100
milli	m	10^{-3}	one thousandth	1/1,000
micro	µ	10^{-6}	one millionth	1/1,000,000
nano	n	10^{-9}	one billionth	1/1,000,000,000
pico	p	10^{-12}	one trillionth	1/1,000,000,000,000

Commonly Used Units

The following table outlines units commonly used in medication orders and medication administration in Canada. You should understand what these units measure and how to convert from one unit of measure to another. Occasionally, you will see measurements given using the US customary system of measure, derived from the British imperial system. You may need to convert between units of the metric and imperial systems of measurement. Refer to the conversion table in this textbook for commonly used conversion factors.

Table 5.2 Common Units

Quantity	Abbreviation	Measure
Amount	i.u.	**international unit**
Amount	mEq	**milliequivalent**
Amount	mmol	**millimole**
Volume	mL	**millilitre**
Volume	L	**litre**
Mass	mcg	**microgram**
Mass	mg	**milligram**
Mass	g	**gram**
Mass	kg	**kilogram**
Length	cm	**centimetre**
Length	m	**metre**

Defining Units

Within the table above, each unit of measure is defined in the glossary of this textbook. Click on the word to view the definition if you cannot define the unit of measure in your own words. If using this book in another format, you can find the glossary at the back of the book.

National Institute of Standards and Technology. (2020, January). *SI units.* https://www.nist.gov/si-redefinition/definitions-si-base-units

Exercises: Unit Abbreviations

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=76#h5p-2

5.2 CONVERTING UNITS FOR MEDICATION AMOUNTS

Learning Objectives

By the end of this chapter, learners will be able to:

- identify when units require conversion when comparing the medication order and medication supply
- convert between common units of measure.

Identify When Units Require conversion When Comparing the Medication Order and Medication Supply

When an order for medication is supplied in an amount with a different unit of measure than the order, you will need to convert units so they match to ensure you are giving the correct dose of medication. Not all orders will require unit conversion.

Example 1

Which of the following orders requires unit conversion before medication administration?

Order A:

- **Medication Order:** prednisone 25 mg PO once daily
- **Medication Supply:** prednisone 5 mg tablets

Order B:

- **Medication Order:** acetaminophen 1 g PO QID prn
- **Medication Supply:** acetaminophen 500 mg tablets

Answer:

Order B requires unit conversion as the order is given in grams and the supply is provided in milligrams. Order A and the supply are both provided in milligrams.

Convert Between Common Units of Measure

To convert from one unit of measure to another, you need to know how many of a particular unit is equal to a single unit of the other type of measure. You can refer to the conversion table for quick reference if you are unfamiliar with how many of one unit would be in another for the units commonly used in medication administration. These amounts are called **conversion factors**. You will then set up an algebraic equation to convert between units, with the conversion factor written as a fraction.

Let's say we need to give 0.5 grams (g) of medication and the supply is in milligrams (mg). How many mg are equal to 0.5 g?

$$? \text{ mg} = 0.5 \text{ g}$$

Start the equation with what you need to know, in this case, how many mg. We use "x"

to represent the unknown amount of mg. Then, we need to use the conversion factor of **1000 mg = 1 g.** When you set up the formula, put the type of units on top that matches the unit you are looking for. In this example, we are trying to find mg, so write in $\frac{1000 \text{ mg}}{1 \text{ g}}$. Finally, we multiply by the known amount. The formula would look like this:

$$x \text{ mg} = \frac{1000 \text{ mg}}{1 \text{ g}} \times 0.5 \text{ g}$$

To solve this equation, complete the calculation using the standard **order of operations**. Some people use the acronym **BEDMAS** to help them remember the order of operations: **B**rackets, **E**xponents, **D**ivision or **M**ultiplication, **A**ddition or **S**ubtraction.

You can always check to see if you are ending up with the correct units by canceling out units that match the numerator and denominator of the equation. In this case, grams in the numerator and denominator cancel out, leaving us with just an amount of mgs, which is exactly what we want!

$$x \text{ mg} = \frac{1000\text{mg}}{1\cancel{\text{g}}} \times 0.5 \cancel{\text{g}}$$

$$x = 500 \text{ mg}$$

Example 2

Sample Medication Order to Convert

Medication Order: Pulmicort 500 mcg twice a day via nebulizer

Medication Supply: Pulmicort 0.25 mg/mL nebule

You can see that the order is written as **mcg** and the supply is measured in **mg/mL.**

First, decide what type of unit you are converting to. This is what you will use to start the setup of your formula. In this example, we need to find out how many milligrams are in 500 micrograms because our supply is available in milligrams.

$$x \text{ mg} =$$

Second, start with what you know about conversion factor:

$$x \text{ mg} = \frac{1 \text{ mg}}{1000 \text{ mcg}}$$

Third, multiply by the amount you need to convert:

$$x \text{ mg} = \frac{1 \text{ mg}}{1000 \text{ mcg}} \times 500 \text{ mcg}$$

You can see the units of mcg cancel out as there is one in the numerator and the denominator:

$$x \text{ mg} = \frac{1 \text{ mg}}{1000 \cancel{\text{mcg}}} \times 500 \cancel{\text{mcg}}$$

Now, complete the calculation:

$$\frac{500}{1000} = 0.5 \text{ mg}$$

Example 3

How many milligrams of ciprofloxacin must be administered?

Medication Order: Ciprofloxacin 0.75 g PO once daily

Medication Supply: Ciprofloxacin 250 mg tablets

Answer:
Set up the formula. Start with what you need to know (x mg). Use the conversion factor, with the number on top in the same units (mg). Multiply by the amount in the order.

$$x \text{ mg} = \frac{1000 \text{ mg}}{1 \text{ g}} \times 0.75 \text{ g}$$

Cancel out units to ensure the formula is set up correctly.

$$x \text{ mg} = \frac{1000 \text{ mg}}{1 \cancel{\text{g}}} \times 0.75 \cancel{\text{g}}$$

Calculate.

$$1000 \text{ mg} \times 0.75 \text{ g} = 750 \text{ mg}$$

Exercises Part A: When to Convert

In the following exercises, identify if any units need to be converted (yes/no answer) and what unit to convert to.

1. A medication is ordered at a single dose of 500 mg. The capsules provided by the pharmacy are 250 mg each.
2. A medication is ordered at a single dose of 1 g. The tablets provided by the pharmacy are 500 mg each.

3. A medication is ordered at 0.15 mg BID. The tablets provided by the pharmacy are 0.75 mcg each.
4. A medication is ordered at 750 mg TID. The tablets provided by the pharmacy are 250 mg each.
5. A medication is ordered at 500 mcg BID. The tablets provided are 1 mg each.
6. A medication is ordered for a single dose of 500 mg at 1,000. The tablets provided are 1,000 mg each.
7. A medication is ordered at 1 g TID. The capsules provided are 500 mg each.
8. A medication is ordered at 500 mcg BID. The capsules provided by the pharmacy are 1 g each.
9. A medication is ordered at a single dose of 150 mg. The tablets provided are 750 mcg each.
10. A medication is ordered at 300 mcg QID. The capsules provided are 200 mcg each.

Odd Answers:

1) No.

3) Yes. Convert 0.15 mg to mcg.

5) Yes. Convert 500 mcg to mg.

7) Yes. Convert 1 g to mg.

9) Yes. Convert 150 mg to mcg.

Exercises Part B: Converting Mass

In each of the following practice questions, you will be given a medication order and a supply provided with an alternate unit of measurement. Convert the order amount so it matches the unit of measurement of the supply.

The answers to this problem set are visible when you click the drop-down button below. When you click the word “Answers” you will see the answers for all ten questions with the answer listed first, followed by how to set up the formula. It is worth mentioning this is not the only way to solve this type of problem, and it is acceptable to use another method to convert between units if you are comfortable with a different method.

Questions:

1. Order: acetaminophen 1 g PO QID
   Supply 500 mg tablets
2. Order: ipratropium 0.5 mg via nebulizer q6h
   Supply 250 mcg nebules
3. Order: lorazepam 500 mcg SL BID prn
   Supply 0.5 mg tablets
4. Order: cloxacillin 0.5 g PO q4h
   Supply 250 mg tablets
5. Order: digoxin 250 mcg PO once daily
   Supply 0.125 mg tablets
6. Order: azithromycin 2 g PO once daily
   Supply 500 mg tablets
7. Order: budesonide 0.4 mg inhaled BID
   Supply 200 mcg per metered dose
8. Order: Synthroid 0.15 mg PO once daily
   Supply 75 mcg tablets
9. Order: ciprofloxacin 0.75 g PO q12h
   Supply 500 mg tablets
10. Order: metronidazole 1.5 g PO
    Supply 500 mg tablets

Odd Answers:

1) 1,000 mg $x \text{ mg} = \frac{1000 \text{ mg}}{1 \text{ g}} \times 1 \text{ g}$

3) 0.5 mg $x \text{ mg} = \frac{1 \text{ mg}}{1000 \text{ mcg}} \times 500 \text{ mcg}$

5) 0.25 mg $x \text{ mg} = \frac{1 \text{ mg}}{1000 \text{ mcg}} \times 250 \text{ mcg}$

7) 400 mcg $x \text{ mcg} = \frac{1000 \text{ mcg}}{1 \text{ mg}} \times 0.4 \text{ mg}$

9) 750 mg $x \text{ mg} = \frac{1000 \text{ mg}}{1 \text{ g}} \times 0.75 \text{ g}$

Exercises Part C: Converting Mass

In each of the following practice questions, you will be given a weight that needs to be converted to an alternate unit of measure, which may be metric or imperial.

1. A baby weighs 2,347 grams. A medication is ordered and the amount is based on how heavy a child is in kilograms. How many kilograms is this baby?
2. A child weighs 35 kilograms. The parent asks how much the child weighs in pounds. How many pounds is this child?
3. A nurse is on light work duty only after returning to work post-injury. Worksafe requirements state they can lift a maximum of 10 kilograms. A box of IV bags is labeled 25 lbs. How many kilograms is this?
4. A baby weighs 1.27 kilograms. How many grams is this?
5. A person weighs 87.5 kilograms. How many pounds is this?
6. A child weighs 32 pounds. How many kilograms is this?
7. A wheelchair is rated for use by a person up to 400 pounds. The person you would like to transfer using the wheelchair is 167 kilograms. How many pounds is this equivalent to?
8. A premature infant weighs 477 grams. How many kilograms is this?
9. An infant warmer in the hospital neonatal intensive care unit has a maximum patient weight of 30 pounds. The baby you are caring for was born weighing 11.8 kilograms. How many pounds is this?
10. A newborn weighs 6 pounds and 4 ounces. How many grams is this?

Odd Answers:

1) 2.35 kg $x \text{ kg} = \frac{1 \text{ kg}}{1000 \text{ g}} \times 2347 \text{ g}$

3) 11.36 kg $x \text{ kg} = \frac{1 \text{ kg}}{2.2 \text{ lbs}} \times 25 \text{ lbs}$

5) 192.5 lb $x \text{ lbs} = \frac{2.2 \text{ lbs}}{1 \text{ kg}} \times 87.5 \text{ kg}$

7) 367.4 lb $x \text{ lbs} = \frac{2.2 \text{ lbs}}{1 \text{ kg}} \times 167 \text{ kg}$

9) 25.96 lb $x \text{ lbs} = \frac{2.2 \text{ lbs}}{1 \text{ kg}} \times 11.8 \text{ kg}$

If you use the method demonstrated in this textbook, but you do not have the conversion factor between grams and pounds, you can answer this question using this method:

1. Convert ounces to pounds $x \text{ lbs} = \frac{1 \text{ lb}}{16 \text{ oz}} \times 4 \text{ oz}$ = 0.25
2. Add converted ounces (0.25 lbs) to the known amount of pounds (6) from the question. =6.25
3. Convert pounds to grams: $x \text{ g} = \frac{1000 \text{ g}}{1 \text{ kg}} \times \frac{1 \text{ kg}}{2.2 \text{ lbs}} \times 6.25 \text{ lbs}$

Alternately, you could use the conversion factor for pounds and grams: 1 lb = 454 g, which would give you a slightly different answer due to rounding error as both conversion factors have been rounded from the most precise conversion amount: 2,837.5 g

Exercises Part D: Converting Volume

In each of the following practice questions, you will be given a measurement that needs to be converted to an alternate unit of measure, which may be metric or imperial.

1. Convert 1.15 litres to millilitres.
2. Convert 237 millilitres to litres.
3. Convert 5,819 millilitres to litres.

4. A medication requires you to mix a package of powdered medication into 1.5 cups of water. How many millilitres is this?
5. Before an ultrasound, the radiology department calls and asks you to have the patient drink 2 cups of water. How many millilitres is this?
6. At the end of your shift, the charge nurse asks you how many litres of intake your patient had today. When you check the fluid balance record you see they have received 1,875 mL of intravenous fluid and 680 mL of fluid from their meal trays.
7. You are recording fluid intake for a client. They reported in the afternoon they had 2.5 cans of flavoured soda water. Each can hold 355 ml. How many mL is this?
8. When caring for a pediatric client, the guardian informs you they gave the child 2.5 teaspoons of children's Tylenol. You weigh the child and determine the appropriate dose based on their weight is 15 mL. Presuming her teaspoon measurements were precise, was the amount given correct?
9. A client has a new prescription for eye drops: One drop in each eye once a day. The client is curious how long the bottle might last so you help them out with the math. The bottle contains 2.5 mL of fluid. A standard eye drop dispenser releases drops of approximately 50 microlitres each. How many days will the bottle likely last for, if the client takes the medication as prescribed and does not waste any drops?
10. While discussing effective treatments for constipation on the night shift, a senior nurse describes their previous success with milk and molasses enemas to you. While you are researching literature to find out if their anecdotal findings have been experienced by others, you come across a recipe for the treatment: Mix 8-16 oz milk with 8-16 oz molasses and instill slowly. Knowing that a large volume enema can be given safely at a volume of 500-1,000 mL, would this recipe fall in the safe range?

Odd Answers:

1) 1,150 mL

$$x \text{ mL} = \frac{1000 \text{ mL}}{1 \text{ L}} \times 1.15 \text{ L}$$

3) 5.819 L

$$x \text{ L} = \frac{1 \text{ L}}{1000 \text{ mL}} \times 5819 \text{ mL}$$

5) 500 mL

$$x \text{ mL} = \frac{250 \text{ mL}}{1 \text{ cup}} \times 2 \text{ cups}$$

7) 887.5 mL

$$x \text{ mL} = \frac{355 \text{ mL}}{1 \text{ can}} \times 2.5 \text{ cans}$$

9) 25 days

$$x \text{ days} = \frac{1 \text{ day}}{2 \text{ gtts}} \times \frac{1 \text{ gtt}}{50 \mu\text{L}} \times \frac{1000 \mu\text{L}}{1 \text{ mL}} \times 2.5 \text{ mL}$$

5.3 CONVERSION TABLE

The following tables outline common conversion amounts used in nursing.

Table 6.3 Common Metric Conversions for Weight

Weight Measurements in Metric	Equivalent Amount in Metric
1,000 microgram (mcg)	1 milligram (mg)
1,000 milligram (mg)	1 gram (g)
1,000 gram (g)	1 kilogram (kg)

Table 6.4 Common Metric Conversions for Volume

Volume Measurements in Metric	Equivalent Amount in Metric
1,000 millilitre (mL)	1 litre (L)
1,000 milliunits (no official abbreviation)	1 unit (U*)
1 cubic centimetre (cc*)	1 millilitre (mL)
1 milliquivalent (mEq)	1 millimole (mmol)

Table 6.5 Conversions Between Metric and Imperial Measurement of Weight

Weight Measurement in Metric	Equivalent Amount in Imperial
1 kilogram (kg)	2.2 pounds
28.35 grams	1 ounces (oz)

Table 6.6 Additional Common Conversions

Additional Measures	Equivalent Amount
1 milliquivalent (mEq)	1 millimole (mmol)
30 centimetres	1 foot (ft or ‘)
2.54 centimetres	1 inch (in or “)
16 ounces	1 pound (lb)
1 millilitre (mL)	1 gram (g)

*symbol denotes a dangerous abbreviation that should not be used. This abbreviation is included in this text as you may see this abbreviation in the health care setting. Clarification of the order is required to ensure patient safety.

5.4 ROMAN NUMERALS

Learning Objectives

By the end of this chapter, learners will be able to:

- describe the Roman numeral system,
- convert Roman numerals to Arabic numbers
- convert Arabic numbers to Roman numerals.

Describe the Roman Numeral System

The basic Roman numeral system is made up of seven letters which represent numerical values. Either upper or lowercase letters can be used. Although not used extensively in the healthcare system, some situations arise where Roman numerals are used instead of typical Arabic numbers. Can you think of any instances where you have seen Roman numerals used in healthcare?

Table 5.7 Roman Numeral Values

Roman Numeral	Arabic Number
I	1
V	5
X	10
L	50
C	100
D	500
M	1,000

Convert Roman Numerals to Arabic numbers

Numbers outside of the values above are represented by using these letters in combination. The combinations can have multiple letters in a row but always follow a particular pattern.

- You add the values of the letters together when they are the same letter or the letter values are in descending order.
  - This pattern is used for all numbers except those including 4 and 9.
  - Examples:
    - XX (10+10) = 20
    - MCCLXXXVIII (1000+100+100+50+10+10+10+5+1+1+1) = 1,288
- For values that include 4 and 9, one must use subtraction of values to determine the number being represented.
  - When a lower-value letter is to the left of a higher-value letter, subtract it from the letter to the right.
  - If a lower-value letter is between two large value letters, subtract it from the letter to the right.
  - Examples:
    - IV (5-1) = 4
    - XLIV [(50-10)+(5-1)] = 44
- If a line is drawn above a letter, multiply the numeral by 1,000, however, it is very unlikely you will see any Roman numerals written in this form in the context of nursing work.

What is the numerical value of XXI?

First, determine each of the letter values: 10 , 10 , 1

Second, determine if the letters should be added or subtracted. To do this, identify if a lower-value letter is to the left of any of the values. If it is, subtract the lower value from the value to the right, then add the remaining values.

In this case, all values to the left of 1 are higher, therefore you can just add all the values together.

10 + 10 + 1 = 21
XXI = 21

Example 1

What is the numerical value of XXIV?

Answer:
XXIV = 24

1. Determine the values of each letter: 10, 10, 1, 5
2. Identify if a lower value is to the left of a larger value: 1 is to the left of 5.
3. Subtract the lower value from the larger value: 5 – 1 = 4
4. Add this number to the remaining numbers: 10 + 10 + 4 = 24

Convert Arabic Numbers as Roman Numerals

When writing numbers, you should follow the rules written in the section above and also note there should never be more than three of the same letter in a row.

How do you write the Arabic number 71 as a Roman numeral?

Generally*, start by using the biggest value Roman numeral without going over the Arabic number: L = 50

Next, add additional letters until the values add up to the correct numerical value: LXXI (50+10+10+1=71)

Sometimes it can be helpful to count as you add the letters in a row: L 50, LX 60, LXX 70, LXXI 71

*Note that in some cases, particularly with numbers ending in 9, you may start with a numeral with a value greater than the final Arabic number.

Example 2

How do you write the Arabic number 53 as a Roman numeral?

Answer:
LIII

1. Identify the Roman numeral with the biggest value without going over: L
2. Add additional letters to obtain the correct numerical value: 50 + 1 + 1 + 1 = 53

Exercises Part A: Converting Roman Numerals to Arabic Numbers

Convert the following Roman numerals into Arabic numbers.

1. XX
2. VI
3. VIII
4. IV
5. IX
6. XXVII

7. II
8. M
9. LV
10. CXXV

Odd Answers:

1) 20

3) 8

5) 9

7) 2

9) 55

Exercises Part B: Converting Arabic Numbers to Roman Numerals

Convert the following Arabic numbers to Roman numerals.

1. 36
2. 21
3. 7
4. 18
5. 14
6. 49
7. 86
8. 35
9. 3
10. 12

Odd Answers:

1) XXXVI

3) VII

5) XIV

7) LXXXVI

9) III

5.5 THE 24-HOUR CLOCK

Learning Objectives

By the end of this chapter, learners will be able to:

- describe and convert the difference between the ante meridiem (a.m.) and post meridiem (p.m) system of time (12 hour clock) and the 24-hour clock

Describe and Convert the Difference Between the Ante Meridiem (a.m.) and Post Meridiem (p.m) System of Time (12 Hour Clock) and the 24-Hour Clock

In healthcare, the 24-hour clock is often used instead of the Latin system using **ante meridiem** (a.m.) and **post meridiem** (p.m.). Using the a.m. and p.m. notation when communicating about time can be confusing and lead to errors in places that function during the night and day. Instead of having time be broken into two periods of 12 hours each, each hour of the day is noted with its number the clock being a 24-hour clock. The 24-hour clock starts counting at 12 a.m., or midnight, at 0000. 1 a.m. is written as 0100, 2 a.m. is written as 0200, 1 p.m. is written as 1300. To determine how to write p.m. times, all you need to do is add 1200 to any notation of p.m. time.

3:00 p.m. = 1500 hours

Eventually, using the 24-hour clock will become habitual, and you will not have to think about how to convert between the two systems at all. You can also refer to the table below as a quick reference to see what each hour of p.m. time is in the 24-hour system. You never have to change how minutes are recorded. Some professions continue to use a colon when writing times using the 24-hour system, but in handwritten healthcare charting the colons are generally left out to keep charting entries clear. Computer systems will often use colons, especially if there are seconds also being recorded.

Table 5.8 Converting P.M. Time to 24-Hour Time

Post Meridiem Time	24-Hour Time
1:00 p.m.	1300 hours
2:00 p.m.	1400 hours
3:00 p.m.	1500 hours
4:00 p.m.	1600 hours
5:00 p.m.	1700 hours
6:00 p.m.	1800 hours
7:00 p.m.	1900 hours
8:00 p.m.	2000 hours
9:00 p.m.	2100 hours
10:00 p.m.	2200 hours
11:00 p.m.	2300 hours
12:00 p.m.	2400 or 0000 hours

Example 1

How do you write 9:45 p.m. using the 24-hour clock format?

Answer:

945 + 1200 = 2145 hours

You could also use the table and see 9 p.m. = 2100 hours to change the 9 to 21, keeping the minutes the same.

Key Concepts

- When converting a.m. and p.m. time to the 24-hour system, add 1200 to any p.m. time.
- Notation of minutes remains the same in the 12 and 24-hour systems.

Exercises Part A: Converting a.m. and p.m. time to the 24-hour clock

Convert the following times from the a.m. and p.m. system to the 24-hour clock.

1. 6:15 a.m.
2. 9:30 p.m.
3. 4:00 p.m.
4. 7:25 p.m.
5. 11:15 p.m.
6. 2:10 p.m.
7. 3:54 a.m.
8. 5:05 p.m.
9. 1:38 p.m.
10. 6:45 p.m.
11. 8:45 p.m.
12. 9:36 a.m.
13. 2:44 p.m.
14. 5:12 p.m.
15. 10:00 p.m.
16. 6:18 p.m.
17. 3:48 p.m.
18. 11:15 p.m.
19. 7:27 p.m.
20. 4:20 a.m.

Odd Answers:

1) 0615 hours

3) 1600 hours

5) 2315 hours

7) 0354 hours

9) 1338 hours

11) 2045 hours

13) 1444 hours

15) 2200 hours

17) 1548 hours

19) 1927 hours

Exercises Part B: Converting from the 24-hour system to a.m. and p.m. time

Convert the following times from the 24-hour system to a.m. and p.m. time.

1. 1915
2. 1435
3. 0642
4. 1724
5. 2317
6. 1604
7. 1303
8. 2159
9. 1522
10. 2220

Odd Answers:

1) 7:15 p.m.

3) 6:42 a.m.

5) 11:17 p.m.

7) 1:03 p.m.

9) 3:22 p.m.

Exercises Part C: Converting time on Analog Clocks to the 24-hour Clock

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https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=81#h5p-3

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=81#h5p-4

Blank-face analog clocks from FreeSVG.org are part of the public domain.

5.6 READING SYRINGES

Learning Objectives

By the end of this chapter, learners will be able to:

- identify the volume of liquid in a syringe
- select the most appropriate syringe to draw up a volume of liquid.

Identify the Volume of Liquid in a Syringe

Syringes come in a variety of sizes and are most often labeled for measurement in millilitres (mL) of liquid. You may see some syringes measuring cubic centimeters (cm^3), but most large companies use only mL. 1 cm^3 is equivalent to 1 mL. Insulin syringes, used only in drawing up insulin, measure units of insulin — not millilitres of insulin. Less commonly, you will see some syringes packaged with particular medications labeled in imperial units, such as teaspoons or tablespoons. For instance, a liquid antibiotic for pediatric patients may be packaged with a syringe measuring teaspoons of liquid.

Three samples of syringes with different measurements are included below:

1 mL syringe

100 Unit Insulin Syringe

3 mL syringe

Reading Syringe Volumes

Attention to detail is required to read syringe volumes accurately.

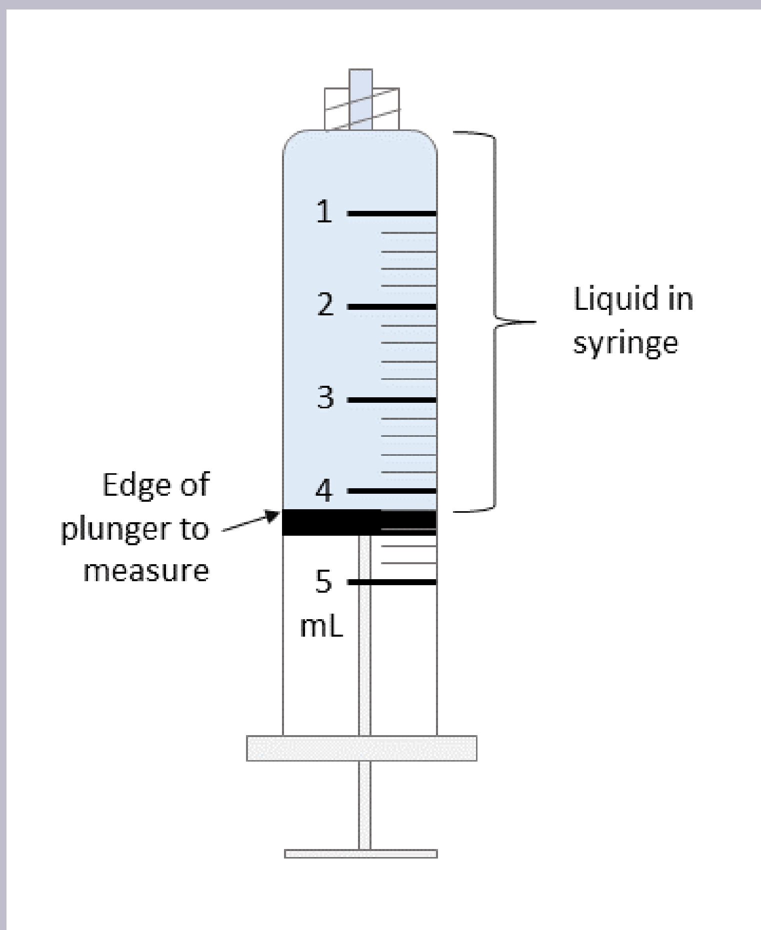
1. Identify the unit of measure for the syringe you are using.
   a. The barrel of the syringe is marked with graduated measurements of a particular unit of measure. Long or possibly bold lines mark whole units, with whole numbers often marked on the syringe barrel.
2. Identify the size of graduated measurements.
   a. Depending on the total capacity of the syringe, the space between each line on the syringes will vary in volume. For instance, in a 3 mL syringe, each space between lines is equal to 0.1 mL. In a 5 mL syringe, each space is equal to 0.2 mL.
3. Identify where the end of the plunger is.
   a. The liquid inside the syringe will take up space up to the edge of the plunger inside the syringe. This edge is what should be compared to the graduated measurements to determine how much liquid is inside the syringe. The plunger inside the syringe is large and usually shows two sharp edges where the rubber is touching the inside of the syringe barrel. The edge you are looking at is always going to be the edge close to the tip of the syringe, not the edge close to where your fingers are holding the flanges and plunger rod.
4. Determine if the volume is accurate.
   a. If bubbles are present, the measurement of volume will not be accurate. Remove the bubbles

before reading the volume of the liquid.

5. Determine the volume of liquid present.
   a. Now you are ready to read the volume of liquid in the syringe. Compare the edge of the plunger to the graduated markings to determine how much liquid is in the syringe.

Example 1

Use the image below to answer the questions and click on "Answers" to check your work.



Questions:

1. What is the capacity of the syringe?
2. How much volume is between each graduated measure on the syringe?
3. Identify the volume of liquid in the pictured syringe.

Answers:

1. 5 mL
   The capacity is the total volume the syringe can measure. This is represented by the largest number on the scale, and will be closest to the flange of the syringe.
2. 0.2 mL
   On this scale, 1 mL is broken up into 5 parts. $\frac{1}{5} = 0.2 \text{ mL}$
3. 4.2 mL
   Make sure to take the measurement from the side of the plunger closest to the syringe tip. You can see liquid represented by the shaded portion from this plunger edge to the tip of the syringe.

Select the Most Appropriate Syringe to Draw up a Volume of Liquid.

Multiple syringes can measure 1 mL of liquid. You will use critical thinking to determine which is the most appropriate syringe to use when drawing up medication. Factors you can consider when choosing a syringe include, but are not limited to:

1. Policy requirements.
   a. Be aware of policy requirements related to the equipment you are using. In particular situations, you will need to select a specific size of syringe.
2. The total volume of liquid required.
   a. You should use a syringe that can draw up the total amount of liquid required whenever possible. Choose a syringe that can hold all of the volume of liquid required.
3. The precision required for the context of the situation.
   a. There are a variety of factors that impact how precise you must be when measuring liquid medications. For instance, the effect of medications on a premature infant versus an adult patient

    can be much more pronounced, and thus require a smaller syringe allowing measurements of volume to additional decimal places.

4. The pressure which will be created when delivering the liquid.
    a. In general, smaller syringes create higher pressure than larger syringes when liquid is delivered from the syringe. Pressure may or may not be a factor that needs to be considered in the particular situation you are working in.
5. The supply is available.
    a. What is available for use at the time you are drawing up liquid? If the syringe you were going to choose is not available, you will need to determine if it is safe to replace it with an alternate syringe type.

Key Concepts

When determining the volume of liquid in a syringe, keep the following in mind:

- Note the unit of measure of the syringe.
- Ensure you are measuring from the correct side of the plunger.
- Remove all bubbles before determining the volume.

Exercises Part A: Reading syringe volumes

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here:
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Exercises Part B: Reading syringe volumes

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=86#h5p-6

Exercises Part C: Reading Syringe Volumes

An interactive H5P element has been excluded from this version of the text. You can view it online here:
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Exercises Part D: Reading Syringe Volumes

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=86#h5p-8

Exercises Part E: Choosing Syringes

Identify the type of syringe you would select to draw up the following volumes of liquid medication.

1. 4.2 mL
2. 0.22 mL
3. 1.6 mL
4. 0.75 mL
5. 3.0 mL
6. 0.1 mL
7. 2.6 mL
8. 1 mL
9. 2.3 mL
10. 7.8 mL

Odd Answers:

1) 5 mL syringe

3) 3 mL syringe (5 mL is also possible)

5) 3 mL or 5 mL syringe

7) 3 mL or 5 mL syringe

9) 3 mL syringe (a 5 mL syringe has measurements in 0.2 mL increments so is not the best choice as odd measurements fall midway between the measurement lines)

PART VI

UNIT 6: MATH FOR MEDICATION ADMINISTRATION

6.1 UNDERSTANDING MEDICATION LABELS

Learning Objectives

By the end of this chapter, learners will be able to:

- describe and Identify the key components of a medication label

Describe and Identify the Key Components of a Medication Label

Medication containers are labeled with various types of information about the particular medication inside. When administering medications, you must understand what the numbers on the label represent to determine the correct amount of medication to give. Medication labels may be formatted differently, but contain particular types of information about the medication. This chapter reviews the components of the label you should be familiar with when determining how much of a medication to give.

When you read the information on the medication label, be sure to note the medication name, the quantity of medication in the container, and the expiry date. The quantity of medication may be measured using a variety of units, such as milligrams (mg), grams (g), or international units (IU). For liquid medications, the total volume and concentration of medication in the container will also be written on the label. It is important not to confuse the concentration with the total amount of medication in the vial. Concentration is often given as a quantity of medication in a particular volume, which is often only part of the total volume in the container. For example, the concentration of a liquid medication could be 5 mg/1 mL. This would indicate there are 5 mg of medication in every 1 mL of liquid.

In addition to numerical information related to the dose, the package may include numbers representing the expiry date, reconstitution information, lot number, and non-medicinal ingredients. Other key information includes the medication name, which may be listed with both the generic and trade name in some cases, what route the medication may be used for, and the pharmaceutical company name. Alternatively, some of this information may be included on the box or wrapper the medication was packaged in or on an information sheet inside the package.

In the image below, note the generic name, the milligrams of medication (amount) per tablet, and the total number of tablets in the package.

Example 1

Answer the questions related to the image of the package containing salbutamol nebules below.

1. What is the volume of a single nebule?
2. What dose of salbutamol is one nebule?

3. What is the concentration of salbutamol?
4. How many nebules are inside the package?

Answers:

1. 2.5 mL. This is written in the upper left corner of the package.
2. 2.5 mg. This is the amount of medication inside the total volume of one nebule. Sometimes, students mix up the total amount of medication with the amount related to the information about concentration. The total amount will always be the amount related to the information listed with the largest volume (the total volume in the container).
3. 1 mg/mL
4. 20. This is written in the upper left corner of the package.

Example 2

Answer the questions related to the image of the heparin vial below.

1. What is the volume of liquid in the vial?
2. What is the total amount of heparin in the vial?
3. What is the concentration of heparin?

Answers:

1. 0.5 mL. This is seen at the top left corner of the vial.
2. 5 000 USP units. There is only 0.5 mL of fluid, therefore 5000 USP units.
3. 10 000 USP units/mL. This is written below the quantity of the vial, in a smaller font.

Critical Thinking Question

When administering liquid medications, should you calculate the volume of liquid (eg. mL) or the amount of medication (eg. mg) first?

Answer:
Amount. The amount of medication is constant. The volume of liquid may change depending on the concentration of solution you are administering. It is important to know the volume you will administer as well, but it is based on the amount of medication you are giving. Ensure you know the amount of medication first, and then calculate the volume to administer, based on the concentration of the medication in the available supply.

Key Concepts

Key components of medication labels when preparing a medication dose:

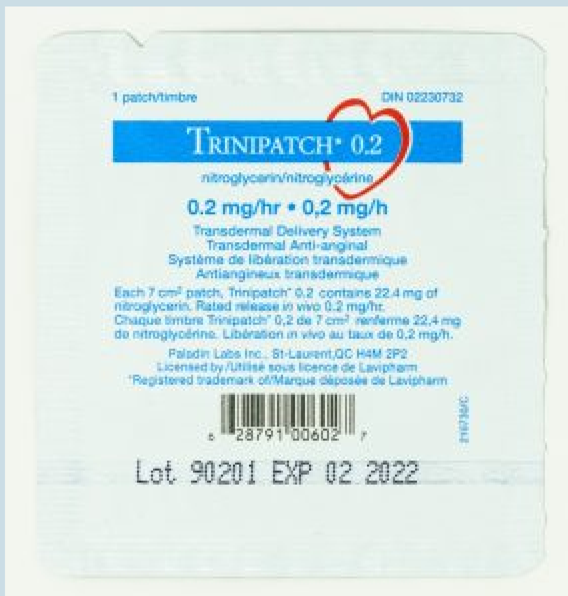
- medication name
- quantity of medication
- volume of liquid (for liquid medications)
- concentration (for liquid medications)
- expiry date

Exercises Part A: Reading Medication Labels

Answer the following questions for each image included in this practice set. To check on the answers for a particular image, click on the word **Answers** below the image.

1. What is the generic name of the medication in the image?
2. What is the total amount in one dose of the medication?

Questions:

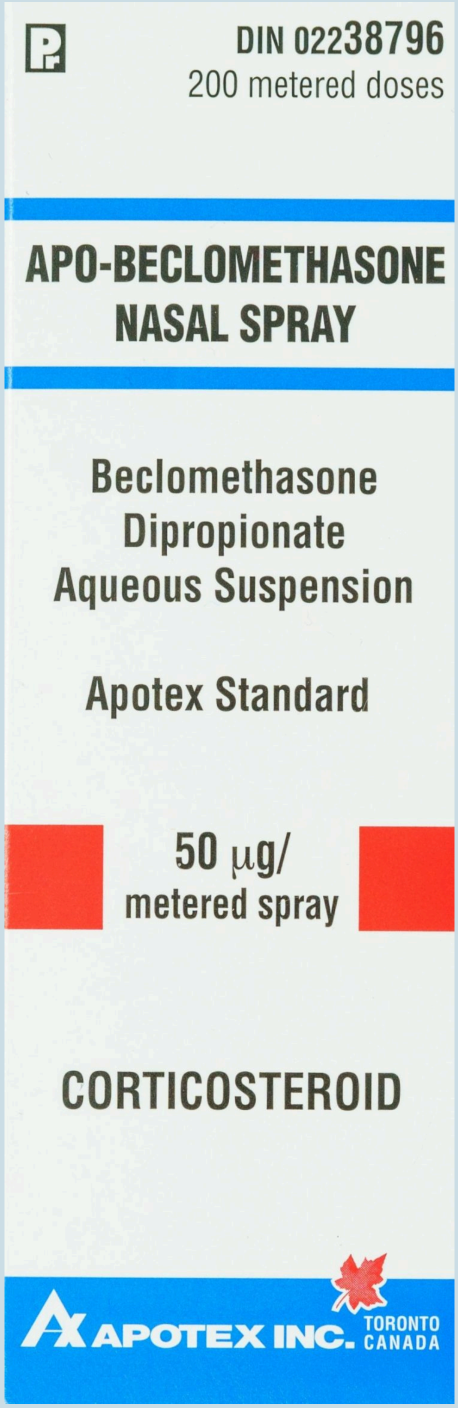


Answers:

1. nitroglycerin
2. 22.4 mg

Answers:

1. dimenhydrinate
2. 50 mg



Answers:

1. beclomethasone dipropionate
2. 50 micrograms (µg)

Answers:

1. cholecalciferol – it is not visible on this bottle
2. 1000 IU (in each tablet)

Answers:

1. budesonide

2. 0.5 mg. The total amount in one whole ampoule is 0.5 mg, as there is 2 mL in one ampoule.

Exercises Part B: Reading Medication Labels

Answer the following questions for each image included in this practice set. To check on the answers for a particular image, click on the word **Answers** below the image.

1. What is the volume of liquid in the vial?
2. What is the total amount of medication in the vial?
3. What is the concentration of liquid?

Questions:

1. methylprednisolone 

   Answers:

   1. 2 mL (once the fluid above the rubber stopper is pushed into the main chamber of the vial)

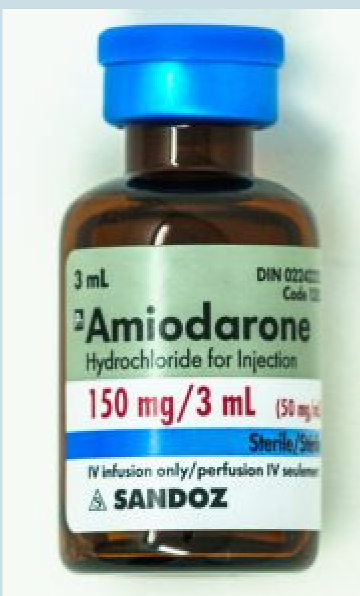
    2. 125 mg
    3. 62.5 mg/mL

2. tobramycin

   Answers:

    1. 2 mL
    2. 80 mg
    3. 40 mg/mL

3. amiodarone



   Answers:

    1. 3 mL
    2. 150 mg
    3. 50 mg/mL. The concentration is in brackets to the right of the total amount, it is hard to read in the image but you can also calculate the number of mg in each mL as you

know the total volume and the total amount (in mg) of medication.

4. phenytoin

Answers:

1. 2 mL
2. 100 mg
3. 50 mg/mL

5. metoclopramide

Answers:

1. 2 mL
2. 10 mg
3. 5 mg/mL

6.2 RECONSTITUTING MEDICATIONS

Learning Objectives

By the end of this chapter, the learner will be able to:

- define reconstitution, identify the correct amount of liquid to be added to dried medications, and identify the final concentration of medication.

Define Reconstitution, Identify the Correct Amount of Liquid to be Added to Dried Medications, and Identify the Final Concentration of Medication.

Many medications are packaged as a dried powder inside a vial or bottle. To administer these medications **parenterally** or as an oral liquid medication, the liquid must be added to the powder which then dissolves into the liquid. Instructions supplied with the medication will direct exactly how much liquid should be added to the container to give a specific concentration of medication. The instructions might be found on the container, on the packaging, on paper inside the package, or in information sent by the pharmacy. In addition, directions as to the specific type of liquid to be used will also be present, most often **normal saline** or sterile water. Careful reading of instructions and use of equipment is required to ensure the right amount of liquid is measured out and added to the container. Additional considerations for the safe preparation of medications such as proper labeling, avoidance of contamination, etc. will not be discussed in the context of this manual. Seek out additional information in a nursing skills textbook, such as Clinical procedures for safer patient care. by Doyle and McCutcheon, 2015.

Practice questions will focus on the following steps in reconstituting a medication:

1. Identifying the amount and type of liquid to reconstitute the powdered medication with.
2. Identifying the final concentration of medication.
3. Calculating the correct volume of medication for a single dose.

Example 1: Reconstituting clindamycin

Refer to the images below of a bottle of clindamycin and two sides of the packaging to answer the questions in this example.

Container | Front of Package | Side of Package

A full transcribed label is provided at the end of the chapter for readability. Click this link to go there: Clindamycin transcribed label.

Questions:

1. What type of liquid is used for reconstitution?
2. How much of this liquid should be added to the container for reconstitution?
3. What is the total volume in the container once reconstituted?
4. What is the concentration of the reconstituted medication?
5. What volume of medication would be poured from the bottle to give a dose of 90 mg?

Answers:

1. demineralized or distilled water (this is found on the side of the package)
2. 75 mL (this is found on the side of the package)

3. 100 mL (this is found on the bottle and the front of the package)
4. 75 mg/5 mL (this is found on the bottle and the front of the package)
5. 6 mL

$$\mathrm{mL} = \frac{5\ \mathrm{mL}}{75\ \mathrm{mg}} \times 90\ \mathrm{mg}$$

$$\mathrm{mL} = 6$$

Exercises Part A: Reconstituting clarithromycin

Refer to the images below, of a bottle of clarithromycin and two sides of the packaging, to answer the questions in this example.

A full transcribed label is provided at the end of the chapter for readability. Click this link to go there: Clarithromycin transcribed label.

Questions:

1. What type of liquid is used for reconstitution?
2. How much of this liquid should be added to the bottle for reconstitution?
3. What is the total volume in the container once reconstituted?
4. What is the concentration of the reconstituted medication?
5. What volume of medication would be poured from the bottle to give a dose of 50 mg?

Answers:

1. water (this is found on the side of the package)
2. 29 mL (this is found on the side of the package)
3. 55 mL (this is found on the bottle and both sides of the package)
4. 125 mg/5 mL (this is found on the bottle and both sides of the package)
5. 2 mL

$$\text{mL} = \frac{5\text{ mL}}{125\text{ mg}} \times 50\text{ mg}$$

$$\mathrm{mL} = 2$$

Exercises Part B: Reconstituting amoxicillin + clavulanate

Refer to the images below, of a bottle of amoxicillin + clavulanate and two sides of the packaging, to answer the questions in this example.

A full transcribed label is provided at the end of the chapter for readability. Click this link to go there: Amoxicillin + clavulanate transcribed label.

Questions:

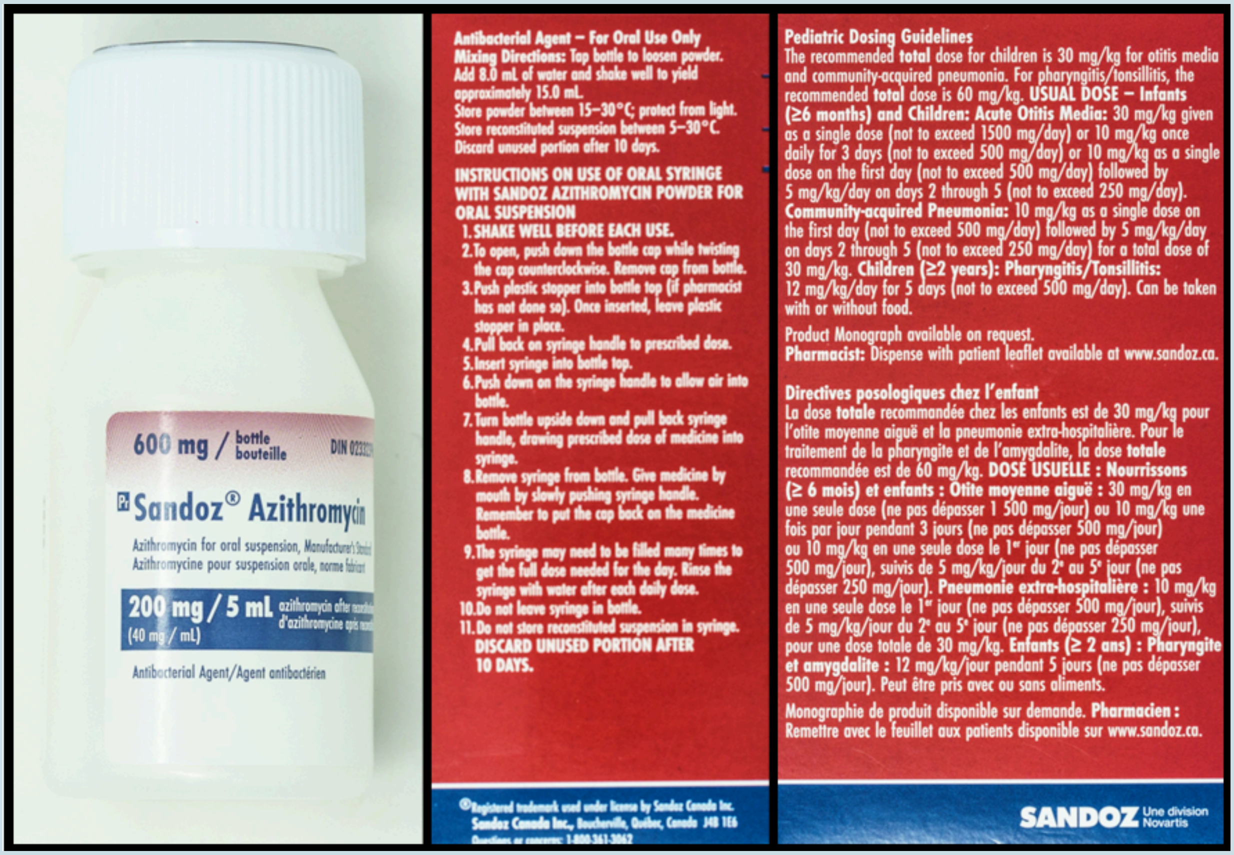
1. How much water should be added to the container for reconstitution?
2. What is the total volume in the container once reconstituted?
3. What is the concentration of the reconstituted medication?

Answers:

1. 92 mL (this is found on the front of the bottle and the package)
2. 100 mL (this is found on the bottle and the front of the package)
3. 125 amoxicillin + 31.25 mg clavulanate/5 mL (this is found on the bottle and the front of the package)

Exercises Part C: Reconstituting azithromycin

Refer to the images below, of a bottle of azithromycin and two sides of the packaging, to answer the questions in this example.



A full transcribed label is provided at the end of the chapter for readability. Click this link to go there: Azithromycin transcribed label.

Questions:

1. How much water should be added to the container for reconstitution?
2. What is the total volume in the container once reconstituted?
3. What is the concentration of the reconstituted medication?
4. What volume of medication would be poured from the bottle to give a dose of 72 mg?

Answers:

1. 8 mL (this is found on the skinny side of the package)
2. 15 mL (this is found on the skinny side of the package)
3. 200 mg/5 mL or 40 mg/mL (this is found on the bottle label)
4. 1.8 mL

$$\text{mL} = \frac{1\ \text{mL}}{40\ \text{mg}} \times 72\ \text{mg}$$

$$\text{mL} = 1.8$$

Exercises Part D: Reconstituting cefixime

Refer to the images below, of a bottle of cefixime and two sides of the packaging, to answer the questions in this example.

A full transcribed label is provided at the end of the chapter for readability. Click this link to go there: Cefixime transcribed label.

Questions:

1. How much water should be added to the bottle for reconstitution?
2. What is the total volume in the container once reconstituted?

3. What is the concentration of the reconstituted medication?
4. What volume of medication would be poured from the bottle to give a dose of 400 mg?

Answers:

1. 33 mL total, split into two portions of 16 and 17 mL (this is found on the side of the package)
2. 50 mL (this is found on the side of the package)
3. 100 mg/5 mL (this is found on the bottle and the front of the package)
4. 20 mL

$$\text{mL} = \frac{5\ \text{mL}}{100\ \text{mg}} \times 400\ \text{mg}$$

$$\text{mL} = 20$$

Exercises Part E: Reconstituting cefprozil

Refer to the images below, of the front and side labels of a bottle of cefprozil, to answer the questions in this example.

A full transcribed label is provided at the end of the chapter for readability. Click this link to go there: Cefprozil transcribed label.

Questions:

1. What volume of water should be added to the container for reconstitution?
2. What is the total volume in the container once reconstituted?
3. What is the concentration of the reconstituted medication?
4. What volume of medication would be poured from the bottle to give a dose of 250 mg?

Answers:

1. 60 mL (this is found on the side of the bottle)
2. 75 mL (this is found on the front of the bottle)
3. 125 mg/5 mL (this is found on the front and side of the bottle
4. 10 mL

$$\text{mL} = \frac{5\text{ mL}}{125\text{ mg}} \times 250\text{ mg}$$

$$\text{mL} = 10$$

Exercises Part F: Reconstituting cephalexin

Refer to the images below, of the front and side labels of a bottle of cephalexin to answer the questions in this example.

A full transcribed label is provided at the end of the chapter for readability. Click this link to go there: Cephalexin transcribed label.

Questions:

1. What volume of water should be added in the first step of mixing?
2. What is the concentration of the reconstituted medication?
3. What volume of medication would be poured from the bottle to give a dose of 87.5 mg?

Answers:

1. 34 mL (this is found on the side of the bottle)
2. 125 mg/5 mL (this is found on the front of the bottle) or 25 mg/mL (side of bottle)
3. 3.5 mL

$$\text{mL} = \frac{1\ \text{mL}}{25\ \text{mg}} \times 87.5\ \text{mg}$$

$$\text{mL} = 3.5$$

Transcribed Labels

Example 1: Clindamycin label

- Front of label:
  - Dalacin C. Flavoured granules.
  - Clindamycin palmitate hydrochloride for oral solution USP.
  - 75 mg clindamycin per 5 mL when reconstituted as directed.
  - 100 mL when reconstituted.
- Side of label:
  - Oral antibiotic.
  - Dosage: Children (over 1 month of age): 8 to 25 mg per kg per day in 3 to 4 divided doses.
  - For prevention of endocarditis:
    - Adult: 300 mg orally, 1 hour before the procedure; then 150 mg, 6 hours after the initial dose.
    - Children: 10 mg per kg (not to exceed 300 mg) orally, 1 hour before the procedure; then 5 mg per kg 6 hours after the initial dose.
  - Reconstitute with a total of 75 mL of demineralized or distilled water. Add water in two potions. Mix well after each addition of water.
  - Store power or solution at room temperature (20-25°C). Discard the solution after 14 days.

Click here to return to Example 1

Exercises Part A: Clarithromycin label

- Front of label:
  - Pediatric Biaxin.
  - 125 mg per 5 mL.
  - Clarithromycin for oral suspension USP.
  - Fruit punch flavour.
  - 55 mL after reconstitution.
- Side of label:
  - Antibiotic
  - To the Pharmacist: Add 29 mL of water and shake well to yield approximately 55 mL.
  - Each 5 mL contains Clarithromycin 125 mg.
  - Usual dose (infants and children): 15 mg per kg per day, in divided doses every 12 hours, not to exceed 1000 mg per day for 5 to 10 days.
  - Shake well before use.
  - Do not refrigerate reconstituted suspension. Discard unused medication after 14 days.
  - Storage: Store granules between 15 and 30°C in a tightly closed bottle. Protect from light.

Click here to return to Exercise A

Exercises Part B: Amoxicillin + clavulanate label

- Front of label:
  - Clavulin. Amoxicillin and clavulanate potassium for oral suspension.
  - 100 mL bottle.
  - 125 mg amoxicillin (as trihydrate) and 31.25 mg clavulanic acid (as clavulanate potassium) per 5 mL when reconstituted with 92 mL water.
- Side of label:
  - Bottle contains 2642.5 mg amoxicillin and 693.7 mg clavulanic acid. Sugar-free, and contains aspartame.
  - Storage: Store powder in a dry place at room temperature (15-25°C); use only if white to off-white. Refrigerate reconstituted suspension; use within 10 days. Keep the bottle tightly closed. Keep out of reach and sight of children.
  - Dosage and other information: See the enclosed leaflet and product monograph available at www.gsk.ca.
  - Shake well before use.

Click here to return to Exercise B

Exercises Part C: Azithromycin label

- Front of label
  - Sandoz® Azithromycin
  - 600 mg per bottle
  - Azithromycin for oral suspension
  - 200 mg per 5 mL (40 mg per mL)
  - Antibacterial agent
- Side 1 of the label:
  - Antibacterial agent – for oral use only.
  - Mixing directions: Top bottle to loosen powder. Add 8.0 mL of water and shake well to yield approximately 15.0 mL.
  - Store powder between 15-30°C. Protect from light. Store reconstituted suspension between 5-30°C. Discard unused portion after 10 days.
  - INSTRUCTIONS ON USE OF ORAL SYRINGE WITH SANDOZ AZITHROMYCIN POWDER FOR ORAL SUSPENSION.

    1. SHAKE WELL BEFORE EACH USE.
    2. To open, push down the bottle cap while twisting the cap counterclockwise. Remove the cap from the bottle.
    3. Push plastic stopper into bottle top (if the pharmacist has not done so). Once inserted, leave the plastic stopper in place.
    4. Pull back on the syringe handle to the prescribed dose.
    5. Insert the syringe into the bottle top.
    6. Push down the syringe handle to allow air into the bottle.
    7. Turn the bottle upside down and pull back the syringe handle, drawing the prescribed dose of medicine into the syringe.
    8. Remove the syringe from the bottle. Give medicine by mouth by slowly pushing the syringe handle. Remember to put the cap back on the medicine bottle.
    9. The syringe may need to be filled many times to get the full dose needed for the day. Rinse the syringe with water and have each daily dose.
    10. Do not leave the syringe in the bottle.
    11. Do not store reconstituted suspension in a syringe. DISCARD UNUSED PORTION AFTER 10 DAYS.

- Side 2 of the label: Pediatric dosing guidelines:
  - The recommended total dose for children is 30 mg per kg for otitis media and community-acquired pneumonia. For pharyngitis/tonsillitis, the recommended total dose is 60 mg per kg.
  - Usual dose – infants (≥6 months) and children: Acute otitis media: 30 mg per kg given as a single

dose (not to exceed 1500 mg per day) or 10 mg per kg once daily for 3 days (not to exceed 500 mg per day) or 10 mg per kg as a single dose on the first day (not to exceed 500 mg per day) followed by 5 mg per kg per day on days 2 through 5 (not to exceed 250 mg per day).
  - Community-acquired pneumonia: 10 mg per kg as a single dose on the first day (not to exceed 500 mg per day) followed by 5 mg per kg per day on days 2 through 5 (not to exceed 250 mg per day) for a total dose of 30 mg per kg.
  - Children (≥2 years): Pharyngitis/tonsillitis: 12 mg per kg per day for 5 days (not to exceed 500 mg per day). Can be taken with or without food.

Click here to return to Exercise C

Exercises Part D: Cefixime label

- Front of label
  - Suprax. Cefixime for oral suspension, Mfr. Std.
  - 100 mg per 5 mL when reconstituted
  - Antibiotic
  - 50 mL
- Side of label:
  - Shake well before use.
  - The bottle contains cefixime as trihydrate, corresponding to 1 g cefixime anhydrous.
  - Dosage – Adults: 400 mg once daily. If necessary, 200 mg twice daily. Urinary tract infections: 400 mg once daily.
  - Children: 8 mg per kg per day once daily. If necessary, 4 mg per kg twice daily. Urinary tract infections: 8 mg per kg per day once daily.
  - Reconstitution: Tap the bottle several times to loosen the powder contents before reconstitution. Add a total volume of 33 mL of water split in TWO POTIONS. Mix well after each addition. Provides 20 mg per mL.
  - Suspension may be kept for 14 days at room temperature or under refrigeration. Discard unused portion. Store powder at controlled room temperature between 15 and 30°C.

Click here to return to Exercise D

Exercises Part E: Cefprozil label

- Front of label:
  - Cefprozil
  - 75 mL
  - Cefprozil for oral suspension USP

- 125mg/5mL
- Antibiotic
- Ranbaxy pharmaceuticals

- Side of label:
  - Each: 5 mL of reconstituted suspension contains 125 mg of cefprozil.
  - Directions for use: Reconstitute with 54 mL of water. Shake well before use.

Click here to return to Exercise E

Exercises Part F: Cephalexin label

- Front of label:
  - Lupin-Cephalexin
  - 100 mL
  - 125 mg per 5 mL.
  - Cephalexin for Oral Suspension, manufacturer standard.
  - Antibiotic.
  - Each bottle contains 25000 mg cephalexin (as monohydrate).
  - Strawberry flavour.
- Side of label:
  - Reconstitution: Add 69 mL of water divided into two potions (34 mL and 35 mL each) to the 50 g dry mixture in the bottle to make 100 mL of the suspension. After reconstitution, 1 mL of suspension contains 25 mg cephalexin.
  - Dosage – Adults: 250 mg every 6 hr.
  - Children: 25-50 mg per kg of body weight per day in equally divided doses at 6-hour intervals. For more severe infections, the dose may be doubled.
  - SHAKE WELL BEFORE USING.
  - Storage: Store the dry powder between 15-30°C. Protect from light. When reconstituted, suspension is stable for 14 days under refrigeration 2-8°C. Do not freeze.

Click here to return to Exercise F

6.3 CALCULATING MEDICATION DOSAGE

Learning Objectives

By the end of this chapter, learners will be able to:

- explain a method for solving medication dose problems, and correctly solve medication dose problems.

Explain a Method for Solving Medication Dose Problems, and Correctly Solve Medication Dose Problems.

There are often multiple ways to solve problems involving math. In this workbook, the process of dimensional analysis is presented as one way to solve all types of medication-related problems. You can continue to use this process when solving practice problems presented in this chapter. Using this form of problem-solving can help you reduce errors if you struggle to remember formulas or figure out which formula to use as you can learn just one method to approach all the calculations.

How to

Steps to Calculate Medication Amounts

1. Read the information carefully. Avoid distractions when preparing medications so you can focus on the problem at hand. What information are you trying to calculate?
2. Set up the formula. When beginning to calculate medication amounts it is helpful to always write down your work so you can check your work and so others can help you find errors if you have made a mistake.
3. Calculate the answer by using a calculator or mental math.
4. Check your work. Always double-check your answer. Does it make sense? If you have determined you need to give 20 tablets for one dose this would be a red flag-an unlikely situation and indicates an error has probably been made. As a student, you should always have a supervising nurse check your math. After you graduate, there are specific high-risk medications (like insulin) that must be double-checked by another nurse. You may also decide that specific situations warrant asking someone to check, such as if you have been feeling tired or if there are distractions during your process of medication administration.

Example 1: Determining the number of tablets required when the dose is more than the supply

How many tablets would you administer for the following medication order?

Order: prednisone 20 mg PO OD
Supply: prednisone 5 mg tablets

Answer:

4 tablets.

$$\text{x tablets} = \frac{\text{1 tablet}}{\text{5 mg}} \times \text{20mg} \qquad \text{x tablets} = \frac{\text{1 tablet}}{5\ \cancel{\text{mg}}} \times 20\ \cancel{\text{mg}} \qquad \text{x tablets} = \frac{\text{20 tablets}}{5} \qquad \text{x tablets} = 4$$

Alternate Formula for Solving Medication Dose Problems

$$\frac{\text{Desired amount}}{\text{Amount in supply}} \times \text{Volume of supply} = \text{Quantity for dose}$$

This may also be written as:

$$\frac{\text{Desired Amount}}{\text{Dose on hand}} \times \text{Volume} = \text{Dose}$$

Or abbreviated as:

$$\frac{\text{D}}{\text{H}} \times \text{V} = \text{Dose}$$

Regardless of the words or letters chosen to represent numbers in the formula, the numbers related to the medication problem are always in the same place.

Here is the same problem from the sample exercise above, using this formula:

Order: prednisone 20 mg PO OD
Supply: prednisone 5 mg tablets

$$\frac{\text{Desired amount}}{\text{Amount in supply}} \times \text{Volume of supply} = \text{Quantity for dose} \qquad \frac{\text{20 mg}}{\text{5 mg}} \times \text{1 tablet} = \text{Quantity for dose} \qquad \frac{20\ \cancel{\text{mg}}}{5\ \cancel{\text{mg}}} \times \text{1 tablet} = \text{Quantity for dose} \qquad \text{4 tablets}$$

Make sure to check your work:

- First, check your formula and make sure it has been set up correctly.
- Do all of the units cancel out based on the way you have set up the formula? If you are not left with units matching on both sides of the equals sign, this is a clue that information has been put into the formula incorrectly.
- Next, look for calculation errors by doing the math again to see if you get the same answer.
- Lastly, have a supervising nurse confirm you have calculated the correct amount.

Example 2: Determining the number of tablets required when the dose is less than the supply

How many tablets would you administer for the following medication order?

Order: metoprolol 12.5 mg PO BID
Supply: metoprolol 50 mg tablets

Answers:

0.25 of one tablet, or $\frac{1}{4}$

$$\text{x tablets} = \frac{1\ \text{tablet}}{50\ \text{mg}} \times 12.5\ \text{mg} \qquad \text{x tablets} = \frac{1\ \text{tablet}}{50\ \cancel{\text{mg}}} \times 12.5\ \cancel{\text{mg}} \qquad \text{x tablets} = \frac{12.5\ \text{tablet}}{50} \qquad \text{x tablets} = 0.25$$

Example 3: Determining the volume of medication required

What volume needs to be drawn up to administer the following medication?

Order: dimenhydrinate 25 mg SC prn q6h
Supply: dimenhydrinate 50 mg/mL

Answers:

0.5 mL

$$\text{x mL} = \frac{1\ \text{mL}}{50\ \text{mg}} \times 25\ \text{mg} \qquad \text{x mL} = \frac{1\ \text{mL}}{50\ \cancel{\text{mg}}} \times 25\ \cancel{\text{mg}} \qquad \text{x mL} = \frac{25\ \text{mL}}{50} \qquad \text{x mL} = 0.5$$

Key Concepts

When calculating dosages, follow the following steps:

- Read the information presented carefully
- Set up the formula
- Ensure units cancel out
- Calculate the amount required
- Check your work

Exercises Part A: Calculating the number of tablets to administer

Calculate how many tablets would you administer for the following medication orders. Click on the word answers to check your work.

Questions:

1. Order: acetaminophen 650 mg PO QID
   Supply: acetaminophen 325 mg tablets
2. Order: sertraline 75 mg PO OD
   Supply: sertraline 25 mg capsules
3. Order: ibuprofen 400 mg PO QID prn
   Supply: ibuprofen 200 mg tablets
4. Order: dimenhydrinate 12.5 mg PO QID prn

Supply: dimenhydrinate 50 mg tablets

5. Order: gabapentin 300 mg PO OD
Supply: gabapentin 100 mg capsules
6. Order: azithromycin 1 g PO
Supply: azithromycin 250 mg tablets
7. Order: digoxin 250 mcg PO OD
Supply: digoxin 0.5 mg tablets
8. Order: furosemide 80 mg PO OD
Supply: furosemide 40 mg tablets
9. Order: glyburide 10 mg PO BID
Supply: glyburide 5 mg tablets
10. Order: diltiazem 45 mg PO BID
Supply: diltiazem 90 mg tablets

Odd Answers:

1) 2 tablets

$\text{x tablets} = 2$

3) 2 tablets

$\text{x tablets} = 2$

5) 3 capsules

$\text{x tablets} = 3$

7) 0.5 tablet

$\text{x tablets} = 0.5$

9) 2 tablets

$\text{x tablets} = 2$

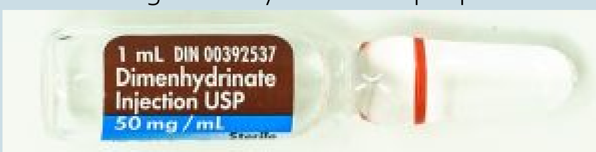
Exercises Part B: Calculating the volume of a medication

dose

Calculate the volume of liquid you would draw up for the following medication orders. Click on the word answers to check your work.

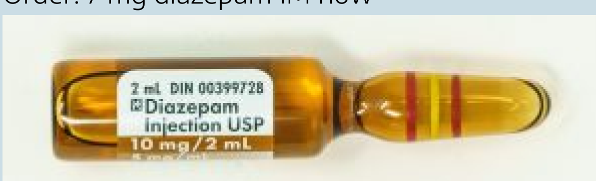
Questions:

1. Order: 25 mg dimenhydrinate IV q6h prn

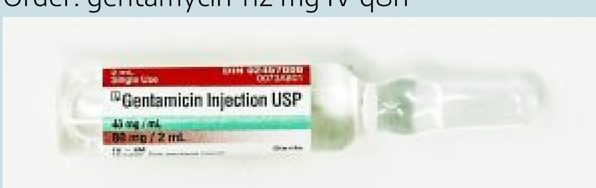


2. Order: hydromorphone 1.5 mg SC q4h prn

3. Order: 7 mg diazepam IM now



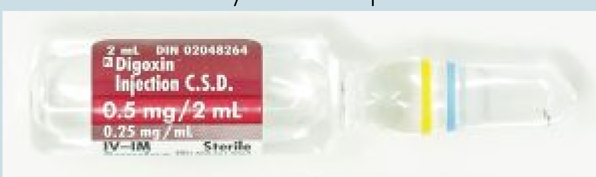
4. Order: gentamycin 112 mg IV q8h



5. Order: hydromorphone 0.5 mg SC q4-6 h prn

6. Order: 50 mcg cyanocobalamin SC OD x 7 days

7. Order: digoxin 10 mcg/kg IV total loading dose; administer 50% initially; then give ¼ the loading dose q8hr twice. Client weighs 74 kg
   How much would you draw up for the first dose of digoxin?



Odd Answers:

1) 0.5 mL

$x \text{ mL} = 0.5$

3) 1.4 mL

$x \text{ mL} = 1.4$

5) 0.25 mL

$x \text{ mL} = 0.25$

7) 1.48 mL

First, calculate the total loading dose based on the client's weight.

Second, calculate the amount of the first portion of the loading dose. (50% = half of the total)

$740 \text{ mcg} \div 2$

$= 370 \text{ mcg}$

Third, calculate the volume to draw up from the medication supply.

$x \text{ mL} = 1.48 \text{ mL}$

Exercises Part C: Calculating the Volume of a Medication Dose and Selecting the Correct Syringe Size

Calculate the volume of liquid you would draw up for the following medication orders. Look at the images and drag and drop an image of a syringe filled with the correct volume into the empty rectangle on the right. Click on the word **check** to see if you have selected a syringe with the correct volume. Click on the word **answers** to check your work for the medication math.

Questions:

1. *An interactive H5P element has been excluded from this version of the text. You can view it online here:* https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-9

2. *An interactive H5P element has been excluded from this version of the text. You can view it online here:* https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-10

3. *An interactive H5P element has been excluded from this version of the text. You can view it online here:* https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-11

4. *An interactive H5P element has been excluded from this version of the text. You can view it online here:* [*https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-12*](https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-12)

5. *An interactive H5P element has been excluded from this version of the text. You can view it online here:* [*https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-18*](https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-18)

6. *An interactive H5P element has been excluded from this version of the text. You can view it online here:* [*https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-13*](https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-13)

7. *An interactive H5P element has been excluded from this version of the text. You can view it online here:* [*https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-14*](https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-14)

8. *An interactive H5P element has been excluded from this version of the text. You can view it online here:* [*https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-15*](https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-15)

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An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-17

Odd Answers:

1) 0.65 mL

$\frac{65}{x} = \frac{100}{1}$

$65 = (100)(x)$

$0.65 = x$

3) 0.5 mL

$\frac{0.5}{x} = \frac{1}{1}$

$0.5 = (1)(x)$

$0.5 = x$

5) 0.7 mL

$\frac{35}{x} = \frac{50}{1}$

$35 = (50)(x)$

$0.7 = x$

7) 0.35 mL

$\frac{1.4}{x} = \frac{4}{1}$

$1.4 = (4)(x)$

$0.35 = x$

9) 1.8 mL

$\frac{7.2}{x} = \frac{4}{1}$

$7.2 = (4)(x)$

$1.8 = x$

Exercises Part D: Calculating the Volume of a Medication Dose and Selecting the Correct Syringe Size

Calculate the volume of liquid you would draw up for the following medication orders. Look at the images and determine which syringes filled with liquid represent the correct volume of medication. There may be more than one correct syringe size option. Drag and drop all syringes with the correct volumes into the open rectangle on the right. Click on the word **check** to see if your selection is correct. Click on the word **answers** to see the work for the medication math.

Questions:

1. *An interactive H5P element has been excluded from this version of the text. You can view it online here:*
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2. *An interactive H5P element has been excluded from this version of the text. You can view it online here:*
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3. *An interactive H5P element has been excluded from this version of the text. You can view it online here:*
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4. *An interactive H5P element has been excluded from this version of the text. You can view it online here:*
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5. *An interactive H5P element has been excluded from this version of the text. You can view it online here:*
https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-23

6. *An interactive H5P element has been excluded from this version of the text. You can view it online here:*
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7. *An interactive H5P element has been excluded from this version of the text. You can view it online here:*
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8. *An interactive H5P element has been excluded from this version of the text. You can view it online here:*
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https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-27

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https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-28

Odd Answers:

1) 2 mL

$\frac{0.4}{x} = \frac{0.2}{1}$

$0.4 = (0.2)(x)$

$2 = x$

3) 0.4 mL

$\frac{50}{x} = \frac{125}{1}$

$50 = (125)(x)$

$0.4 = x$

5) 0.75 mL

$\frac{3}{x} = \frac{4}{1}$

$3 = (4)(x)$

$0.75 = x$

7) 0.8 mL

$\frac{4}{x} = \frac{5}{1}$

$4 = (5)(x)$

$0.8 = x$

9) 2.6 mL

$\frac{5.2}{x} = \frac{2}{1}$

$5.2 = (2)(x)$

$2.6 = x$

Exercises Part E: Calculating the Volume of a Medication Dose and Selecting the Correct Syringe Size

Calculate the volume of liquid you would draw up for the following medication orders. Look at the images and determine which syringes filled with liquid represent the correct volume of medication. There may be more than one correct syringe size option. Drag and drop all syringes with the correct volumes into the open rectangle on the right. Click on the word **check** to see if your selection is correct. Click on the word **answers** to see the work for the medication math.

Questions:

1. A patient exposed to an STI receives an order of ceftriaxone.

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   https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-35

2. A pediatric patient is requiring a dose of glycopyrrolate.

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   https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-36

3. A patient with an infection requires a dose of ampicillin.

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https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-37

4. A pediatric patient has an anaphylactic reaction to a medication and pre-filled syringes are out of stock. Draw up the ordered dose of epinephrine. After the patient stabilizes, draw up diphenhydramine for IM injection.

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https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-38

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-39

5. A physician has ordered dexamethasone for you to draw up and administer for a pediatric patient with cerebral edema.

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https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-40

6. You are drawing up heparin for a pediatric patient's loading dose for heparinization.

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https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-41

7. A prophylactic dose of gentamicin is required for a pre-surgical adult patient weighing 53 kg.

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https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-42

8. A child arrives at the emergency department with a fractured arm and requires morphine for acute pain.

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-43

9. A 45-year-old with right-sided flank pain requires analgesics.

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-44

10. You are caring for a 73-year-old with pulmonary edema.

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://ecampusontario.pressbooks.pub/sccmedicalmath/?p=116#h5p-45

Odd Answers:

1) 2.2 mL

$\frac{785}{x} = \frac{250}{0.7}$

$549.5 = (250)(\mathrm{x})$

$2.2 = \mathrm{x}$

3) 1.7 mL

$\frac{425}{x} = \frac{500}{2}$

$850 = (500)(\mathrm{x})$

$1.7 = \mathrm{x}$

5) 1.5 mL

$\frac{6}{x} = \frac{4}{1}$

$6 = (4)(\mathrm{x})$

$1.5 = \mathrm{x}$

7) 1.9 mL

$\frac{76}{x} = \frac{40}{1}$

$76 = (40)(\mathrm{x})$

$1.9 = \mathrm{x}$

9) 0.45 mL

$\frac{13.5}{x} = \frac{30}{1}$

$13.5 = (30)(\mathrm{x})$

$0.45 = \mathrm{x}$

6.4 CALCULATING MEDICATION DOSES BASED ON WEIGHT

Learning Objectives

By the end of this chapter, learners will be able to:

- calculate medication doses when the quantity is based on the client's weight
- verify safe doses for weight-based medication recommendations

Calculate Medication Doses When the Quantity is Based on the Client's Weight

Some medications require the dose to be determined based on the weight of the client. Orders are written indicating how much medication should be given for each kilogram (kg) the client weighs. You will need to calculate this amount to ensure you are administering the correct amount to the client. You may see weight-based orders written in different ways, for example:

morphine 0.2 mg/kg po q4h

morphine 0.1 – 0.5 mg/kg po q4h

Note in the first example only one dose will be calculated, whereas in the second example, there is a range of doses that are possible. If the client's weight has been recorded in pounds or ounces, you must convert the client's weight into kilograms or grams, as weight-based drug dosages are always given in an amount per kilogram or gram (for some neonatal dosages). When a range is given in the order, you will calculate the smallest and largest amounts which can be given, based on the client's weight.

Example 1

An order is written for a child to receive 0.2 mg/kg of morphine PO q4h prn. The child weighs 14 kg. Calculate the prn dose for this child.

Answer:

$$x \text{ mg} = \frac{0.2 \text{ mg}}{1 \cancel{\text{kg}}} \times 14 \cancel{\text{kg}}$$

$$x \text{ mg} = \frac{2.8 \text{ mg}}{1}$$

$$\text{mg} = 2.8$$

Example 2

An order is written for a child to receive morphine 0.1 – 0.5 mg/kg PO q4h prn. The child weighs 24 kg. Calculate the range of prn doses for this child.

Answer:
The range is 2.4-12 mg of morphine per dose. These represent the smallest and largest doses the child should receive.

$$x \text{ mg} = \frac{0.1 \text{ mg}}{1 \cancel{\text{kg}}} \times 24 \cancel{\text{kg}}$$

$$x \text{ mg} = \frac{2.4 \text{ mg}}{1}$$

$$\text{mg} = 2.4$$

$$x \text{ mg} = \frac{0.5 \text{ mg}}{1 \cancel{\text{kg}}} \times 24 \cancel{\text{kg}}$$

$$x \text{ mg} = \frac{12 \text{ mg}}{1}$$

$$\text{mg} = 12$$

You might have noticed there is a step we can take out of the calculation above. Whenever we are calculating a dose where the amount per dose and the weight of the client have the same unit for weight, the following formula can be used:

$$\text{mg} = \text{amount per dose} \times \text{weight of client}$$

Example 3

An order is written for a child to receive 25 mg/kg of cephazolin IV q12h. The child weighs 37 kg. Calculate the dose for this child.

Answer:

$$\text{mg} = \text{amount per dose} \times \text{weight of client}$$

$$\text{mg} = 25 \times 37$$

$$\text{mg} = 925$$

Verify Safe Doses for Weight-Based Medication Recommendations

Recommended Dosage

To verify if a medication dose is safe for a client, the ordered dose must be compared with the recommended dosage information published by the manufacturer or within a reference manual (drug guide, pharmacy documents, etc).

You will need to read carefully to ensure you are following the recommendations for categories relevant to your client, as there may be recommended dosages for multiple circumstances. For instance, the type of client (often based on age and/or weight) or the reason the medication is being prescribed.

Sample Recommended Dose

The World Health Organization recommends the following dosage options for children ages 2 months to 12 years:

Meropenem 60 mg/kg/day IV divided into 3 doses

or

Meropenem 120 mg/kg/day IV divided into 3 doses (in severe infection)

In this example, the difference in the two doses is related to the reason the medication is being administered.

[Antibiotic Dosing for Children: Draft Expert Recommendations for the 2017 Essential Medicines List for Children (EMLc) [PDF]]

Critical Thinking Questions

1. What is the risk to the client if the dose is higher than the recommended range?
2. What is the risk to the client if the dose is lower than the recommended range?

Answers:

1. There is a risk of the client receiving a toxic dose if a dose of the medication is given above the recommended range. Adverse reactions may also worsen or occur, with the severity depending on the amount of extra medication.
2. There is a risk of the medication not having any or reduced effect. The client's condition could worsen.

Safe Dosage for Weight-Based Medications

To determine if the ordered dose is appropriate for the client, you need to compare the medication order to the information in the reference guide. Often, the recommended dose is given as a total daily amount with a range of possible frequencies, rather than the amount for a single dose. If this is the case, you will need to calculate the recommended doses for the low and high end of the range. You must ensure you are using the right information from the drug reference manual when determining which numbers to use in your calculations. You will determine which information is important by referring back to the information about the client. Relevant information may include:

- client age
- client weight
- route
- frequency
- reason
- presence of kidney or liver disease
- pregnancy status
- other diagnostic results

Determine if the following medication order is safe:

vancomycin 500 mg IV q12h

Drug guide information: vancomycin
Children:
IV: 10-15 mg/kg q6-12h
PO: 40 mg/kg
Adults:
IV: 500-2,000mg q12-24h
PO: 125-500 mg q6-8h

Client information:
11-year-old with severe staphylococcal infection, weight 38kg

To solve the problem:
Step 1: Determine which information in the drug guide is relevant to the client.
In this case, the relevant information is **IV: 10-15 mg/kg q6-12h**
This is the dose for the **age** of the client and matches the **route** in the medication order.
Step 2: Calculate the minimum and maximum recommended doses.

$$\text{min dose} = \frac{10\text{ mg}}{1\text{ }\cancel{\text{kg}}} \times 38\text{ }\cancel{\text{kg}} = 380\text{ mg}$$

$$\text{max dose} = \frac{15\text{ mg}}{1\text{ }\cancel{\text{kg}}} \times 38\text{ }\cancel{\text{kg}} = 570\text{ mg}$$

Step 3: Compare the dose in the order to the minimum and maximum recommended doses. Does the dose fall between these numbers? Yes, 500 mg falls between 380 and 570mg.

Example 4

Order: amoxicillin 250 mg PO TID

Reference manual states: Safe dosage is 20 to 40 mg/kg/day

The client weighs: 20 kg

Problem: Based on the client's weight, is this a safe dose?

Answer:
Yes, this is a safe dose. 250 mg is in the safe range of 133-267 mg per dose.
Since the reference manual gives you a total daily amount, you must calculate the total daily range first, then divide by the order frequency to see if the dose is safe.

$$\text{min daily amount} = \frac{20\text{ mg}}{1\text{ kg}} \times 20\text{ kg} = 400\text{ mg}$$

$$\text{max daily amount} = \frac{40\text{ mg}}{1\text{ kg}} \times 20\text{ kg} = 800\text{ mg}$$

$$\text{min dose} = \frac{400\text{ mg}}{1\text{ day}} \times \frac{1\text{ day}}{3\text{ doses}} = 133\text{ mg/dose}$$

$$\text{max dose} = \frac{800\text{ mg}}{1\text{ day}} \times \frac{1\text{ day}}{3\text{ doses}} = 267\text{ mg/dose}$$

Administering Doses from a Range

When a weight-based medication order includes a range of dose options, you will need to use critical thinking to determine what dose of medication to give after completing a thorough assessment with the client.

Critical Thinking Questions

What factors might impact the dose you choose to give?

Answer:

You may need to consider factors such as:

- has the client received this medication before?
- if given previously, was the medication effective when it was last given?
- does the client experience adverse effects with this medication?
- are there other diagnostic tests you need to review when selecting the dose (creatinine, INR, ptt, etc.)?
- what is the patient's preference?

As a novice student, you will make decisions about what particular dose to give in partnership with the client's primary nurse and/or your nursing instructor.

Exercises Part A: Calculating weight based doses

Calculate the dose for each of the following medication orders:

1. vancomycin 15 mg/kg IV TID, child weighs 32 kg
2. nitrofurantoin 1mg/kg PO q6h, child weighs 44 lb
3. acetaminophen 10-15 mg/kg PO q4h prn, child weighs 16 kg
4. piperacillin-tazobactam 90mg/kg IV q8h, child weighs 27 kg
5. ibuprofen 4-10 mg/kg PO q4-6h prn, child weighs 19 kg
6. clindamycin 2-5 mg/kg PO q6h, child weighs 17 kg
7. meropenem 20 mg/kg IV q8h, child weighs 38.5 lb
8. clarithromycin 7.5mg/kg PO BID, child weighs 12 kg
9. cefotaxime 50 mg/kg IV q8h, child weighs 36 kg

10. ketorolac 0.5 mg/kg IV q6-8h prn, child weighs 70 kg

Odd Answers:

1) 480 mg/dose $\frac{15 \text{ mg}}{1 \text{ kg}} \times 32 \text{ kg}$

3) 160-240 mg/dose $\frac{10 \text{ mg}}{1 \text{ kg}} \times 16 \text{ kg}$, $\frac{15 \text{ mg}}{1 \text{ kg}} \times 16 \text{ kg}$

5) 76-190 mg/dose $\frac{4 \text{ mg}}{1 \text{ kg}} \times 19 \text{ kg}$, $\frac{10 \text{ mg}}{1 \text{ kg}} \times 19 \text{ kg}$

7) 350 mg/dose $\frac{1 \text{ kg}}{2.2 \text{ lbs}} \times 38.5 \text{ lbs} = 17.5 \text{ kg}$, $\text{dose} = \frac{20 \text{ mg}}{1 \text{ kg}} \times 17.5 \text{ kg}$

9) 1800 mg/dose $\frac{50 \text{ mg}}{1 \text{ kg}} \times 36 \text{ kg}$

Exercises Part B: Calculating weight based doses

Calculate the dose for each of the following medication orders:

1. amoxicillin 50 mg/kg PO BID, child weighs 43 kg
2. diphenhydramine 1-2 mg/kg IM q4-8h prn, child weighs 15 kg
3. cephalexin 12.5 mg/kg PO q6h, child weighs 23 kg
4. ketorolac 0.5 mg/kg IV q6-8h prn, child weighs 47 lb
5. ciprofloxacin 15 mg/kg PO q12h, child weighs 31 kg
6. acetaminophen 10-15 mg/kg PO q4h prn, child weighs 35 kg
7. ceftriaxone 80 mg/kg IV once daily, child weighs 92 lb
8. gentamycin 7 mg/kg IV once daily, child weighs 29 kg
9. dimenhydrinate 12.5-25 mg/kg PO q6-8h prn, child weighs 73 lb
10. codeine 0.5-1 mg/kg q4-6h PO prn, child weighs 19 kg

Odd Answers:

1) 2150 mg/dose $\frac{50 \text{ mg}}{1 \text{ kg}} \times 43 \text{ kg}$

3) 287.5 mg/dose $\frac{12.5 \text{ mg}}{1 \text{ kg}} \times 23 \text{ kg}$

5) 465 mg/dose $\frac{15 \text{ mg}}{1 \text{ kg}} \times 31 \text{ kg}$

7) 3345 mg/dose $\frac{1 \text{ kg}}{2.2 \text{ lbs}} \times 92 \text{ lbs} = 41.8 \text{ kg}$, $\text{dose} = \frac{80 \text{ mg}}{1 \text{ kg}} \times 41.8 \text{ kg}$

9) 415 – 830 mg/dose $\frac{1 \text{ kg}}{2.2 \text{ lbs}} \times 73 \text{ lbs} = 33.2 \text{ kg}$, $\text{dose} = \frac{12.5 \text{ mg}}{1 \text{ kg}} \times 33.2 \text{ kg}$, $\text{dose} = \frac{25 \text{ mg}}{1 \text{ kg}} \times 33.2 \text{ kg}$

Example Part C: Calculating weight based doses

Calculate the dose for each of the following medication orders:

1. atropine 0.02 mg/kg/dose IV Q5 min x 2 doses prn, child weighs 15 kg
2. morphine 0.08 mg/kg/dose PO Q3-4H PRN, child weighs 22 lbs
3. naproxen 7 mg/kg/dose PO Q8-12H, child weighs 82 lbs
4. digoxin 10 mcg/kg/24 hr PO once daily, child weighs 31 lbs
5. octreotide 2 mcg/kg/dose IV bolus over 2-5 min, child weighs 25 kgs
6. adenosine initial dose 0.1 mg/kg rapid IV within 1-2 sec, child weighs 16 lbs
7. meropenem 20 mg/kg/dose IV Q8H, child weighs 13 kgs
8. prednisone 0.25 mg/kg/dose PO BID, child weighs 48 lbs
9. piperacillin-tazobactam 75 mg/kg/dose IV Q6H, child weighs 24 kg
10. norepinephrine 0.1 mcg/kg/min, child weighs 14 kgs

Odd Answers:

1) 0.3 mg/dose

$\frac{0.02 \text{ mg}}{1 \text{ kg}} \times 15 \text{ kg} = 0.3 \text{ mg/dose}$

3) 260 mg/dose

$\frac{1 \text{ kg}}{2.2 \text{ lbs}} \times 82 \text{ lbs} = 37.19 \text{ kg} \frac{7 \text{ mg}}{1 \text{ kg}} \times 37.19 \text{ kg} = 260 \text{ mg/dose}$

5) 50 mcg/dose

$\frac{2 \text{ mcg}}{1 \text{ kg}} \times 25 \text{ kg} = 50 \text{ mcg/dose}$

7) 261 mg/dose

$\frac{20 \text{ mg}}{1 \text{ kg}} \times 13 \text{ kg} = 260 \text{ mg/dose}$

9) 1800 mg/dose

$\frac{75 \text{ mg}}{1 \text{ kg}} \times 24 \text{ kg} = 1800 \text{ mg/dose}$

Exercises Part D: Calculating weight based doses with ranges

Calculate the dose for each of the following medication orders:

1. amoxicillin/clavulanic acid 10-15 mg/kg/dose PO TID, child weighs 39 kg
2. cefazolin 35–50 mg/kg/dose IV Q8H, child weighs 48 kg
3. clopidogrel 0.2-1 mg/kg/dose PO once daily, child weighs 27 lbs
4. haloperidol 0.01-0.02 mg/kg/dose PO BID, child weighs 84 lbs
5. diazepam 0.04-0.2 mg/kg/dose PO Q6-8H, child weighs 37 kg
6. ibuprofen 5-10 mg/kg/dose PO Q6-8H, child weighs 72 lbs
7. ketorolac 0.2-0.5 mg/kg/dose IV/IM Q6-8H PRN, child weighs 49 kg
8. metoprolol 0.5-1 mg/kg/dose PO BID, child weighs 49 lbs

9. mannitol 0.25-1 g/kg IV over 20-30 min repeat Q4-8H PRN, child weighs 33 lbs
10. spironolactone 1-1.5 mg/kg/dose BID, child weighs 83 lbs

Odd Answers:

1) 390-585 mg/dose

$$\frac{15\ \text{mg}}{1\ \text{kg}} \times 39\ \text{kg} = 585\ \text{mg/dose}$$

3) 2.45-12.25 mg/dose

$$\frac{1\ \text{mg}}{1\ \text{kg}} \times 12.25 = 12.25\ \text{mg/dose}$$

5) 1.48-7.4 mg/dose

$$\frac{0.2\ \text{mg}}{1\ \text{kg}} \times 37\ \text{kg} = 7.4\ \text{mg/dose}$$

7) 9.8-24.5 mg/dose

$$\frac{0.5\ \text{mg}}{1\ \text{kg}} \times 49\ \text{kg} = 24.5\ \text{mg/dose}$$

9) 3.75-15 g/dose

$$\frac{1\ \text{g}}{1\ \text{kg}} \times 15\ \text{kg} = 15\ \text{g/dose}$$

Exercises Part E: Verifying Dosage Safety – acetaminophen

All of the questions in this practice set use the following dosage information for acetaminophen. For each question, determine if the ordered dose is safe.

Drug guide dosage information:

1-3 months:
10 mg/kg/dose PO Q4H PRN (max 60 mg/kg/day)
20 mg/kg/dose PR Q6H PRN (max 80 mg/kg/day)

3 months of age to adolescents:
10 to 15 mg/kg/dose PO Q4H PRN (max 75 mg/kg/day)
20 mg/kg/dose PR Q6H PRN (max 80 mg/kg/day)

Usual adult dose:
325 to 650 mg/dose PO Q4 to 6H PRN (max 4 g/24 hr)

Questions:

1. **Client information:** 12-year-old weighing 41 kg
   Ordered dose: 500 mg PO Q4H prn
2. **Client information:** 2-year-old weighing 12 kg
   Ordered dose: 250 mg PO Q4H prn
3. **Client information:** 1-month-old weighing 4 kg
   Ordered dose: 80 mg PR Q6H prn
4. **Client information:** 7-year-old weighing 23 kg
   Ordered dose: 300 mg PO Q4H prn
5. **Client information:** 4-year-old weighing 16 kg
   Ordered dose: 125 mg PO Q4H prn

Odd Answers:

1) Yes, this is a safe dose. 500 mg is in the safe range of 410-615 mg.

$\text{min dose} = \frac{10 \text{ mg}}{1 \text{ kg}} \times 41 \text{ kg} = 410 \text{ mg}$ $\text{max dose} = \frac{15 \text{ mg}}{1 \text{ kg}} \times 41 \text{ kg} = 615 \text{ mg}$

3) Yes, this is a safe dose. 80 mg is the correct dose when given rectally (PR).

$\text{dose} = \frac{20 \text{ mg}}{1 \text{ kg}} \times 4 \text{ kg} = 80 \text{ mg}$

5) No, this is not a safe dose. 125 mg is below the safe range of 160-240 mg.

$\text{min dose} = \frac{10 \text{ mg}}{1 \text{ kg}} \times 16 \text{ kg} = 160 \text{ mg}$ $\text{max dose} = \frac{15 \text{ mg}}{1 \text{ kg}} \times 16 \text{ kg} = 240 \text{ mg}$

Exercises Part F: Verifying Dosage Safety – phenoxymethyl penicillin

All of the questions in this practice set use the following dosage information for phenoxymethyl penicillin (Penicillin V). For each question, determine if the ordered dose is safe.

Drug guide dosage information:

Children:
8-17 mg/kg/dose PO Q8H OR 6-13 mg/kg/dose PO Q6H (max: 3 g/day).

Adults:
300-600 mg/dose PO Q6-8H.

Prophylaxis for Asplenia:
>3 months to 5 years old: 150 mg PO BID.
>5 years old: 300 mg PO BID.

Questions:

1. **Client information:** 6-year-old weighing 22 kg
   Ordered dose: 300 mg PO Q8H
2. **Client information:** 16-year-old weighing 63 kg
   Ordered dose: 600 mg PO Q6H
3. **Client information:** 8-year-old weighing 25 kg
   Ordered dose: 600 mg PO Q8H
4. **Client information:** 1-year-old weighing 9 kg with asplenia
   Ordered dose: 300 mg PO BID
5. **Client information:** 4-year-old weighing 13 kg
   Ordered dose: 150 mg PO Q6H

Odd Answers:

1) Yes, this is a safe dose. 300 mg is in the safe range of 176-374 mg.

$$\text{max dose} = \frac{17 \text{ mg}}{1 \text{ kg}} \times 22 \text{ kg} = 374 \text{ mg}$$

3) No, this is not a safe dose. 600 mg is above the safe range of 200-425 mg.

$$\text{max dose} = \frac{17 \text{ mg}}{1 \text{ kg}} \times 25 \text{ kg} = 425 \text{ mg}$$

5) Yes, this is a safe dose. 150 mg is in the safe range of 78-169 mg.

$$\text{max dose} = \frac{13 \text{ mg}}{1 \text{ kg}} \times 13 \text{ kg} = 169 \text{ mg}$$

Exercises Part G: Verifying Dosage Safety –

metoclopramide

All of the questions in this practice set use the following dosage information for metoclopramide. For each question, determine if the ordered dose is safe.

Drug guide dosage information:

GI Hypomotility and GE Reflux:
Children: 0.1-0.2 mg/kg/dose PO/IV/IM up to QID (Maximum: 0.5 mg/kg/24 hr).
Adults: 10-15 mg/dose PO/IV/IM QID.

Chemotherapy Induced Nausea and Vomiting (N&V)*
1mg/kg/dose IV/PO before chemotherapy, then 0.04 mg/kg/dose IV/PO Q6H (Maximum: 10 mg/dose).
Alternate dosing: 1 mg/kg IV/PO Q6H*

Post-Op or Opioid-Induced Nausea and Vomiting:
Children: 0.1-0.2 mg/kg/dose IV Q6-8H PRN. (Maximum 0.5 mg/kg/24 hr)
Children > 14 years and Adults: 10 mg/dose IV Q6-8H PRN

Questions:

1. **Client information:** 15-year-old weighing 50 kg with GE reflux.
   Ordered dose: 5 mg PO QID prn
2. **Client information:** 46-year-old weighing 88 kg with GI hypomotility.
   Ordered dose: 15 mg PO QID
3. **Client information:** 7-year-old weighing 23 kg with post-op N&V.
   Ordered dose: 2.5 mg IV Q8H prn
4. **Client information:** 6-year-old weighing 20 kg with chemo-induced N&V.
   Ordered dose: 0.8 mg PO Q6H
5. **Client information:** 12-year-old weighing 41 kg with post-op N&V.
   Ordered dose: 2.5 mg IV Q6-8H prn

Odd Answers:

1. Yes, this is a safe dose. 5 mg is in the safe range of 5-10 mg and does not exceed the daily maximum if all 4 prn doses were given.

   $\text{min dose} = \frac{0.1 \text{ mg}}{1 \text{ kg}} \times 50 \text{ kg} = 5 \text{ mg}$ $\quad \text{max dose} = \frac{0.2 \text{ mg}}{1 \text{ kg}} \times 50 \text{ kg} = 10 \text{ mg}$

   Max daily total 25mg/day

2. Yes, this is a safe dose as 2.5 mg is in the safe range of 2.3-4.6 mg and does not exceed the daily maximum if all 3 prn doses were given.

   $\text{min dose} = \frac{0.1 \text{ mg}}{1 \text{ kg}} \times 23 \text{ kg} = 2.3 \text{ mg}$ $\quad \text{max dose} = \frac{0.2 \text{ mg}}{1 \text{ kg}} \times 23 \text{ kg} = 4.6 \text{ mg}$

   Max daily total 11.6 mg/day.

3. No, this is not a safe dose. 2.5 mg is below the safe range of 4.1-8.2 mg and therefore the drug will not be in therapeutic range and the child's N&V will persist.

   $\text{min dose} = \frac{0.1 \text{ mg}}{1 \text{ kg}} \times 41 \text{ kg} = 4.1 \text{ mg}$ $\quad \text{max dose} = \frac{0.2 \text{ mg}}{1 \text{ kg}} \times 41 \text{ kg} = 8.2 \text{ mg}$

Exercises Part H: Verifying Dosage Safety – fentanyl

All of the questions in this practice set use the following dosage information for fentanyl. For each question, determine if the ordered dose is safe.

Drug guide dosage information:

Intermittent Dosing:
Infants: 1 to 2 mcg/kg/dose IV Q2-4H PRN (usual max 4 mcg/kg/dose)
Children: 1 to 2 mcg/kg/dose IV Q30-60 minutes PRN (usual adolescent starting dose: 25-50 mcg)
1 to 2 mcg/kg/dose buccal Q30-60 min. PRN. Maximum initial dose 50 mcg

Continuous IV infusion (by Acute Pain Service, ICU, or Palliative Care Specialists only):
Usual dose: 1 -4 mcg/kg/hr. Higher doses may be required in palliative care or end-of-life symptom management with monitored titration.

Intubation Dosage:
2 to 5 mcg/kg/dose IV over 1-2 min.

Questions:

1. **Client information:** 2-year-old weighing 14 kg.
   Ordered dose: 20 mcg IV q30-60min prn (IV intermittent).
2. **Client information:** 6-year-old weighing 38 kg.
   Ordered dose: 190 mcg/hour IV (continuous infusion).
3. **Client information:** 4-year-old weighing 65 lbs.
   Ordered dose: 120 mcg IV over 1-2 min (intubation dose).
4. **Client information:** 7-month-old weighing 9 kgs.
   Ordered dose: 9 mcg IV Q2-4h prn (IV intermittent).
5. **Client information:** 4-month-old weighing 10 lbs.
   Ordered dose: 16 mcg IV Q2-4h prn (IV intermittent).

Odd Answers:

1) Yes, this is a safe dose. 20 mcg is in the safe range of 14-28 mcg.

$\text{min dose} = \frac{1\text{ mcg}}{1\text{ kg}} \times 14\text{ kg} = 14\text{ mcg/dose}$ $\text{max dose} = \frac{2\text{ mcg}}{1\text{ kg}} \times 14\text{ kg} = 28\text{ mcg/dose}$

3) Yes, this is a safe dose. 120 mcg is in the safe range of 59-147.5 mcg.

$\text{weight conversion} = \frac{1\text{ kg}}{2.2\text{ lbs}} \times 65\text{ lbs} = 29.5\text{ kg}$ $\text{min dose} = \frac{2\text{ mcg}}{1\text{ kg}} \times 29.5\text{ kg} = 59\text{ mcg/dose}$ $\text{max dose} = \frac{5\text{ mcg}}{1\text{ kg}} \times 29.5\text{ kg} = 147.5\text{ mcg/dose}$

5) No, this is not a safe dose. 16 mcg is below the safe range of 4.5-9 mcg.

$\text{weight conversion} = \frac{1\text{ kg}}{2.2\text{ lbs}} \times 10\text{ lbs} = 4.5\text{ kg}$ $\text{min dose} = \frac{1\text{ mcg}}{1\text{ kg}} \times 4.5\text{ kg} = 4.5\text{ mcg/dose}$ $\text{max dose} = \frac{2\text{ mcg}}{1\text{ kg}} \times 4.5\text{ kg} = 9\text{ mcg/dose}$

Exercises Part I: Verifying Dosage Safety and Checking Orders – furosemide

All of the questions in this practice set use the following dosage information for infants and children receiving furosemide. For each question, determine if the ordered dose is safe as well as check the orders compared to the drug guide dosage information as you go.

Drug guide dosage information:

Oral Dose
Initial: 0.5-2 mg/kg/dose PO Q6H–Q24H
Maximum PO dose 4 mg/kg/dose

Parenteral Dose:
0.5-2 mg/kg/dose /IV Q6-24H PRN
Usual: 1 mg/kg/dose IV Q6-24H
The maximum single dose is 2 mg/kg/dose IV

Continuous IV Infusion in Critical Care areas:
0.1-0.5 mg/kg/hr

Questions:

1. **Client information:** 3-year-old weighing 43 lbs needing continuous IV infusion of furosemide in the PICU.
   Ordered dose: 9.75 mg/hour continuous IV infusion.
2. **Client information:** 7-year-old weighing 40 kgs, needing oral dosing of furosemide.
   Ordered dose: 20.4 mg/dose IV Q6h.
3. **Client information:** 15-month-old weighing 12 kgs, needing intermittent parenteral dosing of furosemide.
   Ordered dose: 18 mg/dose IV Q6h PRN.
4. **Client information:** 4-year-old weighing 38 lbs, needing oral dosing of furosemide.
   Ordered dose: 76 mg/dose PO Q6h.
5. **Client information:** 3-month-old weighing 9.7 lbs, needing continuous IV infusion of furosemide in the Emergency Department.
   Ordered dose: 2.2 mg/dose IV Q24h.

Odd Answers:

1) Yes, this is a safe dose. 9.75 mg/hr is in the safe range of 1.95-9.75 mg/hr with continuous infusion.

$\text{weight conversion} = \frac{1\text{ kg}}{2.2\text{ lbs}} \times 43\text{ lbs} = 19.5\text{ kg}$ $\text{min dose} = \frac{0.1\text{ mg}}{1\text{ kg}} \times 19.5\text{ kg} = 1.95\text{ mg/hour}$ $\text{max dose} = \frac{0.5\text{ mg}}{1\text{ kg}} \times 19.5\text{ kg} = 9.75\text{ mg/dose}$

3) Yes, this is a safe dose. 18 mg is in the safe range of 6-24 mg/dose.

$\text{min dose} = \frac{0.5\text{ mg}}{1\text{ kg}} \times 12\text{ kg} = 6\text{ mg/dose}$ $\text{max dose} = \frac{2\text{ mg}}{1\text{ kg}} \times 12\text{ kg} = 24\text{ mg/dose}$

5) No, 2.2 mg/dose would be the correct dosing for continuous IV furosemide. The order is wrong

because it is for intermittent IV dosing when the child needs continuous dosing of furosemide.

$$\text{weight conversion} = \frac{1\text{ kg}}{2.2\text{ lbs}} \times 9.7\text{ lbs} = 4.4\text{ kg}$$

$$\text{min dose} = \frac{0.1\text{ mg}}{1\text{ kg}} \times 4.4\text{ kg} = 0.44\text{ mg/dose}$$

$$\text{max dose} = \frac{0.5\text{ mg}}{1\text{ kg}} \times 4.4\text{ kg} = 2.2\text{ mg/dose}$$

Exercises Part J: Verifying Dosage Safety and Checking Orders – methotrimeprazine

All of the questions in this practice set use the following dosage information for methotrimeprazine. For each question, determine if the ordered dose is safe as well as check the orders compared to the drug guide dosage information as you go.

Drug guide dosage information:

Agitation, Aggression, Psychosis:
Children (<12 yrs):
Initial: 0.125 mg/kg/dose PO BID Increase gradually as needed to control agitation/psychosis.
IM: 0.06-0.125 mg/kg/dose IM once daily or div. TID.
Maximum: 40 mg/24 hr.

Adolescents/Adults:
Initial: 2.5-10 mg PO BID-TID Increase gradually as needed to control agitation/psychosis. In severe cases may start with 25 mg PO BID-TID, IM: 25 mg IM BID-QID

Pain, Agitation, Delirium (Palliative Care):
0.05-0.2 mg/kg/dose PO/IV Q6-8H PRN. When titrating IV doses, may give Q30-60 min until stable dose is reached.
Maximum 0.5 mg/kg/dose (50 mg/dose)

Questions:

1. **Client information:** 10-year-old weighing 74.5 lbs, needing intermittent IV administration of methotrimeprazine for delirium symptoms during palliation.
   Ordered dose: 3.7 mg/hour IV, after titration.

2. **Client information:** 5-year-old weighing 18 kgs, needing oral dosing of methotrimeprazine for agitation and aggression.
   Ordered dose: 1.1 mg/dose PO BID.
3. **Client information:** 14-year-old weighing 120 lbs, needing oral dosing of methotrimeprazine to control psychosis and agitation symptoms.
   Ordered dose: 96 mg/dose PO BID.
4. **Client information:** 3-year-old weighing 32 lbs, needing intramuscular dosing of methotrimeprazine for agitation.
   Ordered dose: 1.5 mg/dose IM once daily.
5. **Client information:** 17-month-old weighing 22 lbs, needing intermittent IV administration of methotrimeprazine for pain and delirium symptoms during palliation.
   Ordered dose: 1.5 mg/dose PO Q6h PRN.

Odd Answers:

1) No, this is a safe dose. 3.7 mg is in the safe range of 1.68-6.72 mg but the timing is off. The order is for continuous IV dosing when the patient needs intermittent IV administration.

$\text{weight conversion} = \frac{1\text{ kg}}{2.2\text{ lbs}} \times 74.5\text{ lbs} = 33.9\text{ kg}$ $\text{min dose} = \frac{0.05\text{ mg}}{1\text{ kg}} \times 33.9\text{ kg} = 1.70\text{ mg/dose}$ $\text{max dose} = \frac{0.2\text{ mg}}{1\text{ kg}} \times 33.9\text{ kg} = 6.78\text{ mg/dose}$

3) No, the weight does not matter, 2.5-10 mg PO BID-TID is the generic range.

5) No, the patient needs intermittent IV not oral, like what was ordered. Though 1.5 mg is within the safe range of 0.5-2 mg/dose and Q6-8h PRN is correct.

$\text{weight conversion} = \frac{1\text{ kg}}{2.2\text{ lbs}} \times 22\text{ lbs} = 10\text{ kg}$ $\text{min dose} = \frac{0.05\text{ mg}}{1\text{ kg}} \times 10\text{ kg} = 0.5\text{ mg/dose}$ $\text{max dose} = \frac{0.2\text{ mg}}{1\text{ kg}} \times 10\text{ kg} = 2\text{ mg/dose}$

6.5 IV FLOW RATES

Learning Objectives

By the end of this chapter, learners will be able to:

- define the terms rate, drop factor, and drop rate
- calculate the IV infusion rate
- calculate the IV drop rate
- calculate the IV infusion time

Define the Terms Rate, Drop Factor, and Drop Rate

When administering intravenous (IV) fluids, you will come across a variety of ways prescribers write IV orders. You will need to be familiar with the types of information presented in the order, how to use this information correctly when administering IV fluid, and how to make this information meaningful when discussing the IV treatment with the client. Much of this information is related to the amount of fluid they will receive, either the total amount or the amount infused each hour. The type of equipment you have available will impact how the IV infusion is set up, so it is important to understand how the equipment works and how it can impact how fluid is infused.

Calculate the IV Infusion Rate

The term rate refers to an amount over a period of time. When we consider intravenous (IV) fluid treatments, the rate is typically measured in millilitres/hour (mL/hr). Most of the time, IV pumps will be set to infuse IV fluids in mL/hr as well, but there are some exceptions when we are infusing IV medications. Just like other med math questions, you need to make sure you are always considering the type of units written in the order and what you have available in your supply when preparing medications as you may need to convert units before completing your calculation.

$$\text{rate} = \frac{\text{total volume}}{\text{time in hours}}$$

You can use the formula above to help you quickly calculate a rate if the rate is not specifically written in the order. You can also use dimensional analysis to calculate the rate when unit conversion is necessary.

Example 1

IV infusion order: Run 500 mL of normal saline over 4 hours. What rate would this IV run at?

Answer:

$$\text{rate} = \frac{\text{total volume}}{\text{time in hours}}$$

$$\text{rate} = \frac{500\ \text{mL}}{4\ \text{hours}}$$

$$\text{rate} = 125\ \text{mL/hr}$$

Example 2

IV infusion order: Run 1 L of D5W over 4 hours. What rate would this IV run at?

Answer:

$$\text{rate} = \frac{\text{total volume}}{\text{time in hours}}$$

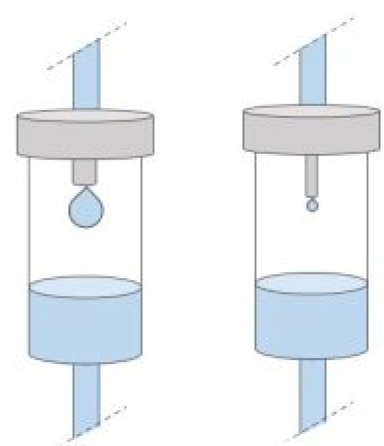
$$\frac{\text{mL}}{\text{hr}} = \frac{1000\ \text{mL}}{1\ \text{L}} \times \frac{1\ \text{L}}{4\ \text{hours}}$$

$$\frac{\text{mL}}{\text{hr}} = \frac{1000\text{ mL}}{1\ \cancel{\text{L}}} \times \frac{1\ \cancel{\text{L}}}{4\text{ hours}}$$

$$\frac{\text{mL}}{\text{hr}} = 250\text{ mL/hr}$$

Calculate the IV Drop Rate

When you watch an IV infuse, you can see drops falling from where the IV tubing enters the drop chamber. The **drop rate** refers to how fast the IV fluid is dripping, measured in drops/minute (gtts/min). When there is no pump available, you must set the rate manually by using the roller clamp to control the flow and watch how quickly the drops are dripping in the drip chamber. In order to determine exactly how fast we should let the IV drip, we need to consider how big each drop is and count how many drops are falling in a minute. Each IV set you use has a specific size drop it produces, some large, others small. The size of the drop is reflected in the **drop factor** which is identified on the IV set packaging. The drop factor tells you how many drops are in 1 mL of fluid, which you can use to help calculate the rate the IV should run. In the image below, you can see the size difference between a large drop of a macro set and a small drop of a micro set.



Macro vs. Micro Drip

Note the drop rate is measured in gtts/min, versus the infusion rate which is measured in mL/hr. To calculate the drop rate you must know the infusion rate and the drop factor of the IV tubing. You can use the formula below, but only if you convert the rate of mL/hr to mL/min. As IV rates are usually given in mL/1 hr, you should be able to use 60 minutes as the denominator in most cases.

Drop Rate = Infusion Rate x Drop Factor

You can see how this works using dimensional analysis:

Consider an IV running at 125mL/hr, using a standard tubing set with a drop factor of 10 gtts/mL. What is the drop rate in gtts/min?

$$\frac{\text{gtts}}{\text{min}} = \frac{10 \text{ gtts}}{1 \text{ mL}} \times \frac{125 \text{ mL}}{1 \text{ hr}} \times \frac{1\text{hr}}{60 \text{ min}}$$

$$\frac{\text{gtts}}{\text{min}} = \frac{10 \text{ gtts}}{1 \cancel{\text{mL}}} \times \frac{125 \cancel{\text{mL}}}{1 \cancel{\text{hr}}} \times \frac{1 \cancel{\text{hr}}}{60 \text{ min}}$$

$$\frac{\text{gtts}}{\text{min}} = \frac{10 \text{ gtts x } 125}{60 \text{ min}}$$

$$\frac{\text{gtts}}{\text{min}} = \frac{1250 \text{ gtts}}{60 \text{ min}}$$

$$\frac{\text{gtts}}{\text{min}} = 20.83$$

$$\frac{\text{gtts}}{\text{min}} = 21$$

*round to the whole number as you cannot count a portion of a drop

Critical Thinking Questions

1. If an IV set creates a large drop, will the patient finish the infusion more quickly than an IV set that creates a small drop?
2. If the drop rate is calculated at 15.5, what do you set the drop rate at when administering the IV?

Answers:

1. If the rate is set in mL/hr, then both infusions should finish at the same time. Macro drips will take fewer drops to fill 1 mL than a micro drip set, so if you compare the drops falling in the drop chamber, the drops per minute in the IV set with small drops will be running at a faster rate (drops/min). If we compare two infusions with the same drop rate, then the infusion running with the macro set would finish faster as more volume per drop is being infused, however, IV infusions are not ordered with a drop/minute rate because of this variability.
2. Drops must be rounded up or down, as you cannot count a portion of a drop. So do you

choose 15 or 16? The actual difference of fluid administered in one hour between a drop rate of 15 and 16 is only 6 mL. When the IV infusion is running by gravity, there are fluctuations in the actual rate of the IV infusion and you will need to keep readjusting the drop rate to keep the IV running at the appropriate rate. So, it is likely that you can choose either drop rate, knowing that you will be frequently readjusting the drop rate to ensure the fluid infuses in the correct amount of time. However, if the extra 6 mL/hour of fluid would be detrimental to your client, then you should find a pump and ensure a more exact rate is delivered to the client. Instances where the infusion is a medication or the client is at risk for fluid overload may be circumstances that require the pump versus gravity.

Example 3

Calculate the drop rate for the following order:

Give NS IV 70 mL/hr.

The IV set has a drop factor of 10 gtts/min.

Answer:

$$\frac{\text{gtts}}{\text{min}} = \frac{10\text{ gtts}}{1\text{ mL}} \times \frac{70\text{ mL}}{1\text{ hr}} \times \frac{1\text{hr}}{60\text{ min}}$$

$$\frac{\text{gtts}}{\text{min}} = \frac{10\text{ gtts x }70}{60\text{ min}}$$

$$\frac{\text{gtts}}{\text{min}} = \frac{700\text{ gtts}}{60\text{ min}}$$

$$\frac{\text{gtts}}{\text{min}} = 11.66$$

$$\frac{\text{gtts}}{\text{min}} = 12$$

Calculate the IV Infusion Time

In some cases, you may want to know how long an IV infusion is going to take. Perhaps you need to know how long the IV will take to infuse to inform the patient how long it will be until they take a shower, or to be discharged home. Most IV pumps will display the remaining time of the infusion, but you may need this information before you have programmed infusion details into the IV pump. Understanding how to calculate infusion time can help you be prepared to plan the timing of infusions and answer questions about the infusion time promptly.

$$\text{infusion time} = \frac{\text{total volume}}{\text{rate}}$$

Example 4

A client has the following order: Infuse 1 L NS at 125 mL/hr.
How long will this take to infuse?

Answer:

$$\text{infusion time} = \frac{\text{total volume}}{\text{rate}}$$

$$\text{infusion time} = \frac{1000}{125}$$

$$\text{infusion time} = 8$$

Here is the same question showing how dimensional analysis is used, and where the units will cancel out:

$$\text{infusion time} = \frac{1\text{ hr}}{125\text{ mL}} \times 1000\text{ mL}$$

$$\text{infusion time} = \frac{1\text{ hr}}{125\ \cancel{\text{mL}}} \times 1000\ \cancel{\text{mL}}$$

$$\text{infusion time} = 8\text{ hrs}$$

Critical Thinking Questions

A patient asks you how much longer their IV will run for. You manually calculate the remaining amount of time and determine the answer is 1.66 hours. How could you communicate this information to the client in a way that is easier to understand?

Answer:
You could report the remaining amount of time in hours and minutes. First, convert 0.66 into minutes (39.6 mins) and then inform the patient the infusion will run for about 1 hour and 40 minutes.

You could inform the client what time the infusion will finish. For instance, if it was 0800 hours, you could report the infusion will be finished around 0940 hours if the infusion is uninterrupted.

Key Concepts

- The infusion rate is measured in mL/hr.
- The drop rate is measured in gtts/hr.
- Micro drip tubing has a drop factor of 60 gtts/mL.
- Standard tubing has a drop factor of 10 gtts/mL.
- Drop rate = Infusion Time x Drop Factor

Exercises Part A: Calculating IV Flow Rate

Calculate the IV flow rate for the following IV infusion orders:

1. Run 500 mL of normal saline over 2 hours.
2. Give 1 L of Ringer's lactate over 12 hours.
3. Run 1 L of normal saline over 4 hours.
4. Administer 1 L bolus of NS over 2 hours.
5. Give 250 ml of 3% sodium chloride over 30 minutes.
6. Run 1 L D5 0.45% NS over 5 hours.
7. Infuse 500 mL D10W over 4 hours.
8. Give 1,000 mL of D5W over 8 hours.
9. Administer 750 mL of Ringer's lactate over 8 hours.
10. Give a 500 mL bolus of NS over an hour and a half.

Odd Answers:

Answers should be in mL/hr. Use dimensional analysis or the formula $\text{rate} = \frac{\text{total volume}}{\text{total time}}$.

1) $\text{rate} = \frac{500 \text{ mL}}{2 \text{ hours}} = 250 \text{ mL/hr}$

3) $\text{rate} = \frac{1000 \text{ mL}}{4 \text{ hours}} = 250 \text{ mL/hr}$

5) $\text{rate} = \frac{250 \text{ mL}}{30 \text{ minutes}} = \frac{250 \text{ mL}}{0.5 \text{ hours}} = 500 \text{ mL/hr}$

7) $\text{rate} = \frac{500 \text{ mL}}{4 \text{ hours}} = 125 \text{ mL/hr}$

9) $\text{rate} = \frac{750 \text{ mL}}{8 \text{ hours}} = 94 \text{ mL/hr}$

Exercises Part B: Calculating IV Flow Rates

Calculate the IV flow rate for the following IV infusion orders:

1. Give 600 mL over 120 minutes.
2. Administer 1 L of NS over 8 hours.
3. Run 750 mL over 4 hours.
4. Run 500 mL over 210 minutes.
5. Give a 1 L Ringer's Lactate bolus over 90 minutes.
6. Administer 800 mL over 6 hours.
7. Administer 500 mL over 2 hours.
8. Run 800 mL NS over 4 hours.
9. Give a 250 mL bolus over 30 minutes.
10. Run 500 mL over 6 hours.

Odd Answers:

1) $\text{rate} = \frac{600 \text{ mL}}{120 \text{ minutes}} = \frac{600 \text{ mL}}{2 \text{ hours}} = 300 \text{ mL/hr}$

3) $\text{rate} = \frac{750 \text{ mL}}{4 \text{ hours}} = 187.5 \text{ mL/hr}$

5) $\text{rate} = \frac{1000 \text{ mL}}{90 \text{ minutes}} = \frac{1000}{1.5 \text{ hours}} = 667 \text{ mL/hr}$

7) $\text{rate} = \frac{500 \text{ mL}}{2 \text{ hours}} = 250 \text{ mL/hr}$

9) $\text{rate} = \frac{250 \text{ mL}}{30 \text{ minutes}} = \frac{250}{0.5 \text{ hours}} = 500 \text{ mL/hr}$

Exercises Part C: Calculating Infusion Rate

Calculate the infusion rate for the following IV orders:

1. You need to administer 10,800 mL of Ringer's lactate over 24 hours to a patient with burns to 45% of their total body surface area. How many mL/hour will you program on the IV pump?
2. A person arrives at the emergency room in hemorrhagic shock. The physician prescribes 1 unit of packed red blood cells (PRBC) to be given over 20 minutes and the unit you collect from blood services has 346 mL of PRBC. What would you program as the infusion rate on the pump?
3. A person is severely hypokalemic but is NPO and cannot take oral potassium. Your order is to give 40 mEq of potassium in 1L lactated Ringer over 4 hours. What would your infusion rate be?
4. You are working on the surgical floor and one of your patients is having severe postoperative nausea after hip replacement surgery. The physician has ordered 4 mg of ondansetron to help with this nausea. Using the parenteral manual you have decided to dilute the medication in a 50 mL minibag and give it over 15 minutes because your patient is over 65 years old but has been tolerating fluids well. What will your infusion rate be?
5. You have a patient on the medical floor who is having an allergic reaction to one of the medications that the doctor has ordered. The doctor has now ordered 50 mg diphenhydramine which you have diluted in a 50 mL minibag that you are going to run over 20 minutes. What will your infusion rate be?
6. A patient comes into the emergency room with the flu severely dehydrated and unable to tolerate fluids by mouth. The physician orders 1 L of Ringer's lactate over four and a half hours. What will your infusion rate be?
7. You have a child who came into the hospital in ventricular fibrillation. The doctor has ordered 160 mg of amiodarone which you have diluted in a 50 mL minibag of D5W and you choose to give this dose over 30 minutes. What will your infusion rate be?

8. A woman who is 26 weeks pregnant was admitted for hyperemesis gravidarum treatment. The midwife has ordered 100 mg diphenhydrinate IV. You have diluted this in 1 L of D5W and are going to give over 10 hours. What will your infusion rate be?
9. You are receiving a postoperative patient from the PAR who is on a ketorolac infusion that is almost complete. The surgeon wants another 10 mg hung and you add the medication into a 50 mL bag to run over 30 minutes. What will your infusion rate be?
10. Your patient has been NPO for a couple of days as their surgery has been postponed. In the meantime, the physician has ordered 1 L over 4 hours. What will the infusion rate be?

Odd Answers:

1) $\text{rate} = \frac{10800 \text{ mL}}{24 \text{ hours}} = 450 \text{ mL/hr}$

3) $\text{rate} = \frac{1000 \text{ mL}}{4 \text{ hours}} = 250 \text{ mL/hr}$

5) $\text{rate} = \frac{50 \text{ mL}}{20 \text{ minutes}} = 150 \text{ mL/hr}$

7) $\text{rate} = \frac{50 \text{ mL}}{30 \text{ minutes}} = \frac{50 \text{ mL}}{0.5 \text{ hours}} = 100 \text{ mL/hr}$

9) $\text{rate} = \frac{50 \text{ mL}}{30 \text{ minutes}} = \frac{50 \text{ mL}}{0.5 \text{ hours}} = 100 \text{ mL/hr}$

Exercises Part D: Calculating IV Drop Rate

Calculate the drop rate for the following IV orders:

1. IV NS 125 ml/hr using standard tubing
2. IV D5W 80 ml/hr using standard tubing
3. IV RL 150 ml/hr using standard tubing
4. IV NS 25 ml/hr using micro drip tubing
5. IV RL 35 ml/hr using micro drip tubing

6. IV 0.45% NS 100 ml/hr using standard tubing
7. IV NS 130 ml/hr using standard tubing
8. IV D5W 15 ml/hr using micro drip tubing
9. IV NS 55 ml/hr using standard tubing
10. IV NS 200 ml/hr using standard tubing

Odd Answers:

1) $\frac{\text{gtts}}{\text{min}} = \frac{\text{10 gtts x 125}}{\text{60 min}} = 21$

3) $\frac{\text{gtts}}{\text{min}} = \frac{\text{10 gtts x 150}}{\text{60 min}} = 25$

5) $\frac{\text{gtts}}{\text{min}} = \frac{\text{60 gtts x 35}}{\text{60 min}} = 35$

7) $\frac{\text{gtts}}{\text{min}} = \frac{\text{10 gtts x 130}}{\text{60 min}} = 22$

9) $\frac{\text{gtts}}{\text{min}} = \frac{\text{10 gtts x 55}}{\text{60 min}} = 9$

Exercises Part E: Calculating IV Drop Rate

Calculate the drop rate for the following IV orders:

1. Run 1L NS over 4 hours, standard tubing
2. IV RL 70 ml/hr, macro drip tubing 20 gtts/min
3. IV NS 25 ml/hr, micro drip tubing
4. IV D5W 175 ml/hr, standard tubing
5. IV RL 76 ml/hr, standard tubing
6. Infuse 500 ml NS over 2 hours, standard tubing

7. IV NS 135 ml/hr, standard tubing
8. Give 1L RL over 8 hours, standard tubing
9. IV NS 40 ml/hr, macro drip tubing 15 gtts/min
10. IV D5W 60ml/hr, standard tubing

Odd Answers:

1) $\text{rate} = \frac{\text{1000 mL}}{\text{4 hours}} = \text{250 mL/hr} = \frac{\text{10 gtts x 250}}{\text{60 min}} = 42$

3) $\frac{\text{gtts}}{\text{min}} = \frac{\text{60 gtts x 25}}{\text{60 min}} = 25$

5) $\frac{\text{gtts}}{\text{min}} = \frac{\text{10 gtts x 75}}{\text{60 min}} = 13$

7) $\frac{\text{gtts}}{\text{min}} = \frac{\text{10 gtts x 135}}{\text{60 min}} = 23$

9) $\frac{\text{gtts}}{\text{min}} = \frac{\text{15 gtts x 40}}{\text{60 min}} = 10$

Exercises Part F: Calculating IV Drop Rate

Calculate the drop rate for the following IV orders:

1. IV RL 50 mL/hr using standard tubing.
2. IV NS 175 mL/hr using microdrip tubing.
3. IV D5W 500 mL over 3 hours using microdrip tubing.
4. IV RL 250 mL/hr using standard tubing.
5. IV NS 1L over 6 hours using standard tubing.
6. IV RL 75 mL/hr using standard tubing.
7. IV D5W 250 mL over 3 hours using standard tubing.

8. IV D5W 100 mL/hr using macrodrip tubing 20 gtts/min.
9. IV NS 30 mL/hr using microdrip tubing.
10. IV NS 50 mL over 30 minutes using macrodrip tubing 15 gtts/min.

Odd Answers:

1) $\text{rate} = 50\text{ mL/hr} = \frac{10\text{ gtts x }50}{60\text{ min}} = 8.3$

3) $\text{rate} = \frac{500\text{ mL}}{3\text{ hours}} = 166\text{ mL/hr} = \frac{60\text{ gtts x }166}{60\text{ min}} = 167$

5) $\text{rate} = \frac{1000\text{ mL}}{6\text{ hours}} = 166.7\text{ mL/hr} = \frac{10\text{ gtts x }166.7}{60\text{ min}} = 27.8$

7) $\text{rate} = \frac{250\text{ mL}}{3\text{ hours}} = 83.3\text{ mL/hr} = \frac{10\text{ gtts x }83.3}{60\text{ min}} = 13.9$

9) $\text{rate} = 30\text{ mL/hr} = \frac{60\text{ gtts x }30}{60\text{ min}} = 30$

Exercises Part G: Calculating Infusion Time

Calculate the infusion time for the following problems:

1. A client has an IV infusing at 70 mL/hr. There is 140 mL left to be infused. How long will it take to infuse the rest of the fluid?
2. A client has an IV infusing at 125 ml/hr. There is 587 mL left to be infused. How long will it take to infuse the rest of the fluid?
3. A client has an IV infusing at 150 mL/hr. There is 225 mL left to be infused. How long will it take until the IV fluid has been infused?
4. A client has an IV infusing at 100 mL/hr. There is 850 mL left to be infused. How long will it take until the IV fluid has been infused?
5. A client has an IV infusing at 80 mL/hr. There is 320 mL left to be infused. How long will it take until the IV fluid has been infused?

6. A client has an IV infusing at 125 mL/hr. There is 725 mL left to be infused. How long will it take until the IV infusion is complete?
7. A client has an IV infusing at 80 mL/hr. There is 409 mL left to be infused and it is currently 1945. What time should you expect to change the IV bag?
8. A client has an IV infusing at 125 mL/hr. They can be discharged when the IV bag is empty. There is currently 340 mL of fluid remaining. How long will it be until they can be discharged?
9. A client has an IV infusing at 100 mL/hr. There is 385 mL left to be infused and it is currently 0735. What time should you expect to change the IV bag?
10. A client has an IV bolus running at 500 mL/hr, with 200 mL already infused. It is currently 1425. When the bolus is complete they will be taken for a CT scan. What time should you ask the porter to pick up the client?

Odd Answers:

1) $\text{infusion time} = \frac{140}{70} = 2 \text{ hrs}$

3) $\text{infusion time} = \frac{225}{150} = 1.5 \text{ hr} = 1 \text{ hr } 30 \text{ min}$

5) $\text{infusion time} = \frac{320}{80} = 4 \text{ hrs}$

7) $\text{infusion time} = \frac{409}{80} = 5.1125 \text{ hrs} = 5 \text{ hr } 7\text{min} = 0052 \text{ hours}$

9) $\text{infusion time} = \frac{385}{100} = 3.85 \text{ hrs} = 3 \text{ h } 51 \text{ min hrs} = 1126 \text{ hours}$

Exercises Part H: Calculating Infusion Time

Calculate the infusion time for the following problems:

1. A client has an IV infusing at 75 mL/hr. There is 346 mL left to be infused. How long until you will need to change the IV bag?

2. A client has an IV infusing at 250 mL/hr. There is 115 mL left to be infused. The patient will be discharged when the infusion is finished, how long until the patient can be discharged?
3. A client has an IV infusing at 33 mL/hr. There is 453 mL left to be infused. It is 0945, at what time should you change the IV bag next?
4. A client has an IV infusing at 80 mL/hr. There is 235 mL left to be infused. How long until you change to the next IV bag?
5. A client has an IV infusing at 125 mL/hr. There is 167 mL left to be infused. It is 1000, when can you change the IV bag next?
6. A client has an IV infusing at 100 mL/hr. There is 245 mL left to be infused. It is 1635, when should the IV bag be changed next?
7. A client has an IV infusing at 55 mL/hr. There is 374 mL left to be infused. It is 0745, the patient will be going to physio after their infusion is complete, when will the physio expect them?
8. A client has an IV infusing at 500 mL/hr. There is 238 mL left to be infused. When the bolus is completed, the patient is to go to have an MRI, when should you tell the MRI staff to expect the patient?
9. A client has an IV infusing at 200 mL/hr. There is 784 mL left to be infused. How long until you change the IV bag?
10. A client has an IV infusing at 83 mL/hr. There is 675 mL left to be infused. Your patient can be discharged when the infusion finishes, how long until the patient can be discharged?

Odd Answers:

1) $\text{infusion time} = \frac{346}{75} = 4.613 \text{ hrs} = 4 \text{ hr } 37 \text{ min}$

3) $\text{infusion time} = \frac{453}{33} = 13.7 \text{ hr} = 13 \text{ hr } 44 \text{ min, change at } 2329 \text{ hrs}$

5) $\text{infusion time} = \frac{167}{125} = 1.336 \text{ hrs} = 1 \text{ hr } 20 \text{ min, change at } 1120 \text{ hrs}$

7) $\text{infusion time} = \frac{374}{55} = 6.8 \text{ hrs} = 6 \text{ hr } 48 \text{ min}$

$07{:}45 + 6 \text{ hr} = 13{:}45 + 48 \text{ min} = 14{:}33 \text{ is when physio should expect the patient}$

9) $\text{infusion time} = \frac{784}{200} = 3.92 \text{ hrs} = 3 \text{ h } 55 \text{ min}$

6.6 ADMINISTERING MEDICATIONS IV DIRECT

Learning Objectives

By the end of this chapter, learners will be able to:

- identify factors that affect the rate of administration of IV direct medications
- create a dose, volume, and timetable for use in the administration of IV direct medications
- label a syringe to help keep track of the medication delivery rate

Identify Factors That Affect the Rate of Administration of IV Direct Medications

It is essential for medications administered directly into an IV line to be given at the correct rate to prevent complications. Information describing the appropriate rate will be available in the health agency's drug reference manual. Ensure all factors about the patient's context are considered when reviewing rate information in the reference manual. For instance, the rate may be different for a neonate versus an adult. In some cases, a variable rate is given and you will need to use critical thinking skills to determine the most appropriate rate, often in discussion with your nursing instructor or the most responsible nurse for the patient.

Create a Dose, Volume, and Timetable For Use in the Administration of IV Direct Medications

To determine how fast to administer an IV direct medication, you will need the following information:

- the medication order
- information about the patient's context (eg. age, the reason for medication order, hepatic function…)
- the administration information from the drug reference manual
- the total volume of medication to be administered

Once the rate of administration of the drug is determined, the actual administration of the drug must be timed to ensure it is given at the correct rate. To administer the drug evenly over a particular period, nurses often calculate the volume of medication to be given over specific time intervals (for example, every 15 seconds) during the administration period. The drug information in the text box below will be used to exemplify this process. It is important to note the information presented here should not be used for direct patient care, always review your agency's drug reference manual for up-to-date information regarding the drug and the administration policy of your institution.

diazepam

Status Epilepticus: 5-10 mg IV/IM q5-10min; not to exceed 30 mg, OR 0.5 mg/kg PR (using the parenteral solution), THEN 0.25 mg/kg in 10 minutes PRN
IV Direct: Administer undiluted over 3 min; no more than 5 mg/min

https://reference.medscape.com/drug/valium-diastat-diazepam-342902

1. Gather the information that will affect the rate of administration.

Medication order: diazepam 10 mg
Rate information: over 3 min; no more than 5mg/min
Supply: the concentration of diazepam in a vial is 5 mg : 1 mL
Calculate the total volume to be administered:

$$x \text{ mL} = \frac{1 \text{ mL}}{5 \text{ mg}} \times 10 \text{ mg}$$

$$x \text{ mL} = \frac{1 \text{ mL}}{5 \cancel{\text{mg}}} \times 10 \cancel{\text{mg}}$$

$$x \text{ mL} = \frac{1 \text{ mL} \times 10}{5}$$

$$x \text{ mL} = 2 \text{ mL}$$

2. Determine what volume of medication will be given at each time interval:

When beginning to administer IV direct medications, it can be hard to ensure you are maintaining an even speed of delivery
over the whole period. To stay within a consistent delivery speed, you can create a table to determine how much of the syringe volume should be given over smaller time intervals to ensure you have an even speed throughout the injection. This can be called a dose, volume, time table. Including the amount, volume, and time interval in the table is helpful when checking to ensure you are meeting the criteria as laid out in the drug reference manual as rate information is often given in an amount (usually mg or mcg) per time interval (usually min or sec).
In this dose, volume, and timetable, the total amount, volume, and time are entered into the second row. Each subsequent value has been cut in half from the value above.

Amount (mg)	**Volume (mL)**	**Time Interval**
10	*2*	*3 min (180 sec)*
5	*1*	*90 sec*
2.5	*0.5*	*45 sec*
1.25	*0.25*	*22.5 sec*

This is not the only way to set up the table. Here is an alternate table in which each value has been divided by 3:

Amount (mg)	**Volume (mL)**	**Time Interval**
10	*2*	*3 min*
3.33	*0.66*	*1 min (60 sec)*
1.11	*0.22*	*20 sec*

And finally, a table where each value has been divided by 5:

Amount (mg)	**Volume (mL)**	**Time Interval**
10	*2*	*3 min (180 sec)*
2 mg	*0.4*	*36 sec*
1 mg	*0.2*	*18 sec*

In this example, clinical judgment must be used to determine which one will give a volume and time interval easiest for a nurse to follow. The middle table might be the easiest to use in practice, as watching a timer and noting how much you have given every 20 seconds is easier to follow than watching for every 22.5 or 18 seconds. You may also find any number you divide by gives a difficult number to measure with the available syringes. In this case, you will need to round the number so the volume can be measured easily on the syringe. For example, if you calculated the final volume to be 1.08 mL you could round to 1 mL. If you are rounding down, you are giving slightly less volume over the allotted time and should be safe for the client to receive. You must be very cautious about rounding up, as the medication would be given slightly faster. Make sure to discuss the rationale for rounding with your preceptor or instructor to ensure you are making appropriate decisions.

Example 1

Complete a dose, volume, and timetable when administering the following medication:

Medication Order: ketorolac 30 mg IV prn q8h
Rate Information: undiluted over 1-2 minutes
Supply: ketorlac 30 mg/mL

Answer:

Amount (mg)	Volume (mL)	Time Interval
30	1	2 min
15	0.5	1 min
7.5	0.25	30 sec

Example 2

Create a dose, volume, and time chart for the following medication order.

Medication Order: furosemide 40 mg IV BID

Rate Information: administer undiluted at a rate of 20 mg/min
Supply: 20 mg/1 mL ampule

Answer:
First, calculate the total volume of the injection.

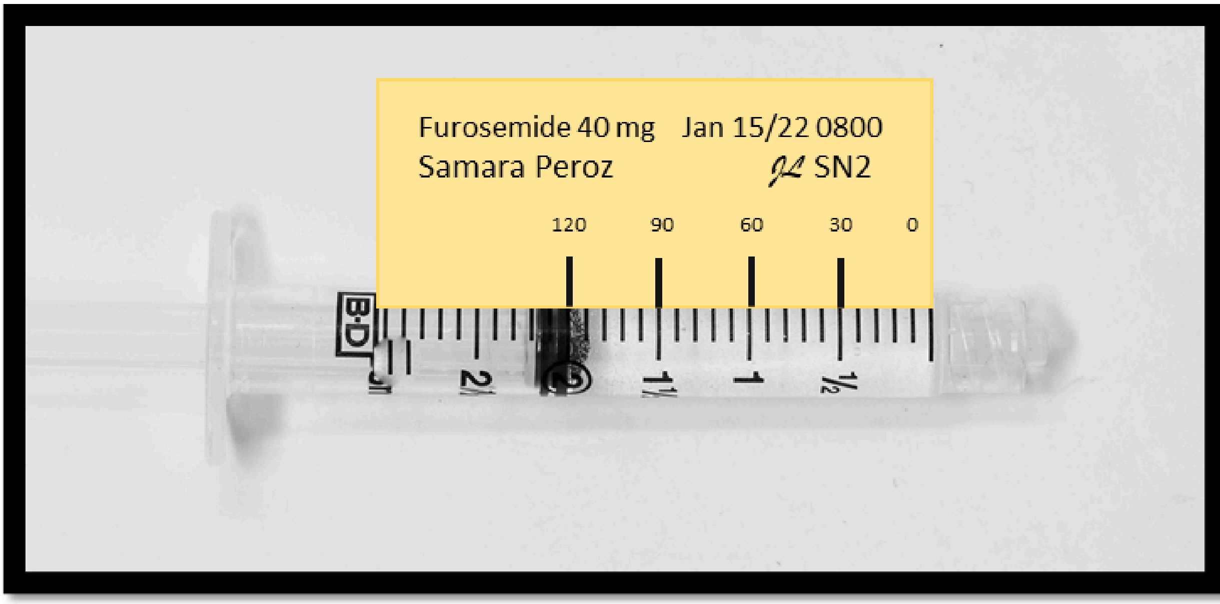
$x\ \text{mL} = \frac{1\ \text{mL}}{20\ \text{mg}} \times 40\ \text{mg}$ $\quad x\ \text{mL} = \frac{1\ \text{mL}}{20\ \text{mg}} \times 40\ \text{mg}$ $\quad x\ \text{mL} = \frac{40\ \text{mL}}{20}$ $\quad x\ \text{mL} = 2\ \text{mL}$

Enter the total amount, volume, and time in the first row. In this case, the recommended rate of 20 mg/min is represented in the third row of the chart and thus meets the criteria for safe administration.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 2.

Amount (mg)	Volume (mL)	Time Interval
40	2	2 min
20	1	1 min
10	0.5	30 sec

Label a Syringe to Help Keep Track of the Medication Delivery Rate.

It can be challenging for some people to focus on the amount of volume being administered in each short time interval while in the practice setting, even while watching a timer. Distractions such as alarms, conversations, and/or questions from the patient or family and other interruptions can break the focus of the nurse during this process. When beginning to administer IV direct medications, it may be helpful to add a visual cue to the syringe label to help keep on track. In the image below, two components have been added to a syringe label for the administration of furosemide. Lines to indicate every 0.5 mL on the syringe and numbers representing the seconds remaining at each interval are added. These correspond to the dose, volume, time table created in the sample exercise above, in which 0.5 mL is given every 30 seconds.



Critical Thinking Question

When comparing the same dose of a particular medication given by IV direct versus an intermittent IV infusion, do you anticipate any differences in the onset, peak, and duration of the medication?

Answer:

The onset occurs when the medication enters the bloodstream and can take effect in the body. It varies with mechanism of action, dosage, and individual patient factors. The IV direct route is often delivered over a total shorter time than the intermittent IV route. If the medication is delivered into the vein more quickly, it is likely the onset will be earlier for IV direct than IV intermittent. It may affect the peak for some medications as well. The duration of the effect is more variable. The duration of IV intermittent may be longer, especially for infusions that are given over longer periods. If you think variations in onset, peak, and duration of various routes will affect the choice of route given to a patient, you should check the parenteral manual for guidelines on specific medications when preparing the medication.

Exercise Part A : Creating Dose, Volume, Time Tables

Create a dose, volume, time table for the following scenarios.

1. Medication Order: dexamethasone 4 mg IV q6h
   Rate Information: administer undiluted, no more than 8mg/min
   Supply: 20 mg/5 mL vial
2. Medication Order: pantoprazole 40 mg IV OD
   Rate Information: administer over 2 minutes
   Supply: 40 mg powder in medication vial, to be reconstituted with 10 mL NS
3. Medication Order: benztropine 2 mg IV now
   Rate Information: administer undiluted over 1 minute
   Supply: 2 mg/2 mL ampule
4. Medication Order: morphine 2 mg IV q2h prn
   Rate Information: dilute with 9 mL NS or SW for injection to give 1 mg/mL and give at a maximum rate of 2 mg/min
   Supply: 10 mg/mL ampule
5. Medication Order: oxytocin 8 units once postpartum bolus
   Rate Information: may dilute with 3 mL NS, given slowly over at least 1 minute
   Supply: 10 units/mL vial
6. Medication Order: haloperidol 2 mg IV push PRN.
   Rate Information: dilute with 5 mL SW and give rapid IV push over at least 1 minute.
   Supply: 5 mg/mL (1 mL) ampule.

Odd Answers:

1) First, calculate the total volume of the injection.

$x\ mL = \frac{5\ mL}{20\ mg} \times 4\ mg$ $x\ mL = \frac{5\ mL}{20\ mg} \times 4\ mg$ $x\ mL = \frac{20\ mL}{20}$ $x\ mL = 1\ mL$ Enter the total amount, volume and time in the first row. Choosing to give 4 mg in 1 minute is giving the medication slower than the maximum rate of 8 mg/min, and meets the criteria for safe administration.

Next, choose an amount to divide each value by. In this answer, each value has been divided by 2.

Amount (mg)	Volume (mL)	Time Interval
4	1	1 min
2	0.5	30 sec
1	0.25	15 sec

3) First, note the total volume of the injection (2 mg in 2 mL).
Enter the total amount, volume, and time in the first row.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 2.

Amount (mg)	Volume (mL)	Time Interval
2	2	1 min
1	1	30 sec
0.5	0.5	15 sec

5) First, calculate the total volume of the injection. $x\text{ mL} = \frac{1\text{ mL}}{10\text{ units}} \times 8\text{ units}$ $x\text{ mL} = \frac{1\text{ mL}}{10\text{ units}} \times 8\text{ units}$ $x\text{ mL} = \frac{8\text{ mL}}{10}$ $x\text{ mL} = 0.8\text{ mL}$ Enter the total amount, volume and time in the first row.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 2.

Amount (units)	Volume (mL)	Time Interval
8	0.8	1 min
4	0.4	30 sec
2	0.2	15 sec

Exercises Part B: Creating Dose, Volume, Time Tables

Create a dose, volume, time table for the following scenarios. Identify the final volume and time interval you would use when administering the medication.

1. Medication Order: dexamethasone 15 mg IV once
   Rate Information: max rate of 8 mg/min
   Supply: 4 mg/mL vial
2. Medication Order: diphenhydrinate 35 mg IV push q4h.
   Rate Information: dilute every 50 mg with 10 mL NS and administer over 2-4 minutes.
   Supply: 50 mg/mL (1 mL) vial.
3. Medication Order: meropenum 600 mg IV q8h.
   Rate Information: reconstitute with 20 mL NS or D5W and give over 3-5 minutes.
   Supply: 1 g (powdered) vial.
4. Medication Order: thiamine 100 mg IV push once daily for 5 days.
   Rate Information: give undiluted over 1 minute.
   Supply: 100 mg/mL (1 mL) vial.
5. Medication Order: morphine 2.5 mg IV push q4h.
   Rate Information: max rate of 2 mg/mL, diluted to 1 mg/mL with NS (10 mL of fluids).
   Supply: 10 mg/mL
6. Medication Order: flumazenil 0.2 mg for suspected benzo overdose, possibly giving additional doses if inadequate results.
   Rate Information: give over 30 seconds.
   Supply: 0.1 mg/mL (5 mL) vial.
7. Medication Order: ampicillin 25 mg/kg/dose, for a 15 kg child, IV push for a mild infection.
   Rate Information: give over 4 minutes at a max rate of 100 mg/minute, diluted to 250 mg/2.5 mL.
   Supply: 250 mg (powdered) vial.
8. Medication Order: ketorolac 20 mg q4h IV push.

Rate Information: give over 1-2 minutes undiluted.
Supply: 30 mg/mL (1 mL) vial.

9. Medication Order: diazepam 8 mg q3h.
Rate Information: give undiluted 2-5 mg/minute.
Supply: 5 mg/mL (2 mL) ampule.

10. Medication Order: midazolam 0.2 mg/kg IV push loading dose, repeating q5 minute until seizure stops. This is for a 65 kg adult.
Rate Information: given at a rate of 2 mg/minute.
Supply: 1 mg/mL

Odd Answers:

1) First, calculate the total volume of the injection. In this answer, in the table set up, each value has been divided by 2.
When you calculate a volume that is not a round decimal number, round to the number that you can measure in the syringe that you are using.

Amount (mg)	Volume (mL)	Rounded Volume (mL)	Time Interval
15	3.75	3.8	2 min
7.5	1.875	1.9	1 min
3.75	0.9375	1.0	30 sec
1.875	0.46875	0.5	15 sec

3) First, calculate the total volume of the injection. Enter the total amount, volume and time in the first row.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 5 to start and then by 2 to get to 15-second increments.

Amount (mg)	Volume (mL)	Time Interval
600	12	5
120	2.4	1
60	1.2	30 sec
30	0.6	15 sec

5) First, calculate the total volume of the injection. Enter the total amount, volume and time in the first row.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 3 to start and then divided by 2 to get to 15-second increments.

Amount (mg)	Volume (mL)	Time Interval
2.5	2.5	1.5 min
0.8	0.8	30 sec
0.4	0.4	15 sec

7) First, calculate the dose in mg from the weight of the patient given. $25\ \text{mg} \times 15\ \text{kg} = 375\ \text{mg/dose}$
Then, calculate the total volume of the injection.
Enter the total amount, volume and time in the first row.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 4 to start and then by 2 to get to 15-second increments.

Amount (mg)	Volume (mL)	Time Interval
375	3.75	4 min
93.75	0.94	1 min
46.9	0.47	30 sec
23.4	0.23	15 sec

From the table, 0.23 is not an easy volume to track over 15 seconds when giving a direct IV. Instead, you can round this number to 0.2 to track the volume you are giving better.

9) First, calculate the total volume of the injection. Enter the total amount, volume and time in the first row.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 2.

Amount (mg)	Volume (mL)	Time Interval
8	1.6	2 min
4	0.8	1 min
2	0.4	30 sec
1	0.2	15 sec

Exercises Part C: Creating Dose, Volume, Time Tables

Create a dose, volume, time table for the following scenarios. Identify the final volume and time interval you would use when administering the medication.

1. Medication Order: diltiazem 0.25 mg/kg IV direct for a 55kg adult.
   Rate Information: give undiluted over 2 minutes.
   Supply: 5 mg/mL (10 mL) vial.
2. Medication Order: hydromorphone 0.5 mg IV direct q6h.
   Rate Information: dilute to 5 mL with NS and administer slowly over 2 minutes.
   Supply: 2 mg/mL vial.
3. Medication Order: labetalol 7 mg IV initial dose in a hypertensive emergency.
   Rate Information: give over 2 minutes.
   Supply: 5 mg/mL (20 mL) vial.
4. Medication Order: propofol 18 mg IV direct bolus.
   Rate Information: given over 3 minutes.
   Supply: 10 mg/mL (20 mL) ampule.
5. Medication Order: metoclopramide 10 mg IV for prophylactic postoperative vomiting.

Rate Information: given over 2 minutes.
Supply: 5 mg/mL (2 mL) vial.

6. Medication Order: hydrocortisone 200 mg IV direct.
   Rate Information: dilute with 2 mL SW (diluted to 125 mg/mL), given over 1.5 minutes.
   Supply: 250 mg vial.
7. Medication Order: ephedrine 20 mg IV repeat q5-10 minutes according to blood pressure response.
   Rate Information: given over 10 mg/minute as a max rate.
   Supply: 50 mg/mL (1 mL) vial.
8. Medication Order: meperidine 15 mg IV direct q4h PRN for severe pain.
   Rate Information: diluted to 10 mg/mL, given over 5 minutes.
   Supply: 50 mg/mL (1 mL) ampule.
9. Medication Order: fentanyl 75 mcg intrapartum q30 minutes IV direct.
   Rate Information: given undiluted over 2 minutes.
   Supply: 50 mcg/mL (5 mL) ampule.
10. Medication Order: octreotide 2 mg/kg/dose IV direct bolus for a 20 kg toddler with a severe gastrointestinal bleed.
    Rate Information: given undiluted over 2 minutes.
    Supply: 100 mcg/mL (1 mL) ampule.

Odd Answers:

1) First, calculate the dose in mg from the weight of the patient given.

$0.25 \text{ mg} \times 55 \text{ kg} = 13.75 \text{ mg}$

Then, calculate the total volume of the injection.

$x \text{ mL} = \frac{1 \text{ mL}}{5 \text{ mg}} \times 13.75 \text{ mg}$ $x \text{ mL} = \frac{1 \text{ mL}}{5 \cancel{\text{mg}}} \times 13.75 \cancel{\text{mg}}$ $x \text{ mL} = \frac{13.75 \text{ mL}}{5}$ $x \text{ mL} = 2.75 \text{ mL}$ Enter the total amount, volume and time in the first row.

Next, choose an amount to divide each value by. In this answer, each value has been divided by 2.

Amount (units)	**Volume (mL)**	**Rounded Volume (mL)**	**Time Interval**
13.75	2.75	2.75	2 min
6.875	1.38	1.4	1 min
3.437	0.69	0.7	30 sec
1.718	0.34	0.3	15 sec

3) First, calculate the total volume of the injection. Enter the total amount, volume and time in the first row.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 2.

Amount (units)	Volume (mL)	Time Interval
7	1.4	2 min
3.5	0.7	1 min
1.75	0.35	30 sec
0.875	0.175	15 sec

From the table, 0.175 is not an easy volume to track over 15 seconds when giving a direct IV. Instead, you can round this number to 0.2 to track the volume you are giving better.

5) First, calculate the total volume of the injection. Enter the total amount, volume and time in the first row.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 2.

Amount (units)	Volume (mL)	Time Interval
10	2	2 min
5	1	1 min
2.5	0.5	30 sec
1.25	0.25	15 sec

7) First, calculate the total volume of the injection. Enter the total amount, volume and period of time in the first row.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 2.

Amount (units)	Volume (mL)	Time Interval
20	0.4	2 min
10	0.2	1 min
5	0.1	30 sec
2.5	0.05	15 sec

9) First, calculate the total volume of the injection. $x\ mL = \frac{1\ mL}{50\ mg} \times 75\ mg$ $x\ mL = \frac{1\ mL}{50\ \cancel{mg}} \times 75\ \cancel{mg}$ $x\ mL = \frac{75\ mL}{50}$ $x\ mL = 1.5\ mL$ Enter the total amount, volume and time in the first row.
Next, choose an amount to divide each value by. In this answer, each value has been divided by 2.

Amount (units)	Volume (mL)	Time Interval
75	1.5	2 min
37.5	0.75	1 min
18.75	0.375	30 sec
9.375	0.1875	15 sec

From the table, 0.1875 is not an easy volume to track over 15 seconds when giving a direct IV. Instead, you can round this number to 0.2 to track the volume you are giving better.

6.7 MEDICAL CALCULATIONS USING BODY SURFACE AREA (BSA)

Learning Objectives

By the end of this chapter, learners will be able to:

- Calculate the BSA of a patient using the Mosteller formula or a nomogram.
- Calculate the dose of a chemotherapy drug based on the patient's BSA and the standard dose per m^2.
- Adjust the dose of a chemotherapy drug based on the patient's renal or hepatic function, if needed.

Calculate the BSA of a Patient Using the Mosteller Formula or a Nomogram.

- BSA-based dosing is a method of prescribing chemotherapy drugs that normalizes the dose according to the patient's body size. This is done to reduce the variability in drug exposure and toxicity among different patients and to optimize the efficacy and safety of the treatment.
- BSA is the external surface area of the human body measured in square meters (m^2). It can be estimated using various formulas that are based on the patient's height and weight. The Mosteller formula is one of the simplest and most commonly used formulas. It is given by:

$$BSA(m^2) = \sqrt{\frac{height(cm) \times weight(kg)}{3600}}$$

where:

- BSA = Body Surface Area in square meters (m^2)
- Height = height of the patient in centimeters (cm)
- Weight = weight of the patient in kilograms (kg)

Calculate the Dose of a Chemotherapy Drug Based on the Patient's BSA and the Standard Dose per m^2.

Once the BSA is calculated, the dose of a chemotherapy drug can be determined by multiplying the BSA by the standard dose per m^2. For example, if the standard dose of a drug is 100 mg/m^2, and the patient's BSA is 1.5 m^2, then the dose of the drug is:

$$\begin{aligned} Dose &= BSA \times StandardDose \\ &= 1.5 \times 100 \\ &= 150mg \end{aligned}$$

Adjust the Dose of a Chemotherapy Drug Based on the Patient's Renal or Hepatic Function, if Needed.

However, some chemotherapy drugs may need to be adjusted based on the patient's renal or hepatic function, as these organs are responsible for metabolizing and eliminating the drugs from the body. If the renal or hepatic function is impaired, drug clearance may be reduced, leading to higher drug levels and increased risk of toxicity. Therefore, some formulas or guidelines have been developed to adjust the dose of certain drugs based on the patient's creatinine clearance (CrCl) or liver enzymes. These formulas or guidelines should be consulted before prescribing the drugs, and the dose should be modified accordingly.

Example 1

Calculate the BSA of a patient who is 175 cm tall and weighs 70 kg using the Mosteller formula.

Solution:

Using the Mosteller formula, the BSA is:

$$\begin{aligned} BSA &= \sqrt{\frac{Height \times Weight}{3600}} \\ &= \sqrt{\frac{175 \times 70}{3600}} \\ &= 1.84m^2 \end{aligned}$$

Example 2

Calculate the dose of doxorubicin for a patient who has a BSA of $1.8m^2$ and a CrCl of 40 mL/min. The standard dose of doxorubicin is 60 mg/m^2, and the dose should be reduced by 50% if the CrCl is less than 50 mL/min.

Solution:

Using the standard dose and the BSA, the dose of doxorubicin is:

$$\begin{aligned} Dose &= BSA \times StandardDose \\ &= 1.8 \times 60 \\ &= 108mg \end{aligned}$$

However, since the patient has a CrCl of 40 mL/min, which is less than 50 mL/min, the dose should be reduced by 50%. Therefore, the adjusted dose of doxorubicin is:

$$\begin{aligned} AdjustedDose &= Dose \times 0.5 \\ &= 108 \times 0.5 \\ &= 54mg \end{aligned}$$

Example 3

Calculate the BSA of a patient who is 5 feet 8 inches tall and weighs 150 lbs using the Mosteller formula.

Solution:

First, we need to convert the height and weight to centimeters and kilograms, respectively. To do this, we can use the following conversion factors:

- 1 inch = 2.54 cm
- 1 foot = 12 inches
- 1 lb = 0.4536 kg

Therefore, the height in centimeters is:

$$\begin{aligned} Height &= (5 \times 12 + 8) \times 2.54 \\ &= 172.72cm \end{aligned}$$

The weight in kilograms is:

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Then, we can use the Mosteller formula to calculate the BSA:

$$\begin{aligned} BSA &= \sqrt{\frac{Height \times Weight}{3600}} \\ &= \sqrt{\frac{172.72 \times 68.04}{3600}} \\ &= 1.81m^2 \end{aligned}$$

Example 4

Calculate the dose of methotrexate for a patient who has a BSA of 1.9m^2. The standard dose of methotrexate is 12 g/m^2.

Solution: Using the standard dose and the BSA, the dose of methotrexate is:

$$\begin{aligned} Dose &= BSA \times StandardDose \\ &= 1.9 \times 12 \\ &= 22.8g \end{aligned}$$

Example 5

Calculate the dose of doxorubicin for a patient who is 5 feet 6 inches tall and weighs 140 lbs, and has a CrCl of 40 mL/min. The standard dose of doxorubicin is 60 mg/m^2, and the dose should be reduced by 50% if the CrCl is less than 50 mL/min.

Solution:

First, we need to convert the height and weight to centimeters and kilograms, respectively. To do this, we can use the following conversion factors:

- 1 inch = 2.54 cm
- 1 foot = 12 inches
- 1 lb = 0.4536 kg

Therefore, the height in centimeters is:

$$\begin{aligned} Height &= (5 \times 12 + 6) \times 2.54 \\ &= 167.64cm \end{aligned}$$

The weight in kilograms is:

$$\begin{aligned} Weight &= 140 \times 0.4536 \\ &= 63.5kg \end{aligned}$$

Then, we can use the Mosteller formula to calculate the BSA:

$$\begin{aligned} BSA &= \sqrt{\frac{Height \times Weight}{3600}} \\ &= \sqrt{\frac{167.64 \times 63.5}{3600}} \\ &= 1.72m^2 \end{aligned}$$

Using the standard dose and the BSA, the dose of doxorubicin is:

$$\begin{aligned} Dose &= BSA \times StandardDose \\ &= 1.72 \times 60 \\ &= 103.2mg \end{aligned}$$

However, since the patient has a CrCl of 40 mL/min, which is less than 50 mL/min, the dose should be reduced by 50%. Therefore, the adjusted dose of doxorubicin is:

$$\begin{aligned} AdjustedDose &= Dose \times 0.5 \\ &= 103.2 \times 0.5 \\ &= 51.6mg \end{aligned}$$

So, the dose of doxorubicin for a patient who is 5 feet 6 inches tall and weighs 140 lbs, and has a CrCl of 40 mL/min, is 51.6 mg.

Exercises

1) Calculate the BSA of a patient who is 160 cm tall and weighs 50 kg using the Mosteller formula.

2) Calculate the dose of cyclophosphamide for a patient who has a BSA of 1.6m^2. The standard dose of cyclophosphamide is 750 mg/m^2.

3) Calculate the dose of cisplatin for a patient who has a BSA of 1.7 m^2 and a CrCl of 60 mL/min. The standard dose of cisplatin is 75 mg/m^2 , and the dose should be reduced by 25% if the CrCl is between 50 and 60 mL/min.

4) Calculate the BSA of a patient who is 180 cm tall and weighs 80 kg using the Mosteller formula.

5) Calculate the dose of fluorouracil for a patient who has a BSA of 1.9 m^2 and a liver enzyme level of 2 times the upper limit of normal (ULN). The standard dose of fluorouracil is 425 mg/m^2, and the dose should be reduced by 20% if the liver enzyme level is more than 1.5 times the ULN.

6) Calculate the BSA of a patient who is 150 cm tall and weighs 45 kg using the Mosteller formula.

7) Calculate the dose of paclitaxel for a patient who has a BSA of 1.5 m^2. The standard dose of paclitaxel is 175 mg/m^2.

8) Calculate the dose of gemcitabine for a patient who has a BSA of 1.8 m^2 and a CrCl of 30 mL/min. The standard dose of gemcitabine is 1000 mg/m^2, and the dose should be reduced by 50% if the CrCl is less than 40 mL/min.

9) Calculate the BSA of a patient who is 165 cm tall and weighs 55 kg using the Mosteller formula.

10) Calculate the dose of etoposide for a patient who has a BSA of 1.7 m^2. The standard dose of etoposide is 100 mg/m^2.

11) Calculate the dose of irinotecan for a patient who has a BSA of 1.6 m^2 and a liver enzyme level of 3 times the ULN. The standard dose of irinotecan is 180 mg/m^2, and the dose should be reduced by 33% if the liver enzyme level is more than 2.5 times the ULN.

12) Calculate the BSA of a patient who is 170 cm tall and weighs 60 kg using the Mosteller formula.

13) Calculate the dose of vinorelbine for a patient who has a BSA of 1.8 m^2. The standard dose of vinorelbine is 25 mg/m^2.

14) Calculate the dose of capecitabine for a patient who has a BSA of 1.9 m^2 and a CrCl of 50 mL/

min. The standard dose of capecitabine is 1250 mg/m^2 twice daily, and the dose should be reduced by 25% if the CrCl is between 30 and 50 mL/min.

15) Calculate the BSA of a patient who is 190 cm tall and weighs 90 kg using the Mosteller formula.

16) Calculate the BSA of a patient who is 6 feet 2 inches tall and weighs 180 lbs using the Mosteller formula.

17) Calculate the dose of bleomycin for a patient who has a BSA of 1.8m^2. The standard dose of bleomycin is 15 units/m^2.

18) Calculate the dose of oxaliplatin for a patient who has a BSA of 1.7 m^2 and a CrCl of 80 mL/min. The standard dose of oxaliplatin is 85 mg/m^2, and the dose should be reduced by 25% if the CrCl is between 30 and 80 mL/min.

19) Calculate the BSA of a patient who is 5 feet 4 inches tall and weighs 120 lbs using the Mosteller formula.

20) Calculate the dose of docetaxel for a patient who has a BSA of 1.6 m^2 and a liver enzyme level of 4 times the ULN. The standard dose of docetaxel is 75 mg/m^2, and the dose should be reduced by 50% if the liver enzyme level is more than 3 times the ULN.

21) Calculate the BSA of a patient who is 5 feet 10 inches tall and weighs 160 lbs using the Mosteller formula.

22) Calculate the dose of dacarbazine for a patient who has a BSA of 1.9 m^2. The standard dose of dacarbazine is 250 mg/m^2.

23) Calculate the dose of pemetrexed for a patient who has a BSA of 1.8 m^2 and a CrCl of 45 mL/min. The standard dose of pemetrexed is 500 mg/m^2, and the dose should be reduced by 50% if the CrCl is between 30 and 45 mL/min.

24) Calculate the BSA of a patient who is 5 feet 6 inches tall and weighs 140 lbs using the Mosteller formula.

25) Calculate the dose of topotecan for a patient who is 5 feet 11 inches and 215 lbs and a CrCl of 55 mL/min. The standard dose of topotecan is 1.5 mg/m^2 daily for 5 days, and the dose should be reduced by 20% if the CrCl is between 40 and 60 mL/min.

Answers to Odd Exercises

1) 1.49 m^2	3) 96 mg	5) 646 mg
7) 263 mg	9) 1.59 m^2	11) 193 mg
13) 45 mg	15) 2.18 m^2	17) 27 units
19) 1.57 m^2	21) 1.89 m^2	23) 450 mg

25) 2.65 mg