The Living Tree of Mathematics: Math Problems through World History and Cultures.

The Living Tree of Mathematics: Math Problems through World History and Cultures.

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About the book

This book was developed at the University of Western Ontario. The development of the book was supported by the Open Educational Resources Grant and Support Program. The program is a partnership between Western Libraries, the Instructional Technology Resource Centre, Centre for Teaching and Learning, and Western Research's Knowledge Exchange and Impact Team.

The book was created by Vera Sarina, a lecturer at the Faculty of Education, with invaluable contributions from many people. My special thanks go to my students, pre-service teachers Andrew Rethazi and Emily Deeb who wrote solutions to the problems in the book. And I cannot thank enough Abby Chapman and Sophie Furtado of Centre for Teaching and Learning for transforming all my materials into this beautiful multimedia book.

The intended audience of the book is current and future teachers of Mathematics.

Solving word problems is considered to be the most challenging and boring part of mathematics learning by many students. To engage students and make them enthusiastic about solving word problems is a goal not easily achieved. An open online multimedia book of historical problems where problems are accompanied by stories introducing the historical/cultural context of the problems could hopefully do the trick in your classroom.

The book aligned with the Grade 7-9 Ontario mathematics curriculum (which is consistent with to the Content Core Standards) that gives teachers an easy-to-use tool to bring history into math classrooms and at the same time engage and motivate students in problem solving.

The book has three parts: Chapters, Solutions and Teacher Supports.

Chapters

Each chapters has four components:

- Map and Timeline. Map and Timeline are not specific for each Chapter. However, all the regions and times where the problems in the book come from are present on Map and Timeline.
- Introduction. The purpose of Introductions is to place the topic of a Chapter into its cultural and historic context.
- Problems. The problems of each Chapter are drawn from various historical sources. Their subject is the topic of the particular Chapter. Problems are numbered and open as drop-down items.
- Stories. Each set of problems is preceded by a story from the geographical and cultural context of the problems. The stories are narrated. Narrations are included in each story.

Solutions

The solutions to problems are sorted by the Chapters. Solutions are numbered as their corresponding problems. Solutions open as drop-down items.

Teacher Supports

Teacher Supports include:

- Connections to Curriculum Expectations.
- Sample Lesson Plans

Introduction

Learning is embedded in memory, history, and story.
First Peoples Principles of Learning

Mathematics is a unique school subject. It leads students through human history by way of mathematical concepts and ideas. Around the world, mathematics curricula roughly follow the historical development of central mathematical concepts. Think, for example, of the number systems: natural numbers, fractions, decimals, integers, irrational numbers, and finally complex numbers. This is the order that we introduce students to the number systems, and in a broad sense it slides along the timeline of the discovery and development of different numbers in mathematics. And although mathematics is deeply rooted in our collective history, it at times feels detached from the real world.

Mathematical word problems is one of the oldest genre of writing tradition. Many of them have very ancient origins, and a very long continuous history of use in the teaching and learning of mathematics. Through stories, riddles and tricks mathematical knowledge traveled around the world. Many word problems "have been disseminated as folk stories through merchant connections and trade routes" (Daroczy, 2015). And the wording of problems have been easily adapted to "suit the cultural context allows problems to cross cultural boundaries." (Daroczy, 2015). That is why we encounter the same problems in diverse cultures and diverse times. The practical, historical context of the problems might be quite different but strip a problem of its cultural attire or reword it in the modern language and we are solving a problem from a textbook of today. But what happens if we leave a problem from Babylon or ancient Egypt or China or medieval Europe or India or Japan of 18 century in its original form for our students to solve it? The answer is an immersive experience. Students not only doing meaningful mathematics but also get a glimpse, feel a touch, experience a flavour of history. Mathematics does not seem detached from the real world anymore.

But let's take one step further in immersing students in the cultural, historical context of the problems they solve. For instance, students are to embark on the calculation of the volume of a canal in Babylon. Why not to let them listen first to the story about *The Code of Hammurabi?* Or students are tasked to tackle the Japanese problem on riders riding in circles. Why not to let them listen first to the story about *Samurai Mathematicians*? Or students are to calculate the distribution of grain to workers in ancient Egypt. Why not to let them listen first to the story about *The girl with the rose-red slippers or Egyptian Cinderella?* A plunge into the Mayan number system can be started by the story of *The Story of Pok-A-Tok, Mayan ancient Ballgame.* The Chinese problem on the Pythagorean Theorem can be introduced by *The Legend of Yu the Great,* a legendary Chinese emperor who conquered annual disastrous flooding caused by Yellow River.

Mathematics and human history come hand-in-hand in the book as they should. "The entire body of mathematical knowledge is very much like a tree." (Kennedy 1995, p. 460). It is a global affair. It has its roots in cultural practices from numerous civilizations. The book gives your students a chance to see mathematics as a part of human history. It gives your students a chance to learn some snippets of human history. And the most important goals of the book is that it gives students an opportunity to feel that they belong to the living tree of mathematics, that history is not dead but lives through the stories and math problems, and it is a part of their own personal stories.

CHAPTERS

Chapter 1 - Numbers

Map

1.1

1.19

Timeline

1.1

1.19

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Introduction

Can you imagine a world without numbers? It's safe to say that society as we know it today would never have developed without numbers. The scientific and technological advances upon which society is built depend upon mathematics, which in turn depends upon numbers.

Despite their importance, the development of numbers remains mostly a mystery. That's because the first ancient prehistoric people who likely developed simple methods of counting didn't leave any records behind to explain themselves.

Common sense and ancient evidence points to the idea that numbers and counting began with the number one. Although they probably didn't call it "one," prehistoric people likely counted by ones and kept track by carving lines on a bone.

Evidence that this occurred as long as tens of thousands of years ago can be found on an ancient artifact known as the Ishango Bone. It is one of the earliest, if not *the* earliest, mathematical manmade object. It was found in 1950 in the then-Belgian colony of the Congo (now the Democratic Republic of Congo). It is the bone of a large mammal, probably a baboon or large cat, and is approximately 25,000 years old. It is 10 cm long and has a distinct, organized series of notches. Looks like as a tally stick, doesn't it?

1.1

1.1

1.1

Though its original purpose remains a subject of debate, the grouping of etchings on the bone suggests more advanced mathematics than just tallying. Can you find any possible number patterns among the numbers representing the groupings of tally notches?

Number Systems

A number or numeral system is a system, writing or oral, for expressing numbers, The system we use now is called a decimal system and has 10 symbols with which we can express any number in the Universe. We are so used to seeing the symbols 1, 2, 3, 4, etc. that it may be somewhat surprising to see that many creative and innovative ways to compute and record numbers have existed and which still exist all over the world. That's not to say our own system is not important or efficient. The fact that it has survived for hundreds of years and shows no sign of going away any time soon suggests that we may have finally found a system that works well and may not need further improvement, but only time will tell that whether or not that conjecture is valid or not.

Babylon – Why and how did humans start writing?

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Have you ever wanted to live forever? What about cloning yourself? The miracle of writing allows you to do both. You can capture your thoughts and ideas eternally by writing them down. You can communicate with a written message to someone on the other side of the world while you are still right here. In a way, your thoughts have been cloned and now live in two places at the same time! If you wanted someone to thank for this subtle superpower you have inherited, you would need a time machine to take you back over five thousand years to the land of ancient Mesopotamia…

Although people in ancient times seem so distant, they still had many of the same needs that we do today. As ancient Mesopotamian society developed and grew more complex, it required a more effective way of communicating than just through human speech. The story of human writing began as a way for people to keep count of objects during trade. They used small clay tokens in a variety of shapes to symbolize different physical goods. Spheres, cones, and discs represented differing measurements of grain, while cylinders stood for livestock.

The tokens were enveloped inside clay balls that had a marking on them to show what type of token was inside. After a while, they realized it was unnecessary to have the token as well as the marking, and chose just to use a clay marking (much like how we use money to represent the value of goods). But, as the nature of goods became more detailed, they needed to create more and more types of markings. And, this exploration into a world of new symbols is how the Mesopotamians embarked on the journey of creating written language.

As time went on they were presented with new challenges. How to record a person's name? It wouldn't be possible to create a new symbol for every single name, so the Mesopotamians devised a strategy to reuse symbols to represent sounds. For example, if they wanted to write down the last name "Sontag," they drew a circle sign in clay – since it already symbolized the sound of the spoken word "sun" – and then drew a label sign in clay to represent the word "tag." This allowed them to pronounce the whole name just by reading the two symbols together. This method of naming is called the *Rebus Principle*. "Rebus," in Latin, means "by things," so the meaning here is that sounds can now be represented by pictograms (pictures of things).

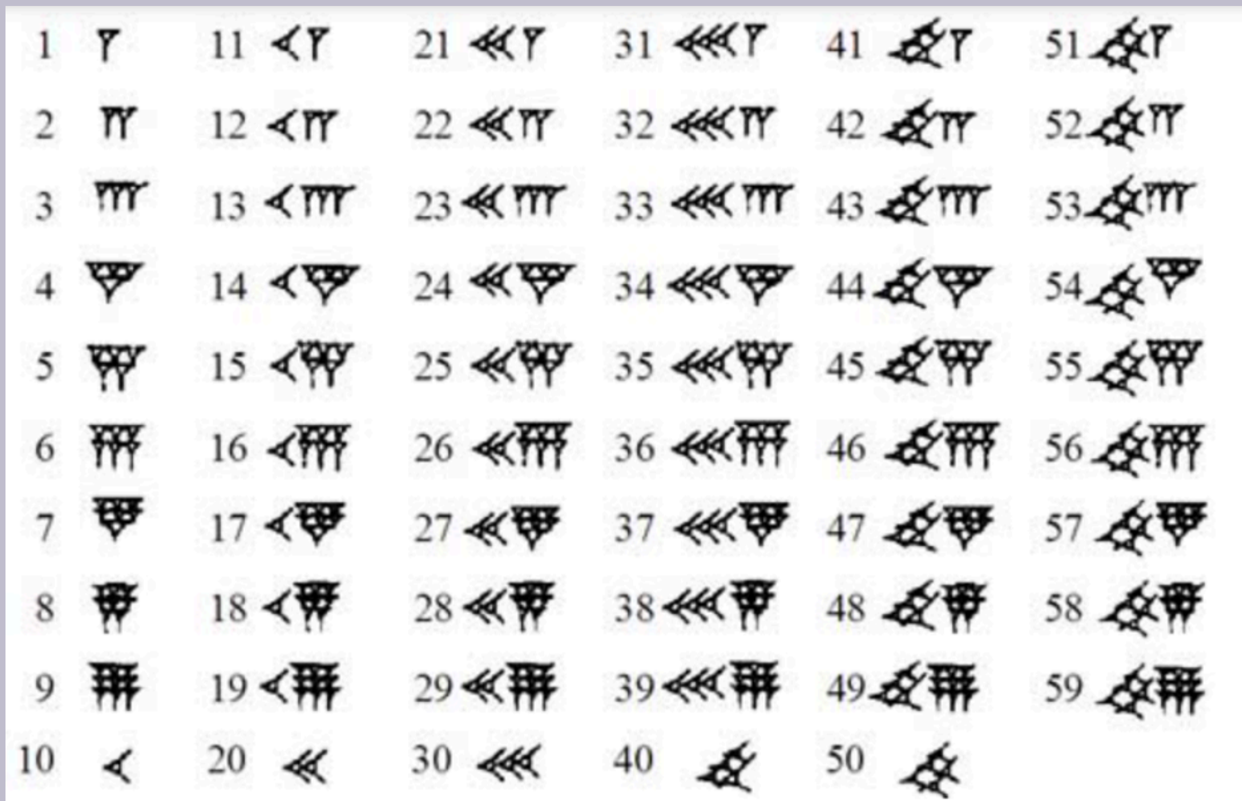
We take writing for granted, but like many of the human achievements we rely on for our modern world to function, we have ancient cultures to thank for them. We truly stand on the shoulders of the giants of the past.

Do you want to see your monogram in Cuneiform, the way an ancient Babylonian might have written it? Click on the link from The University of Pennsylvania Museum of Archaeology and Anthropology https://www.penn.museum/cgi/cuneiform.php

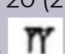
1.2

1.1

1.2



Babylonian numerals are surprisingly easy to decipher. Image: public domain, via sugarfish and Wikimedia Commons.

The Babylonian number system uses base 60 (sexagesimal) instead of 10. Their notation is not terribly hard to decipher, partly because they use a positional notation system, just like we do. To us, the digit 2 can mean 2, 20 (2*10), 200 (2*10*10), and so on, depending on where it appears in a number. Similarly, to Babylonians  can mean 2, 120 (2*60), 7200 (2*60*60) and so on, depending on where it appears in a number.

1) What symbols did the Babylonians use to express numbers?

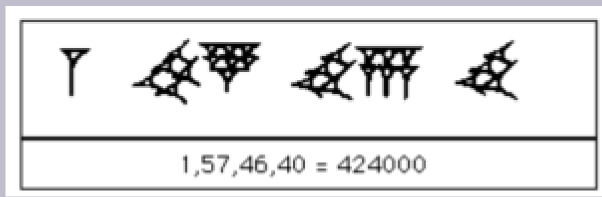
2) Calculate the value of:

a. 

b. 

c. 

Example of Babylonians numbers:



$$1 \times 60^3 + 57 \times 60^2 + 46 \times 60 + 40$$

d. How would Babylonians write 66? 666?

1.3

1.1

1.3

A granary of barley contains 2400 gur, where 1 gur equals 480 sila. If workers are to receive 7 sila of grain for a day's work, how many men can be paid from this granary?

1.4

1.1

1.4

A woman weaves a textile that is to be 48 rods long. In one day, she weaves 1/3 rod. In how many days will she cut the textile from the loom?

Mayan Civilization – The Story of Pok-A-Tok

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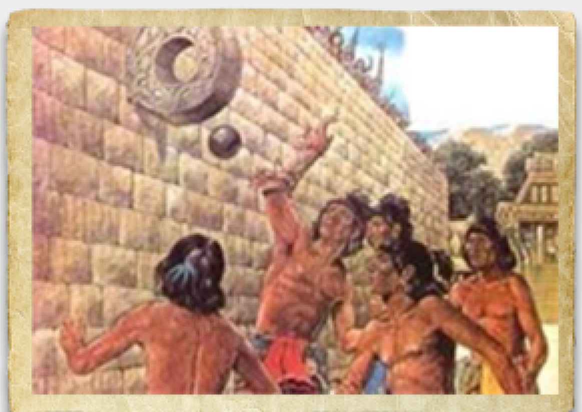
What comes to mind when thinking about the oldest team sport? Is it soccer? Polo? When we talk about ancient civilizations, much of the discussion centers around culture, architecture, famous kings and queens, but rarely do we shine a light on the sports played by the ancients. In fact, the earliest team sport on record comes from Mesoamerica over 3,500 years ago, and is profoundly linked to death, ritual, and the afterlife. The sport is known as *Pok-A-Tok,* and – player beware – losing a game could cost you your life…



When Spanish explorers first landed in the New World and made contact with the Aztec people of Central America, they were fascinated by many things, one of which was an exotic Aztec sport played with a ball made of rubber, a material unknown in Europe at that time. A cross between soccer and basketball, Pok-A-Tok features two teams on a stone court both trying to get the ball up a slope and through a stone hoop. The catch? Players couldn't use their feet or hands. The only way to pass and shoot the ball was with your head, shoulders, elbows, wrists, or most commonly, with your hips. Beyond the difficulty and athleticism of the gameplay itself, what made the sport so intriguing that the Spanish sent a team of Aztec players back to Spain to play for King Charles V himself?

For the Aztecs and their ancestors, the ancient Mayans, Pok-A-Tok was much more than a sport. It symbolized war, hunting, the battle for survival, and was a line of communication for them to the gods of the Mayan religion. Believing that the gods required a tribute of human blood, and human hearts, to keep the sun and moon in orbit, the Mayans would execute the captain, or sometimes the entire losing team, of a Pok-A-Tok game in a human sacrifice ritual! As scary as it seems, the Mayans had an entirely different view of life and death than we do today. Some legends even spoke of winning Pok-A-Tok captains bizarrely offering their own heads for decapitation! While this may sound counter-intuitive, to the Mayans, it was the ultimate honor since it was seen as winning a direct ticket to heaven without having to go through the thirteen steps needed to ascend to a peaceful afterlife.

Aside from the darker aspects of the game, Pok-A-Tok was also played ceremonially, as well as being used to resolve disputes between rival cities instead of resorting to warfare. With over a thousand Pok-A-Tok courts scattered across Mesoamerica, it is no surprise that the game played an integral role in ancient Mayan life. The question is, will it ever regain its popularity? And, if so, will the human sacrifice too?

1.5

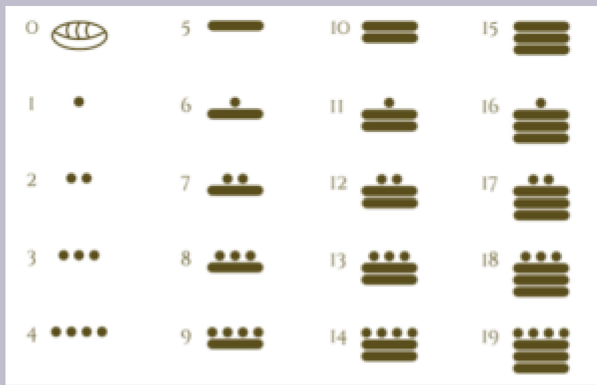
1.1

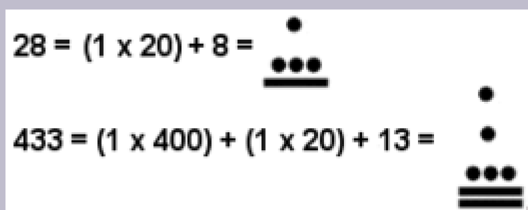
1.5

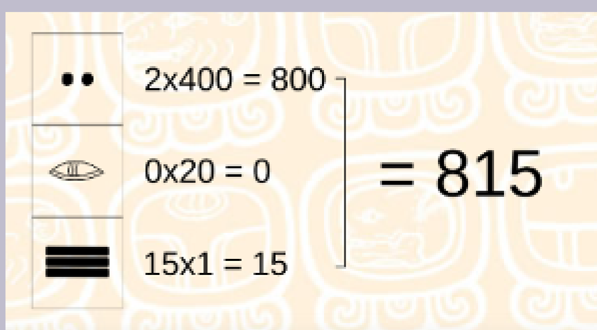
Maya mathematics constituted the most sophisticated mathematical system ever developed in the Americas. The Maya counting system required only three symbols: a dot representing a value of one, a bar representing five, and a shell representing zero. These three symbols were used in various combinations, to keep track of calendar events both past and future, and so that even uneducated people could do the simple arithmetic needed for trade and commerce. That the Maya understood the value of zero is remarkable – most of the world's civilizations had no concept of zero at that time.

The Maya used the vigesimal system for their calculations – a system based on 20 rather than 10. For example to us, the digit 1 can mean 1, 10 (1*10), 100 (1*10*10), and so on, depending on where it appears in a number. Similarly, to Mayans a dot can mean 1, 20 ((1*20), 200 (2*20*20), and so on, depending on where it appears in a number. This means that instead of the 1, 10, 100, 1,000 and 10,000 place values of our number system, the Maya used 1, 20, 400, 8,000 and 160,000 place values.

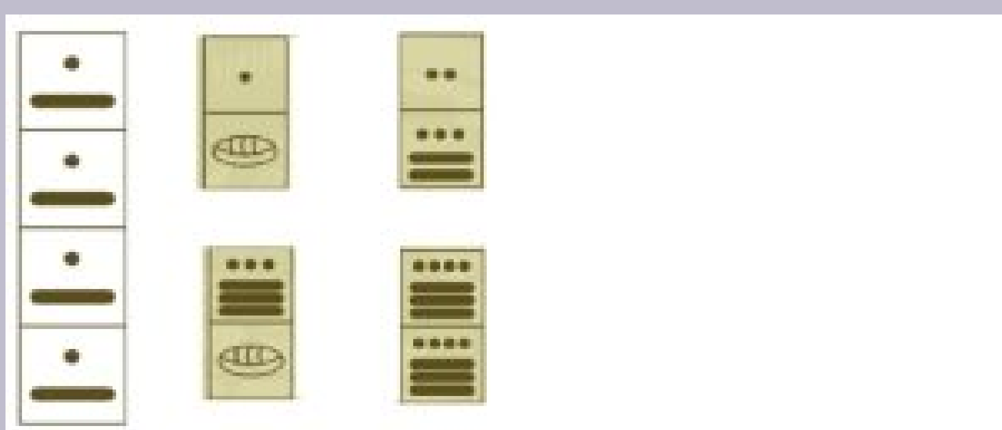
Maya numbers were written from bottom to top, rather than horizontally. As an example of how they worked, three was represented by three dots in a horizontal row; 12 was two bars with two dots on top; and 19 was three bars with four dots on top. Numbers larger than 19 were represented by the same kind of sequence, but a dot was placed above the number for each group of 20. Thirty-two, for example, consisted of the symbols for 12, with a dot on top of the whole thing representing an additional group of 20. The system could thus be extended infinitely.







a. Find the values of the numbers



b. Write numbers 85, 121, 2222 in Mayan numerals

Medieval Europe – The story of prohibited numbers.

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Have you ever wondered where the numbers we use came from? The story behind our numerical system began in the middle ages, journeyed across the world and into the present day – but, did you know there was once a time where using them was forbidden?

Originating in ancient India, our number system made its maiden voyage into the wider world after being discovered by Arabic invaders in the year 711. The Arabs adopted the new system and introduced it into Western Europe, however, European authorities were not interested in using it, and even went as far as to outlaw it entirely for hundreds of years! Even as late as the 12th century, many Italian cities still banned the system's use in contracts and official documents, favouring Roman numerals despite the fact that the Indo-Arabic method was more convenient for representing large numbers and performing multiplication and division. With that in mind, what reason could there have been for Europe to shun such a superior system?

In the middle ages, mathematics was most commonly used in trade and commerce. Since making calculations using Roman numerals could be difficult, European traders used a device called an *abacus* in order to simplify the process.

While the abacus is still used today in East Asia, India, and Russia, the Indo-Arabic system is undoubtedly more effective for a wide range of functions. The European traders came to that same conclusion after their Arabian counterparts crossed the Mediterranean Sea and introduced the new system. The problem was, despite its efficiency, the Indo-Arabic numerical system contained some loopholes for those unfamiliar with it. *Traders could easily write one number and claim later that they had meant some other number*. For example, one could write down the price of an item as the number sixteen, but later claim the price was actually ninety-one by alleging the buyer viewed the number upside-down. The Arabian traders could make out these subtle differences, but the locals were not so familiar with them.

Another reason why the European powers were cautious about changing their current system was that Indo-Arabic numerals contained the number zero. On top of being unknown and difficult to understand for Europeans, the idea of a zero number was also problematic for them because of its connection to the concept of debt. At that time, the use of debt was considered to be immoral in Europe. The fear was that zero was associated with negative numbers (AKA debt), and its use would eventually lead to the use of debt in their society.

During the French Revolution of 1789, the abacus was banned in schools and administrations, which gave an opportunity for Indo-Arabic numerals to be used throughout Europe. Its zigzagging journey took over a thousand years to reach its destination as the most prominent numerical system in the world, but its story is not finished yet. What does the future hold for Indo-Arabic numerals? Could it one day be dethroned by a better system waiting to be discovered?

1.6

1.1

1.6

There is a tree with 100 branches; each branch has 100 nests; each nest, 100 eggs; each egg, 100 birds. How many nests, eggs and birds are there?

1.7

1.1

1.7

Three hundred pigs are to be prepared for a feast. They are to be prepared in three batches on three successive days with an odd number of pigs in each batch. Can this be accomplished?

1.8

1.1

1.8

A leech invited a slug for lunch a leuca away. But he could only crawl an inch a day. How long will it take the slug to get his meal? [1 leuca = 1500 paces; 1 pace = 5 feet.]

1.9

1.1

1.9

A certain man had in his trade four weights with which he could weigh integral pounds from 1 up to 40. How many pounds was each weight?

1.10

1.1

1.10

I have a cloak 100 cubits long and 80 cubits wide. I wish to make small cloaks with it; each small cloak is 5 cubits long, and 4 wide. How many small cloaks can I make

1.11

1.1

1.11

A father, when dying, gave to his sons 30 glass flasks, of which 10 were full of oil, 10 were half full, and the last 10 were empty. Divide the oil and the flasks so that each of the three sons received equally of both glass and oil.

1.12

1.1

1.12

A king ordered his servants to collect an army from 30 manors in such a way that from each manor he would take the same number of men he had collected up until then. The servant went to the first manor alone; to the second he went with one other; to the next he took three with him. How many were collected from the 30 manors?

1.13

1.1

1.13

A gentleman has a household of 30 people and orders that they be given 30 measures of grain. He directs that each man should receive 3 mea¬sures, each woman 2 measures, and each child 1/2 measure. How many men, women, and children are there?

1.14

1.1

1.14

A man wanting to build a house contracted with six builders, five of whom were master builders, and the sixth an apprentice, to build it for him. He agreed to pay them a total of 25 pence a day, with the apprentice to get half the rate of a master builder. How much did each receive per day?

1.15

1.1

1.14

A man in the east wanted to buy 100 assorted animals for 100 shillings. He ordered his servant to pay 5 shillings for a camel, 1 shilling for an ass, and 1 shilling for 20 sheep. How many camels, asses, and sheep did he buy?

Egypt – The girl with the rose-red slippers or Egyptian Cinderella

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=50#audio-50-4

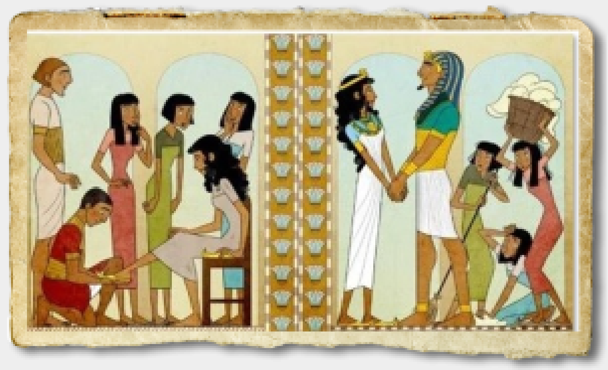
Ancient Egypt is known for its great civilization, pyramids, and art. But, did you know they also have their own version of Cinderella? Known as The Girl with the Rose-Red Slippers, the story follows the unpredictable life of a girl named Rhodopis…

Born in ancient Greece, Rhodopis was carried away by pirates as a child, and sold to a wealthy man on the island of Samos. As the years passed, Rhodopis blossomed into a beautiful young woman, and yearned to break free of the bonds of slavery. Her greedy owner was so astonished at how beautiful she had become, he realized she was now worth at least twice as much as he initially paid. So, he sent her to the wealthy city of Naucratis, in ancient Egypt, to be sold at a slave-auction.

Rhodopis caught the attention of a man named Charaxos, who was so fascinated by her beauty that he bought her before anyone else could. After hearing the heart-wrenching story of her childhood, Charaxos was struck by a deep sense of compassion, and showered her with gifts. He bought her a house with a garden, fancy clothing, jewelry, and her most prized possession of all: a pair of rose-red slippers…

One day as Rhodopis was bathing in her pool, with her misfortunes a distant memory, an eagle swooped down from the sky into her garden, clutched her rose-red slippers in its talons and flew away. Rhodopis wept at the loss of her treasured slippers, and felt even worse losing a gift given by the generous Charaxos.

Far away south, in the valley of the Nile, lived Amaris, the pharaoh of Egypt. One day, while roaming the courtyard of his great palace, the pharaoh was visited by the very same eagle, who flew down and plunked the rose-red slippers at his feet. The pharaoh believed it to be an omen from the gods, since the ancient Egyptian god Horus was a man with the head of a bird. The pharaoh became obsessed with finding the owner of the rose-red slippers, convinced she must be his true love.



He sent out his messengers to scour the land in search of Rhodopis. When they reached Naucratis and heard stories of Rhodopis' beauty, they knew she must be the future queen. When the messengers knocked at her door, she couldn't believe her eyes – her rose-red slippers had miraculously returned!

She was escorted back to meet the pharaoh, who offered his hand in marriage. And, just like that, Rhodopis went from slave to queen of Egypt. The moral of the story is that life is a long, winding road where hardship can sometimes lead to good fortune.

1.16

1.1

1.13

There is an estate that contains 7 houses; each house has 7 cats; each cat catches 7 mice; each mouse eats 7 spelt of seeds; each spelt was capable of producing 7 hekats of grain. How many things were in the estate?

1.17

1.1

1.14

Suppose a scribe tells you that four overseers have drawn 100 great quadruple hekats of grain, and their work gangs consist of 12, 8, 6, and 4 men. How much grain does each overseer receive?

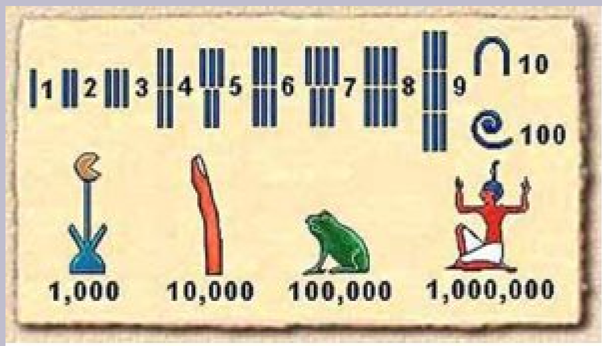
1.18

1.1

1.15

The Egyptians had a number system using seven different symbols:

- 1 is shown by a single stroke.
- 10 is shown by a drawing of a hobble for cattle.
- 100 is represented by a coil of rope.
- 1,000 a drawing of a lotus plant.
- 10,000 is represented by a finger.
- 100,000 a tadpole or frog
- 1,000,000 figure of a god with arms raised above his head.



To read Egyptian numbers you have to add the values of the symbols describing a number:

The scene below depicts a cattle count done by an Egyptian scribe some 4000 years ago. It was found on the walls in one of the ancient Egyptian pyramid which was a tomb of a rich man. The scene would have depicted the wealth of the tomb owner and the depiction on the tomb wall also meant the tomb owner would have these animals with him in the afterlife.

Can you read Egyptian numbers and calculate:

1. a) horned cattle on the left
2. b) animals (cows?) right behind them
3. c) goats on the right in the middle row
4. d) donkeys in the bottom row on the left
5. e) goats in the bottom row on the right

Greece – 300 Spartans and the Battle of Thermopylae

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Greece. 300 Spartans and the Battle of ThermopylaeOne ancient battle above all others has achieved a legacy as the most famous last stand of all time: The Battle of Thermopylae. In this real life David vs. Goliath story, a small army of just a few thousand Greek warriors somehow defied impossible odds to defeat a colossal Persian army of nearly a quarter of a million soldiers! How did they do it? And, what made this Greek army so special that their voices continue to echo in modern times?

The story begins in the ancient Greek city-state of Sparta. This unique society was set apart from its neighbors because of its deep commitment to warrior-culture. In fact, their fighting force was so exceptional, they were the only Greek city to survive without defensive walls. Why were their soldiers so effective that the outside world viewed them as invincible?

Firstly, training began during childhood! Recruited at just the age of seven, young soldiers' educational paths forced them to leave home and join a squad known as *aghéle* where they were all treated equally, however, extremely unpleasantly. Their hair was shaved off, they were forced to walk only in bare feet, and had to live without clothing no matter what the weather conditions were. On top of that, their thirteen year-long boot camp consisted of endless tests of their strength and skill, and absolute obedience to adults – all in the name of molding the perfect citizen.

By the time young Spartans reached the age of twenty, they were a step away from admission into the Spartan army, but first had to pass one final death-defying test: the *krypteìa*. The cruel challenge, meaning "something that takes place secretly," involved one whole year of wandering alone barefoot, and surviving only on what they could hunt, forage, and steal for themselves. Beyond that unimaginable hardship, young Spartans were required to find and kill a member of the Spartan servant class, known as a Helot. Once the test was passed, they were given a red cloak and a shield, permitted to grow their hair back, and were granted membership into a spectacular military order that was unmatched by any other in the ancient world!

The Spartan's reputation for excellence in combat did not, however, stop neighboring kingdoms from attempting to invade their country. In 480 B.C., Xerxes, King of Persia, sent an army of 242,000 troops into ancient Greece, and so began one of the most legendary military clashes of the ancient world. King Leonidas of Sparta knew that his army of 6,700 soldiers needed an excellent strategy to stop the Persian invasion, and so he chose to meet them head on at the pass of Thermopylae, a geographic bottleneck that caused the Persians to file through in a line. This allowed the Spartans to engage them man to man, rather than being swamped by the outnumbering force. Before the attack, Xerxes sent a message to the Spartans to accept defeat and surrender their weapons, to which Leonidas replied *"molon labe,"* which translates to "come and take them." In doing so, King Leonidas knew he was sending himself and his soldiers to their deaths, but a Spartan would never accept defeat, especially when it came to protecting his homeland.

After three days of successfully defending the pass of Thermopylae, tragedy and treachery were visited upon King Leonidas and his men. A local resident named Ephialtes is said to have betrayed his own Greek army by revealing a secret path to Xerxes that led around the pass of Thermopylae. Now outflanked by the Persians, Leonidas realized the battle could not be won, and dismissed the majority of his army while remaining to fight alongside three hundred Spartans. The Persian army successfully defeated the Spartans, but not after suffering many more casualties than their highly skilled opponents.

Despite the loss at Thermopylae, the Greeks were able to regroup and mount a successful attack against the Persian armada under the naval command of Themistocles in the Battle of Salamis in late 480 B.C. Afraid of being trapped in Europe, Xerxes withdrew his army back to Asia, but lost most of them to starvation and disease during the process. With the Persians' losing the upper hand, the Spartans assembled at full strength and defeated the Persians decisively at the Battle of Plataea, bringing the Greco-Persian war to an end, and with it, a conclusion to the Persian empire's advances into Europe.

The story of the Battle of Thermopylae and the famous last stand of its three hundred Spartan soldiers has left an indelible mark on ancient and modern writers alike, even being made into three Hollywood movies! Its ability to capture the determination of the human spirit stands the test of time as an enduring tale of courage and conviction in the face of seemingly insurmountable odds.

1.19

1.1

1.16

Ancient Greece was not a country as we know it today. It was composed of city-states also known as a polis. Among the main poleis were Athens and Sparta. While Spartans were mostly warriors, Athenians loved arts, philosophy and mathematics. They left us many mathematical problems and puzzles. Try to solve one of them.

a and *b* are two numbers. Order the following expressions from least to greatest and defend your solution.

$$M_A = \frac{(a+b)}{2},$$

$$M_G = \sqrt{ab},$$

$$M_H = \frac{2ab}{(a+b)}.$$

1.20

1.1

1.17

Given, four whole numbers where, if added together three at a time, their sums are 20, 22, 24, and 27. What are the numbers?

Chapter 2 - Circles

Map

1.1

1.19

Timeline

1.1

1.19

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Introduction

The circle is a basic shape and a simple plane figure. But, it is a mysterious shape. There is something fantastical about the circle that we perceive on a subconscious level. Circular shapes are foundational geometrical creations of the universe that instill a sense of order, safety, and beauty upon all who observe them.

Round city of Darabgerd in Iran

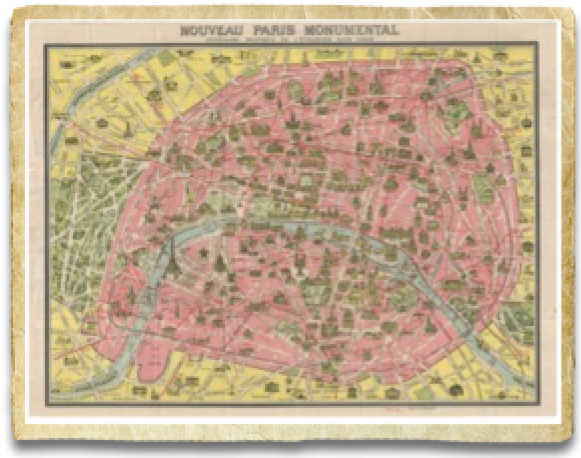
The importance of circles to human cultures is highlighted in many places, but also in some you might not expect, like language. When we refer to our close friends, we say "circle of friends." The word *encyclopedia* comes from ancient Greek words for "circle of learning." Even the fundamental nature of reality demands our attention to circular patterns. For example, if a group of people speak with each other, they are inclined to be face to face in a circle, not a line, or a polygon, or a rectangle. In sacred Indigenous ceremonies, talking circles were typically used to create a structure where each person's voice is given equal importance.

Teuchitlan in Mexico

Most of the early settlements, villages, and cities have followed circular arrangements. Coliseums, circuses, theaters, concert halls, stadiums, and even many present-day and ancient cities and settlements use circles as a blueprint for their design. Try to visualize how a circular layout of homes could forge a strong sense of unity for its inhabitants, as well as serve as protection against danger from all directions. Such circular planning was applied in ancient cities like Arkaim in the Southern Urals of Russia, the round city of Darabgerd in Iran, or Teuchitlan in Mexico. Many modern cities also use the same design in the form of a ring-shaped road encircling them; Paris, Berlin, Brussels, Addis Ababa, Lahore, Hyderabad, and Melbourne, to name a few.

If we journey back 11,000 years into prehistory, we yet again witness the importance of circular shapes in architecture.

Pre-ancient inhabitants of what is now Turkey erected a superstructure of massive stone carvings arranged in circles. Despite having not yet developed writing, agriculture, metal tools or pottery, they were nonetheless fixated on demonstrating their inherent desire to replicate the beauty and symmetry of the circle.



Map of Paris, France 1920

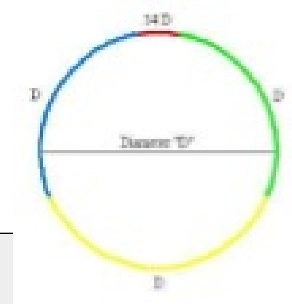
Gobekli Tepe, Turkey. The World First Temple

What is the reason humans are so fascinated by circles and not squares, rhombuses, and other shapes? Are we drawn to seek out symmetries created by the universe? And, if so, why? Researchers theorize three possible explanations. First, angular shapes like thorns, teeth, and jagged rocks, present threats to our safety, whereas round shapes elicit a sense of safety. Secondly, human beings showcase happy emotions in a *curvilinear* way – arcing smiles, rounded cheeks. Thirdly, our eyes see the world through a circular frame, and it's possible we are attracted to aesthetics that blend accordingly with that same shape. Whether it's supergiant stars, human cells, or subatomic particles like electrons, circular shapes appear throughout the universe on the largest and smallest scales imaginable. And, as a species uniquely appreciative of harmony and transcendence, we are forever mesmerized by their symbolic representation of eternity, continuity, perfection, and unity.

Quest for Pi

From the time immemorial people used a straightedge, be it a modern school ruler or measuring rod of ancient Egypt and Babylon, to measure the length of objects. But it is pretty tricky if not impossible to measure the length of circles by straightedges. The only dimension that can be measured by a straightedge in circles is the radius or its double the diameter. Thus all the known mathematics were trying to find the relationship between the diameter of a circle and the perimeter or circumference. If you know the ratio of the diameter of a circle to its circumference you can measure the diameter and quickly calculate the circumference. But how do you know the circumference of a circle in the first place if you can't measure it? Ancient societies were not obsessed with precision, and a good approximation was generally enough for their practical tasks. And what is the best (and easiest) approximation of a circle if not a regular polygon, square, pentagon, hexagon, etc.?

Explore the following problems from different historic sources. What were the values of the ratio C/d that these mathematics used? Which one was the closest to the modern approximation of pi?



Judea – The tumultuous history of the Talmud

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As long as there has been human civilization, there has been war. Ancient, medieval, and modern accounts highlight the complete and utter devastation wrought by armed conflict. How is it possible then, that through the ravages of war, the Jewish people were able to strengthen their tradition, and create one of the most quintessential books of their religious faith?

Most religions have a central book that captures the arc of their beliefs and teachings. For the Jewish people, this book is the Talmud. Serving as the architecture for Jewish religious law, the Talmud is a chronicle of debates held by generations of spiritual leaders and teachers called *rabbis*. In biblical times, these debates were held in the Temple of Jerusalem, focusing on philosophy, law, and how best to interpret the Hebrew Bible. The shocking part is that, back then, none of the debates were recorded – all of the knowledge and wisdom gathered through them was kept alive in the minds of the rabbis, being passed down to further generations only through speech. This is called the *oral tradition*. For centuries, the system of oral tradition preserved the teachings of the Talmud without any problem. However, trouble was on the horizon.

In 66 AD, the Jewish peoples' homeland of Judaea was ruled by the Roman empire, more specifically, the Roman procurator Gessius Florus. After receiving less tax money from Judaea than he felt was owed, Gessius Florus ordered his troops to seize silver from the holiest site in the Jewish world: the Temple of Jerusalem. This act ignited explosive opposition to the Romans, to which Gessius Florus responded by massacring thousands of Judaeans. A full-scale rebellion ensued, and while the Romans troops were undeniably strained by the Jewish resistance, their military might proved too strong to fend off. By 70 AD, most of Judaea had been destroyed, and only Jerusalem was still standing.

Under the direction of the Roman general, Titus, a massive army of sixty-thousand soldiers attacked Jerusalem with the sole purpose of razing it to the ground. Despite being greatly outnumbered, the Jewish resistance was able to keep the Roman onslaught at bay for over five months!

The mixture of Jerusalem's defensive walls and the passionate fighting of the Jewish soldiers made it extremely difficult for the Romans to lay siege to the city. Although they fought courageously, the Roman war machine wore down the Jewish resistance and eventually broke through the walls of the city and burnt Temple. Jerusalem was completely demolished by the ruthless Roman army. Witnesses remarked "that nothing was left that could ever persuade visitors that it had once been a place of habitation."

Widely regarded as one of the most disastrous events in Jewish history, the destruction of their capital city required the Jewish people to reimagine how they preserved their culture. Facing the uncertainty of having no Temple, no central meeting place for the continued debate of the Talmud, rabbis abandoned their oral tradition and began documenting the teachings of the Talmud in writing. This would ensure its survival should their people ever endure being separated from each other.

The story behind the writing of the Talmud is a lesson in overcoming adversity. It is a window into the strange relationship between destruction and creation, and how sometimes a terrible loss can have a silver lining. As the saying goes, necessity is the mother of invention.

2.1

1.1

2.1

One of the geometric rules given in the Talmud is “How much is the square greater than the inscribed circle? A quarter” What was the Talmudic value for pi?

2.2

1.1

2.2

In 4000 year old Egyptian papyrus we read “A circular field has diameter 9 chet. What is its area?” The Ahmes, the scribe of the papyrus, gives the solution: “Subtract 1/9 of the diameter namely 1 chet. The remainder is 8 chet. Multiply 8 by 8; it makes 64. Therefore, it contains 64 square chet of land.”

For the Egyptians a square with the side length 8/9 of the diameter of a circle was a good enough approximation to calculate the area of a circle.

Calculate the ancient Egyptian value of pi.

2.3

1.1

2.3

In Babylonian mathematics a circle was equated to a regular hexagon which side was equal to the radius of a circle. Calculate the Babylonian value of pi.

Medieval Europe – Schools in the Middle Ages

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Reading, writing, and going to school every day is seen by many as more of a chore than a privilege. It's easy to take these monumental opportunities for granted, especially when we don't consider what the alternatives are. Imagine how hard it would be to go through life completely illiterate – seeing nothing more than meaningless letters when looking at a sign, or having to ask someone to help you read an email. It sounds crazy, but did you know that for most of human history the average person couldn't read or write?

In the early Middle Ages, most Europeans weren't able to read and write… The peasant class was the most likely to be illiterate since all of their time was occupied performing physical labour to provide for their families. However, the trend of illiteracy also extended to the noble classes, and even to most kings! The job of reading and writing fell on the *clergymen*, the monks and religious figures who worked in the Roman Catholic Church. Since the printing press wasn't invented until the 15th century, monks spent much of their lives copying ancient texts and manuscripts by hand in order for them to be replicated. As a result, the monasteries where the monks lived boasted incredible libraries, and became centers of education. Many kings sought advice from the educated monks, but the problem was that many monks had taken a *vow of seclusion*, preventing them from sitting on the king's council.

Medieval rulers began to understand that having very few educated citizens threatened the stability of their empires. One king in particular had plans to change that: Charlemagne, King of the Franks, ruling from 768-814, held a strong desire to educate his people, and ordered monasteries and cathedrals to offer free education for all young boys. The new schools were established inside cathedrals or large churches. Their main purpose was to teach boys how to read and write in Latin since it was the language of the Holy Roman Empire. On top of learning Latin grammar, students were also instructed in the subject of *logic* (the art of debate), and *rhetoric* (the art of public speaking). Other studies included music, art, and various forms of physical education such as archery, wrestling, horseshoes, and hammer-throwing. Science and math were also taught, however, they weren't a priority compared to other subjects.

Now that they had designated schools, subjects, teachers, and students, only one thing was missing – textbooks! King Charlemagne invited a well-known monk from England named Alcuin of York to help design them. Born in 735, Alcuin had become a clergyman, scholar, poet, teacher, and was called "the most learned man anywhere to be found." So, naturally, he was a perfect fit for the role. Among his numerous works, Alcuin wrote a collection of math puzzles that stood out as one of the earliest math textbooks for kids. Unfortunately, only one of his books survived into modern times, and, amazingly enough, is still used in math classes over a thousand years after his death! The book is a series of fifty-three math problems called "Problems to Sharpen the Young."

Try to solve one of them.

2.4

1.1

2.4

There is a round field which contains 400 yards in its circumference. How many square yards will its area be? Alcuin obtains 10000 square yards. How does he get that, you might ask? Calculate the value of pi he used.

2.5

1.1

2.5

There is a city which is 8000 feet in circumference. How many houses could the city contain if each house is 30 feet long and 20 feet wide?

Japan – Samurai mathematicians

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=73#audio-73-3

The samurai were the warriors of Japan. For centuries, the *samurai* served as military to the *daimyos*, or "lords" of Japan. Even the word "samurai" roughly translates as "those who serve."

In 1600, Japan was divided into two alliances: the Western Army, commanded by *Toyotomi Hideyoshi* and the Eastern Army, led by *Tokugawa Ieyasu*. Toyotomi and his Western Army, whose soldiers were mostly *samurai*, invaded Korea with the plan to use Korea as the platform to conquer China.

Meanwhile, Tokugawa kept his forces away from the Korean campaign, and *samurai* who did not want to fight in Korea and China rallied under him. When the Eastern Army became strong enough it fought against Toyotomi's army in the Battle of *Sekigahara*, the largest battle of Japanese feudal history Tokugawa and his Eastern Army won.

This victory marks the beginning of the Tokugawa Shogunate, and the beginning of a period of peace in Japan that lasted almost 300 years. During this period, the war-trained samurai became redundant, their warrior skills were not required. and they had to earn their living by working as clerks or other governmental jobs, But the civil jobs were not as demanding as military service. These jobs left samurai with more leisure time on their hands. Many of samurai discovered that there was a lot more in this world rather than battles and wars. They used their time on other pursuits. They studied literature, philosophy, and *mathematics*. Turns out that these studies opened samurai doors to earn money in an unexpected for samurai way. The civil jobs were not paid handsomely, the pay was poor. And the samurai started taking teaching posts at small private schools called *juku*, which taught martial arts, of course, but also mathematics. The *juku* were attended by other samurai, merchants, and farmers. The juku students wanted to be able to solve mathematical problems on their own, some to better calculate business transactions, some to better plan their land and some just for pleasure and entertainment.

And this is the story of how samurai became mathematicians.

2.6

1.1

2.6

A circular road A that is 48 km in circumference touches at point P another circular road B of circumference 36 km. A cow and a horse start walking from the point P along the road A and B, respectively. The cow walks 6 km per day and the horse walks 12 km per day. How many days later days later do the cow and horse meet again at P?

2.7

1.1

2.7

A circle of radius r is inscribed in an isosceles triangle with sides 10 and 12. Find r.

Japan – Sungaku: mathematical offerings to gods and spirits

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=73#audio-73-4

Can you imagine that mathematical problems are used as offerings to gods? Hard to imagine but there was once a Japanese tradition to draw geometrical problems on wooden tablets and place them as offerings at Shinto shrines or Buddhist temples.

The tradition started after the Battle of Sekigahara in 1600. The Eastern Army under the rule of Tokugawa Ieyasu won and Tokugawa began his rule of Japan.

Tokugawa was deeply concerned over the rise of Christianity and other Western influences on Japanese people brought by the ever-increasing trade. He feared the changes the Western influence might bring on the traditional Japanese values and way of living. And thus he decided to close the country to forbid Portuguese, Spanish and other ships to enter Japan. The whole country went to sort of lockdown for 265 years. This Japan vs. the world lockdown was called *sakoku*, which means in Japanese “country in chains” or “lock up of country”). During the centuries long *sakoku* no foreigner could enter or Japanese leave the country on penalty of death.

Interestingly enough, the period of sukoku was one of the most prosperous and productive periods in Japan history. Japan greatly developed many aspects of their culture, including literature, art, and *mathematics*. Many practices were created and revived during this time, one of which was the practice of hanging wooden tablets, called *sangaku*, at shrines temples. The *sangaku* were colorfully painted and inscribed with math problems which, in a Western context, would be like seeing math problems etched on glass windows in a cathedral. While this practice might seem odd and weird, it made perfect sense to the people of that time.

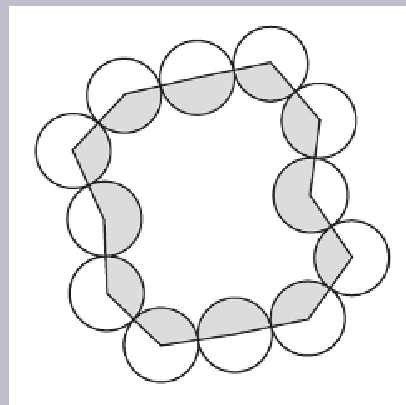
The practice of hanging these wooden tablets could be traced back to the practice of giving offerings to the gods. For instance, the most favorable offering would be a horse, but as most people couldn't afford to offer a horse, they chose to inscribe drawings of a horse on a wooden tablet as a substitute. The *sangaku*, a word that literally means "mathematical tablet", may have been also acts of homage–a thanks to a guiding spirit. Or they may have been cheeky challenges to other worshipers: The *sangaku* were hung up by various mathematic enthusiasts, including samurai, merchants, farmers; men, women and children alike. They would hang as a challenge to other worshippers, as if saying: "solve this if you can!".

2.8

1.1

2.8

The centers of a loop of the circles of radius *r* form the vertices of a polygon, as shown in the figure below. Let *S*1 be the sum of the shaded areas of the circles, and *S*2 the sum of the unshaded areas of the circles. Find *S*2 – *S*1.



Chapter 3 - Area and Volume

Map

1.1

1.19

Timeline

1.1

1.19

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Introduction

On a scale of one to ten, how important do you think measurement is? Without realizing it, you just confirmed the importance of measurement. Imagine living in a world without it. How would you tell the time, keep score of a basketball game, or count your Instagram followers? In fact, the origin of mathematics can be traced back to two fundamental human activities: counting and measuring.

Measurement is how we subdivide and categorize the physical properties of our world into the meaningful symbols we communicate to each other. Without the ability to accurately measure time, size, distance, speed, direction, weight, volume, temperature, pressure, force, sound, light, or energy, to name a few, civilization as we know it would not be able to function!

In Britain and America, the height of horses is traditionally measured in hands.

One of the earliest forms of measurement is *length*. To this day, we still use an age-old unit of measurement for length: *the foot*. Have you ever wondered why a body part was used for measurement? The answer is that, before there were standardized units for measurement, the only objects available for measuring were those produced by nature itself! So, naturally, we used our own body parts to measure the world around us. Archaic as it may be, even the *hand* is still used to measure the height of horses!

Measurement units based on the human body were common to societies across the world. Take the *fathom*, for example. Still is used to estimate the depth of water, a fathom is equal to an average human male's wingspan (the length between one's outstretched arms), and has been used independently by societies in Asia, Northern Europe, and the Mediterranean. The Vikings called it "favn," the Mongolians called it "ald," the Greeks called it "orguia," and the Russians called it "sazhen." Can you estimate what *hands*, *feet*, and *fathoms* are in centimetres?

The problem with using human body parts as units of measurement, is that not everyone's body is the same size. To address this issue, the rulers of ancient and medieval lands aimed to standardize measurements by using their own body parts. According to legend, the 12th century English king, Henry I, proclaimed that the *yard* would be the distance from the tip of his nose to the end of his outstretched thumb. The ancient Egyptian unit of measurement was the *cubit* – the length of the forearm from the elbow to the tip of the middle finger. But, they also had the *royal cubit*: a normal cubit plus the width of the palm of the current Pharaoh.

With all the confusion surrounding body parts as units of measurement, King John, of England, decreed in 1213 that there should be “one measure throughout our whole realm.” Hundreds of years later, we had successfully implemented a global unit of measurement independent of any body part: *the metre.*

Since the dawn of humanity, we have been on a quest to understand the universe we inhabit. This journey has given us mythology, religion, art, and many other cultural treasures. But, where they inspire and transcend, they fail at providing us a concrete and reliable picture of the material world. It is measurement that defines the world around us. As the bedrock of mathematics and science, our tools to create order out of chaos, human societies simply could not exist without it!

Egypt – Joseph and His Brothers

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Unlike in mathematics, there is no all-encompassing formula to solve the problems that we face in life. Very often we find ourselves taking two steps forward and one step back. And, what’s more, we find that life is not a straight line, but an unpredictable zig-zag of events that, if we stay true to our values, ultimately lead us to our destination. In the biblical story of Joseph, we see just how perseverance through hardship can lead us to the light at the end of the tunnel.

The story begins in ancient times in the land of Canaan which is now called Israel. At the age of seventeen, Joseph was given a colorful tunic from his father, Jacob, triggering the jealousy of Joseph’s brothers. To make matters worse, Joseph had a dream that predicted his brothers would one day bow to him. The idea of being inferior to the youngest-born member of the family was too much for them to bear, and they conspired to murder Joseph and steal his multicoloured tunic.

Joseph traveled with his brothers to the land of Dothan, unaware of their plot to kill him. Before they were able to enact their wicked plan, Joseph was blessed by a stroke of good luck as a camel caravan of spice traders crossed their path. Joseph's brother, Judah, proposed that, instead of killing him and stealing the tunic, it would be more profitable to sell Joseph to the traders as a slave. Judah's greed managed to save Joseph's life, but it was only the first twist in Joseph's unpredictable journey.

Joseph sold by his brothers.

The spice traders took Joseph to Egypt and sold him to Potiphar, the captain of the Pharaoh's guard. Potiphar took a liking to Joseph, and put him in charge of his household. However, Potiphar's wife also took a liking to Joseph and repeatedly attempted to seduce him. Sticking to his strong values, Joseph rejected her advances, leading her to suffer such a great embarrassment that she accused Joseph of trying to seduce her! When Potiphar heard the news, Joseph's fate was sealed, and he was sent to prison.

Brothers bow to Joseph.

As the years passed by, Joseph developed a reputation in prison for interpreting the inmates' dreams with remarkable accuracy. As fate would have it, the Pharaoh was puzzled by a dream of his that no sage, magician, or wiseman in all of Egypt could decipher. Word of Joseph's ability to interpret dreams found its way to the Pharaoh, and Joseph was summoned to interpret the Pharaoh's dream. In the dream, the Pharaoh witnessed seven plump, healthy cows emerge from the Nile followed by seven starved cows. Then seven robust heads of grain growing from a single stalk were swallowed up by seven thin, weak heads of grain. Joseph interpreted this to mean seven years of abundant harvest in Egypt followed by seven years of famine. Joseph recommended storing one fifth of all the grain for the next seven years in the granaries, grain storages, to prepare for the famine. The Pharaoh was so impressed by Joseph that he appointed him governor of all the land. When the famine struck, Joseph's plan worked like a charm, and news of Egypt's success traveled far and wide to other regions suffering from the famine...even to the land of Canaan, and Joseph's brothers.

Facing starvation in Canaan, Joseph's brothers set out to Egypt in search of grain. When they arrived, they arranged to meet with the governor of Egypt...their estranged brother Joseph! Upon meeting Joseph, they got down on their knees and bowed to him, just as Joseph's vision had predicted all those years ago...At the end Joseph forgave them and sent cards loaded with grain to his homeland.

3.1

1.1

2.1

Find the volume of a cylindrical granary of diameter 9 cubits and height 10 cubits. (1 cubit = 52 cm)

3.2

1.1

2.7

Find the volume of a cylindrical granary of diameter 10 cubits and height 10 cubits. (1 cubit = 52 cm)

3.3

1.1

2.7

A cylindrical granary of diameter 9 cubits and height 6 cubits. What is the amount of grain that goes into it? (1 cubit = 52 cm. The hekat was an ancient Egyptian volume unit used to measure grain, bread, and beer. It equals 4.8 litres. 30 hekats equals 1 cubic cubit)

3.4

1.1

2.7

A rectangular granary into which there have gone 7500 quadruple hekat of grain. What are its dimensions? (The hekat was an ancient Egyptian volume unit used to measure grain, bread, and beer. It equals 4.8 litres.)

3.5

1.1

2.2

A rectangular granary into which there have gone 2500 quadruple hekat of grain. What are its dimensions? (The hekat was an ancient Egyptian volume unit used to measure grain, bread, and beer. It equals 4.8 litres.)

3.6

1.1

2.3

Suppose it is said to thee. What is the area of a triangle of side 10 khet and of base 4 khet? (1 khet = 100 cubits, 1 cubit = 52 cm. Assume the triangle is isosceles).

3.7

1.1

2.3

Suppose it is said to thee, What is the area of a cut-off (truncated) triangle of land of 20 khet in its side, 6 khet in its base, 4 khet in its cut-off line? (1 khet = 100 cubits, 1 cubit = 52 cm. Assume the triangle is isosceles).

Mongolia – Genghis Khan

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What comes to mind when you think of Genghis Khan? Is he a notorious warlord? A misunderstood visionary? Much like a math problem, a human being is composed of many complex variables, and all must be considered to understand the whole of the equation.

The story of Genghis Khan begins in 12th century Mongolia. Originally named Temujin, he was rejected by his clan at the age of nine, and was taken by his father, Yesukhei, to live with the family of his future bride. On his voyage home, Yesukhei came across a rival Tatar tribe, who tricked him into eating a meal laced with a fatal poison. This left Temujin without a father, without his original clan, but with the plan to one day overcome it all and rule the world…

Temujin slowly developed into a brilliant military strategist. By the age of twenty, he avenged his father's death by demolishing the Tatar army, ordering the death of every Tatar male over three feet tall. Such brutality gave Genghis Khan a reputation that left his enemies trembling with fear.

Genghis Khan went on to conquer all the land from the Asian edge of the Pacific Ocean to modern-day Hungary in Europe in what became the biggest empire to date. But, it wasn't his savagery alone that helped him do it. Genghis Khan's creative vision, unrivaled organizational talents, and his speedy and robust cavalry were all vital aspects of his success. Fear did play a part, however. His army of mounted archers were known to his foes as "the devil's horsemen."

Although it's not possible to know exactly how many people perished at the hands of the Mongol conquests, historians believe that approximately forty million people were killed. Censuses from the Middle Ages show that China's population dropped by tens of millions, and some estimate that up to seventy-five percent of Iran's population disappeared as well. Under Genghis Khan, the global population was reduced by roughly eleven percent.

While it might appear that his reign was focused on bloodshed, Genghis Khan also financed advances in medicine and astronomy, as well as a number of construction projects like the extension of the Grand Canal, the palaces in Shangdu ("Xanadu") and Takht-i-Suleiman, a network of roads and postal stations throughout the empire, and strengthened the vital east-west trade route, "The Silk Road."

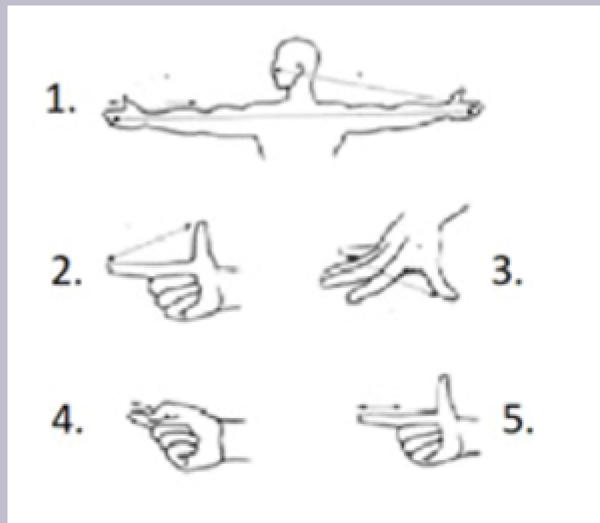
Another remarkable feat of the Mongols was their ability to transform from a nomadic tribe into the administrators of a vast empire in such a short amount of time. Their secret to effective political rule was to allow their conquered territories to operate their everyday affairs as they did before, but with Mongol leaders placed at the top of administrative hierarchies. Beyond his impact on science, infrastructure, war, and politics, Genghis Khan also influenced art and culture. The Mongol empire created a unification of divided lands, allowing artists and craftsmen to travel to different ethnic regions, creating a rich exchange of ideas betweens peoples.

After considering the variables of the human equation that is Genghis Khan, do you see him as a merciless warlord, or as a fearless trailblazer? Or, is it possible that, like in a math problem, one solution can be expressed in a number of different ways?

40-metre statue of Genghis Khan

3.8

1.1



The body of the human and its parts were considered as the most effective scales for measuring in all cultures. The limbs were considered as the best scales for measurement because they allowed instant measurements. You don't need a ruler! Your ruler is always with you. The following are some old Mongolian units of length. Can you match them with the body parts pictured in the diagram above?

- Huruu 1.5–2 cm
- Yamh 3.5 cm
- üzür sööm 18 cm
- ald 1.6 m
- sööm – 16 cm

3.9

1.1

2.5

The most ancient inscription found so far in Mongolian language is carved onto the stone known as

Genghis Khan's stone. It is dated around 1225 and immortalizes one of Genghis Khan's warrior's archery achievement:

"When Chinggis Khan was holding an assembly of Mongolian nobles at Bukha-(S)ochiqai after he had come back from the conquest of the Sartuul people, Yisüngke hit a target at 335 alds."

- How far is the target in meters?
- Do you think that this feat is achievable?

Europe – Mile, Furlong and Metre

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=90#audio-90-3

Imagine it is the year 4024. You are a historian trying to learn about life in Canada two thousand years ago. Where would you start? Would you look at "ancient Canadian" art, music, and architecture? While these would all be very revealing aspects of our culture, did you know that math, and units of measurement also shine a very powerful light onto our society's values?

Think about ancient Rome, for example. Rome was the greatest empire of the ancient world, and it should come as no surprise that one of Rome's units of measurement was the *mile*. Why the mile? The word "mile" comes from the Latin phrase *mille passus* – "one thousand paces of a marching man." So, since Roman culture was, in many ways, defined by their military conquests, it makes perfect sense that this unit of measurement would have been created as the by-product of the many paces taken during a soldier's march.

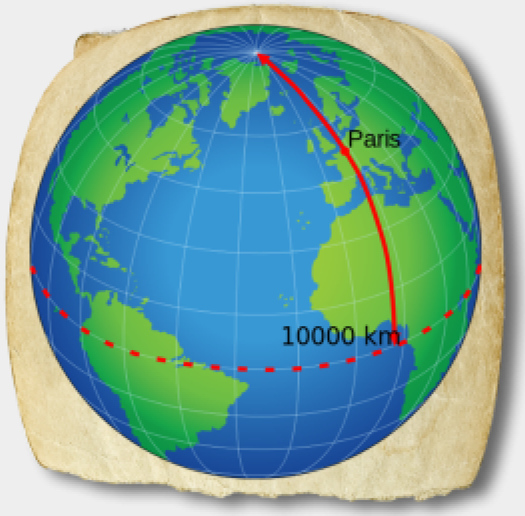
Roman legionaries

Plowing with oxen

Agricultural societies had units of measurement that reflected their values as well. Let's look at Britain between the 5th and 11th centuries. The word "acre" describes a plot of land equal to the area of land that could be farmed by one man and two oxen in a single day. Most people at that time worked on farms, so it makes sense that they used the *acre* as a form of measurement. The same goes for the word *furlong*. Derived from the Old English words *furh* (furrow) and *lang* (long), "furlong" means the length of a furrow: 201 metres, the distance a team of oxen could plough, without resting, in one day. Despite the fact that modern people have lost touch with the origins of these words, "mile," "acre," and "furlong" are still vital units of measurement across the English-speaking world and beyond.

Sometimes, units of measurements can also change as the result of cultural revolutions. Did you know that the *metric system* would never have existed without the French Revolution of 1789? The French revolutionaries made many historic changes: abolishing slavery, giving civil rights to Jews and Muslims, allowing men and women to divorce, and making education available to people of all socioeconomic classes. They even changed their calendar, renamed their months, and made a week ten days instead of seven!

As profound as these changes were, many argue that the most globally significant was the birth of the metric system. In 1790, the French revolutionaries sought to create a new system that was simple, scientific, and a universal constant that would never change. They appointed a commission that invented "the metre," adapting the name from the Greek word *metron*, meaning "a measure." They defined one metre a measurement equal to one ten-millionth of the distance from the equator to the North Pole. In order to figure out this distance, they set out on an expedition in 1792 to determine the "arc" that the earth made in a line between Dunkirk and Barcelona, and after seven long years, finally produced the "*metre*."



What is a metre?

A metre station in Paris

Human cultures cannot resist the temptation to instill their values into every feature of their civilizations. As a result, we manage to conceive new creations that lead to greater prosperity for all mankind. Only time will tell how the cultural values of the future will innovate the world as we know it. The question remains, do superior forms of mathematical measurement lie ahead?

3.10

1.1

2.6

A basilica is 240 feet long and 120 feet wide. The basilica paved with tiles 23 inches long and 12 inches wide. How many tiles are needed to cover the basilica? (There are 12 inches in a foot.)

3.11

1.1

2.7

A wine cellar is 100 feet long and 64 feet wide. How many casks can it hold, given that each cask is seven feet long and four feet wide, and given that there is an aisle four feet wide down the middle of the cellar?

3.12

1.1

2.7

A four-sided town measures 1100 feet on one side and 1000 feet on the other side; on one edge, 600, and on the other edge, 600. I want to cover it with roofs of houses, each of which is to be 40 feet long and 30 feet wide. How many dwellings can I make there?

3.13

1.1

2.7

I have a cloak 100 cubits long and 80 cubits wide. I wish to make small cloaks with it; each small cloak is 5 cubits long, and 4 wide. How many small cloaks can I make? (1 cubit = 50 cm)

3.14

1.1

2.7

A four-sided field measures 30 perches down one side and 32 down the other; it is 34 perches across the top and 32 across the bottom. How many acres are included in this field? (1 perch ≈5 m. 1 acre = 40 perches by 4 perches ≈ 4000 m2)

3.15

1.1

2.7

There is a triangular city which has one side of 1000 feet, another side of 1000 feet, and a third of 900 feet. Inside of this city, I want to build houses each of which is 20 feet in length and 10 feet in width. How many houses can I build in the city?

Babylon – The Code of Hammurabi

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=90#audio-90-4

The Babylonian empire was founded over four thousand years ago. It had many inventions in many fields. It included advanced geometry and astronomy, innovations in irrigation and warfare. The Babylonians built some of the most complex and visually stunning canal systems in the ancient world and their engineers knew to the shovelful how much earth was required to build the ramps that packed dirt to the top of a sieged city's walls. It influenced many elements of the ancient world, and continues to influence modern civilization as well. Many of our beliefs about right and wrong stem from the laws created by the Babylonians. One man, in particular, is credited as the grandfather of the modern legal system. His name is *Hammurabi*, and he reigned as the king of Babylon from 1792 B.C. to 1750 B.C.

In his forty-two years as king, Hammurabi achieved many notable successes including the transformation of Babylon from a small city-state into an empire. However, none of his military achievements were as important as his ultimate gift to civilization, *Hammurabi's Law Code*: a system of two hundred and eighty-two laws carved into a massive finger-shaped black stone pillar called a *stele*. For the first time, laws were written into stone, suggesting they were unchangeable, even by the king himself!

The gigantic stele was on display for all Babylonians to see so that no one could commit a crime and use ignorance as a defense from punishment.

The Code of Hammurabi stele

Here are some examples of laws from Hammurabi's Law Code:

- *"If a man brings an accusation against another man, charging him with murder, but cannot prove it, the accuser shall be put to death."*
- *"If a man bears false witness in a case, or does not establish the testimony that he has given, if that case is a case involving life, that man shall be put to death."*
- *"If a man bears false witness concerning grain or money, he shall himself bear the penalty imposed in the case."*

Notice how Hammurabi's laws were the first to include this fundamental legal principle of our current system: the presumption of innocence. A person is presumed innocent of a crime until proven guilty in a court of law. Can you imagine living in a world where you were considered guilty of a crime and had to prove your innocence? We have Hammurabi to thank that it doesn't work that way. And, we're not alone. 6th century Roman law, and 15th century Islamic law both adopted the same principle.

On top of protecting the rights of innocent citizens, Hammurabi's law code also built in safeguards to ensure judges oversaw court cases fairly. Considering the following law:

- *"If a judge pronounces judgment, renders a decision, delivers a verdict duly signed and sealed, and afterward alters his judgment , they shall call that judge to account for the alteration of the judgment which he has pronounced, and he shall pay twelve-fold the penalty in that judgment; and, in the assembly, they shall expel him from his judgment seat."*

With artwork of Hammurabi decorating the halls of the U.S. Supreme Court, and Capitol building, it's obvious that he's had a profound influence on the past and present. And, although other legal texts written before Hammurabi's Law Code have been discovered, none have had the same lasting impact on global civilization. Like his law code, forever carved in stone, Hammurabi's legacy stands firmly as one of a pioneering lawmaker who aspired to – in the words of his monument – *"prevent the strong from oppressing the weak and to see that justice is done..."*

3.17

The bar-relief of Hammurabi in the U.S. Supreme Court

3.16

1.1

2.6

A little rectangular canal is to be excavated for a length of 5 km. Its width is 2 m, and its depth is 1 m. Each laborer is assigned to remove 4m3 of earth, for which he will be paid one-third of a basket of barley. How many laborers are required for the job, and what are the total wages to be paid?

3.17

1.1

2.7

A canal is 5 rods long, 11/2 rods wide, and 1/2 rod deep. Workers are assigned to dig 10 gin of earth for which task they are paid 6 sila of grain. What is the area of the surface of this canal and its volume? What are the number of workers required and their wages? (1 rod = 12 cubits = 12 x 50 cm= 6 m, 1 sar area = 36 m2, 1 gin = 5 litres, 1 sila = 1 litre)

3.18

1.1

2.7

A siege ramp is to be built to attack a walled city. The volume of earth allowed is 5400 sar. The ramp will have a width of 6 rods, a base length of 40 rods, and a height of 45 cubits. Construction of the ramp is incomplete; an 8-rod gap is left between the end of the ramp and the city wall. The height of the uncompleted ramp is 36 cubits. How much more earth is needed to complete this ramp? (1 sar volume = 1 sar area x 1 cubit =18 m3 , 1 cubit = 50 cm; 1 rod = 12 cubits = 6 m)

Chapter 4 - Fractions

Map

1.1

1.19

Timeline

1.1

1.19

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Introduction

Can you recall a word that sounds like "fraction" and even means the same? It is a word that we use in everyday life. Like when your friend is skiing downhill, falls and … his leg? Right, the word is "to fracture" The word "fraction" comes from the Latin word "fractio," which means "a breaking." It reflects the idea of breaking something into smaller parts, which is what a fraction represents.

The fractions that we use today come to us from ancient Egypt. The ancient Egyptians developed the symbols that represented parts of something like halves, quarters, thirds as opposed to wholes like one, three or four. . And one of the reasons that the Egyptians needed fractions was the fact that for thousands of years there was no real money in Egypt. In ancient world before inventing money such as coins or banknotes people used different objects to represent the cost of things. Cacao beans, cowrie shells, whale teeth, and giant stones were among objects used as money in different cultures around the globe. Early civilizations were especially fond of metals, particularly silver, gold which were used to trade and exchange goods and services. But the Egyptians didn't bother with real objects for money. Instead of money the Egyptians used the system called barter. The barter system is where people trade goods or services for other goods and services without the use of money. For example, someone may trade grain for a fur coat, or someone may trade food in exchange for help with a harvest. Bartering can be tricky because both people must agree that the trade is fair. For most of its history, ancient Egypt's economy operated on a barter system without cash. Instead of cash they had virtual money, believe it or not, similar to modern bitcoins! A virtual coin was called deben and everything was priced in debens. In every given time in Egypt there were known prices for almost every item imaginable, for instance, 1 cake: 1/5 deben, 1 bundle of vegetables: ½ deben, 1 shirt: 2½ deben, 1 goat: 2½ deben, 1 litre of oil: 1 deben, 1 loaf of bread: 1/10 deben, 1 slave girl: 4 deben, 1 ordinary male slave: 3 deben (yes, slaves, don't be surprised – every civilization had slaves. Also, do you know that the word "slave" comes from old French word for slavs the people from Eastern Europe? Back in the time they were the majority of enslaved people.)

Egyptian craftsmen

So the barter worked like that: you compare prices and trade. If a litre of oil costed one *deben*, and a loaf of bread was 1/10 of *deben*, 1 litre of oil could be traded fairly for 10 loaves of bread . In this same way, if one jug of beer costed a 1/3 *deben* and a day's work was worth one *deben* then one would fairly be paid three jugs of beer for one's daily labour. Let's imagine that you are an ancient Egyptian who hired workers to work in your field or build a shed for you.

You and your workers agreed that you would pay them in beer and bread (that was the usual form of payement in ancient Egypt) and they would work for ten days If one jug of beer costed a 1/3 *deben* and a day's work was worth a *deben* then you would pay your workers 3 jugs of beer. Or 100 loaves of bread since one loaf cost 1/10 deben. And then the workers could exchange their beer and bread for what they need, say sandals or oil or papyrus.

And we can safely say that the Egyptians were very practical in their approach to mathematics and their barter trade required that they could deal in fractions.

Egypt. Eye of Horus

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Among all the fractions the Egyptians used there were six fractions considered to be sacred. These sacred fractions, 1/2, 1/4, 1/8, 1/16, 1/32, and 1/64, all with powers of two in their denominators, were used to represent the fractions of *hekat*, the unit measure of capacity for grains. According to a legend, the Egyptians attributed these fractions to the six parts of the eye of the god Horus the pieces were lost in a battle, and restored by the god Thoth.

Horus, the falcon-headed god, was an important god in ancient Egypt. The symbol representing his eye, known as the Eye of Horus, was a powerful symbol used to protect from evil. According to the old myth, in Egyptian mythology, Horus was the son of Isis and Osiris, king of Egypt.

Osiris had a brother named Set, who after years of watching his brother bring prosperity and joy to the people of Egypt became jealous of his brother and killed him to claim his throne. Once he had come of age, Horus sought to reclaim the throne that was rightfully his. In a battle between Horus and Set, Set gouged out Horus's eye and ripped it into six pieces (the fight wasn't too one sided – Set lost a testicle).

The sacred unit fractions attributed to the six parts of the eye of the Horus.

Horus, an ancient Egyptian deity.

Seeing the damage of the rightful heir to Egypt, Thoth, the wise moon god and patron of the sciences and the art of writing, volunteered to put Horus's eye back and heal it. He collected the pieces of Horus's eye and restored them using magic. To do so, he assigned the value of a fraction to each individual part of the eye. A whole eye represented one. the inner corner of the eye stood for 1/2, the pupil for 1/4, the eyebrow for 1/8, the outer corner for 1/16, the curling line for 1/32, and the cheek mark for 1/64. The total should have added up to a whole, but Thoth made sure that it didn't.

He named the parts the way that one fraction was missing. Only he would know the value of the missing fraction and this would give him the ultimate power to protect the world from evil! Can you break Thot's spell and find the missing fraction? The answer is quite simple but requires to know how to add fractions. If you add all the pieces up, you find that the sum comes to 63/64 not 64/64, 1/64 short of 1. So, after all, Thoth's magic was not that magical!

4.1

1.1

5.1

You have to divide 8 loaves among 10 men. What will be the best way to do it?

4.2

1.1

5.1

A quantity and its $\frac{1}{2}$ added together become 16. What is the quantity?

4.3

1.1

5.1

A quantity and its $\frac{2}{3}$ are added together and from the sum $\frac{1}{3}$ of the sum is subtracted, and 10 remains. What is the quantity?

4.4

1.1

5.1

A quantity, its $\frac{1}{2}$ and its $\frac{1}{4}$, added together, become 10. What is the quantity?

4.5

1.1

5.1

A quantity together with its two-thirds has one-third of its sum taken away to yield 10. What is the quantity?

4.6

1.1

5.8

The sum of a certain quantity, together with its two-thirds, its half, and its one-seventh, becomes 37. What is the quantity?

4.7

1.1

5.9

[Given] a quantity: its two-thirds, one-half, and one-seventh are added together, giving 33. What is the quantity?

4.8

1.1

5.10

Egyptian only used unit fractions $\frac{1}{n}$ with the exception of $\frac{2}{3}$. They would write any fraction whose numerator is not 1 as a sum of unit fractions (and $\frac{2}{3}$ if needed), e.g.

$$\frac{15}{40} = \frac{3}{8} = \frac{2}{8} + \frac{1}{8} = \frac{1}{4} + \frac{1}{8}$$

And a scribe would write in his papyrus not $\frac{15}{40}$ but $\frac{1}{4}$ $\frac{1}{8}$.

Another example:

$$\frac{13}{15} = \frac{10}{15} + \frac{3}{15} = \frac{2}{3} + \frac{1}{5}$$

And a scribe would write in his papyrus not $\frac{13}{15}$ but $\frac{2}{3}$ $\frac{1}{5}$.

There is another peculiar feature in the way Egyptians decomposed fractions. They never repeat the same fractions in the sums. For instance, $\frac{3}{5} = \frac{1}{5} + \frac{1}{6} + \frac{1}{30}$. The trick here is to first bring the fraction to a greater denominator $\frac{3}{5} = \frac{12}{30}$ And then decompose the resulting fraction $\frac{12}{30} = \frac{6}{30} + \frac{5}{30} + \frac{1}{30} = \frac{1}{5} + \frac{1}{6} + \frac{1}{30}$.

Can you write the following fractions the way Ahmes-the-scribe wrote in the papyrus in around 1550 BC:

$\frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}$?

Mesopotamia. The Epic of Gilgamesh

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=102#audio-102-2

The Epic of Gilgamesh, while not the oldest written piece of literature, is regarded as the oldest written tale in the world. It's a poem from ancient Mesopotamia (now modern Iraq) and it is over 4,000 years old!

The Epic of Gilgamesh is the work of an anonymous Babylonian poet, about the king of the walled city of Uruk. In the story, King Gilgamesh is claimed to be part god, part human – making him the strongest and most beautiful man in the world, but with the mortality of a human being.

The young Gilgamesh is widely disliked in his kingdom He terrorizes his people, sleeps with the brides of his subjects on their wedding night, and consistently uses force to get his way in all things. The gods decide to humble him by creating the wild man, Enkidu. The gods answer by creating a man – one that is equal to Gilgamesh in strength, yet his opposite. Enkidu was their creation, brought to life from water and clay, and was just as wild as Gilgamesh – but with complete innocence. Enkidu was raised by the animals of the forest, completely ignorant of humans – until a sacred priestess by the name of Shamhat introduces him to the ways of humanity and civilisation.

Once tamed and introduced to civilization, Enkidu is outraged by the stories he hears of Gilgamesh and his arrogance and travels to Uruk to challenge him. Enkidu and Gilgamesh are considered an even match by the people, but after an epic battle, Enkidu is bested. He freely accepts his defeat, and the two become best friends Their friendship tames Gilgamesh a little bit, much to the relief of his people.

In order to make his name immortal, Gilgamesh decided to travel with Enkidu to the Cedar Forest to kill the guardian of the forest monster-demon Humbaba. Humbaba has done nothing wrong and is favored by the gods for his protection of the forest, but this means nothing to Gilgamesh, who is only thinking of himself. Once the two friends have defeated Humbaba, Humbaba cries out for mercy, but Enkidu encourages Gilgamesh to kill him, which he does.

The friends return to Uruk where Gilgamesh prepares to celebrate his victory, putting on his finest clothes. This attracts the attention of the goddess Ishtar. Ishtar is enraged and sends the Bull of Heaven, down to earth to destroy Uruk and Gilgamesh. The two heroes kill the bull and Enkidu flings one of its legs at Ishtar in contempt. For this affront to a deity, as well as his cruelty to Humbaba, the gods decree Enkidu must die.

Enkidu lingers in pain for some time, and when he dies, Gilgamesh falls into deep grief. Recognizing his own mortality through the death of his friend, he questions the meaning of life and the value of human accomplishment in the face of ultimate extinction. He cries:

How can I rest, how can I be at peace? Despair is in my heart. What my brother is now, that shall I be when I am dead. Because I am afraid of death I will go as best I can to find Utnapishtim whom they call the Faraway, for he has entered the assembly of the gods.

Casting away all of his old vanity and pride, Gilgamesh sets out on a quest to find the meaning of life and, finally, some way of defeating death. He travels far, through the mountains and past the Scorpion People, hoping to find Utnapishtim, the man who survived the Great Flood and was rewarded with immortality by the gods. At one point, he meets the goddess of wine-making and brewing Siduri who tells him his quest is in vain and he should accept life as it is and enjoy the pleasures it has to offer. Gilgamesh rejects her advice, however, as he believes life is meaningless if one must eventually lose all that one loves.

Siduri directs him to the ferryman Urshanabi, who takes him across the waters of death to the home of Utnapishtim. Utnapishtim tells him that there is nothing he can do for him. He, Utnapishtim, was granted immortality by the gods, he says, and has no power to do the same for Gilgamesh. Even so, Utnapishtim offers Gilgamesh two chances at eternal life. First, he must show himself worthy by staying awake for six days and nights, which Gilgamesh fails at, and then he is given a magic plant which, in a moment of carelessness, he leaves on the shore while he bathes, and it is eaten by a snake. Having failed in his quest, Gilgamesh has Urshanabi bring him back to Uruk, where he has a dream in which the father of the gods, Enlil tells him: 'You were given the kingship, such was your destiny, everlasting life was not your destiny. Because of this do not be sad at heart, do not be grieved or oppressed; he has given you power to bind and to loose, to be the darkness and the light of mankind. … But do not abuse this power, deal justly with your servants in the palace, deal justly before the face of the Sun.'

And that is how Gilgamesh, the king of Uruk has lived, until his time came to be laid in the tomb.</p.

4.9

1.1

5.7

A woman weaves a textile that is to be 48 rods long. In one day, she weaves $\frac{1}{3}$ rod. In how many days will she cut the textile from the loom [be finished]?

4.10

1.1

5.8

Two-thirds of two-thirds of a certain quantity of barley is taken, and we obtain 100 units. What was the original quantity?

4.11

1.1

5.9

It is known that the digging of a canal becomes more difficult the deeper one goes. In order to compensate for this fact, differential work allotments are computed: a laborer working at the top level is expected to remove $\frac{1}{3}$ sar of earth in one day, while a laborer at the middle level removes $\frac{1}{6}$ sar, and one at the bottom level, $\frac{1}{9}$ sar. If a fixed amount of the earth is to be removed from the canal in one day, how much digging time should be spent at each level?

4.12

1.1

5.10

I have two fields of grain. From the first field, I harvest two-thirds of a bushel of grain per unit area; from the second, one-half a bushel per unit area. The yield of the first exceeds the second by 50 bushels. The total area of the two fields together is 300 units. What is the area of each field?

4.13

1.1

5.8

There are two silver rings; $\frac{1}{7}$ of the first and $\frac{1}{11}$ of the second ring is broken off, so that what is broken off weighs one shekel. The first that is diminished by $\frac{1}{7}$ weighs as much as the second diminished by its $\frac{1}{11}$. What was the weight of the silver rings originally?

Islamic Empire. 1001 Nights or Arabian Nights

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=102#audio-102-3

We have all heard the story of Aladdin and the Magic Lamp. It has sparked our collective fascination for generations whether through Disney's famous animated film or the earliest cinematic depiction from all the way back in 1917. The mystique of distant adventures from exotic lands has enticed us into the worlds of *Ali Baba and the Forty Thieves, Sinbad the Sailor*, and thrust us deep into realms where genies can bring our most impossible dreams to life! But, what do Aladdin, magic lamps, genies, thieves, and sailors all have to do with each other? They all come from one common source! An awe-inspiring collection of folk tales that have roots in Persia, India, Greece, Turkey, Central Asia, Middle East, Egypt. All of them.weaved into one epic book known as *1001 Nights*, or *Arabian Nights*.

The story behind how *Arabian Nights* came to be is equally as interesting as the folk tales themselves! It begins with an ancient Arabian King by the name of Shahrayar. King Shahrayar was betrayed by his wife who had cheated on him with another man. King Shahrayar was driven mad by his wife's actions and hatched a sinister plot to enact his revenge. He swore to marry every eligible bride in his kingdom and have them executed the morning after the wedding before they had the opportunity to betray him like his wife did...

Under the influence of his ferocious desire for revenge, King Shahrayar initiated his plan and began marrying and executing all the unwed women in his dominion. Before long, the king had taken the lives of so many brides that there were none left except the daughter of his grand vizier (an advisor to the king). Her name was Shahrazad, and on top of being beautiful, she was also quick-witted, creative, and a brilliant storyteller! With the announcement that she would be the next bride of King Shahrayar, and well aware of her fate, Shahrazad had very little time to devise a strategy to save her own life...

Shahrazad knew that escape was not an option, so she put all of her faith into distracting the king so that the wedding would be delayed for as long as possible. The only question was how to do it. The night before the wedding, Shahrazad took a leap of faith, and began telling the king a long, captivating story. The king became so mesmerized by the story that Shahrazad kept it going on and on until the sun peeked over the horizon and began to rise. King Shahrayar was so interested in how the story would end that he postponed the wedding for the following day!

At that moment, Shahrazad knew she had found her secret weapon. All she had to do was begin a new story every night and leave it unfinished until the following night, at which point she would begin a new story spanning into the next day and repeat the same process. Shahrazad, a masterful storyteller, was able to keep telling stories for one-thousand and one nights, and by that point, through the magical and therapeutic power of storytelling itself, the king was cured of his paranoia and anger. He abandoned his vengeful bloodlust, married Shahrazad, and they lived happily ever after, leaving behind a trail of one-thousand and one fantastic stories!

Shahrazad and King Shahrayar

The power of storytelling managed to literally save Shahrazad's life, and it has also saved the lives of countless others potential brides. But, listeners beware! The storytelling spell Shahrazad cast on king Shahrayar may have ended happily, but there are some who fall victim to the trance that a well-told story can conjure!

4.14

1.1

5.9

A woman dies, leaving her husband, a son, and three daughters. Calculate the fraction of her estate each will receive.

[Note: The conditions of Islamic law must be followed—that is, the husband must receive one-fourth share, and his son twice as much as a daughter.]

Greece. Archimedes and the Golden Crown

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=102#audio-102-4

For almost a hundred years, the ancient Greek city of Syracuse had been at war with Carthage, an ancient city in Northern Africa. Finally the Syracusan troops, tired of the inefficiencies of their leaders, elected a commander from amongst themselves. It was a young general called Hiero.

After Hiero led the Syracusans to victory against their enemies, the people of Syracuse chose Hiero to be their king.

Hiero was grateful to the gods for his success and good fortune, and to show his gratitude, he decided to place in a certain temple, a golden crown in their honour. Hiero weighed out a precise amount of gold, and appointing a goldsmith, commanded him to fashion out of the gold a crown worthy of the gods.

The goldsmith did as he had been ordered, and on he appointed day, he delivered to the king an exquisitely wrought crown. The crown seemed to weight exactly as much as the gold that the king had given the goldsmith. Hiero was pleased, and paid the goldsmith handsomely. The goldsmith, receiving his payment, went away.

Hiero made preparations for the ceremony to place the crown in the temple. But a few days before the ceremony, he heard rumours that the goldsmith had cheated him, and given him a crown not of pure gold, but of gold that had silver mixed in it. The goldsmith, said the rumours, had replaced some of the gold that Hiero had given him, with an equal weight of silver.

Hiero was furious to learn that he might have been tricked. But he was a fair-minded man and wished to determine the truth before he punished the goldsmith.

If the goldsmith had indeed cheated him and mixed silver into the gold, then the goldsmith would have to be punished, and the crown could no longer be given as an offering to the gods. But if the goldsmith had been honest, then the crown remained what it had been intended to be, a sacred offering, and it would be placed in the temple as planned. So it was important that Hiero find out the truth quickly, before the day fixed for the ceremony, and without damaging the crown in any way.

Hiero believed there was only one man in Syracuse capable of discovering the truth and solving his problem. This was his cousin, Archimedes, a young man of 22, who was already renowned for his work in mathematics, mechanics and physics.

Deep in thought, pondering how best to solve the king's problem, Archimedes walked to the public baths for his daily bath. Still thinking about the golden crown, he stepped into a tub of cool water for a dip. As he began to lower himself into the water, the water in the tub began to spill out over the sides. Curious, Archimedes continued to lower himself slowly into the water, and he noticed that the more his body sank into the water, the more water ran out over the sides of the tub. He realised that he had found the solution to Hiero's problem. He was so excited by his discovery that he jumped out of the tub, and ran all the way home naked without remembering to put his clothes on, and shouting 'Eureka, Eureka!' – which in Greek means, 'I have found it! I have found it!'

What Archimedes had found was a method for measuring the volume of an irregularly-shaped object. He realised that an object, when immersed in water, displaced a volume of water equal to its own volume, and that by measuring the volume of the displaced water, the volume of the object could be determined, regardless of the object's shape.

So, he could measure the volume of the crown by measuring the volume of the water spilled from a container filled with water to the brim when the crown was fully dipped in it.

How then, would this realisation help him to answer Hiero's question – had the goldsmith mixed silver in the golden crown or not? Let us see how Archimedes used his discovery to solve the king's problem.

Archimedes knew that gold was denser than silver – so a piece of gold weighing a certain amount would be smaller than a piece of silver weighing the same:Thus, if the goldsmith had stolen some of the gold the king had given him, and replaced it with an equal weight of silver in the crown, then the total volume of the gold and silver crown would be greater than the volume of the original amount of gold.

So now, all that remained for Archimedes to do was to compare the volume of the crown to the volume of the amount of gold that Hiero had given the goldsmith. He filled the bowl with water to the brim. He lowered the crown into the water. He knew that if the crown was pure gold, its volume would be the same as that of the lump of gold (which he had made sure weighed the same as the crown), regardless of shape, and that it would displace the same amount of water as the gold. But, if the goldsmith had replaced some of the gold with silver, then the volume of the gold and silver crown would be greater than the volume of the gold, and so the crown would displace more water than the gold.

Archimedes found that the crown did, in fact displace more water than the lump of gold of equal weight. Thus he came to the conclusion that the crown was not pure gold, and that the goldsmith had indeed mixed some silver (or other, lighter metal) into the gold and cheated the king. That is how Archimedes is said to have helped King Hiero detect the goldsmith's fraud.

4.15

1.1

5.7

If someone says a workman receives a pay of 10 dirhams per month, how much must he be paid for six days?

1 mina = 100 drachms, the Greek silver coins

4.16

1.1

5.8

The father perished in the shoals of the Syrtis, and this, the eldest of the brothers, came back from that voyage with five talents. To me he gave twice two thirds of his share, on our mother he bestowed two eighths of my share, nor did he sin against divine justice.

1 talent = 6,000 drachmas, the Greek silver coins.

4.17

1.1

5.9

"Best of clocks, how much of the day is past?" There remain twice two thirds of what is gone.

4.18

1.1

5.10

We three Loves stand here pouring out water for the bath, sending streams into the fair flowing tank. I on the right, from my long-winged feet, fill it full in the sixth part of a day; I on the left, from my jar, fill it in four hours; and I in the middle, from my bow, in just half a day. Tell me in what a short time we should fill it, pouring water from wings, bow, and jar all at once.

4.19

1.1

5.8

I am a brazen lion; my spouts are my two eyes, my mouth, and the flat of my right foot. My right eye fills a jar in two days, my left eye in three, and my foot in four. My mouth is capable of filling it in six hours; tell me how long all four together will take to fill it.

4.20

1.1

5.9

Brick makers, I am in a great hurry to erect this house. Today is cloudless, and I do not require many more bricks, but I have all I want but three hundred. Thou alone in one day couldst make as many, but thy son left off working when he had finished two hundred, and thy son in law when he had made two hundred and fifty. Working all together, in how many hours can you make these?

4.21

1.1

5.10

After praying for a just increase in my fortunes of gold, I have nothing. I gave 40 talents of gold under evil auspices to my friends in vain, and I see my enemies in possession of a half, a third, and an eighth of my fortune. How many talents did this man once have?

4.22

1.1

5.9

The inscription on the tomb reads: “This tomb holds Diophantus. Ah, how great a marvel!” The inscription then tells the length of his life as follows: God granted him to be a boy one-sixth of his life, and adding one- twelfth part of this, he clothed his cheeks with down. He lit the light of wedlock after one-seventh part of his life, and after 5 years in his marriage he granted him a son. Alas, late-born child; after reaching one-half the measure of his father’s life, cruel fate took him. After consoling his grief by the science of mathematics for 4 years, Diophantus ended his life. How old was Diophantus when he passed away?

Chapter 5 - Pythagorean Theorem

Map

1.1

1.19

Timeline

1.1

1.19

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=117

Introduction

The Pythagoren theorem is called after Pythagoras, a Greek mathematician who lived 570 to ca. 490 BCE. However, civilizations from Babylon to Egypt discovered the concept behind this theorem about one thousand years before Pythagoras. Apparently, there was no civilization in the world history that didn't know or use the properties of the right-angled triangle, the properties that we now call the Pythagorean Theorem. This theorem represents one of the oldest mathematical developments in human history. We can even say that this theorem is as important and vital to human race as inventing agriculture or domesticating wild animals.

People from many different cultures throughout history had a lot of the same problems we have now–they needed to measure land and distances, build sturdy structures such as pyramids or houses, and dig canals for irrigation. These practical matters are what inspired the ancient mathematicians all around the world to discover the relationship between the sides of a right angled triangle. Digging canals, restoring land borders after flooding. building altars, building temples, measuring slopes of pyramids, measuring depth of rivers, building scaling ladders, and a myriad of other tasks that involved creating and measuring right triangles was impossible without learning about the theorem long before it got the name we recognize today.

China – The Legend of Yu the Great

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How important is Pythagorean theorem? Listen to the 2000 old words from the old Chinese manuscript about the Yu the Great and the Gougu theorem (that's how Pythagorean theorem is called in China).

"Emperor Yǔ quells floods, he deepens rivers and streams, observes the shape of mountains and valleys, surveys the high and low places, relieves the greatest calamities and saves the people from danger. He leads the floods east into the sea and ensures no flooding or drowning. **This is made possible because of the Gōugǔ theorem."**

Yu the Great

The Yellow River is considered by many as the cradle of Chinese civilization. Often referred to as "China's Pride," it spans over five thousand kilometres and is one of the longest river systems in the world. How is it possible then, that the ancient Chinese saw the river as a curse, even going as far as to name it "China's Sorrow"?

Four thousand years ago, during the reign of King Yao, the Yellow River was the cause of mass flooding events that frequently demolished settlements and displaced the ancient residents of the Chinese heartland. King Yao tasked an engineer named Gun with the job of taming the mighty river. Gun spent nine years creating a series of dikes and dams to control the flow of the river, but he was ultimately unsuccessful. When Gun's son, Yu, reached adulthood, he chose to pursue his father's work with the aim of saving the Chinese people from the plague of floods caused by the Yellow River.

Yu teamed up with a mysterious agricultural master named Hou Ji, and together they created a system that actually worked to control the flooding of the river! Rather than directly damming its flow, Yu installed a sequence of irrigation canals that diverted flood water into the surrounding fields in small enough amounts that it wouldn't overwhelm the land. On top of irrigation, Yu also dredged the riverbed of sediment and debris to improve the flow of the waterways. It is said that Yu spent thirteen years dredging the river – eating, sleeping, and labouring alongside the common workers. Their effort was a complete success, ushering in a new era of prosperity free from flooding in ancient China. The project earned Yu great acclaim, and is referred to in Chinese history as "Great Yu Controls the Waters." However, Yu's success was not without sacrifice.

Hou Ji

Beyond the toll the project took on Yu's body, he suffered a much greater burden emotionally. After being married for only four days, duty called on Yu to commit himself to fight the floods. He left home telling his wife he did not know when he would return. In the thirteen years he dedicated to subdue the flooding, he passed by his family's home only three times, but never went inside. The first time, he could hear his wife giving birth to their son. The second time, he overheard his young son crying. The third time, he witnessed his ten-year-old son grieving about life without a father. Despite the pain Yu experienced, he put his duty ahead of himself, believing that he could not enjoy the peace of life with his family while the flooding was destroying the lives of thousands more families.

The emperor of China found Yu's selflessness and dedication to the welfare of the Chinese people above his own needs to be so impressive that he selected him as the next emperor. Yu was uncertain at first, but after overwhelming support from his community he conceded and became the heir to the throne. The Legend of Yu the Great tells the story of how one man devoted his life to the service of others. It is a lesson in duty and self-sacrifice, and the idea that with great power comes great responsibility...

5.1

1.1

5.1

Solve the ancient Chinese math problem and you'll see how the Gougu theorem helped Yu to determine the depth of the rivers and canals.

In the center of a river whose width is 10 chi (*a chi is 1/3 of a metre*) grows a reed whose top reaches 1 chi above the water level. If we pull the reed towards the bank, its top is even with the water's surface. What is the depth of the river?

Egypt – Egyptian Afterlife

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The pyramids of Egypt are iconic symbols of the ancient Egyptians' obsession with the afterlife. These monumental structures were built to house the tombs of the pharaohs, serving as their gateways to heaven. But, did you know that not only pharaohs built tombs for themselves? Even average ancient Egyptians were busy preparing tombs for their journey to the next life. However, passage to the afterlife was not guaranteed, and the road was paved with danger and uncertainty.

In ancient Egypt, death was seen as the beginning of a voyage to a new world. The ancient Egyptians believed that their spirits would live on for forever in a paradise they called the *Field of Reeds.* Departing from the common view of heaven as a place of ecstasy, the Field of Reeds was another version of life on earth. The newly dead were given a plot of land and expected to plant and harvest crops, worship the gods, and live much like they had before they died. But, first they would have to get in. In order to gain admission into eternity they had to do two things: Firstly, while on earth, they had to live a morally upright life where they committed as few sins as possible. Secondly, they had to build a tomb, known to them as "houses of eternity," and, after they died, embark on the harrowing journey to the afterlife.

Following death, a person's soul would travel to the *Hall of Truth* guided by Anubis, the god of the dead. Once there, it would wait in line for the final judgment by the god Osiris. When facing Osiris, the soul would have to testify that it never committed any of the forty-two sins outlined by the gods in a ceremony called the *Negative Confessions*. Some of the sins include:

I have not done people wrong.

I have not impoverished my fellows.

I have not learned false things.

I have not done evil.

I have not deprived a poor man of his property.

I have not caused pain.

I have not created hunger.

I have not caused tears.

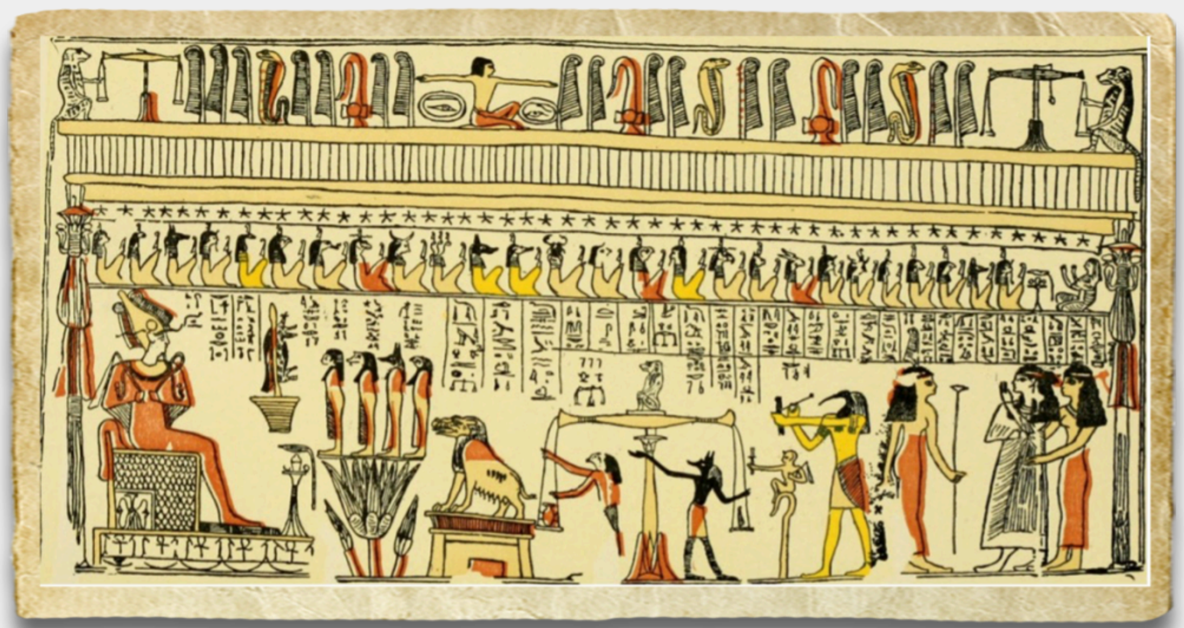
I have not killed.

I have not given orders to kill.

I have not slept around.

I have not created suffering for anyone.

I have not committed fraud.



Hall of Truth

After the Negative Confessions ritual, the gods Osiris, Thoth, Anubis, and the *Forty-Two Judges* would discuss the soul's testimony. If they ruled that the confession was accurate, the soul would make it to the next step and Osiris would weigh its heart on a golden scale against the *feather of truth*. If the soul's heart was lighter than the feather, it would be granted access to eternity in the Field of Reeds. But, if the heart was heavier, it was thrown onto the ground and eaten by Ammut, "the female devourer of the dead," and the soul would undergo "The Great Death" – a state of total non-existence. The ancient Egyptians had no concept of hell, feeling no need for it since non-existence was considered the ultimate punishment.

5.2

1.1

5.2

Given a pyramid 300 cubits (*a royal cubit is 52.5 cm*) high, with the square base 500 cubits on a side, determine the distance from the center of any side to the apex.

5.3

1.1

5.3

A pyramid has a base of 360 cubits (*a royal cubit is 52.5 cm*) and a height of 250 cubits. What is its seqt (the ratio of vertical and horizontal dimensions)?

5.4

1.1

5.4

Find the height of a square pyramid with a seqt (the ratio of vertical and horizontal dimensions) of 21 fingers per cubit (cubit equals 28 fingers) and a base of 140 cubits (*a royal cubit is 52.5 cm*) on one side.

5.5

1.1

5.5

A rectangular plot is 60 cubit (*a common cubit is 40 cm*) square; the diagonal is 13 cubits. How many cubits does it take to make the sides?

5.6

1.1

5.6

An erect pole of 10 cubits (*a common cubit is 40 cm*) has its base moved 6 cubits. Determine the new height and the distance the top of the pole is lowered

India – Lilavati, the Beloved Daughter

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=117#audio-117-3

Can the love for your daughter become a reason to write a mathematical textbook? This the story of the Indian mathematician Bhaskara and his daughter, *Līlāvatī*.

Bhaskara lived 900 years ago and was one of the best mathematicians of his time. He lived with his wife on the banks of a beautiful lotus pond. Bhaskara's wife delivered a baby girl who they named Lilavati which meant charming or graceful. Lilavati was a very beautiful and intelligent child. As she grew up the little girl asked her father many questions and gained a lot of knowledge this way.

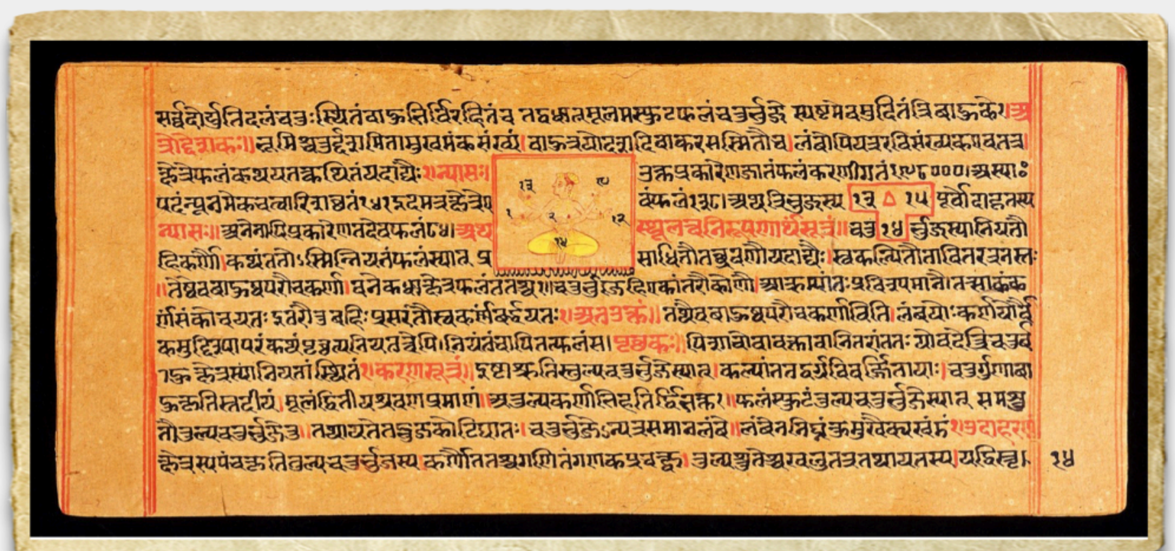
Water clock

When the time came, Lilavati became a bride and the day of the wedding was set. Before the wedding day, Bhaskara inspected Lilavati's horoscope. He was shocked when he found out that Lilavati would not have a happy married life if she did not get married at a particular lucky time. He did not tell Lilavati about this, as he didn't want to spoil his daughter's happy mood. He decided to make all arrangements to make sure that Lilavati would get married at this specific lucky time. In order to make sure that he did not miss this particular time, Bhaskara tuned his water clock the way that it would stop exactly at the lucky time. Hoe the water clock work, you would ask? One small bowl is floating in the second bigger bowl filled also with water.

The small bowl had a tiny hole at the bottom. Water sipped through the hole and when the small bowl fills in and becomes too heavy to float it would sink to the bottom. Bhaskara arranged so that the bowl would sink exactly at the beginning of the destined lucky hour. He asked Lilavati not to go near the clock but didn't tell her the reason.When Bhaskara was not around, Lilavati, could not hold her curiosity and went to see what her father had devised.When Lilavati approached the clock, she bent forward to get a closer look. A little pearl from her nose ring fell into the water. She rushed back in a hurry so that her father would not find out what she was up to.

The little pearl fell into the water and the bowl became heavier and sank before the time set by Bhaskara. The wedding took place, but not at the only hour that could avoid the disaster. And as destined, Lilavati's husband died a few days after the marriage.

Lilavati lost all her joyousness. She sat by the pond looking into nothingness and wept. She remained silent most of the time. Bhaskara found it very difficult to see these changes in his beautiful daughter. One day, Bhaskara had an idea. He posed a mathematical puzzle to her and said, "Lilavati! Why don't you solve this problem? You have always been interested in solving problems. Take it as a challenge and do it."



A page from "Lilavati"

Lilavati agreed. As she got involved in solving the problem, she was totally drawn into it. She felt alive when she solved it. Bhaskara kept posing mathematical problems to Lilavati. Step by step, her dejection and depression disappeared. Lilavati felt like she was born again. Her mind was busy by solving the problems posed by her father and never again she got depressed. And it is believed that the problems posed to Lilavati form the major portion of Bhaskara's book which is named after her.

And this is the story behind "*Lilavati*" – the book that served as the main textbook for arithmetic and geometry in India for many centuries.

5.7

1.1

5.7

There is a hole at the foot of a pillar 9 hastas (*a hasta is 45 cm*) high, and a pet peacock standing on top of it. Seeing a snake returning to its hole at a distance from the pillar equal to three times its height, the peacock swoops down upon the snake slantwise. Say quickly, how far from the pole does the meeting of their paths occur?

5.8

1.1

5.8

A fish is resting at the northeast corner of a rectangular pool. A heron standing at the northwest corner spies the fish. When the fish sees the heron looking at him, he quickly swims towards the south. When he reaches the south side of the pool, he has the unwelcome surprise of meeting the heron who has calmly walked due south along the side and turned at the southwest corner of the pool and proceeded due east, to arrive simultaneously with the fish on the south side. Given that the pool measures 12 units by 6 units, and that the heron walks as quickly as the fish swims, find the distance the fish swam.

5.9

1.1

5.9

One monkey came down a tree of height 100 and went to a pond a distance of 200. Another monkey, leaping some distance above the tree, went diagonally to the same place. If their total distances traveled are equal, tell me quickly, learned one —if you have a thorough understanding of calculation- how much is the height of the leap?

5.10

1.1

5.10

In a certain lake swarming with red geese, the tip of a lotus bud was seen to extend a span [9 inches] above the surface of the water. Forced by the wind, it gradually advanced and was submerged at a distance of 2 cubits [40 inches]. Compute quickly, mathematician, the depth of the pond.

Medieval Europe – Game of Thrones and the War of Roses

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In 15th century England, a decades-long series of battles known as the Wars of the Roses saw two royal families pitted against each other for control of the country. Throughout all the drama, bloodshed, and betrayal, could they have possibly imagined their conflict would serve as the inspiration for a hit TV show centuries later?

The Wars of the Roses was given its name because both of the families responsible for the war used a rose as their emblem: the House of York, a white rose, and the House of Lancaster, a red rose. The Yorks and the Lancasters each felt they had a right to rule the country since they were both descendents of the royal Plantagenet family bloodline.

Splitting of nobles into the factions of York and Lancaster by choosing the white or red roses

At the start of the war, the House of Lancaster ruled England under the reign of King Henry VI, but by 1460, were briefly unseated by the army of Richard, Duke of York, who was later killed in battle. Shortly thereafter, the Yorks had dealt a decisive military blow to the Lancasters, and installed Edward IV as England's new king.

Much of the Yorks' success was owed to the Earl of Warwick. Despite helping Edward IV become king, the Earl of Warwick disagreed with him on many issues, and ultimately double-crossed him, teaming up with Edward's own brother, the Duke of Clarence, to overthrow him and retake power for themselves. Unfortunately, their attempted coup failed, and they were forced to flee to France where they joined forces with the exiled former Queen Margaret of the House of Lancaster. After regrouping, these three unlikely allies summoned their armies and invaded England. Just when victory seemed certain, betrayal reared its ugly head when the Duke of Clarence again changed sides and realigned himself with the House of York! The Earl of Warwick was killed in battle, the former Queen Margaret was captured, and her husband, the previous king Henry VI, and his sons, were all executed.


Battle of Bosworth Field

After being dethroned for a short time in 1470, Edward the IV quickly won back his title as king, and ruled for several years of moderate peace. Following his death in 1483, the carnage of the previous decades reappeared. New contenders for the throne eagerly sought to fill the power vacuum, and within a short time King Edward's heirs were assassinated, and only two candidates for king remained: the Yorkist, Richard III, and the Lancastrian, Henry Tudor. When Richard III attempted to seize power, many of his allies changed sides and pledged allegiance to Henry Tudor. Tudor attacked Richard III's forces on August 22, 1485 at the Battle of Bosworth Field. The bloody confrontation ended in an unquestionable Tudor victory, with Richard III killed during battle from a brutal strike to the head. Tudor was promptly crowned King Henry VII, ushering in a new Tudor Dynasty that prospered until the early 1800s. The peaceful new era was largely attributed to Henry uniting the Yorks and Lancasters by marrying Elizabeth of York, Edward the IV's daughter.

In an attempt to put a symbolic end to the Wars of the Roses, he created a "Tudor rose" emblem by combining the white rose of the Yorks with the red of the Lancasters.

Some of the strongest works of fiction are often adapted from real historic events. It's no surprise then, that the epic drama of the Wars of the Roses, with all of its plot twists, acts of treachery, and sheer brutality, went on to inspire *Game of Thrones*. The question is, do we owe history our thanks? Or is there a more somber moral to the story?

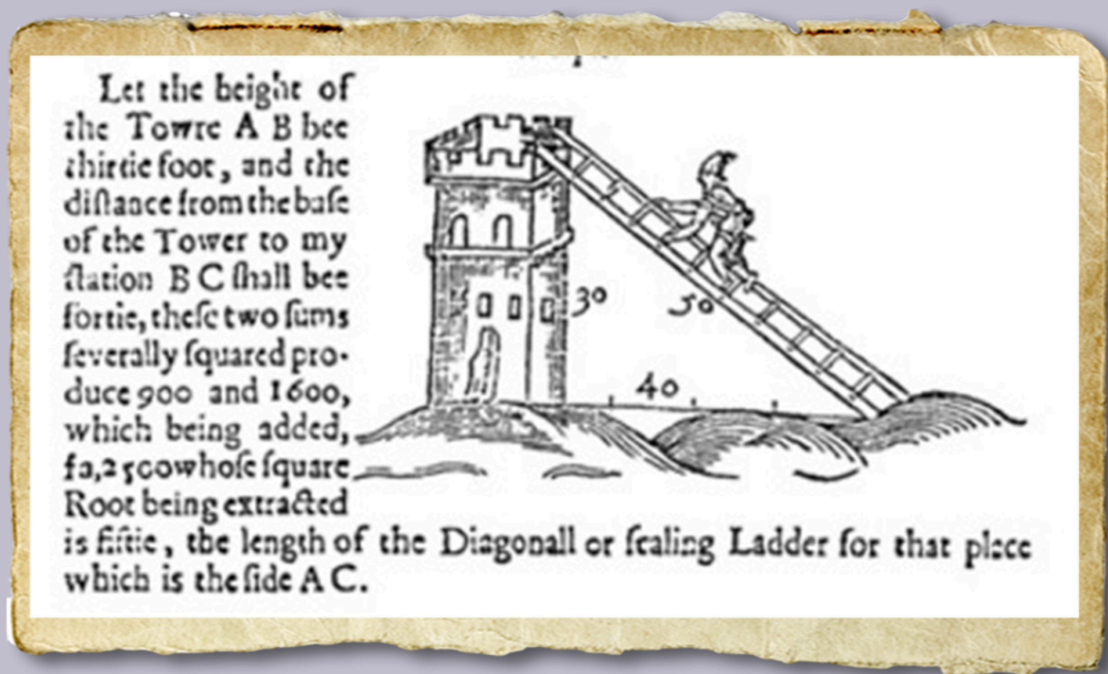

Scaling the wall

5.11

1.1

5.11

A siege, or scaling, ladder would have been used to enable fighters to climb to the top of a fortification wall and gain access to the interior of a defensive position. A siege ladder would need to be tall enough to reach the top of the wall. It would be carried to the base of the wall as closely as possible and tilted over so that the top of it could rest on the top of the wall. At the same time its angle had make it difficult for the defenders to push a ladder backward. All these considerations had to be taken into account when building a siege ladder. *No wonder medieval students were learning how to solve the problems that require the knowledge of the Pythagorean Theorem. For example, their textbooks would have illustrations that have the Pythagorean triangles in them:*


Let the height of the Towre A B bee thirtie foot, and the diſtance from the baſe of the Tower to my ſtation B C ſhall bee fortie, theſe two ſums ſeverally ſquared produce 900 and 1600, which being added, fa,2500 whoſe ſquare Root being extracted is fiftie, the length of the Diagonall or ſcaling Ladder for that place which is the ſide A C.

Two towers, the heights of which are 30 paces and 40 paces, are 50 paces apart. Between the two towers there is a font where two birds, flying down from the two towers at the same speed will arrive at the same time. What is the distance of the font from the two towers?

Chapter 6 - Ratio and Proportion

Map

1.1

1.19

Timeline

1.1

1.19

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Introduction

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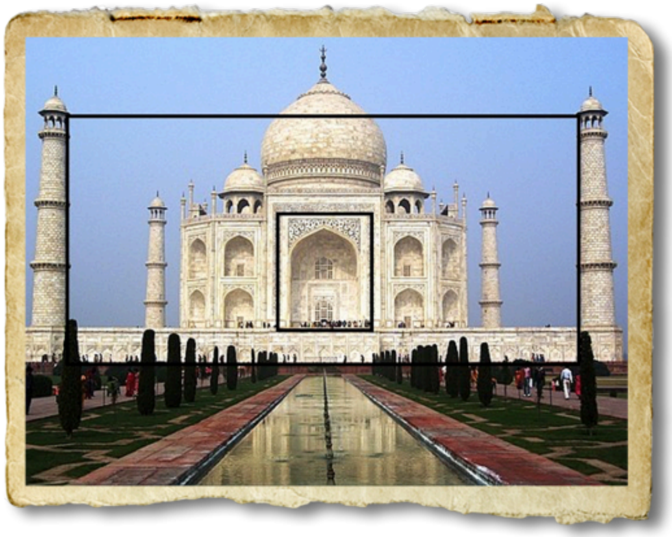
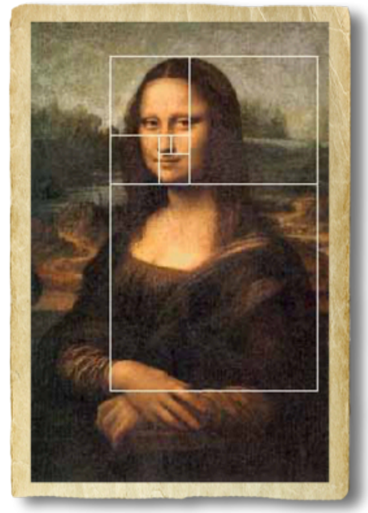
Have you ever wondered why humans see some things as beautiful? What would you say to the idea that there was a mathematical formula that influences what we see as beauty? There is, in fact, a secret ratio, and it is called *The Golden Ratio.*

As you have studied mathematics, you have learned about ratios and proportion. For example, if a ratio of pizzas to celery sticks is 2:5, and there are 20 pizzas, how many celery sticks would there be? 50, right? Perhaps it sounds boring to some, but when we imagine that mathematics is used to decode the fundamental language of the universe, applying it gives us the power to perceive the raw nature of reality in ways we never could before! One of the most interesting uses of ratios and proportions is illuminating our understanding of beauty across the whole of nature. Here's where the Golden Ratio comes in.

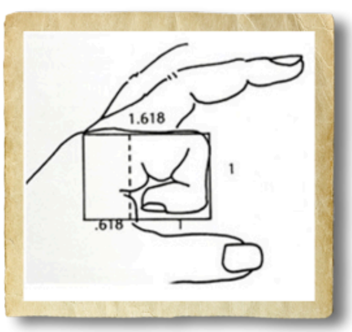
Have you ever wondered why you see a cat's face as beautiful? How about the pyramids of Egypt, the Taj Mahal, or the Mona Lisa? How would you feel knowing that these seemingly different things share a common mathematical connection? That connection is The Golden Ratio. It is a symmetrical relationship so widespread across nature, art, music, architecture, and even human biology, that it quietly guides our concept of what is and is not beautiful. When applied, it creates a natural, harmonious, and visually pleasant experience to all who observe it.

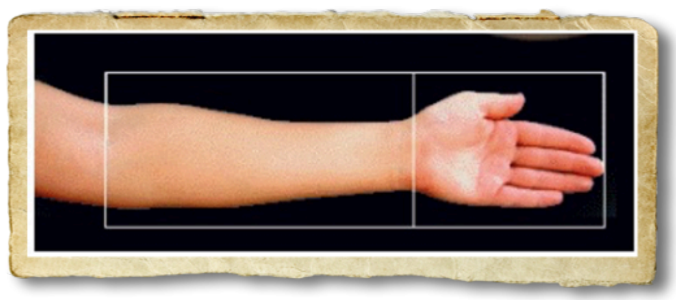




Golden ratio in Taj Mahal and Mona Lisa: length to width of the rectangles is approximately 1:1.6

Now, to the fun part – the math. The Golden Ratio is 1:1.68 – aka the length is 1 and width is 1.68. Let's see how it affects the world right in front of us. Have a look at your index finger. Notice that, from the tip to the base of your wrist, each section is bigger than the last one by a ratio of 1 to 1.68. Using this scale, your fingernail is 1 unit in length. The ratio between your hand and your forearm does the same, abiding by the Golden Ratio.





Golden Ratio in your hand and arm

Although beauty is mysterious, and is ultimately in the eye of the beholder, there are basic characteristics of beauty that are inseparable from the Golden Ratio. Rather than being solely established by cultural beliefs, recent science claims that beauty standards have their roots in mathematics. Proportions and ratios are useful in our everyday lives, but they have a secret, and ancient life of their own as cosmic agents of beauty and harmony across the universe.

Pyramids across the World

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Humanity's architectural achievements are truly astonishing. Looking up at the great monuments of the world like the Eiffel Tower or Taj Mahal can have a truly humbling effect on us. But, have you ever wondered why human civilizations build such tall towers? With a new record-breaking tower erected every decade, from Toronto's CN Tower to the Burj Khalifa in Dubai, there seems to be a never ending competition among mankind to construct taller and taller buildings. Even in the biblical story, The Tower of Babel, the ancient Babylonians built a tower so tall, it was perceived as a threat to God! We know how the builders of today do it, but how did the ancients manage to construct such impressive buildings without the help of modern machinery?

Pyramids in Giza, Egypt

The greatest architectural mystery of the ancient world is by far the Pyramids of Giza. Although the ancient Egyptians are widely known for constructing the most iconic pyramids in the world, they were not the only ancient culture to build them. Archaeological digs have discovered pyramids across the ancient world whether it be in Mesopotamia, China, India, Indonesia, Cambodia, Spain, Italy, Nigeria, and Sudan. Pyramids were even found on the other side of the globe! Mesoamerican cultures like the Incan, Mayan, Aztec, Xelhua, Toltec, Velacruz, and Teotihuacan all excelled at Pyramid building. But, the question remains…how did ancient peoples build such majestic pyramids? What technology did they use?

Pyramid in Chichen Itza, Mexico

Since there is no evidence to answer that question, we can only rely on speculation. Was it strictly a matter of hard labor and dedication? Or were they using a technology we are not aware of? Take the pyramids in Chichen Itza in Mexico, for example. The main pyramid, Kukulkan, stands atop a rectangular platform 55.5 metres wide and 24 meters tall, with a 6 metre temple perched atop it. Witnessing the sheer magnitude of these megalithic structures suggests that manpower alone could not have been solely responsible for their construction. With a missing piece to this puzzle, many have foregone modern scientific reasoning and sought to obtain answers from ancestral legends instead.

Ancient Mayan legends claim their pyramids were built with the help of magic powers. The descendants of the Mayans still hold this idea to be true, but with a key insight: the magic powers were not actually magic, but rather, they were a forgotten technology! Can you try and guess what it was? According to locals in the south of the Yucatán Peninsula, their mystery machine wasn't a machine at all. It was sound! They maintain that the ancient Mayans would whistle in such a way as to harness an extraordinary amount of energy – enough to move megalithic stones! Imagine that scene: gigantic stones and wooden logs floating through the air of a construction site, guided by the whistle of a musical foreman! No hammers, drills, or cranes, just the wonder of enormous objects dancing on the wind to the tune of ancient music.

Whether it's with modern machinery or with an ancient technology long since forgotten, one thing remains clear: humanity's ambition to build towering structures will likely never come to an end. But, where our ambition will lead us is yet to be determined. Can you imagine what the skyscrapers of the future will look like?

6.1

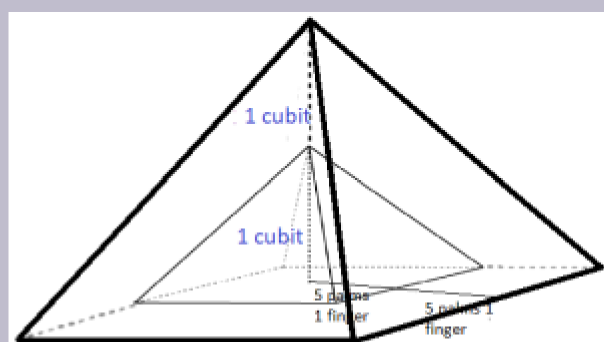
1.1

5.1

The Egyptian pyramids have different slopes. Some pyramids are more high-pitched, others are more low-pitched. Egyptians expressed the steepness of a pyramid by a measure they called “seked”. For instance, they say that a pyramid has to have a seqed of 5 palms 1 finger. What does it mean? First, we have to remember that a cubit, the Egyptian unit of length which is approximately 50 cm, is divided in 28 fingers and each 4 fingers make a palm. So 5 palms 1 finger is 5×4+1= 21 fingers. BTW, can you calculate what is 1 finger in cm?

Egyptian ruler

Now for the seqed of 5 palms 1 finger: It means that when you build a pyramid you have to add for each cubit of its height the length of 5 palms 1 finger to each side of its base. if you build a pyramid 1 cubit tall which is 28 fingers the base should be 5 palms 1 finger wide from each side of its centre. A pyramid of 2 cubits high had to be 10 palms 2 fingers wide from each side of its centre.



Calculation of seqed

How to calculate the seqed of a pyramid that is already built? To calculate the seqed of a pyramid you have to divide the half of its base by its height. For example, the pyramid below is 250 cubits high. Its base is 360 cubits long. The seqed is 180 divide by 250 which is 0.72. It means that for each cubit of its height the length of 0.72 is added to both sides of its base. But the ancient Egyptians didn’t have decimals so they expressed 0.72 of cubit as fingers. 0.72×1 cubit = 0.72×28 = 20.16 or approximately 20 fingers or 5 palms (remember 1 palm = 4 fingers) and 16/100 or 4/25 of finger.

6.1 a)

If a pyramid is 280 cubits high and the side of its base 360 cubits long, what is its seked?

6.1 b)

If the seked of a pyramid is 5 palms 1 finger per cubit and the side of its base 12 cubits long, what is its altitude?

6.1 c)

If the seked of a pyramid is 5 palms 1 finger per cubit and the side of its base 140 cubits, what is its altitude?

6.2

1.1

5.1

In ancient Egypt the quality of products made with grain, such as bread and beer, was measured by a unit called a ***pefsu***. Pefsu shows how many loaves of bread or jugs of beer were made from 1 hekat, which is 5 litres, of grain. The more beer or bread you made from the same amount of grain the weaker and less tasty was your beer or bread.. For example, if you make 20 loafs of bread from 1 hekat the bread is of 20 pefsu value. But if you make 40 loaves of bread your bread is of 40 pefsu value. So, the higher the pefsu, the less valuable bread or beer was. If you had one loaf of bread of pefsu 20 you could exchange it to two loaves of pefsu 40 or to three loaves of pefsu 60.

6.2 a)

100 loaves of pesu 10 are to be exchanged for a certain number of loaves of pesu 45. What is the number?

6.2 b)

1000 loaves of pefsu 5 are to be exchanged, a half for loaves of pefsu 10, and a half for loaves of pefsu 20. How many of each will there be?

6.2 c)

1000 loaves of pefsu 10 are to be exchanged for a number of loaves of pefsu 20 and the same number of pefsu 30. How many of each kind will there be?

6.2 d)

Suppose it is said to thee, 100 loaves of pefsu 10 are to be exchanged for a quantity of beer of pefsu 2. How many des of beer will there be?

6.2 e)

3 1/2 hekat of grain is made into 80 loaves of bread. Let me know the amount of grain in each loaf and what is the pesu. (pefsu?)

6.3

1.1

5.1

If it is said to you, “Have sailcloth made for the ships,” and it is further said, “Allow 1000 cloth cubits for one sail and have the ratio of the height of the sail to its width be 1 to 1 1/2,” what is the height of the sail?

6.4

1.1

5.1

A cobbler can cut leather for ten pairs of shoes in one day. He can finish five pairs of shoes in one day. How many pairs of shoes can he both cut and finish in one day?

China Terracotta Warriors and the Search For Eternal Life

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On March 29th, 1974, a groundbreaking discovery was made in northwestern China. Local peasants digging a well accidentally unearthed fragments of a millennia-old terracotta clay figure. This peculiar finding led archaeologists to excavate the site, revealing an ancient tomb guarded by one of the most significant archaeological discoveries in modern history: the Terracotta Army of China's first Emperor, Qin Shi Huangdi.

The terracotta clay army consisted of more than 8,000 life-sized soldiers and 600 war-horses in full armor and battle formation. Weighing up to 272 kilograms each (600 pounds), and many standing two meters tall (roughly 6 feet), the soldiers showcased remarkable detail, with each clay sculpture characterized by unique facial features, hairstyles, clothing, and posture. But, who was China's First Emperor? And, why did he surround himself with the monumental terracotta army?

Terracotta Warriors

Born Prince Ying Zheng in the year 259 B.C., he became King of the Qin State at the age of thirteen, and by age twenty-two had turned into the emperor that would revolutionize ancient China in a number of ways: uniting China in 221 B.C. by taking over the six independent kingdoms of the late Warring States Period, creating a single system for writing, units of weight, measurement, and coinage; building the Great Wall to strengthen the northern border, and establishing the Ling Canal to connect China's north and south river systems.

However, all of these achievements came at a huge cost to the people of ancient China. The Emperor governed with an iron fist. His tyrannical reign was guided by the philosophy of *Legalism*, which was based on the cynical belief that the average person is motivated by self-interest and, therefore, more likely to do wrong than right. As time passed, the Emperor forbade all other philosophies and ordered the destruction of texts not conforming Legalist views. Anecdotal evidence suggests he may have even executed writers, philosophers, and scholars to stamp out opposing ideologies.

After a career of landmark achievements, China's first emperor set his sights on one final conquest: death itself! Increasingly obsessed with discovering the proverbial fountain of youth, he sought the advice of his alchemists, who fascinated him with tales of magical life-extending herbs found only on the *Islands of the Immortals* in the East China Sea – a mystical place that only allowed pure-hearted children to enter. In 219 B.C., emperor Qin Shi Huangdi dispatched thousands of children to the islands. When they never returned, he sent three alchemists. Just one returned, recounting that the islands were protected by an enormous fish, and that their expedition had failed. Unable to accept his mortality, emperor Qin Shi Huangdi journeyed to find the islands himself, but instead of conquering death, he succumbed to it along the way. Experts believe he died from consuming mercury administered by his alchemists, who themselves believed it to be the elixir of life.

At the same time he was searching for immortality, emperor Qin Shi Huangdi was also planning for the afterlife. Throughout his entire life, he had been gradually constructing an enormous underground burial chamber bursting with his empire of riches, and guarded by his army of terracotta warriors, chariots, and war-horses. After conquering the whole of ancient China, he truly believed he could do the same in the great beyond! The question is, would his terracotta soldiers still serve their emperor in the afterlife?

6.5

1.1

A fox, a raccoon, and a hound pass through customs and together pay 111 coins. The hound says to the raccoon, and the raccoon says to the fox, "Since your fur is worth twice as much as mine, then the tax you pay should be twice as much!" How much should each pay?

6.6

1.1

A cow, a horse, and a goat were in a wheat field and ate some stalks of wheat. Damages of five baskets of grain were asked by the wheat field's owner. If the goat ate one-half the number of stalks as the horse, and the horse ate one-half of what was eaten by the cow, how much should be paid by the owners of the goat, horse, and cow, respectively?

6.7

1.1

If in one day, a person can make 30 arrows or fletch (put the feathers on) 20 arrows, how many arrows can this person both make and fletch in a day?

6.8

1.1

5.5

In general, a fair exchange is 50 bushels of millet for 27 bushels of rice. Here is 21 bushels of millet. How many bushels of rice will we obtain in exchange?

6.9

1.1

5.6

Four counties are required to furnish wagons to transport grain to a depot. There are 10,000 families in the first county, 9500 families in the second county; 12,350 families in the third county; and 12,200 families in the last county. The total number of wagons required is 1000. How many wagons are to be provided by each county according to the size of the population?

Europe Mathematical Duels

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Competition is an inherent feature of the human condition and beyond. Whether it's animals, plants, or microscopic life, every organism on the planet is competing against one another for resources, status, or the highest position in their given hierarchy. It is no wonder then, that sports play such a significant role in human society. Whether it's MMA, basketball, or even professional video gamers, the need to compete and achieve the triumph of success proliferates through many aspects of our lives and even goes beyond sport and into the realm of science. Renaissance Italy boasted one of the most bizarre forms of competition to date: Mathematical duels…the most famous example of these contests revolved around the discovery of a general solution of cubic equations.

In the early 16th century, an Italian mathematician named Scipione Dal Ferro from the University of Bologna secretly devised a solution to cubic equations: $x3+px=n$�3+��=�. A few years after his death, a mathematician from Brescia named Niccolò Pisano, nicknamed *Tartaglia,* meaning "stammerer," revealed to the mathematics community that he had discovered a solution to cubic equations, the key to unlocking a seemingly unsolvable, two-thousand-year-old mathematical problem! Determined to undermine Tartaglia's announcement, a former student of Dal Ferro's named Antonio Maria Fiore publicly challenged Tartaglia to a mathematical duel!

Modeled after knightly duels, the *cartelli di matematica disfida* or “bills of mathematical challenge,” consisted of a series of oral or written mathematical equations posed by each competitor to one another with juries, notaries, and witnesses present to observe the battle. As per this challenge, Tartaglia and Fiore provided each other with thirty problems, to be solved in forty days. Tartaglia shocked the community by solving all thirty equations in two hours! Dal Fiore, however, was unable to solve even a single problem.

Four years later, in 1539, Tartaglia disclosed his secret formula as a twenty-five verse poem to Gerolamo Cardano, a Milanese physician and mathematician, swearing him never to reveal it to the outside world. Cardano did, however, allow his student Ludovico Ferrari to see Tartaglia’s formula for third-degree equations, which Ferrari then used as a necessary tool to create a solution for the next frontier in mathematics: fourth-degree equations. Ferrari wanted to publish his momentous discovery, but doing so would reveal Tartaglia’s equation, and feeling honor-bound to Tartaglia, Cardano refused to allow Ferrari to publish the solution to fourth-degree equations.

Tartaglia and Cardano

As fate would have it, Cardano and Ferrari later stumbled upon the work of Scipione Dal Ferro, and were astonished to learn that he in fact had discovered a solution to cubic equations prior to Tartaglia. With this new information, Cardano asserted that his oath of secrecy to Tartaglia was now voided, and he and Ferrari were free to publish their work on fourth-degree equations!

In 1545, Cardano published *Ars Magna* (the Great Art) which included the formulae for the general solution of equations of the **third** and **fourth** degree. Outraged by the betrayal, Tartaglia traveled to Milan to challenge Cardano to a mathematical duel. Perhaps too ashamed to face Tartaglia himself, Cardano skipped town and his student Ferrari filled in for him, squaring off with Tartaglia in front of a large crowd including the Governor of Milan, Don Ferrante Gonzaga. Unfortunately, for Tartaglia, this was not a written duel, but rather, a spoken one which highlighted Tartaglia's speech impairment. With most of the crowd rooting for Ferrari, Tartaglia was soundly defeated, and humiliated so much so that he later lost his job in Brescia. The solution for cubic equations was then credited to Cardano, and went on to be known as *Cardano's Formulae*.

Who would have thought that the world of mathematics could be so deeply entangled with competition, betrayal, and glory? It is a stark reminder that no matter how high the arc of human achievement rises, it is nonetheless subject to, and perhaps driven by, the ever-present pursuit of individual notoriety. Cynical as it may seem, it begs the question, without the desire for personal glory, how many of the achievements that have propelled humanity towards a brighter future would have even come into existence whatsoever?

6.10

1.1

5.7

A man had four creditors. To the first he owed 624 ducats; to the second, 546; to the third, 492; and to the fourth, 368. It happened that the man defaulted and escaped, and the creditors found that his goods amounted to 830 ducats in all. In what ratio should they divide this, and what will be the share of each?

6.11

1.1

5.8

There were two men, of whom the first had three small loaves of bread and the other two. They walked to a spring, where they sat down and ate; and a soldier joined them and shared the meal, each of the three men eating the same amount. When all the bread was eaten, the soldier departed, leaving 5 bezants to pay for his meal. The first man accepted 3 of these bezants, since he had three loaves; the other took the remaining 2 bezants for his two loaves. Was this division fair?

6.12

1.1

5.9

Suppose I tell you that I bought saffron in Siena for 18 lire a pound and took it to Venice, where I found that 10 ounces Siena weight are equivalent to 12 ounces in Venice, and 10 lire in Siena money are equal to 8 lire Venetian. I sell the saffron for 14 lire Venetian money a pound. I ask how much I gained in percent.

6.13

1.1

5.10

Suppose you have two kinds of wine. A measure of the poorer sort is worth 6 denarii. One of the better sort is worth 13 denarii. I wish to have a measure of wine worth 8 denarii. How much of each wine should I put in the mixture?

6.14

1.1

5.8

A dying man makes a will: if the wife gives birth to a male child, she will have one third of what he leaves, his son the other two parts; and if the wife gives birth to a daughter, the wife will receive two thirds and the girl one third. It now happens that the woman bears twins, one male and one female. The question is how much comes to the woman, the male and female child when the legacy values 70 lire.

6.15

1.1

5.9

The head of a household had 20 servants. He ordered them to be given 20 measures of corn as follows. The men must receive three measures, the women must receive two measures, and the children half a measure each. How many men, women and children servants are there in the household?

6.16

1.1

5.10

A head of a household had 100 servants. He ordered that they be given 100 measures of corn as follows. The men should receive three measures, the women should receive two measures, and the children should receive half a measure each. How many men, women, and children servants are there in the household?

6.17

1.1

5.8

A certain bishop ordered 12 loaves of bread to be divided amongst the clergy. He stipulated that each priest should receive two loaves, each deacon should receive half a loaf and each reader should receive a quarter of a loaf. It turned out that the number of clerics and the number of loaves were the same. How many priests, deacons and readers must there have been?

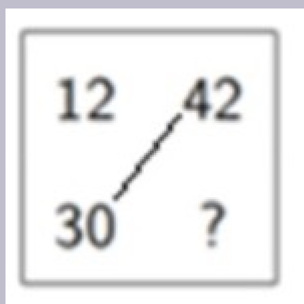
6.18

1.1

5.9

Most of the ratio or proportion problems can be solved by the method called “the rule of three”. The rule of three came to us from medieval India. It was transported to other countries by Arabian voyagers, and, thanks to the Islamic invasion of Spain, the rule made it to Europe. In the 13th century the Italian merchant and mathematician Leonardo de Fibonacci, who frequently traveled to North Africa, learned about the the rule of three and made it popular among European merchants and bankers. Let's look at the example Fibonacci gives:

*If 12 bottles of wine cost 30 denari, what will 42 bottles of wine cost? Fibonacci says, make a square as follows, then multiply the two numbers that lie in the diagonal and then divide by the remaining number: (***30 x 42)/12.** *The result is the price to be paid, 105 denari.*



Ratio Square

6.18 a)

Suppose 100 rolls of cotton cost 40 lira, how much would five rolls cost?

6.18 b)

I have bought five palas of sandalwood for nine rupakas. How much sandalwood, then, should be obtained for one rupaka?

6.19

1.1

5.10

If 12 horses can plow 96 acres in 6 days, how many horses will plow 64 acres in 8 days?

SOLUTIONS

Solutions

Chapter 1 – Numbers

1.1

1.1

2.1

Answers may vary. Some of the patterns are:

The **left column** can be divided in 4 groups, with each group possessing 19, 17, 13, and 11 notches. The sum of these being 60. Those are the 4 successive prime numbers between 10 and 20.

The **central column** is divided in groups of 8. By an approximate count, one can find (in the parenthesis, is the maximum number): 7 (8), 5 (7), 5 (9), 10, 8 (14), 4 (6), 6, 3. The minimal sum is 48, while the maximal sum is 63.

The **right column** is divided into 4 groups, where each group has 9, 19, 21, and 11 notches. The sum of these 4 numbers is 60.

The numbers in the left column were compatible with a numeration system based on 10, since 21 = 20 + 1, 19 = 20 – 1, 11 = 10 +1, and 9 = 10 -1. These numbers are also prime numbers between 10 and 20: 11, 13, 17, 19.

1.2

1.1

2.1

1) How many symbols did the Babylonians use to express numbers?

2 unique symbols in 60 different arrangements.

2) In our decimal system we have 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. How many digits did the Babylonians have?

60 digits

3) Calculate the value of

a) *2 x 60 = 120, 6 x 1 = 6*

126

b) *2 x 60 = 120, 12 x 1 = 12*

132

c) *10 x 3600 = 36000, 10 x 60 = 600, 2 x 1 = 2*

36602

d) How would Babylonians write 65? 615? 665?

1.3

1.1

2.1

A granary of barley contains 2400 *gur,* where 1 gur equals 480 *sila.* If workers are to receive 7 sila of grain for a day's work, how many men can be paid from this granary?

$$1\ gur\ =\ 480\ sila$$

$$2400\ gur\ =\ 2400\ x\ 480\ sila\ =\ 115200\ sila$$

$$divide\ by\ 7\ sila\ per\ wor\mathrm{ker}$$

$$\frac{115200\ sila}{7\ \frac{sila}{wrkr}} \approx 16457.142\ wrkrs$$

1.4

1.1

2.1

A woman weaves a textile that is to be 48 rods long. In one day, she weaves 1/3 rod. In how many days will she cut the textile from the loom?

Length = 48 rods

Rate = ⅓ rod / day

Total time = ? days

$$total\ time\ =\ \frac{total\ length}{rate}$$

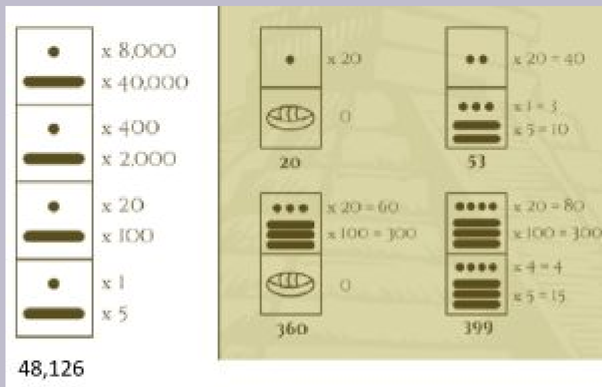
$$=\frac{48\ rods}{\frac{1}{3}rod\ per\ day}=\frac{48\cdot 3}{1}=48\cdot 3=144\ rods$$

1.5

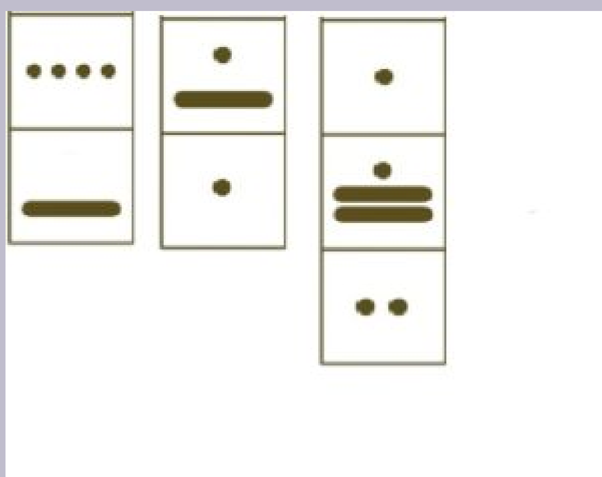
1.1

2.1

a) Find values of the numbers



b) Write numbers 85, 121 and 2222 in Mayan numerals



1.6

1.1

2.1

There is a tree with 100 branches; each branch has 100 nests; each nest, 100 eggs; each egg, 100 birds. How many nests, eggs and birds are there?

100 branches per tree (100)
100 nests per branch (100 x 100=10000 nests)

100 eggs per nest (100 x 10000 = 1000000 eggs)
100 birds per egg (100 x 1000000 = 100000000 birds)
Total nests, eggs, and birds = 10000 + 1000000 + 100000000 = 101010000 total objects

1.7

1.1

2.1

Three hundred pigs are to be prepared for a feast. They are to be prepared in three batches on three successive days with an odd number of pigs in each batch. Can this be accomplished?

Odd + Odd = even
Odd + Even = odd

Three batches: Odd + Odd + Odd
Since Odd + Odd = even, then: Odd + Odd + Odd = Even + Odd = Odd.

Since there are 300 pigs (which is even), no this is not possible.

1.8

1.1

2.1

A leech invited a slug for lunch a leuca away. But he could only crawl an inch a day. How long will it take the slug to get his meal? [1 leuca = 1500 paces; 1 pace = 5 feet.]

1 leuca = 1500 paces
1500 paces = 1500 x 5 feet = 7500 feet
7500 feet = 7500 x 12 inches = 90000 feet
Therefore, it will take 90000 days for the slug to get his meal.
90000 days at 365 days per year = 246 years with a remainder of 210 days.

1.9

1.1

2.1

A certain man had in his trade four weights with which he could weigh integral pounds from 1 up to 40. How many pounds was each weight?

1, 3, 9 and 27.

Weight of object to be measured	Left side	Right side
1	O	1
2	O, 1	3
3	O	3
4	O	3, 1
5	O, 3, 1	9
6	O, 3	9
7	O, 3	9, 1
8	O, 1	9
9	O	9
10	O	9, 1
11	O, 1	9, 3
12	O	9, 3
13	O	9, 3, 1
14	O, 9, 3, 1	27
15	O, 9, 3	27
16	O, 9, 3	27, 1
17	O, 9, 1	27
18	O, 9	27
19	O, 9	27, 1
20	O, 9, 1	27, 3
21	O, 9	27, 3
22	O, 9	27, 3, 1
23	O, 3, 1	27
24	O, 3	27
25	O, 3	27, 1
26	O, 1	27
27	O	27
28	O	27, 1
29	O, 1	27, 3
30	O	27, 3
31	O	27, 3, 1
32	O, 3, 1	27, 9
33	O, 3	27, 9
34	O, 3	27, 9, 1
35	O, 1	27, 9
36	O	27, 9
37	O	27, 9, 1

38	O, 1	27, 9, 3
39	O	27, 9, 3
40	O	27, 9, 3, 1

1.10

1.1

2.1

I have a cloak 100 cubits long and 80 cubits wide. I wish to make small cloaks with it; each small cloak is 5 cubits long, and 4 wide. How many small cloaks can I make?

Since 5 divides into 100 and 4 divides into 80, we can assume that all of the fabric will be used if we orient the small cloaks such that their 5 cubit sides align along the 100 cubit side of the large cloak, and the 4 cubit sides along the 80 cubit.

Along the long side, we can fit 20 (100/5) small cloaks in a row.

Along the short side, we can fit 20 (80/4) small cloaks in a row.

Therefore we have an array of 20 small cloaks by 20 small cloaks. The total is 400 cloaks.

1.11

1.1

2.1

A father, when dying, gave to his sons 30 glass flasks, of which 10 were full of oil, 10 were half full, and the last 10 were empty. Divide the oil and the flasks so that each of the three sons received equally of both glass and oil.

Each son needs to receive 10 actual flasks (glass)
There is a total of 10 x 1 + 10 x 0.5 = 15 flasks worth of oil. Therefore each son needs to receive 5 flasks worth of oil.

One possible solution is to give:

Son 1 – 5 full and 5 empty
Son 2 – 5 full and 5 empty
Son 3 – 10 half-empty

1.12

1.1

2.1

A king ordered his servants to collect an army from 30 manors in such a way that from each manor he would take the same number of men he had collected up until then. The servant went to the first manor alone; to the second he went with one other; to the next he took three with him. How many were collected from the 30 manors?

Manor #	How many brought	Collected at this	New Total
1	0	1	1
2	1	1+1=2	3
3	3	2+3=5	8
4	8	5+8=13	21
5	21	13+21=34	55
6	55	34+55=89	144
7	144	89+144=233	377
8	377	233+377=610	987
9	987	610+987=1597	2584
10	2584	1597+2584=4181	6765
11	6765	4181+6765=10946	17711
12	17711	10946+17711=28657	46368
13	46368	28657+46368=75025	121393
14	121393	75025+121393=196418	317811
15	317811	196418+317811=514229	832040
16	832,040	514229+832040=1,346,269	2,178,309
17	2,178,309	1,346,269+2,178,309= 3,524,578	5,702,887
18	5,702,887	3,524,578+	

1.13

1.1

2.1

A gentleman has a household of 30 people and orders that they be given 30 measures of grain. He directs that each man should receive 3 measures, each woman 2 measures, and each child 1/2 measure. How many men, women, and children are there?

The solution can be found by trial and error method.
3 men, 5 women, 22 children

1.14

1.1

2.1

A man wanting to build a house contracted with six builders, five of whom were master builders, and the sixth an apprentice, to build it for him. He agreed to pay them a total of 25 pence a day, with the apprentice to get half the rate of a master builder. How much did each receive per day?

A master builder got 4.54 and apprentice got 2.27 pence a day.

1.15

1.1

2.1

A man in the east wanted to buy 100 assorted animals for 100 shillings. He ordered his servant to pay 5 shillings for a camel, 1 shilling for an ass, and 1 shilling for 20 sheep. How many camels, asses, and sheep did he buy?

The solution can be found by trial and error method or by making a list.

19 camels, 1 ass and 80 sheep

1.16

1.1

2.1

There is an estate that contains 7 houses; each house has 7 cats; each cat catches 7 mice; each mouse eats 7 spelt of seeds; each spelt was capable of producing 7 hekats of grain. How many things were in the estate?

7 + 49 + 343 + 2401 + 16,807.= 19,607

1.17

1.1

2.1

Suppose a scribe tells you that four overseers have drawn 100 great quadruple hekats of grain, and their work gangs consist of 12, 8, 6, and 4 men. How much grain does each overseer receive?

100 / (12 + 8 + 6 + 4)= 3 1/3
12(3 1/3)= 40
8(3 1/3) = 26 2/3
6(3 1/3)= 20
4(3 1/3)= 13 1/3

1.18

1.1

2.1

In the middle register we see 835 horned cattle on the left, right behind them are some 220 animals (cows?) and on the right 2235 goats. In the bottom register we see 760 donkeys on the left and 974 goats on the right

1.19

1.1

2.1

$$M_A = \frac{(a+b)}{2} > M_G = \sqrt{ab} > M_H = \frac{2ab}{(a+b)}$$

1.20

1.1

2.1

Given, four whole numbers where, if added together three at a time, their sums are 20, 22, 24, and 27. What are the numbers?

Suppose the numbers are a, b, c and d. Then:
a+b+c = 20
b+c+d = 22
a+c+d = 24
a+b+d = 27

If we add all of these equations together we find:
3(a+b+c+d) = 22+24+27+20 = 93

Dividing both ends by 3 we find:
a+b+c+d = 93/3 = 31

Then we can find the values of a, b, c and d:
a = (a+b+c+d) – (b+c+d) = 31-22 = 9
b = (a+b+c+d)-(a+c+d) = 31-24=7
c = (a+b+c+d)-(a+b+d) = 31-27 = 4
d = (a+b+c+d)-(a+b+c) = 31-20 = 11

Chapter 2 – Circles

2.1

1.1

2.1

One of the geometric rules given in the Talmud is "How much is the square greater than the inscribed circle? A quarter" What was the Talmudic value for pi?

Case 1 – Assuming we are discussing area, then:
Let S be the area of the square
Let C be the area of the circle

l (of square) = d = 2r
S = 1.25C

Therefore:

$$l^2 = \pi r^2$$
$$substituting\ 2r\ for\ l : (2r)^2 = \pi r^2$$
$$4r^2 = \pi r^2$$
$$4 = \pi$$

Case 2 – if we are discussing circumference:

$$S = 1.25C$$
$$l = d$$
$$4d = 1.25\pi d$$
$$1.25\pi = 4$$
$$\pi = \frac{4}{1.25} = \frac{16}{5} = 3.2$$

Although the wording of the question seems to suggest area as the measurement being discussed in this question, the calculation for pi in the circumference solution is much closer to the modern value of pi than the area solution.

2.2

1.1

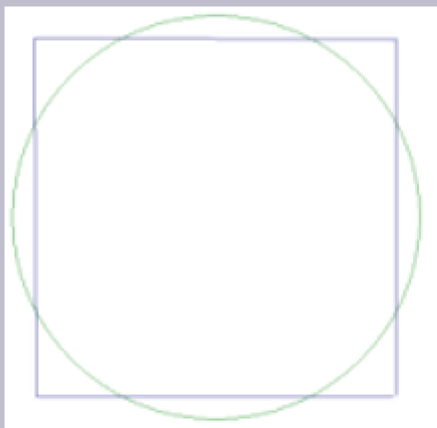
2.1

In 4000 year old Egyptian papyrus we read "A circular field has diameter 9 chet. What is its area?" The Ahmes, the scribe of the papyrus, gives the solution: "Subtract 1/9 of the diameter namely 1

chet. The remainder is 8 chet. Multiply 8 by 8; it makes 64. Therefore, it contains 64 square chet of land"

For the Egyptians a square with the side length 8/9 of the diameter of a circle was a good enough approximation to calculate the area of a circle.

Calculate the ancient Egyptian value of pi.



$$A = \left(\frac{8}{9}d\right)^2$$

If we substitute the modern formula for area of a circle, we can solve for pi:

$$A = \left(\frac{8}{9}d\right)^2$$

$$\pi r^2 = \left(\frac{8}{9}d\right)^2$$

$$d = 2r$$

$$\pi r^2 = \left(\frac{16r}{9}\right)^2$$

$$\pi r^2 = \frac{256r^2}{81}$$

$$\pi = \frac{256r^2}{81r^2} = \frac{256}{81}$$

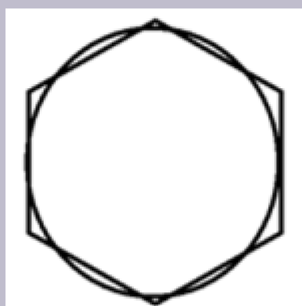
Therefore π = 256/81≈ 3.16

2.3

1.1

2.1

In Babylonian mathematics a circle was equated to a regular hexagon which side was equal to the radius of a circle. Calculate the Babylonian value of pi.



The perimeter of the hexagon is P≅6r. The circumference C=2πr. 2π=6 π=3

2.4

1.1

2.1

There is a round field which contains 400 yards in its circumference. How many square yards will its area be? Alcuin obtains 10000 square yards. How does he get that, you might ask? Calculate the value of pi he used.

We begin with the formulas for Area of a Circle and Circumference of a Circle:

$$A = \pi r^2$$
$$C = 2\pi r$$

(Note the use of the radius-centred circumference formula, rather than the diameter formula. This is done to preserve variables)

We know that C = 400, and A = 10000. Therefore, we can solve for pi.

$$10000 = \pi r^2$$
$$400 = 2\pi r$$
$$r = \frac{400}{2\pi}$$
$$\therefore 10000 = \pi\left(\frac{400}{2\pi}\right)^2$$
$$10000 = \frac{160000\pi}{4\pi^2}$$
$$10000 = \frac{40000}{\pi}$$
$$10000\pi = 40000$$
$$\pi = 4$$

2.5

1.1

2.1

There is a city which is 8000 feet in circumference. How many houses could the city contain if each house is 30 feet long and 20 feet wide?

There are two scenarios here. The first, which is the simplistic scenario, is the assumption that the shape of the houses can be squished into a shape to meet the curve of the circle of the city. Assuming unrealistic city design where each house was connected to each other like bricks and there were no additional buildings or spaces within the city, we can calculate the theoretical maximum number of houses. Each house has a constant area of 600 square feet, notwithstanding some slight variance in the shape of each house to accommodate the curved edges of the city. Therefore the theoretical maximum (TM) is the area of the circle in square feet divided by 600 square feet per house.

$$TM = \frac{A}{600}$$
$$A = \pi r^2$$
$$C = 2\pi r$$
$$If\ C = 8000$$
$$then\ 8000 = 2\pi r$$
$$r = \frac{8000}{2\pi} = \frac{4000}{\pi}$$
$$TM = \frac{\pi r^2}{600}$$
$$= \frac{\pi\left(\frac{4000}{\pi}\right)^2}{600}$$
$$= \frac{16000000}{600\pi}$$
$$= \frac{80000}{3\pi}$$
$$If\ we\ take\ \pi\ \approx 3.1415$$
$$TM \approx 8488.26$$

Therefore the theoretical maximum if every square foot of the city was house (unrealistic) is 8488 houses. Answers will vary, and the solutions need to be justified based on realistic considerations, such as:

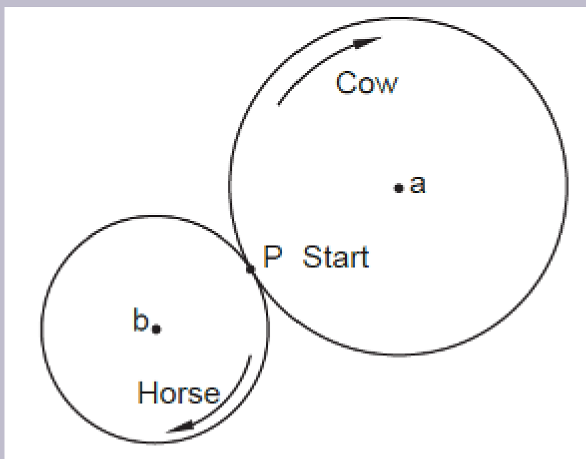
- Other buildings (businesses, markets, schools, community centres, religious buildings, government buildings, utility buildings, etc.)
- Open spaces (paths, roads, squares, etc.)
- Green space (parks, plazas, hiking trails, etc.)
- Attractions (fairs, sports centres, arts centres, etc.)
- Topography (whether there are hills, valleys, rivers, etc.)
- Transportation requirements (stables, bus facilities, trains, airport, etc.)
- How much space people take up in a city when outside their homes
- And perhaps most importantly, the fact that buildings need to have separation from each other, and not be squished together in all directions.

2.6

1.1

2.1

A circular road A that is 48 km in circumference touches at point P another circular road B of circumference 36 km. A cow and a horse start walking from the point P along the road A and B, respectively. The cow walks 6 km per day and the horse walks 12 km per day. How many days later days later do the cow and horse meet again at P?**



The cow walks the full circle A in 48/6 = 8 days. The horse walks the full circle B in 36/12 = 3 days. Finding the lowest common multiple, which is 3×8=24, we find that 24 days later the cow and horse meet again at P

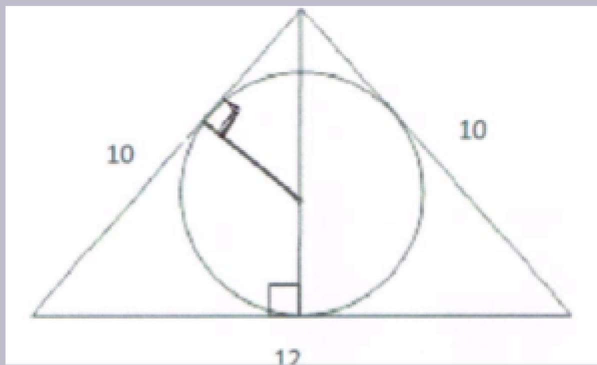
2.7

1.1

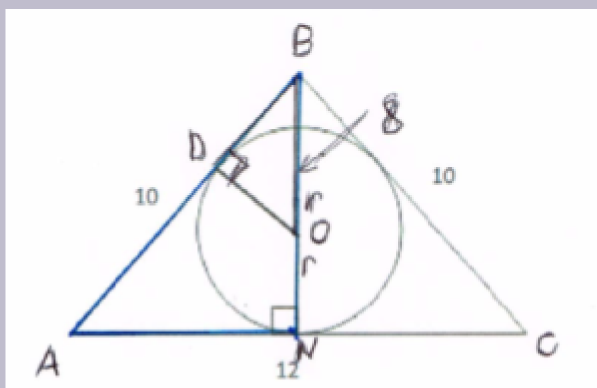
2.1

A circle of radius r is inscribed in an isosceles triangle with sides 10 and 12. Find r.

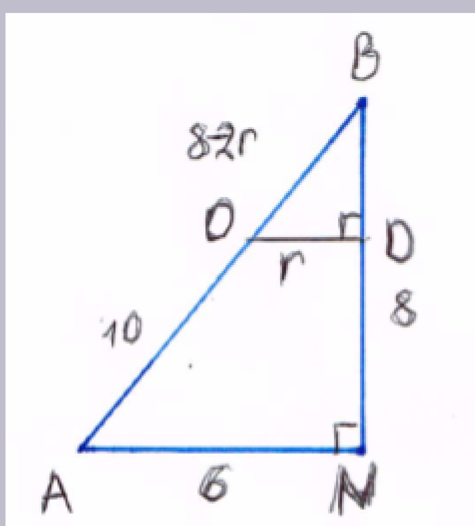
Draw the diagram with some auxiliary lines



Label the vertices. By the Pythagorean theorem BN=8.



We can see now that triangles ABN and ODB are similar (the angles are equal). For better visual we overlap them. Label the sides in the diagram by their length.



Now we can write the ratio AN : AB = OD : DB or 6 : 10 = r : (8-2r).
r=3.

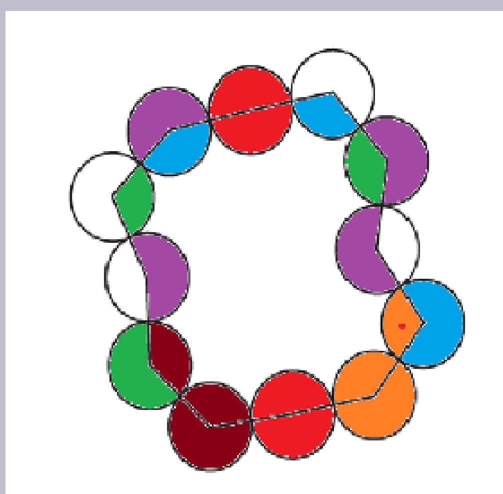
2.8

1.1

2.1

The centers of a loop of the circles of radius r form the vertices of a polygon, as shown in the figure below. Let S1 be the sum of the shaded areas of the circles, and S2 the sum of the unshaded areas of the circles. Find $S2_{.}\quad S1.$

Match the shaded and unshaded pieces. Then count the rest of unshaded pieces. If you combine them you get two full circles. Therefore $S2_{.}\quad S1 = 2\pi r2$



Chapter 3 – Area and Volume

3.1

1.1

2.1

Find the volume of a cylindrical granary of diameter 9 cubits and height 10 cubits. (1 cubit = 52 cm)

If we want to find the volume in cubits:

Let $V =$ the volume of the cylindrical granary in cubits

$$V = \text{area of base} \cdot \text{height}$$
$$V = (\pi \cdot r^2) \cdot h$$
$$V = \pi \cdot (\frac{9}{2})^2 \cdot 10$$
$$V = \pi \cdot 4.5^2 \cdot 10$$
$$V = \pi \cdot 20.25 \cdot 10$$
$$V = \pi \cdot 202.5$$
$$V \approx 636.17$$

Therefore the volume of this cylindrical granary is approximately 636.17 cubic cubits.

If we want to find the volume in centimeters:

1 cubit:52 cm

If there were a cube of 1 cubit, its volume would be calculated as

$$l \cdot w \cot h = 1 \cdot 1 \cdot 1 = 1^3 = 1$$

Therefore its volume would be 1 cubic cubit.

The same cube could be said to be of 52 cm, and the same volume would be calculated as

$$52 \cdot 52 \cdot 52 = 140608$$

Therefore the volume of 140,608 cm ^{3} is equal to 1 cubic cubit.

1 cubic cubit:140,608 cm ^{3}

Let $V =$ the volume of the cylindrical granary in cubic centimeters.

$$1 \text{ cubic cubit} : 140,608 \text{ cm}^3$$
$$202.5\pi \text{ cubic cubits} : V \text{ cm}^3$$

$$V = 140608 * 202.5\pi$$
$$= 89450944.62$$

Therefore the volume of this cylindrical granary is approximately 89,450,944.62 cm ^{3}

3.2

1.1

2.1

Find the volume of a cylindrical granary of diameter 10 cubits and height 10 cubits. (1 cubit = 52 cm)

If we want the volume in cubic centimeters, we can start by converting the measure of diameter and height from cubits to cm:

Let $d =$ the diameter in cm.

$$1 \text{ cubit} : 52 \text{ cm}$$
$$10 \text{ cubits} : d \text{ cm}$$

$$d = 52 * 10$$
$$= 520$$

Therefore the diameter is 520 cm.

Let $h =$ the height in cm.

$$1 \text{ cubit} : 52 \text{ cm}$$
$$10 \text{ cubits} : h \text{ cm}$$
$$h = 52 * 10$$
$$= 520$$

Therefore the height is 520 cm.

The volume of a cylindrical container is the product of the container's height and the area of its circular base.

Let $V =$ the volume of the cylindrical granary in cubic centimeters.

$$V = \text{Area of base} \cdot \text{height}$$
$$V = (\pi \cdot r^2) \cdot h$$
$$V = \pi \cdot (\frac{520}{2})^2 \cdot 520$$
$$V = \pi \cdot 260^2 \cdot 520$$
$$V = \pi \cdot 67600 \cdot 520$$
$$V = \pi \cdot 35152000$$
$$V \approx 110433264.96$$

Therefore the volume of this cylindrical granary is approximately 110,433,264.96 cm ^{3}

If we want to convert this into cubic cubits:

Let $V =$ the volume of the cylindrical granary in cubic cubits.

From 3.1, we know that:

$$1 \text{ cubic cubit} : 140,608cm^3$$
$$V \text{ cubic cubits} : 35,152,000\pi \text{ cubic centimeters}$$

$$V = \frac{35152000\pi}{140608}$$
$$V \approx 785.40$$

Therefore, the volume of this cylindrical granary is approximately 785.40 cubic cubits.

3.3

1.1

2.1

A cylindrical granary of diameter 9 cubits and height 6 cubits. What is the amount of grain that goes into it? (1 cubit = 52 cm. The hekat was an ancient Egyptian volume unit used to measure grain, bread, and beer. It equals 4.8 litres. 30 hekats equals 1 cubic cubit)

By asking for the amount of grain that fits within a cylindrical granary, this question is asking for this granary's volume; the volume of a cylindrical container is the product of the container's height and the area of its circular base.

$$V = \text{area of base} \cdot \text{height}$$
$$V = (\pi \cdot r^2) \cdot h$$
$$V = \pi \cdot (\tfrac{9}{2})^2 \cdot 6$$
$$V = \pi \cdot 4.5^2 \cdot 6$$
$$V = \pi \cdot 20.25 \cdot 6$$
$$V = \pi \cdot 121.5$$
$$V \approx 381.70$$

Therefore the volume of this cylindrical granary is approximately 381.70 cubic cubits.

As 30 hekats equals 1 cubic cubit, we can multiply this volume by 30 to get the amount of grain being stored.

Let $q =$ the quantity of grain being stored in the 381.70 cubic cubits cylindrical granary.

$$q = 381.70 \cdot 30$$
$$q = 11451.10$$

Therefore there is approximately 11,451.10 hekat of grain being stored in this cylindrical granary.

3.4

1.1

2.1

A rectangular granary into which there have gone 7500 quadruple hekat of grain. What are its dimensions? (The hekat was an ancient Egyptian volume unit used to measure grain, bread, and beer. It equals 4.8 litres.)

First we convert the quantity of grain into the measure of volume of the container.

From Question 3.3, we know that 1 cubic cubit of volume is equal to 30 hekat of grain.
We want to know the volume in cubic cubits that corresponds with 7500 quadruple hekat of grain.

Let $x =$ the amount of 7500 quadruple hekat in units of hekat.

$7500 : x$

$1 : 4$

$\frac{7500}{x} = \frac{1}{4}$

$7500 = x \cdot \frac{1}{4}$

$7500 \cdot 4 = x$

$30000 = x$

Therefore 7500 quadruple hekat – where 1 quadruple hekat is equivalent to 4 hekat – is equal to 30,000 hekat.

Let $v =$ the volume of the rectangular granary in cubic cubits.

$1 : 30$

$v : 30000$

$\frac{1}{30} = \frac{v}{30000}$

$30000 \cdot \frac{1}{30} = v$

$1000 = v$

Therefore the volume of the rectangular granary is 1000 cubic cubits.

The volume of a rectangular container is the product of the container's height and the area of its base.

$$V = \text{area of base} \cdot \text{height}$$
$$V = (l \cdot w) \cdot h$$
$$1000 = l \cdot w \cdot h$$

There are multiple possible dimensions for this rectangular granary, as there are multiple dimensions that meet the requirement of being 10,000 cubic cubits in volume.

We can solve for a set of possible dimensions if we make additional assumptions. E.g., the rectangular granary is actually a perfect cube where its length, width, and height are all equal.

$l = w = h$

therefore:

$1000 = l \cdot w \cdot h$

$1000 = w \cdot w \cdot w$

$1000 = w^3$

$\sqrt[3]{1000} = \sqrt[3]{w^3}$

$10 = w$

Therefore, based on the additional assumption, we have the dimensions of a cube granary with side length of 10 cubits.

3.5

1.1

2.1

A rectangular granary into which there have gone 2500 quadruple hekat of grain. What are its dimensions? (The hekat was an ancient Egyptian volume unit used to measure grain, bread, and beer. It equals 4.8 litres.)

First we convert the quantity of grain into the measure of volume of the container.

From Question 3.3, we know that 1 cubic cubit of volume is equal to 30 hekat of grain.
We want to know the volume in cubic cubits that corresponds with 2500 quadruple hekat of grain.

Let $y =$ the amount of 2500 quadruple hekat in units of hekat.

$$2500 : y$$

$$1 : 4$$

$$\frac{2500}{y} = \frac{1}{4}$$

$$2500 = y \cdot \frac{1}{4}$$

$$2500 \cdot 4 = y$$

$$10000 = y$$

2500 quadruple hekat – where 1 quadruple hekat is equivalent to 4 hekat – is equal to 10,000 hekat.

Let $v =$ the volume of the rectangular granary in cubic cubits.

$$1 : 30 = v : 10,000$$

$$\frac{1}{30} = v10000$$

$$10000 \cdot \frac{1}{30} = v$$

$$333\frac{1}{3} = v$$

Therefore the volume of the rectangular granary is $333\frac{1}{3}$ cubic cubits.

The volume of a rectangular container is the product of the container's height and the area of its base.

$$V = \text{area of base} \cdot \text{height}$$
$$V = (l * w) * h$$
$$333\frac{1}{3} = l * w * h$$

We can solve for a set of possible dimensions if we make additional assumptions. E.g., the rectangular granary is actually a perfect cube where its length, width, and height are all equal.

$$l = w = h$$

$$333\frac{1}{3} = l \cdot w \cdot h$$
$$333\frac{1}{3} = w \cdot w \cdot w$$
$$333\frac{1}{3} = w^3$$
$$\sqrt[3]{333\frac{1}{3}} = \sqrt[3]{w^3}$$
$$w \approx 6.93$$

Therefore, based on our additional assumption, we have the dimensions of a cube granary with side length of approximately 6.93 cubits.

3.6

1.1

2.1

Suppose it is said to thee. What is the area of a triangle of side 10 khet and of base 4 khet? (1 khet = 100 cubits, 1 cubit = 52 cm. Assume the triangle is isosceles).

Let $b =$ the measure of the base of the triangle in khet, let $s =$ the measure of the side length of the triangle in khet, and

let $A =$ the area of the triangle in square khet.

The area of an isosceles triangle of side 10 khet and base 4 khet is half of the area of a rectangle of side 10 khet and base 4 khet. The area of the rectangle is the product of the base and the side length:

$$\frac{b \cdot s}{2}$$

$b \cdot s$. Therefore the area of the triangle is

$$A = \frac{b \cdot s}{2}$$
$$A = \frac{4 \cdot 10}{2}$$
$$A = \frac{40}{2}$$
$$A = 20$$

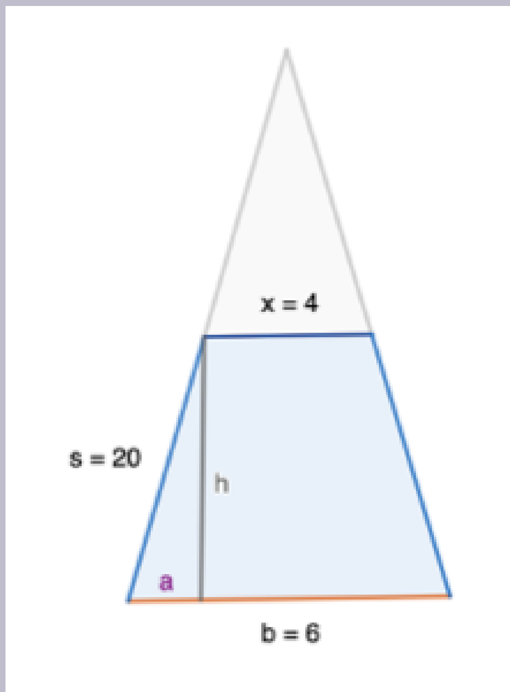
Therefore the area of the triangle is 20 square khet.

3.7

1.1

2.1

Suppose it is said to thee, What is the area of a cut-off (truncated) triangle of land of 20 khet in its side, 6 khet in its base, 4 khet in its cut-off line? (1 khet = 100 cubits, 1 cubit = 52 cm. Assume the triangle is isosceles).



$$\text{Area of a Trapezoid} = \frac{x+b}{2} \cdot h$$

To determine h, we can use Pythagorean Theorem on the right-angle triangle made by a vertical line connecting the two horizontal lines, x and b.

$$a^2 + b^2 = c^2$$

$$a^2 + h^2 = s^2$$

$$a^2 + h^2 = 20^2$$

a is the measure between the far left point to where the height line meets the base, b. The same length also occurs on the right side of the triangle. Therefore, this measurement is half of the difference between the two lines:

$a = \frac{b-x}{2}$

$a = \frac{6-4}{2}$

$a = \frac{2}{2}$

$a = 1$

$a^2 + h^2 = 20^2$

$1^2 + h^2 = 20^2$

$1 + h^2 = 400$

$h^2 = 400 - 1$

$h = \sqrt{399}$

$\text{Area of trapezoid} = \frac{x+b}{2} \cdot h$

$= \frac{4+6}{2} \cdot \sqrt{399}$

$= \frac{10}{2} \cdot \sqrt{399}$

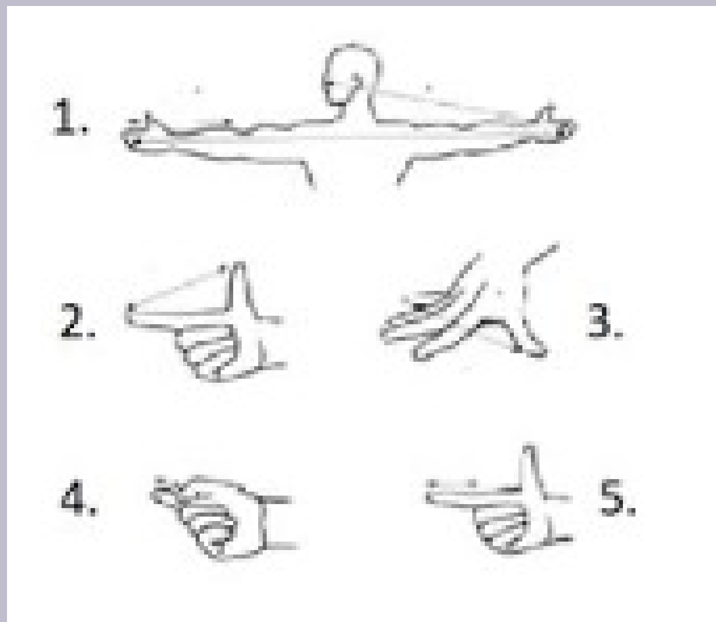
$= 5 \cdot \sqrt{399}$

Therefore the area of the land is $5 \cdot \sqrt{(399)}$ square khet.

3.8

1.1

2.1



The body of the human and its parts were considered as the most effective scales for measuring in all cultures. The limbs were considered as the best scales for measurement because they allowed instant measurements. You don't need a ruler! Your ruler is always with you. The following are some old Mongolian units of length. Can you match them with the body parts pictured in the diagram above?

- Huruu 1.5–2 cm
- Yamh 3.5 cm
- üzür sööm 18 cm
- ald 1.6 m
- sööm – 16 cm

Unit	In Metric	Pic Number	Reasoning
Huruu	1.5– 2cm	4. Finger nail	smallest
Yamh	3.5cm	5. Finger tip	second smallest
üzür sööm	18cm	3. Hand span	second largest
ald	1.6m	1. Wing span	largest
sööm	16cm	2. Pointer to thumb	process of elimination

3.9

1.1

2.1

The most ancient inscription found so far in Mongolian language is carved onto the stone known as Genghis Khan's stone. It is dated around 1225 and immortalizes one of Genghis Khan's warrior's archery achievement:
"When Chinggis Khan was holding an assembly of Mongolian nobles at Bukha-(S)ochiqai after he had come back from the conquest of the Sartuul people, Yisüngke hit a target at 335 alds."

- **How far is the target in meters?**
- **Do you think that this feat is achievable?**

Let $d =$ the distance of the target in meters.

$$335 \text{ ald} : d \text{ meters}$$

$$\text{From 3.8, we know that 1 ald is equal to 1.6 meters.}$$

$$1 \text{ ald} : 1.6 \text{ m}$$

$$\frac{335}{1} = \frac{d}{1.6}$$

$$1.6 \cdot 335 = d$$

$$536 = d$$

Therefore, 335 ald is equivalent to 536 meters.

Do I think this feat is achievable? Hitting a target more than half a kilometer away? No. I think that the target would be difficult to discern at that distance, let alone be able to hit it accurately! And that is not to mention the quality needs of the bow to travel that distance – with precision, no less!

3.10

1.1

2.1

A basilica is 240 feet long and 120 feet wide. The basilica paved with tiles 23 inches long and 12 inches wide. How many tiles are needed to cover the basilica? (There are 12 inches in a foot.)

Let $t =$ the length of the basilica tiles in feet.

$$\text{12 inches : 1 foot}$$
$$\text{23 inches : } t \text{ feet}$$
$$\frac{12}{23} = \frac{1}{t}$$
$$t = \frac{1 \cdot 23}{12}$$
$$t = \frac{23}{12}$$
$$t = 1\frac{11}{12}$$
$$\text{23 inches} = 1\frac{11}{12} \text{ feet}$$

In feet:

Let $=$ the area of the basilica in sqft.

Let $=$ the area of each tile in sqft.

Let $=$ the number of tiles needed to fill the basilica.

$$B = \text{length} \cdot \text{width}$$
$$= 240 \cdot 120$$
$$= 28800$$

Therefore, the area of the basilica is 28,800 sqft.

$$T = \text{length} \cdot \text{width}$$
$$= 1\frac{11}{12} \cdot 1$$
$$= 1\frac{11}{12}$$

Therefore, the area of each tile is $\frac{11}{12}$ sqft.

$$N = \frac{B}{T}$$
$$= \frac{28800}{1\frac{11}{12}}$$
$$\approx 15026.09$$

Therefore, we need 15,027 tiles.

In inches:

Let $=$ the area of the basilica in square inches.

Let $=$ the area of each tile in square inches.

Let $=$ the number of tiles needed to fill the basilica.

$$240 \text{ ft} \cdot 12 \frac{\text{in}}{\text{ft}} = 2880\text{in}$$
$$120 \text{ ft} \cdot 12 \frac{\text{in}}{\text{ft}} = 1440 \text{ in}$$
$$\text{Therefore, the dimensions of the basilica are 2880 inches by 1440 inches.}$$

$$B = 2880 \cdot 1440$$
$$= 4147200$$
$$\text{Therefore the area of the basilica is 4,147,200 square inches.}$$

$$T = 23 \cdot 12 = 276\text{Therefore, the area of each tile is 276 square inches.}$$

$$N = \frac{4147200}{276}$$
$$\approx 15026.09$$
$$\text{Therefore, 15,027 tiles are needed to cover the entire basilica.}$$

3.11

1.1

2.1

A wine cellar is 100 feet long and 64 feet wide. How many casks can it hold, given that each cask is seven feet long and four feet wide, and given that there is an aisle four feet wide down the middle of the cellar?

Let $W =$ the area of the wine cellar.

Let $A =$ the area of the aisle in the cellar.

Let $S =$ the area of the storage area of the wine cellar.

Let $C =$ the area of each cask.

Let $N =$ the number of casks that can be stored in this cellar.

$$W = 100 \cdot 64$$
$$= 6400$$

$$A = 100 \cdot 4$$
$$= 400$$

$$S = W - A$$
$$= 6400 - 400$$
$$= 6000$$

$$C = 7 * 4$$
$$= 28$$

$$N = \frac{S}{C}$$
$$= \frac{6000}{28}$$
$$= \frac{1500}{7}$$
$$\approx 214.29$$

Because we cannot store a fraction of a cask, we can store a maximum of 214 casks.

3.12

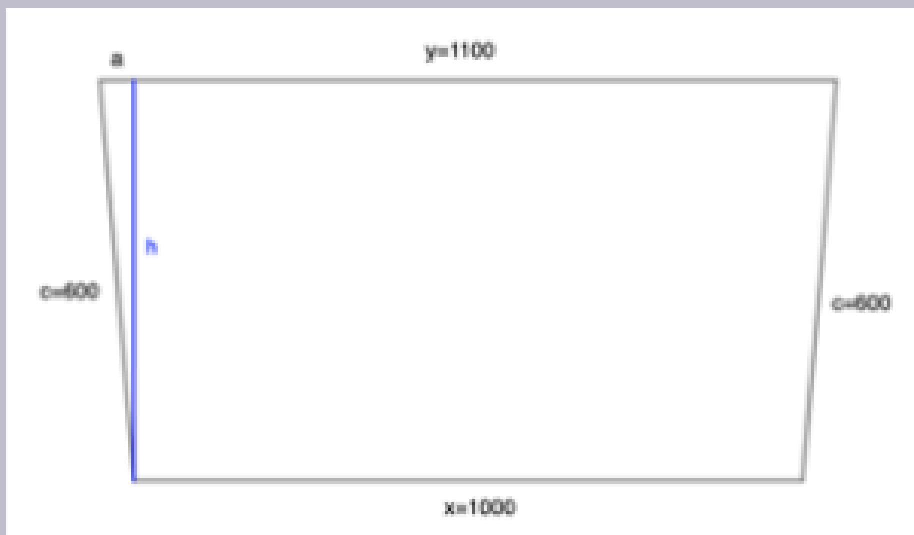
1.1

2.1

A four-sided town measures 1100 feet on one side and 1000 feet on the other side; on one edge, 600, and on the other edge, 600. I want to cover it with roofs of houses, each of which is to be 40 feet long and 30 feet wide. How many dwellings can I make there?

https://www.geogebra.org/calculator/tmdr4ej4

Because this shape has two sides of equal length and two sides of unequal length, we know that the two side of unequal length must be parallel so as to intersect the two sides of equal length.



$$\text{Area of trapezoid} = \frac{x+y}{2}c\dot{h}$$

We can solve for h by using Pythagorean Theorem.

The base of the triangle, x, is half of the difference between the two horizontal lines.

$$a = \frac{y-x}{2}$$
$$= \frac{1100-1000}{2}$$
$$= \frac{100}{2}$$
$$= 50$$

$$a^2 + b^2 = c^2$$
$$50^2 + h^2 = 600^2$$
$$h^2 = 600^2 - 50^2$$
$$h = \sqrt{357500}$$
$$h = 50 \cdot \sqrt{143}$$
$$h \approx 597.91$$

$$\text{Area of trapezoid} = \frac{x+y}{2} \cdot h$$
$$= \frac{1000+1100}{2} \cdot 50 \cdot \sqrt{143}$$
$$= \frac{2100}{2} \cdot 50 \cdot \sqrt{143}$$
$$= 1050 \cdot 50 \cdot \sqrt{143}$$
$$= 52500 \cdot \sqrt{143}$$
$$\approx 627808.69$$

Therefore the area of the town is $52,500 \cdot \sqrt{143}$ sqft.

$$\text{Area of each house } = 40 \cdot 30$$
$$= 1200 \text{ sqft}$$

$$\text{Number of houses} = \frac{\text{Area of town}}{\text{Area of each house}}$$
$$= \frac{52500 \cdot \sqrt{143}}{1200}$$
$$= \frac{175 \cdot \sqrt{143}}{4}$$
$$\approx 523.17$$

Therefore 523 houses of dimensions 40ft by 30ft can be made in this town.

3.13

1.1

2.1

I have a cloak 100 cubits long and 80 cubits wide. I wish to make small cloaks with it; each small cloak is 5 cubits long, and 4 wide. How many small cloaks can I make? (1 cubit = 50 cm)

$$\text{Area of large cloak} = 100 \cdot 80$$
$$= 8000 \text{ square cubits}$$

$$\text{Area of each small cloak} = 5 \cdot 4$$
$$= 20 \text{ square cubits}$$

$$\text{Number of small cloaks} = \frac{\text{Area of large cloak}}{\text{Area of each small cloak}}$$
$$= \frac{8000}{20}$$
$$= 400$$

Therefore we can make 400 small cloaks.

3.14

1.1

2.1

A four-sided field measures 30 perches down one side and 32 down the other; it is 34 perches across the top and 32 across the bottom. How many acres are included in this field? (1 perch ≈ 5 m. 1 acre = 40 perches by 4 perches ≈ 4000 m ^{2})

https://www.geogebra.org/calculator/d4du8dv9

Note that this is not necessarily a trapezoid – there is no indication that any sides are parallel. However, there are multiple solutions if we treat this as any irregular quadrilateral. Therefore, I have to make an assumption to be able to solve.

(If Trapezoid:)
(If we made the top and bottom parallel: https://www.geogebra.org/calculator/cwyuhfmc)
(If we made the sides parallel: https://www.geogebra.org/calculator/uzwzpt7d)

I am assuming that this is a cyclic quadrilateral (has all 4 vertices lying on a circle) and therefore will use Brahmagupta's formula for a cyclic quadrilateral:

$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$

Where s the semi-perimeter of the quadrilateral $= \frac{a+b+c+d}{2}$

A is the area, and $a,\ b,\ c,$ and d are side lengths of the quadrilateral.

$a = 30$
$b = 34$
$c = 32$
$d = 32$

$s = \frac{a+b+c+d}{2}$
$= \frac{30+34+32+32}{2}$
$= 64$

$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$
$= \sqrt{(64-30)(64-34)(64-32)(64-32)}$
$= \sqrt{1044480}$
$= 64 \cdot \sqrt{255}$
≈ 1022.00 square perches

1 acre $= 40$ perches by 4 perches $= 160$ square perches

$\frac{64 \cdot \sqrt{255} \text{ square perches}}{160 \text{ square perches per acre}}$
$= \frac{2 \cdot \sqrt{255}}{5}$
≈ 6.39

Therefore there are approximately 6.39 acres on this field.

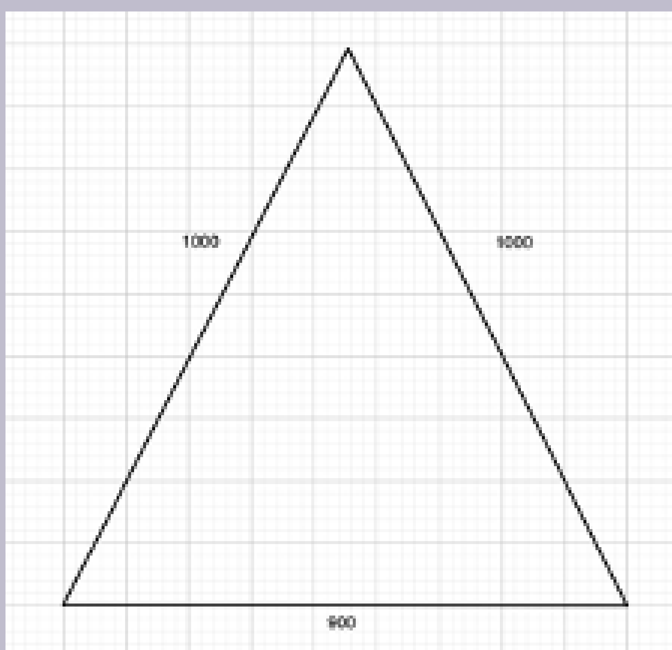
3.15

1.1

2.1

There is a triangular city which has one side of 1000 feet, another side of 1000 feet, and a third of 900 feet. Inside of this city, I want to build houses each of which is 20 feet in length and 10 feet in width. How many houses can I build in the city?

This is an Isosceles Triangle as two sides are the same length.



To solve this problem, I will first set design assumptions, and then use a data program to solve the model.

In this city, I imagine one road running North to South, and then rows of houses on either side of the street. There will be horizontal streets separating the front doors of houses, but they will all share back and side walls.

The main, vertical road should be the width of 2 horse-drawn buggies: approximately 8 feet.
Each side road can be the width of 1 horse-drawn buggy: approximately 4 feet.

Starting with the base, we cannot fit houses in the vertical road, nor can we fit rectangular houses in neither the first nor last 10 feet of the row given the angles of the space. Therefore, the amount of useable space on each side of the triangle is:
(900/2)-(8/2)-10

=450-4-10
=436 feet

At 20 feet per house:
436/20
=21.8

Therefore we can fit 21 houses on each side of the road along the base; 42 houses along the base of our city.

The next row of houses will be above these 42, 10 feet wide houses with an additional 4 feet street inbetween.

We know that the base is 900, but its height is not explicitly stated. The height of this triangle can be determined using Pythagorean Theorem. The height is a vertical line going through the top point through the middle of the base. Therefore, the base of this right-angle triangle is half of the base of the full triangle.

$$a^2 + b^2 = c^2$$
$$(\frac{900}{2})^2 + h^2 = 1000^2$$
$$h^2 = 1000^2 - 450^2$$
$$\sqrt(h^2) = \sqrt{797500}$$
$$h = \sqrt{797500}$$
$$h = 50 \cdot \sqrt{319}$$
$$h \approx 893.03$$

$$\text{Therefore the height of the triangle is approximately 893.03 feet.}$$

Moving upwards by 14 feet, the next row of houses start at a height of 879 feet. Similarly to the previous row, we cannot fit houses in the first 10 feet of the base due to the angle of the edge of the city, nor in the 4 feet of one side's vertical road. Therefore, the amount of base we are working with is 450 ft less the difference due to the change in height (but not the actual height value), less the 10 feet that is on an angle, less the 4 feet for the vertical road. To determine the change in base, we can use the ratio of the base to the height as a fixed ratio and use the known change in height to determine the change in base.

Using an Excel Spreadsheet, I can continue this pattern through the entire triangle and determine that, based on my design assumptions, 1554 rectangular houses can fit within this area.

3.16

1.1

2.1

A little rectangular canal is to be excavated for a length of 5 km. Its width is 2 m, and its depth is 1 m. Each laborer is assigned to remove 4m ^{3} of earth, for which he will be paid one-third of a basket of barley. How many laborers are required for the job, and what are the total wages to be paid?

The section being excavated is of dimensions 5000 m by 2 m by 1 m.
The volume of earth being excavated is:

$$5000 \cdot 2 \cdot 1$$
$$= 10000\ m^3$$
$$= 10\ km^3$$

$$\text{Total number of laborers needed} = \frac{\text{total volume of earth to be excavated}}{\text{volume assigned to each laborer}}$$
$$= \frac{10000m^3}{4m^3 \text{ per laborer}}$$
$$= 2500 \text{ laborers}$$

Therefore 2,500 laborers are needed for this job.

$$\text{Total wages} = \text{Wage per laborer} \cdot \text{Number of laborers}$$
$$= \frac{1}{3} \cdot 2500$$
$$= \frac{2500}{3}$$
$$= 833\frac{1}{3}$$

Therefore, $833\frac{1}{3}$ baskets of barley will be the total wages paid for this job.

3.17

1.1

2.1

A canal is 5 rods long, 11/2 rods wide, and 1/2 rod deep. Workers are assigned to dig 10 gin of earth for which task they are paid 6 sila of grain. What is the area of the surface of this canal and its volume? What are the number of workers required and their wages? (1 rod = 12 cubits = 12 x 50 cm= 6 m, 1 sar area = 36 m ^{2} , 1 gin = 5 litres, 1 sila = 1 litre]

Given: $1 \text{ rod} = 12 \text{ cubits} = 6\ m$

Therefore: $5 \text{ rods} = 60 \text{ cubits} = 30\ m$ (multiply everything by 5)

$\frac{11}{2} \text{ rods} = 66 \text{ cubits} = 33\ m$

$\frac{1}{2} \text{ rod} = 6 \text{ cubits} = 3\ m$

Volume of dig site $= 30\ m \cdot 33\ m \cdot 3\ m$

$= \frac{55}{4}$ cube rods

$= 2970\ m^3$

Therefore the volume is $2970\ m^3 //$

Surface Area $=$ (4 sides $\cdot$ area of each side) $+$ (2 ends $\cdot$ area of each end)

$= 4 \cdot (30 \cdot 33) + 2 \cdot (33 \cdot 3)$

$= 4 \cdot 990 + 2 \cdot 99$

$= 4158\ m^2$

Therefore the surface area is $4158\ m^2$

Given: $1 \text{ gin} = 5 \text{ litres} = 5 \text{ sila}$

Note: $1000 \text{ litre} = 1$ cubic metres

Therefore: $10 \text{ gin} = 50 \text{ litres} = 50 \text{ sila} = \frac{1}{20}$ cubic metres (divide by 1000)

Number of workers $= \frac{\text{total volume of dig site}}{\text{volume per worker}}$

$= \frac{2970}{\frac{1}{20}}$

$= 59400$

Therefore 59,400 workers are to be hired for this job.

Total wages $=$ wage per worker $\cdot$ number of workers

$= 6 \cdot 59400$

$= 356400$

Therefore 356,400 sila, which is 356,400 litres or 71,280 gin of grain, is needed for total wages.

3.18

1.1

2.1

A siege ramp is to be built to attack a walled city. The volume of earth allowed is 5400 sar. The ramp will have a width of 6 rods, a base length of 40 rods, and a height of 45 cubits. Construction of the ramp is incomplete; an 8-rod gap is left between the end of the ramp and the city wall. The height of the uncompleted ramp is 36 cubits. How much more earth is needed to complete this ramp? (1 sar volume = 1 sar area x 1 cubit =18 m ^{3} , 1 cubit = 50 cm; 1 rod = 12 cubits = 6 m.)

Total ramp dimensions:
– Max 5400 sar of earth
– Length = 40 rods
– Width = 6 rods
– Height = 45 cubits

Given: 1 rod = 12 cubits
Let $h =$ the height of the ramp in rods.

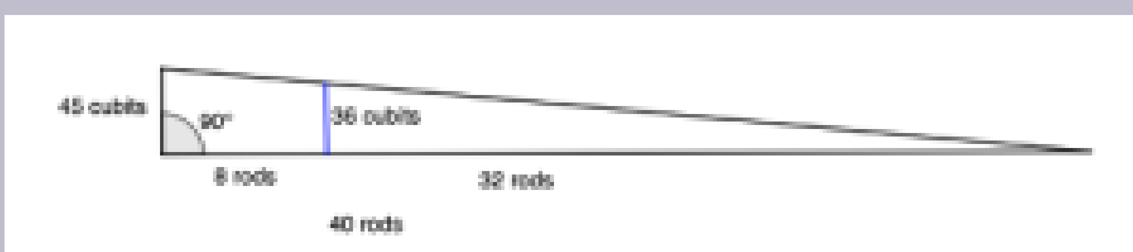
$$h \text{ rods} : 45 \text{ cubits}$$

$$h = \frac{45}{12} \text{ rods}$$

$$\text{Volume of ramp} = \frac{40 \cdot 6 \cdot \frac{45}{12}}{2}$$

$$= \frac{900}{2}$$

$$= 450 \text{ cubic rods}$$



Note: 36 cubits = 3 rods

$$\text{Volume of amount completed} = \frac{(40-8)\cdot 3\cdot 6}{2}$$
$$= \frac{576}{2}$$
$$= 288$$

Therefore 288 cubic rods of earth have been completed.

$$\text{Amount remaining} = \text{Total} - \text{Amount completed}$$
$$= 450 - 288$$
$$= 162$$

Therefore 162 cubic rods of earth left.

To measure this in sar of earth, we first converts the rods to metres.

Given: 1 rod = 6 m.

$$1 \text{ cubic rod} = 6\ m \cdot 6\ m \cdot 6\ m = 216\ m^3$$
$$\text{volume of earth in } m^3:$$
$$162 \text{ cubic rods} \cdot 216 \text{ cubic metres per rod}$$
$$= 34992$$

Therefore there are 34,992 cubic metres of earth needed to complete this ramp.

Given: 1 sar volume = 18 cubic metres.

$$\text{volume in sar} = \frac{34992}{18}$$
$$= 1944 \text{ sar}$$

Therefore 1944 sar of earth is needed to complete the ramp.

Chapter 4 – Fractions

4.1

1.1

2.1

You have to divide 8 loaves among 10 men. What will be the best way to do it?

We have to make sure that every man gets as big pieces as possible. First, give each man a half. Now we have 3 loaves left. Divide each loaf into 4 parts. Give one part to each worker. Now we left with two quarters of a loaf. Divide each quarter in five parts and distribute them to workers. Therefore each worker will get $\frac{1}{2} + \frac{1}{4} + \frac{1}{20} = \frac{16}{20} = \frac{8}{10}$ part of a loaf but in the largest pieces possible.

4.2

1.1

2.1

A quantity and its $\frac{1}{2}$ added together become 16. What is the quantity?

Problems 4.2 – 4.7 can be solved either by writing an equation or bar models. The equation solutions are provided.

Let quantity be x. Then

$$x + \frac{1}{2}x = 16$$

$$x = 10\frac{2}{3}$$

4.3

1.1

2.1

A quantity and its $\frac{2}{3}$ are added together and from the sum $\frac{1}{3}$ of the sum is subtracted, and 10 remains. What is the quantity?

Let quantity be x. Then

$$x + \frac{2}{3}x - \frac{1}{3}(x + \frac{2}{3}x) = 10$$

$$x = 9$$

4.4

1.1

2.1

A quantity, its $\frac{1}{2}$ and its $\frac{1}{4}$, added together, become 10. What is the quantity?

Let quantity be x. Then

$$x + \frac{1}{2}x + \frac{1}{4}x = 10$$

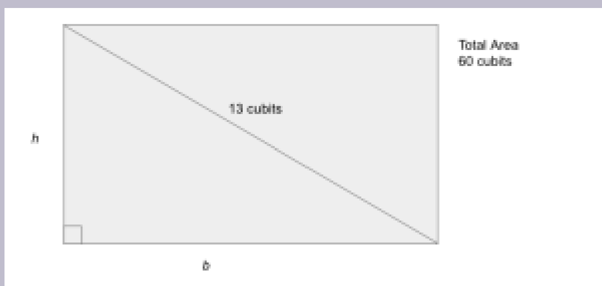
$$x = \frac{7}{4}$$

4.5

1.1

2.1

A quantity together with its two-thirds has one-third of its sum taken away to yield 10. What is the quantity?



Let quantity be x. Then

Let quantity be x. Then

$$x+\frac{2}{3}x-\frac{1}{3}\left(x+\frac{2}{3}x\right)=10$$

$$x=9$$

4.6

1.1

2.1

The sum of a certain quantity, together with its two-thirds, its half, and its one-seventh, becomes 37. What is the quantity?

Let quantity be x. Then

$$x+\frac{2}{3}x+\frac{1}{2}x+\frac{1}{7}x=37$$

$$x=16\frac{2}{97}$$

4.7

1.1

2.1

[Given] a quantity: its two-thirds, one-half, and one-seventh are added together, giving 33. What is the quantity?

Let quantity be x. Then

$$x + \frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x = 33$$

$$x = 14\frac{28}{97}$$

4.8

1.1

2.1

Egyptian only used unit fractions $\frac{1}{n}$ with the exception of $\frac{2}{3}$. They would write any fraction whose numerator is not 1 as a sum of unit fractions (and $\frac{2}{3}$ if needed), e.g.

$\frac{15}{40}$ **=** $\frac{3}{8}$ **=** $\frac{2}{8}$ **+** $\frac{1}{8}$ **=** $\frac{1}{4}$ **+** $\frac{1}{8}$

And a scribe would write in his papyrus not $\frac{15}{40}$ but $\frac{1}{4}$ $\frac{1}{8}$.

Another example:

$\frac{13}{15} = \frac{10}{15} + \frac{3}{15} = \frac{2}{3} + \frac{1}{5}$

And a scribe would write in his papyrus not $\frac{13}{15}$ but $\frac{2}{3}$ $\frac{1}{5}$.

There is another peculiar feature in the way Egyptians decomposed fractions. They never repeat the same fractions in the sums. For instance, $\frac{3}{5} = \frac{1}{5} + \frac{1}{6} + \frac{1}{30}$. The trick here is to first bring the fraction to a greater denominator $\frac{3}{5} = \frac{12}{30}$ And then decompose the resulting fraction $\frac{12}{30} = \frac{6}{30} + \frac{5}{30} + \frac{1}{30} = \frac{1}{5} + \frac{1}{6} + \frac{1}{30}$.

Can you write the following fractions the way Ahmes-the-scribe wrote in the papyrus in around 1550 BC:

$\frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}$ **?**

One of the possible solutions:

$\frac{2}{10} = \frac{1}{5}$

$\frac{3}{10} = \frac{2}{10} + \frac{1}{10} = \frac{1}{5} + \frac{1}{10}$

$\frac{4}{10} = \frac{3}{10} + \frac{1}{10} = \frac{1}{5} + \frac{1}{10} + \frac{3}{30} = \frac{1}{5} + \frac{1}{10} + \frac{1}{15} + \frac{1}{30}$

$\frac{5}{10} = \frac{1}{2}$

$\frac{6}{10} = \frac{1}{2} + \frac{1}{10}$

$\frac{7}{10} = \frac{1}{2} + \frac{2}{10} = \frac{1}{2} + \frac{1}{5}$

$\frac{8}{10} = \frac{1}{2} + \frac{3}{10} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10}$

$\frac{9}{10} = \frac{1}{2} + \frac{4}{10} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{15} + \frac{1}{30}$

Another is found in Rhind or Ahmes papyrus

Note that a bar above a number indicates 1/

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Table 3
Table of divisions in pRhind

1 : 10	$\overline{10}$
2 : 10	$\overline{5}$
3 : 10	$\overline{5}\ \overline{10}$
4 : 10	$\overline{3}\ \overline{15}$
5 : 10	$\overline{2}$
6 : 10	$\overline{2}\ \overline{10}$
7 : 10	$\overline{\overline{3}}\ \overline{30}$
8 : 10	$\overline{\overline{3}}\ \overline{10}\ \overline{30}$
9 : 10	$\overline{\overline{3}}\ \overline{5}\ \overline{30}$

4.9

1.1

2.1

A woman weaves a textile that is to be 48 rods long. In one day, she weaves $\frac{1}{3}$ rod. In how many days will she cut the textile from the loom [be finished]?

48 multiply by $\frac{1}{3}$ is $\frac{48}{3}$. She cut the textile from the loom in 16 days

4.10

1.1

2.1

Two-thirds of two-thirds of a certain quantity of barley is taken, and we obtain 100 units. What was the original quantity?

$\frac{2}{3} \times \frac{2}{3}x = 100$

$\frac{4}{9}x = 100$

$x = \frac{900}{4} = 225$ days

4.11

1.1

2.1

It is known that the digging of a canal becomes more difficult the deeper one goes. In order to compensate for this fact, differential work allotments are computed: a laborer working at the top level is expected to remove $\frac{1}{3}$ sar of earth in one day, while a laborer at the middle level removes $\frac{1}{6}$ sar, and one at the bottom level, $\frac{1}{9}$ sar. If a fixed amount of the earth is to be removed from the canal in one day, how much digging time should be spent at each level?

To answer this question you don't need to know to know what amount of earth is to be removed. Since all laborers work together for one day then a top laborer spends $\frac{1}{3}$ of a day, a middle laborer spends $\frac{1}{6}$ of a day, and a bottom laborer spends $\frac{1}{9}$ of a day digging. The exact time would depend on how long was a working day in Babylon.

4.12

1.1

2.1

I have two fields of grain. From the first field, I harvest two-thirds of a bushel of grain per unit area; from the second, one-half a bushel per unit area. The yield of the first exceeds the second by 50 bushels. The total area of the two fields together is 300 units. What is the area of each field?

Let x be the yield of the second field. Then the yield of the first field can be expressed as x + 50

$$x + x + 50 = 300$$

$$x = 125$$

The yield of the second field is 125 bushels. The yield of the first field is 175 bushels.

The area of the second field is $125 \div \frac{2}{3} = 187.5$ units. The area of the first field is $175 \div \frac{1}{2} = 150$ units.

4.13

1.1

2.1

There are two silver rings; $\frac{1}{7}$ of the first and $\frac{1}{11}$ of the second ring is broken off, so that what is broken off weighs one shekel. The first that is diminished by $\frac{1}{7}$ weighs as much as the second diminished by its $\frac{1}{11}$. What was the weight of the silver rings originally?

If after cutting the rings weight the same then $\frac{6}{7}$ of the first ring weighs the same as $\frac{10}{11}$ of the second ring. It means that the first ring's original weight is $\frac{70}{66}$ of the original weight of the second ring.

Let x be the original weight of the second ring. Then we can express the combined weight of broken pieces as

$$\frac{1}{7} \times \frac{70}{66}x + \frac{1}{11}x = 1$$

$$\frac{113}{462}x = 1$$

$$x = \frac{462}{112} = 4.125$$

The second ring was 4.125 shekels and the first is $\frac{70}{66} \cdot 4.125 = 4.375$ shekels

4.14

1.1

2.1

A woman dies, leaving her husband, a son, and three daughters. Calculate the fraction of her estate each will receive.

[Note: The conditions of Islamic law must be followed—that is, the husband must receive one-fourth share, and his son twice as much as a daughter.]

The kids will get $\frac{3}{4}$ of the estate. This should be divide into 5 parts in order to distribute the wealth lawfully. Each part is $frac320$ of the estate. Therefore each daughter will receive $\frac{3}{20}$ of the estate and the sun will receive $\frac{6}{20}$.

4.15

1.1

2.1

If someone says a workman receives a pay of 10 dirhams per month, how much must he be paid for six days?

He gets $\frac{10}{30}$ or $\frac{1}{3}$ dirham a day. So in 6 day he gets 2 dirhams

4.16

1.1

2.1

The father perished in the shoals of the Syrtis, and this, the eldest of the brothers, came back from that voyage with five talents. To me he gave twice two thirds of his share, on our mother he bestowed two eighths of my share, nor did he sin against divine justice. How much did he leave to himself?

1 talent is 6,000 drachmas or denarii, the Greek and Roman silver coins.

My share is $\frac{10}{3}$ of talents or 3 talents and 2000 drachmas

My mother's share is $\frac{20}{24}$ of talents or 5000 drachmas

He left to himself 5 talents – (3 talents + 2000 drachmas + 5000 drachmas) = 1 talent 5000 drachmas

4.17

1.1

2.1

“Best of clocks, how much of the day is past?” There remain twice two thirds of what is gone.

Let the past part is x, then the remaining part is $\frac{4}{3}x$

$$x + \frac{4}{3}x = 24$$

$x = \frac{72}{7}$ or 10 hours and $\frac{2}{3}$ of hour

10 hours and 40 minutes of the day is past.

4.18

1.1

2.1

We three Loves stand here pouring out water for the bath, sending streams into the fair flowing tank. I on the right, from my long-winged feet, fill it full in the sixth part of a day; I on the left, from my jar, fill it in four hours; and I in the middle, from my bow, in just half a day. Tell me in what a short time we should fill it, pouring water from wings, bow, and jar all at once.

Let’s calculate how much water each Love can pour in hour.

First Love will pour 6 jars per day or $\frac{1}{2}$ jar per hour, second Love will pour 3 jars per day or $\frac{1}{4}$ jar per hour, and third Love will pour 2 jars per day or $\frac{1}{6}$ jar per hour

Together they will pour $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$ jar per hour

Then to figure out how long the jar will be full with the combined effort of all 3 Loves

$1\div\frac{11}{12}=\frac{12}{11}$ of hour or 1 hour and approximately 5-6 minutes.

4.19

1.1

2.1

I am a brazen lion; my spouts are my two eyes, my mouth, and the flat of my right foot. My right eye fills a jar in two days, my left eye in three, and my foot in four.

My mouth is capable of filling it in six hours; tell me how long all four together will take to fill it. (The Greeks consider a day being 12 hours)

The right eye fills a jar in two days. A day was divided into 12 hours in the ancient world (as is explained in the problem), so the flow of the right eye is one jar every 24 hours, or 1 24 jar per hour. The left eye requires three days, or 36 hours, to fill a jar, so its flow rate is 1 36 jar per hour. The foot can fill a jar in four days, or 48 hours, so its flow rate is 1 48 jar per hour. Finally, the mouth can fill a jar in just six hours, so it has a flow of 1 6 jar per hour. The total flow of the fountain, then, is the sum of these four individual flows. To add these fractions, we need a common denominator; the least common denominator of 24, 36, 48, and 16 is 144, so we have $\frac{1}{24}+\frac{1}{36}+\frac{1}{48}+\frac{1}{6}=\frac{37}{144}$. Therefore, the total flow of the fountain is 37 144 jar per hour. We have a description of the flow of the fountain in jars per hour, but we would like a description in hours per jar (because we are interested in knowing how long it will take to fill one jar). Hours per jar is simply the reciprocal of jars per hour; so an equivalent description of the flow of the fountain is that it can fill jars at the rate of $\frac{144}{37}$ hours per jar. In other words, all four spouts together can fill one jar in $\frac{144}{37}=3\frac{33}{37}$ hours, or about 3 hours 54 minutes.

4.20

1.1

2.1

Brick makers, I am in a great hurry to erect this house. Today is cloudless, and I do not require many more bricks, but I have all I want but three hundred. Thou alone in one day couldst make as many, but thy son left off working when he had finished two hundred, and thy son in law when he had made two hundred and fifty. Working all together, in how many hours can you make these?

Let's assume that a working day in ancient Greece was 10 hours. Then the main bricklayer can make 30 bricks\hour, his son =20 bricks\hour, and his son-in-law =25 bricks\hour. Working together they can make 30+20+25=75 bricks\hour. Therefore they need $300 \div 75 = 4$ hours to finish the house.

4.21

1.1

2.1

2.1

After praying for a just increase in my fortunes of gold, I have nothing. I gave 40 talents of gold

under evil auspices to my friends in vain, and I see my enemies in possession of a half, a third, and an eighth of my fortune. How many talents did this man once have?

Let x be the number of talents this man once had.

Then

$$40 + \frac{1}{2}x + \frac{1}{3}x + \frac{1}{8}x = x40 = \frac{1}{24}xx = 480$$

Therefore, this man used to have 480 talents.

4.22

1.1

2.1

The inscription on the tomb reads: "This tomb holds Diophantus. Ah, how great a marvel!" The inscription then tells the length of his life as follows: God granted him to be a boy one-sixth of his life, and adding one- twelfth part of this, he clothed his cheeks with down. He lit the light of wed-lock after one-seventh part of his life, and after 5 years in his marriage he granted him a son. Alas, late-born child; after reaching one-half the measure of his father's life, cruel fate took him. After consoling his grief by the science of mathematics for 4 years, Diophantus ended his life. How old was Diophantus when he passed away?

Let x be Diophantus' age at death. Then we can express all the periods in Diophantus' life

childhood: $\frac{1}{6}x$

adolescence: $\frac{1}{12}x$

bachelorhood: $\frac{1}{7}x$

childless marriage: 5

age of child at death: $\frac{1}{2}x$

life after child's death: 4

We now add the lengths of those periods, set their sum equal to his total age, and solve:

$$\frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + x = x40 = \frac{1}{24}xx = 84$$

Diaphanous lived 84 years.

Chapter 5 – Pythagorean Theorem

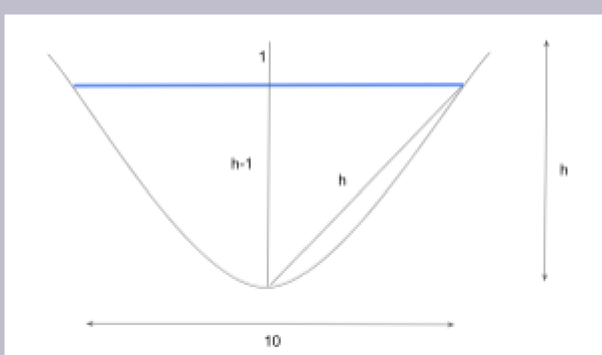
5.1

1.1

2.1

In the center of a river whose width is 10 chi grows a reed whose top reaches 1 chi above the water level. If we pull the reed towards the bank, its top is even with the water's surface. What is the depth of the river?

Let's set up a model showing the reed with height h that, when growing vertically, extends 1 above the water. Therefore the depth of the river is equal to h – 1.



Since the width of the river is 10, the width of half the river is 5. Therefore, we are solving a Pythagorean triangle with side lengths 5 and h-1, and hypotenuse h

$$5^2+(h-1)^2=h^2$$
$$5^2+h^2-2h+1=h^2$$
$$25+h^2-2h+1=h^2$$
$$25-2h+1=0$$
$$26=2h$$
$$h=13$$

Therefore, the depth, or h-1, equals 12 chi.

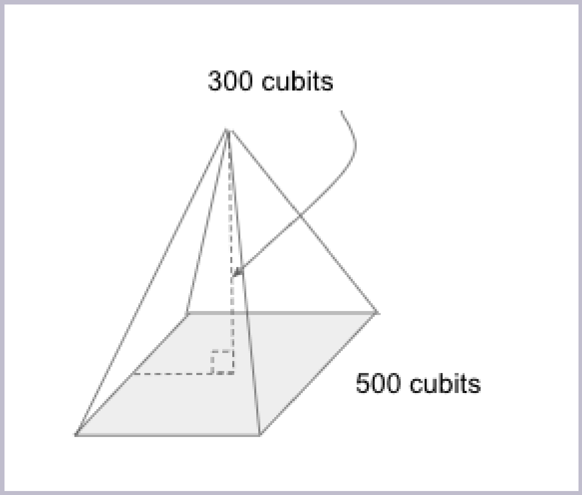
5.2

1.1

2.1

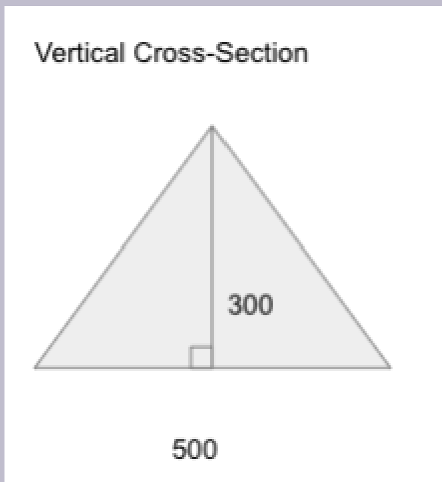
Given a pyramid 300 cubits high, with the square base 500 cubits on a side, determine the distance from the center of any side to the apex.

Given a pyramid 300 cubits high, with a square base 500 cubits on a side, determine the distance from the centre of any side to the apex



When we consider a vertical cross-section of the pyramid that intersects with the top vertex, we notice that it is an isosceles triangle with base of 500 cubits and height of 300 cubits. If we draw a height line down from the top vertex, we have a line that bisects the base, creating 2 congruent right triangles with base 250 cubits and height 300 cubits.

The point at which the cross-section intersects the edge of the base is the centre of the side. Therefore, the distance being asked is the hypotenuse of the right triangles in the diagram below:



$$a^2 + b^2 = hyp^2$$
$$250^2 + 300^2 = hyp^2$$
$$62500 + 90000 = hyp^2$$
$$hyp^2 = 152500$$
$$hyp = \sqrt{152500} = 50\sqrt{61} \approx 390.5125$$

Therefore, the distance from the centre of each side to the apex is approximately 390.5125 cubits.

5.3

1.1

2.1

A pyramid has a base of 360 cubits and a height of 250 cubits. What is its seqt (the ratio of vertical and horizontal dimensions)?

The ratio of vertical and horizontal dimensions or seqt is $250/(360/2) = 25/18$

5.4

1.1

2.1

Find the height of a square pyramid with a seqt (the ratio of vertical and horizontal dimensions) of 21 fingers per cubit (cubit equals 28 fingers) and a base of 140 cubits on one side. Find the length of the slope.

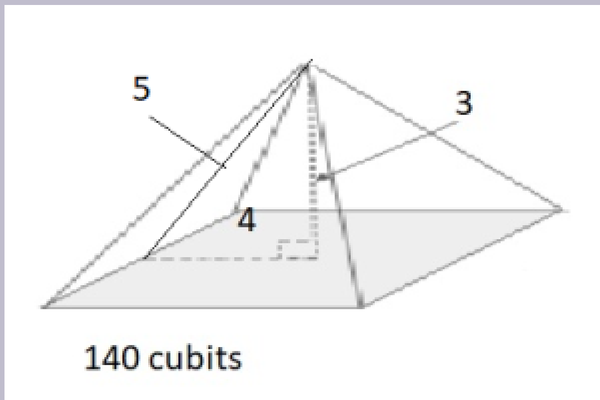
The seqt or the ratio of vertical and horizontal dimensions is $21/28 = 3/4$

Thus if the horizontal distance is $(140/2) = 70$ cubits then the vertical is $70/4 \times 3 = 52.5$ cubits or 52 cubits 14 fingers.

You can calculate the slope by two methods:

1) Since the legs of the right triangle in the diagram are in 3 to 4 ratio then the hypotenuse is in 5 to 4 to 3 ratio (a Pythagorean triplet). The length of the slope is $70/4 \times 5 = 87.5$ cubits or 87 cubits and 14 fingers

2) By the Pythagorean theorem the length of the slope is a square root of $70^2 + 52.5^2 = 87.5$

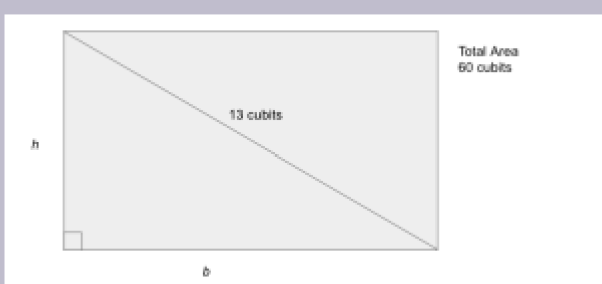


5.5

1.1

2.1

A rectangular plot is 60 cubit square; the diagonal is 13 cubits. How many cubits does it take to make the sides?



You can solve the problem by three methods.

Method 1. One of the Pythagorean triples is 5, 12, 13. Thus the sides are 5 and 12 cubits.

Method 2. Quadratic equation,

Since we know that d = 13 and A = 60

$$bh = 60 \quad b = \frac{60}{h} \quad \therefore 13 = \left(\frac{60}{h}\right)^2$$

By substitution :

$$
\begin{aligned}
13^2 &= b^2 + h^2 \\
13^2 &= \left(\frac{60}{h}\right)^2 + h^2 \\
169 &= \frac{3600}{h^2} + h^2 \\
169h^2 &= 3600 + h^4 \\
h^4 - 169h^2 + 3600 &= 0 \\
\left(h^2\right)^2 - 169\left(h^2\right) + 3600 &= 0
\end{aligned}
$$

Use Quadratic Formula with h^2 as the variable :

$$
\begin{aligned}
h^2 &= \frac{169 \pm \sqrt{(-169)^2 - 4(3600)}}{2} \\
&= \frac{169 \pm \sqrt{28561 - 14400}}{2} \\
&= \frac{169 \pm \sqrt{14161}}{2} \\
&= \frac{169 \pm 119}{2} \\
h^2 &= 144 \text{ or } 25 \\
h &= \pm 12 \text{ or } \pm 5
\end{aligned}
$$

Since the side lengths must be positive values, these values must be 12 cubits and 5 cubits respectively.

Method 3. Trial and error

Students may try a trial and error with different side lengths that result in an area of 60, as laid out in the following table (as well as the opposite order):

$$
\begin{array}{ll}
1 \times 60 = 60,\ d \approx 60.01 & 4 \times 15 = 60,\ d \approx 15.52 \\
2 \times 30 = 60,\ d \approx 30.07 & 5 \times 12 = 60,\ d = 13 \\
3 \times 20 = 60,\ d \approx 20.22 & 6 \times 10 = 60,\ d \approx 11.66
\end{array}
$$

5.6

1.1

2.1

An erect pole of 10 cubits has its base moved 6 cubits. Determine the new height and the distance the top of the pole is lowered.

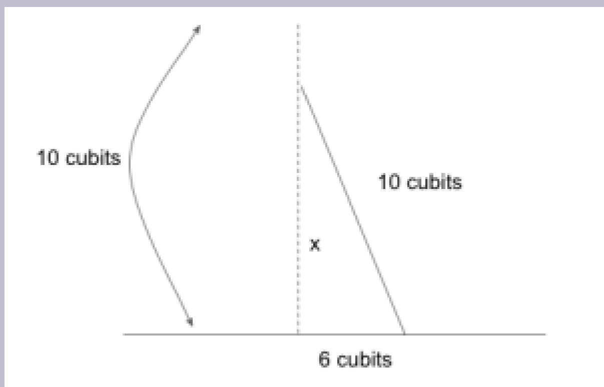
An erect pole, 10 cubits in length, has its base moved outwards, 6 cubits. Determine the new height and the distance the top of the pole has been lowered.

Consider that the original height is the length of the pole, or 10 cubits. With this consideration, we can model the problem:

Animation:

One or more interactive elements has been excluded from this version of the text. You can view them online here: https://ecampusontario.pressbooks.pub/thelivingtreeofmathematicsmathproblems/?p=170#video-170-1

Diagram:



Method 1 – Pythagorean theorem
The hypotenuse (the pole) is known. Therefore:

$$x^2 + 6^2 = 10^2$$

$$x = \sqrt{10^2 - 6^2}$$

$$x = \sqrt{100 - 36}$$

$$x = \sqrt{64}$$

$$x = 8$$

Method 2 – Pythagorean triples
The lengths of the sides form a Pythagorean Triple 6-8-10. The two sides are 6 and 10 (with 10 as the hypotenuse), therefore the missing side is 8 cubits.

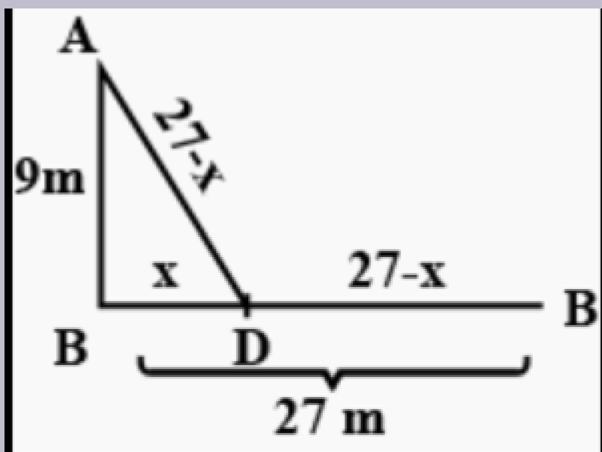
5.7

1.1

2.1

There is a hole at the foot of a pillar 9 hastas high, and a pet peacock standing on top of it. Seeing a snake returning to its hole at a distance from the pillar equal to three times its height, the peacock swoops down upon the snake slantwise. Say quickly, how far from the pole does the meeting of their paths occur?

Let the peacock and snake meet at x m from the pillar as shown in the above figure.



Again we can solve the question by the Pythagorean triples. In this case it is 9, 12, 15 triple. So the peacock and snake are going to meet at a distance of 12 feet from the pillar. Or we can write and solve the quadratic equation.

5.8

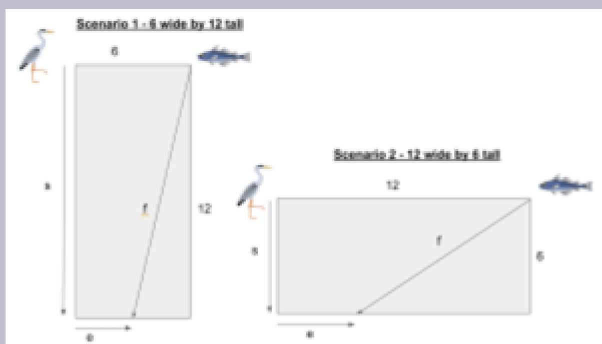
1.1

2.1

A fish is resting at the northeast corner of a rectangular pool. A heron standing at the northwest corner spies the fish. When the fish sees the heron looking at him, he quickly swims towards the south. When he reaches the south side of the pool, he has the unwelcome surprise of meeting the

heron who has calmly walked due south along the side and turned at the southwest corner of the pool and proceeded due east, to arrive simultaneously with the fish on the south side. Given that the pool measures 12 units by 6 units, and that the heron walks as quickly as the fish swims, find the distance the fish swam.

We begin by drawing out a model. There are two possible orientations for the 12×6 pool, one in which the 12-unit length is oriented North-South and the second that is oriented east-west. We will call these Scenario 1 and Scenario 2, respectively. These are indicated below:



In all cases:
f is the distance travelled by the fish and h is the distance travelled by the heron

h can be separated into the southbound portion (labelled s) and the eastbound portion (labelled e). Therefore h = f = s + e.

SCENARIO 1

\begin{array}{l}f^2=12^2+\left(6-e\right)^2\\
h=s+e=12+e\\
Since\ h=f,\ then\ f=h=12+e\\
Substituting:\\
\left(12+e\right)^2=12^2+\left(6-e\right)^2\\
144+24e+e^2=144+36-2e+e^2\\
24e=36-2e\\
26e=36\\
e=\frac{36}{26}=1\ \frac{10}{26}=1\ \frac{5}{13}\\
f=12+e\\
f=12+1\ \frac{5}{13}\\
f=13\frac{5}{13}\end{array}

Therefore, each animal travelled $13\frac{5}{13}$ units

SCENARIO 2
\begin{array}{l}f^2=6^2+\left(12-e\right)^2\\
h=s+e=12+e\\
Since\ h=f,\ then\ f=h=12+e\\
Substituting:\\
\left(12+e\right)^2=6^2+\left(12-e\right)^2\\
144+24e+e^2=36+144-24e+e^2\\

24e=36-24e\\
48e=36\\
e=\frac{36}{48}=\frac{3}{4}\\
h=12+e\\
h=12+\frac{3}{4}\\
h=12\frac{3}{4}\end{array}

Therefore, each animal travelled $12\frac{3}{4}$ units.

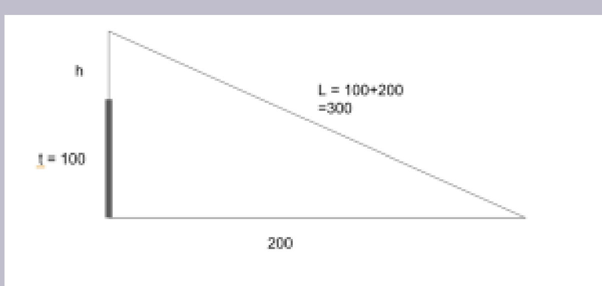
5.9

1.1

2.1

One monkey came down a tree of height 100 and went to a pond a distance of 200. Another monkey, leaping some distance above the tree, went diagonally to the same place. If their total distances traveled are equal, tell me quickly, learned one —if you have a thorough understanding of calculation- how much is the height of the leap?

Let h be the height of the leap above the tree. Since the diagonal length of the leap is equal to the height of the tree plus the ground distance (100+200), which is 300 units. Therefore, the model looks like:



This creates a right triangle with side lengths 200, 300 and h + 100

$$(h + 100)^2 + 200^2 = 300^2$$
$$(h + 100)^2 + 40000 = 90000$$
$$(h + 100)^2 = 50000$$
$$h + 100 = \sqrt{50000}$$
$$h = \sqrt{50000} - 100 \approx 123.6$$

Therefore, the height of the jump is approximately 123.6

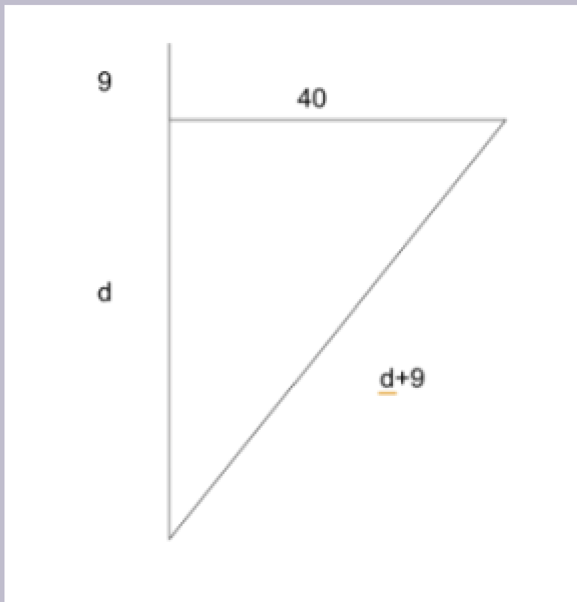
5.10

1.1

2.1

In a certain lake swarming with red geese, the tip of a lotus bud was seen to extend a span [9 inches] above the surface of the water. Forced by the wind, it gradually advanced and was submerged at a distance of 2 cubits [40 inches]. Compute quickly, mathematician, the depth of the pond.

Let d be the depth of the river. Assuming that the lotus is rooted in a consistent place in the riverbed, then the length of the stem is d+9.



$$d^2 + 40^2 = (d+9)^2$$
$$40^2 = (d+9)^2 - d^2$$
$$1600 = d^2 + 18d + 81 - d^2$$
$$1600 = 18d + 81$$
$$1600 - 81 = 18d$$
$$1519 = 18d$$
$$84.3\overline{8} = d$$

5.11

1.1

2.1

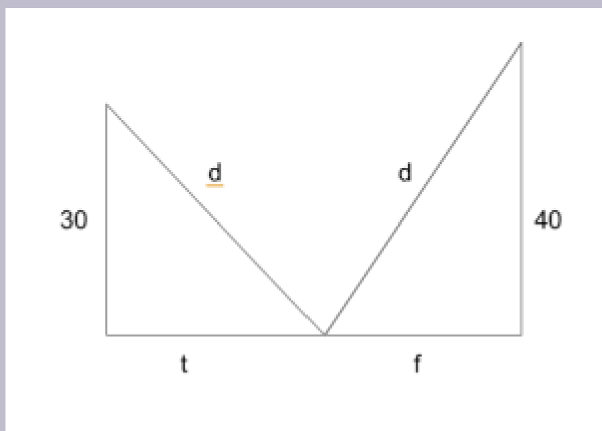
Two towers, the heights of which are 30 paces and 40 paces, have a 50 paces distance. Between

the two towers there is a font where two birds, flying down from the two towers at the same speed will arrive at the same time. What is the distance of the font from the two towers?

Let d be the distance that each bird flies. If the speed is the same, and the time of departure and arrival is the same, then the distance must also be the same.

Let t be the distance of the font from the thirty-pace tower.

Let f be the distance of the font from the forty-pace tower.



Using the Pythagorean Theorem, and the given information that the towers are 50 paces apart:

$$t^2 + 30^2 = d^2$$
$$f^2 + 40^2 = d^2$$
$$t + f = 50$$

$$f = 50 - t$$
$$Substitute:$$
$$t^2 + 30^2 = d^2$$
$$(50 - t)^2 + 40^2 = d^2$$
$$Substituting\ again:$$
$$t^2 + 30^2 = (50 - t)^2 + 40^2$$
$$t^2 + 900 = 2500 - 100t + t^2 + 1600$$
$$900 = 2500 - 100t + 1600$$
$$100t = 2500 + 1600 - 900$$
$$100t = 3200$$
$$t = 32$$

Therefore, the font is 32 paces from the thirty-pace tower and 50-32 or 18 paces from the 40-pace tower.

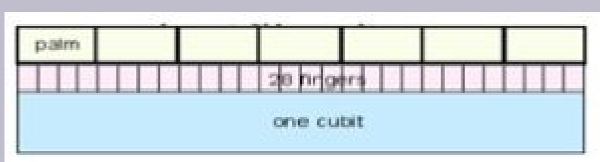
Chapter 6 – Ratio and Proportion

6.1

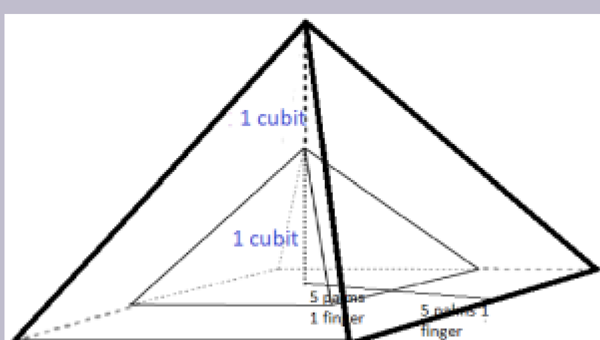
1.1

2.1

The Egyptian pyramids have different slopes. Some pyramids are more high-pitched, others are more low-pitched. Egyptians expressed the steepness of a pyramid by a measure they called "seked". For instance, they say that a pyramid has to have a seqed of 5 palms 1 finger. What does it mean? First, we have to remember that a cubit, the Egyptian unit of length which is approximately 50 cm, is divided in 28 fingers and each 4 fingers make a palm. So 5 palms 1 finger is 5×4+1= 21 fingers. BTW, can you calculate what is 1 finger in cm?



Now for the seqed of 5 palms 1 finger: It means that when you build a pyramid you have to add for each cubit of its height the length of 5 palms 1 finger to each side of its base. if you build a pyramid 1 cubit tall which is 28 fingers the base should be 5 palms 1 finger wide from each side of its centre. A pyramid of 2 cubits high had to be 10 palms 2 fingers wide from each side of its centre.



How to calculate the seqed of a pyramid that is already built? To calculate the seqed of a pyramid you have to divide the half of its base by its height. For example, the pyramid below is 250 cubits high. Its base is 360 cubits long. The seqed is 180 divide by 250 which is 0.72. It means that for each cubit of its height the length of 0.72 is added to both sides of its base. But the ancient Egyptians

didn't have decimals so they expressed 0.72 of cubit as fingers. 0.72×1 cubit = 0.72×28 = 20.16 or approximately 20 fingers or 5 palms (remember 1 palm = 4 fingers) and 16/100 or 4/25 of finger.

6.1 a) If a pyramid is 280 cubits high and the side of its base 360 cubits long, what is its seked ?

We are given: *To calculate the seqed of a pyramid you divide the half of its base by its height.*

Let $S =$ the seked of the pyramid.
Let $B =$ the base of the pyramid in cubits.
Let $H =$ the height of the pyramid in cubits.

$$S = \frac{\frac{1}{2} \cdot B}{H}$$
$$= \frac{\frac{1}{2} \cdot 360}{280}$$
$$= \frac{180}{280}$$
$$= \frac{9}{14}$$
$$\approx 0.64286 \text{ of a cubit}$$

Except the ancient Egyptians didn't have decimals, so they would express it in terms of palms and fingers:

$$\frac{9}{14} \text{ of a cubit}$$
$$= \frac{9}{14} \text{ of a cubit} \cdot 28 \frac{\text{fingers}}{\text{cubit}}$$
$$= 18 \text{ fingers}$$
$$\text{Remembering that there are 4 fingers per palm:}$$
$$= \frac{18 \text{ fingers}}{4 \frac{\text{fingers}}{\text{palm}}}$$
$$= 4\frac{1}{2} \text{ palms}$$
$$= 4 \text{ palms and } 2 \text{ fingers}$$

Therefore, the seked of this pyramid is 4 palms and 2 fingers.

6.1 b) If the seked of a pyramid is 5 palms 1 finger per cubit and the side of its base 12 cubits long, what is its altitude?

Let $S =$ the seked of the pyramid.
Let $B =$ the base of the pyramid in cubits.
Let $H =$ the height of the pyramid in cubits.

$$
\begin{aligned}
S &= 5 \text{ palms } + 1 \text{ finger} \\
&= 5 \text{ palms } \cdot 4 \frac{\text{fingers}}{\text{palm}} + 1 \text{ finger} \\
&= 20 \text{ fingers} + 1 \text{ finger} \\
&= 21 \text{ fingers} \\
&= \frac{21 \text{ fingers}}{28 \frac{\text{fingers}}{\text{cubit}}} \\
&= \frac{3}{4} \text{ of a cubit} \\
&= 0.75 \text{ of a cubit}
\end{aligned}
$$

$$
\begin{aligned}
S &= \frac{\frac{1}{2} \cdot B}{H} \\
\frac{3}{4} &= \frac{\frac{1}{2} \cdot 12}{H} \\
\frac{3}{4} &= \frac{6}{H} \\
\frac{3}{4} \cdot H &= 6 \\
H &= 6 \cdot \frac{4}{3} \\
H &= 8
\end{aligned}
$$

Therefore, the height (or altitude) of this pyramid is 8 cubits.

6.1 c) If the seked of a pyramid is 5 palms 1 finger per cubit and the side of its base 140 cubits, what is its altitude?

Let $S =$ the seked of the pyramid.

Let $B =$ the base of the pyramid in cubits.

Let $H =$ the height of the pyramid in cubits.

$$
\begin{aligned}
S &= 5 \text{ palms } + 1 \text{ finger} \\
&= 5 \text{ palms } \cdot 4 \frac{\text{fingers}}{\text{palm}} + 1 \text{ finger} \\
&= 20 \text{ fingers} + 1 \text{ finger} \\
&= 21 \text{ fingers} \\
&= \frac{21 \text{ fingers}}{28 \frac{\text{fingers}}{\text{cubit}}} \\
&= \frac{3}{4} \text{ of a cubit} \\
&= 0.75 \text{ of a cubit}
\end{aligned}
$$

6.2

1.1

2.1

In ancient Egypt the quality of products made with grain, such as bread and beer, was measured by a unit called a *pefsu*. Pefsu shows how many loaves of bread or jugs of beer were made from 1 hekat, which is 5 litres, of grain. The more beer or bread you made from the same amount of grain the weaker and less tasty was your beer or bread.. For example, if you make 20 loafs of bread from 1 hekat the bread is of 20 pefsu value. But if you make 40 loaves of bread your bread is of 40 pefsu value. So, the higher the pefsu, the less valuable bread or beer was. If you had one loaf of bread of pefsu 20 you could exchange it to two loaves of pefsu 40 or to three loaves of pefsu 60.

6.2 a) 100 loaves of pesu 10 are to be exchanged for a certain number of loaves of pesu 45. What is the number?

We know that the number will be greater than 100 as loaves of a higher pefsu are worth less, and therefore would require more quantity to be of equal value. We will use this observation to help confirm that our response makes sense given the provided context.

Pefsu Value	Number of Loaves
20	1
40	2
60	3

We will refer to ratios in terms of Quantity to Pefsu Value; note that the ratio can be reversed (Pefsu Value:Quantity).

Given the ratio of $1:20$, it is easy to see how we double both sides to get to the equivalent ratio of $2:40$. Likewise, we triple the original ratio of $1:20$ to get the ratio of $3:60$.

Given the ratio of $100:10$, we can find the ratio $x:45$ – where x is the number of loaves of pefsu 45 that can be exchanged for 100 loaves of pefsu 10.

To go from a pefsu value of 10 to 45, we multiply 10 by a factor of 4.5. Therefore, to go from a quantity of 100 to x, we multiply 100 by the same factor of 4.5 and we get $x = 450$.

Therefore, we can exchange 100 loaves of pefsu 10 for 450 loaves of pefsu 45.

6.2 b) 1000 loaves of pefsu 5 are to be exchanged, a half for loaves of pefsu 10, and a half for loaves of pefsu 20. How many of each will there be?

We are given the ratio of $1000:5$ representing 1000 loaves of pefsu 5.

Half of the 1000 loaves will be exchanged for loaves of pefsu 10, and the other half for loaves of pefsu 20. Half of 1000 loaves is 500 loaves.

Let $y =$ the number of loaves of pefsu 10 from exchanging 500 loaves of pefsu 5.
Let $z =$ the number of loaves of pefsu 20 from exchanging 500 loaves of pefsu 5.

We have (i) $500:5$ to be exchanged for $y:10$
and (ii) $500:5$ is to be exchanged for $z:20$

(i) $500:5 = y:10$

To go from a pefsu value of 5 to 10, we multiply 5 by a factor of 2. Therefore, to go from a quantity of 500 to y, we multiply 500 by the same factor of 2.

$$500 \cdot 2 = y$$
$$500 \cdot 2 = 1000$$
$$\text{Therefore, } 500:5 = 1000:10$$

Therefore, 500 loaves of pefsu 5 can be exchanged for 1000 loaves of pefsu 10.

(ii) $500:5 = z:20$

To go from a pefsu value of 5 to 20, we multiply 5 by 4. Therefore, to go from 500 to z, we multiply 500 by the same factor: 4.

$$500 \cdot 4 = z$$

$$500 \cdot 4 = 2000$$

$$\text{Therefore, } 500 : 5 = 2000 : 20$$

Therefore, 500 loaves of pefsu 5 can be exchanged for 2000 loaves of pefsu 20.

6.2 c) 1000 loaves of pefsu 10 are to be exchanged for a number of loaves of pefsu 20 and the same number of pefsu 30. How many of each kind will there be?

We want the same number of loaves of pefsu 20 and pefsu 30.

Let $z =$ the number of loaves of each pefsu (20 and 30) being exchanged for a total of 1000 loaves of pefsu 10.

$$1000 : 10 = a : 20 + a : 30$$

We can rewrite the ratios as fractions, and substitute the arrow with an equal sign.

$$\frac{1000}{10} = \frac{a}{20} + \frac{a}{30}$$

$$100 = \frac{30}{30} \cdot \frac{a}{20} + \frac{20}{20} \cdot \frac{a}{30}$$

$$100 = \frac{30 \cdot a}{30 \cdot 20} + \frac{20 \cdot a}{20 \cdot 30}$$

$$100 = \frac{30a + 20a}{20 \cdot 30}$$

$$100 = \frac{50a}{600}$$

$$100 = \frac{5a}{60}$$

$$100 = \frac{a}{12}$$

$$100 \cdot 12 = a$$

$$1200 = a$$

$$\text{Therefore, } 1000 : 10 = 1200 : 20 + 1200 : 30$$

Therefore we will be exchanging 1000 loaves of pefsu 10 for 1200 loaves of pefsu 20 and 1200 loaves of pefsu 30.

6.2 d) Suppose it is said to thee, 100 loaves of pefsu 10 are to be exchanged for a quantity of beer of pefsu 2. How many des of beer will there be?

Note: there will be less than 100 beers of pefsu 2 as the higher quality beer will be more valuable than the loaves of bread and therefore will require less quantity to be of equal value.

Let $b =$ the number of beers of pefsu 2 being exchanged for 100 loaves of pefsu 10.

$$100 : 10 = b : 2$$
$$\frac{100}{10} = \frac{b}{2}$$
$$10 = \frac{b}{2}$$
$$20 = b$$
$$\text{Therefore, } 100 : 10 = 20 : 2$$

Therefore 100 loaves of pefsu 10 can be exchanged for 20 des of beer of pefsu 2.

6.2 e) 3 $\frac{1}{2}$ hekat of grain is made into 80 loaves of bread. Let me know the amount of grain in each loaf and what is the pefsu.

We are given the information that 80 loaves of bread are made from 3 $\frac{1}{2}$ hekat of grain, and we want to determine the equivalent proportion of hekat of grain per loaf of bread.

Let $x =$ the amount of grain in hekat in each loaf of bread.

$$80 : 3\tfrac{1}{2} = 1 : x$$
$$\frac{80}{3\frac{1}{2}} = \frac{1}{x}$$
$$x = \frac{3\frac{1}{2}}{80}$$
$$x \approx 0.04375$$
$$\text{Therefore, } 80 : 3\tfrac{1}{2} \approx 1 : 0.04375$$

Therefore 0.04375 hekat of grain is used per loaf of bread.

To determine the pefsu value, we again refer back to the information provided: we know that the pefsu value is that amount of beer or loaves of bread made from 1 hekat of grain. In this case, we are talking about loaves of bread.

We know 3 $\frac{1}{2}$ hekat of grain was used to make 80 loaves. We need to know the number of loaves made per hekat of grain used.

Let $y =$ the number of loaves made per hekat of grain used.

$$80 : 3\tfrac{1}{2} = y : 1$$
$$\frac{80}{3\frac{1}{2}} = \frac{y}{1}$$
$$22\tfrac{6}{7} = y$$
$$\text{Therefore, } 80 : 3\tfrac{1}{2} = 22\tfrac{6}{7} : 1$$

Therefore, 22 $\frac{6}{7}$ loaves were made per hekat of grain used.

As the pefsu value is the amount of loaves made per hekat of grain used, we have determined that the pefsu value is 22 $\frac{6}{7}$.

6.3

1.1

2.1

If it is said to you, "Have sailcloth made for the ships," and it is further said, "Allow 1000 cloth cubits for one sail and have the ratio of the height of the sail to its width be 1 to 1 1/2," what is the height of the sail?

The area of a triangle is:

$A = \frac{b \cdot h}{2}$

And the ratio of the height to its width (base) be $1 : 1\frac{1}{2}$

$h : b = 1 : 1\frac{1}{2}$

$\frac{b}{h} = \frac{1\frac{1}{2}}{1}$

$\frac{b}{h} = 1\frac{1}{2}$

$b = 1\frac{1}{2} \cdot h$

$A = 1000$ and $A = \frac{b \cdot h}{2}$

$1000 = \frac{(1\frac{1}{2} \cdot h) \cdot h}{2}$

$1000 = \frac{\frac{3}{2} \cdot h^2}{2}$

$1000 = \frac{3}{4} \cdot h^2$

$1000 \cdot \frac{4}{3} = h^2$

$\frac{4000}{3} = h^2$

$\sqrt{\frac{4000}{3}} = h$

$20\sqrt{\frac{10}{3}} = h$

$h \approx 36.51484$

Therefore, the height of the triangular sailcloth is approximately 36.51 cubits.

6.4

1.1

2.1

A cobbler can cut leather for ten pairs of shoes in one day. He can finish five pairs of shoes in one day. How many pairs of shoes can he both cut and finish in one day?

Let $C =$ the time it takes a cobbler to cut a pair of shoes.

Let $F =$ the time it takes a cobbler to finish a pair of shoes.

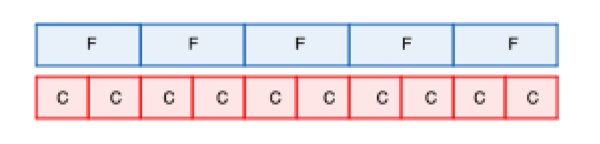
$$C : F = 10 : 5$$

$$10 : 5 = 2 : 1$$

$$C : F = 2 : 1$$

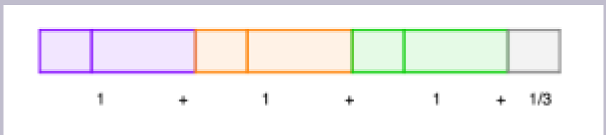
The cobbler can cut twice as many pairs of shoes as they can finish in the same time span.

Using Bar Models, we can express the work that would fill the cobbler's day the follow ways:



We can assume that the cobbler would need to first cut and then finish a shoe, repeatedly:

We can count the number of pairs of shoes that are both cut and finished within the time span of a day:



Therefore, in a day, the cobbler can both cut and finish $3\frac{1}{3}$ pairs of shoes.

Algebraically, we can express a day's worth of work in terms of cutting and finishing a pair of shoes:

Let $D =$ a day's worth of work.

Let $p =$ the pairs of shoes a cobbler can both cut and finish in a day.

Three (3) ways to express a day's worth of work are:

$$\text{(i) } D = 10 \cdot C$$

$$\text{(ii) } D = 5 \cdot F$$

$$\text{(iii) } D = p \cdot (C + F)$$

Using (i) and (ii):

$$10 \cdot C = 5 \cdot F$$

$$2 \cdot C = F$$

Substituting into (iii):

$$10 \cdot C = p \cdot (C + F)$$
$$10 \cdot C = p \cdot (C + 2 \cdot C)$$
$$10 \cdot C = p \cdot (3 \cdot C)$$
$$10 \cdot C = p \cdot 3 \cdot C$$
$$\text{Divide both sides by C:}$$
$$10 = 3 \cdot p$$
$$\frac{10}{3} = p$$
$$p = 3\frac{1}{3}$$

Therefore, the cobbler can cut and finish 3 1/3 pairs of shoes per day.

6.5

1.1

2.1

A fox, a raccoon, and a hound pass through customs and together pay 111 coins. The hound says to the raccoon, and the raccoon says to the fox, "Since your fur is worth twice as much as mine, then the tax you pay should be twice as much!" How much should each pay?

Let $f =$ the number of coins that the fox should pay.
Let $r =$ the number of coins that the raccoon should pay.
Let $h =$ the number of coins that the hound should pay.

We are given:
(i) $f + r + h = 111$ The total number of coins being paid by the group is 111.
(ii) $2h = r$ The raccoon should pay twice as much as the hound.
(iii) $2r = f$ The fox should pay twice as much as the raccoon.

Substituting (iii) into (i) and (ii):

$$f + r + h = 111$$
$$2r + r + h = 111$$
$$2(2h) + 2h + h = 111$$
$$4h + 2h + h = 111$$
$$7h = 111$$
$$h = \frac{111}{7}$$
$$r = 2h$$
$$= 2 \cdot \frac{111}{7}$$
$$= \frac{222}{7}$$
$$f = 2r$$
$$= 2 \cdot \frac{222}{7}$$
$$= \frac{444}{7}$$

Therefore, the fox should pay $\frac{444}{7}$ coins, the raccoon should pay $\frac{222}{7}$ coins, and the raccoon should pay $\frac{111}{7}$ coins.

6.6

1.1

2.1

A cow, a horse, and a goat were in a wheat field and ate some stalks of wheat. Damages of five baskets of grain were asked by the wheat field's owner. If the goat ate one-half the number of stalks as the horse, and the horse ate one-half of what was eaten by the cow, how much should be paid by the owners of the goat, horse, and cow, respectively?

Let $C =$ the amount of wheat stalks eaten by the cow.

Let $H =$ the amount of wheat stalks eaten by the horse.
Let $G =$ the amount of wheat stalks eaten by the goat.

Given:

$$\text{(i) } G = \frac{1}{2} \cdot H$$

$$\text{(ii) } H = \frac{1}{2} \cdot C$$

$$\text{(iii) } G + H + C = 5$$

We can use equations (i) and (ii) to substitute the variables in (iii) to be one variable, and thus solvable.

$G = \frac{1}{2} \cdot H$

$G = \frac{1}{2} \cdot (\frac{1}{2} \cdot C)$

$G = \frac{1}{4} \cdot C$

$G + H + C = 5$

$(\frac{1}{4} \cdot C) + (\frac{1}{2} \cdot C) + C = 5$

$\frac{1}{4} \cdot C + \frac{1}{2} \cdot C + C = 5$

$\frac{7}{4} \cdot C = 5$

$C = \frac{5}{\frac{7}{4}}$

$C = \frac{5 \cdot 4}{7}$

$C = \frac{20}{7}$

$C = 2\frac{6}{7}$

$H = \frac{1}{2} \cdot C$

$H = \frac{1}{2} \cdot \frac{20}{7}$

$H = \frac{20}{14}$

$H = \frac{10}{7}$

$H = 1\frac{3}{7}$

$G = \frac{1}{4} \cdot C$

$G = \frac{1}{4} \cdot \frac{20}{7}$

$G = \frac{20}{28}$

$G = \frac{5}{7}$

Therefore the cow's owner should pay 2 $\frac{6}{7}$ baskets, the horse's owner should pay 1 $\frac{3}{7}$ baskets, and the goat's owner should pay $\frac{5}{7}$ baskets.

6.7

1.1

2.1

If in one day, a person can make 30 arrows or fletch (put the feathers on) 20 arrows, how many arrows can this person both make and fletch in a day?

Let $D =$ the time available for this person to work in a day.

Let $A =$ the time it takes for this person to make one arrow.

Let $F =$ the time it takes for this person to fletch one arrow.

Let $p =$ the number of arrows this person can make and fletch in a day.

Given:

$$(\text{i})\ D = 30 \cdot A$$

$$(\text{ii})\ D = 20 \cdot F$$

$$(\text{iii})\ D = p(A + F)$$

Equate (i) and (ii):

$$30 \cdot A = 20 \cdot F$$

$$A = \frac{20 \cdot F}{30}$$

$$A = \frac{20}{30} \cdot F$$

$$A = \frac{2}{3} \cdot F$$

Substitute into (iii):

$$D = p(A + F)$$

$$20 \cdot F = p(\tfrac{2}{3} \cdot F + F)$$

$$20 \cdot F = p(\tfrac{5}{3} \cdot F)$$

$$20 \cdot F = p \cdot \tfrac{5}{3} \cdot F$$

Divide both sides by F

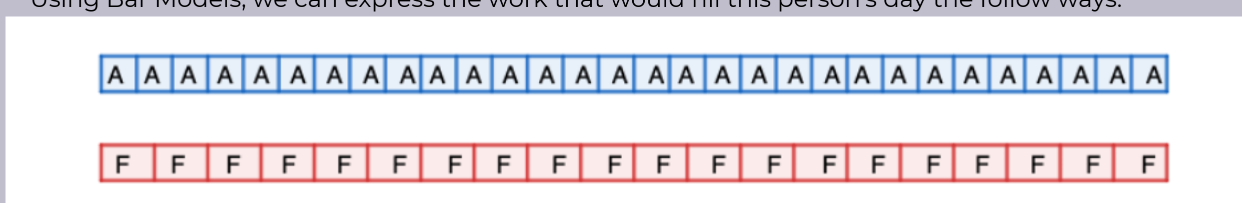
$$20 = \tfrac{5}{3} \cdot p$$

$$\tfrac{3}{5} \cdot 20 = p$$

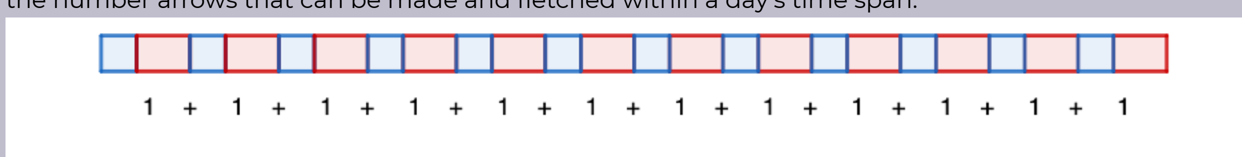
$$12 = p$$

Therefore, this person can make and fletch 12 arrows in a day.

Using Bar Models, we can express the work that would fill this person's day the follow ways:



We can assume that person would first make an arrow and then fletch it, repeatedly, and then count the number arrows that can be made and fletched within a day's time span.



And we get the same answer: this person can make and fletch 12 arrows in a day.

6.8

1.1

2.1

In general, a fair exchange is 50 bushels of millet for 27 bushels of rice. Here is 21 bushels of millet. How many bushels of rice will we obtain in exchange?

The value of 50 bushels of millet is equal to 27 bushels of rice.

We want to know the value of 21 bushels of millet in terms of bushels of rice.

As there is to be a consistent exchange ratio between millet and rice, we know that the ratio of $50:27$ will have the same proportionality of $21:x$, where x is the value of 21 bushels of millet in terms of bushels of rice, and is what is to be solved.

$$50:27=21:x$$
$$\frac{50}{27}=\frac{21}{x}$$
$$x\cdot\frac{50}{27}=21$$
$$x=21\cdot\frac{27}{50}$$
$$x=\frac{567}{50}$$
$$x=11\frac{17}{50}$$
$$\text{Therefore, the ratio of 50:27 is proportional to the ratio of 21:11}\frac{17}{50}$$

Therefore, 21 bushels of millet can be exchanged for 11 $\frac{17}{50}$ bushels of rice.

6.9

1.1

2.1

Four counties are required to furnish wagons to transport grain to a depot. There are 10,000 families in the first county, 9500 families in the second county; 12,350 families in the third county; and 12,200 families in the last county. The total number of wagons required is 1000. How many wagons are to be provided by each county according to the size of the population?

Let $A =$ the number of wagons to be provided by the first county.

Let $B =$ the number of wagons to be provided by the second county.

Let $C =$ the number of wagons to be provided by the third county.

Let $D =$ the number of wagons to be provided by the fourth county.

$$A + B + C + D = 1000$$

$$\text{The total number of people living across all 4 counties are:}$$
$$10,000 + 9,500 + 12,350 + 12,200 = 44,050$$

County	Number of Families	Proportion of Total Population
1	$10,000$	$\frac{10{,}000}{44{,}050} = \frac{200}{881}$
2	9500	$\frac{9500}{44{,}050} = \frac{190}{881}$
3	$12,350$	$\frac{12{,}350}{44{,}050} = \frac{247}{881}$
4	$12,200$	$\frac{12{,}200}{44{,}050} = \frac{244}{881}$

Each County's Proportion of Total Population can be used as the same factor to determine the proportion of wagons each county is to provide:

$$A = \frac{200}{881} \cdot 1000$$
$$A \approx 227$$
$$B = \frac{190}{881} \cdot 1000$$
$$B \approx 216$$
$$C = \frac{247}{881} \cdot 1000$$
$$C \approx 280$$
$$D = \frac{244}{881} \cdot 1000$$
$$D \approx 277$$

Therefore, the first county should supply 227 wagons, the second county should supply 216 wagons, the third county should supply 280 wagons, and the fourth county should supply 277 wagons for a total of 1000 wagons, according to the size of their population.

6.10

1.1

2.1

A man had four creditors. To the first he owed 624 ducats; to the second, 546; to the third, 492; and to the fourth, 368. It happened that the man defaulted and escaped, and the creditors found that his goods amounted to 830 ducats in all. In what ratio should they divide this, and what will be the share of each?

Let $A =$ ducats to be shared to the first creditor.
Let $B =$ ducats to be shared to the second creditor.
Let $C =$ ducats to be shared to the third creditor.
Let $D =$ ducats to be shared to the fourth creditor.

$$A + B + C + D = 830$$

The total amount owed across all 4 creditors are:
$624 + 546 + 492 + 368 = 2030$

Creditor	Amount Owed	Proportion of Total Owed
1	624	$\frac{624}{2030} = \frac{312}{1015}$
2	546	$\frac{546}{2030} = \frac{39}{145}$
3	492	$\frac{492}{2030} = \frac{246}{1015}$
4	368	$\frac{368}{2030} = \frac{184}{1015}$

Each creditor's Proportion of Total Owed can be used as the same factor to determine the proportion of ducats each creditor should collect:

$$A = \frac{312}{1015} \cdot 830$$
$$A \approx 255.133$$
$$B = \frac{39}{145} \cdot 830$$
$$B \approx 223.241$$
$$C = \frac{246}{1015} \cdot 830$$
$$C \approx 201.163$$
$$D = \frac{184}{1015} \cdot 830$$
$$D \approx 150.463$$

If we were to round all the values, the total value would sum to 829 ducats. Therefore, we look at the decimals and see that the fourth creditor is closest to the next largest whole number and should be awarded the remaining ducat.

Therefore, the first creditor should be given 255 ducats, the second creditor should be given 223 ducats, the third creditor should be given 201 ducats, and the fourth creditor should be given 151 ducats for a total of 830 ducats, as proportional to the amount owing to each creditor.

6.11

7.1

2.1

There were two men, of whom the first had three small loaves of bread and the other two. They walked to a spring, where they sat down and ate; and a soldier joined them and shared the meal, each of the three men eating the same amount. When all the bread was eaten, the soldier departed, leaving 5 bezants to pay for his meal. The first man accepted 3 of these bezants, since he had three loaves; the other took the remaining 2 bezants for his two loaves. Was this division fair?

The three men ate an equal share of 5 loaves of bread.

They each ate $\frac{5}{3}$ loaves.

5 loaves = $\frac{15}{3}$ loaves.

The first man brought 3 loaves.

3 loaves = $\frac{9}{3}$ loaves.

The second man brought 2 loaves.

2 loaves = $\frac{6}{3}$ loaves.

So the first man brought enough for himself, and ($\frac{9}{3}$ – $\frac{5}{3}$ = $\frac{4}{3}$) nearly enough for Ali.

The 2nd man brought enough for himself, and ($\frac{6}{3}$ – $\frac{5}{3}$ = $\frac{1}{3}$) $\frac{1}{3}$ to Ali.

So first man gave up $\frac{4}{3}$

Second man gave up $\frac{1}{3}$

Of the bread given to Ali, the first man supplied $\frac{\frac{4}{3}}{\frac{5}{3}} = \frac{4}{5}$

Of the bread given to Ali, the second man supplied $\frac{\frac{1}{3}}{\frac{5}{3}} = \frac{1}{5}$

Therefore, this division of bezants is not fair. Instead, of the 5 bezants, 4 should go to the first man and 1 should go to the second man.

6.12

1.1

2.1

Suppose I tell you that I bought saffron in Siena for 18 lire a pound and took it to Venice, where I found that 10 ounces Siena weight are equivalent to 12 ounces in Venice, and 10 lire in Siena money are equal to 8 lire Venetian. I sell the saffron for 14 lire Venetian money a pound. I ask how much I gained in percent.

Note: there are 16 ounces (oz) in a pound (lb).

10 oz Siena weight : 12 oz Venice weight
10 lire Siena money : 8 lire Venetian money

Saffron sells for 18 lire/lb in Siena
= 18 lire for 16 oz (Siena weight)
Saffron sells for 14 lire/lb in Venice
= 14 lire for 16 oz (Venice weight)

Converting 1 lb Siena weight to Venice weight:

Recall: 10 oz Siena weight = 12 oz Venice weight

1 oz Siena weight = $\frac{12}{10}$ oz Venice weight

Therefore, to go from Siena weight to Venice weight, we multiply by a factor of $\frac{12}{10}$ = 1.2

1 lb (16 oz) in Siena weight would be:
16 oz in Siena weight $\cdot 1.2 = 19.2$ oz in Venice weight

Therefore, 18 lire Siena money buys 19.2 ounces Venice weight.

Converting 18 lire Siena money to Venetian money:
10 lire Siena money = 8 lire Venetian money
1 lire Siena money = $\frac{8}{10}$ lire Venetian money

Therefore, to go from Siena money to Venetian money, we multiple by a factor of $\frac{8}{10}$ =0.8
18 lire Siena $\cdot 0.8 = 14.4$ lire Venetian

Therefore, 14.4 lire Venetian money buys 19.2 ounces Venice weight.

Determining the equivalent exchange rate for 1 lb (16 oz) Venice weight:
Given: 14.4 lire per 19.2 oz Venice weight
Want: how much lire per 16 oz Venice weight?

Let x = the amount of lire for 16 oz Venice weight

$$\frac{14.4}{19.2} = \frac{x}{16}$$

$$16 \cdot \frac{14.4}{19.2} = x$$

$$12 = x$$

Therefore, 12 lire Venetian money buys 1 lb (16 oz) Venice weight.

Calculate the percentage gain:
The saffron was sold for 14 lire Venetian money per pound (16 oz Venice weight).

Therefore, the additional profit was (14 – 12 =) 2 lire Venetian money per lb.

Percentage gain:

$$\frac{2}{12} \cdot 100\%$$
$$= \frac{1}{6} \cdot 100\%$$
$$= 0.16667 \cdot 100\%$$
$$\approx 16.667\%$$

Therefore, you gained approximately 16.67% on the saffron transaction.

6.13

1.1

2.1

Suppose you have two kinds of wine. A measure of the poorer sort is worth 6 denarii. One of the better sort is worth 13 denarii. I wish to have a measure of wine worth 8 denarii. How much of each wine should I put in the mixture?

Let assume we make 1 litre of 8 denari wine Let x be a part of 6 denari wine in a mixture. Then the part of 8 denari wine in a mixture is be 1-x.

$$6x + 13(1 - x) = 8$$
$$x = \frac{5}{8}$$

You should put $\frac{5}{7}$ part a of the cheap wine and $\frac{2}{7}$ part of the better sort.

6.14

1.1

2.1

A dying man makes a will: if the wife gives birth to a male child, she will have one third of what he leaves, his son the other two parts; and if the wife gives birth to a daughter, the wife will receive two thirds and the girl one third. It now happens that the woman bears twins, one male and one female. The question is how much comes to the woman, the male and female child when the legacy values 70 lire.

Let assume the wife gets x lire. Then the son should get twice the amount, 2x. And the daughter would get a half of what the son gets. We can write an equation

$$x + 2x + 0.5x = 70$$

$$x = 20$$

Therefore, the wife gets 20 lires, the son gets 40 lires and the daughter gets 10 lires.

6.15

1.1

2.1

The head of a household had 20 servants. He ordered them to be given 20 measures of corn as follows. The men must receive three measures, the women must receive two measures, and the children half a measure each. How many men, women and children servants are there in the household?

Start with random number of man. Let say there are 2 men in the household. Then there will be 18 women and children (remember there were many children in families back in the day). And there will be 14 measures of corn left for them. Let x be number of the children. Then there will be 18 -x number of the women. We can now write an equation

$$0.5x + 2(18 - x) = 14$$
$$1.5x = 22$$

The value of x is decimal. Let's then try 1 men.

$$0.5x + 2(19 - x) = 17$$
$$x = 14$$

Therefore there were 14 kids, 5 women and 1 men in the household.

6.16

1.1

2.1

A head of a household had 100 servants. He ordered that they be given 100 measures of corn as follows. The men should receive three measures, the women should receive two measures, and the children should receive half a measure each. How many men, women, and children servants are there in the household?

Start with random number of man. Let say there are 10 men in the household. Then there will be 90 women and children (remember there were many children in families back in the day). And there will be 70 measures of corn left for them. Let x be number of the children. Then there will be 90-x number of the women. We can now write an equation

$$0.5x + 2(90 - x) = 70$$
$$1.5x = 110$$

The value of x is decimal. Let's then try 11 men.

$$0.5x + 2(89 - x) = 67$$
$$x = 74$$

Therefore there were 74 kids, 15 women and 11 men in the household.

6.17

1.1

2.1

A certain bishop ordered 12 loaves of bread to be divided amongst the clergy. He stipulated that each priest should receive two loaves, each deacon should receive half a loaf and each reader should receive a quarter of a loaf. It turned out that the number of clerics and the number of loaves were the same. How many priests, deacons and readers must there have been?

The problem yields to guess and check strategy. But first we have to observe that since the number of clerics and the number of loaves were the same that there should be an even number of readers since their share of bread is ¼ of a loaf.

If we start with 2 readers very soon we will find that 5 priests, 7 deacons and 2 readers satisfy the requirements of the problem:

$5+7+2=14$ and $5\cdot 2+7\cdot\frac{1}{2}+2\cdot\frac{1}{4}=14$

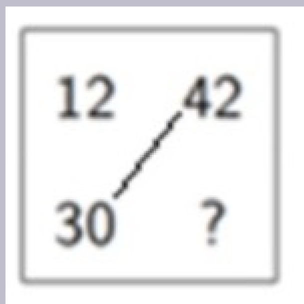
6.18

1.1

2.1

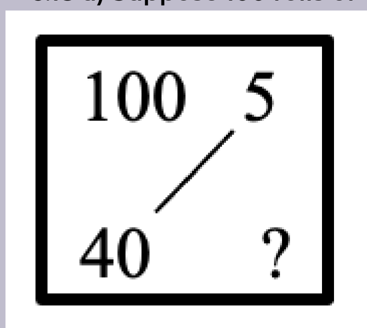
Most of the ratio or proportion problems can be solved by the method called "the rule of three".The rule of three came to us from medieval India. It was transported to other countries by Arabian voyagers, and, thanks to the Islamic invasion of Spain, the rule made it to Europe. In the 13th century the Italian merchant and mathematician Leonardo de Fibonacci, who frequently traveled to North Africa, learned about the the rule of three and made it popular among European merchants and bankers. Let's look at the example Fibonacci gives:

If 12 bottles of wine cost 30 denari, what will 42 bottles of wine cost? Fibonacci says, make a square as follows, then multiply the two numbers that lie in the diagonal and then divide by the remaining number: \[(30\times 42)\div 12\]. The result is the price to be paid, 105 denari.



Ratio Sqaure

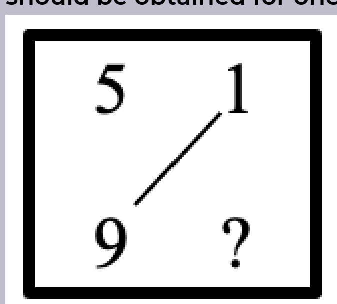
6.18 a) Suppose 100 rolls of cottoncost 40 lira, how much would five rolls cost?



$40 \cdot \frac{5}{100} = 2$

Five rolls cost 2 liras

6.18 b) I have bought five palas of sandalwood for nine rupakas. How much sandalwood, then, should be obtained for one rupaka?



$9 \cdot \frac{1}{5} = frac95or1\frac{4}{5}$

$1\frac{4}{5}$ palas of sandalwood can be obtained for one rupaka

6.19

1.1

2.1

If 12 horses can plow 96 acres in 6 days, how many horses will plow 64 acres in 8 days?

First let’s find how much 1 horse can plow in a day.

12 horses can plow $\frac{96}{6} = 16$ acres a day

1 horse can plow $\frac{96}{12} = 1\frac{1}{3}$ acres a day

In 8 days one horse can plow $\frac{16}{12} \cdot 8 = 10\frac{2}{3}$ acres a day

If we divide 64 by $\frac{32}{3}$ we find that 6 horses are needed plow 64 acres in 8 days.

TEACHER SUPPORTS

Curriculum Connections

Each state in the USA and province in Canada has its own mathematics curriculum. However, the core of the North American curricula is the same despite the local differences. We suggest the connections to the Ontario Mathematics curriculum since it is our hometown province. But we emphasize the fact that the Ontario Mathematics curriculum is consistent with the Content Core Standards.

The main intended Grade band is 7-9. However, you can use the book in other Grades depending on the needs and knowledge base of your students.

Overarching Curriculum Expectations

Ancient and medieval mathematics were mostly concerned with practical applications of mathematical knowledge. The problems from old texts were about practical matters, such as areas of fields of various shapes, exchange of commodities at different rates, pricing, volumes of granaries and siege ramps of various shapes and many other real world activities. By solving these problems modern students can see mathematics as an integral and utmost necessary part of people's lives. Thus, the following expectations are overarching expectations for almost all of the problems included in the book.

Grade 9

- make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics

Grade 7 and 8

- use knowledge of numbers and operations to solve mathematical problems encountered in everyday lifeSpecific Curriculum Expectations

Specific Curriculum Expectations

Grade 9

1.1
Make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics

Number

- *research a number concept to tell a story about its development and use in a specific culture, and describe its relevance in a current context*
    - Problems 1.1, 1.2, 1.5, 1.18
- *analyse, through the use of patterning, the relationship between the sign and size of an exponent and the value of a power, and use this relationship to … evaluate powers*
    - Problems 1.6, 1.12, 1.16
- *apply an understanding of rational numbers … to solve problems*
    - Problems 1.3, 1.4, 1.7, 1.8, 1.9, 1.10, 1.11, 1.13, 1.14, 1.15, 1.17, 1.20
- *pose and solve problems involving rates, percent, and proportions in various contexts, including contexts connected to real-life applications of data, measurement, geometry, linear relations, and financial literacy*
    - Problems: 6.1-6.20
- *apply an understanding of unit fractions and their relationship to other fractional amounts, in various contexts, including the use of measuring tools*
    - Problems: 4.1, 4.8

Algebra

- create and solve equations for various contexts, and verify their solutions
    - Problems: 4.2-4.7, 4.12, 4.13, 4.17, 4.22, 4.21

Geometry and Measurement

- *research a geometric concept or a measurement system to tell a story about its development and use in a specific culture or community, and describe its relevance in connection to careers and to other disciplines*
    - Problems: 2.1, 2.2, 2.3, 2.4, 3.8, 3.9
- *solve problems involving the side-length relationship for right triangles in real-life situations, including problems that involve composite shapes*
    - Problems: 5.1 – 5.11

- *solve problems involving different units within a measurement system and between measurement systems, including those from various cultures or communities, using various representations and technology, when appropriate*
  - Problems: 2.4, 2.5, 2.6, 2.7
- *create and analyse designs involving geometric relationships and circle and triangle properties, using various tools*
  - Problems: 2.8
- *solve problems involving different units within a measurement system and between measurement systems, including those from various cultures or communities*
  - Problems: 3.1 -3.6, 3.10 – 3.18, 6.13 – 6.16

Grade 8

1.1

Number

- *use knowledge of numbers and operations to solve mathematical problems encountered in everyday life*
- *use the properties and order of operations, and the relationships between operations, to solve problems involving rational numbers, ratios, rates, and percents, including those requiring multiple steps or multiple operations*
  - Problems 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 1.10, 1.11, 1.13, 1.14, 1.15, 1.17, 1.20
- *add and subtract fractions, using appropriate strategies, in various contexts*
  - Problems: 4.1, 4.8
- *multiply and divide fractions by fractions, as well as by whole numbers and mixed numbers, in various contexts*
  - Problems: 1.17, 4.9, 4.10, 4.14, 4.15, 4.16, 4.18, 4.19

- *compare proportional situations and determine unknown values in proportional situations, and apply proportional reasoning to solve problems in various contexts*
    - Problems: 6.1- 6.20

Spatial Sense (Geometry and Measurement)

- *solve problems involving the perimeter, circumference, area, volume, and surface areaof composite two-dimensional shapes and three-dimensional objects, using appropriate formulas*
    - Problems: 2.1-2.8, 3.1 – 3.7, 3.10 – 3.16, 3.17, 3.18
- *describe the Pythagorean relationship using various geometric models, and apply the theorem to solve problems involving an unknown side length for a given right triangle*
    - Problems: 5.1 – 5.11

Algebra

- *create and describe patterns to illustrate relationships among rational numbers*
    - Problems: 1.1
- *demonstrate an understanding of variables, expressions, equations, and inequalities, and apply this understanding in various contexts*
    - Problems: 1.19, 6.15, 6.16
- *Mathematical Modelling: apply the process of mathematical modelling to represent, analyse, make predictions, and provide insight into real-life situations*
    - Problems: 3.12, 3.15

Grade 7

1.1

Number

- *use the properties and order of operations, and the relationships between operations, to solve problems involving whole numbers, decimal numbers, fractions, ratios, rates, and percents, including those requiring multiple steps or multiple operations*
    - Problems 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 1.10, 1.11, 1.13, 1.14, 1.15, 1.17, 1.20
- *add and subtract fractions, including by creating equivalent fractions, in various contexts*
    - Problems: 4.1, 4.8
- *multiply and divide fractions by fractions, using tools in various contexts*
    - Problems: 1.17, 4.9, 4.10, 4.14, 4.15, 4.16, 4.18, 4.19
- *identify proportional and non-proportional situations and apply proportional reasoning to solve problems*
    - Problems: 4.1, 4.8

Spatial Sense (Geometry and Measurement)

- *use the relationships between the radius, diameter, and circumference of a circle to explain the formula for finding the circumference*
    - Problems: 2.1, 2.2, 2.3
- *use the relationships between the radius, diameter, and circumference of a circle … to solve related problems*
    - Problems: 2.6
- *show the relationships between the radius, diameter, and area of a circle, and use these relationships to explain the formula for measuring the area of a circle and to solve related problems*
    - Problems: 2.4, 2.5
- *solve problems involving perimeter, area, and volume that require converting from one metric unitof measurement to another*
    - Problems: 3.1 – 3.6, 3.10, 3.11, 3.13, 3.15, 3.16

Algebra

- *identify, describe, extend, create, and make predictions about a variety of patterns, including those found in real-life contexts*
    - Problems: 1.1
- *solve equations that involve multiple terms, whole numbers, and decimal numbers in various contexts, and verify solutions*
    - Problems: 6.15, 6.16

LESSON PLANS

Lesson Plan - Babylonian Numeral System

Curriculum Expectations

Grade 9

Development and Use of Numbers
B1.1 research a number concept to tell a story about its development and use in a specific culture, and describe its relevance in a current context

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Learning Goals of the Lesson

Students will

- Get familiar with different ways people record numbers
- Learn how to write numbers in the Babylonian numeral systems

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Getting Started

- Open Chapter 1 – Numbers and project it on the big screen.
- Explain to students what “numeral system” means by reading, *Number Systems* paragraph
- Tell students that today they will learn to write numbers as Babylonians did
- Find Babylon on the Map and Timeline
- Let students listen to *Why and how did humans start writing?*

Working on It

- Assign 1.2
- Let students work in pairs or groups.
- If your students are familiar in the Thinking Classroom let them work on vertical non-permanent surfaces (VNPS)
- When students are done let them share their answers on the boards or chart paper or VNPS

Consolidating and Connecting (After)

- Ask students to describe how they got their answers.
- Ask them what they think about the Babylonian system? Is it more effective than the one we use? What are the differences, what are the similarities (it is a place value system but with a different base, not 10, but 60)?
- Ask students whether we still use Babylonian numbers (1 hour=60 minutes, 1 minute=60 seconds)
- Assign 1.3 and 1.4 problems and tell students to express their answers in Babylonian numerals

Assessment tools

Exit ticket: Assign students to write 126 in Babylonian numbers

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Materials https://ecampusontario.pressbooks.pub/oersamplechapter/chapter/greece/

Note: the same lesson format can be used to explore the Mayan numeral system or Egyptian numeral system (See Chapter 1 – Numbers)

Lesson Plan - Unit Fractions in Ancient Egypt

Curriculum Expectations

Grade 9

B3. Number Sense and Operations
B3.2 apply an understanding of unit fractions and their relationship to other fractional amounts, in various contexts, including the use of measuring tools

Grade 8

B2. Operations
B2.5 add and subtract fractions, using appropriate strategies, in various contexts

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Learning Goals of the Lesson

Students will

- Learn what unit fractions are
- Learn to decompose common fractions into sums of unit fractions
- Solve problems using unit fractions

Getting Started

- Open Chapter 4 – Fractions and project it on the big screen.
- Tell students that today they will learn what Egyptian unit fractions are and solve some 4 thousand year old Egyptian problems
- Find and show Egypt on the Map and Timeline
- Let students listen to *Egypt. Eye of Horus*
- Read and solve 4.8 with the class in a class discussion format. Leave the solution on the board

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Working on It

- Assign 4.1. Let students work in pairs or groups. If your students are familiar to the Thinking Classroom format let them work on vertical non-permanent surfaces (VNPS)
- When students are done let them write their answers on the boards or chart paper or VNPS

Consolidating and Connecting (After)

- Discuss which solution makes larger pieces when loaves are cut (1/2+1/3+1/15). This is an example of practical use of unit fractions.
- Ask students to come up with examples when unit fractions might be useful in real world
- Assign 4.2 and 4.4 either individually or in pairs. Students have to check their solutions with the teacher or they work in the Thinking Classroom format on the VNPS

Extension:

Problems 4.3, 4.5 and 4.7 can be assigned for those who finish earlier

Assessment tools

Observation: Students check their answers with a teacher or they work on VNPS

Materials https://ecampusontario.pressbooks.pub/oersamplechapter/chapter/greece/

Note: Bar modeling method might be a greater tool to solve the above problems especially in Grade 8.

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World History Encyclopedia https://www.worldhistory.org/
https://en.wikipedia.org/