College Physics

COLLEGE PHYSICS

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Changes from Adapted Text

The following changes were made from the adapted OpenStax text:

- • Section 4.9 Friction (originally section 5.1) added to Chapter 4, with figures, tables, and examples renumbered accordingly.
- Chapter 4 Dynamics: Force and Newton’s Laws of Motion is a combination of OpenStax Chapters 4 & 5.
- Chapter 5 Uniform Circular Motion and Gravitation was reordered from OpenStax Chapter 6, and some sub-sections (6.4 and 6.6) were removed.
- Chapter 6 Work, Energy, and Energy Resources was reordered from OpenStax Chapter 7.

- Chapter 7 Statics and Torque was reordered from OpenStax Chapter 9.
- Chapter 8 Rotational Motion and Angular Momentum was reordered from OpenStax Chapter 10, and some sub-sections (10.6-10.7) were removed.
- Chapter 9 Fluid Statics was reordered from OpenStax Chapter 11.
- Chapter 10 Fluid Dynamics and Its Biological and Medical Applications, was reordered from OpenStax Chapter 12, and sub-sections (12.5-12.7) were removed.
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- Chapter 12 Heat and Heat Transfer Methods was reordered from OpenStax Chapter 14
- Chapter 13 Oscillatory Motion and Waves was reordered from OpenStax Chapter 16
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- Chapter 15 Electric Charge and Electric Field was reordered from OpenStax Chapter 18
- Chapter 16 Electric Current, Resistance, and Ohm's Law is a combination of OpenStax Chapters 20 & 21
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ABOUT THIS BOOK

The content of this book is organized to align with and supplement the syllabi of the Selected Topics in Physics and College Qualifying Physics courses at Fanshawe College. These courses have been designed to provide students with a comprehensive background in college-level physics. They also reinforce the ubiquitousness of physics in all fields of scientific inquiry.

The outlines and the in-depth syllabi for these courses, in their present forms, have been developed over several semesters of first-hand delivery by Dr. Iftekhar Haque, professor of science and mathematics at Fanshawe College

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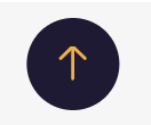


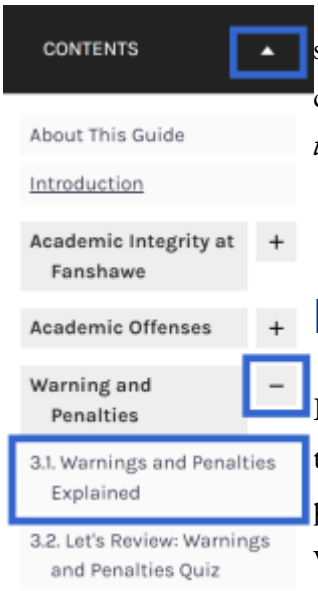
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INTRODUCTION: THE NATURE OF SCIENCE AND PHYSICS



Figure 1.1 The laws of physics describe the smallest and largest forces and structures, such as the Veil Nebula, a supernova remnant from a star that was 20 times larger than our Sun. Stretching over 110 light years across, the spectacular image is created by the stellar explosion's blast wave impacting clouds of gas. Scientists use successive observations of the still-moving wave to learn about nebula formation in ways they can apply to other systems. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature's apparent complexity. Image by NASA Hubble Space Telescope, Unsplash License

Chapter Outline

1.1 Physics: An Introduction
1.2 Physical Quantities and Units
1.3 Accuracy, Precision, and Significant Figures
1.4 Approximation
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

What is your first reaction when you hear the word "physics"? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

Consider the Veil Nebula, a cloud of heated dust and gas located about 2,400 light years from Earth (a light year is the distance light travels in one year, or approximately 9.5 trillion kilometers). The unique structure is the ongoing result of a supernova that occurred 8,000 years ago. The shock wave from the explosion is colliding with a cloud of gas and dust, creating the rope-like filaments and sheet-like appearance. Scientists compare newer images, such as the one above, with detailed images taken by the Hubble Space Telescope in 1997 in order to understand nebula expansion and other properties of astronomical objects. The forces that cause the supernova remnant to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply heating a new home. Although the scale is much different, the interaction of gasses in the Veil Nebula resembles those on Earth; and the vibrant color combinations are created by the familiar glows of hydrogen, sulfur, oxygen, and similar elements that make up everything we know. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater understanding of the interconnectedness of everything we can see and know in this universe.

Humans have created and manufactured millions of different objects over the history of our species. Successive technological periods (often referred to as the Stone Age, the Bronze Age, the Iron Age, and so on)

were marked by our knowledge of the physical properties of certain materials and our ability to manipulate them. This knowledge all stems from physics, whether it's the way a rock would flake when constructing a spear point, the effect of integrating carbon with iron in South Indian and Sri Lankan furnaces to create the earliest high-quality steel, or the proper way to combine perfectly ground and polished pieces of glass to create optical instruments. Our current technological age, the Information Age, builds on all that knowledge and can be traced to critical innovations made by people from all backgrounds working together. Mohamed M. Atalla and Dawon Kahng, for example, invented the MOSFET (metal-oxide-semiconductor field-effect transistor). Although unknown to most people, this tiny device, created in 1959 by an Egyptian-born scientist and Korean-born scientist working in a lab in New Jersey, is the basis for modern electronics. More MOSFETs have been produced than any other object in human history. They are used in computers, smart phones, microwave ovens, automotive controls, medical instruments, and nearly every other electronic device.

Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, "invisibility cloaks" that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient thinkers from the Middle East and the Mediterranean, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

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1.1 PHYSICS: AN INTRODUCTION

Learning Objectives

By the end of this section, you will be able to:

- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.



Figure 1.2 The flight formations of migratory birds such as Canada geese are governed by the laws of physics. Image by Jocelyn Anderson, CC-BY 4.0

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore

and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. Physics is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone (Figure 1.3). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to

travel that distance. GPS relies on precise calculations that account for variations in the Earth's landscapes, the exact distance between orbiting satellites, and even the effect of a complex occurrence of time dilation. Most of these calculations are founded on algorithms developed by Gladys West, a mathematician and computer scientist who programmed the first computers capable of highly accurate remote sensing and positioning. When you use a GPS device, it utilizes these algorithms to recognize where you are and how your position relates to other objects on Earth.

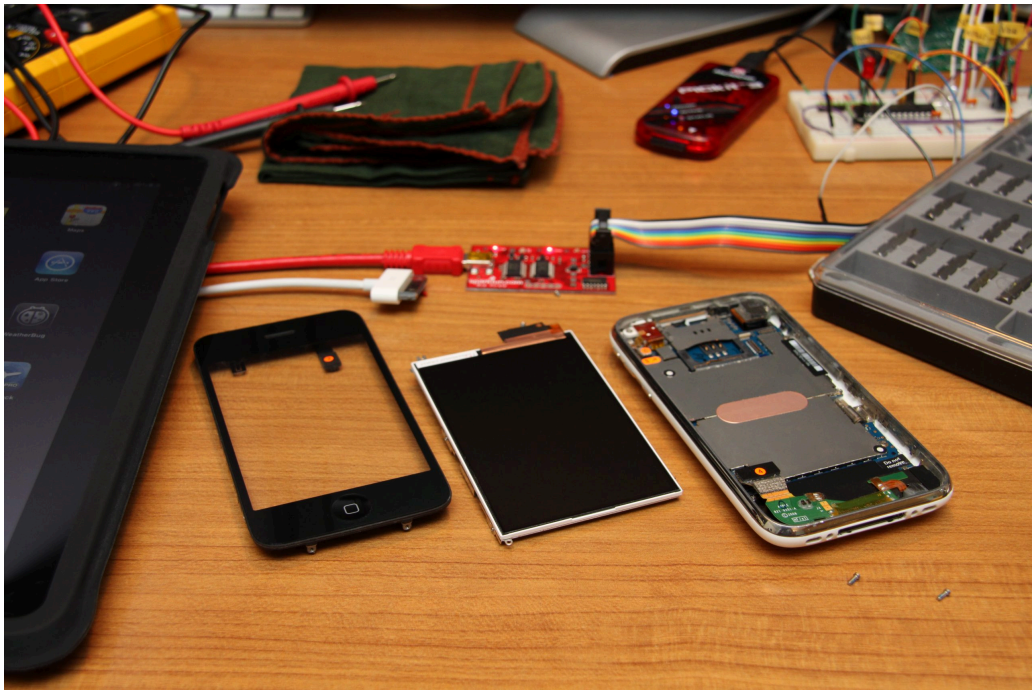


Figure 1.3 The Apple iPhone is a common smart phone with a GPS function, sophisticated camera, haptic (vibration) capabilities, and many other functions. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. Image by Yutaka Tsutano, CC-BY 2.0

Applications of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See Figure 1.4 and Figure 1.5.) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous system are much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes (Figure 1.6 and Figure 1.7). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.



Figure 1.4 The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. Image by MoneyBlogNewz, Wikimedia Commons License

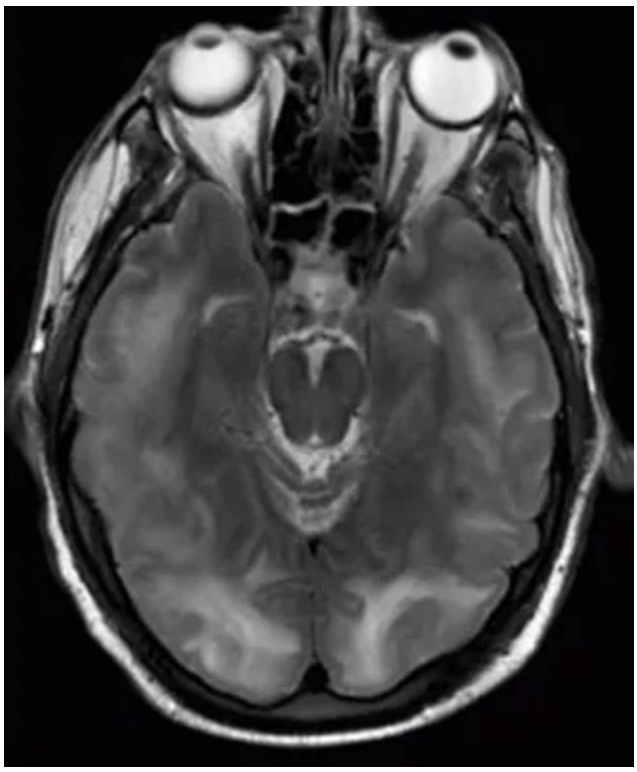


Figure 1.5 These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. Image by Rashmi Chawla, Daniel Smith and Paul E Marik, Wikimedia Commons License

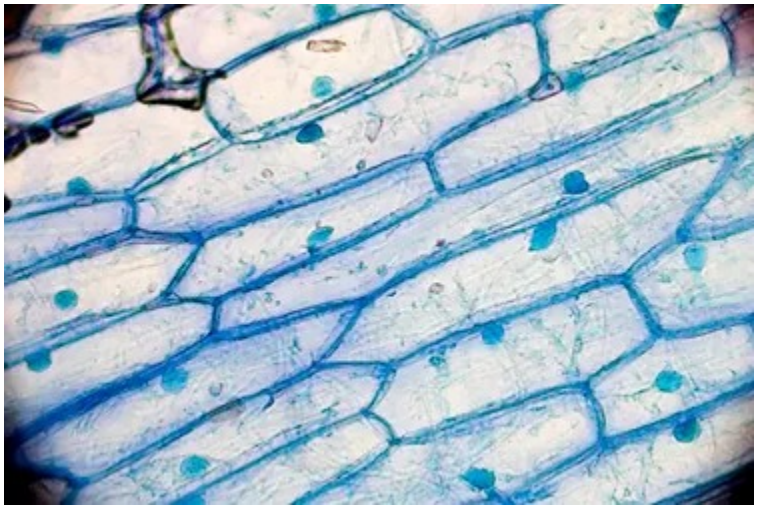


Figure 1.6 Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. Image by kaibara87, Wikimedia Commons License

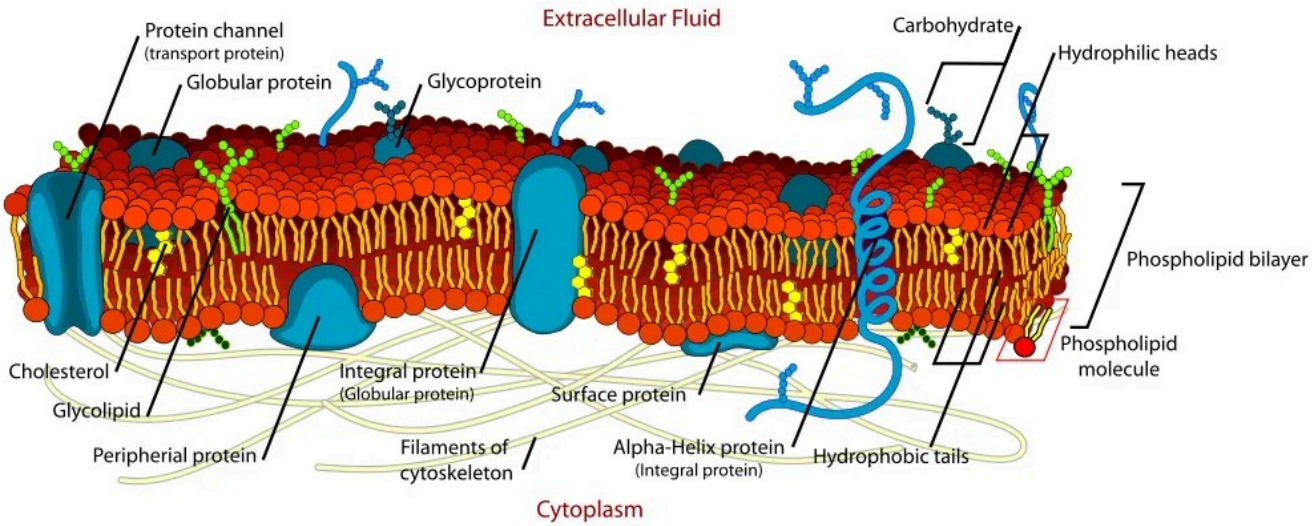


Figure 1.7 An artist's rendition of the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. Image by LadyofHats, Wikimedia Commons License

Image Description

The image is an illustrative diagram of a cell membrane, labeled as a phospholipid bilayer. The main components are:

- *Extracellular Fluid* is labeled at the top of the image above the membrane.
- *Phospholipid bilayer:* Consists of double rows of phospholipid molecules. These have *hydrophilic heads* that face outward towards the extracellular fluid and cytoplasm, and *hydrophobic tails* that face inward, creating a lipid barrier.
- *Proteins:* Various proteins are embedded within and along the membrane, including:
  - *Integral proteins:* Span the membrane, with *protein channels* for substance transport.
  - *Peripherical proteins:* Located on the cytoplasm side associated with the inner surface of the membrane.
  - *Surface proteins* are attached to the membrane surface.
  - *Alpha-Helix protein* positioned across the phospholipid bilayer.
- *Glycoprotein and glycolipid:* Attached to the exterior of the membrane, these molecules have carbohydrate chains that extend into the extracellular fluid.
- *Cholesterol:* Interspersed within the phospholipid bilayer, providing structural stability to the membrane.
- *Filaments of cytoskeleton:* Found within the cytoplasm near the inner membrane surface, providing

structural support.

- *Carbohydrate:* A carbohydrate chain can be seen extending from a glycoprotein.

The bottom of the image is labeled *Cytoplasm*, indicating the interior environment of the cell. Arrows and labels are used throughout the diagram to point to specific structures and provide additional context.

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See Figure 1.8 and Figure 1.9.) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.

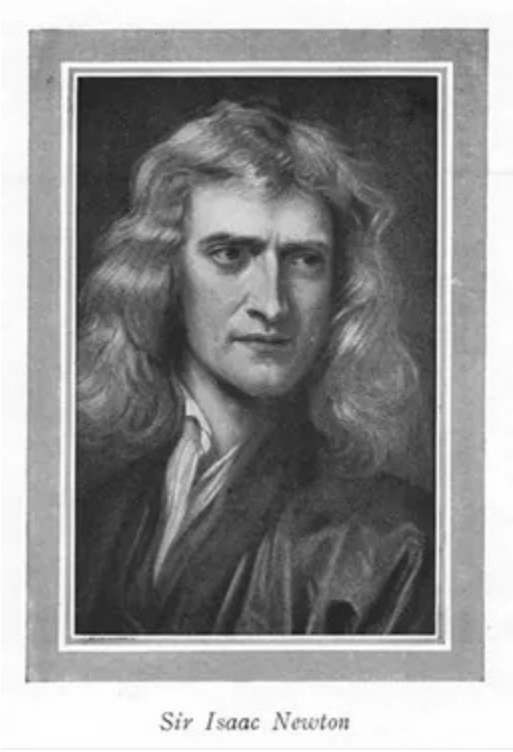


Figure 1.8 ***Isaac Newton*** *(1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. Image by Arthur E. Shipley: Britain's Heritage of Science. London, 1917, Wikimedia Commons License*



Figure 1.9 ***Marie Curie*** *(1867–1934) sacrificed monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. Image by Unknown Author, Wikimedia Commons License*

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

A model is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. (See Figure 1.10.) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation.

The word theory means something different to scientists than what is often meant when the word is used in everyday conversation. In particular, to a scientist a theory is not the same as a “guess” or an “idea” or even a

"hypothesis." The phrase "it's just a theory" seems meaningless and silly to scientists because science is founded on the notion of theories. To a scientist, a theory is a testable explanation for patterns in nature supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. Although models are meant only to describe certain aspects of a physical system accurately, a theory should describe all aspects of any system that falls within its domain of applicability. In particular, any experimentally testable implication of a theory should be verified. If an experiment ever shows an implication of a theory to be false, then the theory is either thrown out or modified suitably (for example, by limiting its domain of applicability).

Some theories include models to help visualize phenomena, whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A law uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $\mathbf{F} = m\mathbf{a}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal's principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.

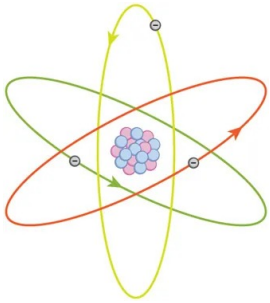


Figure 1.10 What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small. Image from OpenStax College Physics 2e, CC-BY 4.0

Models, Theories, and Laws

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved.* These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

The Scientific Method

Ibn al-Haytham (sometimes referred to as Alhazen), a 10th-11th century scientist working in Cairo, significantly advanced the understanding of optics and vision. But his contributions go much further. In demonstrating that previous approaches were incorrect, he emphasized that scientists must be ready to reject existing knowledge and become "the enemy" of everything they read; he expressed that scientists must trust only objective evidence. Al-Haytham emphasized repeated experimentation and validation, and acknowledged that senses and predisposition could lead to poor conclusions. His work was a precursor to the scientific method that we use today.

As scientists inquire and gather information about the world, they follow a process called the scientific method. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called "natural philosophy." From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See Figure 1.11, Figure 1.12, and Figure 1.13.) Physics as it developed from the Renaissance to the end of the 19th century is called classical physics. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

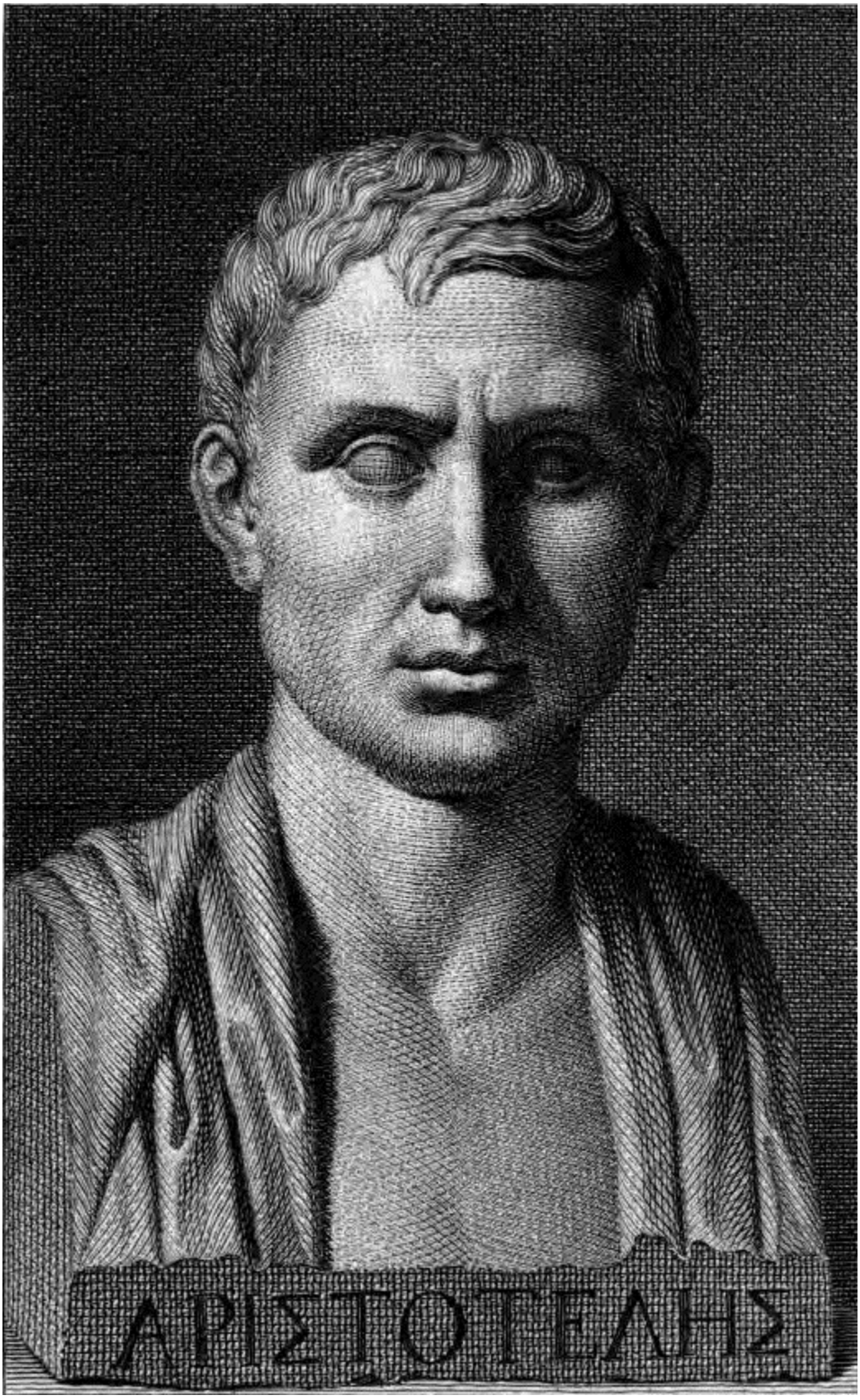


Figure 1.11 Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher

Aristotle *(384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. Image by Unidentified engraver, Wikimedia Commons License*



Figure 1.12 Ibn al-Haytham (965–1040) and Galileo Galilei (1564–1642) were so critical to the advancement of scientific practice and to the fields of mathematics, optics, physics, and astronomy that a later prominent scientist featured them on the cover of his major work. Image by Adolph Boÿ, engraved by Jeremias Falck, Wikimedia Commons License



Figure 1.13 ***Niels Bohr*** *(1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. Image by <a href="https:// authorUnknown author, Wikimedia Commons License*

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually "picture" the atom.

Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.

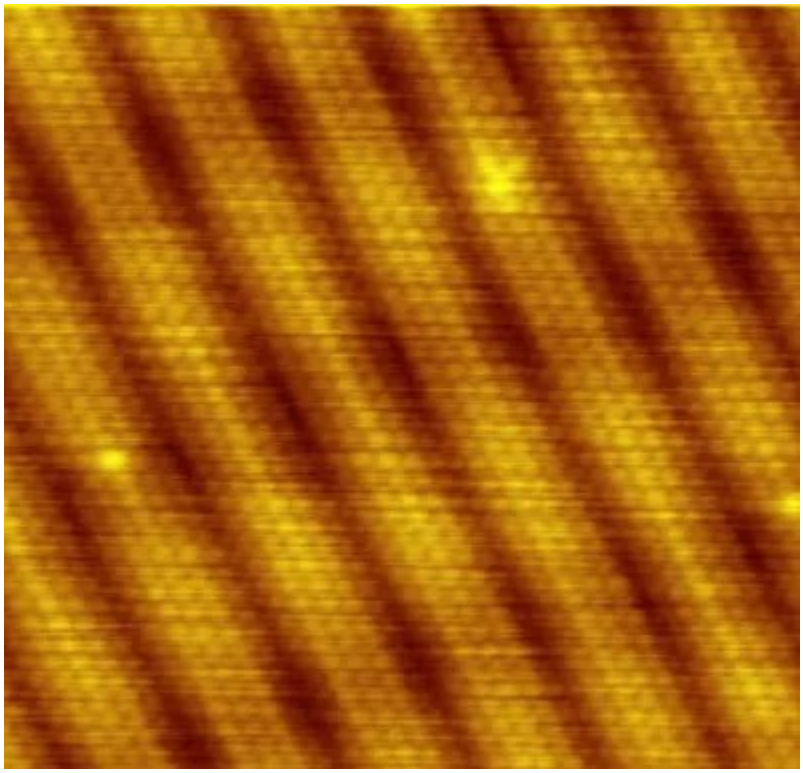


Figure 1.14 Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. Image by Erwinrossen, Wikimedia Commons License

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is

the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. Relativity must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. Quantum mechanics must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics,* and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

Check Your Understanding

A friend tells you they have learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you they had learned about a scientific theory rather than a law?

Click for Solution

Solution

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

PhET Explorations

Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

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1.2 PHYSICAL QUANTITIES AND UNITS

Learning Objectives

By the end of this section, you will be able to:

- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

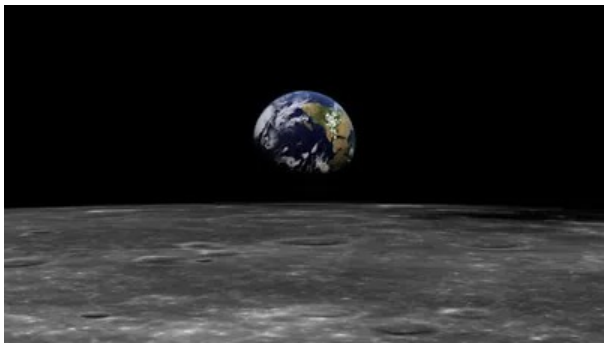


Figure 1.15 The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. Image from OpenStax College Physics 2e, CC-BY 4.0

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane

discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—most physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a physical quantity either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of units, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See Figure 1.16.)

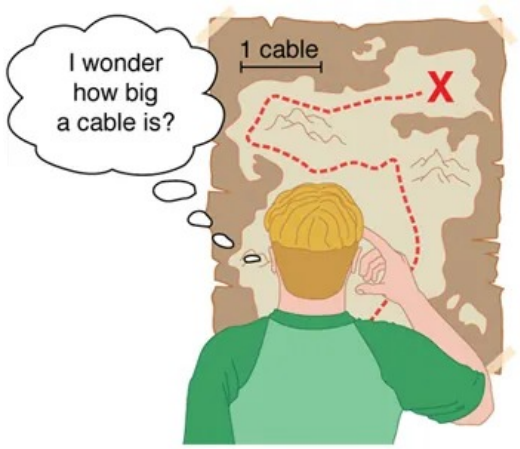


Figure 1.16 Distances given in unknown units are maddeningly useless. Image from OpenStax College Physics 2e, CC-BY 4.0

There are two major systems of units used in the world: SI units (also known as the metric system) and English units (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym "SI" is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

Table 1.1 gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Table 1.1 Fundamental SI Units

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called fundamental units. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called derived units.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the second (abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth's rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See Figure 1.17.) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.



Figure 1.17 An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! Image by Steve Jurvetson from Menlo Park, USA, Wikimedia Commons License

The Meter

The SI unit for length is the meter (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See Figure 1.18.) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

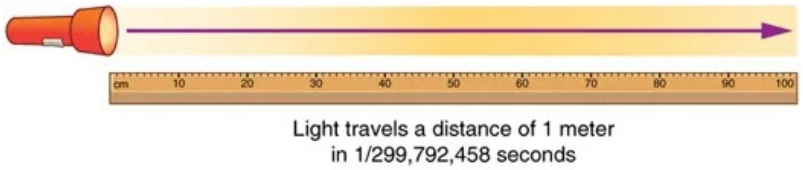


Figure 1.18 The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time. Image from OpenStax College Physics 2e, CC-BY 4.0

The Kilogram

The SI unit for mass is the kilogram (abbreviated kg); it was previously defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the previously defined kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses could be ultimately traced to a comparison with the standard mass. Even though the platinum-iridium cylinder was resistant to corrosion, airborne contaminants were able to adhere to its surface, slightly changing its mass over time. In May 2019, the scientific community adopted a more stable definition of the kilogram. The kilogram is now defined in terms of the second, the meter, and Planck's constant, h (a quantum mechanical value that relates a photon's energy to its frequency).

Electric current and its accompanying unit, the ampere, will be introduced in Electric Current, Resistance, and Ohm's Law when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the metric system. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. Table 1.2 gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers

are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term order of magnitude refers to the scale of a value expressed in the metric system. Each power of 10, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Thus, the numbers 800 and 450 are of the same order of magnitude: 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of 10^{-10} m, while the diameter of the Sun is on the order of 10^9 m.

The Quest for Microscopic Standards for Basic Units

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

Table 1.2 Metric Prefixes for Powers of 10 and their Symbols

Prefix	Symbol	Value	Example (some are approximate)			
exa	E	10^{18}	exameter	Em	10^{18} m	distance light travels in a century
peta	P	10^{15}	petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	terawatt	TW	10^{12} W	powerful laser output
giga	G	10^{9}	gigahertz	GHz	10^{9} Hz	a microwave frequency
mega	M	10^{6}	megacurie	MCi	10^{6} Ci	high radioactivity
kilo	k	10^{3}	kilometer	km	10^{3} m	about 6/10 mile
hecto	h	10^{2}	hectoliter	hL	10^{2} L	26 gallons
deka	da	10^{1}	dekagram	dag	10^{1} g	teaspoon of butter
—	—	10^{0} (=1)				
deci	d	10^{-1}	deciliter	dL	10^{-1} L	less than half a soda
centi	c	10^{-2}	centimeter	cm	10^{-2} m	fingertip thickness
milli	m	10^{-3}	millimeter	mm	10^{-3} m	flea at its shoulders
micro	μ	10^{-6}	micrometer	μm	10^{-6} m	detail in microscope
nano	n	10^{-9}	nanogram	ng	10^{-9} g	small speck of dust
pico	p	10^{-12}	picofarad	pF	10^{-12} F	small capacitor in radio
femto	f	10^{-15}	femtometer	fm	10^{-15} m	size of a proton
atto	a	10^{-18}	attosecond	as	10^{-18} s	time light crosses an atom

Known Ranges of Length, Mass, and Time

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times in Table 1.3. Examination of this table will give you some feeling for the range of possible topics and numerical values. (See Figure 1.19 and Figure 1.20.)

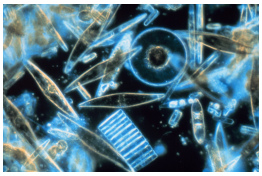


Figure 1.19 Tiny phytoplankton swims among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. Image by Prof. Gordon T. Taylor, Stony Brook University, Wikimedia Commons License



Figure 1.20 Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. Image by X-ray: NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al., Wikimedia Commons License

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a conversion factor relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

$$80 \cancel{\text{m}} \times \frac{1 \text{ km}}{1000 \cancel{\text{m}}} = 0.080 \text{ km}.$$

1.1

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Table 1.3 Approximate Values of Length, Mass, and Time

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron $(9.11 \times 10^{-31}\ \text{kg})$	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom $(1.67 \times 10^{-27}\ \text{kg})$	10^{-22}	Mean life of an extremely unstable nucleus
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day $(8.64 \times 10^{4}\text{s})$
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (y) $(3.16 \times 10^{7}\text{s})$
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of the Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from the Earth to the Sun	10^{23}	Mass of the Moon $(7.35 \times 10^{22}\ \text{kg})$	10^{17}	Age of the Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of the Earth $(5.97 \times 10^{24}\ \text{kg})$	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way galaxy	10^{30}	Mass of the Sun $(1.99 \times 10^{30}\ \text{kg})$		
10^{22}	Distance from the Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from the Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Example 1.1

Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your school to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}.$$

1.2

(2) Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0\ \text{km}}{20.0\ \text{min}} = 0.500\frac{\text{km}}{\text{min}}.$$

1.3

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is $60\ \text{min}/\text{hr}$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

$$\frac{\text{km}}{\text{min}} \times \frac{1\ \text{hr}}{60\ \text{min}} = \frac{1}{60}\frac{\text{km} \cdot \text{hr}}{\text{min}^2},$$

1.4

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

$$\text{Average speed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1\text{h}}{3,600\text{ s}} \times \frac{1,000\text{m}}{1\text{ km}},$$

1.5

$$\text{Average speed} = 8.33\frac{\text{m}}{\text{s}}.$$

1.6

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module Accuracy, Precision, and Significant Figures will help you answer these questions.

Nonstandard Units

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different "weights and measures." Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

Check Your Understanding

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Click for Solution

Solution

The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Check Your Understanding

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Click for Solution

Solution

The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

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1.3 ACCURACY, PRECISION, AND SIGNIFICANT FIGURES

Learning Objectives

By the end of this section, you will be able to:

- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.



Figure 1.21A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The "known masses" are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. Image from OpenStax College Physics 2e, CC-BY 4.0

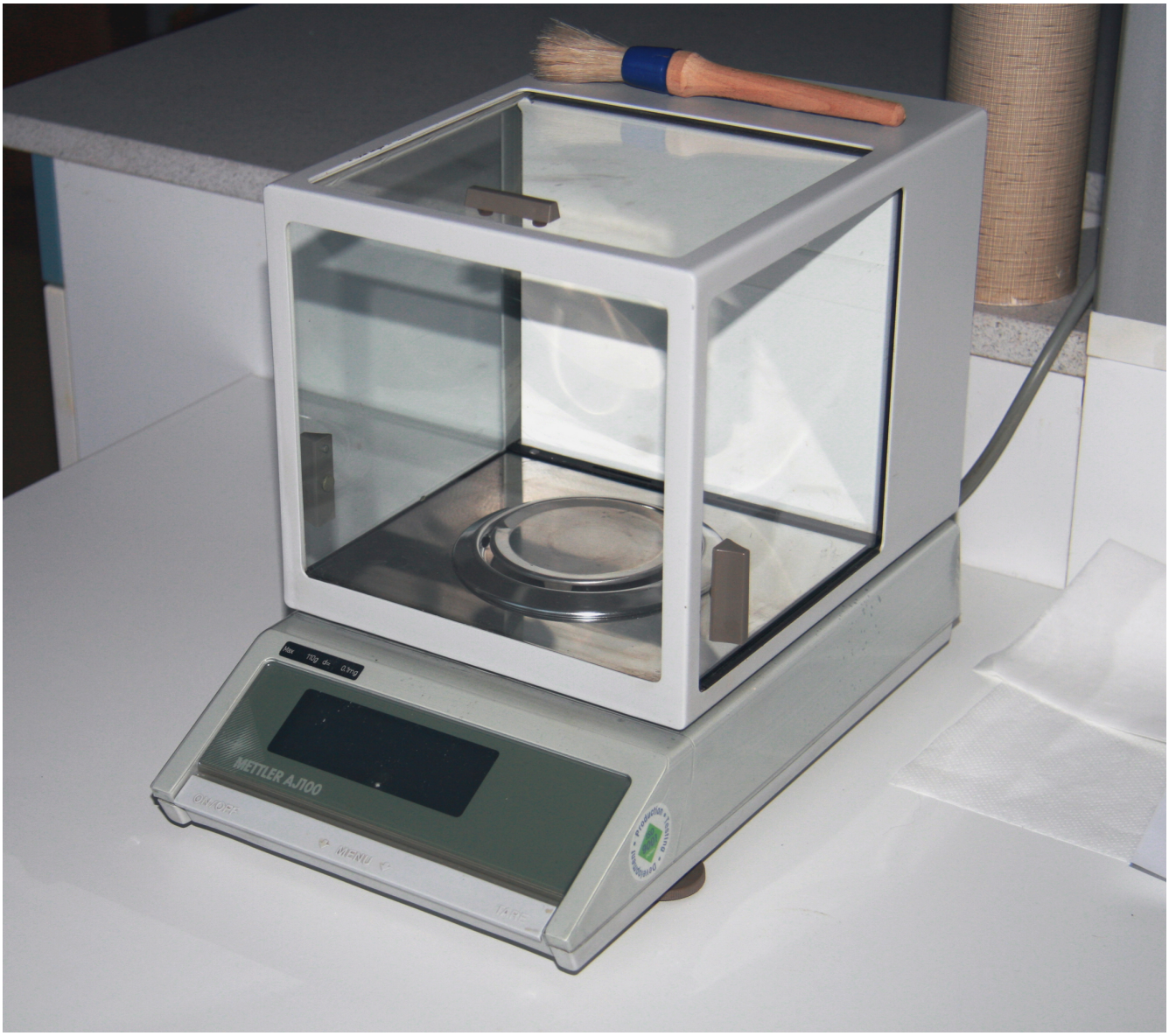


Figure 1.22 Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. Image by Karelj, Wikimedia Commons License

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. Accuracy is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1

in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The precision of a measurement system refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In Figure 1.23, you can see that the GPS measurements are spread out far apart from each other, but they are centered close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 1.24, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.

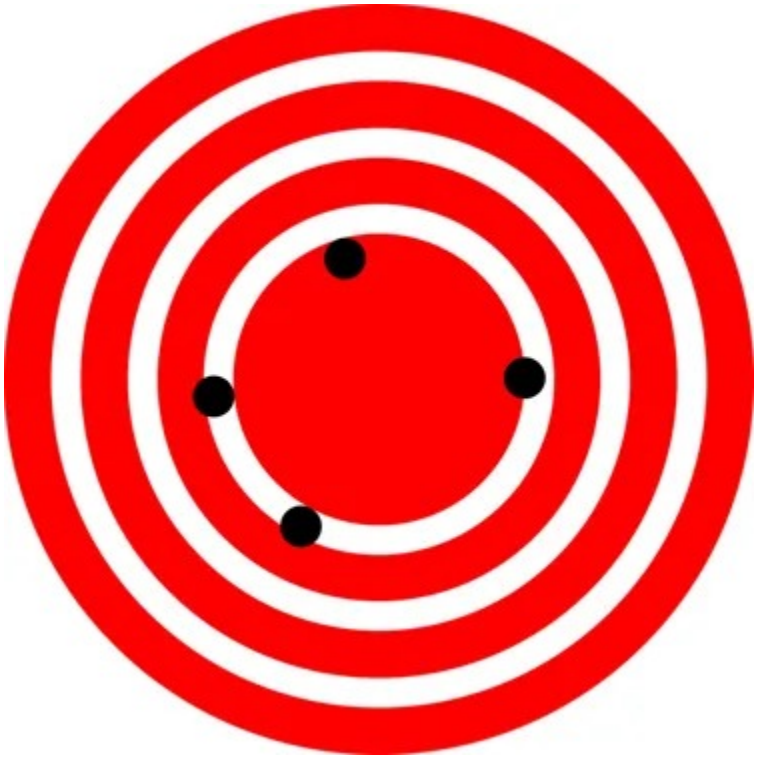


Figure 1.23 A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. Image by DarkEvil, Wikimedia Commons License



Figure 1.24 In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. Image by DarkEvil, Wikimedia Commons License

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the uncertainty in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A, is often denoted as δA ("delta A"), so the measurement result would be recorded as

$A \pm \delta A$. In our paper example, the length of the paper could be expressed as $11 \text{ in.} \pm 0.2$.

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object being measured,
4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Making Connections

Real-World Connections – Fevers or Chills?

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check their temperature with a thermometer. What if the uncertainty of the thermometer were $3.0^{\circ}\ \mathrm{C}$? If the child's temperature reading was $37.0^{\circ}\ \mathrm{C}$ (which is normal body temperature), the "true" temperature could be anywhere from a hypothermic $34.0^{\circ}\ \mathrm{C}$ to a dangerously high $40.0^{\circ}\ \mathrm{C}$. A thermometer with an uncertainty of $3.0^{\circ}\ \mathrm{C}$ would be useless.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty, δA, the percent uncertainty (%unc) is defined to be

$$\%\ \text{unc} = \frac{\delta A}{A} \times 100\%.$$

1.7

Example 1.2

Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells $5 - \text{lb}$ bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: $4.8\ \text{lb}$
- Week 2 weight: $5.3\ \text{lb}$
- Week 3 weight: $4.9\ \text{lb}$
- Week 4 weight: $5.4\ \text{lb}$

You have previously determined, using additional measurements, that the uncertainty in the average weight of the $5 - \text{lb}$ bag is $\pm 0.4\text{lb}$. What is the average weight and percent uncertainty of the bag's weight, given these 4 measurements?

Strategy

The average of the four measured weights is 5.1 lb. Observe that the measured average agrees with the nominal weight of 5 lb within the uncertainty of 0.4 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

$$\%\ \text{unc} = \frac{\delta A}{A} \times 100\%.$$

1.8

Solution

Plug the known values into the equation:

$$\%\ \text{unc} = \frac{0.4\ \text{lb}}{5.1\ \text{lb}} \times 100\% = 8\%.$$

1.9

Discussion

We can conclude that the weight of the apple bag is $5\text{lb} \pm 8\%$. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainties in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the method of adding percents can be used for multiplication or division. This method says that *the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation*. For example, if a floor has a length of $4.00\ \text{m}$ and a width of $3.00\ \text{m}$, with uncertainties of 2% and 1%, respectively, then the area of the floor is 12.0m^2 and has an uncertainty of 3%. (Expressed as an area this is 0.36m^2, which we round to 0.4m^2 since the area of the floor is given to a tenth of a square meter.)

Check Your Understanding

A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of ±0.05 s. The team's top sprinter clocked a 100 meter sprint at 12.04 seconds last week and at 11.96 seconds this week. Can we conclude that this week's time was faster?

Click for Solution

Solution

No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7cm. You could not express this value as 36.71cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6cm and 36.7cm, and they must estimate the value of the last digit. Using the method of significant figures, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count

the number of digits through the last digit written on the right. For example, the measured value 36.7cm has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers*.

Check Your Understanding

Determine the number of significant figures in the following measurements:

a. 0.0009
b. 15,450.0
c. 6×10^3
d. 87.990
e. 30.42

Click for Solution

Solution

(a) 1; the zeros in this number are placekeepers that indicate the decimal point

(b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) 1; the value 10^3 signifies the decimal place, not the number of measured values

(d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(e) 4; any zeros located in between significant figures in a number are also significant

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

1. For multiplication and division: *The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation*. For example, the area of a circle can be

calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area has if the radius has only two—say, $r = 1.2\text{m}$. Then,

$$A = \pi r^2 = (3.1415927\ldots) \times (1.2\text{m})^2 = 4.5238934\text{m}^2$$
1.10

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated quantity to two significant figures or

$$A = 4.5\text{m}^2,$$
1.11

even though π is good to at least eight digits.

2. For addition and subtraction: *The answer can contain no more decimal places than the least precise measurement.* Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$\begin{array}{r} 7.56\ \text{kg} \\ -6.052\ \text{kg} \\ +13.7\ \text{kg} \\ \hline 15.208\ \text{kg} = 15.2\ \text{kg} \end{array}$$
1.12

Next, we identify the least precise measurement: 13.7 kg. Ths measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than

three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

Check Your Understanding

Perform the following calculations and express your answer using the correct number of significant digits.

(a) A woman has three bags: two of the bags weigh 13.5 pounds each, and one bag weighs of 10.2 pounds. What is the total weight of the bags?

(b) The force F on an object is equal to its mass m multiplied by its acceleration a. If a wagon with mass 55 kg accelerates at a rate of $0.0255\ \mathrm{m/s^2}$, what is the force on the wagon? (The unit of force is called the newton, and it is expressed with the symbol N.)

Click for Solution

Solution

(a) 23.7 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.

(b) 1.4 N; Because the value 55 kg has only two significant figures, the final value must also contain two significant figures.

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1.4 APPROXIMATION

Learning Objectives

By the end of this section, you will be able to:

- Make reasonable approximations based on given data.

On many occasions, physicists, other scientists, and engineers need to make approximations or "guesstimates" for a particular quantity. What is the distance to a certain destination? What is the approximate density of a given item? About how large a current will there be in a circuit? Many approximate numbers are based on formulae in which the input quantities are known only to a limited accuracy. As you develop problem-solving skills (that can be applied to a variety of fields through a study of physics), you will also develop skills at approximating. You will develop these skills through thinking more quantitatively, and by being willing to take risks. As with any endeavor, experience helps, as well as familiarity with units. These approximations allow us to rule out certain scenarios or unrealistic numbers. Approximations also allow us to challenge others and guide us in our approaches to our scientific world. Let us do two examples to illustrate this concept.

Example 1.3

Approximate the Height of a Building

Can you approximate the height of one of the buildings on your campus, or in your neighborhood? Let us make an approximation based upon the height of a person. In this example, we will calculate the height of a 39-story building.

Strategy

Think about the average height of an adult male. We can approximate the height of the building by scaling up from the height of a person.

Solution

Based on information in the example, we know there are 39 stories in the building. If we use the fact that the height of one story is approximately equal to about the length of two adult humans (each human is about 2-m tall), then we can estimate the total height of the building to be

$$\frac{2\ \text{m}}{1\ \text{person}} \times \frac{2\ \text{person}}{1\ \text{story}} \times 39\ \text{stories} = 156\ \text{m}.$$

1.13

Discussion

You can use known quantities to determine an approximate measurement of unknown quantities. If your hand measures 10 cm across, how many hand lengths equal the width of your desk? What other measurements can you approximate besides length?

Example 1.4

Approximating Vast Numbers: a Trillion Dollars



Figure 1.25 A bank stack contains one-hundred $100 bills, and is worth $10,000. How many bank stacks make up a trillion dollars? Image from OpenStax College Physics 2e, CC-BY 4.0

The U.S. federal debt in 2021 was a little more than $28 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $100 bills, such as you might see in movies or at a bank. Since this is an

easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

(1) Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is:

$$\begin{aligned}&\text{volume of stack} = \text{length} \times \text{width} \times \text{height},\\&\text{volume of stack} = 6\text{ in.} \times 3\text{ in.} \times 0.5\text{ in.},\\&\text{volume of stack} = 9\text{ in.}^3.\end{aligned}$$

1.14

(2) Calculate the number of stacks. Note that a trillion dollars is equal to $\$1 \times 10^{12}$, and a stack of one-hundred $\$100$ bills is equal to $\$10,000$, or $\$1 \times 10^4$. The number of stacks you will have is:

$$\frac{\$1 \times 10^{12}\,(\text{a trillion dollars})}{\$1 \times 10^4 \text{per stack}} = 1 \times 10^8 \text{stacks.}$$

1.15

(3) Calculate the area of a football field in square inches. The area of a football field is $100\text{ yd} \times 50\text{ yd}$, which gives $5,000\text{ yd}^2$. Because we are working in inches, we need to convert square yards to square inches:

$$\begin{aligned}\text{Area} &= 5,000\text{ yd}^2 \times \frac{3\text{ft}}{1\text{ yd}} \times \frac{3\text{ft}}{1\text{ yd}} \times \frac{12\text{in.}}{1\text{ ft}} \times \frac{12\text{in.}}{1\text{ ft}}\\&= 6,480,000\text{ in.}^2,\\\text{Area} &\approx 6 \times 10^6 \text{in.}^2.\end{aligned}$$

1.16

This conversion gives us $6 \times 10^6 \text{in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

(4) Calculate the total volume of the bills. The volume of all the $\$100$-bill stacks is $9\text{in.}^3/\text{stack} \times 10^8 \text{ stacks} = 9 \times 10^8 \text{in.}^3$.

(5) Calculate the height. To determine the height of the bills, use the equation:

$$\text{volume of bills} = \text{area of field} \times \text{height of money}:$$

$$\text{Height of money} = \frac{\text{volume of bills}}{\text{area of field}},$$

$$\text{Height of money} = \frac{9 \times 10^8 \text{ in.}^3}{6 \times 10^6 \text{ in.}^2} = 1.33 \times 10^2 \text{ in.},$$

$$\text{Height of money} \approx 1 \times 10^2 \text{ in.} = 100 \text{ in.}$$

1.17

The height of the money will be about 100 in. high. Converting this value to feet gives

$$100 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = 8.33 \text{ ft} \approx 8 \text{ ft}.$$

1.18

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough “guesstimates” versus carefully calculated approximations?

Check Your Understanding

Using mental math and your understanding of fundamental units, approximate the area of a regulation basketball court. Describe the process you used to arrive at your final approximation.

Click for Solution

Solution

An average male is about two meters tall. It would take approximately 15 males laid out end to end to cover the length, and about 7 to cover the width. That gives an approximate area of $420\ \mathrm{m}^2$.

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GLOSSARY

accuracy
: the degree to which a measured value agrees with correct value for that measurement

approximation
: an estimated value based on prior experience and reasoning

classical physics
: physics that was developed from the Renaissance to the end of the 19th century

conversion factor
: a ratio expressing how many of one unit are equal to another unit

derived units
: units that can be calculated using algebraic combinations of the fundamental units

English units
: system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units
: units that can only be expressed relative to the procedure used to measure them

kilogram
: the SI unit for mass, abbreviated (kg)

law
: a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated experiments

meter
: the SI unit for length, abbreviated (m)

method of adding percents
: the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

metric system
: a system in which values can be calculated in factors of 10

model
: representation of something that is often too difficult (or impossible) to display directly

modern physics
: the study of relativity, quantum mechanics, or both

order of magnitude

refers to the size of a quantity as it relates to a power of 10

percent uncertainty

the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

physical quantity

a characteristic or property of an object that can be measured or calculated from other measurements

physics

the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

precision

the degree to which repeated measurements agree with each other

quantum mechanics

the study of objects smaller than can be seen with a microscope

relativity

the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

scientific method

a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

second

the SI unit for time, abbreviated (s)

SI units

the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

significant figures

express the precision of a measuring tool used to measure a value

theory

an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

uncertainty

a quantitative measure of how much your measured values deviate from a standard or expected value

units

a standard used for expressing and comparing measurements

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CHAPTER SUMMARY

1.1 Physics: An Introduction

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

1.2 Physical Quantities and Units

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

1.3 Accuracy, Precision, and Significant Figures

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

1.4 Approximation

Scientists often approximate the values of quantities to perform calculations and analyze systems.

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CONCEPTUAL QUESTIONS

Conceptual Questions

1.1 Physics: An Introduction

1. Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?
2. How does a model differ from a theory?
3. If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?
4. What determines the validity of a theory?
5. Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?
6. Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?
7. Classical physics is a good approximation to modern physics under certain circumstances. What are they?
8. When is it *necessary* to use relativistic quantum mechanics?
9. Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

1.2 Physical Quantities and Units

10. Identify some advantages of metric units.

1.3 Accuracy, Precision, and Significant Figures

11. What is the relationship between the accuracy and uncertainty of a measurement?

12. Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

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PROBLEMS & EXERCISES

Problems & Exercises

1.2 Physical Quantities and Units

1. The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?
2. A car is traveling at a speed of $33\ \mathrm{m/s}$. (a) What is its speed in kilometers per hour? (b) Is it exceeding the $90\ \mathrm{km/h}$ speed limit?
3. Show that $1.0\ \mathrm{m/s} = 3.6\ \mathrm{km/h}$. Hint: Show the explicit steps involved in converting $1.0\ \mathrm{m/s} = 3.6\ \mathrm{km/h}$.
4. American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)
5. Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)
6. What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)
7. Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)
8. The speed of sound is measured to be $342\ \mathrm{m/s}$ on a certain day. What is this in km/h?
9. Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?
10. (a) Refer to Table 1.3 to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

1.3 Accuracy, Precision, and Significant Figures

Express your answers to problems in this section to the correct number of significant figures and proper units.

11. Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?
12. A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?
13. (a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads 90km/h? (b) Convert this range to miles per hour. $(1\ \text{km} = 0.6214\ \text{mi})$
14. An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?
15. (a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats do they have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?
16. A can contains 375 mL of soda. How much is left after 308 mL is removed?
17. State how many significant figures are proper in the results of the following calculations: (a) $(106.7)(98.2)/(46.210)(1.01)$ (b) $(18.7)^2$ (c) $\left(1.60 \times 10^{-19}\right)(3712)$.
18. (a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?
19. (a) If your speedometer has an uncertainty of 2.0km/h at a speed of 90km/h, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads $60\ \text{km/h}$, what is the range of speeds you could be going?
20. (a) A person's blood pressure is measured to be $120 \pm 2\text{mm Hg}$. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80mm Hg?
21. A person measures their heart rate by counting the number of beats in 30s. If 40 ± 1 beats are counted in $30.0 \pm 0.5\text{s}$, what is the heart rate and its

uncertainty in beats per minute?

22. What is the area of a circle 3.102cm in diameter?
23. If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?
24. A marathon runner completes a $42.188 - \text{km}$ course in 2h, 30 min, and 12s. There is an uncertainty of 25m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?
25. The sides of a small rectangular box are measured to be $1.80 \pm 0.01\text{cm}$, $2.05 \pm 0.02\text{cm}$, and $3.1 \pm 0.1\text{cm}$ long. Calculate its volume and uncertainty in cubic centimeters.
26. When non-metric units were used in the United Kingdom, a unit of mass called the pound-mass (lbm) was employed, where $1\text{lbm} = 0.4539\text{kg}$. (a) If there is an uncertainty of 0.0001kg in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?
27. The length and width of a rectangular room are measured to be $3.955 \pm 0.005\text{m}$ and $3.050 \pm 0.005\ \text{m}$. Calculate the area of the room and its uncertainty in square meters.
28. A car engine moves a piston with a circular cross section of $7.500 \pm 0.002\text{cm}$ diameter a distance of $3.250 \pm 0.001\text{cm}$ to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

1.4 Approximation

29. How many heartbeats are there in a lifetime?
30. A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?
31. How many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human? (Hint: The lifetime of an unstable atomic

nucleus is on the order of 10^{-22} s.)

32. Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of 10^{-27} kg and the mass of a bacterium is on the order of 10^{-15} kg.)



Figure 1.26 Salmonella typhimurium. This color-enhanced photo shows (red) attacking human cells. These bacteria are commonly known for causing foodborne illness. Can you estimate the number of atoms in each bacterium? Image from OpenStax College Physics 2e, CC-BY 4.0

33. Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?(a) What fraction of Earth's diameter is the greatest ocean depth? (b) The greatest mountain height?
34. (a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?
35. Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

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KINEMATICS



Figure 2.1 The motion of an American kestrel through the air can be described by the bird's displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. Image by Vince Maidens, Wikimedia Commons License

Chapter Outline

2.1 Displacement
2.2 Vectors, Scalars, and Coordinate Systems
2.3 Time, Velocity, and Speed
2.4 Acceleration
2.5 Motion Equations for Constant Acceleration in One Dimension
2.6 Problem-Solving Basics for One-Dimensional Kinematics
2.7 Falling Objects
2.8 Graphical Analysis of One-Dimensional Motion
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with kinematics which is defined as the *study of motion without considering its causes*. The word "kinematics" comes from a Greek term meaning motion and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). In one-dimensional kinematics and Two-Dimensional Kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In Two-Dimensional Kinematics, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve. Access multimedia content

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2.1 DISPLACEMENT

Learning Objectives

By the end of this section, you will be able to:

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.



Figure 2.2 These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. Image from OpenStax College Physics 2e, CC-BY 4.0

Position

In order to describe the motion of an object, you must first be able to describe its position—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. (See Figure 2.3.) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See Figure 2.4.)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as displacement. The word "displacement" implies that an object has moved, or has been displaced.

Displacement

Displacement is the *change in position* of an object:

$$\Delta x = x_{\mathrm{f}} - x_0,$$

2.1

where Δx is displacement, x_{f} is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means "change in" whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_{f}.

Note that the SI unit for displacement is the meter (m) (see 1.2 Physical Quantities and Units), but sometimes

kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

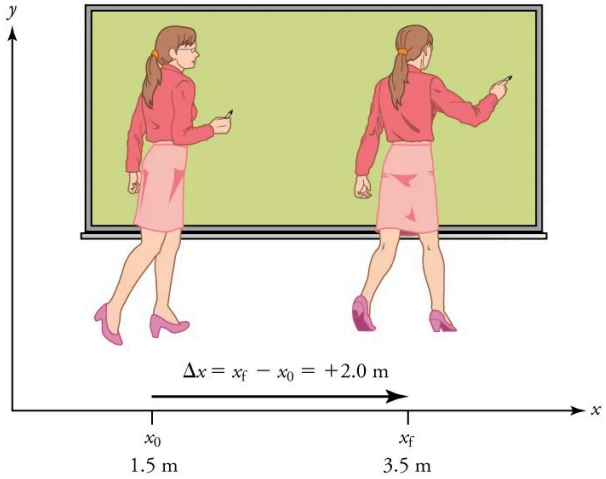


Figure 2.3 A professor paces left and right while lecturing. Her position relative to Earth is given by x*. The* $+2.0\,\text{m}$ *displacement of the professor relative to Earth is represented by an arrow pointing to the right. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows two illustrations of a woman in front of a chalkboard, indicating a depiction of motion along the x-axis.

– Left Side: The woman is facing to the right, standing in front of a chalkboard. She is holding a pen or marker in her right hand and is wearing a pink skirt and a red blouse. Her initial position, labeled as x_0, is marked at 1.5 meters along the x-axis.

– Right Side: The woman is in the same outfit but now turned slightly towards the board, indicating her final position. She is holding the pen or marker up to the board. Her final position, labeled as x_f, is marked at 3.5 meters along the x-axis.

– Background: A large green chalkboard is behind her, framed in black.

– Mathematics: Below the woman are mathematical equations depicting the change in position Δx, which is calculated as $x_f - x_0 = +2.0\,\text{m}$. An arrow points to the right, indicating the direction of motion.

– Axes: The x-axis is labeled at the bottom, with arrows on both ends pointing horizontally. The y-axis points vertically upwards on the left side.

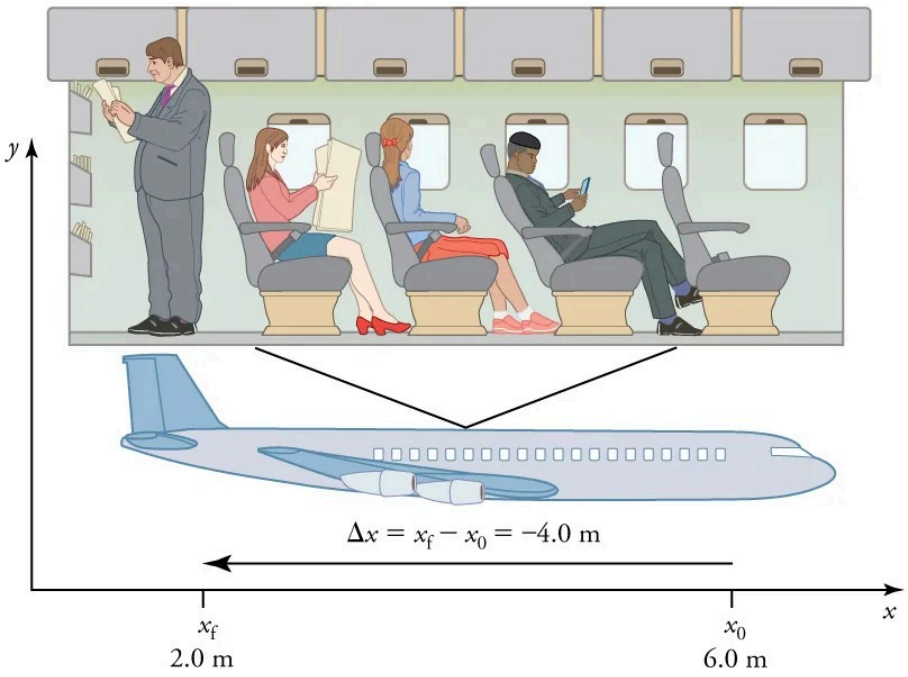


Figure 2.4 A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0 m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in . Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a cross-section of an airplane cabin viewed from the side. Inside the cabin, there are four passengers and one flight attendant.

- The flight attendant is standing at the front of the cabin, facing forward. There are storage compartments above.
- Four passengers are seated in a row:
  - The first passenger is reading a newspaper.
  - The second is using a phone.
  - The third passenger is a woman with red hair, also using a phone.
  - The fourth passenger, a man wearing a hat, is looking at a tablet.

Below the cabin is an illustration of a complete airplane showing its exterior profile. Two lines extend from the outside of the cabin to the ends of the airplane, depicting the horizontal distance.

$\Delta x = x_f - x_0 = -4.0$ m
$x_f = 2.0$ m
$x_0 = 6.0$ m

Axes are shown with *y* pointing upwards and *x* pointing towards the right, indicating horizontal and vertical orientation.

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $x_0 = 1.5\text{m}$ and her final position is $x_\text{f} = 3.5\text{m}$. Thus her displacement is

$$\Delta x = x_f - x_0 = 3.5 \text{ m} - 1.5m = +2.0 \text{ m}.$$
2.2

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0m$ and his final position is $x_f = 2.0m$, so his displacement is

$$\Delta x = x_f - x_0 = 2.0m - 6.0m = -4.0m.$$
2.3

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. Distance is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. Distance traveled is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Misconception Alert: Distance Traveled vs. Magnitude of Displacement

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Check Your Understanding

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is their displacement? (b) What distance do they ride? (c) What is the magnitude of their displacement?

Click for Solution

Solution

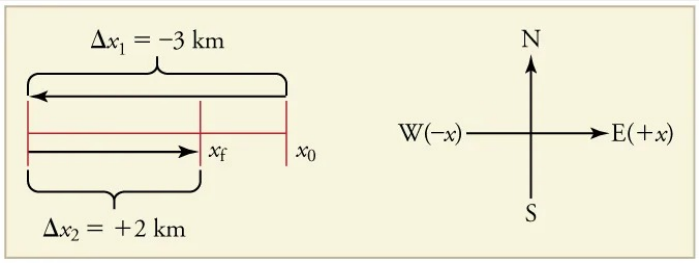


Figure 2.5 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two main parts:

1. Left Side:
– There is a horizontal line segment labeled with the endpoints x_0 and x_f.
– Two distances are indicated along this line:
– The section from x_0 to a point on the left is labeled as $\Delta x_1 = -3\,\text{km}$ with an arrow pointing left. A curly bracket shows this distance.
– The section from a point on the left to x_f is labeled as $\Delta x_2 = +2\,\text{km}$ with an arrow pointing right. Another curly bracket shows this distance.

2. Right Side:
– A compass rose indicates directions with arrows pointing:
– Upward for North (N)

– Right for East (E) labeled as $(+x)$
– Downward for South (S)
– Left for West (W) labeled as $(-x)$

(a) The rider's displacement is
$\Delta x = x_{\mathrm{f}} - x_0 = -1\ \mathrm{km}$.
(The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is $3\ \mathrm{km}\ +\ 2\ \mathrm{km}\ =\ 5\ \mathrm{km}$.

(c) The magnitude of the displacement is $1km$.

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2.2 VECTORS, SCALARS, AND COORDINATE SYSTEMS

Learning Objectives

By the end of this section, you will be able to:

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.



What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A vector is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus $(+)$ or minus $(-)$ sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A scalar is any quantity that has a magnitude, but no direction. For example, a $20^{\circ}\ \mathrm{C}$ temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a $-20^{\circ}\ \mathrm{C}$ temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in Figure 2.6, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

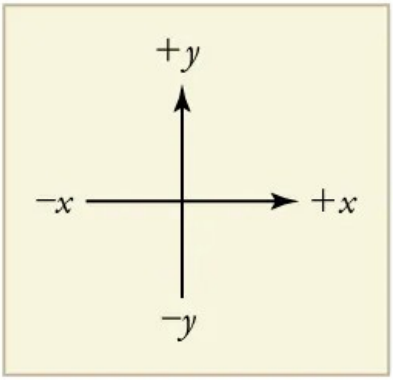


Figure 2.7 It is usually convenient to consider motion upward or to the right as positive $(+)$ *and motion downward or to the left as negative* $(-)$. *Image from College Physics 2e, CC-BY 4.0*

Check Your Understanding

A person's speed can stay the same as they round a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Click for Solution

Solution

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

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2.3 TIME, VELOCITY, AND SPEED

Learning Objectives

By the end of this section, you will be able to:

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.



Figure 2.8 The motion of these racing snails can be described by their speeds and their velocities. Image from OpenStax College Physics 2e, CC-BY 4.0

There is more to motion than distance and displacement. Questions such as, "How long does a foot race

take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

As discussed in 1.2 Physical Quantities and Units, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple—time is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. Elapsed time Δt is the difference between the ending time and beginning time,

$$\Delta t = t_f - t_0,$$
2.4

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then $\Delta t = t_f \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero $(t_0 = 0)$
- the symbol t is used for elapsed time unless otherwise specified $(\Delta t = t_f \equiv t)$

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Average Velocity

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0},$$

2.5

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}.$$

2.6

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4\text{m}}{5\text{s}} = -0.8\ \text{m/s}.$$

2.7

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

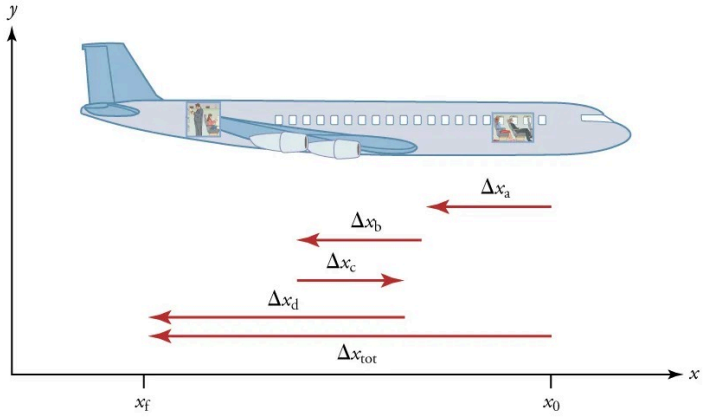


Figure 2.9 A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a side view of an airplane flying from left to right. The airplane is depicted with a simple illustration style, and there are two highlighted sections with figures visible through the windows. Below the airplane, there is a graph-like illustration with the horizontal and vertical axes labeled as "x" and "y," respectively. The horizontal axis is marked with "x_0" at the right end and "x_f" at the left.

Several red arrows are drawn horizontally across the bottom part of the image, each labeled with a different Greek character followed by an "x" (e.g., Δx_a, Δx_b). These arrows represent distances or displacements, varying in length and direction. The longest arrow is labeled Δx_tot, signifying the total displacement from x_0 to x_f. The labels all appear to correspond to different segments of the total distance the airplane travels.

Each arrow points in the direction of the displacement it represents, with most pointing to the left.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) Instantaneous velocity v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. Average speed is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than the magnitude of displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

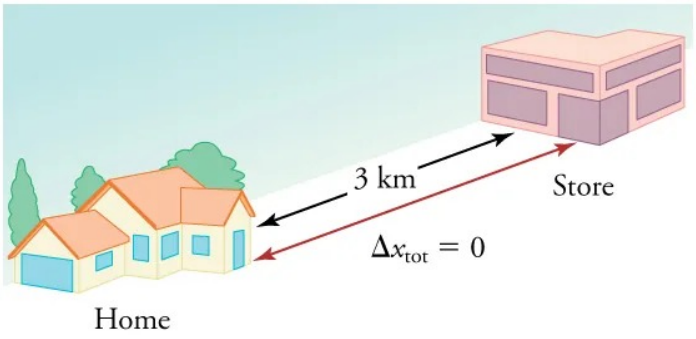


Figure 2.10 During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a simple diagram with two main elements: a house labeled “Home” on the left and a building labeled “Store” on the right. Between these two elements, there is a horizontal line indicating the distance. The line is annotated with “3 km” indicating the distance from Home to Store. Below this line, another annotation, “$\Delta x_{tot} = 0''$, is present, suggesting some kind of total change or displacement is zero. This might imply a round trip, where the starting and ending points are the same, making the total displacement zero. The background is a light blue gradient, and there are a few stylized trees near the house.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in Figure 2.11. (Note that these graphs depict a very simplified model of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we’ll probably stop at the store. But for simplicity’s sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)

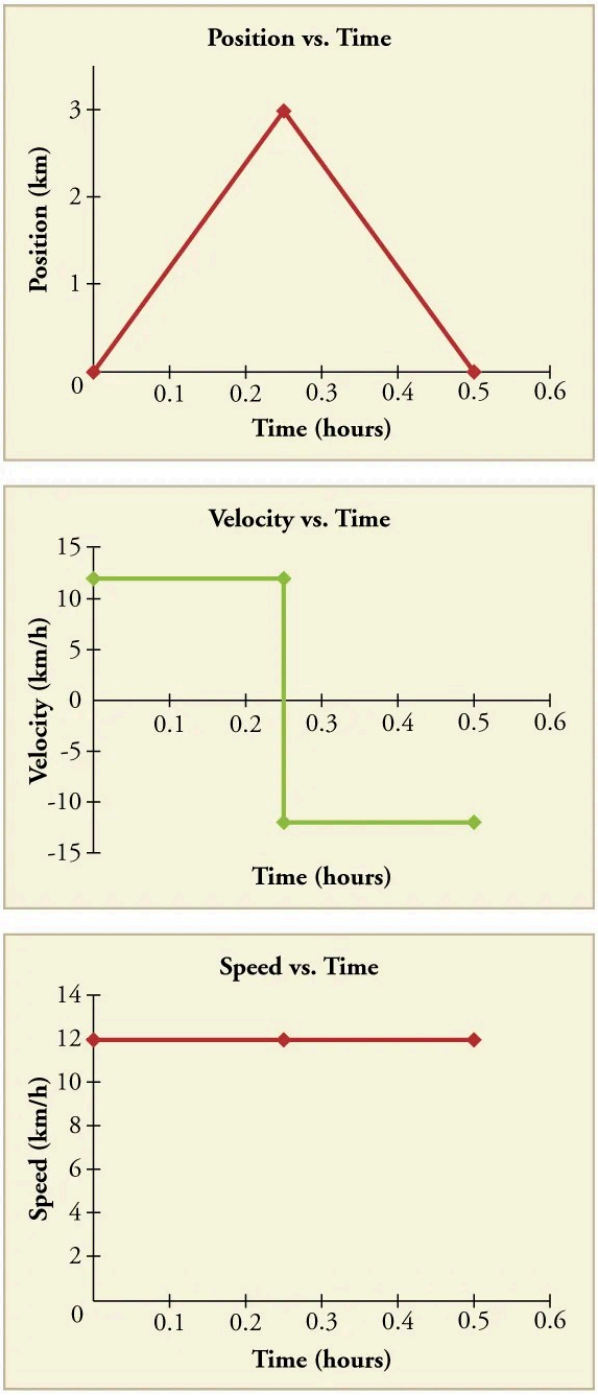


Figure 2.11 Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three graphs, each depicting different relationships over a time span of 0 to 0.6 hours.

1. Position vs. Time Graph:
– Content: A graph with position on the y-axis (in kilometers) and time on the x-axis (in hours).
– Description: The graph shows a triangular shape. Starting at the origin (0,0), the position increases linearly to 3 kilometers at 0.25 hours and then decreases back to 0 kilometers at 0.5 hours.

2. Velocity vs. Time Graph:
– Content: A graph with velocity on the y-axis (in kilometers per hour) and time on the x-axis (in hours).
– Description: The graph shows a step function. The velocity is constant at 12 km/h from 0 to 0.25 hours. At 0.25 hours, it drops sharply to -12 km/h and remains constant until 0.5 hours.

3. Speed vs. Time Graph:
– Content: A graph with speed on the y-axis (in kilometers per hour) and time on the x-axis (in hours).
– Description: The graph displays a constant speed. The speed is constant at 12 km/h throughout the entire duration from 0 to 0.5 hours.

These graphs illustrate the dynamics of motion in terms of position, velocity, and speed over a short time period.

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Check Your Understanding

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Click for Solution

Solution

(a) The average velocity of the train is zero because $x_f = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{\text{distance}}{\text{time}} = \frac{80 \text{ miles}}{105 \text{ minutes}}$$

2.8

$$\frac{80 \text{ miles}}{105 \text{ minutes}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 20 \text{ m/s}$$

2.9

"2.3 Time, Velocity, and Speed" from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

2.4 ACCELERATION

Learning Objectives

By the end of this section, you will be able to:

- Define and distinguish between instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.



Figure 2.12A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. Image from OpenStax College Physics 2e, CC-BY 4.0

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the acceleration, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Average Acceleration

Average Acceleration is *the rate at which velocity changes*,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0},$$

2.10

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are $\mathrm{m/s^2}$, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

Acceleration as a Vector

Acceleration is a vector in the same direction as the *change* in velocity, Δv. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as deceleration.



Figure 2.13 A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. Image from OpenStax College Physics 2e, CC-BY 4.0

Misconception Alert: Deceleration vs. Negative Acceleration

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider Figure 2.14.

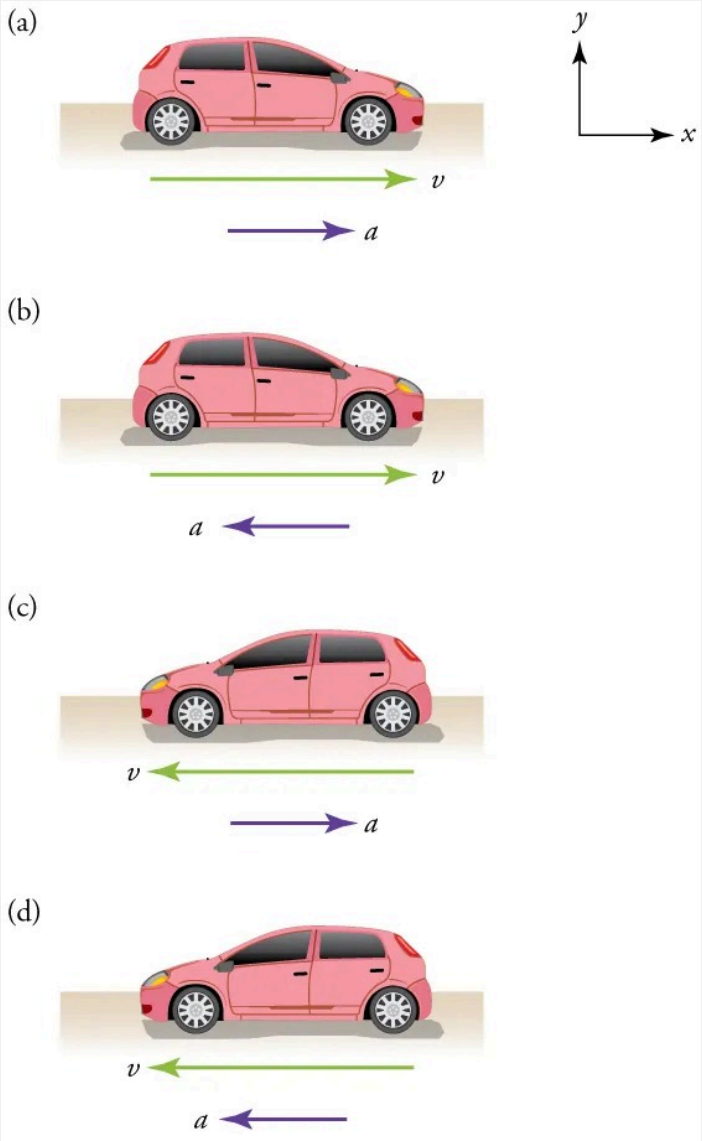


Figure 2.14(a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However,

because its acceleration is in the same direction as its motion, it is speeding up (not decelerating) Image *from* OpenStax College Physics 2e, CC-BY 4.0

Example 2.1

Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?



Figure 2.15 Image by The original uploader was Kevin Saff at English Wikipedia., Wikimedia Commons License

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

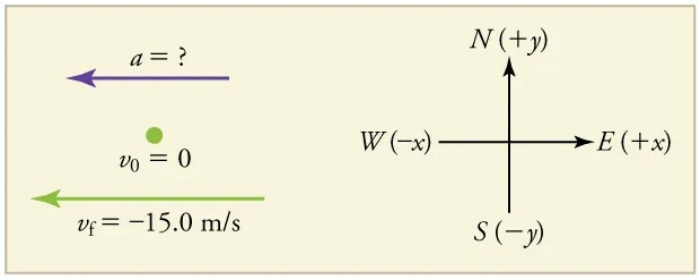


Figure 2.16 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is divided into two sections.

Left Section:

– There is a horizontal arrow pointing to the left, labeled with a purple label: "a = ?" This indicates an unknown acceleration directed to the left.

– Below this, there is a green dot with the label "v^0 = 0," indicating an initial velocity of zero.

– Beneath the dot, another horizontal arrow points to the left, labeled with a green label: "v◈ = -15.0 m/s," indicating a final velocity of -15.0 meters per second.

Right Section:

– A coordinate system is shown with four arrows:

– "N (+y)" indicates an upward direction (positive y-axis).

– "E (+x)" indicates a rightward direction (positive x-axis).

– "S (-y)" indicates a downward direction (negative y-axis).

– "W (-x)" indicates a leftward direction (negative x-axis).

We can solve this problem by identifying Δv and Δt from the given information and

then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0\ \text{m/s}$ (the negative sign indicates direction toward the west), $\Delta t = 1.80\ \text{s}$.

2. Find the change in velocity. Since the horse is going from zero to $-15.0\ \text{m/s}$, its change in velocity equals its final velocity: $\Delta v = v_f = -15.0\ \text{m/s}$.

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\ \text{m/s}}{1.80\ \text{s}} = -8.33\ \text{m/s}^2.$$

2.11

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of $8.33\ \text{m/s}^2$ due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as $8.33\ \text{m/s}^2$. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

Instantaneous acceleration a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in 2.3 Time, Velocity, and Speed—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. Figure 2.17 shows graphs of instantaneous acceleration versus time for two very different motions. In Figure 2.17(a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in

this case about $1.8 m/s^2$). In Figure 2.17(b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0 m/s^2$ and $-2.0 m/s^2$, respectively.

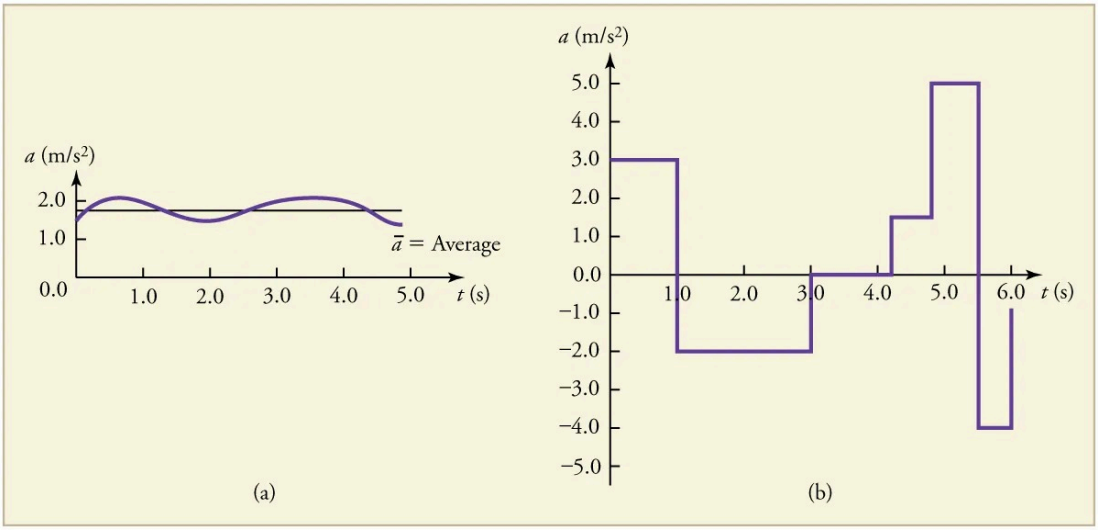


Figure 2.17Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two graphs labeled (a) and (b), side by side.

Graph (a):
– Axis Labels: The vertical axis is labeled as "a (m/s²)" representing acceleration in meters per second squared. The horizontal axis is labeled as "t (s)" representing time in seconds.
– Data Representation: The graph shows a sinusoidal wave fluctuating around a horizontal line labeled "ā = Average." The wave oscillates between approximately 0 and 2 m/s² over 6 seconds.
– Average Line: A horizontal line indicating the average value, running at approximately 1.0 m/s².

Graph (b):
– Axis Labels: Similar to graph (a), the vertical axis is labeled "a (m/s²)" and the horizontal axis is labeled "t (s)."

– Data Representation: This graph features a step function with varying values of acceleration over time:
– From 0 to 1 second, the acceleration is 3 m/s².
– From 1 to 3 seconds, the acceleration drops to -3 m/s².
– From 3 to 4 seconds, the acceleration is 0 m/s².
– From 4 to 5 seconds, the acceleration increases to 2 m/s².
– From 5 to 6 seconds, the acceleration is -5 m/s².

Both graphs depict acceleration versus time relationships but in different forms; one is a smooth wave, and the other is a series of steps.

The next several examples consider the motion of the subway train shown in Figure 2.18. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

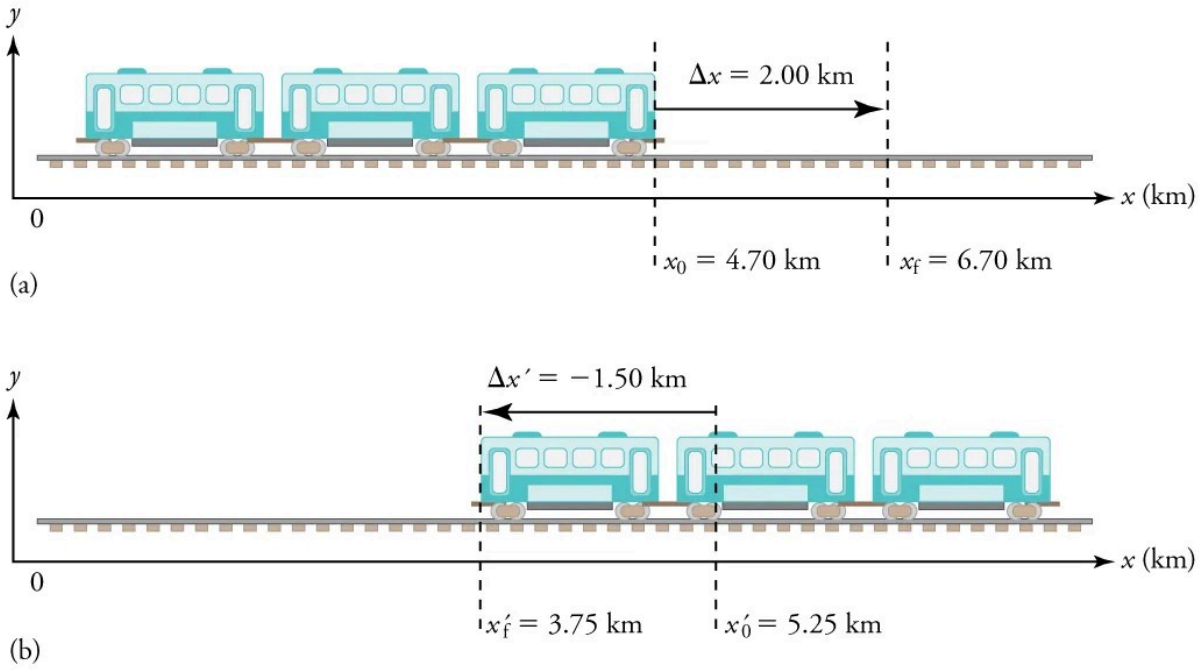


Figure 2.18One-dimensional motion of a subway train considered in , , , , , and . Here we have chosen the x-axis so that + means to the right and — means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 tox_f. Its displacement Δx is +2.0 km. (b) The train moves to the left from $(x')_0$ to $(x')_f$. Its displacement $\Delta x'$ is -1.5 km. Image from OpenStax College Physics 2e, CC-BY 4.0

Example 2.2

Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of Figure 2.18?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_f - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_f = 6.70\ \text{km}$ and $x_0 = 4.70\ \text{km}$ for part (a), and $(x')_f = 3.75\ \text{km}$ and $(x')_0 = 5.25\ \text{km}$ for part (b).

2. Solve for displacement in part (a).

$$\begin{aligned}\Delta x &= x_f - x_0 \\ &= 6.70\ \text{km} - 4.70\ \text{km} \\ &= +2.00\ \text{km}\end{aligned}$$

2.12

3. Solve for displacement in part (b).

$$\begin{aligned}\Delta x' &= (x')_f - (x')_0 \\ &= 3.75\ \text{km} - 5.25\ \text{km} \\ &= -1.50\ \text{km}\end{aligned}$$

2.13

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

Example 2.3

Comparing Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in Figure 2.18?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in Example 2.2. Distance traveled is the total length of the path traveled between the two positions. (See 2.1 Displacement.) In the case of the subway train shown in Figure 2.18, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was $-1.5\ \text{km}$. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

Example 2.4

Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in Figure 2.18(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:

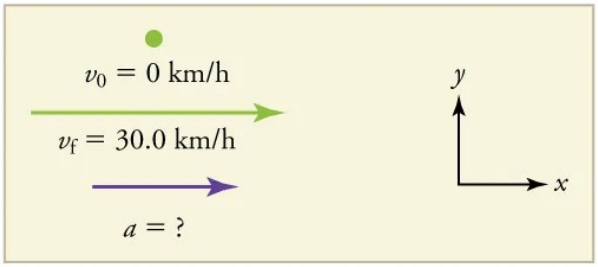


Figure 2.19 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a physics problem involving motion. On the left side, there are three expressions:

- v_0 = *0 km/h* with a green arrow pointing to the right.
- v_f = *30.0 km/h* with a purple arrow pointing to the right.
- *a = ?* indicating that acceleration is unknown, represented with a purple arrow.

There is a green dot above the expression for v_0.

On the right side, an x-y coordinate system is shown with axes labeled: 'x' for the horizontal axis pointing to the right, and 'y' for the vertical axis pointing upwards.

This problem involves three steps. First we must determine the change in velocity, then

we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_f = 30.0\ \text{km/h}$, and $\Delta t = 20.0\ \text{s}$.

2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0\ \text{km/h}$, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0\ \text{km/h}}{20.0 s}$$

2.14

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See 1.2 Physical Quantities and Units for more guidance.)

$$\bar{a} = \left(\frac{+30\ \text{km/h}}{20.0\ \text{s}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 0.417\ \text{m/s}^2$$

2.15

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

Example 2.5

Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in Figure 2.18(a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy

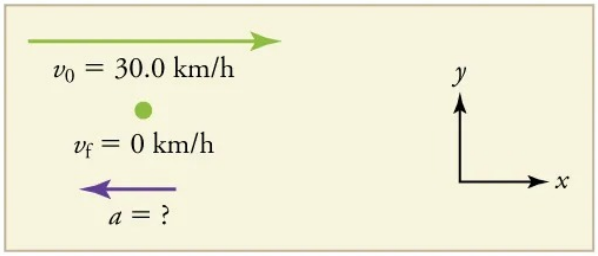


Figure 2.20 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image features a diagram with the following components:

– On the left side, there is a green horizontal arrow pointing to the right, labeled as $v_0 = 30.0$ *km/h*. This indicates the initial velocity.
– Below that, a green dot is placed, alongside which is written $v_f = 0$ *km/h* representing the final velocity.
– Further below, there is a purple horizontal arrow pointing to the left, labeled as $a = ?$. This suggests an unknown acceleration.
– On the right side of the diagram, there is a set of axes. The y-axis points upwards, and the x-axis points to the right. They are labeled with their respective directions.

The overall background of the image is a soft beige color, framed with a light border.

In this case, the train is decelerating and its acceleration is negative because it is toward

the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0\ \text{km/h}$, $v_f = 0km/h$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00\ \text{s}$.

2. Solve for the change in velocity, Δv.

$$\Delta v = v_f - v_0 = 0 - 30.0\ \text{km/h} = -30.0\ \text{km/h}$$

2.16

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0\ \text{km/h}}{8.00\ \text{s}}$$

2.17

4. Convert the units to meters and seconds.

$$\begin{aligned}\bar{a} &= \frac{\Delta v}{\Delta t} \\ &= \left(\frac{-30.0\ \text{km/h}}{8.00\ \text{s}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) \\ &= -1.04\ \text{m/s}^2.\end{aligned}$$

2.18

Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in Example 2.4 and Example 2.5 are

displayed in Figure 2.21. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)

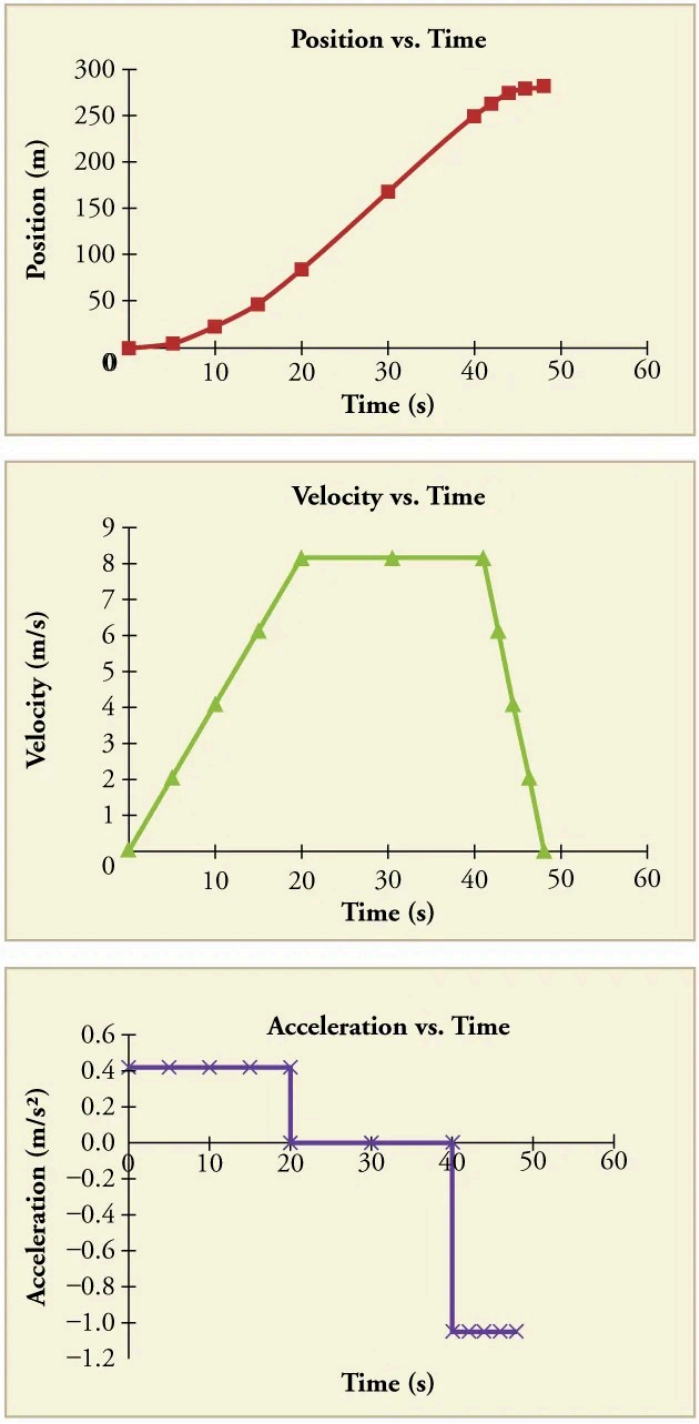


Figure 2.21(a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at

constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains three line graphs stacked vertically, each illustrating different aspects of motion over time.

1. Top Graph: Position vs. Time
– Description: This graph is a plot of position (in meters) against time (in seconds). The graph shows a red line with square markers that starts near the origin (0,0) and curves upwards, indicating an increase in position over time. The line has sections with varying steepness, suggesting changes in speed.
– X-axis: Labeled "Time (s)" and ranges from 0 to 60.
– Y-axis: Labeled "Position (m)" and ranges from 0 to 300.

2. Middle Graph: Velocity vs. Time
– Description: This graph shows velocity (in meters per second) plotted against time. A green line with triangular markers illustrates the velocity starting at zero, increasing to a peak around 5 m/s, remaining steady, and eventually decreasing back to zero.
– X-axis: Labeled "Time (s)" and ranges from 0 to 60.
– Y-axis: Labeled "Velocity (m/s)" and ranges from 0 to 9.

3. Bottom Graph: Acceleration vs. Time
– Description: The graph plots acceleration (in meters per second squared) against time. A purple line with cross markers shows segments of constant acceleration, beginning at 0.4, dropping to zero, and eventually decreasing to -1.0 before stabilizing.
– X-axis: Labeled "Time (s)" and ranges from 0 to 60.
– Y-axis: Labeled "Acceleration (m/s^2)" and ranges from -1.2 to 0.6.

Each graph provides a visual representation of different physical quantities describing motion as functions of time.

Example 2.6

Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of Example 2.2, and shown again below, if it takes 5.00 min to make its trip?

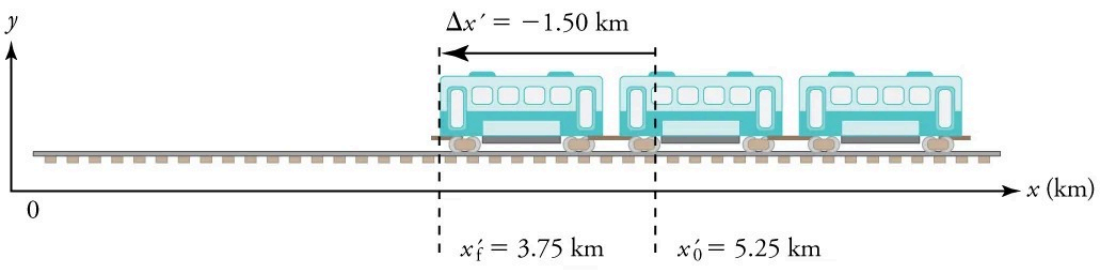


Figure 2.22 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of a train on a straight railway track. The train consists of three connected light blue carriages, and it is shown moving along the track in a negative x-direction.

– Axis Information:
– The horizontal axis is labeled with “x (km)” with an arrow pointing to the right, indicating increasing x-values.
– The vertical axis is labeled with “y” with an upward arrow marked from 0, indicating increasing y-values.

– Train Positioning:
– The train’s motion indicates a change in position from an initial to a final state.
– The initial position of the train is marked at $x'_0 = 5.25$ km.
– The final position of the train is marked at $x'_f = 3.75$ km.
– The change in position is marked as $\Delta x' = -1.50$ km, indicating a movement backward along the track.

– Additional Details:
– The positions x'_0 and x'_f are marked on the track with dashed vertical lines extending upwards to highlight the positions of the train.

The diagram serves to visually represent the displacement of the train along the track over a specific distance.

Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns. $\left(x'\right)_f = 3.75\ \text{km}$, $\left(x'\right)_0 = 5.25\ \text{km}$, $\Delta t = 5.00\ \text{min}$.

2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be $-1.5\ \text{km}$ in Example 2.2.

3. Solve for average velocity.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50\ \text{km}}{5.00\ \text{min}}$$

2.19

4. Convert units.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \left(\frac{-1.50\ \text{km}}{5.00\ \text{min}}\right)\left(\frac{60\ \text{min}}{1h}\right) = -18.0\ \text{km/h}$$

2.20

Discussion

The negative velocity indicates motion to the left.

Example 2.7

Calculating Deceleration: The Subway Train

Finally, suppose the train in Figure 2.22 slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:

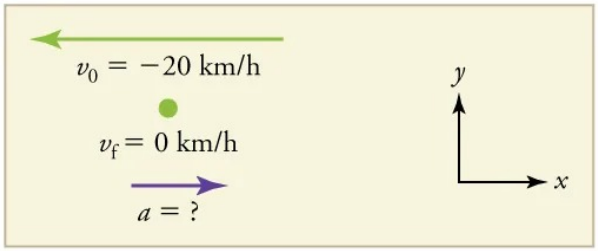


Figure 2.23 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of the following elements:

– Vectors and Equations:

– A green arrow pointing to the left labeled with the initial velocity equation: $v_0 = -20\,\mathrm{km/h}$.

– A point labeled $v_f = 0\,\mathrm{km/h}$, indicating the final velocity.

– A purple arrow pointing to the right labeled with the acceleration equation: $a = ?$.

– Coordinate Axes:

– On the right side, there is a set of coordinate axes. The horizontal axis is labeled as x and the vertical axis is labeled as y.

As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20.0\ \text{km/h}$,
$v_f = 0 km/h$,
$\Delta t = 10.0 s$.

2. Calculate Δv. The change in velocity here is actually positive, since

$$\begin{aligned}\Delta v &= v_f - v_0 \\ &= 0 - (-20\ \text{km/h}) \\ &= +20.0\ \text{km/h}.\end{aligned}$$

2.21

3. Solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0\text{km/h}}{10.0\text{s}}$$

2.22

4. Convert units.

$$\begin{aligned}\bar{a} &= \left(\frac{+20.0\text{km/h}}{10.0\text{s}}\right)\left(\frac{10^3 m}{1 km}\right)\left(\frac{1 h}{3600\ \text{s}}\right) \\ &= +0.556\text{m/s}^2\end{aligned}$$

2.23

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in Example 2.5, this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in Example 2.7, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in Figure 2.22 is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

Check Your Understanding

An airplane lands on a runway traveling east. Describe its acceleration.

Click for Solution

Solution

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

PhET Explorations

Moving Man Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

Click to view content.

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2.5 MOTION EQUATIONS FOR CONSTANT ACCELERATION IN ONE DIMENSION

Learning Objectives

By the end of this section, you will be able to:

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.



Figure 2.24 Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. Image from OpenStax College Physics 2e, CC-BY 4.0

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_f - t_0$, taking $t_0 = 0$ means that $\Delta t = t_f$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 *is the initial position* and v_0 *is the initial velocity*. We put no subscripts on the final values. That is, t *is the final time*, x *is the final position*, and v *is the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\begin{aligned} \Delta t &= t \\ \Delta x &= x - x_0 \\ \Delta v &= v - v_0 \end{aligned}$$

2.24

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = \text{constant},$$

2.25

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

Solving for Displacement (Δx) and Final Position (x) from Average Velocity when Acceleration (a) is Constant

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

2.26

Substituting the simplified notation for Δx and Δt yields

$$\bar{v} = \frac{x - x_0}{t}.$$

2.27

Solving for x yields

$$x = x_0 + \bar{v}t,$$

2.28

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2} (\text{constant } a).$$

2.29

The equation $\bar{v} = \frac{v_0 + v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30 \text{ km/h} + 60 \text{ km/h}}{2} = 45 \text{ km/h},$$

2.30

which seems logical.

Example 2.8

Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.

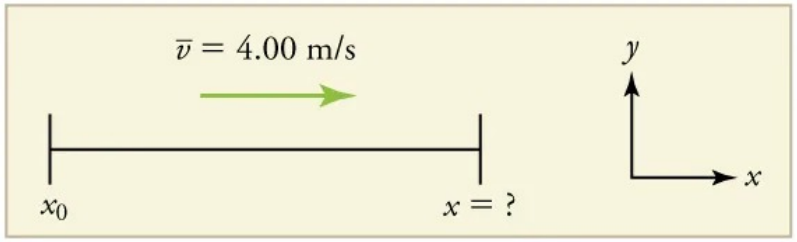


Figure 2.25 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram representing motion along a straight path, depicted in a horizontal plane with an x-y coordinate system for orientation.

On the left side of the diagram, there is a horizontal line segment, representing a path with two endpoints:

- The left endpoint is labeled x_0, indicating the initial position.
- The right endpoint is labeled $x = ?$, symbolizing the final position which is unknown.

An arrow is drawn above the line segment pointing from left to right, symbolizing motion in the positive x-direction. Above the arrow, the velocity is specified as $\bar{v} = 4.00$ *m/s*, indicating constant speed.

To the right of the path, there is an x-y coordinate system. The x-axis is parallel to the path, extending horizontally to the right, and the y-axis is vertical, pointing upwards.

The final position x is given by the equation

$$x = x_0 + \bar{v}t.$$

2.31

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v} = 4.00\ \text{m/s}$, $\Delta t = 2.00\text{min}$, and $x_0 = 0\ \text{m}$.

2. Enter the known values into the equation.

$$x = x_0 + \bar{v}t = 0 + (4.00\ \text{m/s})\,(120\ \text{s}) = 480\ \text{m}$$

2.32

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\left(\bar{v}\right)^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.

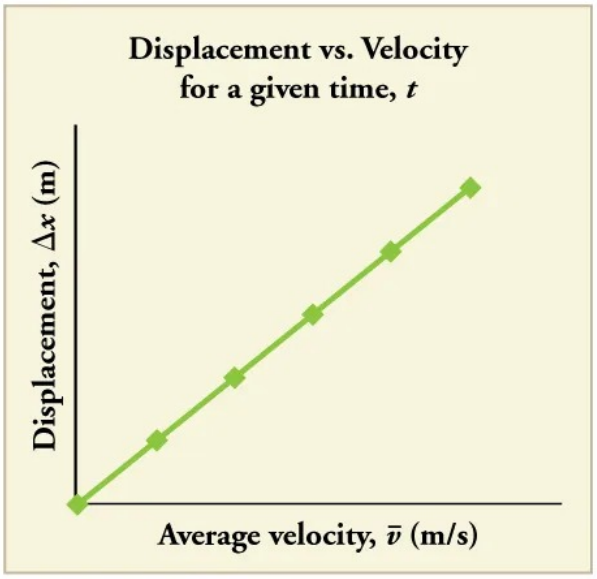


Figure 2.26 There is a linear relationship between displacement and average velocity. For a given time t*, an object moving twice as fast as another object will move twice as far as the other object. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a graph titled "Displacement vs. Velocity for a given time, t". The graph has two axes. The vertical axis is labeled "Displacement, Δx (m)" and the horizontal axis is labeled "Average velocity, $\bar{v}$(m/s)".

The graph shows a linear relationship between displacement and average velocity, represented by a green line that passes through a series of green diamond-shaped data points. The line starts from the origin at the lower-left corner and slopes upwards to the right, indicating a direct proportionality between the two variables.

Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

$$a = \frac{\Delta v}{\Delta t}$$

2.33

Substituting the simplified notation for Δv and Δt gives us

$$a = \frac{v - v_0}{t} (\text{constant } a).$$

2.34

Solving for v yields

$$v = v_0 + \text{at } (\text{constant } a).$$

2.35

Example 2.9

Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at $1.50\ \mathrm{m/s^2}$ for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.

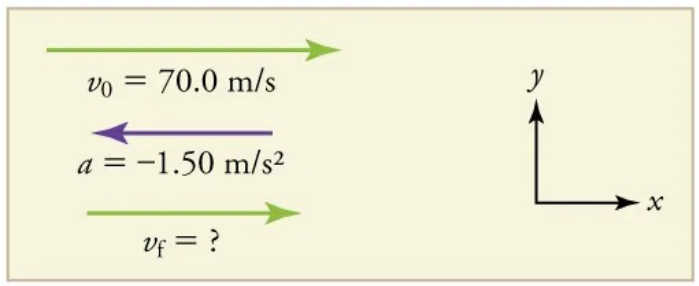


Figure 2.27 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating a physics problem involving motion. It contains the following elements:

- A green arrow pointing to the right with the text $v_0 = 70.0$ *m/s* next to it, indicating the initial velocity of an object is 70.0 meters per second.
- A purple arrow pointing to the left with the text $a = -1.50$ *m/s*^{2} next to it, indicating the acceleration of the object is -1.50 meters per second squared.
- A green arrow pointing to the right with the text $v_f = ?$, indicating the final velocity is unknown.

On the right side of the image, there is a small coordinate system with the axes labeled *x* for horizontal and *y* for vertical, both arrows pointing in the positive direction.

Solution

1. Identify the knowns. $v_0 = 70.0\ \text{m/s}$, $a = -1.50\ \text{m/s}^2$, $t = 40.0\text{s}$.

2. Identify the unknown. In this case, it is final velocity, v_f.

3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + \text{at}$.

4. Plug in the known values and solve.

$$\begin{aligned} v &= v_0 + \text{at} \\ &= 70.0\ \text{m/s} + \left(-1.50\ \text{m/s}^2\right)(40.0\ \text{s}) \\ &= 10.0\ \text{m/s} \end{aligned}$$

2.36

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.



Figure 2.28 The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts two illustrations of an airplane showing its motion with corresponding initial and final velocities, along with acceleration.

– On the left side, the airplane is shown with a label indicating the initial time, $t_0 = 0$. The initial velocity is labeled as $v_0 = 70.0$ m/s, with a green arrow pointing to the right, indicating the direction of motion. A purple arrow is pointing to the left, labeled with an acceleration of $a = -1.5$ m/s^2, indicating deceleration.

– On the right side, the same airplane is shown at a later time, labeled as $t = 40.0$ s. The final velocity is labeled as $v = 10.0$ m/s with a green arrow still pointing to the right. A similar purple arrow pointing to the left indicates the same deceleration with $a = -1.5$ m/s^2.

Overall, the illustration demonstrates a decelerating motion of an airplane from a higher initial velocity to a lower final velocity over a specific time period.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity $(v = v_0)$, as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

Making Connections: Real-World Connection



Endeavor

Figure 2.29 The Space Shuttle blasts off from the Kennedy Space Center in February 2010. Image from OpenStax College Physics 2e, CC-BY 4.0

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

Solving for Final Position When Velocity is Not Constant ($a \neq 0$)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v = v_0 + \mathrm{at}.$$

2.37

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}\mathrm{at}.$$

2.38

Since $\frac{v_0 + v}{2} = \bar{v}$ for constant acceleration, then

$$\bar{v} = v_0 + \frac{1}{2}\mathrm{at}.$$

2.39

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding

$$x = x_0 + v_0 t + \frac{1}{2}\mathrm{at}^2 \,(\text{constant } a\,).$$

2.40

Example 2.10

Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of $26.0\ \mathrm{m/s^2}$. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?



Figure 2.30 U.S. Army Top Fuel pilot Tony "The Sarge" Schumacher begins a race with a controlled burnout. Image by Lt. Col. William Thurmond. Photo Courtesy of U.S. Army., Wikimedia Commons License

Strategy

Draw a sketch.

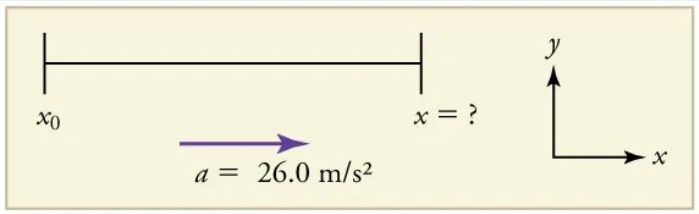


Figure 2.31 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a physics problem involving motion. It includes a horizontal line with two vertical ticks at both ends. The left tick is labeled x_0, and the right tick is labeled $x = ?$. An arrow pointing to the right is positioned below the line, indicating acceleration. The equation $a = 26.0\ m/s^2$ is written beside the arrow, specifying the acceleration value. On the right side of the image, a coordinate system is shown with y indicated as the vertical axis and x as the horizontal axis.

We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0t + \frac{1}{2}\text{at}^2$once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as 26.0m/s^2 and t is given as 5.56 s.

2. Plug the known values into the equation to solve for the unknown x:

$$x = x_0 + v_0t + \frac{1}{2}\text{at}^2.$$

2.41

Since the initial position and velocity are both zero, this simplifies to

$$x = \frac{1}{2}\text{at}^2.$$

2.42

Substituting the identified values of a and t gives

$$x = \frac{1}{2}\left(26.0\ \text{m/s}^2\right)\left(5.56\ \text{s}\right)^2,$$

2.43

yielding

$$x = 402\ \text{m}.$$

2.44

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2}\text{at}^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In Example 2.10, the dragster covers only one fourth of the total distance in the first half of the elapsed time
- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0 t + \frac{1}{2}\text{at}^2$ becomes $x = x_0 + v_0 t$

Solving for Final Velocity when Velocity Is Not Constant ($a \neq 0$)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + \mathbf{at}$ for t, we get

$$t = \frac{v - v_0}{a}.$$

2.45

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

$$v^2 = v_0^2 + 2a\,(x - x_0)\,(\mathbf{constant}a\,).$$

2.46

Example 2.11

Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in Example 2.10 without using information about time.

Strategy

Draw a sketch.

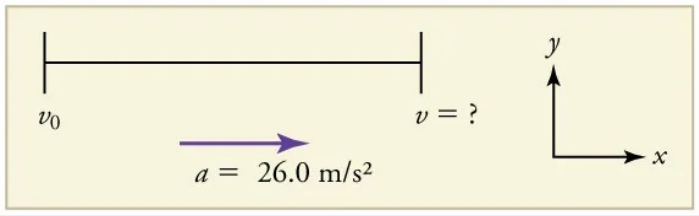


Figure 2.32 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains a physics diagram showing an object's motion. On the left, there's an initial velocity denoted as v_0, represented with a short vertical line. To its right, there's a longer horizontal line ending with another vertical line, indicating the final velocity v, which is unknown. Below the horizontal line, there's an arrow pointing to the right, labeled with the acceleration $a = 26.0\,\mathrm{m/s^2}$. On the right side of the image, there is a set of axes with "x" labeled on the horizontal axis and "y" on the vertical axis. The background is light-colored.

The equation $v^2 = v_0^2 + 2a\,(x - x_0\,)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402\ \text{m}$ (this was the answer in Example 2.10). Finally, the average acceleration was given to be $a = 26.0\ \text{m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a\left(x - x_0\right)$ and solve for v.

$$v^2 = 0 + 2\left(26.0\ \text{m/s}^2\right)(402\ \text{m}).$$

2.47

Thus

$$v^2 = 2.09 \times 10^4 \text{m}^2/\text{s}^2.$$

2.48

To get v, we take the square root:

$$v = \sqrt{2.09 \times 10^4\ \text{m}^2/\text{s}^2} = 145\ \text{m/s}.$$

2.49

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a\left(x - x_0\right)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

Summary of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t$$

2.50

$$\bar{v} = \frac{v_0 + v}{2}$$

2.51

$$v = v_0 + \mathrm{at}$$

2.52

$$x = x_0 + v_0 t + \frac{1}{2}\mathrm{at}^2$$

2.53

$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$

2.54

Example 2.12

Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of $7.00\ \mathrm{m/s^2}$, whereas on wet concrete it can decelerate at only $5.00\ \mathrm{m/s^2}$. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.

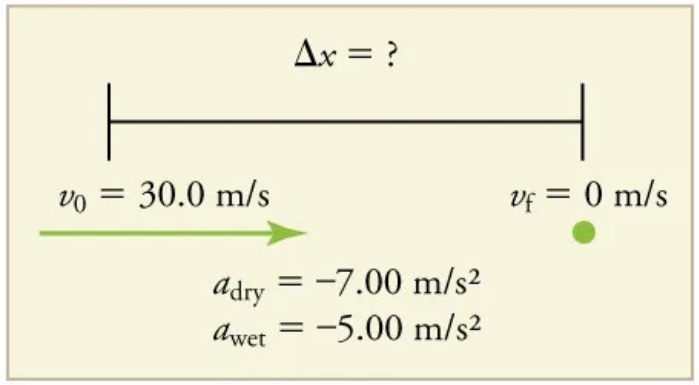


Figure 2.33 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a physics problem involving motion and acceleration. It shows an object with initial velocity and final velocity, asking for the displacement (Δx).

The elements in the image are as follows:

- Initial velocity (v_0) is 30.0 m/s.

- Final velocity (v_f) is 0 m/s.
- Acceleration on dry surface (a_{dry}) is -7.00 m/s2.
- Acceleration on wet surface (a_{wet}) is -5.00 m/s2.

There is an arrow pointing leftward, indicating the direction of motion, aligned with the initial velocity.

In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0\ \mathrm{m/s}$; $v = 0$; $a = -7.00 \mathrm{m/s^2}$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

$$v^2 = v_0^2 + 2a\,(x - x_0).$$

2.55

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x.

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

2.56

4. Enter known values.

$$x - 0 = \frac{0^2 - (30.0\ \mathrm{m/s})^2}{2\,(-7.00\ \mathrm{m/s^2})}$$

2.57

Thus,

$x = 64.3 \text{ m on dry concrete.}$

2.58

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is -5.00 m/s^2. The result is

$x_{\text{wet}} = 90.0 \text{ m on wet concrete.}$

2.59

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0 \text{ m/s}$; $t_{\text{reaction}} = 0.500\text{s}$; $a_{\text{reaction}} = 0$. We take $x_{0-\text{reaction}}$ to be 0. We are looking for x_{reaction}.

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.

3. Plug in the knowns to solve the equation.

$x = 0 + (30.0 \text{ m/s})(0.500 \text{ s}) = 15.0 \text{ m.}$

2.60

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}}$$

2.61

a. 64.3 m + 15.0 m = 79.3 m when dry
b. 90.0 m + 15.0 m = 105 m when wet

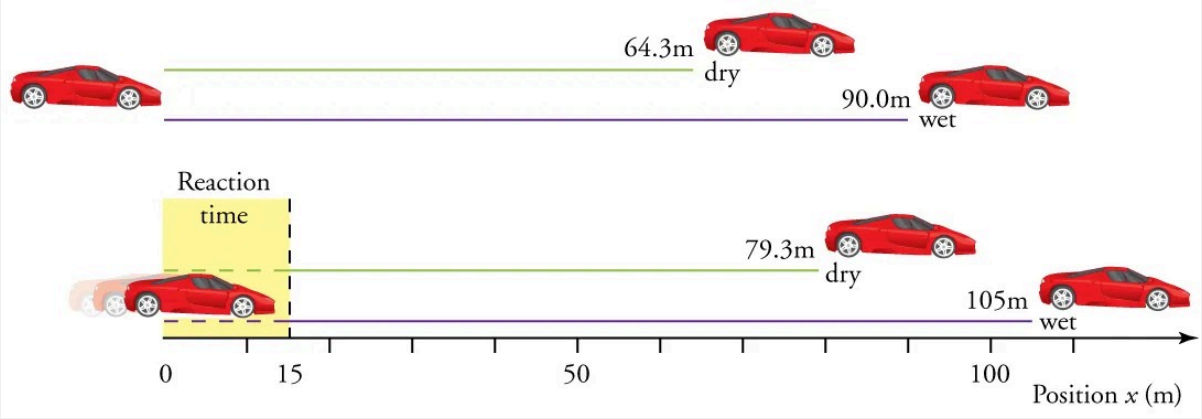


Figure 2.34 The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration showing the stopping distances of a red sports car on dry and wet roads. It depicts two scenarios with labeled stopping distances in meters.

In the upper part of the image:

- Stops at 64.3 meters on a dry road.
- Stops at 90.0 meters on a wet road.

In the lower part of the image:

- Stops at 79.3 meters on a dry road.
- Stops at 105 meters on a wet road.

The bottom of the image shows an axis marked as “Position x (m)” ranging from 0 to 100 meters. There is also a section labeled “Reaction time” at the start, depicted with a yellow box next to a transparent red car moving forward to a solid image of the car at the beginning of the measurement line.

Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

Example 2.13

Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at $2.00\ \mathrm{m/s^2}$, how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

Draw a sketch.

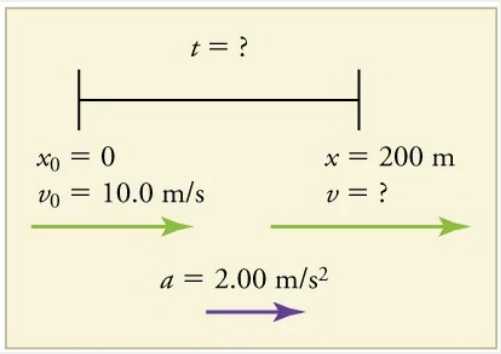


Figure 2.35 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a physics problem illustration involving motion. It is set against a light beige background and includes several equations and annotations:

– At the top center, there is a horizontal line bracket labeled with “t = ?” to indicate that the time is unknown.
– On the left side of the line, it is labeled with:
– $x_0 = 0$
– $v_0 = 10.0\ \mathrm{m/s}$
– This section also features a green arrow pointing to the right, symbolizing initial

velocity.
– On the right side, it is labeled with:
– $x = 200\,\text{m}$
– $v = ?$
– This portion also has a green arrow pointing to the right, depicting motion towards the final position.
– Centered below the line is the acceleration given by:
– $a = 2.00\,\text{m/s}^2$
– An accompanying purple arrow pointing to the right illustrates the direction of acceleration.

The problem is asking to determine the unknown final velocity (v) and the time (t) while the object moves from position x_0 to position x under constant acceleration.

We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

Solution

1. Identify the knowns and what we want to solve for. We know that $v_0 = 10\,\text{m/s}$; $a = 2.00\,\text{m/s}^2$; and $x = 200\,\text{m}$.

2. We need to solve for t. Choose the best equation. $x = x_0 + v_0 t + \frac{1}{2}\text{at}^2$ works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

$$200\,\text{m} = 0\,\text{m} + (10.0\,\text{m/s})t + \frac{1}{2}\left(2.00\,\text{m/s}^2\right)t^2$$

2.62
4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t = t\text{s}$, where t is the magnitude of time and s is the unit. Doing so leaves

$$200 = 10t + t^2.$$

2.63

5. Use the quadratic formula to solve for t.

(a) Rearrange the equation to get 0 on one side of the equation.

$$t^2 + 10t - 200 = 0$$

2.64

This is a quadratic equation of the form

$$\mathrm{at}^2 + \mathrm{bt} + c = 0,$$

2.65

where the constants are $a = 1.00, b = 10.0, \mathrm{and} c = -200$.

(b) Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4\mathrm{ac}}}{2a}.$$

2.66

This yields two solutions for t, which are

$$t = 10.0 \mathrm{and} -20.0.$$

2.67

In this case, then, the time is $t = t$ in seconds, or

$$t = 10.0 \mathrm{sand} -20.0 \mathrm{s}.$$

2.68

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

$$t = 10.0 \mathrm{s}.$$

2.69

Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. Problem-Solving Basics discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v/\Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

Check Your Understanding

A rocket accelerates at a rate of $20\ \mathrm{m/s^2}$ during launch. How long does it take the rocket to reach a velocity of 400 m/s?

Click for Solution

Solution

To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.

$$v = v_0 + \mathrm{at}$$

2.70

Rearrange to solve for t.

$$t = \frac{v - v_0}{a} = \frac{400\ \mathrm{m/s} - 0\ \mathrm{m/s}}{20\ \mathrm{m/s^2}} = 20\ \mathrm{s}$$

2.71

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2.6 PROBLEM-SOLVING BASICS FOR ONE-DIMENSIONAL KINEMATICS

Learning Objectives

By the end of this section, you will be able to:

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.

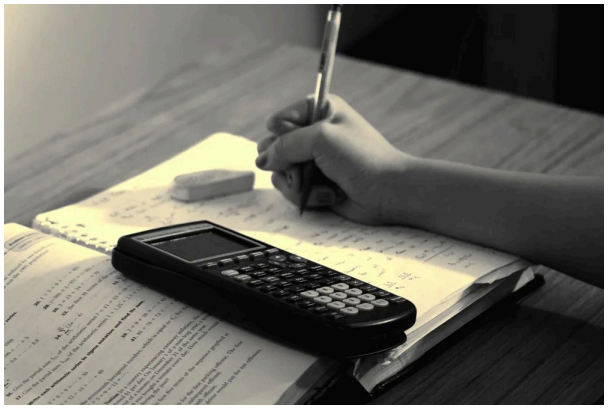


Figure 2.36 Problem-solving skills are essential to your success in Physics. Image from OpenStax College Physics 2e, CC-BY 4.0

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and

problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, "stopped" means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns

must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at $0.40\ \text{m/s}^2$ for 100 s, his final speed will be 40 m/s (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable

does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$v = v_0 + \text{at} = 0 + \left(0.40\text{m/s}^2\right)(100\text{s}) = 40\text{m/s}.$$
2.72

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, ...? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$\left(\frac{40\text{ m}}{s}\right)\left(\frac{3.28\text{ ft}}{m}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{60\text{ s}}{\text{min}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = 89\text{ mph}$$
2.73

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at 0.40 m/s^2, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of 0.40 m/s^2 for 100 s (almost two minutes).

"2.6 Problem-Solving Basics for One-Dimensional Kinematics" from College Physics 2e by OpenStax is

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2.7 FALLING OBJECTS

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.

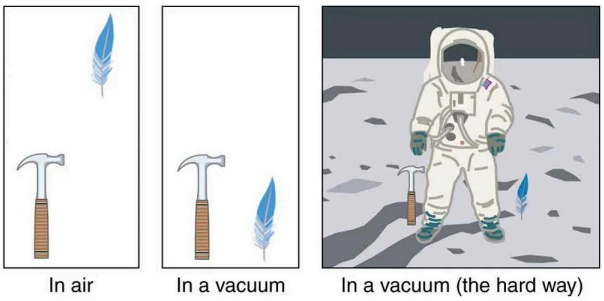


Figure 2.37 A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only $1.67\ \mathrm{m/s^2}$. *Image from OpenStax College Physics 2e, CC-BY 4.0*

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in free-fall.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the acceleration due to gravity. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, g. It is constant at any given location on Earth and has the average value

$$g = 9.80\ \mathrm{m/s^2}.$$
2.74

Although g varies from $9.78\ \mathrm{m/s^2}$ to $9.83\ \mathrm{m/s^2}$, depending on latitude, altitude, underlying geological formations, and local topography, the average value of $9.80\ \mathrm{m/s^2}$ will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80\ \mathrm{m/s^2}$, and if we define the downward direction as positive, then $a = g = 9.80\ \mathrm{m/s^2}$.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

Kinematic Equations for Objects in Free-Fall where Acceleration $= -g$

$$v = v_0 - \mathrm{gt}$$

2.75

$$y = y_0 + v_0 t - \frac{1}{2}\mathrm{gt}^2$$

2.76

$$v^2 = v_0^2 - 2g\left(y - y_0\right)$$

2.77

Example 2.14

Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.

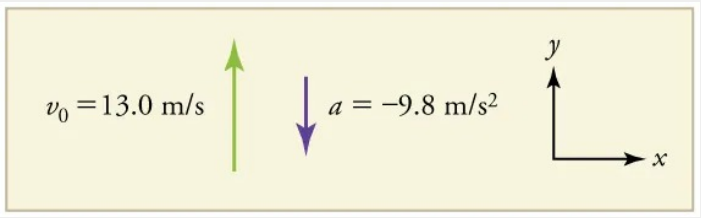


Figure 2.38 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration displaying initial velocity and acceleration in a two-dimensional coordinate system. On the left, there's a mathematical notation indicating the initial velocity:

$v_0 = 13.0$ m/s

accompanied by a green arrow pointing upwards, representing the positive initial velocity direction. Next to it, there's an equation for acceleration:

$a = -9.8$ m/s^2

paired with a purple arrow pointing downwards, illustrating the negative acceleration due to gravity. On the right side of the image, there's a two-dimensional coordinate system with horizontal and vertical axes labeled as *x* and *y* respectively. The *y*-axis is vertical, and the *x*-axis is horizontal.

We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as
y_1 and v_1; y_2 and
v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$;
$a = -g = -9.80\ \text{m/s}^2$; and
$t = 1.00\ \text{s}$.

2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2}\text{at}^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.

3. Plug in the known values and solve for y_1.

$$\begin{aligned} y_1 &= 0 + (13.0\ \text{m/s})\,(1.00\ \text{s}) + \frac{1}{2}\left(-9.80\text{m/s}^2\right)(1.00\ \text{s})^2 \\ &= 8.10\text{m} \end{aligned}$$

2.78

Discussion

The rock is 8.10 m above its starting point at $t = 1.00$ s, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity

v_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$; $a = -g = -9.80\ \text{m/s}^2$; and $t = 1.00\ \text{s}$. We also know from the solution above that $y_1 = 8.10\ \text{m}$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - \text{gt}$ (from $v = v_0 + \text{at}$, where $a = \text{gravitational acceleration} = -g$).

3. Plug in the knowns and solve.

$$\begin{aligned} v_1 &= v_0 - \text{gt} \\ &= 13.0\ \text{m/s} - \left(9.80\ \text{m/s}^2\right)(1.00\ \text{s}) \\ &= 3.20\ \text{m/s} \end{aligned}$$

2.79

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00\text{s}$. However, it has slowed from its original 13.0 m/s, as expected.

Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00\text{s}$ and $3.00\ \text{s}$ are the same as those above. The results are summarized in Table 2.1 and illustrated in Figure 2.39.

Table 2.1 Results

Time, t	Position, y	Velocity, v	Acceleration, a
$1.00\ \text{s}$	$8.10\ \text{m}$	$3.20\ \text{m/s}$	$-9.80\ \text{m/s}^2$
$2.00\ \text{s}$	$6.40\ \text{m}$	$-6.60\ \text{m/s}$	$-9.80\ \text{m/s}^2$
$3.00\ \text{s}$	$-5.10\ \text{m}$	$-16.4\ \text{m/s}$	$-9.80\ \text{m/s}^2$

Graphing the data helps us understand it more clearly.

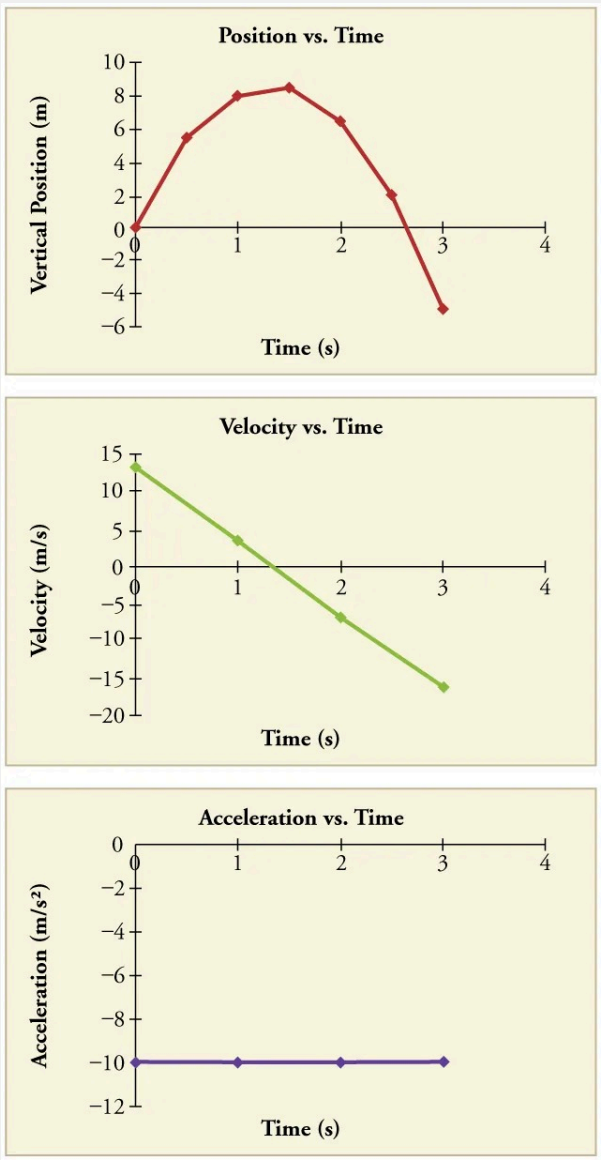


Figure 2.39 Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. Misconception Alert! Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is time, not space. The actual path of the rock in space is straight up, and straight down. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains three separate graphs, each depicting a different aspect of motion over time. Below is a description of each graph:

1. Position vs. Time Graph
– The vertical axis represents vertical position in meters, ranging from -6 to 10.
– The horizontal axis represents time in seconds, ranging from 0 to 4.
– The graph shows a red curve starting at position 0 at time 0, peaking at approximately 9 meters at around 1.5 seconds, and then returning to 0 at around 3 seconds, before moving downwards to around -6 meters at 4 seconds.

2. Velocity vs. Time Graph
– The vertical axis represents velocity in meters per second (m/s), ranging from -20 to 15.
– The horizontal axis represents time in seconds, ranging from 0 to 4.
– The graph shows a green straight line descending from 15 m/s at 0 seconds, reaching 0 m/s at 3 seconds, and continuing to -15 m/s at 4 seconds.

3. Acceleration vs. Time Graph
– The vertical axis represents acceleration in meters per second squared (m/s_2), ranging from -12 to 0.
– The horizontal axis represents time in seconds, ranging from 0 to 4.
– The graph shows a constant purple line at approximately -10 m/s_2 throughout the time period from 0 to 4 seconds.

These graphs illustrate an object in motion, with changes in its position, velocity, and constant acceleration over a span of 4 seconds.

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.33 s), its velocity is zero, but its acceleration is still $-9.80\ \mathrm{m/s^2}$. Its acceleration is

$-9.80\ \mathrm{m/s^2}$ for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

Making Connections: Take-Home Experiment—Reaction Time

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

Example 2.15

Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.

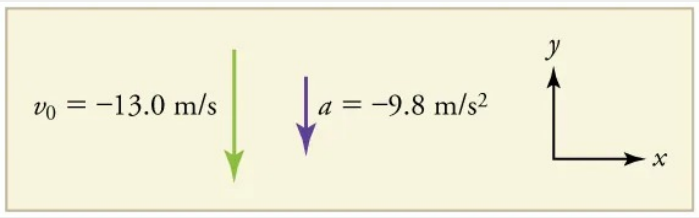


Figure 2.40 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a physics problem involving motion. On the left, there are two equations:

- $v_0 = -13.0$ *m/s*, with an accompanying green downward arrow, representing the initial velocity.
- $a = -9.8$ *m/s*^{2}, with an accompanying purple downward arrow, representing the acceleration, likely due to gravity.

On the right side, there is a coordinate system with *x* and *y* axes. The *y*-axis points

upward, and the *x*-axis points to the right. Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0$;
$y_1 = -5.10\ \text{m}$;
$v_0 = -13.0\ \text{m/s}$;
$a = -g = -9.80\ \text{m/s}^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a\,(y - y_0\,)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

$$\begin{aligned} v^2 &= (-13.0\ \text{m/s})^2 + 2\left(-9.80\ \text{m/s}^2\right)(-5.10\ \text{m} - 0\ \text{m}) \\ &= 268.96\ \text{m}^2/\text{s}^2, \end{aligned}$$

2.80

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

$$v = \pm 16.4\ \text{m/s}.$$

2.81

The negative root is chosen to indicate that the rock is still heading down. Thus,

$$v = -16.4\ \text{m/s}.$$

2.82

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed.* (See Example 2.14 and Figure 2.41(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from Example 2.14) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20\ \mathrm{m/s}$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.

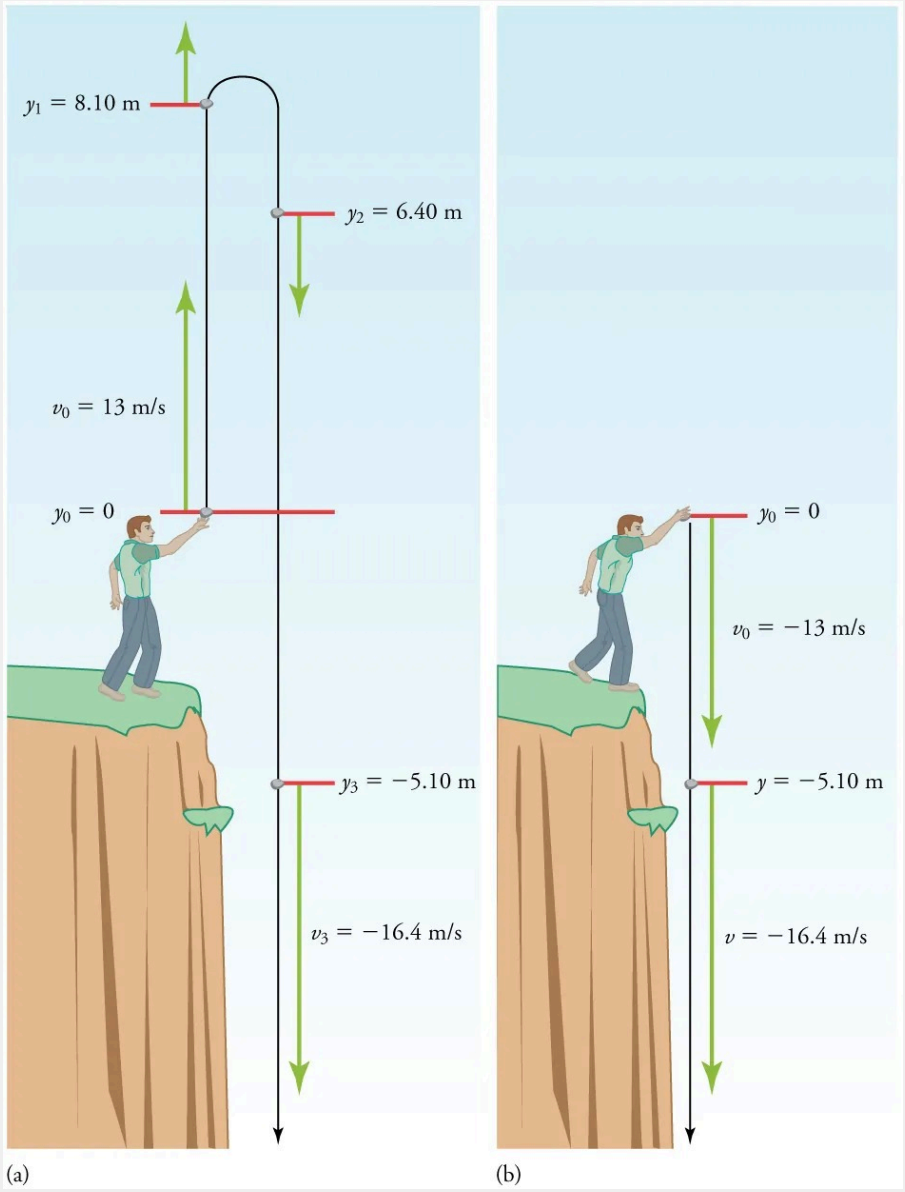


Figure 2.41 (a) A person throws a rock straight up, as explored in . The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in . Note that at the same distance below the point of release, the rock has the same velocity in both cases. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two panels, labeled (a) and (b), depicting the motion of a ball

thrown from the edge of a cliff. Both panels illustrate different scenarios of projectile motion.

Panel (a):

– A person stands at the edge of a cliff and throws a ball upwards.
– The ball follows a path that forms an arc, going upwards and then downwards.
– Initial position at the cliff edge is marked as $y_0 = 0$.
– The ball is thrown with an initial velocity $v_0 = 13\,\mathrm{m/s}$ upwards.
– The highest point of the ball's arc is labeled as $y_1 = 8.10\,\mathrm{m}$.
– The ball descends to a height of $y_2 = 6.40\,\mathrm{m}$ before continuing its downward trajectory.
– The ball reaches a point below the cliff at $y_3 = -5.10\,\mathrm{m}$, with a velocity $v_3 = -16.4\,\mathrm{m/s}$ downwards.

Panel (b):

– A person stands at the edge of the same cliff and throws a ball directly downwards.
– The initial position at the cliff edge is marked as $y_0 = 0$.
– The ball is thrown with an initial velocity $v_0 = -13\,\mathrm{m/s}$ downwards.
– The ball continues its downward path directly reaching $y = -5.10\,\mathrm{m}$, with a velocity $v = -16.4\,\mathrm{m/s}$ downwards.

Both panels illustrate the concept of vertical motion with initial velocities and different trajectories. Arrow indicators show the direction of the ball's motion, with annotations providing details about height and velocity.

Another way to look at it is this: In Example 2.14, the rock is thrown up with an initial velocity of $13.0\ \mathrm{m/s}$. It rises and then falls back down. When its position is $y = 0$ on its way back down, its velocity is $-13.0\ \mathrm{m/s}$. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10\ \mathrm{m}$ to be the same whether we have thrown it upwards at $+13.0\ \mathrm{m/s}$ or thrown it downwards at $-13.0\ \mathrm{m/s}$. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

Example 2.16

Find *g* from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, Figure 2.42. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.

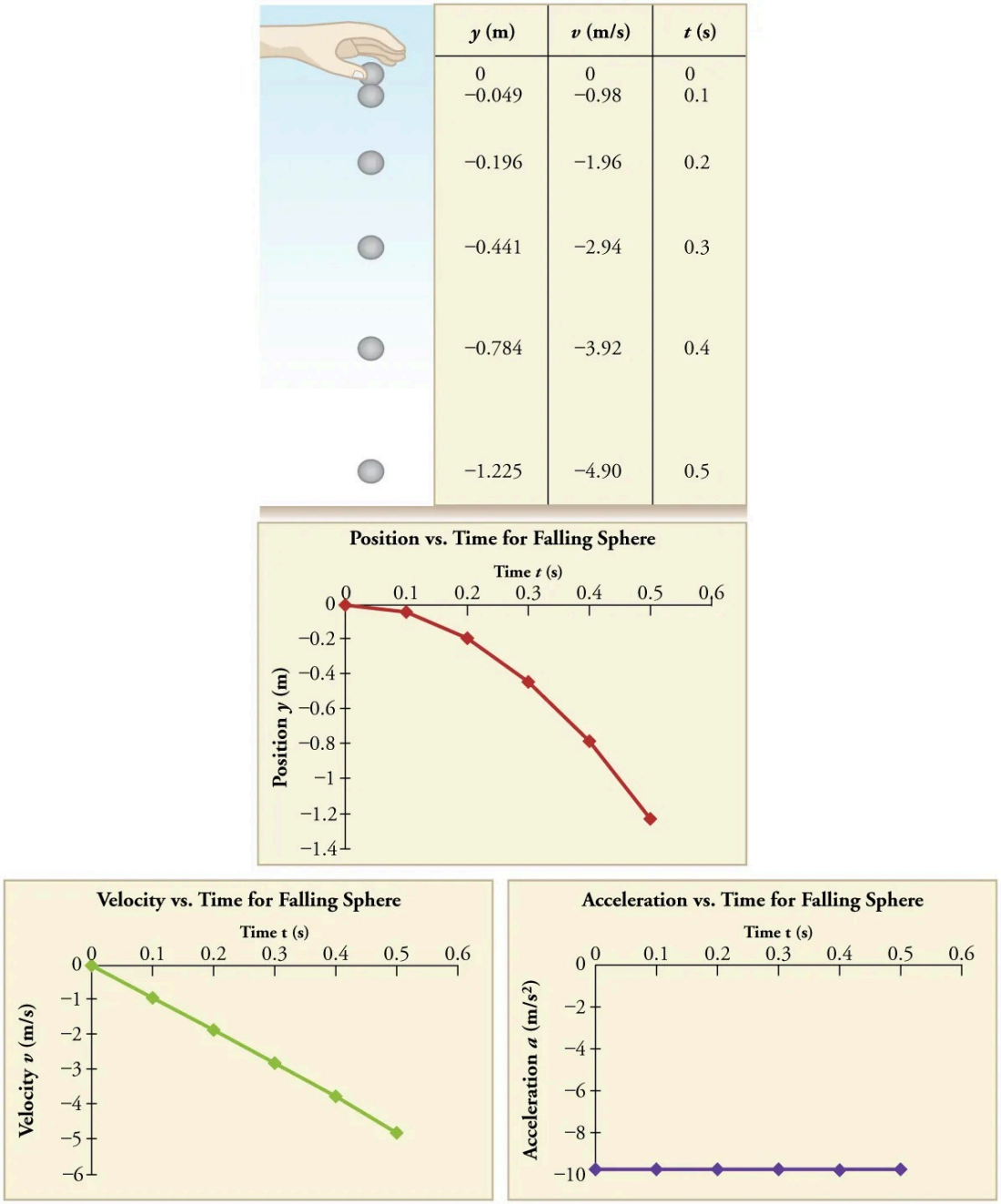


Figure 2.42 Positions and velocities of a metal ball released from rest when air resistance
is negligible. Velocity is seen to increase linearly with time while displacement increases

with time squared. Acceleration is a constant and is equal to gravitational acceleration. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of four parts: a diagram, a table, and three graphs.

1. Diagram:

– A hand is depicted holding and dropping a sphere, illustrating the motion of the sphere as it falls, shown in sequential downward positions.

2. Table:

– This table displays data of a falling sphere with three columns:

–

y (m)	v (m/s)	t (s)
0	0	0
-0.049	-0.98	0.1
-0.196	-1.96	0.2
-0.441	-2.94	0.3
-0.784	-3.92	0.4
-1.225	-4.90	0.5

3. Graphs:

– Position vs. Time for Falling Sphere:

– This graph has “Time t (s)” on the horizontal axis ranging from 0 to 0.6 seconds and “Position y (m)” on the vertical axis ranging from -1.4 to 0 meters. The graph shows a downward curve indicating the position of the sphere over time.

– Velocity vs. Time for Falling Sphere:

– This graph has “Time t (s)” on the horizontal axis and “Velocity v (m/s)” on the vertical axis ranging from -6 to 0 meters per second. The graph displays a linear downward trend, representing the increasing speed of the falling sphere.

– Acceleration vs. Time for Falling Sphere:
– This graph has “Time t (s)” on the horizontal axis and “Acceleration a (m/s_2)” on the vertical axis ranging from -10 to 0 meters per second squared. It shows a horizontal line at -9.8, indicating constant acceleration due to gravity.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.

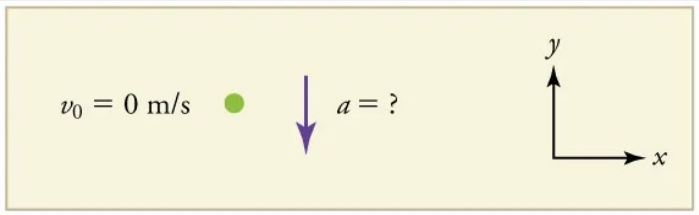


Figure 2.43 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description
The image contains a diagram with the following elements:

– On the left side, there is the text: “v^0 = 0 m/s,” indicating an initial velocity of zero meters per second.
– To the right of this text, there is a small green circle.
– A purple arrow points downward from the circle, with the text “a = ?” next to the arrow, questioning what the acceleration is.
– On the right side of the image, there is a coordinate system with ‘x’ and ‘y’ axes, where the ‘x’ axis points horizontally to the right and the ‘y’ axis points vertically upward.

The background of the image is a light, beige color. We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$;
$y = -1.0000\ \text{m}$;
$t = 0.45173$; $v_0 = 0$.

2. Choose the equation that allows you to solve for a using the known values.

$$y = y_0 + v_0 t + \frac{1}{2}\text{a}t^2$$

2.83

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

$$y = y_0 + \frac{1}{2}\text{a}t^2.$$

2.84

Solving for a gives

$$a = \frac{2\,(y - y_0)}{t^2}.$$

2.85

4. Substitute known values yields

$$a = \frac{2\,(-1.0000\ \text{m} - 0\,)}{(0.45173\ \text{s})^2} = -9.8010\ \text{m/s}^2,$$

2.86

so, because $a = -g$ with the directions we have chosen,

$$g = 9.8010\ \text{m/s}^2.$$

2.87

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as

expected. We expect the value to be somewhere around the average value of $9.80\ \text{m/s}^2$, so $9.8010\ \text{m/s}^2$ makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of $9.80\ \text{m/s}^2$; it represents the local value for the acceleration due to gravity.

Check Your Understanding

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Click for Solution

Solution

We know that initial position $y_0 = 0$, final position $y = -30.0\ \text{m}$, and $a = -g = -9.80\ \text{m/s}^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}\text{at}^2$ to solve for t. Inserting $a = -g$, we obtain

y &= &0 + 0 - \frac{1}{2} \text{gt}^{2} \\ t^{2} &= &\frac{2 y}{- g} \\ t &= &\pm \sqrt{\frac{2 y}{- g}} = \pm \sqrt{\frac{2 \left(\right. - \text{30}.\text{0 m} \left.\right)}{- 9.80 m /\text{s}^{2}}} = \pm \sqrt{\text{6}.\text{12} s^{2}} = \text{2}.\text{47 s} \approx \text{2}.\text{5 s}

2.88

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

PhET Explorations

Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = \mathrm{bx}$) to see how they add to generate the polynomial curve.

Access multimedia content

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2.8 GRAPHICAL ANALYSIS OF ONE-DIMENSIONAL MOTION

Learning Objectives

By the end of this section, you will be able to:

- Describe a straight-line graph in terms of its slope and *y*-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of position, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an independent variable and the vertical axis a dependent variable. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in Figure 2.44, a straight-line graph has the general form

$$y = \mathrm{mx} + b.$$
2.89

Here m is the slope, defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the *y*-intercept, which is the point at which the line crosses the vertical axis.

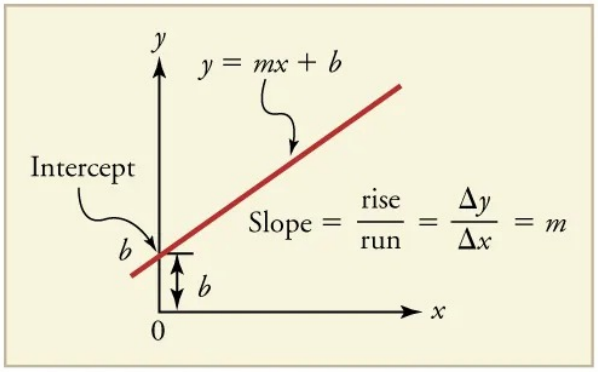


Figure 2.44 A straight-line graph. The equation for a straight line is $y = \mathrm{mx} + b$. *Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a graphical representation of a linear equation in the slope-intercept form, which is expressed as $y = mx + b$. Here's a detailed description:

– Axes: The graph features an x-axis (horizontal) and a y-axis (vertical), both labeled accordingly. The origin is marked at the intersection of these axes with the point labeled as "0."

– Line: A red line is drawn on the graph, demonstrating a positive slope. The line is labeled with the equation $y = mx + b$.

– Intercept: The point where the line intersects the y-axis is marked and labeled as the "Intercept," denoted by b.

– Slope Representation: The slope of the line is illustrated alongside the line and noted as:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = m$$

This represents how the slope m is calculated as the ratio of the change in y (rise) to the change in x (run).

– Visual Indicators: Arrows are used to show the rise and run, providing a visual demonstration of how the slope is determined.

This graph provides an illustrative explanation of how a linear equation is composed in the slope-intercept form and what each component represents in the equation.

Graph of Position vs. Time (a = 0, so v is constant)

Time is usually an independent variable that other quantities, such as position, depend upon. A graph of position versus time would, thus, have x on the vertical axis and t on the horizontal axis. Figure 2.45 is just such a straight-line graph. It shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.

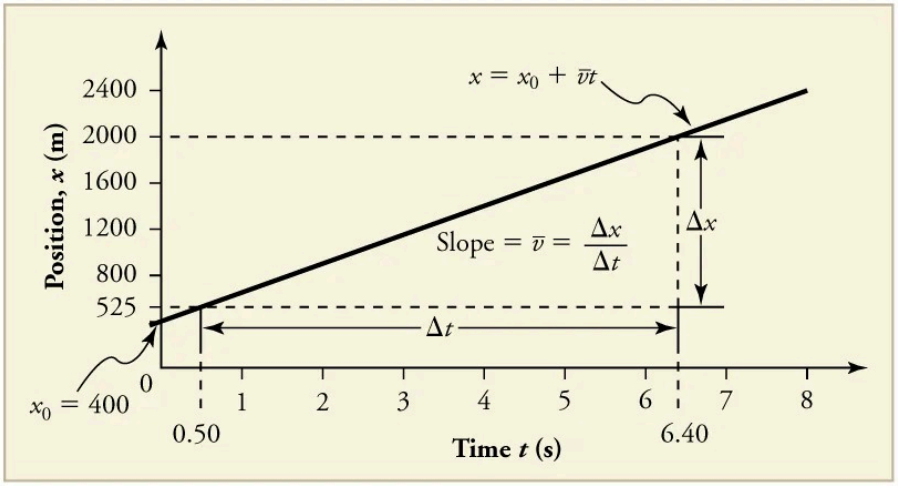


Figure 2.45 Graph of position versus time for a jet-powered car on the Bonneville Salt Flats. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph illustrating the relationship between position and time, showing the motion of an object. Here's a detailed description:

– X-Axis (Horizontal): Represents the time, t, measured in seconds (s). It ranges from 0 to 8 seconds.
– Y-Axis (Vertical): Represents the position, x, measured in meters (m). It ranges from 0 to 2400 meters.
– Graph Line: A straight line with a positive slope, indicating constant velocity. The line begins around the coordinates (0.50, 400) and extends to approximately (6.40, 2000).
– Starting Position: The initial position, x_0, is marked as 400 meters.
– Equation: The line on the graph is defined by the equation $x = x_0 + \bar{v}t$.
– Slope Annotation: The slope of the line, which represents the average velocity $\bar{v}$, is shown as $\Delta x/\Delta t$.
– Dotted Lines and Arrows:
– Horizontal dotted lines indicate the change in position Δx from approximately 525 m to 2000 m.

– Vertical dotted line indicates Δt on the x-axis from 0.50 s to 6.40 s.
– Horizontal double-ended arrow marks the time interval Δt.
– Vertical double-ended arrow marks the change in position Δx.

The background color is a pale yellow, and the graph lines are in solid black.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is position at time zero—that is, x_0. Substituting these symbols into $y = \mathrm{mx} + b$ gives

$$x = \bar{v}t + x_0$$
2.90

or

$$x = x_0 + \bar{v}t.$$
2.91

Thus a graph of position versus time gives a general relationship among displacement(change in position), velocity, and time, as well as giving detailed numerical information about a specific situation.

The Slope of x vs. t

The slope of the graph of position x vs. time t is velocity v.

$$\text{slope} = \frac{\Delta x}{\Delta t} = v$$
2.92

Notice that this equation is the same as that derived algebraically from other motion equations in 2.5 Motion Equations for Constant Acceleration in One Dimension.

From the figure we can see that the car has a position of 525 m at 0.50 s and 2000 m at 6.40 s. Its position at

other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Example 2.17

Determining Average Velocity from a Graph of Position versus Time: Jet Car

Find the average velocity of the car whose position is graphed in Figure 2.45.

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in position and run = change in time, so that

$$\text{slope} = \frac{\Delta x}{\Delta t} = \bar{v}.$$

2.93

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000\ \text{m} - 525\ \text{m}}{6.4\ \text{s} - 0.50\ \text{s}},$$

2.94

yielding

$$\bar{v} = 250 \text{ m/s}.$$

2.95

Discussion

This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when a is constant but $a \neq 0$

The graphs in Figure 2.46 below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the position and velocity are initially 200 m and 15 m/s, respectively.

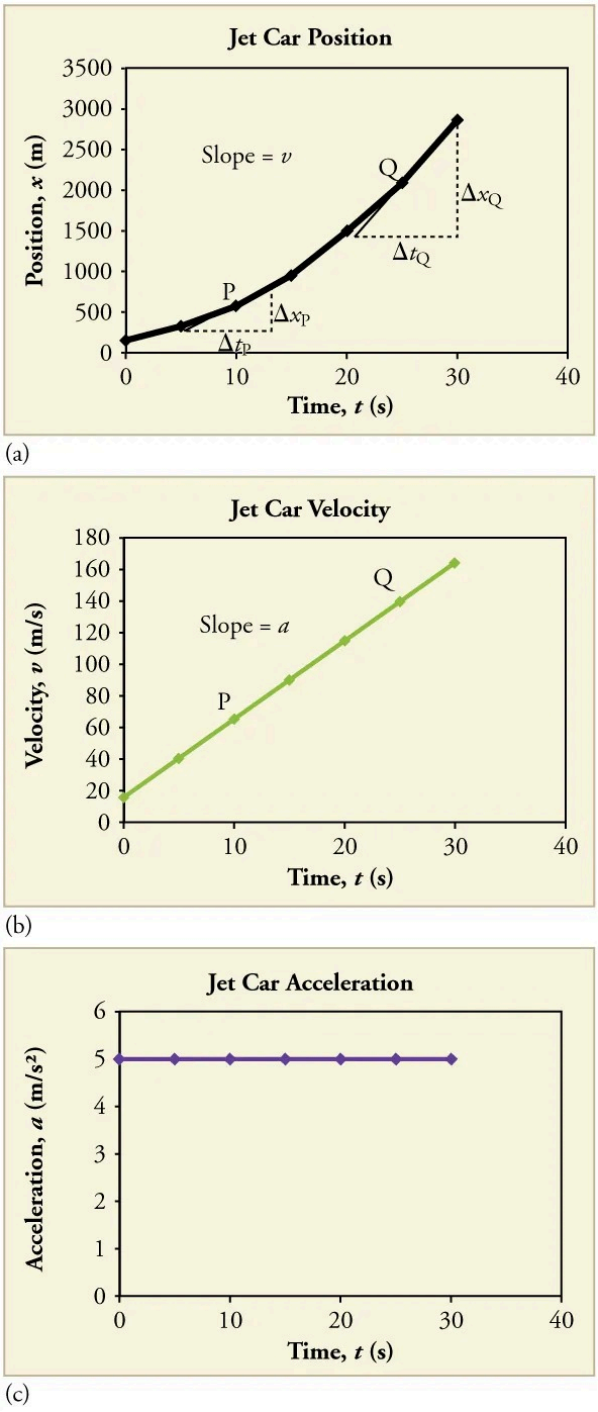


Figure 2.46 Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an x vs. t graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the v vs. t graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of $5.0\ \mathrm{m/s^2}$ over the time interval plotted. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains three separate graphs related to a jet car's motion:

1. Jet Car Position (Graph a):
– Title: Jet Car Position
– Axes:
– Y-axis: Position, x (m)
– X-axis: Time, t (s)
– Description: The graph shows a curve that starts at the origin and rises steeply as time progresses, indicating an increase in position over time. Points P and Q are marked on the graph, and the slope of the tangent at these points is represented as v (velocity). Dotted lines show the changes Δx_P and Δt_P, as well as Δx_Q and Δt_Q.

2. Jet Car Velocity (Graph b):
– Title: Jet Car Velocity
– Axes:
– Y-axis: Velocity, v (m/s)
– X-axis: Time, t (s)
– Description: This graph shows a straight line with a positive slope, starting near the origin and rising steadily, indicating an increase in velocity over time. Points P and Q are marked on the line. The slope is labeled as a (acceleration).

3. Jet Car Acceleration (Graph c):
– Title: Jet Car Acceleration
– Axes:
– Y-axis: Acceleration, a (m/s^2)
– X-axis: Time, t (s)
– Description: The graph plots a horizontal line, indicating constant acceleration over time. The line is at approximately 5 m/s^2.

Each graph illustrates different aspects of the jet car's motion over a 40-second period, showing position, velocity, and acceleration.



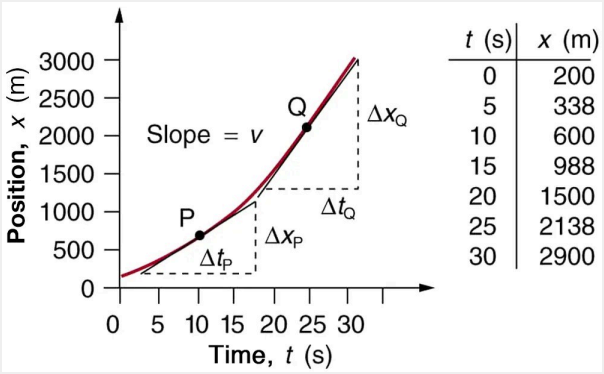
Figure 2.47 A U.S. Air Force jet car speeds down a track. Image from OpenStax College Physics 2e, CC-BY 4.0

The graph of position versus time in Figure 2.46(a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure 2.46(a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in Figure 2.46(b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in Figure 2.46(c).

Example 2.18

Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the x vs. t graph in the graph below.



t (s)	x (m)
0	200
5	338
10	600
15	988
20	1500
25	2138
30	2900

Figure 2.48 The slope of an x vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains a graph and a table along with annotations that explain a concept related to motion.

Graph Description

– The graph plots Position (x in meters) on the vertical axis against Time (t in seconds) on the horizontal axis.

– The x-axis ranges from 0 to 30 seconds, and the y-axis ranges from 0 to 3000 meters.

– A curve is drawn showing the relationship between time and position. It begins at (0, 200) and ends at (30, 2900).

– Two points, P and Q, are highlighted on the curve, each with lines forming right

triangles.
– P is positioned at approximately (10, 600) and **Q** at approximately (20, 1500).
– The slope of the line connecting P and Q is labeled as “Slope = v”, indicating velocity.
– Horizontal lines labeled ΔtP and ΔtQ represent the change in time, while vertical lines labeled ΔxP and ΔxQ represent the change in position.

Table
A table on the right side provides specific data points:

t (s)	x (m)
0	200
5	338
10	600
15	988
20	1500
25	2138
30	2900

This visual aids in understanding motion, demonstrating how position changes over time, with a focus on the slope as velocity.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in Figure 2.48, where Q is the point at $t = 25\ \text{s}$.

Solution

1. Find the tangent line to the curve at $t = 25\ \text{s}$.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, v.

$$\text{slope} = v_Q = \frac{(\Delta x)_Q}{(\Delta t)_Q} = \frac{(3120\ \text{m} - 1300\ \text{m})}{(32\ \text{s} - 19\ \text{s})}$$

2.96

Thus,

$$v_Q = \frac{1820\ \text{m}}{13\ \text{s}} = 140\ \text{m/s}.$$

2.97

Discussion

This is the value given in this figure's table for v at $t = 25\ \text{s}$. The value of 140 m/s for v_Q is plotted in Figure 2.48. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

The Slope of v vs. t

The slope of a graph of velocity v vs. time t is acceleration a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = a$$

2.98

Since the velocity versus time graph in Figure 2.46(b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in Figure 2.46(c).

Additional general information can be obtained from Figure 2.48 and the expression for a straight line, $y = \mathrm{mx} + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields

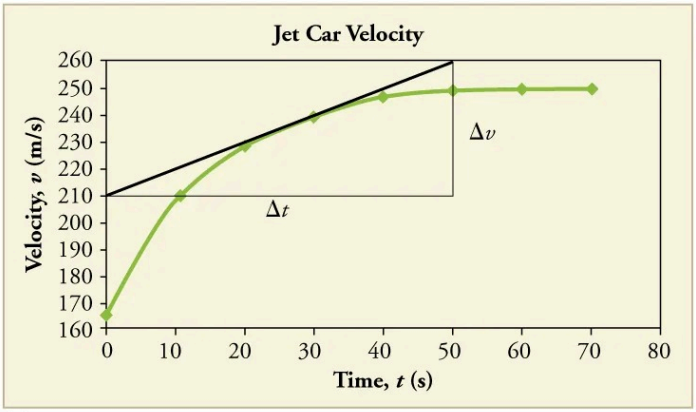
$$v = v_0 + \mathrm{at}.$$
2.99

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in 2.5 Motion Equations for Constant Acceleration in One Dimension.

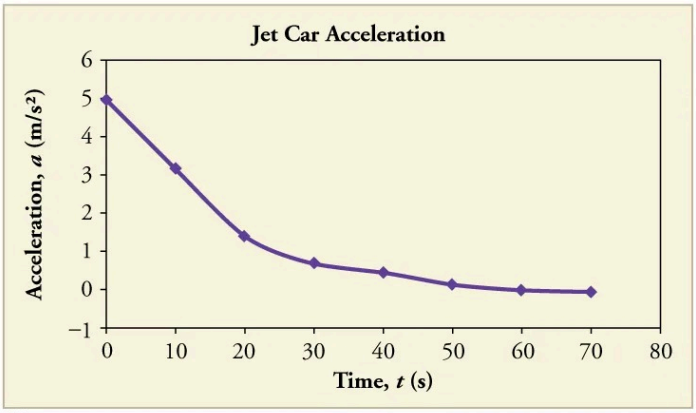
It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in Figure 2.49. Time again starts at zero, and the initial velocity is 165 m/s. (This was the final velocity of the car in the motion graphed in Figure 2.46.) Acceleration gradually decreases from $5.0\ \mathrm{m/s^2}$ to zero when the car hits 250 m/s. The velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.



(a)



(b)

Figure 2.49 Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in ends. (a) The velocity gradually approaches its top value. The slope of this graph is acceleration. It is plotted in the final graph. (b) Acceleration gradually declines to zero when velocity becomes constant. Notice in each of the three graphs that the acceleration drops down to zero and the velocity levels out. This results in a position-time graph that is almost linear. A close-up of the position time graph would show a slight curvature, as indicated in the velocity graph. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two graphs related to a jet car's velocity and acceleration over time:

(a) *Jet Car Velocity*: This graph shows the velocity of the jet car on the y-axis, labeled as "Velocity, v (m/s)," ranging from 160 to 260 meters per second. The x-axis represents time in seconds, labeled as "Time, t (s)," with values from 0 to 80 seconds. A green curved line begins around 160 m/s at 0 seconds, rising sharply and then flattening towards 260 m/s beyond 70 seconds. A black straight line indicates a constant rate of increase over a segment, with Δv and Δt marked for change in velocity and time.

(b) *Jet Car Acceleration*: This graph displays the acceleration on the y-axis, labeled as "Acceleration, a (m/s^2),"

ranging from -1 to 6 meters per second squared. The x-axis again shows time, from 0 to 80 seconds. A purple line starts at around 5 m/s^2 at time 0, decreasing gradually and approaching 0 m/s^2 as time increases, showing a deceleration trend.

Example 2.19

Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in Figure 2.49(a).

Strategy

The slope of the curve at $t = 25\ \text{s}$ is equal to the slope of the line tangent at that point, as illustrated in Figure 2.49(a).

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = \frac{(260\ \text{m/s} - 210\ \text{m/s})}{(51\ \text{s} - 1.0s)}$$

2.100

$$a = \frac{50\ \text{m/s}}{50\ \text{s}} = 1.0 m/\text{s}^2.$$

2.101

Discussion

Note that this value for a is consistent with the value plotted in Figure 2.49(b) at $t = 25\ \text{s}$.

A graph of position versus time can be used to generate a graph of velocity versus time, and a graph of velocity

versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

Check Your Understanding

A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship's acceleration look like?

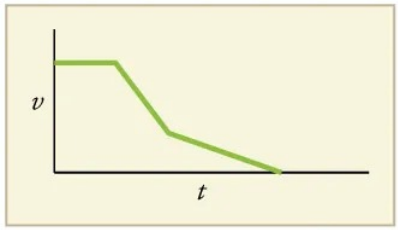


Figure 2.50 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph on a light yellow background. It depicts a velocity-time relationship, where the horizontal axis is labeled "t" for time, and the vertical axis is labeled "v" for velocity. The graph consists of three distinct segments:

1. Horizontal Line: The graph starts with a horizontal line at a constant velocity level.
2. Descending Line: The line then slopes downward, indicating a decrease in velocity over time. This section is depicted in a green color.
3. Further Descending Line: Another green sloped line follows, descending at a different angle until it reaches the horizontal axis, where velocity becomes zero and is constant.

The plot demonstrates a reduction in velocity over time, eventually reaching a plateau.

Click for Solution

Solution

(a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

(b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

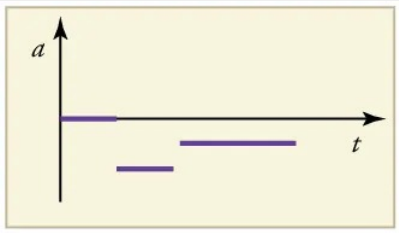


Figure 2.51 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a graph on a light beige background. The graph has two axes: the vertical axis is labeled “a” (most likely representing acceleration) and the horizontal axis is labeled “t” (likely representing time). The origin of the graph is at the bottom left corner.

Along the time axis, there are three horizontal purple line segments of varying lengths:

1. The first line segment starts from the vertical axis, indicating acceleration for a short time period.
2. The second line segment is positioned slightly lower on the vertical axis and starts later along the horizontal axis, extending for a medium length.
3. The third line segment is aligned at the same vertical height as the second, starting even further to the right along the time axis, and it extends parallel to the previous segments.

All three segments are parallel to the time axis, suggesting constant acceleration values during each period.

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GLOSSARY

acceleration
: the rate of change in velocity; the change in velocity over time

acceleration due to gravity
: acceleration of an object as a result of gravity

average acceleration
: the change in velocity divided by the time over which it changes

average speed
: distance traveled divided by time during which motion occurs

average velocity
: displacement divided by time over which displacement occurs

deceleration
: acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

dependent variable
: the variable that is being measured; usually plotted along the y-axis

displacement
: the change in position of an object

distance
: the magnitude of displacement between two positions

distance traveled
: the total length of the path traveled between two positions

elapsed time
: the difference between the ending time and beginning time

free-fall
: the state of movement that results from gravitational force only

independent variable
: the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

instantaneous acceleration
: acceleration at a specific point in time

instantaneous speed
: magnitude of the instantaneous velocity

instantaneous velocity
: velocity at a specific instant, or the average velocity over an infinitesimal time interval

kinematics
: the study of motion without considering its causes

model
: simplified description that contains only those elements necessary to describe the physics of a physical situation

position
: the location of an object at a particular time

scalar
: a quantity that is described by magnitude, but not direction

slope
: the difference in y-value (the rise) divided by the difference in x-value (the run) of two points on a straight line

time
: change, or the interval over which change occurs

vector
: a quantity that is described by both magnitude and direction

y-intercept
: the y—value when $x = 0$, or when the graph crosses the y-axis

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CHAPTER SUMMARY

2.1 Displacement

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement Δx is defined to be
  $\Delta x = x_f - x_0,$
  where x_0 is the initial position and x_f is the final position. In this text, the Greek letter Δ (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.
- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

2.2 Vectors, Scalars, and Coordinate Systems

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

2.3 Time, Velocity, and Speed

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is
  $\Delta t = t_f - t_0$,
  where t_f is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is
  $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_\mathrm{f} - x_0}{t_\mathrm{f} - t_0}$.
- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

2.4 Acceleration

- Acceleration is the rate at which velocity changes. In symbols, average acceleration $\bar{a}$ is $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.
- The SI unit for acceleration is $\mathrm{m/s^2}$.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

2.5 Motion Equations for Constant Acceleration in One Dimension

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,
  \left\begin{matrix} Δ t & = & t \\ Δ x & = & x - x_{0} \\ Δ v & = & v - v_{0} \end{matrix}\right}
- The following kinematic equations for motion with constant a are useful:

  $$x = x_0 + \bar{v}t$$
  $$\bar{v} = \frac{v_0 + v}{2}$$
  $$v = v_0 + \mathrm{at}$$
  $$x = x_0 + v_0 t + \frac{1}{2}\mathrm{at}^2$$
  $$v^2 = v_0^2 + 2a\left(x - x_0\right)$$
- In vertical motion, y is substituted for x.

2.6 Problem-Solving Basics for One-Dimensional Kinematics

The six basic problem solving steps for physics are:

- *Step 1.* Examine the situation to determine which physical principles are involved.
- *Step 2.* Make a list of what is given or can be inferred from the problem as stated (identify the knowns).
- *Step 3.* Identify exactly what needs to be determined in the problem (identify the unknowns).
- *Step 4.* Find an equation or set of equations that can help you solve the problem.
- *Step 5.* Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.
- *Step 6.* Check the answer to see if it is reasonable: Does it make sense?

2.7 Falling Objects

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g, which averages $g = 9.80\ \text{m/s}^2$.
- Whether the acceleration *a* should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80\ \text{m/s}^2$ is negative. In the opposite case, $a = +g = 9.80\ \text{m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

2.8 Graphical Analysis of One-Dimensional Motion

- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

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CONCEPTUAL QUESTIONS

Conceptual Questions

2.1 Displacement

1. Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.
2. Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?
3. Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\ \mu\text{m/s}\ \left(50 \times 10^{-6}\ \text{m/s}\right)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

2.2 Vectors, Scalars, and Coordinate Systems

4. A student writes, "A bird that is diving for prey has a speed of $-10m/s$." What is wrong with the student's statement? What has the student actually described? Explain.
5. What is the speed of the bird in Exercise 2.4?
6. Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.
7. A weather forecast states that the temperature is predicted to be $-5^{\circ}\ C$ the following day. Is this temperature a vector or a scalar quantity? Explain.

2.3 Time, Velocity, and Speed

8. Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

9. There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.
10. Does a car's odometer measure distance traveled or displacement? Does its speedometer measure speed or velocity?
11. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?
12. How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

2.4 Acceleration

13. Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.
14. Is it possible for velocity to be constant while acceleration is not zero? Explain.
15. Give an example in which velocity is zero yet acceleration is not.
16. If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?
17. Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

2.6 Problem-Solving Basics for One-Dimensional Kinematics

18. What information do you need in order to choose which equation or equations to use to solve a problem? Explain.
19. What is the last thing you should do when solving a problem? Explain.

2.7 Falling Objects

20. What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?
21. An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?
22. Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.
23. If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?
24. The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?
25. How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

2.8 Graphical Analysis of One-Dimensional Motion

26. (a) Explain how you can use the graph of position versus time in Figure 2.52 to describe the change in velocity over time. Identify (b) the time (t_a, t_b, t_c, t_d, or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.

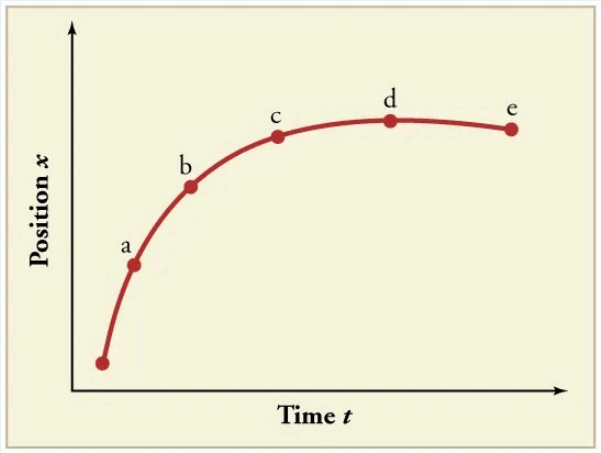


Figure 2.52 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph with a light background, showing a curved line that represents a relationship between time and position.

– Axes:
– The horizontal axis is labeled “Time t.”
– The vertical axis is labeled “Position x.”

– Graph Line:
– The red curved line starts at the bottom left and curves upward, then levels off, moving towards the right.

– Points:
– Five marked points labeled from “a” to “e” are placed along the curve at intervals, representing different times and positions.
– Point “a” is positioned lowest on the curve, and point “e” is highest.

The curve suggests that as time progresses, the position increases rapidly at first, then the rate of increase slows down, eventually leveling off. This curve may represent deceleration or an approach to a maximum position, illustrating a common type of dynamic behavior.

27. (a) Sketch a graph of velocity versus time corresponding to the graph of position versus time given in Figure 2.53. (b) Identify the time or times (t_a,

t_b,
t_c, etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?

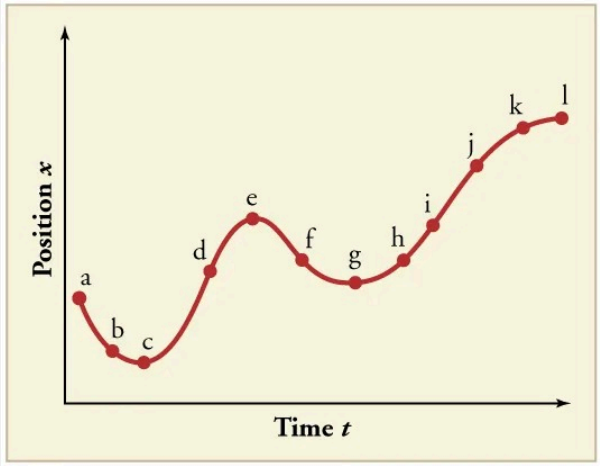


Figure 2.53 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph illustrating the position of an object over time. The x-axis is labeled "Time t" and the y-axis is labeled "Position x". The graph is plotted with a smooth red curve that shows changes in position at different time intervals.

Key points along the curve are marked with lowercase letters from 'a' to 'l', with points spaced along the curve. The shape of the curve indicates fluctuations, showing peaks at points 'd', 'e', 'j', and 'l', and troughs at points 'c', 'f', and 'h'.

The graph depicts a general upward trend over time despite the fluctuations, indicating increasing position as time progresses. The background is a light beige color, and the axes are represented by black arrows indicating the direction of increase for both time and position.

28. (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in Figure 2.54. (b) Based on the graph, how does acceleration change over time?

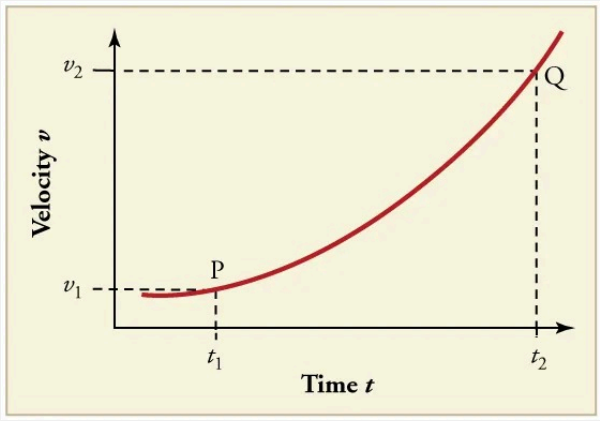


Figure 2.54 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a graph with labeled axes, depicting the relationship between velocity and time. The x-axis is labeled "Time *t*" and the y-axis is labeled "Velocity *v*." The graph displays a curve starting at a point labeled *P* and ending at a point labeled *Q*.

The significant points on the axes are marked: the y-axis has values labeled as v_1 (lower) and v_2 (higher), while the x-axis has points t_1 and t_2 corresponding to specific time values.

The curve starts at point *P* with velocity v_1 at time t_1, showing an upward trend and gaining steepness as it approaches point *Q*, where it reaches velocity v_2 at time t_2. Dashed lines extend from the points on the curve to the axes, showing the correspondence of each point to its velocity and time values.

29. (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure 2.55. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?

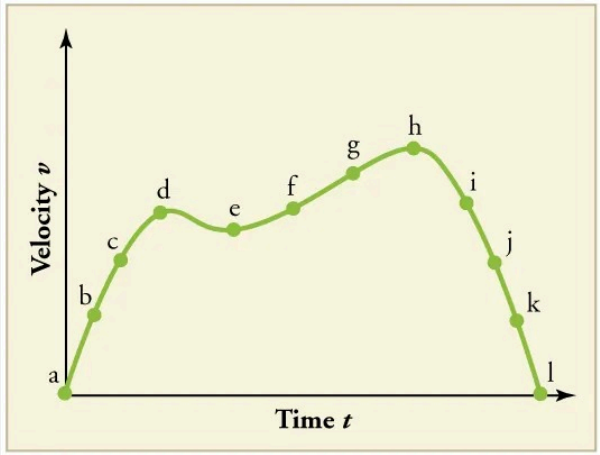


Figure 2.55 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph illustrating the relationship between velocity and time. The horizontal axis is labeled "Time t," and the vertical axis is labeled "Velocity v."

The graph is a curved green line depicting changes in velocity over time with various labeled points from "a" to "l":

– Point "a" is at the origin (0,0), indicating zero velocity at time zero.
– From "a" to "b," the velocity increases.
– From "b" to "d," the velocity continues to rise but at a slower rate, showing a decrease in acceleration.
– Points "d" and "e" are peaks where the velocity is relatively stable.
– From "e" to "f," there is a slight decrease, followed by an increase in velocity to point "g."
– Point "h" is the highest point on the graph.
– From "h" to "i," the velocity starts to decrease.
– After point "i," from "j" to "k," the velocity decreases sharply.
– Finally, from "k" to "l," the velocity returns to zero, similar to the starting point "a."

The overall shape of the graph resembles a hill, where the peak is at point "h."

30. Consider the velocity vs. time graph of a person in an elevator shown in Figure 2.56. Suppose the elevator is initially at rest. It then accelerates for 3 seconds,

maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from 2.5 Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

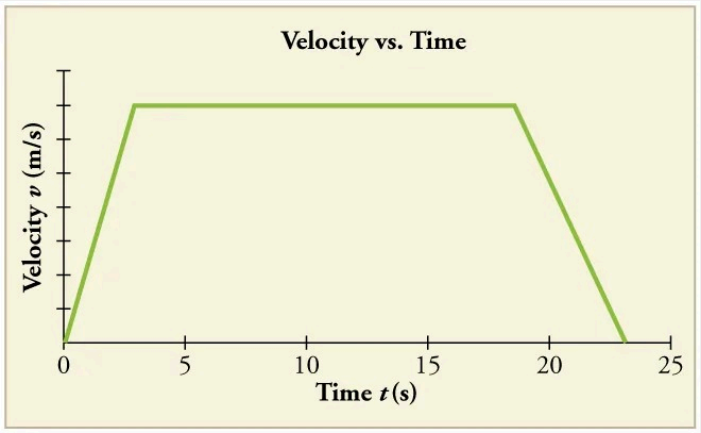


Figure 2.56 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph titled "Velocity vs. Time." It plots velocity (v) in meters per second (m/s) on the vertical y-axis against time (t) in seconds (s) on the horizontal x-axis. The graph is set on a light beige background and is bordered in light brown.

The y-axis, labeled "Velocity v (m/s)," ranges from 0 to 40 m/s with markings at intervals of 5 m/s. The x-axis, labeled "Time t (s)," ranges from 0 to 25 seconds with markings at intervals of 5 seconds.

The graph consists of a green line that illustrates the change in velocity over time. The line starts at the origin (0,0) and increases steeply until it reaches a maximum velocity around 5 seconds. Then, it maintains this constant velocity until roughly 20 seconds, after which it decreases steeply back to 0 m/s at 25 seconds, forming a trapezoidal shape.

30. A cylinder is given a push and then rolls up an inclined plane. If the origin is the

starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

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PROBLEMS & EXERCISES

Problems & Exercises

2.1 Displacement

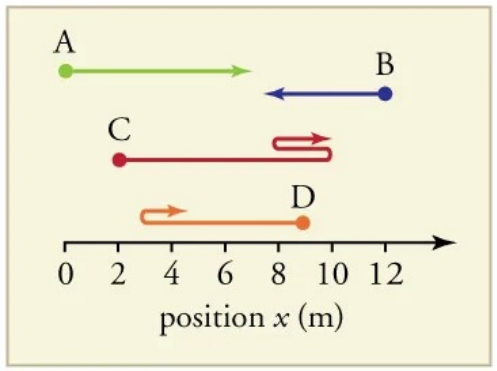


Figure 2.57 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the movements of four objects, labeled A, B, C, and D, along a horizontal axis, which represents position x in meters.

– Object A: Represented by a green line with an arrow pointing right, starting at a position slightly greater than 0 meters and ending near 4 meters.

– Object B: Represented by a blue line with an arrow pointing left, starting near 7 meters and ending around 11 meters.

– Object C: Represented by a red line with a loop in the middle. It starts near 3 meters, loops back around 5 meters, and ends around 9 meters.

– Object D: Represented by an orange line with an arrow pointing right, starting near 2 meters and ending around 8 meters.

Below these lines, there's a horizontal axis marked from 0 to 12 meters, labeled as

"position x (m)." An arrow at the end of the axis indicates increasing direction to the right.

1. Find the following for path A in Figure 2.57: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.
2. Find the following for path B in Figure 2.57: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.
3. Find the following for path C in Figure 2.57: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.
4. Find the following for path D in Figure 2.57: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2.3 Time, Velocity, and Speed

5. (a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?
6. A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?
7. The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?
8. Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?
9. On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set

the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

10. Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84 \times 10^6 m$ (1%)?
11. A student drove to the university from their home and noted that the odometer reading of their car increased by 12.0 km. The trip took 18.0 min. (a) What was their average speed? (b) If the straight-line distance from their home to the university is 10.3 km in a direction 25.0° south of east, what was their average velocity? (c) If they returned home by the same path 7 h 30 min after they left, what were their average speed and velocity for the entire trip?
12. The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?
13. Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light $(3.00 \times 10^8 \text{ m/s})$.
14. A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.
15. The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest

atom, as having a single electron in a circular orbit $1.06 \times 10^{-10}\ \text{m}$ in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20 \times 10^{6}\ \text{m/s}$, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity per revolution?

2.4 Acceleration

16. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?
17. **Professional Application:** Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of g $(9.80\ \text{m/s}^2)$ by taking its ratio to the acceleration of gravity.
18. A commuter backs her car out of her garage with an acceleration of $1.40\ \text{m/s}^2$. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?
19. Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of g $(9.80\ \text{m/s}^2)$?

2.5 Motion Equations for Constant Acceleration in One Dimension

20. An Olympic-class sprinter starts a race with an acceleration of $4.50\ \text{m/s}^2$. (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.
21. A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10 \times 10^{4}\ \text{m/s}^2$, and 1.85 ms $(1\ \text{ms} = 10^{-3}\ \text{s})$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?
22. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at

an average rate of $6.20 \times 10^5 \text{ m/s}^2$ for $8.10 \times 10^{-4} \text{ s}$. What is its muzzle velocity (that is, its final velocity)?

23. (a) A light-rail commuter train accelerates at a rate of 1.35 m/s^2. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of 1.65 m/s^2. How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s^2?
24. While entering a freeway, a car accelerates from rest at a rate of 2.40 m/s^2 for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.
25. At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of 2.00 m/s^2. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?
26. **Professional Application:** Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?
27. In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes $3.33 \times 10^{-2} \text{ s}$, calculate the distance over which the puck accelerates.
28. A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?
29. Freight trains can produce only relatively small accelerations and decelerations. (a)

What is the final velocity of a freight train that accelerates at a rate of $0.0500\ \mathrm{m/s^2}$ for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of $0.550\ \mathrm{m/s^2}$, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

30. A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.
31. A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of $0.350\ \mathrm{m/s^2}$, how far will it travel before becoming airborne? (b) How long does this take?
32. **Professional Application:** A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in $\mathrm{m/s^2}$ and in multiples of $g\ (g = 9.80\ \mathrm{m/s^2})$. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g?
33. An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?
34. In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.
35. Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b)

If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

36. An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of $0.150\ \mathrm{m/s^2}$ as it goes through. The station is 210 m long. (a) How long did the nose of the train stay in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?
37. Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in Example 2.10 and Example 2.11. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? Hint: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.
38. A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of $0.500\ \mathrm{m/s^2}$ for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?
39. In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?
40. (a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time

of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

2.7 Falling Objects

Assume air resistance is negligible unless otherwise stated.

1. Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.
2. Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.
3. A basketball referee tosses the ball straight up for the starting tip-off. At what minimum velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?
4. A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.
5. A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the

answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

6. A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?
7. (a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?
8. A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?
9. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?
10. A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?
11. Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?
12. An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.
13. There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which

would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

14. A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball's initial velocity? Hint: First consider only the distance along the window, and solve for the ball's velocity at the bottom of the window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.
15. Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.
16. A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms $(8.00 \times 10^{-5}\ \text{s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?
17. A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.
18. A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50 \times 10^{-3}\ \text{s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

2.8 Graphical Analysis of One-Dimensional Motion

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

59. (a) By taking the slope of the curve in Figure 2.58, verify that the velocity of the jet car is 115 m/s at $t = 20$ s. (b) By taking the slope of the curve at any point in Figure 2.59, verify that the jet car's acceleration is $5.0\ \mathrm{m/s^2}$.

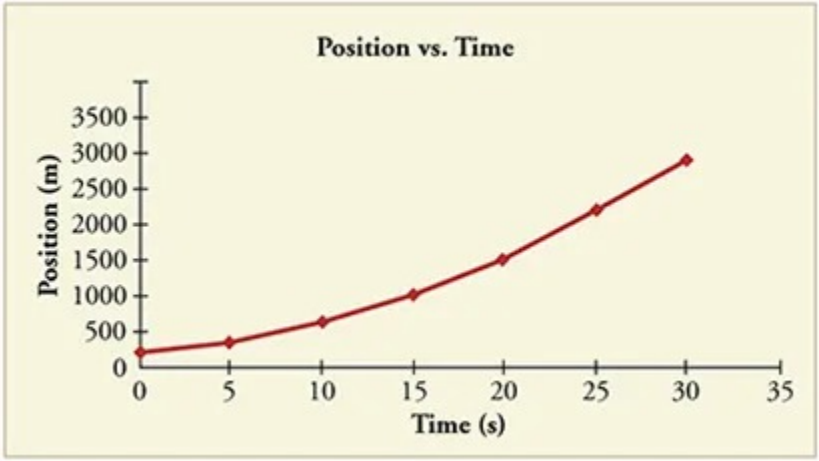


Figure 2.58 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a line graph titled "Position vs. Time." It displays the relationship between position in meters (m) and time in seconds (s).

The horizontal axis represents time, labeled "Time (s)," and ranges from 0 to 35 seconds with intervals of 5 seconds. The vertical axis represents position, labeled "Position (m)," and ranges from 0 to 3500 meters, with intervals of 500 meters.

A red line with data points plotted as red diamonds shows an increasing pattern. The line starts near the origin, indicating a position close to 0 meters at 0 seconds, and increases non-linearly upwards to approximately 3500 meters at 30 seconds, suggesting acceleration over time.

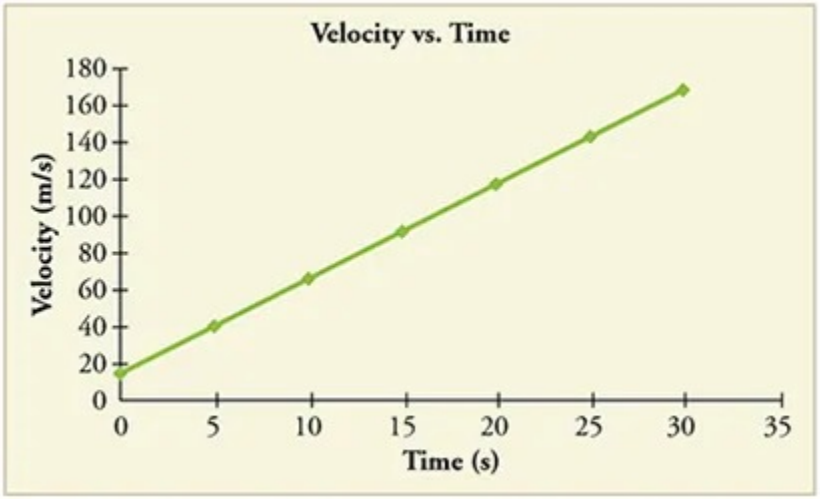


Figure 2.59 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a line graph titled "Velocity vs. Time." It represents velocity on the vertical axis, labeled as "Velocity (m/s)", and time on the horizontal axis, labeled as "Time (s)".

The graph features a straight, diagonal green line with dots at intervals, indicating data points. The line starts at the origin (0,0) and progresses upwards and to the right, indicating a constant increase in velocity over time. The axes are marked with intervals:

– Time (s): Marked at 0, 5, 10, 15, 20, 25, 30, and 35.
– Velocity (m/s): Marked at 0, 20, 40, 60, 80, 100, 120, 140, 160, and 180.

The line suggests that velocity increases at a constant rate with respect to time.

60. Using approximate values, calculate the slope of the curve in Figure 2.60 to verify that the velocity at $t = 30.0\ \text{s}$ is approximately 0.24 m/s. Assume all values are known to 2 significant figures.

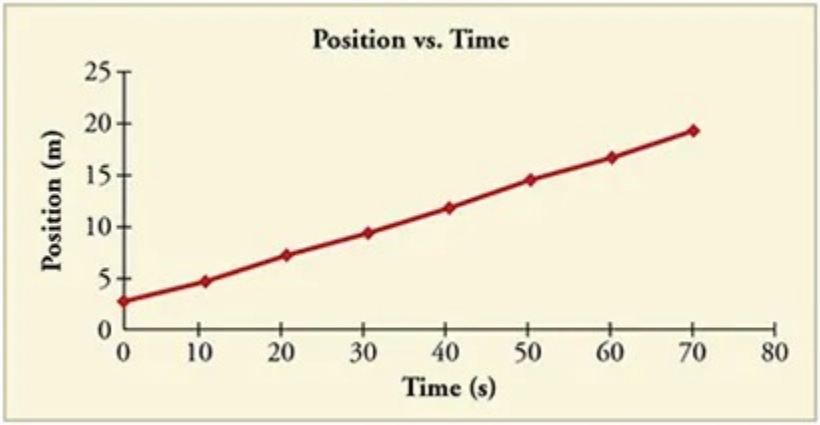


Figure 2.60 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph titled “Position vs. Time.” It has an x-axis labeled “Time (s)” ranging from 0 to 80 seconds, and a y-axis labeled “Position (m)” ranging from 0 to 25 meters. The graph displays data points connected with a red line, indicating a linear increase in position over time. The graph shows how position changes consistently, suggesting constant speed.

61. Using approximate values, calculate the slope of the curve in Figure 2.60 to verify that the velocity at $t = 30.0\ \text{s}$ is approximately 0.24 m/s.
62. By taking the slope of the curve in Figure 2.61, verify that the acceleration is $3.2 m/\text{s}^2$ at $t = 10\ \text{s}$.

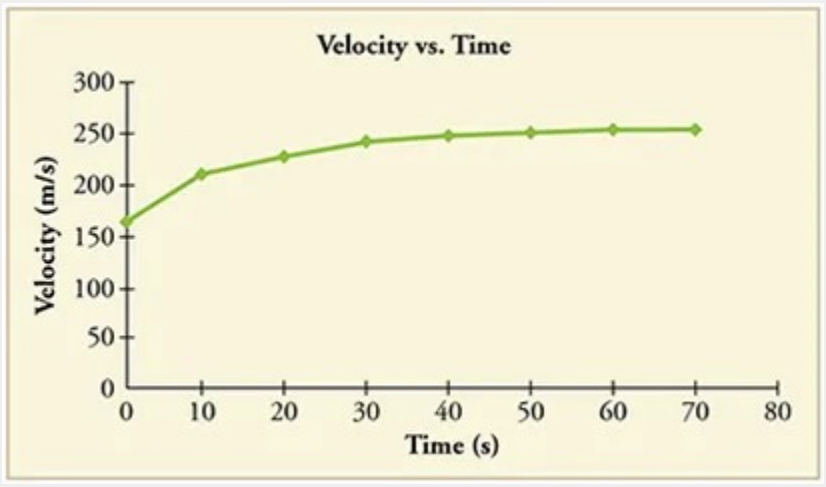


Figure 2.61 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph titled "Velocity vs. Time." It shows a line graph on a rectangular grid with the x-axis labeled "Time (s)" and the y-axis labeled "Velocity (m/s)".

– The x-axis ranges from 0 to 80 seconds.
– The y-axis ranges from 0 to 300 meters per second.

The data points are plotted and connected with a green line, indicating changes in velocity over time. The trend starts at the origin, quickly rises, and then begins to level off, forming a curve that flattens as time progresses. The velocity appears to increase rapidly at first, then gradually approaches a constant value around 250 m/s from approximately 60 seconds onwards.

63. Construct the position graph for the subway shuttle train as shown in Figure 2.18(a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.
64. (a) Take the slope of the curve in Figure 2.62 to find the jogger's velocity at $t = 2.5s$. (b) Repeat at 7.5 s. These values must be consistent with the graph in Figure 2.63.

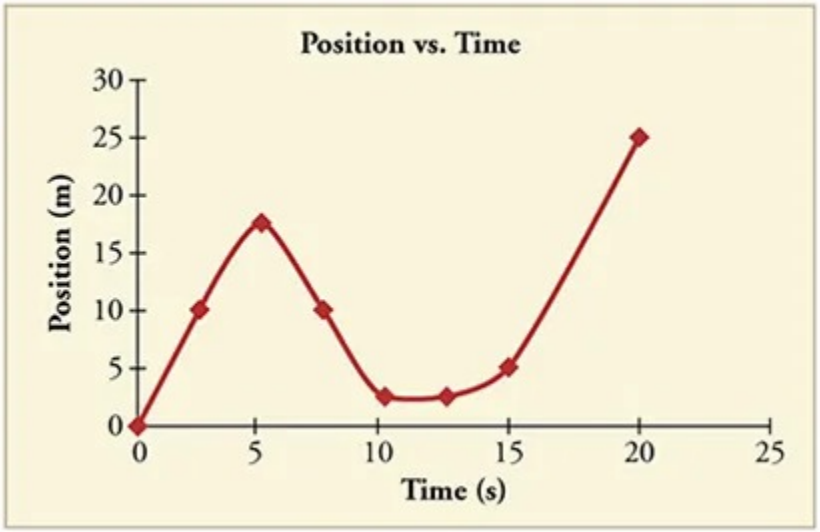


Figure 2.62 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a line graph titled "Position vs. Time." It shows a red line with diamond markers illustrating the relationship between position and time.

Axes Details:

- The x-axis represents "Time (s)" and is labeled with values from 0 to 25 in increments of 5 seconds.
- The y-axis represents "Position (m)" and is labeled with values from 0 to 30 in increments of 5 meters.

Graph Description:

The graph begins at the origin (0,0), meaning the initial position is 0 meters at 0 seconds. It then follows these movements:

1. Increases to around 12 meters at 5 seconds.
2. Peaks slightly above 15 meters before dropping to below 5 meters at 10 seconds.
3. Rises slightly above 10 meters at 15 seconds.
4. Rises sharply again, reaching approximately 25 meters at 20 seconds.

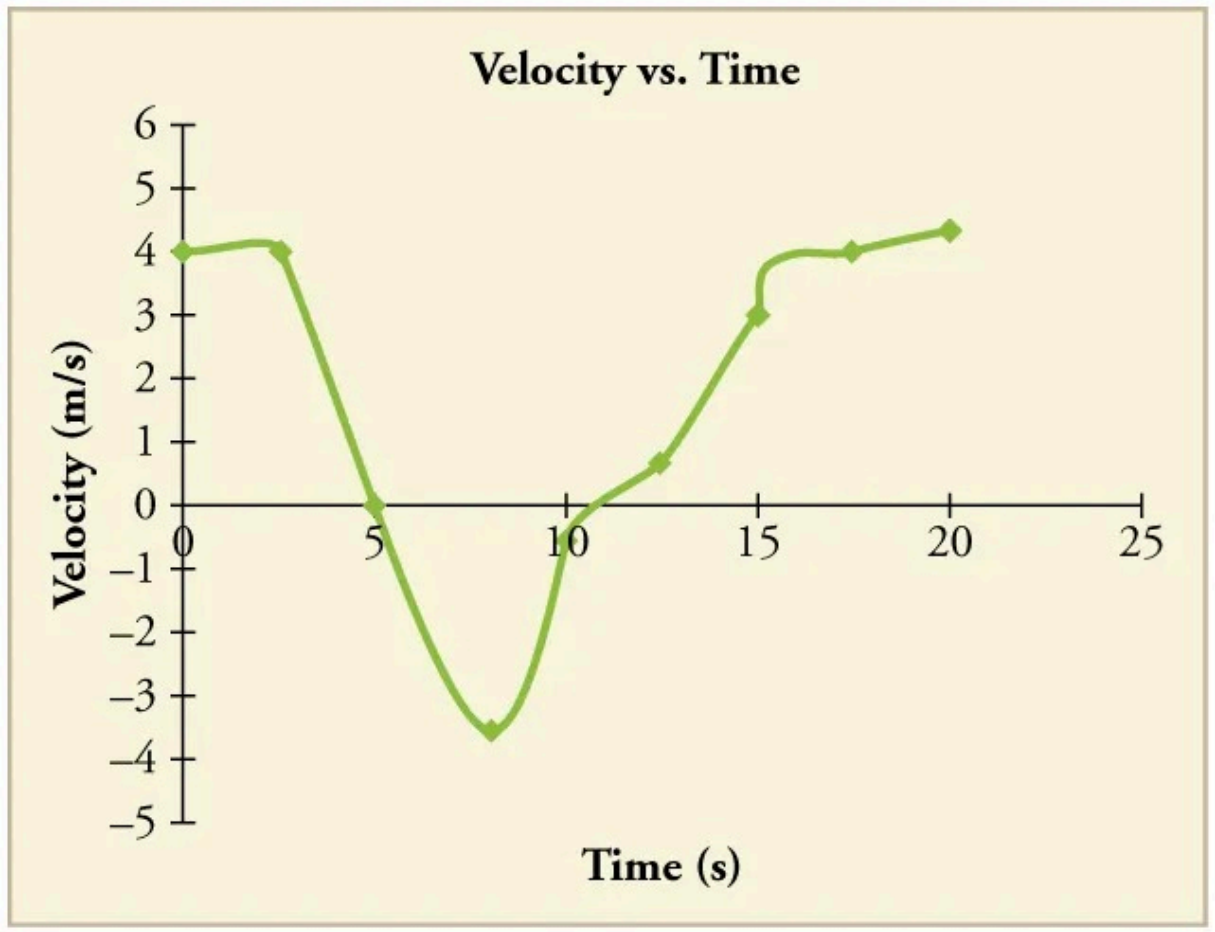


Figure 2.63 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph depicting "Velocity vs. Time". It displays the relationship between velocity, measured in meters per second (m/s), and time, measured in seconds (s).

– The horizontal axis represents time, ranging from 0 to 25 seconds, marked with intervals of 5 seconds.
– The vertical axis represents velocity, ranging from -5 to 6 meters per second, marked with intervals of 1 meter per second.

The graph shows a green line plot with data points marked by small diamonds. Here's a summary of the plot's trajectory:

1. Starting at time 0 seconds, the velocity is approximately 4 m/s.

2. At around 2 seconds, the velocity decreases to about 3 m/s, and continues decreasing.
3. By 5 seconds, the velocity drops to 0 m/s.
4. From 5 to 10 seconds, the velocity dips to a minimum of -5 m/s around 8 seconds, followed by an increase back to approximately 0 m/s at 10 seconds.
5. From 10 seconds onward, the velocity is shown to increase steadily, reaching about 4 m/s at 15 seconds.
6. The velocity plateaus slightly just over 3 m/s at around 20 seconds.
7. Finally, from 20 to 25 seconds, the velocity remains stable at a little over 3 m/s.

The overall motion described is a decrease in velocity, a dip into negative velocity indicating a change in direction, followed by a return to positive velocity.

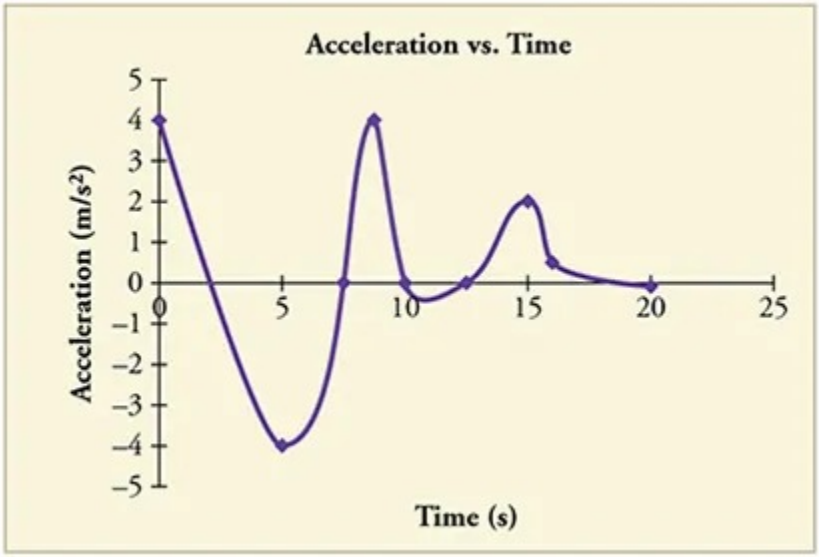


Figure 2.64 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph titled “Acceleration vs. Time.” It shows a line graph on a coordinate plane where the x-axis represents time in seconds (ranging from 0 to 25) and the y-axis represents acceleration in meters per second squared (ranging from -5 to 5).

The plot of the graph consists of several peaks and valleys:

- At time 0 seconds, the acceleration starts at 4 m/s_2.

- At about 4 seconds, there is a steep decline to approximately -5 m/s$_2$ around 5 seconds.
- The graph then ascends sharply, reaching a peak of approximately 4.5 m/s$_2$ at around 8 seconds.
- Following this, there is a decline to 0 m/s$_2$ around 11 seconds.
- Another peak occurs at about 13 seconds with an acceleration of around 2 m/s$_2$.
- The graph experiences a slight dip and small fluctuations before flattening out near 0 m/s$_2$ as it approaches 20 seconds and beyond.

65. A graph of $v\,(t)$ is shown for a world-class track sprinter in a 100-m race. (See Figure 2.65). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at $t = 5s$? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?

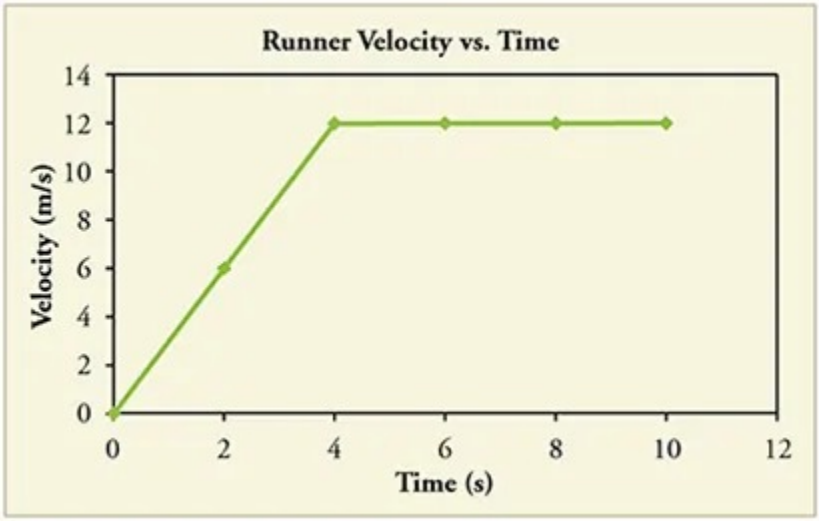


Figure 2.65 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a line graph titled “Runner Velocity vs. Time.” It shows the velocity of a runner over a period of time in seconds.

The x-axis is labeled “Time (s)” and ranges from 0 to 12 seconds. The y-axis is labeled “Velocity (m/s)” and ranges from 0 to 14 meters per second.

The graph line is green and consists of three segments:

- From 0 to 2 seconds, the velocity increases linearly from 0 to 12 meters per second.
- From 2 to 10 seconds, the velocity remains constant at 12 meters per second.

66. Figure 2.66 shows the position graph for a particle for 6 s. (a) Draw the corresponding Velocity vs. Time graph. (b) What is the acceleration between 0 s and 2 s? (c) What happens to the acceleration at exactly 2 s?

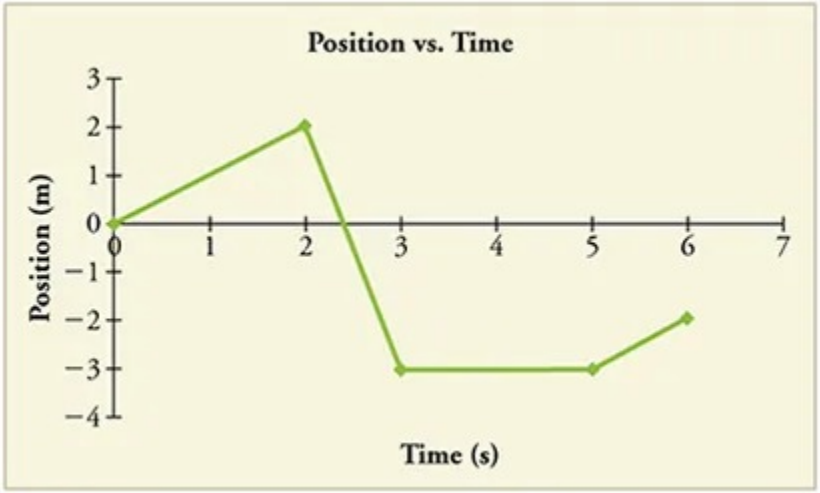


Figure 2.66 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph titled “Position vs. Time”. It depicts a line graph with position in meters on the y-axis and time in seconds on the x-axis.

– The graph’s x-axis ranges from 0 to 7 seconds, and the y-axis ranges from -4 to 3 meters.
– At time 0 seconds, the position is at 0 meters.
– From 0 to 2 seconds, the line rises to a position of 3 meters.
– From 2 to 3 seconds, the line drops sharply to a position of 0 meters.
– From 3 to 5 seconds, the line declines further to a position of -3 meters.
– At 5 seconds, the line flattens out at -3 meters, continuing until 6 seconds.
– From 6 to 7 seconds, the line rises to a position of -2 meters.

The line is colored green and marked with data points at each second interval.

67. Critical Thinking Two cars are side by side on a path where friction is negligible. Car A starts from rest and has a positive, constant acceleration while car B travels at a constant speed. The cars reach the same check point down the road at the same time. (a) Which car is traveling faster at the check point? Briefly explain your reasoning. (b) Based on experimental data, an equation that fits the data is suggested for the speed of car A at the check point, *V*, which may not be correct: $V = KV_0$, where *K* is a constant with appropriate units and V_0 is the initial velocity of car B. Is this equation consistent with your answer from part a? Explain why or why not. Does this equation make sense? Explain why or why not. (c) Graph the velocity of car A and the velocity of car B vs. distance along the path, starting when the cars are side by side.

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TWO-DIMENSIONAL KINEMATICS



Figure 3.1 Hitting a tennis ball is difficult enough when it is traveling at high speeds in a relatively straight line. However, most motion is in curved, rather than straight-line, paths. Tennis players must account for the arc of the ball and the impact of its spin, while they also predict how it will bounce and plan the location of their shot. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

3.1 Kinematics in Two Dimensions: An Introduction
3.2 Vector Addition and Subtraction: Graphical Methods
3.3 Vector Addition and Subtraction: Analytical Methods
3.4 Projectile Motion
3.5 Addition of Velocities
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.
Access multimedia content

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3.1 KINEMATICS IN TWO DIMENSIONS: AN INTRODUCTION

Learning Objectives

By the end of this section, you will be able to:

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.



Figure 3.2 Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. Image from OpenStax College Physics 2e, CC-BY 4.0

Two-Dimensional Motion: Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in Figure 3.3.

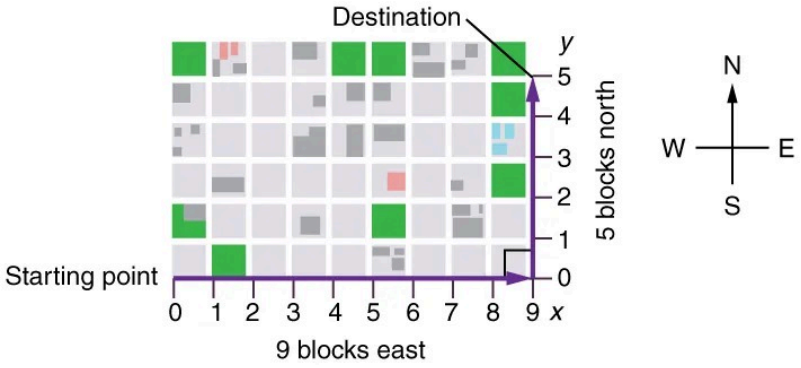


Figure 3.3 A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a grid map representing a route through a city with labeled axes and a compass rose for orientation.

	x-axis (9 blocks east)									
y-axis	0	1	2	3	4	5	6	7	8	9
5 blocks north						Destination				
4										
3										
2										
1										
0										Starting point

The grid is composed of several squares, some filled with colors. A thick purple line runs along the x-axis from 0 to 9 and up the y-axis from 0 to 5, marking a path from the "Starting point" at the lower right corner (0,0) to the "Destination" at the upper right corner (9,5), traveling east and then north. The map includes a compass indicating north, south, east, and west directions, positioned on the right side of the grid.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to

take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2 + b^2 = c^2$, can be used to find the straight-line distance.

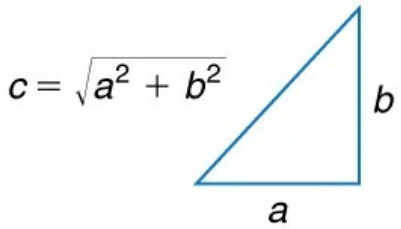


Figure 3.4 The Pythagorean theorem relates the length of the legs of a right triangle, labeled a and b, with the hypotenuse, labeled c. The relationship is given by: $a^2 + b^2 = c^2$. This can be rewritten, solving for c: $c = \sqrt{a^2 + b^2}$. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a right-angled triangle next to a mathematical equation. The triangle is positioned with one angle at the bottom left and the right angle at the bottom right.

The sides are labeled as follows:
– The horizontal side at the bottom is labeled “a”.
– The vertical side on the right is labeled “b”.
– The hypotenuse, which is the slanted side, does not have a label on the triangle itself but is implied in the equation.

To the left of the triangle, there is the equation:

$$c = \sqrt{a^2 + b^2}$$

This equation represents the Pythagorean theorem, describing the relationship between the lengths of the sides of a right-angled triangle. It states that the square of the hypotenuse c is equal to the sum of the squares of the other two sides a and b.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(9 \text{ blocks})^2 + (5 \text{ blocks})^2} = 10.3 \text{ blocks}$, considerably shorter than the 14 blocks you walked.

(Note that we are using three significant figures in the answer. Although it appears that "9" and "5" have only one significant digit, they are discrete numbers. In this case "9 blocks" is the same as "9.0 or 9.00 blocks." We have decided to use three significant figures in the answer in order to show the result more precisely.)

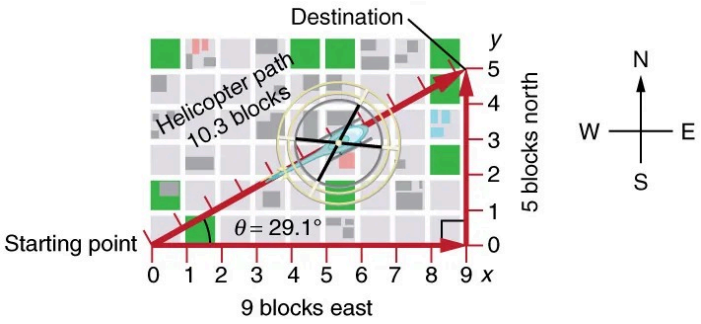


Figure 3.5 The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a map diagram illustrating the path of a helicopter over a grid-like city layout. Here's a detailed description:

– Grid Layout: The map shows a city grid with blocks marked by gray and white rectangles, intersected by streets. There are also green squares representing parks or open spaces scattered throughout the grid.
– Axes: The horizontal axis (x-axis) is labeled at the bottom, ranging from 0 to 9, representing blocks east. The vertical axis (y-axis) is on the right side, ranging from 0 to 5, representing blocks north.
– Path: A thick red arrow represents the helicopter's path, starting at the bottom left labeled as "Starting point" and ending at the top right labeled as "Destination." The path covers 9 blocks east and 5 blocks north.
– Measurements: The path has a diagonal distance labeled "Helicopter path 10.3 blocks." The angle θ is marked at the starting point as 29.1° from the horizontal axis.
– Compass Rose: To the right of the grid, there is a compass rose indicating cardinal directions with "N" for north pointing upwards, "E" for east pointing to the right, "S" for south pointing downwards, and "W" for west pointing left.

This image visually demonstrates vector navigation on a grid, showing direction, distance, and angle.

The fact that the straight-line distance (10.3 blocks) in Figure 3.5 is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that vectors are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in Figure 3.3 and Figure 3.5. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in Figure 3.5. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods.)

The Independence of Perpendicular Motions

The person taking the path shown in Figure 3.5 walks east and then north (two perpendicular directions). How far they walk east is only affected by their motion eastward. Similarly, how far they walk north is only affected by their motion northward.

Independence of Motion

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown

horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

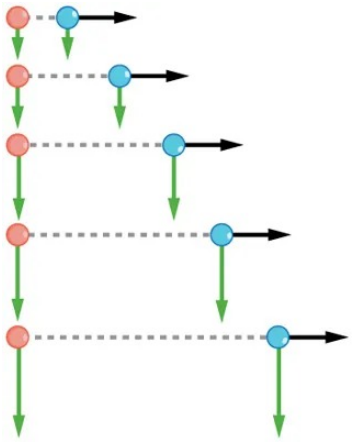


Figure 3.6 This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram representing a process flow with colored circles connected by arrows. Here's a textual description:

– The diagram consists of two types of circles: red and blue.
– Each red circle is connected to a blue circle by a dashed horizontal line.
– From each blue circle, a black arrow points to the right.
– From each red circle, a green arrow points downwards.
– The sequence starts at the top with a red circle linked horizontally to a blue circle, followed by a right-pointing black arrow.
– The process repeats multiple times down the diagram, with each level following the same pattern:
– A red circle connected to a blue circle horizontally,
– A green arrow pointing down from each red circle,
– A black arrow pointing right from each blue circle.

The diagram visually represents a step-by-step flow from left to right, while progress continues downward from step to step.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods. We will find such techniques to be useful in many areas of physics.

PhET Explorations

Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

Click to view content.

"3.1 Kinematics in Two Dimensions: An Introduction" from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

3.2 VECTOR ADDITION AND SUBTRACTION: GRAPHICAL METHODS

Learning Objectives

By the end of this section, you will be able to:

- Understand the rules of vector addition, subtraction, and multiplication by scalars.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.

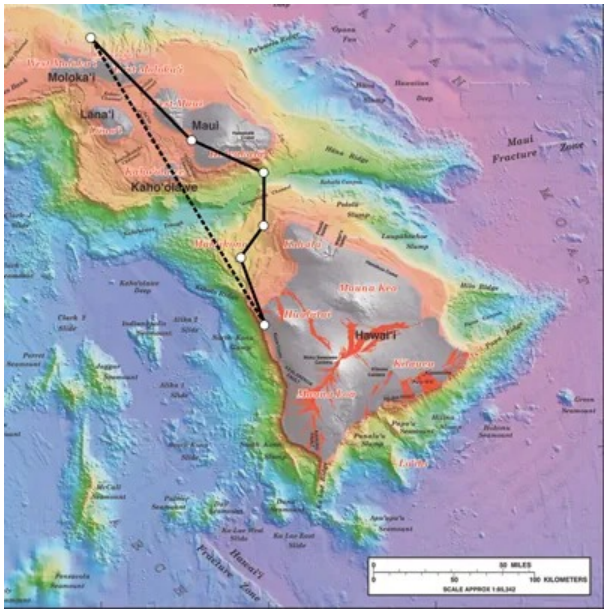


Figure 3.7 Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. Image from OpenStax College Physics 2e, CC-BY 4.0

Vectors in Two Dimensions

A vector is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

Figure 3.8 shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Kinematics in Two Dimensions: An Introduction. We shall use the notation that a boldface symbol, such as $\mathbf{D}$, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by θ.

Vectors in this Text

In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector $\mathbf{F}$, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle θ.

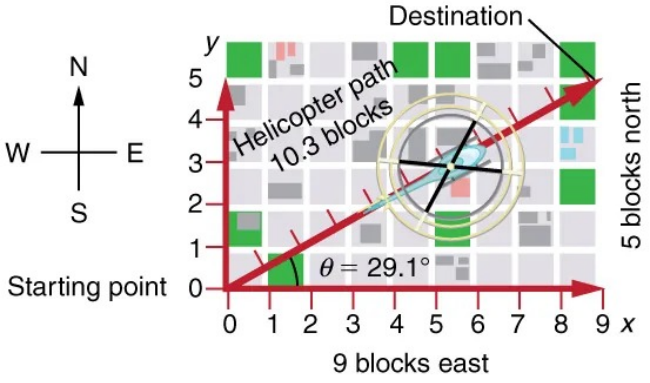


Figure 3.8 A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle 29.1° *north of east. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows a grid map with streets in a city layout. The map is overlaid with arrows, a compass, and labels to illustrate a path taken by a helicopter.

Below is a detailed description of the image elements:

– Compass Rose: Located on the left side of the image, it indicates north, south, east, and west directions with arrows pointing in each cardinal direction. The starting point is marked directly south.

– Grid Map: The grid is marked with blocks and numbered axes. The x-axis runs horizontally at the bottom, labeled from 0 to 9, with the direction indicated as "9 blocks east." The y-axis runs vertically on the left, labeled from 0 to 5, with the direction marked as "5 blocks north."

– Path and Arrows:
– A thick red arrow extends diagonally from the bottom left to the top right, representing the helicopter's path. The path is labeled "Helicopter path 10.3 blocks" along the arrow.
– Another label near the beginning of the red arrow indicates an angle, "θ = 29.1°," marking the angle of the path relative to the eastward direction.
– Spaced along the path are several smaller, parallel red arrows marking the intended direction of travel.

– Destination and Starting Point:
– The path begins at the "Starting point" at the bottom left (0,0) and extends to the "Destination" in the top right.

– City Blocks: The gray grid represents city blocks with random green squares, which could represent parks or important landmarks scattered throughout.

This map visualizes a helicopter's flight path over a city grid, indicating its intended travel direction, distance, and angle from the starting point to the destination.

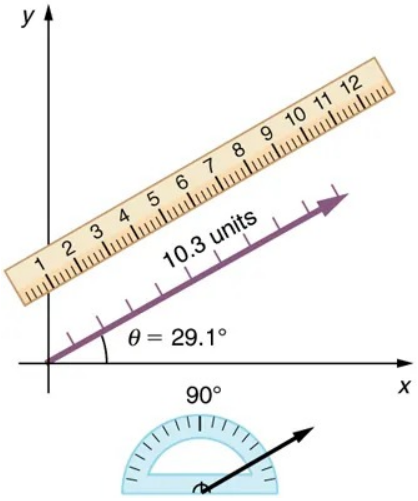


Figure 3.8

Figure 3.9 To describe the resultant vector for the person walking in a city considered in graphically, draw an arrow to represent the total displacement vector D*. Using a protractor, draw a line at an angle* θ *relative to the east-west axis. The length* D *of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude* D *of the vector is 10.3 units, and the direction* θ *is* 29.1° *north of east. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image features a visual representation of a vector on a coordinate plane. Here's a detailed description:

– Coordinate Axes:
– Includes a vertical y-axis and a horizontal x-axis.
– The axes intersect at the origin point (0,0).

– Ruler:
– Placed diagonally aligning from point (1,1) going upwards towards positive y and x directions.
– The ruler measures 12 units and helps illustrate another vector beneath it.

– Vector:
– A line represents the vector, drawn from the origin in a diagonal direction towards the upper right, mirroring the direction of the ruler.
– Marked with an arrowhead pointing toward the x-axis, along the line.
– The vector is labeled as 10.3 units long.

– Angle:
– The vector creates an angle of 29.1 degrees ($\theta = 29.1°$) with the positive x-axis.
– A small arc near the origin highlights this angle.

– Protractor:
– Located below the coordinate axes to measure the angle.
– The protractor has a marked arrow pointing towards the direction of the vector, aligned with the angle measurement.

This image is likely used to demonstrate vector measurement, angle calculation, and the relationship between angles and coordinate positions.

Vector Addition: Head-to-Tail Method

The head-to-tail method is a graphical way to add vectors, described in Figure 3.10 below and in the steps following. The tail of the vector is the starting point of the vector, and the head (or tip) of a vector is the final, pointed end of the arrow.

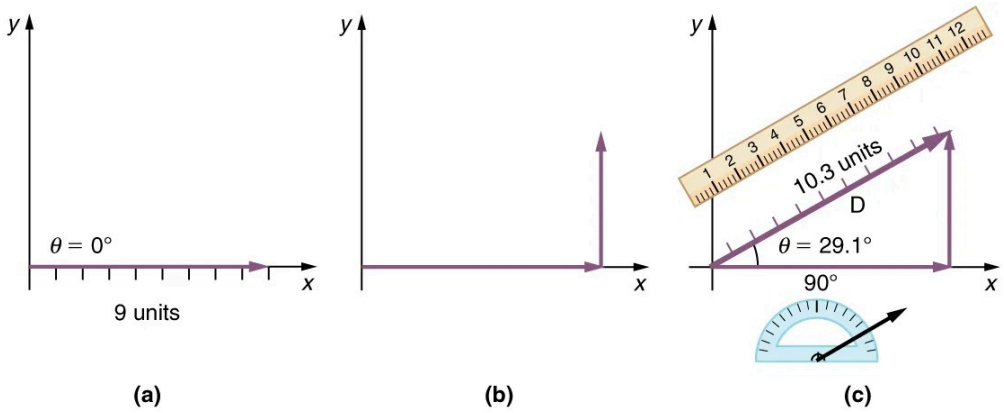


Figure 3.10 ***Head-to-Tail Method:*** *The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in . (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or resultant vector* D*. The length of the arrow* D *is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis)* θ *is measured with a protractor to be* 29.1° *. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is composed of three separate panels labeled (a), (b), and (c), each showing a coordinate system with an x-axis and y-axis.

– Panel (a):
– Displays a vector along the x-axis.
– The vector is horizontal, moving right with an angle θ = 0°.
– The length of the vector is labeled as 9 units.

– Panel (b):
– Shows two vectors forming a right angle.
– One vector is horizontal along the x-axis pointing right.
– The other vector is vertical along the y-axis pointing upwards.

– Panel (c):
– Features a vector forming a triangle with the horizontal x-axis.
– The vector is diagonal, labeled as 10.3 units in length.
– The angle between the vector and the x-axis is θ = 29.1°.

– The triangle has a right angle, with the 90° highlighted.
– There is a protractor above the triangle for angle measurement and a ruler along the vector.

Each panel illustrates different properties and orientations of vectors using coordinate systems.

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor.*

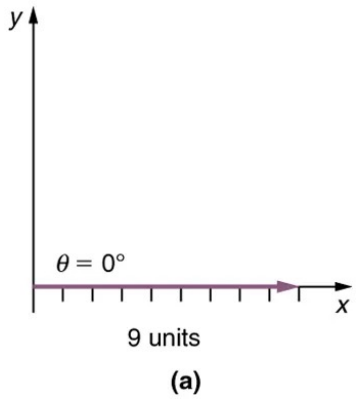


Figure 3.11 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a coordinate plane with an x-axis and y-axis. There is a horizontal arrow along the x-axis pointing to the right, representing a vector. The vector is labeled as "9 units" in length and has an angle θ equal to 0 degrees with respect to the x-axis. The point where the vector starts is at the origin of the coordinate plane. Below the vector's label, it is marked as "(a)".

Step 2. Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector.*

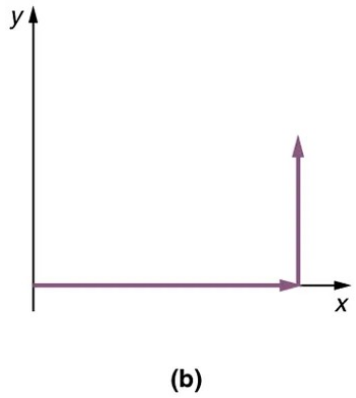


Figure 3.12 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a 2D coordinate system with two axes labeled “x” and “y”. The “x” axis runs horizontally from left to right, and the “y” axis runs vertically from bottom to top. Both axes are illustrated with arrows to indicate positive directions. There is a purple vector starting at the origin (0,0) extending first horizontally right along the “x” axis, then turning vertically upwards along the “y” axis. The vector forms a right angle, resembling an “L” shape, moving initially in the positive x direction and then in the positive y direction. Below the graph, there is a label “(b)”.

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail*.

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the resultant, or the sum, of the other vectors.

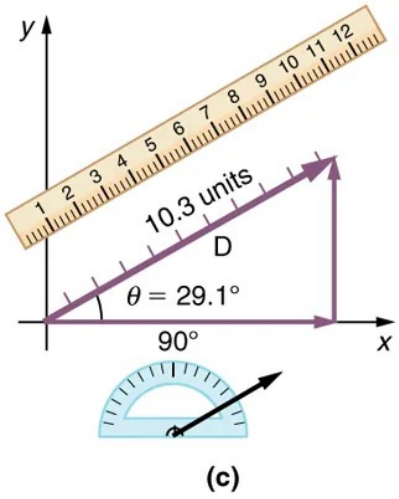


Figure 3.13 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a diagram with coordinate axes and a geometric representation of a vector. The *x*-axis is horizontal and extends to the right, while the *y*-axis is vertical and extends upward. A vector labeled *D* is drawn diagonally from the origin at an angle, with a magnitude of 10.3 units. The angle between vector *D* and the *x*-axis is indicated as 29.1 degrees.

A protractor is positioned below vector *D*, measuring the angle from the horizontal axis. The protractor also shows a 90-degree angle marked between the *x*– and *y*-axes. Additionally, a ruler is placed parallel to vector *D*, showing measurement markings from 0 to 12. The vector extends from the origin, implying a scale relevant to the ruler’s markings.

Step 5. To get the magnitude of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the direction of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Example 3.1

Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction 49.0° north of east. Then, she walks 23.0 m heading 15.0° north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

Strategy

Represent each displacement vector graphically with an arrow, labeling the first $\mathbf{A}$, the second $\mathbf{B}$, and the third $\mathbf{C}$, making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted $\mathbf{R}$.

Solution

(1) Draw the three displacement vectors.

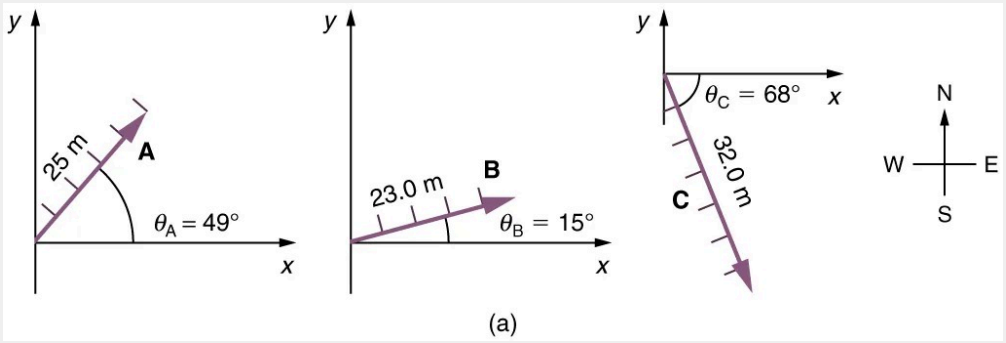


Figure 3.14 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three vector diagrams labeled A, B, and C, along with a compass rose on the right. Each vector is positioned in a coordinate system with x and y axes.

– Vector A:
– The vector has a magnitude of 25 meters.
– It is oriented at an angle of 49 degrees above the positive x-axis.

– Vector B:
– The vector has a magnitude of 23 meters.
– It is oriented at an angle of 15 degrees above the positive x-axis.

– Vector C:
– The vector has a magnitude of 32 meters.
– It is oriented at an angle of 68 degrees below the negative y-axis, parallel to the negative x-axis.

– Compass Rose:
– The compass rose shows the cardinal directions: North (N), East (E), South (S), and West (W).

(2) Place the vectors head to tail retaining both their initial magnitude and direction.

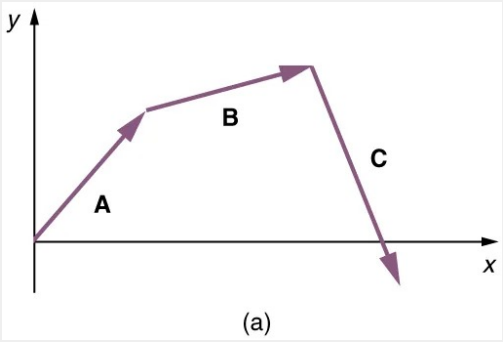


Figure 3.15 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph with an x-axis labeled “x” and a y-axis labeled “y”. Three vectors labeled A, B, and C are represented by arrows forming a path breaking at two points.

– Vector A: Starts at the origin and points upward and to the right, forming an angle

with the positive x-axis.
– Vector B: Begins where vector A ends, continuing upward and to the right with a smaller angle.
– Vector C: Begins at the end of vector B and points downward to the right, crossing the x-axis.

Each vector is shown with a purple arrow indicating the direction. The sequence of vectors resembles a sawtooth pattern on the graph. The label "(a)" appears at the bottom center of the image.

(3) Draw the resultant vector, R.

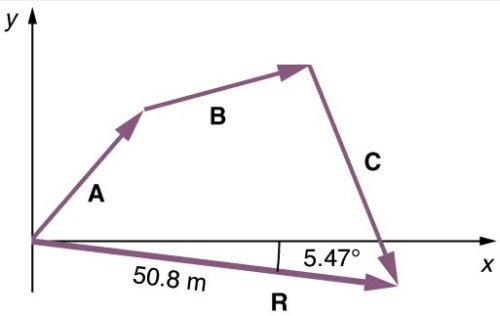


Figure 3.16 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a vector diagram on a coordinate plane with x and y axes. The diagram includes three vectors labeled A, B, and C, connected in a path. Each vector is depicted with an arrow indicating its direction:

– Vector A starts from the origin and points upward and slightly to the right.
– Vector B continues from the endpoint of Vector A, moving in a more horizontal direction to the right with upward ascent.
– Vector C starts from the endpoint of Vector B, heading downward with a steep angle towards the positive x-axis.

The resulting vector, labeled R, is shown pointing horizontally to the right from the origin, forming the base of the triangular path created by the vectors A, B, and C.

There are two annotations on the diagram:

– The length of the horizontal resultant vector R is marked as 50.8 meters.
– The angle between vector R and the positive x-axis is labeled as $5.47°$.

Both the x and y axes have arrows indicating positive directions, with the y-axis pointing upwards and the x-axis pointing towards the right.

(4) Use a ruler to measure the magnitude of $\mathbf{R}$, and a protractor to measure the direction of $\mathbf{R}$. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.

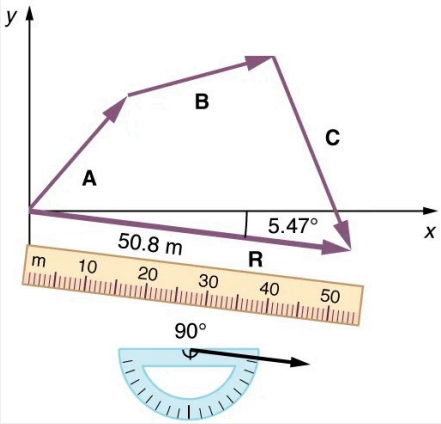


Figure 3.17 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram showing a vector addition problem and involves a coordinate system along with measurement tools.

– There is a coordinate system with an x-axis and y-axis.
– Three vectors labeled A, B, and C form a closed triangular path.
– Vector A points upward and to the left.
– Vector B points slightly downward and to the right.
– Vector C points downward and to the left.
– Each vector is represented with an arrow showing its direction and relative magnitude.
– A vector labeled R represents the resultant vector and points horizontally along the

x-axis.
– The angle between vector C and the x-axis is marked as 5.47 degrees.
– A ruler below the vectors shows a measurement of 50.8 meters, positioned parallel to the x-axis.
– A protractor is shown below, indicating a 90-degree angle alignment.

The diagram visually combines geometry and measurement concepts to demonstrate how the vectors add up to form the resultant vector R.

In this case, the total displacement $\mathbf{R}$ is seen to have a magnitude of 50.8 m and to lie in a direction 5.47° south of east. By using its magnitude and direction, this vector can be expressed as $R = 50.8\ \text{m}$ and $\theta = 5.47^{\circ}$ south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in Figure 3.18 and we will still get the same solution.

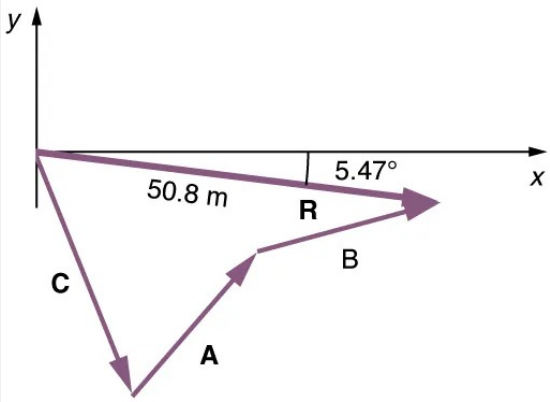


Figure 3.18 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram showing a vector addition problem on a two-dimensional Cartesian coordinate system. Here's a detailed description of the elements:

– There are two axes: the horizontal axis labeled "x" and the vertical axis labeled "y".

– Three vectors are labeled “A”, “B”, and “C”, each represented by arrows.
– Vector “A” points downward and slightly to the left.
– Vector “B” connects to the tip of vector “A” and points upwards and to the right.
– Vector “C” starts at the origin and points downward.
– A resultant vector, labeled “R”, is shown as the diagonal of the parallelogram formed with vectors “B” and “C”. It points approximately horizontally to the right.
– The angle between the resultant vector “R” and the positive x-axis is labeled as “5.47°”.
– The magnitude of the resultant vector “R” is labeled as “50.8 m”.

Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is commutative. Vectors can be added in any order.

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}.$$

3.1

(This is true for the addition of ordinary numbers as well—you get the same result whether you add $\mathbf{2} + \mathbf{3}$ or $\mathbf{3} + \mathbf{2}$, for example).

Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract $\mathbf{B}$ from $\mathbf{A}$, written $\mathbf{A}–\mathbf{B}$, we must first define what we mean by subtraction. The *negative* of a vector $\mathbf{B}$ is defined to be $–\mathbf{B}$; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in Figure 3.19. In other words, $\mathbf{B}$ has the same length as $–\mathbf{B}$, but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.

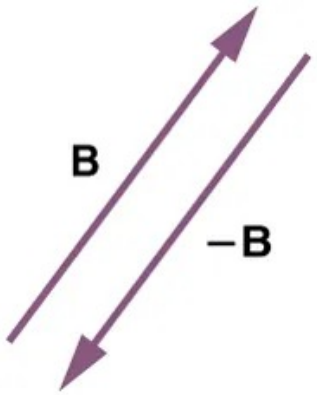


Figure 3.19 The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So $\mathbf{B}$ *is the negative of* $-\mathbf{B}$*; it has the same length but opposite direction. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows two diagonal arrows, each pointing in opposite directions. The arrows are parallel to each other. The arrow on the left points upward to the right, labeled "B." The arrow on the right points downward to the left, labeled "-B." Both arrows are colored purple. The label "B" represents the magnetic field direction, while "-B" represents the opposite direction of the magnetic field.

The *subtraction* of vector $\mathbf{B}$ from vector $\mathbf{A}$ is then simply defined to be the addition of $-\mathbf{B}$ to $\mathbf{A}$. Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

$$\mathrm{A} - \mathrm{B} = \mathrm{A} + (-\mathrm{B}).$$
3.2

This is analogous to the subtraction of scalars (where, for example, $5 - 2 = 5 + (-2)$). Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

Example 3.2

Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.

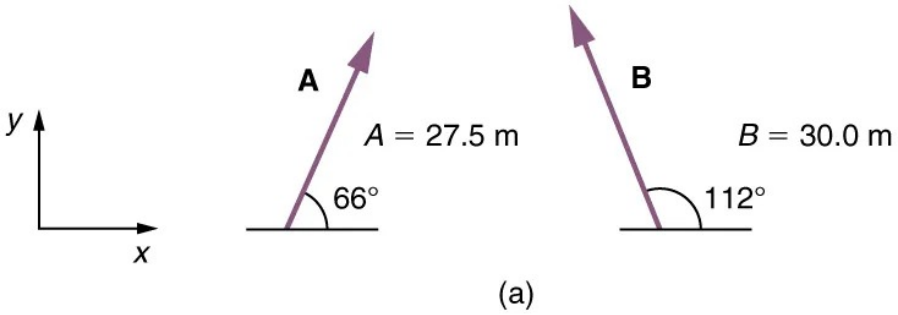


Figure 3.20 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two vectors labeled A and B, shown in a two-dimensional plane with coordinate axes.

– Vector A:
– Magnitude: 27.5 meters
– Direction: The arrow is directed upwards and to the right, forming an angle of 66 degrees with the horizontal axis.

– Vector B:
– Magnitude: 30.0 meters
– Direction: The arrow is directed upwards and to the left, forming an angle of 112 degrees with the horizontal axis.

– The coordinate axes are labeled with x (horizontal) and y (vertical) directions. The arrows representing the vectors are colored and have arrows at one end to indicate direction.

Strategy

We can represent the first leg of the trip with a vector $\mathbf{A}$, and the second leg of the trip with a vector $\mathbf{B}$. The dock is located at a location $\mathbf{A} + \mathbf{B}$. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance B (30.0 m) in the direction $180^\circ - 112^\circ = 68^\circ$ south of east. We represent this as $-\mathbf{B}$, as shown below. The vector $-\mathbf{B}$ has the same magnitude as $\mathbf{B}$ but is in the opposite direction. Thus, she will end up at a location $\mathbf{A} + (-\mathbf{B})$, or $\mathbf{A}-\mathbf{B}$.

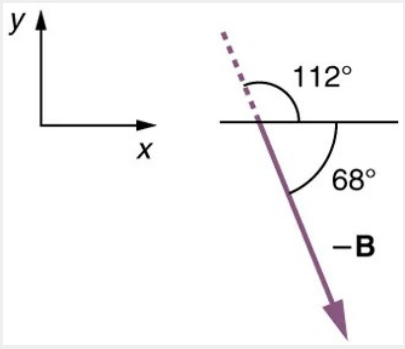


Figure 3.21 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a two-dimensional coordinate system with x and y axes. A vector labeled "-B" is positioned in the plane. The vector is angled downward to the right, creating a 68-degree angle with the positive x-axis. There is an additional angle labeled 112 degrees between a dashed line extending the vector backward and the positive x-axis. The vector arrow is depicted as pointing in the downward direction to the right.

We will perform vector addition to compare the location of the dock, $\mathbf{A} + \mathbf{B}$, with the location at which the woman mistakenly arrives, $\mathbf{A} + (-\mathbf{B})$.

Solution

(1) To determine the location at which the woman arrives by accident, draw vectors $\mathbf{A}$ and $-\mathbf{B}$.

(2) Place the vectors head to tail.

(3) Draw the resultant vector $\mathbf{R}$.

(4) Use a ruler and protractor to measure the magnitude and direction of $\mathbf{R}$.

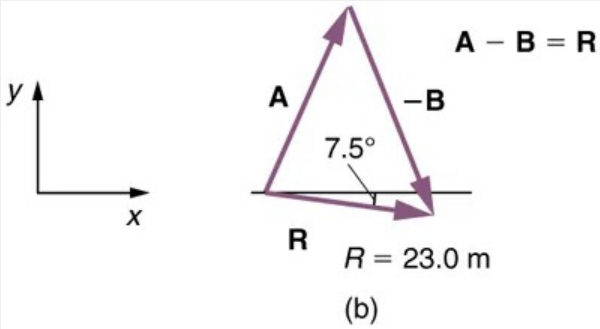


Figure 3.22 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two main components:

1. A set of axes on the left, labeled *x* and *y*, representing a Cartesian coordinate system.
2. A vector diagram on the right. This diagram consists of three vectors that form a triangle. The vectors are labeled as *A*, *-B*, and *R*.
    - Vector *A* is pointing upwards to the right.
    - Vector *-B* is pointing downwards to the left and forms a 7.5-degree angle with the horizontal line extended from *R*.
    - Vector *R* is pointing horizontally to the right and is labeled as *R = 23.0 m*.

    The equation *A – B = R* is written above the triangle diagram.

In this case, $R = 23.0\ \text{m}$ and $\theta = 7.5^{\circ}$ south of east.

(5) To determine the location of the dock, we repeat this method to add vectors $\mathbf{A}$ and $\mathbf{B}$. We obtain the resultant vector $\mathbf{R}'$:

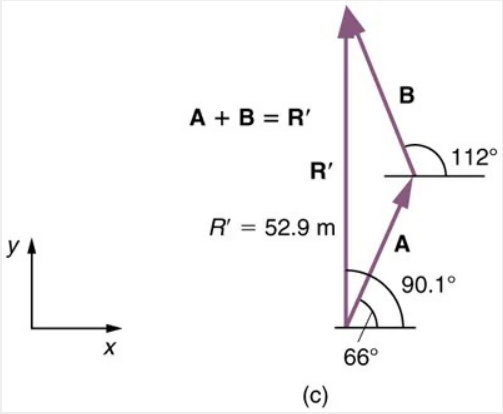


Figure 3.23 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a vector addition diagram on a two-dimensional plane with an x-y axis. There are two vectors, labeled A and B, which are being added to form a resultant vector R'. Here's a detailed description:

- The x-y axis is shown on the left side of the image, with arrows indicating the positive x-direction pointing to the right and the positive y-direction pointing upwards.
- Vector A is drawn starting at a horizontal line, making an angle of 66 degrees with the x-axis. Its angle relative to the horizontal is marked as 90.1 degrees. It points upwards and to the left.
- Vector B is drawn starting from the tip of vector A, making an angle of 112 degrees with the resultant vector R'. It points upwards and to the right.
- The resultant vector R' is drawn vertically upwards from the starting point of vector A. The magnitude of R' is labeled as 52.9 meters.
- The equation A + B = R' is written beside the vectors, indicating that the vectors A and B are added to produce the resultant vector R'.

In this case $R = 52.9\ \mathrm{m}$ and $\theta = 90.1^{\circ}$ north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk $3 \times 27.5\ \text{m}$, or 82.5 m, in a direction 66.0° north of east. This is an example of multiplying a vector by a positive scalar. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite* direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector $\mathbf{A}$ is multiplied by a scalar c,

- the magnitude of the vector becomes the absolute value of cA,
- if c is positive, the direction of the vector does not change,
- if c is negative, the direction is reversed.

In our case, $c = 3$ and $A = 27.5m$. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular components of a single vector, for example the *x*– *and y*-components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a

direction 29.0° north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in 3.4 Projectile Motion, and much more when we cover **forces** in Dynamics: Newton's Laws of Motion. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in Vector Addition and Subtraction: Analytical Methods are ideal for finding vector components.

PhET Explorations

Maze Game

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.

Click to view content.

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3.3 VECTOR ADDITION AND SUBTRACTION: ANALYTICAL METHODS

Learning Objectives

By the end of this section, you will be able to:

- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like $\mathbf{A}$ in Figure 3.24, we may wish to find which two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it.

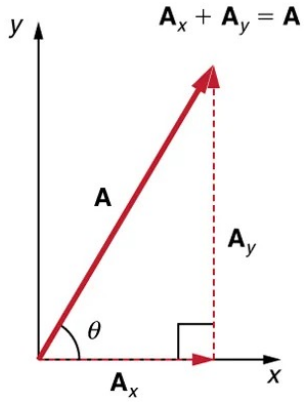


Figure 3.24 The vector $\mathbf{A}$, with its tail at the origin of an x, y-coordinate system, is shown together with its x– and y-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle. The analytical relationships among these vectors are summarized below. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a vector diagram on a Cartesian coordinate system. There is a right triangle formed by two components and the resultant vector:

- A represents the resultant vector, shown as a red arrow at an angle. The vector A is positioned from the origin and extends into the first quadrant.
- The vector has a magnitude labeled as A.
- The horizontal component, A_x, is shown as a dashed red line extending along the x-axis.
- The vertical component, A_y, is shown as a dashed red line extending parallel to the y-axis.
- An angle θ (theta) is formed between the x-axis and the vector A.
- A right angle is marked between A_x and A_y using a small square.
- Text at the top of the image states: $A_x + A_y = A$
- Both x and y axes are labeled accordingly.

$\mathbf{A}_x$ and $\mathbf{A}_y$ are defined to be the components of $\mathbf{A}$ along the x– and y-axes. The three vectors $\mathbf{A}$, $\mathbf{A}_x$, and $\mathbf{A}_y$ form a right triangle:

$$\mathbf{A}_x + \mathrm{A}_y = \mathrm{A}.$$

3.3

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $\mathbf{A}_x = 3m$ east, $\mathbf{A}_y = 4m$ north, and $\mathbf{A} = 5m$ north-east, then it is

true that the vectors $\mathbf{A}_x + \mathbf{A}_y = \mathbf{A}$. However, it is *not* true that the sum of the magnitudes of the vectors is also equal. That is,

$$3\ \text{m} + 4\ \text{m} \neq 5\ \text{m}$$
3.4

Thus,

$$A_x + A_y \neq A$$
3.5

If the vector $\mathbf{A}$ is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its *x*– and *y*-components, we use the following relationships for a right triangle.

$$A_x = A\cos\theta$$
3.6

and

$$A_y = A\sin\theta.$$
3.7

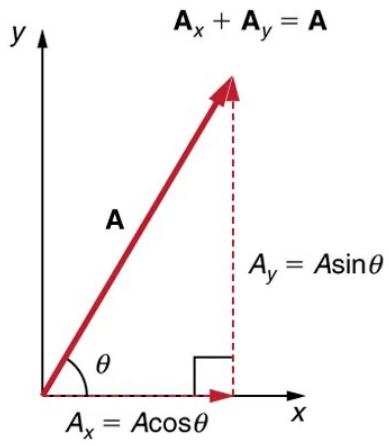


Figure 3.25 The magnitudes of the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$ can be related to the resultant vector $\mathbf{A}$ and the angle θ with trigonometric identities. Here we see that $A_x = A\cos\theta$ and $A_y = A\sin\theta$. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating vector decomposition in a two-dimensional coordinate system. It shows:

– A two-dimensional Cartesian plane with an x-axis and y-axis.
– A vector A represented by a red arrow pointing diagonally from the origin, creating an angle **θ** with the positive x-axis.
– The vector A is decomposed into two components:
– A_x, lying along the x-axis, calculated as **Acosθ**.
– A_y, lying along the y-axis, calculated as **Asinθ**.
– A dashed line alongside **A_y**, perpendicular to the x-axis, emphasizes the right triangle formed by **A**, **A_x**, and **A_y**.
– There is a mathematical expression at the top: **$A_x + A_y = A$**, indicating the composition of vector **A** from its components.

Suppose, for example, that $\mathbf{A}$ is the vector representing the total displacement of the person walking in a city considered in 3.1 Kinematics in Two Dimensions: An Introduction and 3.2 Vector Addition and Subtraction: Graphical Methods.

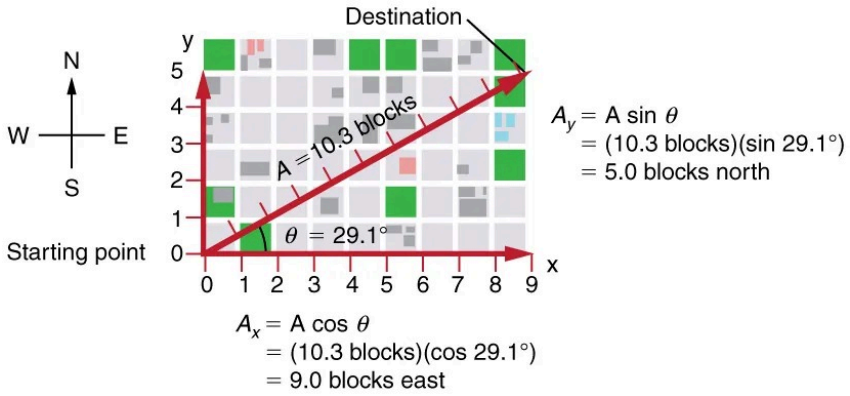


Figure 3.26 We can use the relationships $A_x = A\cos\theta$ *and* $A_y = A\sin\theta$ *to determine the magnitude of the horizontal and vertical component vectors in this example. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a diagram illustrating the components of a vector on a grid map. It includes the following elements:

- A compass rose in the top left corner pointing to the cardinal directions: North (N) is upwards, East (E) is to the right, South (S) is downwards, and West (W) is to the left. The label “Starting point” is near this compass.
- A grid is displayed on the main part of the image, representing city blocks. Each block is a square, some of which are colored green or gray, indicating different types of locations or obstacles.
- The x-axis runs horizontally along the bottom from 0 to 9, and the y-axis runs vertically along the left

side from 0 to 5. Both axes have red arrows indicating their directions, with the x-axis pointing east and the y-axis pointing north.

- A red vector arrow extends from the starting point at the origin (0,0) to a destination at (8,5). This vector is labeled as $A = 10.3$ *blocks* and forms an angle θ = 29.1° with the x-axis.
- The vector is decomposed into horizontal and vertical components:
  - The horizontal component, A_x, is calculated as:
    - $A_x = A \cos \theta$
    - *(10.3 blocks)(cos 29.1°)*
    - = 9.0 blocks east
  - The vertical component, A_y, is calculated as:
    - $A_y = A \sin \theta$
    - *(10.3 blocks)(sin 29.1°)*
    - = 5.0 blocks north

Then $A = 10.3$ blocks and $\theta = 29.1^\circ$, so that

$$A_x = A\cos\theta = (10.3 \text{ blocks})\,(\cos 29.1^\circ\ ) = 9.0 \text{ blocks}$$
3.8

$$A_y = A\sin\theta = (10.3 \text{ blocks})\,(\sin 29.1^\circ\ ) = 5.0 \text{ blocks}.$$
3.9

Calculating a Resultant Vector

If the perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$ of a vector $\mathbf{A}$ are known, then $\mathbf{A}$ can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$, relative to the *x*-axis, we use the following relationships:

$$A = \sqrt{A_x^2 + A_y^2}$$
3.10

$$\theta = \tan^{-1}(A_y/A_x).$$
3.11

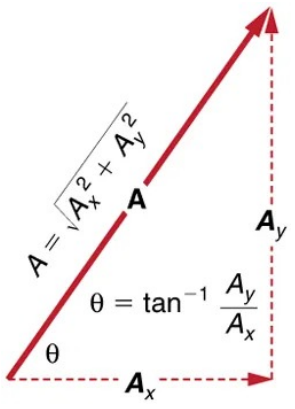


Figure 3.27 The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components A_x and A_y have been determined. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of a vector represented in a two-dimensional coordinate system. The vector is shown as a red arrow pointing diagonally upwards and to the right. Here's a breakdown of the image:

– The vector is labeled as "A" and is drawn from the origin to an endpoint, creating a triangle with the horizontal and vertical components.
– The horizontal component of the vector, labeled as "A_x", is represented by a dashed red arrow along the x-axis.
– The vertical component, labeled as "A_y", is represented by a dashed red arrow along the y-axis.
– An angle θ (theta) is shown between the horizontal component and the vector, indicating the direction of the vector.
– There are two mathematical equations:
– The magnitude of the vector is given by: $A = \sqrt{(A_x^2 + A_y^2)}$.
– The angle θ is given by: $\theta = \tan^{-1}(A_y/A_x)$.

The image is designed to illustrate how the magnitude and direction of vector A can be determined using its components.

Note that the equation $A = \sqrt{A_x^2 + A_y^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if A_x and A_y are 9 and 5 blocks, respectively, then $A = \sqrt{9^2 + 5^2} = 10.3$ blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta = \tan^{-1}(5/9) = 29.1^\circ$, as before.

Determining Vectors and Vector Components with Analytical Methods

Equations $A_x = A\cos\theta$ and $A_y = A\sin\theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to A_x and A_y. Equations $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}\left(A_y/A_x\right)$ are used to find a vector from its perpendicular components—that is, to go from A_x and A_y to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

Adding Vectors Using Analytical Methods

To see how to add vectors using perpendicular components, consider Figure 3.28, in which the vectors $\mathbf{A}$ and $\mathbf{B}$ are added to produce the resultant $\mathbf{R}$.

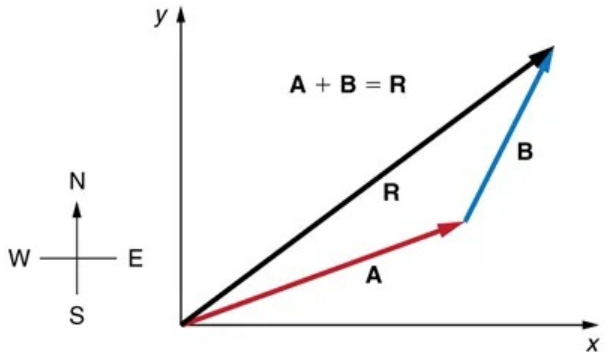


Figure 3.28 Vectors $\mathbf{A}$ *and* $\mathbf{B}$ *are two legs of a walk, and* $\mathbf{R}$ *is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of* $\mathbf{R}$. *Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image contains a diagram illustrating vector addition on a two-dimensional coordinate system, with an x-axis and a y-axis.

– On the right side, the diagram shows three arrows representing vectors.

– Vector A is depicted as a red arrow starting from the origin (0,0) and pointing towards the positive x-axis.
– Vector B is depicted as a blue arrow that starts at the head of Vector A and points diagonally upwards towards the positive y-axis.
– Resultant Vector R is shown as a black arrow starting from the origin and ending at the head of Vector B, representing the vector sum of A and B (A + B = R).

– The x-axis is horizontal, pointing right, and the y-axis is vertical, pointing upwards.

– On the left side of the image, there is a compass rose indicating the cardinal directions, with 'N' at the top for north, 'S' at the bottom for south, 'E' on the right for east, and 'W' on the left for west.

If $\mathbf{A}$ and $\mathbf{B}$ represent two legs of a walk (two displacements), then $\mathbf{R}$ is the total displacement. The person taking the walk ends up at the tip of $\mathbf{R}$. There are many ways to arrive at the same point. In particular, the person could have walked first in the *x*-direction and then in the *y*-direction. Those paths are the *x*– and *y*-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find R and θ using the equations $A = \sqrt{(A_x)^2 + (A_y)^2}$ and $\theta = \tan^{-1}(A_y/A_x)$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

Step 1. *Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes.* Use the equations $A_x = A\cos\theta$ and $A_y = A\sin\theta$ to find the components. In Figure 3.29, these components are A_x, A_y, B_x, and B_y. The angles that vectors $\mathbf{A}$ and $\mathbf{B}$ make with the *x*-axis are θ_A and θ_B, respectively.

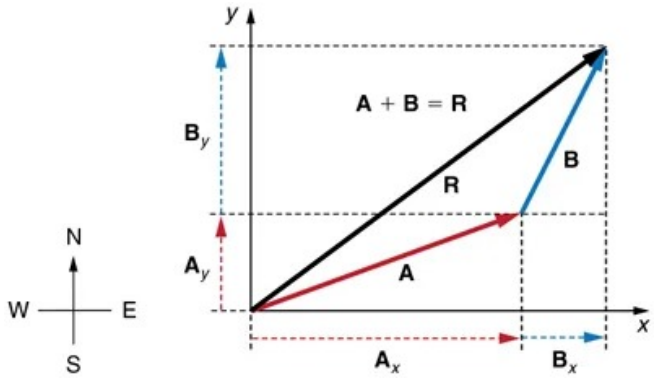


Figure 3.29 To add vectors $\mathbf{A}$ *and* $\mathbf{B}$*, first determine the horizontal and vertical components of each vector. These are the dotted vectors* $\mathbf{A}_x$*,* $\mathbf{A}_y$*,* $\mathbf{B}_x$ *and* $\mathbf{B}_y$ *shown in the image. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image illustrates a vector addition diagram on a coordinate plane.

The diagram consists of three vectors labeled as A, B, and R, where $A + B = R$. All vectors are positioned on an x–y coordinate system.

- A is drawn as a red arrow originating from the origin (0,0) and points in a diagonal direction. Its horizontal component is labeled as A_x and the vertical component as A_y.
- B is a blue arrow originating from the endpoint of vector A and also forms a diagonal. Its components are labeled similarly as B_x and B_y.
- R is a black arrow representing the resultant vector, stretching from the origin to the endpoint of vector B and is in a diagonal orientation.

Dashed lines are present to show the components of each vector along the x and y axes.

To the left of the coordinate system, there is a compass rose indicating directions: North (N), South (S), East (E), and West (W).

Step 2. *Find the components of the resultant along each axis by adding the components of the individual vectors along that axis.* That is, as shown in Figure 3.30,

$$R_x = A_x + B_x$$
3.12

and

$$R_y = A_y + B_y.$$
3.13

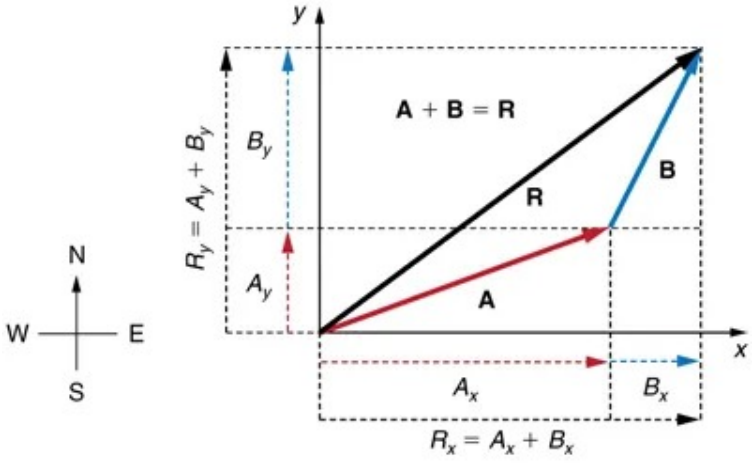


Figure 3.30 The magnitude of the vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude R_y of the resultant vector in the vertical direction. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a vector addition diagram on a Cartesian coordinate system. It features two vectors, A and B, represented as arrows, and their resultant vector, R, also shown as an arrow. The vector A is drawn in red, vector B in blue, and the resultant vector R in black.

The vectors are positioned such that they follow the rule for vector addition (head-to-tail method), where the tail of vector B is at the head of vector A. The resultant vector R is drawn from the tail of A to the head of B.

The diagram includes the horizontal and vertical components of each vector, as follows:

- A_x and A_y are the horizontal and vertical components of vector A.
- B_x and B_y are the horizontal and vertical components of vector B.
- R_x and R_y are the horizontal and vertical components of the resultant vector R.

Equations shown in the diagram:

- $A + B = R$, indicating the vector addition.
- $R_x = A_x + B_x$
- $R_y = A_y + B_y$

To the left of the vector addition diagram is a compass indicating directions with labels: N (North), S (South), E (East), and W (West).

Components along the same axis, say the x-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the y-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of $\mathbf{R}$ are known, its magnitude and direction can be found.

Step 3. *To get the magnitude R of the resultant, use the Pythagorean theorem:*

$$R = \sqrt{R_x^2 + R_y^2}.$$

3.14

Step 4. *To get the direction of the resultant relative to the x-axis:*

$$\theta = \tan^{-1}\left(R_y/R_x\right).$$

3.15

The following example illustrates this technique for adding vectors using perpendicular components.

Example 3.3

Adding Vectors Using Analytical Methods

Add the vector $\mathbf{A}$ to the vector $\mathbf{B}$ shown in Figure 3.31, using perpendicular components along the *x*– and *y*-axes. The *x*– and *y*-axes are along the east–west and north–south directions, respectively. Vector $\mathbf{A}$ represents the first leg of a walk in which a person walks $53.0\ \text{m}$ in a direction 20.0° north of east. Vector $\mathbf{B}$ represents the second leg, a displacement of $34.0\ \text{m}$ in a direction 63.0° north of east.

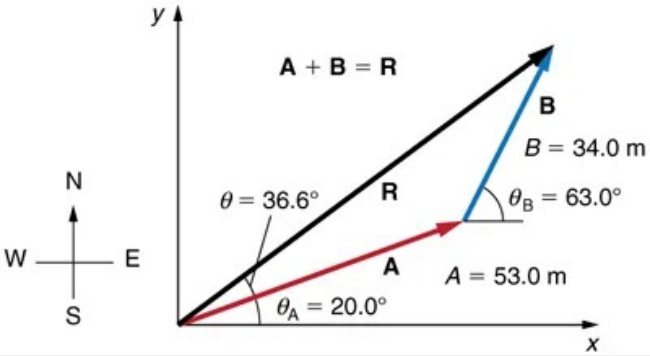


Figure 3.31 Vector $\mathbf{A}$ has magnitude $53.0\ \text{m}$ and direction 20.0° north of the x-axis. Vector $\mathbf{B}$ has magnitude $34.0\ \text{m}$ and direction 63.0° north of the x-axis. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a vector diagram on an x-y coordinate system illustrating the addition of two vectors, *A* and *B*, resulting in a resultant vector *R*. Here is a detailed description:

- The x-axis is horizontal, and the y-axis is vertical, forming a right-angle coordinate system.
- Vector *A* is represented by a red arrow, pointing at an angle of 20.0° above the positive x-axis, with a magnitude of 53.0 meters.

- Vector *B* is a blue arrow, pointing at an angle of 63.0° above the line where vector *A* ends, with a magnitude of 34.0 meters.
- Vectors *A* and *B* are added together to form the resultant vector *R*, represented as a black arrow. This resultant vector *R* points at an angle of 36.6° from the positive x-axis.
- The compass rose in the lower-left corner indicates directions with cardinal points: North (N), South (S), East (E), and West (W). North is upwards, South is downwards, East is to the right, and West is to the left.

Strategy

The components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*– and *y*-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

Solution

Following the method outlined above, we first find the components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*– and *y*-axes. Note that $A = 53.0m$, $\theta_{\text{A}} = 20.0^{\text{o}}$, $B = 34.0m$, and $\theta_{\text{B}} = 63.0^{\text{o}}$. We find the *x*-components by using $A_x = A\cos\theta$, which gives

$$\begin{aligned} A_x &= A\cos\theta_A = (53.0m)\,(\cos\,20.0^{\text{o}}\,) \\ &= (53.0m)\,(0.940) = 49.8m \end{aligned}$$

3.16

and

$$\begin{aligned} B_x &= B\cos\theta_B = (34.0m)\,(\cos\,63.0^{\text{o}}\,) \\ &= (34.0m)\,(0.454) = 15.4m. \end{aligned}$$

3.17

Similarly, the *y*-components are found using $A_y = A\sin\theta_A$:

$$\begin{aligned} A_y &= A\sin\theta_A = (53.0m)\,(\sin\,20.0^{\text{o}}\,) \\ &= (53.0m)\,(0.342) = 18.1m \end{aligned}$$

3.18

and

$$\begin{aligned} B_y &= B\sin\theta_B = (34.0m)\,(\sin 63.0o) \\ &= (34.0m)\,(0.891) = 30.3m. \end{aligned}$$

3.19

The *x*– and *y*-components of the resultant are thus

$$R_x = A_x + B_x = 49.8m + 15.4m = 65.2m$$

3.20

and

$$R_y = A_y + B_y = 18.1m + 30.3m = 48.4m.$$

3.21

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2\ \mathrm{m}}$$

3.22

so that

$$R = 81.2m.$$

3.23

Finally, we find the direction of the resultant:

$$\theta = \tan^{-1}(R_y/R_x) = +\tan^{-1}(48.4/65.2).$$

3.24

Thus,

$$\theta = \tan^{-1}(0.742) = 36.6o.$$

3.25

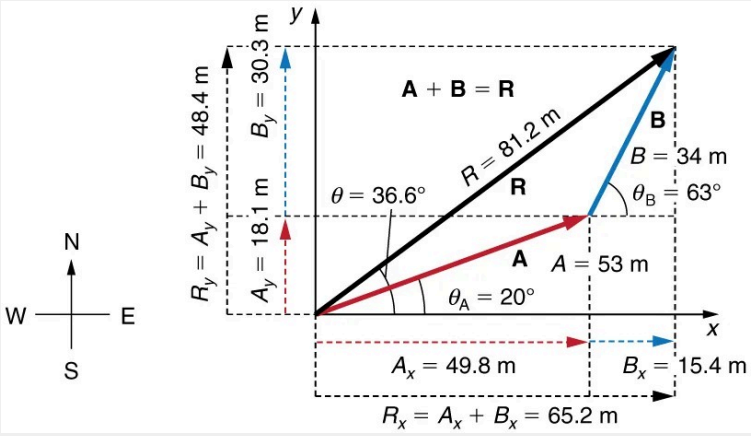


Figure 3.32 Using analytical methods, we see that the magnitude of $\mathbf{R}$ *is* $81.2\ \text{m}$ *and its direction is* 36.6° *north of east. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a vector diagram illustrating the addition of two vectors, A and B, to produce a resultant vector R on a two-dimensional plane with an x-y coordinate system.

– Vectors:
– A: Magnitude of 53 meters, forming an angle $\theta_A = 20^\circ$ with the x-axis.
– B: Magnitude of 34 meters, forming an angle $\theta_B = 63^\circ$ with the x-axis.
– R: Resultant vector, magnitude of 81.2 meters, forming an angle $\theta = 36.6^\circ$ with the x-axis.

– Vector Components:
– A:
– Horizontal component $A_x = 49.8$ meters.
– Vertical component $A_y = 18.1$ meters.
– B:
– Horizontal component $B_x = 15.4$ meters.
– Vertical component $B_y = 30.3$ meters.
– R:
– Horizontal component $R_x = A_x + B_x = 65.2$ meters.
– Vertical component $R_y = A_y + B_y = 48.4$ meters.

– Coordinate System:

– A Cartesian coordinate system is shown with the x-axis pointing to the right and the y-axis pointing upwards.
– There's also a reference to the cardinal directions: North (N), South (S), East (E), and West (W).

This diagram helps visualize how vectors can be broken down into their components and combined to form a resultant vector.

Discussion

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is, $\mathbf{A} - \mathbf{B} \equiv \mathbf{A} + (-\mathbf{B})$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition*. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The *x*– and *y*-components of the resultant $\mathbf{A} - \mathrm{B} = \mathrm{R}$ are thus

$$R_x = A_x + (-B_x)$$

3.26

and

$$R_y = A_y + (-B_y)$$

3.27

and the rest of the method outlined above is identical to that for addition. (See Figure 3.33.)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, 3.4 Projectile Motion, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.

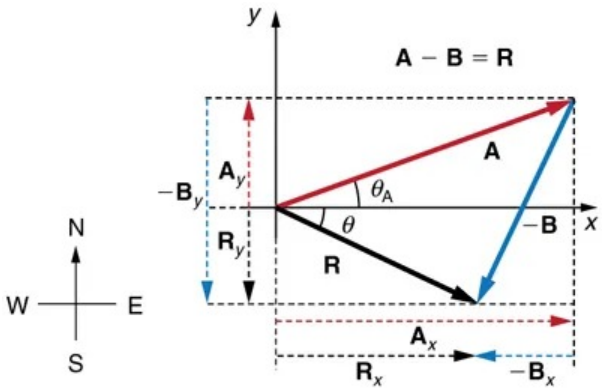


Figure 3.33 The subtraction of the two vectors shown in . The components of $-\mathbf{B}$ *are the negatives of the components of* $\mathbf{B}$*. The method of subtraction is the same as that for addition. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a diagram demonstrating vector subtraction and its components in a two-dimensional Cartesian coordinate system. The main axes are labeled *x* and *y*, with positive and negative directions clearly indicated.

There are three main vectors shown:

- A (red vector) directed from the origin with components A_x (horizontal component) and A_y (vertical component). The angle θ_A is the angle between vector A and its x-component.
- B (blue vector) directed downward and to the right with components B_x (horizontal component) and B_y (vertical component).
- R (black vector) represents the resultant of subtracting vector B from vector A, with components R_x and R_y.

The subtraction operation is expressed as $A - B = R$.

Each component has a corresponding line indicating its magnitude along the respective axis:

- A_x and B_x are horizontal components along the x-axis.
- A_y and B_y are vertical components along the y-axis.
- R_x and R_y represent the resultant's components.

A compass rose is included to indicate directions with N (North), S (South), E (East), and W (West) marked.

PhET Explorations

Vector Addition

Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.

Access multimedia content

"3.3 Vector Addition and Subtraction: Analytical Methods" from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

3.4 PROJECTILE MOTION

Learning Objectives

By the end of this section, you will be able to:

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

Projectile motion is the motion of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a projectile, and its path is called its trajectory. The motion of falling objects, as covered in 2.6 Problem-Solving Basics for One-Dimensional Kinematics, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which air resistance *is negligible*.

The most important fact to remember here is that *motions along perpendicular axes are independent* and thus can be analyzed separately. This fact was discussed in 3.1 Kinematics in Two Dimensions: An Introduction, where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the x-axis and the vertical axis the y-axis. Figure 3.34 illustrates the notation for displacement, where $\mathbf{s}$ is defined to be the total displacement and $\mathbf{x}$ and $\mathbf{y}$ are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are s, x, and y. (Note that in the last section we used

the notation $\mathbf{A}$ to represent a vector with components $\mathbf{A}_x$ and $\mathbf{A}_y$. If we continued this format, we would call displacement $\mathbf{s}$ with components $\mathbf{s}_x$ and $\mathbf{s}_y$. However, to simplify the notation, we will simply represent the component vectors as $\mathbf{x}$ and $\mathbf{y}$.)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the *x*– and *y*-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple:

$a_y = -g = -9.80 m/\text{s}^2$. (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical,

$a_x = 0$. Both accelerations are constant, so the kinematic equations can be used.

Review of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t$$

3.28

$$\bar{v} = \frac{v_0 + v}{2}$$

3.29

$$v = v_0 + \text{at}$$

3.30

$$x = x_0 + v_0 t + \frac{1}{2}\text{at}^2$$

3.31

$$v^2 = v_0^2 + 2a\,(x - x_0\,).$$

3.32

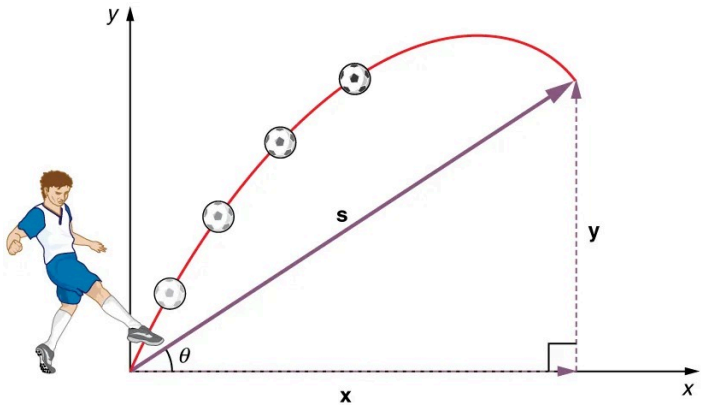


Figure 3.34 The total displacement **s** *of a soccer ball at a point along its path. The vector* **s** *has components* **x** *and* **y** *along the horizontal and vertical axes. Its magnitude is* s*, and it makes an angle* θ *with the horizontal. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a diagram illustrating the parabolic trajectory of a soccer ball being kicked. A soccer player dressed in a blue and white uniform is shown on the left, kicking the ball. The ball follows a curved, red path, moving upward then downward, with several positions of the ball marked along the trajectory to show its motion through the air.

There are two axes in the diagram:

- The vertical axis is labeled *y* and indicates the height of the ball.
- The horizontal axis is labeled *x* and represents the distance traveled by the ball.

An angle, θ, is shown at the point where the player kicks the ball, highlighting the initial angle of the ball's trajectory relative to the horizontal axis. A straight, purple line labeled *s* indicates the distance of the projectile's path from the kicking point to the apex of its path. At the endpoint of the path, a dashed line connects from the ball to both the horizontal and vertical axes, illustrating the maximum height (*y*) and horizontal distance (*x*) the ball achieves.

Given these assumptions, the following steps are then used to analyze projectile motion:

Step 1. *Resolve or break the motion into horizontal and vertical components along the x- and y-axes.* These axes are perpendicular, so

$A_x = A\cos\theta$ and

$A_y = A\sin\theta$ are used. The magnitude of the components of displacement

$\mathbf{s}$ along these axes are x and

y. The magnitudes of the components of the velocity $\mathbf{v}$ are

$v_x = v\cos\theta$ and

$v_y = v\sin\theta$, where

v is the magnitude of the velocity and θ is its direction, as shown in Figure 3.35. Initial values are denoted with a subscript 0, as usual.

Step 2. *Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations for horizontal and vertical motion take the following forms:

$$\text{Horizontal Motion } (a_x = 0\,)$$
3.33

$$x = x_0 + v_x t$$
3.34

$$v_x = v_{0x} = v_x = \text{velocity is a constant.}$$
3.35

$$\text{Vertical Motion (if positive is up} \quad a_y = -g = -9.80\text{m/s}^2\,)$$
3.36

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y\,)t$$
3.37

$$v_y = v_{0y} - \text{gt}$$
3.38

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
3.39

$$v_y^2 = v_{0y}^2 - 2g\,(y - y_0\,).$$
3.40

Step 3. *Solve for the unknowns in the two separate motions—one horizontal and one vertical.* Note that the only common variable between the motions is time t. The problem solving procedures here are the same as for one-dimensional kinematics and are illustrated in the solved examples below.

Step 4. *Recombine the two motions to find the total displacement* $\mathbf{s}$ *and velocity* $\mathbf{v}$. Because the x – and y -motions are perpendicular, we determine these vectors by using the techniques outlined in the 3.3 Vector Addition and Subtraction: Analytical Methods and employing $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}\left(A_y/A_x\right)$ in the following form, where θ is the direction of the displacement $\mathbf{s}$ and θ_v is the direction of the velocity $\mathbf{v}$:

Total displacement and velocity

$$s = \sqrt{x^2 + y^2}$$
3.41

$$\theta = \tan^{-1}\left(y/x\right)$$
3.42

$$v = \sqrt{v_x^2 + v_y^2}$$
3.43

$$\theta_v = \tan^{-1}\left(v_y/v_x\right).$$
3.44

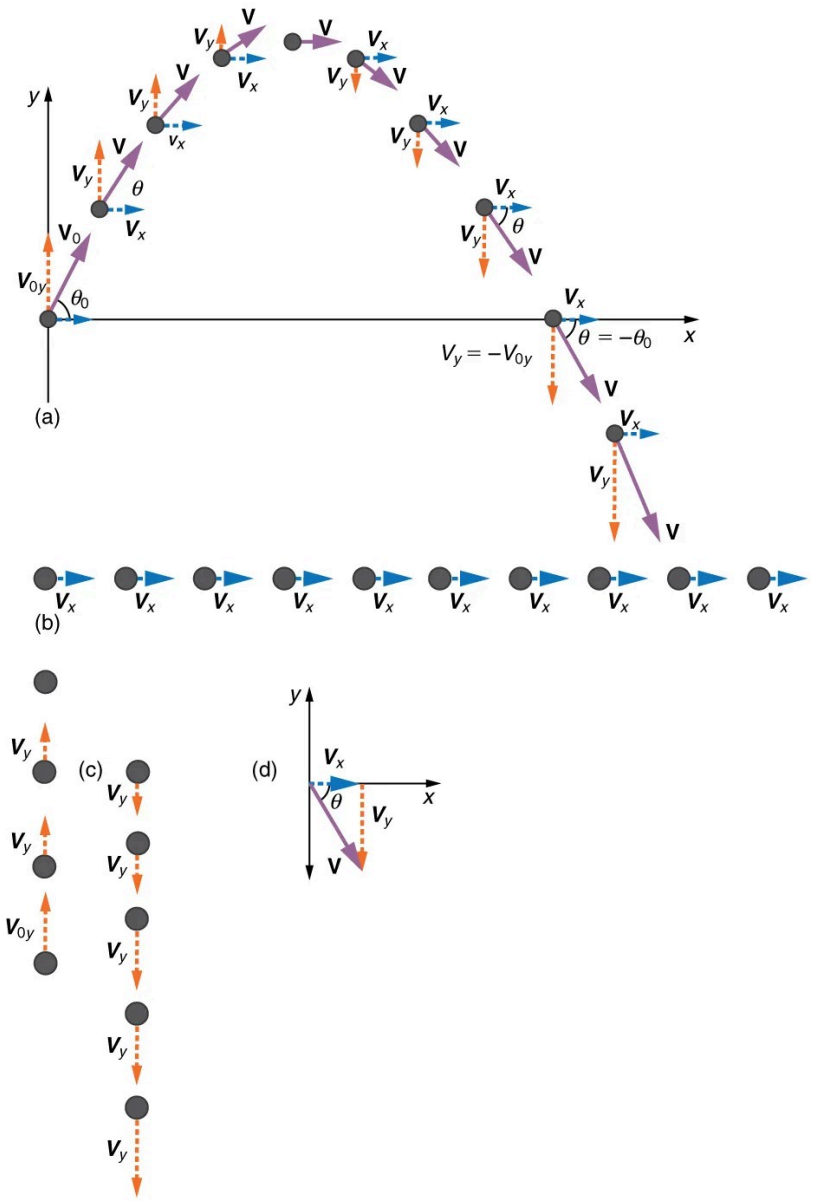


Figure 3.35 (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ *and* v_x *is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The x – and y -motions are recombined to give the total velocity at any given point on the trajectory. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of four labeled diagrams (a, b, c, and d) illustrating the components of motion:

1. Diagram (a):
– Shows a projectile motion trajectory in a parabolic path.
– The projectile starts at an origin point on the x-y axis, launching at angle θ_0 with initial velocity v_0.
– Horizontal velocity v_x is constant (depicted by blue arrows), while vertical velocity v_y (orange arrows) changes direction and magnitude along the trajectory.
– The resulting velocity v is shown as purple arrows, with varying direction and magnitude.
– The angle of velocity θ changes from launch to landing.

2. Diagram (b):
– Displays several identical projectiles moving horizontally.
– All projectiles have a constant horizontal velocity v_x indicated by blue arrows, showing no vertical component of motion.

3. Diagram (c):
– Illustrates projectiles moving vertically, affected by gravity.
– Projectiles are shown ascending and descending, with initial vertical velocity v_{0y} at ascent and just v_y at other points.

4. Diagram (d):
– A detailed breakdown of velocity components.
– Angle θ is measured between the horizontal axis and velocity vector v.
– Velocity is split into horizontal v_x (blue arrow) and vertical v_y (orange arrow) components.
– All vectors originate from a common point on the x-y axis.

Each diagram provides a different perspective on velocity components in projectile motion, highlighting the effects of gravity and trajectory changes.

Example 3.4

A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal, as illustrated in Figure 3.36. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

Strategy

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which $a_x = 0$ and $a_y = -g$. We can then define x_0 and y_0 to be zero and solve for the desired quantities.

Solution for (a)

By "height" we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when $v_y = 0$. Since we know the initial and final velocities as well as the initial position, we use the following equation to find y:

$$v_y^2 = v_{0y}^2 - 2g\,(y - y_0\,).$$

3.45

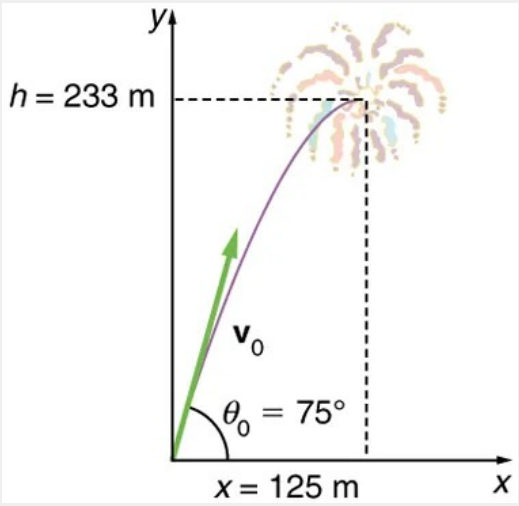


Figure 3.36 The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a trajectory of a firework projectile launched at an angle. Here is a detailed description:

– The x-y coordinate system is shown, with x representing the horizontal axis and y representing the vertical axis.
– The initial launch angle, labeled as $\theta^0 = 75°$, is marked with an arrow starting from the origin (0,0) and pointing upwards.
– The initial velocity is labeled as v^0 alongside the green arrow indicating the projectile's launch direction.
– A curved trajectory path is drawn from the origin, peaking at the coordinates where a firework explodes.
– The firework explosion is visually represented at the upper right side of the graph with colorful bursts.
– The horizontal distance from the origin to the explosion point is labeled as x = 125 m.
– The vertical height from the origin to the explosion point is labeled as h = 233 m.
– Dashed lines from both the horizontal and vertical axes intersect at the explosion point, indicating the maximum height and distance reached by the firework.

Because y_0 and v_y are both zero, the equation simplifies to

$$0 = v_{0y}^2 - 2gy.$$

3.46

Solving for y gives

$$y = \frac{v_{0y}^2}{2g}.$$

3.47

Now we must find

v_{0y}, the component of the initial velocity in the *y*-direction. It is given by

$v_{0y} = v_0 \sin\theta$, where

v_{0y} is the initial velocity of 70.0 m/s, and

$\theta_0 = 75.0^\circ$ is the initial angle. Thus,

$$v_{0y} = v_0 \sin\theta_0 = (70.0 \text{ m/s})(\sin 75^\circ) = 67.6 \text{ m/s}.$$

3.48

and y is

$$y = \frac{(67.6 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)},$$

3.49

so that

$$y = 233 \text{ m}.$$

3.50

Discussion for (a)

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with

a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

Solution for (b)

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$. Because y_0 is zero, this equation reduces to simply

$$y = \frac{1}{2}(v_{0y} + v_y)t.$$

3.51

Note that the final vertical velocity, v_y, at the highest point is zero. Thus,

$$t = \frac{2y}{(v_{0y} + v_y)} = \frac{2\,(233\text{ m})}{(67.6\text{ m/s})} = 6.90\text{ s}.$$

3.52

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using $y = y_0 + v_{0y}t - \frac{1}{2}\mathrm{gt}^2$, and solving the quadratic equation for t.)

Solution for (c)

Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where x_0 is equal to zero:

$$x = v_x t,$$

3.53

where v_x is the *x*-component of the velocity, which is given by $v_x = v_0 \cos\theta_0$. Now,

$$v_x = v_0 \cos\theta_0 = (70.0 m/s)\,(\cos 75.0^\circ) = 18.1 m/s.$$

3.54

The time t for both motions is the same, and so x is

$$x = (18.1 m/s)\,(6.90\ \text{s}) = 125\ \text{m}.$$

3.55

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for y is valid for any projectile motion where air resistance is negligible. Call the maximum height $y = h$; then,

$$h = \frac{v_{0y}^2}{2g}.$$

3.56

This equation defines the *maximum height of a projectile* and depends only on the vertical component of the initial velocity.

Defining a Coordinate System

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_0 = 0$ and $y_0 = 0$. It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, $a_y = -g$, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, $a_y = g$ takes a positive value.

Example 3.5

Calculating Projectile Motion: Hot Rock Projectile

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle 35.0° above the horizontal, as shown in Figure 3.37. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?

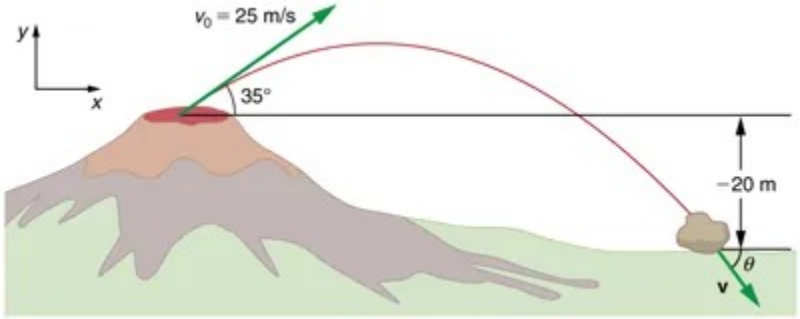


Figure 3.37 The trajectory of a rock ejected from the Kilauea volcano. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of a volcanic eruption scenario with a projectile motion path. It shows a volcano on the left side, with the top emitting a projectile. The projectile is moving in a parabolic trajectory.

- **Projectile Information:**
  - Initial velocity (v_0): 25 m/s
  - Launch angle: 35 degrees above the horizontal

- **Path:**
    - The projectile follows a red, curved path, indicating its trajectory.
- **Final Point:**
    - The projectile lands to the right of the volcano, 20 meters below the launch point.
    - The final velocity vector (v) is shown at the landing point.
- **Axes:**
    - A coordinate system is shown, with an x-axis (horizontal) and y-axis (vertical).

Strategy

Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. We will solve for t first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain v and θ_v at the final time t determined in the first part of the example.

Solution for (a)

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$y = y_0 + v_{0y}t - \frac{1}{2}\mathrm{gt}^2.$$

3.57

If we take the initial position y_0 to be zero, then the final position is $y = -20.0\ \mathrm{m}$. Now

the initial vertical velocity is the vertical component of the initial velocity, found from $v_{0y} = v_0 \sin\theta_0$ = $(25.0\ \text{m/s})(\sin\ 35.0^\circ)$ = $14.3\ \text{m/s}$. Substituting known values yields

$$-20.0m = (14.3m/s)t - \left(4.90\ \text{m/s}^2\right)t^2.$$

3.58

Rearranging terms gives a quadratic equation in t:

$$\left(4.90\ \text{m/s}^2\right)t^2 - (14.3\ \text{m/s})t - (20.0\ \text{m}) = 0.$$

3.59

This expression is a quadratic equation of the form $at^2 + bt + c = 0$, where the constants are $a = 4.90$, $b = -14.3$, and $c = -20.0$. Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4\text{ac}}}{2\text{a}}.$$

3.60

This equation yields two solutions:

$t = 3.96$ and $t = -1.03$. (It is left as an exercise for the reader to verify these solutions.) The time is $t = 3.96\text{s}$ or -1.03s. The negative value of time implies an event before the start of motion, and so we discard it. Thus,

$$t = 3.96\ \text{s}.$$

3.61

Discussion for (a)

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Solution for (b)

From the information now in hand, we can find the final horizontal and vertical velocities

v_x and v_y and combine them to find the total velocity v and the angle θ_0 it makes with the horizontal. Of course, v_x is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

$$v_x = v_0 \cos\theta_0 = (25.0 m/s)\,(\cos 35^\circ) = 20.5 m/s.$$

3.62

The final vertical velocity is given by the following equation:

$$v_y = v_{0y} - \mathrm{gt},$$

3.63

where v_{0y} was found in part (a) to be $14.3\ \mathrm{m/s}$. Thus,

$$v_y = 14.3 m/s - (9.80\ \mathrm{m/s^2})\,(3.96\ \mathrm{s})$$

3.64

so that

$$v_y = -24.5 m/s.$$

3.65

To find the magnitude of the final velocity v we combine its perpendicular components, using the following equation:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5 m/s)^2 + (-24.5 m/s)^2},$$

3.66

which gives

$$v = 31.9 m/s.$$

3.67

The direction θ_v is found from the equation:

$$\theta_v = \tan^{-1}(v_y/v_x)$$

3.68

so that

$$\theta_v = \tan^{-1}(-24.5/20.5) = \tan^{-1}(-1.19).$$

3.69

Thus,

$$\theta_v = -50.1o.$$

3.70

Discussion for (b)

The negative angle means that the velocity is 50.1° below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See Figure 3.37.)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define range to be the horizontal distance R traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.

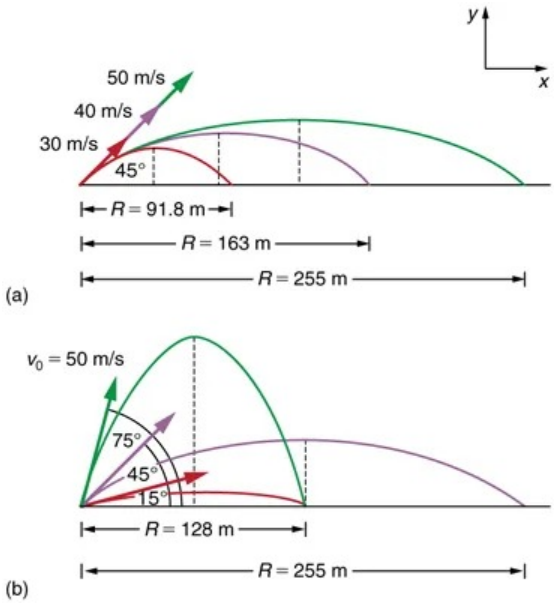


Figure 3.38 Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for 15° and 75°, although the maximum heights of those paths are different. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams showing projectile motion on a graph with different launch angles. Each diagram includes vectors, angle measurements, and horizontal distances.

Diagram (a):
– Three projectiles are launched with velocities of 30 m/s, 40 m/s, and 50 m/s from the same point.
– The projectile paths are represented in three different colors, each following a parabolic trajectory.
– The angles of launch for each projectile are marked as 45°, which indicates optimal launch angle for maximum range.
– Horizontal distances are labeled as:
– First distance: $R = 91.8\,\text{m}$
– Second distance: $R = 163\,\text{m}$
– Total distance to farthest point of impact: $R = 255\,\text{m}$

Diagram (b):
– A single projectile is launched with an initial velocity (v_0) of 50 m/s.
– The launch angles are varied: 75°, 45°, and 15°, each resulting in different trajectories.

– Horizontal distance for one marked trajectory: $R = 128\,\text{m}$
– Total distance for the farthest trajectory: $R = 255\,\text{m}$

Additional Features:
– Both diagrams include an x–y axis to indicate direction and position.
– The angles and trajectories visually compare how different launch heights and angles affect the range of a projectile.

This image illustrates the principles of projectile motion in physics, highlighting how velocity and angle of launch affect the range of a projectile.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in Figure 3.38(a). The initial angle θ_0 also has a dramatic effect on the range, as illustrated in Figure 3.38(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\theta_0 = 45^\text{o}$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately 38° . Interestingly, for every initial angle except 45° , there are two angles that give the same range—the sum of those angles is 90° . The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on *level ground* for which air resistance is negligible is given by

$$R = \frac{v_0^2 \sin(2\theta)_0}{g},$$

3.71

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See Figure 3.39.) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In 3.5 Addition of Velocities, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.

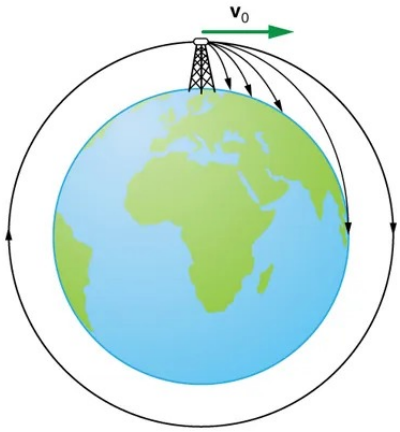


Figure 3.39 Hypothetical projectile to satellite. From this theoretical tower, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the concept of projectile motion and orbital paths around the Earth. It shows a representation of Earth, which is a predominantly blue circle with some green landmasses indicating continents.

At the top of the diagram, there is a small tower-like structure, positioned at the edge of Earth's atmosphere. From this structure, multiple curved arrows are depicted. Each arrow represents a different trajectory that an object might take if launched from this point with varying initial velocities.

– The arrows are labeled with the velocities, starting with a small arc that quickly descends back to Earth, and progressively longer and higher arcs.
– The final arrow forms a complete circle around the Earth, suggesting a trajectory that achieves orbit.
– A green arrow labeled "v_0" points horizontally from the tower, indicating the initial velocity direction needed to achieve these paths.

The arcs visually demonstrate how increasing the launch velocity can change an object's trajectory from a short-range path to a long-range or even orbital path around the planet.

PhET Explorations

Projectile Motion

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.

Access multimedia content

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3.5 ADDITION OF VELOCITIES

Learning Objectives

By the end of this section, you will be able to:

- Apply principles of vector addition to determine relative velocity.
- Explain the significance of the observer in the measurement of velocity.

Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves *diagonally* relative to the shore, as in Figure 3.40. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in Figure 3.41. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.

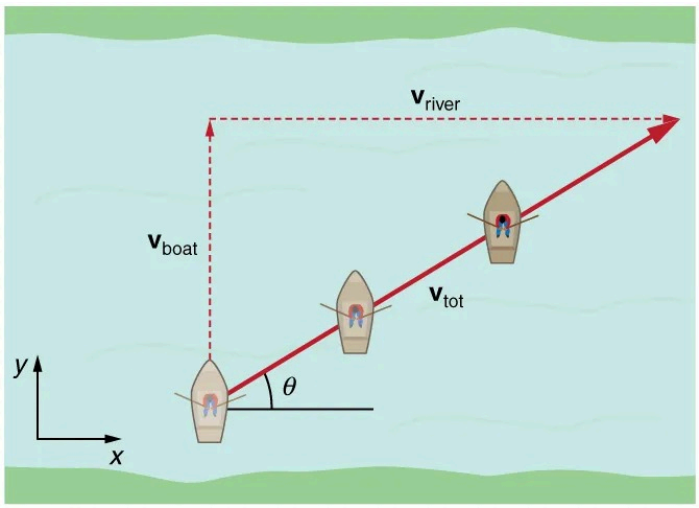


Figure 3.40 A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of a boat navigating a river. The background shows a blue river bordered by green banks at the top and bottom. There are three boats depicted along a diagonal path across the river.

Here are the elements illustrated in the diagram:

- There is a labeled v_{boat} vector shown as a dashed red arrow pointing upwards from the first boat, indicating the boat's velocity relative to the water.
- A v_{river} vector is also present, shown as a horizontal dashed red arrow indicating the velocity of the river.
- A resultant velocity vector, labeled v_{tot}, is drawn as a solid red arrow extending diagonally from the first boat, passing through the subsequent boats, indicating the total velocity of the boat as it moves across the river.
- An angle θ is marked between the v_{boat} vector and the horizontal line, demonstrating the angle at which the boat is heading relative to the riverbank.

In the bottom-left corner, there is an x and y axis that indicates the coordinate system for the river flow direction.

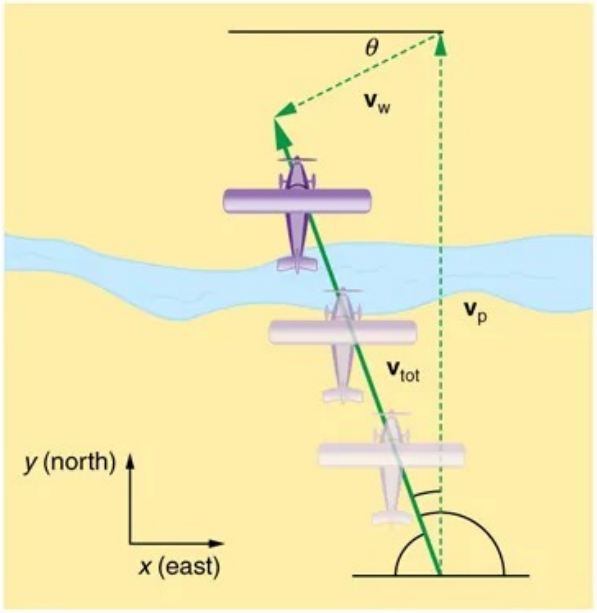


Figure 3.41 An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram used to illustrate the vector addition of velocities in physics. It features three planes shown at different positions over a water body, indicating the path of motion.

– A river is shown running horizontally across the image, colored in light blue.
– Three airplanes are depicted, representing different stages of motion, starting from the bottom and ascending diagonally to the top left.
– A green arrow labeled v_{tot} shows the total velocity of the airplane relative to the ground.
– A vertical green arrow labeled v_w depicts the velocity component of wind, which is directed north.
– A slanted arrow labeled v_p represents the airplane's velocity relative to the air.
– An angle θ is formed between the total velocity vector and the wind's velocity vector, illustrating the angle of drift.
– At the bottom left of the image, a coordinate axis shows the directions: *y* (north) and *x* (east).

The diagram aims to explain how the resultant velocity is affected by both wind and the plane's velocity.

In each of these situations, an object has a velocity relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object *relative to the observer* is the sum of these velocity vectors, as indicated in Figure 3.40 and Figure 3.41. These situations are only two of many in

which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of vector addition discussed in 3.2 Vector Addition and Subtraction: Graphical Methods and 3.3 Vector Addition and Subtraction: Analytical Methods apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at $5\ \mathrm{m/s}$ straight toward the goal and drives the ball in the same direction with a velocity of $30\ \mathrm{m/s}$ relative to her body, then the velocity of the ball is $35\ \mathrm{m/s}$ relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the *x*– and *y*-axes of an appropriately chosen coordinate system:

$$v_x = v\cos\theta$$
3.72

$$v_y = v\sin\theta$$
3.73

$$v = \sqrt{v_x^2 + v_y^2}$$
3.74

$$\theta = \tan^{-1}\left(v_y/v_x\right).$$
3.75

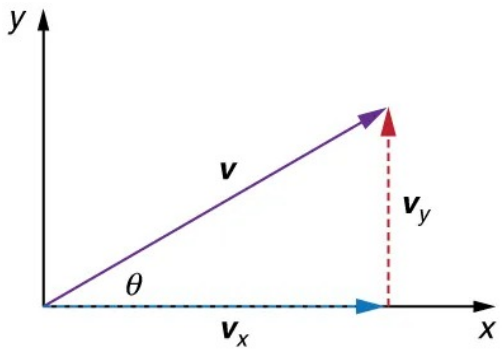


Figure 3.42 The velocity, v, of an object traveling at an angle θ to the horizontal axis is the sum of component vectors $\mathbf{v}_x$ and $\mathbf{v}_y$. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram depicting a vector on a two-dimensional Cartesian coordinate system. Here's a description of its components:

– The horizontal axis is labeled x, and the vertical axis is labeled y.
– There is a vector v represented by a purple arrow, extending from the origin toward the first quadrant.
– The vector v forms an angle θ with the horizontal axis, which is marked as a diagonal line extending from the origin.
– The horizontal component of the vector, labeled v_x, is shown as a blue arrow on the x-axis extending to the right from the origin.
– The vertical component of the vector, labeled v_y, is shown as a red dashed line extending upwards from the end of the horizontal component to meet the tip of the vector v.
– The tip of the vector v is indicated by a small red triangle where the purple, blue, and red lines converge.

The diagram illustrates how a vector can be decomposed into its horizontal and vertical components using vector resolution.

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

Take-Home Experiment: Relative Velocity of a Boat

Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up immediately opposite? Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

Example 3.6

Adding Velocities: A Boat on a River

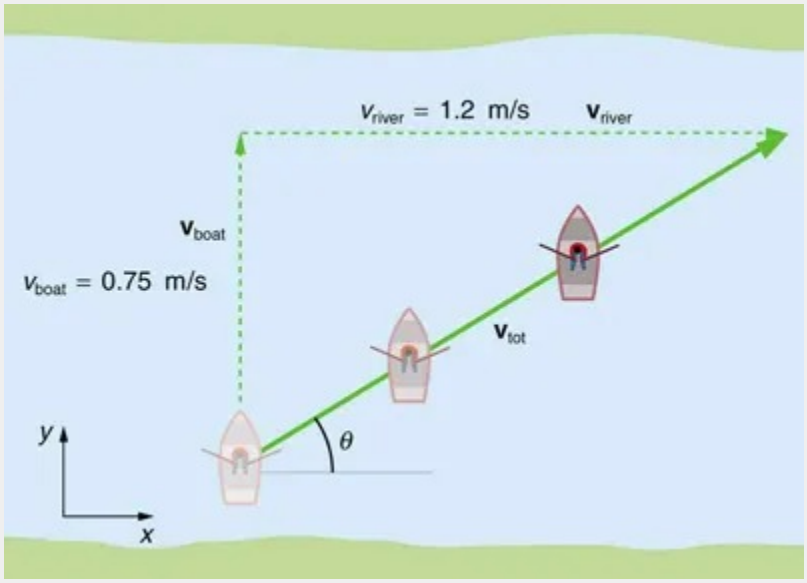


Figure 3.43 A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. Image *from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a diagram illustrating the motion of a boat crossing a river. It demonstrates the concept of vector addition and the impact of different velocities involved in the scenario.

The diagram shows a blue river with a represented flow from left to right. There are three positions of a boat depicted moving across the river.

- The boat's own velocity (v_{boat}) is 0.75 meters per second, represented by a solid black arrow pointing upward, perpendicular to the river's flow.
- The river's current velocity (v_{river}) is 1.2 meters per second, represented by a

dashed green arrow pointing right along the flow of the river.
- The total velocity of the boat (v_{tot}) is shown as a diagonal arrow in solid green, demonstrating the result of combining the boat and river velocities.

The diagram also includes an angle (θ) between the boat's velocity and the resulting vector, indicating the deviation due to the river's influence.

In the bottom left corner, an *x*-*y* coordinate system is displayed with *y* pointing upward and *x* pointing to the right.

Refer to Figure 3.43, which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, $\mathbf{v}_{\text{tot}}$. The velocity of the boat, $\mathbf{v}_{\text{boat}}$, is 0.75 m/s in the y-direction relative to the river and the velocity of the river, $\mathbf{v}_{\text{river}}$, is 1.20 m/s to the right.

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river, as shown in Figure 3.43. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel to the y-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}\left(v_y/v_x\right)$ directly.

Solution

The magnitude of the total velocity is

$$v_{\text{tot}} = \sqrt{v_x^2 + v_y^2},$$

3.76

where

$$v_x = v_{\text{river}} = 1.20\ \text{m/s}$$

3.77

and

$$v_y = v_{\text{boat}} = 0.750 \text{ m/s}.$$

3.78

Thus,

$$v_{\text{tot}} = \sqrt{(1.20 \text{ m/s})^2 + (0.750 \text{ m/s})^2}$$

3.79

yielding

$$v_{\text{tot}} = 1.42 \text{ m/s}.$$

3.80

The direction of the total velocity θ is given by:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(0.750/1.20).$$

3.81

This equation gives

$$\theta = 32.0^{\circ}.$$

3.82

Discussion

Both the magnitude v and the direction θ of the total velocity are consistent with Figure 3.43. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only 32.0°) the total velocity has relative to the riverbank.

Example 3.7

Calculating Velocity: Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in Figure 3.44. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction 20.0° west of north.

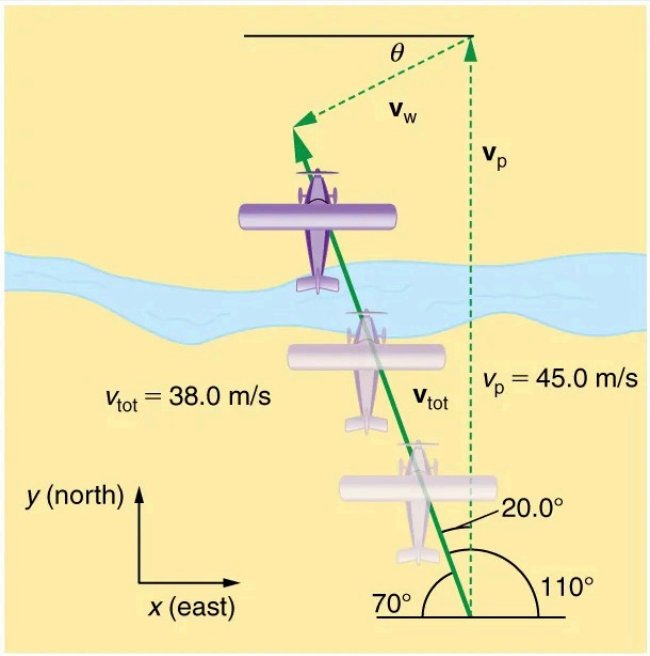


Figure 3.44 An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind? Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of an airplane flying over a river, used to illustrate vector addition. It shows the path of the airplane and associated velocity vectors. Here's a description of the elements:

– A light purple airplane is shown flying in the sky, above a river that runs horizontally.
– Three vectors are illustrated:
– v_{p}: Representing the plane's velocity vector (45.0 m/s). It is vertical in the diagram.
– v_{w}: Representing the wind's velocity vector, shown at an angle θ.
– v_{tot}: The total velocity vector (38.0 m/s), depicted at an oblique angle.
– The vectors form a triangle, where the sum of the plane's velocity and wind's velocity results in the total velocity.
– An angle of 20° is marked between the plane's velocity vector and the total velocity vector.
– A coordinate system is shown in the bottom left corner, indicating north (y-axis) and east (x-axis).
– Several angles are annotated:
– 70°, representing the angle between the x-axis (east) and the total velocity vector.
– 110°, being the supplementary angle between the x-axis and the wind vector.

The image visually represents the relationship between the airplane's velocity, the wind's effect, and the resulting path of the airplane using vector addition principles.

Strategy

In this problem, somewhat different from the previous example, we know the total velocity $\mathbf{v}_{\mathrm{tot}}$ and that it is the sum of two other velocities, $\mathbf{v}_{\mathrm{w}}$ (the wind) and $\mathbf{v}_{\mathrm{p}}$ (the plane relative to the air mass). The quantity $\mathbf{v}_{\mathrm{p}}$ is known, and we are asked to find $\mathbf{v}_{\mathrm{w}}$. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of $\mathbf{v}_{\mathrm{w}}$, then we can combine them to solve for its magnitude and direction. As shown in Figure 3.44, we choose a coordinate system with its *x*-axis due east and its *y*-axis due north (parallel to $\mathbf{v}_{\mathrm{p}}$). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in 3.3 Vector Addition and Subtraction: Analytical Methods.)

Solution

Because $\mathbf{v}_{\mathrm{tot}}$ is the vector sum of the $\mathbf{v}_{\mathrm{w}}$ and $\mathbf{v}_{\mathrm{p}}$, its *x*– and *y*-components are the sums

of the *x*– and *y*-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{px} = 0$ and $v_{py} = v_{\mathrm{p}}$. That is,

$$v_{\mathrm{tot}x} = v_{\mathrm{w}x}$$

3.83

and

$$v_{\mathrm{tot}y} = v_{\mathrm{w}y} + v_{\mathrm{p}}.$$

3.84

We can use the first of these two equations to find $v_{\mathrm{w}x}$:

$$v_{\mathrm{w}x} = v_{\mathrm{tot}x} = v_{\mathrm{tot}} \cos\ 110^{\mathrm{o}}\ .$$

3.85

Because $v_{\mathrm{tot}} = 38.0 m/\mathrm{s}$ and $\cos\ 110^{\mathrm{o}}\ = –0.342$ we have

$$v_{\mathrm{w}x} = (38.0\ \mathrm{m/s}\ )\ (–0.342\ ) = –13\ \mathrm{m/s}.$$

3.86

The minus sign indicates motion west which is consistent with the diagram.

Now, to find $v_{\mathrm{w}y}$ we note that

$$v_{\mathrm{tot}y} = v_{\mathrm{w}y} + v_{\mathrm{p}}$$

3.87

Here $v_{\mathrm{tot}y} = v_{\mathrm{tot}} \sin\ 110^{\mathrm{o}}$; thus,

$$\begin{aligned} v_{\mathrm{w}y} &= (38.0 m/s\ )\ (0.940\ ) - 45.0 m/s \\ &= -9.29\ \mathrm{m/s}. \end{aligned}$$

3.88

This minus sign indicates motion south which is consistent with the diagram.

Now that the perpendicular components of the wind velocity $v_{\mathrm{w}x}$ and $v_{\mathrm{w}y}$ are known, we can find the magnitude and direction of $\mathbf{v}_{\mathrm{w}}$. First, the magnitude is

$$v_{\mathrm{w}} = \sqrt{v_{\mathrm{w}x}^2 + v_{\mathrm{w}y}^2}$$
$$= \sqrt{(-13.0 m/s)^2 + (-9.29\ \mathrm{m/s})^2}$$

3.89

so that

$$v_{\mathrm{w}} = 16.0 m/s.$$

3.90

The direction is:

$$\theta = \tan^{-1}\left(v_{\mathrm{w}y}/v_{\mathrm{w}x}\right) = \tan^{-1}\left(-9.29/-13.0\right)$$

3.91

giving

$$\theta = 35.6^{\circ}\ .$$

3.92

Discussion

The wind's speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in Figure 3.44. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the *velocity is relative to some reference frame*.

These velocities are called relative velocities. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of relativity, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his *modern* theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton. Classical relativity is limited to situations where speeds are less than about 1% of the speed of light—that is, less than $3,000\ \mathrm{km/s}$. Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops their binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (See Figure 3.45.) To the observer on shore, the binoculars and the ship have the *same* horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the curved path shown in Figure 3.45. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

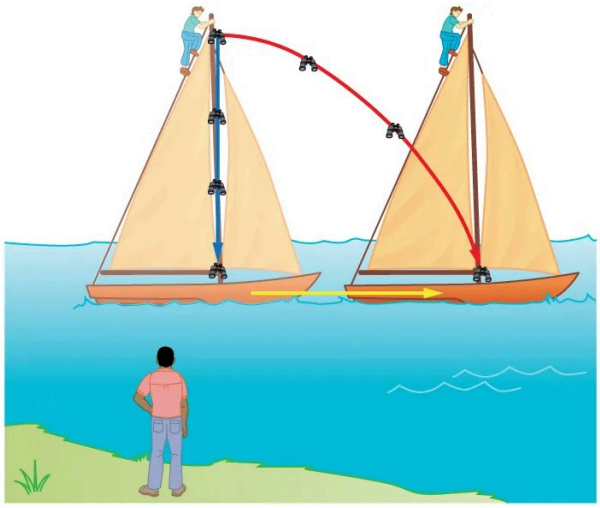


Figure 3.45 Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates two sailboats on a body of water. Both sailboats have tan sails and brown hulls.

– Left Sailboat:
– A person is perched at the top of the mast.
– A blue arrow runs vertically down the mast to indicate downward movement.
– A yellow arrow underneath the boat indicates its forward movement toward the right.

– Right Sailboat:
– Another person is at the top of its mast.
– A red curving arrow points from the top of the mast of the left sailboat to the mast of the right sailboat, suggesting a transfer or movement between the masts.
– A yellow arrow indicates the rightward movement under this boat as well.

A person stands on the grassy shore in the foreground, observing the scene. The sky is clear, and there are small wave patterns on the water.

Example 3.8

Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?



Figure 3.46 The motion of a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two parts, labeled (a) and (b).

– Part (a): This section shows the interior of an airplane cabin. Inside, there are two passenger seats. A person is seated on the left, with a red arrow pointing straight down from his position, indicating the path of a coin falling from the point of view of an observer in the airplane's cabin.

– Part (b): This section shows an exterior side view of an airplane flying in the sky. Below the airplane stands a person on the ground looking up at it. The individual is shown from the back, wearing a red shirt and blue pants, against a clear sky background. A red arrow pointing to the right (the direction the plane is flying) and angle slightly down indicates the coin's path as observed from the ground.

Together, these sections illustrate a concept combining aspects of passenger awareness inside an aircraft and observation from the ground.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

$$v_y^2 = v_{0,y}^2 - 2g\,(y - y_0\,).$$

3.93

Substituting known values into the equation, we get

$$\begin{aligned} v_y^2 &= 0^2 - 2\,(9.80\ \mathrm{m/s^2}\,)\,(\,-1.50\mathrm{m} - 0m\,) \\ &= 29.4\mathrm{m^2/s^2} \end{aligned}$$

3.94

yielding

$$v_y = -5.42 \text{ m/s}.$$

3.95

We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42\text{m/s}$, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and $v_x = 260 \text{ m/s}$. The *x*– and *y*-components of velocity can be combined to find the magnitude of the final velocity:

$$v = \sqrt{v_x^2 + v_y^2}.$$

3.96

Thus,

$$v = \sqrt{(260 \text{ m/s})^2 + (-5.42 \text{ m/s})^2}$$

3.97

yielding

$$v = 260.06 \text{ m/s}.$$

3.98

The direction is given by:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(-5.42/260)$$

3.99

so that

$$\theta = \tan^{-1}(-0.0208) = -1.19^{\circ}\ .$$

3.100

Discussion

In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is *not* $(260 - 5.42)\ \mathrm{m/s}$; rather, it is $260.06\ \mathrm{m/s}$. The velocity's magnitude had to be calculated to five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see *very* different paths. (See Figure 3.46.) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

Making Connections: Relativity and Einstein

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

PhET Explorations

Motion in 2D

Try the "Motion in 2D" simulation to Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

Click to view content.

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GLOSSARY

air resistance
: a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

analytical method
: the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

classical relativity
: the study of relative velocities in situations where speeds are less than about 1% of the speed of light—that is, less than 3000 km/s

commutative
: refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

component (of a 2-d vector)
: a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

direction (of a vector)
: the orientation of a vector in space

head (of a vector)
: the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the "tip"

head-to-tail method
: a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

kinematics
: the study of motion without regard to mass or force

magnitude (of a vector)
: the length or size of a vector; magnitude is a scalar quantity

motion
: displacement of an object as a function of time

projectile
: an object that travels through the air and experiences only acceleration due to gravity

projectile motion
: the motion of an object that is subject only to the acceleration of gravity

range
: the maximum horizontal distance that a projectile travels

relative velocity
: the velocity of an object as observed from a particular reference frame

relativity
: the study of how different observers moving relative to each other measure the same phenomenon

resultant
: the sum of two or more vectors

resultant vector
: the vector sum of two or more vectors

scalar
: a quantity with magnitude but no direction

tail
: the start point of a vector; opposite to the head or tip of the arrow

trajectory
: the path of a projectile through the air

vector
: a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

vector addition
: the rules that apply to adding vectors together

velocity
: speed in a given direction

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CHAPTER SUMMARY

3.1 Kinematics in Two Dimensions: An Introduction

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

3.2 Vector Addition and Subtraction: Graphical Methods

- The **graphical method of adding vectors** $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector$\mathbf{R}$ is defined such that$\mathbf{A} + \mathbf{B} = \mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** $\mathbf{B}$ from $\mathbf{A}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A}–\mathbf{B} = \mathbf{A} + (–\mathrm{B}) = \mathrm{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is commutative such that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.
- The head-to-tail method of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector $\mathbf{A}$ is multiplied by a scalar quantity c, the magnitude of the product is given by $c\mathrm{A}$. If c is positive, the direction of the product points in the same direction as $\mathbf{A}$; if c is negative, the direction of the product points in the opposite direction as $\mathbf{A}$.

3.3 Vector Addition and Subtraction: Analytical Methods

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors $\mathbf{A}$ and $\mathbf{B}$ using the analytical method are as follows:Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

$$A_x = A\cos\theta$$
$$B_x = B\cos\theta$$

and

$$A_y = A\ \sin\theta$$
$$B_y = B\ \sin\ \theta.$$

Step 2: Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, $\mathbf{R}$:

$$R_x = A_x + B_x$$

and

$$R_y = A_y + B_y.$$

Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector $\mathbf{R}$:

$$R = \sqrt{R_x^2 + R_y^2}.$$

Step 4: Use a trigonometric identity to determine the direction, θ, of $\mathbf{R}$:

$$\theta = \tan^{-1}\left(R_y/R_x\ \right).$$

3.4 Projectile Motion

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:

1. Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position $\mathbf{s}$ are given by the quantities x and y, and the components of the velocity $\mathbf{v}$ are given by $v_x = v\cos\theta$ and $v_y = v\sin\theta$, where v is the magnitude of the velocity and θ is its direction.
2. Analyze the motion of the projectile in the horizontal direction using the following equations:
   $\text{Horizontal motion } (a_x = 0)$
   $x = x_0 + v_x t$
   $v_x = v_{0x} = \mathbf{v}_\text{x} = \text{velocity is a constant.}$
3. Analyze the motion of the projectile in the vertical direction using the following equations:
   $\text{Vertical Motion (assuming positive is up} a_y = -g = -9.80 \text{m/s}^2)$
   $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$
   $v_y = v_{0y} - \text{gt}$
   $y = y_0 + v_{0y}t - \frac{1}{2}\text{gt}^2$
   $v_y^2 = v_{0y}^2 - 2g(y - y_0).$
4. Recombine the horizontal and vertical components of location and/or velocity using the following equations:
   $s = \sqrt{x^2 + y^2}$
   $\theta = \tan^{-1}(y/x)$
   $v = \sqrt{v_x^2 + v_y^2}$
   $\theta_\text{v} = \tan^{-1}(v_y/v_x).$

- The maximum height h of a projectile launched with initial vertical velocity v_{0y} is given by
  $h = \frac{v_{0y}^2}{2g}$.
- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by
  $R = \frac{v_0^2 \sin(2\theta)_0}{g}$.

3.5 Addition of Velocities

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as
  $v_x = v\cos\theta$
  $v_y = v\sin\theta$
  $v = \sqrt{v_x^2 + v_y^2}$
  $\theta = \tan^{-1}(v_y/v_x)$.
- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

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CONCEPTUAL QUESTIONS

Conceptual Questions

3.2 Vector Addition and Subtraction: Graphical Methods

1. Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?
2. Give a specific example of a vector, stating its magnitude, units, and direction.
3. What do vectors and scalars have in common? How do they differ?
4. Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?

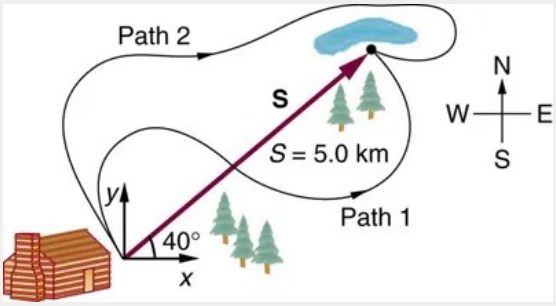


Figure 3.47 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram showing two paths from a log cabin to a lake.

– Log Cabin: Located in the bottom left corner.
– Lake: Shown at the top right corner, represented by a blue shape.
– Paths: There are two winding routes labeled as "Path 1" and "Path 2". Both paths start from the cabin and end at the lake, taking different, curved trajectories.
– Direct Path (S): A direct straight line marked in red with an arrow pointing from the

cabin to the lake. This line is labeled “S = 5.0 km” and crosses through a forest area represented by green tree icons. The angle between the direct path and the horizontal axis (x-axis) is marked as “40°”.
– Axes: An x-y coordinate system is depicted with the x-axis running horizontally and the y-axis vertically.
– Compass: Located on the right side, showing the cardinal directions: North (N), South (S), East (E), and West (W).

The diagram illustrates distance, direction, and angle for navigation from the cabin to the lake.

5. If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in Figure 3.48. What other information would he need to get to Sacramento?



Figure 3.48 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a map detailing a specific area of Northern California, focusing on the cities of San Francisco and Sacramento. Here’s a detailed description:

– Geographical Coverage: The map displays a section of Northern California, showcasing

cities such as San Francisco, Sacramento, Oakland, San Jose, Santa Rosa, Stockton, Modesto, and nearby areas.

– Circle and Line: A large circle is drawn on the map, centered near Berkeley, with a line extending from Sacramento to a point on the circle in the direction of San Francisco. This line is labeled with "S = 123 km."

– Direction and Angle: There is a dotted line representing the East direction from Berkeley, marked with "East" and forming a 45-degree angle with the line from Sacramento.

– Compass Rose: A compass rose is positioned on the map, identifying the cardinal directions: North (N), East (E), South (S), and West (W).

– Additional Details: Cities and landmarks are marked with small dots, and highways or main roads appear as lines connecting them. The circle seems to represent a boundary or radius from a certain point near Berkeley.

This map visually demonstrates the spatial relationship between Sacramento and the San Francisco Bay Area, highlighting directional details and distances.

6. Suppose you take two steps $\mathbf{A}$ and $\mathbf{B}$ (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point $\mathbf{A} + \mathbf{B}$ the sum of the lengths of the two steps?
7. Explain why it is not possible to add a scalar to a vector.
8. If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

3.3 Vector Addition and Subtraction: Analytical Methods

9. Suppose you add two vectors $\mathbf{A}$ and $\mathbf{B}$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with

the smallest magnitude? What is the minimum magnitude?

10. Give an example of a nonzero vector that has a component of zero.
11. Explain why a vector cannot have a component greater than its own magnitude.
12. If the vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, what is the component of $\mathbf{A}$ along the direction of $\mathbf{B}$? What is the component of $\mathbf{B}$ along the direction of $\mathbf{A}$?

3.4 Projectile Motion

13. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at $t = 0$? (d) Can the speed ever be the same as the initial speed at a time other than at $t = 0$?
14. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?
15. For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?
16. During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

3.5 Addition of Velocities

17. What frame or frames of reference do you instinctively use when driving a car?

When flying in a commercial jet airplane?

18. A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn't he need to keep his eyes on the ball?
19. If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?
20. The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger's frame of reference. Draw its path as viewed by a stationary observer.
21. A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. What is the direction of its velocity relative to the truck just before it hits? Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

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PROBLEMS & EXERCISES

Problems & Exercises

3.2 Vector Addition and Subtraction: Graphical Methods

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

1. Find the following for path A in Figure 3.49: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

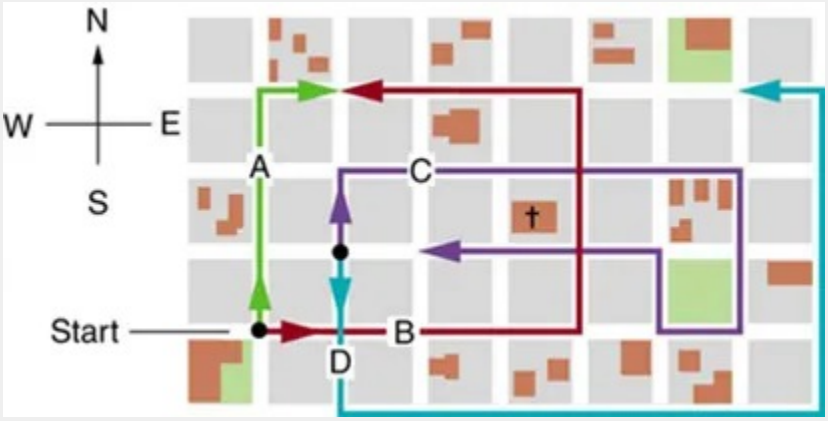


Figure 3.49 The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of a city grid with multiple routes marked in different colors. In the top left corner, there is a compass rose indicating the cardinal directions: North (N), East (E), South (S), and West (W).

The city grid is represented by gray squares, which likely signify buildings or blocks, interspersed with some small colored areas that might indicate parks or notable locations. There is a path starting point labeled 'Start' at the bottom left.

The routes are marked as follows:

- Green Path: Starts at ‘Start’, moves north to point A, then proceeds further east.
- Blue Path: Begins at ‘Start’, goes east to point D, continues north to B, and then heads east.
- Purple Path: Begins at point D, moves north to B, continues east to C, then travels north and then heads east and finally south.
- Red Path: Starts at point A, moves east to C, and then continues in a loop, cutting through the grid.

The diagram includes various intersections labeled with letters A, B, C, and D, indicating key waypoints along the paths. In the grid’s center, there is a small icon resembling a cross, possibly denoting a church or a significant landmark.

2. Find the following for path B in Figure 3.49: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.
3. Find the north and east components of the displacement for the hikers shown in Figure 3.47.
4. Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 3.50, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

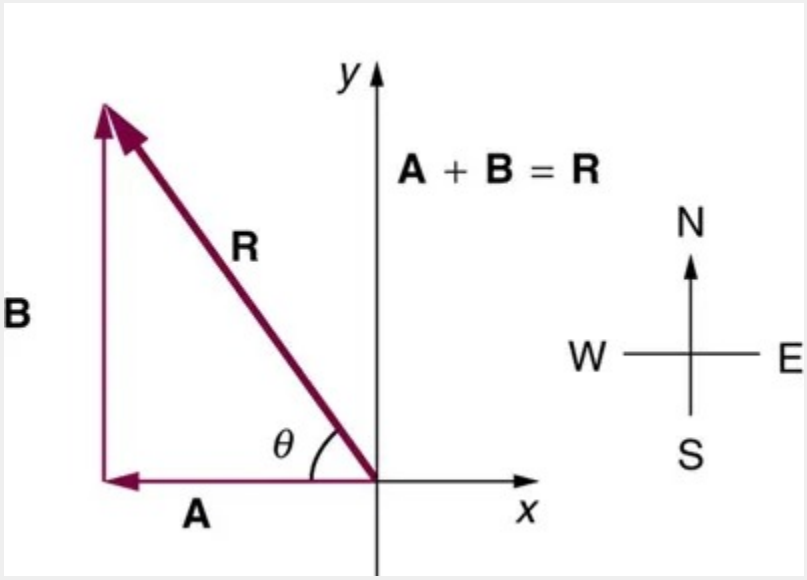


Figure 3.50 The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a vector diagram with coordinate axes and a compass rose for direction. Here's a description of the elements:

- *Vector Components:*
  - The vector `R` is represented as a diagonal arrow pointing upwards and to the left, forming the hypotenuse of a right triangle.
  - The horizontal component `A` is shown as an arrow along the `x-axis` pointing to the right.
  - The vertical component `B` is an arrow along the `y-axis` pointing upwards.
- *Angle:* The angle θ is located between the x-axis and the vector R. It is enclosed within the triangle at the origin.
- *Equation:* Above the x-axis, the equation "A + B = R" is written to depict how the vector R is the resultant of the vector addition.
- *Coordinate Axes:*
  - `x-axis` is the horizontal axis, and `y-axis` is the vertical axis, both

intersecting at the origin (0,0).

- *Compass Rose:* On the right side of the image, a compass rose indicates cardinal directions:
    - N (North) is upwards, S (South) is downwards, E (East) is to the right, and W (West) is to the left.

5. Suppose you first walk 12.0 m in a direction 20° west of north and then 20.0 m in a direction 40.0° south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 3.51, then this problem finds their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

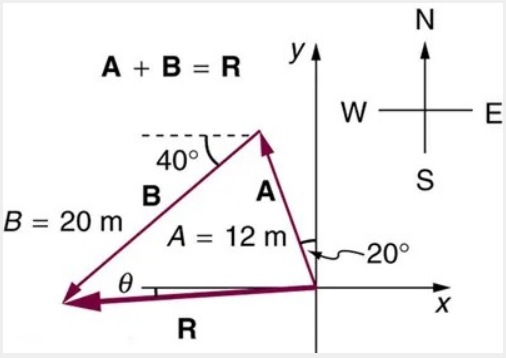


Figure 3.51 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a vector diagram with two vectors, A and B, and their resultant vector R. The diagram is set on a Cartesian coordinate plane with x and y axes marked. A compass rose is also present, indicating the cardinal directions: North (N), East (E), South (S), and West (W).

– Vector A is represented by an arrow, directed up-right, labeled as 12 meters, making an angle of 20 degrees with the positive x-axis.

– Vector B is represented by another arrow, directed upward and slightly to the left, labeled as 20 meters, making an angle of 40 degrees with Vector A.

– Resultant Vector R is represented as the diagonal of the parallelogram, directed towards the left, pointing downwards from the tip of Vector B to the horizontal component of Vector A.

– The diagram is labeled with the equation `A + B = R`.

– The angle between Vector R and the x-axis is marked as theta (θ).

– An additional directional compass at the top right indicates West and East on the horizontal plane, and North and South on the vertical plane.

6. Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg $\mathbf{B}$, which is 20.0 m in a direction exactly 40° south of west, and then leg $\mathbf{A}$, which is 12.0 m in a direction exactly 20° west of north. (This problem shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.)
7. (a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction 40.0° north of east, (which is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction 40.0° east of south (which is equivalent to subtracting $\mathbf{A}$ from $\mathbf{B}$ —that is, to finding $\mathbf{R}'' = \mathbf{B} - \mathbf{A} = -\mathbf{R}'$). Show that this is the case.
8. Show that the order of addition of three vectors does not affect their sum. Show this property by choosing any three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, all having different lengths and directions. Find the sum $\mathrm{A} + \mathrm{B} + \mathrm{C}$ then find their sum when added in a different order and show the result is the same. (There are five other orders in which $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ can be added; choose only one.)
9. Show that the sum of the vectors discussed in Example 3.2 gives the result shown in Figure 3.23.
10. Find the magnitudes of velocities v_{A} and v_{B} in Figure 3.52

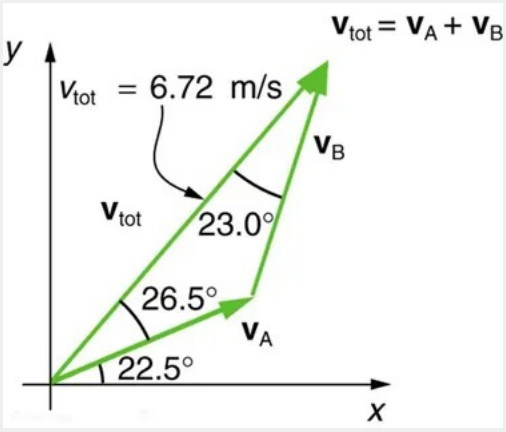


Figure 3.52 The two velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ add to give a total $\mathbf{v}_{tot}$. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a vector diagram with a coordinate system labeled with *x* and *y* axes. There are three vectors shown:

- **Vector v_A**: Starts at the origin and is marked at an angle of 22.5° above the *x*-axis.
- **Vector v_B**: Starts at the tip of vector v_A and is marked at an angle of 26.5° relative to vector v_A.
- **Vector v_{tot}**: Starts at the origin and follows the combined path of vectors v_A and v_B, marked at an angle of 23.0° from the resultant vector v_A and labeled with a magnitude of 6.72 m/s.

The equation shown in the image is $v_{tot} = v_A + v_B$, indicating that v_{tot} is the resultant vector from adding vectors v_A and v_B.

11. Find the components of v_{tot} along the x- and y-axes in Figure 3.52.
12. Find the components of v_{tot} along a set of perpendicular axes rotated 30° counterclockwise relative to those in Figure 3.52.

3.3 Vector Addition and Subtraction: Analytical Methods

13. Find the following for path C in Figure 3.53: (a) the total distance traveled and (b)

the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

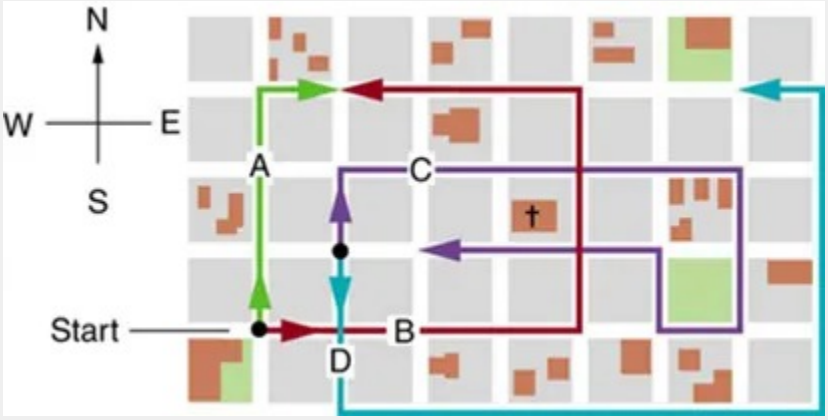


Figure 3.53 The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a map illustrating various routes through a grid-like layout, with directions marked by colored arrows. In the top left corner, a compass rose indicates directions: North (up), South (down), East (right), and West (left).

The map background consists of blocks representing a city layout, with gray squares for buildings and smaller colored shapes representing different areas within the blocks. Some blocks have green patches possibly indicating parks or open spaces, and others have smaller brown patches suggesting smaller buildings or houses.

The path details are as follows:

- Green Path: Starting from the bottom left marked “Start,” it heads north to point A.
- Blue Path: Starts at point A, moves east, then south to point B, and continues to the east, eventually turning north.
- Purple Path: Begins at point B, moves north to point C, then heads east and loops around to connect back towards the south.

- Red Path: Initiates at point A, heading east through the center of the map, turns south to D, and makes a loop back at point C, then drives east towards the edge of the map.

A cross symbol is shown near the center, possibly indicating a significant landmark or point of interest.

14. Find the following for path D in Figure 3.53: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.
15. Find the north and east components of the displacement from San Francisco to Sacramento shown in Figure 3.54.



Figure 3.54 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a map highlighting a geographical area in California, including cities and various geographical markers. Key features include:

- A large circle is drawn, centered on San Francisco, covering nearby regions.
- The circle indicates a radius labeled as *S = 123 km*.

- The map highlights a line from San Francisco to Sacramento, forming an angle marked as *45°* with the horizontal line labeled “East”.
- Major cities marked on the map include San Francisco, Sacramento, Vallejo, Oakland, Berkeley, San Jose, Santa Cruz, and Monterey.
- Other minor cities and towns such as Healdsburg, Santa Rosa, Stockton, Modesto, Gilroy, and Salinas are also marked.
- A north-south-east-west directional compass is located in the lower left corner of the map for orientation.
- Major roads and rivers are illustrated in blue and black lines, indicating connections between different cities.

The map is a representation of the San Francisco Bay Area extending towards Sacramento, showing major transportation and geographical directions.

16. Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 3.55, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

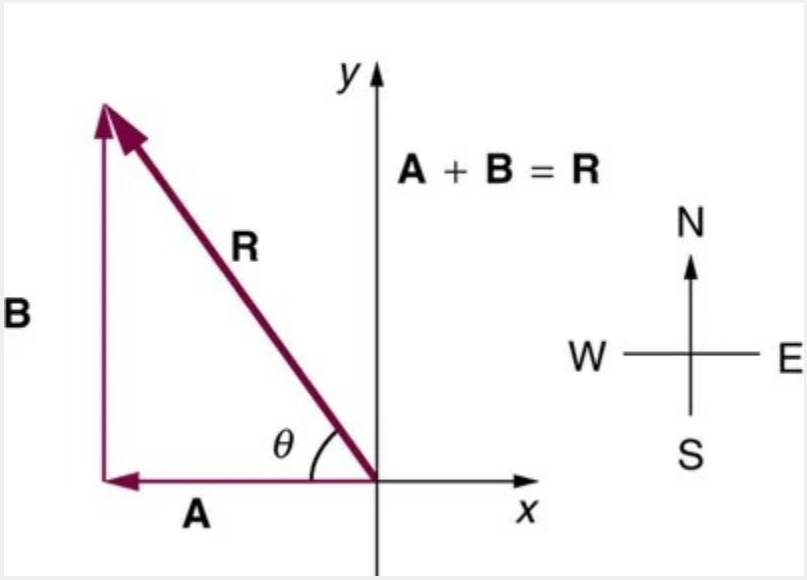


Figure 3.55 The two displacements $\mathbf{A}$ *and* $\mathbf{B}$ *add to give a total displacement* $\mathbf{R}$ *having magnitude* R *and direction* θ*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a diagram illustrating vector addition. It features a right triangle formed by two vectors and their resultant:

- *Vector A* is a horizontal line pointing to the right, along the x-axis.
- *Vector B* is a vertical line pointing upwards, along the y-axis.
- The hypotenuse, labeled *R*, is a vector combining *A* and *B*, pointing diagonally upward and to the right, creating an angle labeled as θ with the x-axis.

The relationship is expressed as "A + B = R".

To the right of the triangle, there is a compass rose indicating directions: *N* (North) pointing upwards, *S* (South) pointing downwards, *E* (East) pointing to the right, and *W* (West) pointing to the left.

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

17. Repeat Exercise 3.16 using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows

that adding them in reverse order gives the same result—that is, $\mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B}$.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking your other path.

18. You drive $7.50\ \text{km}$ in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.) (b) Show that you still arrive at the same point if the east and north legs are reversed in order.
19. Do Exercise 3.16 again using analytical techniques and change the second leg of the walk to $25.0\ \text{m}$ straight south. (This is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$) (b) Repeat again, but now you first walk $25.0\ \text{m}$ north and then $18.0\ \text{m}$ east. (This is equivalent to subtract $\mathbf{A}$ from $\mathbf{B}$ —that is, to find $\mathbf{A} = \mathbf{B} + \mathbf{C}$. Is that consistent with your result?)
20. A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors $\mathbf{A}$ from $\mathbf{B}$ in Figure 3.56. She then correctly calculates the length and orientation of the third side $\mathbf{C}$. What is her result?

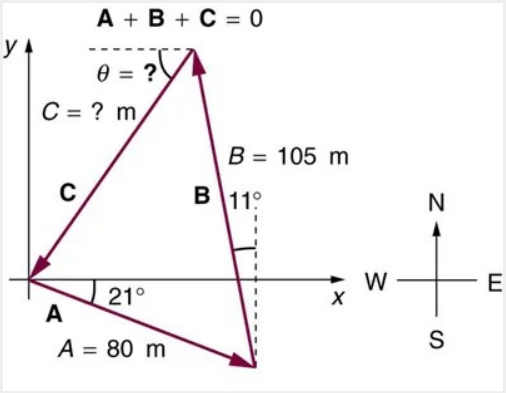


Figure 3.56 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a vector diagram involving three vectors, A, B, and C, that form a closed triangle, illustrating vector addition where A + B + C = 0.

– Vectors:
– A: Labeled as 80 meters, positioned along the x-axis at an angle of 21 degrees from the horizontal.
– B: Labeled as 105 meters, extends upwards from vector A, making an 11-degree angle with the vertical.
– C: Listed with an unknown magnitude (? m) and unknown angle (θ), closing the triangle with an upward left diagonal direction towards the y-axis.

– Coordinate System:
– An x-y coordinate plane is shown with the x-axis aligned horizontally and the y-axis vertically. There is also a compass indicating directions: North (N) up, South (S) down, East (E) to the right, and West (W) to the left.

– Arrows indicate the direction of vectors, with vector C completing the triangle as it angles back to the start of vector A, ensuring the summation equals zero.

21. You fly $32.0\ \text{km}$ in a straight line in still air in the direction 35.0° south of west. (a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.) (b) Find the distances you would have to fly first in a direction 45.0° south of west and then in a direction 45.0° west of north. These are the components of the displacement along a different set of axes—one rotated 45°.
22. A farmer wants to fence off a four-sided plot of flat land. They measure the first three sides, shown as $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ in Figure 3.57, and then correctly calculate the length and orientation of the fourth side $\mathbf{D}$.
    What is their result?

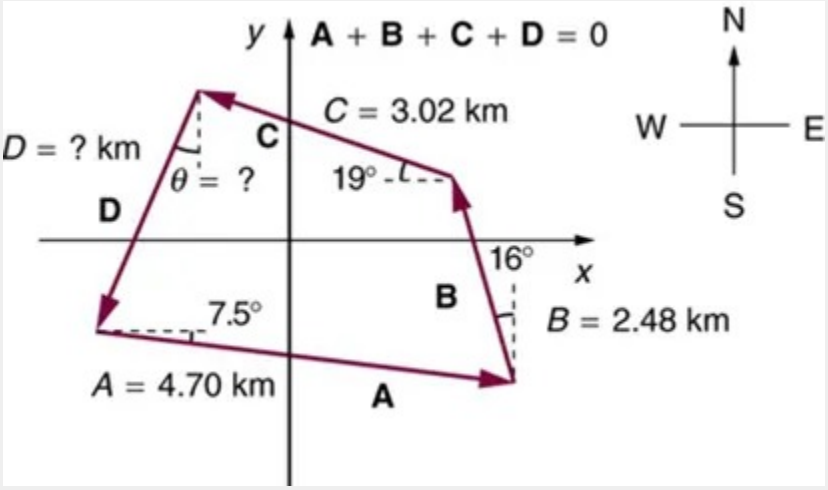


Figure 3.57 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a vector diagram on a coordinate plane with four vectors labeled as A, B, C, and D, forming a closed polygonal path.

– Vector A is directed mostly leftwards and slightly upwards with a magnitude of 4.70 km, at an angle of 7.5° from the negative x-axis.
– Vector B is directed rightwards and slightly upwards with a magnitude of 2.48 km, at an angle of 16° from the positive x-axis.
– Vector C is directed leftwards and slightly downwards with a magnitude of 3.02 km, at an angle of 19° from the negative x-axis.
– Vector D completes the polygon, though its magnitude and angle (θ) are unspecified, with the notation indicating both are unknown.

The sum of the vectors is given as $A + B + C + D = 0$, indicating the path forms a closed loop.

There is also a compass rose in the top right corner of the image, indicating the cardinal directions: North (N), East (E), South (S), and West (W).

The coordinate system includes both x and y axes, with positive directions indicated.

23. In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight

lines: $2.50\ \text{km}\ 45.0^\circ$ north of west; then $4.70\ \text{km}\ 60.0^\circ$ south of east; then $1.30\ \text{km}\ 25.0^\circ$ south of west; then $5.10\ \text{km}$ straight east; then $1.70\ \text{km}\ 5.00^\circ$ east of north; then $7.20\ \text{km}\ 55.0^\circ$ south of west; and finally $2.80\ \text{km}\ 10.0^\circ$ north of east. What is his final position relative to the island?

24. Suppose a pilot flies $40.0\ \text{km}$ in a direction 60° north of east and then flies $30.0\ \text{km}$ in a direction 15° north of east as shown in Figure 3.58. Find her total distance R from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.

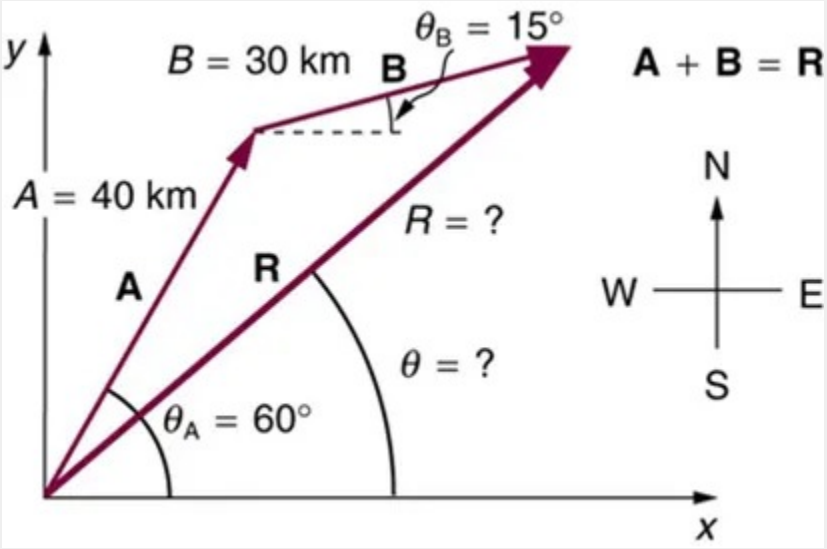


Figure 3.58 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a vector diagram showing the addition of two vectors, *A* and *B*, resulting in vector *R*. Here's a detailed description:

- Vector *A* is depicted with an arrow pointing upwards and slightly to the left, with a magnitude labeled as "40 km". It makes an angle, θ_A, of 60 degrees with the horizontal axis.
- Vector *B* is represented with an arrow pointing upwards and slightly to the right,

with a magnitude labeled as "30 km". It makes an angle, θ_B, of 15 degrees above the horizontal line marked for Vector *A*.

- The resultant vector *R* is shown as a diagonal arrow pointing to the right. The angle θ and the magnitude *R* are unknown and labeled as "*R* = ?" and "θ = ?".
- The vectors are arranged in a vector addition format where *A* + *B* = *R*.
- The coordinate plane has *x* and *y* axes, with a compass rose on the right denoting directions as North (*N*), South (*S*), East (*E*), and West (*W*).

3.4 Projectile Motion

25. A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?
26. A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction. (a) At what speed does the ball hit the ground? (b) For how long does the ball remain in the air? (c)What maximum height is attained by the ball?
27. A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance. (a) How long is the ball in the air? (b) What must have been the initial horizontal component of the velocity? (c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?
28. (a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a 32° ramp at a speed of $40.0\ \text{m/s}\ (144\ \text{km/h})$. How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long? (b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)
29. An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is

at same height as the release height of the arrow. (a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems. (b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

30. A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?
31. Verify the ranges for the projectiles in Figure 3.38(a) for $\theta = 45^{\circ}$ and the given initial velocities.
32. Verify the ranges shown for the projectiles in Figure 3.38(b) for an initial velocity of 50 m/s at the given initial angles.
33. The cannon on a battleship can fire a shell a maximum distance of 32.0 km. (a) Calculate the initial velocity of the shell. (b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.) (c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is $6.37 \times 10^3 \text{ km}$. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?
34. An arrow is shot from a height of 1.5 m toward a cliff of height H. It is shot with a velocity of 30 m/s at an angle of 60° above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?
35. In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the

acceleration due to gravity, g. How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

36. The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.
37. Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle θ below the horizontal. The base line is 11.9 m from the net, which is 0.91 m high. What is the angle θ such that the ball just crosses the net? Will the ball land in the service box, whose service line is 6.40 m from the net?
38. A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. (a) If the ball is thrown at an angle of 25° relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground? (b) How long does it take to get to the receiver? (c) What is its maximum height above its point of release?
39. Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range. (a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s. (b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.
40. An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.
41. An owl is carrying a mouse to the chicks in its nest. Its position at that time is 4.00 m west and 12.0 m above the center of the 30.0 cm diameter nest. The owl is flying east at 3.50 m/s at an angle 30.0º below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen 12.0 m.
42. Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground,

given the initial direction to be 40° above the horizontal.

43. Can a goalkeeper at her/ his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.
44. The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 8.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.
45. In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of 38.0° above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at 45° when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus, 38° will give a longer range than 45° in the shot put.)
46. A basketball player is running at $5.00\ \mathrm{m/s}$ directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?
47. A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?
48. Prove that the trajectory of a projectile is parabolic, having the form $y = \mathrm{ax} + \mathrm{bx}^2$. To obtain this expression, solve the equation $x = v_{0x}t$ for t and substitute it into the expression for $y = v_{0y}t - (1/2)\mathrm{gt}^2$ (These equations describe the x and y positions of a projectile that starts at the origin.) You should obtain an equation of the form $y = \mathrm{ax} + \mathrm{bx}^2$ where a and b are constants.

49. Derive $R = \frac{v_0^2 \sin 2\theta_0}{g}$ for the range of a projectile on level ground by finding the time t at which y becomes zero and substituting this value of t into the expression for $x - x_0$, noting that $R = x - x_0$
50. Unreasonable Results (a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s. (b) What is unreasonable about the range you found? (c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer. (d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.
51. Construct Your Own Problem Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

3.5 Addition of Velocities

52. Bryan Allen pedaled a human-powered aircraft across the English Channel from the cliffs of Dover to Cap Gris-Nez on June 12, 1979. (a) He flew for 169 min at an average velocity of 3.53 m/s in a direction 45° south of east. What was his total displacement? (b) Allen encountered a headwind averaging 2.00 m/s almost precisely in the opposite direction of his motion relative to the Earth. What was his average velocity relative to the air? (c) What was his total displacement relative to the air mass?
53. A seagull flies at a velocity of 9.00 m/s straight into the wind. (a) If it takes the bird 20.0 min to travel 6.00 km relative to the Earth, what is the velocity of the wind? (b) If the bird turns around and flies with the wind, how long will he take to return 6.00 km? (c) Discuss how the wind affects the total round-trip time

compared to what it would be with no wind.

54. Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s. (a) What is the velocity of the second runner relative to the first? (b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity? (c) What distance ahead will the winner be when she crosses the finish line?
55. Verify that the coin dropped by the airline passenger in the Example 3.8 travels 144 m horizontally while falling 1.50 m in the frame of reference of the Earth.
56. A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. The ball is thrown at an angle of 25.0° relative to the ground and is caught at the same height as it is released. What is the initial velocity of the ball relative to the quarterback ?
57. A ship sets sail from Rotterdam, The Netherlands, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0° north of east. What is the velocity of the ship relative to the Earth?
58. (a) A jet airplane flying from Darwin, Australia, has an air speed of 260 m/s in a direction 5.0° south of west. It is in the jet stream, which is blowing at 35.0 m/s in a direction 15° south of east. What is the velocity of the airplane relative to the Earth? (b) Discuss whether your answers are consistent with your expectations for the effect of the wind on the plane's path.
59. (a) In what direction would the ship in Exercise 3.57 have to travel in order to have a velocity straight north relative to the Earth, assuming its speed relative to the water remains $7.00\ \mathrm{m/s}$? (b) What would its speed be relative to the Earth?
60. (a) Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20° south of east (as in Exercise 3.58). Its direction of motion relative to the Earth is 45.0° south of west, while its direction of travel relative to the air is 5.00° south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?
61. A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship: (a) relative to the ship and (b) relative to a stationary observer on shore. (c)

Discuss how the answers give a consistent result for the position at which the sandal hits the deck.

62. The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction 30.0° east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of 50.0° south of west relative to the Earth. What is the velocity of the wind relative to the water?
63. The great astronomer Edwin Hubble discovered that all distant galaxies are receding from our Milky Way Galaxy with velocities proportional to their distances. It appears to an observer on the Earth that we are at the center of an expanding universe. Figure 3.59 illustrates this for five galaxies lying along a straight line, with the Milky Way Galaxy at the center. Using the data from the figure, calculate the velocities: (a) relative to galaxy 2 and (b) relative to galaxy 5. The results mean that observers on all galaxies will see themselves at the center of the expanding universe, and they would likely be aware of relative velocities, concluding that it is not possible to locate the center of expansion with the given information.

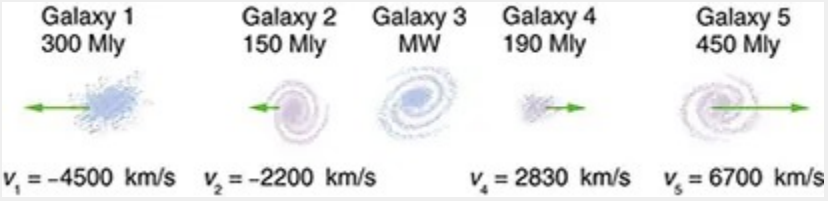


Figure 3.59 Five galaxies on a straight line, showing their distances and velocities relative to the Milky Way (MW) Galaxy. The distances are in millions of light years (Mly), where a light year is the distance light travels in one year. The velocities are nearly proportional to the distances. The sizes of the galaxies are greatly exaggerated; an average galaxy is about 0.1 Mly across. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows five different galaxies, each labeled with a name, distance, and velocity. The galaxies are represented with illustrations and are accompanied by green arrows indicating direction.

– Galaxy 1: Located 300 million light-years (Mly) away. Its velocity is indicated as $v_1 = -4500$ km/s, suggesting it's moving towards us.
– Galaxy 2: Located 150 Mly away with a velocity of $v_2 = -2200$ km/s, also moving towards us.
– Galaxy 3: Labeled "MW" (presumably the Milky Way), no distance provided. No velocity is shown.
– Galaxy 4: 190 Mly away, with a velocity of $v_4 = 2830$ km/s, moving away.
– Galaxy 5: 450 Mly away with a velocity of $v_5 = 6700$ km/s, moving away.

An HTML table can be used to summarize the information:

Galaxy	Distance	Velocity
Galaxy 1	300 Mly	-4500 km/s
Galaxy 2	150 Mly	-2200 km/s
Galaxy 3 (MW)	—	—
Galaxy 4	190 Mly	2830 km/s
Galaxy 5	450 Mly	6700 km/s

The light-years (Mly) indicate distance, and the velocity values in km/s indicate the speed and direction relative to Earth, with negative values showing approaching motion.

64. (a) Use the distance and velocity data in Figure 3.59 to find the rate of expansion as a function of distance. (b) If you extrapolate back in time, how long ago would all of the galaxies have been at approximately the same position? The two parts of this problem give you some idea of how the Hubble constant for universal expansion and the time back to the Big Bang are determined, respectively.
65. An athlete crosses a 25-m-wide river by swimming perpendicular to the water current at a speed of 0.5 m/s relative to the water. They reach the opposite side at a distance 40 m downstream from the starting point. How fast is the water in the river flowing with respect to the ground? What is the speed of the swimmer with respect to a friend at rest on the ground?
66. A ship sailing in the Gulf Stream is heading 25.0° west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is $4.80 m/s\ 5.00^{\circ}$

west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)

67. An ice hockey player is moving at 8.00 m/s when they hit the puck toward the goal. The speed of the puck relative to the player is 29.0 m/s. The line between the center of the goal and the player makes a 90.0° angle relative to their path as shown in Figure 3.60. What angle must the puck's velocity make relative to the player (in their frame of reference) to hit the center of the goal?

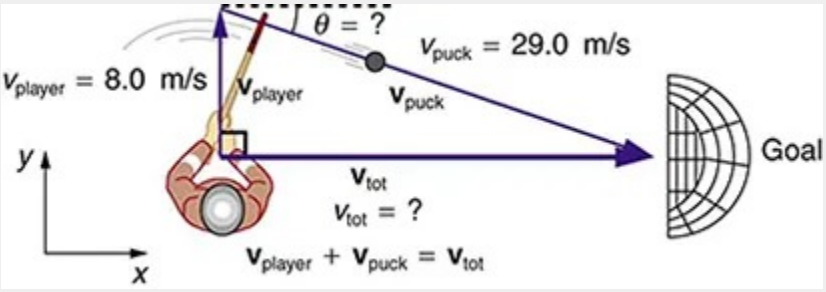


Figure 3.60 An ice hockey player moving across the rink must shoot backward to give the puck a velocity toward the goal. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the trajectory and velocity components involved in a hockey puck being shot towards a goal. Here is a detailed description:

– Player: Positioned on the left side, facing the right. The player is holding a hockey stick.
– Vector Labels:
– v_{player}: 8.0 m/s, shown as a purple arrow pointing to the right, indicating the player's movement direction.
– v_{puck}: 29.0 m/s, shown as a diagonal purple arrow pointing towards the goal from the puck's initial position.
– $\theta = ?$: Angle of the puck's path relative to the horizontal direction, marked at the junction of the player's direction and the puck's path.
– $v_{\text{tot}} = ?$: The resulting velocity of the puck after being hit, pointing towards the goal.
– The equation $v_{\text{player}} + v_{\text{puck}} = v_{\text{tot}}$ is written, explaining the relationship between velocities.

– Axis:
– The diagram includes an x and y-axis on the lower left side, with x pointing right (horizontal) and y pointing up (vertical).

– Goal: Illustrated on the right with a net pattern and labeled “Goal.”

The diagram is meant to illustrate the physics of the puck’s movement and the calculation of its total velocity when combining the player’s velocity with the puck’s velocity.

1. Unreasonable Results Suppose you wish to shoot supplies straight up to astronauts in an orbit 36,000 km above the surface of the Earth. (a) At what velocity must the supplies be launched? (b) What is unreasonable about this velocity? (c) Is there a problem with the relative velocity between the supplies and the astronauts when the supplies reach their maximum height? (d) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.
2. Unreasonable Results A commercial airplane has an air speed of $280\ \mathrm{m/s}$ due east and flies with a strong tailwind. It travels 3000 km in a direction 5° south of east in 1.50 h. (a) What was the velocity of the plane relative to the ground? (b) Calculate the magnitude and direction of the tailwind’s velocity. (c) What is unreasonable about both of these velocities? (d) Which premise is unreasonable?
3. Construct Your Own Problem Consider an airplane headed for a runway in a cross wind. Construct a problem in which you calculate the angle the airplane must fly relative to the air mass in order to have a velocity parallel to the runway. Among the things to consider are the direction of the runway, the wind speed and direction (its velocity) and the speed of the plane relative to the air mass. Also calculate the speed of the airplane relative to the ground. Discuss any last minute maneuvers the pilot might have to perform in order for the plane to land with its wheels pointing straight down the runway.
4. Critical Thinking Two identical spring-loaded launchers are fixed to the end of a table. Launcher A is inclined 40° above the horizontal and launcher B is inclined 50° above the horizontal. (a) Which of these launchers will fire the ball the greatest vertical height above the launcher before it crashes into the ground? (b) Based on experimental data, an equation that fits the data is suggested for the

vertical height above the launcher, y, which may not be correct: $y = KV_0^2 sin\theta$, where K is a constant with appropriate units, V_0 is the initial velocity of the launcher, and θ is the angle relative to the x-axis of the launcher. Is this equation consistent with your answer from part a? Explain why or why not. Does this equation make sense? Explain why or why not. (c) Graph the magnitude of the velocity vs. vertical height for both launcher A and B.

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DYNAMICS: FORCE AND NEWTON'S LAWS OF MOTION



Figure 4.1 Newton's laws of motion describe the motion of the dolphin's path. Image by gcardinal from Norway, Wikimedia Commons License

Chapter Outline

4.1 Development of Force Concept
4.2 Newton's First Law of Motion: Inertia
4.3 Newton's Second Law of Motion: Concept of a System
4.4 Newton's Third Law of Motion: Symmetry in Forces
4.5 Normal, Tension, and Other Examples of Forces
4.6 Problem-Solving Strategies
4.7 Further Applications of Newton's Laws of Motion
4.8 Extended Topic: The Four Basic Forces—An Introduction
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Conceptual Questions
Problems & Exercises

Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. Dynamics considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

Isaac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the many great thinkers who contributed to this change were Newton and Galileo.

Cover page of the first edition of a book, Philosophiae Naturalis Principia Mathematica, written by Isaac Newton.

Figure 4.2 Isaac Newton's monumental work, Philosophiae Naturalis Principia Mathematica, was published in 1687. It proposed scientific laws that are still used today to describe the motion of objects. Image by Isaac Newton, Wikimedia Commons License

Galileo was instrumental in establishing *observation* as the absolute determinant of truth, rather than "logical" argument. Galileo's use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe, and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formation of what is now called Newton's first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made with Newton working alone, without the benefit of the usual interactions that take place among scientists today.

It was not until the advent of modern physics early in the 20th century that it was discovered that Newton's laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the size of most molecules (about $10^{-9}\ \text{m}$ in diameter). These constraints define the realm of classical mechanics, as discussed in Introduction: the Nature of Science and Physics. At the beginning of the 20th century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, developed quantum theory. This theory does not have the constraints present in classical physics. All of the situations we consider in this chapter, and all those preceding the introduction of relativity, are in the realm of classical physics.

Making Connections: Past and Present Philosophy

The importance of observation and the concept of *cause and effect* were not always so entrenched in human thinking. This realization was a part of the evolution of modern physics from natural philosophy. The achievements of Galileo, Newton, Einstein, and others were key milestones in the history of scientific thought. Most of the scientific theories that are described in this book descended from the work of these scientists.

Access multimedia content

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4.1 DEVELOPMENT OF FORCE CONCEPT

Learning Objectives

By the end of this section, you will be able to:

- Understand the definition of force.

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of force—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in Figure 4.3, we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors, as illustrated in Figure 4.3(a) for two ice skaters. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods. These ideas were developed in Two-Dimensional Kinematics.

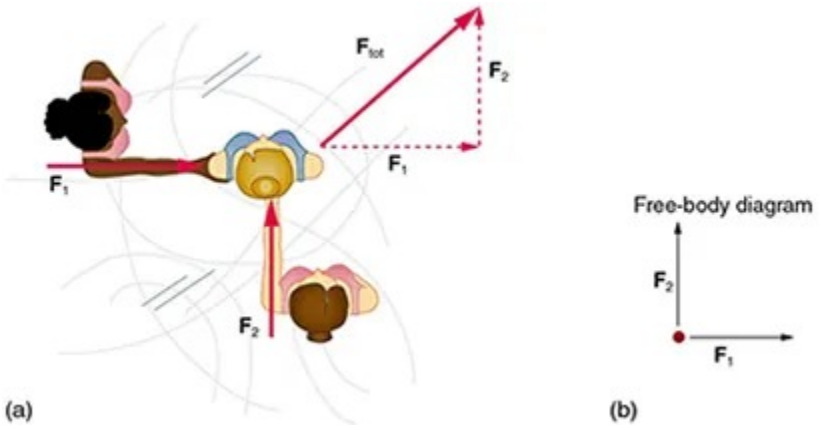


Figure 4.3 Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two parts labeled (a) and (b).

(a) This section shows a diagram of three people interacting with forces involved. The people are positioned in a top-down view. The diagram includes force vectors:
– F1: A horizontal force vector pointing left, applied by the person on the left.
– F2: A vertical force vector pointing upward, applied by the person at the bottom.
– F_tot: The resultant force vector depicted as a diagonal line in the upward-right direction, formed by combining F1 and F2.

The vectors are represented with arrows, indicating both direction and magnitude.

(b) This section displays a simplified free-body diagram:
– The diagram includes a dot representing an object.
– A horizontal arrow labeled F1 points to the right, indicating a horizontal force.
– A vertical arrow labeled F2 points upward, indicating a vertical force.
– Labels indicate the axes and respective forces.

Overall, the image illustrates the concept of vector addition and free-body diagrams in physics through a top-down view of people applying forces and a corresponding simplified diagram.

Figure 4.3(b) is our first example of a free-body diagram, which is a technique used to illustrate all the external forces acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting *on* the body from the outside (external forces) are shown. (These forces are the only ones shown,

because only external forces acting on the body affect its motion. We can ignore any internal forces within the body.) Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton's laws of motion.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in Figure 4.4, and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. (One that we will encounter in Magnetism is the magnetic force between two wires carrying electric current.) Some alternative definitions of force will be given later in this chapter.

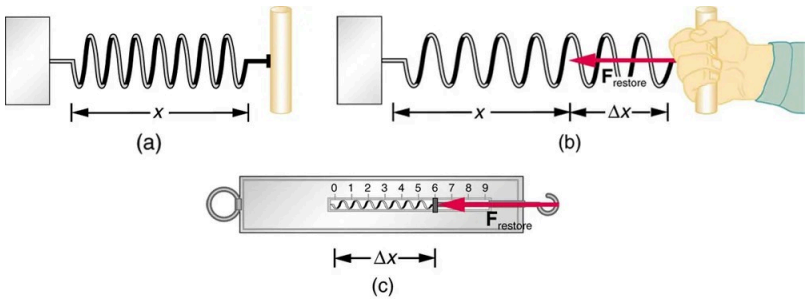


Figure 4.4 The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length x when undistorted. (b) When stretched a distance Δx, the spring exerts a restoring force, $\mathbf{F}_{\text{restore}}$, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force $\mathbf{F}_{\text{restore}}$ is exerted on whatever is attached to the hook. Here $\mathbf{F}_{\text{restore}}$ has a magnitude of 6 units in the force standard being employed. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams demonstrating the concept of spring force and Hooke's Law.

1. Diagram (a):
– Shows a spring in its natural, uncompressed state.
– The spring is attached between two blocks.
– A distance labeled as "x" is indicated, measuring from the left block to the right block.
– There's no external force applied.

2. Diagram (b):
– Illustrates the spring in a stretched state.
– A hand is pulling on the right end of the spring, extending it.
– The extension is denoted as "Δx" from the original position "x."

– A red arrow labeled "F restore" indicates the restoring force of the spring, pointing leftward, opposing the extension.

3. Diagram (c):
– Depicts a spring scale.
– The scale's face is marked from 0 to 9.
– Inside the scale, a spring is shown stretched, with "Δx" indicating the extension from its unstretched position.
– A red arrow labeled "F restore" points leftwards within the scale, signifying the spring's restoring force.

These diagrams collectively illustrate how a spring exerts a restoring force when stretched, with this force being proportional to the extension of the spring, as per Hooke's Law.

Take-Home Experiment: Force Standards

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

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4.2 NEWTON'S FIRST LAW OF MOTION: INERTIA

Learning Objectives

By the end of this section, you will be able to:

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What Newton's first law of motion states, however, is the following:

Newton's First Law of Motion

A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb "remains." We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, Newton's first law of motion states that there must be a *cause*

(which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton's first law). The object would not slow down at all if friction were completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, "What is the cause?" Thinking in terms of cause and effect is a worldview fundamentally different from the typical ancient Greek approach when questions such as "Why does a tiger have stripes?" would have been answered in Aristotelian fashion, "That is the nature of the beast." True perhaps, but not a useful insight.

Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called inertia. Newton's first law is often called the law of inertia. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its mass. Roughly speaking, mass is a measure of the amount of "stuff" (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

Check Your Understanding

Which has more mass: a kilogram of cotton balls or a kilogram of gold?

Click for Solution

Solution

They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

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4.3 NEWTON'S SECOND LAW OF MOTION: CONCEPT OF A SYSTEM

Learning Objectives

By the end of this section, you will be able to:

- Define net force, external force, and system.
- Understand Newton's second law of motion.
- Apply Newton's second law to determine the weight of an object.

Newton's second law of motion is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an acceleration. Newton's first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an external force acts from outside the system (object or collection of objects) of interest. For example, in Figure 4.5(a) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at Figure 4.5(a), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the*

boundaries of the system before you can determine which forces are external. Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept will be revisited many times on our journey through physics.

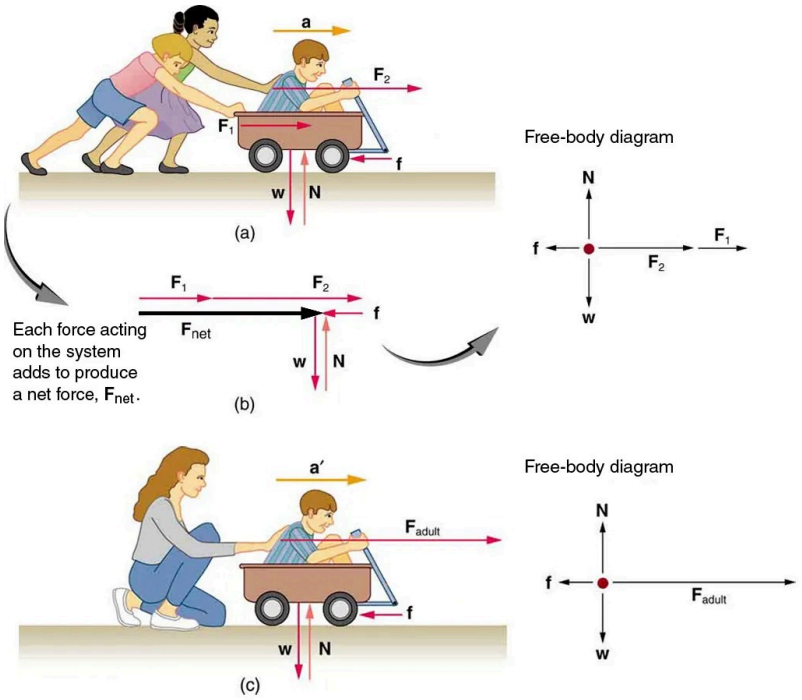


Figure 4.5 Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight $\mathbf{w}$ *of the system and the support of the ground* $\mathbf{N}$ *are also shown for completeness and are assumed to cancel. The vector* $\mathbf{f}$ *represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force,* $\mathbf{F}_{\text{net}}$*. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger acceleration (*$\mathbf{a}\prime > \mathbf{a}$*) when an adult pushes the child. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of three diagrams depicting the forces acting on a child in a wagon in different scenarios. Here's a detailed description:

1. Diagram (a):
– Features two children, a boy sitting in a wagon and a girl pushing him.
– Arrows indicate forces:

– F_1 is the force applied by the girl.
– F_2 is a smaller force exerted by the boy.
– w represents the weight of the wagon.
– N is the normal force.
– f is the frictional force.
– An arrow labeled a indicates acceleration.
– A labeled "Free-body diagram" on the right shows force vectors F_1, F_2, f, w, and N.

2. Diagram (b):
– Shows the forces with an emphasis on the net force F_{net}.
– An arrow description indicates that each force adds up to produce this net force.

3. Diagram (c):
– Depicts an adult pushing the boy in the wagon.
– Arrows indicate forces:
– F_{adult} is the force applied by the adult.
– w continues to represent weight.
– N remains the normal force.
– f continues as the frictional force.
– An arrow labeled a' indicates acceleration.
– Another "Free-body diagram" on the right shows force vectors: F_{adult}, f, w, and N.

In each scenario, the direction and magnitude of the forces are visually represented with arrows, providing insight into the dynamics of the situation.

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in Figure 4.5. In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight $\mathbf{w}$ and the support of the ground $\mathbf{N}$, and the horizontal force $\mathbf{f}$ represents the force of friction. These will be discussed in more detail in later sections. For now, we will define friction as a force that opposes the motion past each other of objects that are touching. Figure 4.5(b) shows how vectors representing the external forces add together to produce a net force, $\mathbf{F}_{\text{net}}$. To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality

$$\mathbf{a} \propto \mathbf{F}_{\text{net}},$$
4.1

where the symbol $\propto$ means "proportional to," and $\mathbf{F}_{\text{net}}$ is the net external force. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method, or analytically, using components. The techniques are the same as for the addition of other vectors, and are covered in Two-Dimensional Kinematics.) This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in Figure 4.6, the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

$$\mathbf{a} \propto \frac{1}{m}$$
4.2

where m is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.

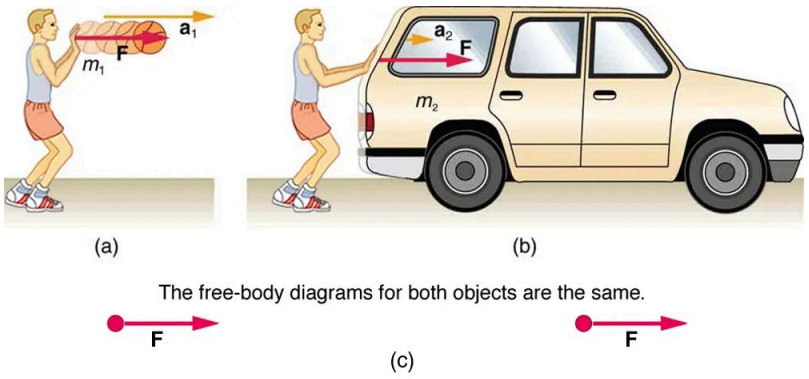


Figure 4.6 The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams labeled (a), (b), and (c).

(a) Shows a person pushing a large loaf of bread. The person has their arms extended, with force F in the same direction as the person's push. The bread loaf is marked as having mass m_1, and it's accelerating to the right with acceleration a_1.

(b) Depicts the same person in a similar stance, but now they are pushing against the rear of a car. The car is labeled with a mass m_2, and it is accelerating to the right with acceleration a_2. Force F is applied in the opposite direction to the acceleration, indicating the force of the person's push.

(c) Contains two simple free-body diagrams. Both diagrams show a red dot with an arrow labeled F pointing to the right. Below these diagrams is the text: "The free-body diagrams for both objects are the same."

It has been found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

Newton's Second Law of Motion

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}.$$

4.3

This is often written in the more familiar form

$$\mathbf{F}_{\text{net}} = m\mathbf{a}.$$

4.4

When only the magnitude of force and acceleration are considered, this equation is simply

$$F_{\text{net}} = \text{ma}.$$

4.5

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

Units of Force

$\mathbf{F}_{\text{net}} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of $1\ \text{m/s}^2$. That is, since $\mathbf{F}_{\text{net}} = m\mathbf{a}$,

$$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2.$$

4.6

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its weight $\mathbf{w}$. Weight can be denoted as a vector $\mathbf{w}$ because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as w. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration g. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass m falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude w. Newton's second law states that the magnitude of the net external force on an object is $F_{\text{net}} = \text{ma}$.

Since the object experiences only the downward force of gravity, $F_{\text{net}} = w$. We know that the acceleration of an object due to gravity is g, or $a = g$. Substituting these into Newton's second law gives

Weight

This is the equation for *weight*—the gravitational force on a mass m:

$$w = \mathrm{mg}.$$

4.7

Since $g = 9.80\ \mathrm{m/s^2}$ on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

$$w = \mathrm{mg} = (1.0\ \mathrm{kg})\,(9.80\mathrm{m/s^2}) = 9.8\mathrm{N}.$$

4.8

Recall that g can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in free-fall. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity g varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only $1.625\mathrm{m/s^2}$. A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of "weightlessness" and "microgravity," they are really referring to the phenomenon we call "free-fall" in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are

closely related. Mass is the quantity of matter (how much "stuff") and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our "weight" in kilograms, but never in the correct units of newtons.

Common Misconceptions: Mass vs. Weight

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the "slug" in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (m) multiplied by the acceleration due to gravity (g). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object *can change* when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is $1.625\ \mathrm{m/s^2}$ (which is much less than the acceleration due to gravity on Earth, $9.80\ \mathrm{m/s^2}$). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you "weigh" much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are "losing weight," they really mean that they are losing "mass" (which in turn causes them to weigh less).

Take-Home Experiment: Mass and Weight

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same "mass" on Earth as on the Moon?

Example 4.1

What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration?

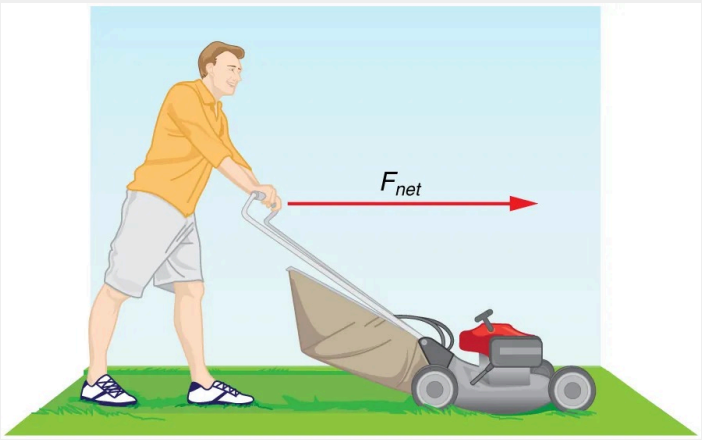


Figure 4.7 The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right? Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a person pushing a lawn mower on a grassy area. The individual is wearing a long-sleeve yellow shirt, light gray shorts, and white sneakers with blue details. The lawn mower has a bag attached at the back for collecting grass clippings and features a red engine at the top. An arrow labeled "F_{net}" is shown pointing horizontally in the direction the person is pushing, indicating the net force applied to the lawn mower. The sky in the background is clear and blue, suggesting a calm day.

Strategy

Since $\mathbf{F}_{\text{net}}$ and m are given, the acceleration can be calculated directly from Newton's second law as stated in $\mathbf{F}_{\text{net}} = m\mathbf{a}$.

Solution

The magnitude of the acceleration a is $a = \frac{F_{\text{net}}}{m}$. Entering known values gives

$$a = \frac{51 \text{ N}}{24 \text{ kg}}$$

4.9

Substituting the units $\text{kg} \cdot \text{m/s}^2$ for N yields

$$a = \frac{51 \text{ kg} \cdot \text{m/s}^2}{24 \text{ kg}} = 2.1 \text{ m/s}^2.$$

4.10

Discussion

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

Example 4.2

What Rocket Thrust Accelerates This Sled?

Prior to space flights carrying astronauts, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust $\mathbf{T}$, for the four-rocket propulsion system shown in Figure 4.8. The sled's initial acceleration is $49\ \mathrm{m/s^2}$, the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N.

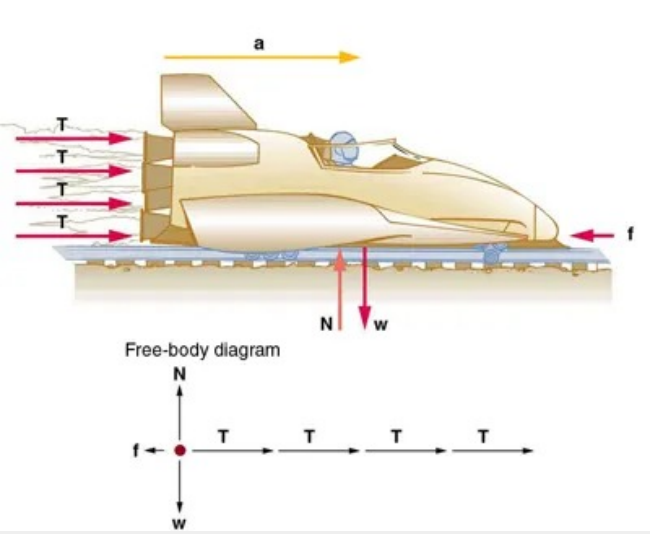


Figure 4.8 A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust $\mathbf{T}$*. As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force* $\mathbf{N}$ *on the system that is equal in magnitude and opposite in direction to its weight,* $\mathbf{w}$*. The system here is the sled, its rockets, and rider, so none of the forces between these objects are considered. The arrow representing friction (*$\mathbf{f}$*) is drawn larger than scale. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows a side view of a high-speed vehicle, possibly representing a simplified model of a rocket car or similar vehicle. It is depicted moving from left to right.

Above the vehicle:

– Several labeled arrows indicate forces acting on the vehicle:
– A yellow horizontal arrow labeled "a" points to the right, representing acceleration.
– A red horizontal arrow labeled "f" points to the left, representing friction.
– Several red horizontal arrows labeled "T" point to the left, representing thrust.

Below the vehicle:

– Two vertical arrows indicate other forces:
– A red downward arrow labeled "w" represents weight.
– A red upward arrow labeled "N" represents the normal force.

Below the vehicle's illustration, there is a free-body diagram:

– It consists of a dot with arrows extending from it:
– An upward arrow labeled "N" represents the normal force.
– A downward arrow labeled "w" represents weight.
– A leftward arrow labeled "f" represents friction.
– Several rightward arrows labeled "T" represent thrust.

This diagram shows a simplified representation of the forces acting on the vehicle during motion.

Strategy

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

Solution

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

$$F_{\text{net}} = \text{ma},$$

4.11

where F_{net} is the net force along the horizontal direction. We can see from Figure 4.8 that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

$$F_{\text{net}} = 4T - f.$$

4.12

Substituting this into Newton's second law gives

$$F_{\text{net}} = \text{ma} = 4T - f.$$

4.13

Using a little algebra, we solve for the total thrust $4T$:

$$4T = \text{ma} + f.$$

4.14

Substituting known values yields

$$4T = \text{ma} + f = (2100\ \text{kg})\,(49\ \text{m/s}^2) + 650\ \text{N}.$$

4.15

So the total thrust is

$$4T = 1.0 \times 10^5\ \text{N},$$

4.16

and the individual thrusts are

$$T = \frac{1.0 \times 10^5 \text{ N}}{4} = 2.6 \times 10^4 \text{ N}.$$

4.17

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 g's. (Recall that g, the acceleration due to gravity, is 9.80 m/s^2. When we say that an acceleration is 45 g's, it is $45 \times 9.80 \text{ m/s}^2$, which is approximately 440 m/s^2.) While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law of motion is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

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4.4 NEWTON'S THIRD LAW OF MOTION: SYMMETRY IN FORCES

Learning Objectives

By the end of this section, you will be able to:

- Understand Newton's third law of motion.
- Apply Newton's third law to define systems and solve problems of motion.

Baseball relief pitcher Mariano Rivera was so highly regarded that during his retirement year, opposing teams conducted farewell presentations when he played at their stadiums. The Minnesota Twins offered a unique gift: A chair made of broken bats. Any pitch can break a bat, but with Rivera's signature pitch—known as a cutter—the ball and the bat frequently came together at a point that shattered the hardwood. Typically, we think of a baseball or softball hitter exerting a force on the incoming ball, and baseball analysts focus on the resulting "exit velocity" as a key statistic. But the force of the ball can do its own damage. This is exactly what happens whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or pushing off the floor during a jump, confirm this. It is precisely stated in Newton's third law of motion.

Newton's Third Law of Motion

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as "action-reaction," where the force exerted is the action and the force experienced as a consequence is the reaction. Newton's third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton's third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in Figure 4.9. She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then $\mathbf{F}_{\text{wall on feet}}$ is an external force on this system and affects its motion. The swimmer moves in the direction of $\mathbf{F}_{\text{wall on feet}}$. In contrast, the force $\mathbf{F}_{\text{feet on wall}}$ acts on the wall and not on our system of interest. Thus $\mathbf{F}_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\mathbf{F}_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to that in which she wishes to move. The reaction to her push is thus in the desired direction.

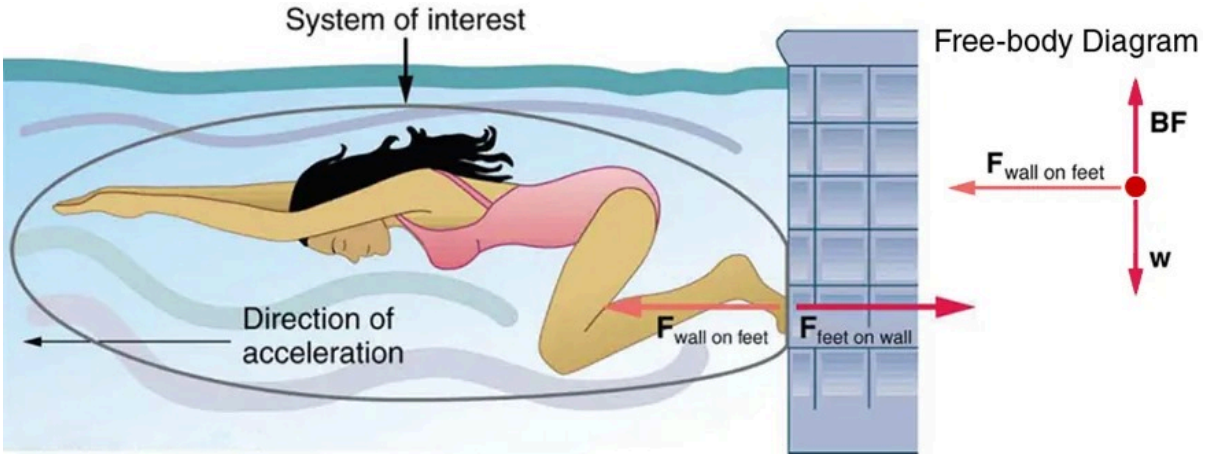


Figure 4.9 When the swimmer exerts a force $\mathbf{F}_{\text{feet on wall}}$ *on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to* $\mathbf{F}_{\text{feet on wall}}$*. This opposition occurs because, in accordance with Newton's third law of motion, the wall exerts a force* $\mathbf{F}_{\text{wall on feet}}$ *on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that* $\mathbf{F}_{\text{feet on wall}}$ *does not act on this system (the swimmer) and, thus, does not cancel* $\mathbf{F}_{\text{wall on feet}}$*. Thus the free-body diagram shows only* $\mathbf{F}_{\text{wall on feet}}$*,* $\mathbf{w}$*, the gravitational force, and* $\mathbf{BF}$*, the buoyant force of the water supporting the swimmer's weight. The vertical forces* $\mathbf{w}$ *and* $\mathbf{BF}$ *cancel since there is no vertical motion. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a swimmer in a pool, positioned horizontally and pushing against the wall with her feet. She is wearing a pink swimsuit and has her arms extended forward. The area around her is labeled "System of interest," with an oval shape surrounding her body.

An arrow labeled "Direction of acceleration" points from right to left, indicating the swimmer's intended movement direction.

To the right, a "Free-body Diagram" is shown, illustrating the forces acting on the swimmer. The diagram includes:

– An arrow labeled "$F_{\text{wall on feet}}$" pointing left, representing the force exerted by the wall on the swimmer's feet.
– An arrow labeled "$F_{\text{feet on wall}}$" pointing right, indicating the force exerted by the swimmer's feet on the wall.
– An arrow labeled "BF" pointing upwards.
– An arrow labeled "w" pointing downwards, representing the weight force.

The wall is depicted with a tiled pattern, and the swimmer's body is streamlined, emphasizing movement.

Other examples of Newton's third law are easy to find. As a professor walks in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. In another example, rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket. This reaction force is called thrust. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases. Helicopters similarly create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski. Boxers and other martial arts fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent's body.

Example 4.3

Getting Up To Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in Figure 4.10. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.

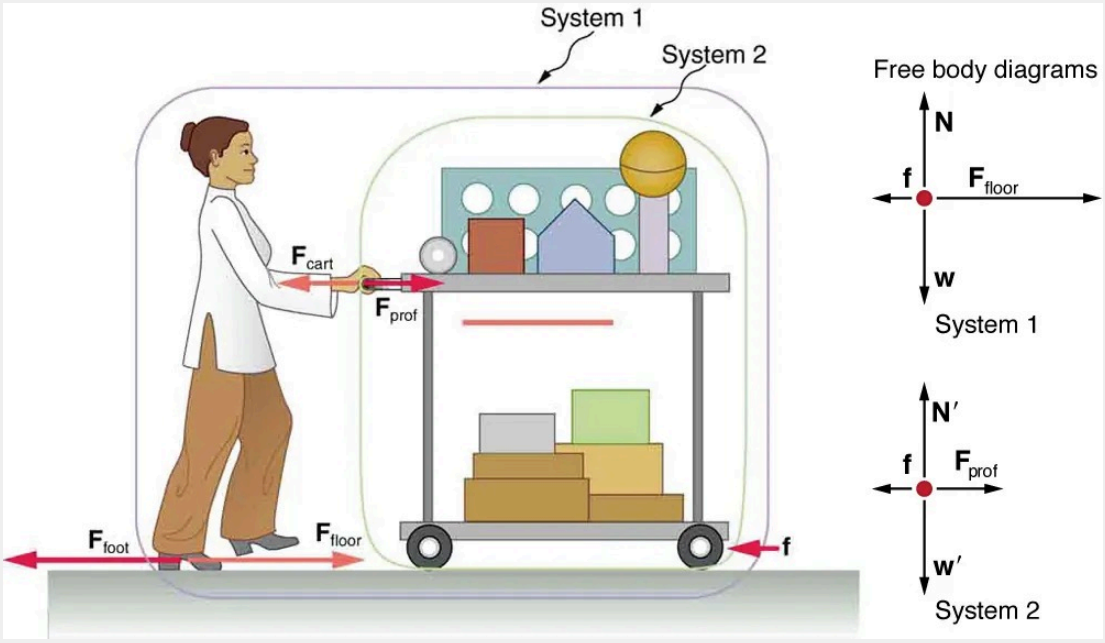


Figure 4.10 A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for $\mathbf{f}$*, since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for this example, since it asks for the acceleration of the entire group of objects. Only* $\mathbf{F}_{\text{floor}}$ *and* $\mathbf{f}$ *are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for so that* $\mathbf{F}_{\text{prof}}$ *will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

This image illustrates a physics concept involving forces and free-body diagrams. It shows a person pushing a cart. Various forces are labeled with arrows to explain the dynamics:

– The person is pushing the cart with a force labeled as "F_{cart}".
– The cart is loaded with boxes and shapes, illustrating a system labeled "System 1".
– The cart's wheels are shown with a smaller circle, and the force of friction is labeled as "f".

– The floor exerts a force labeled "F_{floor}" against the cart in the opposite direction of "F_{cart}".
– A force labeled "F_{foot}" is acting on the person's foot in the left direction.
– Another system, "System 2", is labeled around the cart.
– Two free-body diagrams are shown on the right side:
– System 1 diagram shows forces "N" (normal force), "w" (weight), "f" (friction), and "F_{floor}" (floor force).
– System 2 diagram shows forces "N'" (normal force), "w'" (weight), "f" (friction), and "F_{prof}" (professor's force).

The person is shown with a neutral expression, wearing a white top and brown pants, and is actively engaging in pushing the cart forward.

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in Figure 4.10. The professor pushes backward with a force $\mathbf{F}_{\text{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward reaction force $\mathbf{F}_{\text{floor}}$ of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the horizontal direction. As noted, $\mathbf{f}$ opposes the motion and is thus in the opposite direction of $\mathbf{F}_{\text{floor}}$. Note that we do not include the forces $\mathbf{F}_{\text{prof}}$ or $\mathbf{F}_{\text{cart}}$ because these are internal forces, and we do not include $\mathbf{F}_{\text{foot}}$ because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton's second law is given by

$$a = \frac{F_{\text{net}}}{m}.$$

4.18

The net external force on System 1 is deduced from Figure 4.10 and the discussion above to be

$$F_{\text{net}} = F_{\text{floor}} - f = 150\ \text{N} - 24.0\ \text{N} = 126\ \text{N}.$$

4.19

The mass of System 1 is

$$m = (65.0\ +\ 12.0\ +\ 7.0\,)\ \text{kg}\ =\ 84\ \text{kg}.$$

4.20

These values of F_{net} and m produce an acceleration of

$$a = \frac{F_{\text{net}}}{m}, a = \frac{126\ \text{N}}{84\ \text{kg}} = \ 1.5\ \text{m/s}^2.$$

4.21

Discussion

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

Example 4.4

Force on the Cart—Choosing a New System

Calculate the force the professor exerts on the cart in Figure 4.10 using data from the previous example if needed.

Strategy

If we now define the system of interest to be the cart plus equipment (System 2 in Figure 4.10), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, $\mathbf{F}_{\text{prof}}$, is an external force acting on System 2. $\mathbf{F}_{\text{prof}}$ was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2.

Solution

Newton's second law can be used to find $\mathbf{F}_{\text{prof}}$. Starting with

$$a = \frac{F_{\text{net}}}{m}$$

4.22

and noting that the magnitude of the net external force on System 2 is

$$F_{\text{net}} = F_{\text{prof}} - f,$$

4.23

we solve for F_{prof}, the desired quantity:

$$F_{\text{prof}} = F_{\text{net}} + f.$$

4.24

The value of f is given, so we must calculate net F_{net}. That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

$$F_{\text{net}} = \text{ma},$$

4.25

where the mass of System 2 is 19.0 kg (m= 12.0 kg + 7.0 kg) and its acceleration was found to be $a = 1.5\ \text{m/s}^2$ in the previous example. Thus,

$$F_{\text{net}} = \text{ma},$$

4.26

$$F_{\text{net}} = (19.0\ \text{kg})\,(1.5\ \text{m/s}^2) = 29\ \text{N}.$$

4.27

Now we can find the desired force:

$$F_{\text{prof}} = F_{\text{net}} + f,$$

4.28

$$F_{\text{prof}} = 29\ \text{N} + 24.0\ \text{N} = 53\ \text{N}.$$

4.29

Discussion

It is interesting that this force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

PhET Explorations

Gravity Force Lab

Visualize the gravitational force that two objects exert on each other. Change properties of the objects in order to see how it changes the gravity force.

Access multimedia content

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4.5 NORMAL, TENSION, AND OTHER EXAMPLES OF FORCES

Learning Objectives

By the end of this section, you will be able to:

- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

Normal Force

Weight (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in Figure 4.11(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in Figure 4.11(b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly,

and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

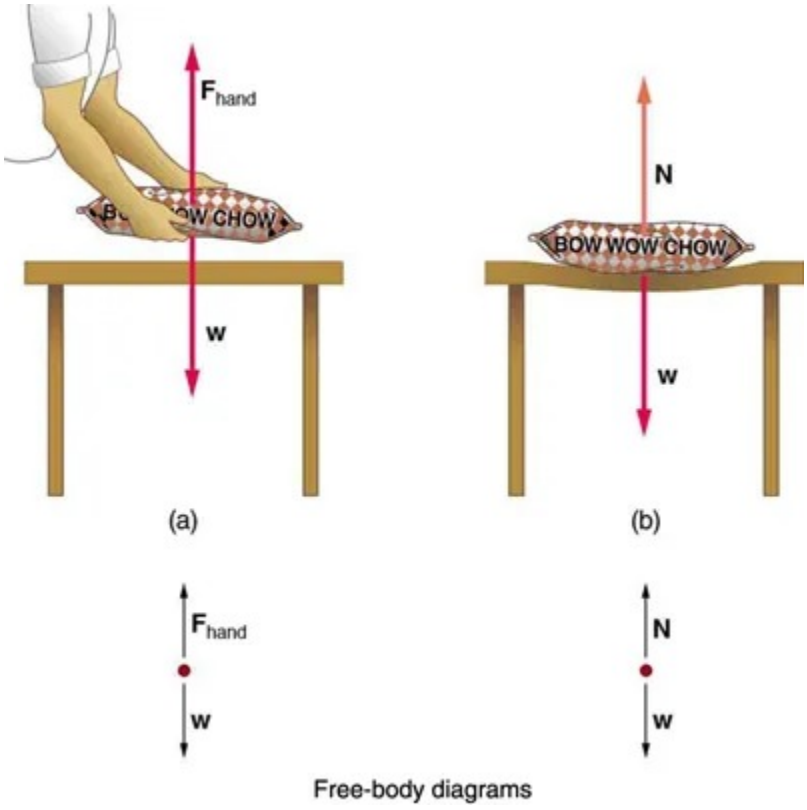


Figure 4.11 (a) The person holding the bag of dog food must supply an upward force $\mathbf{F}_{\text{hand}}$ *equal in magnitude and opposite in direction to the weight of the food* $\mathbf{w}$*. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force* $\mathbf{N}$ *equal in magnitude and opposite in direction to the weight of the load. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image features two diagrams demonstrating the concept of forces applied to a bag. Both diagrams show a bag labeled "BOW WOW CHOW" placed on a table.

– Diagram (a):
– A pair of hands is pressing down on the bag, causing it to compress.
– An arrow labeled "F_{hand}" points upwards, representing the force applied by the hands.
– An arrow labeled "w" points downwards, indicating the weight of the bag.
– Below the table, a free-body diagram shows a dot with an upward arrow labeled "F_{hand}" and a downward arrow labeled "w."

– Diagram (b):
– The bag is shown without any external force being applied, allowing it to remain uncompressed but slightly deformed by its own weight.
– An arrow labeled "N" points upwards, which represents the normal force exerted by the table on the bag.
– An arrow labeled "w" points downwards, indicating the weight of the bag.
– Below the table, a free-body diagram shows a dot with an upward arrow labeled "N" and a downward arrow labeled "w."

Below both diagrams, the text reads "Free-body diagrams."

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a normal force and here is given the symbol $\mathbf{N}$. (This is not the unit for force N.) The word *normal* means perpendicular to a surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

Common Misconception: Normal Force (N) vs. Newton (N)

In this section we have introduced the quantity normal force, which is represented by the variable $\mathbf{N}$. This should not be confused with the symbol for the newton, which is also represented by the letter N. These symbols are particularly important to distinguish because the units of a normal force ($\mathbf{N}$) happen to be newtons (N). For example, the normal force $\mathbf{N}$ that the floor exerts on a chair might be $\mathbf{N} = 100\ \mathrm{N}$. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work (W) and the unit watts (W).

Example 4.5

Weight on an Incline, a Two-Dimensional Problem

Consider the skier on a slope shown in Figure 4.12. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be 45.0 N?

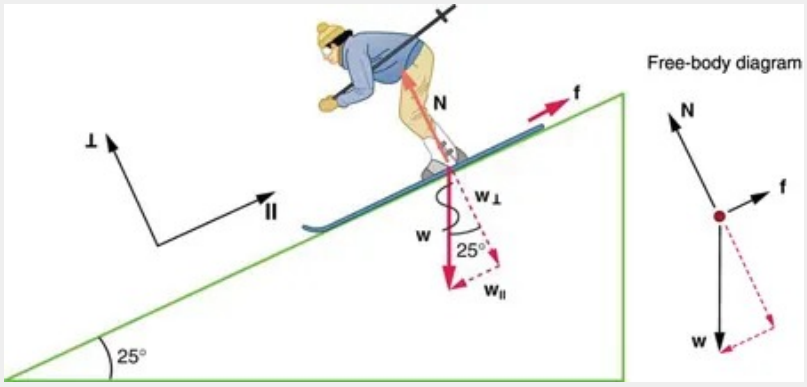


Figure 4.12 Since motion and friction are parallel to the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). $\mathbf{N}$ *is perpendicular to the slope and* $\mathbf{f}$ *is parallel to the slope, but* $\mathbf{w}$ *has components along both axes, namely* $\mathbf{w}_{\perp}$ *and* $\mathbf{w}_{\parallel}$. $\mathbf{N}$ *is equal in magnitude to* $\mathbf{w}_{\perp}$, *so that there is no motion perpendicular to the slope, but* f *is less than* $w_{\parallel}$, *so that there is a downslope acceleration Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows a skier descending a slope inclined at 25 degrees. The skier is leaning forward and using ski poles for balance. The diagram is labeled as a “Free-body diagram.”

To the left, there is a depiction of the slope with three arrows indicating forces. The arrow labeled “N” represents the normal force perpendicular to the slope. The force of friction, labeled “f,” acts parallel to the slope and pointed uphill. The weight of the skier, labeled “W,” is shown with a vector pointing straight down. The weight vector is decomposed into two components: “W⊥” is perpendicular to the slope and “W ||” is parallel to the slope.

On the right, a simplified free-body diagram shows these forces without the skier: “N” points perpendicularly from the slope, “W” points downward, and “f” points up the slope. The weight vector “W” is shown decomposed into its components with “W⊥” and “W ||” forming a 25-degree angle with each other.

These diagrams visually represent the forces acting on the skier as they move down the slope.

Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in two-dimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating *two* connected *one*-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular axes are independent.) We use the symbols $\perp$ and $\parallel$ to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier’s weight, friction, and the support of the slope, respectively labeled $\mathbf{w}$, $\mathbf{f}$, and $\mathbf{N}$ in Figure 4.12. $\mathbf{N}$ is always perpendicular to the slope, and $\mathbf{f}$ is parallel to it. But $\mathbf{w}$ is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining $w_{\parallel}$ to be the component of weight parallel to the slope and $w_{\perp}$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of the weight parallel to the slope is $w_{\parallel} = w\sin(25^{\circ}) = mg\sin(25^{\circ})$, and the magnitude of the component of the weight perpendicular to the slope is $w_{\perp} = w\cos(25^{\circ}) = mg\cos(25^{\circ})$.

(a) Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the amount of the skier's weight parallel to the slope $w_{\parallel}$ and friction f. Using Newton's second law, with subscripts to denote quantities parallel to the slope, $a_{\parallel} = \frac{F_{\text{net}\parallel}}{m}$

4.30

where $F_{\text{net}\parallel} = w_{\parallel} = \text{mgsin}(25^{\circ})$, assuming no friction for this part, so that

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m} = \frac{\text{mgsin}(25^{\circ})}{m} = g\sin(25^{\circ})$$

4.31

$$(9.80\ \text{m/s}^2)(0.4226) = 4.14\text{m/s}^2$$

4.32

is the acceleration.

(b) Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$F_{\text{net}\parallel} = w_{\parallel} - f,$$

4.33

and substituting this into Newton's second law, $a_{\parallel} = \frac{F_{\text{net}\parallel}}{m}$, gives

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m} = \frac{w_{\parallel} - f}{m} = \frac{\text{mgsin}(25^{\circ}) - f}{m}.$$

4.34

We substitute known values to obtain

$$a_{\parallel} = \frac{(60.0\ \text{kg})(9.80\ \text{m/s}^2)(0.4226) - 45.0\ \text{N}}{60.0\ \text{kg}},$$

4.35

which yields

$$a_{\parallel} = 3.39\ \text{m/s}^2,$$

4.36

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. In fact, it is a general result that if friction on an incline is negligible, then the acceleration down the incline is $a = g\sin\theta$, *regardless of mass*. This is related to the previously discussed fact that all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

Resolving Weight into Components

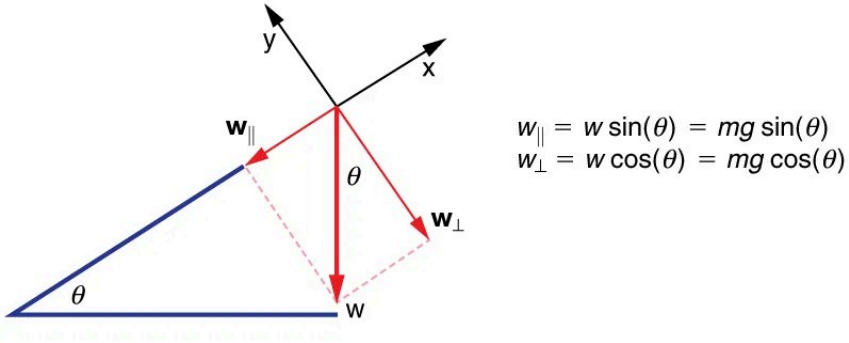


*Figure 4.13 An object rests on an incline that makes an angle **θ** with the horizontal. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a diagram illustrating the decomposition of a weight vector *w* on an inclined plane. The inclined plane is represented as a blue triangle with the angle ***θ*** at the bottom left corner. The weight vector *w* is shown as a red arrow pointing downwards, perpendicular to the horizontal ground.

There are two components of the weight vector depicted:

- $w_{||}$: Parallel to the inclined plane, shown as a red arrow pointing down the plane.
- $w_{\perp}$: Perpendicular to the inclined plane, shown as a red arrow pointing directly into the inclined surface.

The axis is labeled with *x* pointing up the incline and *y* perpendicular to it.

To the right of the diagram, there are equations representing the components of the weight vector:

- $w_{||} = w\sin(\theta) = mg\sin(\theta)$
- $w_{\perp} = w\cos(\theta) = mg\cos(\theta)$

Here, w is the weight of the object, m is the mass, and g is the acceleration due to gravity.

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane, $\mathbf{w}_{\perp}$, and a force acting parallel to the plane, $\mathbf{w}_{\parallel}$. The perpendicular force of weight, $\mathbf{w}_{\perp}$, is typically equal in magnitude and opposite in direction to the normal force, $\mathbf{N}$. The force acting parallel to the plane, $\mathbf{w}_{\parallel}$, causes the object to accelerate down the incline. The force of friction, $\mathbf{f}$, opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$w_{\parallel} = w\sin(\theta) = \text{mgsin}(\theta)$$

4.37

and

$$w_{\perp} = w\cos(\theta) = \text{mgcos}(\theta).$$

4.38

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle θ of the incline is the same as the angle formed between $\mathbf{w}$ and $\mathbf{w}_{\perp}$. Knowing this property, you can use trigonometry to determine the magnitude of the weight components:

$$\cos(\theta) = \frac{w_{\perp}}{w}$$
$$w_{\perp} = w\cos(\theta) = \text{mgcos}(\theta)$$

4.39

$$\sin(\theta) = \frac{w_{\parallel}}{w}$$
$$w_{\parallel} = w\sin(\theta) = \text{mgsin}(\theta)$$

4.40

Take-Home Experiment: Force Parallel

To investigate how a force parallel to an inclined plane changes, find a rubber band, some objects to hang from the end of the rubber band, and a board you can position at different angles. How much does the rubber band stretch when you hang the object from the end of the board? Now place the board at an angle so that the object slides off when placed on the board. How much does the rubber band extend if it is lined up parallel to the board and used to hold the object stationary on the board? Try two more angles. What does this show?

Tension

A tension is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word "tension" comes from a Latin word meaning "to stretch." Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: "You can't push a rope." The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in Figure 4.14.

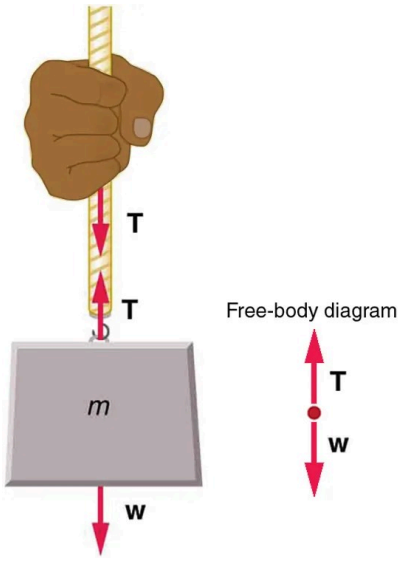


Figure 4.14 When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force **T**, *that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows a hand gripping a rope that is connected to a rectangular block labeled with "m". There are arrows and labels indicating forces acting on the system. The following elements are present:

– A hand is grasping a rope, which is vertical.
– The rope is attached to the top of a rectangular block with the mass labeled as "m".
– Two upward arrows, labeled "T", are drawn along the rope, signifying the tension force exerted by the rope at different points.
– A downward arrow labeled "w" is shown at the bottom of the block, representing the weight of the block.
– On the right side of the image, there is a free-body diagram:
– A small dot represents the object (the block).
– An upward arrow labeled "T" shows the tension force.
– A downward arrow labeled "w" shows the weight force.
– The term "Free-body diagram" is written next to the simplified representation to explain the forces acting on the block.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $\mathbf{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight $\mathbf{w}$ and the tension $\mathbf{T}$ supplied by the rope. Thus,

$$F_{\text{net}} = T - w = 0,$$
4.41

where T and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

$$T = w = \text{mg}.$$
4.42

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

$$T = \text{mg} = (5.00\ \text{kg})\,(9.80\ \text{m/s}^2) = 49.0\ \text{N}.$$
4.43

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in Figure 4.15 (a) and (b).

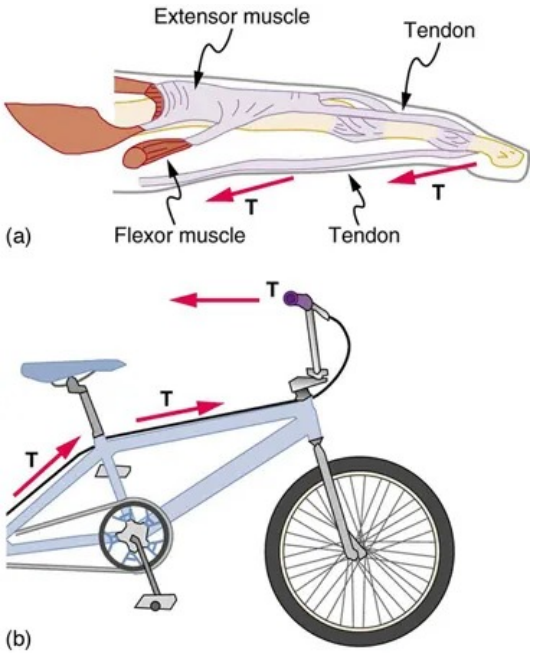


Figure 4.15 (a) Tendons in the finger carry force $\mathbf{T}$ *from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension* $\mathbf{T}$ *from the handlebars to the brake mechanism. Again, the direction but not the magnitude of* $\mathbf{T}$ *is changed. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two diagrams labeled (a) and (b), illustrating concepts related to tension.

—

Diagram (a): Anatomy of Muscles and Tendons

– The diagram shows a side view of a human arm focusing on the muscles and tendons.
– Extensor Muscle: Located at the top, indicated by a label.
– Flexor Muscle: Found below the extensor muscle, also labeled.
– Tendons: Connect muscles to the bones, with arrows indicating the direction of tension.
– Tension (T): Represented by red arrows showing the forces at work in the muscles and tendons.

—

Diagram (b): Bicycle Structure

– The diagram displays a side view of a bicycle.
– Main components shown:

– Handlebars: Labeled, with an arrow pointing to the right.
– Frame: Includes the top tube, seat tube, and down tube.
– Pedals and Chainwheel: Situated towards the bottom of the frame.
– Tension (T): Highlighted by red arrows indicating the direction of tension along the frame's components.

—

Both diagrams use red arrows to represent the concept of tension within their respective contexts, emphasizing how forces are at play in both biological and mechanical systems.

Example 4.6

What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in Figure 4.16.

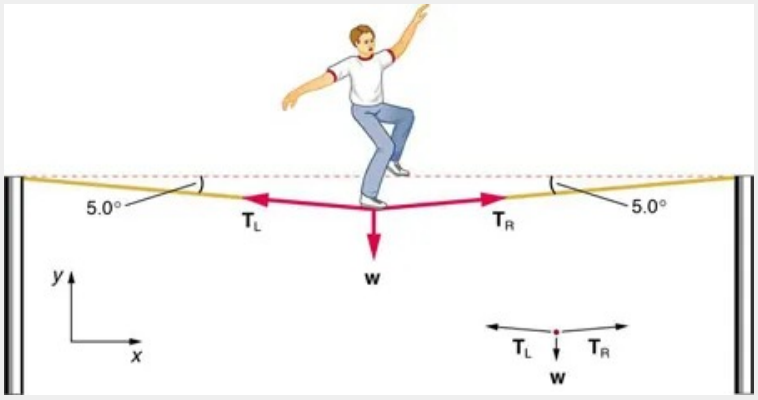


Figure 4.16 The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates the physics of a person balancing on a tightrope. Here's a breakdown of the elements in the image:

– Person on Tightrope: A person is standing in the center of a tightrope, arms outstretched for balance.

– Tightrope: The tightrope is depicted as a straight line with slight angles at both ends where it connects to supports. The angles are labeled as 5.0 degrees.

– Forces:
– There are three forces shown acting on the rope at the point where the person stands: two tensile forces and the weight of the person.

– Left Tension (TL): An arrow is labeled TL pointing diagonally upwards to the left.
– Right Tension (TR): An arrow is labeled TR pointing diagonally upwards to the right.
– Weight (w): An arrow labeled w points directly downwards towards the tightrope.

– Angle and Direction:
– Both sides of the rope are angled upward from the horizontal by 5.0 degrees, with arrows indicating the direction of tension forces.

– Graphical Elements:
– A small coordinate system is shown at the bottom left corner with x and y axes labeled.
– Beside the person balancing, there is a small diagram repeating the three force arrows (TL, TR, and w) to indicate their directions and interactions.

This visual representation helps in understanding the equilibrium and balance involved in tightrope walking, focusing on the tension and gravitational forces at play.

Strategy

As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight $\mathbf{w}$ and the two tensions $\mathbf{T}_{\mathrm{L}}$ (left tension) and $\mathbf{T}_{\mathrm{R}}$ (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions T_{L} and T_{R} must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are T_{L} and T_R. Thus, the magnitude of those forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and

project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the x-axis and the vertical the y-axis.

Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.

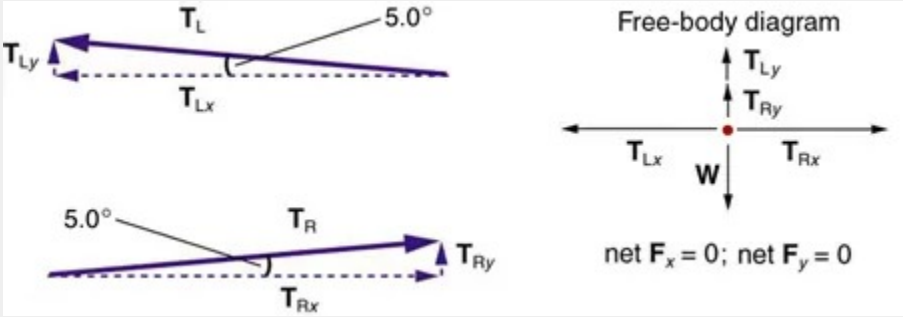


Figure 4.17 When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is stationary. The small angle results in T being much greater than w. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two sets of diagrams related to physics, specifically forces and tension angles.

The left side of the image shows two diagrams with inclined arrows representing forces on a plane tilted at a 5-degree angle. Each diagram includes:

- The top diagram displays the force T_L (left tension) with its vertical component T_{Ly} pointing upwards and its horizontal component T_{Lx} pointing to the right along the plane.
- The bottom diagram shows the force T_R (right tension) with its vertical component T_{Ry} pointing upwards and its horizontal component T_{Rx} pointing to the left along the plane.

The right side of the image features a labeled free-body diagram that summarizes the forces in a single coordinate system:

- An upward pointing arrow labeled T_{Ly}.
- A downward pointing arrow labeled W (weight).
- A rightward pointing arrow labeled T_{Lx}.
- A leftward pointing arrow labeled T_{Rx}.
- An upward pointing arrow labeled T_{Ry}.

Below the free-body diagram, there are notes indicating the conditions:

net $F_X = 0$*; net* $F_Y = 0$, signifying that the net forces in both the x and y directions are zero, implying equilibrium.

Consider the horizontal components of the forces (denoted with a subscript x):

$$F_{\text{net } x} = T_{\text{L}x} - T_{\text{R}x}.$$

4.44

The net external horizontal force $F_{\text{net } x} = 0$, since the person is stationary. Thus,

$$\begin{aligned} F_{\text{net } x} = 0 &= T_{\text{L}x} - T_{\text{R}x} \\ T_{\text{L}x} &= T_{\text{R}x}. \end{aligned}$$

4.45

Now, observe Figure 4.17. You can use trigonometry to determine the magnitude of T_{L} and T_{R}. Notice that:

$$\begin{aligned} \cos\left(5.0^{\text{o}}\right) &= \frac{T_{\text{L}x}}{T_{\text{L}}} \\ T_{\text{L}x} &= T_{\text{L}}\cos\left(5.0^{\text{o}}\right) \\ \cos\left(5.0^{\text{o}}\right) &= \frac{T_{\text{R}x}}{T_{\text{R}}} \\ T_{\text{R}x} &= T_{\text{R}}\cos\left(5.0^{\text{o}}\right). \end{aligned}$$

4.46

Equating $T_{\text{L}x}$ and $T_{\text{R}x}$:

$$T_{\text{L}}\cos\left(5.0^{\text{o}}\right) = T_{\text{R}}\cos\left(5.0^{\text{o}}\right).$$

4.47

Thus,

$$T_{\mathrm{L}} = T_{\mathrm{R}} = T,$$

4.48

as predicted. Now, considering the vertical components (denoted by a subscript y), we can solve for T. Again, since the person is stationary, Newton's second law implies that net $F_y = 0$. Thus, as illustrated in the free-body diagram in Figure 4.17,

$$F_{\mathrm{net}\ y} = T_{\mathrm{L}y} + T_{\mathrm{R}y} - w = 0.$$

4.49

Observing Figure 4.17, we can use trigonometry to determine the relationship between $T_{\mathrm{L}y}$, $T_{\mathrm{R}y}$, and T. As we determined from the analysis in the horizontal direction, $T_{\mathrm{L}} = T_{\mathrm{R}} = T$:

$$\begin{aligned} \sin\left(5.0^{\circ}\right) &= \frac{T_{\mathrm{L}y}}{T_{\mathrm{L}}} \\ T_{\mathrm{L}y} = T_{\mathrm{L}}\sin\left(5.0^{\circ}\right) &= T\sin\left(5.0^{\circ}\right) \\ \sin\left(5.0^{\circ}\right) &= \frac{T_{\mathrm{R}y}}{T_{\mathrm{R}}} \\ T_{\mathrm{R}y} = T_{\mathrm{R}}\sin\left(5.0^{\circ}\right) &= T\sin\left(5.0^{\circ}\right). \end{aligned}$$

4.50

Now, we can substitute the values for $T_{\mathrm{L}y}$ and $T_{\mathrm{R}y}$, into the net force equation in the vertical direction:

$$\begin{aligned} F_{\mathrm{net}\ y} = T_{\mathrm{L}y} + T_{\mathrm{R}y} - w &= 0 \\ T\sin\left(5.0^{\circ}\right) + T\sin\left(5.0^{\circ}\right) - w &= 0 \\ 2T\sin\left(5.0^{\circ}\right) - w &= 0 \\ 2T\sin\left(5.0^{\circ}\right) &= w \end{aligned}$$

4.51

and

$$T = \frac{w}{2\sin\left(5.0^{\circ}\right)} = \frac{\mathrm{mg}}{2\sin\left(5.0^{\circ}\right)},$$

4.52

so that

$$T = \frac{(70.0\ \text{kg})\,(9.80\ \text{m/s}^2)}{2\,(0.0872)},$$

4.53

and the tension is

$$T = 3900\ \text{N}.$$

4.54

Discussion

Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in Figure 4.18. As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the roped related to the weight of the tightrope walker in the following way:

$$T = \frac{w}{2\sin(\theta)}.$$

4.55

We can extend this expression to describe the tension T created when a perpendicular force ($\mathbf{F}_\perp$) is exerted at the middle of a flexible connector:

$$T = \frac{F_\perp}{2\sin(\theta)}.$$

4.56

Note that θ is the angle between the horizontal and the bent connector. In this case, T becomes very large as θ approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., $\theta = 0$ and $\sin\theta = 0$). (See Figure 4.18.)

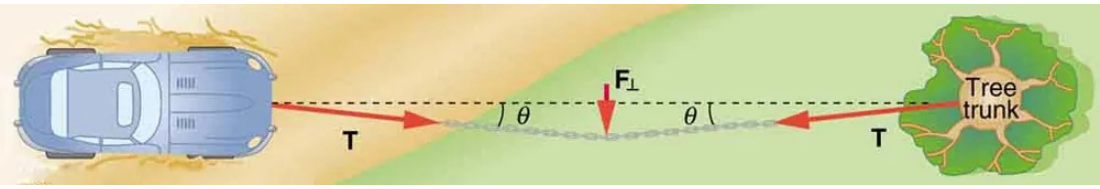


Figure 4.18 We can create a very large tension in the chain by pushing on it perpendicular to its length, as shown. Suppose we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as nearly straight as possible. The tension in the chain is given by $T = \frac{F_\perp}{2\sin(\theta)}$; *since* θ *is small,* T *is very large. This situation is analogous to the tightrope walker shown in , except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where* $\mathbf{F}_\perp$ *is applied. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a diagram of a blue car stuck in sand, connected by a chain to a tree trunk. The car is on the left side, and the tree is on the right. The chain is under tension and forms two equal angles, labeled as θ (theta), with the ground. There are red arrows labeled "T" representing tension forces pointing along the chain from both the car and the tree trunk. At the midpoint of the chain, there is a downward arrow labeled "F_1" representing a force. The ground surrounding the tree trunk is green, while the area around the car is sandy.



Figure 4.19 Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape. Image by Leaflet, Wikimedia Commons License

Extended Topic: Real Forces and Inertial Frames

There is another distinction among forces in addition to the types already mentioned. Some forces are real, whereas others are not. *Real forces* are those that have some physical origin, such as the gravitational pull. Contrastingly, *fictitious forces* are those that arise simply because an observer is in an accelerating frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is heading due north above Earth's northern hemisphere, then to an observer on Earth it will appear to experience a force to the west that has no physical origin. Of course, what is happening here is that Earth is rotating toward the east and moves east under the satellite. In Earth's frame this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). An inertial frame of reference is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On the large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are considered in inertial frames.

All the forces discussed in this section are real forces, but there are a number of other real forces, such as lift and thrust, that are not discussed in this section. They are more specialized, and it is not necessary to discuss every type of force. It is natural, however, to ask where the basic simplicity we seek to find in physics is in the long list of forces. Are some more basic than others? Are some different manifestations of the same underlying force? The answer to both questions is yes, as will be seen in the next (extended) section and in the treatment of modern physics later in the text.

PhET Explorations

Forces in 1 Dimension

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

Click to view content.

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4.6 PROBLEM-SOLVING STRATEGIES

Learning Objectives

By the end of this section, you will be able to:

- Understand and apply a problem-solving procedure to solve problems using Newton's laws of motion.

Success in problem solving is obviously necessary to understand and apply physical principles, not to mention the more immediate need of passing exams. The basics of problem solving, presented earlier in this text, are followed here, but specific strategies useful in applying Newton's laws of motion are emphasized. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, and so the following techniques should reinforce skills you have already begun to develop.

Problem-Solving Strategy for Newton's Laws of Motion

Step 1. As usual, it is first necessary to identify the physical principles involved. *Once it is determined that Newton's laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation*. Such a sketch is shown in Figure 4.20(a). Then, as in Figure 4.20(b), use arrows to represent all forces, label them carefully, and make their lengths and directions correspond to the forces they represent (whenever sufficient information exists).

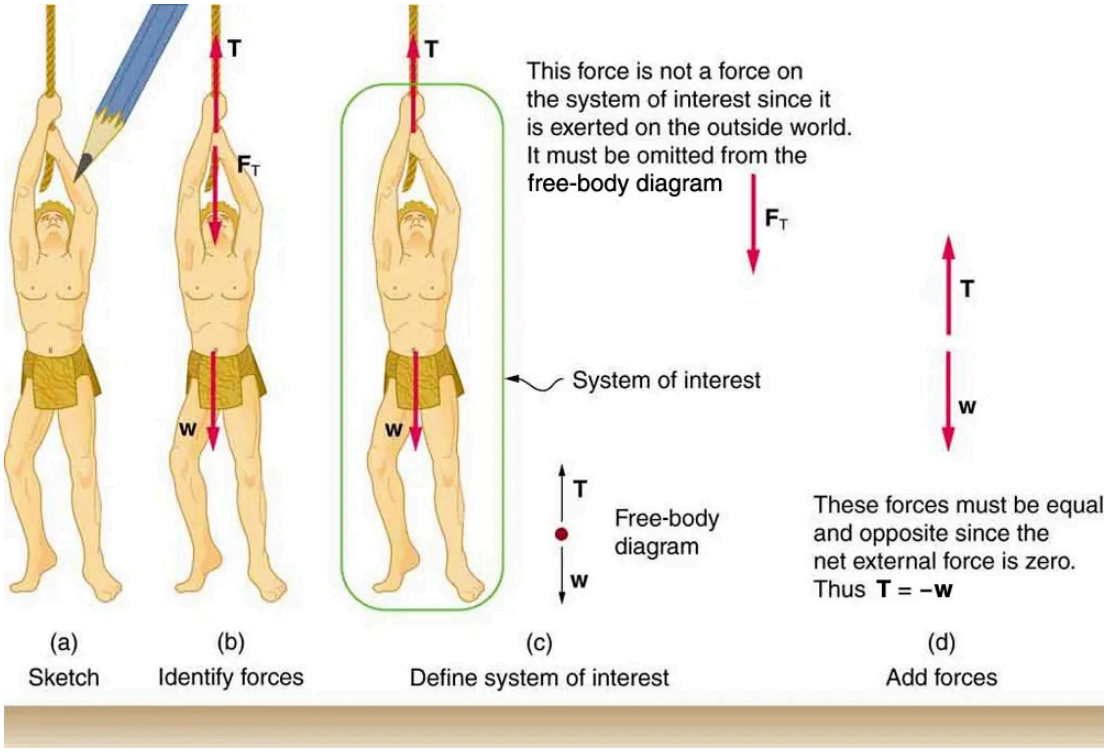


Figure 4.20 (a) A sketch of Tarzan hanging from a vine. (b) Arrows are used to represent all forces. $\mathbf{T}$ *is the tension in the vine above Tarzan,* $\mathbf{F}_{\mathrm{T}}$ *is the force he exerts on the vine, and* $\mathbf{w}$ *is his weight. All other forces, such as the nudge of a breeze, are assumed negligible. (c) Suppose we are given the ape man's mass and asked to find the tension in the vine. We then define the system of interest as shown and draw a free-body diagram.* $\mathbf{F}_{\mathrm{T}}$ *is no longer shown, because it is not a force acting on the system of interest; rather,* $\mathbf{F}_{\mathrm{T}}$ *acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that* $\mathbf{T} = -\mathbf{w}$*, if Tarzan is stationary. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a step-by-step guide demonstrating how to create a free-body diagram for a person hanging by a rope. It comprises four main sections labeled (a) to (d).

(a) Sketch: An illustration of a person hanging vertically from a rope. The person is depicted with arms raised, gripping the rope.

(b) Identify forces: The same illustration as (a), but with two force vectors added. There is an upward force labeled "T" (tension) directed along the rope and a downward force labeled "W" (weight) directed towards the ground.

(c) Define system of interest: The same illustration as (b), but the person is outlined with a green rectangle labeled "System of interest." To the right, a simplified free-body diagram shows a dot representing the person,

with two opposing arrows: "T" pointing upwards and "W" pointing downwards. A note explains that the "FT" force is not considered because it acts on the outside world and is omitted from the diagram.

(d) Add forces: A detailed free-body diagram is shown again, indicating the forces. Text states that the forces "T" and "W" must be equal and opposite since there is no net external force, resulting in the equation "T = -W."

Step 2. Identify what needs to be determined and what is known or can be inferred from the problem as stated. That is, make a list of knowns and unknowns. *Then carefully determine the system of interest*. This decision is a crucial step, since Newton's second law involves only external forces. Once the system of interest has been identified, it becomes possible to determine which forces are external and which are internal, a necessary step to employ Newton's second law. (See Figure 4.20(c).) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated earlier in this chapter, the system of interest depends on what question we need to answer. This choice becomes easier with practice, eventually developing into an almost unconscious process. Skill in clearly defining systems will be beneficial in later chapters as well.

A diagram showing the system of interest and all of the external forces is called a free-body diagram. Only forces are shown on free-body diagrams, not acceleration or velocity. We have drawn several of these in worked examples. Figure 4.20(c) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Step 3. Once a free-body diagram is drawn, *Newton's second law can be applied to solve the problem*. This is done in Figure 4.20(d) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then they add like scalars. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. This is done by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known.

Applying Newton's Second Law

Before you write net force equations, it is critical to determine whether the system is accelerating in a particular direction. If the acceleration is zero in a particular direction, then the net force is zero in that direction. Similarly, if the acceleration is nonzero in a particular direction, then the net force is described by the equation: $F_{\text{net}} = \text{ma}$.

For example, if the system is accelerating in the horizontal direction, but it is not accelerating in the vertical direction, then you will have the following conclusions:

$$F_{\text{net } x} = \text{ma},$$

4.57

$$F_{\text{net } y} = 0.$$

4.58

You will need this information in order to determine unknown forces acting in a system.

Step 4. As always, *check the solution to see whether it is reasonable*. In some cases, this is obvious. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving, and with experience it becomes progressively easier to judge whether an answer is reasonable. Another way to check your solution is to check the units. If you are solving for force and end up with units of m/s, then you have made a mistake.

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4.7 FURTHER APPLICATIONS OF NEWTON'S LAWS OF MOTION

Learning Objectives

By the end of this section, you will be able to:

- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

Example 4.7

Drag Force on a Barge

Suppose two tugboats push on a barge at different angles, as shown in Figure 4.21. The first tugboat exerts a force of 2.7×10^5 N in the *x*-direction, and the second tugboat exerts a force of 3.6×10^5 N in the *y*-direction.

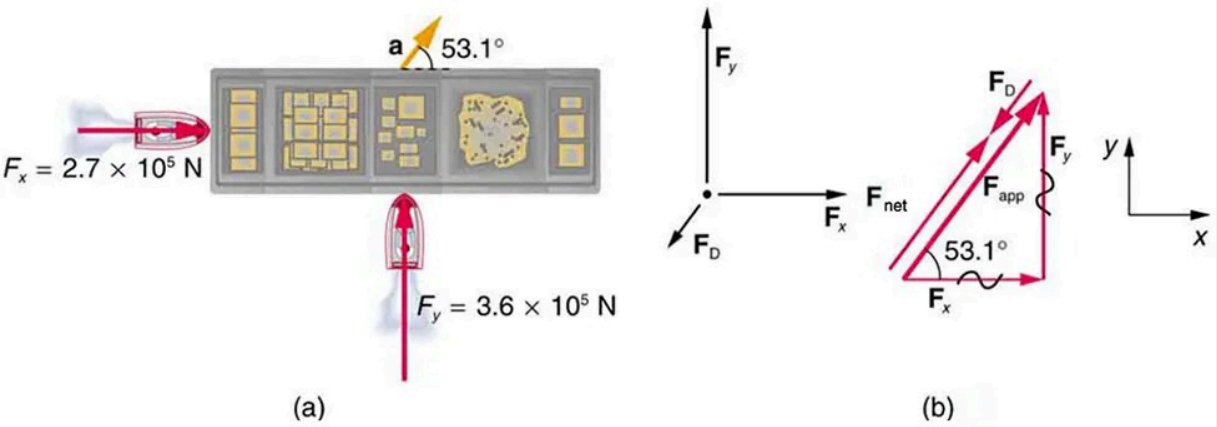


Figure 4.21 (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the x– and y-axes are in the same direction as $\mathbf{F}_x$ *and* $\mathbf{F}_y$*. The problem quickly becomes a one-dimensional problem along the direction of* $\mathbf{F}_{\text{app}}$*, since friction is in the direction opposite to* $\mathbf{F}_{\text{app}}$*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image contains two diagrams labeled (a) and (b).

(a)

– A top view of a barge with a rectangular shape divided into three sections.

– Two forces are acting on it:

– A horizontal force labeled $F_x = 2.7 \times 10^5\ \mathrm{N}$ pointing to the right.
– A vertical force labeled $F_y = 3.6 \times 10^5\ \mathrm{N}$ pointing upwards.
– The forces appear to originate from tugboats represented by icons at the side of the barge.
– An angle a is marked as 53.1° at the top right of the barge, showing a direction related to the resultant force.

(b)
– A vector diagram showing a breakdown of forces in the x and y directions.
– The vectors include:
– F_x pointing right.
– F_y pointing up.
– F_{net} (net force) and F_{app} (applied force), represented diagonally.
– The angle 53.1° is marked between F_x and F_{app}.
– Additional vectors include:
– F_D, showing drag force, opposite to F_{app}.
– The diagram includes x and y axes indicating the coordinate directions.

If the mass of the barge is $5.0 \times 10^6\ \mathrm{kg}$ and its acceleration is observed to be $7.5 \times 10^{-2}\ \mathrm{m/s^2}$ in the direction shown, what is the drag force of the water on the barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)

Strategy

The directions and magnitudes of acceleration and the applied forces are given in Figure 4.21**(a)**. We will define the total force of the tugboats on the barge as $\mathbf{F}_{\mathrm{app}}$ so that:

$$\mathbf{F}_{\mathrm{app}} = \mathbf{F}_x + \mathbf{F}_y$$

4.59

Since the barge is flat bottomed, the drag of the water $\mathbf{F}_{\mathrm{D}}$ will be in the direction opposite to $\mathbf{F}_{\mathrm{app}}$, as shown in the free-body diagram in Figure 4.21(b). The system of interest here is the barge, since the forces on *it* are given as well as its acceleration. Our

strategy is to find the magnitude and direction of the net applied force $\mathbf{F}_{\text{app}}$, and then apply Newton's second law to solve for the drag force $\mathbf{F}_{\text{D}}$.

Solution

Since $\mathbf{F}_x$ and $\mathbf{F}_y$ are perpendicular, the magnitude and direction of $\mathbf{F}_{\text{app}}$ are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

$$F_{\text{app}} = \sqrt{\mathrm{F}_x^2 + \mathrm{F}_y^2}$$
$$F_{\text{app}} = \sqrt{(2.7 \times 10^5 \text{ N})^2 + (3.6 \times 10^5 \text{ N})^2}$$
$$F_{\text{app}} = 4.5 \times 10^5 \text{ N}.$$

4.60

The angle is given by

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$
$$\theta = \tan^{-1}\left(\frac{3.6 \times 10^5 \text{ N}}{2.7 \times 10^5 \text{ N}}\right)$$
$$\theta = 53^{\text{o}} \text{ ,}$$

4.61

which we know, because of Newton's first law, is the same direction as the acceleration. $\mathbf{F}_{\text{D}}$ is in the opposite direction of $\mathbf{F}_{\text{app}}$, since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as $\mathbf{F}_{\text{app}}$, but its magnitude is slightly less than $\mathbf{F}_{\text{app}}$. The problem is now one-dimensional. From Figure 4.21(b), we can see that

$$F_{\text{net}} = F_{\text{app}} - F_{\text{D}}.$$

4.62

But Newton's second law states that

$$F_{\text{net}} = \text{ma}.$$

4.63

Thus,

$$F_{\text{app}} - F_{\text{D}} = \text{ma}.$$

4.64

This can be solved for the magnitude of the drag force of the water F_{D} in terms of known quantities:

$$F_{\text{D}} = F_{\text{app}} - \text{ma}.$$

4.65

Substituting known values gives

$$\text{F}_{\text{D}} = (4.5 \times 10^5\ \text{N}) - (5.0 \times 10^6\ \text{kg})\,(7.5 \times 10^{-2}\ \text{m/s}^2)$$
$$\text{F}_{\text{D}} = 7.5 \times 10^4\ \text{N}.$$

4.66

The direction of $\mathbf{F}_{\text{D}}$ has already been determined to be in the direction opposite to $\mathbf{F}_{\text{app}}$, or at an angle of 53° south of west.

Discussion

The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where F_{D} is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal. Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

Example 4.8

Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in Figure 4.22. Find the tension in each wire, neglecting the masses of the wires.

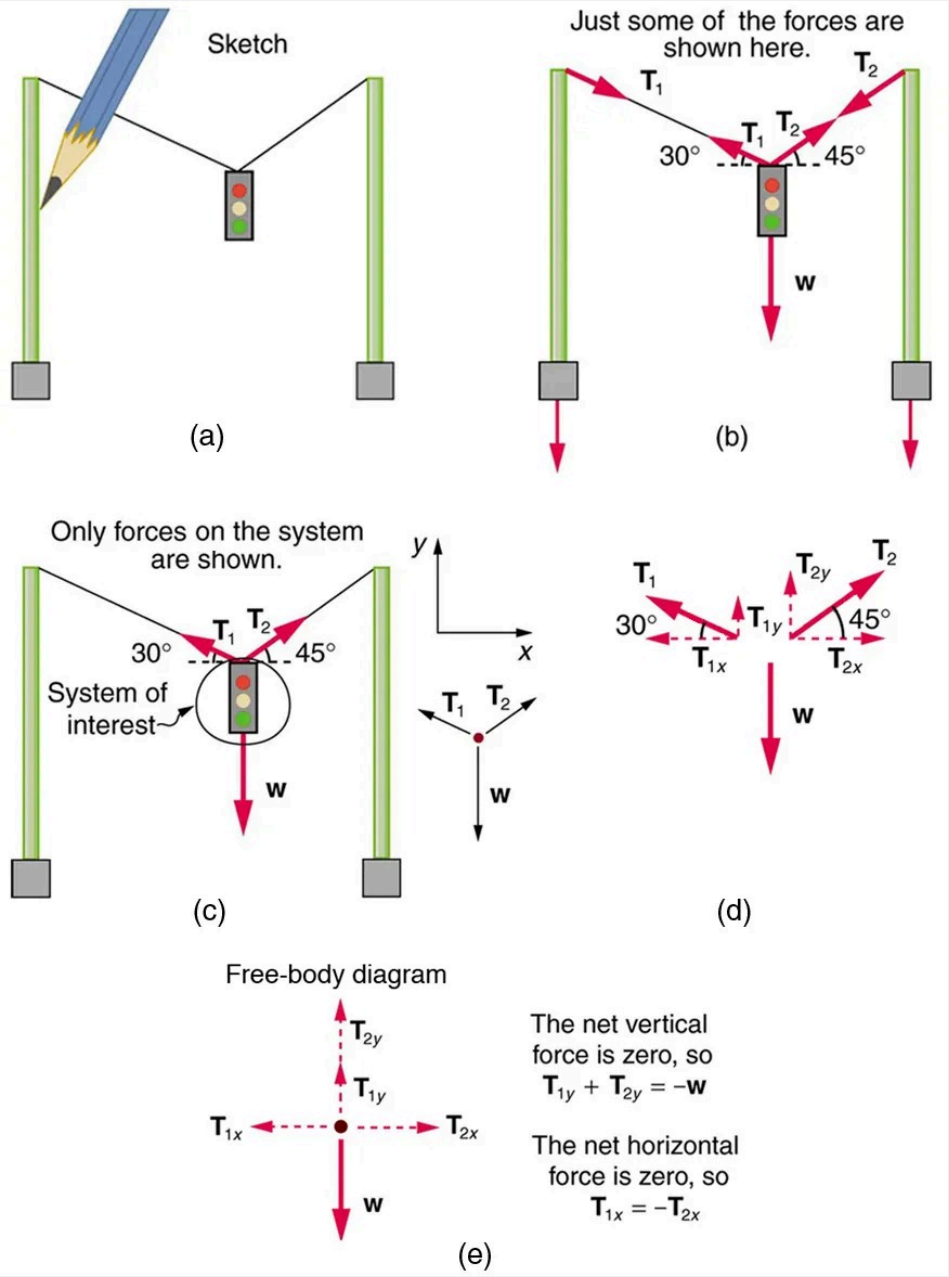


Figure 4.22 A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (y) and horizontal (x) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The

free-body diagram shows the vertical and horizontal forces acting on the traffic light. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of five sub-images labeled (a) to (e), illustrating a physics problem involving forces.

– (a) Sketch: Shows a traffic light hanging at the intersection of two cables between two poles. A pencil is drawing the diagram.

– (b) Just some of the forces: Labels the tensions as T_1 and T_2 at angles 30° and 45° with the horizontal, respectively. The weight w of the traffic light is shown with a downward arrow.

– (c) System of interest: Similar to (b), focusing on the forces on the traffic light. A coordinate system with axes x and y is included, and T_1 and T_2 vectors are drawn separately next to it, pointing away from a dot representing the object.

– (d) Decomposition of forces: Breaks the tensions T_1 and T_2 into horizontal (T_{1x}, T_{2x}) and vertical (T_{1y}, T_{2y}) components with angles.

– (e) Free-body diagram: Summarizes the forces acting on the object, showing $T_{1x}, T_{2x}, T_{1y}, T_{2y}$, and w. Text explains that the net vertical force is zero, $T_{1y} + T_{2y} = -w$, and the net horizontal force is zero, $T_{1x} = -T_{2x}$.

Strategy

The system of interest is the traffic light, and its free-body diagram is shown in Figure 4.22(c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem (T_1 and T_2), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

Solution

First consider the horizontal or *x*-axis:

$$F_{\text{net } x} = T_{2x} - T_{1x} = 0.$$

4.67

Thus, as you might expect,

$$T_{1x} = T_{2x}.$$

4.68

This gives us the following relationship between T_1 and T_2:

$$T_1 \cos(30^\circ) = T_2 \cos(45^\circ).$$

4.69

Thus,

$$T_2 = (1.225)T_1.$$

4.70

Note that T_1 and T_2 are not equal in this case, because the angles on either side are not equal. It is reasonable that T_2 ends up being greater than T_1, because it is exerted more vertically than T_1.

Now consider the force components along the vertical or *y*-axis:

$$F_{\text{net } y} = T_{1y} + T_{2y} - w = 0.$$

4.71

This implies

$$T_{1y} + T_{2y} = w.$$

4.72

Substituting the expressions for the vertical components gives

$$T_1 \sin(30^\circ) + T_2 \sin(45^\circ) = w.$$

4.73

There are two unknowns in this equation, but substituting the expression for T_2 in terms of T_1 reduces this to one equation with one unknown:

$$T_1\,(0.500\,) + (1.225T_1\,)\,(0.707\,) = w = \mathrm{mg},$$

4.74

which yields

$$(1.366)T_1 = (15.0\ \mathrm{kg}\,)\,(9.80\ \mathrm{m/s^2}\,).$$

4.75

Solving this last equation gives the magnitude of T_1 to be

$$T_1 = 108\ \mathrm{N}.$$

4.76

Finally, the magnitude of T_2 is determined using the relationship between them, T_2 = 1.225 T_1, found above. Thus we obtain

$$T_2 = 132\ \mathrm{N}.$$

4.77

Discussion

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

Example 4.9

What Does the Bathroom Scale Read in an Elevator?

Figure 4.23 shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of $1.20\ \mathrm{m/s^2}$, and (b) if the elevator moves upward at a constant speed of 1 m/s.

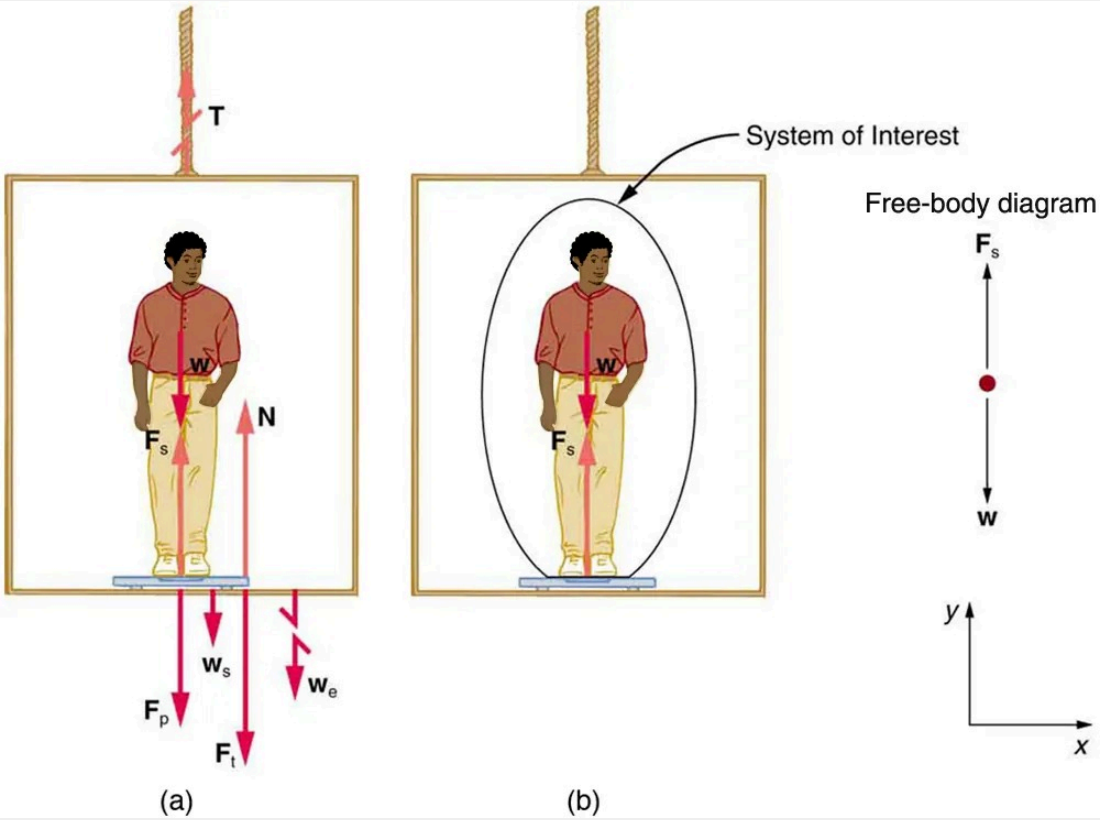


Figure 4.23 (a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. $\mathbf{T}$ *is the upward force on the elevator,* $\mathbf{w}_s$ *is the weight of the scale,* $\mathbf{w}_e$ *is the weight of the elevator,* $\mathbf{F}_s$ *is the force of the scale on the person,* $\mathbf{F}_p$ *is the force of the person on the scale,* $\mathbf{F}_t$ *is the force of the scale on the floor of the elevator, and* $\mathbf{N}$ *is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is composed of two main panels labeled (a) and (b), along with a free-body diagram.

Panel (a):

– Shows a person standing inside a box suspended by a rope.

– Forces acting on the person and the box are indicated by arrows labeled with letters.
– The upward arrow on the person represents the support force, labeled "N."
– Two downward arrows on the person: one for the gravitational force labeled "W" and the other for the force due to the tension "Fs."
– Additional forces on the box include:
– A downward arrow labeled "Fp" for the force due to the person.
– A downward arrow labeled "Ws" for the weight of the system.
– A downward arrow labeled "We" for the weight of the external force.
– An upward arrow labeled "Ft" for the tension in the rope.

Panel (b):
– Shows the same person in the box but now labeled as a "System of Interest."
– The forces acting on the person inside the system are the same as in panel (a) with labels "W" and "Fs."

Free-Body Diagram:
– Depicts a simple diagram with two arrows:
– One pointing upward labeled "Fs."
– One pointing downward labeled "W."
– The axes are labeled "y" for the vertical axis and "x" for the horizontal axis.

Each component illustrates the forces involved in the system, with arrows showing direction and labels indicating force types.

Strategy

If the scale is accurate, its reading will equal F_{p}, the magnitude of the force the person exerts downward on it. Figure 4.23(a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn as in Figure 4.23(b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight $\mathbf{w}$ and the upward force of the scale $\mathbf{F}_{\mathrm{s}}$. According to Newton's third law $\mathbf{F}_{\mathrm{p}}$ and $\mathbf{F}_{\mathrm{s}}$ are equal

in magnitude and opposite in direction, so that we need to find F_{s} in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

$$F_{\mathrm{net}} = \mathrm{ma}.$$

4.78

From the free-body diagram we see that $F_{\mathrm{net}} = F_{\mathrm{s}} - w$, so that

$$F_{\mathrm{s}} - w = \mathrm{ma}.$$

4.79

Solving for F_{s} gives an equation with only one unknown:

$$F_{\mathrm{s}} = \mathrm{ma} + w,$$

4.80

or, because $w = mg$, simply

$$F_{\mathrm{s}} = \mathrm{ma} + \mathrm{mg}.$$

4.81

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.

Solution for (a)

In this part of the problem, $a = 1.20\ \mathrm{m/s^2}$, so that

$$F_{\mathrm{s}} = (75.0\ \mathrm{kg})\,(1.20\ \mathrm{m/s^2}) + (75.0\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2}),$$

4.82

yielding

$$F_{\mathrm{s}} = 825\ \mathrm{N}.$$

4.83

Discussion for (a)

This is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

$$\begin{aligned} F_{\text{net}} &= \text{ma} = 0 = F_{\text{s}} - w \\ F_{\text{s}} &= w = \text{mg} \\ F_{\text{s}} &= (75.0\ \text{kg})\,(9.80\ \text{m/s}^2) \\ F_{\text{s}} &= 735\ \text{N}. \end{aligned}$$

4.84

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

Solution for (b)

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \dfrac{\Delta v}{\Delta t}$, and $\Delta v = 0$.

Thus,

$$F_{\text{s}} = \text{ma} + \text{mg} = 0 + \text{mg}.$$

4.85

Now

$$F_{\text{s}} = (75.0\ \text{kg})\,(9.80\ \text{m/s}^2),$$

4.86

which gives

$$F_{\text{s}} = 735\ \text{N}.$$

4.87

Discussion for (b)

The scale reading is 735 N, which equals the person's weight. This will be the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, a is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person's weight. If the elevator is in free-fall and accelerating downward at g, then the scale reading will be zero and the person will *appear* to be weightless.

Integrating Concepts: Newton's Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

Problem-Solving Strategy

Step 1. *Identify which physical principles are involved*. Listing the givens and the quantities to be calculated will allow you to identify the principles involved.

Step 2. *Solve the problem using strategies outlined in the text*. If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

Example 4.10

What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.

Strategy

To solve an *integrated concept problem*, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example considers *acceleration* along a straight line. This is a topic of *kinematics*. Part (b) deals with *force*, a topic of *dynamics* found in this chapter.

The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

Solution for (a)

We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is $\Delta v = 8.00 m/s$. We are given the elapsed time, and so $\Delta t = 2.50 s$. The unknown is acceleration, which can be found from its definition:

$$a = \frac{\Delta v}{\Delta t}.$$

4.88

Substituting the known values yields

$$a = \frac{8.00 m/s}{2.50 s}$$
$$= 3.20 \text{ m/s}^2.$$

4.89

Discussion for (a)

This is an attainable acceleration for an athlete in good condition.

Solution for (b)

Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player's acceleration and are given his mass, we can use Newton's second law to find the force exerted. That is,

$$F_{\text{net}} = \text{ma}.$$

4.90

Substituting the known values of m and a gives

$$F_{\text{net}} = (70.0 \text{ kg}) (3.20 \text{ m/s}^2)$$
$$= 224 \text{ N}.$$

4.91

Discussion for (b)

This is about 50 pounds, a reasonable average force.

This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these

techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

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4.8 EXTENDED TOPIC: THE FOUR BASIC FORCES—AN INTRODUCTION

Learning Objectives

By the end of this section, you will be able to:

- Understand the four basic forces that underlie the processes in nature.

One of the most remarkable simplifications in physics is that only four distinct forces account for all known phenomena. In fact, nearly all of the forces we experience directly are due to only one basic force, called the electromagnetic force. (The gravitational force is the only force we experience directly that is not electromagnetic.) This is a tremendous simplification of the myriad of *apparently* different forces we can list, only a few of which were discussed in the previous section. As we will see, the basic forces are all thought to act through the exchange of microscopic carrier particles, and the characteristics of the basic forces are determined by the types of particles exchanged. Action at a distance, such as the gravitational force of Earth on the Moon, is explained by the existence of a force field rather than by "physical contact."

The *four basic forces* are the gravitational force, the electromagnetic force, the weak nuclear force, and the strong nuclear force. Their properties are summarized in Table 4.1. Since the weak and strong nuclear forces act over an extremely short range, the size of a nucleus or less, we do not experience them directly, although they are crucial to the very structure of matter. These forces determine which nuclei are stable and which decay, and they are the basis of the release of energy in certain nuclear reactions. Nuclear forces determine not only the stability of nuclei, but also the relative abundance of elements in nature. The properties of the nucleus of an atom determine the number of electrons it has and, thus, indirectly determine the chemistry of the atom. More will be said of all of these topics in later chapters.

Concept Connections: The Four Basic Forces

The four basic forces will be encountered in more detail as you progress through the text. The gravitational force is defined in Uniform Circular Motion and Gravitation, electric force in Electric Charge and Electric Field, and magnetic force in Magnetism. On a macroscopic scale, electromagnetism and gravity are the basis for all forces. The nuclear forces are vital to the substructure of matter, but they are not directly experienced on the macroscopic scale.

Table 4.1 Properties of the Four Basic Forces

Force	Approximate Relative Strengths	Range	Attraction/Repulsion	Carrier Particle
Gravitational	10^{-38}	∞	attractive only	Graviton[1]
Electromagnetic	10^{-2}	∞	attractive and repulsive	Photon
Weak nuclear	10^{-6}	10^{-18}m	attractive and repulsive	$\text{W}^+, \text{W}^-, \text{Z}^0$
Strong nuclear	1	∞	attractive and repulsive	gluons

The gravitational force is surprisingly weak—it is only because gravity is always attractive that we notice it at all. Our weight is the gravitational force due to the *entire* Earth acting on us. On the very large scale, as in astronomical systems, the gravitational force is the dominant force determining the motions of moons, planets, stars, and galaxies. The gravitational force also affects the nature of space and time. As we shall see later in the study of general relativity, space is curved in the vicinity of very massive bodies, such as the Sun, and time actually slows down near massive bodies.

Electromagnetic forces can be either attractive or repulsive. They are long-range forces, which act over extremely large distances, and they nearly cancel for macroscopic objects. (Remember that it is the *net* external

1. The graviton is a proposed particle, though it has not yet been observed by scientists. See the discussion of gravitational waves later in this section. The particles W^+, W^-, and Z^0 are called vector bosons; these were predicted by theory and first observed in 1983. There are eight types of gluons proposed by scientists, and their existence is indicated by meson exchange in the nuclei of atoms

force that is important.) If they did not cancel, electromagnetic forces would completely overwhelm the gravitational force. The electromagnetic force is a combination of electrical forces (such as those that cause static electricity) and magnetic forces (such as those that affect a compass needle). These two forces were thought to be quite distinct until early in the 19th century, when scientists began to discover that they are different manifestations of the same force. This discovery is a classical case of the *unification of forces*. Similarly, friction, tension, and all of the other classes of forces we experience directly (except gravity, of course) are due to electromagnetic interactions of atoms and molecules. It is still convenient to consider these forces separately in specific applications, however, because of the ways they manifest themselves.

Concept Connections: Unifying Forces

Attempts to unify the four basic forces are discussed in relation to elementary particles later in this text. By "unify" we mean finding connections between the forces that show that they are different manifestations of a single force. Even if such unification is achieved, the forces will retain their separate characteristics on the macroscopic scale and may be identical only under extreme conditions such as those existing in the early universe.

Physicists are now exploring whether the four basic forces are in some way related. Attempts to unify all forces into one come under the rubric of Grand Unified Theories (GUTs), with which there has been some success in recent years. It is now known that under conditions of extremely high density and temperature, such as existed in the early universe, the electromagnetic and weak nuclear forces are indistinguishable. They can now be considered to be different manifestations of one force, called the *electroweak* force. So the list of four has been reduced in a sense to only three. Further progress in unifying all forces is proving difficult—especially the inclusion of the gravitational force, which has the special characteristics of affecting the space and time in which the other forces exist.

While the unification of forces will not affect how we discuss forces in this text, it is fascinating that such underlying simplicity exists in the face of the overt complexity of the universe. There is no reason that nature must be simple—it simply is.

Action at a Distance: Concept of a Field

All forces act at a distance. This is obvious for the gravitational force. Earth and the Moon, for example, interact without coming into contact. It is also true for all other forces. Friction, for example, is an electromagnetic force between atoms that may not actually touch. What is it that carries forces between objects? One way to answer this question is to imagine that a force field surrounds whatever object creates the force. A second object (often called a *test object*) placed in this field will experience a force that is a function of location and other variables. The field itself is the "thing" that carries the force from one object to another. The field is defined so as to be a characteristic of the object creating it; the field does not depend on the test object placed in it. Earth's gravitational field, for example, is a function of the mass of Earth and the distance from its center, independent of the presence of other masses. The concept of a field is useful because equations can be written for force fields surrounding objects (for gravity, this yields $w = \mathrm{mg}$ at Earth's surface), and motions can be calculated from these equations. (See Figure 4.24.)

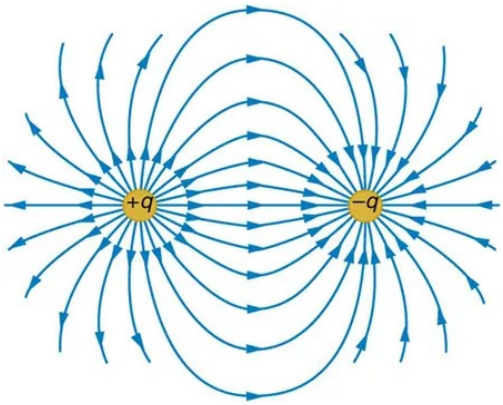


Figure 4.24 The electric force field between a positively charged particle and a negatively charged particle. When a positive test charge is placed in the field, the charge will experience a force in the direction of the force field lines. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts an electric field diagram illustrating the interaction between two point charges. On the left side, there is a positive charge labeled "+q" represented by a yellow circle. On the right side, there is a negative charge labeled "-q" also represented by a yellow circle. Blue lines with arrows represent the electric field lines originating from the positive charge and terminating at the negative charge. The lines are denser near the charges, indicating stronger electric fields. The arrows on the lines point away from the positive charge and toward the negative charge, illustrating the direction of the electric field. This arrangement shows how opposite charges attract each other.

Concept Connections: Force Fields

The concept of a *force field* is also used in connection with electric charge and is presented in Electric Charge and Electric Field. It is also a useful idea for all the basic forces. Fields help us to visualize forces and how they are transmitted, as well as to describe them with precision and to link forces with subatomic carrier particles.

The field concept has been applied very successfully; we can calculate motions and describe nature to high precision using field equations. As useful as the field concept is, however, it leaves unanswered the question of what carries the force. It has been proposed in recent decades, starting in 1935 with Hideki Yukawa's (1907–1981) work on the strong nuclear force, that all forces are transmitted by the exchange of elementary particles. We can visualize particle exchange as analogous to macroscopic phenomena such as two people passing a basketball back and forth, thereby exerting a repulsive force without touching one another. (See Figure 4.25.)

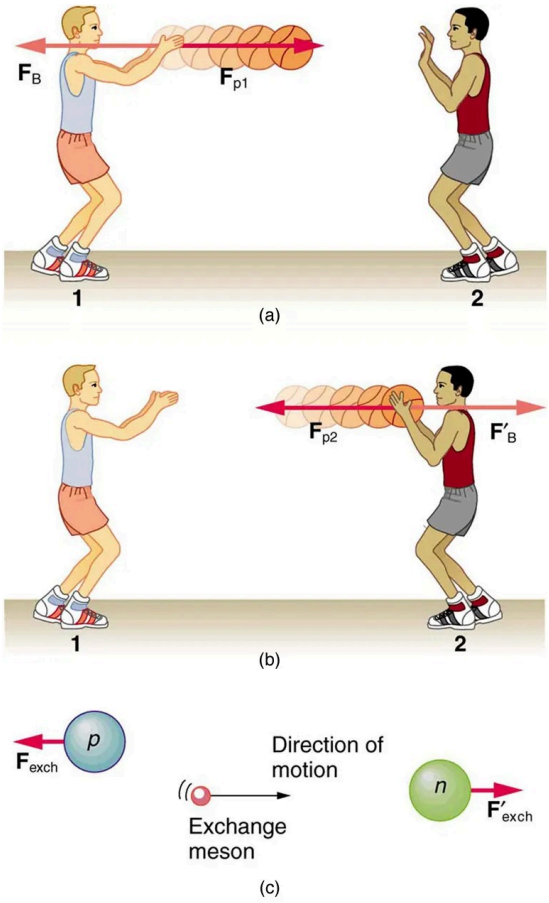


Figure 4.25 The exchange of masses resulting in repulsive forces. (a) The person throwing the basketball exerts a force $\mathbf{F}_{p1}$ *on it toward the other person and feels a reaction force* $\mathbf{F}_{B}$ *away from the second person. (b) The person catching the basketball exerts a force* $\mathbf{F}_{p2}$ *on it to stop the ball and feels a reaction force* $\mathbf{F}'_{B}$ *away from the first person. (c) The analogous exchange of a meson between a proton and a neutron carries the strong nuclear forces* $\mathbf{F}_{exch}$ *and* $\mathbf{F}'_{exch}$ *between them. An attractive force can also be exerted by the exchange of a mass—if person 2 pulled the basketball away from the first person as he tried to retain it, then the force between them would be attractive. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of three parts labeled (a), (b), and (c), illustrating a conceptual model involving forces and motion.

– Figure (a): Two individuals are shown facing each other. The person on the left, labeled "1," is throwing a ball towards the person on the right, labeled "2." The ball is shown in multiple positions along its path, indicating its motion. Arrows labeled "F_B" and "F_{P1}" point from person 1 towards person 2, representing the force applied by the ball in motion towards person 2.

– Figure (b): The same two individuals are shown facing each other, with the ball returning from the right to the left. The person on the right, labeled "2," exerts a force labeled "F'_B" while the ball exerts a force labeled "F_{P2}" towards the left.

– Figure (c): This part illustrates a physics concept with symbolic representations. A small sphere labeled "p" on the left has an arrow labeled "F_{exch}" pointing left, and a sphere labeled "n" on the right has an arrow labeled "F'_{exch}" pointing right. Between the two spheres, there's a smaller circle labeled "Exchange meson" with an arrow indicating the "Direction of motion" moving from left to right.

Overall, the image demonstrates the exchange of forces between two individuals throwing a ball back and forth and connects this concept to particle physics through the depiction of exchange mesons.

This idea of particle exchange deepens rather than contradicts field concepts. It is more satisfying philosophically to think of something physical actually moving between objects acting at a distance. Table 4.1 lists the exchange or carrier particles, both observed and proposed, that carry the four forces. But the real fruit of the particle-exchange proposal is that searches for Yukawa's proposed particle found it *and* a number of others that were completely unexpected, stimulating yet more research. All of this research eventually led to the proposal of quarks as the underlying substructure of matter, which is a basic tenet of GUTs. If successful, these theories will explain not only forces, but also the structure of matter itself. Yet physics is an experimental science, so the test of these theories must lie in the domain of the real world. As of this writing, scientists at the CERN laboratory in Switzerland are starting to test these theories using the world's largest particle accelerator: the Large Hadron Collider. This accelerator (27 km in circumference) allows two high-energy proton beams, traveling in opposite directions, to collide. An energy of 14 trillion electron volts will be available. It is anticipated that some new particles, possibly force carrier particles, will be found. (See Figure 4.26.) One of the force carriers of high interest that researchers hope to detect is the Higgs boson. The observation of its properties might tell us why different particles have different masses.

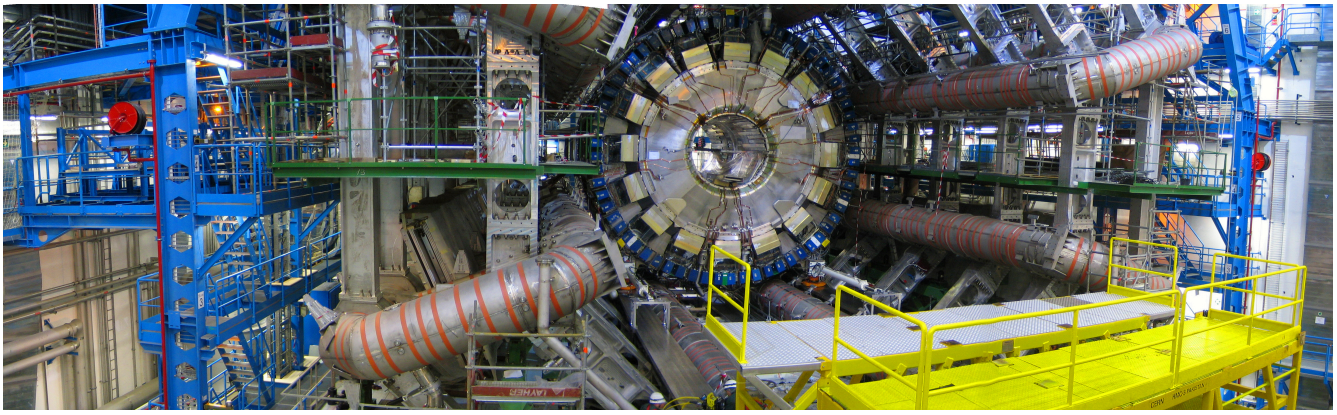


Figure 4.26 The world's largest particle accelerator spans the border between Switzerland and France. Two beams, traveling in opposite directions close to the speed of light, collide in a tube similar to the central tube shown here. External magnets determine the beam's path. Special detectors will analyze particles created in these collisions. Questions as broad as what is the origin of mass and what was matter like the first few seconds of our universe will be explored. This accelerator began preliminary operation in 2008. Image by Freanki~commonswiki assumed (based on copyright claims)., Wikimedia Commons License

Tiny particles also have wave-like behavior, something we will explore more in a later chapter. To better understand force-carrier particles from another perspective, let us consider gravity. The search for gravitational waves has been going on for a number of years. Over 100 years ago, Einstein predicted the existence of these waves as part of his general theory of relativity. Gravitational waves are created during the collision of massive stars, in black holes, or in supernova explosions—like shock waves. These gravitational waves will travel through space from such sites much like a pebble dropped into a pond sends out ripples—except these waves move at the speed of light. A detector apparatus has been built in the U.S., consisting of two large installations nearly 3000 km apart—one in Washington state and one in Louisiana! The facility is called the Laser Interferometer Gravitational-Wave Observatory (LIGO). Each installation is designed to use optical lasers to examine any slight shift in the relative positions of two masses due to the effect of gravity waves. The two sites allow simultaneous measurements of these small effects to be separated from other natural phenomena, such as earthquakes. Initial operation of the detectors began in 2002, and work is proceeding on increasing their sensitivity. Similar installations have been built in Italy (VIRGO), Germany (GEO600), and Japan (TAMA300) to provide a worldwide network of gravitational wave detectors.

In September, 2015, LIGO fulfilled its promise and helped prove Einstein's predictions. The system detected the first gravitational waves arising from the merger of two black holes—one 29 times the mass of our Sun and the other 36 times the mass of our Sun—that occurred 1.3 billion years ago. About 3 times the mass of the Sun was converted into gravitational waves in a fraction of a second—with a peak power output about 50 times that of the whole visible universe. Due to the 7 millisecond delay in detection, researchers established that the merger occurred on the southern hemisphere side of Earth. Since then, LIGO and VIRGO have

combined to detect about a dozen similar events, with better and more precise measurements. Waves from neutron star mergers and different-sized black holes have deepened our understanding of these objects and their impact on the universe.

International collaboration in this area is moving into space with the joint EU/US project LISA (Laser Interferometer Space Antenna). Earthquakes and other Earthly noises will be no problem for these monitoring spacecraft. LISA will complement LIGO by looking at much more massive black holes through the observation of gravitational-wave sources emitting much larger wavelengths. Three satellites will be placed in space above Earth in an equilateral triangle (with 5,000,000-km sides) (Figure 4.27). The system will measure the relative positions of each satellite to detect passing gravitational waves. Accuracy to within 10% of the size of an atom will be needed to detect any waves. The launch of this project will likely be in the 2030s.

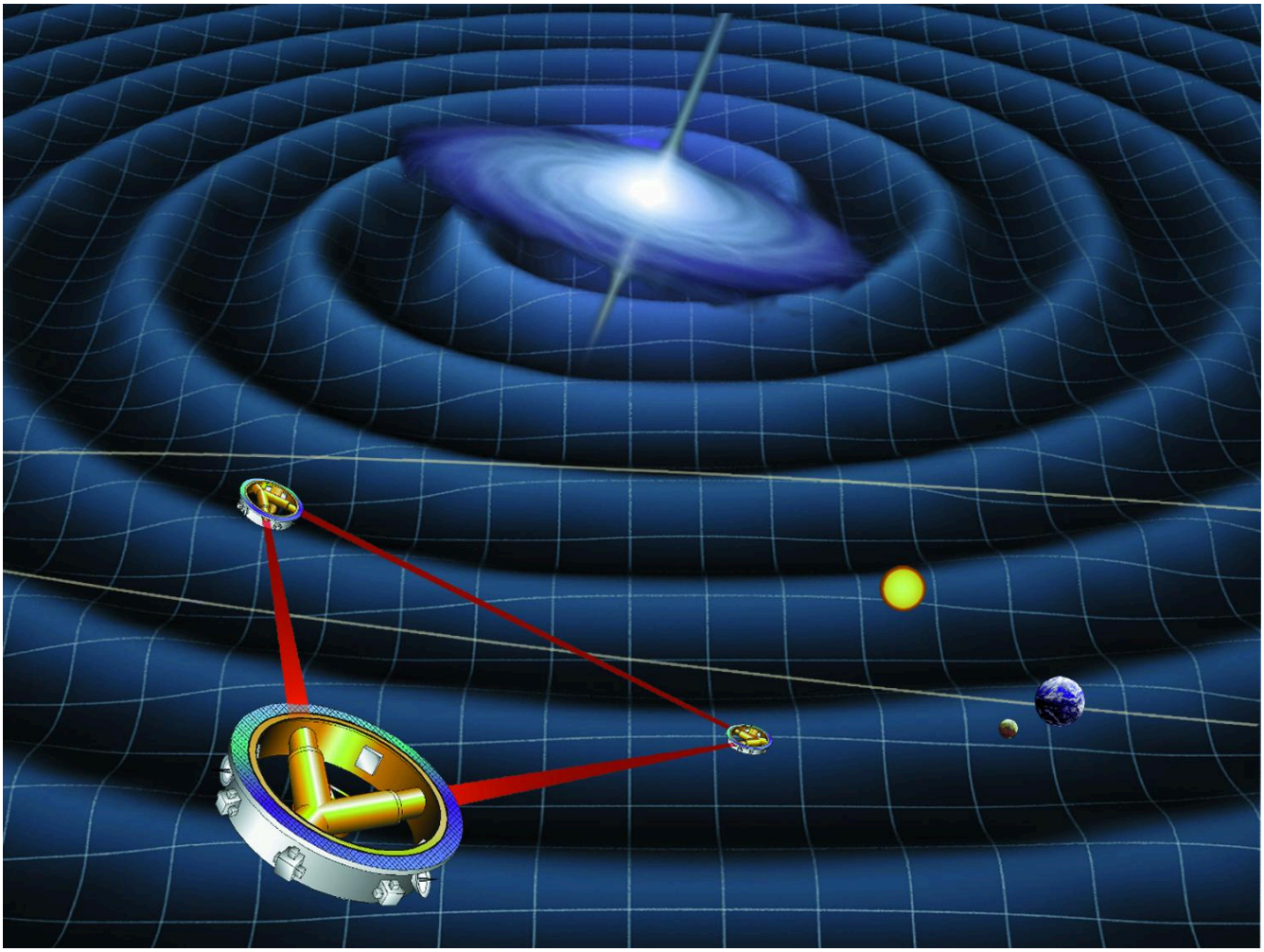


Figure 4.27 Space-based future experiments for the measurement of gravitational waves. Shown here is a drawing of LISA's orbit. Each satellite of LISA will consist of a laser source and a mass. The lasers will transmit a signal to measure the distance between each satellite's test mass. The relative motion of these masses will provide information about passing gravitational waves.
Image by NASA, Wikimedia Commons License

As you can see above, some of the most groundbreaking developments in physics are made with a relatively long gap from theoretical prediction to experimental detection. This pattern continues the process of science from its earliest days, where early thinkers and researchers made discoveries that only led to more questions. Einstein was unique in many ways, but he was not unique in that later scientists, building on his and each other's work, would prove his theories. Evidence for black holes became more and more concrete as scientists developed new and better ways to look for them. Some of the most prominent have been Roger Penrose, who developed new mathematical models related to black holes, as well as Reinhard Genzel and Andrea Ghez, who independently used telescope observations to identify a region of our galaxy where a massive unseen gravity source (4 million times the size of our Sun) was pulling on stars. And soon after, collaborators on the Event Horizon Telescope project produced the first actual image of a black hole.

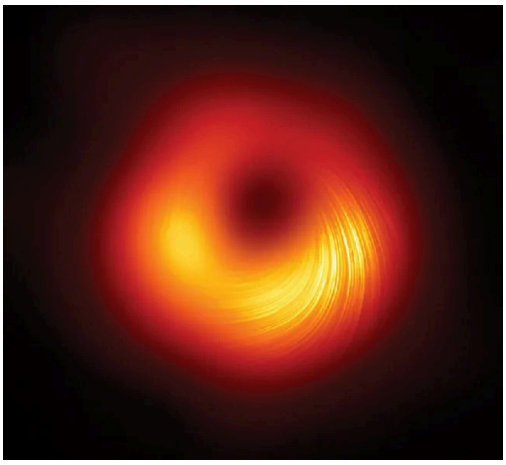


Figure 4.28 This updated image of the supermassive black hole at the center of the M87 galaxy shows the polarization created by the powerful magnetic field that surrounds the object. The image was created by the Event Horizon Telescope, which combines data from telescopes around the world. The project involves over 200 researchers and produced the first image of a black hole in 2019. Image from OpenStax College Physics 2e, CC-BY 4.0

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4.9 FRICTION

Learning Objectives

By the end of this section, you will be able to:

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

Friction is a force that is around us all the time that opposes relative motion between surfaces in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

Friction

Friction is a force that opposes relative motion between surfaces in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between surfaces and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two surfaces are in contact and moving relative to one another, then the friction between them is called kinetic

friction. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, static friction can act between them; the static friction is usually greater than the kinetic friction between the surfaces.

Kinetic Friction

If two surfaces are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

Figure 4.29 is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

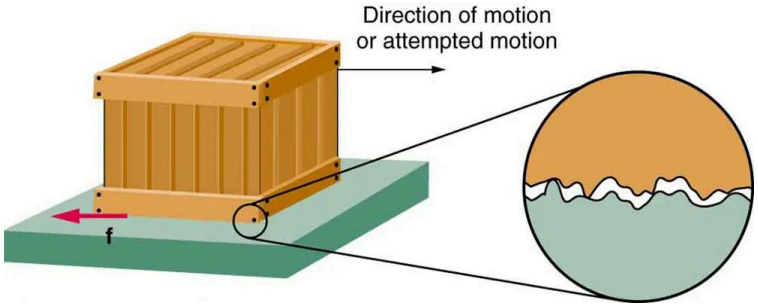


Figure 4.29 Frictional forces, such as $\boldsymbol{f}$*, always oppose motion or attempted motion between surfaces in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows a diagram illustrating the concept of friction. There is a rectangular wooden crate resting on a flat, green surface. An arrow labeled “Direction of motion or attempted motion” points to the right, indicating the intended or actual movement of the crate. Another arrow labeled “f” points to the left, representing the force of friction opposing the motion.

On the right side of the image, there is an enlarged circular view that zooms in on the contact area between the crate and the surface. This close-up shows a rough, uneven interaction between the surfaces, highlighting the microscopic irregularities that contribute to friction.

The diagram helps to explain how friction works by showing both the direction of the applied force and the opposing frictional force, as well as the surface roughness involved.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the magnitude of static friction $\mathbf{f}_{\mathrm{s}}$ is

$$f_s \leq \mu_s N,$$
5.1

where μ_s is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

Magnitude of Static Friction

Magnitude of static friction f_s is

$$f_s \leq \mu_s N,$$

4.93

where μ_s is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\mu_s N$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{s(max)}$, the object will move. Thus

$$f_{s(max)} = \mu_s N.$$

4.94

Once an object is moving, the magnitude of kinetic friction $\mathbf{f}_k$ is given by

$$f_k = \mu_k N,$$

4.95

where μ_k is the coefficient of kinetic friction. A system in which $f_k = \mu_k N$is described as a system in which *friction behaves simply*.

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_k is given by

$$f_k = \mu_k N,$$

4.96

where μ_k is the coefficient of kinetic friction.

As seen in Table 4.2, the coefficients of kinetic friction are less than their static counterparts. That values of μ in Table 4.2 are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

System	Static friction μ_s	Kinetic friction μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.04	0.02

Table 4.2

Coefficients of Static and Kinetic FrictionThe equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg = (100\ \text{kg})\,(9.80\text{m/s}^2) = 980\ \text{N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than

$f_{\text{s(max)}} = \mu_{\text{s}} N = (0.45)\,(980\text{N}) = 440\text{N}$to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N
$(f_{\text{k}} = \mu_{\text{k}} N = (0.30)\,(980\text{N}) = 290\text{N})$
would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unitless quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

Take-Home Experiment

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table, simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (Figure 4.30). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.



Figure 4.30 Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op X-rays of the right knee joint replacement. Image from OpenStax College Physics 2e, CC-BY 4.0

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to move freely over the skin.

Example 4.11

Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force
N as $f_{\mathrm{k}} = \mu_{\mathrm{k}} N$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in Figure 4.32.)

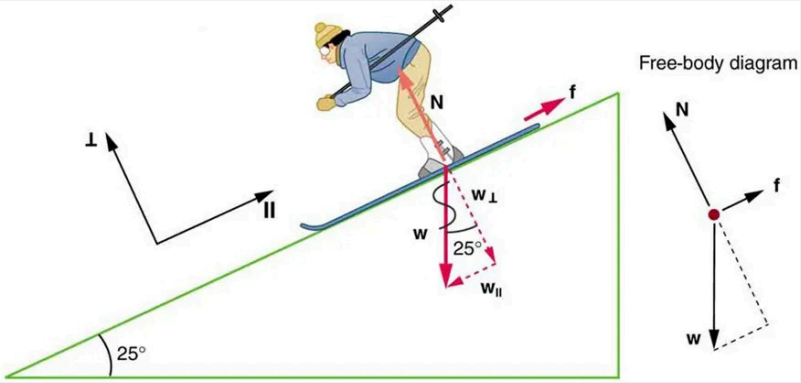


Figure 4.32 The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). $\mathbf{N}$ *(the normal force) is perpendicular to the slope, and* $\mathbf{f}$ *(the friction) is parallel to the slope, but* $\mathbf{w}$ *(the skier's weight) has components along both axes, namely* $\mathbf{w}_{\perp}$ *and* $\mathbf{W}_{//}$. N *is equal in magnitude to* $w_{\perp}$*, so there is no motion perpendicular to the slope. However,* f *is less than* $W_{//}$ *in magnitude, so there is acceleration down the slope* Image *from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a skier on a slope inclined at 25 degrees, along with a free-body diagram illustrating the forces acting on the skier.

– Skier Illustration:
– The skier is leaning forward, using ski poles for stability, and is positioned on skis sliding down the slope.
– The slope is represented by a green line inclined at an angle of 25 degrees.

– Forces on the Skier:
– N: The normal force is shown perpendicular to the slope, indicated by an arrow labeled "N."
– f: The frictional force is depicted as an arrow pointing uphill along the slope, labeled "f."
– w (Weight): The weight of the skier is directed vertically downward, labeled as "w."
– The weight is decomposed into two components:

– w⊥: Perpendicular component of weight, directed into the slope.
– w ‖ : Parallel component of weight, directed down the slope.

– **Axes**:
– There are two sets of axes marked on the image:
– ⊥: Perpendicular to the slope.
– ‖ : Parallel to the slope.

– Free-Body Diagram:
– Located to the right of the main skier illustration.
– It shows the same forces, with arrows indicating:
– Normal force (N) directed outwards and perpendicular.
– Frictional force (f) directed uphill.
– Weight (w) directed vertically downward.
– The diagram includes dashed lines denoting the components of the weight.

This setup illustrates the dynamics of the skier as they move down an inclined plane, emphasizing the interplay of gravitational, normal, and frictional forces.

That is,

$$N = w_{\perp} = w\cos 25^{\circ} \ = \mathrm{mg}\cos 25^{\circ} \ .$$

4.97

Substituting this into our expression for kinetic friction, we get

$$f_{\mathrm{k}} = \mu_{\mathrm{k}}\mathrm{mg}\cos 25^{\circ} \ ,$$

4.98

which can now be solved for the coefficient of kinetic friction μ_{k}.

Solution

Solving for μ_k gives

$$\mu_{\mathrm{k}} = \frac{f_{\mathrm{k}}}{N} = \frac{f_{\mathrm{k}}}{w\cos 25^{\circ}} = \frac{f_{\mathrm{k}}}{\mathrm{mg}\cos 25^{\circ} \ .}$$

4.99

Substituting known values on the right-hand side of the equation,

$$\mu_{\text{k}} = \frac{45.0\ \text{N}}{(62\ \text{kg})\,(9.80\ \text{m/s}^2)\,(0.906)} = 0.082.$$

4.100

Discussion

This result is a little smaller than the coefficient listed in Table 4.2 for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_{\text{k}} = \mu_{\text{k}}\text{mgcos}\theta$. All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

Take-Home Experiment

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in Example 4.11, the kinetic friction on a slope $f_{\mathrm{k}} = \mu_{\mathrm{k}}\mathrm{mgcos}\theta$. The component of the weight down the slope is equal to $\mathrm{mgsin}\theta$ (see the free-body diagram in Figure 5.32). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

$$f_{\mathrm{k}} = \mathrm{mg}_{\mathrm{x}}$$

4.101

$$\mu_{\mathrm{k}}\mathrm{mgcos}\theta = \mathrm{mgsin}\theta.$$

4.102

Solving for μ_{k}, we find that

$$\mu_{\mathrm{k}} = \frac{\mathrm{mgsin}\theta}{\mathrm{mgcos}\theta} = \mathrm{tan}\theta.$$

4.103

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_{k}. Note that the coin will not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for μ_{k} and its uncertainty.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

Making Connections: Submicroscopic Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

Figure 4.33 illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.

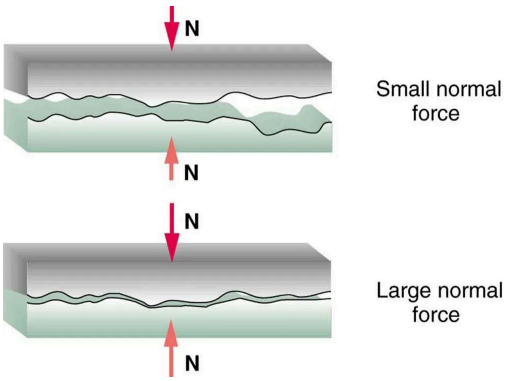


Figure 4.33 Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two side-by-side diagrams depicting surfaces under normal forces:

1. Top Diagram:
– Two surfaces are in contact, represented with a zigzag line to illustrate roughness.
– A downward red arrow labeled “N” indicates a small normal force is applied.
– An upward red arrow labeled “N” reflects equal reaction force.
– The caption on the right reads “Small normal force.”

2. Bottom Diagram:
– Similar depiction of two surfaces in contact with a zigzag line to show roughness.
– A larger downward red arrow labeled “N” signifies a large normal force.
– Correspondingly, a large upward red arrow labeled “N” represents the reaction force.
– The caption on the right reads “Large normal force.”

These diagrams illustrate the concept of normal force and its effect on contact surfaces.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. Figure 4.34 shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of 10^{12}
) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.

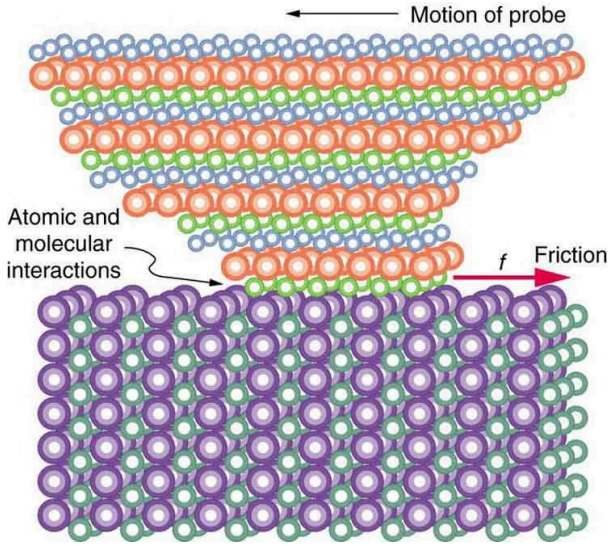


Figure 4.34 The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating atomic and molecular interactions between two surfaces, commonly used in explanations of friction at a microscopic level.

– Layers and Colors:
– The top part of the diagram consists of multiple rows of circles in different colors, including blue, orange, and green, representing atoms or molecules.
– The bottom layer consists of rows of circles colored in purple and green.

– Motion and Friction Arrows:
– An arrow labeled "Motion of probe" points to the left, indicating the direction of movement for the upper layers.
– Another arrow labeled "Friction" with the letter "f" points to the right, showing the direction of the frictional force acting against the movement of the probe.

– Labels:
– There is a label pointing to the interaction zone between the upper and lower layers, stating "Atomic and molecular interactions."

This diagram is a schematic portrayal of how friction arises from the interaction between atomic or molecular layers when one surface moves over another.

PhET Explorations

Forces and Motion

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

Access multimedia content

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GLOSSARY

acceleration
: the rate at which an object's velocity changes over a period of time

carrier particle
: a fundamental particle of nature that is surrounded by a characteristic force field; photons are carrier particles of the electromagnetic force

dynamics
: the study of how forces affect the motion of objects and systems

external force
: a force acting on an object or system that originates outside of the object or system

force
: a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

force field
: a region in which a test particle will experience a force

free-body diagram
: a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

free-fall
: a situation in which the only force acting on an object is the force due to gravity

friction
: a force past each other of objects that are touching; examples include rough surfaces and air resistance

inertia
: the tendency of an object to remain at rest or remain in motion

inertial frame of reference
: a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

kinetic friction
: a force that opposes the motion of two systems that are in contact and moving relative to one another

law of inertia
: see Newton's first law of motion

magnitude of kinetic friction
: $f_k = \mu_k N$, where

μ_k is the coefficient of kinetic friction

magnitude of static friction

$f_s \leq \mu_s N$, where

μ_s is the coefficient of static friction and

N is the magnitude of the normal force

mass

the quantity of matter in a substance; measured in kilograms

net external force

the vector sum of all external forces acting on an object or system; causes a mass to accelerate

Newton's first law of motion

a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

Newton's second law of motion

the net external force

$\mathbf{F}_{\text{net}}$ on an object with mass

m is proportional to and in the same direction as the acceleration of the object,

$\mathbf{a}$, and inversely proportional to the mass; defined mathematically as

$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$

Newton's third law of motion

whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts

normal force

the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

static friction

a force that opposes the motion of two systems that are in contact and are not moving relative to one another

system

defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

tension

the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a

tension force

thrust

a reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

weight

the force

$\mathbf{w}$due to gravity acting on an object of mass

m; defined mathematically as:

$\mathbf{w} = m\mathbf{g}$, where

$\mathbf{g}$is the magnitude and direction of the acceleration due to gravity

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CHAPTER SUMMARY

4.1 Development of Force Concept

- Dynamics is the study of how forces affect the motion of objects.
- Force is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- External forces are any outside forces that act on a body. A free-body diagram is a drawing of all external forces acting on a body.

4.2 Newton's First Law of Motion: Inertia

- Newton's first law of motion states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the law of inertia.
- Inertia is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- Mass is the quantity of matter in a substance.

4.3 Newton's Second Law of Motion: Concept of a System

- Acceleration, $\mathbf{a}$, is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is $\mathbf{a} = \dfrac{\mathbf{F}_{\text{net}}}{m}$.
- This is often written in the more familiar form: $\mathbf{F}_{\text{net}} = m\mathbf{a}$.
- The weight $\mathbf{w}$ of an object is defined as the force of gravity acting on an object of mass m. The object experiences an acceleration due to gravity $\mathbf{g}$: $\mathbf{w} = m\mathbf{g}$.
- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

4.4 Newton's Third Law of Motion: Symmetry in Forces

- Newton's third law of motion represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A thrust is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

4.5 Normal, Tension, and Other Examples of Forces

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, $\mathbf{N}$.
- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object:
  $N = \mathrm{mg}.$
- When objects rest on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be resolved into components that act perpendicular ($\mathbf{w}_{\perp}$) and parallel ($\mathbf{w}_{\parallel}$) to the surface of the plane. These components can be calculated using:
  $w_{\parallel} = w\sin(\theta) = \mathrm{mg}\sin(\theta)$
  $w_{\perp} = w\cos(\theta) = \mathrm{mg}\cos(\theta).$
- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, $\mathbf{T}$. When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:
  $T = \mathrm{mg}.$
- In any inertial frame of reference (one that is not accelerated or rotated), Newton's laws have the simple forms given in this chapter and all forces are real forces having a physical origin.

4.6 Problem-Solving Strategies

- To solve problems involving Newton's laws of motion, follow the procedure described:

    1. Draw a sketch of the problem.
    2. Identify known and unknown quantities, and identify the system of interest. Draw a free-body diagram, which is a sketch showing all of the forces acting on an object. The object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot. If vectors act in directions that are not horizontal or vertical, resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
    3. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the x-direction) then $F_{\text{net}x} = 0$. If the object does accelerate in that direction, $F_{\text{net } x} = \text{ma}$.
    4. Check your answer. Is the answer reasonable? Are the units correct?

4.7 Further Applications of Newton's Laws of Motion

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $F_{\text{net}} = \text{ma}$ or $F_{\text{net}} = 0$.
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

4.8 Extended Topic: The Four Basic Forces—An Introduction

- The various types of forces that are categorized for use in many applications are all manifestations of the *four basic forces* in nature.
- The properties of these forces are summarized in Table 4.1.
- Everything we experience directly without sensitive instruments is due to either electromagnetic forces or gravitational forces. The nuclear forces are responsible for the submicroscopic structure of matter, but they are not directly sensed because of their short ranges. Attempts are being made to show all four forces are different manifestations of a single unified force.
- A force field surrounds an object creating a force and is the carrier of that force.

4.9 Friction

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force N pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction f_{s} between systems stationary relative to one another is given by $f_{\text{s}} \leq \mu_{\text{s}} N$, where μ_{s} is the coefficient of static friction, which depends on both of the materials.
- The kinetic friction force f_{k} between systems moving relative to one another is given by $f_{\text{k}} = \mu_{\text{k}} N$, where μ_{k} is the coefficient of kinetic friction, which also depends on both materials.

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CONCEPTUAL QUESTIONS

Conceptual Questions

4.1 Development of Force Concept

1. Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.
2. What properties do forces have that allow us to classify them as vectors?

4.2 Newton's First Law of Motion: Inertia

3. How are inertia and mass related?
4. What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

4.3 Newton's Second Law of Motion: Concept of a System

5. Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.
6. Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?
7. Explain how the choice of the "system of interest" affects which forces must be considered when applying Newton's second law of motion.
8. Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.
9. A system can have a nonzero velocity while the net external force on it is zero. Describe such a situation.
10. A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory
11. (a) Give an example of different net external forces acting on the same system to

produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

12. If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.
13. If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?
14. The gravitational force on the basketball in Figure 4.6 is ignored. When gravity is taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

4.4 Newton's Third Law of Motion: Symmetry in Forces

15. When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)
16. A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the "ballistocardiograph." What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?
17. Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton's laws of motion apply?
18. Why does an ordinary rifle recoil (kick backward) when fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton's third law applies when one is fired. Can you safely stand close behind one when it is fired?
19. An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton's laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

20. Newton's third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the "system of interest" affects whether one such pair of forces cancels.

4.5 Normal, Tension, and Other Examples of Forces

21. If a leg is suspended by a traction setup as shown in Figure 4.35, what is the tension in the rope?

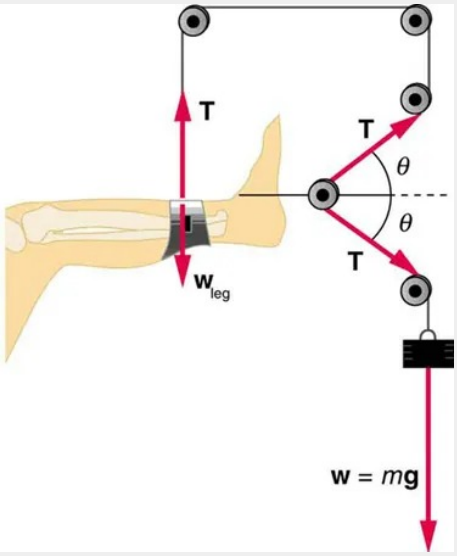


Figure 4.35 A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force T without changing its magnitude. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a mechanical system involving a leg and a pulley system. Here's a detailed description:

– On the left side, there is a diagram of a human leg shown horizontally, with a skeletal structure visible inside.
– A strap or band is wrapped around the leg, which is labeled as having a weight force acting downward, denoted as "w_leg."
– The strap is connected to a cable system depicted on the right, which goes through

several pulleys.
– The pulleys redirect the cable in different directions.
– Tension in the cable is marked with a force vector "T" at multiple points along the cables.
– The cable system includes angles labeled as θ (theta) at strategic points where the direction changes, indicating the angle between the cable path and the horizontal or vertical direction.
– At the bottom right of the system, a weight is suspended by the cable, indicated by "w = mg," representing the weight as a product of mass (m) and gravitational acceleration (g).
– All force vectors are represented by red arrows, showing direction and points of application of the forces throughout the system.

22. In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See Figure 4.35.) (Note that the tibia is the shin bone shown in this image.)

4.7 Further Applications of Newton's Laws of Motion

23. To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g. Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?
24. A cartoon shows the hat coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

4.8 Extended Topic: The Four Basic Forces—An Introduction

25. Explain, in terms of the properties of the four basic forces, why people notice the gravitational force acting on their bodies if it is such a comparatively weak force.
26. What is the dominant force between astronomical objects? Why are the other

three basic forces less significant over these very large distances?

27. Give a detailed example of how the exchange of a particle can result in an attractive force. (For example, consider one child pulling a toy out of the hands of another.)

4.9 Friction

28. Define normal force. What is its relationship to friction when friction behaves simply?
29. The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.
30. When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

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PROBLEMS & EXERCISES

Problems & Exercises

4.3 Newton's Second Law of Motion: Concept of a System

You may assume data taken from illustrations is accurate to three digits.

1. A 63.0-kg sprinter starts a race with an acceleration of $4.20\ \mathrm{m/s^2}$. What is the net external force on him?
2. If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?
3. A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.
4. Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut's acceleration is measured to be $0.893\ \mathrm{m/s^2}$. (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method in which recoil of the vehicle is avoided.
5. In Figure 4.7, the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the motion is 24 N, what force F (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force F is removed. How far will the mower go before stopping?
6. The same rocket sled drawn in Figure 4.36 is decelerated at a rate of $196\ \mathrm{m/s^2}$. What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2100 kg.

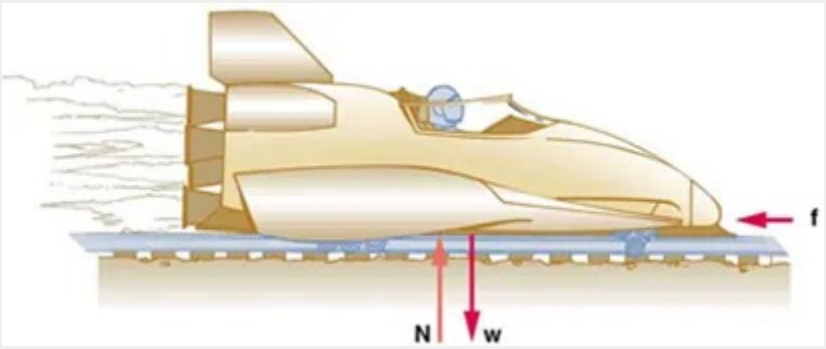


Figure 4.36 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a side view of a streamlined, futuristic train-like vehicle traveling along a track. The vehicle appears aerodynamic, with smooth curves and a sleek design. It has a large tail fin, suggesting high speed. A person is visible inside the cockpit of the vehicle.

There are several arrows indicating forces acting on the vehicle:

– A red arrow pointing to the right is labeled “f”, representing the forward driving force.
– Two arrows pointing downward towards the track are labeled “N” and “w”. “N” likely represents the normal force exerted by the track, and “w” represents the weight of the vehicle.
– A wispy, smoky trail is shown behind the vehicle, indicating movement or speed.

Overall, the image presents a dynamic depiction of motion and forces acting on the vehicle as it moves along the track.

7. (a) If the rocket sled shown in Figure 4.37 starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust T is 2.59×10^4 N, and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

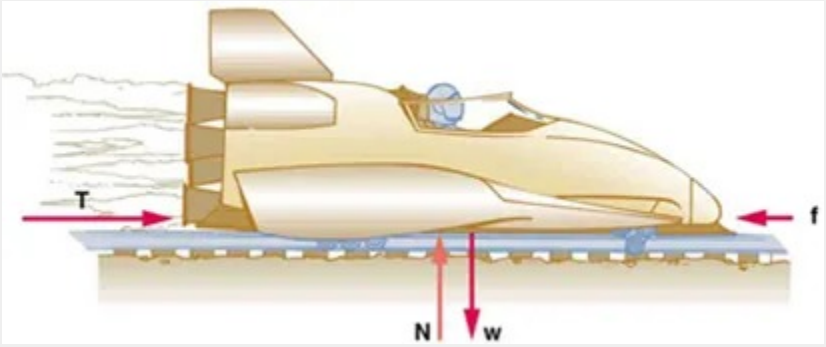


Figure 4.37 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a schematic side view of a streamlined vehicle, resembling a bobsled, moving along a track. The vehicle has a sleek design and is shown with indicators pointing to various forces acting upon it. These forces are represented by arrows with labels:

– An arrow labeled “T” points to the left, indicating thrust or a propelling force.
– An arrow labeled “f” points to the right, indicating friction acting against the motion of the vehicle.
– Two arrows labeled “N” and “w” point downward and upward respectively, indicating the normal force and the weight of the vehicle.

The vehicle is depicted in motion, with lines suggesting speed and airflow over the surface.

8. What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

9.
   a. What is the system of interest if the acceleration of the child in the wagon is to be calculated?
   b. Draw a free-body diagram, including all forces acting on the system.
   c. Calculate the acceleration.
   d. What would the acceleration be if friction were 15.0 N?Suppose two children push horizontally, but in exactly opposite directions, on a third child

in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg.

10. A powerful motorcycle can produce an acceleration of $3.50\ \mathrm{m/s^2}$ while traveling at 90.0 km/h. At that speed the forces resisting motion, including friction and air resistance, total 400 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?
11. The rocket sled shown in Figure 4.38 accelerates at a rate of $49.0\ \mathrm{m/s^2}$. Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

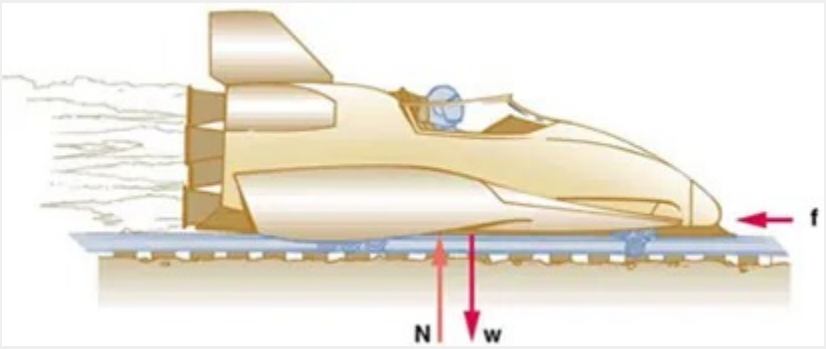


Figure 4.38 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a streamlined, futuristic vehicle that resembles a high-speed train or a rocket-powered car. The vehicle is depicted from the side and is moving to the right. It has a sleek, aerodynamic shape with a pointed nose and fin-like structures at the rear. A trail of exhaust or vapor is shown behind the vehicle, indicating high speed.

A person is seated inside the vehicle, positioned near the front. The vehicle is on a set of

rails or a track, suggesting it's designed for guided travel. Below the vehicle, three arrows indicate forces acting upon it:

1. A red arrow pointing to the right labeled "f" represents the forward force or thrust pushing the vehicle forward.
2. A downward red arrow labeled "w" represents the weight of the vehicle.
3. An upward red arrow labeled "N" represents the normal force exerted by the track on the vehicle.

The visual suggests the dynamics of motion and the forces involved in keeping the vehicle on its track.

12. Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of $201\ \mathrm{m/s^2}$. In this problem, the forces are exerted by the seat and restraining belts.
13. The weight of an astronaut plus their space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?
14. Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

4.4 Newton's Third Law of Motion: Symmetry in Forces

15. What net external force is exerted on a 1100-kg artillery shell fired from a battleship if the shell is accelerated at $2.40 \times 10^4\ \mathrm{m/s^2}$? What is the magnitude of the force exerted on the ship by the artillery shell?
16. A rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at $1.20\ \mathrm{m/s^2}$ backward. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110 kg? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and write the net force

equation.

4.5 Normal, Tension, and Other Examples of Forces

17. Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?
18. What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at $7.50\ \mathrm{m/s^2}$? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.
19. (a) Calculate the tension in a vertical strand of spider web if a spider of mass $8.00 \times 10^{-5}\ \mathrm{kg}$ hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in Figure 4.16. The strand sags at an angle of 12° below the horizontal. Compare this with the tension in the vertical strand (find their ratio).
20. Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if they climb at a constant speed? (b) What is the tension in the rope if they accelerate upward at a rate of $1.50\ \mathrm{m/s^2}$?
21. Show that, as stated in the text, a force $\mathbf{F}_{\perp}$ exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in Figure 4.16) gives rise to a tension of magnitude
22. $T = \dfrac{F_{\perp}}{2\sin(\theta)}$.
23. Consider the baby being weighed in Figure 4.39. (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension T_1 in the cord attaching the baby to the scale? (c) What is the tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part.

The masses of the cords are negligible.

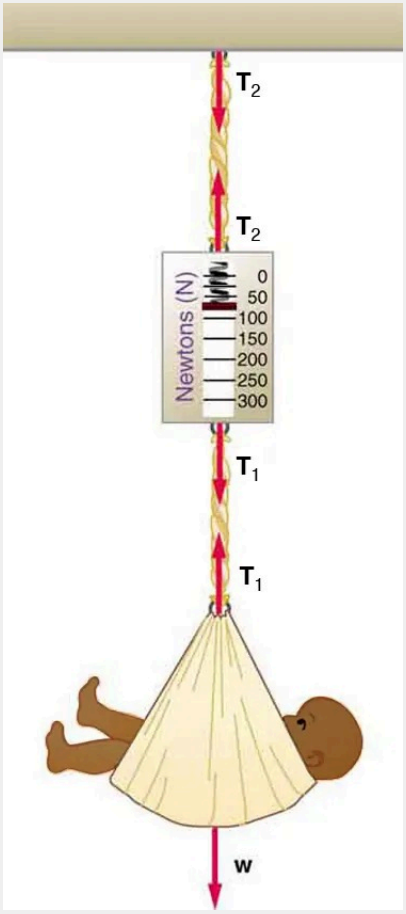


Figure 4.39 A baby is weighed using a spring scale. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a diagram depicting a baby lying in a hammock-like cloth suspended from a ceiling. The setup illustrates forces in physics, specifically focusing on tension. Here's a detailed description:

– A small child, depicted as an illustration, is lying in a triangular hammock or cloth sling.
– The cloth is connected to a rope, which is tied above to a point on the ceiling.
– The rope is divided into two sections, labeled as T1 and T2, indicating the tension in the rope.
– These tension forces (T1 and T2) are visualized with red arrows pointing upwards

along the rope, indicating the direction of the tension forces.
– Between the segments of the rope, there is a spring scale showing the force in Newtons, acting as a measuring device. The scale shows measurements ranging from 0 to 300 Newtons, marked in increments of 50.
– The reading shown on the scale is around 150 Newtons.
– Below the child, a downward arrow marked "w" indicates the weight of the child, representing the force of gravity acting on the child's mass.

This setup is commonly used to illustrate concepts of tension and equilibrium in physics, showing how tension varies along a rope when supporting a weight.

4.6 Problem-Solving Strategies

24. A $5.00 \times 10^5 - \text{kg}$ rocket is accelerating straight up. Its engines produce $1.250 \times 10^7\ \text{N}$ of thrust, and air resistance is $4.50 \times 10^6\ \text{N}$. What is the rocket's acceleration? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.
25. The wheels of a midsize car exert a force of 2100 N backward on the road to accelerate the car in the forward direction. If the force of friction including air resistance is 250 N and the acceleration of the car is $1.80\ \text{m/s}^2$, what is the mass of the car plus its occupants? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. For this situation, draw a free-body diagram and write the net force equation.
26. Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.
27. When landing after a spectacular somersault, a 40.0-kg gymnast decelerates by pushing straight down on the mat. Calculate the force she must exert if her deceleration is 7.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.
28. A freight train consists of two $8.00 \times 10^4 - \text{kg}$ engines and 45 cars with average masses of $5.50 \times 10^4\text{kg}$. (a) What force must each engine exert backward on the

track to accelerate the train at a rate of $5.00 \times 10^{-2}\ \mathrm{m/s^2}$ if the force of friction is $7.50 \times 10^{5}\ \mathrm{N}$, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

29. Commercial airplanes are sometimes pushed out of the passenger loading area by a tractor. (a) An 1800-kg tractor exerts a force of $1.75 \times 10^{4}\ \mathrm{N}$ backward on the pavement, and the system experiences forces resisting motion that total 2400 N. If the acceleration is $0.150\ \mathrm{m/s^2}$, what is the mass of the airplane? (b) Calculate the force exerted by the tractor on the airplane, assuming 2200 N of the friction is experienced by the airplane. (c) Draw two sketches showing the systems of interest used to solve each part, including the free-body diagrams for each.
30. A 1100-kg car pulls a boat on a trailer. (a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900-N force on the road and produces an acceleration of $0.550\ \mathrm{m/s^2}$? The mass of the boat plus trailer is 700 kg. (b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer?
31. (a) Find the magnitudes of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ that add to give the total force $\mathbf{F}_{\mathrm{tot}}$ shown in Figure 4.40. This may be done either graphically or by using trigonometry. (b) Show graphically that the same total force is obtained independent of the order of addition of $\mathbf{F}_1$ and $\mathbf{F}_2$. (c) Find the direction and magnitude of some other pair of vectors that add to give $\mathbf{F}_{\mathrm{tot}}$. Draw these to scale on the same drawing used in part (b) or a similar picture.

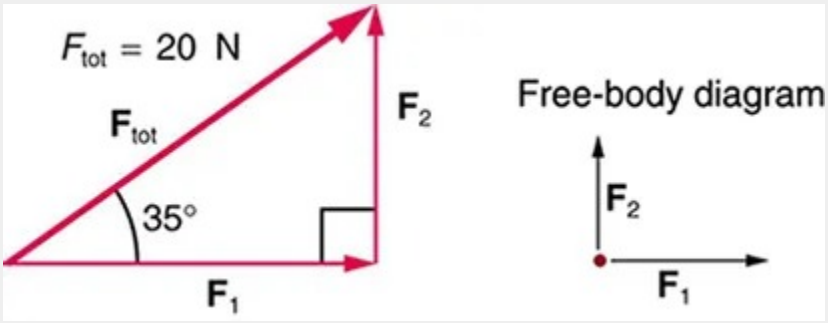


Figure 4.40 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams related to forces:

1. Triangle Diagram on the Left:
– There is a right triangle with a hypotenuse labeled as F_{tot}.
– $F_{\text{tot}} = 20\,\text{N}$ is written above the hypotenuse.
– The angle between the hypotenuse and the horizontal side is labeled 35°.
– The horizontal side of the triangle is labeled F_1.
– The vertical side of the triangle is labeled F_2.
– Both F_1 and F_2 have arrows indicating direction, with F_1 pointing right and F_2 pointing upward.

2. Free-body Diagram on the Right:
– This diagram consists of two arrows originating from the same point.
– One arrow is labeled F_1 and points horizontally to the right.
– The other arrow is labeled F_2 and points vertically upward.
– The label “Free-body diagram” is present near this setup.

Both diagrams are used to demonstrate the components of a resultant force using vector representation.

31. Two children pull a third child on a snow saucer sled exerting forces $\mathbf{F}_1$ and $\mathbf{F}_2$ as shown from above in Figure 4.41. Find the acceleration of the 49.00-kg sled and child system. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of $\mathbf{F}_1$ and $\mathbf{F}_2$.

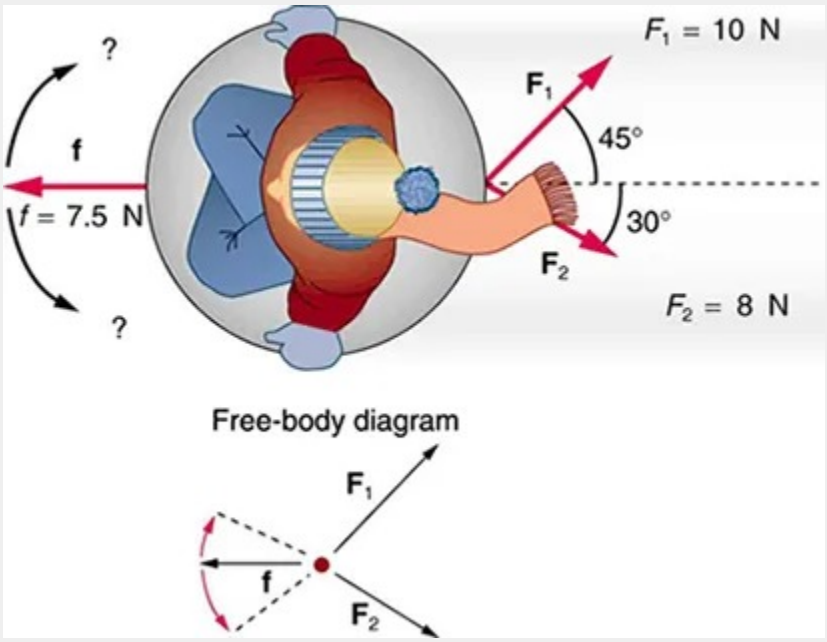


Figure 4.41 An overhead view of the horizontal forces acting on a child's snow saucer sled. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a top-down view of a person sitting on a frictionless rotating stool, holding weights in each hand. The person is wearing a blue shirt and a yellow hat with a blue pom-pom. There are several forces and angles annotated around the person:

– Two forces, labeled as F_1 and F_2, are acting outward from the person's hands. F_1 is 10 N and is at a 45-degree angle to the right of the vertical axis. F_2 is 8 N and is at a 30-degree angle to the left of the vertical axis.
– A frictional force f of 7.5 N acts against the direction of motion.
– The diagram shows suggested directions for movement with question marks.

Below this, there is a separate "Free-body diagram" showing the vectors of forces. Here, F_1, F_2, and f are represented with dashed lines, illustrating their directions.

The overall image appears to illustrate the concept of forces acting on a person rotating on a frictionless surface, highlighting the balances and interactions between different forces.

32. Suppose your car was mired deeply in the mud and you wanted to use the method illustrated in Figure 4.42 to pull it out. (a) What force would you have to exert perpendicular to the center of the rope to produce a force of 12,000 N on the car if the angle is 2.00°? In this part, explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. (b) Real ropes stretch under such forces. What force would be exerted on the car if the angle increases to 7.00° and you still apply the force found in part (a) to its center?



Figure 4.42 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram showing a scenario where a vehicle is stuck and is being pulled using a winch connected to a tree trunk. The diagram includes:

– A blue car on the left side, positioned on a dirt surface.
– A tree trunk on the right side, depicted in green.
– A dotted line representing a cable extending from the tree trunk to the car.
– The cable is slightly sagging in the middle, forming an angle denoted as θ.
– Red arrows indicating tension (labeled as T) are directed outward from both the car and the tree trunk along the cable.
– In the middle of the cable, there's a red vertical arrow pointing downwards, labeled as Fg, representing the force of gravity acting on the cable.

The background shows a gradient of colors suggesting an outdoor setting, possibly with grass and dirt.

33. What force is exerted on the tooth in Figure 4.43 if the tension in the wire is 25.0 N? Note that the force applied to the tooth is smaller than the tension in the wire, but this is necessitated by practical considerations of how force can be applied in the mouth. Explicitly show how you follow steps in the Problem-Solving Strategy for Newton's laws of motion.

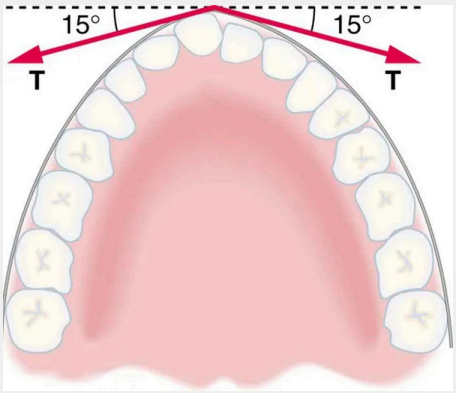


Figure 4.43 Braces are used to apply forces to teeth to realign them. Shown in this figure are the tensions applied by the wire to the protruding tooth. The total force applied to the tooth by the wire, $\mathbf{F}_{\text{app}}$, points straight toward the back of the mouth. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of the upper jaw, viewed from above, showing the arrangement of teeth. The upper dental arch is depicted with the teeth arranged in a semi-circular formation. Above this, there are two arrows pointing outward from the center with a 15-degree angle labeled between them. These arrows are marked with the letter "T." The teeth appear to be outlined, and there is some shading to suggest the gums. The focus of the image is likely on the 15-degree angle between the arrows as they relate to the alignment or force applied to the dental arch.

34. Figure 4.44 shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible. (a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope. (b) Find the tension in the rope above Superhero. (c) Find the tension in the rope between Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.



Figure 4.44 Superhero and Trusty Sidekick hang motionless on a rope as they try to figure out what to do next. Will the tension be the same everywhere in the rope? Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a scene where two individuals dressed as superheroes are engaged in an action sequence. The superhero at the top is wearing a purple cape and boots with a gray costume. This character is holding onto a horizontal bar and hanging upside down. Below this character, another superhero is hanging on to the first individual's ankles. The second superhero is dressed in a yellow and blue costume with teal boots and gloves. Both characters are positioned vertically, with the purple-caped superhero at the top and the yellow-and-blue-costumed superhero beneath, as if participating in a rescue or climbing scenario.

35. A nurse pushes a cart by exerting a force on the handle at a downward angle 35.0° below the horizontal. The loaded cart has a mass of 28.0 kg, and the force of friction is 60.0 N. (a) Draw a free-body diagram for the system of interest. (b) What force must the nurse exert to move at a constant velocity?
36. Construct Your Own Problem Consider the tension in an elevator cable during the time the elevator starts from rest and accelerates its load upward to some cruising

velocity. Taking the elevator and its load to be the system of interest, draw a free-body diagram. Then calculate the tension in the cable. Among the things to consider are the mass of the elevator and its load, the final velocity, and the time taken to reach that velocity.

37. Construct Your Own Problem Consider two people pushing a toboggan with four children on it up a snow-covered slope. Construct a problem in which you calculate the acceleration of the toboggan and its load. Include a free-body diagram of the appropriate system of interest as the basis for your analysis. Show vector forces and their components and explain the choice of coordinates. Among the things to be considered are the forces exerted by those pushing, the angle of the slope, and the masses of the toboggan and children.
38. Unreasonable Results (a) Repeat Exercise 4.29, but assume an acceleration of $1.20\ \mathrm{m/s^2}$ is produced. (b) What is unreasonable about the result? (c) Which premise is unreasonable, and why is it unreasonable?
39. Unreasonable Results (a) What is the initial acceleration of a rocket that has a mass of $1.50 \times 10^6\ \mathrm{kg}$ at takeoff, the engines of which produce a thrust of $2.00 \times 10^6\ \mathrm{N}$? Do not neglect gravity. (b) What is unreasonable about the result? (This result has been unintentionally achieved by several real rockets.) (c) Which premise is unreasonable, or which premises are inconsistent? (You may find it useful to compare this problem to the rocket problem earlier in this section.)

4.7 Further Applications of Newton's Laws of Motion

40. A flea jumps by exerting a force of $1.20 \times 10^{-5}\ \mathrm{N}$ straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of $0.500 \times 10^{-6}\ \mathrm{N}$ on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is $6.00 \times 10^{-7}\ \mathrm{kg}$. Do not neglect the gravitational force.
41. Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in Figure 4.45. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?

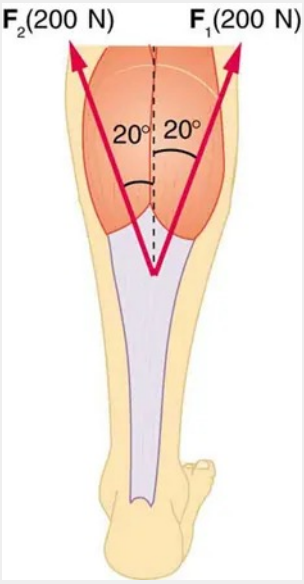


Figure 4.45 Achilles tendon Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an anatomical diagram of a leg viewed from the back, focusing on the muscles around the thigh and lower leg. It highlights the division of muscular forces in the leg.

– The thigh is depicted with two large muscles, and the diagram focuses on the forces exerted by these muscles.
– Two arrows, labeled as F_1 and F_2, represent forces of 200 Newtons each.
– These arrows point diagonally outward, away from each other and form a symmetrical "V" shape.
– Each arrow originates from the center of the thigh and indicates the direction of force exerted by the muscles.
– The angle between each force line (arrow) and the midline of the thigh is marked as 20 degrees.
– The forces diverge from a central point at the top of the thigh.

Overall, the image is a schematic representation illustrating how forces are distributed in the muscles of the leg, specifically around the thigh area.

42. A 76.0-kg person is being pulled away from a burning building as shown in Figure

4.46. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

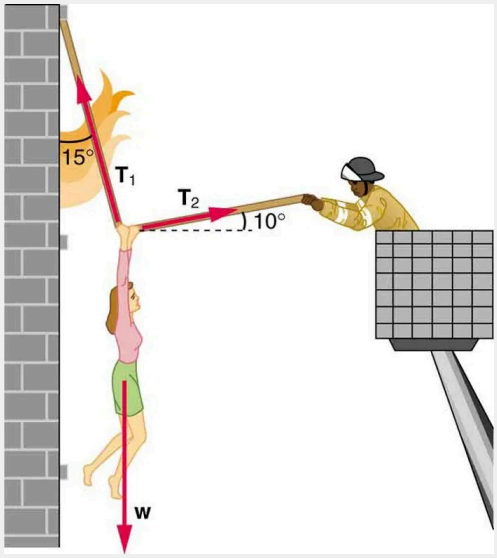


Figure 4.46 The force $\mathbf{T}_2$ needed to hold steady the person being rescued from the fire is less than her weight and less than the force $\mathbf{T}_1$ in the other rope, since the more vertical rope supports a greater part of her weight Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a rescue scene involving a firefighter and a person being rescued from a burning building.

– On the left, there is a brick wall with flames visible, suggesting a fire. A person is hanging from the wall, holding onto a horizontal bar.
– The person, a woman, is suspended in mid-air and appears to be held by two forces represented by arrows labeled T1 and T2.
– T1 is angled at 15 degrees from the vertical, pointing upwards toward the wall.
– T2 is angled at 10 degrees from the horizontal, pointing towards the right where the firefighter is located.
– The woman's body is aligned with the direction of arrow T1, indicating tension or force in that direction.
– A downward arrow labeled "w" points straight down from the woman, indicating her

weight or the force of gravity acting on her.
– To the right, a firefighter is standing in a basket attached to a ladder, reaching out with a stick or pole to assist the person.
– The firefighter is wearing protective gear, including a helmet.
– The basket is part of an extendable platform, positioning the firefighter at the same height as the hanging woman.

This illustration appears to be a physics-based diagram illustrating tension, force, and angle relationships in the context of a practical rescue scenario.

43. Integrated Concepts A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)
44. Integrated Concepts When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?
45. Integrated Concepts A large rocket has a mass of 2.00×10^6 kg at takeoff, and its engines produce a thrust of 3.50×10^7 N. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.
46. Integrated Concepts A basketball player jumps straight up for a ball. To do this, they lower their body 0.300 m and then accelerate through this distance by forcefully straightening their legs. This player leaves the floor with a vertical velocity sufficient to carry the player 0.900 m above the floor. (a) Calculate the player's velocity when they leave the floor. (b) Calculate their acceleration while they are straightening their legs. They go from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force they exert on the floor to do this, given that their mass is 110 kg.
47. Integrated Concepts A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption,

but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

48. Integrated Concepts Repeat Exercise 4.47 for a shell fired at an angle 10.0° from the vertical.
49. Integrated Concepts An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of $1.20\ \mathrm{m/s^2}$ for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of $0.600\ \mathrm{m/s^2}$ for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?
50. Unreasonable Results (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of $0.400\ \mathrm{m/s^2}$ for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?
51. Unreasonable Results A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

4.8 Extended Topic: The Four Basic Forces—An Introduction

52. (a) What is the strength of the weak nuclear force relative to the strong nuclear force? (b) What is the strength of the weak nuclear force relative to the electromagnetic force? Since the weak nuclear force acts at only very short distances, such as inside nuclei, where the strong and electromagnetic forces also act, it might seem surprising that we have any knowledge of it at all. We have such knowledge because the weak nuclear force is responsible for beta decay, a

type of nuclear decay not explained by other forces.

53. (a) What is the ratio of the strength of the gravitational force to that of the strong nuclear force? (b) What is the ratio of the strength of the gravitational force to that of the weak nuclear force? (c) What is the ratio of the strength of the gravitational force to that of the electromagnetic force? What do your answers imply about the influence of the gravitational force on atomic nuclei?
54. What is the ratio of the strength of the strong nuclear force to that of the electromagnetic force? Based on this ratio, you might expect that the strong force dominates the nucleus, which is true for small nuclei. Large nuclei, however, have sizes greater than the range of the strong nuclear force. At these sizes, the electromagnetic force begins to affect nuclear stability. These facts will be used to explain nuclear fusion and fission later in this text.
55. Critical Thinking Two boxes of equal mass slide start from rest and slide with different horizontal applied forces across frictionless surfaces to reach the same distance away. Box A has a horizontal force of three times that of box B. (a) Which box is traveling faster at this distance away from the starting point? Briefly explain your reasoning. (b) Based on experimental data, an equation that fits the data is suggested for the final speed, V, which may not be correct: $V = KF$, where K is a constant with appropriate units and F is the horizontal applied force. Is this equation consistent with your answer from part a? Explain why or why not. Does this equation make sense? Explain why or why not. (c) Now the applied force can be any value and the distance from the starting point is measured when the speed gets to a consistent value. Graph the distance the box gets to at a given speed as a function of the applied force.

4.9 Friction

56. A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?
57. (a) When rebuilding her car's engine, a physics major must exert 300 N of force to

insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

58. (a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.
59. Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?
60. (a) If half of the weight of a small $1.00 \times 10^3 \text{kg}$utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.
61. A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.
62. Consider the 65.0-kg ice skater being pushed by two others shown in Figure 4.47. (a) Find the direction and magnitude of $\mathbf{F}_{\text{tot}}$, the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of $\mathbf{F}_{\text{tot}}$? (c) What is her acceleration assuming she is already moving in the direction of $\mathbf{F}_{\text{tot}}$? (Remember that friction

always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)

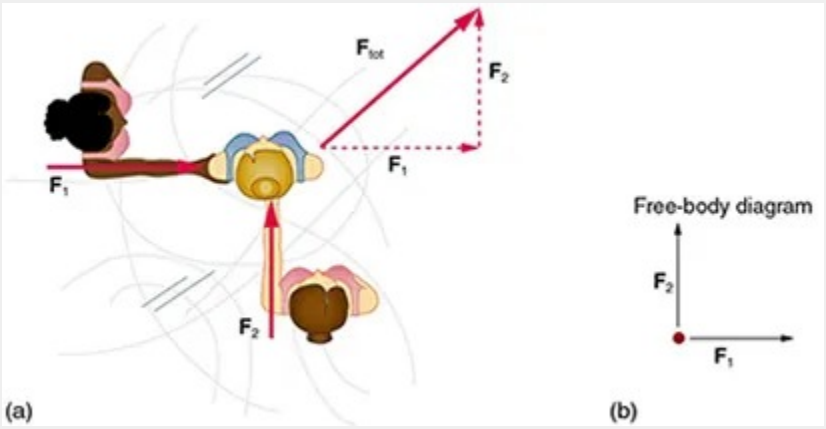


Figure 4.47 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two parts labeled (a) and (b).

Part (a):
– Shows a top-down view of three individuals engaged in a tug-of-war on a frictionless platform.
– The individuals are positioned in a triangular formation.
– There are two forces labeled F_1 and F_2 with arrows showing direction and magnitude.
– F_1 is directed towards the right, and F_2 is directed upwards.
– A resultant force F_{tot} is indicated by a dashed arrow pointing diagonally upwards and to the right.

Part (b):
– Depicts a simplified free-body diagram.
– Shows a red point with two perpendicular arrows.
– The horizontal arrow is labeled F_1 (pointing right), and the vertical arrow is labeled F_2 (pointing up).
– The label “Free-body diagram” is positioned above these arrows.

The image illustrates the concept of combining forces and creating a vector diagram to represent the interaction visually.

63. Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g\sin\theta$. (Note that this acceleration is independent of mass.)
64. Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_{\mathrm{k}} = \mu_{\mathrm{k}}N$) is $a = g\left(\sin\theta - \mu_{\mathrm{k}}\cos\theta\right)$. Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small $(\mu_{\mathrm{k}} = 0)$.
65. Calculate the deceleration of a snow boarder going up a 5.0° , slope assuming the coefficient of friction for waxed wood on wet snow. The result of Exercise 4.64 may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in 4.6 Problem-Solving Strategies.
66. (a) Calculate the acceleration of a skier heading down a 10.0° slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of Exercise 4.64 to be useful. Explicitly show how you follow the steps in 4.6 Problem-Solving Strategies.
67. If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is $\theta = \tan^{-1}\mu_{\mathrm{s}}$. You may use the result of the previous problem. Assume that $a = 0$ and that static friction has reached its maximum value.
68. Calculate the maximum deceleration of a car that is heading down a 6° slope (one that makes an angle of 6° with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.
69. Calculate the maximum acceleration of a car that is heading up a 4° slope (one that makes an angle of 4° with the horizontal) under the following road conditions. Assume that only

half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

70. Repeat Exercise 4.69 for a car with four-wheel drive.
71. A freight train consists of two $8.00 \times 10^5 - \text{kg}$ engines and 45 cars with average masses of 5.50×10^5 kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00 \times 10^{-2} m/s^2$ if the force of friction is $7.50 \times 10^5 N$, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?
72. Consider the 52.0-kg mountain climber in Figure 4.48. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?

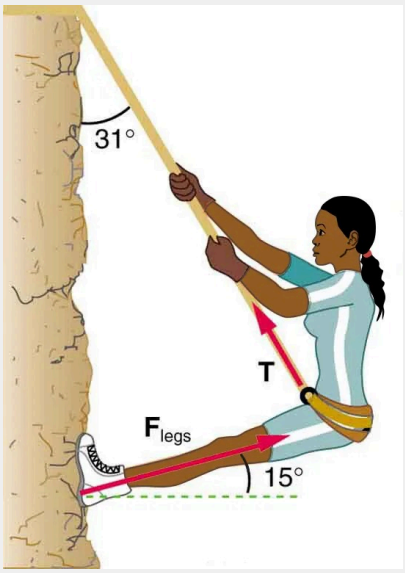


Figure 4.48 Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a person rock climbing with two key angles and force vectors depicted. The climber, wearing athletic clothing, is pulling on a rope with both hands. The rope is oriented at a 31-degree angle relative to the rock wall. Two arrows indicating force are shown: one labeled "T" points along the rope towards the climber, representing tension in the rope, and the other labeled "F_{legs}" extends from the climber's feet, pushing against the wall to maintain position. The force from the climber's legs forms a 15-degree angle relative to the horizontal line below their feet. The rock wall has a textured appearance with cracks and ledges, suggesting a natural climbing environment.

73. A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in Figure 5.21(a). (a) Calculate the minimum force F he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?
74. Repeat Exercise 4.73 with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in Figure 4.49(b).

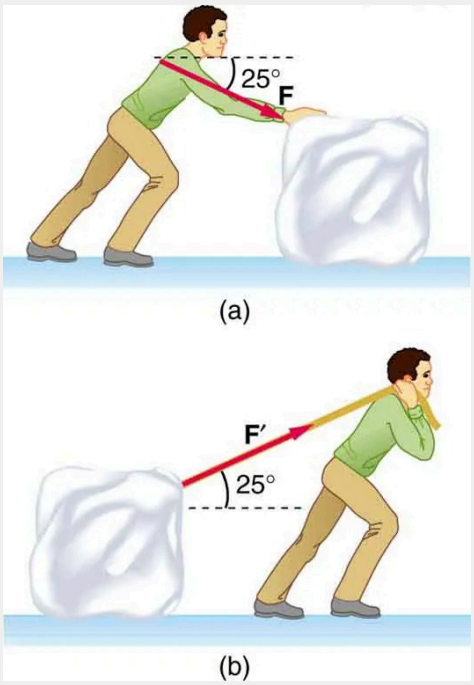


Figure 4.49 Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal? Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams, labeled (a) and (b), illustrating a person moving a large object that resembles a block of ice.

– Diagram (a): A person is pushing the block. The diagram shows the person leaning forward, with their hands placed on the back of the block. An arrow labeled “F” is drawn from the person’s shoulders to the block, making an angle of 25 degrees downward from the horizontal. This suggests that the force is applied with an angle towards the floor.

– Diagram (b): A person is pulling the block using a rope or similar object. The diagram shows the person leaning backward with the pulling object wrapped around their shoulder. An arrow labeled “F'” extends from the block to the person’s shoulders, making an angle of 25 degrees upward from the horizontal. This indicates that the pulling force is directed slightly upwards.

Both diagrams demonstrate the angles and direction of force application in pushing and pulling scenarios.

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UNIFORM CIRCULAR MOTION AND GRAVITATION



Figure 5.1 This Australian Grand Prix Formula 1 race car moves in a circular path as it makes the turn. Its wheels also spin rapidly—the latter completing many revolutions, the former only part of one (a circular arc). The same physical principles are involved in each. Image by Richard Munckton from Windsor, Melbourne, Australia, Wikimedia Commons License

Chapter Outline

5.1 Rotation Angle and Angular Velocity
5.2 Centripetal Acceleration
5.3 Centripetal Force
5.4 Newton's Universal Law of Gravitation
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

Many motions, such as the arc of a bird's flight or Earth's path around the Sun, are curved. Recall that Newton's first law tells us that motion is along a straight line at constant speed unless there is a net external force. We will therefore study not only motion along curves, but also the forces that cause it, including gravitational forces. In some ways, this chapter is a continuation of Dynamics: Force and Newton's Laws of Motion as we study more applications of Newton's laws of motion.

This chapter deals with the simplest form of curved motion, uniform circular motion, motion in a circular path at constant speed. Studying this topic illustrates most concepts associated with rotational motion and leads to the study of many new topics we group under the name *rotation*. Pure *rotational motion* occurs when points in an object move in circular paths centered on one point. Pure *translational motion* is motion with no rotation. Some motion combines both types, such as a rotating hockey puck moving along ice.

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5.1 ROTATION ANGLE AND ANGULAR VELOCITY

Learning Objectives

By the end of this section, you will be able to:

- Define arc length, rotation angle, radius of curvature and angular velocity.
- Calculate the angular velocity of a car wheel spin.

In Kinematics, we studied motion along a straight line and introduced such concepts as displacement, velocity, and acceleration. Two-Dimensional Kinematics dealt with motion in two dimensions. Projectile motion is a special case of two-dimensional kinematics in which the object is projected into the air, while being subject to the gravitational force, and lands a distance away. In this chapter, we consider situations where the object does not land but moves in a curve. We begin the study of uniform circular motion by defining two angular quantities needed to describe rotational motion.

Rotation Angle

When objects rotate about some axis—for example, when the CD (compact disc) in Figure 5.2 rotates about its center—each point in the object follows a circular arc. Consider a line from the center of the CD to its edge. Each pit used to record sound along this line moves through the same angle in the same amount of time. The rotation angle is the amount of rotation and is analogous to linear distance. We define the rotation angle $\Delta\theta$ to be the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta s}{r}.$$

5.1



Figure 5.2 All points on a CD travel in circular arcs. The pits along a line from the center to the edge all move through the same angle $\Delta\theta$ *in a time* Δt*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows a graphic representation of a CD (compact disc). The CD displays its typical circular shape with a reflective surface that exhibits a rainbow-like spectrum. Part of the CD is removed to reveal an internal section which is illustrated with a dotted pattern.

Within this cutaway section, there are angular lines meeting at a point creating a small triangular angle marked with the symbol Δθ (Delta Theta). This symbol represents a change in the angle. The diagram appears to be illustrating a concept related to angular measurement or perhaps data encoding on a CD.

The reflective surface gives off a rainbow effect due to the light diffraction, a characteristic of CDs. The center contains the familiar circular hole typical in all CDs for placement onto a drive spindle.

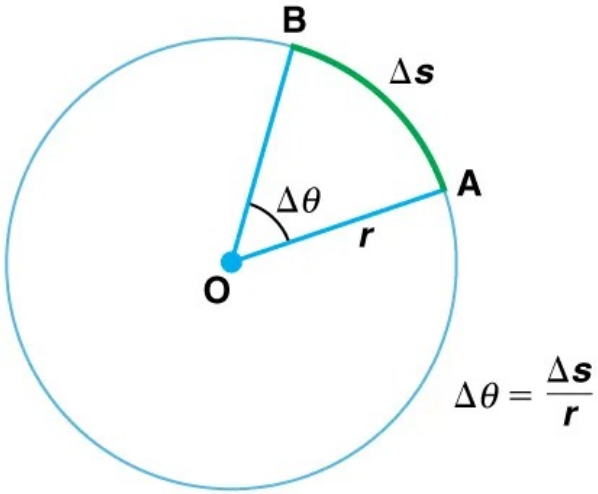


Figure 5.3 The radius of a circle is rotated through an angle $\Delta\theta$*. The arc length* Δs *is described on the circumference. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a circle with center labeled as *O*. A sector of the circle is highlighted with an arc between points *A* and *B*. The following details describe the elements in the image:

- *O* is the center of the circle.
- The radius of the circle from *O* to *A* is labeled as *r*.
- The arc length between points *A* and *B* is labeled as *Δs*, highlighted in green.
- The angle at *O* subtended by the arc *AB* is labeled as *Δθ*.
- A formula is provided: *Δθ = Δs / r*.

This image illustrates the relationship between the arc length, the radius, and the angle measured in radians.

The arc length Δs is the distance traveled along a circular path as shown in Figure 6.3. Note that r is the radius of curvature of the circular path.

We know that for one complete revolution, the arc length is the circumference of a circle of radius r. The circumference of a circle is $2\pi r$. Thus for one complete revolution the rotation angle is

$$\Delta\theta = \frac{2\pi r}{r} = 2\pi.$$

5.2

This result is the basis for defining the units used to measure rotation angles, $\Delta\theta$ to be radians (rad), defined so that

$$2\pi\text{rad} = 1 \text{ revolution}.$$
5.3

A comparison of some useful angles expressed in both degrees and radians is shown in Table 5.1.

Table 5.1 Comparison of Angular Units

Degree Measures	Radian Measure
30°	$\frac{\pi}{6}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$
120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
180°	π

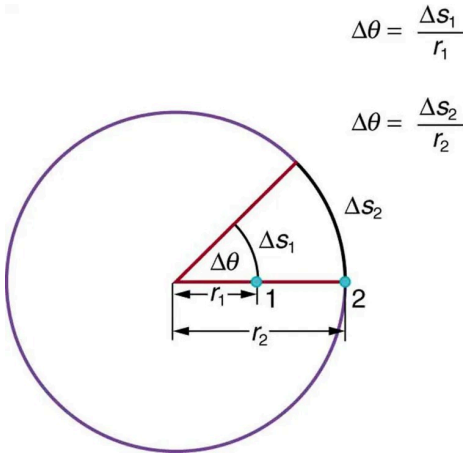


Figure 5.4 Points 1 and 2 rotate through the same angle ($\Delta\theta$), but point 2 moves through a greater arc length (Δs) because it is at a greater distance from the center of rotation (r). Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of a circle with two radii and arcs, illustrating angular displacement.

– The circle is outlined in purple, containing a sector highlighted in red.
– Inside the sector, there are two points labeled “1” and “2” along the arcs.
– The angle at the center of the circle, denoted as Δθ (delta theta), is formed between two radii.
– The radius extending to point “1” is labeled as r_1, and the arc segment associated with it is labeled as Δs_1.
– Similarly, the radius extending to point “2” is labeled as r_2, and the corresponding arc segment is Δs_2.
– Two formulae are shown on the right side of the image:
- $\Delta\theta = \frac{\Delta s_1}{r_1}$
- $\Delta\theta = \frac{\Delta s_2}{r_2}$

This diagram is likely illustrating the relationship between arc length, radius, and angular displacement in radians.

If $\Delta\theta = 2\pi$ rad, then the CD has made one complete revolution, and every point on the CD is back at its original position. Because there are 360° in a circle or one revolution, the relationship between radians and degrees is thus

$$2\pi\text{rad} = 360^\circ$$
5.4

so that

$$1\text{rad} = \frac{360^\circ}{2\pi} \approx 57.3^\circ \ .$$
5.5

Angular Velocity

How fast is an object rotating? We define angular velocity ω as the rate of change of an angle. In symbols, this is

$$\omega = \frac{\Delta\theta}{\Delta t},$$
5.6

where an angular rotation $\Delta\theta$ takes place in a time Δt. The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).

Angular velocity ω is analogous to linear velocity v. To get the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length Δs in a time Δt, and so it has a linear velocity

$$v = \frac{\Delta s}{\Delta t}.$$
5.7

From $\Delta\theta = \frac{\Delta s}{r}$ we see that $\Delta s = r\Delta\theta$. Substituting this into the expression for v gives

$$v = \frac{r\Delta\theta}{\Delta t} = r\omega.$$
5.8

We write this relationship in two different ways and gain two different insights:

$$v = r\omega \text{ or } \omega = \frac{v}{r}.$$
5.9

The first relationship in $v = r\omega$ or $\omega = \frac{v}{r}$ states that the linear velocity v is proportional to the distance from the center of rotation, thus, it is largest for a point on the rim (largest r), as you might expect. We can also call this linear speed v of a point on the rim the *tangential speed*. The second relationship in $v = r\omega$ or $\omega = \frac{v}{r}$ can be illustrated by considering the tire of a moving car. Note that the speed of a point on the rim of the tire is the same as the speed v of the car. See Figure 5.5. So the faster the car moves, the faster the tire spins—large v means a large ω, because $v = r\omega$. Similarly, a larger-radius tire rotating at the same angular velocity (ω) will produce a greater linear speed (v) for the car.

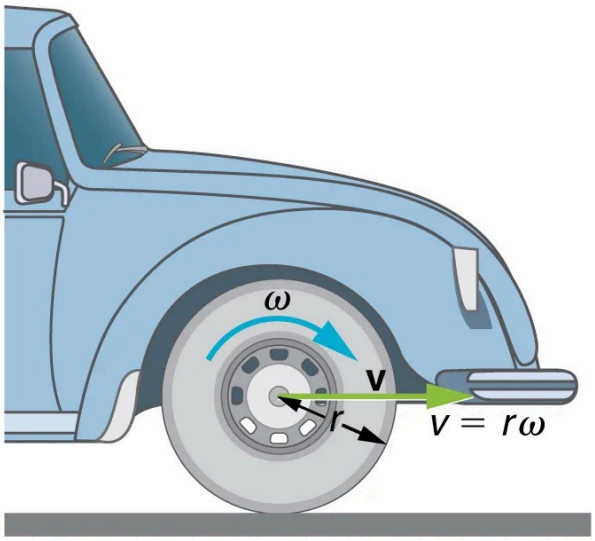


Figure 5.5 A car moving at a velocity v to the right has a tire rotating with an angular velocity ω.The speed of the tread of the tire relative to the axle is v, the same as if the car were jacked up. Thus the car moves forward at linear velocity $v = r\omega$, where r is the tire radius. A larger angular velocity for the tire means a greater velocity for the car. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of the front left side of a blue car, focusing on the wheel. The car is facing to the right. The main elements depicted in the image are:

– Car Body: A simplistic, partial side view of a blue car, showing details like the front bumper, a side mirror, and part of the front wheel well.
– Wheel: The wheel of the car is prominently featured. An arrow labeled with the Greek letter ω (omega) indicates the angular velocity or rotational speed of the wheel, shown in a counterclockwise direction.
– Radius: A line labeled 'r' is shown from the center of the wheel to its edge, representing the radius of the wheel.
– Velocity Vector: An arrow labeled 'v' extends horizontally from the center of the wheel toward the right, indicating linear velocity. Another arrow from outside the wheel to the right is labeled with the formula 'v = rω', relating linear velocity, radius, and angular velocity.

The illustration visually explains the relationship between angular velocity, linear velocity, and radius in a rotating wheel of a car.

Example 5.1

How Fast Does a Car Tire Spin?

Calculate the angular velocity of a 0.300 m radius car tire when the car travels at 15.0m/s (about 54km/h). See Figure 5.5.

Strategy

Because the linear speed of the tire rim is the same as the speed of the car, we have $v = 15.0\ \text{m/s}$.

The radius of the tire is given to be $r = 0.300\ \text{m}$. Knowing v and r, we can use the second relationship in $v = r\omega, \omega = \frac{v}{r}$ to calculate the angular velocity.

Solution

To calculate the angular velocity, we will use the following relationship:

$$\omega = \frac{v}{r}.$$

5.10

Substituting the knowns,

$$\omega = \frac{15.0\text{m/s}}{0.300\text{m}} = 50.0\text{rad/s}.$$

5.11

Discussion

When we cancel units in the above calculation, we get 50.0/s. But the angular velocity must have units of rad/s. Because radians are actually unitless (radians are defined as a ratio of distance), we can simply insert them into the answer for the angular velocity. Also note that if an earth mover with much larger tires, say 1.20 m in radius, were moving at the same speed of 15.0 m/s, its tires would rotate more slowly. They would have an angular velocity

$$\omega = (15.0\text{m/s})/(1.20\text{m}) = 12.5\text{rad/s}.$$

5.12

Both ω and v have directions (hence they are angular and linear *velocities*, respectively). Angular velocity has only two directions with respect to the axis of rotation—it is either clockwise or counterclockwise. Linear velocity is tangent to the path, as illustrated in Figure 5.6.

Take-Home Experiment

Tie an object to the end of a string and swing it around in a horizontal circle above your head (swing at your wrist). Maintain uniform speed as the object swings and measure the angular velocity of the motion. What is the approximate speed of the object? Identify a point close to your hand and take appropriate measurements to calculate the linear speed at this point. Identify other circular motions and measure their angular velocities.

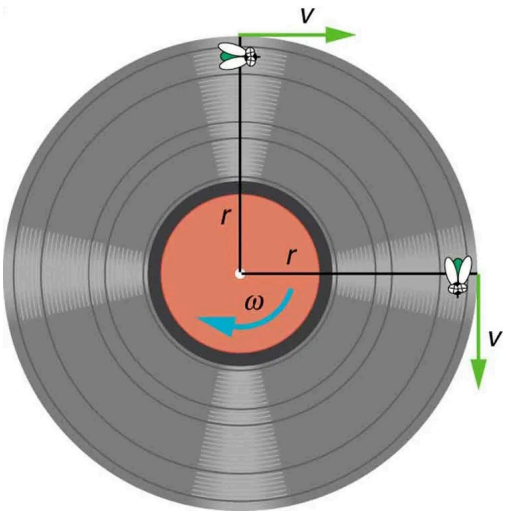


Figure 5.6 As an object moves in a circle, here a fly on the edge of an old-fashioned vinyl record, its instantaneous velocity is always tangent to the circle. The direction of the angular velocity is clockwise in this case. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of a vinyl record with two cartoon flies positioned on opposite sides of the record's surface. The record is shown from a top view and contains the following elements:

– The vinyl record is circular, with alternating light and dark gray stripes indicating grooves.
– At the center is an orange circular label area.
– A blue arrow within the orange section indicates rotation in a counterclockwise direction, labeled with the Greek letter "ω" (omega), signifying angular velocity.
– Two straight black lines extend from the center to the edges of the record, each labeled with "r" for radius.
– At the end of each radial line is a cartoon fly, each positioned on the outer edge of the record.
– Each fly has an associated green arrow pointing tangentially along the edge of the record, representing linear velocity, labeled "v".
– The fly at the top of the image has an arrow pointing to the right, while the fly on the right side of the image has an arrow pointing downward.

Ladybug Revolution

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

Click to view content.

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5.2 CENTRIPETAL ACCELERATION

Learning Objectives

By the end of this section, you will be able to:

- Establish the expression for centripetal acceleration.
- Explain the centrifuge.

We know from kinematics that acceleration is a change in velocity, either in its magnitude or in its direction, or both. In uniform circular motion, the direction of the velocity changes constantly, so there is always an associated acceleration, even though the magnitude of the velocity might be constant. You experience this acceleration yourself when you turn a corner in your car. (If you hold the wheel steady during a turn and move at constant speed, you are in uniform circular motion.) What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we examine the direction and magnitude of that acceleration.

Figure 5.7 shows an object moving in a circular path at constant speed. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the centripetal acceleration(a_c); centripetal means “toward the center” or “center seeking.”

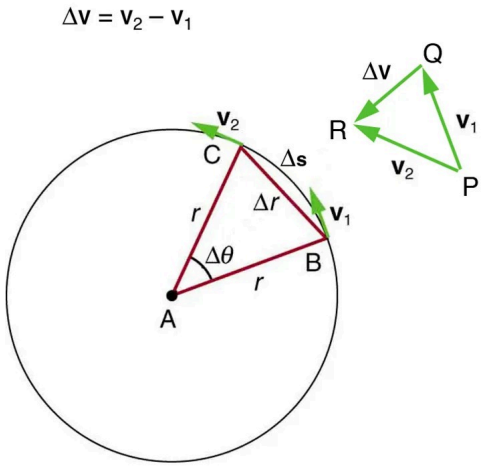


Figure 5.7 The directions of the velocity of an object at two different points are shown, and the change in velocity $\Delta \mathbf{v}$ *is seen to point directly toward the center of curvature. (See small inset.) Because* $\mathbf{a}_c = \Delta \mathbf{v}/\Delta t$*, the acceleration is also toward the center;* $\mathbf{a}_c$ *is called centripetal acceleration. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a diagram illustrating the concept of change in velocity in circular motion. The key elements are as follows:

– A circle with a center marked as point A.
– Two radii, AC and AB, are drawn from the center A to points C and B on the circumference.
– The radius AC is labeled as vector v_2 and AB as vector v_1.
– The angle between radii AC and AB is labeled $\Delta\theta$.
– The arc between points B and C on the circle's circumference is labeled as Δs.
– A line segment from B to C, representing the chord, is labeled Δr.
– A green arrow labeled $\Delta v = v_2 - v_1$ indicates the change in velocity.
– Nearby, an additional vector diagram with arrows represents the velocities and their change:
– Arrow PQ represents v_1.
– Arrow QR represents v_2.
– Arrow RP represents Δv.

The diagram reflects how a change in direction of velocity occurs during circular motion.

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? Note that the triangle formed by the velocity vectors and the one formed by the radii r and Δs are similar. Both

the triangles ABC and PQR are isosceles triangles (two equal sides). The two equal sides of the velocity vector triangle are the speeds $v_1 = v_2 = v$. Using the properties of two similar triangles, we obtain

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}.$$
5.13

Acceleration is $\Delta v/\Delta t$, and so we first solve this expression for Δv:

$$\Delta v = \frac{v}{r}\Delta s.$$
5.14

Then we divide this by Δt, yielding

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta s}{\Delta t}.$$
5.15

Finally, noting that $\Delta v/\Delta t = a_\text{c}$ and that $\Delta s/\Delta t = v$, the linear or tangential speed, we see that the magnitude of the centripetal acceleration is

$$a_\text{c} = \frac{v^2}{r},$$
5.16

which is the acceleration of an object in a circle of radius r at a speed v. So, centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that a_c is proportional to speed squared, implying, for example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that a_c is greater for tighter turns, as you have probably noticed.

It is also useful to express a_c in terms of angular velocity. Substituting $v = r\omega$ into the above expression, we find $a_\text{c} = (r\omega)^2/r = r\omega^2$. We can express the magnitude of centripetal acceleration using either of two equations:

$$a_\text{c} = \frac{v^2}{r}; a_\text{c} = r\omega^2.$$
5.17

Recall that the direction of a_c is toward the center. You may use whichever expression is more convenient, as illustrated in examples below.

A centrifuge (see Figure 5.8b) is a rotating device used to separate specimens of different densities. High centripetal acceleration significantly decreases the time it takes for separation to occur, and makes separation possible with small samples. Centrifuges are used in a variety of applications in science and medicine, including the separation of single cell suspensions such as bacteria, viruses, and blood cells from a liquid medium and the separation of macromolecules, such as DNA and protein, from a solution. Centrifuges are often rated in terms of their centripetal acceleration relative to acceleration due to gravity (g); maximum centripetal acceleration of several hundred thousand g is possible in a vacuum. Human centrifuges, extremely large centrifuges, have been used to test the tolerance of astronauts to the effects of accelerations larger than that of Earth's gravity.

Example 5.2

How Does the Centripetal Acceleration of a Car Around a Curve Compare with That Due to Gravity?

What is the magnitude of the centripetal acceleration of a car following a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h)? Compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed. See Figure 5.8(a).

Strategy

Because v and r are given, the first expression in $a_{\mathrm{c}} = \dfrac{v^2}{r}; a_{\mathrm{c}} = r\omega^2$ is the most convenient to use.

Solution

Entering the given values of $v = 25.0\mathrm{m/s}$ and $r = 500\ \mathrm{m}$ into the first expression for a_{c} gives

$$a_{\mathrm{c}} = \frac{v^2}{r} = \frac{(25.0\mathrm{m/s})^2}{500\ \mathrm{m}} = 1.25\mathrm{m/s}^2.$$

5.18

Discussion

To compare this with the acceleration due to gravity $(g = 9.80\mathrm{m/s}^2)$, we take the ratio of $a_{\mathrm{c}}/g = \left(1.25\mathrm{m/s}^2\right)/\left(9.80\mathrm{m/s}^2\right) = 0.128$. Thus, $a_{\mathrm{c}} = 0.128\ \mathrm{g}$ and is noticeable especially if you were not wearing a seat belt.

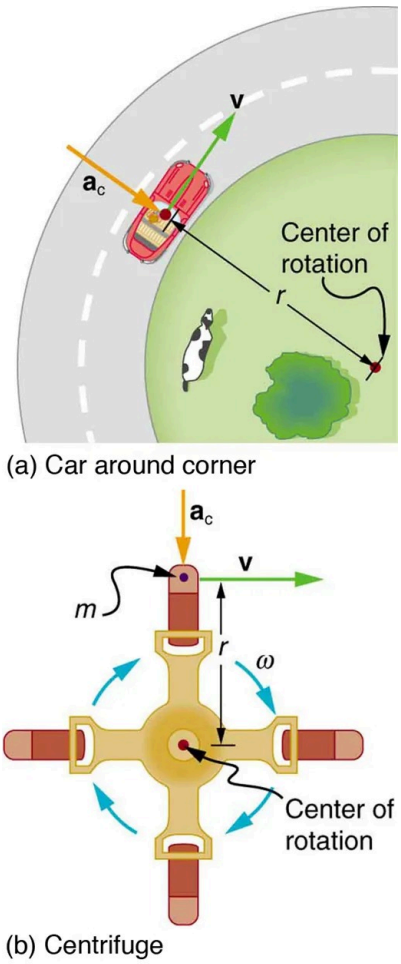


Figure 5.8 (a) The car following a circular path at constant speed is accelerated perpendicular to its velocity, as shown. The magnitude of this centripetal acceleration is found in . (b) A particle of mass in a centrifuge is rotating at constant angular velocity . It must be accelerated perpendicular to its velocity or it would continue in a straight line. The magnitude of the necessary acceleration is found in . Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two separate diagrams labeled (a) and (b).

Diagram (a): Car around corner
– A car is depicted moving on a curved road.
– The road has a dashed line indicating the lane divider.
– There is a cow and a small patch of grass in the green area beside the road.
– The car is shown with vectors: a green vector labeled "v" indicating velocity, an orange vector labeled "a_c" indicating centripetal acceleration, and a black dashed line extending from the car to a point labeled "Center

of rotation."
– The distance from the car to the center of rotation is labeled "r."

Diagram (b): Centrifuge
– The diagram shows a rotating centrifuge with a cross-shaped structure.
– At the end of each arm is a brown tube represented with a small mass labeled "m."
– An orange vector labeled "a_c" represents centripetal acceleration, and a green vector labeled "v" represents velocity.
– A black arrow labeled "ω" indicates angular velocity.
– The distance from the mass to the center is marked by "r," and the central point is labeled "Center of rotation."
– Four blue arrows on each arm depict the direction of rotation.

Example 5.3

How Big Is the Centripetal Acceleration in an Ultracentrifuge?

Calculate the centripetal acceleration of a point 7.50 cm from the axis of an ultracentrifuge spinning at

$$7.5 \times 10^4 \text{ rev/min.}$$

Determine the ratio of this acceleration to that due to gravity. See Figure 5.8(b).

Strategy

The term rev/min stands for revolutions per minute. By converting this to radians per second, we obtain the angular velocity ω. Because r is given, we can use the second expression in the equation

$$a_c = \frac{v^2}{r}; a_c = r\omega^2$$

to calculate the centripetal acceleration.

Solution

To convert $7.\ 50\ \times 10^{\ 4}\text{rev}\ /\text{min}$ to radians per second, we use the facts that one revolution is $2\pi\text{rad}$ and one minute is 60.0 s. Thus,

$$\omega = 7.50 \times 10^4 \frac{\text{rev}}{\text{min}} \times \frac{2\pi\text{rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60.0 \text{ s}} = 7854 \text{ rad/s.}$$

5.19

Now the centripetal acceleration is given by the second expression in $a_c = \frac{v^2}{r}; a_c = r\omega^2$ as

$$a_{\mathrm{c}} = r\omega^2.$$

5.20

Converting 7.50 cm to meters and substituting known values gives

$$a_{\mathrm{c}} = (0.0750\ \mathrm{m})\,(7854\ \mathrm{rad/s})^2 = 4.63 \times 10^6\ \mathrm{m/s^2}.$$

5.21

Note that the unitless radians are discarded in order to get the correct units for centripetal acceleration. Taking the ratio of a_{c} to g yields

$$\frac{a_{\mathrm{c}}}{g} = \frac{4.63 \times 10^6}{9.80} = 4.72 \times 10^5.$$

5.22

Discussion

This last result means that the centripetal acceleration is 472,000 times as strong as g. It is no wonder that such high ω centrifuges are called ultracentrifuges. The extremely large accelerations involved greatly decrease the time needed to cause the sedimentation of blood cells or other materials.

Of course, a net external force is needed to cause any acceleration, just as Newton proposed in his second law of motion. So a net external force is needed to cause a centripetal acceleration. In 5.3 Centripetal Force, we will consider the forces involved in circular motion.

PhET Explorations

Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

Click to view content.

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5.3 CENTRIPETAL FORCE

Learning Objectives

By the end of this section, you will be able to:

- Calculate coefficient of friction on a car tire.
- Calculate ideal speed and angle of a car on a turn.

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a centripetal force. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: net $\mathrm{F} = \mathrm{ma}$. For uniform circular motion, the acceleration is the centripetal acceleration— $a = a_c$. Thus, the magnitude of centripetal force $\mathrm{F_c}$ is

$$\mathrm{F_c} = (m\mathrm{a})_\mathrm{c}.$$
5.23

By using the expressions for centripetal acceleration a_c from $a_c = \frac{v^2}{r}; a_c = r\omega^2$, we get two expressions for the centripetal force $\mathrm{F_c}$ in terms of mass, velocity, angular velocity, and radius of curvature:

$$F_c = m\frac{v^2}{r}; F_c = \mathrm{mr}\omega^2.$$
5.24

You may use whichever expression for centripetal force is more convenient. Centripetal force F_c is always

perpendicular to the path and pointing to the center of curvature, because $\mathbf{a}_c$ is perpendicular to the velocity and pointing to the center of curvature.

Note that if you solve the first expression for r, you get

$$r = \frac{mv^2}{F_c}.$$
5.25

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.

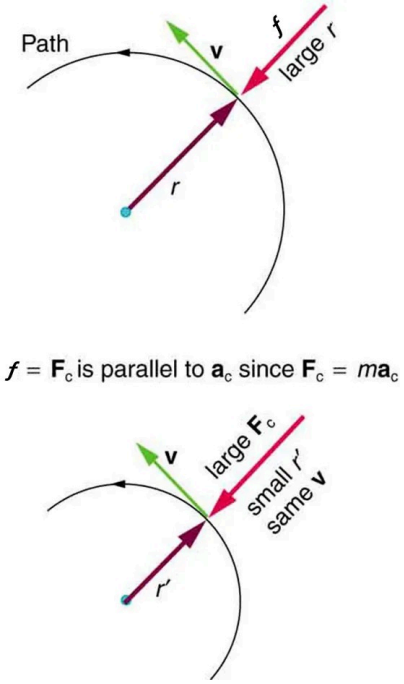


Figure 5.9 The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the $\mathrm{F_c}$*, the smaller the radius of curvature* r *and the sharper the curve. The second curve has the same* v*, but a larger* $\mathrm{F_c}$ *produces a smaller* r'*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two diagrams illustrating centripetal force in circular motion.

Top Diagram:
– A circle is depicted, with a point on its circumference.
– A vector labeled “v” is shown in green, pointing tangentially to the circle, indicating velocity.

– A vector labeled “f” is shown in red, pointing radially inward towards the circle’s center, indicating force.
– A vector labeled “r” is pointing from the circle’s center to the point on its circumference.
– Text labels this situation as “large r.”

Bottom Diagram:
– Another circle, with a smaller radius than the first, is depicted with a point on its circumference.
– A green vector labeled “v” points tangentially to the circle, indicating velocity, with the note “same v.”
– A red vector labeled “large Fc” points radially inward, indicating a larger centripetal force compared to the first diagram.
– A vector labeled “r′” points from the circle’s center to the point on its circumference.
– Text labels this as “small r’.”

Additional Text:
– Between the diagrams, there is an explanation: “f = Fc is parallel to ac since Fc = mac”, indicating the relationship between centripetal force and acceleration.

Example 5.4

What Coefficient of Friction Do Car Tires Need on a Flat Curve?

(a) Calculate the centripetal force exerted on a 900 kg car that negotiates a 500 m radius curve at 25.0 m/s.

(b) Assuming an unbanked curve, find the minimum static coefficient of friction, between the tires and the road, static friction being the reason that keeps the car from slipping (see Figure 5.10).

Strategy and Solution for (a)

We know that $F_{\mathrm{c}} = \dfrac{mv^2}{r}$. Thus,

$$F_{\mathrm{c}} = \frac{mv^2}{r} = \frac{(900\ \mathrm{kg})\,(25.0\ \mathrm{m/s})^2}{(500\ \mathrm{m})} = 1125\ \mathrm{N}.$$

5.26

Strategy for (b)

Figure 5.10 shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is $\mu_{\mathrm{s}}N$, where μ_{s} is the static coefficient of friction and N is the normal force. The normal force equals the car's weight on level ground, so that

$N = mg$. Thus the centripetal force in this situation is

$$F_{\mathrm{c}} = f = \mu_{\mathrm{s}}N = \mu_{\mathrm{s}}\mathrm{mg}.$$

5.27

Now we have a relationship between centripetal force and the coefficient of friction. Using the first expression for F_{c} from the equation

$$\left.\begin{aligned} F_{\mathrm{c}} &= m\frac{v^2}{r} \\ F_{\mathrm{c}} &= \mathrm{mr}\omega^2 \end{aligned}\right\},$$

5.28

$$m\frac{v^2}{r} = \mu_{\mathrm{s}}\mathrm{mg}.$$

5.29

We solve this for μ_{s}, noting that mass cancels, and obtain

$$\mu_{\mathrm{s}} = \frac{v^2}{\mathrm{rg}}.$$

5.30

Solution for (b)

Substituting the knowns,

$$\mu_{\mathrm{s}} = \frac{(25.0\ \mathrm{m/s})^2}{(500\ \mathrm{m})\,(9.80\ \mathrm{m/s^2})} = 0.13.$$

5.31

(Because coefficients of friction are approximate, the answer is given to only two digits.)

Discussion

We could also solve part (a) using the first expression in $\left.\begin{aligned} F_{\mathrm{c}} &= m\frac{v^2}{r} \\ F_{\mathrm{c}} &= \mathrm{mr}\omega^2 \end{aligned}\right\}$, because m,v, and r are given. The coefficient of friction found in part (b) is much smaller than is typically found between tires and roads. The car will still negotiate the curve if the

coefficient is greater than 0.13, because static friction is a responsive force, being able to assume a value less than but no more than $\mu_s N$. A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less as will be discussed below.

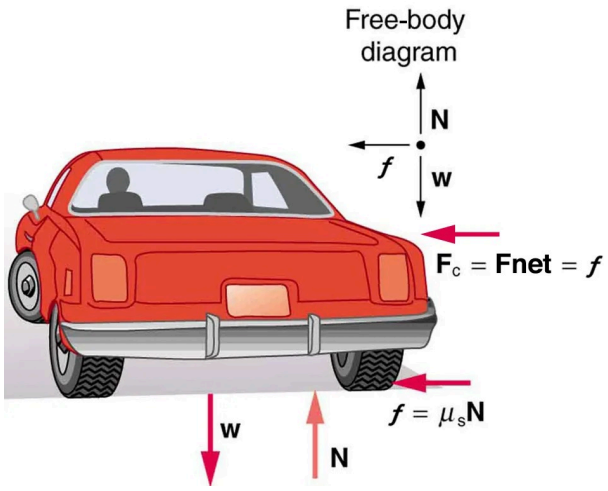


Figure 5.10 This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a red car viewed from the rear. Alongside the car, there is a free-body diagram illustrating the various forces acting on it.

– Forces and Labels:
– An upward arrow labeled “N” represents the normal force.
– A downward arrow labeled “w” indicates the weight of the car.
– A leftward arrow labeled “f” represents friction.

– To the right of the car, the equation "$F_c = F_{net} = f$" is noted, highlighting the net force.
– Another notation in the lower right explains "$f = \mu_s N$", where "μ_s" is the coefficient of static friction and "N" is the normal force.

– Additional Details:
– The car has a visible rear bumper, taillights, and tires.
– There is a side mirror on the car's left side.
– The car appears to be on a flat surface.

The diagram serves to illustrate the physics of forces involved, particularly focusing on friction, weight, and normal force.

Let us now consider banked curves, where the slope of the road helps you negotiate the curve. See Figure 5.11. The greater the angle θ, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for θ for an ideally banked curve and consider an example related to it.

For ideal banking, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force N in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

Figure 5.11 shows a free body diagram for a car on a frictionless banked curve. If the angle θ is ideal for the speed and radius, then the net external force will equal the necessary centripetal force. The only two external forces acting on the car are its weight $\mathbf{w}$ and the normal force of the road $\mathbf{N}$. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude mv^2/r. Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, and so this must equal the centripetal force—that is,

$$N\sin\theta = \frac{mv^2}{r}.$$

5.32

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From the figure, we see that the vertical component of the normal force is $N\cos\theta$, and the only other vertical force is the car's weight. These must be equal in magnitude; thus,

$$N\cos\theta = \text{mg}.$$
5.33

Now we can combine the last two equations to eliminate N and get an expression for θ, as desired. Solving the second equation for $N = \text{mg}/\left(\cos\theta\right)$, and substituting this into the first yields

$$\text{mg}\frac{\sin\theta}{\cos\theta} = \frac{\text{mv}^2}{r}$$
5.34

$$\text{mgtan}\left(\theta\right) = \frac{mv^2}{r}$$
$$\tan\theta = \frac{v^2}{\text{rg}}.$$
5.35

Taking the inverse tangent gives

$$\theta = \tan^{-1}\left(\frac{v^2}{\text{rg}}\right) \text{(ideally banked curve, no friction)}.$$
5.36

This expression can be understood by considering how θ depends on v and r. A large θ will be obtained for a large v and a small r. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve is frictionless. Note that θ does not depend on the mass of the vehicle.

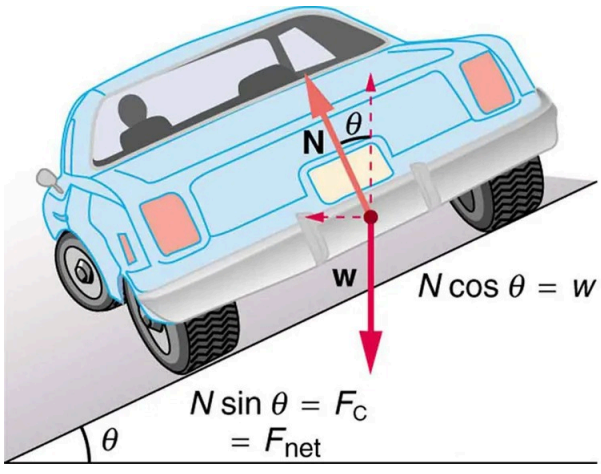


Figure 5.11 The car on this banked curve is moving away and turning to the left. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a blue car positioned on an inclined surface. The car is viewed slightly from the rear and side, with two visible passengers inside. The rear left tire is clearly visible, as the car is tilted downward along the slope.

Several vector arrows and mathematical notations are overlaid on the image to depict forces acting on the car:

- A red dashed arrow labeled N extends vertically upward from the center of the car, and a red solid arrow labeled w points downward, indicating the weight.
- An angular notation θ is shown where the vertical dashed line of N extends from the car, depicting the angle between the normal force and the inclined plane.
- A horizontal red arrow is directed to the left, labeled with equations: $N \sin \theta = F_c = F_{net}$, showing the component of force parallel to the incline.
- Two other equations are included: $N \cos \theta = w$, showing the perpendicular component of the weight, in line with the normal force.
- An arc labeled θ is shown at the bottom left, representing the incline's angle relative to the horizontal.

The diagram illustrates the forces and components acting on a car on an inclined plane, focusing on the normal force and weight relationship.

Example 5.5

What Is the Ideal Speed to Take a Steeply Banked Tight Curve?

Curves on some test tracks and race courses, such as the Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100 m radius curve banked at 65.0° should be driven if the road is frictionless.

Strategy

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

Solution

Starting with

$$\tan\theta = \frac{v^2}{rg}$$

5.37

we get

$$v = (rg\tan\theta)^{1/2}.$$

5.38

Noting that tan 65.0º = 2.14, we obtain

$$v = \left[(100\ \text{m})\,(9.80 m/\text{s}^2)\,(2.14)\right]^{1/2}$$
$$= 45.8\ \text{m/s}.$$

5.39

Discussion

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Calculations similar to those in the preceding examples can be performed for a host of interesting situations in which centripetal force is involved—a number of these are presented in this chapter's Problems and Exercises.

Take-Home Experiment

Ask a friend or relative to swing a golf club or a tennis racquet. Take appropriate measurements to estimate the centripetal acceleration of the end of the club or racquet. You may choose to do this in slow motion.

PhET Explorations

Gravity and Orbits

Move the sun, earth, moon and space station to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies, and turn off gravity to see what would happen without it!

Access multimedia content

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5.4 NEWTON'S UNIVERSAL LAW OF GRAVITATION

Learning Objectives

By the end of this section, you will be able to:

- Explain Earth's gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Discuss weightlessness in space.
- Examine the Cavendish experiment

What do aching feet, a falling apple, and the orbit of the Moon have in common? Each is caused by the gravitational force. Our feet are strained by supporting our weight—the force of Earth's gravity on us. An apple falls from a tree because of the same force acting a few meters above Earth's surface. And the Moon orbits Earth because gravity is able to supply the necessary centripetal force at a distance of hundreds of millions of meters. In fact, the same force causes planets to orbit the Sun, stars to orbit the center of the galaxy, and galaxies to cluster together. Gravity is another example of underlying simplicity in nature. It is the weakest of the four basic forces found in nature, and in some ways the least understood. It is a force that acts at a distance, without physical contact, and is expressed by a formula that is valid everywhere in the universe, for masses and distances that vary from the tiny to the immense.

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See Figure 5.12. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause. Some of Newton's contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form

and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph—it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others. Other prominent scientists and mathematicians of the time, particularly those outside of England, were reluctant to accept Newton's principles. It took the work of another prominent philosopher, writer, and scientist, Émilie du Châtelet, to establish the Newtonian gravitation as the accurate and overarching law. Du Châtelet, who had earlier laid the foundation for the understanding of conservation of energy as well as the principle that light had no mass, translated and augmented Newton's key work. She also utilized calculus to explain gravity, which helped lead to its acceptance.



Figure 5.12 According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton's apple is a part of worldwide folklore and may even be based in fact. Newton's universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Like many revolutionary discoveries, it was not immediately accepted. Prominent French scientist and philosopher Émilie du Châtelet helped establish Newton's theory in France and mainland Europe. Image from OpenStax College Physics 2e, CC-BY 4.0

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, Newton's universal law of gravitation states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

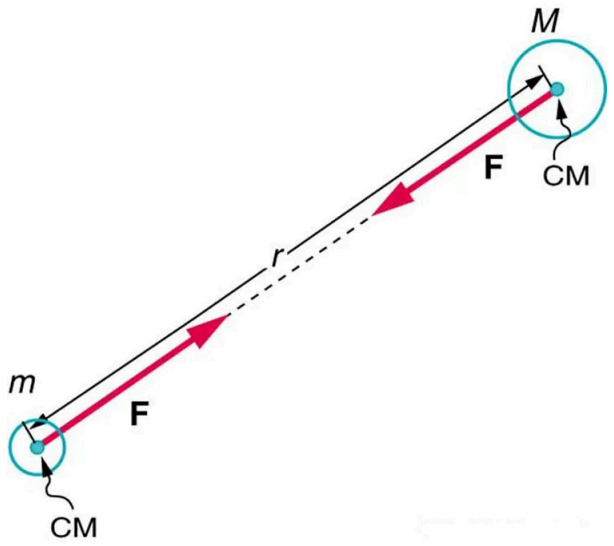


Figure 5.13 Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton's third law. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the gravitational interaction between two masses. There are two circles labeled "CM" to denote the center of mass. The first mass, labeled "m," is on the left, and the second mass, labeled "M," is on the right. Both have their centers of mass indicated.

A dashed line connects the centers of mass of the two objects, marked with the letter "r" to represent the distance between them. Two red arrows labeled "F" point towards the centers of each mass, indicating the direction of the gravitational force each mass exerts on the other.

The diagram visually represents Newton's Law of Universal Gravitation, showing how two bodies pull towards each other with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between their centers of mass.

Misconception Alert

The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.

The bodies we are dealing with tend to be large. To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the center of mass (CM). The concept of center of mass applies both to a single, extended object and to a system of objects that may be moving relative to each other. The center of mass is the average location of the mass if we divide the object (or system) into pieces of equal mass. The center of mass of a system of objects will respond to external forces as if all of the mass of the system were concentrated there. For two bodies having masses m and M with a distance r between their centers of mass, the equation for Newton's universal law of gravitation is

$$F = G\frac{\mathrm{mM}}{r^2},$$

5.40

where F is the magnitude of the gravitational force and G is a proportionality factor called the gravitational constant. G is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be

$$G = 6.674 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$$

5.41

in SI units. Note that the units of G are such that a force in newtons is obtained from $F = G\frac{\mathrm{mM}}{r^2}$, when considering masses in kilograms and distance in meters. For example, two 1.000 kg masses separated by 1.000 m will experience a gravitational attraction of $6.674 \times 10^{-11}\mathrm{N}$. This is an extraordinarily small force. The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us. In fact, our body weight is the force of attraction of the *entire Earth* on us with a mass of $6 \times 10^{24}\mathrm{kg}$.

Recall that the acceleration due to gravity g is about $9.80 m/\mathrm{s}^2$ on Earth. We can now determine why this is

so. The weight of an object *mg* is the gravitational force between it and Earth. Substituting *mg* for F in Newton's universal law of gravitation gives

$$\mathrm{mg} = G\frac{\mathrm{mM}}{r^2},$$

5.42

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See Figure 5.14. The mass m of the object cancels, leaving an equation for g:

$$g = G\frac{M}{r^2}.$$

5.43

Substituting known values for Earth's mass and radius (to three significant figures),

$$g = \left(6.67 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right) \times \frac{5.98 \times 10^{24}\mathrm{kg}}{\left(6.38 \times 10^{6}\mathrm{m}\right)^2},$$

5.44

and we obtain a value for the acceleration of a falling body:

$$g = 9.80\mathrm{m/s^2}.$$

5.45

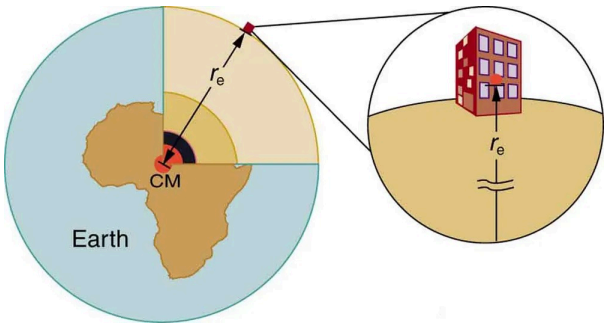


Figure 5.14 The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram with two main sections. On the left, there is a side-view cross-section of the Earth,

primarily showing the African continent. The cross-section highlights the Earth's core, marked by a small red circle labeled "CM" at the center. The diagram shows an angular section extending from the core to Earth's surface, creating a wedge shape with two lines. One line extends vertically from the Earth's surface to "CM," denoting a radius, labeled as r_e.

On the right side, there is a magnified view of a multi-story red building on Earth's surface. A black arrow points from the building's base to its top, again labeled as r_e, indicating the building's height. The whole setup suggests a comparison or relation between the Earth's core, surface, and the building, potentially illustrating gravitational concepts or radius measurements.

This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

Take-Home Experiment

Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

Making Connections

Attempts are still being made to understand the gravitational force. Modern physics is exploring the connections of gravity to other forces, space, and time. General relativity alters our view of gravitation, leading us to think of gravitation as bending space and time.

In the following example, we make a comparison similar to one made by Newton himself. He noted that if the gravitational force caused the Moon to orbit Earth, then the acceleration due to gravity should equal the centripetal acceleration of the Moon in its orbit. Newton found that the two accelerations agreed "pretty nearly."

Example 5.6

Earth's Gravitational Force Is the Centripetal Force Making the Moon Move in a Curved Path

(a) Find the acceleration due to Earth's gravity at the distance of the Moon.

(b) Calculate the centripetal acceleration needed to keep the Moon in its orbit (assuming a circular orbit about a fixed Earth), and compare it with the value of the acceleration due to Earth's gravity that you have just found.

Strategy for (a)

This calculation is the same as the one finding the acceleration due to gravity at Earth's surface, except that $r$is the distance from the center of Earth to the center of the Moon. The radius of the Moon's nearly circular orbit is $3.84 \times 10^8 \text{m}$.

Solution for (a)

Substituting known values into the expression for g found above, remembering that M is the mass of Earth not the Moon, yields

$$g = G\frac{M}{r^2} = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \times \frac{5.98 \times 10^{24} \text{kg}}{(3.84 \times 10^8 \text{m})^2}$$
$$= 2.70 \times 10^{-3} \text{m/s.}^2$$

5.46

Strategy for (b)

Centripetal acceleration can be calculated using either form of

$$\left.\begin{aligned} a_c &= \frac{v^2}{r} \\ a_c &= r\omega^2 \end{aligned}\right\}.$$

5.47

We choose to use the second form:

$$a_c = r\omega^2,$$

5.48

where ω is the angular velocity of the Moon about Earth.

Solution for (b)

Given that the period (the time it takes to make one complete rotation) of the Moon's orbit is 27.3 days, (d) and using

$$1d \times 24\frac{\text{hr}}{\text{d}} \times 60\frac{min}{\text{hr}} \times 60\frac{s}{\text{min}} = 86,400 \text{ s}$$

5.49

we see that

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{(27.3 \text{ d})\,(86,400 \text{ s/d})} = 2.66 \times 10^{-6}\frac{\text{rad}}{\text{s}}.$$

5.50

The centripetal acceleration is

$$\begin{aligned} a_c &= r\omega^2 = (3.84 \times 10^8 \text{m})\,(2.66 \times 10^{-6}\text{rad/s})^2 \\ &= 2.72 \times 10^{-3}\text{m/s.}^2 \end{aligned}$$

5.51

The direction of the acceleration is toward the center of the Earth.

Discussion

The centripetal acceleration of the Moon found in (b) differs by less than 1% from the acceleration due to Earth's gravity found in (a). This agreement is approximate because

the Moon's orbit is slightly elliptical, and Earth is not stationary (rather the Earth-Moon system rotates about its center of mass, which is located some 1700 km below Earth's surface). The clear implication is that Earth's gravitational force causes the Moon to orbit Earth.

Why does Earth not remain stationary as the Moon orbits it? This is because, as expected from Newton's third law, if Earth exerts a force on the Moon, then the Moon should exert an equal and opposite force on Earth (see Figure 5.15). We do not sense the Moon's effect on Earth's motion, because the Moon's gravity moves our bodies right along with Earth but there are other signs on Earth that clearly show the effect of the Moon's gravitational force.

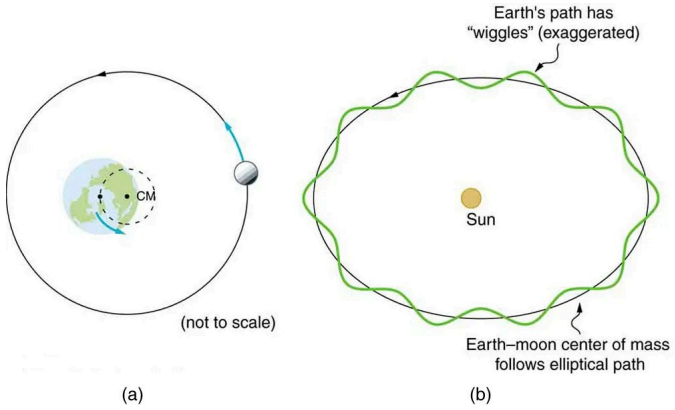


Figure 5.15 (a) Earth and the Moon rotate approximately once a month around their common center of mass. (b) Their center of mass orbits the Sun in an elliptical orbit, but Earth's path around the Sun has "wiggles" in it. Similar wiggles in the paths of stars have been observed and are considered direct evidence of planets orbiting those stars. This is important because the planets' reflected light is often too dim to be observed. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is composed of two diagrams labeled (a) and (b), illustrating the orbital dynamics of the Earth and Moon system.

Diagram (a):

– This diagram represents the Earth and Moon orbiting their common center of mass (CM), which is marked with a small black dot inside a dashed circle.

– The Earth is depicted in green and white, with the CM located beneath its surface.
– The Moon is shown as a smaller gray sphere positioned along a circular path around the CM.
– Arrows indicate the direction of the Moon's orbit around the Earth.
– The label "not to scale" indicates that the sizes and distances are not proportionally accurate.

Diagram (b):

– This diagram shows the elliptical orbit of the Earth-Moon system around the Sun.
– A yellow circle represents the Sun at the center of the ellipse.
– The path of the Earth is depicted with green waves superimposed on the ellipse, illustrating the "wiggles" in its trajectory due to the Moon's gravitational influence.
– Black arrows indicate the direction of this elliptical path.
– Labels describe this path as the Earth's with exaggerated "wiggles" and state that the Earth-Moon center of mass follows this elliptical trajectory.

These diagrams together illustrate the complexity of the Earth-Moon system's movement relative to both their own center of mass and the Sun.

Tides

Ocean tides are one very observable result of the Moon's gravity acting on Earth. Figure 5.16 is a simplified drawing of the Moon's position relative to the tides. Because water easily flows on Earth's surface, a high tide is created on the side of Earth nearest to the Moon, where the Moon's gravitational pull is strongest. Why is there also a high tide on the opposite side of Earth? The answer is that Earth is pulled toward the Moon more than the water on the far side, because Earth is closer to the Moon. So the water on the side of Earth closest to the Moon is pulled away from Earth, and Earth is pulled away from water on the far side. As Earth rotates, the tidal bulge (an effect of the tidal forces between an orbiting natural satellite and the primary planet that it orbits) keeps its orientation with the Moon. Thus there are two tides per day (the actual tidal period is about 12 hours and 25.2 minutes), because the Moon moves in its orbit each day as well).

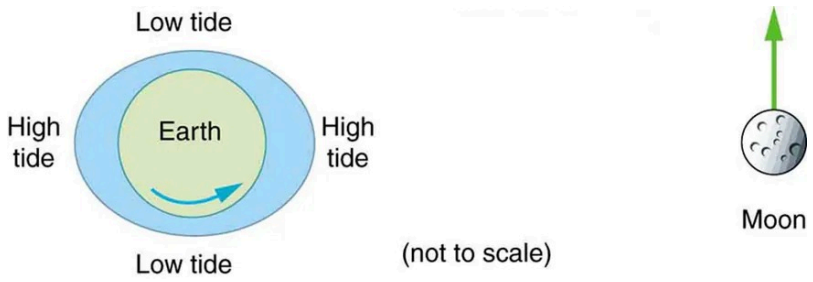


Figure 5.17 The Moon causes ocean tides by attracting the water on the near side more than Earth, and by attracting Earth more than the water on the far side. The distances and sizes are not to scale. For this simplified representation of the Earth-Moon system, there are two high and two low tides per day at any location, because Earth rotates under the tidal bulge. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the effect of the Moon's gravitational pull on Earth's tides. It shows a side view of Earth surrounded by water, with two labeled zones: "High tide" and "Low tide."

– Earth (Center): Depicted as a green circle.
– Water (Around Earth): Shown as a blue oval surrounding Earth.
– High Tide Areas: Indicated at the top and bottom edges of the blue oval.
– Low Tide Areas: Indicated on the left and right sides of the blue oval.
– Arrow: A blue arrow is inside the blue oval pointing in a counterclockwise direction to represent Earth's rotation.

To the right of Earth, there is an illustration of the Moon with a green arrow pointing towards Earth. The arrow represents the gravitational pull of the Moon, which causes the tides. Below the diagram, the text "(not to scale)" is shown, indicating that the drawing does not represent actual sizes or distances.

The Sun also affects tides, although it has about half the effect of the Moon. However, the largest tides, called spring tides, occur when Earth, the Moon, and the Sun are aligned. The smallest tides, called neap tides, occur when the Sun is at a 90° angle to the Earth-Moon alignment.

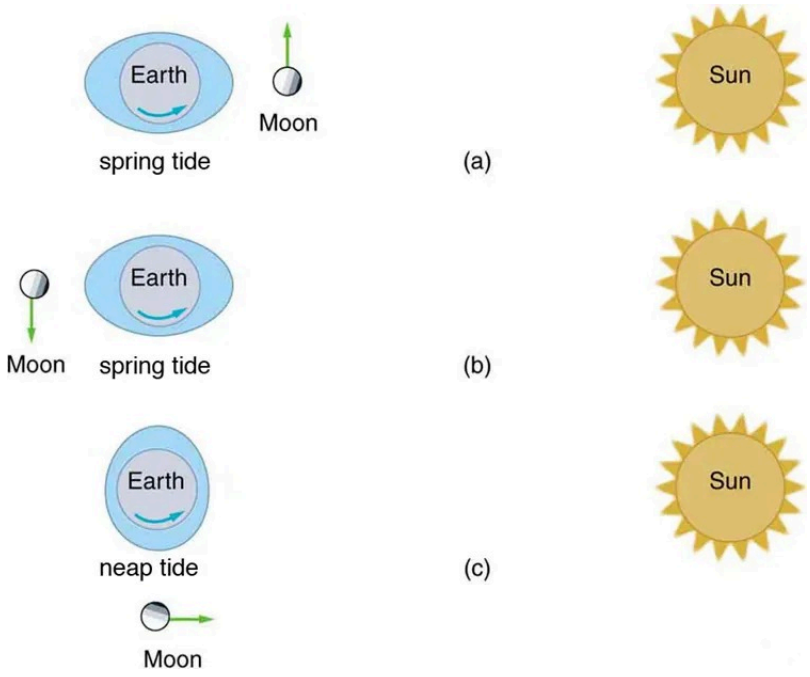


Figure 5.18 (a, b) Spring tides: The highest tides occur when Earth, the Moon, and the Sun are aligned. (c) Neap tide: The lowest tides occur when the Sun lies at 90° *to the Earth-Moon alignment. Note that this figure is not drawn to scale. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of three diagrams, each illustrating different positions of the Earth, Moon, and Sun and their effects on tides. The diagrams are labeled (a), (b), and (c) from top to bottom:

1. **(a) Spring Tide**:

- An image showing the Earth with a large blue oval around it, representing the tidal bulge.
- The Moon is positioned directly above the Earth. A green arrow points up from the Moon, indicating its direction or force.
- The Sun is depicted to the right of the Earth-Moon system with rays extending outward.

2. **(b) Spring Tide**:

- Similar to (a), showing the Earth with a blue tidal bulge around it.
- The Moon is positioned directly to the left of the Earth. A green arrow points down from the Moon.
- The Sun is again depicted to the right, aligned with the Earth and Moon.

3. **(c) Neap Tide**:

- The Earth is shown with a smaller tidal bulge compared to spring tides.
- The Moon is positioned directly below the Earth. A green arrow points right from the Moon.
- The Sun remains to the right, but not aligned with the Earth and Moon.

The Sun is consistently shown as a large circle with rays on the right side of each diagram, symbolizing its influence on the tides.

Tides are not unique to Earth but occur in many astronomical systems. The most extreme tides occur where the gravitational force is the strongest and varies most rapidly, such as near black holes (see Figure 5.19). A few likely candidates for black holes have been observed in our galaxy. These have masses greater than the Sun but have diameters only a few kilometers across. The tidal forces near them are so great that they can actually tear matter from a companion star.

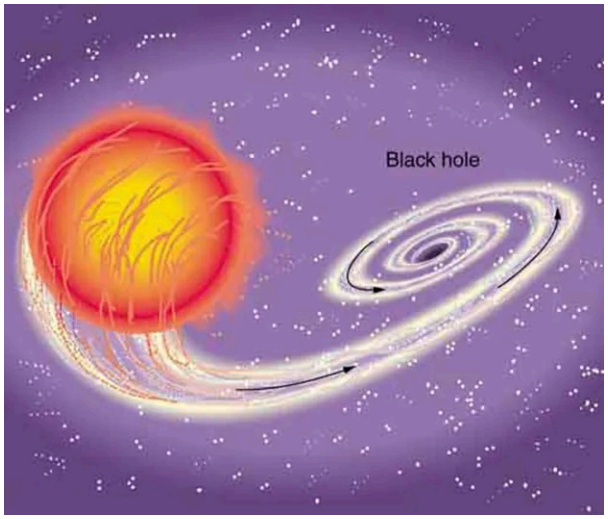


Figure 5.19 A black hole is an object with such strong gravity that not even light can escape it. This black hole was created by the supernova of one star in a two-star system. The tidal forces created by the black hole are so great that it tears matter from the companion star. This matter is compressed and heated as it is sucked into the black hole, creating light and X-rays observable from Earth. Image from OpenStax College Physics 2e, CC-BY 4.0

"Weightlessness" and Microgravity

In contrast to the tremendous gravitational force near black holes is the apparent gravitational field experienced by astronauts orbiting Earth. What is the effect of "weightlessness" upon an astronaut who is in orbit for months? Or what about the effect of weightlessness upon plant growth? Weightlessness doesn't mean that an astronaut is not being acted upon by the gravitational force. There is no "zero gravity" in an astronaut's orbit. The term just means that the astronaut is in free-fall, accelerating with the acceleration due

to gravity. If an elevator cable breaks, the passengers inside will be in free fall and will experience weightlessness. You can experience short periods of weightlessness in some rides in amusement parks.



Figure 5.20 Astronauts experiencing weightlessness on board the International Space Station. Image from Wikimedia Commons, Public Domain

Microgravity refers to an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface. Many interesting biology and physics topics have been studied over the past three decades in the presence of microgravity. Of immediate concern is the effect on astronauts of extended times in outer space, such as at the International Space Station. Researchers have observed that muscles will atrophy (waste away) in this environment. There is also a corresponding loss of bone mass. Study continues on cardiovascular adaptation to space flight. On Earth, blood pressure is usually higher in the feet than in the head, because the higher column of blood exerts a downward force on it, due to gravity. When standing, 70% of your blood is below the level of the heart, while in a horizontal position, just the opposite occurs. What difference does the absence of this pressure differential have upon the heart?

Some findings in human physiology in space can be clinically important to the management of diseases back on Earth. On a somewhat negative note, spaceflight is known to affect the human immune system, possibly making the crew members more vulnerable to infectious diseases. Experiments flown in space also have shown that some bacteria grow faster in microgravity than they do on Earth. However, on a positive note, studies indicate that microbial antibiotic production can increase by a factor of two in space-grown cultures. One hopes to be able to understand these mechanisms so that similar successes can be achieved on the ground. In another area of physics space research, inorganic crystals and protein crystals have been grown in outer space that have much higher quality than any grown on Earth, so crystallography studies on their structure can yield much better results.

Plants have evolved with the stimulus of gravity and with gravity sensors. Roots grow downward and shoots

grow upward. Plants might be able to provide a life support system for long duration space missions by regenerating the atmosphere, purifying water, and producing food. Some studies have indicated that plant growth and development are not affected by gravity, but there is still uncertainty about structural changes in plants grown in a microgravity environment.

The Cavendish Experiment: Then and Now

As previously noted, the universal gravitational constant G is determined experimentally. This definition was first done accurately by Henry Cavendish (1731–1810), an English scientist, in 1798, more than 100 years after Newton published his universal law of gravitation. The measurement of G is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most), using apparatus like that in Figure 5.21. Remarkably, his value for G differs by less than 1% from the best modern value.

One important consequence of knowing G was that an accurate value for Earth's mass could finally be obtained. This was done by measuring the acceleration due to gravity as accurately as possible and then calculating the mass of Earth M from the relationship Newton's universal law of gravitation gives

$$\mathrm{mg} = G\frac{\mathrm{mM}}{r^2},$$

5.52

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See Figure 5.21. The mass m of the object cancels, leaving an equation for g:

$$g = G\frac{M}{r^2}.$$

5.53

Rearranging to solve for M yields

$$M = \frac{gr^2}{G}.$$

5.54

So M can be calculated because all quantities on the right, including the radius of Earth r, are known from direct measurements. Knowing G also allows for the determination of astronomical masses. Interestingly, of all the fundamental constants in physics, G is by far the least well determined.

The Cavendish experiment is also used to explore other aspects of gravity. One of the most interesting questions is whether the gravitational force depends on substance as well as mass—for example, whether one kilogram of lead exerts the same gravitational pull as one kilogram of water. A Hungarian scientist named Roland von Eötvös pioneered this inquiry early in the 20th century. He found, with an accuracy of five parts per billion, that the gravitational force does not depend on the substance. Such experiments continue today, and have improved upon Eötvös' measurements. Cavendish-type experiments such as those of Eric Adelberger and others at the University of Washington, have also put severe limits on the possibility of a fifth force and have verified a major prediction of general relativity—that gravitational energy contributes to rest mass. Ongoing measurements there use a torsion balance and a parallel plate (not spheres, as Cavendish used) to examine how Newton's law of gravitation works over sub-millimeter distances. On this small-scale, do gravitational effects depart from the inverse square law? So far, no deviation has been observed.

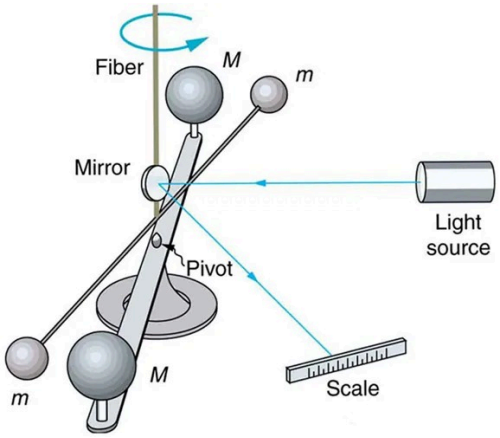


Figure 5.21 Cavendish used an apparatus like this to measure the gravitational attraction between the two suspended spheres (m) and the two on the stand (M) by observing the amount of torsion (twisting) created in the fiber. Distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of a torsion balance apparatus, used to measure very small forces such as gravitational forces.

– At the center, there is an arm balanced on a pivot. This arm has two larger spheres labeled "M" at each end and two smaller spheres labeled "m" positioned near the larger spheres.
– The entire setup is suspended by a fiber, which can twist. There is an arrow indicating a twisting motion of the fiber.
– A mirror is attached to the pivot point on the arm, reflecting a light beam from a light source situated on the right.

– The reflected light strikes a scale, allowing for measurement based on the arm's rotation.
– The arrowed lines illustrate the path of light from the light source to the mirror and then to the scale.

The setup visually represents the experimental arrangement used to detect small force interactions, often associated with gravitational measurements.

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GLOSSARY

angular velocity
: ω, the rate of change of the angle with which an object moves on a circular path

arc length
: Δs, the distance traveled by an object along a circular path

banked curve
: the curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

center of mass
: the point where the entire mass of an object or of a system of objects can be thought to be concentrated

centrifugal force
: a fictitious force that tends to throw an object off when the object is rotating in a non-inertial frame of reference

centripetal acceleration
: the acceleration of an object moving in a circle, directed toward the center

centripetal force
: any net force causing uniform circular motion

Coriolis force
: the fictitious force causing the apparent deflection of moving objects when viewed in a rotating frame of reference

fictitious force
: a force having no physical origin

gravitational constant, *G*
: a proportionality factor used in the equation for Newton's universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

ideal angle
: the angle at which a car can turn safely on a steep curve, which is in proportion to the ideal speed

ideal banking
: the sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

ideal speed
: the maximum safe speed at which a vehicle can turn on a curve without the aid of friction between the tire and the road

microgravity
an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface

Newton's universal law of gravitation
every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

non-inertial frame of reference
an accelerated frame of reference

pit
a tiny indentation on the spiral track moulded into the top of the polycarbonate layer of CD

radians
a unit of angle measurement

radius of curvature
radius of a circular path

rotation angle
the ratio of the arc length to the radius of curvature on a circular path:
$\Delta\theta = \frac{\Delta s}{r}$

ultracentrifuge
a centrifuge optimized for spinning a rotor at very high speeds

uniform circular motion
the motion of an object in a circular path at constant speed

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CHAPTER SUMMARY

5.1 Rotation Angle and Angular Velocity

- Uniform circular motion is motion in a circle at constant speed. The rotation angle $\Delta\theta$ is defined as the ratio of the arc length to the radius of curvature:
$$\Delta\theta = \frac{\Delta s}{r},$$
where arc length Δs is distance traveled along a circular path and r is the radius of curvature of the circular path. The quantity $\Delta\theta$ is measured in units of radians (rad), for which $2\pi \text{rad} = 360^\circ = 1 \text{ revolution.}$
- The conversion between radians and degrees is $1\text{rad} = 57.3^\circ$.
- Angular velocity ω is the rate of change of an angle, $\omega = \frac{\Delta\theta}{\Delta t}$, where a rotation $\Delta\theta$ takes place in a time Δt. The units of angular velocity are radians per second (rad/s). Linear velocity v and angular velocity ω are related by $v = r\omega$ or $\omega = \frac{v}{r}$.

5.2 Centripetal Acceleration

- Centripetal acceleration a_{c} is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity v and has the magnitude $a_{\mathrm{c}} = \frac{v^2}{r}; a_{\mathrm{c}} = r\omega^2.$
- The unit of centripetal acceleration is $\mathrm{m/s^2}$.

5.3 Centripetal Force

- Centripetal force $\mathbf{F}_{\mathrm{c}}$ is any force causing uniform circular motion. It is a "center-seeking" force that always points toward the center of rotation. It is perpendicular to linear velocity v and has magnitude $F_{\mathrm{c}} = \mathrm{ma_c}$, which can also be expressed as \left\begin{matrix} F_{\text{c}} = m \frac{v^{2}}{r} \\ \begin{matrix}\text{or} \\ F_{\text{c}} = \text{mr} \omega^{2}\end{matrix} , \end{matrix}\right}

5.4 Newton's Universal Law of Gravitation

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is $F = G\frac{\mathrm{mM}}{r^2}$, where F is the magnitude of the gravitational force. G is the gravitational constant, given by $G = 6.674 \times 10^{-11}\mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$.
- Newton's law of gravitation applies universally.

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CONCEPTUAL QUESTIONS

Conceptual Questions

5.1 Rotation Angle and Angular Velocity

1. There is an analogy between rotational and linear physical quantities. What rotational quantities are analogous to distance and velocity?

5.2 Centripetal Acceleration

2. Can centripetal acceleration change the speed of circular motion? Explain.

5.3 Centripetal Force

3. If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.
4. Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?
5. If centripetal force is directed toward the center, why do you feel that you are 'thrown' away from the center as a car goes around a curve? Explain.
6. Race car drivers routinely cut corners as shown in Figure 5.22. Explain how this allows the curve to be taken at the greatest speed.

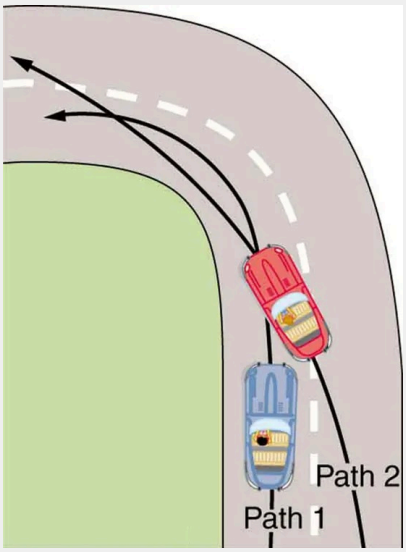


Figure 5.22 Two paths around a race track curve are shown. Race car drivers will take the inside path (called cutting the corner) whenever possible because it allows them to take the curve at the highest speed. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a curved road with two cars and two possible driving paths.

– Road Details: The road bends to the left with a grassy area on the inner side.
– Cars:
– There is a red car positioned on the outer side of the curve.
– A blue car is positioned behind the red car and closer to the grass.
– Paths:
– Path 1: Indicated as a solid line, it starts from the position of the blue car and follows a tighter curve close to the inner grassy edge.
– Path 2: Indicated as the path followed by the red car, depicted with a wider line that moves further from the grassy edge, taking a wider arc around the bend.

The image illustrates different trajectories a vehicle can take while navigating a curve.

7. A number of amusement parks have rides that make vertical loops like the one shown in Figure 5.23. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity

alone will supply the centripetal force. What other force acts and what is its direction if:(a) The car goes over the top at faster than this speed?(b)The car goes over the top at slower than this speed?

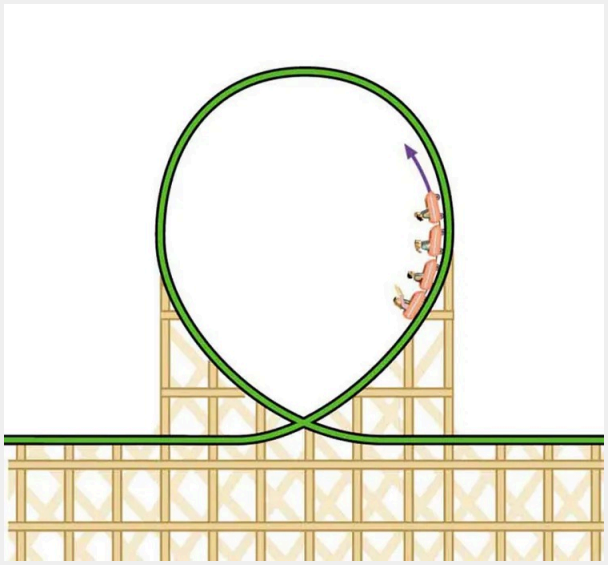


Figure 5.23 Amusement rides with a vertical loop are an example of a form of curved motion. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a diagram of a roller coaster loop. The track is depicted in green and forms a vertical loop, similar in shape to an inverted tear drop. The roller coaster cars, illustrated with three cars in sequence, are positioned on the right side of the loop, ascending upward in a clockwise direction. There is a purple arrow pointing upwards, indicating the direction of movement. The structure supporting the track is wooden and crisscrossed, resembling a classic wooden roller coaster design. The background is plain white, highlighting the roller coaster structure.

8. What is the direction of the force exerted by the car on the passenger as the car goes over the top of the amusement ride pictured in Figure 6.30 under the following circumstances:
   (a) The car goes over the top at such a speed that the gravitational force is the only force acting?

(b) The car goes over the top faster than this speed?
(c) The car goes over the top slower than this speed?

9. As a skater forms a circle, what force is responsible for making her turn? Use a free body diagram in your answer.
10. Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown in Figure 5.24 will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.

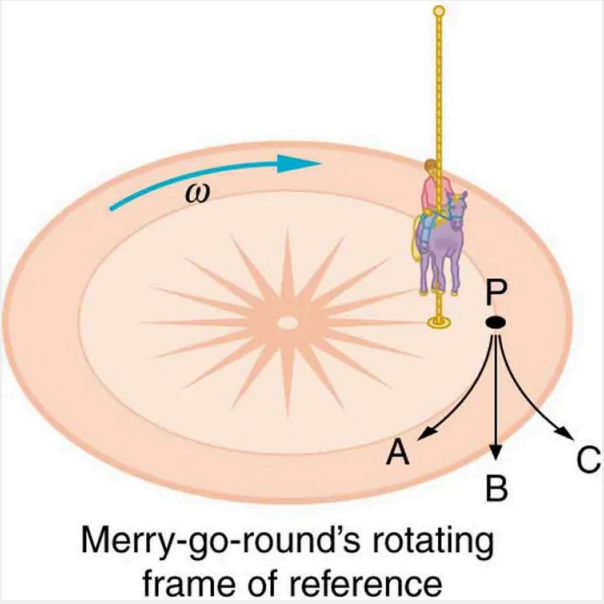


Figure 5.24 A child riding on a merry-go-round releases her lunch box at point P. This is a view from above the clockwise rotation. Assuming it slides with negligible friction, will it follow path A, B, or C, as viewed from Earth's frame of reference? What will be the shape of the path it leaves in the dust on the merry-go-round? Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image illustrates a merry-go-round from a top-down perspective, focusing on the concept of a rotating frame of reference. The merry-go-round is depicted as a circular

platform with an outer rim. There is a stylized star pattern in the center of the platform. On the right side, a horse with a rider is fixed to a vertical pole on the edge of the platform.

An arrow labeled with the Greek letter "ω" (omega) indicates the direction of rotation, going clockwise. Below the merry-go-round, there are labeled points with arrows emanating from point "P", directed towards "A", "B", and "C", representing vectors.

The text at the bottom of the image reads "Merry-go-round's rotating frame of reference."

11. Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?
12. Suppose a mass is moving in a circular path on a frictionless table as shown in figure. In the Earth's frame of reference, there is no centrifugal force pulling the mass away from the centre of rotation, yet there is a very real force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.

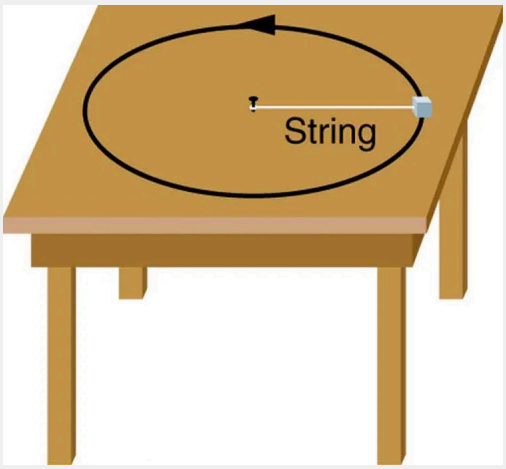


Figure 5.25 A mass attached to a nail on a frictionless table moves in a circular path. The force stretching the string is real and not fictional. What is the physical origin of the force on the string? Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram showing a top view of a wooden table with a physics experiment setup. A circular track is drawn on the table, represented by a thick black line. At the center of this circle, a small pin is inserted into the table. A string is attached to the pin and extends to the edge of the circle, where it is connected to a small blue block. The string runs parallel to one side of the table and appears to allow the block to move in a circular path around the pin. An arrow drawn along the circumference of the circle indicates the direction of motion. The word "String" is labeled on the string itself, suggesting the experimental setup involves circular motion.

5.4 Newton's Universal Law of Gravitation

13. Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?
14. Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not $9.80 m/\mathrm{s}^2$. Who do you agree with and why?
15. Draw a free body diagram for a satellite in an elliptical orbit showing why its speed increases as it approaches its parent body and decreases as it moves away.
16. Newton's laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

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PROBLEMS & EXERCISES

Problems & Exercises

6.1 Rotation Angle and Angular Velocity

1. Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?
2. Microwave ovens rotate at a rate of about 6 rev/min. What is this in revolutions per second? What is the angular velocity in radians per second?
3. An automobile with 0.260 m radius tires travels 80,000 km before wearing them out. How many revolutions do the tires make, neglecting any backing up and any change in radius due to wear?
4. (a) What is the period of rotation of Earth in seconds? (b) What is the angular velocity of Earth? (c) Given that Earth has a radius of $6.4 \times 10^6 \text{m}$ at its equator, what is the linear velocity at Earth's surface?
5. A baseball pitcher brings his arm forward during a pitch, rotating the forearm about the elbow. If the velocity of the ball in the pitcher's hand is 35.0 m/s and the ball is 0.300 m from the elbow joint, what is the angular velocity of the forearm?
6. In lacrosse, a ball is thrown from a net on the end of a stick by rotating the stick and forearm about the elbow. If the angular velocity of the ball about the elbow joint is 30.0 rad/s and the ball is 1.30 m from the elbow joint, what is the velocity of the ball?
7. A truck with 0.420-m-radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?
8. **Integrated Concepts:** When kicking a football, the kicker rotates his leg about the hip joint. (a) If the velocity of the tip of the kicker's shoe is 35.0 m/s and the hip joint is 1.05 m from the tip of the shoe, what is the shoe tip's angular velocity?

(b) The shoe is in contact with the initially stationary 0.500 kg football for 20.0 ms. What average force is exerted on the football to give it a velocity of 20.0 m/s? (c) Find the maximum range of the football, neglecting air resistance.

9. **Construct Your Own Problem:** Consider an amusement park ride in which participants are rotated about a vertical axis in a cylinder with vertical walls. Once the angular velocity reaches its full value, the floor drops away and friction between the walls and the riders prevents them from sliding down. Construct a problem in which you calculate the necessary angular velocity that assures the riders will not slide down the wall. Include a free body diagram of a single rider. Among the variables to consider are the radius of the cylinder and the coefficients of friction between the riders' clothing and the wall.

6.2 Centripetal Acceleration

10. A fairground ride spins its occupants inside a flying saucer-shaped container. If the horizontal circular path the riders follow has an 8.00 m radius, at how many revolutions per minute will the riders be subjected to a centripetal acceleration whose magnitude is 1.50 times that due to gravity?
11. A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If the runner completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of their centripetal acceleration as they run the curved portion of the track?
12. Taking the age of Earth to be about 4×10^9 years and assuming its orbital radius of

    1.5×10^{11}

    m has not changed and is circular, calculate the approximate total distance Earth has traveled since its birth (in a frame of reference stationary with respect to the Sun).
13. The propeller of a World War II fighter plane is 2.30 m in diameter. (a) What is its angular velocity in radians per second if it spins at 1200 rev/min? (b) What is the linear speed of its tip at this angular velocity if the plane is stationary on the

tarmac? (c) What is the centripetal acceleration of the propeller tip under these conditions? Calculate it in meters per second squared and convert to multiples of g.

14. An ordinary workshop grindstone has a radius of 7.50 cm and rotates at 6500 rev/min. (a) Calculate the magnitude of the centripetal acceleration at its edge in meters per second squared and convert it to multiples of g. (b) What is the linear speed of a point on its edge?
15. Helicopter blades withstand tremendous stresses. In addition to supporting the weight of a helicopter, they are spun at rapid rates and experience large centripetal accelerations, especially at the tip. (a) Calculate the magnitude of the centripetal acceleration at the tip of a 4.00 m long helicopter blade that rotates at 300 rev/min. (b) Compare the linear speed of the tip with the speed of sound (taken to be 340 m/s).
16. Olympic ice skaters are able to spin at about 5 rev/s. (a) What is their angular velocity in radians per second? (b) What is the centripetal acceleration of the skater's nose if it is 0.120 m from the axis of rotation? (c) An exceptional skater named Dick Button was able to spin much faster in the 1950s than anyone since—at about 9 rev/s. What was the centripetal acceleration of the tip of his nose, assuming it is at 0.120 m radius? (d) Comment on the magnitudes of the accelerations found. It is reputed that Button ruptured small blood vessels during his spins.
17. What percentage of the acceleration at Earth's surface is the acceleration due to gravity at the position of a satellite located 300 km above Earth?
18. Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating: (a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min. (b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth's orbit and approximate it as being circular).
19. A rotating space station is said to create "artificial gravity"—a loosely-defined term used for an acceleration that would be crudely similar to gravity. The outer wall of the rotating space station would become a floor for the astronauts, and centripetal acceleration supplied by the floor would allow astronauts to exercise

and maintain muscle and bone strength more naturally than in non-rotating space environments. If the space station is 200 m in diameter, what angular velocity would produce an "artificial gravity" of 9.80m/s^2 at the rim?

20. At takeoff, a commercial jet has a 60.0 m/s speed. Its tires have a diameter of 0.850 m. (a) At how many rev/min are the tires rotating? (b) What is the centripetal acceleration at the edge of the tire? (c) With what force must a determined $1.00 \times 10^{-15} \text{kg}$ bacterium cling to the rim? (d) Take the ratio of this force to the bacterium's weight.
21. **Integrated Concepts:** Riders in an amusement park ride shaped like a Viking ship hung from a large pivot are rotated back and forth like a rigid pendulum. Sometime near the middle of the ride, the ship is momentarily motionless at the top of its circular arc. The ship then swings down under the influence of gravity. The speed at the bottom of the arc is 23.4 m/s. The radius of the arc is 14.0 m. (a) What is the centripetal acceleration at the bottom of the arc? (b) Draw a free body diagram of the forces acting on a rider at the bottom of the arc. (c) Find the force exerted by the ride on a 60.0 kg rider and compare it to her weight. (d) Discuss whether the answer seems reasonable.
22. **Unreasonable Results:** A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child's center of mass. (a) What is the magnitude of the centripetal acceleration of the child at the low point? (b) What is the magnitude of the force the child exerts on the seat if his mass is 18.0 kg? (c) What is unreasonable about these results? (d) Which premises are unreasonable or inconsistent?

6.3 Centripetal Force

23. (a) A 22.0 kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force must she exert to stay on if she is 1.25 m from its center? (b) What centripetal force does she need to stay on an amusement park merry-go-round that rotates at 3.00 rev/min if she is 8.00 m from its center? (c) Compare each force with her weight.
24. Calculate the centripetal force on the end of a 100 m (radius) wind turbine blade

that is rotating at 0.5 rev/s. Assume the mass is 4 kg.

25. What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?
26. What is the ideal speed to take a 100 m radius curve banked at a 20.0° angle?
27. (a) What is the radius of a bobsled turn banked at 75.0° and taken at 30.0 m/s, assuming it is ideally banked? (b) Calculate the centripetal acceleration. (c) Does this acceleration seem large to you?
28. Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen in Figure 5.26. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force), and the vertical normal force (which must equal the system's weight).
29. (a) Show that θ (as defined in the figure) is related to the speed v and radius of curvature r of the turn in the same way as for an ideally banked roadway—that is, $\theta = \tan^{-1} v^2/rg$. (b) Calculate θ for a 12.0 m/s turn of radius 30.0 m (as in a race).

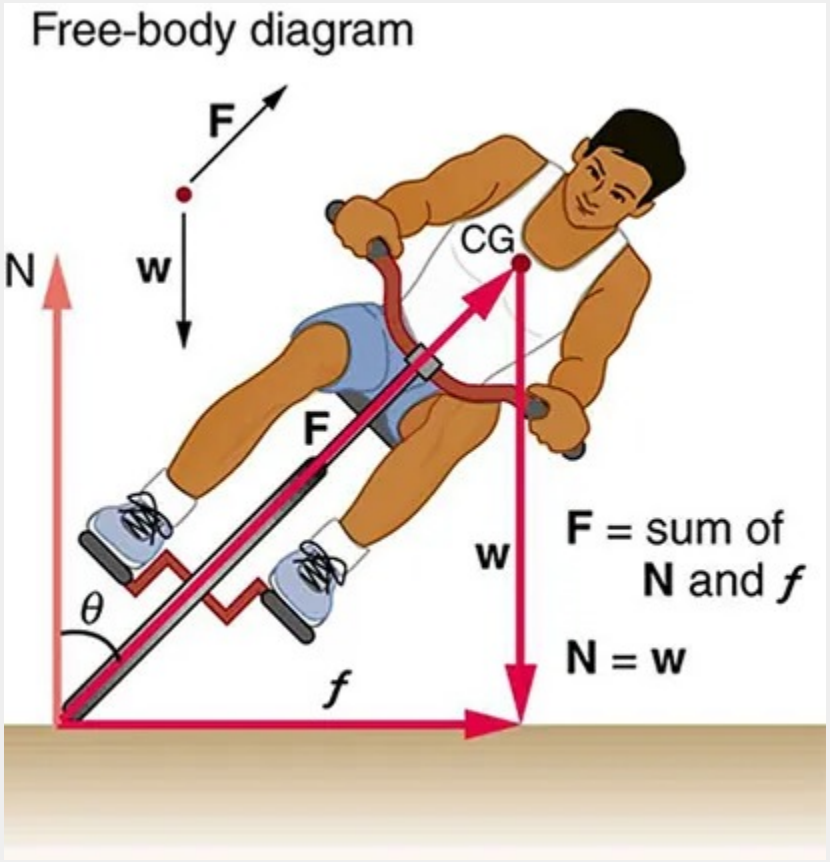


Figure 5.26 A bicyclist negotiating a turn on level ground must lean at the correct angle—the ability to do this becomes instinctive. The force of the ground on the wheel needs to be on a line through the center of gravity. The net external force on the system is the centripetal force. The vertical component of the force on the wheel cancels the weight of the system while its horizontal component must supply the centripetal force. This process produces a relationship among the angle θ, the speed v, and the radius of curvature r of the turn similar to that for the ideal banking of roadways. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

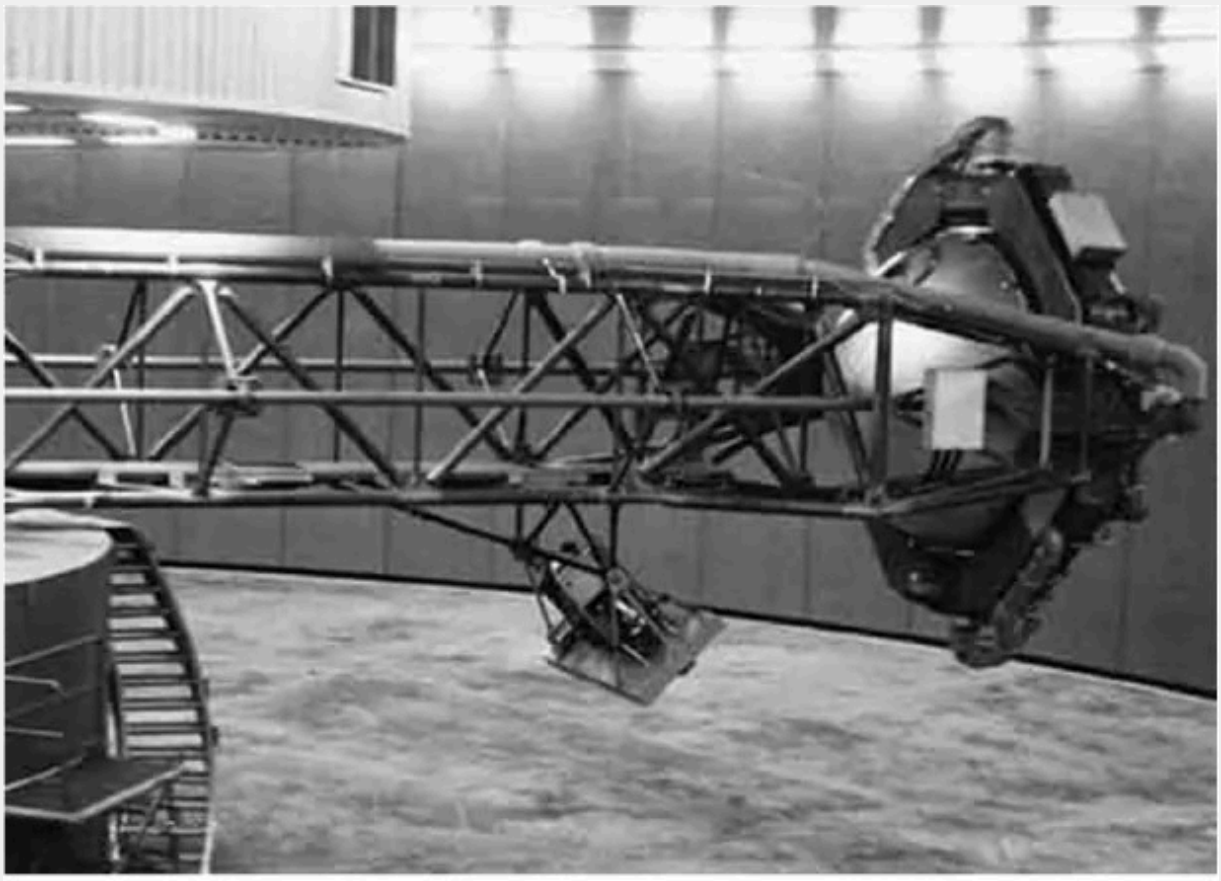
This image is a diagram labeled “Free-body diagram” showing a person riding a bicycle while tilted sideways. The diagram illustrates various forces acting on the cyclist and the bicycle.

- The person is wearing a sleeveless shirt and shorts, and they are hunched over

the handlebars.

- The bicycle is depicted as tilted to the left at an angle, labeled with the Greek letter "θ" (theta).
- Several vectors are drawn to indicate different forces:
    - An upward arrow labeled "N" represents the normal force exerted by the ground.
    - A downward arrow labeled "w" represents the weight of the cyclist and bicycle.
    - A horizontal arrow pointing to the right at the point of contact with the ground is labeled "f," representing the frictional force.
- A diagonal arrow going through the cyclist's center of gravity (CG) is labeled "F," representing the sum of the normal force "N" and frictional force "f." This arrow points upward and to the left, aligning with the center of gravity.
- Additional text explains that "F = sum of N and f" and "N = w."

29. A large centrifuge, like the one shown in Figure 5.27(a), is used to expose aspiring astronauts to accelerations similar to those experienced in rocket launches and atmospheric reentries. (a) At what angular velocity is the centripetal acceleration $10g$ if the rider is 15.0 m from the center of rotation? (b) The rider's cage hangs on a pivot at the end of the arm, allowing it to swing outward during rotation as shown in Figure 5.27(b). At what angle θ below the horizontal will the cage hang when the centripetal acceleration is $10g$? (Hint: The arm supplies centripetal force and supports the weight of the cage. Draw a free body diagram of the forces to see what the angle θ should be.)



(a)

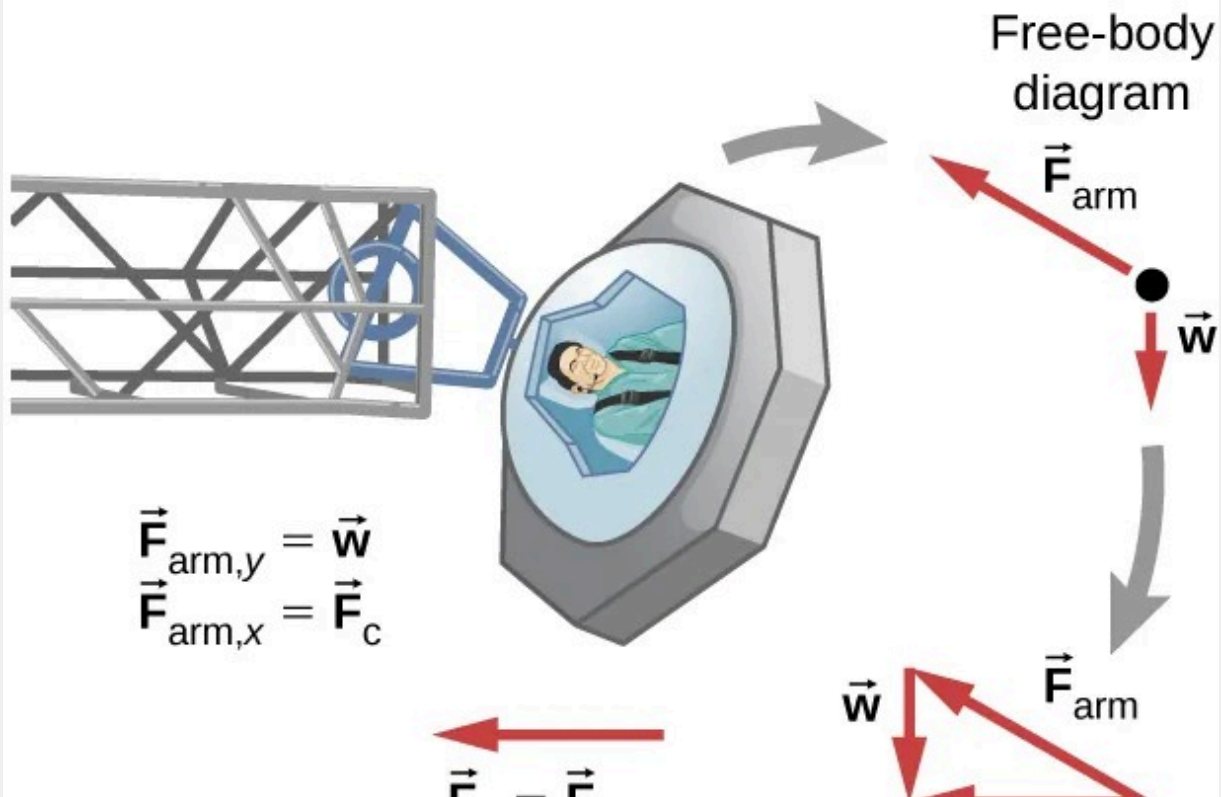

Free-body diagram
$\vec{F}_{arm}$
$\vec{w}$
$\vec{F}_{arm,y} = \vec{w}$
$\vec{F}_{arm,x} = \vec{F}_c$
$\vec{w}$
$\vec{F}_{arm}$

Figure 5.27 (a) NASA centrifuge used to subject trainees to accelerations similar to those experienced in rocket launches and reentries. (credit: NASA) (b) Rider in cage showing how the cage pivots outward during rotation. This allows the total force exerted on the rider by the cage to be along its axis at all times. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two parts labeled (a) and (b):

– Image (a): Displays a large, mechanical structure resembling a centrifuge arm in a laboratory setting. The arm is a metal framework attached to a circular device where a person or object could be seated or secured. The environment appears to be industrial, with smooth walls and ceiling lights above.

– Image (b): An illustration showing a schematic of the centrifuge arm with a person inside a capsule at the end of the arm. It includes labeled forces illustrating physics concepts, specifically a free-body diagram:
– F_{arm}: Indicates the force exerted by the arm.
– F_c: Denotes the centripetal force.
– $\overline{w}$: Represents weight.

Arrows illustrate the direction of these forces. In this schematic, various mathematical notations and arrows are used to explain the balance of forces involved in the system. The depiction shows the arm rotating in circular motion with a central pivot point.

30. **Integrated Concepts:** If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside of the curve (a real problem on icy mountain roads). (a) Calculate the ideal speed to take a 100 m radius curve banked at 15.0º. (b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at 20.0 km/h?
31. Modern roller coasters have vertical loops like the one shown in Figure 5.28. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. What is the speed

of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50 g?

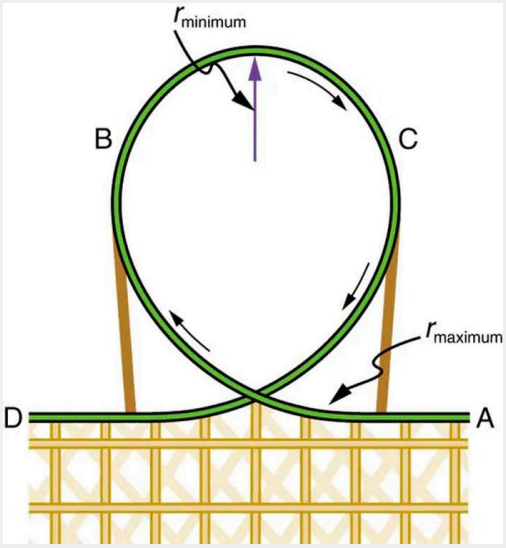


Figure 5.28 Teardrop-shaped loops are used in the latest roller coasters so that the radius of curvature gradually decreases to a minimum at the top. This means that the centripetal acceleration builds from zero to a maximum at the top and gradually decreases again. A circular loop would cause a jolting change in acceleration at entry, a disadvantage discovered long ago in railroad curve design. With a small radius of curvature at the top, the centripetal acceleration can more easily be kept greater than g *so that the passengers do not lose contact with their seats nor do they need seat belts to keep them in place. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a diagram depicting a roller coaster loop. The loop is vertically oriented and resembles a teardrop shape. Here are some details about the diagram:

– The loop begins at point A, which is situated on the horizontal track at the bottom right of the diagram.
– As the track moves upward, it reaches a maximum curvature labeled as *r maximum*.
– The track continues to curve into a loop, ascending to point B at the top of the loop.
– At point B, the curvature is labeled as *r minimum*, indicating the point with the smallest radius of curvature at the top.

– A purple arrow points upwards through the center of the loop, suggesting the direction of the force or motion.
– The track continues from point B downwards to point C, and finally returns to the horizontal track at point D.
– Arrows along the track indicate the direction of travel, moving clockwise from A to D.
– The entire structure is supported by vertical beams on either side of the loop.

The diagram is used to illustrate the physics of roller coaster motion, focusing on the radii of curvature at different points in the loop.

32. **Unreasonable Results:** (a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s. (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

6.5 Newton's Universal Law of Gravitation

33. Earth is not a perfect sphere. Its radius as well as the value of g vary with latitude. (a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is $9.807 m/\mathrm{s}^2$ and the radius of the Earth at the pole is 6356.8 km. (b) Compare this with the accepted value of $5.972 \times 10^{24}\mathrm{kg}$.
34. (a) Calculate the magnitude of the acceleration due to gravity on the surface of Earth due to the Moon. (b) Calculate the magnitude of the acceleration due to gravity at Earth due to the Sun. (c) Take the ratio of the Moon's acceleration to the Sun's and comment on why the tides are predominantly due to the Moon in spite of this number.
35. (a) What is the acceleration due to gravity on the surface of the Moon? (b) On the surface of Mars? The mass of Mars is $6.418 \times 10^{23}\mathrm{kg}$ and its radius is $3.38 \times 10^{6}\mathrm{m}$.
36. (a) Calculate the acceleration due to gravity on the surface of the Sun. (b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)
37. The Moon and Earth rotate about their common center of mass, which is located about 4700 km from the center of Earth. (This is 1690 km below the surface.) (a)

Calculate the magnitude of the acceleration due to the Moon's gravity at that point. (b) Calculate the magnitude of the centripetal acceleration of the center of Earth as it rotates about that point once each lunar month (about 27.3 d) and compare it with the acceleration found in part (a). Comment on whether or not they are equal and why they should or should not be.

38. Solve part (b) of Example 5.6 using $a_c = v^2/r$.
39. (a) Calculate the magnitude of the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child). (b) Calculate the magnitude of the force on the baby due to Jupiter if it is at its closest distance to Earth, some $6.29 \times 10^{11}\text{m}$ away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)
40. The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune: (a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are $4.50 \times 10^{12}\text{m}$ apart, as they are at present. The mass of Pluto is $1.4 \times 10^{22}\text{kg}$. (b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about $2.50 \times 10^{12}\text{m}$ apart, and compare it with that due to Pluto. The mass of Uranus is $8.62 \times 10^{25}\ \text{kg}$.
41. (a) The Sun orbits the Milky Way galaxy once each $2.60\ \text{x}\ 10^8\ \text{y}$, with a roughly circular orbit averaging $3.00\ \text{x}\ 10^4$ light years in radius. (A light year is the distance traveled by light in 1 y.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun? (b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?
42. **Unreasonable Result:** A mountain 10.0 km from a person exerts a gravitational

force on him equal to 2.00% of his weight. (a) Calculate the mass of the mountain. (b) Compare the mountain's mass with that of Earth. (c) What is unreasonable about these results? (d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)

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WORK, ENERGY, AND ENERGY RESOURCES



Figure 6.1 How many forms of energy can you identify in this photograph of a wind farm in Iowa? Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

6.1 Work: The Scientific Definition
6.2 Kinetic Energy and the Work-Energy Theorem
6.3 Gravitational Potential Energy
6.4 Conservative Forces and Potential Energy
6.5 Nonconservative Forces
6.6 Conservation of Energy
6.7 Power
6.8 Work, Energy, and Power in Humans
6.9 World Energy Use
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

Conservation of energy (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. For example, scientists Willem 's Gravesande and Émilie du Châtelet undertook (separate) experiments where they dropped heavy lead balls into beds of clay. Du Châtelet showed that the balls that hit the clay with twice the velocity penetrated four times as deep into the clay; those with three times the velocity reached a depth nine times greater. This led her to develop a more accurate concept of energy conservation, expressed as $E = \frac{1}{2}mv^2$. Even as scientists discovered new forms of energy,

conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation $E = mc^2$).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define energy as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.
Access multimedia content

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6.1 WORK: THE SCIENTIFIC DEFINITION

Learning Objectives

By the end of this section, you will be able to:

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be displacement in the direction of the force.

Formally, the work done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = | \mathbf{F} | (\cos\theta) | \mathbf{d} |,$$

6.1

where W is work, $\mathbf{d}$ is the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$, as in Figure 6.2. We can also write this as

$$W = \mathrm{F}\mathrm{d}\ \cos\theta.$$

6.2

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

What is Work?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = \mathrm{F}\mathrm{d}\cos\theta,$$

6.3

where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$.

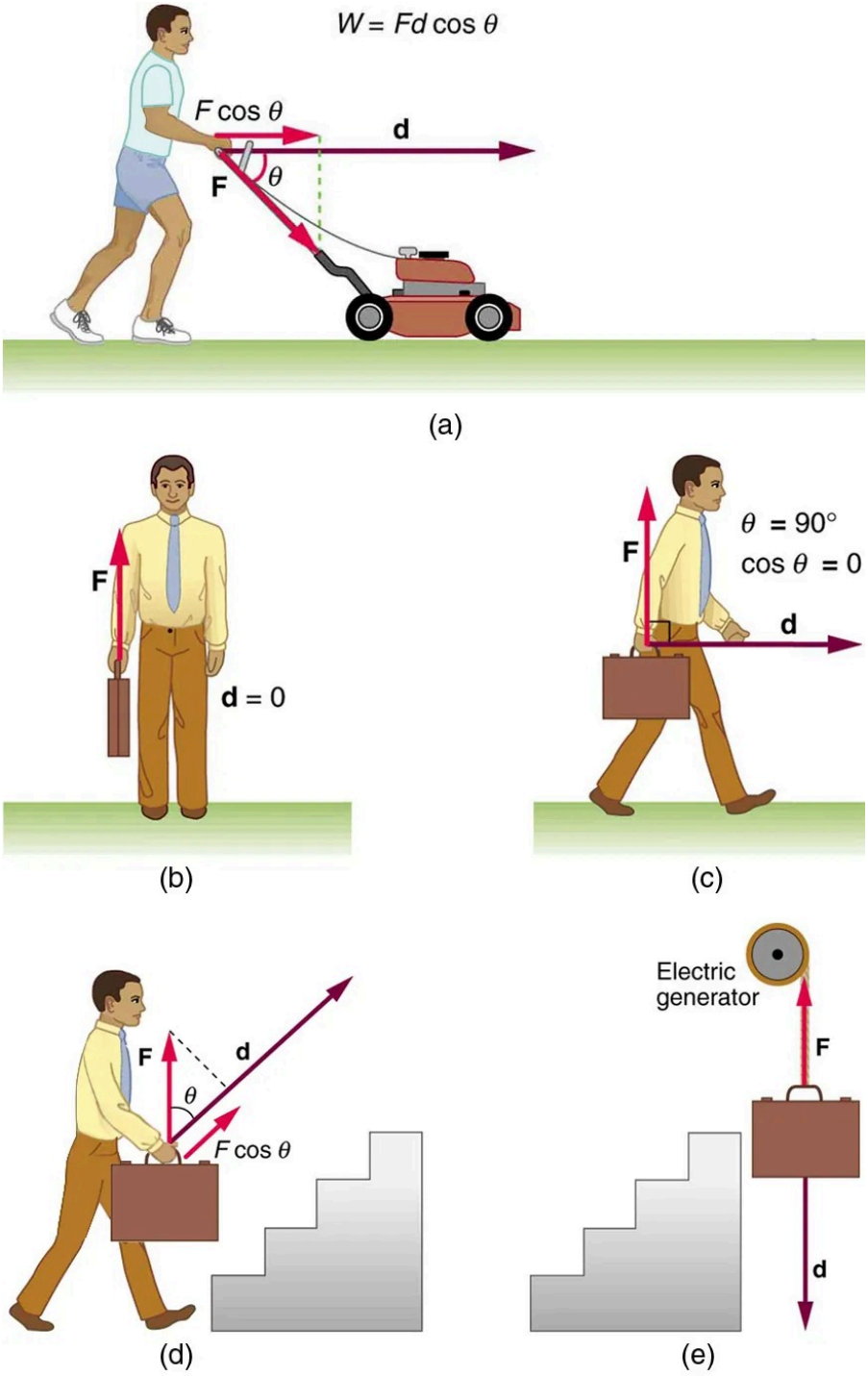


Figure 6.2 Examples of work. (a) The work done by the force $\mathbf{F}$ *on this lawn mower is* $\mathrm{F d cos}\theta$. *Note that* $F\cos\theta$ *is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no displacement. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force* $\mathbf{F}$ *in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the*

briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because $\mathbf{F}$ *and* $\mathbf{d}$ *are in opposite directions. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image contains five illustrations, labeled (a) to (e), demonstrating concepts of work, force, and distance in physics. Here's a description of each:

– (a): A person is pushing a lawnmower. An arrow labeled "F" represents the force applied, which is at an angle "θ" to the horizontal. A horizontal arrow shows "F cos θ", and another shows the distance "d". The equation "W = Fd cos θ" is written above.

– (b): A person is standing still holding a briefcase. The force "F" is shown as an upward arrow, and "d = 0" is indicated, implying no distance moved.

– (c): A person is walking while holding a briefcase. The force "F" is vertical, and the distance "d" is horizontal. It shows "θ = 90°" and "cos θ = 0", indicating no work done in the direction of travel.

– (d): A person is carrying a briefcase up a flight of stairs. The force "F" is shown with an angle "θ", and there are arrows for both "F cos θ" and distance "d" along the incline, demonstrating work done against gravity.

– (e): An illustration of an electric generator being used to lift an object vertically with force "F" and distance "d" against gravity, illustrating the concept of work in a mechanical system.

To examine what the definition of work means, let us consider the other situations shown in Figure 6.2. The person holding the briefcase in Figure 6.2(b) does no work, for example. Here $d = 0$, so $W = 0$. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the "briefcase-Earth system"—see 6.3 Gravitational Potential Energy for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in Figure 6.2(c) does no work on it, because the force is perpendicular to the motion. That is, $\cos 90^\circ = 0$, and so $W = 0$.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in Figure 6.2(d), work *is* done—energy is transferred to the briefcase. Finally, in Figure 6.2(e), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase's weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter,

with the force from the generator upward on the briefcase, and the displacement downward. This makes $\theta = 180^\circ$, and $\cos 180^\circ = -1$; therefore, W is negative.

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in newton-meters. A newton-meter is given the special name joule (J), and $1\ \text{J} = 1\ \text{N} \cdot \text{m} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2$. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Example 6.1

Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in Figure 6.2(a) if he exerts a constant force of 75.0N at an angle 35° below the horizontal and pushes the mower 25.0m on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of $10,000\text{kJ}$ (about 2400kcal) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1° C, and is equivalent to 4.186J, while one *food calorie* (1 kcal) is equivalent to 4186J.

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = \text{Fdcos}\theta$. The force, angle, and displacement are given, so that only the work W is unknown.

Solution

The equation for the work is

$$W = \text{Fdcos}\theta.$$

6.4

Substituting the known values gives

$$\begin{aligned} W &= (75.0\text{ N})\,(25.0\text{ m})\cos\,(35.0^\circ\,) \\ &= 1536\text{ J} = 1.54 \times 10^3\text{J}. \end{aligned}$$

6.5

Converting the work in joules to kilocalories yields

$W = (1536\text{J})\,(1\text{kcal}/4186\text{J}) = 0.367\text{kcal}$. The ratio of the work done to the daily consumption is

$$\frac{W}{2400\text{kcal}} = 1.53 \times 10^{-4}.$$

6.6

Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we "work" all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

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6.2 KINETIC ENERGY AND THE WORK-ENERGY THEOREM

Learning Objectives

By the end of this section, you will be able to:

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in Figure 6.2(a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in Figure 6.2(d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in Figure 6.2(e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in Dynamics: Force and Newton's Laws of Motion that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work on an object. The net work can be written in terms of the net force on an object. $\mathbf{F}_{\text{net}}$. In equation form, this is $W_{\text{net}} = F_{\text{net}} d\cos\theta$ where θ is the angle between the force vector and the displacement vector.

Figure 6.3(a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $F\cos\theta$ vs. d graph. In this case, $F\cos\theta$ is constant. You can see that the area under the graph is $Fd\cos\theta$, or the work done. Figure 6.3(b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $(F\cos\theta)_{i(\text{ave})}$. The work done is $(F\cos\theta)_{i(\text{ave})} d_i$ for each strip, and the total work done is the sum of the W_i. Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

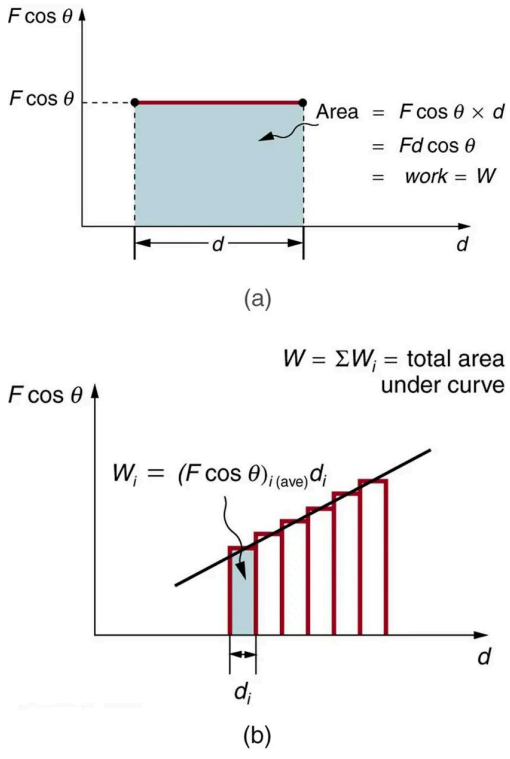


Figure 6.3 (a) A graph of $F\cos\theta$ vs. d, when $F\cos\theta$ is constant. The area under the curve represents the work done by the force. (b) A graph of $F\cos\theta$ vs. d in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b), illustrating concepts related to work done by force in physics.

Diagram (a):
– Depicts a graph with a horizontal axis labeled “d” and a vertical axis labeled “F cos θ.”
– A shaded rectangle is shown with one side along the “F cos θ” line and the other along the “d” axis.
– The area of the rectangle is labeled with equations:
– Area = F cos θ × d
– = Fd cos θ
– = work = W

Diagram (b):
– Also features a graph with horizontal axis “d” and vertical axis “F cos θ.”
– Displays a series of adjacent vertical bars under a sloped line.
– A single bar with height “(F cos θ)_i(ave)” and width “di” is highlighted, indicating the work done on that segment.
– The bar’s area is labeled as:
– W_i = (F cos θ)_i(ave) di
– Alongside is an equation:
– W = ΣW_i = total area under curve

Both diagrams illustrate the relationship between force, distance, and work, showing the concepts of constant and variable force.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in Figure 6.4.

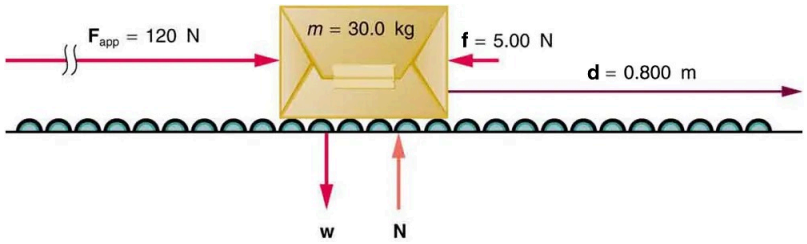


Figure 6.4 A package on a roller belt is pushed horizontally through a distance **d**. *Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a physics problem illustrating forces acting on a box that is sliding on rollers. The key elements in the diagram are as follows:

- A box with a mass of $m = 30.0\ kg$ sits on a surface with rollers.
- An applied force, denoted as $F_{app} = 120\ N$, is shown as a red arrow pointing horizontally to the right, acting on the box.
- A frictional force, denoted as $f = 5.00\ N$, opposes the applied force and is shown as an arrow pointing to the left.
- The displacement of the box is illustrated by $d = 0.800\ m$, also indicated by an arrow pointing to the right.
- The weight (w) of the box is represented by a downward arrow.
- A normal force (N) is depicted as an upward arrow, equal in magnitude and opposite to the weight.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force $\mathbf{F}_{\text{app}}$ and the horizontal friction force $\mathbf{f}$. Thus, as expected, the net force is parallel to the displacement, so that $\theta = 0^{\text{o}}$ and $\cos\theta = 1$, and the net work is given by

$$W_{\text{net}} = F_{\text{net}}d.$$
6.7

The effect of the net force $\mathbf{F}_{\text{net}}$ is to accelerate the package from v_0 to v. The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See Example 6.2.) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F_{\text{net}} = \text{ma}$ from Newton's second law gives

$$W_{\text{net}} = \text{mad}.$$
6.8

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in 2.5 Motion Equations for Constant Acceleration in One Dimension for the change in speed over a distance d if the acceleration has the constant value a; namely, $v^2 = (v_0)^2 + 2\text{ad}$ (note that a appears in the expression for the net work). Solving for acceleration gives $a = \dfrac{v^2 - (v_0)^2}{2d}$. When a is substituted into the preceding expression for W_{net}, we obtain

$$W_{\text{net}} = m\left(\frac{v^2 - (v_0)^2}{2d}\right)d.$$

6.9

The d cancels, and we rearrange this to obtain

$$W_\text{net} = \frac{1}{2}\text{mv}^2 - \frac{1}{2}\text{mv}_0^2.$$

6.10

This expression is called the work-energy theorem, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}\text{mv}^2$. This quantity is our first example of a form of energy.

The Work-Energy Theorem

The net work on a system equals the change in the quantity $\frac{1}{2}\text{mv}^2$.

$$W_{\text{net}} = \frac{1}{2}\text{mv}^2 - \frac{1}{2}\text{mv}_0^2$$

6.11

The quantity $\frac{1}{2}\text{mv}^2$ in the work-energy theorem is defined to be the translational kinetic energy (KE) of a mass m moving at a speed v. (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$\text{KE} = \frac{1}{2}\text{mv}^2,$$

6.12

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in Figure 6.4, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Example 6.2

Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in Figure 6.4 is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation $\mathrm{KE} = \frac{1}{2}\mathrm{mv}^2$.

Solution

The kinetic energy is given by

$$\mathrm{KE} = \frac{1}{2}\mathrm{mv}^2.$$

6.13

Entering known values gives

$$\mathrm{KE} = 0.5\,(30.0\ \mathrm{kg})\,(0.500\ \mathrm{m/s})^2,$$

6.14

which yields

$$\mathrm{KE} = 3.75\ \mathrm{kg}\cdot m^2/\mathrm{s}^2 = 3.75\ \mathrm{J}.$$

6.15

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as

mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example 6.3

Determining the Work to Accelerate a Package

Suppose that you push on the 30.0-kg package in Figure 6.4 with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See Figure 6.4.) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or $F_{\text{net}} = 120\ \text{N} - 5.00\ \text{N} = 115\ \text{N}$. Thus the net work is

$$W_{\text{net}} = F_{\text{net}}d = (115\ \text{N})\,(0.800\ \text{m})$$
$$= 92.0\ \text{N} \cdot m = 92.0\ \text{J}.$$

6.16

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes

some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

$$\begin{aligned} W_{\text{app}} &= F_{\text{app}} d \cos\left(0^{\circ}\right) = F_{\text{app}} d \\ &= (120\ \text{N})\,(0.800\ \text{m}) \\ &= 96.0\ \text{J} \end{aligned}$$

6.17

The friction force and displacement are in opposite directions, so that $\theta = 180^{\circ}$, and the work done by friction is

$$\begin{aligned} W_{\text{fr}} &= F_{\text{fr}} d \cos\left(180^{\circ}\right) = -F_{\text{fr}} d \\ &= -(5.00\ \text{N})\,(0.800\ \text{m}) \\ &= -4.00\ \text{J}. \end{aligned}$$

6.18

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$\begin{aligned} W_{\text{gr}} &= 0, \\ W_N &= 0, \\ W_{\text{app}} &= 96.0\ \text{J}, \\ W_{\text{fr}} &= -4.00\ \text{J}. \end{aligned}$$

6.19

The total work done as the sum of the work done by each force is then seen to be

$$W_{\text{total}} = W_{\text{gr}} + W_N + W_{\text{app}} + W_{\text{fr}} = 92.0\ \text{J}.$$

6.20

Discussion for (b)

The calculated total work W_{total} as the sum of the work by each force agrees, as expected, with the work W_{net} done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

Example 6.4

Determining Speed from Work and Energy

Find the speed of the package in Figure 6.4 at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work, W_{net}, and the initial kinetic energy, $\frac{1}{2}(mv_0)^2$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}\mathrm{mv}^2$, and thus the final speed v.

Solution

The work-energy theorem in equation form is

$$W_{\text{net}} = \frac{1}{2}\mathrm{mv}^2 - \frac{1}{2}(mv_0)^2.$$

6.21

Solving for $\frac{1}{2}\mathrm{mv}^2$ gives

$$\frac{1}{2}\mathrm{mv}^2 = W_{\text{net}} + \frac{1}{2}(mv_0)^2.$$

6.22

Thus,

$$\frac{1}{2}\mathrm{mv}^2 = 92.0J + 3.75\ \mathrm{J} = 95.75\ \mathrm{J}.$$

6.23

Solving for the final speed as requested and entering known values gives

$$v = \sqrt{\frac{2(95.75\ \text{J})}{m}} = \sqrt{\frac{191.5\ \text{kg} \cdot m^2/\text{s}^2}{30.0\ \text{kg}}}$$
$$= 2.53\ \text{m/s}.$$

6.24

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

Example 6.5

Work and Energy Can Reveal Distance, Too

How far does the package in Figure 6.4 coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so $\theta = 180^{\circ}$. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus $W_{\text{fr}} = -95.75\ \text{J}$. Furthermore, $W_{\text{fr}} = fd'\cos\theta = -fd'$, where d' is the distance it takes to stop. Thus,

$$d' = -\frac{W_{\text{fr}}}{f} = -\frac{-95.75\ \text{J}}{5.00\ \text{N}},$$

6.25

and so

$$d' = 19.2\ \text{m}.$$

6.26

Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

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6.3 GRAVITATIONAL POTENTIAL ENERGY

Learning Objectives

By the end of this section, you will be able to:

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by $\mathrm{PE_g = mgh}$.
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

Work Done Against Gravity

Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h, such as in Figure 6.5, near Earth's surface. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then $\mathrm{W = Fd = mgh}$. We define this to be the gravitational potential energy $(\mathrm{PE_g})$ put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the $\mathrm{PE_g}$ gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word "system"? Potential energy is a property of a system rather than of a single object—due to its physical position. An object's gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because

gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth's surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of $\mathrm{PE_g}$ to KE without explicitly considering the intermediate step of work. (See Example 6.7.) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.

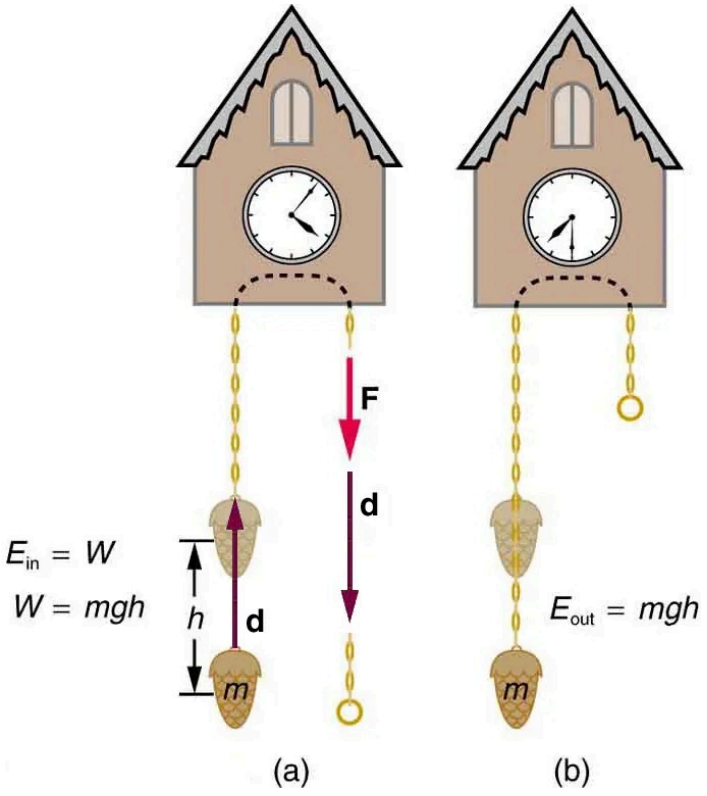


Figure 6.5 (a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled as (a) and (b) depicting a cuckoo clock system with weights and chains. Here's a detailed description:

In both diagrams, there are clock houses at the top with a circular clock face showing a time of 5:00. Each clock has a roof with a triangular shape. Below each clock, there is a hanging chain with a weight shaped like an acorn.

- **Diagram (a):**
  - The chain moves upward pulling the weight (acorn-shaped) upward. The distance moved upwards is marked as "d".
  - There are arrows indicating the directions of force: an upward force at the acorn and a downward force labeled "F" closer to the clock, with the same distance "d".
  - Text on this diagram includes equations:
    - $E_{\text{in}} = W$
    - $W = mgh$

    Another text box indicates the height as "h" and the weight as "m".
- **Diagram (b):**
  - The chain is in a static position, with the acorn-shaped weight hanging downward.
  - Text accompanying this diagram shows the equation:
  - $E_{\text{out}} = mgh$

Both diagrams illustrate the concept of potential energy in the context of a cuckoo clock, with energy transfer represented by the equations.

More precisely, we define the *change* in gravitational potential energy ΔPE_g to be

$$\Delta\text{PE}_\text{g} = \text{mgh},$$

6.27

where, for simplicity, we denote the change in height by h rather than the usual Δh. Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

$$\begin{aligned}\text{mgh} &= (0.500\ \text{kg})\left((9.80\text{m/s})^2\right)(1.00\ \text{m}) \\ &= 4.90\ \text{kg} \cdot m^2/\text{s}^2 \ = \ 4.90\ \text{J}.\end{aligned}$$

6.28

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other

forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation $\Delta \mathrm{PE_g} = \mathrm{mgh}$ applies for any path that has a change in height of h, not just when the mass is lifted straight up, as long as h is small compared to the radius of Earth. Note that, as we learned in Uniform Circular Motion and Gravitation, the force of Earth's gravity does decrease with distance from Earth. The change is negligible for small changes in distance, but if you want to use potential energy in problems involving travel to the moon or even further, Newton's universal law of gravity must be taken into account. This more complete treatment is beyond the scope of this text and is not necessary for the problems we consider here. (See Figure 6.6.) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.

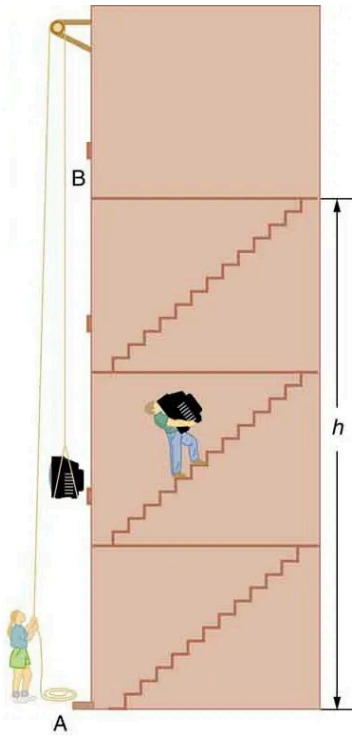


Figure 6.6 The change in gravitational potential energy $(\Delta \mathrm{PE_g})$ *between points A and B is independent of the path.* $\Delta \mathrm{PE_g} = \mathrm{mgh}$ *for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a side view of a building with three floors, connected by two flights of stairs. The building is partitioned into three levels, with each level connected by a set of stairs. On the right side of the image, there is a labeled vertical arrow with the letter "h" at the bottom to indicate the height of one level of the building. On the left side, there is a person at the bottom (labeled "A") holding a rope that runs over a pulley at the top of the building. The pulley is mounted on the roof and the rope is connected to a large, rectangular object, similar to a black box, that is being lifted from the ground. Another person inside the building is seen ascending the stairs with an identical object on their shoulder. The top of the building is labeled with the letter "B." The setup illustrates a comparison between lifting the object via the pulley system and carrying it manually up the stairs.

Example 6.6

The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial $\mathrm{PE_g}$ is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

Solution

The work done on the person by the floor as he stops is given by

$$W = \mathrm{Fdcos}\theta = -\mathrm{Fd},$$

6.29

with a minus sign because the displacement while stopping and the force from floor are in opposite directions $(\mathrm{cos}\theta = \mathrm{cos180^o} = -1)$. The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height h:

$$\mathrm{KE} = -\Delta\mathrm{PE_g} = -\mathrm{mgh},$$

6.30

The distance d that the person's knees bend is much smaller than the height h of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work W done by the floor on the person stops the person and brings the person's kinetic energy to zero:

$$W = -\mathrm{KE} = \mathrm{mgh}.$$

6.31

Combining this equation with the expression for W gives

$$-\mathrm{Fd} = \mathrm{mgh}.$$

6.32

Recalling that h is negative because the person fell *down*, the force on the knee joints is given by

$$\begin{aligned} F &= -\frac{\mathrm{mgh}}{d} \\ &= -\frac{(60.0\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(-3.00\ \mathrm{m})}{5.00 \times 10^{-3}\ \mathrm{m}} \\ &= 3.53 \times 10^5 N. \end{aligned}$$

6.33

Discussion

Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See Figure 6.7.)



Figure 6.7 The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two stacked photos of a kangaroo in motion, set in a grassy outdoor environment with trees in the background.

– Top Image: A kangaroo is captured mid-hop, with its body parallel to the ground. Its long tail is extended behind for balance, and its forelegs are tucked in. The kangaroo is moving from the left to the right of the frame. The background features grass and several trees in partial shadow.

– Bottom Image: Another view of the kangaroo, slightly more advanced in its hop. The kangaroo's legs are extended, and its body appears slightly higher off the ground compared to the top image. It continues in the same direction, against a backdrop of grass and leafy trees under diffused light.

Example 6.7

Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in Figure 6.8 if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?

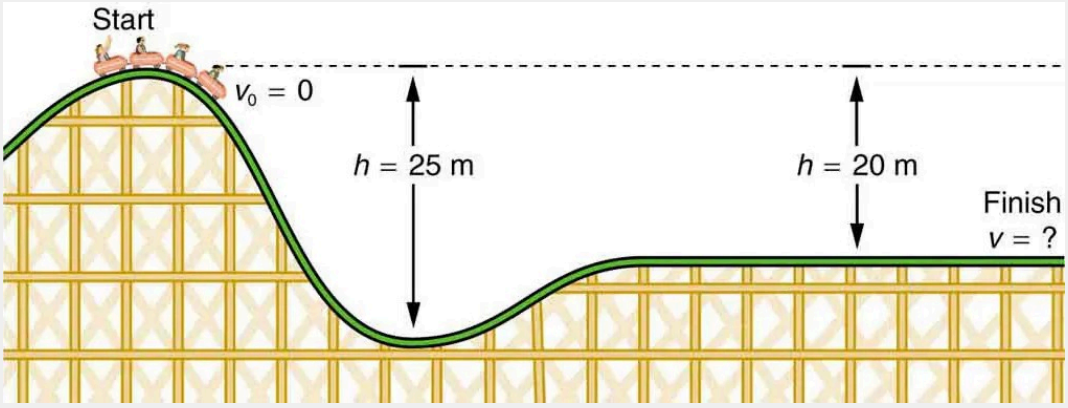


Figure 6.8 The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all $\Delta \mathrm{PE}_\mathrm{g}$ *is converted to* KE*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a physics concept related to roller coasters and potential energy. It shows a roller coaster track with two distinct sections.

On the left side of the image, a roller coaster car with several people sitting in it is at the start position, labeled as "Start." The car is situated on top of a hill, and the initial velocity is indicated as $v_0 = 0$. The height of this hill is labeled $h = 25\,\mathrm{m}$.

The track then descends into a valley and rises up again to a second hill on the right side.

This hill is labeled with a height of $h = 20\,\mathrm{m}$. At the top of this second hill, the track is labeled as "Finish" with a velocity $v = ?$, indicating that the velocity at this point is unknown and needs to be calculated.

The diagram is illustrated with structural supports beneath the track and a dashed line at the level of the starting point to visually emphasize the differences in height between the two hills.

Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta \mathrm{PE_g} = \Delta \mathrm{KE}$. Using the equations for $\mathrm{PE_g}$ and KE, we can solve for the final speed v, which is the desired quantity.

Solution for (a)

Here the initial kinetic energy is zero, so that $\Delta \mathrm{KE} = \frac{1}{2}\mathrm{mv}^2$. The equation for change in potential energy states that $\Delta \mathrm{PE_g} = \mathrm{mgh}$. Since h is negative in this case, we will rewrite this as $\Delta \mathrm{PE_g} = -\mathrm{mg} \mid h \mid$ to show the minus sign clearly. Thus,

$$-\Delta \mathrm{PE_g} = \Delta \mathrm{KE}$$

6.34

becomes

$$\mathrm{mg} \mid h \mid = \frac{1}{2}\mathrm{mv}^2.$$

6.35

Solving for v, we find that mass cancels and that

$$v = \sqrt{2g \mid h \mid}.$$

6.36

Substituting known values,

$$v = \sqrt{2\,(9.80\ \text{m/s}^2)\,(20.0\ \text{m})}$$
$$= 19.8\ \text{m/s}.$$

6.37

Solution for (b)

Again $-\Delta\text{PE}_\text{g} = \Delta\text{KE}$. In this case there is initial kinetic energy, so $\Delta\text{KE} = \frac{1}{2}mv^2 - \frac{1}{2}m(v_0)^2$. Thus,

$$\text{mg} \mid h \mid = \frac{1}{2}\text{mv}^2 - \frac{1}{2}m(v_0)^2.$$

6.38

Rearranging gives

$$\frac{1}{2}\text{mv}^2 = \text{mg} \mid h \mid + \frac{1}{2}m(v_0)^2.$$

6.39

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

$$v = \sqrt{2g \mid h \mid + (v_0)^2}.$$

6.40

This equation is very similar to the kinematics equation $v = \sqrt{(v_0)^2 + 2\text{ad}}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

$$v = \sqrt{2\,(9.80\ \text{m/s}^2\,)\,(20.0\ \text{m}\,) + (5.00\ \text{m/s})^2}$$
$$= 20.4\ \text{m/s}.$$

6.41

Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in Falling Objects that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see Figure 6.9). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.

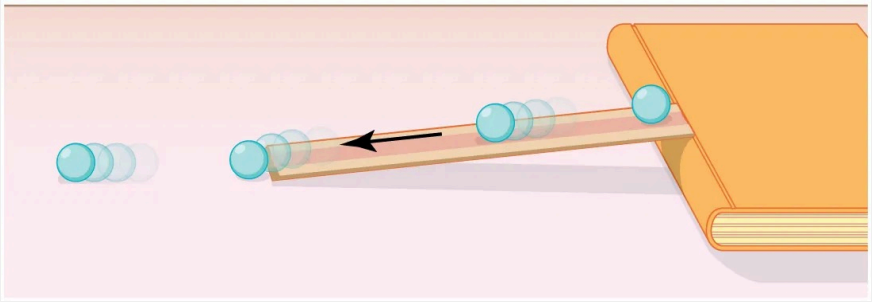


Figure 6.9 A marble rolls down a ruler, and its speed on the level surface is measured. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a side view of a physics experiment involving a wooden strip, glass marbles, and an orange book. The setup is as follows:

– A thin, flat strip of wood is resting with one end elevated by an orange book.
– The strip extends horizontally from the book towards the left side of the image.
– Three glass marbles are positioned on the strip. One is positioned near the end of the

strip close to the book, another is midway, and one is rolling off the strip towards the left.
– A solitary glass marble is in mid-air to the left, showing motion trails indicating it has rolled off the strip.
– An arrow is placed on the strip, pointing from right to left, indicating the direction of motion of the marbles.

The background is a soft gradient from light pink to light peach, and the image showcases the principle of motion and gravity as the marbles move from the elevated end downward and off the strip.

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6.4 CONSERVATIVE FORCES AND POTENTIAL ENERGY

Learning Objectives

By the end of this section, you will be able to:

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A conservative force is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a potential energy (PE) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration depends on the configuration, not the path followed, and is the potential energy added.

Potential Energy of a Spring

First, let us obtain an expression for the potential energy stored in a spring (PE_s). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law states that the magnitude of force F on the spring and the resulting deformation ΔL are proportional, $F = k\Delta L$.) (See Figure 6.10.) For our spring, we will replace ΔL (the amount of deformation produced by a force F) by the distance x that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude $\mathrm{F} = \mathrm{kx}$, where k is the spring's force constant. The force increases linearly from 0 at the start to kx in the fully stretched position. The average force is $\mathrm{kx}/2$. Thus the work done in stretching or compressing the spring is $W_s = \mathrm{Fd} = \left(\frac{\mathrm{kx}}{2}\right)x = \frac{1}{2}\mathrm{kx}^2$. Alternatively, we noted in 6.2 Kinetic Energy and the Work-Energy Theorem that the area under a graph of F vs. x is the work done by the force. In Figure 6.10(c) we see that this area is also $\frac{1}{2}\mathrm{kx}^2$. We therefore define the potential energy of a spring, PE_s, to be

$$\mathrm{PE_s} = \frac{1}{2}\mathrm{kx}^2,$$

6.42

where k is the spring's force constant and x is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or

compressing it a distance x. The potential energy of the spring PE_s does not depend on the path taken; it depends only on the stretch or squeeze x in the final configuration.

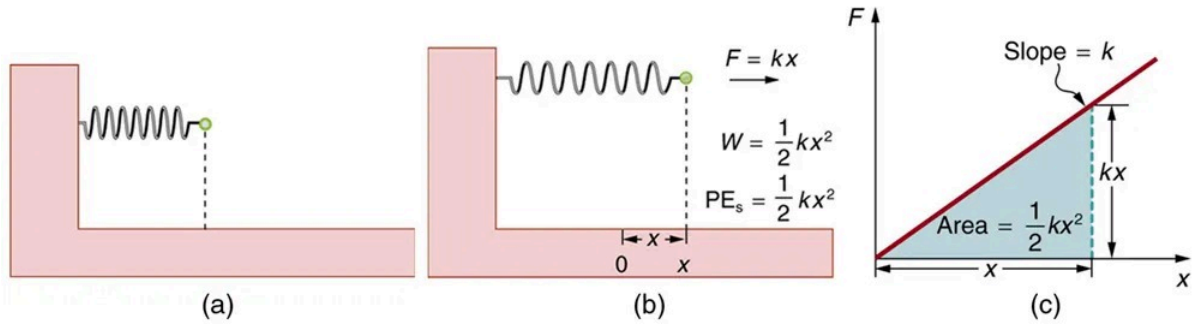


Figure 6.10 (a) An undeformed spring has no PE_s *stored in it. (b) The force needed to stretch (or compress) the spring a distance* x *has a magnitude* $F = \mathrm{kx}$*, and the work done to stretch (or compress) it is* $\frac{1}{2}\mathrm{kx}^2$*. Because the force is conservative, this work is stored as potential energy* (PE_s) *in the spring, and it can be fully recovered. (c) A graph of* F *vs.* x *has a slope of* k*, and the area under the graph is* $\frac{1}{2}\mathrm{kx}^2$*. Thus the work done or potential energy stored is* $\frac{1}{2}\mathrm{kx}^2$*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of three panels labeled (a), (b), and (c), illustrating the concept of potential energy in a spring.

Panel (a) shows a spring attached to a wall on the left side, in an unstressed state. The spring is depicted horizontally, with a dashed vertical line indicating its position at rest.

Panel (b) illustrates the same spring extended by a distance "x" to the right. There are annotations indicating:

- $F = kx$: the force exerted by the spring, where "k" is the spring constant, and "x" is the displacement.
- $W = ½kx^2$: the work done on the spring.
- $PE_s = ½kx^2$: the potential energy stored in the spring.

The reference point (marked as "0") is depicted near the original position of the spring.

Panel (c) contains a graph plotting "Force" (F) on the vertical axis and "Displacement" (x) on the horizontal axis. The graph includes:

- A straight line with a positive slope "k".
- An annotated point along the line labeled *kx*, representing the force at displacement "x".

- The area under the line, shaded in blue, labeled *Area* = ½*kx*², representing the potential energy of the spring.
- The slope of the line, described as *Slope* = *k*.

The equation $\mathrm{PE}_s = \frac{1}{2}\mathrm{kx}^2$ has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of potential energy is energy due to position, shape, or configuration. For shape or position deformations, stored energy is $\mathrm{PE}_s = \frac{1}{2}\mathrm{kx}^2$, where k is the force constant of the particular system and x is its deformation. Another example is seen in Figure 6.11 for a guitar string.

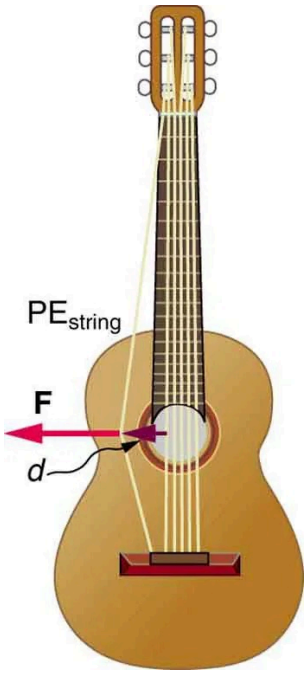


Figure 6.11 Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of a guitar. The focus is on the guitar strings and a few labeled elements that seem to explain a physics concept related to tension or elasticity.

– Guitar: It is a classical acoustic guitar with six strings, a sound hole, and tuning pegs at the top.
– Strings: One string is highlighted, stretching from the tuning pegs at the top to the guitar's bridge at the

bottom.
– Labels:
– PE_{string}: This label likely refers to the potential energy of the string.
– F: This label indicates a force applied to the string, represented by a red arrow pointing horizontally to the left.
– d: This represents a displacement or distance, indicated by a black line with a double-headed arrow pointing horizontally along with the red arrow for force.

The illustration is likely used to demonstrate how force and displacement relate to the tension or energy in a guitar string.

Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

$$W_{\text{net}} = \frac{1}{2}\text{mv}^2 - \frac{1}{2}(\text{mv}_0)^2 = \Delta\text{KE}.$$
6.43

If only conservative forces act, then

$$W_{\text{net}} = W_{\text{c}},$$
6.44

where W_c is the total work done by all conservative forces. Thus,

$$W_{\text{c}} = \Delta\text{KE}.$$
6.45

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is, $W_{\text{c}} = -\Delta\text{PE}$. Therefore,

$$-\Delta\text{PE} = \Delta\text{KE}$$
6.46

or

$$\Delta\text{KE} + \Delta\text{PE} = 0.$$

6.47

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\left.\begin{array}{c} \mathrm{KE} + \mathrm{PE} = \text{constant} \\ \text{or} \\ \mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f} \end{array}\right\} (\text{conservative forces only}),$$

6.48

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the conservation of mechanical energy principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its mechanical energy, $(\mathrm{KE} + \mathrm{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE, with the total energy remaining constant.

Example 6.8

Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in Figure 6.12. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.

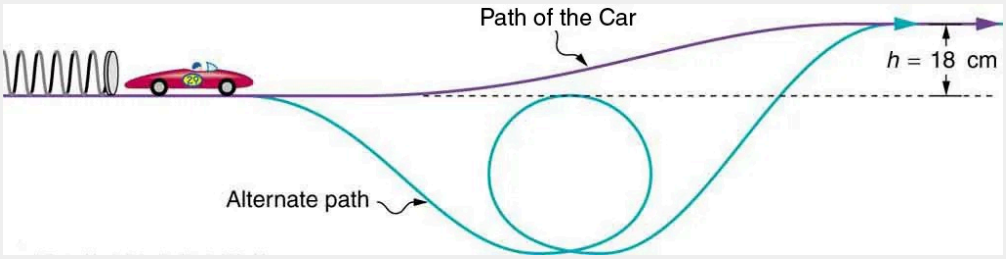


Figure 6.12 A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a simple physics illustration involving a car, two paths, and a spring:

– On the left side, there is a red toy car positioned against a compressed spring, which is ready to push it forward.
– The main path is shown as a purple line that gradually inclines from left to right and reaches a peak height marked as “h = 18 cm.”
– A dotted line horizontally extends across the image at the peak height indicating the

maximum height reached by the car.
– Below the main path, an “Alternate path” is indicated by a blue line. This path features a downward slope into a circular loop before joining the main path again as it ascends.
– The “Path of the Car” is labeled next to the main path, and the “Alternate path” is labeled beside the blue line.
– Arrows at the ends of the paths indicate the direction of motion.

Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}$$

6.49

or

$$\frac{1}{2}(\mathrm{mv}_i)^2 + \mathrm{mgh}_i + \frac{1}{2}(\mathrm{kx}_i)^2 = \frac{1}{2}(\mathrm{mv}_f)^2 + \mathrm{mgh}_f + \frac{1}{2}(\mathrm{kx}_f)^2,$$

6.50

where h is the height (vertical position) and x is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both h_i and h_f are zero. Furthermore, the initial speed v_i is zero and the final compression of the spring x_f is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

$$\frac{1}{2}(\mathrm{kx}_i)^2 = \frac{1}{2}(\mathrm{mv}_f)^2.$$

6.51

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

$$\begin{aligned} v_f &= \sqrt{\frac{k}{m}} x_i \\ &= \sqrt{\frac{250.0\ \text{N/m}}{0.100\ \text{kg}}} (0.0400\ \text{m}) \\ &= 2.00\ \text{m/s}. \end{aligned}$$

6.52

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\frac{1}{2}\text{kx}_\text{i}^2 = \frac{1}{2}\text{mv}_\text{f}^2 + \text{mgh}_\text{f}.$$

6.53

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for v_f and substituting known values gives

$$\begin{aligned} v_f &= \sqrt{\frac{(\text{kx}_i)^2}{m} - 2\text{gh}_f} \\ &= \sqrt{\left(\frac{250.0\ \text{N/m}}{0.100\ \text{kg}}\right)(0.0400\ \text{m})^2 - 2\,(9.80\ \text{m/s}^2)\,(0.180\ \text{m})} \\ &= 0.687\ \text{m/s}. \end{aligned}$$

6.54

Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in Example 6.8. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

PhET Explorations

Energy Skate Park

Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

Access multimedia content

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6.5 NONCONSERVATIVE FORCES

Learning Objectives

By the end of this section, you will be able to:

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.

Nonconservative Forces and Friction

Forces are either conservative or nonconservative. Conservative forces were discussed in 6.4 Conservative Forces and Potential Energy. A nonconservative force is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in Figure 6.13, work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. Friction, for example, creates thermal energy that dissipates, removing energy from the system. Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.

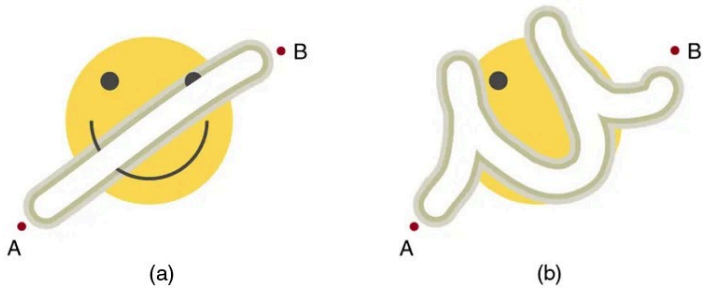


Figure 6.13 The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two parts, (a) and (b), each depicting a smiley face with a path running across it.

(a): The smiley face is yellow with a black curved line for a mouth and two black circles for eyes. A smooth, wavy path that looks like a thick line runs diagonally from the bottom left (point A) to the top right (point B) across the face. Both point A and point B are marked with small red dots.

(b): Similarly, the smiley face is yellow with the same features. The path, however, is more intricate, looping in a complex manner across the face, starting at point A on the left and ending at point B on the right, both marked with red dots.

In both parts, the smiley's facial features are partially obscured by the paths.

How Nonconservative Forces Affect Mechanical Energy

Mechanical energy *may* not be conserved when nonconservative forces act. For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy. Figure 6.14 compares the effects of conservative and nonconservative forces. We often choose to understand simpler systems such as that described in Figure 6.14(a) first before studying more complicated systems as in Figure 6.14(b).

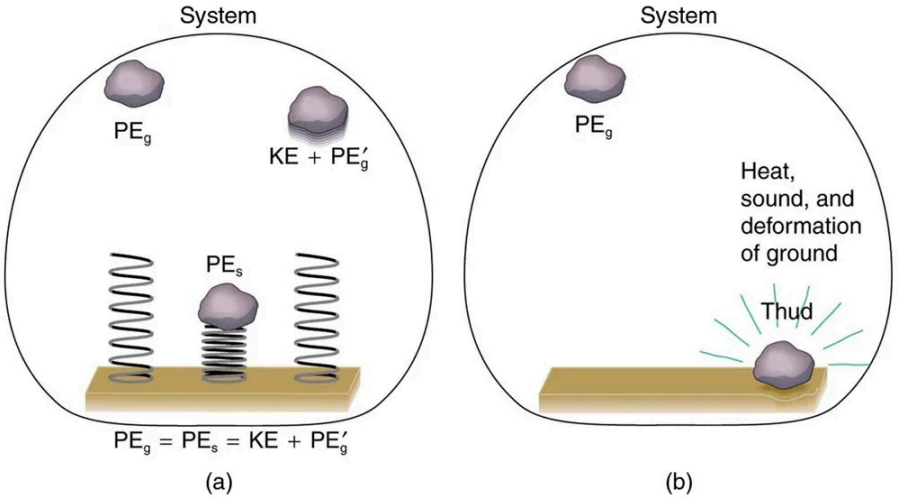


Figure 6.14 Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image consists of two diagrams labeled (a) and (b), each representing a different system involving a falling object. Both systems are enclosed by a line labeled "System."

Diagram (a):
– The falling object starts with gravitational potential energy, labeled "PE_g."
– As it falls, it converts potential energy into kinetic energy and gravitational potential energy, labeled "KE + PE'_g."
– The object lands on a spring, compressing it, resulting in stored elastic potential energy, labeled "PE_s."
– There is a balance equation at the bottom: "$PE_g = PE_s = KE + PE'_g$."

Diagram (b):
– The falling object begins with gravitational potential energy, labeled "PE_g."
– Upon impact with the ground, energy is transformed into heat, sound, and deformation of the ground, annotated with the word "Thud."

Each system shows different outcomes of energy transformation when an object falls and interacts with other surfaces.

How the Work-Energy Theorem Applies

Now let us consider what form the work-energy theorem takes when both conservative and nonconservative forces act. We will see that the work done by nonconservative forces equals the change in the mechanical energy of a system. As noted in 6.2 Kinetic Energy and the Work-Energy Theorem, the work-energy theorem states that the net work on a system equals the change in its kinetic energy, or $W_{\text{net}} = \Delta\text{KE}$. The net work is the sum of the work by nonconservative forces plus the work by conservative forces. That is,

$$W_{\text{net}} = W_{\text{nc}} + W_{\text{c}},$$
6.55

so that

$$W_{\text{nc}} + W_c = \Delta\text{KE},$$
6.56

where W_{nc} is the total work done by all nonconservative forces and W_{c} is the total work done by all conservative forces.

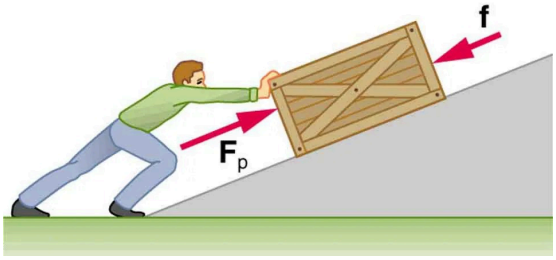


Figure 6.15 A person pushes a crate up a ramp, doing work on the crate. Friction and gravitational force (not shown) also do work on the crate; both forces oppose the person's push. As the crate is pushed up the ramp, it gains mechanical energy, implying that the work done by the person is greater than the work done by friction. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a person pushing a wooden crate up an inclined surface. The person is leaning forward, with one foot on the ground and the other foot slightly lifted, indicating exertion in pushing the crate. The crate has visible wooden planks and is secured with nails. The inclination implies that the surface is at an angle from the horizontal ground.

There are two arrows labeled with forces: a large red arrow pointing to the right labeled F_{p}, indicating the

force applied by the person, and a smaller red arrow pointing to the left at the crate labeled *f*, indicating frictional force resisting the motion. These arrows represent the dynamics of force involved in this scenario.

Consider Figure 6.15, in which a person pushes a crate up a ramp and is opposed by friction. As in the previous section, we note that work done by a conservative force comes from a loss of gravitational potential energy, so that $W_{\text{c}} = -\Delta\text{PE}$. Substituting this equation into the previous one and solving for W_{nc} gives

$$W_{\text{nc}} = \Delta\text{KE} + \Delta\text{PE}.$$
6.57

This equation means that the total mechanical energy $(\text{KE} + \text{PE})$ changes by exactly the amount of work done by nonconservative forces. In Figure 6.15, this is the work done by the person minus the work done by friction. So even if energy is not conserved for the system of interest (such as the crate), we know that an equal amount of work was done to cause the change in total mechanical energy.

We rearrange $W_{\text{nc}} = \Delta\text{KE} + \Delta\text{PE}$ to obtain

$$\text{KE}_{\text{i}} + \text{PE}_{\text{i}} + W_{\text{nc}} = \text{KE}_{\text{f}} + \text{PE}_{\text{f}}.$$
6.58

This means that the amount of work done by nonconservative forces adds to the mechanical energy of a system. If W_{nc} is positive, then mechanical energy is increased, such as when the person pushes the crate up the ramp in Figure 6.15. If W_{nc} is negative, then mechanical energy is decreased, such as when the rock hits the ground in Figure 6.14(b). If W_{nc} is zero, then mechanical energy is conserved, and nonconservative forces are balanced. For example, when you push a lawn mower at constant speed on level ground, your work done is removed by the work of friction, and the mower has a constant energy.

Applying Energy Conservation with Nonconservative Forces

When no change in potential energy occurs, applying $\text{KE}_{\text{i}} + \text{PE}_{\text{i}} + W_{\text{nc}} = \text{KE}_{\text{f}} + \text{PE}_{\text{f}}$ amounts to applying the work-energy theorem by setting the change in kinetic energy to be equal to the net work done on the system, which in the most general case includes both conservative and nonconservative forces. But when seeking instead to find a change in total mechanical energy in situations that involve changes in both potential and kinetic energy, the previous equation

$$\text{KE}_{\text{i}} + \text{PE}_{\text{i}} + W_{\text{nc}} = \text{KE}_{\text{f}} + \text{PE}_{\text{f}}$$

says that you can start by finding the change in mechanical energy that would have resulted from just the

conservative forces, including the potential energy changes, and add to it the work done, with the proper sign, by any nonconservative forces involved.

Example 6.9

Calculating Distance Traveled: How Far a Baseball Player Slides

Consider the situation shown in Figure 6.16, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.

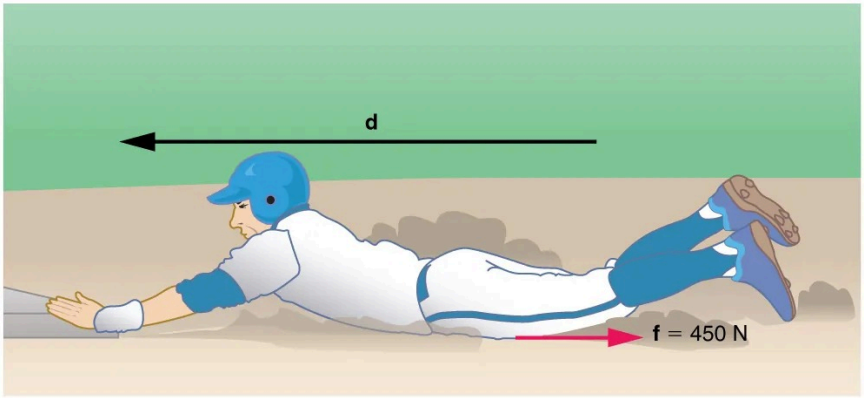


Figure 6.16 The baseball player slides to a stop in a distance d. In the process, friction removes the player's kinetic energy by doing an amount of work fd equal to the initial kinetic energy. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a baseball player sliding head-first into a base. The player is wearing a white uniform with blue accents and a blue helmet, and is extending their body forward, arms outstretched towards the base, which is partially visible on the left side of the image. The player's legs are bent, and cleats are visible. There is a graphic arrow pointing to the left labeled "d," indicating the direction of the slide. Another arrow pointing to the right labeled "f = 450 N" denotes the force of friction acting against the slide. The

background shows a gradient from green to beige, representing the field and dirt. Dust is illustrated around the player to emphasize movement.

Strategy

Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction, which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because $\mathbf{f}$ is in the opposite direction of the motion (that is, $\theta = 180^{\circ}$, and so $\cos\theta = -1$). Thus $W_{\text{nc}} = -\text{fd}$. The equation simplifies to

$$\frac{1}{2}(\text{mv}_i)^2 - \text{fd} = 0$$

6.59

or

$$\text{fd} = \frac{1}{2}(\text{mv}_i)^2.$$

6.60

This equation can now be solved for the distance d.

Solution

Solving the previous equation for d and substituting known values yields

$$\begin{aligned} d &= \frac{(\text{mv}_i)^2}{2f} \\ &= \frac{(65.0\ \text{kg})\,(6.00\ \text{m/s})^2}{(2)\,(450\ \text{N})} \\ &= 2.60\ \text{m}. \end{aligned}$$

6.61

Discussion

The most important point of this example is that the amount of nonconservative work

equals the change in mechanical energy. For example, you must work harder to stop a truck, with its large mechanical energy, than to stop a mosquito.

Example 6.10

Calculating Distance Traveled: Sliding Up an Incline

Suppose that the player from Example 6.9 is running up a hill having a 5.00° incline upward with a surface similar to that in the baseball stadium. The player slides with the same initial speed, and the frictional force is still 450 N. Determine how far he slides.

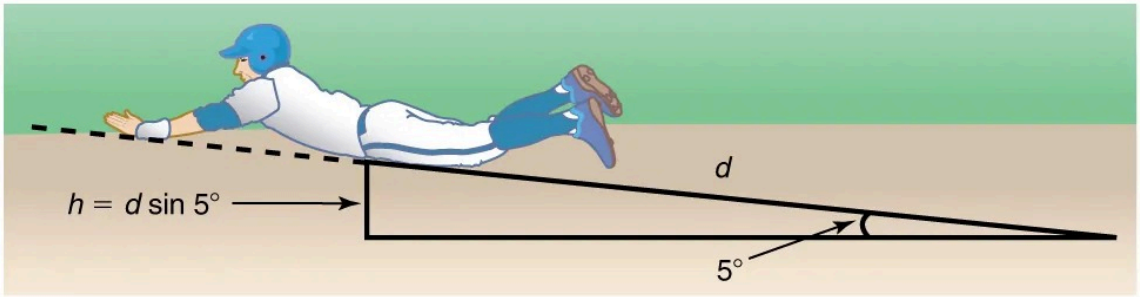


Figure 6.17 The same baseball player slides to a stop on a 5.00° slope. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a diagram of a person wearing a baseball uniform, including a helmet, performing a head-first slide along a surface. The illustration is overlaid with geometric lines and a mathematical equation.

– The person is sliding on a surface represented by a long, sloped line. This line is labeled as “d,” which signifies distance.
– The slope forms a 5-degree angle with the horizontal base.
– A perpendicular line extends from the base to the sloped line, representing height “h.”
– The mathematical equation $h = d \sin 5^{\circ}$ is displayed, indicating the relationship between the height “h,” the distance “d,” and the angle of incline.
– The background consists of a simple gradient with an earth-toned foreground and a greenish sky.

Strategy

In this case, the work done by the nonconservative friction force on the player reduces the mechanical energy he has from his kinetic energy at zero height, to the final mechanical energy he has by moving through distance d to reach height h along the hill, with $h = d\sin 5.00^{\circ}$. This is expressed by the equation

$$\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} = \mathrm{KE_f} + \mathrm{PE_f}.$$

6.62

Solution

The work done by friction is again $W_{\mathrm{nc}} = -\mathrm{fd}$; initially the potential energy is $\mathrm{PE}_i = \mathrm{mg} \cdot 0 = 0$ and the kinetic energy is $\mathrm{KE}_i = \frac{1}{2}(\mathrm{mv}_i)^2$; the final energy contributions are $\mathrm{KE}_f = 0$ for the kinetic energy and $\mathrm{PE}_f = \mathrm{mgh} = \mathrm{mgdsin}\theta$ for the potential energy.

Substituting these values gives

$$\frac{1}{2}(\mathrm{mv}_i)^2 + 0 + (-\mathrm{fd}) = 0 + \mathrm{mgdsin}\theta.$$

6.63

Solve this for d to obtain

$$\begin{aligned} d &= \frac{\left(\frac{1}{2}\right)(\mathrm{mv_i})^2}{f + \mathrm{mgsin}\theta} \\ &= \frac{(0.5)\,(65.0\ \mathrm{kg})\,(6.00\ \mathrm{m/s})^2}{450\ \mathrm{N} + (65.0\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2})\sin\,(5.00^{\circ})} \\ &= 2.31\ \mathrm{m}. \end{aligned}$$

6.64

Discussion

As might have been expected, the player slides a shorter distance by sliding uphill. Note

that the problem could also have been solved in terms of the forces directly and the work energy theorem, instead of using the potential energy. This method would have required combining the normal force and force of gravity vectors, which no longer cancel each other because they point in different directions, and friction, to find the net force. You could then use the net force and the net work to find the distance d that reduces the kinetic energy to zero. By applying conservation of energy and using the potential energy instead, we need only consider the gravitational potential energy mgh, without combining and resolving force vectors. This simplifies the solution considerably.

Making Connections: Take-Home Investigation—Determining Friction from the Stopping Distance

This experiment involves the conversion of gravitational potential energy into thermal energy. Use the ruler, book, and marble from Take-Home Investigation—Converting Potential to Kinetic Energy. In addition, you will need a foam cup with a small hole in the side, as shown in Figure 6.18. From the 10-cm position on the ruler, let the marble roll into the cup positioned at the bottom of the ruler. Measure the distance d the cup moves before stopping. What forces caused it to stop? What happened to the kinetic energy of the marble at the bottom of the ruler? Next, place the marble at the 20-cm and the 30-cm positions and again measure the distance the cup moves after the marble enters it. Plot the distance the cup moves versus the initial marble position on the ruler. Is this relationship linear?

With some simple assumptions, you can use these data to find the coefficient of kinetic friction μ_k of the cup on the table. The force of friction f on the cup is $\mu_k N$, where the normal force N is just the weight of the cup plus the marble. The normal force and force of gravity do no work because they are perpendicular to the displacement of the cup, which moves horizontally. The work done by friction is fd. You will need the mass of the marble as well to calculate its initial kinetic energy.

It is interesting to do the above experiment also with a steel marble (or ball bearing). Releasing it from the same positions on the ruler as you did with the glass marble, is the velocity of this steel marble the same as the velocity of the marble at the bottom of the ruler? Is the distance the cup moves proportional to the mass of the steel and glass marbles?

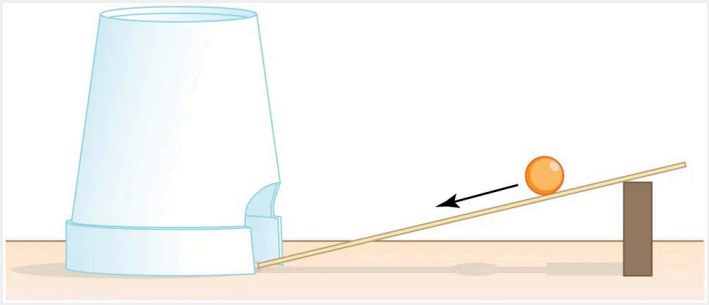


Figure 6.18 Rolling a marble down a ruler into a foam cup. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a simple machine setup designed to launch a ball into a cup. A tilted ramp is created with a narrow plank resting on a small support, forming an incline. At the top right of the ramp is a small orange ball. An arrow points down the slope, indicating the direction the ball will roll. At the bottom of the ramp, on the left, there is an overturned cup with a small cutout in its side, positioned to catch the ball as it descends. The setup is placed on a flat surface, and the ball is poised to roll down the ramp and enter the cup through the cutout opening.

PhET Explorations

The Ramp

Explore forces, energy and work as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces acting on the file cabinet. Graphs show forces, energy and work.

Click to view content.

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6.6 CONSERVATION OF ENERGY

Learning Objectives

By the end of this section, you will be able to:

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The law of conservation of energy can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy $(\mathrm{KE} + \mathrm{PE})$ and energy transferred via work done by nonconservative forces (W_{nc}). But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called other energy (OE). Then we can state the conservation of energy in equation form as

$$\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} + \mathrm{OE_i} = \mathrm{KE_f} + \mathrm{PE_f} + \mathrm{OE_f}.$$
6.65

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is KE, work done by a conservative force is represented by PE, work done by nonconservative forces is W_{nc}, and all other energies are included as OE. This equation applies to all previous examples; in those situations OE was constant, and so it subtracted out and was not directly considered.

Making Connections: Usefulness of the Energy Conservation Principle

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does OE play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of OE).

Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. Electrical energy is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry chemical energy that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as radiant energy, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. Nuclear energy comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power

plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called thermal energy, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Table 6.1 gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

Problem-Solving Strategies for Energy

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

Step 2. Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

Step 3. If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}.$$

6.66

Step 4. If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

$$\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} + \mathrm{OE_i} = \mathrm{KE_f} + \mathrm{PE_f} + \mathrm{OE_f}.$$

6.67

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate W_c, the work done by conservative forces; it is already incorporated in the PE terms.

Step 5. You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose $h = 0$ at either the initial or final point, so that PE_{g} is zero there. Then solve for the unknown in the customary manner.

Step 6. *Check the answer to see if it is reasonable*. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see Figure 6.19) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

A solar-powered aircraft flying over the sea. Solar cells are on the upper surface of the wings, where they are exposed to sunlight.

Figure 6.19 Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. Image by NASA, Wikimedia Commons License

Table 6.1 Energy of Various Objects and Phenomena

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Energy released in a supernova	10^{44}
Fusion of all the hydrogen in Earth's oceans	10^{34}
Annual world energy use	4×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
1 kg hydrogen (fusion to helium)	6.4×10^{14}
1 kg uranium (nuclear fission)	8.0×10^{13}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
90,000-metric ton aircraft carrier at 30 knots	1.1×10^{10}
1 barrel crude oil	5.9×10^{9}
1 ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily home electricity use (developed countries)	7×10^{7}
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
1 g fat (9.3 kcal)	3.9×10^{4}
ATP hydrolysis reaction	3.2×10^{4}
1 g carbohydrate (4.1 kcal)	1.7×10^{4}
1 g protein (4.1 kcal)	1.7×10^{4}
Tennis ball at 100 km/h	22
Mosquito $(10^{-2} gat 0.5 m/s)$	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}

Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The efficiency Eff of an energy conversion process is defined as

$$\text{Efficiency (Eff)} = \frac{\text{useful energy or work output}}{\text{total energy input}} = \frac{W_{\text{out}}}{E_{\text{in}}}.$$

6.68

Table 6.2 lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Table 6.2 Efficiency of the Human Body and Mechanical Devices

Activity/device	**Efficiency (%)**[1]
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

1. Representative values

PhET Explorations

Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

Access multimedia content

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6.7 POWER

Learning Objectives

By the end of this section, you will be able to:

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in Figure 6.20.



Figure 6.20 This powerful rocket on the Space Shuttle Endeavor did work and consumed energy at a very high rate. Image by NASA, Wikimedia Commons License

These images of power have in common the rapid performance of work, consistent with the scientific definition of power (P) as the rate at which work is done.

Power

Power is the rate at which work is done.

$$P = \frac{W}{t}$$

6.69

The SI unit for power is the watt (W), where 1 watt equals 1 joule/second ($1\ \mathrm{W} = 1 J/s$).

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Example 6.11

Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See Figure 7.21.)

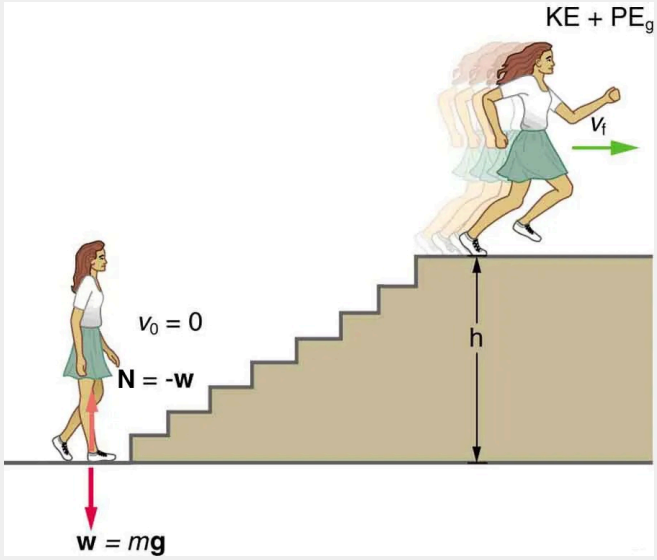


Figure 6.21 When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram illustrating the concepts of kinetic energy, potential energy, and force. It shows a person at the base of a staircase and then ascending to the top.

– On the left side, a person is standing still before the stairs.
– An arrow labeled “w = mg” points downward, indicating the weight force due to gravity.

– A vertical arrow labeled “N = -w” points upward, representing the normal force balancing the weight.
– There’s a horizontal arrow labeled “v^0 = 0,” indicating initial velocity is zero.

– To the right, the person is shown in multiple superimposed positions, depicting motion ascending the stairs.
– An arrow labeled “v_f” points horizontally to the right, indicating final velocity at the top.
– The vertical height “h” is marked beside the stairs, representing gravitational potential energy gained as the person ascends.

– The top of the diagram is labeled “KE + PE^9,” signifying the combination of kinetic energy and potential energy at the top of the stairs.

Strategy and Concept

The work going into mechanical energy is $W = \mathrm{KE} + \mathrm{PE}$. At the bottom of the stairs, we take both KE and
PE_g as initially zero; thus,

$$W = \mathrm{KE}_f + \mathrm{PE}_g = \frac{1}{2}(mv_f)^2 + \mathrm{mgh},$$

where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.

Solution

Substituting the expression for W into the definition of power given in the previous equation, $P = W/t$ yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}(\mathrm{mv}_f)^2 + \mathrm{mgh}}{t}.$$

6.70
Entering known values yields

$$P = \frac{0.5\,(60.0\ \text{kg})(2.00\ \text{m/s})^2 + (60.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)(3.00\ \text{m})}{3.50\ \text{s}}$$
$$= \frac{120\ \text{J} + 1764\ \text{J}}{3.50\ \text{s}}$$
$$= 538\ \text{W}.$$

6.71

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 horsepower $(1\ \text{hp} = 746\ \text{W})$! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

Making Connections: Take-Home Investigation—Measure Your Power Rating

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See Table 6.3 for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter $(\mathrm{kW/m^2})$. A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is 10^6 W of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See Figure 6.22.)



Figure 6.22 Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. Image by Kleineolive, Wikimedia Commons License

Table 6.3 Power Output or Consumption

Object or Phenomenon	Power in Watts
Supernova (at peak)	5×10^{37}
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^{9}
Aircraft carrier (total useful and heat transfer)	10^{8}
Dragster (total useful and heat transfer)	2×10^{6}
Car (total useful and heat transfer)	8×10^{4}
Football player (total useful and heat transfer)	5×10^{3}
Clothes dryer	4×10^{3}
Person at rest (all heat transfer)	100
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	10^{-3}

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P = W/t = E/t$, where E is the energy supplied by the electricity company. So the energy consumed over a time t is

$$E = \mathrm{Pt}.$$
6.72

Electricity bills state the energy used in units of kilowatt-hours $(\mathrm{kW} \cdot h)$, which is the product of power in

kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

Example 6.12

Calculating Energy Costs

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is \$0.120 per $\mathrm{kW} \cdot h$?

Strategy

Cost is based on energy consumed; thus, we must find E from $E = \mathrm{Pt}$ and then calculate the cost. Because electrical energy is expressed in $\mathrm{kW} \cdot h$, at the start of a problem such as this it is convenient to convert the units into kW and hours.

Solution

The energy consumed in $\mathrm{kW} \cdot \mathrm{h}$ is

$$E = \mathrm{Pt} = (0.200\mathrm{kW})\,(6.00\mathrm{h/d})\,(30.0\mathrm{d}) \\ = 36.0\ \mathrm{kW} \cdot \mathrm{h},$$

6.73

and the cost is simply given by

$$\mathrm{cost} = (36.0\ \mathrm{kW} \cdot h)\,(\$0.120\ \mathrm{per\ kW} \cdot h) \\ = \$4.32\ \mathrm{per\ month}.$$

6.74

Discussion

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. The potential for energy to produce useful work has been "degraded" in the energy transformation.

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6.8 WORK, ENERGY, AND POWER IN HUMANS

Learning Objectives

By the end of this section, you will be able to:

- Explain the human body's consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See Figure 6.23.) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.

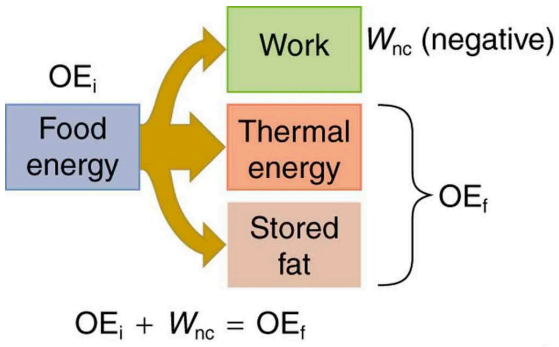


Figure 6.23 Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the flow and conversion of food energy.

– On the left side, there is a blue rectangle labeled "OE_i Food energy."
– Arrows from this rectangle divide the energy into three categories on the right:
– A green rectangle labeled "Work" with a notation "W_{nc} (negative)"
– An orange rectangle labeled "Thermal energy"
– A red rectangle labeled "Stored fat"

– To the right of the rectangles, the diagram groups "Thermal energy" and "Stored fat" under "OE_f".

– At the bottom, there is an equation: "$OE_i + W_{nc} = OE_f$".

This diagram represents the concept of energy balance, showing how food energy is allocated to work, heat, and fat storage.

Power Consumed at Rest

The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate. The total energy conversion rate of a person *at rest* is called the basal metabolic rate (BMR) and is divided among various systems in the body, as shown in Table 6.4. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Table 6.4 Basal Metabolic Rates (BMR)

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Liver & spleen	23	67	27
Brain	16	47	19
Skeletal muscle	15	45	18
Kidney	9	26	10
Heart	6	17	7
Other	16	48	19
Totals	85 W	250 mL/min	100%

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See Figure 6.24.) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. Table 6.5 shows energy and oxygen consumption rates (power expended) for a variety of activities.

Power of Doing Useful Work

Work done by a person is sometimes called useful work, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy $(\text{KE} + \text{PE})$ of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball's kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as Example 6.13 illustrates.

Example 6.13

Calculating Weight Loss from Exercising

If a person who normally requires an average of 12,000 kJ (3000 kcal) of food energy per day consumes 13,000 kJ per day, they will steadily add body fat. How much bicycling per day is required to work off this extra 1000 kJ?

Solution

Table 6.5 states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

$$\text{Time} = \frac{\text{energy}}{\left(\frac{\text{energy}}{\text{time}}\right)} = \frac{1000\ \text{kJ}}{400\ \text{W}} = 2500\ \text{s} = 42\ \text{min}.$$

6.75

Discussion

If this person uses more energy than they consume, the person's body will obtain the needed energy by metabolizing body fat. If the person uses 13,000 kJ but consumes only 12,000 kJ, then the amount of fat loss will be

$$\text{Fat loss} = (1000\ \text{kJ}\,)\left(\frac{1.0\ \text{g fat}}{39\ \text{kJ}}\right) = 26\ \text{g},$$

6.76

assuming the energy content of fat to be 39 kJ/g.

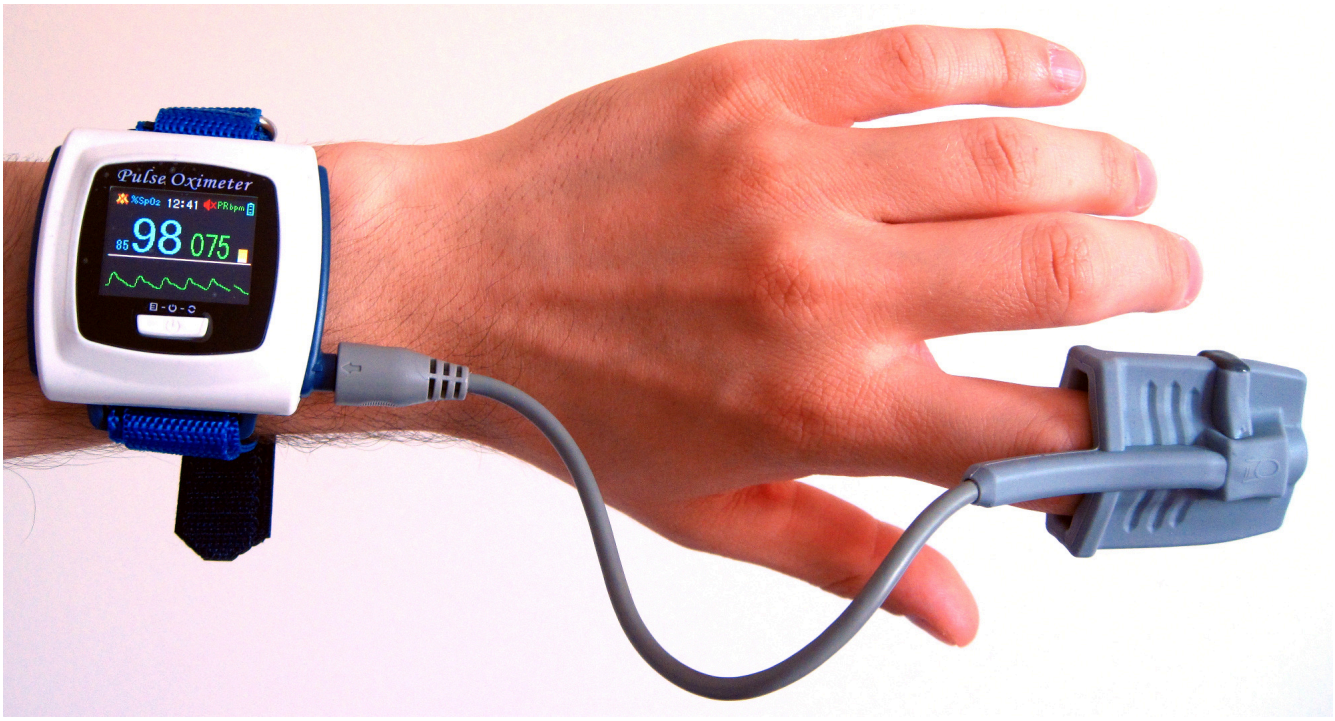


Figure 6.24 A pulse oxymeter is an apparatus that measures the amount of oxygen in blood. A knowledge of oxygen and carbon dioxide levels indicates a person's metabolic rate, which is the rate at which food energy is converted to another form. Such measurements can indicate the level of athletic conditioning as well as certain medical problems. Image by UusiAjaja, Wikimedia Commons License

Table 6.5 Energy and Oxygen Consumption Rates[1] (Power)

Activity	Energy consumption in watts	Oxygen consumption in liters O_2/min
Sleeping	83	0.24
Sitting at rest	120	0.34
Standing relaxed	125	0.36
Sitting in class	210	0.60
Walking (5 km/h)	280	0.80
Cycling (13–18 km/h)	400	1.14
Shivering	425	1.21
Playing tennis	440	1.26
Swimming breaststroke	475	1.36
Ice skating (14.5 km/h)	545	1.56
Climbing stairs (116/min)	685	1.96
Cycling (21 km/h)	700	2.00
Running cross-country	740	2.12
Playing basketball	800	2.28
Cycling, professional racer	1855	5.30
Sprinting	2415	6.90

All bodily functions, from thinking to lifting weights, require energy. (See Figure 6.25.) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all is that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.

1. for a 76-kg male with typical metabolic functions

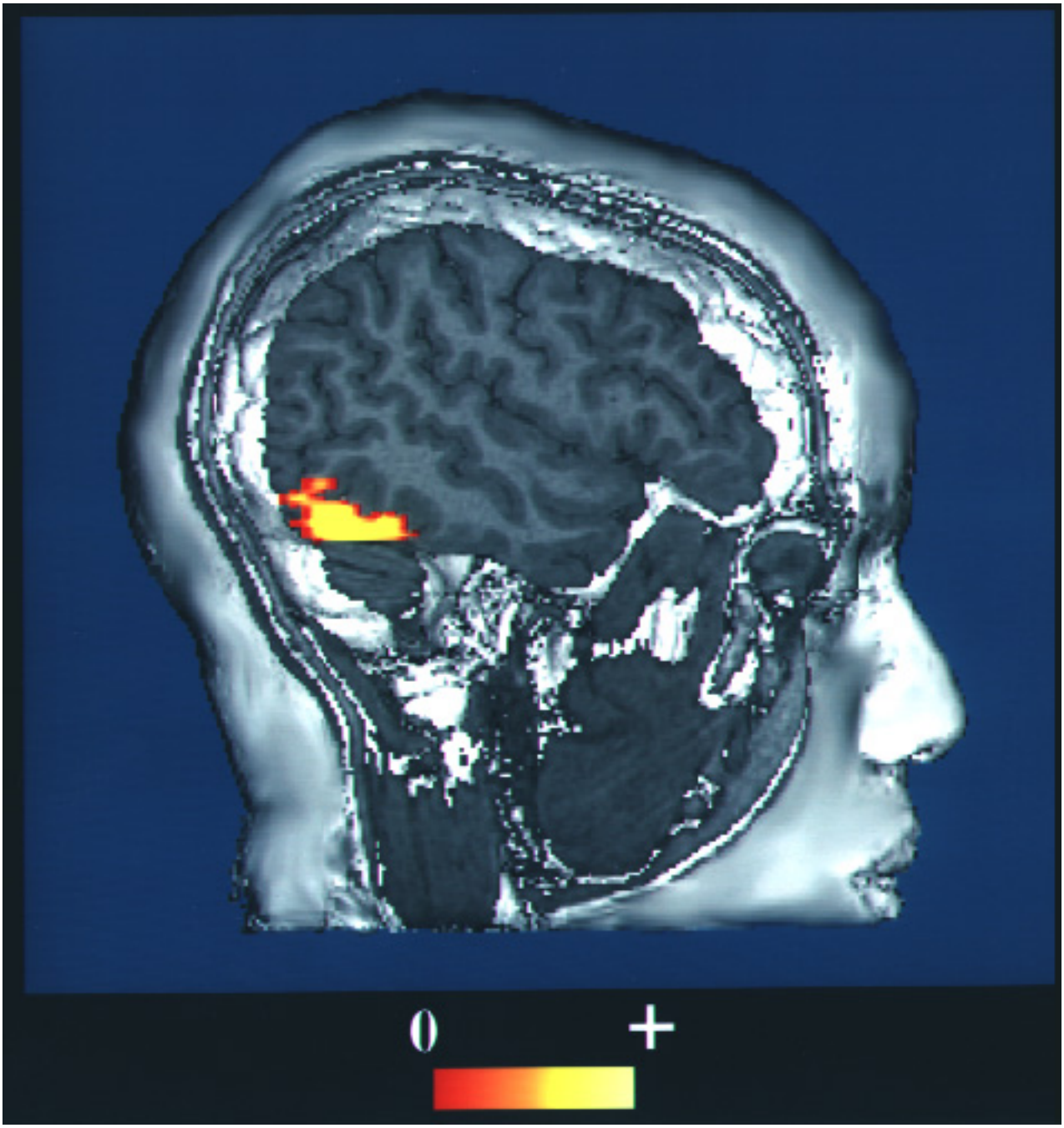


Figure 6.25 This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being asked to recognize faces. Image by NIH, Wikimedia Commons License

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6.9 WORLD ENERGY USE

Learning Objectives

By the end of this section, you will be able to:

- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

Energy is an important ingredient in all phases of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. But current levels of energy consumption and production are not sustainable. Depending on the data source, estimates indicate that about 31–35% of the world's energy comes from oil, and much of that goes to transportation uses. This is a reduction by a few percentage points from ten years ago. Oil prices are dependent as much upon new (or foreseen) discoveries as they are upon political events and situations around the world. The U.S., with 4.25% of the world's population, consumes 21% of the world's oil production per year.

Renewable and Nonrenewable Energy Sources

The principal energy resources used in the world are shown in Figure 6.26. The fuel mix has changed over the years but now is dominated by oil, although natural gas and solar contributions are increasing. Renewable forms of energy are those sources that cannot be used up, such as water, wind, solar, and biomass. About 85% of our energy comes from nonrenewable fossil fuels—oil, natural gas, coal. The likelihood of a link between global warming and fossil fuel use, with its production of carbon dioxide through combustion, has made, in the eyes of many scientists, a shift to non-fossil fuels of utmost importance—but it will not be easy.

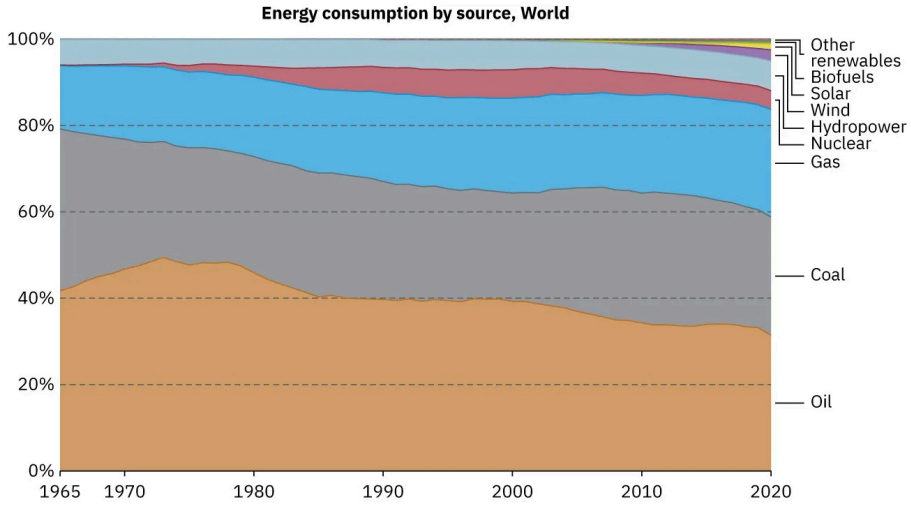


Figure 6.26 World energy consumption by source, in billions of kilowatt-hours: 2006. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an area chart titled "Energy consumption by source, World" showing the global energy consumption percentages from 1965 to 2020. The y-axis represents percentages from 0% to 100%, while the x-axis shows years from 1965 to 2020. Different energy sources are represented by colored areas stacked on top of each other. Below is the breakdown of the areas and their corresponding colors:

– Oil (bottom layer, brown): This area is the largest throughout the timeline, indicating oil's dominance in energy consumption.
– Coal (next layer up, gray): This remains substantial but shows a slight decrease over time.
– Gas (middle layer, blue): Maintains a steady share, slightly increasing.
– Nuclear (thin layer, pink): Remains relatively consistent but small compared to fossil fuels.
– Hydropower (thin layer, turquoise): Consistent small percentage of energy consumption.
– Wind, Solar, Biofuels, and Other renewables (very thin top layers, various colors): These new renewables show slight growth over the years but remain a small part of the total energy mix.

On the right side, there is a legend identifying each color with its respective energy source. Overall, fossil fuels (oil, coal, and gas) dominate the chart, with renewables and nuclear energy showing minimal shares.

The World's Growing Energy Needs

World energy consumption continues to rise, especially in the developing countries. (See Figure 6.27.) Global demand for energy has tripled in the past 50 years and might triple again in the next 30 years. While much of this growth will come from the rapidly booming economies of China and India, many of the developed

countries, especially those in Europe, are hoping to meet their energy needs by expanding the use of renewable sources. Although presently only a small percentage, renewable energy is growing very fast, especially wind energy. For example, Germany plans to meet 65% of its power and 30% of its overall energy needs with renewable resources by the year 2030. (See Figure 6.28.) Energy is a key constraint in the rapid economic growth of China and India. In 2003, China surpassed Japan as the world's second largest consumer of oil. However, over 1/3 of this is imported. Unlike most Western countries, coal dominates the commercial energy resources of China, accounting for 2/3 of its energy consumption. In 2009 China surpassed the United States as the largest generator of CO_2. In India, the main energy resources are biomass (wood and dung) and coal. Half of India's oil is imported. About 70% of India's electricity is generated by highly polluting coal. Yet there are sizeable strides being made in renewable energy. India has a rapidly growing wind energy base, and it has the largest solar cooking program in the world. China has invested substantially in building solar collection farms as well as hydroelectric plants.

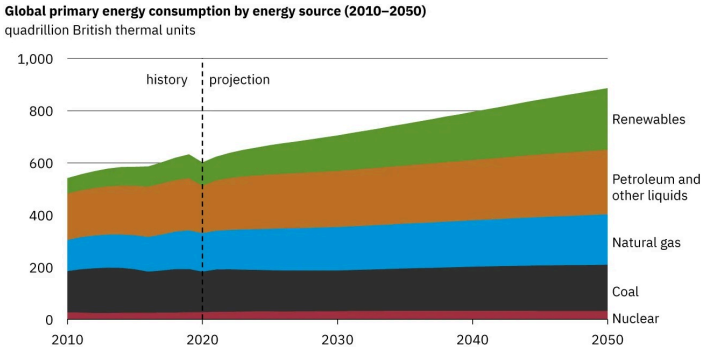


Figure 6.27 Past and projected world energy use Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a stacked area chart titled "Global primary energy consumption by energy source (2010–2050)." It shows the projected and historical consumption of energy in quadrillion British thermal units from 2010 to 2050. The time period is split into "history" and "projection" sections by a vertical dashed line around the year 2020.

The x-axis represents the years from 2010 to 2050. The y-axis represents energy consumption in quadrillion British thermal units, ranging from 0 to 1,000.

The chart displays consumption from different energy sources, with each source represented by a colored band:

– Green band for Renewables, showing a steady increase over time.
– Brown band for Petroleum and other liquids, maintaining a consistent width with a slight increase.
– Blue band for Natural gas, showing steady growth.

– Black band for Coal, with the width decreasing slightly over time.
– Red band for Nuclear, maintaining a thin, consistent width.

The chart visually indicates that Renewables are expected to see significant growth by 2050, while other sources show varying levels of growth or decline.



Figure 6.28 Solar cell arrays at a power plant in California Image from OpenStax College Physics 2e, CC-BY 4.0

Table 6.6 displays the 2020 commercial energy mix by country for some of the prime energy users in the world. While non-renewable sources dominate, some countries get a sizeable percentage of their electricity from renewable resources. For example, about two-thirds of New Zealand's electricity demand is met by hydroelectric. Only 10% of the U.S. electricity is generated by renewable resources, primarily hydroelectric. It is difficult to determine total sources and consumers of energy in many countries, and estimates vary somewhat by data source and type of measurement.

Table 6.6 Energy Consumption—Selected Countries (2020)

Country	Consumption, in EJ (10^{18} J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables
Australia	5.6	1.8	1.5	1.7	0	0.1	0.5
Brazil	12	4.6	1.2	0.6	0.1	3.5	2
China	145.5	28.5	11.9	82.3	3.3	11.7	7.8
Egypt	3.7	1.3	2.1	0.0%	0	0.1	0.1
Germany	12.1	4.2	3.1	1.9	0.6	0.2	2.2
India	31.99	9	2.2	17.5	0.4	1.5	1.4
Indonesia	8.1	2.8	1.5	3.3	0	0.2	0.4
Japan	17	6.5	3.8	4.6	0.4	0.7	1.1
United Kingdom	6.9	2.4	2.6	0.2	0.5	0.1	1.2
Russia	28.3	6.4	14.8	3.2	1.9	1.9	0.5
U.S.	87.8	32.5	30	9.2	7.4	2.6	6.2
World	**557.1**	**174.2**	**137.6**	**151.4**	**24**	**38.2**	**31.7**

Energy and Economic Well-being

Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (gross domestic product) per capita, are matched by higher levels of energy consumption per capita. This is borne out in Figure 6.29. Increased efficiency of energy use will change this dependency. A global problem is balancing energy resource development against the harmful effects upon the environment in its extraction and use.New and diversified energy sources do, however, greatly increase economic opportunity and stability. First, the extensive employment opportunities in renewable energy make it one of the most sustainable and secure fields to enter. Second, renewable energy provides countries and localities with increased levels of resiliency in the face of natural disasters, conflict, or other disruptions. The 21st century has already seen major economic impacts from energy disruptions: Hurricane Katrina, Superstorm Sandy, various wildfires, Hurricane Maria, and the 2021 Texas Winter Storm demonstrate the vulnerability of United States power systems. Diversifying energy sources through renewables and other fossil-fuel alternatives brings power grids and transportation systems back online much more quickly, saving lives and enabling a more swift return to economic operations. And as critical emerging information infrastructure, such as data centers, requires more of the world's energy, supplying those growing systems during normal operations and crises will be increasingly important.

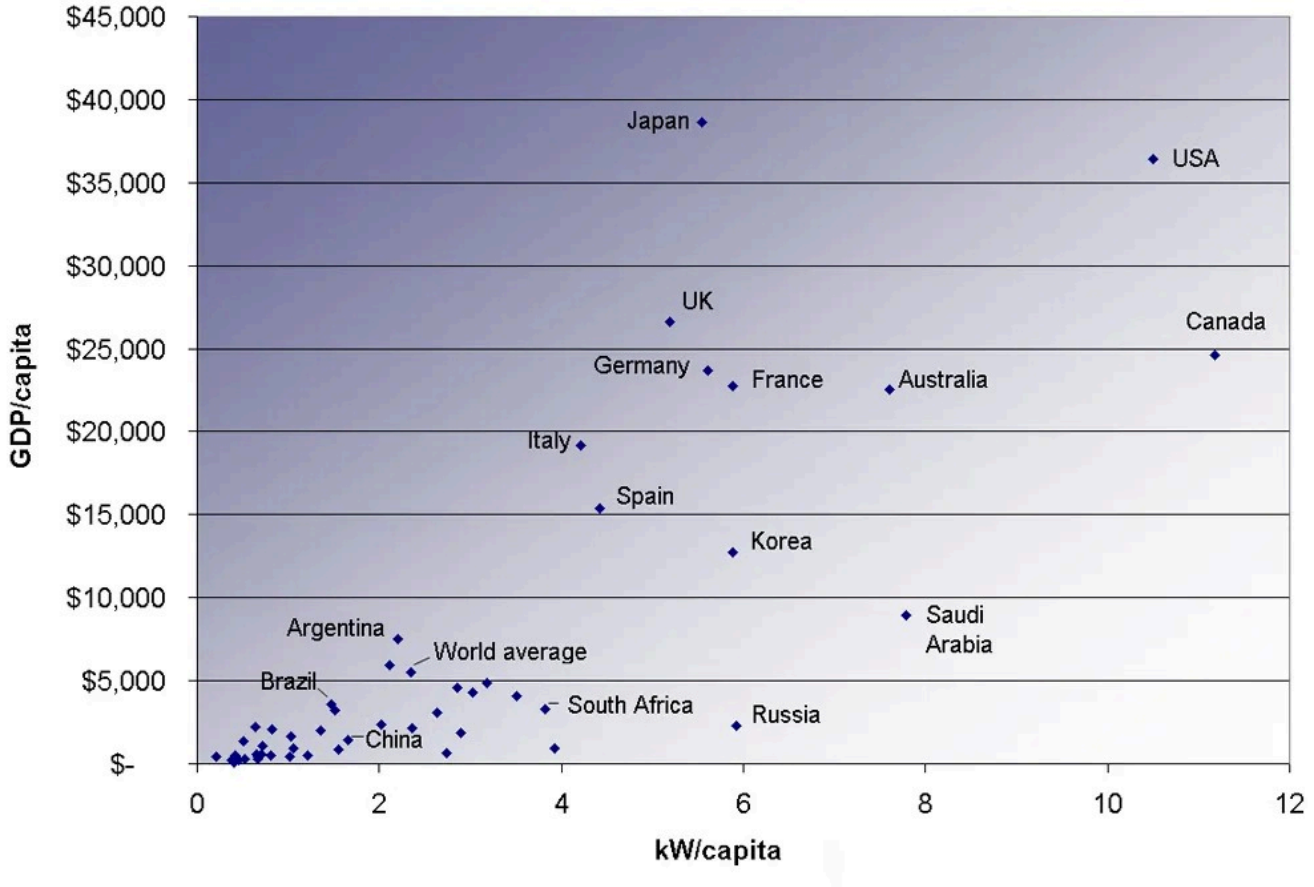


Figure 6.29 Power consumption per capita versus GDP per capita for various countries. Note the increase in energy usage with increasing GDP. Image by Mierlo at English Wikipedia, Wikimedia Commons License

Image Description

The image is a scatter plot graph that compares GDP per capita to energy consumption per capita (kW/capita) for various countries. The background is gradient with a light to dark blue color from bottom to top. The x-axis represents "kW/capita" ranging from 0 to 12, and the y-axis represents "GDP/capita" ranging from $0 to $45,000.

Here is a description of the data points represented as an HTML table:

Country	kW/capita	GDP/capita ($)
USA	11	Approx. 43,000
Japan	7	Approx. 37,000
Canada	10	Approx. 38,000
UK	5.5	Approx. 32,000
Germany	6	Approx. 30,000
France	5.5	Approx. 30,000
Australia	6.5	Approx. 30,000
Italy	4.5	Approx. 22,000
Spain	3.5	Approx. 18,000
Korea	5	Approx. 18,000
Saudi Arabia	8	Approx. 14,000
Argentina	1.5	Approx. 8,000
Brazil	1.5	Approx. 5,000
World average	2	Approx. 7,000
China	1.5	Approx. 4,000
South Africa	2.5	Approx. 5,000
Russia	5.5	Approx. 7,000

The graph shows a general trend that countries with higher energy consumption per capita also tend to have a higher GDP per capita.

Conserving Energy

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As has been mentioned elsewhere, the "law of the conservation of energy" is a very useful principle in analyzing physical processes. It is a statement that cannot be proven from basic principles, but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been "degraded" in the energy transformation.

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GLOSSARY

basal metabolic rate
: the total energy conversion rate of a person at rest

chemical energy
: the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

conservation of mechanical energy
: the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

conservative force
: a force that does the same work for any given initial and final configuration, regardless of the path followed

efficiency
: a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

electrical energy
: the energy carried by a flow of charge

energy
: the ability to do work

fossil fuels
: oil, natural gas, and coal

friction
: the force between surfaces that opposes one sliding on the other; friction changes mechanical energy into thermal energy

gravitational potential energy
: the energy an object has due to its position in a gravitational field

horsepower
: an older non-SI unit of power, with $1\ \text{hp} = 746\ \text{W}$

joule
: SI unit of work and energy, equal to one newton-meter

kilowatt-hour
: $(\text{kW} \cdot h)$ unit used primarily for electrical energy provided by electric utility companies

kinetic energy
the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

law of conservation of energy
the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

mechanical energy
the sum of kinetic energy and potential energy

metabolic rate
the rate at which the body uses food energy to sustain life and to do different activities

net work
work done by the net force, or vector sum of all the forces, acting on an object

nonconservative force
a force whose work depends on the path followed between the given initial and final configurations

nuclear energy
energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

potential energy
energy due to position, shape, or configuration

potential energy of a spring
the stored energy of a spring as a function of its displacement; when Hooke's law applies, it is given by the expression $\frac{1}{2}kx^2$ where x is the distance the spring is compressed or extended and k is the spring constant

power
the rate at which work is done

radiant energy
the energy carried by electromagnetic waves

renewable forms of energy
those sources that cannot be used up, such as water, wind, solar, and biomass

thermal energy
the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

useful work
work done on an external system

watt
(W) SI unit of power, with $1\ \text{W} = 1\ \text{J/s}$

work

the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

work-energy theorem

the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy

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CHAPTER SUMMARY

6.1 Work: The Scientific Definition

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force $\mathbf{F}$ does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols,

  $$W = \mathrm{F d cos}\theta.$$

- The SI unit for work and energy is the joule (J), where $1\mathrm{J} = 1\mathrm{N} \cdot \mathrm{m} = 1\ \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

6.2 Kinetic Energy and the Work-Energy Theorem

- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $\text{KE} = \frac{1}{2}\text{mv}^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{\text{net}} = \frac{1}{2}\text{mv}^2 - \frac{1}{2}(mv_0)^2$.

6.3 Gravitational Potential Energy

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, $\Delta\text{PE}_{\text{g}}$, is $\Delta\text{PE}_g = \text{mgh}$, with h being the increase in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, ΔPE_g, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta\text{KE} = \ -\Delta\text{PE}_{\text{g}}$.

6.4 Conservative Forces and Potential Energy

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined PE_g for the gravitational force.
- The potential energy of a spring is $\mathrm{PE}_s = \frac{1}{2}\mathrm{kx}^2$, where k is the spring's force constant and x is the displacement from its undeformed position.
- Mechanical energy is defined to be $\mathrm{KE} + \mathrm{PE}$ for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,
  \left\begin{matrix} & \text{KE } + \text{ PE } = \text{ constant}\textrm{ }\textrm{ }\textrm{ }\textrm{ } \\ \text{or} & \\ & \text{KE}_{\text{i}} + \text{PE}_{\text{i}} = \text{KE}_{\text{f}} + \text{PE}_{\text{f}} \end{matrix}\right\} (\text{conservative}\textrm{ }\text{forces}\textrm{ }\text{only}),
  where i and f denote initial and final values. This is known as the conservation of mechanical energy.

6.5 Nonconservative Forces

- A nonconservative force is one for which work depends on the path.
- Friction is an example of a nonconservative force that changes mechanical energy into thermal energy.
- Work W_{nc} done by a nonconservative force changes the mechanical energy of a system. In equation form, $W_{\text{nc}} = \Delta\text{KE} + \Delta\text{PE}$ or, equivalently, $\text{KE}_{\text{i}} + \text{PE}_{\text{i}} + W_{\text{nc}} = \text{KE}_{\text{f}} + \text{PE}_{\text{f}}$.
- When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws.

6.6 Conservation of Energy

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as $\text{KE}_i + \text{PE}_i + W_{\text{nc}} + \text{OE}_i = \text{KE}_f + \text{PE}_f + \text{OE}_f$, where OE is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency Eff of a machine or human is defined to be $\text{Eff} = \dfrac{W_{\text{out}}}{E_{\text{in}}}$, where W_{out} is useful work output and E_{in} is the energy consumed.

6.7 Power

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t, $P = W/t$.
- The SI unit for power is the watt (W), where $1\ \text{W} = 1\ \text{J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1\ \text{hp} = 746\ \text{W}$.

6.8 Work, Energy, and Power in Humans

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR)
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

6.9 World Energy Use

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing.
- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

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CONCEPTUAL QUESTIONS

Conceptual Questions

6.1 Work: The Scientific Definition

1. Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.
2. Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.
3. Describe a situation in which a force is exerted for a long time but does no work. Explain.

6.2 Kinetic Energy and the Work-Energy Theorem

4. The person in Figure 6.30 does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?

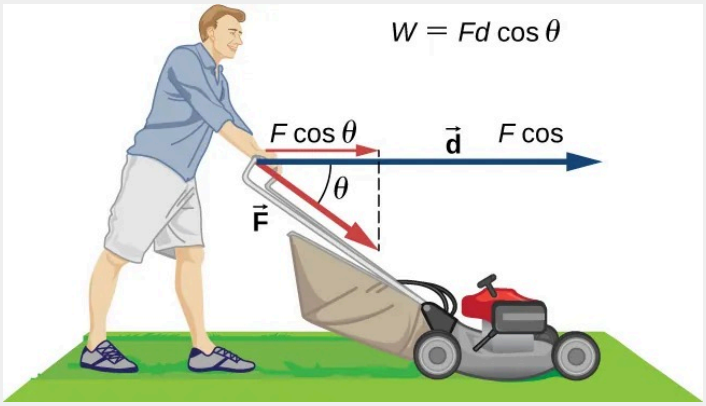


Figure 6.30 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of a person pushing a lawnmower on grass, demonstrating the concept of work in physics. The following elements are present:

– A man in a blue shirt and white shorts is standing on the grass, pushing a lawnmower.
– The lawnmower has a red engine and four wheels.
– A bold red arrow labeled “F” represents the force applied by the man on the lawnmower handle.
– The angle between the force and the horizontal is labeled as “θ” (theta).
– A blue arrow labeled “d” indicates the direction of displacement along the grass, parallel to the ground.
– A horizontal red arrow labeled “F cos θ” shows the horizontal component of the force, aligned with the direction of displacement “d”.
– Above the illustration, there is an equation: “W = Fd cos θ,” which represents the formula for work done where “W” is work, “F” is force, “d” is displacement, and “θ” is the angle between the force and direction of displacement.

5. Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.
6. When solving for speed in Example 6.4, we kept only the positive root. Why?

6.3 Gravitational Potential Energy

7. In Example 6.7, we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed at the bottom of the hill of 5 m/s uphill instead, and it coasted 20 m uphill, stopped, and then rolled back down to a final point at the same elevation as it started. We would find in that case that its final speed is the same as its initial speed. Explain in terms of conservation of energy.
8. Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

6.4 Conservative Forces and Potential Energy

9. What is a conservative force?
10. The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential

energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

11. Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?
12. What is the relationship of potential energy to conservative force?

6.6 Conservation of Energy

13. Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See Figure 7.31.)

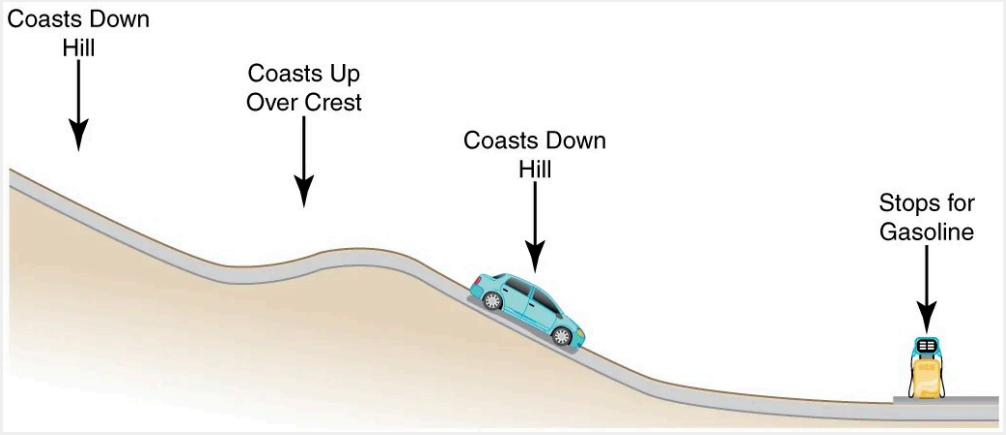


Figure 6.31 A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of a person pushing a lawnmower on grass, demonstrating the concept of work in physics. The following elements are present:

– A man in a blue shirt and white shorts is standing on the grass, pushing a lawnmower.
– The lawnmower has a red engine and four wheels.
– A bold red arrow labeled “F” represents the force applied by the man on the lawnmower handle.
– The angle between the force and the horizontal is labeled as “θ” (theta).
– A blue arrow labeled “d” indicates the direction of displacement along the grass, parallel to the ground.
– A horizontal red arrow labeled “F cos θ” shows the horizontal component of the force, aligned with the direction of displacement “d”.
– Above the illustration, there is an equation: “W = Fd cos θ,” which represents the formula for work done where “W” is work, “F” is force, “d” is displacement, and “θ” is the angle between the force and direction of displacement.

14. Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.
15. Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.
16. List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.
17. List the energy conversions that occur when riding a bicycle.

6.7 Power

18. Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.
19. Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?
20. A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

6.8 Work, Energy, and Power in Humans

21. Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?
22. Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?
23. Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?
24. Discuss the relative impact of dieting and exercise, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal).

6.9 World Energy Use

25. What is the difference between energy conservation and the law of conservation of energy? Give some examples of each.
26. If the efficiency of a coal-fired electrical generating plant is 35%, then what do we mean when we say that energy is a conserved quantity?

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PROBLEMS & EXERCISES

Problems & Exercises

6.1 Work: The Scientific Definition

1. How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.
2. A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task. (Neglect friction in your calculations.)
3. (a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the elevator car by the gravitational force in this process? (c) What is the total work done on the elevator car?
4. Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See Table 6.1 for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?
5. Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See Figure 6.32.) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate and on his body to get up the ramp.

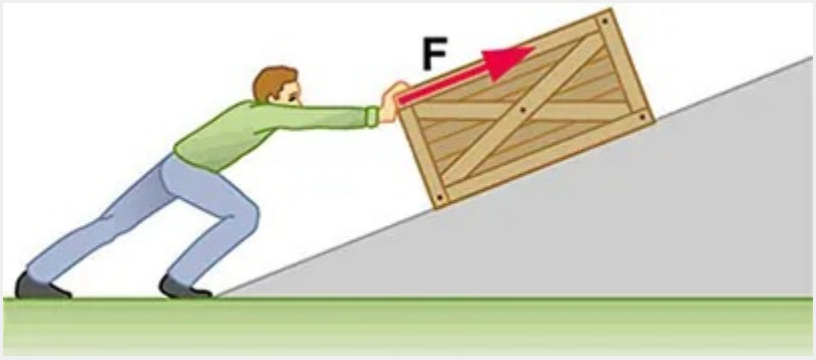


Figure 6.32 A man pushes a crate up a ramp. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a person pushing a large wooden crate up a slope. The person is shown leaning forward, exerting force on the crate. There is a red arrow labeled “F” pointing towards the crate, indicating the direction of the force being applied. The slope is inclined, suggesting the effort needed to push the crate upwards against gravity. The ground is green, indicating grass or some similar surface. The crate is illustrated with diagonal wooden beams forming an “X” pattern on the visible side.

6. How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in Figure 6.33? Assume no friction acts on the wagon.

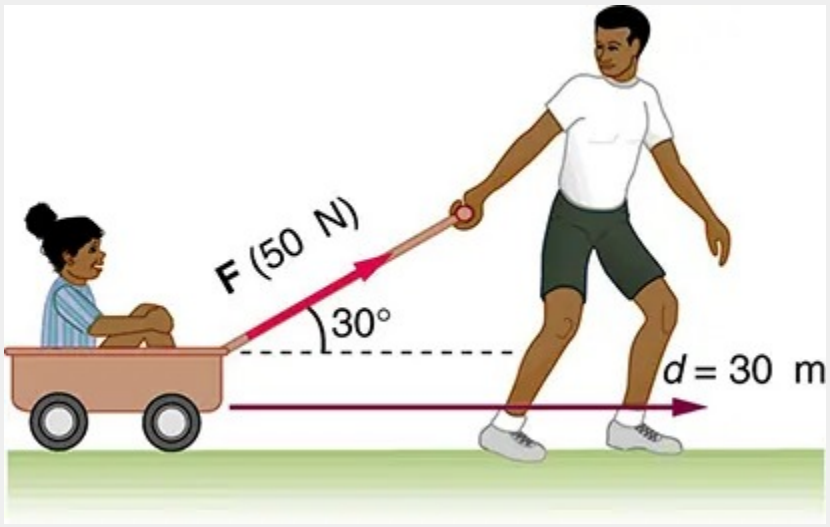


Figure 6.33 The boy does work on the system of the wagon and the child when he pulls them as shown. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a person pulling a small wagon on a grassy surface. The wagon contains a child seated inside. The adult pulling the wagon is wearing a white t-shirt, green shorts, and white shoes. A red arrow labeled “F (50 N)” stretches from the adult’s hand to the wagon, indicating the force applied. This arrow is angled at 30 degrees relative to the horizontal ground. Alongside the wagon, a horizontal dashed line represents the horizontal component of the force. Another horizontal arrow indicates the distance, labeled “d = 30 m,” representing the path covered by the wagon. The scene visually demonstrates a force applied at an angle to move an object over a specific distance.

7. A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?
8. Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown in Figure 6.34. The

coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?

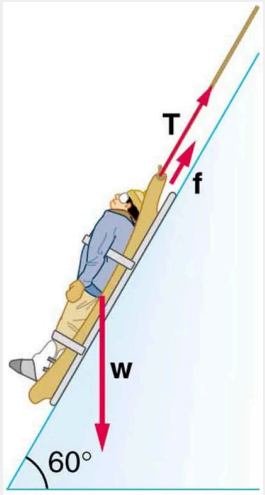


Figure 6.34 A rescue sled and victim are lowered down a steep slope. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a person secured on a stretcher, which is positioned on an inclined plane at a 60-degree angle from the horizontal. The stretcher is pointing upwards along the slope. Three vectors are shown:

- *w*: A vector pointing vertically downward, representing the weight of the person on the stretcher.
- *T*: A vector directed up and along the incline, representing the tension in the rope pulling the stretcher upwards.
- *f*: A vector directed along the incline but downwards, indicating the force of friction opposing the motion.

The person appears to be secured with straps to prevent movement. The angle at the bottom of the incline is explicitly labeled as 60 degrees.

6.2 Kinetic Energy and the Work-Energy Theorem

9. Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.
10. (a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.
11. Confirm the value given for the kinetic energy of an aircraft carrier in Table 6.1. You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).
12. (a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).
13. A car's bumper is designed to withstand a 4.0-km/h (1.12-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.12 m/s.
14. Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?
15. Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

6.3 Gravitational Potential Energy

16. A hydroelectric power facility (see Figure 6.35) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume $50.0 \mathrm{km}^3$ ($\text{mass} = 5.00 \times 10^{13} \mathrm{kg}$), given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.



Figure 6.35 Hydroelectric facility Image by Denis Belevich, Wikimedia Commons License

17. (a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about $7 \times 10^9 \mathrm{~kg}$ and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person?
18. Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?

19. In Example 6.7, we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta \mathrm{PE} >> \mathrm{KE}_{\mathrm{i}}$. Confirm this statement by taking the ratio of $\Delta \mathrm{PE}$ to KE_{i}. (Note that mass cancels.)
20. A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in Figure 6.36. Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.



Figure 6.36 A toy car moves up a sloped track. Image from OpenStax College Physics 2e, CC-BY 4.0

0. In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skis 70.0 m along a 30° slope neglecting friction: (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

6.4 Conservative Forces and Potential Energy

22. A $5.00 \times 10^{5} - \mathrm{kg}$ subway train is brought to a stop from a speed of 0.500 m/s

in 0.400 m by a large spring bumper at the end of its track. What is the force constant k of the spring?

23. A pogo stick has a spring with a force constant of $2.50 \times 10^4 \text{ N/m}$, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the Problem-Solving Strategies for Energy.

6.5 Nonconservative Forces

24. A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in Figure 6.37. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)

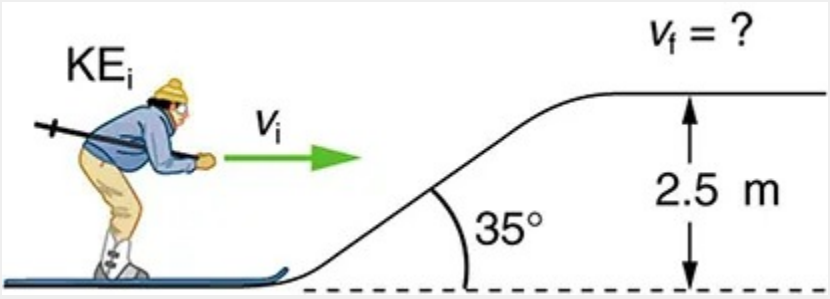


Figure 6.37 The skier's initial kinetic energy is partially used in coasting to the top of a rise. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a skier on a slope. The skier is positioned at the beginning of the slope, represented at a 35-degree angle. An arrow labeled "v_i" points to the right, indicating the initial velocity of the skier. The term "KE_i" is written next to the skier, suggesting initial kinetic energy. At the top of the slope, which has a height of 2.5 meters, there is a question mark next to "v_f = ?", indicating the unknown final velocity at that point.

25. (a) How high a hill can a car coast up (engine disengaged) if work done by friction

is negligible and its initial speed is 110 km/h? (b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction? (c) What is the average force of friction if the hill has a slope 2.5° above the horizontal?

6.6 Conservation of Energy

26. Using values from Table 6.1, how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)
27. Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.
28. If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from Table 6.1)? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.
29. (a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from Table 6.1. To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy. (b) How does this time compare with historically significant events, such as the duration of stable economic systems?

6.7 Power

30. The Crab Nebula (see Figure 6.38) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from Table 6.3, calculate the approximate factor

by which the power output of this astronomical object has declined since its explosion.



Figure 6.38 Crab Nebula Image from Jmencisom, CC-BY 4.0

31. Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from Table 6.3: (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of 10^{11} observable galaxies, the average brightness of which is somewhat less than our own galaxy.
32. A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?
33. What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is \$0.0900 per $\mathrm{kW} \cdot h$?
34. A large household air conditioner may consume 15.0 kW of power. What is the

cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is \$0.110 per $\mathrm{kW} \cdot h$?

35. (a) What is the average power consumption in watts of an appliance that uses $5.00\ \mathrm{kW} \cdot h$ of energy per day? (b) How many joules of energy does this appliance consume in a year?
36. (a) What is the average useful power output of a person who does $6.00 \times 10^6 \mathrm{J}$ of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)
37. A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?
38. (a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?
39. (a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is \$0.0900 per $\mathrm{kW} \cdot h$?
40. (a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply $8.00 \times 10^4 \mathrm{J}$ run a pocket calculator that consumes energy at the rate of $1.00 \times 10^{-3}\ \mathrm{W}$?
41. (a) How long would it take a 1.50×10^5-kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)
42. Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N.

Explicitly show how you follow the steps in the Problem-Solving Strategies for Energy.

43. (a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be $4.00 \times 10^{26}\ \text{W}$.) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of $1.30\ \text{kW/m}^2$ reaches Earth's surface. Calculate the area in km^2 of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs $(1.05 \times 10^{20}\ \text{J})$? Australia's energy needs $(5.4 \times 10^{18}\ \text{J})$? China's energy needs $(6.3 \times 10^{19}\ \text{J})$? (These energy consumption values are from 2006.)

6.8 Work, Energy, and Power in Humans

44. (a) How long can you rapidly climb stairs (116/min) on the 93.0 kcal of energy in a 10.0-g pat of butter? (b) How many flights is this if each flight has 16 stairs?
45. (a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s? (b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time?
46. Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)



Figure 6.39 Shot putter at the Dornoch Highland Gathering in 2007. Image from OpenStax College Physics 2e, CC-BY 4.0

47. (a) What is the efficiency of an out-of-condition person who does 2.10×10^5 J of useful work while metabolizing 500 kcal of food energy? (b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?
48. Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10,000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from Table 6.5 for the energy consumption rates of these activities.
49. Using data from Table 6.5, calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)
50. What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See Table 6.5.)
51. Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W. (a) What is her useful power output? (b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.) (c) How much waste heat transfer in kilojoules will she

generate in the process?

52. Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.
53. Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs. (a) Calculate the magnitude of the force needed to stop the downward motion of a jogger's leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger's body.) (b) Compare this force with the weight of the jogger.
54. (a) Calculate the energy in kJ used by a 55.0-kg person who does 50 deep knee bends in which their center of mass is lowered and raised 0.400 m. (They do work in both directions.) You may assume their efficiency is 20%. (b) What is the average power consumption rate in watts if they do this in 3.00 min?
55. Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the Daedalus 88, an aircraft powered by a bicycle-type drive mechanism (see Figure 6.40). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from Table 6.2, calculate the food energy in kilojoules he metabolized during the flight.



Figure 6.40 The Daedalus 88 in flight. Image by National Aeronautics and Space Administration (en:NASA) / Beasley, Wikimedia Commons License

56. The swimmer shown in Figure 6.41 exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke. (a) What is his work output in each stroke? (b) Calculate the power output of his arms if he does 120 strokes per minute.

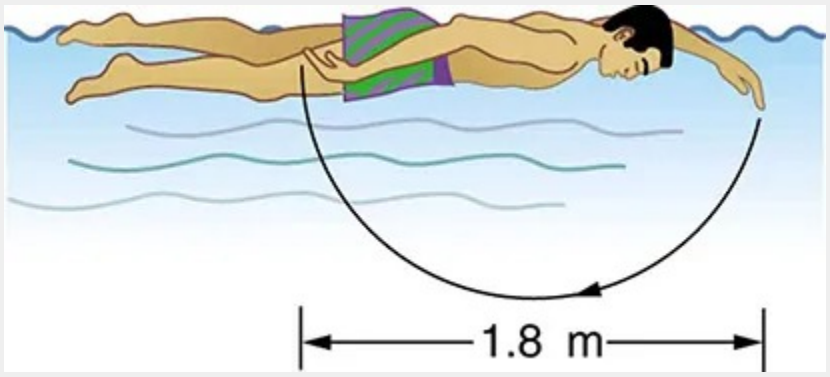


Figure 6.41 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of a person swimming, portrayed in a side view. The

swimmer is engaging in a freestyle stroke, in which one arm is extended forward in the water while the other arm is behind, preparing to stroke forward. The swimmer's face is turned to the side, presumably to breathe. The surrounding water is depicted with wavy lines in shades of blue and aqua.

A curved line with an arrow indicates the motion of the swimmer's arm from the back to the forward position. Below this curved path, there is a horizontal line with arrows at both ends representing a span of 1.8 meters. This measurement illustrates the total reach of the swimmer's arm movement from back to front. The swimmer is wearing swimming trunks with purple and green stripes. The illustration emphasizes the range of motion in swimming and the distance covered per stroke.

57. Mountain climbers carry bottled oxygen when at very high altitudes. (a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled. (b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude? (c) What is his efficiency for the 10.0-h climb?
58. The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about $7 \times 10^9 \text{kg}$. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see Figure 6.42), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own

bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)

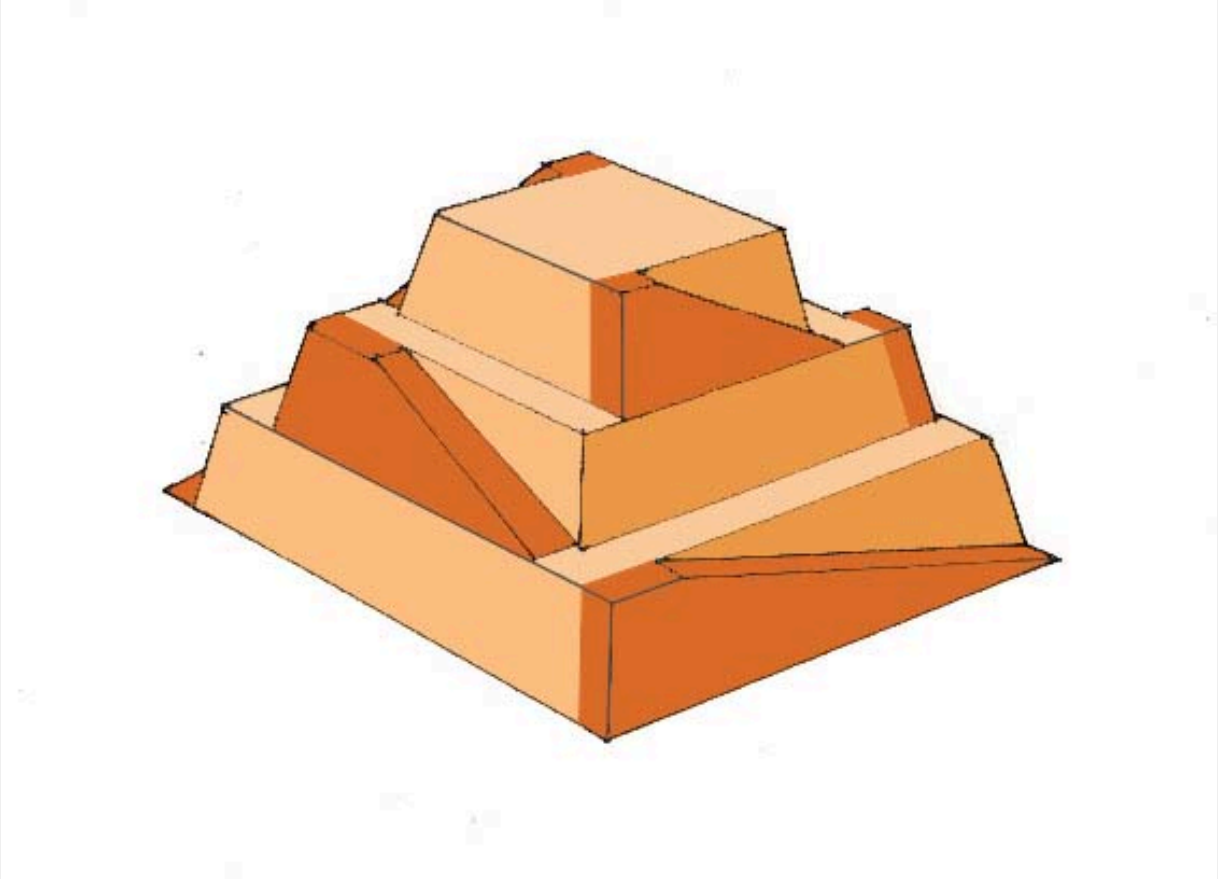


Figure 6.42 Ancient pyramids were probably constructed using ramps as simple machines. Image by Franck Monnier, Wikimedia Commons License

Image Description

The image depicts a 3D illustration of a stepped pyramid structure. The pyramid has multiple layers, each with progressively smaller rectangular shapes stacked on top of each other. The sides of the pyramid are angled, and the structure appears to be made up of four main levels, including the base. The image uses shades of orange and brown

to highlight the different layers and edges, creating a sense of depth and dimension. The background is plain white, which accentuates the geometric shape of the pyramid.

59. (a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar? (b) Does this seem like a long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

6.9 World Energy Use

60. Integrated Concepts: (a) Calculate the force the woman in Figure 6.43 exerts to do a push-up at constant speed, taking all data to be known to three digits. (b) How much work does she do if her center of mass rises 0.240 m? (c) What is her useful power output if she does 25 push-ups in 1 min? (Should work done lowering her body be included? See the discussion of useful work in 6.8 Work, Energy, and Power in Humans.

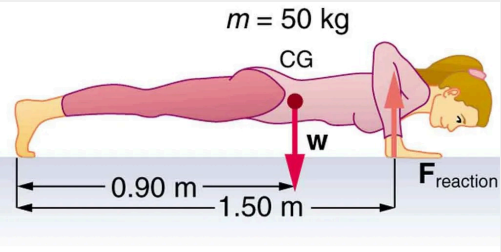


Figure 6.43 Forces involved in doing push-ups. The woman's weight acts as a force exerted downward on her center of gravity Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of a person in a plank position, often used in physics to demonstrate concepts of center of gravity and force distribution. The person is depicted in a side view, wearing a pink outfit, with their body fully extended parallel to a surface. There are several labels and measurements in the image:

- *m = 50 kg:* Indicates the mass of the person.
- *CG:* Denotes the center of gravity, marked with a red dot on the person's body

roughly at the midsection.

- *w:* Represents the force of weight acting vertically downward from the center of gravity.
- *0.90 m:* The horizontal distance from where the feet touch the surface to the center of gravity.
- *1.50 m:* The horizontal distance from the center of gravity to the position of the hands.
- F_{reaction}*:* Marks the upward reaction force at the hands.

61. **Integrated Concepts:** A 75.0-kg cross-country skier is climbing a 3.0° slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. Find his power output for work done against the gravitational force and air resistance. (b) What average force does he exert backward on the snow to accomplish this? (c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?
62. **Integrated Concepts:** The 70.0-kg swimmer in Figure 6.41 starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m long stroke. (a) What is his initial acceleration if water resistance is 45.0 N? (b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of 2.50 m/s? (c) Discuss whether water resistance seems to increase linearly with velocity.
63. **Integrated Concepts:** A toy gun uses a spring with a force constant of 300 N/m to propel a 10.0-g steel ball. If the spring is compressed 7.00 cm and friction is negligible: (a) How much force is needed to compress the spring? (b) To what maximum height can the ball be shot? (c) At what angles above the horizontal may a child aim to hit a target 3.00 m away at the same height as the gun? (d) What is the gun's maximum range on level ground?
64. **Integrated Concepts:** (a) What force must be supplied by an elevator cable to produce an acceleration of $0.800\ \text{m/s}^2$ against a 200-N frictional force, if the mass of the loaded elevator is 1500 kg? (b) How much work is done by the cable in lifting the elevator 20.0 m? (c) What is the final speed of the elevator if it starts from rest? (d) How much work went into thermal energy?

65. **Unreasonable Results:** A car advertisement claims that its 900-kg car accelerated from rest to 30.0 m/s and drove 100 km, gaining 3.00 km in altitude, on 1.0 gal of gasoline. The average force of friction including air resistance was 700 N. Assume all values are known to three significant figures. (a) Calculate the car's efficiency. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?
66. **Unreasonable Results:** Body fat is metabolized, supplying 9.30 kcal/g, when dietary intake is less than needed to fuel metabolism. The manufacturers of an exercise bicycle claim that you can metabolize 0.500 kg of fat per day by vigorously exercising for 2.00 h per day on their machine. (a) How many kcal are supplied by the metabolization of 0.500 kg of fat? (b) Calculate the kcal/min that you would have to utilize to metabolize fat at the rate of 0.500 kg in 2.00 h. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?
67. **Construct Your Own Problem:** Consider a person climbing and descending stairs. Construct a problem in which you calculate the long-term rate at which stairs can be climbed considering the mass of the person, his ability to generate power with his legs, and the height of a single stair step. Also consider why the same person can descend stairs at a faster rate for a nearly unlimited time in spite of the fact that very similar forces are exerted going down as going up. (This points to a fundamentally different process for descending versus climbing stairs.)
68. **Construct Your Own Problem:** Consider humans generating electricity by pedaling a device similar to a stationary bicycle. Construct a problem in which you determine the number of people it would take to replace a large electrical generation facility. Among the things to consider are the power output that is reasonable using the legs, rest time, and the need for electricity 24 hours per day. Discuss the practical implications of your results.
69. **Integrated Concepts:** A 105-kg basketball player crouches down 0.400 m while waiting to jump. After exerting a force on the floor through this 0.400 m, their feet leave the floor and their center of gravity rises 0.950 m above its normal standing erect position. (a) Using energy considerations, calculate the player's velocity when they leave the floor. (b) What average force did the player exert on

the floor? (Do not neglect the force to support their weight as well as that to accelerate them.) (c) What was the player's power output during the acceleration phase?

70. **Critical Thinking:** Two boxes of the same mass are pressed against pistons that compress and release the boxes on frictionless surfaces. Box A's piston has a constant force F over a distance x before the box is released while box B starts at zero force and increases linearly to a force of 2F before releasing the box at a distance of 1/2d. (a) Which of these boxes has the greatest speed after releasing from the piston? (b) Based on experimental data for trials similar to box A, an equation that fits the data is suggested for the change in kinetic energy, ΔKE, which may not be correct: $\Delta KE = KF^{1/2}x^{1/2}$, where K is a constant with appropriate units, F is the applied force, and x is the distance the piston moves in contact with the box. Is this equation consistent with your answer from part a? Explain why or why not. Does this equation make sense? Explain why or why not. (c) If the applied force of the piston is proportional to the distance in contact with the box, graph the work done on the box vs. the distance in contact with the piston and box.

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STATICS AND TORQUE



Figure 7.1 On a short time scale, rocks like these in Australia's Kalbarri National Park are static, or motionless relative to the Earth. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

7.1 The First Condition for Equilibrium
7.2 The Second Condition for Equilibrium
7.3 Stability
7.4 Applications of Statics, Including Problem-Solving Strategies
7.5 Simple Machines
7.6 Forces and Torques in Muscles and Joints
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

What might desks, bridges, buildings, trees, and mountains have in common—at least in the eyes of a physicist? The answer is that they are ordinarily motionless relative to the Earth. Furthermore, their acceleration is zero because they remain motionless. That means they also have something in common with a car moving at a constant velocity, because anything with a constant velocity also has an acceleration of zero. Now, the important part—Newton's second law states that net $\mathbf{F} = \mathbf{ma}$, and so the net external force is zero for all stationary objects and for all objects moving at constant velocity. There are forces acting, but they are balanced. That is, they are in *equilibrium*.

Statics

Statics is the study of forces in equilibrium, a large group of situations that makes up a special case of Newton's second law. We have already considered a few such situations; in this chapter, we cover the topic more thoroughly, including consideration of such possible effects as the rotation and deformation of an object by the forces acting on it.

How can we guarantee that a body is in equilibrium and what can we learn from systems that are in equilibrium? There are actually two conditions that must be satisfied to achieve equilibrium. These conditions are the topics of the first two sections of this chapter.
Access multimedia content

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7.1 THE FIRST CONDITION FOR EQUILIBRIUM

Learning Objectives

By the end of this section, you will be able to:

- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.

The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

$$\text{net}\mathbf{F} = 0$$
7.1

Note that if net $\mathbf{F}$ is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical x– and y-axes are zero. This is written as

$$\text{net } F_x = 0 \text{and net} F_y = 0$$
7.2

Figure 7.2 and Figure 7.3 illustrate situations where $\text{net}\mathbf{F} = 0$ for both static equilibrium (motionless), and dynamic equilibrium (constant velocity).

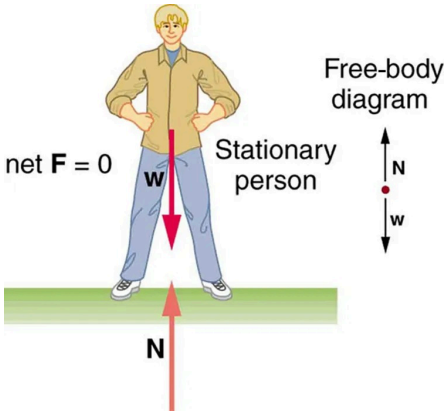


Figure 7.2 This motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram depicting a stationary person standing upright on a surface. The person is shown with their arms akimbo, wearing a beige shirt and blue pants. There are arrows and labels indicating forces acting on the person, emphasizing the concept of equilibrium.

– A vertical arrow labeled "w" points downward, representing the weight force acting on the person.
– Another arrow, labeled "N," points upward from the surface, representing the normal force exerted by the ground on the person.
– The labels "net F = 0" and "Stationary person" indicate that the net force on the person is zero, meaning they are in equilibrium and not accelerating.

To the right of the person, there is a simplified free-body diagram showing:

– A small red dot with an upward arrow labeled "N" and a downward arrow labeled "w," visually demonstrating the forces acting on the person and reinforcing that these forces are balanced.

The setting conveys basic physics concepts of force, equilibrium, and free-body diagrams.

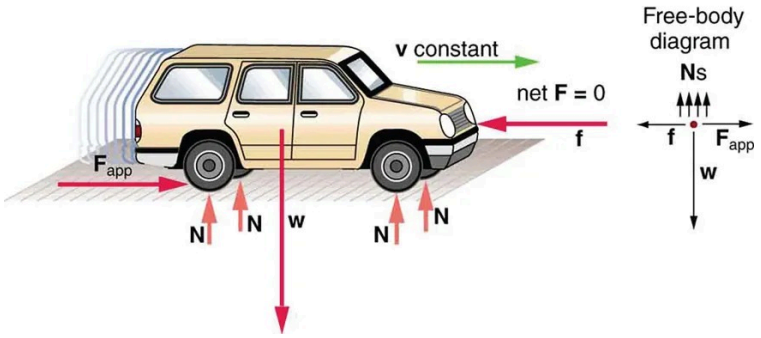


Figure 7.3 This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force $\mathbf{F}_{\text{app}}$ *between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows a side view of a car on a flat surface, illustrating various forces acting on it.

– Car Description: The car is beige with four doors and is oriented to the right. It is on a slightly textured surface.

– Forces on the Car:
– Fapp (Applied Force): A horizontal red arrow pointing to the right, starting from the car's rear.
– f (Friction Force): A horizontal red arrow pointing to the left, at the front of the car.
– w (Weight): A vertical red arrow pointing downward from the car's center.
– N (Normal Force): Four vertical red arrows pointing upward beneath the car's tires, balancing the weight.

– Velocity Indicator: A green arrow labeled "v constant" indicating constant velocity, pointing to the right in front of the car.

– Free-Body Diagram:
– Shown on the right side of the image.
– Consists of a dot representing the car, with arrows indicating forces:
– Ns (Normal Force): Multiple upward arrows.
– f (Friction Force): A leftward arrow.
– Fapp (Applied Force): A rightward arrow.
– w (Weight): A downward arrow.

– Additional Notes:

– The text “net F = 0” is next to the free-body diagram, indicating that the net force acting on the car is zero, resulting in a balanced state.

However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in Figure 7.4 and Figure 7.5 where forces are applied to an ice hockey stick lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. In Figure 7.4, the ice hockey stick remains motionless. But in Figure 7.5, with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved. This will be explored further in the next section.

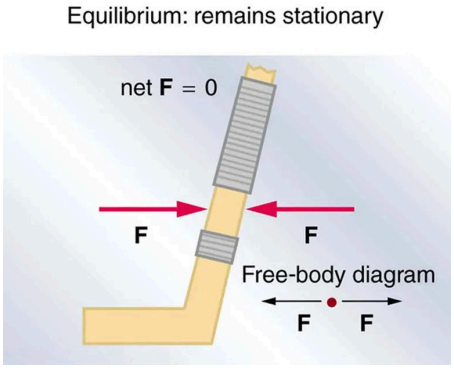


Figure 7.4 An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus, $\text{net}\mathbf{F} = 0$. *Equilibrium is achieved, which is static equilibrium in this case. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image illustrates a concept called equilibrium in physics using a free-body diagram. The setup includes a beige-colored object that is bent at a right angle, with two springs wrapped around it. The springs demonstrate forces acting in opposite directions.

– Two red arrows, labeled “F,” point towards the center from opposite directions, indicating equal forces.
– A text at the top states “Equilibrium: remains stationary,” explaining that the net force is zero.
– “Net F = 0” is written above the object.
– Below the object, a small free-body diagram shows a dot with two arrows pointing left and right, both labeled “F,” corroborating that the forces are balanced.

The object remains stationary as the forces cancel each other out, illustrating equilibrium.

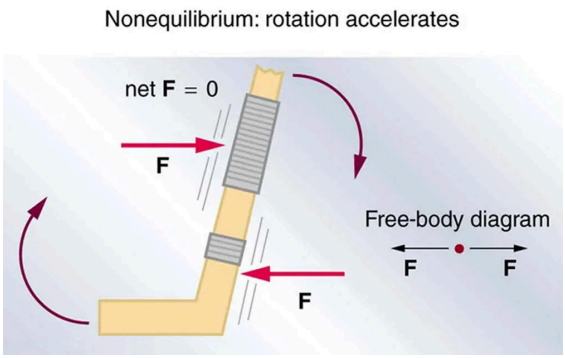


Figure 7.5 The same forces are applied at other points and the stick rotates—in fact, it experiences an accelerated rotation. Here $\text{net}\mathbf{F} = 0$ *but the system is not at equilibrium. Hence, the* $\text{net}\mathbf{F} = 0$ *is a necessary—but not sufficient—condition for achieving equilibrium. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

This image is a physics diagram titled "Nonequilibrium: rotation accelerates". It illustrates a scenario with forces acting on an object. The object is depicted as a bent rod. Two forces labeled *F* are applied horizontally in opposite directions, one at the top and another near the bottom of the rod. These forces are equal, creating a condition where the net force (*net F = 0*) is zero.

Despite the net force being zero, the object is shown experiencing rotational acceleration, indicated by curved arrows. The top and bottom of the rod are depicted with rotational arrows arching in opposite directions around the midsection of the rod, which suggests they are causing the rod to rotate.

To the right of the rod, a "Free-body diagram" is drawn, showing a single point with horizontal forces labeled *F* acting in opposite directions. This emphasizes that no linear acceleration occurs, only rotational acceleration.

PhET Explorations

Torque

Investigate how torque causes an object to rotate. Discover the relationships between angular acceleration, moment of inertia, angular momentum and torque.

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7.2 THE SECOND CONDITION FOR EQUILIBRIUM

Learning Objectives

By the end of this section, you will be able to:

- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics.

Torque

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity). A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it. To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.

Several familiar factors determine how effective you are in opening the door. See Figure 7.6. First of all, the larger the force, the more effective it is in opening the door—obviously, the harder you push, the more rapidly

the door opens. Also, the point at which you push is crucial. If you apply your force too close to the hinges, the door will open slowly, if at all. Most people have been embarrassed by making this mistake and bumping up against a door when it did not open as quickly as expected. Finally, the direction in which you push is also important. The most effective direction is perpendicular to the door—we push in this direction almost instinctively.

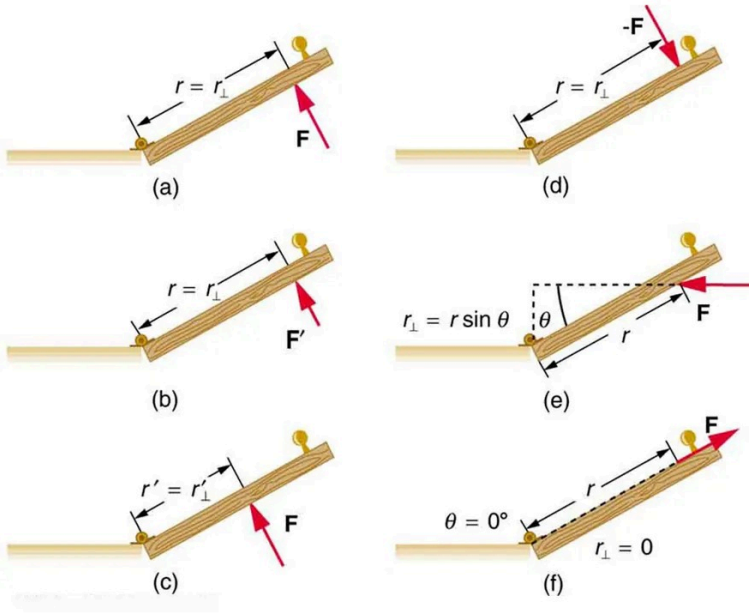


Figure 7.6 Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction. (a) Counterclockwise torque is produced by this force, which means that the door will rotate in a counterclockwise due to $\mathbf{F}$*. Note that* $r_\perp$ *is the perpendicular distance of the pivot from the line of action of the force. (b) A smaller counterclockwise torque is produced by a smaller force* $\mathbf{F}\prime$ *acting at the same distance from the hinges (the pivot point). (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) The same force as in (a), but acting in the opposite direction, produces a clockwise torque. (e) A smaller counterclockwise torque is produced by the same magnitude force acting at the same point but in a different direction. Here,* θ *is less than* 90° *. (f) Torque is zero here since the force just pulls on the hinges, producing no rotation. In this case,* $\theta = 0^\circ$ *. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image displays six diagrams illustrating the concept of torque on a lever system. Each sub-image (labeled a-f) shows a lever with different configurations of force and distance. Below are descriptions of each scenario:

– (a): A lever is inclined with a force labeled F applied perpendicular to the lever at a distance from the pivot. The distance is labeled r = r⊥.

– (b): Similar to (a), but the force is labeled F’ and the direction of the force remains perpendicular to the lever.

– (c): The lever is inclined with a revised perpendicular distance r’ and force F applied. The lever is shown pivoting from a different point.

– (d): A force -F is applied opposite to the direction in (a), perpendicular to the lever at the same distance r = $r_\perp$.

– (e): The lever is inclined, and a force F is applied at an angle θ. The perpendicular component of the distance is shown as $r_\perp = r \sin \theta$.

– (f): The lever is aligned horizontally with θ = 0°, and a force F is applied parallel to the lever. The perpendicular distance is noted as $r_\perp = 0$.

Each diagram demonstrates different aspects of torque, illustrating how the angle and point of force application affect the lever’s motion.

The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque. Torque is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time). In equation form, the magnitude of torque is defined to be

$$\tau = \mathrm{rF}\sin\theta$$
7.3

where τ (the Greek letter tau) is the symbol for torque,

r is the distance from the pivot point to the point where the force is applied,
F is the magnitude of the force, and θ is the angle between the force and the vector directed from the point of application to the pivot point, as seen in Figure 7.6 and Figure 7.7. An alternative expression for torque is given in terms of the perpendicular lever arm $r_\perp$ as shown in Figure 7.6 and Figure 7.7, which is defined as

$$r_\perp = r\sin\theta$$
7.4

so that

$$\tau = r_\perp F.$$
7.5

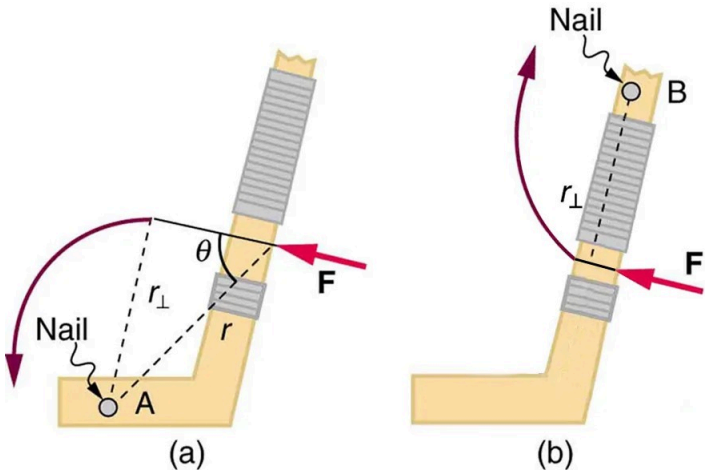


Figure 7.7 A force applied to an object can produce a torque, which depends on the location of the pivot point. (a) The three factors **r**, **F**, *and* θ *for pivot point A on a body are shown here—***r** *is the distance from the chosen pivot point to the point where the force* **F** *is applied, and* θ *is the angle between* **F** *and the vector directed from the point of application to the pivot point. If the object can rotate around point A, it will rotate counterclockwise. This means that torque is counterclockwise relative to pivot A. (b) In this case, point B is the pivot point. The torque from the applied force will cause a clockwise rotation around point B, and so it is a clockwise torque relative to B. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two diagrams, labeled (a) and (b), depicting the use of a wrench or a similar tool to apply torque to a nail via applied force.

Diagram (a):
– The diagram shows the tool at an angle, attached to a nail labeled “A”.
– A force, labeled “F”, is applied perpendicular to the wrench, represented by a red arrow pointing left.
– The wrench forms an angle “θ” with a reference line extending from the nail.
– The distance from the nail to the point where the force is applied is labeled “r”, while the perpendicular distance from the line of action of the force to the nail is labeled “r_⍰”.
– There is a curved arrow indicating the direction of rotation, moving counterclockwise.

Diagram (b):
– In this diagram, the tool is positioned differently, attached to the nail labeled “B”.
– Similarly, a force “F” is applied, indicated by a red arrow pointing to the left.
– The distance from the nail to the force application point is along the wrench, and the perpendicular distance is labeled “r_⍰”.
– A curved arrow shows the direction of rotation, moving clockwise.

In both diagrams, the tool appears to be wrapped in a tape-like material near the nail for reinforcement or stability.

The perpendicular lever arm $r_\perp$ is the shortest distance from the pivot point to the line along which $\mathbf{F}$ acts; it is shown as a dashed line in Figure 7.6 and Figure 7.7. Note that the line segment that defines the distance $r_\perp$ is perpendicular to $\mathbf{F}$, as its name implies. It is sometimes easier to find or visualize $r_\perp$ than to find both r and θ. In such cases, it may be more convenient to use $\tau = r_\perp \mathbf{F}$ rather than $\tau = \mathrm{rF}\sin\theta$ for torque, but both are equally valid.

The SI unit of torque is newtons times meters, usually written as $\mathrm{N \cdot m}$. For example, if you push perpendicular to the door with a force of 40 N at a distance of 0.800 m from the hinges, you exert a torque of 32 N·m(0.800 m × 40 N × sin 90º) relative to the hinges. If you reduce the force to 20 N, the torque is reduced to $16\ \mathrm{N \cdot m}$, and so on.

The torque is always calculated with reference to some chosen pivot point. For the same applied force, a different choice for the location of the pivot will give you a different value for the torque, since both r and θ depend on the location of the pivot. Any point in any object can be chosen to calculate the torque about that point. The object may not actually pivot about the chosen "pivot point."

Note that for rotation in a plane, torque has two possible directions. Torque is either clockwise or counterclockwise relative to the chosen pivot point, as illustrated for points B and A, respectively, in Figure 7.7. If the object can rotate about point A, it will rotate counterclockwise, which means that the torque for the force is shown as counterclockwise relative to A. But if the object can rotate about point B, it will rotate clockwise, which means the torque for the force shown is clockwise relative to B. Also, the magnitude of the torque is greater when the lever arm is longer.

Now, *the second condition necessary to achieve equilibrium* is that *the net external torque on a system must be zero*. An external torque is one that is created by an external force. You can choose the point around which the torque is calculated. The point can be the physical pivot point of a system or any other point in space—but it must be the same point for all torques. If the second condition (net external torque on a system is zero) is satisfied for one choice of pivot point, it will also hold true for any other choice of pivot point in or out of the system of interest. (This is true only in an inertial frame of reference.) The second condition necessary to achieve equilibrium is stated in equation form as

$$\mathrm{net}\tau = 0$$
7.6

where net means total. Torques, which are in opposite directions are assigned opposite signs. A common convention is to call counterclockwise (ccw) torques positive and clockwise (cw) torques negative.

When two children balance a seesaw as shown in Figure 7.8, they satisfy the two conditions for equilibrium. Most people have perfect intuition about seesaws, knowing that the lighter child must sit farther from the pivot and that a heavier child can keep a lighter one off the ground indefinitely.

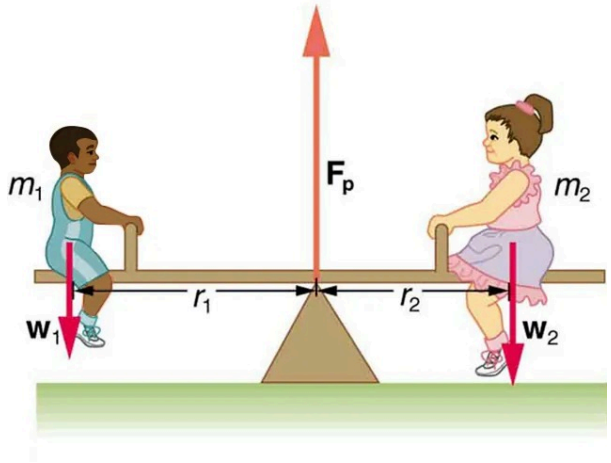


Figure 7.8 Two children balancing a seesaw satisfy both conditions for equilibrium. The lighter child sits farther from the pivot to create a torque equal in magnitude to that of the heavier child. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image displays a seesaw balanced on a triangular fulcrum with two children sitting on opposite ends. The child on the left is labeled with "m_1," and the child on the right is labeled with "m_2."

– The child on the left is depicted as sitting and facing toward the center, with an arrow labeled "w_1" pointing downward, indicating weight. The distance from the center (fulcrum) to this child is labeled "r_1."
– The child on the right is also facing toward the center, with a similar downward arrow labeled "w_2" representing weight. The distance from the fulcrum to this child is labeled "r_2."
– An upward arrow labeled "F_p" is positioned at the fulcrum, representing the force exerted by the pivot point.

The background shows a green base, indicating the ground beneath the seesaw. The image illustrates concepts of balance and force with labeled measurements.

Example 7.1

She Saw Torques On A Seesaw

The two children shown in Figure 7.8 are balanced on a seesaw of negligible mass. (This assumption is made to keep the example simple—more involved examples will follow.) The first child has a mass of 26.0 kg and sits 1.60 m from the pivot.(a) If the second child has a mass of 32.0 kg, how far is she from the pivot? (b) What is F_{p}, the supporting force exerted by the pivot?

Strategy

Both conditions for equilibrium must be satisfied. In part (a), we are asked for a distance; thus, the second condition (regarding torques) must be used, since the first (regarding only forces) has no distances in it. To apply the second condition for equilibrium, we first identify the system of interest to be the seesaw plus the two children. We take the supporting pivot to be the point about which the torques are calculated. We then identify all external forces acting on the system.

Solution (a)

The three external forces acting on the system are the weights of the two children and the supporting force of the pivot. Let us examine the torque produced by each. Torque is defined to be

$$\tau = \mathrm{rF}\sin\theta.$$

7.7

Here $\theta = 90^{\circ}$, so that $\sin\theta = 1$ for all three forces. That means $r_{\perp} = r$ for all three. The torques exerted by the three forces are first,

$$\tau_1 = r_1 w_1$$

7.8

second,

$$\tau_2 = (-r)_2 w_2$$

7.9

and third,

$$\begin{aligned} \tau_\mathrm{p} &= r_\mathrm{p} F_\mathrm{p} \\ &= 0 \cdot F_\mathrm{p} \\ &= 0. \end{aligned}$$

7.10

Note that a minus sign has been inserted into the second equation because this torque is clockwise and is therefore negative by convention. Since F_p acts directly on the pivot point, the distance r_p is zero. A force acting on the pivot cannot cause a rotation, just as pushing directly on the hinges of a door will not cause it to rotate. Now, the second condition for equilibrium is that the sum of the torques on both children is zero. Therefore

$$\tau_2 = (-\tau)_1,$$

7.11

or

$$r_2 w_2 = r_1 w_1.$$

7.12

Weight is mass times the acceleration due to gravity. Entering mg for w, we get

$$r_2 m_2 g = r_1 m_1 g.$$

7.13

Solve this for the unknown r_2:

$$r_2 = r_1 \frac{m_1}{m_2}.$$

7.14

The quantities on the right side of the equation are known; thus, r_2 is

$$r_2 = (1.60\ \text{m})\frac{26.0\ \text{kg}}{32.0\ \text{kg}} = 1.30\ \text{m}.$$

7.15

As expected, the heavier child must sit closer to the pivot (1.30 m versus 1.60 m) to balance the seesaw.

Solution (b)

This part asks for a force F_p. The easiest way to find it is to use the first condition for equilibrium, which is

$$\text{net}\mathbf{F} = 0.$$

7.16

The forces are all vertical, so that we are dealing with a one-dimensional problem along the vertical axis; hence, the condition can be written as

$$\text{net } F_y = 0$$

7.17

where we again call the vertical axis the *y*-axis. Choosing upward to be the positive direction, and using plus and minus signs to indicate the directions of the forces, we see that

$$F_\text{p} – w_1 – w_2 = 0.$$

7.18

This equation yields what might have been guessed at the beginning:

$$F_\text{p} = w_1 + w_2.$$

7.19

So, the pivot supplies a supporting force equal to the total weight of the system:

$$F_\text{p} = m_1 g + m_2 g.$$

7.20

Entering known values gives

$$F_{\text{p}} = (26.0\ \text{kg})\left(9.80\ \text{m/s}^2\right) + (32.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)$$
$$= 568\ \text{N}.$$

7.21

Discussion

The two results make intuitive sense. The heavier child sits closer to the pivot. The pivot supports the weight of the two children. Part (b) can also be solved using the second condition for equilibrium, since both distances are known, but only if the pivot point is chosen to be somewhere other than the location of the seesaw's actual pivot!

Several aspects of the preceding example have broad implications. First, the choice of the pivot as the point around which torques are calculated simplified the problem. Since F_{p} is exerted on the pivot point, its lever arm is zero. Hence, the torque exerted by the supporting force F_{p} is zero relative to that pivot point. The second condition for equilibrium holds for any choice of pivot point, and so we choose the pivot point to simplify the solution of the problem.

Second, the acceleration due to gravity canceled in this problem, and we were left with a ratio of masses. *This will not always be the case*. Always enter the correct forces—do not jump ahead to enter some ratio of masses.

Third, the weight of each child is distributed over an area of the seesaw, yet we treated the weights as if each force were exerted at a single point. This is not an approximation—the distances r_1 and r_2 are the distances to points directly below the center of gravity of each child. As we shall see in the next section, the mass and weight of a system can act as if they are located at a single point.

Finally, note that the concept of torque has an importance beyond static equilibrium. *Torque plays the same role in rotational motion that force plays in linear motion.* We will examine this in the next chapter.

Take-Home Experiment

Take a piece of modeling clay and put it on a table, then mash a cylinder down into it so that a ruler can balance on the round side of the cylinder while everything remains still. Put a penny 8 cm away from the pivot. Where would you need to put two pennies to balance? Three pennies?

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7.3 STABILITY

Learning Objectives

By the end of this section, you will be able to:

- State the types of equilibrium.
- Describe stable and unstable equilibriums.
- Describe neutral equilibrium.

It is one thing to have a system in equilibrium; it is quite another for it to be stable. The toy doll perched on the man's hand in Figure 7.9, for example, is not in stable equilibrium. There are *three types of equilibrium*: *stable*, *unstable*, and *neutral*. Figures throughout this module illustrate various examples.

Figure 7.9 presents a balanced system, such as the toy doll on the man's hand, which has its center of gravity (cg) directly over the pivot, so that the torque of the total weight is zero. This is equivalent to having the torques of the individual parts balanced about the pivot point, in this case the hand. The cgs of the arms, legs, head, and torso are labeled with smaller type.

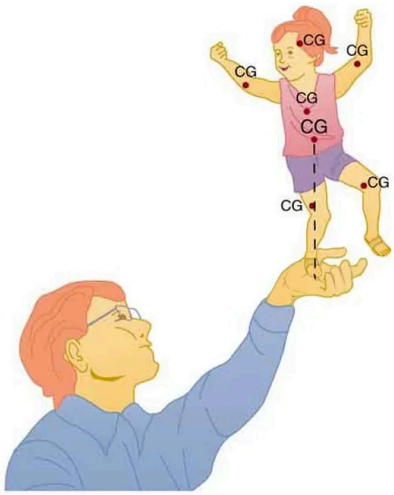


Figure 7.9 A man balances a toy doll on one hand. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts an illustration of a person holding a child on the palm of their hand. The person appears to be looking upwards at the child while they steady them. The child, standing on one foot, has their arms raised in a balancing pose. Several red dots labeled "CG" are marked along the child's body, indicating the concept of "center of gravity." These dots are shown on various points of the child's body: the head, arms, torso, and legs. A dashed line runs through the labeled points, suggesting the alignment of the center of gravity. The illustration conveys a balance or physics concept, emphasizing how the center of gravity allows the child to balance on the person's hand.

A system is said to be in stable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite to the direction of the displacement. For example, a marble at the bottom of a bowl will experience a *restoring* force when displaced from its equilibrium position. This force moves it back toward the equilibrium position. Most systems are in stable equilibrium, especially for small displacements. For another example of stable equilibrium, see the pencil in Figure 9.10.

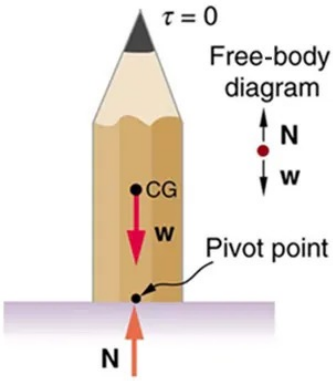


Figure 7.10 This pencil is in the condition of equilibrium. The net force on the pencil is zero and the total torque about any pivot is zero. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a diagram of a pencil representing a physics concept. The pencil is oriented vertically on a surface. Various labels and symbols are used to illustrate the forces and points of interest:

- The pencil is shaded light brown with a sharpened graphite tip at the top.
- At the center of the pencil, there is a black dot labeled "CG," indicating the center of gravity.
- A red arrow pointing downward from "CG" is labeled "w," representing the weight of the pencil.
- There is a small black dot at the bottom of the pencil, labeled as the "Pivot point."
- An orange arrow pointing upward from the "Pivot point" is labeled "N," indicating the normal force exerted by the surface.
- To the right of the pencil, there is a simple free-body diagram with two vertical arrows: one arrow pointing upward labeled "N" and one pointing downward labeled "w."
- At the top left, the Greek letter "τ" is shown followed by "= 0," indicating that the torque is zero.

A system is in unstable equilibrium if, when displaced, it experiences a net force or torque in the *same* direction as the displacement from equilibrium. A system in unstable equilibrium accelerates away from its equilibrium position if displaced even slightly. An obvious example is a ball resting on top of a hill. Once displaced, it accelerates away from the crest. See the next several figures for examples of unstable equilibrium.

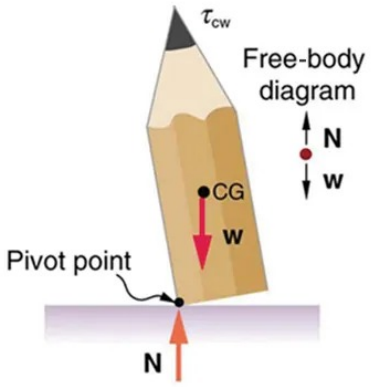


Figure 7.11 If the pencil is displaced slightly to the side (counterclockwise), it is no longer in equilibrium. Its weight produces a clockwise torque that returns the pencil to its equilibrium position. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of a pencil standing vertically on a flat surface, illustrating the concepts of force and torque in a free-body diagram.

– The pencil is slightly tilted and is in contact with the surface at a point labeled “Pivot point.”
– The center of gravity (CG) of the pencil is marked with a black dot and labeled “CG.”
– A red arrow pointing downward from the CG indicates the weight of the pencil, labeled “w.”
– An upward red arrow from the pivot point represents the normal force, labeled “N.”
– A curved arrow near the tip of the pencil indicates clockwise torque, labeled “τ_{cw}.”
– To the side of the pencil, a simplified free-body diagram is shown, consisting of two vertical arrows: an upward arrow for “N” and a downward arrow for “w.”

Overall, the diagram demonstrates how forces and torque act on the pencil, with a focus on the balance of these forces around the pivot point.

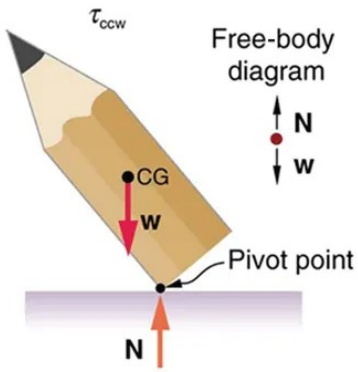


Figure 7.12 If the pencil is displaced too far, the torque caused by its weight changes direction to counterclockwise and causes the displacement to increase. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram depicting the forces acting on a pencil that is balanced on its tip, labeled as a "Free-body diagram."

– Pencil: The pencil is oriented diagonally, pointing upward to the left, and resting on a pivot point.
– Pivot Point: Located at the bottom where the pencil meets a surface.
– Center of Gravity (CG): Marked on the pencil with a black dot labeled "CG."
– Forces: Two forces are acting on the pencil:
– Weight (w): Represented by a downward arrow from the center of gravity, labeled "w."
– Normal Force (N): Represented by an upward arrow at the pivot point, labeled "N."
– Torque (τccw): There is a label τ_{ccw} near the top of the pencil indicating a counterclockwise torque.
– There is a small free-body diagram to the side showing:
– A red dot with two arrows, one pointing up labeled "N" and one pointing down labeled "w," demonstrating the balance of forces.

Overall, this image illustrates the concept of forces and torque in a balanced object scenario.

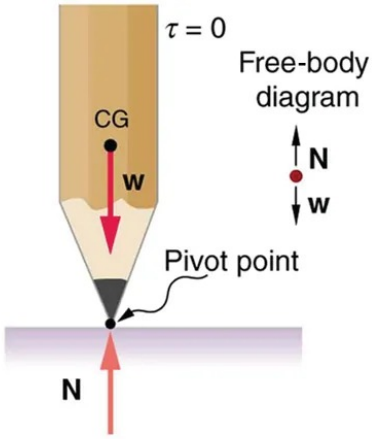


Figure 7.13 This figure shows unstable equilibrium, although both conditions for equilibrium are satisfied. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram showing the physics of a pencil balanced on its tip, illustrating concepts of torque and forces. The pencil is positioned vertically with its tip resting on a surface, acting as the pivot point.

A red arrow labeled "w" is directed downward from the pencil's center of gravity (CG), representing the weight of the pencil. At the base, near the tip, another red arrow labeled "N" points upward, representing the normal force exerted by the surface. These arrows illustrate the balance of forces at play.

There is a small section to the top right containing a simplified "Free-body diagram." It shows two arrows: one pointing upward labeled "N" and another pointing downward labeled "w," reinforcing the direction of the forces acting on the pencil.

The notation "$\tau = 0$" indicates that the torque is zero, highlighting the equilibrium condition where the pencil balances due to the equal weight and normal forces. A curved arrow points to the tip of the pencil, indicating the pivot point.

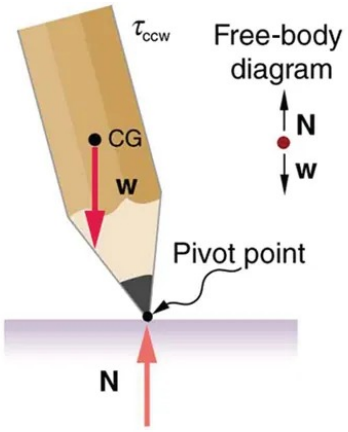


Figure 7.14 If the pencil is displaced even slightly, a torque is created by its weight that is in the same direction as the displacement, causing the displacement to increase. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a simple physics diagram demonstrating the concept of torque and balance with a pencil.

– Pencil Illustration: A pencil is shown resting on a surface at an angle. The eraser end is elevated while the point touches the surface, designated as the "Pivot point."

– Center of Gravity (CG): A black dot with the label "CG" is marked on the pencil, indicating the center of gravity with a red arrow labeled "w" pointing downwards, signifying the weight force acting through the center of gravity.

– Normal Force (N): A red arrow labeled "N" points upwards from the pivot point, representing the normal force exerted by the surface on the pencil.

– Torque (τ_ccw): Above the pencil, the symbol "τ_ccw" is shown, suggesting counterclockwise torque acting on the pencil around the pivot point.

– Free-body Diagram: Located to the right of the pencil, a simplified free-body diagram illustrates the forces. It includes:
– A dot with a small arrow pointing upwards labeled "N" for the normal force.
– A dot with a small arrow pointing downwards labeled "w" for the weight.

This image demonstrates the balance of forces and the creation of torque as the pencil rests at an angle on a surface.

A system is in neutral equilibrium if its equilibrium is independent of displacements from its original

position. A marble on a flat horizontal surface is an example. Combinations of these situations are possible. For example, a marble on a saddle is stable for displacements toward the front or back of the saddle and unstable for displacements to the side. Figure 7.15 shows another example of neutral equilibrium.

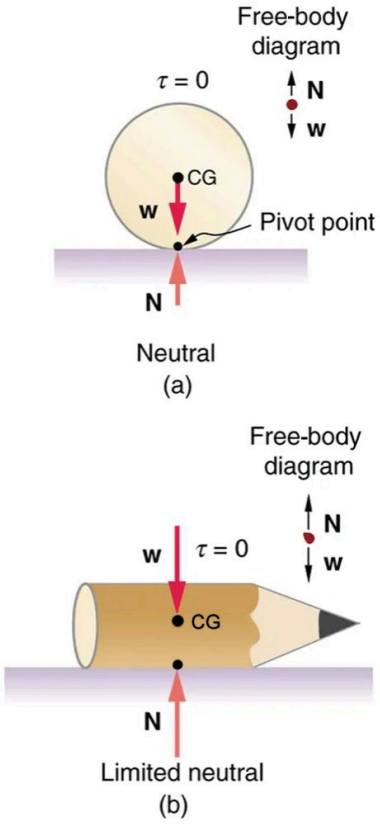


Figure 7.15 (a) Here we see neutral equilibrium. The cg of a sphere on a flat surface lies directly above the point of support, independent of the position on the surface. The sphere is therefore in equilibrium in any location, and if displaced, it will remain put. (b) Because it has a circular cross section, the pencil is in neutral equilibrium for displacements perpendicular to its length. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b), depicting free-body diagrams of different objects on a surface.

Diagram (a) – Neutral:

– Object: A circle is balanced on a surface with a pivot point at the bottom.
– Labels:
– “CG” marks the center of gravity inside the circle.
– “w” is a downward arrow representing weight, pointing from the CG to the surface.

– "N" is an upward arrow representing the normal force, acting through the pivot point.
– "$\tau = 0$" indicates zero torque.
– A small sub-diagram to the side shows two arrows: "N" pointing up and "w" pointing down.

Diagram (b) – Limited Neutral:

– Object: A pencil-shaped object is balanced horizontally on a surface.
– Labels:
– "CG" marks the center of gravity inside the pencil.
– "w" is a downward arrow representing weight, pointing from the CG to the surface.
– "N" is an upward arrow representing the normal force, acting through the base of the pencil.
– "$\tau = 0$" indicates zero torque.
– A small sub-diagram to the side similar to the first diagram shows two arrows: "N" pointing up and "w" pointing down.

Both diagrams illustrate the balance of forces and torque where the weight and normal force are equal in magnitude and opposite in direction, resulting in no torque.

When we consider how far a system in stable equilibrium can be displaced before it becomes unstable, we find that some systems in stable equilibrium are more stable than others. The pencil in Figure 7.10 and the person in Figure 7.16(a) are in stable equilibrium, but become unstable for relatively small displacements to the side. The critical point is reached when the cg is no longer *above* the base of support. Additionally, since the cg of a person's body is above the pivots in the hips, displacements must be quickly controlled. This control is a central nervous system function that is developed when we learn to hold our bodies erect as infants. For increased stability while standing, the feet should be spread apart, giving a larger base of support. Stability is also increased by lowering one's center of gravity by bending the knees, as when a football player prepares to receive a ball or braces themselves for a tackle. A cane, a crutch, or a walker increases the stability of the user, even more as the base of support widens. Usually, the cg of a female is lower (closer to the ground) than a male. Young children have their center of gravity between their shoulders, which increases the challenge of learning to walk.

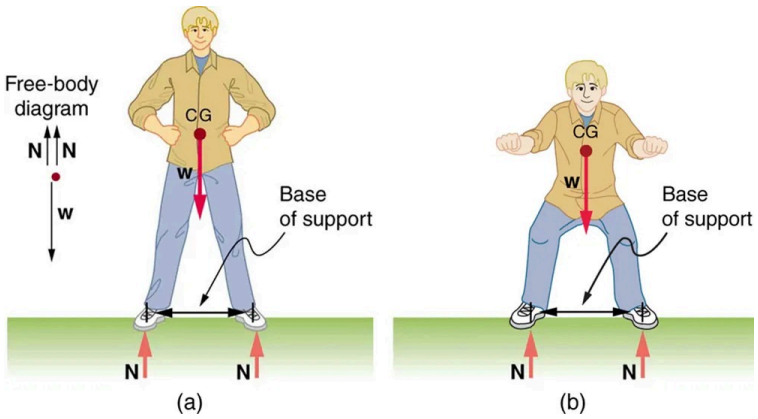


Figure 7.16 (a) The center of gravity of an adult is above the hip joints (one of the main pivots in the body) and lies between two narrowly-separated feet. Like a pencil standing on its eraser, this person is in stable equilibrium in relation to sideways displacements, but relatively small displacements take his cg outside the base of support and make him unstable. Humans are less stable relative to forward and backward displacements because the feet are not very long. Muscles are used extensively to balance the body in the front-to-back direction. (b) While bending in the manner shown, stability is increased by lowering the center of gravity. Stability is also increased if the base is expanded by placing the feet farther apart. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two diagrams labeled (a) and (b), both depicting a person standing and demonstrating concepts related to center of gravity and base of support.

Diagram (a):
– The person is standing upright with their feet shoulder-width apart.
– A red dot marked “CG” indicates the center of gravity near the stomach area.
– A red downward arrow labeled “w” represents the weight of the person, originating from the center of gravity.
– Black arrows labeled “N” originate from the feet, pointing upwards, representing the normal force from the ground.
– A horizontal double-headed arrow between the feet marks the “Base of support.”
– A separate “Free-body diagram” is shown on the left with a red dot and the same vertical arrows for weight (w) and normal force (N).

Diagram (b):
– The person is bending slightly with feet spread further apart than in (a).
– The center of gravity (CG) and weight (w) are indicated similarly as in (a), with the weight arrow pointing down from the center of gravity.

– The base of support is shown as wider than in (a) with the horizontal double-headed arrow between the feet.
– Upward arrows labeled “N” are again shown at the feet to depict the normal force.

The image illustrates how altering foot positioning can affect stability by changing the base of support, while maintaining the center of gravity concept in both scenarios.

Animals such as chickens have easier systems to control. Figure 7.17 shows that the cg of a chicken lies below its hip joints and between its widely separated and broad feet. Even relatively large displacements of the chicken’s cg are stable and result in restoring forces and torques that return the cg to its equilibrium position with little effort on the chicken’s part. Not all birds are like chickens, of course. Some birds, such as the flamingo, have balance systems that are almost as sophisticated as that of humans.

Figure 7.17 shows that the cg of a chicken is below the hip joints and lies above a broad base of support formed by widely-separated and large feet. Hence, the chicken is in very stable equilibrium, since a relatively large displacement is needed to render it unstable. The body of the chicken is supported from above by the hips and acts as a pendulum between the hips. Therefore, the chicken is stable for front-to-back displacements as well as for side-to-side displacements.

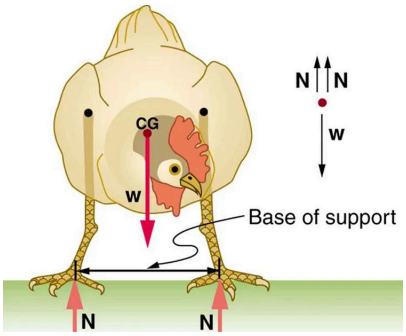


Figure 7.17 The center of gravity of a chicken is below the hip joints. The chicken is in stable equilibrium. The body of the chicken is supported from above by the hips and acts as a pendulum between them. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image illustrates a chicken and demonstrates the concept of balance and support. The chicken’s body is shown with its legs spread apart, standing on a flat green surface.

– A red dot labeled “CG” indicates the center of gravity inside the chicken’s body.
– A downward arrow labeled “W” points from the CG, representing the weight force acting downwards.
– Two upward arrows labeled “N” are positioned under each foot, showing the normal force exerted by the

ground on the chicken's feet.
– The phrase "Base of support" is marked by a double-headed arrow that spans the distance between the chicken's feet.
– To the right of the chicken, another diagram shows the same forces: "N" for normal force directed upward and "w" for weight directed downward.

This visualization explains how the chicken maintains balance by aligning the center of gravity over its base of support, counteracting its weight with normal forces from the ground.

Engineers and architects strive to achieve extremely stable equilibriums for buildings and other systems that must withstand wind, earthquakes, and other forces that displace them from equilibrium. Although the examples in this section emphasize gravitational forces, the basic conditions for equilibrium are the same for all types of forces. The net external force must be zero, and the net torque must also be zero.

Take-Home Experiment

Stand straight with your heels, back, and head against a wall. Bend forward from your waist, keeping your heels and bottom against the wall, to touch your toes. Can you do this without toppling over? Explain why and what you need to do to be able to touch your toes without losing your balance. Is it easier for a woman to do this?

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7.4 APPLICATIONS OF STATICS, INCLUDING PROBLEM-SOLVING STRATEGIES

Learning Objectives

By the end of this section, you will be able to:

- Discuss the applications of Statics in real life.
- State and discuss various problem-solving strategies in Statics.

Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We begin with a discussion of problem-solving strategies specifically used for statics. Since statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in 4.6 Problem-Solving Strategies, still apply.

Problem-Solving Strategy: Static Equilibrium Situations

1. The first step is to determine whether or not the system is in static equilibrium. This condition is always the case when the *acceleration of the system is zero and accelerated rotation does not occur*.
2. It is particularly important to *draw a free body diagram for the system of interest*. Carefully label all forces, and note their relative magnitudes, directions, and points of application whenever these are known.
3. Solve the problem by applying either or both of the conditions for equilibrium (represented by the equations $\text{net}\mathbf{F} = 0$ and $\text{net } \tau = 0$, depending on the list of known and unknown factors. If the second condition is involved, *choose the pivot point to simplify the solution*. Any pivot point can be chosen, but the most useful ones cause torques by unknown forces to be zero. (Torque is zero if the force is applied at the pivot (then $r = 0$), or along a line through the pivot point (then $\theta = 0$)). Always choose a convenient coordinate system for projecting forces.
4. *Check the solution to see if it is reasonable* by examining the magnitude, direction, and units of the answer. The importance of this last step never diminishes, although in unfamiliar applications, it is usually more difficult to judge reasonableness. These judgments become progressively easier with experience.

Now let us apply this problem-solving strategy for the pole vaulter shown in the three figures below. The pole is uniform and has a mass of 5.00 kg. In Figure 7.18, the pole's cg lies halfway between the vaulter's hands. It seems reasonable that the force exerted by each hand is equal to half the weight of the pole, or 24.5 N. This obviously satisfies the first condition for equilibrium $(\text{net}\mathbf{F} = 0)$. The second condition $(\text{net}\tau = 0)$ is also satisfied, as we can see by choosing the cg to be the pivot point. The weight exerts no torque about a pivot point located at the cg, since it is applied at that point and its lever arm is zero. The equal forces exerted by the hands are equidistant from the chosen pivot, and so they exert equal and opposite torques. Similar arguments hold for other systems where supporting forces are exerted symmetrically about the cg. For example, the four legs of a uniform table each support one-fourth of its weight.

In Figure 7.18, a pole vaulter holding a pole with its cg halfway between his hands is shown. Each hand exerts a force equal to half the weight of the pole, $F_R = F_L = w/2$. (b) The pole vaulter moves the pole to his left, and the forces that the hands exert are no longer equal. See Figure 7.18. If the pole is held with its cg to the left of the person, then he must push down with his right hand and up with his left. The forces he exerts are larger here because they are in opposite directions and the cg is at a long distance from either hand.

Similar observations can be made using a meter stick held at different locations along its length.

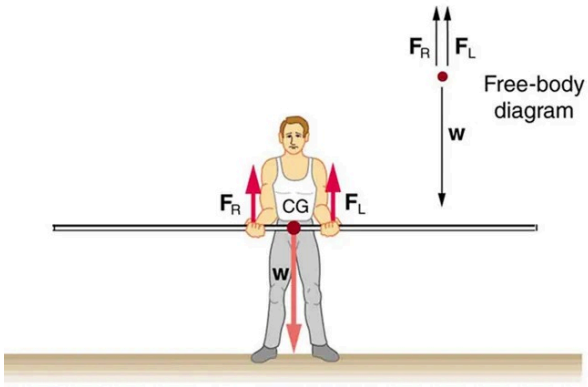


Figure 7.18 A pole vaulter holds a pole horizontally with both hands. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram showing a person standing and holding a horizontal bar across their shoulders. The bar is labeled with forces acting on it. The person is depicted with their arms at their sides, gripping the bar. Three main forces are illustrated:

- *FR*: An upward force on the right side of the bar.
- *FL*: An upward force on the left side of the bar.
- *w*: A downward force representing weight at the center, labeled as CG (center of gravity).

On the right side of the image, a simplified 'Free-body diagram' is shown. It includes:

- An upward arrow labeled *FR*.
- An upward arrow labeled *FL*.
- A single downward arrow labeled *w*.

Both the main image and the free-body diagram are intended to show the balance of forces acting on the bar and the person supporting it, highlighting the concepts of equilibrium and center of gravity.

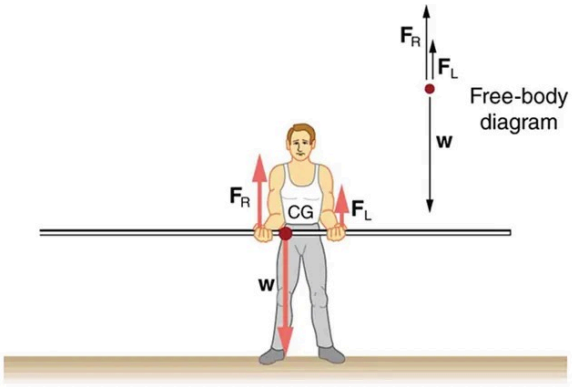


Figure 7.19 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is near his right hand. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a person standing on a flat surface, holding a long horizontal barbell. The person is wearing a white tank top and gray pants. The focus is on explaining the forces acting on the person and the barbell. The image includes a free-body diagram to illustrate these forces:

- *CG* – Center of Gravity: A point on the person's body indicating the center of gravity, marked on the shirt.
- F_R – Force Right: An upward arrow from the right hand, indicating the force exerted by the right hand upwards on the barbell.
- F_L – Force Left: An upward arrow from the left hand, indicating the force exerted by the left hand upwards on the barbell.
- *w* – Weight: Two downward arrows; one is centered on the person's body, and another on the free-body diagram beside, indicating the force of gravity acting downward on the person and the barbell.

Adjacent to the person, a simplified free-body diagram shows how these forces interact on a single point. The diagram has:

- An upward arrow labeled F_R on the right and F_L on the left, representing the upward forces from the hands.
- A downward arrow labeled *w*, representing the weight acting through the center of gravity.

The simplified diagram visually reinforces the balance of forces required to keep the barbell steady.

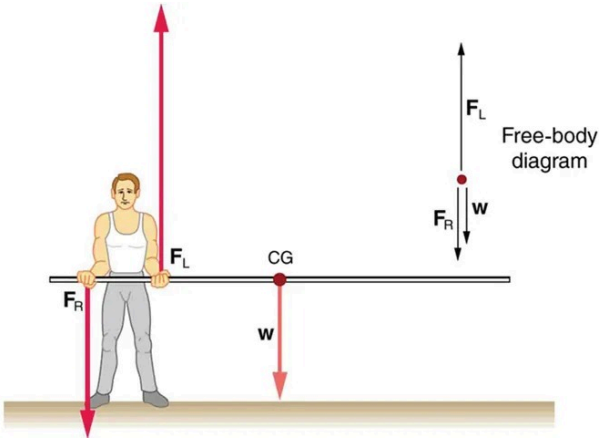


Figure 7.20 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is to the left side of the vaulter. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a person performing a balance-related exercise. The person, wearing a tank top and pants, is holding a long horizontal pole at chest level with both hands. The person's legs and feet are positioned on the ground for stability.

The pole has a red dot labeled "CG" at its center, indicating the center of gravity. There are three arrows originating from the pole at different points:

- A downward arrow labeled "w" at the center, representing weight.
- An upward arrow labeled "F_L" near the left hand, indicating a force exerted upward.
- A downward arrow labeled "F_R" near the right hand, indicating a force exerted downward.

On the right side of the image, a free-body diagram is drawn. It includes:

- A downward arrow labeled "w".
- An upward arrow labeled "F_L".
- A downward arrow labeled "F_R".

The arrows in the free-body diagram are aligned vertically, with the dot from which they originate in the center, representing the center of gravity.

If the pole vaulter holds the pole as shown in Figure 7.19, the situation is not as simple. The total force he exerts is still equal to the weight of the pole, but it is not evenly divided between his hands. (If $F_L = F_R$, then the torques about the cg would not be equal since the lever arms are different.) Logically, the right hand

should support more weight, since it is closer to the cg. In fact, if the right hand is moved directly under the cg, it will support all the weight. This situation is exactly analogous to two people carrying a load; the one closer to the cg carries more of its weight. Finding the forces F_L and F_R is straightforward, as the next example shows.

If the pole vaulter holds the pole from near the end of the pole (Figure 7.20), the direction of the force applied by the right hand of the vaulter reverses its direction.

Example 7.2

What Force Is Needed to Support a Weight Held Near Its CG?

For the situation shown in Figure 7.19, calculate: (a) F_R , the force exerted by the right hand, and (b) F_L, the force exerted by the left hand. The hands are 0.900 m apart, and the cg of the pole is 0.600 m from the left hand.

Strategy

Figure 7.19 includes a free body diagram for the pole, the system of interest. There is not enough information to use the first condition for equilibrium $(\text{net}\mathbf{F} = 0)$, since two of the three forces are unknown and the hand forces cannot be assumed to be equal in this case. There is enough information to use the second condition for equilibrium $(\text{net}\tau = 0)$ if the pivot point is chosen to be at either hand, thereby making the torque from that hand zero. We choose to locate the pivot at the left hand in this part of the problem, to eliminate the torque from the left hand.

Solution for (a)

There are now only two nonzero torques, those from the gravitational force (τ_{w}) and from the push or pull of the right hand (τ_R). Stating the second condition in terms of clockwise and counterclockwise torques,

$$\text{net } \tau_{\text{cw}} = -\text{net}\tau_{\text{ccw}}.$$

7.22

or the algebraic sum of the torques is zero.

Here this is

$$\tau_R = -\tau_{\mathrm{w}}$$

7.23

since the weight of the pole creates a counterclockwise torque and the right hand counters with a clockwise torque. Using the definition of torque, $\tau = \mathrm{rFsin}\theta$, noting that $\theta = 90^{\circ}$, and substituting known values, we obtain

$$(0.900\ \mathrm{m})\,(F_R) = (0.600\ \mathrm{m})\,(mg).$$

7.24

Thus,

$$\begin{aligned} F_R &= (0.667)\,(5.00\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right) \\ &= 32.7\ \mathrm{N}. \end{aligned}$$

7.25

Solution for (b)

The first condition for equilibrium is based on the free body diagram in the figure. This implies that by Newton's second law:

$$F_L + F_R - \mathrm{mg} = 0$$

7.26

From this we can conclude:

$$F_L + F_R = w = \mathrm{mg}$$

7.27

Solving for F_L, we obtain

$$\begin{aligned} F_L &= mg - F_R \\ &= mg - 32.7N \\ &= (5.00\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right) - 32.7\ \mathrm{N} \\ &= 16.3\ \mathrm{N} \end{aligned}$$

7.28

Discussion

F_L is seen to be exactly half of F_R, as we might have guessed, since F_L is applied twice as far from the cg as F_R.

If the pole vaulter holds the pole as he might at the start of a run, shown in Figure 7.20, the forces change again. Both are considerably greater, and one force reverses direction.

Take-Home Experiment

This is an experiment to perform while standing in a bus or a train. Stand facing sideways. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Now stand facing forward. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Why is it easier and safer to stand facing sideways rather than forward? Note: For your safety (and those around you), make sure you are holding onto something while you carry out this activity!

PhET Explorations

Balancing Act

Play with objects on a teeter totter to learn about balance. Test what you've learned by trying the Balance Challenge game.

Access multimedia content

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7.5 SIMPLE MACHINES

Learning Objectives

By the end of this section, you will be able to:

- Describe different simple machines.
- Calculate the mechanical advantage.

Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we apply the force. The word for "machine" comes from the Greek word meaning "to help make things easier." Levers, gears, pulleys, wedges, and screws are some examples of machines. Energy is still conserved for these devices because a machine cannot do more work than the energy put into it. However, machines can reduce the input force that is needed to perform the job. The ratio of output to input force magnitudes for any simple machine is called its mechanical advantage (MA).

$$\text{MA} = \frac{F_\text{o}}{F_\text{i}}$$

7.29

One of the simplest machines is the lever, which is a rigid bar pivoted at a fixed place called the fulcrum. Torques are involved in levers, since there is rotation about a pivot point. Distances from the physical pivot of the lever are crucial, and we can obtain a useful expression for the MA in terms of these distances.

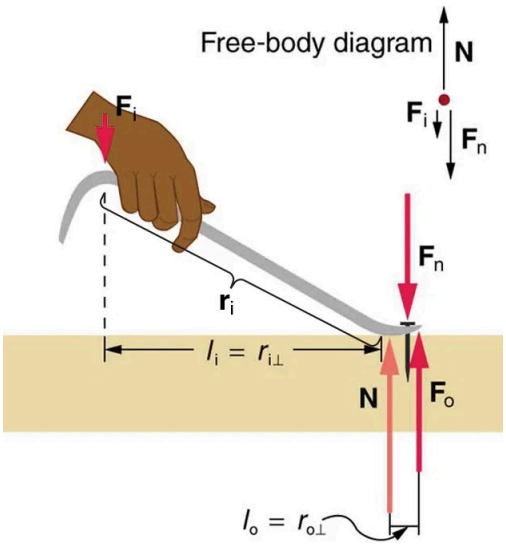


Figure 7.21 A nail puller is a lever with a large mechanical advantage. The external forces on the nail puller are represented by solid arrows. The force that the nail puller applies to the nail ($\mathbf{F}_o$) is not a force on the nail puller. The reaction force the nail exerts back on the puller ($\mathbf{F}_n$) is an external force and is equal and opposite to $\mathbf{F}_o$. The perpendicular lever arms of the input and output forces are l_i and l_o. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the physics of a lever in use, labeled as a "free-body diagram." It depicts a hand applying a downward force on one end of a crowbar, which is used as a lever. The descriptions and labels in the image are as follows:

- A hand holds the lever, applying a force labeled as F_i, represented by a downward arrow near the hand.
- The lever pivots on a point on a surface, with two arrows near the pivot point labeled F_n (downward) and N (upward). These represent the normal force and the ouput force, respectively.
- The force exerted by the lever on the surface is labeled F_o (upward arrow). It is opposing the downward force of F_n.
- The length from the hand to the pivot is labeled $l_i = r_{i\perp}$.
- The length from the pivot to the point where the force F_o is applied is labeled $l_o = r_{o\perp}$.
- There is a smaller diagram in the top right corner that simplifies the forces into arrows, showing F_i and F_n acting downwards, N acting upwards.

Figure 7.21 shows a lever type that is used as a nail puller. Crowbars, seesaws, and other such levers are all analogous to this one. $\mathbf{F}_i$ is the input force and $\mathbf{F}_o$ is the output force. There are three vertical forces acting

on the nail puller (the system of interest) – these are $\mathbf{F}_{\mathrm{i}}$, $\mathbf{F}_{\mathrm{n}}$, and $\mathbf{N}$. $\mathbf{F}_{\mathrm{n}}$ is the reaction force back on the system, equal and opposite to $\mathbf{F}_{\mathrm{o}}$. (Note that $\mathbf{F}_{\mathrm{o}}$ is not a force on the system.)

$\mathbf{N}$ is the normal force upon the lever, and its torque is zero since it is exerted at the pivot. The torques due to $\mathbf{F}_{\mathrm{i}}$ and $\mathbf{F}_{\mathrm{n}}$ must be equal to each other if the nail is not moving, to satisfy the second condition for equilibrium $(\text{net } \tau = 0)$. (In order for the nail to actually move, the torque due to $\mathbf{F}_{\mathrm{i}}$ must be ever-so-slightly greater than torque due to $\mathbf{F}_{\mathrm{n}}$.) Hence,

$$l_{\mathrm{i}}F_{\mathrm{i}} = l_{\mathrm{o}}F_{\mathrm{o}}$$
7.30

Notice that $\mathbf{r}_{\mathrm{i}}$ is the distance from the pivot point to the point where the input force $\mathbf{F}_{\mathrm{i}}$ is applied, and $\mathbf{r}_{\mathrm{o}}$ (not labeled on the diagram) is the distance from the pivot point to the point where the output force $\mathbf{F}_{\mathrm{o}}$ is applied. The distances l_{i} and l_{o} are the perpendicular components of the distances from where the input and output forces are applied to the pivot, as shown in the figure. Rearranging the last equation gives

$$\frac{F_{\mathrm{o}}}{F_{\mathrm{i}}} = \frac{l_{\mathrm{i}}}{l_{\mathrm{o}}}.$$
7.31

What interests us most here is that the magnitude of the force exerted by the nail puller, F_{o}, is much greater than the magnitude of the input force applied to the puller at the other end, F_{i}. For the nail puller,

$$\mathrm{MA} = \frac{F_{\mathrm{o}}}{F_{\mathrm{i}}} = \frac{l_{\mathrm{i}}}{l_{\mathrm{o}}}.$$
7.32

This equation is true for levers in general. For the nail puller, the MA is certainly greater than one. The longer the handle on the nail puller, the greater the force you can exert with it.

Two other types of levers that differ slightly from the nail puller are a wheelbarrow and a shovel, shown in Figure 9.22. All these lever types are similar in that only three forces are involved – the input force, the output force, and the force on the pivot – and thus their MAs are given by $\mathrm{MA} = \frac{F_{\mathrm{o}}}{F_{\mathrm{i}}}$ and $\mathrm{MA} = \frac{d_1}{d_2}$, with distances being measured relative to the physical pivot. The wheelbarrow and shovel differ from the nail puller because both the input and output forces are on the same side of the pivot.

In the case of the wheelbarrow, the output force or load is between the pivot (the wheel's axle) and the input or applied force. In the case of the shovel, the input force is between the pivot (at the end of the handle) and the load, but the input lever arm is shorter than the output lever arm. In this case, the MA is less than one.

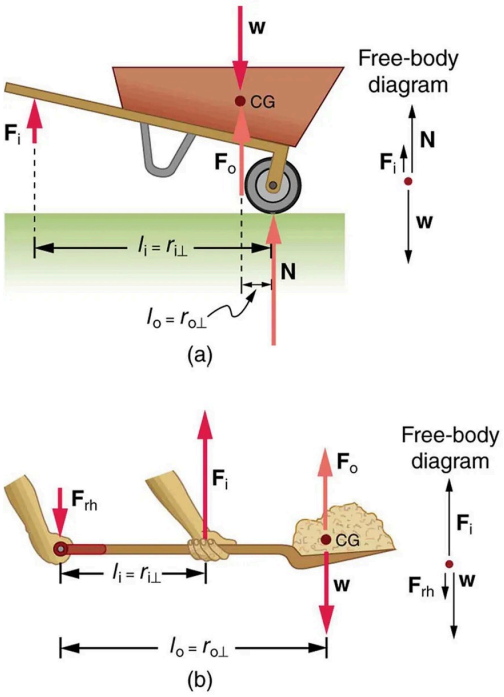


Figure 7.22 (a) In the case of the wheelbarrow, the output force or load is between the pivot and the input force. The pivot is the wheel's axle. Here, the output force is greater than the input force. Thus, a wheelbarrow enables you to lift much heavier loads than you could with your body alone. (b) In the case of the shovel, the input force is between the pivot and the load, but the input lever arm is shorter than the output lever arm. The pivot is at the handle held by the right hand. Here, the output force (supporting the shovel's load) is less than the input force (from the hand nearest the load), because the input is exerted closer to the pivot than is the output. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image contains two diagrams, each illustrating a free-body diagram for different systems involving forces and torques.

Diagram (a):

- A wheelbarrow is shown with a labeled center of gravity (CG).
- The forces acting on it include the following:
  - W: Weight of the load acting downward at CG.
  - N: Normal force acting upward at the wheel.
  - F_o: Force at the wheel.
  - F_i: Input force applied upward by the person.
- Distances are labeled between forces:

- $l_i = r_{i\perp}$: Distance from the input force to the pivot point (wheel).
- $l_o = r_{o\perp}$: Distance from the output force to the pivot point.

- A free-body diagram to the right shows arrows representing these forces, balancing around the wheel pivot.

Diagram (b):

- A shovel filled with a load is shown being held by two hands, with forces labeled:
  - W: Weight of the load acting downward at CG.
  - F_i: Input force applied upward by the person's hands.
  - F_{rh}: Reaction force at the hand furthest from the load, acting downward.
  - F_o: Force at the hand closest to the load, acting upward.
- Distances are labeled between forces:
  - $l_i = r_{i\perp}$: Distance from the input force to the nearest hand.
  - $l_o = r_{o\perp}$: Total distance from the load to the reaction force.
- A free-body diagram to the right shows arrows representing these forces, balancing around the pivot point at the hand.

Example 7.3

What is the Advantage for the Wheelbarrow?

In the wheelbarrow of Figure 7.22, the load has a perpendicular lever arm of 7.50 cm, while the hands have a perpendicular lever arm of 1.02 m. (a) What upward force must you exert to support the wheelbarrow and its load if their combined mass is 45.0 kg? (b) What force does the wheelbarrow exert on the ground?

Strategy

Here, we use the concept of mechanical advantage.

Solution

(a) In this case, $\frac{F_{\text{o}}}{F_{\text{i}}} = \frac{l_i}{l_o}$ becomes

$$F_{\text{i}} = F_{\text{o}}\frac{l_o}{l_i}.$$

7.33

Adding values into this equation yields

$$F_{\text{i}} = (45.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)\frac{0.075\ \text{m}}{1.02\ \text{m}} = 32.4\ \text{N}.$$

7.34

The free-body diagram (see Figure 7.22) gives the following normal force: $F_i + N = W$. Therefore,

$N = (45.0\ \text{kg})\left(9.80\ \text{m/s}^2\right) - 32.4\ \text{N} = 409\ \text{N}$. N is the normal force acting on the wheel; by Newton's third law, the force the wheel exerts on the ground is $409\ \text{N}$.

Discussion

An even longer handle would reduce the force needed to lift the load. The MA here is $\text{MA} = 1.02/0.0750 = 13.6$.

Another very simple machine is the inclined plane. Pushing a cart up a plane is easier than lifting the same cart straight up to the top using a ladder, because the applied force is less. However, the work done in both cases (assuming the work done by friction is negligible) is the same. Inclined lanes or ramps were probably used during the construction of the Egyptian pyramids to move large blocks of stone to the top.

A crank is a lever that can be rotated 360° about its pivot, as shown in Figure 7.23. Such a machine may not look like a lever, but the physics of its actions remain the same. The MA for a crank is simply the ratio of the radii r_{i}/r_0. Wheels and gears have this simple expression for their MAs too. The MA can be greater than 1, as it is for the crank, or less than 1, as it is for the simplified car axle driving the wheels, as shown. If the axle's radius is $2.0cm$ and the wheel's radius is $24.0\ \text{cm}$, then $\text{MA} = 2.0/24.0 = 0.083$ and the axle would have to exert a force of $12,000\ \text{N}$ on the wheel to enable it to exert a force of $1000\ \text{N}$ on the ground.

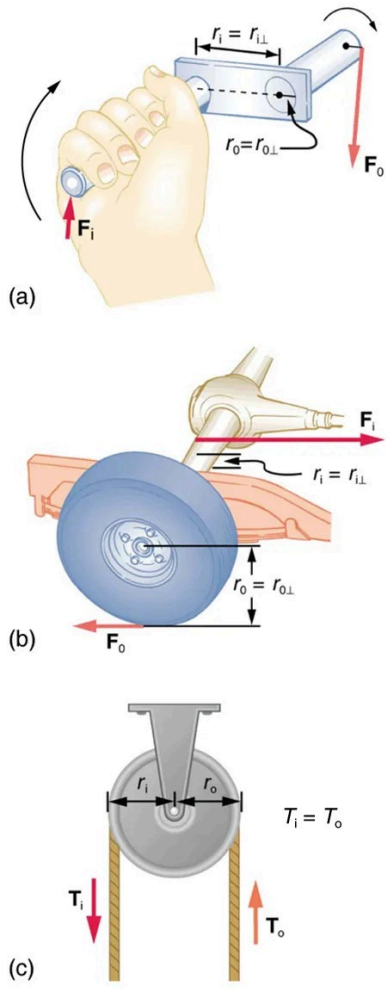


Figure 7.23 (a) A crank is a type of lever that can be rotated 360° *about its pivot. Cranks are usually designed to have a large MA. (b) A simplified automobile axle drives a wheel, which has a much larger diameter than the axle. The MA is less than 1. (c) An ordinary pulley is used to lift a heavy load. The pulley changes the direction of the force* T *exerted by the cord without changing its magnitude. Hence, this machine has an MA of 1. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of three diagrams labeled as (a), (b), and (c), illustrating different mechanical systems involving forces and radii:

(a) A hand gripping a wrench, depicted as applying an input force (F_i) upwards on a bolt. The wrench applies an output force (F_o) downwards. The diagram highlights two radii: r_i for the input and r_o for the output.

(b) A car wheel attached to a rear axle. An input force (F_i) is directed rightward along the axle. An output

force (F_o) acts leftward on the wheel. Radii r_i and r_o are indicated, with r_i corresponding to the axle and r_o to the wheel.

(c) A pulley system with two ropes. The input tension (T_i) acts on one side of the pulley, and the output tension (T_o) acts on the opposite side. The radii, r_i and r_o, are marked. The tensions are shown with downward and upward arrows, respectively, and it states "$T_i = T_o$".

An ordinary pulley has an MA of 1; it only changes the direction of the force and not its magnitude. Combinations of pulleys, such as those illustrated in Figure 9.24, are used to multiply force. If the pulleys are friction-free, then the force output is approximately an integral multiple of the tension in the cable. The number of cables pulling directly upward on the system of interest, as illustrated in the figures given below, is approximately the MA of the pulley system. Since each attachment applies an external force in approximately the same direction as the others, they add, producing a total force that is nearly an integral multiple of the input force T.

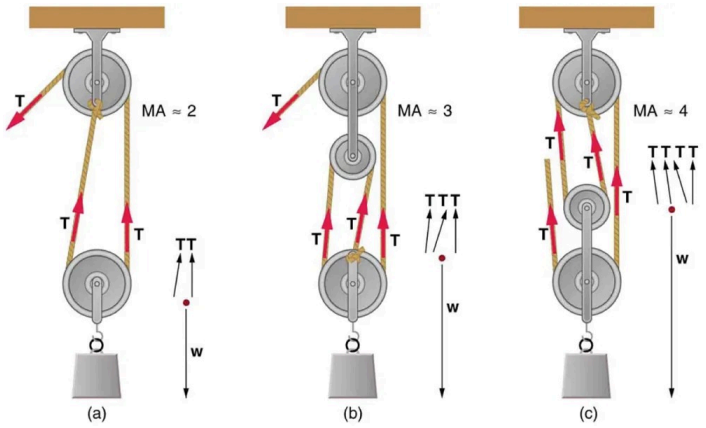


Figure 7.24 (a) The combination of pulleys is used to multiply force. The force is an integral multiple of tension if the pulleys are frictionless. This pulley system has two cables attached to its load, thus applying a force of approximately $2T$. This machine has $\text{MA} \approx 2$. (b) Three pulleys are used to lift a load in such a way that the mechanical advantage is about 3. Effectively, there are three cables attached to the load. (c) This pulley system applies a force of $4T$, so that it has $\text{MA} \approx 4$. Effectively, four cables are pulling on the system of interest. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a series of diagrams illustrating three different pulley systems, each demonstrating varying mechanical advantages (MA). Each diagram highlights the forces involved and how they interact with the pulleys.

1. Diagram (a):

– Shows a pulley system with a mechanical advantage of approximately 2.
– Consists of two pulleys and a rope.
– The rope passes over both pulleys with three sections labeled "T" pointing in opposite directions, indicating tension forces.
– A weight labeled "w" is attached at the bottom of the rope.

2. Diagram (b):
– Shows a pulley system with a mechanical advantage of approximately 3.
– Consists of three pulleys and a rope.
– The rope wraps around the pulleys with four sections labeled "T", depicting the tension forces working in the system.
– The weight "w" hangs from the bottom.

3. Diagram (c):
– Shows a pulley system with a mechanical advantage of approximately 4.
– Includes four pulleys and a rope.
– The rope creates five sections labeled "T", illustrating the distributed tension forces in use.
– A weight "w" is suspended at the bottom of the setup.

Each diagram is visually distinct with forces clearly marked by arrows, showing how different configurations distribute tension to provide mechanical advantage.

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7.6 FORCES AND TORQUES IN MUSCLES AND JOINTS

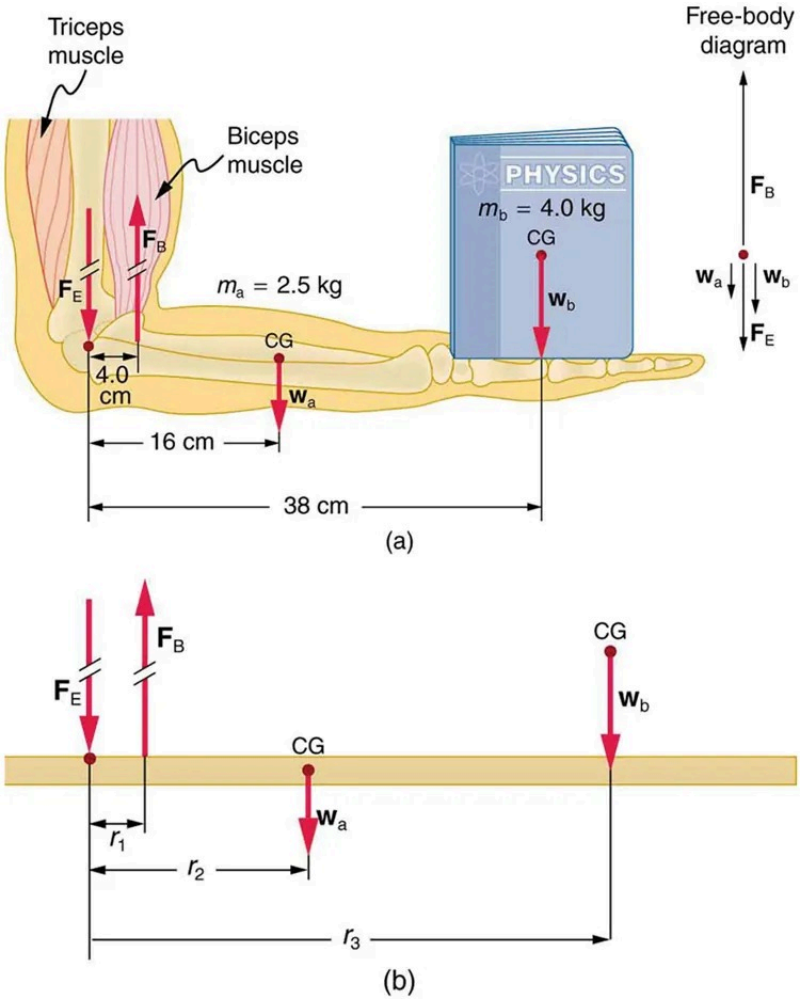
Learning Objectives

By the end of this section, you will be able to:

- Explain the forces exerted by muscles.
- State how a bad posture causes back strain.
- Discuss the benefits of skeletal muscles attached close to joints.
- Discuss various complexities in the real system of muscles, bones, and joints.

Muscles, bones, and joints are some of the most interesting applications of statics. There are some surprises. Muscles, for example, exert far greater forces than we might think. Figure 7.25 shows a forearm holding a book and a schematic diagram of an analogous lever system. The schematic is a good approximation for the forearm, which looks more complicated than it is, and we can get some insight into the way typical muscle systems function by analyzing it.

Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor—that is, it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs apply to the outside world. The reason is clear once we realize that most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen in Figure 7.25.



Example 7.4

Figure 7.25 (a) The figure shows the forearm of a person holding a book. The biceps exert a force $\mathbf{F}_B$ *to support the weight of the forearm and the book. The triceps are assumed to be relaxed. (b) Here, you can view an approximately equivalent mechanical system with the pivot at the elbow joint as seen in . Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two diagrams illustrating the forearm as a lever system, a free-body diagram, and related forces, aimed at explaining the biomechanics of the arm holding an object.

In diagram (a), there is a labeled illustration of a forearm holding a book. Key elements labeled include:

- **Triceps muscle:** Positioned at the back of the arm.
- **Biceps muscle:** Located at the front of the upper arm, exerting force F_B upward.

- **$\mathbf{F_E}$:** The force exerted at the elbow, depicted as a downward arrow.
- **$\mathbf{W_a}$:** The weight of the arm (2.5 kg), with its center of gravity labeled as "CG", placed downward.
- **$\mathbf{W_b}$:** The weight of the book (4.0 kg), with its center of gravity labeled as "CG", also placed downward.
- Distances are labeled: 4.0 cm from the elbow to the biceps force, 16 cm from the elbow to the arm's center of gravity, and 38 cm from the elbow to the book's center of gravity.

The free-body diagram depicts the forces acting on the forearm as a lever:

- **$\mathbf{F_B}$:** Force from the biceps is shown upward.
- **$\mathbf{F_E}$:** Elbow force, shown downward.
- **$\mathbf{W_a}$ and $\mathbf{W_b}$:** Weights of the arm and book are shown downward.

In diagram (b), a simplified lever diagram is provided to represent the arm:

- Distances labeled as r_1, r_2, and r_3, corresponding to the forces F_E, W_a, and W_b, respectively.
- $\mathbf{F_E}$ and $\mathbf{F_B}$ are depicted with arrows for their respective directions.
- Weight forces W_a and W_b are shown with downward arrows at their respective points on the lever.

Example 7.4

Muscles Exert Bigger Forces Than You Might Think

Calculate the force the biceps muscle must exert to hold the forearm and its load as shown in Figure 7.25, and compare this force with the weight of the forearm plus its load. You may take the data in the figure to be accurate to three significant figures.

Strategy

There are four forces acting on the forearm and its load (the system of interest). The magnitude of the force of the biceps is F_{B}; that of the elbow joint is F_{E}; that of the weights of the forearm is w_{a}, and its load is w_{b}. Two of these are unknown (F_{B} and F_{E}), so that the first condition for equilibrium cannot by itself yield F_{B}. But if we use the second condition and choose the pivot to be at the elbow, then the torque due to F_{E} is zero, and the only unknown becomes F_{B}.

Solution

The torques created by the weights are clockwise relative to the pivot, while the torque created by the biceps is counterclockwise; thus, the second condition for equilibrium $(\mathrm{net}\tau = 0)$ becomes

$$r_2 w_{\mathrm{a}} + r_3 w_{\mathrm{b}} = r_1 F_{\mathrm{B}}.$$

7.35

Note that $\sin\theta = 1$ for all forces, since $\theta = 90^{\circ}$ for all forces. This equation can easily be solved for F_{B} in terms of known quantities, yielding

$$F_{\mathrm{B}} = \frac{r_2 w_{\mathrm{a}} + r_3 w_{\mathrm{b}}}{r_1}.$$

7.36

Entering the known values gives

$$F_{\mathrm{B}} = \frac{(0.160\ \mathrm{m})(2.50\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right) + (0.380\ \mathrm{m})(4.00\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)}{0.0400\ \mathrm{m}}$$

7.37

which yields

$$F_{\mathrm{B}} = 470\ \mathrm{N}.$$

7.38

Now, the combined weight of the arm and its load is $(6.50\ \mathrm{kg})\left(9.80\mathrm{m/s^2}\right) = 63.7\ \mathrm{N}$, so that the ratio of the force exerted by the biceps to the total weight is

$$\frac{F_{\mathrm{B}}}{w_{\mathrm{a}} + w_{\mathrm{b}}} = \frac{470}{63.7} = 7.38.$$

7.39

Discussion

This means that the biceps muscle is exerting a force 7.38 times the weight supported.

In the above example of the biceps muscle, the angle between the forearm and upper arm is 90°. If this angle changes, the force exerted by the biceps muscle also changes. In addition, the length of the biceps muscle changes. The force the biceps muscle can exert depends upon its length; it is smaller when it is shorter than when it is stretched.

Very large forces are also created in the joints. In the previous example, the downward force F_{E} exerted by the humerus at the elbow joint equals 407 N, or 6.38 times the total weight supported. (The calculation of

F_{E} is straightforward and is left as an end-of-chapter problem.) Because muscles can contract, but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions and thus subtract. (In the above example, the upward force of the muscle minus the downward force of the joint equals the weight supported—that is,

$470\ \mathrm{N}–407\ \mathrm{N} = 63\ \mathrm{N}$, approximately equal to the weight supported.) Forces in muscles and joints are largest when their load is a long distance from the joint, as the book is in the previous example.

In racquet sports such as tennis the constant extension of the arm during game play creates large forces in this way. The mass times the lever arm of a tennis racquet is an important factor, and many players use the heaviest racquet they can handle. It is no wonder that joint deterioration and damage to the tendons in the elbow, such as "tennis elbow," can result from repetitive motion, undue torques, and possibly poor racquet selection in such sports. Various tried techniques for holding and using a racquet or bat or stick not only increases sporting prowess but can minimize fatigue and long-term damage to the body. For example, tennis balls correctly hit at the "sweet spot" on the racquet will result in little vibration or impact force being felt in the racquet and the body—less torque. Twisting the hand to provide top spin on the ball or using an extended rigid elbow in a backhand stroke can also aggravate the tendons in the elbow.

Training coaches and physical therapists use the knowledge of relationships between forces and torques in the treatment of muscles and joints. In physical therapy, an exercise routine can apply a particular force and torque which can, over a period of time, revive muscles and joints. Some exercises are designed to be carried out under water, because this requires greater forces to be exerted, further strengthening muscles. However, connecting tissues in the limbs, such as tendons and cartilage as well as joints are sometimes damaged by the large forces they carry. Often, this is due to accidents, but heavily muscled athletes, such as weightlifters, can tear muscles and connecting tissue through effort alone.

The back is considerably more complicated than the arm or leg, with various muscles and joints between vertebrae, all having mechanical advantages less than 1. Back muscles must, therefore, exert very large forces, which are borne by the spinal column. Discs crushed by mere exertion are very common. The jaw is somewhat exceptional—the masseter muscles that close the jaw have a mechanical advantage greater than 1 for the back teeth, allowing us to exert very large forces with them. A cause of stress headaches is persistent clenching of teeth where the sustained large force translates into fatigue in muscles around the skull.

Figure 7.26 shows how bad posture causes back strain. In part (a), we see a person with good posture. Note that her upper body's cg is directly above the pivot point in the hips, which in turn is directly above the base of support at her feet. Because of this, her upper body's weight exerts no torque about the hips. The only force needed is a vertical force at the hips equal to the weight supported. No muscle action is required, since the bones are rigid and transmit this force from the floor. This is a position of unstable equilibrium, but only small forces are needed to bring the upper body back to vertical if it is slightly displaced. Bad posture is shown in part (b); we see that the upper body's cg is in front of the pivot in the hips. This creates a clockwise torque around the hips that is counteracted by muscles in the lower back. These muscles must exert large forces, since they have typically small mechanical advantages. (In other words, the perpendicular lever arm for the muscles is much smaller than for the cg.) Poor posture can also cause muscle strain for people sitting at their desks using computers. Special chairs are available that allow the body's CG to be more easily situated above the seat, to reduce back pain. Prolonged muscle action produces muscle strain. Note that the cg of the entire body is still directly above the base of support in part (b) of Figure 7.26. This is compulsory; otherwise the

person would not be in equilibrium. We lean forward for the same reason when carrying a load on our backs, to the side when carrying a load in one arm, and backward when carrying a load in front of us, as seen in Figure 7.27.

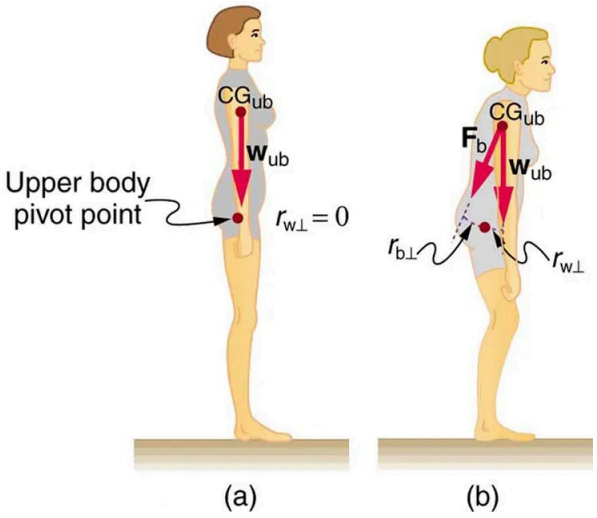


Figure 7.26 (a) Good posture places the upper body's cg over the pivots in the hips, eliminating the need for muscle action to balance the body. (b) Poor posture requires exertion by the back muscles to counteract the clockwise torque produced around the pivot by the upper body's weight. The back muscles have a small effective perpendicular lever arm, $r_{b\perp}$, and must therefore exert a large force $\mathbf{F}_b$. Note that the legs lean backward to keep the cg of the entire body above the base of support in the feet. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b), illustrating the concept of body mechanics.

Diagram (a):
– A person is shown standing upright on a flat surface.
– The person's upper body pivot point is marked.
– A vertical arrow labeled "W_{ub}" (representing the weight of the upper body) points downward from the center of the upper body.
– The center of gravity is indicated by "CG_{ub}".
– The notation "$r_{w\perp} = 0$" signifies that there is no lateral distance from the pivot point.

Diagram (b):
– A person is shown bending forward at the hips.
– The upper body pivot point and the center of gravity "CG_{ub}" are marked.
– Two arrows are present: "W_{ub}" pointing downwards and "F_b" pointing upwards, indicating forces acting on the body.

– The diagram includes notations "$r_{b\perp}$" and "$r_{w\perp}$", representing the perpendicular distances from the pivot point to various forces.

Both diagrams demonstrate the effects of posture on the forces and pivot points acting on the body.

You have probably been warned against lifting objects with your back. This action, even more than bad posture, can cause muscle strain and damage discs and vertebrae, since abnormally large forces are created in the back muscles and spine.

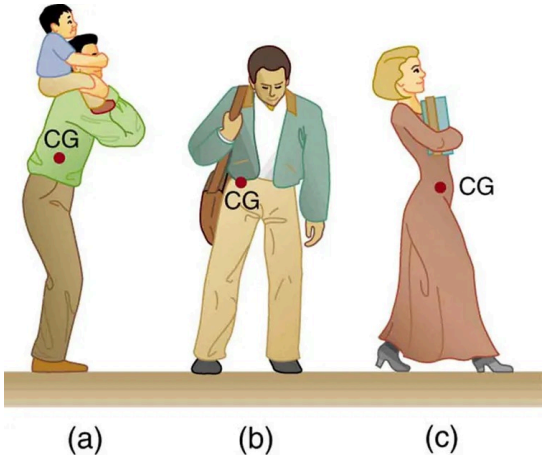


Figure 7.27 People adjust their stance to maintain balance. (a) A father carrying his son piggyback leans forward to position their overall cg above the base of support at his feet. (b) A student carrying a shoulder bag leans to the side to keep the overall cg over their feet. (c) Another student carrying a load of books in her arms leans backward for the same reason. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three cartoon-style illustrations of people, each labeled with "CG," representing the center of gravity.

1. Left (a): A person holds a child on their shoulders. The "CG" is positioned near the midsection of the person, slightly forward, indicating the combined center of gravity with the added weight of the child.

2. Center (b): A person is slouched forward, carrying a shoulder bag. The "CG" is shown slightly forward and lower on the body, suggesting the shift due to the posture and weight of the bag.

3. Right (c): A person stands upright, holding books against their chest. The "CG" is located near the waist area, balanced, reflecting the upright posture and central weight distribution.

Example 7.5

Do Not Lift with Your Back

Consider the person lifting a heavy box with his back, shown in Figure 7.28. (a) Calculate the magnitude of the force F_{B}– in the back muscles that is needed to support the upper body plus the box and compare this with his weight. The mass of the upper body is 55.0 kg and the mass of the box is 30.0 kg. (b) Calculate the magnitude and direction of the force $\mathbf{F}_{\mathrm{V}}$– exerted by the vertebrae on the spine at the indicated pivot point. Again, data in the figure may be taken to be accurate to three significant figures.

Strategy

By now, we sense that the second condition for equilibrium is a good place to start, and inspection of the known values confirms that it can be used to solve for F_{B}– if the pivot is chosen to be at the hips. The torques created by $\mathbf{w}_{\mathrm{ub}}$ and $\mathbf{w}_{\mathrm{box}}$– are clockwise, while that created by $\mathbf{F}_{\mathrm{B}}$– is counterclockwise.

Solution for (a)

Using the perpendicular lever arms given in the figure, the second condition for equilibrium $(\mathrm{net}\ \tau = 0)$ becomes

$$(0.350\ \mathrm{m})\,(55.0\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2}) + (0.500\ \mathrm{m})\,(30.0\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2}) = (0.0800\ \mathrm{m})F_{\mathrm{B}}.$$

7.40

Solving for F_{B} yields

$$F_{\mathrm{B}} = 4.20 \times 10^3 \mathrm{N}.$$

7.41

The ratio of the force the back muscles exert to the weight of the upper body plus its load is

$$\frac{F_{\mathrm{B}}}{w_{\mathrm{ub}} + w_{\mathrm{box}}} = \frac{4200\ \mathrm{N}}{833\ \mathrm{N}} = 5.04.$$

7.42

This force is considerably larger than it would be if the load were not present.

Solution for (b)

More important in terms of its damage potential is the force on the vertebrae $\mathbf{F}_{\mathrm{V}}$. The first condition for equilibrium ($\mathrm{net}\mathbf{F} = 0$) can be used to find its magnitude and direction. Using y for vertical and x for horizontal, the condition for the net external forces along those axes to be zero

$$\mathrm{net}F_y = 0 \mathrm{and} \mathrm{net}F_x = 0.$$

7.43

Starting with the vertical (y) components, this yields

$$F_{\mathrm{V}y} - w_{\mathrm{ub}} - w_{\mathrm{box}} - F_{\mathrm{B}} \sin\ 29.0^{\circ}\ = 0.$$

7.44

Thus,

$$\begin{aligned} F_{\mathrm{V}y} &= w_{\mathrm{ub}} + w_{\mathrm{box}} + F_{\mathrm{B}} \sin\ 29.0^{\circ} \\ &= \ 833\ \mathrm{N} + (4200\ \mathrm{N}) \sin\ 29.0^{\circ} \end{aligned}$$

7.45

yielding

$$F_{\mathrm{V}y} = 2.87 \times 10^{3} \mathrm{N}.$$

7.46

Similarly, for the horizontal (x) components,

$$F_{\mathrm{V}x} - F_{\mathrm{B}} \cos\ 29.0^{\circ}\ = 0$$

7.47

yielding

$$F_{\mathrm{V}x} = 3.67 \times 10^3 \text{ N}.$$

7.48

The magnitude of $\mathbf{F}_{\mathrm{V}}$ is given by the Pythagorean theorem:

$$F_{\mathrm{V}} = \sqrt{F_{\mathrm{V}x}^2 + F_{\mathrm{V}y}^2} = 4.66 \times 10^3 \text{N}.$$

7.49

The direction of $\mathbf{F}_{\mathrm{V}}$ is

$$\theta = \tan^{-1}\left(\frac{F_{\mathrm{V}y}}{F_{\mathrm{V}x}}\right) = 38.0^{\circ}\ .$$

7.50

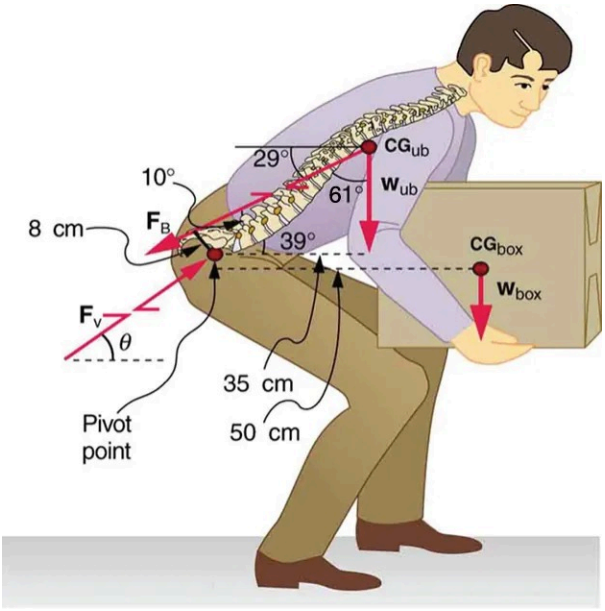
Note that the ratio of F_{V} to the weight supported is

$$\frac{F_{\mathrm{V}}}{w_{\mathrm{ub}} + w_{\mathrm{box}}} = \frac{4660 \text{ N}}{833 \text{ N}} = 5.59.$$

7.51

Discussion

This force is about 5.6 times greater than it would be if the person were standing erect. The trouble with the back is not so much that the forces are large—because similar forces are created in our hips, knees, and ankles—but that our spines are relatively weak. Proper lifting, performed with the back erect and using the legs to raise the body and load, creates much smaller forces in the back—in this case, about 5.6 times smaller.



Example 7.5

Figure 7.28 This figure shows that large forces are exerted by the back muscles and experienced in the vertebrae when a person lifts with their back, since these muscles have small effective perpendicular lever arms. The data shown here are analyzed in the preceding example, . Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a diagram of a person bending over to lift a box. Several measurements, force vectors, and angles are labeled to show the biomechanics involved in lifting.

- The person is shown with a transparent view of their spine, bending at an angle forward to pick up the box.
- The CG_{ub} is labeled at the upper back with an arrow w_{ub} pointing downward, indicating the center of gravity and weight of the upper body.
- The arms are extending forward, grasping a box. A label CG_{box} denotes the center of gravity of the box, with an arrow w_{box} pointing downward.
- An arrow labeled F_B points to the spine at the 10-degree angle.
- Another force vector F_V is directed at an angle θ on the lower back, intersecting at a pivot point.
- The distances from the pivot point to the CG_{ub} and CG_{box} are indicated as 35 cm and 50 cm, respectively.
- The spine is annotated with angles: 29 degrees at the lower back, 61 degrees at the upper back, and 39 degrees near the box.
- A measurement of 8 cm is noted near the forces acting at the hip area.

This diagram is likely meant to explain the mechanics and forces involved when lifting an object to highlight proper lifting techniques or illustrate biomechanical principles.

What are the benefits of having most skeletal muscles attached so close to joints? One advantage is speed because small muscle contractions can produce large movements of limbs in a short period of time. Other advantages are flexibility and agility, made possible by the large numbers of joints and the ranges over which they function. For example, it is difficult to imagine a system with biceps muscles attached at the wrist that would be capable of the broad range of movement we vertebrates possess.

There are some interesting complexities in real systems of muscles, bones, and joints. For instance, the pivot point in many joints changes location as the joint is flexed, so that the perpendicular lever arms and the mechanical advantage of the system change, too. Thus the force the biceps muscle must exert to hold up a book varies as the forearm is flexed. Similar mechanisms operate in the legs, which explain, for example, why there is less leg strain when a bicycle seat is set at the proper height. The methods employed in this section give a reasonable description of real systems provided enough is known about the dimensions of the system. There are many other interesting examples of force and torque in the body—a few of these are the subject of end-of-chapter problems.

“9.6 Forces and Torques in Muscles and Joints” from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

GLOSSARY

center of gravity
the point where the total weight of the body is assumed to be concentrated

dynamic equilibrium
a state of equilibrium in which the net external force and torque on a system moving with constant velocity are zero

mechanical advantage
the ratio of output to input forces for any simple machine

neutral equilibrium
a state of equilibrium that is independent of a system's displacements from its original position

perpendicular lever arm
the shortest distance from the pivot point to the line along which $\mathbf{F}$ lies

SI units of torque
newton times meters, usually written as N·m

stable equilibrium
a system, when displaced, experiences a net force or torque in a direction opposite to the direction of the displacement

static equilibrium
a state of equilibrium in which the net external force and torque acting on a system is zero

static equilibrium
equilibrium in which the acceleration of the system is zero and accelerated rotation does not occur

torque
turning or twisting effectiveness of a force

unstable equilibrium
a system, when displaced, experiences a net force or torque in the same direction as the displacement from equilibrium

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CHAPTER SUMMARY

7.1 The First Condition for Equilibrium

- Statics is the study of forces in equilibrium.
- Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration.
- The first condition necessary to achieve equilibrium is that the net external force on the system must be zero, so that $\mathrm{net}\mathbf{F} = 0$.

7.2 The Second Condition for Equilibrium

- The second condition assures those torques are also balanced. Torque is the rotational equivalent of a force in producing a rotation and is defined to be $\tau = \mathrm{rF}\sin\theta$ where τ is torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between $\mathbf{F}$ and the vector directed from the point where the force acts to the pivot point. The perpendicular lever arm $r_\perp$ is defined to be $r_\perp = r\sin\theta$ so that $\tau = r_\perp F$.
- The perpendicular lever arm $r_\perp$ is the shortest distance from the pivot point to the line along which F acts. The SI unit for torque is newton-meter $(\mathrm{N}\cdot\mathrm{m})$. The second condition necessary to achieve equilibrium is that the net external torque on a system must be zero:$\mathrm{net}\ \tau = 0$ By convention, counterclockwise torques are positive, and clockwise torques are negative.

7.3 Stability

- A system is said to be in stable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite the direction of the displacement.
- A system is in unstable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in the same direction as the displacement from equilibrium.
- A system is in neutral equilibrium if its equilibrium is independent of displacements from its original position.

7.4 Applications of Statics, Including Problem-Solving Strategies

- Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We have discussed the problem-solving strategies specifically useful for statics. Statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in Problem-Solving Strategies, still apply.

7.5 Simple Machines

- Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we have to apply the force.
- The ratio of output to input forces for any simple machine is called its mechanical advantage
- A few simple machines are the lever, nail puller, wheelbarrow, crank, etc.

7.6 Forces and Torques in Muscles and Joints

- Statics plays an important part in understanding everyday strains in our muscles and bones.
- Many lever systems in the body have a mechanical advantage of significantly less than one, as many of our muscles are attached close to joints.
- Someone with good posture stands or sits in such a way that the person's center of gravity lies directly above the pivot point in the hips, thereby avoiding back strain and damage to disks.

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CONCEPTUAL QUESTIONS

Conceptual Questions

7.1 The First Condition for Equilibrium

1. What can you say about the velocity of a moving body that is in dynamic equilibrium? Draw a sketch of such a body using clearly labeled arrows to represent all external forces on the body.
2. Under what conditions can a rotating body be in equilibrium? Give an example.

7.2 The Second Condition for Equilibrium

3. What three factors affect the torque created by a force relative to a specific pivot point?
4. A wrecking ball is being used to knock down a building. One tall unsupported concrete wall remains standing. If the wrecking ball hits the wall near the top, is the wall more likely to fall over by rotating at its base or by falling straight down? Explain your answer. How is it most likely to fall if it is struck with the same force at its base? Note that this depends on how firmly the wall is attached at its base.
5. Mechanics sometimes put a length of pipe over the handle of a wrench when trying to remove a very tight bolt. How does this help? (It is also hazardous since it can break the bolt.)

7.3 Stability

6. A round pencil lying on its side as in Figure 7.12 is in neutral equilibrium relative to displacements perpendicular to its length. What is its stability relative to displacements parallel to its length?
7. Explain the need for tall towers on a suspension bridge to ensure stable equilibrium.

7.4 Applications of Statics, Including Problem-Solving Strategies

8. When visiting some countries, you may see a person balancing a load on the head. Explain why the center of mass of the load needs to be directly above the person's neck vertebrae.

7.5 Simple Machines

9. Scissors are like a double-lever system. Which of the simple machines in Figure 7.21 and Figure 7.22 is most analogous to scissors?
10. Suppose you pull a nail at a constant rate using a nail puller as shown in Figure 7.21. Is the nail puller in equilibrium? What if you pull the nail with some acceleration – is the nail puller in equilibrium then? In which case is the force applied to the nail puller larger and why?
11. Explain the mechanical advantage of a wheelbarrow and how it explains why moving a heavy load is easier with a wheelbarrow.
12. How does a pulley enable a person to lift a load as heavy as a piano with little effort?

7.6 Forces and Torques in Muscles and Joints

13. Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?
14. Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces?
15. Certain types of dinosaurs were bipedal (walked on two legs). What is a good reason that these creatures invariably had long tails if they had long necks?
16. Swimmers and athletes during competition need to go through certain postures at the beginning of the race. Consider the balance of the person and why start-offs are so important for races.

17. If the maximum force the biceps muscle can exert is 1000 N, can we pick up an object that weighs 1000 N? Explain your answer.
18. Suppose the biceps muscle was attached through tendons to the upper arm close to the elbow and the forearm near the wrist. What would be the advantages and disadvantages of this type of construction for the motion of the arm?
19. Explain one of the reasons why pregnant women often suffer from back strain late in their pregnancy.

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PROBLEMS & EXERCISES

Problems & Exercises

7.2 The Second Condition for Equilibrium

1. (a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?
2. When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt. (a) How much torque are you exerting in newton × meters (relative to the center of the bolt)? (b) Convert this torque to footpounds.
3. Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.
4. Use the second condition for equilibrium $(\text{net } \tau = 0)$ to calculate F_{p} in Example 7.1, employing any data given or solved for in part (a) of the example.
5. Repeat the seesaw problem in Example 7.1 with the center of mass of the seesaw 0.160 m to the left of the pivot (on the side of the lighter child) and assuming a mass of 12.0 kg for the seesaw. The other data given in the example remain unchanged. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium.

7.3 Stability

6. Suppose a horse leans against a wall as in Figure 7.29, emulating a scene in the comedy movie *Cat Ballou*. Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal in magnitude and

opposite in direction to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.

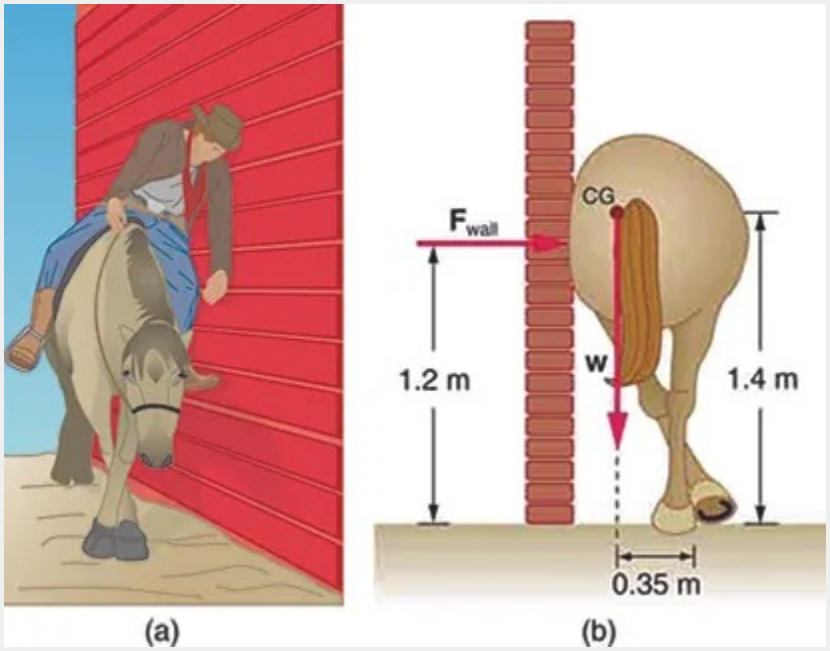


Figure 7.29 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a composite of two illustrations labeled as (a) and (b).

Illustration (a) depicts a person riding a horse near a tall vertical red wall. The person is leaning to the left, away from the horse, with one hand resting on the wall for balance.

Illustration (b) is a side view diagram of a horse against a wall. It includes measurements and forces labeled on the diagram. The horse's back is against a brick wall, and the tail is shown in detail. The vertical distance from the ground to the top of the horse is marked as 1.2 meters, indicating the height of the wall. Another line indicates the height of the horse's center of gravity (CG) at 1.4 meters. A force labeled "F_{wall}" is acting horizontally on the horse at the height of the wall. A downward force, "w," indicates the weight of the horse, acting from the center of gravity. The horizontal distance from the CG to the horse's hind leg is labeled as 0.35 meters.

7. Two children of mass 20.0 kg and 30.0 kg sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of 3.00 m, at what distance from the pivot point is the small child sitting in order to maintain the balance?
8. (a) Calculate the magnitude and direction of the force on each foot of the horse in Figure 7.29 (two are on the ground), assuming the center of mass of the horse is midway between the feet. The total mass of the horse and rider is 500kg. (b) What is the minimum coefficient of friction between the hooves and ground? Note that the force exerted by the wall is horizontal.
9. A person carries a plank of wood 2.00 m long with one hand pushing down on it at one end with a force $\mathbf{F}_1$ and the other hand holding it up at .500 m from the end of the plank with force $\mathbf{F}_2$. If the plank has a mass of 20.0 kg and its center of gravity is at the middle of the plank, what are the magnitudes of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$?
10. A 17.0-m-high and 11.0-m-long wall under construction and its bracing are shown in Figure 7.30. The wall is in stable equilibrium without the bracing but can pivot at its base. Calculate the force exerted by each of the 10 braces if a strong wind exerts a horizontal force of 650 N on each square meter of the wall. Assume that the net force from the wind acts at a height halfway up the wall and that all braces exert equal forces parallel to their lengths. Neglect the thickness of the wall.

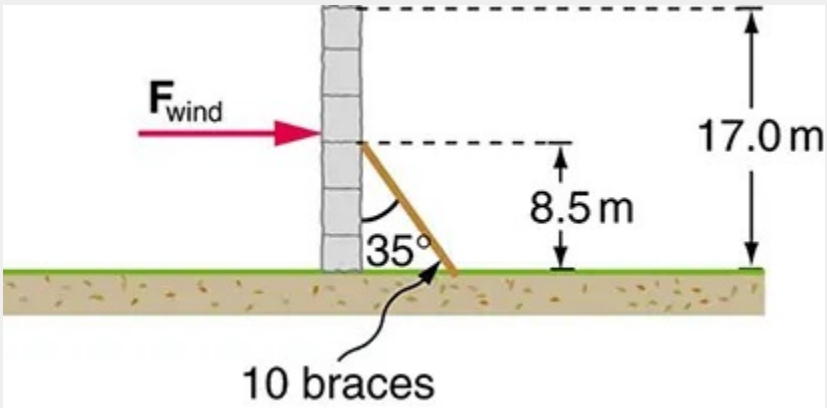


Figure 7.30 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the stabilization of a vertical wall structure against wind force. The wall is shown on the left side with a force labeled "F_{wind}" represented by an arrow pointing from left to right, indicating the direction of the wind force. A diagonal brace is positioned against the wall at an angle of 35 degrees. This brace runs from the base of the wall to the ground. The total height of the wall is marked as 17.0 meters. The point where the brace makes contact with the wall is labeled, showing it is 8.5 meters above the ground. The ground is depicted as a horizontal green line with the label "10 braces" beneath it, indicating the number of braces used.

11. (a) What force must be exerted by the wind to support a 2.50-kg chicken in the position shown in Figure 9.31? (b) What is the ratio of this force to the chicken's weight? (c) Does this support the contention that the chicken has a relatively stable construction?

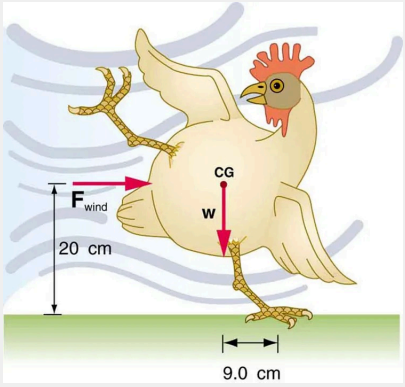


Figure 7.31 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the physics of a chicken experiencing forces while standing on one leg. The chicken is depicted with one leg raised and the other firmly on the ground. The diagram includes several labeled elements:

- **F_{wind}**: A red horizontal arrow pointing to the left, representing the force of the wind acting on the chicken. The arrow is labeled with "F_{wind}" and is positioned at a height of 20 centimeters above the ground.
- **W**: A red vertical arrow pointing downward from the chicken's body, labeled "W"

to represent the weight of the chicken.

- **CG**: A point labeled "CG" (Center of Gravity) indicating the chicken's center of mass located slightly above the leg that is on the ground.
- Measurements: The distance from the ground to the position of the "F_{wind}" arrow is noted as 20 centimeters. The horizontal distance from the chicken's standing leg to a point directly below the center of gravity is labeled as 9.0 centimeters.

The background displays dynamic, wavy lines symbolizing the wind. The chicken is oriented to the right, with its wings spread slightly for balance.

12. Suppose the weight of the drawbridge in Figure 7.32 is supported entirely by its hinges and the opposite shore, so that its cables are slack. The mass of the bridge is 2500 kg. (a) What fraction of the weight is supported by the opposite shore if the point of support is directly beneath the cable attachments? (b) What is the direction and magnitude of the force the hinges exert on the bridge under these circumstances?

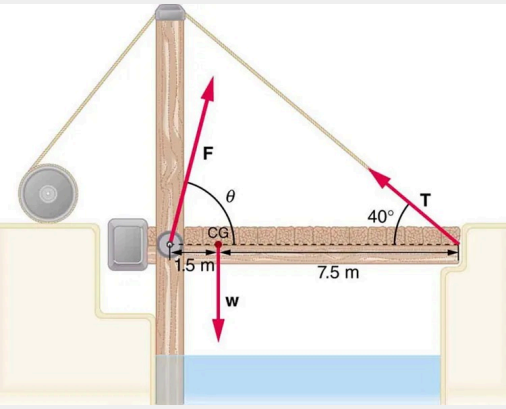


Figure 7.32 A small drawbridge, showing the forces on the hinges ($\mathbf{F}$), its weight ($\mathbf{w}$), and the tension in its wires Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a mechanical setup involving a bridge-like structure, a vertical beam, a pulley system, and vectors representing forces and angles. Here's a detailed description:

- A vertical beam is situated close to the edge of a water body.
- An extended horizontal platform or bridge is attached to the beam and stretches across the water, measuring 7.5 meters in length from the base of the beam.
- The center of gravity (CG) of the bridge is depicted, situated 1.5 meters from the beam.
- Three main force vectors are illustrated:
    - Vector w is pointing downward from the center of gravity, representing the weight.
    - Vector T originates from the endpoint of the bridge and is angled upwards at 40 degrees from the horizontal, representing tension.
    - Vector F is positioned at the base of the beam and angled upward, indicating a force applied at an angle θ.
- A pulley is mounted at the top of the beam, connecting to T via a rope.

13. Suppose a 900-kg car is on the bridge in Figure 7.32 with its center of mass halfway between the hinges and the cable attachments. (The bridge is supported by the cables and hinges only.) (a) Find the force in the cables. (b) Find the direction and magnitude of the force exerted by the hinges on the bridge.
14. A sandwich board advertising sign is constructed as shown in Figure 7.33. The sign's mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewalk. (b) What force is exerted by each side on the hinge?

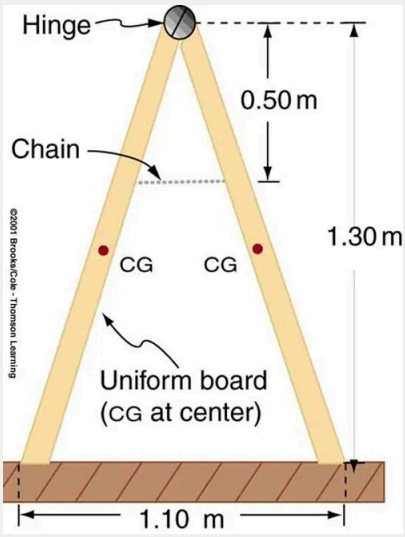


Figure 7.33 A sandwich board advertising sign demonstrates tension. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of a structure resembling an inverted V or an A-frame, supported by a horizontal base. The following components and measurements are noted:

- Two symmetrical boards are shown, joined at the top by a circular hinge.
- A chain is connected near the top of the left board, providing stability.
- The distance from the hinge to the horizontal base is marked as 1.30 meters.
- The distance from the hinge to the chain is marked as 0.50 meters.
- The base of the structure, where the boards meet the horizontal element, measures 1.10 meters wide.
- Each board has a point marked “CG” (center of gravity) indicating that the boards are uniform in weight distribution.
- The label “Uniform board (CG at center)” accompanies the boards, indicating uniformity in design.

15. (a) What minimum coefficient of friction is needed between the legs and the ground to keep the sign in Figure 7.33 in the position shown if the chain breaks? (b) What force is exerted by each side on the hinge?

16. An athlete is attempting to perform splits. From the information given in Figure 7.34, calculate the magnitude and direction of the force exerted on each foot by the floor.

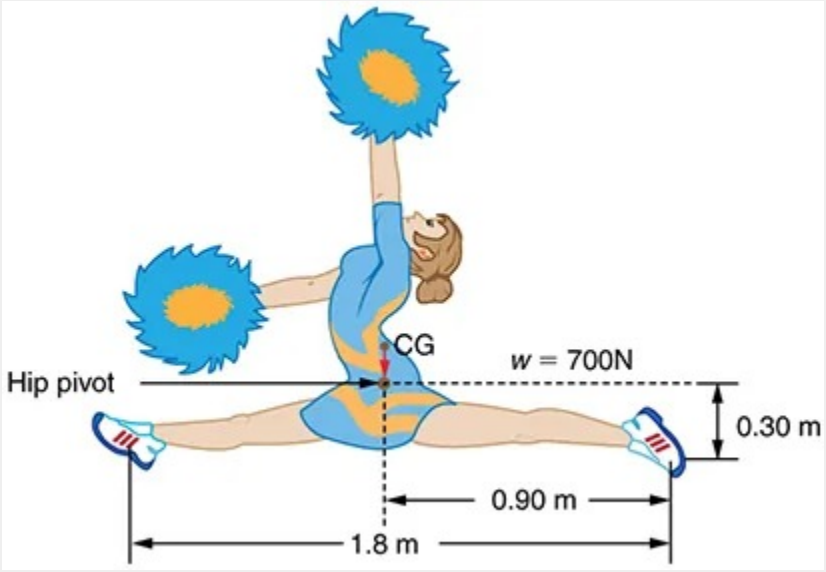


Figure 7.34 An athlete performs full split. The center of gravity and the various distances from it are shown. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of a cheerleader performing a split leap. The cheerleader is shown in mid-air with both legs extended horizontally, each adorned with a sneaker. She holds large pom-poms in each hand, raised above her head. The body is slightly arched back in a dynamic pose.

Measurements and labels in the diagram indicate specific dimensions and forces:

- The distance from the left foot to the hip pivot is *1.8 meters*.
- The distance from the hip pivot to the right foot is *0.9 meters*.
- The vertical distance from the hip pivot to the ground level is *0.3 meters*.
- The label 'CG' indicates the center of gravity near the cheerleader's waist.
- The symbol 'w = 700N' represents the weight, indicating a force of *700 Newtons*.

7.4 Applications of Statics, Including Problem-Solving Strategies

17. To get up on the roof, a person (mass 70.0 kg) places a 6.00-m aluminum ladder (mass 10.0 kg) against the house on a concrete pad with the base of the ladder 2.00 m from the house. The ladder rests against a plastic rain gutter, which we can assume to be frictionless. The center of mass of the ladder is 2 m from the bottom. The person is standing 3 m from the bottom. What are the magnitudes of the forces on the ladder at the top and bottom?
18. In Figure 7.20, the cg of the pole held by the pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by (a) his right hand and (b) his left hand. (c) If each hand supports half the weight of the pole in Figure 7.18, show that the second condition for equilibrium $(\text{net}\tau = 0)$ is satisfied for a pivot other than the one located at the center of gravity of the pole. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium described above.

7.5 Simple Machines

19. What is the mechanical advantage of a nail puller—similar to the one shown in Figure 7.21 —where you exert a force $45\ \text{cm}$ from the pivot and the nail is $1.8\ \text{cm}$ on the other side? What minimum force must you exert to apply a force of $1250\ \text{N}$ to the nail?
20. Suppose you needed to raise a 250-kg mower a distance of 6.0 cm above the ground to change a tire. If you had a 2.0-m long lever, where would you place the fulcrum if your force was limited to 300 N?
21. a) What is the mechanical advantage of a wheelbarrow, such as the one in Figure 7.22, if the center of gravity of the wheelbarrow and its load has a perpendicular lever arm of 5.50 cm, while the hands have a perpendicular lever arm of 1.02 m? (b) What upward force should you exert to support the wheelbarrow and its load if their combined mass is 55.0 kg? (c) What force does the wheel exert on the ground?

22. A typical car has an axle with $1.10\ \text{cm}$ radius driving a tire with a radius of $27.5\ \text{cm}$. What is its mechanical advantage assuming the very simplified model in Figure 7.23(b)?
23. What force does the nail puller in Exercise 7.19 exert on the supporting surface? The nail puller has a mass of 2.10 kg.
24. If you used an ideal pulley of the type shown in Figure 7.24(a) to support a car engine of mass $115\ \text{kg}$, (a) What would be the tension in the rope? (b) What force must the ceiling supply, assuming you pull straight down on the rope? Neglect the pulley system's mass.
25. Repeat Exercise 7.24 for the pulley shown in Figure 7.24(c), assuming you pull straight up on the rope. The pulley system's mass is $7.00\ \text{kg}$.

7.6 Forces and Torques in Muscles and Joints

26. Verify that the force in the elbow joint in Example 7.4 is 407 N, as stated in the text.
27. Two muscles in the back of the leg pull on the Achilles tendon as shown in Figure 7.35. What total force do they exert?

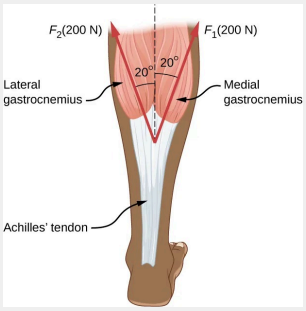


Figure 7.35 The Achilles tendon of the posterior leg serves to attach plantaris, gastrocnemius, and soleus muscles to calcaneus bone. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the anatomy of the lower leg, specifically focusing on the muscles and tendons at the back of the leg. It shows:

– Two main muscles labeled as “Lateral gastrocnemius” and “Medial gastrocnemius.” These muscles are represented as rounded, pinkish areas on either side of the leg, positioned to show symmetry.
– Each muscle is pulling with a force of 200 N (Newtons) as indicated by the arrows pointing upward and labeled as “F^2(200 N)” for the lateral side and “F^1(200 N)” for the medial side.
– The arrows form an angle of 20 degrees with a central vertical dashed line that runs down the center of the leg.
– The “Achilles’ tendon” is shown as a white structure below the muscles, attaching them to the heel. It converges from the two muscles creating a V-shaped junction.
– The angles and forces suggest a biomechanical explanation of how the muscles and tendons work together to exert force on the Achilles’ tendon and enable movement, such as walking or running.

28. The upper leg muscle (quadriceps) exerts a force of 1250 N, which is carried by a tendon over the kneecap (the patella) at the angles shown in Figure 7.36. Find the direction and magnitude of the force exerted by the kneecap on the upper leg bone (the femur).

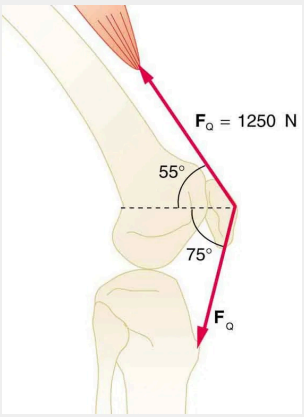


Figure 7.36 The knee joint works like a hinge to bend and straighten the lower leg. It permits a person to sit, stand, and pivot. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of a human knee joint illustrating the forces and angles involved in a biomechanical context. It displays:

– A representation of the femur and tibia, the two bones forming the knee joint, along with muscle fibers above the knee.
– Two angles marked at the joint:
– One angle measuring 55 degrees.
– Another angle measuring 75 degrees.
– A red arrow representing force, labeled as F_Q.
– The magnitude of the force F_Q is given as 1250 Newtons, shown as $F_Q = 1250\,\mathrm{N}$.

The diagram is likely used to study or demonstrate mechanical or muscular forces related to the knee joint.

29. A device for exercising the upper leg muscle is shown in Figure 7.37, together with a schematic representation of an equivalent lever system. Calculate the force exerted by the upper leg muscle to lift the mass at a constant speed. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium in 7.4 Applications of Statics, Including Problem-Solving Strategies.

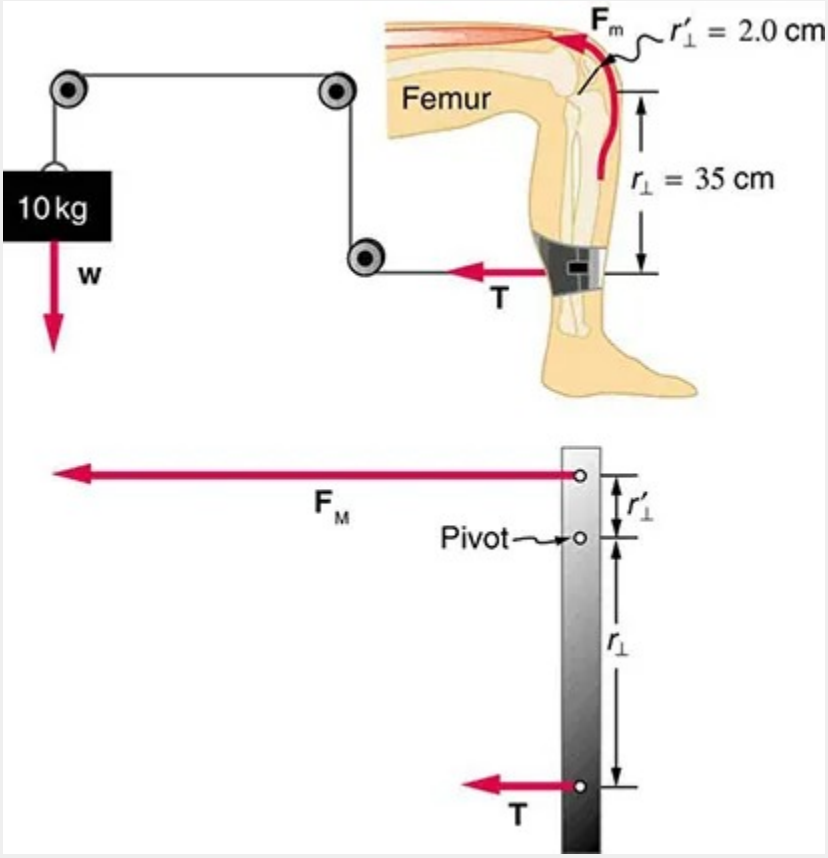


Figure 7.37 A mass is connected by pulleys and wires to the ankle in this exercise device. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams illustrating the mechanics of lifting a weight using a leg, specifically focusing on the forces involved.

Upper Diagram:

- Shows the side view of a human leg bent at the knee.
- A muscle force (F_m) is acting upwards on the femur.
- The femur is labeled, and the muscle connects to it, forming an angle with the leg.
- Two distances are marked: $r'_1 = 2.0\ cm$ from the muscle to the pivot, and $r_1 = 35\ cm$ from the pivot to where the force T acts on the lower leg.
- A weight of 10 kg is being lifted, generating a downward force labeled w.

Lower Diagram:

- Shows a simplified representation of the leg and forces.
- A horizontal bar represents the leg with a pivot point marked.
- The force F_M is shown acting horizontally, while the tension T acts on the opposite side of the pivot.
- Distances are marked similar to the upper diagram: r'_1 and r_1.

The diagrams illustrate the principles of levers and moments, demonstrating how muscle forces are used to lift weights through the mechanics of the leg. The forces and distances are critical in understanding the torque generated in this system.

30. A person working at a drafting board may hold her head as shown in Figure 7.38, requiring muscle action to support the head. The three major acting forces are shown. Calculate the direction and magnitude of the force supplied by the upper vertebrae $\mathbf{F}_V$ to hold the head stationary, assuming that this force acts along a line through the center of mass as do the weight and muscle force.

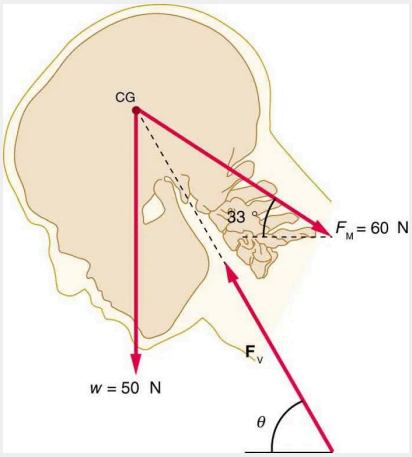


Figure 7.38 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image is a diagram of a human head in profile, illustrating forces acting on it. The head is shown in a beige outline with some internal anatomy visible. Three red arrows represent different forces.

- The first arrow, labeled "CG," points vertically downwards, representing the center of gravity with a force of *w = 50 N*.
- The second arrow, labeled "F_M," points diagonally from the back of the head forward, indicating a force of $F_M = 60$ *N*.
- The third arrow, labeled "F_V," points diagonally downward and forward. The diagram includes a dashed line connecting these forces, forming an angle inside the head.

An angle of 33 degrees is marked between the horizontal dashed line and the direction of the force F_M. Another angle, labeled as theta (θ), is marked below the force F_V arrow.

31. We analyzed the biceps muscle example with the angle between forearm and upper arm set at 90° . Using the same numbers as in Example 7.4, find the force exerted by the biceps muscle when the angle is 120° and the forearm is in a downward position.
32. Even when the head is held erect, as in Figure 7.39, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?

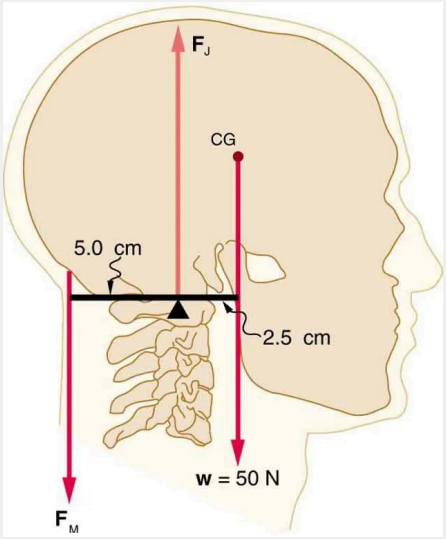
33. 

Figure 7.39 The center of mass of the head lies in front of its major point of support, requiring muscle action to hold the head erect. A simplified lever system is shown. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a sagittal section of a human head focusing on the biomechanics, specifically the forces acting on the skull and neck. It illustrates forces, weights, and measurements with arrows and annotations.

- The head is shown in profile, displaying the skull, neck vertebrae, and associated measurements.
- Two vertical arrows, labeled F_M and F_J, indicate forces acting on the head and neck.
- The arrow F_M points upwards at the back, representing muscle force.
- The arrow F_J points downwards at the front, representing joint force.
- Another arrow pointing downwards is labeled with the weight $w = 50$ N, showing the gravitational force acting through the center of gravity (CG).
- A horizontal line is used to measure distances: 5.0 cm from F_M to the fulcrum (pivot point), and 2.5 cm from the fulcrum to the CG point.
- The vertebrae in the neck are shown with anatomical accuracy to depict where forces are applied.

33. A 75-kg man stands on his toes by exerting an upward force through the Achilles tendon, as in Figure 9.40. (a) What is the force in the Achilles tendon if he stands on one foot? (b) Calculate the force at the pivot of the simplified lever system shown—that force is representative of forces in the ankle joint.

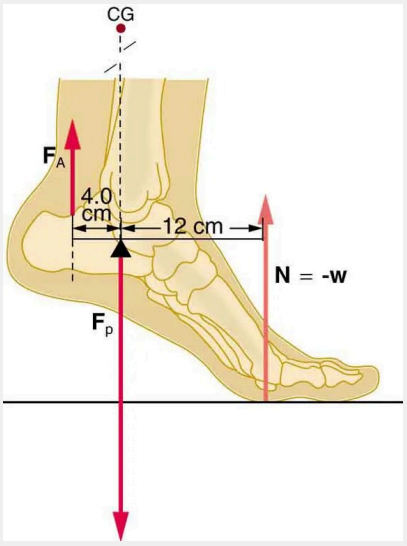


Figure 7.40 The muscles in the back of the leg pull the Achilles tendon when one stands on one's toes. A simplified lever system is shown. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image illustrates a biomechanical diagram of a human foot. The foot is shown in a side view with bones visible. Several forces and measurements are indicated:

- Force labeled as F_A is directed upwards from the heel.
- Force labeled as F_P points downward through the point identified as the center of gravity (CG) above the heel.
- There is an indication of a reaction force labeled N and equal to $-w$ (weight), directed upwards near the toes.

Measurements are provided on the foot:

- Distance of 4.0 cm from the heel to where the vertical force CG is applied.

- Distance of 12 cm from the heel to the point where the opposing force *N* is applied.

Arrows and labels provide visual indicators of the direction and location of the forces acting on the foot.

34. A father lifts his child as shown in Figure 7.41. What force should the upper leg muscle exert to lift the child at a constant speed?

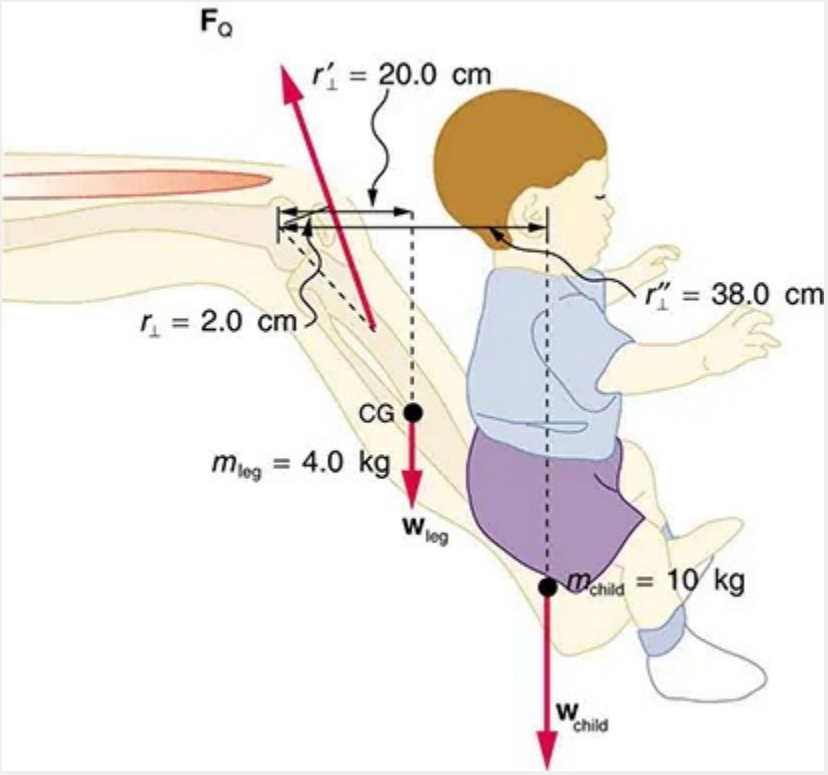


Figure 7.41 A child being lifted by a father's lower leg. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the forces and measurements involved when a person holds a child on their leg. The diagram depicts a side view of a child's body seated on an adult's leg. Various forces and measurements are labeled throughout the diagram:

- An arrow labeled F_Q points upward from near the hip area.

- The upper leg is shown, along with an arrow for $m_{leg} = 4.0\ kg$ directed downward, representing the weight of the leg.
- Two horizontal distances from the hip joint are marked: $r_{\perp} = 2.0\ cm$ and $r'_1 = 20.0\ cm$, both with arrows indicating measurement directions.
- The child is sitting at the edge of the adult's knee, with an arrow labeled w_{child}, pointing downward, representing the child's weight.
- The child's weight is given as $w_{child} = 10\ kg$.
- A vertical measurement from the hip to the child's center of mass indicates $r'_1 = 38.0\ cm$.
- The child's torso is illustrated, with lines marking w_{leg} and w_{child} at the locations corresponding to their respective centers of gravity (CG).

This diagram illustrates the physics of balancing a child on an adult's knee, including vectors and measurements of force and mass positioning. It serves as a visual aid for understanding torques and forces in a practical scenario.

35. Unlike most of the other muscles in our bodies, the masseter muscle in the jaw, as illustrated in Figure 7.42, is attached relatively far from the joint, enabling large forces to be exerted by the back teeth. (a) Using the information in the figure, calculate the force exerted by the lower teeth on the bullet. (b) Calculate the force on the joint.

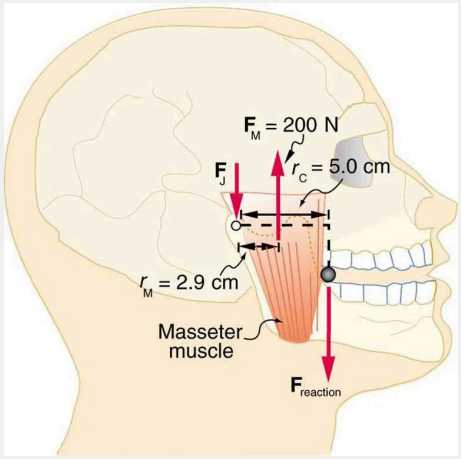


Figure 7.42 A person clenching a bullet between his teeth. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of a human skull in profile, focusing on the jaw and surrounding muscles. It highlights the forces acting on the jaw with specific labels and arrows.

- The *Masseter muscle* is depicted in red, showing its position on the side of the jaw.
- Two radii are labeled: r_M with a length of 2.9 cm, and r_C with a length of 5.0 cm.
- A force F_M is marked as 200 N and shows an upward arrow from the bottom of the masseter muscle.
- A force F_J is indicated with a downward arrow near the top of the jaw.
- A downward force labeled $F_{Reaction}$ is shown at the point where the jaw connects.

The diagram is meant to illustrate the biomechanical forces involved in the jaw's movement, particularly focusing on the masseter muscle's contribution to closing the jaw.

36. **Integrated Concepts:** Suppose we replace the 4.0-kg book in Exercise 7.31 of the biceps muscle with an elastic exercise rope that obeys Hooke's Law. Assume its force constant $k = 600\text{N/m}$. (a) How much is the rope stretched (past equilibrium) to provide the same force F_B as in this example? Assume the rope is

held in the hand at the same location as the book. (b) What force is on the biceps muscle if the exercise rope is pulled straight up so that the forearm makes an angle of 25° with the horizontal? Assume the biceps muscle is still perpendicular to the forearm.

37. (a) What force should the woman in Figure 7.43 exert on the floor with each hand to do a push-up? Assume that she moves up at a constant speed. (b) The triceps muscle at the back of her upper arm has an effective lever arm of 1.75 cm, and she exerts force on the floor at a horizontal distance of 20.0 cm from the elbow joint. Calculate the magnitude of the force in each triceps muscle, and compare it to her weight. (c) How much work does she do if her center of mass rises 0.240 m? (d) What is her useful power output if she does 25 pushups in one minute?

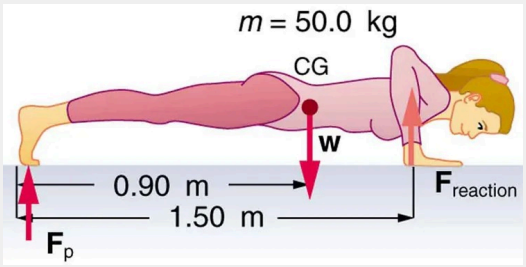


Figure 7.43 A woman doing pushups. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a person in a plank position with a focus on physics concepts related to forces and equilibrium. The person, dressed in a pink outfit, lies horizontally facing downwards, supported by their forearms and toes. Several annotations and symbols are included to illustrate the physics involved:

- The mass of the person is labeled as “m = 50.0 kg”.
- The center of gravity (labeled as “CG”) is marked on the person’s body with a dot and a downward arrow labeled “w”, representing the weight force.
- A horizontal line extends from the feet to the forearms, marked with distances: “0.90 m” from the feet to the center of gravity and “1.50 m” from the center of gravity to the forearms.

- A vertical arrow labeled “Fp” points upwards from the feet, indicating a force acting at the feet.
- Another vertical arrow labeled “Freaction” points upwards from the forearms, representing a reaction force.

38. You have just planted a sturdy 2-m-tall palm tree in your front lawn for your mother’s birthday. Your brother kicks a 500 g ball, which hits the top of the tree at a speed of 5 m/s and stays in contact with it for 10 ms. The ball falls to the ground near the base of the tree and the recoil of the tree is minimal. (a) What is the force on the tree? (b) The length of the sturdy section of the root is only 20 cm. Furthermore, the soil around the roots is loose and we can assume that an effective force is applied at the tip of the 20 cm length. What is the effective force exerted by the end of the tip of the root to keep the tree from toppling? Assume the tree will be uprooted rather than bend. (c) What could you have done to ensure that the tree does not uproot easily?
39. **Unreasonable Results:** Suppose two children are using a uniform seesaw that is 3.00 m long and has its center of mass over the pivot. The first child has a mass of 30.0 kg and sits 1.40 m from the pivot. (a) Calculate where the second 18.0 kg child must sit to balance the seesaw. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?
40. **Construct Your Own Problem:** Consider a method for measuring the mass of a person’s arm in anatomical studies. The subject lies on her back, extends her relaxed arm to the side and two scales are placed below the arm. One is placed under the elbow and the other under the back of her hand. Construct a problem in which you calculate the mass of the arm and find its center of mass based on the scale readings and the distances of the scales from the shoulder joint. You must include a free body diagram of the arm to direct the analysis. Consider changing the position of the scale under the hand to provide more information, if needed. You may wish to consult references to obtain reasonable mass values.

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ROTATIONAL MOTION AND ANGULAR MOMENTUM



Figure 8.1 The mention of a tornado conjures up images of raw destructive power. Tornadoes blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw. They descend from clouds in funnel-like shapes that spin violently, particularly at the bottom where they are most narrow, producing winds as high as 500 km/h. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

8.1 Angular Acceleration
8.2 Kinematics of Rotational Motion
8.3 Dynamics of Rotational Motion: Rotational Inertia
8.4 Rotational Kinetic Energy: Work and Energy Revisited
8.5 Angular Momentum and Its Conservation
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

Why do tornadoes spin at all? And why do tornadoes spin so rapidly? The answer is that air masses that produce tornadoes are themselves rotating, and when the radii of the air masses decrease, their rate of rotation increases. An ice skater increases her spin in an exactly analogous manner as seen in Figure 8.2. The skater starts her rotation with outstretched limbs and increases her spin by pulling them in toward her body. The same physics describes the exhilarating spin of a skater and the wrenching force of a tornado.

Clearly, force, energy, and power are associated with rotational motion. These and other aspects of rotational motion are covered in this chapter. We shall see that all important aspects of rotational motion either have already been defined for linear motion or have exact analogs in linear motion. First, we look at angular acceleration—the rotational analog of linear acceleration.



Figure 8.2 This figure skater increases her rate of spin by pulling her arms and her extended leg closer to her axis of rotation. Image by Luu, Wikimedia Commons License

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8.1 ANGULAR ACCELERATION

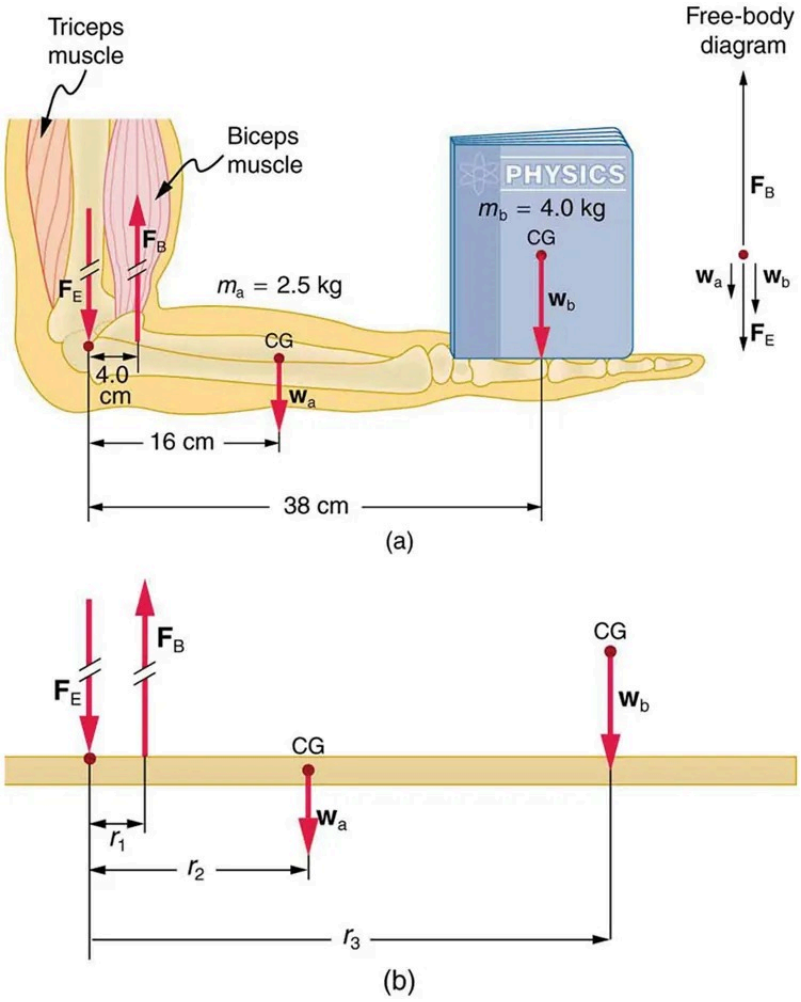
Learning Objectives

By the end of this section, you will be able to:

- Explain the forces exerted by muscles.
- State how a bad posture causes back strain.
- Discuss the benefits of skeletal muscles attached close to joints.
- Discuss various complexities in the real system of muscles, bones, and joints.

Muscles, bones, and joints are some of the most interesting applications of statics. There are some surprises. Muscles, for example, exert far greater forces than we might think. Figure 7.25 shows a forearm holding a book and a schematic diagram of an analogous lever system. The schematic is a good approximation for the forearm, which looks more complicated than it is, and we can get some insight into the way typical muscle systems function by analyzing it.

Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor—that is, it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs apply to the outside world. The reason is clear once we realize that most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen in Figure 7.25.



Example 7.4

Figure 7.25 (a) The figure shows the forearm of a person holding a book. The biceps exert a force $\mathbf{F}_B$ *to support the weight of the forearm and the book. The triceps are assumed to be relaxed. (b) Here, you can view an approximately equivalent mechanical system with the pivot at the elbow joint as seen in . Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two diagrams illustrating the forearm as a lever system, a free-body diagram, and related forces, aimed at explaining the biomechanics of the arm holding an object.

In diagram (a), there is a labeled illustration of a forearm holding a book. Key elements labeled include:

- **Triceps muscle:** Positioned at the back of the arm.
- **Biceps muscle:** Located at the front of the upper arm, exerting force F_B upward.

- **F_E:** The force exerted at the elbow, depicted as a downward arrow.
- **W_a:** The weight of the arm (2.5 kg), with its center of gravity labeled as "CG", placed downward.
- **W_b:** The weight of the book (4.0 kg), with its center of gravity labeled as "CG", also placed downward.
- Distances are labeled: 4.0 cm from the elbow to the biceps force, 16 cm from the elbow to the arm's center of gravity, and 38 cm from the elbow to the book's center of gravity.

The free-body diagram depicts the forces acting on the forearm as a lever:

- **F_B:** Force from the biceps is shown upward.
- **F_E:** Elbow force, shown downward.
- **W_a and W_b:** Weights of the arm and book are shown downward.

In diagram (b), a simplified lever diagram is provided to represent the arm:

- Distances labeled as r_1, r_2, and r_3, corresponding to the forces F_E, W_a, and W_b, respectively.
- **F_E** and **F_B** are depicted with arrows for their respective directions.
- Weight forces W_a and W_b are shown with downward arrows at their respective points on the lever.

Example 7.4

Muscles Exert Bigger Forces Than You Might Think

Calculate the force the biceps muscle must exert to hold the forearm and its load as shown in Figure 7.25, and compare this force with the weight of the forearm plus its load. You may take the data in the figure to be accurate to three significant figures.

Strategy

There are four forces acting on the forearm and its load (the system of interest). The magnitude of the force of the biceps is F_{B}; that of the elbow joint is F_{E}; that of the weights of the forearm is w_{a}, and its load is w_{b}. Two of these are unknown (F_{B} and F_{E}), so that the first condition for equilibrium cannot by itself yield F_{B}. But if we use the second condition and choose the pivot to be at the elbow, then the torque due to F_{E} is zero, and the only unknown becomes F_{B}.

Solution

The torques created by the weights are clockwise relative to the pivot, while the torque created by the biceps is counterclockwise; thus, the second condition for equilibrium $(\mathrm{net}\tau = 0)$ becomes

$$r_2 w_{\mathrm{a}} + r_3 w_{\mathrm{b}} = r_1 F_{\mathrm{B}}.$$

7.35

Note that $\sin\theta = 1$ for all forces, since $\theta = 90^{\circ}$ for all forces. This equation can easily be solved for F_{B} in terms of known quantities, yielding

$$F_{\mathrm{B}} = \frac{r_2 w_{\mathrm{a}} + r_3 w_{\mathrm{b}}}{r_1}.$$

7.36

Entering the known values gives

$$F_{\mathrm{B}} = \frac{(0.160\ \mathrm{m})(2.50\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right) + (0.380\ \mathrm{m})(4.00\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)}{0.0400\ \mathrm{m}}$$

7.37

which yields

$$F_{\mathrm{B}} = 470\ \mathrm{N}.$$

7.38

Now, the combined weight of the arm and its load is $(6.50\ \mathrm{kg})\left(9.80\mathrm{m/s^2}\right) = 63.7\ \mathrm{N}$, so that the ratio of the force exerted by the biceps to the total weight is

$$\frac{F_{\mathrm{B}}}{w_{\mathrm{a}} + w_{\mathrm{b}}} = \frac{470}{63.7} = 7.38.$$

7.39

Discussion

This means that the biceps muscle is exerting a force 7.38 times the weight supported.

In the above example of the biceps muscle, the angle between the forearm and upper arm is 90°. If this angle changes, the force exerted by the biceps muscle also changes. In addition, the length of the biceps muscle changes. The force the biceps muscle can exert depends upon its length; it is smaller when it is shorter than when it is stretched.

Very large forces are also created in the joints. In the previous example, the downward force F_{E} exerted by the humerus at the elbow joint equals 407 N, or 6.38 times the total weight supported. (The calculation of

F_{E} is straightforward and is left as an end-of-chapter problem.) Because muscles can contract, but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions and thus subtract. (In the above example, the upward force of the muscle minus the downward force of the joint equals the weight supported—that is,

$470\ \mathrm{N}–407\ \mathrm{N} = 63\ \mathrm{N}$, approximately equal to the weight supported.) Forces in muscles and joints are largest when their load is a long distance from the joint, as the book is in the previous example.

In racquet sports such as tennis the constant extension of the arm during game play creates large forces in this way. The mass times the lever arm of a tennis racquet is an important factor, and many players use the heaviest racquet they can handle. It is no wonder that joint deterioration and damage to the tendons in the elbow, such as "tennis elbow," can result from repetitive motion, undue torques, and possibly poor racquet selection in such sports. Various tried techniques for holding and using a racquet or bat or stick not only increases sporting prowess but can minimize fatigue and long-term damage to the body. For example, tennis balls correctly hit at the "sweet spot" on the racquet will result in little vibration or impact force being felt in the racquet and the body—less torque. Twisting the hand to provide top spin on the ball or using an extended rigid elbow in a backhand stroke can also aggravate the tendons in the elbow.

Training coaches and physical therapists use the knowledge of relationships between forces and torques in the treatment of muscles and joints. In physical therapy, an exercise routine can apply a particular force and torque which can, over a period of time, revive muscles and joints. Some exercises are designed to be carried out under water, because this requires greater forces to be exerted, further strengthening muscles. However, connecting tissues in the limbs, such as tendons and cartilage as well as joints are sometimes damaged by the large forces they carry. Often, this is due to accidents, but heavily muscled athletes, such as weightlifters, can tear muscles and connecting tissue through effort alone.

The back is considerably more complicated than the arm or leg, with various muscles and joints between vertebrae, all having mechanical advantages less than 1. Back muscles must, therefore, exert very large forces, which are borne by the spinal column. Discs crushed by mere exertion are very common. The jaw is somewhat exceptional—the masseter muscles that close the jaw have a mechanical advantage greater than 1 for the back teeth, allowing us to exert very large forces with them. A cause of stress headaches is persistent clenching of teeth where the sustained large force translates into fatigue in muscles around the skull.

Figure 7.26 shows how bad posture causes back strain. In part (a), we see a person with good posture. Note that her upper body's cg is directly above the pivot point in the hips, which in turn is directly above the base of support at her feet. Because of this, her upper body's weight exerts no torque about the hips. The only force needed is a vertical force at the hips equal to the weight supported. No muscle action is required, since the bones are rigid and transmit this force from the floor. This is a position of unstable equilibrium, but only small forces are needed to bring the upper body back to vertical if it is slightly displaced. Bad posture is shown in part (b); we see that the upper body's cg is in front of the pivot in the hips. This creates a clockwise torque around the hips that is counteracted by muscles in the lower back. These muscles must exert large forces, since they have typically small mechanical advantages. (In other words, the perpendicular lever arm for the muscles is much smaller than for the cg.) Poor posture can also cause muscle strain for people sitting at their desks using computers. Special chairs are available that allow the body's CG to be more easily situated above the seat, to reduce back pain. Prolonged muscle action produces muscle strain. Note that the cg of the entire body is still directly above the base of support in part (b) of Figure 7.26. This is compulsory; otherwise the

person would not be in equilibrium. We lean forward for the same reason when carrying a load on our backs, to the side when carrying a load in one arm, and backward when carrying a load in front of us, as seen in Figure 7.27.

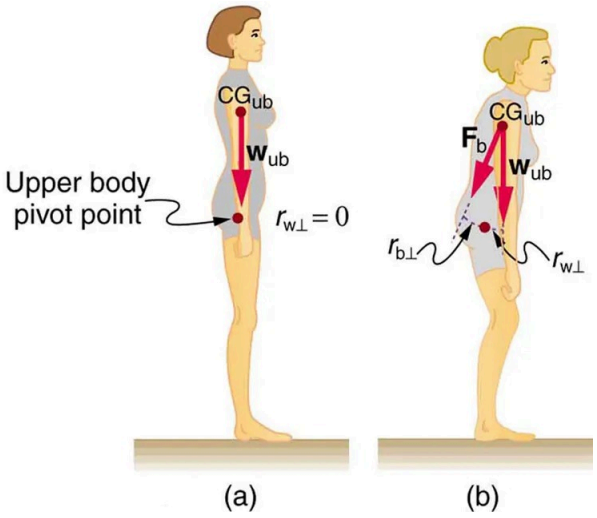


Figure 7.26 (a) Good posture places the upper body's cg over the pivots in the hips, eliminating the need for muscle action to balance the body. (b) Poor posture requires exertion by the back muscles to counteract the clockwise torque produced around the pivot by the upper body's weight. The back muscles have a small effective perpendicular lever arm, $r_{b\perp}$, and must therefore exert a large force $\mathbf{F}_b$. Note that the legs lean backward to keep the cg of the entire body above the base of support in the feet. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b), illustrating the concept of body mechanics.

Diagram (a):
– A person is shown standing upright on a flat surface.
– The person's upper body pivot point is marked.
– A vertical arrow labeled "W_{ub}" (representing the weight of the upper body) points downward from the center of the upper body.
– The center of gravity is indicated by "CG_{ub}".
– The notation "$r_{w\perp} = 0$" signifies that there is no lateral distance from the pivot point.

Diagram (b):
– A person is shown bending forward at the hips.
– The upper body pivot point and the center of gravity "CG_{ub}" are marked.
– Two arrows are present: "W_{ub}" pointing downwards and "F_b" pointing upwards, indicating forces acting on the body.

– The diagram includes notations "$r_{b\perp}$" and "$r_{w\perp}$", representing the perpendicular distances from the pivot point to various forces.

Both diagrams demonstrate the effects of posture on the forces and pivot points acting on the body.

You have probably been warned against lifting objects with your back. This action, even more than bad posture, can cause muscle strain and damage discs and vertebrae, since abnormally large forces are created in the back muscles and spine.

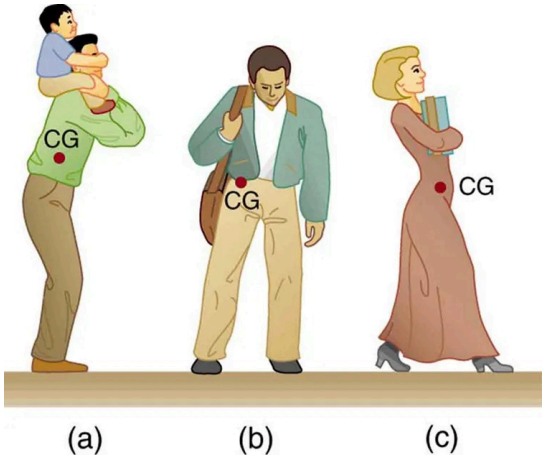


Figure 7.27 People adjust their stance to maintain balance. (a) A father carrying his son piggyback leans forward to position their overall cg above the base of support at his feet. (b) A student carrying a shoulder bag leans to the side to keep the overall cg over their feet. (c) Another student carrying a load of books in her arms leans backward for the same reason. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three cartoon-style illustrations of people, each labeled with "CG," representing the center of gravity.

1. Left (a): A person holds a child on their shoulders. The "CG" is positioned near the midsection of the person, slightly forward, indicating the combined center of gravity with the added weight of the child.

2. Center (b): A person is slouched forward, carrying a shoulder bag. The "CG" is shown slightly forward and lower on the body, suggesting the shift due to the posture and weight of the bag.

3. Right (c): A person stands upright, holding books against their chest. The "CG" is located near the waist area, balanced, reflecting the upright posture and central weight distribution.

Example 7.5

Do Not Lift with Your Back

Consider the person lifting a heavy box with his back, shown in Figure 7.28. (a) Calculate the magnitude of the force F_{B}– in the back muscles that is needed to support the upper body plus the box and compare this with his weight. The mass of the upper body is 55.0 kg and the mass of the box is 30.0 kg. (b) Calculate the magnitude and direction of the force $\mathbf{F}_{\mathrm{V}}$– exerted by the vertebrae on the spine at the indicated pivot point. Again, data in the figure may be taken to be accurate to three significant figures.

Strategy

By now, we sense that the second condition for equilibrium is a good place to start, and inspection of the known values confirms that it can be used to solve for F_{B}– if the pivot is chosen to be at the hips. The torques created by $\mathbf{w}_{\mathrm{ub}}$ and $\mathbf{w}_{\mathrm{box}}$– are clockwise, while that created by $\mathbf{F}_{\mathrm{B}}$– is counterclockwise.

Solution for (a)

Using the perpendicular lever arms given in the figure, the second condition for equilibrium $(\mathrm{net}\ \tau = 0)$ becomes

$$(0.350\ \mathrm{m})\,(55.0\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2}) + (0.500\ \mathrm{m})\,(30.0\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2}) = (0.0800\ \mathrm{m})F_{\mathrm{B}}.$$

7.40

Solving for F_{B} yields

$$F_{\mathrm{B}} = 4.20 \times 10^3 \mathrm{N}.$$

7.41

The ratio of the force the back muscles exert to the weight of the upper body plus its load is

$$\frac{F_{\mathrm{B}}}{w_{\mathrm{ub}} + w_{\mathrm{box}}} = \frac{4200\ \mathrm{N}}{833\ \mathrm{N}} = 5.04.$$

7.42

This force is considerably larger than it would be if the load were not present.

Solution for (b)

More important in terms of its damage potential is the force on the vertebrae $\mathbf{F}_{\mathrm{V}}$. The first condition for equilibrium ($\mathrm{net}\mathbf{F} = 0$) can be used to find its magnitude and direction. Using y for vertical and x for horizontal, the condition for the net external forces along those axes to be zero

$$\mathrm{net}F_y = 0 \mathrm{and} \mathrm{net}F_x = 0.$$

7.43

Starting with the vertical (y) components, this yields

$$F_{\mathrm{V}y} - w_{\mathrm{ub}} - w_{\mathrm{box}} - F_{\mathrm{B}} \sin 29.0^{\circ} = 0.$$

7.44

Thus,

$$\begin{aligned} F_{\mathrm{V}y} &= w_{\mathrm{ub}} + w_{\mathrm{box}} + F_{\mathrm{B}} \sin 29.0^{\circ} \\ &= 833\ \mathrm{N} + (4200\ \mathrm{N}) \sin 29.0^{\circ} \end{aligned}$$

7.45

yielding

$$F_{\mathrm{V}y} = 2.87 \times 10^3 \mathrm{N}.$$

7.46

Similarly, for the horizontal (x) components,

$$F_{\mathrm{V}x} - F_{\mathrm{B}} \cos 29.0^{\circ} = 0$$

7.47

yielding

$$F_{\mathrm{V}x} = 3.67 \times 10^3 \text{ N}.$$

7.48

The magnitude of $\mathbf{F}_{\mathrm{V}}$ is given by the Pythagorean theorem:

$$F_{\mathrm{V}} = \sqrt{F_{\mathrm{V}x}^2 + F_{\mathrm{V}y}^2} = 4.66 \times 10^3 \text{N}.$$

7.49

The direction of $\mathbf{F}_{\mathrm{V}}$ is

$$\theta = \tan^{-1}\left(\frac{F_{\mathrm{V}y}}{F_{\mathrm{V}x}}\right) = 38.0^{\circ}\ .$$

7.50

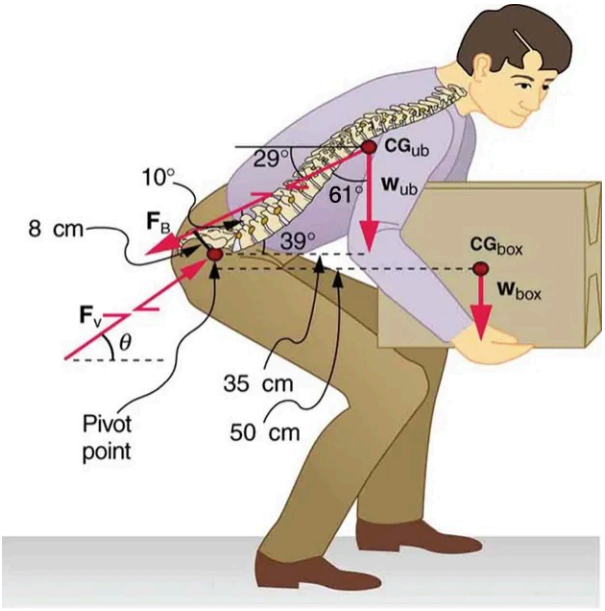
Note that the ratio of F_{V} to the weight supported is

$$\frac{F_{\mathrm{V}}}{w_{\mathrm{ub}} + w_{\mathrm{box}}} = \frac{4660 \text{ N}}{833 \text{ N}} = 5.59.$$

7.51

Discussion

This force is about 5.6 times greater than it would be if the person were standing erect. The trouble with the back is not so much that the forces are large—because similar forces are created in our hips, knees, and ankles—but that our spines are relatively weak. Proper lifting, performed with the back erect and using the legs to raise the body and load, creates much smaller forces in the back—in this case, about 5.6 times smaller.



Example 7.5

Figure 7.28 This figure shows that large forces are exerted by the back muscles and experienced in the vertebrae when a person lifts with their back, since these muscles have small effective perpendicular lever arms. The data shown here are analyzed in the preceding example, . Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a diagram of a person bending over to lift a box. Several measurements, force vectors, and angles are labeled to show the biomechanics involved in lifting.

- The person is shown with a transparent view of their spine, bending at an angle forward to pick up the box.
- The CG_{ub} is labeled at the upper back with an arrow w_{ub} pointing downward, indicating the center of gravity and weight of the upper body.
- The arms are extending forward, grasping a box. A label CG_{box} denotes the center of gravity of the box, with an arrow w_{box} pointing downward.
- An arrow labeled F_B points to the spine at the 10-degree angle.
- Another force vector F_V is directed at an angle θ on the lower back, intersecting at a pivot point.
- The distances from the pivot point to the CG_{ub} and CG_{box} are indicated as 35 cm and 50 cm, respectively.
- The spine is annotated with angles: 29 degrees at the lower back, 61 degrees at the upper back, and 39 degrees near the box.
- A measurement of 8 cm is noted near the forces acting at the hip area.

This diagram is likely meant to explain the mechanics and forces involved when lifting an object to highlight proper lifting techniques or illustrate biomechanical principles.

What are the benefits of having most skeletal muscles attached so close to joints? One advantage is speed because small muscle contractions can produce large movements of limbs in a short period of time. Other advantages are flexibility and agility, made possible by the large numbers of joints and the ranges over which they function. For example, it is difficult to imagine a system with biceps muscles attached at the wrist that would be capable of the broad range of movement we vertebrates possess.

There are some interesting complexities in real systems of muscles, bones, and joints. For instance, the pivot point in many joints changes location as the joint is flexed, so that the perpendicular lever arms and the mechanical advantage of the system change, too. Thus the force the biceps muscle must exert to hold up a book varies as the forearm is flexed. Similar mechanisms operate in the legs, which explain, for example, why there is less leg strain when a bicycle seat is set at the proper height. The methods employed in this section give a reasonable description of real systems provided enough is known about the dimensions of the system. There are many other interesting examples of force and torque in the body—a few of these are the subject of end-of-chapter problems.

“9.6 Forces and Torques in Muscles and Joints” from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

8.2 KINEMATICS OF ROTATIONAL MOTION

Learning Objectives

By the end of this section, you will be able to:

- Observe the kinematics of rotational motion.
- Derive rotational kinematic equations.
- Evaluate problem solving strategies for rotational kinematics.

Just by using our intuition, we can begin to see how rotational quantities like θ, ω, and α are related to one another. For example, if a motorcycle wheel has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotates through many revolutions. In more technical terms, if the wheel's angular acceleration α is large for a long period of time t, then the final angular velocity ω and angle of rotation θ are large. The wheel's rotational motion is exactly analogous to the fact that the motorcycle's large translational acceleration produces a large final velocity, and the distance traveled will also be large.

Kinematics is the description of motion. The kinematics of rotational motion describes the relationships among rotation angle, angular velocity, angular acceleration, and time. Let us start by finding an equation relating ω, α, and t. To determine this equation, we recall a familiar kinematic equation for translational, or straight-line, motion:

$$v = v_0 + \mathrm{at} \quad (\text{constant } a)$$
8.17

Note that in rotational motion $a = a_{\mathrm{t}}$, and we shall use the symbol a for tangential or linear acceleration from now on. As in linear kinematics, we assume a is constant, which means that angular acceleration α is also a constant, because $a = r\alpha$. Now, let us substitute $v = r\omega$ and $a = r\alpha$ into the linear equation above:

$$r\omega = r\omega_0 + r\alpha t.$$
8.18

The radius r cancels in the equation, yielding

$$\omega = \omega_0 + \alpha t \quad \text{(constant } \alpha\text{)},$$
8.19

where ω_0 is the initial angular velocity. This last equation is a *kinematic relationship* among ω, α, and t—that is, it describes their relationship without reference to forces or masses that may affect rotation. It is also precisely analogous in form to its translational counterpart.

Making Connections

Kinematics for rotational motion is completely analogous to translational kinematics, first presented in Kinematics. Kinematics is concerned with the description of motion without regard to force or mass. We will find that translational kinematic quantities, such as displacement, velocity, and acceleration have direct analogs in rotational motion.

Starting with the four kinematic equations we developed in Kinematics, we can derive the following four rotational kinematic equations (presented together with their translational counterparts):

Table 8.1 Rotational Kinematic Equations

Rotational	Translational	
$\theta = \bar{\omega}t$	$x = \bar{v}t$	
$\omega = \omega_0 + \alpha t$	$v = v_0 + \text{at}$	(constant α, a)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}\text{at}^2$	(constant α, a)
$\omega^2 = (\omega_0)^2 + 2\alpha\theta$	$v^2 = (v_0)^2 + 2\text{ax}$	(constant α, a)

In these equations, the subscript 0 denotes initial values (θ_0, x_0, and t_0 are initial values), and the average angular velocity $\bar{\omega}$ and average velocity $\bar{v}$ are defined as follows:

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} \text{ and } \bar{v} = \frac{v_0 + v}{2}.$$

8.20

The equations given above in Table 8.2 can be used to solve any rotational or translational kinematics problem in which a and α are constant.

Problem-Solving Strategy for Rotational Kinematics

1. *Examine the situation to determine that rotational kinematics (rotational motion) is involved.* Rotation must be involved, but without the need to consider forces or masses that affect the motion.
2. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A sketch of the situation is useful.
3. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).*
4. *Solve the appropriate equation or equations for the quantity to be determined (the unknown).* It can be useful to think in terms of a translational analog because by now you are familiar with such motion.
5. *Substitute the known values along with their units into the appropriate equation, and obtain numerical solutions complete with units.* Be sure to use units of radians for angles.
6. *Check your answer to see if it is reasonable: Does your answer make sense*?

Example 8.3

Calculating the Acceleration of a Fishing Reel

A deep-sea fisherman hooks a big fish that swims away from the boat pulling the fishing line from his fishing reel. The whole system is initially at rest and the fishing line unwinds from the reel at a radius of 4.50 cm from its axis of rotation. The reel is given an angular acceleration of $110\text{rad}/\text{s}^2$ for 2.00 s as seen in Figure 8.7.

(a) What is the final angular velocity of the reel?

(b) At what speed is fishing line leaving the reel after 2.00 s elapses?

(c) How many revolutions does the reel make?

(d) How many meters of fishing line come off the reel in this time?

Strategy

In each part of this example, the strategy is the same as it was for solving problems in linear kinematics. In particular, known values are identified and a relationship is then sought that can be used to solve for the unknown.

Solution for (a)

Here α and t are given and ω needs to be determined. The most straightforward equation to use is $\omega = \omega_0 + \alpha t$ because the unknown is already on one side and all other terms are known. That equation states that

$$\omega = \omega_0 + \alpha t.$$

8.21

We are also given that $\omega_0 = 0$ (it starts from rest), so that

$$\omega = 0 + \left(110 \text{ rad/s}^2\right)(2.00 \text{ s}) = 220 \text{rad/s}.$$

8.22

Solution for (b)

Now that ω is known, the speed v can most easily be found using the relationship

$$v = r\omega,$$

8.23

where the radius r of the reel is given to be 4.50 cm; thus,

$$v = (0.0450 \text{ m})(220 \text{ rad/s}) = 9.90 \text{ m/s}.$$

8.24

Note again that radians must always be used in any calculation relating linear and angular quantities. Also, because radians are dimensionless, we have $\text{m} \times \text{rad} = \text{m}$.

Solution for (c)

Here, we are asked to find the number of revolutions. Because $1 \text{ rev} = 2\pi \text{ rad}$, we can find the number of revolutions by finding θ in radians. We are given α and t, and we know ω_0 is zero, so that θ can be obtained using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$.

$$\begin{aligned}\theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ &= 0 + (0.500)\left(110 \text{ rad/s}^2\right)(2.00 \text{ s})^2 \\ &= 220 \text{ rad}.\end{aligned}$$

8.25

Converting radians to revolutions gives

$$\theta = (220 \text{ rad})\frac{1 \text{ rev}}{2\pi \text{ rad}} = 35.0 \text{ rev}.$$

8.26

Solution for (d)

The number of meters of fishing line is x, which can be obtained through its relationship with θ:

$$x = r\theta = (0.0450\ \text{m})\,(220\ \text{rad}) = 9.90\ \text{m}.$$

8.27

Discussion

This example illustrates that relationships among rotational quantities are highly analogous to those among linear quantities. We also see in this example how linear and rotational quantities are connected. The answers to the questions are realistic. After unwinding for two seconds, the reel is found to spin at 220 rad/s, which is 2100 rpm. (No wonder reels sometimes make high-pitched sounds.) The amount of fishing line played out is 9.90 m, about right for when the big fish bites.

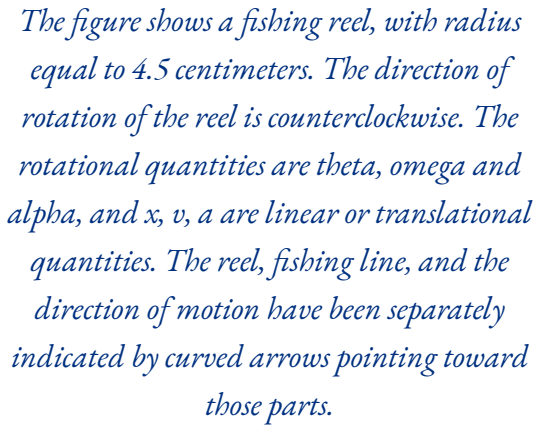


Figure 8.7 Fishing line coming off a rotating reel moves linearly. and consider relationships between rotational and linear quantities associated with a fishing reel. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of a fishing reel depicting various components and measurements. The central part of the reel, which holds the fishing line, is labeled as the “Reel.” A line extends out from it, labeled as

"Fishing line." The left side of the reel shows linear or translational quantities labeled as "x, v, a." On the right side, rotational quantities are displayed as "θ, ω, α." An arrow indicates the "Direction of rotation." The radius of the reel is given as "r = 4.5 cm." Additionally, there are knobs on the right side, commonly used for handling the reel. Each component is pointed out with marker lines and labels for clarity.

Example 8.4

Calculating the Duration When the Fishing Reel Slows Down and Stops

Now let us consider what happens if the fisherman applies a brake to the spinning reel, achieving an angular acceleration of $-300\text{rad}/\text{s}^2$. How long does it take the reel to come to a stop?

Strategy

We are asked to find the time t for the reel to come to a stop. The initial and final conditions are different from those in the previous problem, which involved the same fishing reel. Now we see that the initial angular velocity is $\omega_0 = 220\ \text{rad}/\text{s}$ and the final angular velocity ω is zero. The angular acceleration is given to be $\alpha = -300\text{rad}/\text{s}^2$. Examining the available equations, we see all quantities but t are known in $\omega = \omega_0 + \alpha t$, making it easiest to use this equation.

Solution

The equation states

$$\omega = \omega_0 + \alpha t.$$

8.28

We solve the equation algebraically for t, and then substitute the known values as usual, yielding

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 220\ \text{rad}/\text{s}}{-300\text{rad}/\text{s}^2} = 0.733\ \text{s}.$$

8.29

Discussion

Note that care must be taken with the signs that indicate the directions of various quantities. Also, note that the time to stop the reel is fairly small because the acceleration is rather large. Fishing lines sometimes snap because of the accelerations involved, and fishermen often let the fish swim for a while before applying brakes on the reel. A tired fish will be slower, requiring a smaller acceleration.

Example 8.5

Calculating the Slow Acceleration of Trains and Their Wheels

Large freight trains accelerate very slowly. Suppose one such train accelerates from rest, giving its 0.350-m-radius wheels an angular acceleration of 0.250rad/s^2. After the wheels have made 200 revolutions (assume no slippage): (a) How far has the train moved down the track? (b) What are the final angular velocity of the wheels and the linear velocity of the train?

Strategy

In part (a), we are asked to find x, and in (b) we are asked to find ω and v. We are given the number of revolutions θ, the radius of the wheels r, and the angular acceleration α.

Solution for (a)

The distance x is very easily found from the relationship between distance and rotation angle:

$$\theta = \frac{x}{r}.$$

8.30

Solving this equation for x yields

$$x = r\theta.$$

8.31

Before using this equation, we must convert the number of revolutions into radians, because we are dealing with a relationship between linear and rotational quantities:

$$\theta = (200\text{rev})\frac{2\pi\text{rad}}{1\text{ rev}} = 1257\text{rad}.$$

8.32

Now we can substitute the known values into $x = r\theta$ to find the distance the train moved down the track:

$$x = r\theta = (0.350m)\,(1257\text{ rad}) = 440\text{ m}.$$

8.33

Solution for (b)

We cannot use any equation that incorporates t to find ω, because the equation would have at least two unknown values. The equation $\omega^2 = (\omega_0)^2 + 2\alpha\theta$ will work, because we know the values for all variables except ω:

$$\omega^2 = (\omega_0)^2 + 2\alpha\theta$$

8.34

Taking the square root of this equation and entering the known values gives

$$\begin{aligned}\omega &= \left[0 + 2\,(0.250\text{ rad/s}^2\,)\,(1257\text{ rad}\,)\right]^{1/2}\\ &= 25.1\text{ rad/s}.\end{aligned}$$

8.35

We can find the linear velocity of the train, v, through its relationship to ω:

$$\begin{aligned}v &= r\omega\\ &= (0.350m)\,(25.1\text{ rad/s})\\ &= 8.77\text{ m/s}.\end{aligned}$$

8.36

Discussion

The distance traveled is fairly large and the final velocity is fairly slow (just under 32 km/h).

There is translational motion even for something spinning in place, as the following example illustrates. Figure 8.8 shows a fly on the edge of a rotating microwave oven plate. The example below calculates the total distance it travels.

The figure shows a fly that has landed on the rotating plate of the microwave. The direction of rotation of the plate, omega, is counterclockwise and is shown with an arrow.

Figure 8.8 The image shows a microwave plate. The fly makes revolutions while the food is heated Image *from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

This image is an illustration of the interior of a microwave oven. The microwave door is shown open, revealing the turntable inside. On the turntable, there is a piece of food that appears to be a slice of meat or fish. An arrow labeled "Direction of rotation" is drawn, indicating a clockwise movement of the turntable. Alongside the arrow, there is a notation "ω" (omega), symbolizing angular velocity. There is also a small drawing of a bug on the turntable, near the edge, to illustrate scale or possibly a point of reference.

Example 8.6

Calculating the Distance Traveled by a Fly on the Edge of a Microwave Oven Plate

A person decides to use a microwave oven to reheat some lunch. In the process, a fly accidentally flies into the microwave and lands on the outer edge of the rotating plate and remains there. If the plate has a radius of 0.15 m and rotates at 6.0 rpm, calculate the total distance traveled by the fly during a 2.0-min cooking period. (Ignore the start-up and slow-down times.)

Strategy

First, find the total number of revolutions θ, and then the linear distance x traveled. $\theta = \bar{\omega}t$ can be used to find θ because $\bar{\omega}$ is given to be 6.0 rpm.

Solution

Entering known values into $\theta = \bar{\omega}t$ gives

$$\theta = \bar{\omega}t = (6.0\ \text{rpm})\,(2.0\ \text{min}) = 12\ \text{rev}.$$

8.37

As always, it is necessary to convert revolutions to radians before calculating a linear quantity like x from an angular quantity like θ:

$$\theta = (12\ \text{rev})\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 75.4\ \text{rad}.$$

8.38

Now, using the relationship between x and θ, we can determine the distance traveled:

$$x = r\theta = (0.15\ \text{m})\,(75.4\ \text{rad}) = 11\ \text{m}.$$

8.39

Discussion

Quite a trip (if it survives)! Note that this distance is the total distance traveled by the fly. Displacement is actually zero for complete revolutions because they bring the fly back to its original position. The distinction between total distance traveled and displacement was first noted in Kinematics.

Check Your Understanding

Rotational kinematics has many useful relationships, often expressed in equation form. Are these relationships laws of physics or are they simply descriptive? (Hint: the same question applies to linear kinematics.)

Click for Solution

Solution

Rotational kinematics (just like linear kinematics) is descriptive and does not represent laws of nature. With kinematics, we can describe many things to great precision but kinematics does not consider causes. For example, a large angular acceleration describes a very rapid change in angular velocity without any consideration of its cause.

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8.3 DYNAMICS OF ROTATIONAL MOTION: ROTATIONAL INERTIA

Learning Objectives

By the end of this section, you will be able to:

- Understand the relationship between force, mass and acceleration.
- Study the turning effect of force.
- Study the analogy between force and torque, mass and moment of inertia, and linear acceleration and angular acceleration.

If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity as seen in Figure 10.9. In fact, your intuition is reliable in predicting many of the factors that are involved. For example, we know that a door opens slowly if we push too close to its hinges. Furthermore, we know that the more massive the door, the more slowly it opens. The first example implies that the farther the force is applied from the pivot, the greater the angular acceleration; another implication is that angular acceleration is inversely proportional to mass. These relationships should seem very similar to the familiar relationships among force, mass, and acceleration embodied in Newton's second law of motion. There are, in fact, precise rotational analogs to both force and mass.

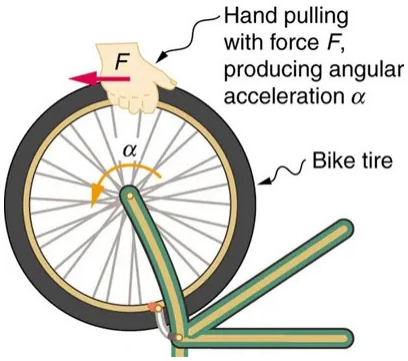


Figure 8.9 Force is required to spin the bike wheel. The greater the force, the greater the angular acceleration produced. The more massive the wheel, the smaller the angular acceleration. If you push on a spoke closer to the axle, the angular acceleration will be smaller. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of a bicycle wheel with a force applied to it. A hand is shown pulling the wheel with a labeled force, *F*, in the direction indicated by an arrow. This action produces an angular acceleration, represented by the symbol α, indicated by a curved arrow along the inside of the wheel. The wheel is labeled as a "Bike tire." The hand applying the force is located at the top of the wheel, pulling horizontally. The angular acceleration arrow is shown moving counterclockwise. Overall, the diagram illustrates the concept of how an applied force generates angular acceleration in a wheel.

To develop the precise relationship among force, mass, radius, and angular acceleration, consider what happens if we exert a force F on a point mass m that is at a distance r from a pivot point, as shown in Figure 8.10. Because the force is perpendicular to r, an acceleration $a = \frac{F}{m}$ is obtained in the direction of F. We can rearrange this equation such that $F = \mathrm{ma}$ and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = \mathrm{ma}$, yielding

$$F = \mathrm{mr}\alpha.$$
8.40

Recall that torque is the turning effectiveness of a force. In this case, because $\mathbf{F}$ is perpendicular to r, torque is simply $\tau = Fr$. So, if we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,

$$\mathrm{rF} = \mathrm{mr}^2\alpha$$
8.41

or

$$\tau = \mathrm{mr}^2\alpha.$$
8.42

This last equation is the rotational analog of Newton's second law ($F = \mathrm{ma}$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia). The quantity mr^2 is called the rotational inertia or moment of inertia of a point mass m a distance r from the center of rotation.

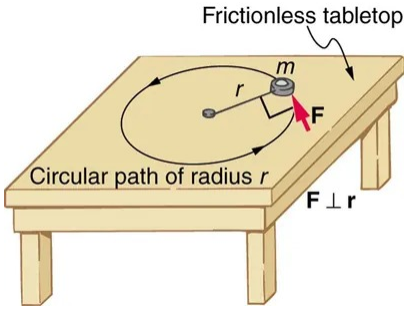


Figure 8.10 An object is supported by a horizontal frictionless table and is attached to a pivot point by a cord that supplies centripetal force. A force F is applied to the object perpendicular to the radius r, causing it to accelerate about the pivot point. The force is kept perpendicular to r. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of an object moving in a circular path on a frictionless tabletop. There is a table with a circular path drawn on its surface, labeled as having a radius 'r'. A small object with mass 'm' is shown on the circular path. An arrow labeled 'F' represents the force acting on the object, pointing towards the center of the circle. The force 'F' is perpendicular to the radius 'r', as indicated by another label. Overall, the diagram illustrates the concept of centripetal force required to maintain an object in uniform circular motion on a frictionless surface.

Making Connections: Rotational Motion Dynamics

Dynamics for rotational motion is completely analogous to linear or translational dynamics. Dynamics is concerned with force and mass and their effects on motion. For rotational motion, we will find direct analogs to force and mass that behave just as we would expect from our earlier experiences.

Rotational Inertia and Moment of Inertia

Before we can consider the rotation of anything other than a point mass like the one in Figure 8.10, we must extend the idea of rotational inertia to all types of objects. To expand our concept of rotational inertia, we define the moment of inertia

I of an object to be the sum of mr^2 for all the point masses of which it is composed. That is, $I = \sum \mathrm{mr}^2$. Here I is analogous to m in translational motion. Because of the distance r, the moment of inertia for any object depends on the chosen axis. Actually, calculating I is beyond the scope of this text except for one simple case—that of a hoop, which has all its mass at the same distance from its axis.

A hoop's moment of inertia around its axis is therefore MR^2, where M is its total mass and R its radius. (We use M and R for an entire object to distinguish them from m and r for point masses.) In all other cases, we must consult Figure 8.11 (note that the table is piece of artwork that has shapes as well as formulae) for formulas for I that have been derived from integration over the continuous body. Note that I has units of mass multiplied by distance squared ($\mathrm{kg} \cdot \mathrm{m}^2$), as we might expect from its definition.

The general relationship among torque, moment of inertia, and angular acceleration is

$$\text{net } \tau = I\alpha$$
8.43

or

$$\alpha = \frac{\text{net } \tau}{I},$$

8.44

where net τ is the total torque from all forces relative to a chosen axis. For simplicity, we will only consider torques exerted by forces in the plane of the rotation. Such torques are either positive or negative and add like ordinary numbers. The relationship in $\tau = I\alpha, \alpha = \dfrac{\text{net } \tau}{I}$ is the rotational analog to Newton's second law and is very generally applicable. This equation is actually valid for *any* torque, applied to *any* object, relative to *any* axis.

As we might expect, the larger the torque is, the larger the angular acceleration is. For example, the harder a child pushes on a merry-go-round, the faster it accelerates. Furthermore, the more massive a merry-go-round, the slower it accelerates for the same torque. The basic relationship between moment of inertia and angular acceleration is that the larger the moment of inertia, the smaller is the angular acceleration. But there is an additional twist. The moment of inertia depends not only on the mass of an object, but also on its *distribution* of mass relative to the axis around which it rotates. For example, it will be much easier to accelerate a merry-go-round full of children if they stand close to its axis than if they all stand at the outer edge. The mass is the same in both cases, but the moment of inertia is much larger when the children are at the edge.

Take-Home Experiment

Cut out a circle that has about a 10 cm radius from stiff cardboard. Near the edge of the circle, write numbers 1 to 12 like hours on a clock face. Position the circle so that it can rotate freely about a horizontal axis through its center, like a wheel. (You could loosely nail the circle to a wall.) Hold the circle stationary and with the number 12 positioned at the top, attach a lump of blue putty (sticky material used for fixing posters to walls) at the number 3. How large does the lump need to be to just rotate the circle? Describe how you can change the moment of inertia of the circle. How does this change affect the amount of blue putty needed at the number 3 to just rotate the circle? Change the circle's moment of inertia and then try rotating the circle by using different amounts of blue putty. Repeat this process several times.

Problem-Solving Strategy for Rotational Dynamics

1. *Examine the situation to determine that torque and mass are involved in the rotation.* Draw a careful sketch of the situation.
2. *Determine the system of interest.*
3. *Draw a free body diagram.* That is, draw and label all external forces acting on the system of interest.
4. *Apply* $net\tau = I\alpha, \alpha = \frac{\text{net } \tau}{I}$, *the rotational equivalent of Newton's second law, to solve the problem.* Care must be taken to use the correct moment of inertia and to consider the torque about the point of rotation.
5. *As always, check the solution to see if it is reasonable.*

Making Connections

In statics, the net torque is zero, and there is no angular acceleration. In rotational motion, net torque is the cause of angular acceleration, exactly as in Newton's second law of motion for rotation.

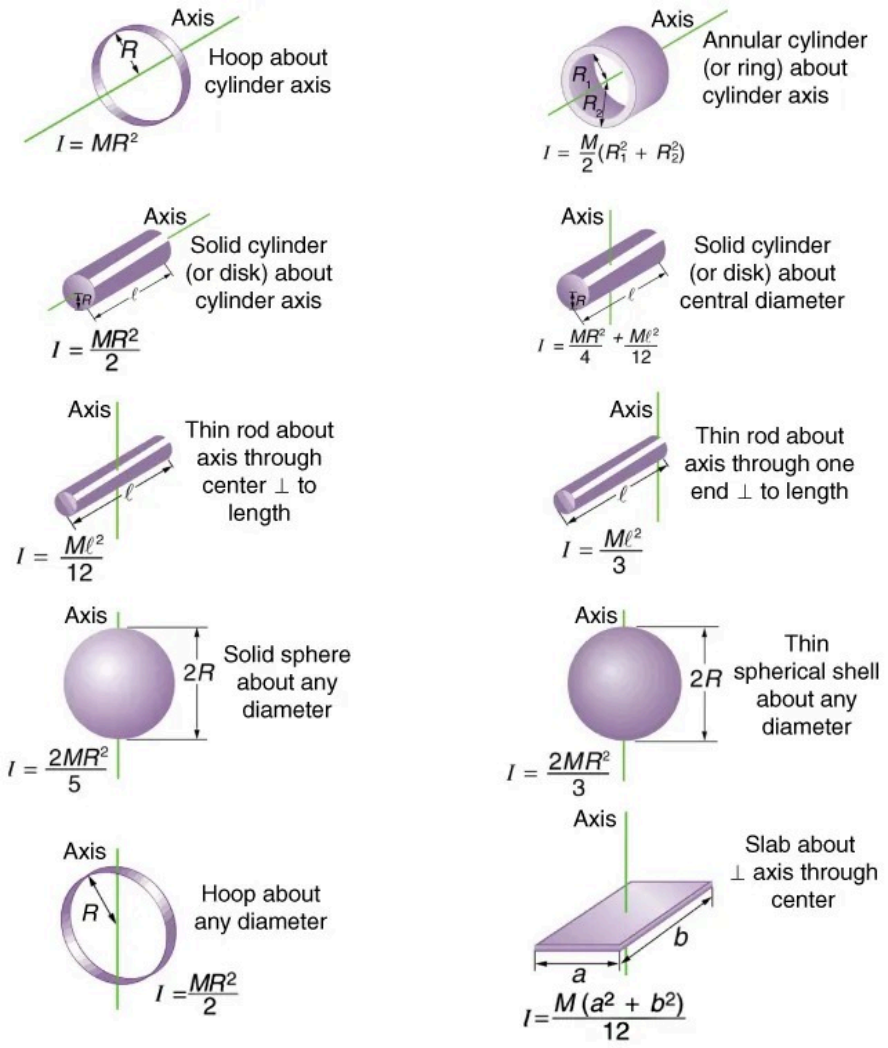


Figure 8.11 Some rotational inertias. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image displays diagrams of various geometric objects with their respective moments of inertia, each in relation to an axis. The objects and their moments of inertia are described as follows:

Object	**Moment of Inertia (I)**	**Description**
Hoop about cylinder axis	$I = MR^2$	A circular hoop with the axis along the center line of the hoop.
Annular cylinder (or ring) about cylinder axis	$I = (M/2)(R_1^2 + R_2^2)$	A hollow cylinder with two radii, the axis along the center line.
Solid cylinder (or disk) about cylinder axis	$I = (MR^2)/2$	A full solid cylinder or disk rotating about its central axis.
Solid cylinder (or disk) about central diameter	$I = (MR^2)/4 + (Ml^2)/12$	A solid cylinder or disk with rotation about its central diameter.
Thin rod about axis through center ⊥ to length	$I = (Ml^2)/12$	A thin rod rotating about an axis perpendicular to its length through the center.
Thin rod about axis through one end ⊥ to length	$I = (Ml^2)/3$	A thin rod with rotation about an axis at one end, perpendicular to its length.
Solid sphere about any diameter	$I = (2MR^2)/5$	A solid sphere rotating about any diameter.
Thin spherical shell about any diameter	$I = (2MR^2)/3$	A thin spherical shell rotating about any diameter.
Hoop about any diameter	$I = (MR^2)/2$	A hoop rotating about any of its diameters.
Slab about axis through center	$I = M(a^2 + b^2)/12$	A rectangular slab with rotation about an axis through its center.

Example 8.7

Calculating the Effect of Mass Distribution on a Merry-Go-Round

Consider the father pushing a playground merry-go-round in Figure 8.12. He exerts a force of 250 N at the edge of the 50.0-kg merry-go-round, which has a 1.50 m radius. Calculate the angular acceleration produced (a) when no one is on the merry-go-round and (b) when an 18.0-kg child sits 1.25 m away from the center. Consider the merry-go-round itself to be a uniform disk with negligible retarding friction.

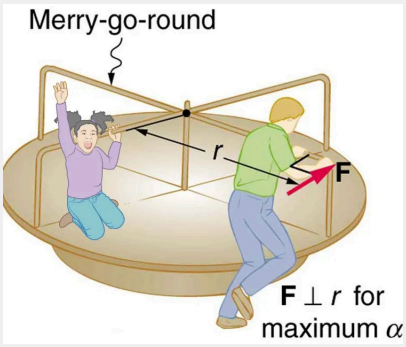


Figure 8.12 A father pushes a playground merry-go-round at its edge and perpendicular to its radius to achieve maximum torque. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a diagram of a merry-go-round. A person is standing on the edge, pushing the structure, depicted by an arrow labeled “F” indicating the force. An arrow perpendicular to “r” is drawn from the center towards the person, labeled “F ⊥ r for maximum α,” suggesting the direction of application for optimal angular acceleration. A child is on their knees inside the merry-go-round with raised arms, appearing to enjoy the ride. The merry-go-round is circular with vertical bars and a central pivot point. The force is demonstrated to be applied tangentially for effectiveness in rotation.

Strategy

Angular acceleration is given directly by the expression $\alpha = \frac{\text{net } \tau}{I}$:

$$\alpha = \frac{\tau}{I}.$$

8.45

To solve for α, we must first calculate the torque τ (which is the same in both cases) and moment of inertia I (which is greater in the second case). To find the torque, we note that the applied force is perpendicular to the radius and friction is negligible, so that

$$\tau = rF\sin\theta = (1.50\ \text{m})\,(250\ \text{N}) = 375\ \text{N}\cdot\text{m}.$$

8.46

Solution for (a)

The moment of inertia of a solid disk about this axis is given in Figure 8.11 to be

$$\frac{1}{2}\text{MR}^2,$$

8.47

where $M = 50.0\ \text{kg}$ and $R = 1.50\ \text{m}$, so that

$$\begin{aligned} I &= (0.500)\,(50.0\ \text{kg})\,(1.50\ \text{m})^2 \\ &= 56.25\ \text{kg}\cdot\text{m}^2. \end{aligned}$$

8.48

Now, after we substitute the known values, we find the angular acceleration to be

$$\alpha = \frac{\tau}{I} = \frac{375\ \text{N}\cdot\text{m}}{56.25\ \text{kg}\cdot\text{m}^2} = 6.67\frac{\text{rad}}{\text{s}^2}.$$

8.49

Solution for (b)

We expect the angular acceleration for the system to be less in this part, because the moment of inertia is greater when the child is on the merry-go-round. To find the total moment of inertia I, we first find the child's moment of inertia I_c by considering the child to be equivalent to a point mass at a distance of 1.25 m from the axis. Then,

$$\begin{aligned} I_c &= MR^2 \\ &= (18.0\ \text{kg})\,(1.25\ \text{m})^2 \\ &= 28.13\ \text{kg} \cdot \text{m}^2. \end{aligned}$$

8.50

The total moment of inertia is the sum of moments of inertia of the merry-go-round and the child (about the same axis). To justify this sum to yourself, examine the definition of I:

$$\begin{aligned} I &= 28.13\ \text{kg} \cdot \text{m}^2 + 56.25\ \text{kg} \cdot \text{m}^2 \\ &= 84.38\ \text{kg} \cdot \text{m}^2. \end{aligned}$$

8.51

Substituting known values into the equation for α gives

$$\alpha = \frac{\tau}{I} = \frac{375\ \text{N} \cdot \text{m}}{84.38\ \text{kg} \cdot \text{m}^2} = 4.44 \frac{\text{rad}}{\text{s}^2}.$$

8.52

Discussion

The angular acceleration is less when the child is on the merry-go-round than when the merry-go-round is empty, as expected. The angular accelerations found are quite large, partly due to the fact that friction was considered to be negligible. If, for example, the father kept pushing perpendicularly for 2.00 s, he would give the merry-go-round an angular velocity of 13.3 rad/s when it is empty but only 8.89 rad/s when the child is on it. In terms of revolutions per second, these angular velocities are 2.12 rev/s and 1.41 rev/s, respectively. The father would end up running at about 50 km/h in the first case.

Summer Olympics, here he comes! Confirmation of these numbers is left as an exercise for the reader.

Check Your Understanding

Torque is the analog of force and moment of inertia is the analog of mass. Force and mass are physical quantities that depend on only one factor. For example, mass is related solely to the numbers of atoms of various types in an object. Are torque and moment of inertia similarly simple?

Click for Solution

Solution

No. Torque depends on three factors: force magnitude, force direction, and point of application. Moment of inertia depends on both mass and its distribution relative to the axis of rotation. So, while the analogies are precise, these rotational quantities depend on more factors.

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8.4 ROTATIONAL KINETIC ENERGY: WORK AND ENERGY REVISITED

Learning Objectives

By the end of this section, you will be able to:

- Derive the equation for rotational work.
- Calculate rotational kinetic energy.
- Demonstrate the Law of Conservation of Energy.

In this module, we will learn about work and energy associated with rotational motion. Figure 8.13 shows a worker using an electric grindstone propelled by a motor. Sparks are flying, and noise and vibration are created as layers of steel are pared from the pole. The stone continues to turn even after the motor is turned off, but it is eventually brought to a stop by friction. Clearly, the motor had to work to get the stone spinning. This work went into heat, light, sound, vibration, and considerable rotational kinetic energy.



Figure 8.13 The motor works in spinning the grindstone, giving it rotational kinetic energy. That energy is then converted to heat, light, sound, and vibration. Image from OpenStax College Physics 2e, CC-BY 4.0

Work must be done to rotate objects such as grindstones or merry-go-rounds. Work was defined in Uniform Circular Motion and Gravitation for translational motion, and we can build on that knowledge when considering work done in rotational motion. The simplest rotational situation is one in which the net force is exerted perpendicular to the radius of a disk (as shown in Figure 8.14) and remains perpendicular as the disk starts to rotate. The force is parallel to the displacement, and so the net work done is the product of the force times the arc length traveled:

$$\text{net } W = (\text{net } F)\Delta s.$$
8.53

To get torque and other rotational quantities into the equation, we multiply and divide the right-hand side of the equation by r, and gather terms:

$$\text{net} W = (r \text{ net } F)\frac{\Delta s}{r}.$$
8.54

We recognize that $r \text{ net } F = \text{ net } \tau$ and $\Delta s/r = \theta$, so that

$$\text{net } W = (\text{net } \tau)\theta.$$
8.55

This equation is the expression for rotational work. It is very similar to the familiar definition of translational work as force multiplied by distance. Here, torque is analogous to force, and angle is analogous to distance. The equation $\text{net } W = (\text{net } \tau)\theta$ is valid in general, even though it was derived for a special case.

To get an expression for rotational kinetic energy, we must again perform some algebraic manipulations. The first step is to note that $\text{net } \tau = I\alpha$, so that

$$\text{net } W = I\alpha\theta.$$
8.56

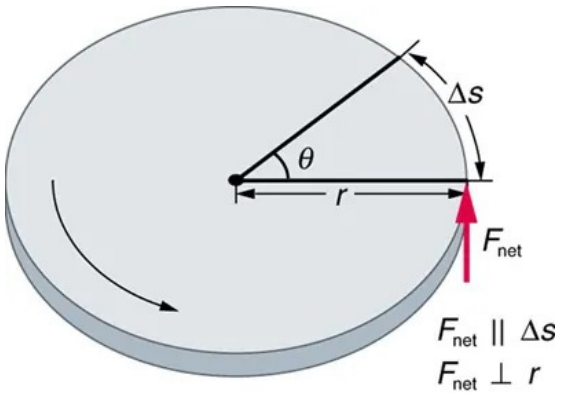


Figure 8.14 The net force on this disk is kept perpendicular to its radius as the force causes the disk to rotate. The net work done is thus $(\text{net } F)\Delta s$*. The net work goes into rotational kinetic energy. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows a diagram of a circular disk with an arrow indicating rotation in the counterclockwise direction. In the center of the disk, a radius labeled "r" is drawn from the center to the edge. At the tip of the radius, there is an angle labeled "θ," and a small arc labeled "Δs" at the edge, indicating a section of the circle.

A red arrow labeled "F_net" points outward from the edge of the circle, perpendicular to the radius. This represents the net force acting on the edge of the disk.

Below the disk, there are two mathematical expressions:

1. F_net ∥ Δs
2. F_net ⊥ r

These expressions mean that the net force is parallel to the arc segment Δs and perpendicular to the radius r.

Making Connections

Work and energy in rotational motion are completely analogous to work and energy in translational motion, first presented in Uniform Circular Motion and Gravitation.

Now, we solve one of the rotational kinematics equations for $\alpha\theta$. We start with the equation

$$\omega^2 = \omega_0^2 + 2\alpha\theta.$$
8.57

Next, we solve for $\alpha\theta$:

$$\alpha\theta = \frac{\omega^2 - \omega_0^2}{2}.$$
8.58

Substituting this into the equation for net W and gathering terms yields

$$\text{net } W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2.$$
8.59

This equation is the work-energy theorem for rotational motion only. As you may recall, net work changes the kinetic energy of a system. Through an analogy with translational motion, we define the term $\left(\frac{1}{2}\right)I\omega^2$ to be rotational kinetic energy KE_{rot} for an object with a moment of inertia I and an angular velocity ω:

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2.$$
8.60

The expression for rotational kinetic energy is exactly analogous to translational kinetic energy, with I being analogous to m and ω to v. Rotational kinetic energy has important effects. Flywheels, for example, can be used to store large amounts of rotational kinetic energy in a vehicle, as seen in Figure 8.15.



Figure 8.15 Experimental vehicles, such as this bus, have been constructed in which rotational kinetic energy is stored in a large flywheel. When the bus goes down a hill, its transmission converts its gravitational potential energy into KE_{rot}. *It can also convert translational kinetic energy, when the bus stops, into* KE_{rot}. *The flywheel's energy can then be used to accelerate, to go up another hill, or to keep the bus from slowing down due to friction. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a cartoon illustration of a yellow bus with multiple windows and a door at the front. The body of the bus has two horizontal black stripes. A cut-out section on the side of the bus reveals a spinning disc with an arrow on top, labeled with the Greek letter omega (ω), indicating rotation. The disc also has blue circular arrows showing the direction of spin. The bus appears to be on a grey road surface.

Example 8.8

Calculating the Work and Energy for Spinning a Grindstone

Consider a person who spins a large grindstone by placing her hand on its edge and exerting a force through part of a revolution as shown in Figure 8.16. In this example, we verify that the work done by the torque she exerts equals the change in rotational energy. (a) How much work is done if she exerts a force of 200 N through a rotation of $1.00\ \text{rad}\ (57.3^\circ)$? The force is kept perpendicular to the grindstone's 0.320-m radius at the point of application, and the effects of friction are negligible. (b) What is the final angular velocity if the grindstone has a mass of 85.0 kg? (c) What is the final rotational kinetic energy? (It should equal the work.)

Strategy

To find the work, we can use the equation $\text{net } W = (\text{net } \tau)\theta$. We have enough information to calculate the torque and are given the rotation angle. In the second part, we can find the final angular velocity using one of the kinematic relationships. In the last part, we can calculate the rotational kinetic energy from its expression in $\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2$.

Solution for (a)

The net work is expressed in the equation

$$\text{net } W = (\text{net } \tau)\theta,$$

8.61

where net τ is the applied force multiplied by the radius (rF) because there is no retarding friction, and the force is perpendicular to r. The angle θ is given. Substituting the given values in the equation above yields

$$\begin{aligned}\text{net } W &= \text{rF}\theta = (0.320\ \text{m})\,(200\ \text{N})\,(1.00\ \text{rad}) \\ &= 64.0\ \text{N} \cdot \text{m}.\end{aligned}$$

8.62

Noting that $1N \cdot \text{m} = 1J$,

$$\text{net } W = 64.0\ \text{J}.$$

8.63

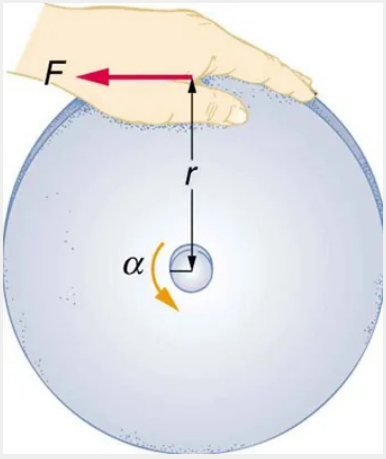


Figure 8.16 A large grindstone is given a spin by a person grasping its outer edge. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image depicts a diagram illustrating the concept of torque. It shows a hand applying a force (*F*) tangentially to the edge of a circular disk. The force is represented by a red arrow pointing to the right. The center of the disk is marked, and there is a vertical line labeled *r* extending from the center to the edge, indicating the radius. An orange arrow curves around the center, labeled with the symbol ***α***, representing angular acceleration. The overall setup visually explains how force applied at a distance from the pivot point causes rotational movement.

Solution for (b)

To find ω from the given information requires more than one step. We start with the kinematic relationship in the equation

$$\omega^2 = \omega_0^2 + 2\alpha\theta.$$

8.64

Note that $\omega_0 = 0$ because we start from rest. Taking the square root of the resulting equation gives

$$\omega = (2\alpha\theta)^{1/2}.$$

8.65

Now we need to find α. One possibility is

$$\alpha = \frac{\text{net } \tau}{I},$$

8.66

where the torque is

$$\text{net } \tau = \text{rF} = (0.320 \text{ m})\,(200 \text{ N}) = 64.0 \text{ N} \cdot \text{m}.$$

8.67

The formula for the moment of inertia for a disk is found in Figure 8.11:

$$\begin{aligned} I &= \frac{1}{2}\text{MR}^2 \\ &= 0.5\,(85.0 \text{ kg})(0.320 \text{ m})^2 \\ &= 4.352 \text{ kg} \cdot \text{m}^2. \end{aligned}$$

8.68

Substituting the values of torque and moment of inertia into the expression for α, we obtain

$$\alpha = \frac{64.0 \text{ N} \cdot \text{m}}{4.352 \text{ kg} \cdot \text{m}^2} = 14.7 \frac{\text{rad}}{\text{s}^2}.$$

8.69

Now, substitute this value and the given value for θ into the above expression for ω:

$$\begin{aligned}\omega &= (2\alpha\theta)^{1/2} \\ &= \left[2\left(14.7\frac{\text{rad}}{\text{s}^2}\right)(1.00\ \text{rad})\right]^{1/2} \\ &= 5.42\frac{\text{rad}}{\text{s}}.\end{aligned}$$

8.70

Solution for (c)

The final rotational kinetic energy is

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2.$$

8.71

Both I and ω were found above. Thus,

$$\begin{aligned}\text{KE}_{\text{rot}} &= (0.5)\left(4.352\ \text{kg}\cdot\text{m}^2\right)(5.42\ \text{rad/s})^2 \\ &= 64.0\ \text{J}.\end{aligned}$$

8.72

Discussion

The final rotational kinetic energy equals the work done by the torque, which confirms that the work done went into rotational kinetic energy. We could, in fact, have used an expression for energy instead of a kinematic relation to solve part (b). We will do this in later examples.

Helicopter pilots are quite familiar with rotational kinetic energy. They know, for example, that a point of no return will be reached if they allow their blades to slow below a critical angular velocity during flight. The blades lose lift, and it is impossible to immediately get the blades spinning fast enough to regain it. Rotational kinetic energy must be supplied to the blades to get them to rotate faster, and enough energy cannot be supplied in time to avoid a crash. Because of weight limitations, helicopter engines are too small to supply both the energy needed for lift and to replenish the rotational kinetic energy of the blades once they have slowed down. The rotational kinetic energy is put into them before takeoff and must not be allowed to drop

below this crucial level. One possible way to avoid a crash is to use the gravitational potential energy of the helicopter to replenish the rotational kinetic energy of the blades by losing altitude and aligning the blades so that the helicopter is spun up in the descent. Of course, if the helicopter's altitude is too low, then there is insufficient time for the blade to regain lift before reaching the ground.

Problem-Solving Strategy for Rotational Energy

1. *Determine that energy or work is involved in the rotation.*
2. *Determine the system of interest.* A sketch usually helps.
3. *Analyze the situation to determine the types of work and energy involved.*
4. *For closed systems, mechanical energy is conserved.* That is, $\mathbf{KE_i + PE_i = KE_f + PE_f}$. Note that $\mathbf{KE_i}$ and $\mathbf{KE_f}$ may each include translational and rotational contributions.
5. *For open systems*, mechanical energy may not be conserved, and other forms of energy (referred to previously as $\mathbf{OE}$), such as heat transfer, may enter or leave the system. Determine what they are, and calculate them as necessary.
6. *Eliminate terms wherever possible to simplify the algebra.*
7. *Check the answer to see if it is reasonable.*

Example 8.9

Calculating Helicopter Energies

A typical small rescue helicopter, similar to the one in Figure 8.17, has four blades, each is 4.00 m long and has a mass of 50.0 kg. The blades can be approximated as thin rods that rotate about one end of an axis perpendicular to their length. The helicopter has a total loaded mass of 1000 kg. (a) Calculate the rotational kinetic energy in the blades when they rotate at 300 rpm. (b) Calculate the translational kinetic energy of the helicopter when it flies at 20.0 m/s, and compare it with the rotational energy in the blades. (c) To what height could the helicopter be raised if all of the rotational kinetic energy could be used to lift it?

Strategy

Rotational and translational kinetic energies can be calculated from their definitions. The last part of the problem relates to the idea that energy can change form, in this case from rotational kinetic energy to gravitational potential energy.

Solution for (a)

The rotational kinetic energy is

$$\mathrm{KE}_{\mathrm{rot}} = \frac{1}{2} I\omega^2.$$

8.73

We must convert the angular velocity to radians per second and calculate the moment of inertia before we can find $\mathrm{KE}_{\mathrm{rot}}$. The angular velocity ω is

$$\omega = \frac{300 \text{ rev}}{1.00 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1.00 \text{ min}}{60.0 \text{ s}} = 31.4 \frac{\text{rad}}{\text{s}}.$$

8.74

The moment of inertia of one blade will be that of a thin rod rotated about its end, found in Figure 8.11. The total I is four times this moment of inertia, because there are four blades. Thus,

$$I = 4\frac{M\ell^2}{3} = 4 \times \frac{(50.0\ \text{kg})(4.00\ \text{m})^2}{3} = 1067\ \text{kg} \cdot \text{m}^2.$$

8.75

Entering ω and I into the expression for rotational kinetic energy gives

$$\begin{aligned} \text{KE}_{\text{rot}} &= 0.5\,(1067\ \text{kg} \cdot \text{m}^2\,)(31.4\ \text{rad/s})^2 \\ &= 5.26 \times 10^5\ \text{J} \end{aligned}$$

8.76

Solution for (b)

Translational kinetic energy was defined in Uniform Circular Motion and Gravitation. Entering the given values of mass and velocity, we obtain

$$\begin{aligned} \text{KE}_{\text{trans}} &= \frac{1}{2}mv^2 \\ &= (0.5)\,(1000\ \text{kg})(20.0\ \text{m/s})^2 \\ &= 2.00 \times 10^5\ \text{J}. \end{aligned}$$

8.77

To compare kinetic energies, we take the ratio of translational kinetic energy to rotational kinetic energy. This ratio is

$$\frac{2.00 \times 10^5\ \text{J}}{5.26 \times 10^5\ \text{J}} = 0.380.$$

8.78

Solution for (c)

At the maximum height, all rotational kinetic energy will have been converted to gravitational energy. To find this height, we equate those two energies:

$$\mathrm{KE_{rot}} = \mathrm{PE_{grav}}$$

8.79

or

$$\frac{1}{2}I\omega^2 = \mathrm{mgh}.$$

8.80

We now solve for h and substitute known values into the resulting equation

$$h = \frac{\left(\frac{1}{2}I\omega\right)^2}{\mathrm{mg}} = \frac{5.26 \times 10^5\ \mathrm{J}}{(1000\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2})} = 53.7\ \mathrm{m}.$$

8.81

Discussion

The ratio of translational energy to rotational kinetic energy is only 0.380. This ratio tells us that most of the kinetic energy of the helicopter is in its spinning blades—something you probably would not suspect. The 53.7 m height to which the helicopter could be raised with the rotational kinetic energy is also impressive, again emphasizing the amount of rotational kinetic energy in the blades.

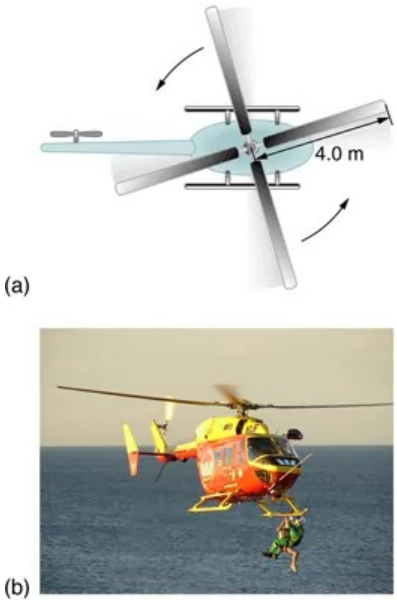


Figure 8.17 The first image shows how helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades. The second image shows a helicopter from the Auckland Westpac Rescue Helicopter Service. Over 50,000 lives have been saved since its operations beginning in 1973. Here, a water rescue operation is shown. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two parts, labeled (a) and (b):

(a) This is a diagram of a helicopter rotor system. It shows a top view of the rotor blades, which are rotating counterclockwise. The rotor system includes four blades, each marked with a black arrow indicating the direction of rotation. A label indicates that the blades have a diameter of 4.0 meters.

(b) This is a photograph of a rescue helicopter in action over a body of water. The helicopter is yellow and red, with its side door open. A person wearing a harness is being lowered from the helicopter by a winch, descending towards the water. The ocean is calm, and the sky is clear, indicating a rescue operation is in progress.

Making Connections

Conservation of energy includes rotational motion, because rotational kinetic energy is another form of KE. Uniform Circular Motion and Gravitation has a detailed treatment of conservation of energy.

How Thick Is the Soup? Or Why Don't All Objects Roll Downhill at the Same Rate?

One of the quality controls in a tomato soup factory consists of rolling filled cans down a ramp. If they roll too fast, the soup is too thin. Why should cans of identical size and mass roll down an incline at different rates? And why should the thickest soup roll the slowest?

The easiest way to answer these questions is to consider energy. Suppose each can starts down the ramp from rest. Each can starting from rest means each starts with the same gravitational potential energy $\mathrm{PE}_{\mathrm{grav}}$, which is converted entirely to KE, provided each rolls without slipping. KE, however, can take the form of $\mathrm{KE}_{\mathrm{trans}}$ or $\mathrm{KE}_{\mathrm{rot}}$, and total KE is the sum of the two. If a can rolls down a ramp, it puts part of its energy into rotation, leaving less for translation. Thus, the can goes slower than it would if it slid down. Furthermore, the thin soup does not rotate, whereas the thick soup does, because it sticks to the can. The thick soup thus puts more of the can's original gravitational potential energy into rotation than the thin soup, and the can rolls more slowly, as seen in Figure 8.18.

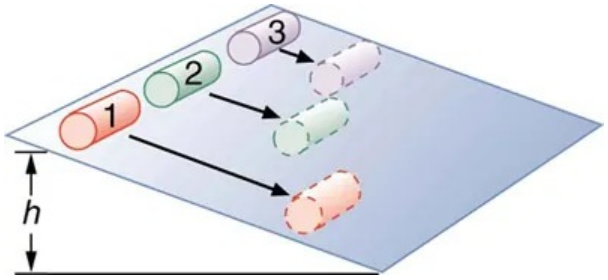


Figure 8.18 Three cans of soup with identical masses race down an incline. The first can has a low friction coating and does not roll but just slides down the incline. It wins because it converts its entire PE into translational KE. The second and third cans both roll down the incline without slipping. The second can contains thin soup and comes in second because part of its initial PE goes into rotating the can (but not the thin soup). The third can contains thick soup. It comes in third because the soup rotates along with the can, taking even more of the initial PE for rotational KE, leaving less for translational KE. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a sequence of three cylindrical objects moving across a surface. Each cylinder is numbered and has a corresponding colored outline:

- Cylinder 1: Positioned on the left, is red and filled in.
- Cylinder 2: Positioned in the middle, is green and filled in.
- Cylinder 3: Positioned on the right, is purple and filled in.

Each cylinder has an arrow pointing towards a faint, dashed outline of itself ahead on the surface, indicating movement in that direction. The outlines suggest the new positions of the cylinders along the plane.

The letter "h" with a bidirectional arrow indicates a vertical measurement from the surface, denoting height or separation distance from another reference point not visible in this image.

Assuming no losses due to friction, there is only one force doing work—gravity. Therefore the total work done is the change in kinetic energy. As the cans start moving, the potential energy is changing into kinetic energy. Conservation of energy gives

$$\mathrm{PE_i} = \mathrm{KE_f}.$$
8.82

More specifically,

$$\mathrm{PE_{grav}} = \mathrm{KE_{trans}} + \mathrm{KE_{rot}}$$

8.83

or

$$\mathrm{mgh} = \frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}I\omega^2.$$

8.84

So, the initial mgh is divided between translational kinetic energy and rotational kinetic energy; and the greater I is, the less energy goes into translation. If the can slides down without friction, then $\omega = 0$ and all the energy goes into translation; thus, the can goes faster.

Take-Home Experiment

Locate several cans each containing different types of food. First, predict which can will win the race down an inclined plane and explain why. See if your prediction is correct. You could also do this experiment by collecting several empty cylindrical containers of the same size and filling them with different materials such as wet or dry sand.

Example 8.10

Calculating the Speed of a Cylinder Rolling Down an Incline

Calculate the final speed of a solid cylinder that rolls down a 2.00-m-high incline. The cylinder starts from rest, has a mass of 0.750 kg, and has a radius of 4.00 cm.

Strategy

We can solve for the final velocity using conservation of energy, but we must first express rotational quantities in terms of translational quantities to end up with v as the only unknown.

Solution

Conservation of energy for this situation is written as described above:

$$\text{mgh} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

8.85

Before we can solve for v , we must get an expression for I from Figure 8.11. Because v and ω are related (note here that the cylinder is rolling without slipping), we must also substitute the relationship $\omega = v/R$ into the expression. These substitutions yield

$$\text{mgh} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right).$$

8.86

Interestingly, the cylinder's radius R and mass m cancel, yielding

$$\text{gh} = \frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2.$$

8.87

Solving algebraically, the equation for the final velocity v gives

$$v = \left(\frac{4gh}{3}\right)^{1/2}.$$

8.88

Substituting known values into the resulting expression yields

$$v = \left[\frac{4\left(9.80\ \text{m/s}^2\right)\left(2.00\ \text{m}\right)}{3}\right]^{1/2} = 5.11\ \text{m/s}.$$

8.89

Discussion

Because m and R cancel, the result $v = \left(\frac{4}{3}gh\right)^{1/2}$ is valid for any solid cylinder, implying that all solid cylinders will roll down an incline at the same rate independent of their masses and sizes. (Rolling cylinders down inclines is what Galileo actually did to show that objects fall at the same rate independent of mass.) Note that if the cylinder slid without friction down the incline without rolling, then the entire gravitational potential energy would go into translational kinetic energy. Thus, $\frac{1}{2}mv^2 = mgh$ and $v = (2gh)^{1/2}$, which is 22% greater than $(4gh/3)^{1/2}$. That is, the cylinder would go faster at the bottom.

Check Your Understanding

Analogy of Rotational and Translational Kinetic EnergyIs rotational kinetic energy completely analogous to translational kinetic energy? What, if any, are their differences? Give an example of each type of kinetic energy.

Click for Solution

Solution

Yes, rotational and translational kinetic energy are exact analogs. They both are the energy of motion involved with the coordinated (non-random) movement of mass relative to some reference frame. The only difference between rotational and translational kinetic energy is that translational is straight line motion while rotational is not. An example of both kinetic and translational kinetic energy is found in a bike tire while being ridden down a bike path. The rotational motion of the tire means it has rotational kinetic energy while the movement of the bike along the path means the tire also has translational kinetic energy. If you were to lift the front wheel of the bike and spin it while the bike is stationary, then the wheel would have only rotational kinetic energy relative to the Earth.

"10.4 Rotational Kinetic Energy: Work and Energy Revisited" from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

8.5 ANGULAR MOMENTUM AND ITS CONSERVATION

Learning Objectives

By the end of this section, you will be able to:

- Understand the analogy between angular momentum and linear momentum.
- Observe the relationship between torque and angular momentum.
- Apply the law of conservation of angular momentum.

Why does Earth keep on spinning? What started it spinning to begin with? And how does an ice skater manage to spin faster and faster simply by pulling her arms in? Why does she not have to exert a torque to spin faster? Questions like these have answers based in angular momentum, the rotational analog to linear momentum.

By now the pattern is clear—every rotational phenomenon has a direct translational analog. It seems quite reasonable, then, to define angular momentum L as

$$L = I\omega.$$
8.90

This equation is an analog to the definition of linear momentum as $p = \mathrm{mv}$. Units for linear momentum are $\mathrm{kg} \cdot \mathrm{m/s}$ while units for angular momentum are $\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}$. As we would expect, an object that has a large moment of inertia I, such as Earth, has a very large angular momentum. An object that has a large angular velocity ω, such as a centrifuge, also has a rather large angular momentum.

Making Connections

Angular momentum is completely analogous to linear momentum, first presented in Uniform Circular Motion and Gravitation. It has the same implications in terms of carrying rotation forward, and it is conserved when the net external torque is zero. Angular momentum, like linear momentum, is also a property of the atoms and subatomic particles.

Example 8.11

Calculating Angular Momentum of the Earth

Strategy

No information is given in the statement of the problem; so we must look up pertinent data before we can calculate $L = I\omega$. First, according to Figure 10.11, the formula for the moment of inertia of a sphere is

$$I = \frac{2\mathrm{MR}^2}{5}$$

8.91

so that

$$L = I\omega = \frac{2\mathrm{MR}^2\omega}{5}.$$

8.92

Earth's mass M is $5.979 \times 10^{24}\ \mathrm{kg}$ and its radius R is $6.376 \times 10^{6}\mathrm{m}$. The Earth's angular velocity ω is, of course, exactly one revolution per day, but we must covert ω to radians per second to do the calculation in SI units.

Solution

Substituting known information into the expression for L and converting ω to radians per second gives

$$\begin{aligned} L &= 0.4\left(5.979 \times 10^{24}\mathrm{kg}\right)\left(6.376 \times 10^{6}\ \mathrm{m}\right)^2\left(\frac{1\mathrm{rev}}{\mathrm{d}}\right) \\ &= 9.72 \times 10^{37}\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{rev/d}. \end{aligned}$$

8.93

Substituting 2π rad for 1 rev and $8.64 \times 10^{4}\mathrm{s}$ for 1 day gives

$$L = \left(9.72 \times 10^{37} \text{kg} \cdot \text{m}^2\right) \left(\frac{2\pi \text{ rad/rev}}{8.64 \times 10^4 \text{ s/d}}\right) (1 \text{ rev/d})$$
$$= 7.07 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}.$$

8.94

Discussion

This number is large, demonstrating that Earth, as expected, has a tremendous angular momentum. The answer is approximate, because we have assumed a constant density for Earth in order to estimate its moment of inertia.

When you push a merry-go-round, spin a bike wheel, or open a door, you exert a torque. If the torque you exert is greater than opposing torques, then the rotation accelerates, and angular momentum increases. The greater the net torque, the more rapid the increase in L. The relationship between torque and angular momentum is

$$\text{net } \tau = \frac{\Delta L}{\Delta t}.$$

8.95

This expression is exactly analogous to the relationship between force and linear momentum, $F = \Delta p/\Delta t$. The equation $\text{net } \tau = \dfrac{\Delta L}{\Delta t}$ is very fundamental and broadly applicable. It is, in fact, the rotational form of Newton's second law.

Example 8.12

Calculating the Torque Putting Angular Momentum Into a Rotating Food Tray

Figure 8.19 shows a rotating food tray, often called a lazy Susan, being turned by a person in quest of sustenance. Suppose the person exerts a 2.50 N force perpendicular to the lazy Susan's 0.260-m radius for 0.150 s. (a) What is the final angular momentum of the lazy Susan if it starts from rest, assuming friction is negligible? (b) What is the final angular velocity of the lazy Susan, given that its mass is 4.00 kg and assuming its moment of inertia is that of a disk?

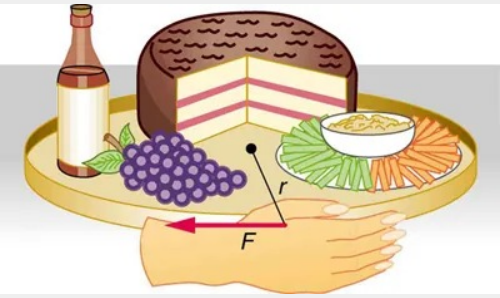


Figure 8.19 A partygoer exerts a torque on a lazy Susan to make it rotate. The equation $\text{net } \tau = \frac{\Delta L}{\Delta t}$ *gives the relationship between torque and the angular momentum produced. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a colorful illustration of a circular serving tray with various food items and a hand applying force to the tray. On the tray, there is a bottle on the left, a bunch of grapes, a large layered cake with chocolate frosting and cream layers (partially cut to show layers), and a plate containing carrot and celery sticks with a small bowl of dip in the center. A hand is shown underneath the tray with a red arrow labeled "F" pointing to the left, indicating the direction of the force applied. Another line labeled "r" extends from the center of the tray to the point where the force is applied.

Strategy

We can find the angular momentum by solving $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for ΔL, and using the given information to calculate the torque. The final angular momentum equals the change in angular momentum, because the lazy Susan starts from rest. That is, $\Delta L = L$. To find the final velocity, we must calculate ω from the definition of L in $L = I\omega$.

Solution for (a)

Solving $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for ΔL gives

$$\Delta L = (\text{net } \tau)\Delta t.$$

8.96

Because the force is perpendicular to r, we see that $\text{net } \tau = \text{rF}$, so that

$$\begin{aligned} L = \text{rF}\Delta t &= (0.260 \text{ m})\,(2.50N)\,(0.150s) \\ &= 9.75 \times 10^{-2}\text{kg} \cdot \text{m}^2/\text{s}. \end{aligned}$$

8.97

Solution for (b)

The final angular velocity can be calculated from the definition of angular momentum,

$$L = I\omega.$$

8.98

Solving for ω and substituting the formula for the moment of inertia of a disk into the resulting equation gives

$$\omega = \frac{L}{I} = \frac{L}{\frac{1}{2}MR^2}.$$

8.99

And substituting known values into the preceding equation yields

$$\omega = \frac{9.75 \times 10^{-2}\ \text{kg} \cdot \text{m}^2/\text{s}}{(0.500)\,(4.00\ \text{kg})(0.260\ \text{m})^2} = 0.721\ \text{rad/s}.$$

8.100

Discussion

Note that the imparted angular momentum does not depend on any property of the object but only on torque and time. The final angular velocity is equivalent to one revolution in 8.71 s (determination of the time period is left as an exercise for the reader), which is about right for a lazy Susan.

Example 8.13

Calculating the Torque in a Kick

The person whose leg is shown in Figure 8.20 kicks his leg by exerting a 2000-N force with his upper leg muscle. The effective perpendicular lever arm is 2.20 cm. Given the moment of inertia of the lower leg is $1.25 kg \cdot \mathrm{m}^2$, (a) find the angular acceleration of the leg. (b) Neglecting the gravitational force, what is the rotational kinetic energy of the leg after it has rotated through 57.3° (1.00 rad)?

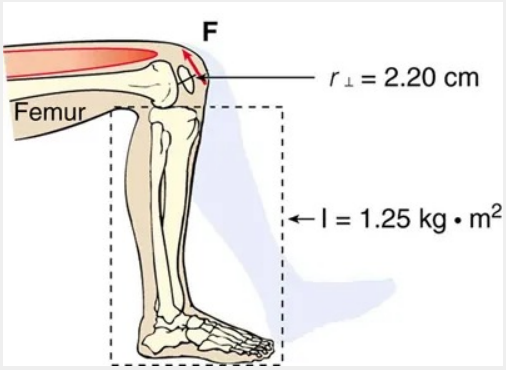


Figure 8.20 The muscle in the upper leg gives the lower leg an angular acceleration and imparts rotational kinetic energy to it by exerting a torque about the knee. $\mathbf{F}$ *is a vector that is perpendicular to* $\boldsymbol{r}$*. This example examines the situation. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a diagram of a human leg, specifically focusing on the knee and femur. It shows the leg extended forward at a right angle from the body, displaying the skeletal structure of the femur and lower leg bones. The knee joint is highlighted as a pivot point. A force, labeled as "F," is indicated with a red arrow at the knee. The diagram includes two measurements: one represents the perpendicular distance from the pivot point to the line of force, depicted as r = 2.20 cm, and the other indicates the moment of

inertia, I = 1.25 kg·m$_2$. The femur is labeled in the upper part of the leg. A dotted line encloses the lower leg, suggesting the area of focus or measurement.

Strategy

The angular acceleration can be found using the rotational analog to Newton's second law, or $\alpha = \text{net } \tau / I$. The moment of inertia I is given and the torque can be found easily from the given force and perpendicular lever arm. Once the angular acceleration α is known, the final angular velocity and rotational kinetic energy can be calculated.

Solution to (a)

From the rotational analog to Newton's second law, the angular acceleration α is

$$\alpha = \frac{\text{net } \tau}{I}.$$

8.101

Because the force and the perpendicular lever arm are given and the leg is vertical so that its weight does not create a torque, the net torque is thus

$$\begin{aligned} \text{net } \tau &= r_{\perp} F \\ &= (0.0220 \text{ m})\,(2000 \text{ N}) \\ &= 44.0 \text{ N} \cdot \text{m}. \end{aligned}$$

8.102

Substituting this value for the torque and the given value for the moment of inertia into the expression for α gives

$$\alpha = \frac{44.0 \text{ N} \cdot \text{m}}{1.25 \text{ kg} \cdot \text{m}^2} = 35.2 \text{ rad/s}^2.$$

8.103

Solution to (b)

The final angular velocity can be calculated from the kinematic expression

$$\omega^2 = (\omega_0)^2 + 2\alpha\theta$$

8.104

or

$$\omega^2 = 2\alpha\theta$$

8.105

because the initial angular velocity is zero. The kinetic energy of rotation is

$$\text{KE}_{\text{rot}} = \frac{1}{2} I\omega^2$$

8.106

so it is most convenient to use the value of ω^2 just found and the given value for the moment of inertia. The kinetic energy is then

$$\begin{aligned} \text{KE}_{\text{rot}} &= 0.5\left(1.25\ \text{kg}\cdot\text{m}^2\right)\left(70.4\ \text{rad}^2/\text{s}^2\right) \\ &= 44.0\ \text{J}. \end{aligned}$$

8.107

Discussion

These values are reasonable for a person kicking his leg starting from the position shown. The weight of the leg can be neglected in part (a) because it exerts no torque when the center of gravity of the lower leg is directly beneath the pivot in the knee. In part (b), the force exerted by the upper leg is so large that its torque is much greater than that created by the weight of the lower leg as it rotates. The rotational kinetic energy given to the lower leg is enough that it could give a ball a significant velocity by transferring some of this energy in a kick.

Making Connections: Conservation Laws

Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

Conservation of Angular Momentum

We can now understand why Earth keeps on spinning. As we saw in the previous example, $\Delta L = (\text{net}\tau)\Delta t$. This equation means that, to change angular momentum, a torque must act over some period of time. Because Earth has a large angular momentum, a large torque acting over a long time is needed to change its rate of spin. So what external torques are there? Tidal friction exerts torque that is slowing Earth's rotation, but tens of millions of years must pass before the change is very significant. Recent research indicates the length of the day was 18 h some 900 million years ago. Only the tides exert significant retarding torques on Earth, and so it will continue to spin, although ever more slowly, for many billions of years.

What we have here is, in fact, another conservation law. If the net torque is *zero*, then angular momentum is constant or *conserved*. We can see this rigorously by considering $\text{net } \tau = \dfrac{\Delta L}{\Delta t}$ for the situation in which the net torque is zero. In that case,

$$\text{net } \tau = 0$$
8.108

implying that

$$\frac{\Delta L}{\Delta t} = 0.$$
8.109

If the change in angular momentum ΔL is zero, then the angular momentum is constant; thus,

$$L = \text{constant } (\text{net } \tau = 0)$$

8.110

or

$$L = L' \, (\text{net } \tau = 0).$$
8.111

These expressions are the law of conservation of angular momentum. Conservation laws are as scarce as they are important.

An example of conservation of angular momentum is seen in Figure 8.21, in which an ice skater is executing a spin. The net torque on her is very close to zero, because there is relatively little friction between her skates and the ice and because the friction is exerted very close to the pivot point. (Both F and r are small, and so τ is negligibly small.) Consequently, she can spin for quite some time. She can do something else, too. She can increase her rate of spin by pulling her arms and legs in. Why does pulling her arms and legs in increase her rate of spin? The answer is that her angular momentum is constant, so that

$$L = L'.$$
8.112

Expressing this equation in terms of the moment of inertia,

$$I\omega = I'\omega',$$
8.113

where the primed quantities refer to conditions after she has pulled in her arms and reduced her moment of inertia. Because I' is smaller, the angular velocity ω' must increase to keep the angular momentum constant. The change can be dramatic, as the following example shows.

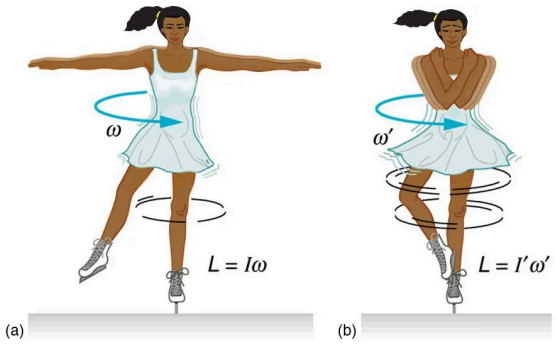


Figure 8.21 (a) An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small. In the next image, her rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image consists of two panels labeled (a) and (b), depicting an ice skater demonstrating the concept of angular momentum.

(a) The first panel shows the skater with arms and one leg extended outward, spinning with an angular velocity represented by the Greek letter omega (ω). The rotational motion is illustrated by arrows around the skater. The equation L = Iω is displayed, indicating the relationship between angular momentum (L), moment of inertia (I), and angular velocity (ω).

(b) The second panel illustrates the skater with arms and legs drawn in closer to the body, increasing her spin speed, indicated by omega prime (ω'). The rotation arrows are more compact, signifying faster rotation. The equation L = I'ω' is shown, indicating the conservation of angular momentum, where I' is the reduced moment of inertia and ω' is the increased angular velocity.

Example 8.14

Calculating the Angular Momentum of a Spinning Skater

Suppose an ice skater, such as the one in Figure 8.21, is spinning at 0.800 rev/ s with her arms extended. She has a moment of inertia of $2.34\text{kg} \cdot \text{m}^2$ with her arms extended and of $0.363\text{kg} \cdot \text{m}^2$with her arms close to her body. (These moments of inertia are based on reasonable assumptions about a 60.0-kg skater.) (a) What is her angular velocity in revolutions per second after she pulls in her arms? (b) What is her rotational kinetic energy before and after she does this?

Strategy

In the first part of the problem, we are looking for the skater's angular velocity ω' after she has pulled in her arms. To find this quantity, we use the conservation of angular momentum and note that the moments of inertia and initial angular velocity are given. To find the initial and final kinetic energies, we use the definition of rotational kinetic energy given by

$$\text{KE}_{\text{rot}} = \frac{1}{2} I\omega^2.$$

8.114

Solution for (a)

Because torque is negligible (as discussed above), the conservation of angular momentum given in $I\omega = I'\omega'$ is applicable. Thus,

$$L = L'$$

8.115

or

$$I\omega = I'\omega'$$

8.116

Solving for $\omega'$and substituting known values into the resulting equation gives

$$\omega' = \frac{I}{I'}\omega = \left(\frac{2.34\ \text{kg} \cdot m^2}{0.363\ \text{kg} \cdot m^2}\right)(0.800\ \text{rev/s})$$
$$= 5.16\ \text{rev/s}.$$

8.117

Solution for (b)

Rotational kinetic energy is given by

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2.$$

8.118

The initial value is found by substituting known values into the equation and converting the angular velocity to rad/s:

$$\text{KE}_{\text{rot}} = (0.5)\left(2.34\ \text{kg} \cdot \text{m}^2\right)((0.800\ \text{rev/s})\,(2\pi\ \text{rad/rev}))^2$$
$$= 29.6\ \text{J}.$$

8.119

The final rotational kinetic energy is

$$\text{KE}'_{\text{rot}} = \frac{1}{2}I'\left(\omega'\right)^2.$$

8.120

Substituting known values into this equation gives

$$KE'_{\text{rot}} = (0.5)\left(0.363\ \text{kg} \cdot m^2\right)[(5.16\ \text{rev/s})\,(2\pi rad/rev)]^2$$
$$= 191\ \text{J}.$$

8.121

Discussion

In both parts, there is an impressive increase. First, the final angular velocity is large, although most world-class skaters can achieve spin rates about this great. Second, the final kinetic energy is much greater than the initial kinetic energy. The increase in rotational kinetic energy comes from work done by the skater in pulling in her arms. This work is internal work that depletes some of the skater's food energy.

There are several other examples of objects that increase their rate of spin because something reduced their moment of inertia. Tornadoes are one example. Storm systems that create tornadoes are slowly rotating. When the radius of rotation narrows, even in a local region, angular velocity increases, sometimes to the furious level of a tornado. Earth is another example. Our planet was born from a huge cloud of gas and dust, the rotation of which came from turbulence in an even larger cloud. Gravitational forces caused the cloud to contract, and the rotation rate increased as a result. (See Figure 8.22.)

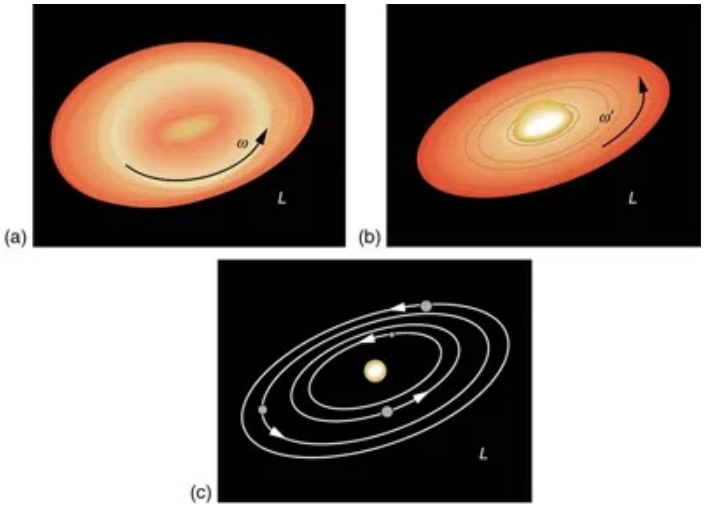


Figure 8.22 The Solar System coalesced from a cloud of gas and dust that was originally rotating. The orbital motions and spins of the planets are in the same direction as the original spin and conserve the angular momentum of the parent cloud. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three illustrations depicting the formation and dynamics of a protoplanetary disk around a star, labeled (a), (b), and (c).

– Image (a): Shows a disk with a red-orange hue, representing a rotating cloud of gas and dust. An arrow curving to the left indicates the direction of rotation, marked by the symbol "ω" for angular momentum. The symbol "L" is shown outside the disk, representing angular momentum.

– Image (b): Similar to the first image but displaying a more concentrated and flattened disk. The rotation is depicted by a more pronounced and curved arrow, again labeled with "ω". The "L" symbol is also shown, indicating angular momentum.

– Image (c): Illustrates a simplified model of a planetary system with a central yellow star and several white elliptical orbits encircling it. Small circles on the orbits depict planets. Arrows along the orbits indicate the direction of planetary movement around the star. The "L" symbol is present, denoting angular momentum of the system.

These images collectively demonstrate the evolution of a rotating disk of material into a planetary system under the influence of angular momentum.

In case of human motion, one would not expect angular momentum to be conserved when a body interacts with the environment as its foot pushes off the ground. Astronauts floating in space aboard the International Space Station have no angular momentum relative to the inside of the ship if they are motionless. Their bodies will continue to have this zero value no matter how they twist about as long as they do not give themselves a push off the side of the vessel.

Check Your Understanding

Is angular momentum completely analogous to linear momentum? What, if any, are their differences?

Click for Solution

Solution

Yes, angular and linear momentums are completely analogous. While they are exact analogs they have different units and are not directly inter-convertible like forms of energy are.

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GLOSSARY

angular acceleration
: the rate of change of angular velocity with time

angular momentum
: the product of moment of inertia and angular velocity

change in angular velocity
: the difference between final and initial values of angular velocity

kinematics of rotational motion
: describes the relationships among rotation angle, angular velocity, angular acceleration, and time

law of conservation of angular momentum
: angular momentum is conserved, i.e., the initial angular momentum is equal to the final angular momentum when no external torque is applied to the system

moment of inertia
: mass times the square of perpendicular distance from the rotation axis; for a point mass, it is $I = \mathrm{mr}^2$ and, because any object can be built up from a collection of point masses, this relationship is the basis for all other moments of inertia

right-hand rule
: direction of angular velocity ω and angular momentum L in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation

rotational inertia
: resistance to change of rotation. The more rotational inertia an object has, the harder it is to rotate

rotational kinetic energy
: the kinetic energy due to the rotation of an object. This is part of its total kinetic energy

tangential acceleration
: the acceleration in a direction tangent to the circle at the point of interest in circular motion

torque
: the turning effectiveness of a force

work-energy theorem
: if one or more external forces act upon a rigid object, causing its kinetic energy to change from KE_1 to KE_2, then the work W done by the net force is equal to the change in kinetic energy

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CHAPTER SUMMARY

8.1 Angular Acceleration

- Uniform circular motion is the motion with a constant angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$.
- In non-uniform circular motion, the velocity changes with time and the rate of change of angular velocity (i.e. angular acceleration) is $\alpha = \frac{\Delta\omega}{\Delta t}$.
- Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction, given as $a_{\mathrm{t}} = \frac{\Delta v}{\Delta t}$.
- For circular motion, note that $v = r\omega$, so that $a_{\mathrm{t}} = \frac{\Delta\left(r\omega\right)}{\Delta t}$.
- The radius r is constant for circular motion, and so $\Delta\left(r\omega\right) = r\Delta\omega$. Thus, $a_{\mathrm{t}} = r\frac{\Delta\omega}{\Delta t}$.
- By definition, $\Delta\omega/\Delta t = \alpha$. Thus,

$$a_{\mathrm{t}} = r\alpha$$

or

$$\alpha = \frac{a_{\mathrm{t}}}{r}.$$

8.2 Kinematics of Rotational Motion

- Kinematics is the description of motion.
- The kinematics of rotational motion describes the relationships among rotation angle, angular velocity, angular acceleration, and time.
- Starting with the four kinematic equations we developed in Kinematics, we can derive the four rotational kinematic equations (presented together with their translational counterparts) seen in Table 8.1.
- In these equations, the subscript 0 denotes initial values (x_0 and t_0 are initial values), and the average angular velocity $\bar{\omega}$ and average velocity $\bar{v}$ are defined as follows:

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} \text{ and } \bar{v} = \frac{v_0 + v}{2}.$$

8.3 Dynamics of Rotational Motion: Rotational Inertia

- The farther the force is applied from the pivot, the greater is the angular acceleration; angular acceleration is inversely proportional to mass.
- If we exert a force F on a point mass m that is at a distance r from a pivot point and because the force is perpendicular to r, an acceleration $\mathrm{a} = \mathrm{F/m}$ is obtained in the direction of F. We can rearrange this equation such that
  $F = ma,$
  and then look for ways to relate this expression to expressions for rotational quantities. We note that
  $a = r\alpha$, and we substitute this expression into $F = ma$, yielding
  $F = mr\alpha$
- Torque is the turning effectiveness of a force. In this case, because F is perpendicular to r, torque is simply $\tau = rF$. If we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,
  $$\mathrm{rF} = \mathrm{mr}^2\alpha$$
  or
  $$\tau = \mathrm{mr}^2\alpha.$$
- The moment of inertia I of an object is the sum of MR^2 for all the point masses of which it is composed. That is,
  $I = \sum \mathrm{mr}^2.$
- The general relationship among torque, moment of inertia, and angular acceleration is
  $$\tau = I\alpha$$
  or
  $$\alpha = \frac{\text{net } \tau}{I}.$$

8.4 Rotational Kinetic Energy: Work and Energy Revisited

- The rotational kinetic energy $\mathrm{KE}_{\mathrm{rot}}$ for an object with a moment of inertia I and an angular velocity ω is given by $\mathrm{KE}_{\mathrm{rot}} = \frac{1}{2} I\omega^2$.
- Helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades.
- Work and energy in rotational motion are completely analogous to work and energy in translational motion.
- The equation for the work-energy theorem for rotational motion is $\mathrm{net}\ W = \frac{1}{2} I\omega^2 - \frac{1}{2} I\omega_0^2$.

8.5 Angular Momentum and Its Conservation

- Every rotational phenomenon has a direct translational analog , likewise angular momentum L can be defined as $L = I\omega$.
- This equation is an analog to the definition of linear momentum as $p = \mathrm{mv}$. The relationship between torque and angular momentum is $\mathrm{net}\ \tau = \frac{\Delta L}{\Delta t}$.
- Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

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CONCEPTUAL QUESTIONS

Conceptual Questions

8.1 Angular Acceleration

1. Analogies exist between rotational and translational physical quantities. Identify the rotational term analogous to each of the following: acceleration, force, mass, work, translational kinetic energy, linear momentum, impulse.
2. Explain why centripetal acceleration changes the direction of velocity in circular motion but not its magnitude.
3. In circular motion, a tangential acceleration can change the magnitude of the velocity but not its direction. Explain your answer.
4. Suppose a piece of food is on the edge of a rotating microwave oven plate. Does it experience nonzero tangential acceleration, centripetal acceleration, or both when: (a) The plate starts to spin? (b) The plate rotates at constant angular velocity? (c) The plate slows to a halt?

8.3 Dynamics of Rotational Motion: Rotational Inertia

5. The moment of inertia of a long rod spun around an axis through one end perpendicular to its length is $ML^2/3$. Why is this moment of inertia greater than it would be if you spun a point mass M at the location of the center of mass of the rod (at $L/2$)? (That would be $ML^2/4$.)
6. Why is the moment of inertia of a hoop that has a mass M and a radius R greater than the moment of inertia of a disk that has the same mass and radius? Why is the moment of inertia of a spherical shell that has a mass M and a radius R greater than that of a solid sphere that has the same mass and radius?
7. Give an example in which a small force exerts a large torque. Give another example in which a large force exerts a small torque.
8. While reducing the mass of a racing bike, the greatest benefit is realized from reducing the mass of the tires and wheel rims. Why does this allow a racer to

achieve greater accelerations than would an identical reduction in the mass of the bicycle's frame?



Figure 8.23 The image shows a side view of a racing bicycle. Can you see evidence in the design of the wheels on this racing bicycle that their moment of inertia has been purposely reduced? Image from OpenStax College Physics 2e, CC-BY 4.0

9. A ball slides up a frictionless ramp. It is then rolled without slipping and with the same initial velocity up another frictionless ramp (with the same slope angle). In which case does it reach a greater height, and why?

8.4 Rotational Kinetic Energy: Work and Energy Revisited

10. Describe the energy transformations involved when a yo-yo is thrown downward and then climbs back up its string to be caught in the user's hand.
11. What energy transformations are involved when a dragster engine is revved, its clutch let out rapidly, its tires spun, and it starts to accelerate forward? Describe the source and transformation of energy at each step.
12. The Earth has more rotational kinetic energy now than did the cloud of gas and dust from which it formed. Where did this energy come from?

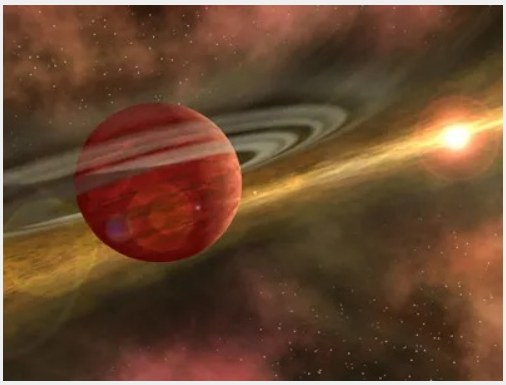


Figure 8.24 An immense cloud of rotating gas and dust contracted under the influence of gravity to form the Earth and in the process rotational kinetic energy increased. Image from OpenStax College Physics 2e, CC-BY 4.0

8.5 Angular Momentum and Its Conservation

13. When you start the engine of your car with the transmission in neutral, you notice that the car rocks in the opposite sense of the engine's rotation. Explain in terms of conservation of angular momentum. Is the angular momentum of the car conserved for long (for more than a few seconds)?
14. Suppose a child walks from the outer edge of a rotating merry-go round to the inside. Does the angular velocity of the merry-go-round increase, decrease, or remain the same? Explain your answer.

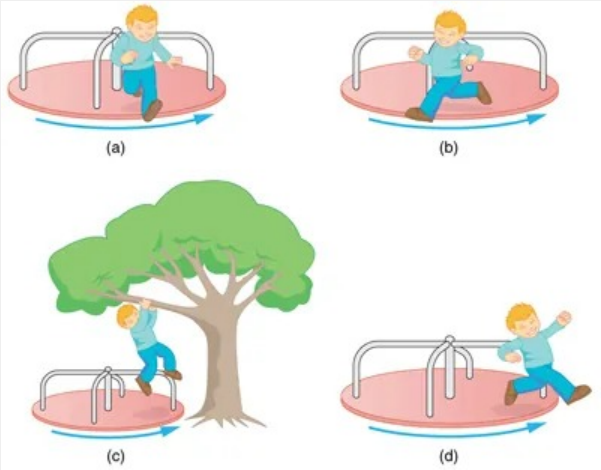


Figure 8.25 A child may jump off a merry-go-round in a variety of directions. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of four panels labeled (a), (b), (c), and (d), showing a boy interacting with a merry-go-round, demonstrating the concept of angular momentum and rotation dynamics.

– Panel (a): The boy is standing on a stationary merry-go-round, leaning slightly forward with one foot extended, about to step on it. Arrows indicate the intended rotation direction.

– Panel (b): The boy is now sitting on the merry-go-round that is partially rotating. He holds onto the railings with both hands, and the rotation appears slower compared to other panels.

– Panel (c): The boy has climbed a tree next to the merry-go-round. He is holding a branch, and the merry-go-round below is rotating more significantly in the opposite direction than panels (a) and (b).

– Panel (d): The boy jumps off the rotating merry-go-round with arms raised and legs bent, indicating rapid motion. The rotation of the merry-go-round is indicated in the same direction as the previous panels.

Arrows on each merry-go-round show the direction of rotation. The tree is only present in panel (c).

15. Suppose a child gets off a rotating merry-go-round. Does the angular velocity of the merry-go-round increase, decrease, or remain the same if: (a) He jumps off radially? (b) He jumps backward to land motionless? (c) He jumps straight up and hangs onto an overhead tree branch? (d) He jumps off forward, tangential to the edge? Explain your answers. (Refer to Figure 8.25).
16. Helicopters have a small propeller on their tail to keep them from rotating in the opposite direction of their main lifting blades. Explain in terms of Newton's third law why the helicopter body rotates in the opposite direction to the blades.
17. Whenever a helicopter has two sets of lifting blades, they rotate in opposite directions (and there will be no tail propeller). Explain why it is best to have the blades rotate in opposite directions.
18. Describe how work is done by a skater pulling in her arms during a spin. In particular, identify the force she exerts on each arm to pull it in and the distance each moves, noting that a component of the force is in the direction moved. Why is angular momentum not increased by this action?
19. When there is a global heating trend on Earth, the atmosphere expands and the length of the day increases very slightly. Explain why the length of a day increases.
20. Nearly all conventional piston engines have flywheels on them to smooth out engine vibrations caused by the thrust of individual piston firings. Why does the flywheel have this effect?
21. Jet turbines spin rapidly. They are designed to fly apart if something makes them seize suddenly, rather than transfer angular momentum to the plane's wing, possibly tearing it off. Explain how flying apart conserves angular momentum without transferring it to the wing.
22. An astronaut tightens a bolt on a satellite in orbit. He rotates in a direction opposite to that of the bolt, and the satellite rotates in the same direction as the bolt. Explain why. If a handhold is available on the satellite, can this counter-rotation be prevented? Explain your answer.
23. Competitive divers pull their limbs in and curl up their bodies when they do flips. Just before entering the water, they fully extend their limbs to enter straight down. Explain the effect of both actions on their angular velocities. Also explain

the effect on their angular momenta.

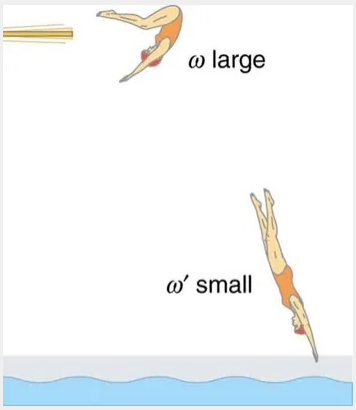


Figure 8.26 The diver spins rapidly when curled up and slows when she extends her limbs before entering the water. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a sequence of a diver performing a dive. The diver is wearing an orange swimsuit and is depicted in two key positions. At the top, the diver is in a tucked position with knees bent and arms extended forward. This position is labeled as "ω large," indicating a large angular velocity. Below, the diver is shown in a straight, vertical position entering the water headfirst. This position is labeled as "ω' small," indicating a smaller angular velocity. The water is depicted at the bottom with a wavy blue line.

24. Draw a free body diagram to show how a diver gains angular momentum when leaving the diving board.
25. In terms of angular momentum, what is the advantage of giving a football or a rifle bullet a spin when throwing or releasing it?

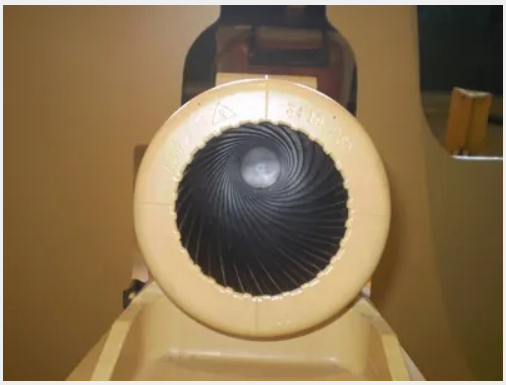


Figure 8.27 The image shows a view down the barrel of a cannon, emphasizing its rifling. Rifling in the barrel of a canon causes the projectile to spin just as is the case for rifles (hence the name for the grooves in the barrel). Image from OpenStax College Physics 2e, CC-BY 4.0

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PROBLEMS & EXERCISES

Problems & Exercises

8.1 Angular Acceleration

1. At its peak, a tornado is 60.0 m in diameter and carries 500 km/h winds. What is its angular velocity in revolutions per second?
2. **Integrated Concepts:** An ultracentrifuge accelerates from rest to 100,000 rpm in 2.00 min. (a) What is its angular acceleration in $\mathrm{rad/s^2}$? (b) What is the tangential acceleration of a point 9.50 cm from the axis of rotation? (c) What is the radial acceleration in $\mathrm{m/s^2}$ and multiples of g of this point at full rpm?
3. **Integrated Concepts:** You have a grindstone (a disk) that is 90.0 kg, has a 0.340-m radius, and is turning at 90.0 rpm, and you press a steel axe against it with a radial force of 20.0 N. (a) Assuming the kinetic coefficient of friction between steel and stone is 0.20, calculate the angular acceleration of the grindstone. (b) How many turns will the stone make before coming to rest?
4. **Unreasonable Results:** You are told that a basketball player spins the ball with an angular acceleration of $100\ \mathrm{rad/s^2}$. (a) What is the ball's final angular velocity if the ball starts from rest and the acceleration lasts 2.00 s? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

8.2 Kinematics of Rotational Motion

5. With the aid of a string, a gyroscope is accelerated from rest to 32 rad/s in 0.40 s. (a) What is its angular acceleration in rad/s2? (b) How many revolutions does it go through in the process?
6. Suppose a piece of dust finds itself on a CD. If the spin rate of the CD is 500 rpm, and the piece of dust is 4.3 cm from the center, what is the total distance traveled by the dust in 3 minutes? (Ignore accelerations due to getting the CD rotating.)
7. A gyroscope slows from an initial rate of 32.0 rad/s at a rate of $0.700\ \mathrm{rad/s^2}$. (a)

How long does it take to come to rest? (b) How many revolutions does it make before stopping?

8. During a very quick stop, a car decelerates at $7.00\ \mathrm{m/s^2}$. (a) What is the angular acceleration of its 0.280-m-radius tires, assuming they do not slip on the pavement? (b) How many revolutions do the tires make before coming to rest, given their initial angular velocity is $95.0\ \mathrm{rad/s}$? (c) How long does the car take to stop completely? (d) What distance does the car travel in this time? (e) What was the car's initial velocity? (f) Do the values obtained seem reasonable, considering that this stop happens very quickly?



Figure 8.28 Yo-yos are amusing toys that display significant physics and are engineered to enhance performance based on physical laws. Image from OpenStax College Physics 2e, CC-BY 4.0

9. **Everyday application:** Suppose a yo-yo has a center shaft that has a 0.250 cm radius and that its string is being pulled. (a) If the string is stationary and the yo-yo accelerates away from it at a rate of $1.50\ \mathrm{m/s^2}$, what is the angular acceleration of the yo-yo? (b) What is the angular velocity after 0.750 s if it starts from rest? (c) The outside radius of the yo-yo is 3.50 cm. What is the tangential acceleration of a point on its edge?

8.3 Dynamics of Rotational Motion: Rotational Inertia

10. This problem considers additional aspects of Example 8.7: Calculating the Effect of Mass Distribution on a Merry-Go-Round. (a) Consider part b of that example, when the child is sitting on the merry-go-round. How long does it take the father

to give the merry-go-round an angular velocity of 1.50 rad/s? (b) How many revolutions must he go through to generate this velocity? (c) If he exerts a slowing force of 300 N at a radius of 1.35 m, how long would it take him to stop them?

11. Calculate the moment of inertia of a skater given the following information. (a) The 60.0-kg skater is approximated as a cylinder that has a 0.110-m radius. (b) The skater with arms extended is approximately a cylinder that is 52.5 kg, has a 0.110-m radius, and has two 0.900-m-long arms which are 3.75 kg each and extend straight out from the cylinder like rods rotated about their ends.
12. The triceps muscle in the back of the upper arm extends the forearm. This muscle in a professional boxer exerts a force of $2.00 \times 10^3 \text{N}$ with an effective perpendicular lever arm of 3.00 cm, producing an angular acceleration of the forearm of 120rad/s^2. What is the moment of inertia of the boxer's forearm?
13. A soccer player extends her lower leg in a kicking motion by exerting a force with the muscle above the knee in the front of her leg. She produces an angular acceleration of $30.00 rad/\text{s}^2$ and her lower leg has a moment of inertia of $0.750 \text{ kg} \cdot \text{m}^2$. What is the force exerted by the muscle if its effective perpendicular lever arm is 1.90 cm?
14. Suppose you exert a force of 180 N tangential to a 0.280-m-radius 75.0-kg grindstone (a solid disk). (a)What torque is exerted? (b) What is the angular acceleration assuming negligible opposing friction? (c) What is the angular acceleration if there is an opposing frictional force of 20.0 N exerted 1.50 cm from the axis?
15. Consider the 12.0 kg motorcycle wheel shown in Figure 8.29. Assume it to be approximately an annular ring with an inner radius of 0.280 m and an outer radius of 0.330 m. The motorcycle is on its center stand, so that the wheel can spin freely. (a) If the drive chain exerts a force of 2200 N at a radius of 5.00 cm, what is the angular acceleration of the wheel? (b) What is the tangential acceleration of a point on the outer edge of the tire? (c) How long, starting from rest, does it take to reach an angular velocity of 80.0 rad/s?

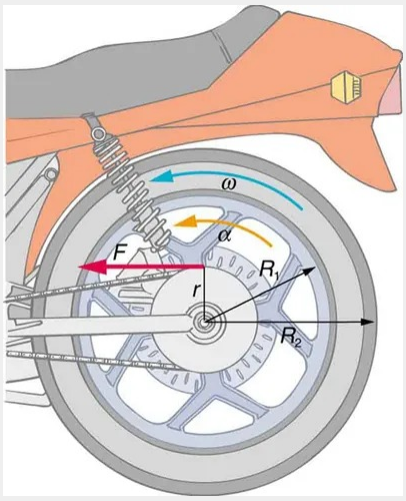


Figure 8.29 A motorcycle wheel has a moment of inertia approximately that of an annular ring. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of a motorcycle's rear wheel, showcasing various mechanical forces and components. Here's a description of the elements:

– The wheel is depicted with a suspension system, illustrated by a coil spring connected to the motorcycle frame.
– There are various arrows indicating forces and directions:
– A red arrow labeled "F" pointing to the left, representing an applied force.
– A yellow arrow indicating an angle "α" pointing in a clockwise direction, suggesting angular motion or force.
– A blue arrow around the wheel showing "ω," representing angular velocity, moving in a clockwise direction.
– Two black arrows labeled "R^1" and "R^2" extending from the wheel's center, denoting radii or distances related to the wheel's rotation.
– The wheel appears to be engaged in motion, with the forces and directions illustrating the dynamics involved.
– The motorcycle's frame is shown in an orange shade with a portion of a seat visible, emphasizing the placement of the suspension and rear wheel.

This diagram likely serves to illustrate the mechanics of wheel movement and the forces acting upon it in a mechanical or engineering context.

16. Zorch, an archenemy of Superman, decides to slow Earth's rotation to once per 28.0 h by exerting an opposing force at and parallel to the equator. Superman is not immediately concerned, because he knows Zorch can only exert a force of $4.00 \times 10^7 \text{N}$ (a little greater than a Saturn V rocket's thrust). How long must Zorch push with this force to accomplish his goal? (This period gives Superman time to devote to other villains.) Explicitly show how you follow the steps found in Problem-Solving Strategy for Rotational Dynamics.
17. An automobile engine can produce 200 N · m of torque. Calculate the angular acceleration produced if 95.0% of this torque is applied to the drive shaft, axle, and rear wheels of a car, given the following information. The car is suspended so that the wheels can turn freely. Each wheel acts like a 15.0 kg disk that has a 0.180 m radius. The walls of each tire act like a 2.00-kg annular ring that has inside radius of 0.180 m and outside radius of 0.320 m. The tread of each tire acts like a 10.0-kg hoop of radius 0.330 m. The 14.0-kg axle acts like a rod that has a 2.00-cm radius. The 30.0-kg drive shaft acts like a rod that has a 3.20-cm radius.
18. Starting with the formula for the moment of inertia of a rod rotated around an axis through one end perpendicular to its length $(I = M\ell^2/3)$, prove that the moment of inertia of a rod rotated about an axis through its center perpendicular to its length is $I = M\ell^2/12$. You will find the graphics in Figure 8.11 useful in visualizing these rotations.
19. **Unreasonable Results:** A gymnast doing a forward flip lands on the mat and exerts a 500-N · m torque to slow their angular velocity to zero. Their initial angular velocity is 10.0 rad/s, and their moment of inertia is $0.050 \text{kg} \cdot \text{m}^2$. (a) What time is required for the gymnast to exactly stop their spin? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?
20. **Unreasonable Results:** An advertisement claims that an 800-kg car is aided by its 20.0-kg flywheel, which can accelerate the car from rest to a speed of 30.0 m/s. The flywheel is a disk with a 0.150-m radius. (a) Calculate the angular velocity the flywheel must have if 95.0% of its rotational energy is used to get the car up to

speed. (b) What is unreasonable about the result? (c) Which premise is unreasonable or which premises are inconsistent?

8.4 Rotational Kinetic Energy: Work and Energy Revisited

21. This problem considers energy and work aspects of Example 8.7 — use data from that example as needed. (a) Calculate the rotational kinetic energy in the merry-go-round plus child when they have an angular velocity of 20.0 rpm. (b) Using energy considerations, find the number of revolutions the father will have to push to achieve this angular velocity starting from rest. (c) Again, using energy considerations, calculate the force the father must exert to stop the merry-go-round in two revolutions
22. What is the final velocity of a hoop that rolls without slipping down a 5.00-m-high hill, starting from rest?
23. (a) Calculate the rotational kinetic energy of Earth on its axis. (b) What is the rotational kinetic energy of Earth in its orbit around the Sun?
24. Calculate the rotational kinetic energy in the motorcycle wheel (Figure 8.29) if its angular velocity is 120 rad/s. Assume M = 12.0 kg, R1 = 0.280 m, and R2 = 0.330 m.
25. A baseball pitcher throws the ball in a motion where there is rotation of the forearm about the elbow joint as well as other movements. If the linear velocity of the ball relative to the elbow joint is 20.0 m/s at a distance of 0.480 m from the joint and the moment of inertia of the forearm is $0.500\ \text{kg} \cdot \text{m}^2$, what is the rotational kinetic energy of the forearm?
26. While punting a football, a kicker rotates her leg about the hip joint. The moment of inertia of the leg is $3.75\ \text{kg} \cdot \text{m}^2$ and its rotational kinetic energy is 175 J. (a) What is the angular velocity of the leg? (b) What is the velocity of tip of the punter's shoe if it is 1.05 m from the hip joint? (c) Explain how the football can be given a velocity greater than the tip of the shoe (necessary for a decent kick distance).
27. A bus contains a 1500 kg flywheel (a disk that has a 0.600 m radius) and has a total mass of 10,000 kg. (a) Calculate the angular velocity the flywheel must have to contain enough energy to take the bus from rest to a speed of 20.0 m/s,

assuming 90.0% of the rotational kinetic energy can be transformed into translational energy. (b) How high a hill can the bus climb with this stored energy and still have a speed of 3.00 m/s at the top of the hill? Explicitly show how you follow the steps in the Problem-Solving Strategy for Rotational Energy.

28. A ball with an initial velocity of 8.00 m/s rolls up a hill without slipping. Treating the ball as a spherical shell, calculate the vertical height it reaches. (b) Repeat the calculation for the same ball if it slides up the hill without rolling.
29. While exercising in a fitness center, a man lies face down on a bench and lifts a weight with one lower leg by contacting the muscles in the back of the upper leg. (a) Find the angular acceleration produced given the mass lifted is 10.0 kg at a distance of 28.0 cm from the knee joint, the moment of inertia of the lower leg is $0.900\ \text{kg} \cdot \text{m}^2$, the muscle force is 1500 N, and its effective perpendicular lever arm is 3.00 cm. (b) How much work is done if the leg rotates through an angle of 20.0° with a constant force exerted by the muscle?
30. To develop muscle tone, a woman lifts a 2.00-kg weight held in her hand. She uses her biceps muscle to flex the lower arm through an angle of 60.0°. (a) What is the angular acceleration if the weight is 24.0 cm from the elbow joint, her forearm has a moment of inertia of $0.250\ \text{kg} \cdot \text{m}^2$, and the net force she exerts is 750 N at an effective perpendicular lever arm of 2.00 cm? (b) How much work does she do?
31. Consider two cylinders that start down identical inclines from rest except that one is frictionless. Thus one cylinder rolls without slipping, while the other slides frictionlessly without rolling. They both travel a short distance at the bottom and then start up another incline. (a) Show that they both reach the same height on the other incline, and that this height is equal to their original height. (b) Find the ratio of the time the rolling cylinder takes to reach the height on the second incline to the time the sliding cylinder takes to reach the height on the second incline. (c) Explain why the time for the rolling motion is greater than that for the sliding motion.
32. What is the moment of inertia of an object that rolls without slipping down a 2.00-m-high incline starting from rest, and has a final velocity of 6.00 m/s? Express the moment of inertia as a multiple of MR^2, where M is the mass of the

object and R is its radius.

33. Suppose a 200-kg motorcycle has two wheels like, the one described in Problem 8.15 and is heading toward a hill at a speed of 30.0 m/s. (a) How high can it coast up the hill, if you neglect friction? (b) How much energy is lost to friction if the motorcycle only gains an altitude of 35.0 m before coming to rest?
34. In softball, the pitcher throws with the arm fully extended (straight at the elbow). In a fast pitch the ball leaves the hand with a speed of 139 km/h. (a) Find the rotational kinetic energy of the pitcher's arm and ball together given that the arm's moment of inertia is $0.720\ \text{kg} \cdot \text{m}^2$ and the ball leaves the hand at a distance of 0.600 m from the pivot at the shoulder. (b) What force did the muscles exert to cause the arm to rotate if their effective perpendicular lever arm is 4.00 cm and the ball is 0.156 kg?
35. **Construct Your Own Problem:** Consider the work done by a spinning skater pulling his arms in to increase his rate of spin. Construct a problem in which you calculate the work done with a "force multiplied by distance" calculation and compare it to the skater's increase in kinetic energy.

8.5 Angular Momentum and Its Conservation

36. (a) Calculate the angular momentum of the Earth in its orbit around the Sun. (b) Compare this angular momentum with the angular momentum of Earth on its axis.
37. (a) What is the angular momentum of the Moon in its orbit around Earth? (b) How does this angular momentum compare with the angular momentum of the Moon on its axis? Remember that the Moon keeps one side toward Earth at all times. (c) Discuss whether the values found in parts (a) and (b) seem consistent with the fact that tidal effects with Earth have caused the Moon to rotate with one side always facing Earth.
38. Suppose you start an antique car by exerting a force of 300 N on its crank for 0.250 s. What angular momentum is given to the engine if the handle of the crank is 0.300 m from the pivot and the force is exerted to create maximum torque the entire time?

39. A playground merry-go-round has a mass of 120 kg and a radius of 1.80 m and it is rotating with an angular velocity of 0.500 rev/s. What is its angular velocity after a 22.0-kg child gets onto it by grabbing its outer edge? The child is initially at rest.
40. Three children are riding on the edge of a merry-go-round that is 100 kg, has a 1.60-m radius, and is spinning at 20.0 rpm. The children have masses of 22.0, 28.0, and 33.0 kg. If the child who has a mass of 28.0 kg moves to the center of the merry-go-round, what is the new angular velocity in rpm?
41. (a) Calculate the angular momentum of an ice skater spinning at 6.00 rev/s given his moment of inertia is $0.400\text{kg} \cdot \text{m}^2$. (b) He reduces his rate of spin (his angular velocity) by extending his arms and increasing his moment of inertia. Find the value of his moment of inertia if his angular velocity decreases to 1.25 rev/s. (c) Suppose instead he keeps his arms in and allows friction of the ice to slow him to 3.00 rev/s. What average torque was exerted if this takes 15.0 s?
42. Construct Your Own Problem Consider the Earth-Moon system. Construct a problem in which you calculate the total angular momentum of the system including the spins of the Earth and the Moon on their axes and the orbital angular momentum of the Earth-Moon system in its nearly monthly rotation. Calculate what happens to the Moon's orbital radius if the Earth's rotation decreases due to tidal drag. Among the things to be considered are the amount by which the Earth's rotation slows and the fact that the Moon will continue to have one side always facing the Earth.

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FLUID STATICS



Figure 9.1 The fluid essential to all life has a beauty of its own. It also helps support the weight of this swimmer. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

9.1 What Is a Fluid?
9.2 Density
9.3 Pressure
9.4 Variation of Pressure with Depth in a Fluid
9.5 Pascal's Principle
9.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement
9.7 Archimedes' Principle
9.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action
9.9 Pressures in the Body
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

Much of what we value in life is fluid: a breath of fresh winter air; the hot blue flame in our gas cooker; the water we drink, swim in, and bathe in; the blood in our veins. What exactly is a fluid? Can we understand fluids with the laws already presented, or will new laws emerge from their study? The physical characteristics of static or stationary fluids and some of the laws that govern their behavior are the topics of this chapter. Fluid Dynamics and Its Biological and Medical Applications explores aspects of fluid flow.
Access multimedia content

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9.1 WHAT IS A FLUID?

Learning Objectives

By the end of this section, you will be able to:

- State the common phases of matter.
- Explain the physical characteristics of solids, liquids, and gases.
- Describe the arrangement of atoms in solids, liquids, and gases.

Matter most commonly exists as a solid, liquid, gas, or plasma; these states are known as the common *phases of matter*. Solids have a definite shape and a specific volume, liquids have a definite volume but their shape changes depending on the container in which they are held, gases have neither a definite shape nor a specific volume as their molecules move to fill the container in which they are held, and plasmas also have neither definite shape nor volume. (See Figure 9.2.) Liquids, gases, and plasmas are considered to be fluids because they yield to shearing forces, whereas solids resist them. Note that the extent to which fluids yield to shearing forces (and hence flow easily and quickly) depends on a quantity called the viscosity which is discussed in detail in 10.4 Viscosity and Laminar Flow; Poiseuille's Law. We can understand the phases of matter and what constitutes a fluid by considering the forces between atoms that make up matter in the three phases.

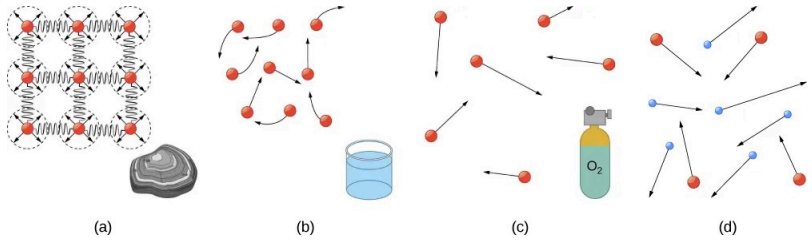


Figure 9.2 (a) Atoms in a solid always have the same neighbors, held near home by forces represented here by springs. These atoms are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms together. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. Water can flow, but it also remains in an open container because of the forces between its atoms. (c) Atoms in a gas are separated by distances that are considerably larger than the size of the atoms themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out freely. (d) A plasma is composed of electrons, protons, and ions that, like gases, are spaced far apart and move about freely. Image from OpenStax College Physics 2e, CC-BY 4.0

Atoms in *solids* are in close contact, with forces between them that allow the atoms to vibrate but not to change positions with neighboring atoms. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.) Thus a solid *resists* all types of stress. A solid cannot be easily deformed because the atoms that make up the solid are not able to move about freely. Solids also resist compression, because their atoms form part of a lattice structure in which the atoms are a relatively fixed distance apart. Under compression, the atoms would be forced into one another. Most of the examples we have studied so far have involved solid objects which deform very little when stressed.

In contrast, *liquids* deform easily when stressed and do not spring back to their original shape once the force is removed because the atoms are free to slide about and change neighbors—that is, they *flow* (so they are a type of fluid), with the molecules held together by their mutual attraction. When a liquid is placed in a container with no lid on, it remains in the container (providing the container has no holes below the surface of the liquid!). Because the atoms are closely packed, liquids, like solids, resist compression.

Atoms in *gases* and charged particles in *plasmas* are separated by distances that are large compared with the size of the particles. The forces between the particles are therefore very weak, except when they collide with one another. Gases and plasmas thus not only flow (and are therefore considered to be fluids) but they are relatively easy to compress because there is much space and little force between the particles. When placed in an open container gases, unlike liquids, will escape. The major distinction is that gases are easily compressed, whereas liquids are not. Plasmas are difficult to contain because they have so much energy. When discussing how substances flow, we shall generally refer to both gases and liquids simply as fluids, and make a distinction between them only when they behave differently.

PhET Explorations

States of Matter—Basics

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.

Access multimedia content

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9.2 DENSITY

Learning Objectives

By the end of this section, you will be able to:

- Define density.
- Calculate the mass of a reservoir from its density.
- Compare and contrast the densities of various substances.

Which weighs more, a ton of feathers or a ton of bricks? This old riddle plays with the distinction between mass and density. A ton is a ton, of course; but bricks have much greater density than feathers, and so we are tempted to think of them as heavier. (See Figure 9.3.)

Density, as you will see, is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid. Density is the mass per unit volume of a substance or object. In equation form, density is defined as

$$\rho = \frac{m}{V},$$

9.1

where the Greek letter ρ (rho) is the symbol for density, m is the mass, and V is the volume occupied by the substance.

Density

Density is mass per unit volume.

$$\rho = \frac{m}{V},$$

9.2

where ρ is the symbol for density, m is the mass, and V is the volume occupied by the substance.

In the riddle regarding the feathers and bricks, the masses are the same, but the volume occupied by the feathers is much greater, since their density is much lower. The SI unit of density is $\mathrm{kg/m^3}$, representative values are given in Table 9.1. The metric system was originally devised so that water would have a density of $\mathrm{1 g/cm^3}$, equivalent to $\mathrm{10^3 kg/m^3}$. Thus the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of 1000 $\mathrm{cm^3}$.

Table 9.1 Densities of Various Substances

Substance	ρ ($\times 10^3 \text{kg/m}^3$ **or** g/mL)	**Substance**	ρ (10^3kg/m^3 **or** g/mL)	**Substance**	ρ (10^3kg/m^3 **or** g/mL)
Solids		**Liquids**		**Gases**	
Aluminum	2.7	Water (4ºC)	1.000	Air	1.29×10^{-3}
Brass	8.44	Blood	1.05	Carbon dioxide	1.98×10^{-3}
Copper (average)	8.8	Sea water	1.025	Carbon monoxide	1.25×10^{-3}
Gold	19.32	Mercury	13.6	Hydrogen	0.090×10^{-3}
Iron or steel	7.8	Ethyl alcohol	0.79	Helium	0.18×10^{-3}
Lead	11.3	Gasoline	0.68	Methane	0.72×10^{-3}
Polystyrene	0.10	Glycerin	1.26	Nitrogen	1.25×10^{-3}
Tungsten	19.30	Olive oil	0.92	Nitrous oxide	1.98×10^{-3}
Uranium	18.70			Oxygen	1.43×10^{-3}
Concrete	2.30–3.0			Steam (100° C)	0.60×10^{-3}
Cork	0.24				
Glass, common (average)	2.6				
Granite	2.7				
Earth’s crust	3.3				
Wood	0.3–0.9				
Ice (0°C)	0.917				
Bone	1.7–2.0				
Silver	10.49				

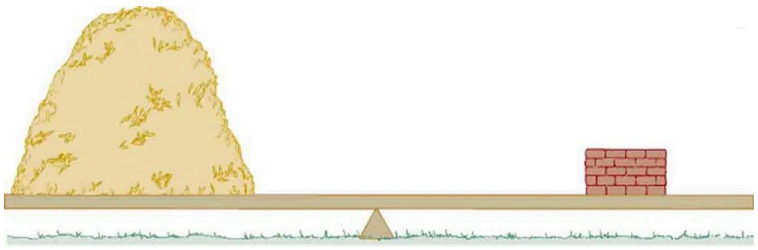


Figure 9.3 A ton of feathers and a ton of bricks have the same mass, but the feathers make a much bigger pile because they have a much lower density. Image from OpenStax College Physics 2e, CC-BY 4.0

As you can see by examining Table 9.1, the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space.

Take-Home Experiment Sugar and Salt

A pile of sugar and a pile of salt look pretty similar, but which weighs more? If the volumes of both piles are the same, any difference in mass is due to their different densities (including the air space between crystals). Which do you think has the greater density? What values did you find? What method did you use to determine these values?

Example 9.1

Calculating the Mass of a Reservoir From Its Volume

A reservoir has a surface area of 50.0km^2 and an average depth of 40.0 m. What mass of water is held behind the dam? (See Figure 9.4 for a view of a large reservoir—the Three Gorges Dam site on the Yangtze River in central China.)

Strategy

We can calculate the volume V of the reservoir from its dimensions, and find the density of water ρ in Table 9.1. Then the mass m can be found from the definition of density

$$\rho = \frac{m}{V}.$$

9.3

Solution

Solving equation $\rho = m/V$ for m gives

$$m = \rho V.$$

The volume V of the reservoir is its surface area A times its average depth h:

$$\begin{aligned} V &= \text{Ah} = \left(50.0\text{km}^2\right)\left(40.0\text{m}\right) \\ &= \left[\left(50.0\ \text{km}^2\right)\left(\frac{10^3\text{m}}{1\text{km}}\right)^2\right]\left(40.0\ \text{m}\right) = 2.00 \times 10^9\text{m}^3 \end{aligned}$$

9.4

The density of water ρ from Table 9.1 is $1.000 \times 10^3\text{kg/m}^3$. Substituting V and ρ into the expression for mass gives

$$m = \left(1.00 \times 10^3 \text{kg/m}^3\right)\left(2.00 \times 10^9 \text{m}^3\right)$$
$$= 2.00 \times 10^{12} \text{kg}.$$

9.5

Discussion

A large reservoir contains a very large mass of water. In this example, the weight of the water in the reservoir is $\text{mg} = 1.96 \times 10^{13} \text{N}$, where g is the acceleration due to the Earth's gravity (about 9.80m/s^2). It is reasonable to ask whether the dam must supply a force equal to this tremendous weight. The answer is no. As we shall see in the following sections, the force the dam must supply can be much smaller than the weight of the water it holds back.



Figure 9.4 Three Gorges Dam in central China. When completed in 2008, this became the world's largest hydroelectric plant, generating power equivalent to that generated by 22 average-sized nuclear power plants. The concrete dam is 181 m high and 2.3 km across. The reservoir made by this dam is 660 km long. Over 1 million people were displaced by the creation of the reservoir. Image from OpenStax College Physics 2e, CC-BY 4.0

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9.3 PRESSURE

Learning Objectives

By the end of this section, you will be able to:

- Define pressure.
- Explain the relationship between pressure and force.
- Calculate force given pressure and area.

You have no doubt heard the word pressure being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids. Pressure P is defined as

$$P = \frac{F}{A}$$
9.6

where F is a force applied to an area A that is perpendicular to the force.

Pressure

Pressure is defined as the force divided by the area perpendicular to the force over which the force is applied, or

$$P = \frac{F}{A}.$$

9.7

A given force can have a significantly different effect depending on the area over which the force is exerted, as shown in Figure 9.5. The SI unit for pressure is the *pascal*, where

$$1Pa = 1\text{N}/\text{m}^2.$$

9.8

In addition to the pascal, there are many other units for pressure that are in common use. In meteorology, atmospheric pressure is often described in units of millibar (mb), where

$$100\text{ mb} = 1 \times 10^4\text{Pa}.$$

9.9

Pounds per square inch $\left(\text{lb}/\text{in}^2\text{orpsi}\right)$ is still sometimes used as a measure of tire pressure, and millimeters of mercury (mm Hg) is still often used in the measurement of blood pressure. Pressure is defined for all states of matter but is particularly important when discussing fluids.

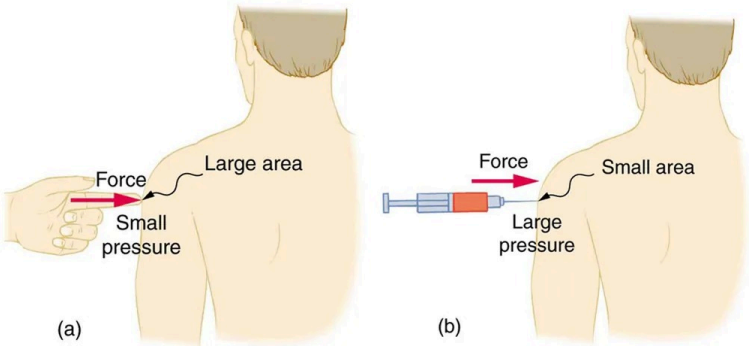


Figure 9.5 (a) While the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two illustrations labeled (a) and (b). Both depict the back view of a person's upper body, specifically focusing on the shoulder area.

Illustration (a):

A hand applies force to the person's shoulder with a flat surface. The force is directed with a red arrow pointing forward. The contact area is labeled "Large area," and the resulting effect is described as "Small pressure."

Illustration (b):

A syringe applies force to the person's shoulder with the needle. The force is also indicated by a red arrow. The contact area is labeled "Small area," and the resulting effect is described as "Large pressure."

Example 9.2

Calculating Force Exerted by the Air: What Force Does a Pressure Exert?

An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank reads $6.90 \times 10^6 \text{Pa}$. What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk 0.150 m in diameter?

Strategy

We can find the force exerted from the definition of pressure given in $P = \frac{F}{A}$, provided we can find the area A acted upon.

Solution

By rearranging the definition of pressure to solve for force, we see that

$$F = \text{PA}.$$

9.10

Here, the pressure P is given, as is the area of the end of the cylinder A, given by $A = \pi r^2$. Thus,

$$\begin{aligned} F &= \left(6.90 \times 10^6 \text{N/m}^2\right) (3.14)(0.0750 \text{ m})^2 \\ &= 1.22 \times 10^5 \text{N}. \end{aligned}$$

9.11

Discussion

Wow! No wonder the tank must be strong. Since we found $F = \mathrm{PA}$, we see that the force exerted by a pressure is directly proportional to the area acted upon as well as the pressure itself.

The force exerted on the end of the tank is perpendicular to its inside surface. This direction is because the force is exerted by a static or stationary fluid. We have already seen that fluids cannot *withstand* shearing (sideways) forces; they cannot *exert* shearing forces, either. Fluid pressure has no direction, being a scalar quantity. The forces due to pressure have well-defined directions: they are always exerted perpendicular to any surface. (See the tire in Figure 9.6, for example.) Finally, note that pressure is exerted on all surfaces. Swimmers, as well as the tire, feel pressure on all sides. (See Figure 9.7.)

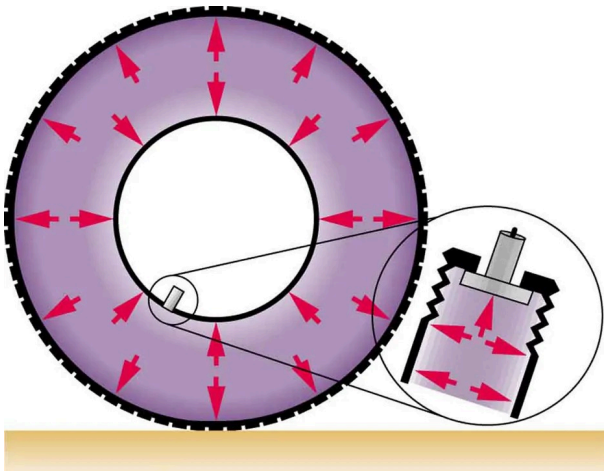


Figure 9.6 Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows give representative directions and magnitudes of the forces exerted at various points. Note that static fluids do not exert shearing forces. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a cross-section of a tire with red arrows pointing inward, indicating the force or pressure being applied uniformly across the inner surface of the tire. There is a magnified inset showing a closer view of the tire valve, with further red arrows emphasizing the pressure inside. The background is a gradient from yellow to white, and the tire has a circular shape with a light purple hue.

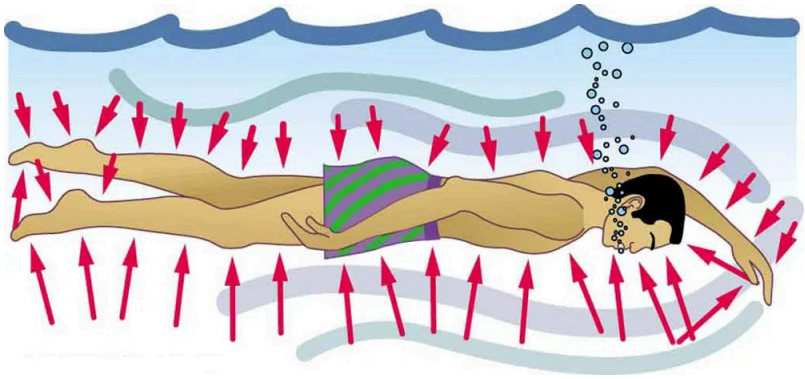


Figure 9.7 Pressure is exerted on all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force that is balanced by the weight of the swimmer. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of a person swimming in water. The swimmer is depicted in a horizontal position, wearing striped swim trunks with green and purple colors. The swimmer's face is underwater, and bubbles are shown around their head, indicating exhalation. Their arms are extended, with one arm reaching forward and the other positioned by their side. The body is streamlined, suggesting a swimming motion.

Above and surrounding the swimmer are large blue waves representing the water's surface and movement. Red arrows are positioned around the swimmer, both above and below the body. The arrows point inwards towards the swimmer, illustrating the buoyant force acting upwards from below the body and some resistance acting from above.

PhET Explorations

Gas Properties

Pump gas molecules to a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other.

Access multimedia content

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9.4 VARIATION OF PRESSURE WITH DEPTH IN A FLUID

Learning Objectives

By the end of this section, you will be able to:

- Define pressure in terms of weight.
- Explain the variation of pressure with depth in a fluid.
- Calculate density given pressure and altitude.

If your ears have ever popped on a plane flight or ached during a deep dive in a swimming pool, you have experienced the effect of depth on pressure in a fluid. At the Earth's surface, the air pressure exerted on you is a result of the weight of air above you. This pressure is reduced as you climb up in altitude and the weight of air above you decreases. Under water, the pressure exerted on you increases with increasing depth. In this case, the pressure being exerted upon you is a result of both the weight of water above you *and* that of the atmosphere above you. You may notice an air pressure change on an elevator ride that transports you many stories, but you need only dive a meter or so below the surface of a pool to feel a pressure increase. The difference is that water is much denser than air, about 775 times as dense.

Consider the container in Figure 9.8. Its bottom supports the weight of the fluid in it. Let us calculate the pressure exerted on the bottom by the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):

$$P = \frac{\mathrm{mg}}{A}.$$

9.12

We can find the mass of the fluid from its volume and density:

$$m = \rho V.$$
9.13

The volume of the fluid V is related to the dimensions of the container. It is

$$V = \mathrm{Ah},$$
9.14

where A is the cross-sectional area and h is the depth. Combining the last two equations gives

$$m = \rho \mathrm{Ah}.$$
9.15

If we enter this into the expression for pressure, we obtain

$$P = \frac{(\rho \mathrm{Ah})g}{A}.$$
9.16

The area cancels, and rearranging the variables yields

$$P = h\rho g.$$
9.17

This value is the *pressure due to the weight of a fluid*. The equation has general validity beyond the special conditions under which it is derived here. Even if the container were not there, the surrounding fluid would still exert this pressure, keeping the fluid static. Thus the equation $P = h\rho g$ represents the pressure due to the weight of any fluid of *average density* ρ at any depth h below its surface. For liquids, which are nearly incompressible, this equation holds to great depths. For gases, which are quite compressible, one can apply this equation as long as the density changes are small over the depth considered. Example 9.4 illustrates this situation.

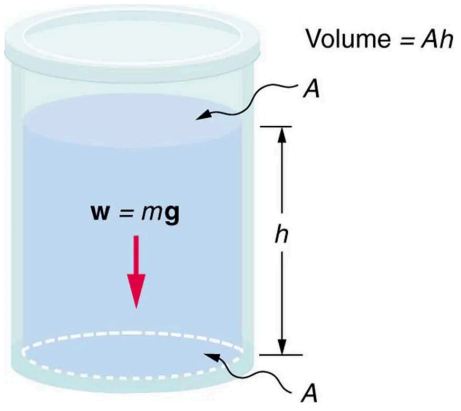


Figure 9.8 The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), and so the bottom must support it all. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of a cylindrical container filled with a liquid. The following elements are indicated:

- The cylindrical container is shown with a light blue liquid inside.
- There is a downward pointing red arrow labeled “w = mg”, representing the weight of the liquid.
- The top and bottom surfaces of the container are labeled as “A”, representing the area of these surfaces.
- The height of the liquid is labeled “h”.
- The right side of the image states “Volume = Ah”, indicating the volume of the liquid inside the cylinder is calculated by multiplying the area of the base by the height.

Example 9.3

Calculating the Average Pressure and Force Exerted: What Force Must a Dam Withstand?

In Example 9.1, we calculated the mass of water in a large reservoir. We will now consider the pressure and force acting on the dam retaining water. (See Figure 9.9.) The dam is 500 m wide, and the water is 80.0 m deep at the dam. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam and compare it with the weight of water in the dam (previously found to be $1.96 \times 10^{13}\text{N}$).

Strategy for (a)

The average pressure $\bar{P}$ due to the weight of the water is the pressure at the average depth $\bar{h}$ of 40.0 m, since pressure increases linearly with depth.

Solution for (a)

The average pressure due to the weight of a fluid is

$$\bar{P} = \bar{h}\rho g.$$

9.18

Entering the density of water from Table 9.1 and taking $\bar{h}$ to be the average depth of 40.0 m, we obtain

$$\begin{aligned}\bar{P} &= (40.0\ \text{m})\left(10^3\frac{\text{kg}}{\text{m}^3}\right)\left(9.80\frac{\text{m}}{\text{s}^2}\right) \\ &= 3.92 \times 10^5\frac{\text{N}}{\text{m}^2} = 392\ \text{kPa}.\end{aligned}$$

9.19

Strategy for (b)

The force exerted on the dam by the water is the average pressure times the area of contact:

$$F = \bar{P}A.$$

9.20

Solution for (b)

We have already found the value for $\bar{P}$. The area of the dam is $A = 80.0\ \text{m} \times 500\ \text{m} = 4.00 \times 10^4 \text{m}^2$, so that

$$\begin{aligned} F &= (3.92 \times 10^5 \text{N/m}^2)\,(4.00 \times 10^4 \text{m}^2) \\ &= 1.57 \times 10^{10} \text{N}. \end{aligned}$$

9.21

Discussion

Although this force seems large, it is small compared with the $1.96 \times 10^{13}\text{N}$ weight of the water in the reservoir—in fact, it is only

0.0800% of the weight. Note that the pressure found in part (a) is completely independent of the width and length of the lake—it depends only on its average depth at the dam. Thus the force depends only on the water's average depth and the dimensions of the dam, *not* on the horizontal extent of the reservoir. In the diagram, the thickness of the dam increases with depth to balance the increasing force due to the increasing pressure.

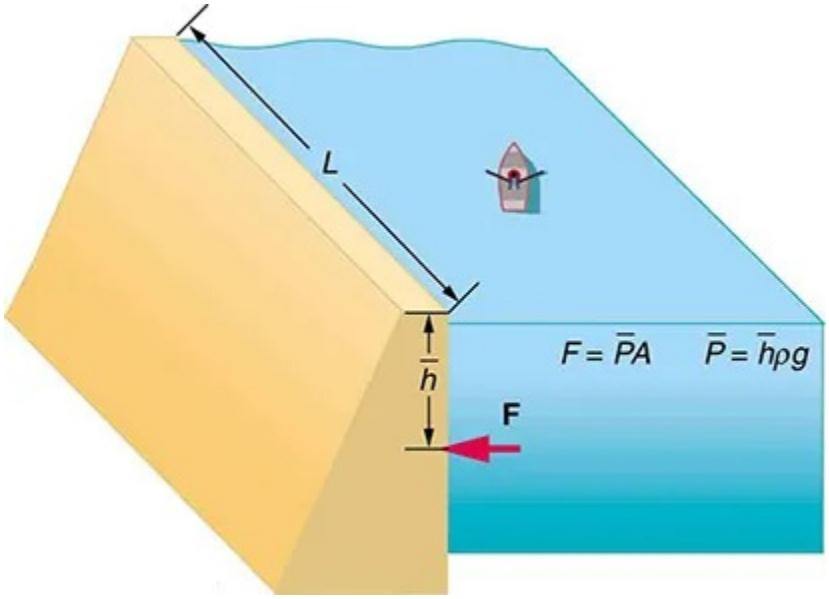


Figure 9.9 The dam must withstand the force exerted against it by the water it retains. This force is small compared with the weight of the water behind the dam. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of a dam and a body of water. On the left side, there is a triangular cross-section of a dam shown in yellow, which is slanted towards the right. The body of water is shown in blue against the right side of the dam.

The image includes the following labeled elements:

- A line labeled "L" showing the length of the water against the dam.
- A vertical line labeled "h" indicating the depth of water from the surface to the base of the dam.
- An arrow marked "F" pointing horizontally from the water towards the dam, indicating the force exerted by the water on the dam.

The image also contains two equations:

- $F = \overline{P}A$, representing the force as the product of pressure and area.
- $\overline{P} = hpg$, indicating pressure as the product of height, density of the fluid (p), and acceleration due to gravity (g).

A small boat is depicted floating on the surface of the water, slightly to the right of the dam structure.

Atmospheric pressure is another example of pressure due to the weight of a fluid, in this case due to the weight

of *air* above a given height. The atmospheric pressure at the Earth's surface varies a little due to the large-scale flow of the atmosphere induced by the Earth's rotation (this creates weather "highs" and "lows"). However, the average pressure at sea level is given by the *standard atmospheric pressure* P_{atm}, measured to be

$$1 \text{ atmosphere (atm)} = P_{\text{atm}} = 1.01 \times 10^5 \text{N/m}^2 = 101 \text{ kPa}. \tag{9.22}$$

This relationship means that, on average, at sea level, a column of air above 1.00m^2 of the Earth's surface has a weight of $1.01 \times 10^5\text{N}$, equivalent to 1 atm. (See Figure 9.10.)

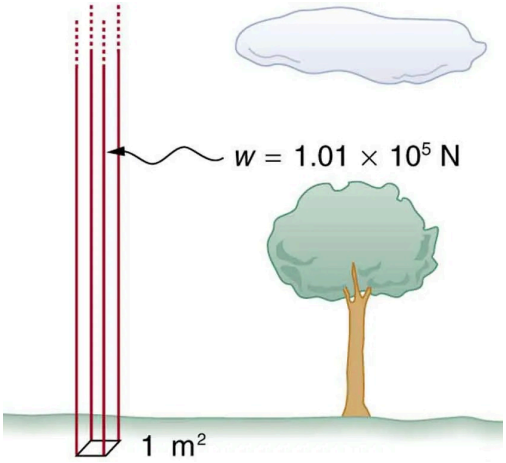


Figure 9.10 Atmospheric pressure at sea level averages $1.01 \times 10^5\text{Pa}$ *(equivalent to 1 atm), since the column of air over this* 1m^2*, extending to the top of the atmosphere, weighs* 1.01×10^5 N. *Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a diagram illustrating atmospheric pressure. On the left side, there is a vertical stack of red lines, representing the weight of the atmosphere over a specific area. Below this stack is a 3D box drawn on the ground, labeled as covering an area of "1 m^2". An arrow points from these lines to an equation, indicating that the weight (w) is equal to "1.01 × 10^5 N" (Newtons).

On the right side of the image, there is a tree and a cloud. The presence of these natural elements suggests that the concept relates to the point that atmospheric pressure is exerted over all surfaces, including trees and the ground.

Example 9.4

Calculating Average Density: How Dense Is the Air?

Calculate the average density of the atmosphere, given that it extends to an altitude of 120 km. Compare this density with that of air listed in Table 9.1.

Strategy

If we solve $P = h\rho g$ for density, we see that

$$\bar{\rho} = \frac{P}{\text{hg}}.$$

9.23

We then take P to be atmospheric pressure, h is given, and g is known, and so we can use this to calculate $\bar{\rho}$.

Solution

Entering known values into the expression for $\bar{\rho}$ yields

$$\bar{\rho} = \frac{1.01 \times 10^5 \text{N/m}^2}{(120 \times 10^3 \text{m})\,(9.80 \text{m/s}^2)} = 8.59 \times 10^{-2} \text{kg/m}^3.$$

9.24

Discussion

This result is the average density of air between the Earth's surface and the top of the Earth's atmosphere, which essentially ends at 120 km. The density of air at sea level is given in Table 9.1 as 1.29kg/m^3 —about 15 times its average value. Because air is so

compressible, its density has its highest value near the Earth's surface and declines rapidly with altitude.

Example 9.5

Calculating Depth Below the Surface of Water: What Depth of Water Creates the Same Pressure as the Entire Atmosphere?

Calculate the depth below the surface of water at which the pressure due to the weight of the water equals 1.00 atm.

Strategy

We begin by solving the equation $P = h\rho g$ for depth h:

$$h = \frac{P}{\rho g}.$$

9.25

Then we take P to be 1.00 atm and ρ to be the density of the water that creates the pressure.

Solution

Entering the known values into the expression for h gives

$$h = \frac{1.01 \times 10^5 \text{N/m}^2}{(1.00 \times 10^3 \text{kg/m}^3)\,(9.80 \text{m/s}^2)} = 10.3 \text{m}.$$

9.26

Discussion

Just 10.3 m of water creates the same pressure as 120 km of air. Since water is nearly incompressible, we can neglect any change in its density over this depth.

What do you suppose is the *total* pressure at a depth of 10.3 m in a swimming pool? Does the atmospheric pressure on the water's surface affect the pressure below? The answer is yes. This seems only logical, since both the water's weight and the atmosphere's weight must be supported. So the *total* pressure at a depth of 10.3 m is 2 atm—half from the water above and half from the air above. We shall see in 9.5 Pascal's Principle that fluid pressures always add in this way.

"11.4 Variation of Pressure with Depth in a Fluid" from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

9.5 PASCAL'S PRINCIPLE

Learning Objectives

By the end of this section, you will be able to:

- Define pressure.
- State Pascal's principle.
- Understand applications of Pascal's principle.
- Derive relationships between forces in a hydraulic system.

Pressure is defined as force per unit area. Can pressure be increased in a fluid by pushing directly on the fluid? Yes, but it is much easier if the fluid is enclosed. The heart, for example, increases blood pressure by pushing directly on the blood in an enclosed system (valves closed in a chamber). If you try to push on a fluid in an open system, such as a river, the fluid flows away. An enclosed fluid cannot flow away, and so pressure is more easily increased by an applied force.

What happens to a pressure in an enclosed fluid? Since atoms in a fluid are free to move about, they transmit the pressure to all parts of the fluid and to the walls of the container. Remarkably, the pressure is transmitted *undiminished*. This phenomenon is called Pascal's principle, because it was first clearly stated by the French philosopher and scientist Blaise Pascal (1623–1662): A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Pascal's Principle

A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Pascal's principle, an experimentally verified fact, is what makes pressure so important in fluids. Since a change in pressure is transmitted undiminished in an enclosed fluid, we often know more about pressure than other physical quantities in fluids. Moreover, Pascal's principle implies that *the total pressure in a fluid is the sum of the pressures from different sources*. We shall find this fact—that pressures add—very useful.

Blaise Pascal had an interesting life in that he was home-schooled by his father who removed all of the mathematics textbooks from his house and forbade him to study mathematics until the age of 15. This, of course, raised the boy's curiosity, and by the age of 12, he started to teach himself geometry. Despite this early deprivation, Pascal went on to make major contributions in the mathematical fields of probability theory, number theory, and geometry. He is also well known for being the inventor of the first mechanical digital calculator, in addition to his contributions in the field of fluid statics.

Application of Pascal's Principle

One of the most important technological applications of Pascal's principle is found in a *hydraulic system*, which is an enclosed fluid system used to exert forces. The most common hydraulic systems are those that operate car brakes. Let us first consider the simple hydraulic system shown in Figure 9.11.

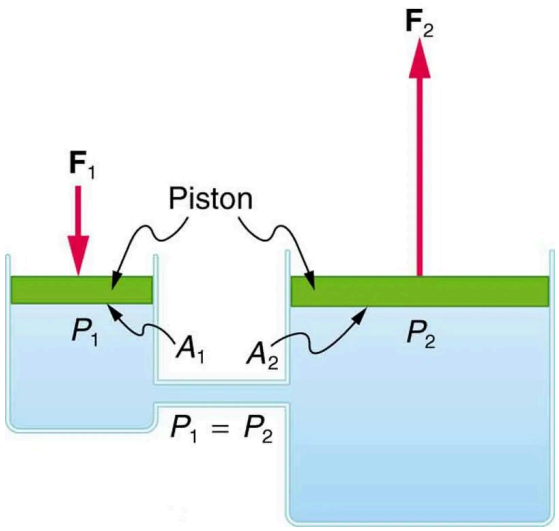


Figure 9.11 A typical hydraulic system with two fluid-filled cylinders, capped with pistons and connected by a tube called a hydraulic line. A downward force $\mathbf{F}_1$ *on the left piston creates a pressure that is transmitted undiminished to all parts of the enclosed fluid. This results in an upward force* $\mathbf{F}_2$ *on the right piston that is larger than* $\mathbf{F}_1$ *because the right piston has a larger area. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a cross-section of a hydraulic system illustrating Pascal's Principle. There are two connected chambers filled with fluid. Each chamber has a piston on top, represented by green rectangles.

The left chamber shows a force labeled "F1" with an arrow pointing downwards onto the piston with the area labeled "A1". The pressure in this chamber is labeled "P1".

The right chamber shows a force labeled "F2" with an arrow pointing upwards from the piston with the area labeled "A2". The pressure here is labeled "P2".

The equation "P1 = P2" indicates the principle that pressure is equal throughout the fluid system. Arrows indicate the flow and equalization of pressure from one chamber to the other.

Relationship Between Forces in a Hydraulic System

We can derive a relationship between the forces in the simple hydraulic system shown in Figure 9.11 by applying Pascal's principle. Note first that the two pistons in the system are at the same height, and so there will be no difference in pressure due to a difference in depth. Now the pressure due to F_1 acting on area A_1 is simply $P_1 = \frac{F_1}{A_1}$, as defined by $P = \frac{F}{A}$. According to Pascal's principle, this pressure is transmitted

undiminished throughout the fluid and to all walls of the container. Thus, a pressure P_2 is felt at the other piston that is equal to P_1. That is $P_1 = P_2$.

But since $P_2 = \frac{F_2}{A_2}$, we see that $\frac{F_1}{A_1} = \frac{F_2}{A_2}$.

This equation relates the ratios of force to area in any hydraulic system, providing the pistons are at the same vertical height and that friction in the system is negligible. Hydraulic systems can increase or decrease the force applied to them. To make the force larger, the pressure is applied to a larger area. For example, if a 100-N force is applied to the left cylinder in Figure 9.11 and the right one has an area five times greater, then the force out is 500 N. Hydraulic systems are analogous to simple levers, but they have the advantage that pressure can be sent through tortuously curved lines to several places at once.

Example 9.6

Calculating Force of Wheel Cylinders: Pascal Puts on the Brakes

Consider the automobile hydraulic system shown in Figure 9.12.

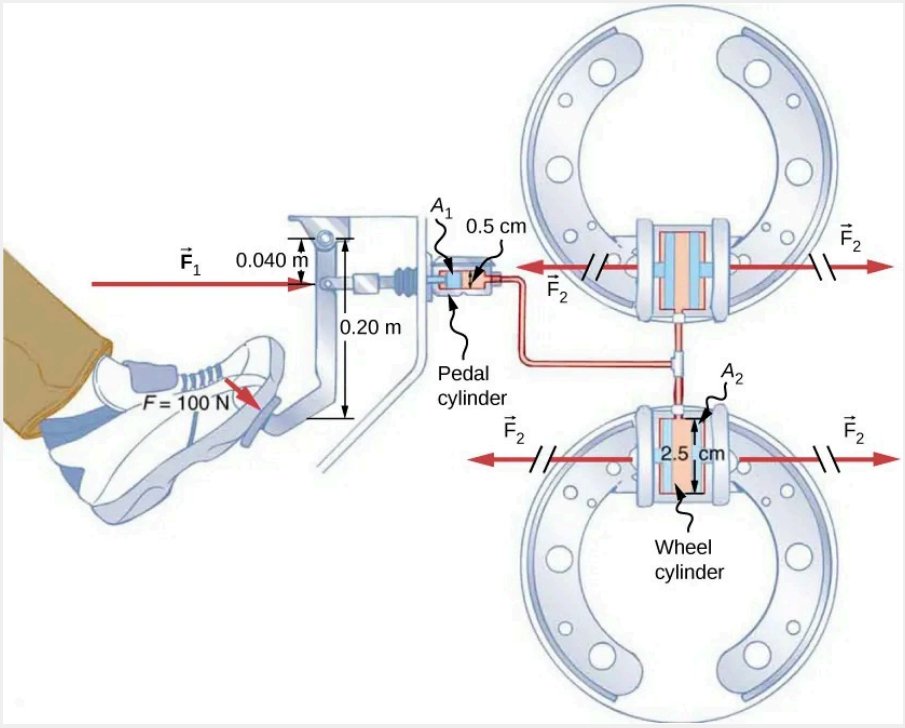


Figure 9.12 Hydraulic brakes use Pascal's principle. The driver exerts a force of 100 N on the brake pedal. This force is increased by the simple lever and again by the hydraulic system. Each of the identical wheel cylinders receives the same pressure and, therefore, creates the same force output F_2. The circular cross-sectional areas of the pedal and wheel cylinders are represented by A_1 and A_2, respectively Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a hydraulic braking system in operation. On the left side of the image,

a foot is pressing down on a brake pedal, applying a force of F = 100 N. This force is represented by a red arrow labeled F_1, pointing to the right. A lever mechanism is shown, where the distance from the pedal to the pivot point is marked as 0.20 meters, and the distance from the pivot to the hydraulic connection is 0.040 meters.

The force F_1 is transmitted through a pedal cylinder with a cross-sectional area of A_1 = 0.5 cm. A line connects this pedal cylinder to a wheel cylinder. The wheel cylinder has a larger cross-sectional area of A_2 = 2.5 cm and is positioned near two drums, each of which represents a wheel brake.

Force from the pedal cylinder is converted and applied as force F_2 (also represented by red arrows) at the wheel brakes. The image illustrates how the hydraulic system amplifies the force applied by the foot to increase the braking power on the wheels.

A force of 100 N is applied to the brake pedal, which acts on the pedal cylinder through a lever. A force of 500 N is exerted on the pedal cylinder. (The reader can verify that the force is 500 N using techniques of statics from 7.4 Applications of Statics, Including Problem-Solving Strategies.) Pressure created in the pedal cylinder is transmitted to four wheel cylinders. The pedal cylinder has a diameter of 0.500 cm, and each wheel cylinder has a diameter of 2.50 cm. Calculate the force F_2 created at each of the wheel cylinders.

Strategy

We are given the force F_1 that is applied to the pedal cylinder. The cross-sectional areas A_1 and A_2 can be calculated from their given diameters. Then $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ can be used to find the force F_2. Manipulate this algebraically to get F_2 on one side and substitute known values:

Solution

Pascal's principle applied to hydraulic systems is given by $\frac{F_1}{A_1} = \frac{F_2}{A_2}$:

$$F_2 = \frac{A_2}{A_1}$$

$$\begin{aligned} F_1 &= \frac{\pi r_2^2}{\pi r_1^2} \\ &= \frac{(1.25cm)^2}{(0.250cm)^2} \times 500 \text{ N} \\ &= 1.25 \times 10^4 \text{N}. \end{aligned}$$

9.27

Discussion

This value is the force exerted by each of the four wheel cylinders. Note that we can add as many wheel cylinders as we wish. If each has a 2.50-cm diameter, each will exert $1.25 \times 10^4 \text{N}$.

A simple hydraulic system, such as a simple machine, can increase force but cannot do more work than done on it. Work is force times distance moved, and the wheel cylinder moves through a smaller distance than the pedal cylinder. Furthermore, the more wheels added, the smaller the distance each moves. Many hydraulic systems—such as power brakes and those in bulldozers—have a motorized pump that actually does most of the work in the system. The movement of the legs of a spider is achieved partly by hydraulics. Using hydraulics, a jumping spider can create a force that makes it capable of jumping 25 times its length!

Making Connections: Conservation of Energy

Conservation of energy applied to a hydraulic system tells us that the system cannot do more work than is done on it. Work transfers energy, and so the work output cannot exceed the work input. Power brakes and other similar hydraulic systems use pumps to supply extra energy when needed.

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9.6 GAUGE PRESSURE, ABSOLUTE PRESSURE, AND PRESSURE MEASUREMENT

Learning Objectives

By the end of this section, you will be able to:

- Define gauge pressure and absolute pressure.
- Understand the working of aneroid and open-tube barometers.

If you limp into a gas station with a nearly flat tire, you will notice the tire gauge on the airline reads nearly zero when you begin to fill it. In fact, if there were a gaping hole in your tire, the gauge would read zero, even though atmospheric pressure exists in the tire. Why does the gauge read zero? There is no mystery here. Tire gauges are simply designed to read zero at atmospheric pressure and positive when pressure is greater than atmospheric.

Similarly, atmospheric pressure adds to blood pressure in every part of the circulatory system. (As noted in 9.5 Pascal's Principle, the total pressure in a fluid is the sum of the pressures from different sources—here, the heart and the atmosphere.) But atmospheric pressure has no net effect on blood flow since it adds to the pressure coming out of the heart and going back into it, too. What is important is how much *greater* blood pressure is than atmospheric pressure. Blood pressure measurements, like tire pressures, are thus made relative to atmospheric pressure.

In brief, it is very common for pressure gauges to ignore atmospheric pressure—that is, to read zero at atmospheric pressure. We therefore define gauge pressure to be the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

Gauge Pressure

Gauge pressure is the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

In fact, atmospheric pressure does add to the pressure in any fluid not enclosed in a rigid container. This happens because of Pascal's principle. The total pressure, or absolute pressure, is thus the sum of gauge pressure and atmospheric pressure: $P_{\text{abs}} = P_{\text{g}} + P_{\text{atm}}$ where P_{abs} is absolute pressure, P_{g} is gauge pressure, and P_{atm} is atmospheric pressure. For example, if your tire gauge reads 34 psi (pounds per square inch), then the absolute pressure is 34 psi plus 14.7 psi (P_{atm} in psi), or 48.7 psi (equivalent to 336 kPa).

Absolute Pressure

Absolute pressure is the sum of gauge pressure and atmospheric pressure.

For reasons we will explore later, in most cases the absolute pressure in fluids cannot be negative. Fluids push rather than pull, so the smallest absolute pressure is zero. (A negative absolute pressure is a pull.) Thus the smallest possible gauge pressure is $P_{\text{g}} = -P_{\text{atm}}$ (this makes P_{abs} zero). There is no theoretical limit to how large a gauge pressure can be.

There are a host of devices for measuring pressure, ranging from tire gauges to blood pressure cuffs. Pascal's principle is of major importance in these devices. The undiminished transmission of pressure through a fluid allows precise remote sensing of pressures. Remote sensing is often more convenient than putting a measuring device into a system, such as a person's artery.

Figure 9.13 shows one of the many types of mechanical pressure gauges in use today. In all mechanical pressure gauges, pressure results in a force that is converted (or transduced) into some type of readout.

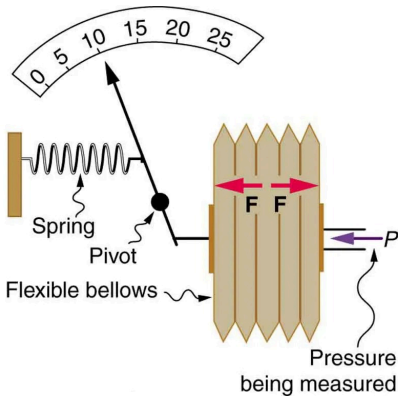


Figure 9.13 This aneroid gauge utilizes flexible bellows connected to a mechanical indicator to measure pressure. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a schematic diagram of a pressure gauge mechanism using a flexible bellows system. Key components are labeled as follows:

- A flexible bellows on the right side, which expands and contracts when pressure is applied, labeled "Pressure being measured" with an arrow pointing to the right labeled "P".
- Two red arrows on the bellows indicated as "F" show the direction of force upon the bellows due to pressure.
- A pivot point, at which the bellows is connected to a lever arm. The pivot translates the motion of the bellows to a spring mechanism.
- The spring is located on the left side and is under tension. It is labeled "Spring" and is connected to the pivoted arm.
- An arrow labeled "Pivot" points to the point of rotation or the pivot itself.
- A lever arm extends upwards from the pivot point to a semi-circular dial gauge. The arm's motion causes a needle to move across the gauge.
- The dial gauge at the top is marked from 0 to 25 in increments of 5, indicating the pressure reading.

An entire class of gauges uses the property that pressure due to the weight of a fluid is given by $P = h\rho g$. Consider the U-shaped tube shown in Figure 9.14, for example. This simple tube is called a *manometer*. In Figure 9.14(a), both sides of the tube are open to the atmosphere. Atmospheric pressure therefore pushes down on each side equally so its effect cancels. If the fluid is deeper on one side, there is a greater pressure on the deeper side, and the fluid flows away from that side until the depths are equal.

Let us examine how a manometer is used to measure pressure. Suppose one side of the U-tube is connected to some source of pressure P_{abs} such as the toy balloon in Figure 9.14(b) or the vacuum-packed peanut jar shown in Figure 9.14(c). Pressure is transmitted undiminished to the manometer, and the fluid levels are no longer equal. In Figure 9.14(b), P_{abs} is greater than atmospheric pressure, whereas in Figure 9.14(c), P_{abs} is less than atmospheric pressure. In both cases, P_{abs} differs from atmospheric pressure by an amount $h\rho g$, where ρ is the density of the fluid in the manometer. In Figure 9.14(b), P_{abs} can support a column of fluid of height h, and so it must exert a pressure $h\rho g$ greater than atmospheric pressure (the gauge pressure P_{g} is positive). In Figure 9.14(c), atmospheric pressure can support a column of fluid of height h, and so P_{abs} is less than atmospheric pressure by an amount $h\rho g$ (the gauge pressure P_{g} is negative). A manometer with one side open to the atmosphere is an ideal device for measuring gauge pressures. The gauge pressure is $P_{\text{g}} = h\rho g$ and is found by measuring h.

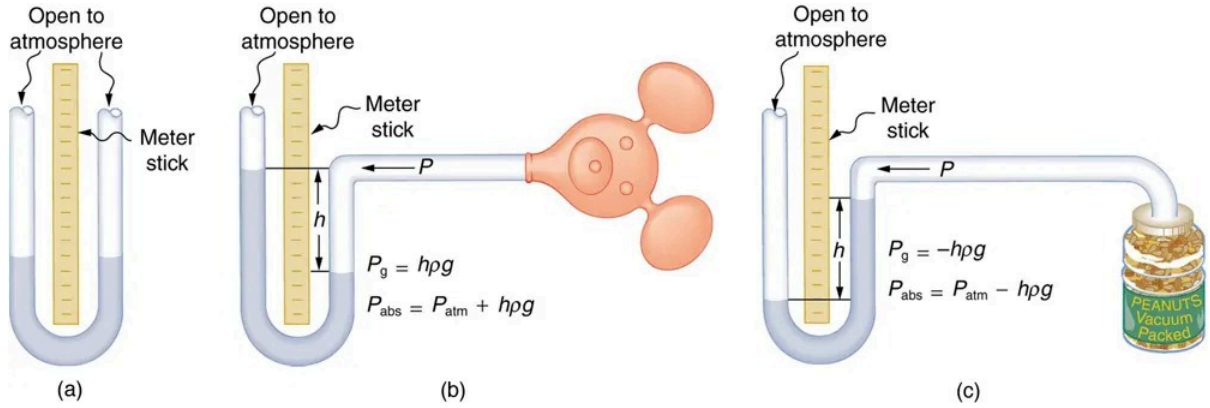


Figure 9.14 An open-tube manometer has one side open to the atmosphere. (a) Fluid depth must be the same on both sides, or the pressure each side exerts at the bottom will be unequal and there will be flow from the deeper side. (b) A positive gauge pressure $P_{\text{g}} = h\rho g$ transmitted to one side of the manometer can support a column of fluid of height h. (c) Similarly, atmospheric pressure is greater than a negative gauge pressure P_{g} by an amount $h\rho g$. The jar's rigidity prevents atmospheric pressure from being transmitted to the peanuts. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts three diagrams (a, b, and c) illustrating the concept of pressure measurement using a manometer, with different setups:

(a) A simple U-tube manometer. The tube is open to the atmosphere on both sides. A meter stick is placed next to the U-tube, measuring the height difference *h* of the liquid levels in the two arms.

(b) A manometer connected to a bulb on one side. The left side remains open to the atmosphere with a meter

stick measuring height h. The diagram shows the pressure equation: $P_g = h\rho g$ and $P_{abs} = P_{atm} + h\rho g$, indicating gauge and absolute pressures, respectively.

(c) A manometer connected to a vacuum-packed jar of peanuts. The left side is open to the atmosphere, again with a meter stick and height h noted. The pressure equations are $P_g = -h\rho g$ and $P_{abs} = P_{atm} - h\rho g$, showing how gauge and absolute pressures differ in this setup.

Mercury manometers are often used to measure arterial blood pressure. An inflatable cuff is placed on the upper arm as shown in Figure 9.15. By squeezing the bulb, the person making the measurement exerts pressure, which is transmitted undiminished to both the main artery in the arm and the manometer. When this applied pressure exceeds blood pressure, blood flow below the cuff is cut off. The person making the measurement then slowly lowers the applied pressure and listens for blood flow to resume. Blood pressure pulsates because of the pumping action of the heart, reaching a maximum, called systolic pressure, and a minimum, called diastolic pressure, with each heartbeat. Systolic pressure is measured by noting the value of h when blood flow first begins as cuff pressure is lowered. Diastolic pressure is measured by noting h when blood flows without interruption. The typical blood pressure of a young adult raises the mercury to a height of 120 mm at systolic and 80 mm at diastolic. This is commonly quoted as 120 over 80, or 120/80. The first pressure is representative of the maximum output of the heart; the second is due to the elasticity of the arteries in maintaining the pressure between beats. The density of the mercury fluid in the manometer is 13.6 times greater than water, so the height of the fluid will be 1/13.6 of that in a water manometer. This reduced height can make measurements difficult, so mercury manometers are used to measure larger pressures, such as blood pressure. The density of mercury is such that $1.0 mmHg = 133 \text{Pa}$.

Systolic Pressure

Systolic pressure is the maximum blood pressure.

Diastolic Pressure

Diastolic pressure is the minimum blood pressure.



Figure 9.15 In routine blood pressure measurements, an inflatable cuff is placed on the upper arm at the same level as the heart. Blood flow is detected just below the cuff, and corresponding pressures are transmitted to a mercury-filled manometer. Image from OpenStax College Physics 2e, CC-BY 4.0

Example 9.7

Calculating Height of IV Bag: Blood Pressure and Intravenous Infusions

Intravenous infusions are usually made with the help of the gravitational force. Assuming that the density of the fluid being administered is 1.00 g/ml, at what height should the IV bag be placed above the entry point so that the fluid just enters the vein if the blood pressure in the vein is 18 mm Hg above atmospheric pressure? Assume that the IV bag is collapsible.

Strategy for (a)

For the fluid to just enter the vein, its pressure at entry must exceed the blood pressure in the vein (18 mm Hg above atmospheric pressure). We therefore need to find the height of fluid that corresponds to this gauge pressure.

Solution

We first need to convert the pressure into SI units. Since $1.0 mmHg = 133\ \text{Pa}$,

$$P = 18\ \text{mm Hg} \times \frac{133\ \text{Pa}}{1.0 mmHg} = 2400\ \text{Pa}.$$

9.28

Rearranging $P_{\text{g}} = h\rho g$ for h gives $h = \frac{P_{\text{g}}}{\rho g}$. Substituting known values into this equation gives

$$h = \frac{2400\ \text{N/m}^2}{\left(1.0 \times 10^3 \text{kg/m}^3\right)\left(9.80 \text{m/s}^2\right)} = 0.24\ \text{m}.$$

9.29

Discussion

The IV bag must be placed at 0.24 m above the entry point into the arm for the fluid to just enter the arm. Generally, IV bags are placed higher than this. You may have noticed that the bags used for blood collection are placed below the donor to allow blood to flow easily from the arm to the bag, which is the opposite direction of flow than required in the example presented here.

A *barometer* is a device that measures atmospheric pressure. A mercury barometer is shown in Figure 9.16. This device measures atmospheric pressure, rather than gauge pressure, because there is a nearly pure vacuum above the mercury in the tube. The height of the mercury is such that $h\rho g = P_{\text{atm}}$. When atmospheric pressure varies, the mercury rises or falls, giving important clues to weather forecasters. The barometer can also be used as an altimeter, since average atmospheric pressure varies with altitude. Mercury barometers and manometers are so common that units of mm Hg are often quoted for atmospheric pressure and blood pressures. Table 9.2 gives conversion factors for some of the more commonly used units of pressure.

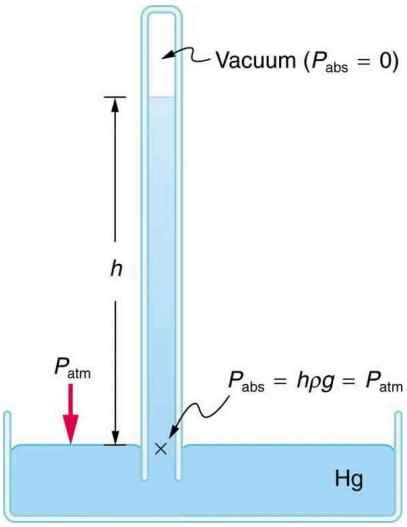


Figure 9.16 A mercury barometer measures atmospheric pressure. The pressure due to the mercury's weight, $h\rho g$, equals atmospheric pressure. The atmosphere is able to force mercury in the tube to a height h because the pressure above the mercury is zero. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a mercury barometer used to measure atmospheric pressure. It consists of a tall, inverted glass tube filled with mercury and partially emptied at the top to create a vacuum. The diagram includes the following elements:

- The glass tube contains mercury with the open end submerged in a pool of mercury labeled “Hg.”
- The top portion of the tube contains a vacuum, labeled “Vacuum ($P_{abs} = 0$).” P_{abs} refers to the absolute pressure.
- The height of the mercury column inside the tube is marked “h.”
- An arrow and label “P_{atm}” indicate atmospheric pressure, pressing down on the mercury reservoir outside the tube.
- A label inside the tube at the mercury surface says “$P_{abs} = hp_g = P_{atm}$,” showing the relationship between pressure, gravity, and height.

Table 9.2 Conversion Factors for Various Pressure Units

Conversion to N/m^2 (Pa)	Conversion from atm
$1.0 atm = 1.013 \times 10^5 \text{N/m}^2$	$1.0 atm = 1.013 \times 10^5 \text{N/m}^2$
$1.0 \text{dyne/cm}^2 = 0.10 \text{N/m}^2$	$1.0 \text{atm} = 1.013 \times 10^6 \text{dyne/cm}^2$
$1.0 \text{kg/cm}^2 = 9.8 \times 10^4 \text{N/m}^2$	$1.0 \text{atm} = 1.013 \text{kg/cm}^2$
$1.0 \text{lb/in.}^2 = 6.90 \times 10^3 \text{N/m}^2$	$1.0 \text{atm} = 14.7 \text{lb/in.}^2$
$1.0 mmHg = 133 \text{N/m}^2$	$1.0 \text{atm} = 760 \text{ mm Hg}$
$1.0 cmHg = 1.33 \times 10^3 \text{N/m}^2$	$1.0 \text{atm} = 76.0 cmHg$
$1.0 cmwater = 98.1 \text{N/m}^2$	$1.0 \text{atm} = 1.03 \times 10^3 \text{cm water}$
$1.0 bar = 1.000 \times 10^5 \text{N/m}^2$	$1.0 \text{atm} = 1.013 bar$
$1.0 millibar = 1.000 \times 10^2 \text{N/m}^2$	$1.0 atm = 1013 \text{ millibar}$

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9.7 ARCHIMEDES' PRINCIPLE

Learning Objectives

By the end of this section, you will be able to:

- Define buoyant force.
- State Archimedes' principle.
- Understand why objects float or sink.
- Understand the relationship between density and Archimedes' principle.

When you rise from lounging in a warm bath, your arms feel strangely heavy. This is because you no longer have the buoyant support of the water. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected? (See Figure 9.17.)

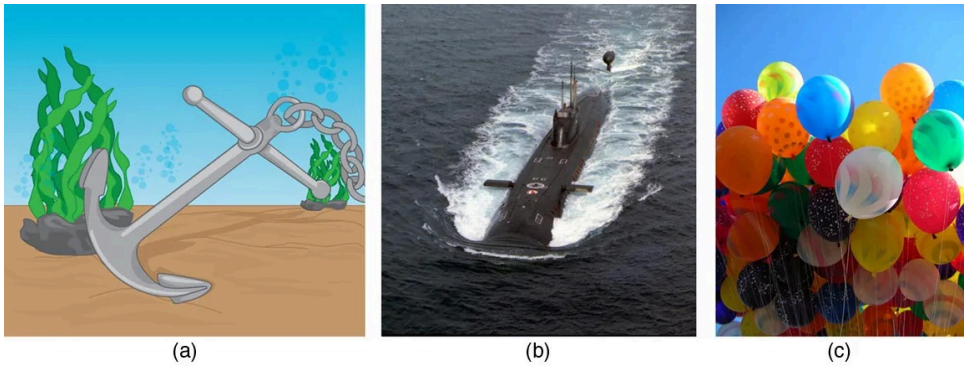


Figure 9.17 (a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (credit: Allied Navy) (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. Image from OpenStax College Physics 2e, CC-BY 4.0

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward, or buoyant force on any object in any fluid. (See Figure 9.18.) If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.

Buoyant Force

The buoyant force is the net upward force on any object in any fluid.

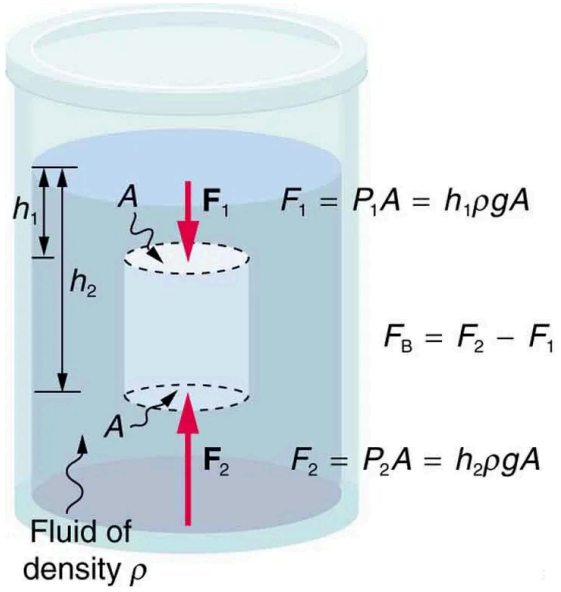


Figure 9.18 Pressure due to the weight of a fluid increases with depth since $P = h\rho g$. *This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. Their difference is the buoyant force* $\mathbf{F}_{\mathrm{B}}$. *Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image illustrates a cylindrical object submerged in a fluid within a container, demonstrating buoyancy

concepts. The fluid has a density denoted by ρ. Two arrows represent forces acting on the object: F_1, a downward force, and F_2, an upward force.

The following variables and equations are present in the image:

- h_1 – represents the height from the surface of the fluid to the top of the object.
- h_2 – represents the height from the surface of the fluid to the bottom of the object.
- A – represents the cross-sectional area of the object.
- The equation for force F_1: $F_1 = P_1A = h_1\rho gA$, where g is the acceleration due to gravity.
- The equation for force F_2: $F_2 = P_2A = h_2\rho gA$.
- The buoyant force equation is given as: $F_B = F_2 - F_1$.

Just how great is this buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in Figure 9.19.

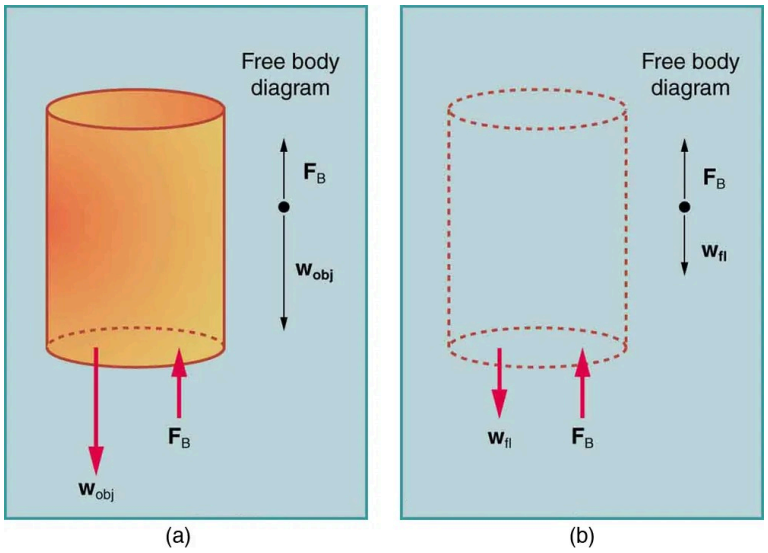


Figure 9.19 (a) An object submerged in a fluid experiences a buoyant force F_B. *If* F_B *is greater than the weight of the object, the object will rise. If* F_B *is less than the weight of the object, the object will sink. (b) If the object is removed, it is replaced by fluid having weight* w_{fl}. *Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced. That is,* $F_B = w_{fl}$,*a statement of Archimedes' principle. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image contains two parts labeled (a) and (b), illustrating free body diagrams:

– Image (a): A cylindrical object is depicted, filled with an orange gradient to indicate it is solid. Two forces

act on the object:
– A downward arrow labeled "w_{obj}" representing the weight of the object.
– An upward arrow labeled "F_B" indicating the buoyant force.

– Image (b): Represents an imaginary cylinder with dashed lines indicating that it is filled with fluid, rather than being solid. Two forces are present:
– A downward arrow labeled "w_{fl}" denoting the weight of the fluid.
– An upward arrow labeled "F_B" for the buoyant force.

Both diagrams have the label "Free body diagram" with arrows indicating vertical forces either acting upwards or downwards on the cylinder.

The space it occupied is filled by fluid having a weight w_{fl}. This weight is supported by the surrounding fluid, and so the buoyant force must equal w_{fl}, the weight of the fluid displaced by the object. It is a tribute to the genius of the Greek mathematician and inventor Archimedes (ca. 287–212 B.C.) that he stated this principle long before concepts of force were well established. Stated in words, Archimedes' principle is as follows: The buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_{\text{B}} = w_{\text{fl}},$$

9.30

where F_{B} is the buoyant force and w_{fl} is the weight of the fluid displaced by the object. Archimedes' principle is valid in general, for any object in any fluid, whether partially or totally submerged.

Archimedes' Principle

According to this principle the buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_{\text{B}} = w_{\text{fl}},$$

9.31

where F_{B} is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

Humm ... High-tech body swimsuits were introduced in 2008 in preparation for the Beijing Olympics. One concern (and international rule) was that these suits should not provide any buoyancy advantage. How do you think that this rule could be verified?

Making Connections: Take-Home Investigation

The density of aluminum foil is 2.7 times the density of water. Take a piece of foil, roll it up into a ball and drop it into water. Does it sink? Why or why not? Can you make it sink?

Floating and Sinking

Drop a lump of clay in water. It will sink. Then mold the lump of clay into the shape of a boat, and it will float. Because of its shape, the boat displaces more water than the lump and experiences a greater buoyant force. The same is true of steel ships.

Example 9.8

Calculating buoyant force: dependency on shape

(a) Calculate the buoyant force on 10,000 metric tons $(1.00 \times 10^7 \text{kg})$ of solid steel completely submerged in water, and compare this with the steel's weight. (b) What is the maximum buoyant force that water could exert on this same steel if it were shaped into a boat that could displace $1.00 \times 10^5 \text{m}^3$ of water?

Strategy for (a)

To find the buoyant force, we must find the weight of water displaced. We can do this by using the densities of water and steel given in Table 9.1. We note that, since the steel is completely submerged, its volume and the water's volume are the same. Once we know the volume of water, we can find its mass and weight.

Solution for (a)

First, we use the definition of density $\rho = \frac{m}{V}$ to find the steel's volume, and then we substitute values for mass and density. This gives

$$V_{\text{st}} = \frac{m_{\text{st}}}{\rho_{\text{st}}} = \frac{1.00 \times 10^7 \text{kg}}{7.8 \times 10^3 \text{kg/m}^3} = 1.28 \times 10^3 \text{m}^3.$$

9.32

Because the steel is completely submerged, this is also the volume of water displaced, V_{w}. We can now find the mass of water displaced from the relationship between its volume and density, both of which are known. This gives

$$\begin{aligned} m_w &= \rho_w V_w \\ &= (1.000 \times 10^3 \text{kg/m}^3)\,(1.28 \times 10^3 m^3) \\ &= 1.28 \times 10^6 \text{kg}. \end{aligned}$$

9.33

By Archimedes' principle, the weight of water displaced is $m_{\mathrm{w}}g$, so the buoyant force is

$$\begin{aligned} F_B &= w_w \\ &= m_w g \\ &= \left(1.28 \times 10^6 \mathrm{kg}\right)\left(9.80 \mathrm{m/s^2}\right) \\ &= 1.3 \times 10^7 \mathrm{N}. \end{aligned}$$

9.34

The steel's weight is $m_{\mathrm{s}}w = 9.80 \times 10^7 \mathrm{N}$, which is much greater than the buoyant force, so the steel will remain submerged. Note that the buoyant force is rounded to two digits because the density of steel is given to only two digits.

Strategy for (b)

Here we are given the maximum volume of water the steel boat can displace. The buoyant force is the weight of this volume of water.

Solution for (b)

The mass of water displaced is found from its relationship to density and volume, both of which are known. That is,

$$\begin{aligned} m_w &= \rho_w V_w \\ &= \left(1.000 \times 10^3 \mathrm{kg/m^3}\right)\left(1.00 \times 10^5 m^3\right) \\ &= 1.00 \times 10^8 \mathrm{kg}. \end{aligned}$$

9.35

The maximum buoyant force is the weight of this much water, or

$$\begin{aligned} F_B &= w_w \\ &= m_w g \\ &= \left(1.00 \times 10^8 \mathrm{kg}\right)\left(9.80 \mathrm{m/s^2}\right) \\ &= 9.80 \times 10^8 \mathrm{N}. \end{aligned}$$

9.36

Discussion

The maximum buoyant force is ten times the weight of the steel, meaning the ship can carry a load nine times its own weight without sinking.

Making Connections: Take-Home Investigation

A piece of household aluminum foil is 0.016 mm thick. Use a piece of foil that measures 10 cm by 15 cm. (a) What is the mass of this amount of foil? (b) If the foil is folded to give it four sides, and paper clips or washers are added to this "boat," what shape of the boat would allow it to hold the most "cargo" when placed in water? Test your prediction.

Density and Archimedes' Principle

Density plays a crucial role in Archimedes' principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object's density is related to that of the fluid. In Figure 9.20, for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

$$\text{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{\text{fl}}}{V_{\text{obj}}}.$$

9.37

The volume submerged equals the volume of fluid displaced, which we call V_{fl}. Now we can obtain the relationship between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}}/\rho_{\text{fl}}}{m_{\text{obj}}/(\bar{\rho})_{\text{obj}}},$$

9.38

where $(\bar{\rho})_{\text{obj}}$ is the average density of the object and ρ_{fl} is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{(\bar{\rho})_{\text{obj}}}{\rho_{\text{fl}}}.$$

9.39

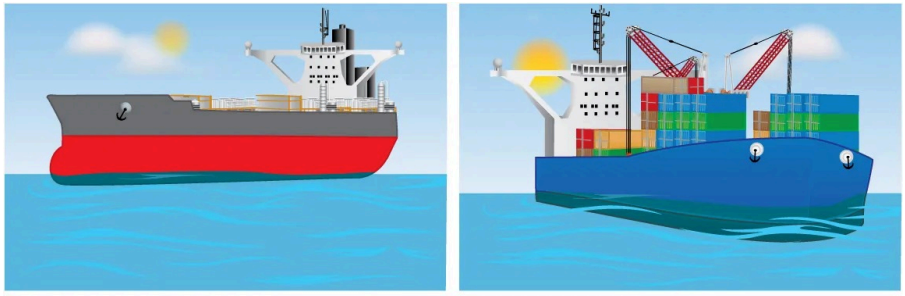


Figure 9.20 An unloaded ship (a) floats higher in the water than a loaded ship Image from OpenStax College Physics 2e, CC-BY 4.0

We use this last relationship to measure densities. This is done by measuring the fraction of a floating object that is submerged—for example, with a hydrometer. It is useful to define the ratio of the density of an object to a fluid (usually water) as specific gravity:

$$\text{specific gravity} = \frac{\bar{\rho}}{\rho_{\text{w}}},$$

9.40

where $\bar{\rho}$ is the average density of the object or substance and ρ_{w} is the density of water at 4.00°C. Specific gravity is dimensionless, independent of whatever units are used for ρ. If an object floats, its specific gravity is less than one. If it sinks, its specific gravity is greater than one. Moreover, the fraction of a floating object that is submerged equals its specific gravity. If an object's specific gravity is exactly 1, then it will remain suspended

in the fluid, neither sinking nor floating. Scuba divers try to obtain this state so that they can hover in the water. We measure the specific gravity of fluids, such as battery acid, radiator fluid, and urine, as an indicator of their condition. One device for measuring specific gravity is shown in Figure 9.21.

Specific Gravity

Specific gravity is the ratio of the density of an object to a fluid (usually water).

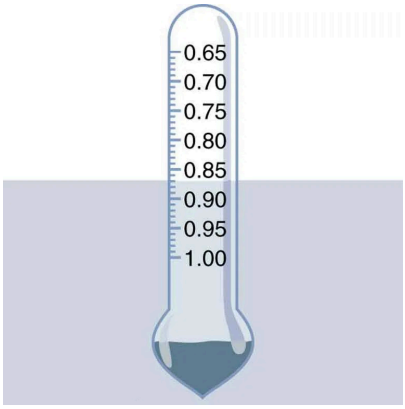


Figure 9.21 This hydrometer is floating in a fluid of specific gravity 0.87. The glass hydrometer is filled with air and weighted with lead at the bottom. It floats highest in the densest fluids and has been calibrated and labeled so that specific gravity can be read from it directly. Image from OpenStax College Physics 2e, CC-BY 4.0

Example 9.9

Calculating Average Density: Floating Woman

Suppose a 60.0-kg woman floats in freshwater with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

Strategy

We can find the woman's density by solving the equation

$$\text{fraction submerged} = \frac{(\bar{\rho})_{\text{obj}}}{\rho_{\text{fl}}}$$

9.41

for the density of the object. This yields

$$(\bar{\rho})_{\text{obj}} = (\bar{\rho})_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}.$$

9.42

We know both the fraction submerged and the density of water, and so we can calculate the woman's density.

Solution

Entering the known values into the expression for her density, we obtain

$$(\bar{\rho})_{\text{person}} = 0.970 \cdot \left(10^3 \frac{\text{kg}}{\text{m}^3}\right) = 970 \frac{\text{kg}}{\text{m}^3}.$$

9.43

Discussion

Her density is less than the fluid density. We expect this because she floats. Body density is one indicator of a person's percent body fat, of interest in medical diagnostics and athletic training. (See Figure 9.22.)

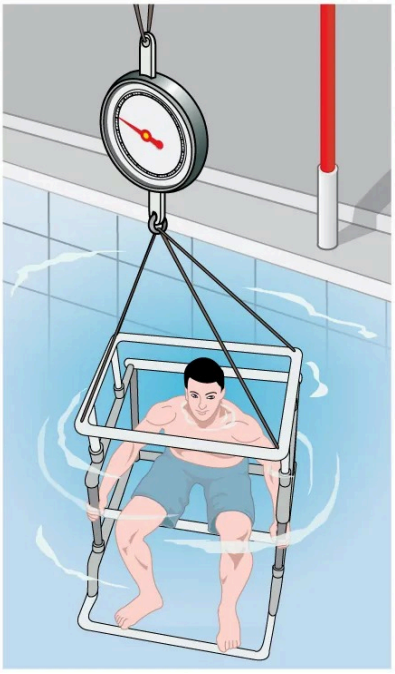


Figure 9.22 Subject in a hydrostatis weighing device, where they are weighed while completely submerged as part of a body density determination. The subject must completely empty their lungs and hold a metal weight in order to sink. Corrections are made for the residual air in the lungs (measured separately) and the metal weight. Their corrected submerged weight, their weight in air, and pinch tests of strategic fatty areas are used to calculate the percent body fat. Image from OpenStax College Physics 2e, CC-BY 4.0

There are many obvious examples of lower-density objects or substances floating in higher-density fluids—oil on water, a hot-air balloon, a bit of cork in wine, an iceberg, and hot wax in a "lava lamp," to name a few. Less obvious examples include lava rising in a volcano and mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

More Density Measurements

One of the most common techniques for determining density is shown in Figure 9.23.

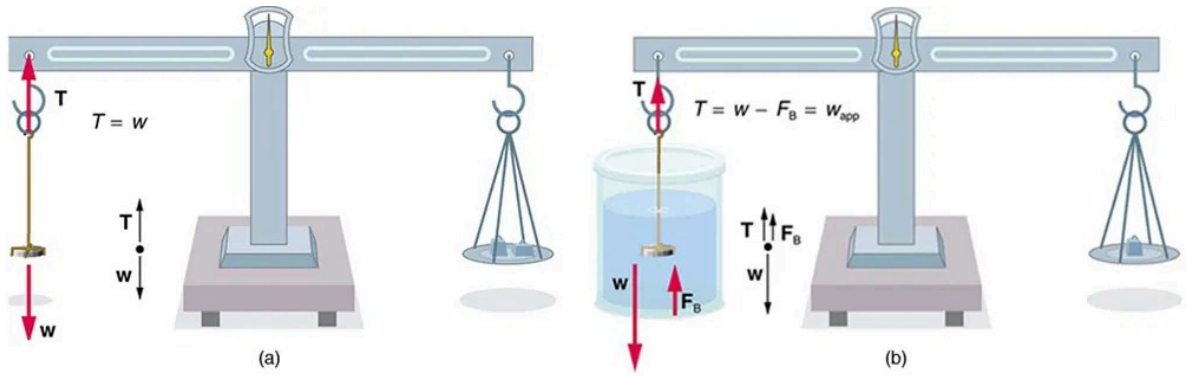


Figure 9.23 (a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image consists of two illustrations side by side, labeled (a) and (b), depicting a balance mechanism with a scale and an object hanging on one side.

Illustration (a): A balance scale with a crossbeam supports two pans. The left pan has an object hanging from it, represented by a downward arrow labeled "w" for weight. The upward tension in the string is labeled "T". The text on the image reads "T = w", indicating the tension equals the weight. The balance is shown in equilibrium.

Illustration (b): Similar balance scale setup as in illustration (a), but the object is submerged in a liquid within a container on the left pan. An additional upward force, labeled "F_B", represents the buoyant force. The tension is labeled "T", and the object's weight is "w". The text reads "$T = w - F_B = w_{app}$", indicating the apparent weight equals the weight minus the buoyant force. The balance remains in equilibrium.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. This same technique can also be used to determine the density of the fluid if the density of the coin is known. All of these calculations are based on Archimedes' principle.

Archimedes' principle states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object *appears* to weigh less when submerged; we call this measurement the

object's *apparent weight*. The object suffers an *apparent weight loss* equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an *apparent mass loss* equal to the mass of fluid displaced. That is

$$\text{apparent weight loss} = \text{weight of fluid displaced}$$
9.44

or

$$\text{apparent mass loss} = \text{mass of fluid displaced.}$$
9.45

The next example illustrates the use of this technique.

Example 9.10

Calculating Density: Is the Coin Authentic?

The mass of an ancient Greek coin is determined in air to be 8.630 g. When the coin is submerged in water as shown in Figure 9.23, its apparent mass is 7.800 g. Calculate its density, given that water has a density of 1.000g/cm^3 and that effects caused by the wire suspending the coin are negligible.

Strategy

To calculate the coin's density, we need its mass (which is given) and its volume. The volume of the coin equals the volume of water displaced. The volume of water displaced V_w can be found by solving the equation for density $\rho = \frac{m}{V}$ for V.

Solution

The volume of water is $V_\text{w} = \frac{m_\text{w}}{\rho_\text{w}}$ where m_w is the mass of water displaced. As noted, the mass of the water displaced equals the apparent mass loss, which is $m_\text{w} = 8.630 \text{ g} - 7.800 \text{ g} = 0.830 \text{ g}$. Thus the volume of water is $V_\text{w} = \frac{0.830 \text{ g}}{1.000 \text{ g/cm}^3} = 0.830 \text{cm}^3$. This is also the volume of the coin, since it is completely submerged. We can now find the density of the coin using the definition of density:

$$\rho_\text{c} = \frac{m_\text{c}}{V_c} = \frac{8.630 \text{ g}}{0.830 \text{ cm}^3} = 10.4 g/\text{cm}^3.$$

9.46

Discussion

You can see from Table 9.1 that this density is very close to that of pure silver, appropriate for this type of ancient coin. Most modern counterfeits are not pure silver.

This brings us back to Archimedes' principle and how it came into being. As the story goes, the king of Syracuse gave Archimedes the task of determining whether the royal crown maker was supplying a crown of pure gold. The purity of gold is difficult to determine by color (it can be diluted with other metals and still look as yellow as pure gold), and other analytical techniques had not yet been conceived. Even ancient peoples, however, realized that the density of gold was greater than that of any other then-known substance. Archimedes purportedly agonized over his task and had his inspiration one day while at the public baths, pondering the support the water gave his body. He came up with his now-famous principle, saw how to apply it to determine density, and ran naked down the streets of Syracuse crying "Eureka!" (Greek for "I have found it"). Similar behavior can be observed in contemporary physicists from time to time!

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9.8 COHESION AND ADHESION IN LIQUIDS: SURFACE TENSION AND CAPILLARY ACTION

Learning Objectives

By the end of this section, you will be able to:

- Understand cohesive and adhesive forces.
- Define surface tension.
- Understand capillary action.

Cohesion and Adhesion in Liquids

Children blow soap bubbles and play in the spray of a sprinkler on a hot summer day. (See Figure 9.24.) An underwater spider keeps his air supply in a shiny bubble he carries wrapped around him. A technician draws blood into a small-diameter tube just by touching it to a drop on a pricked finger. A premature infant struggles to inflate her lungs. What is the common thread? All these activities are dominated by the attractive forces between atoms and molecules in liquids—both within a liquid and between the liquid and its surroundings.

Attractive forces between molecules of the same type are called cohesive forces. Liquids can, for example, be held in open containers because cohesive forces hold the molecules together. Attractive forces between molecules of different types are called adhesive forces. Such forces cause liquid drops to cling to window panes, for example. In this section we examine effects directly attributable to cohesive and adhesive forces in liquids.

Cohesive Forces

Attractive forces between molecules of the same type are called cohesive forces.

Adhesive Forces

Attractive forces between molecules of different types are called adhesive forces.

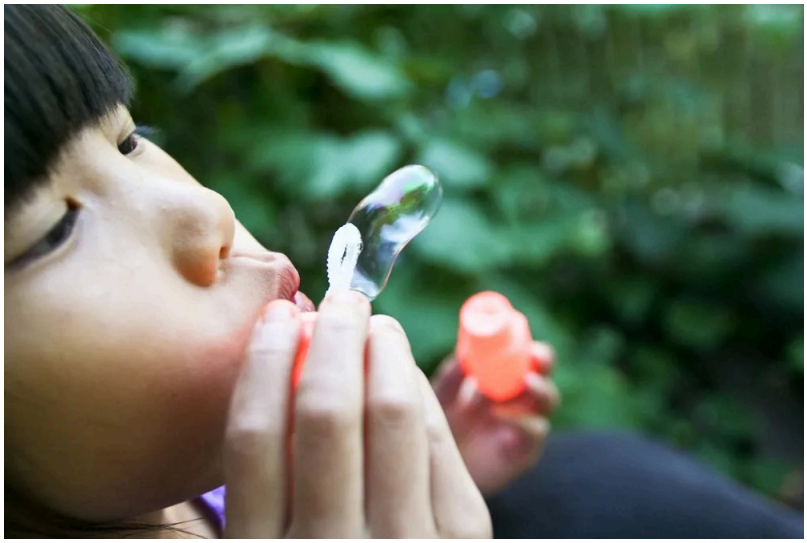


Figure 9.24 The soap bubbles in this photograph are caused by cohesive forces among molecules in liquids. Image from OpenStax College Physics 2e, CC-BY 4.0

Surface Tension

Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension. Molecules on the surface are pulled inward by cohesive forces, reducing the surface area. Molecules inside the liquid experience zero net force, since they have neighbors on all sides.

Surface Tension

Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.

Making Connections: Surface Tension

Forces between atoms and molecules underlie the macroscopic effect called surface tension. These attractive forces pull the molecules closer together and tend to minimize the surface area. This is another example of a submicroscopic explanation for a macroscopic phenomenon.

The model of a liquid surface acting like a stretched elastic sheet can effectively explain surface tension effects. For example, some insects can walk on water (as opposed to floating in it) as we would walk on a trampoline—they dent the surface as shown in Figure 9.25(a). Figure 9.25(b) shows another example, where a needle rests on a water surface. The iron needle cannot, and does not, float, because its density is greater than that of water. Rather, its weight is supported by forces in the stretched surface that try to make the surface smaller or flatter. If the needle were placed point down on the surface, its weight acting on a smaller area would break the surface, and it would sink.

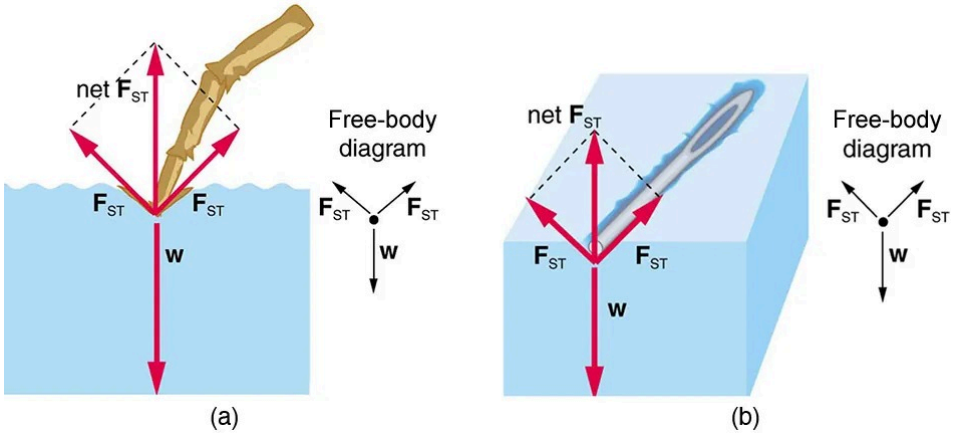


Figure 9.25 Surface tension supporting the weight of an insect and an iron needle, both of which rest on the surface without penetrating it. They are not floating; rather, they are supported by the surface of the liquid. (a) An insect leg dents the water surface. F_{ST} *is a restoring force (surface tension) parallel to the surface. (b) An iron needle similarly dents a water surface until the restoring force (surface tension) grows to equal its weight. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two diagrams labeled (a) and (b), each illustrating the concept of surface tension with the use of arrows and free-body diagrams.

Diagram (a):

- A leaf is placed on the surface of a liquid, with arrows indicating forces acting on it.
- The arrows labeled F_{ST} point outward and upward from the leaf, indicating surface tension forces.
- An arrow labeled w points downwards, indicating the weight of the leaf.
- A dashed arrow labeled net F_{ST} points vertically upward, representing the net surface tension force.
- The accompanying free-body diagram shows three arrows: two labeled F_{ST} diagonally upward and outward, and one labeled w downward.

Diagram (b):

- A liquid surface is depicted with a disturbance, possibly created by an object.
- Similar to diagram (a), arrows labeled F_{ST} point outwards and upwards from the contact line.
- An arrow labeled w points downwards, showing the weight of the object or disturbance.
- A dashed arrow labeled net F_{ST} points vertically upward.
- The free-body diagram mirrors the one in (a) with the same force labels and directions.

Surface tension is proportional to the strength of the cohesive force, which varies with the type of liquid. Surface tension γ is defined to be the force F per unit length L exerted by a stretched liquid membrane:

$$\gamma = \frac{F}{L}.$$
9.47

Table 9.3 lists values of γ for some liquids. For the insect of Figure 9.25(a), its weight w is supported by the upward components of the surface tension force: $w = \gamma L \sin\theta$, where L is the circumference of the insect's foot in contact with the water. Figure 9.26 shows one way to measure surface tension. The liquid film exerts a force on the movable wire in an attempt to reduce its surface area. The magnitude of this force depends on the surface tension of the liquid and can be measured accurately.

Surface tension is the reason why liquids form bubbles and droplets. The inward surface tension force causes bubbles to be approximately spherical and raises the pressure of the gas trapped inside relative to atmospheric pressure outside. It can be shown that the gauge pressure P inside a spherical bubble is given by

$$P = \frac{4\gamma}{r},$$
9.48

where r is the radius of the bubble. Thus the pressure inside a bubble is greatest when the bubble is the smallest. Another bit of evidence for this is illustrated in Figure 9.27. When air is allowed to flow between two balloons of unequal size, the smaller balloon tends to collapse, filling the larger balloon.

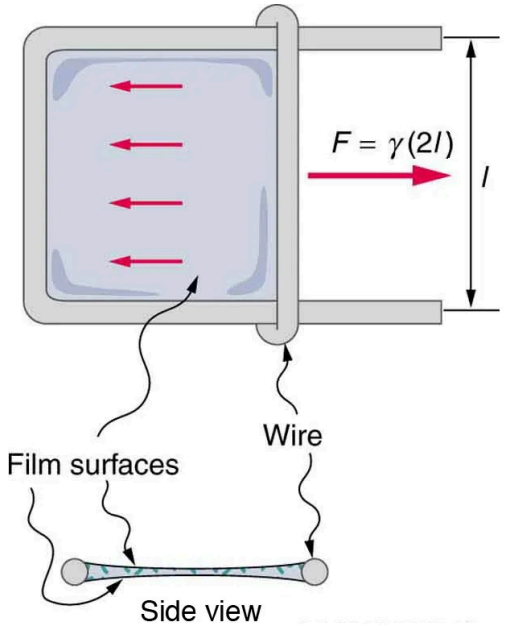


Figure 9.26 Sliding wire device used for measuring surface tension; the device exerts a force to reduce the film's surface area. The force needed to hold the wire in place is $F = \gamma L = \gamma\,(2l\,)$*, since there are two liquid surfaces attached to the wire. This force remains nearly constant as the film is stretched, until the film approaches its breaking point. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

This image depicts a scientific diagram focusing on concepts related to surface tension and force. The main part of the diagram shows a rectangular frame with a movable wire at the top. Inside the frame, there is a liquid film illustrated with a light gray color and several red arrows indicating force direction. This section is annotated with the formula "F = γ(2l)," where "F" represents force, "γ" represents surface tension, and "l" is the length of the wire. The arrows indicate that the force is being applied horizontally towards the right.

Below the main diagram, there is a side view illustration. This part shows the wire and the film surfaces from the side, further explaining the setup. The term "Film surfaces" is labeled alongside two thin lines representing the liquid film. The "Wire" is marked, showing its position in relation to the film.

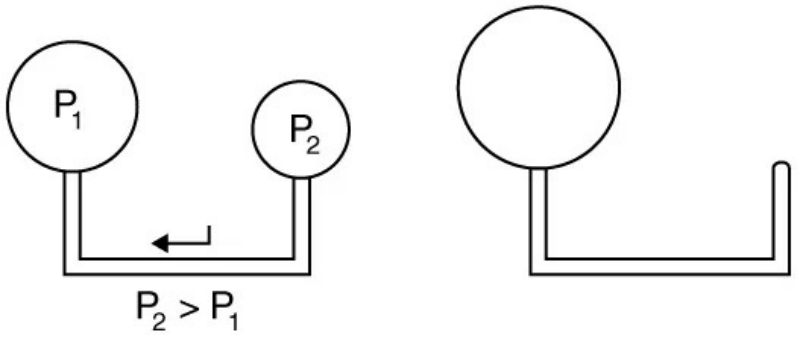


Figure 9.27 With the valve closed, two balloons of different sizes are attached to each end of a tube. Upon opening the valve, the smaller balloon decreases in size with the air moving to fill the larger balloon. The pressure in a spherical balloon is inversely proportional to its radius, so that the smaller balloon has a greater internal pressure than the larger balloon, resulting in this flow. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams illustrating fluid pressure concepts:

1. Left Diagram:
– Two circles labeled “P1” and “P2” are connected by a pipe in a U-shape.
– “P2” is on the right and is higher than “P1,” indicating a higher pressure (P2 > P1).
– An arrow shows the direction of flow from “P2” to “P1” along the pipe.

2. Right Diagram:
– A single larger circle is connected to a U-shaped pipe.
– This part of the illustration represents a single reservoir but does not indicate pressure differences or flow direction.

Table 9.3 Surface Tension of Some Liquids[1]

Liquid	Surface tension γ(N/m)
Water at 0° C	0.0756
Water at 20° C	0.0728
Water at 100° C	0.0589
Soapy water (typical)	0.0370
Ethyl alcohol	0.0223
Glycerin	0.0631
Mercury	0.465
Olive oil	0.032
Tissue fluids (typical)	0.050
Blood, whole at 37° C	0.058
Blood plasma at 37° C	0.073
Gold at 1070° C	1.000
Oxygen at -193° C	0.0157
Helium at -269° C	0.00012

1. At 20ºC unless otherwise stated.

Example 9.11

Surface Tension: Pressure Inside a Bubble

Calculate the gauge pressure inside a soap bubble $2.00 \times 10^{-4}\text{m}$ in radius using the surface tension for soapy water in Table 9.3. Convert this pressure to mm Hg.

Strategy

The radius is given and the surface tension can be found in Table 9.3, and so P can be found directly from the equation $P = \frac{4\gamma}{r}$.

Solution

Substituting r and γ into the equation $P = \frac{4\gamma}{r}$, we obtain

$$P = \frac{4\gamma}{r} = \frac{4\,(0.037N/m\,)}{2.00 \times 10^{-4}\text{m}} = 740\text{N/m}^2 = 740\text{Pa}.$$

9.49

We use a conversion factor to get this into units of mm Hg:

$$P = \left(740\ \text{N/m}^2\right)\frac{1.00mmHg}{133\text{N/m}^2} = 5.56mmHg.$$

9.50

Discussion

Note that if a hole were to be made in the bubble, the air would be forced out, the bubble would decrease in radius, and the gauge pressure would reduce to zero, and the absolute pressure inside would *decrease* to atmospheric pressure (760 mm Hg).

Our lungs contain hundreds of millions of mucus-lined sacs called *alveoli*, which are very similar in size, and about 0.1 mm in diameter. (See Figure 9.28.) You can exhale without muscle action by allowing surface tension to contract these sacs. Medical patients whose breathing is aided by a positive pressure respirator have air blown into the lungs, but are generally allowed to exhale on their own. Even if there is paralysis, surface tension in the alveoli will expel air from the lungs. Since pressure increases as the radii of the alveoli decrease, an occasional deep cleansing breath is needed to fully reinflate the alveoli. Respirators are programmed to do this and we find it natural, as do our companion dogs and cats, to take a cleansing breath before settling into a nap.

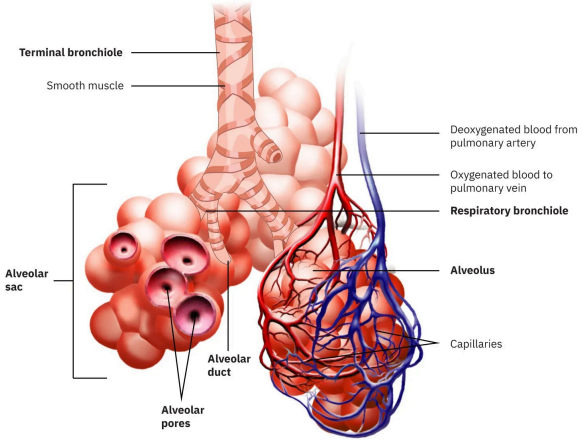


Figure 9.28 Bronchial tubes in the lungs branch into ever-smaller structures, finally ending in alveoli. The alveoli act like tiny bubbles. The surface tension of their mucous lining aids in exhalation and can prevent inhalation if too great. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a detailed medical illustration of the respiratory system's alveolar region. It depicts the microscopic structures involved in the exchange of gases in the lungs. Key components are labeled to show their function and position.

- **Terminal bronchiole:** A small airway that leads to the alveolar structures, shown at the top left. It has smooth muscle indicated around it.
- **Alveolar sac:** A cluster of alveoli, which are small air sacs where gas exchange occurs. This sac is shown on the left side of the image.
- **Alveolar duct:** Connects the respiratory bronchioles to the alveolar sacs. It is depicted proceeding into the alveolar sac.
- **Alveolar pores:** Small openings between alveoli allowing for equalization of pressure and gas exchange between alveoli. These are highlighted within the alveolar sac.

- **Deoxygenated blood from pulmonary artery:** Shown as blue vessels, it represents the blood coming to the lungs to receive oxygen.
- **Oxygenated blood to pulmonary vein:** Shown as red vessels, it represents the blood leaving the lungs after oxygenation.
- **Respiratory bronchiole:** Located at the center right, this structure connects bronchioles to alveoli.
- **Alveolus:** The individual air sacs within the alveolar sac are labeled to show the site of gas exchange.
- **Capillaries:** Thin blood vessels shown entangled with the alveoli, facilitating the exchange of gases.

The illustration uses color coding to differentiate between oxygenated and deoxygenated blood, with red and blue colors respectively, and highlights the intricate network of capillaries surrounding the alveoli.

The tension in the walls of the alveoli results from the membrane tissue and a liquid on the walls of the alveoli containing a long lipoprotein that acts as a surfactant (a surface-tension reducing substance). The need for the surfactant results from the tendency of small alveoli to collapse and the air to fill into the larger alveoli making them even larger (as demonstrated in Figure 9.27). During inhalation, the lipoprotein molecules are pulled apart and the wall tension increases as the radius increases (increased surface tension). During exhalation, the molecules slide back together and the surface tension decreases, helping to prevent a collapse of the alveoli. The surfactant therefore serves to change the wall tension so that small alveoli don't collapse and large alveoli are prevented from expanding too much. This tension change is a unique property of these surfactants, and is not shared by detergents (which simply lower surface tension). (See Figure 9.29.)

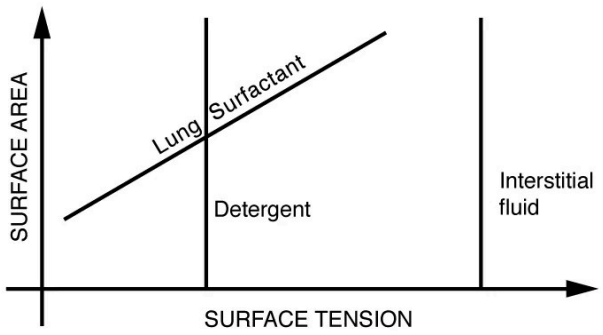


Figure 9.29 Surface tension as a function of surface area. The surface tension for lung surfactant decreases with decreasing area. This ensures that small alveoli don't collapse and large alveoli are not able to over expand. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image is a graph depicting the relationship between surface area and surface tension. The x-axis is labeled "SURFACE TENSION" and the y-axis is labeled "SURFACE AREA." There are three lines shown, each with a different slope, representing different substances:

- **Lung Surfactant:** A diagonal line with a positive slope, indicating an increase in surface area as surface tension increases.
- **Detergent:** A vertical line, showing no change in surface area with a change in surface tension.
- **Interstitial Fluid:** Another vertical line on the right, similar to the detergent line, representing a constant surface area with varying surface tension.

If water gets into the lungs, the surface tension is too great and you cannot inhale. This is a severe problem in resuscitating drowning victims. A similar problem occurs in newborn infants who are born without this surfactant—their lungs are very difficult to inflate. This condition is known as *hyaline membrane disease* and is a leading cause of death for infants, particularly in premature births. Some success has been achieved in treating hyaline membrane disease by spraying a surfactant into the infant's breathing passages. Emphysema produces the opposite problem with alveoli. Alveolar walls of emphysema victims deteriorate, and the sacs combine to form larger sacs. Because pressure produced by surface tension decreases with increasing radius, these larger sacs produce smaller pressure, reducing the ability of emphysema victims to exhale. A common test for emphysema is to measure the pressure and volume of air that can be exhaled.

Making Connections: Take-Home Investigation

(1) Try floating a sewing needle on water. In order for this activity to work, the needle needs to be very clean as even the oil from your fingers can be sufficient to affect the surface properties of the needle. (2) Place the bristles of a paint brush into water. Pull the brush out and notice that for a short while, the bristles will stick together. The surface tension of the water surrounding the bristles is sufficient to hold the bristles together. As the bristles dry out, the surface tension effect dissipates. (3) Place a loop of thread on the surface of still water in such a way that all of the thread is in contact with the water. Note the shape of the loop. Now place a drop of detergent into the middle of the loop. What happens to the shape of the loop? Why? (4) Sprinkle pepper onto the surface of water. Add a drop of detergent. What happens? Why? (5) Float two matches parallel to each other and add a drop of detergent between them. What happens? Note: For each new experiment, the water needs to be replaced and the bowl washed to free it of any residual detergent.

Adhesion and Capillary Action

Why is it that water beads up on a waxed car but does not on bare paint? The answer is that the adhesive forces between water and wax are much smaller than those between water and paint. Competition between the forces of adhesion and cohesion are important in the macroscopic behavior of liquids. An important factor in studying the roles of these two forces is the angle θ between the tangent to the liquid surface and the surface. (See Figure 9.30.) The contact angle θ is directly related to the relative strength of the cohesive and adhesive forces. The larger the strength of the cohesive force relative to the adhesive force, the larger θ is, and the more the liquid tends to form a droplet. The smaller θ is, the smaller the relative strength, so that the adhesive force is able to flatten the drop. Table 9.4 lists contact angles for several combinations of liquids and solids.

Contact Angle

The angle θ between the tangent to the liquid surface and the surface is called the contact angle.

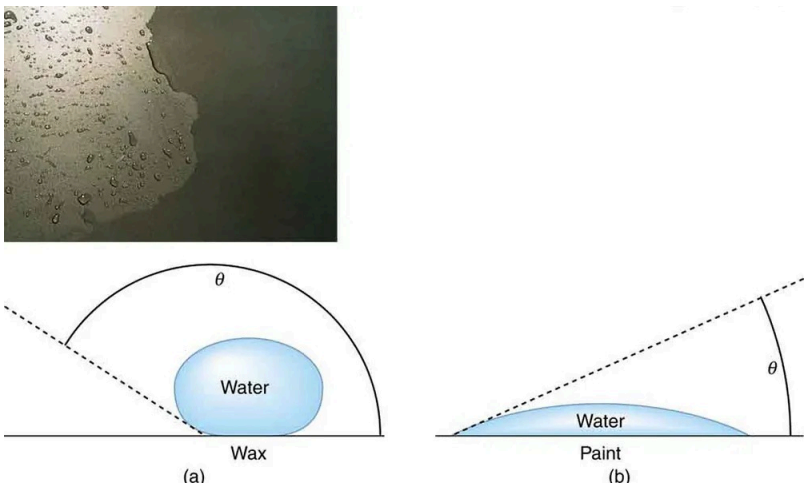


Figure 9.30 In the photograph, water beads on the waxed car paint and flattens on the unwaxed paint. (a) Water forms beads on the waxed surface because the cohesive forces responsible for surface tension are larger than the adhesive forces, which tend to flatten the drop. (b) Water beads on bare paint are flattened considerably because the adhesive forces between water and paint are strong, overcoming surface tension. The contact angle θ is directly related to the relative strengths of the cohesive and adhesive forces. The larger θ is, the larger the ratio of cohesive to adhesive forces. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two parts.

The top section shows a surface with droplets of water scattered across it. The surface appears textured and reflective, possibly made of a material like granite or metal that causes the water to bead up.

The bottom section consists of two separate diagrams labeled (a) and (b).

– Diagram (a) illustrates a drop of water sitting on a wax surface. The water droplet is rounded and forms a high contact angle with the wax, signifying poor wettability. The angle is labeled as θ.

– Diagram (b) shows a drop of water on a paint surface. The water droplet is flatter and spreads more, indicating a smaller contact angle with the paint. This demonstrates better wettability compared to the wax. The angle is also labeled as θ.

These diagrams illustrate the concept of contact angles and wettability on different surfaces.

One important phenomenon related to the relative strength of cohesive and adhesive forces is capillary action—the tendency of a fluid to be raised or suppressed in a narrow tube, or *capillary tube*. This action causes blood to be drawn into a small-diameter tube when the tube touches a drop.

Capillary Action

The tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube, is called capillary action.

If a capillary tube is placed vertically into a liquid, as shown in Figure 9.31, capillary action will raise or suppress the liquid inside the tube depending on the combination of substances. The actual effect depends on the relative strength of the cohesive and adhesive forces and, thus, the contact angle θ given in the table. If θ is less than 90° , then the fluid will be raised; if θ is greater than 90° , it will be suppressed. Mercury, for example, has a very large surface tension and a large contact angle with glass. When placed in a tube, the surface of a column of mercury curves downward, somewhat like a drop. The curved surface of a fluid in a tube is called a **meniscus**. The tendency of surface tension is always to reduce the surface area. Surface tension thus flattens the curved liquid surface in a capillary tube. This results in a downward force in mercury and an upward force in water, as seen in Figure 9.31.

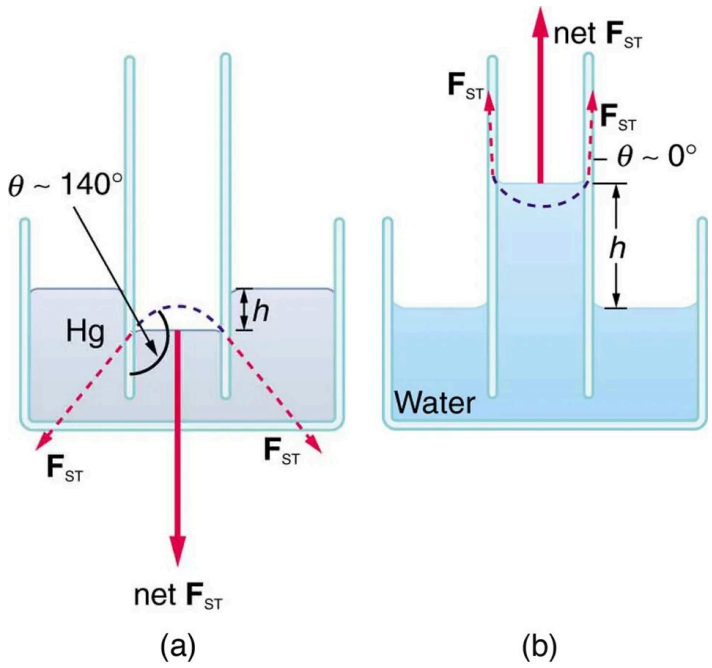


Figure 9.31 (a) Mercury is suppressed in a glass tube because its contact angle is greater than 90° *. Surface tension exerts a downward force as it flattens the mercury, suppressing it in the tube. The dashed line shows the shape the mercury surface would have without the flattening effect of surface tension. (b) Water is raised in a glass tube because its contact angle is nearly* 0° *. Surface tension therefore exerts an upward force when it flattens the surface to reduce its area. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image displays two diagrams illustrating capillary action in narrow tubes immersed in two different liquids: mercury (Hg) and water.

(a) The diagram on the left shows mercury (Hg) in a container with narrow tubes. The surface of the mercury is convex, forming a contact angle (θ) of approximately 140 degrees with the glass. The liquid level in the tube is lower than in the container, indicating the repellent nature of the mercury. Arrows labeled F_{ST} represent the surface tension forces acting at the liquid-air interface. The net force (*net* F_{ST}) is directed downward outside the tube, illustrating the downward capillary action.

(b) The diagram on the right illustrates water in a similar setup. The water forms a concave surface within the tube, indicating adhesion to the glass surface. The contact angle (θ) is approximately 0 degrees. The liquid level is higher in the tube than in the container, demonstrating the cohesive attraction of water. Arrows labeled F_{ST} represent the surface tension forces, with the net force (*net* F_{ST}) directed upward, signifying upward capillary action. Both diagrams are labeled with height (h) to indicate the difference in liquid levels.

Table 9.4 Contact Angles of Some Substances

Interface	Contact angle Θ
Mercury–glass	140°
Water–glass	0°
Water–paraffin	107°
Water–silver	90°
Organic liquids (most)–glass	0°
Ethyl alcohol–glass	0°
Kerosene–glass	26°

Capillary action can move liquids horizontally over very large distances, but the height to which it can raise or suppress a liquid in a tube is limited by its weight. It can be shown that this height h is given by

$$h = \frac{2\gamma \cos\theta}{\rho \mathrm{gr}}.$$

9.51

If we look at the different factors in this expression, we might see how it makes good sense. The height is directly proportional to the surface tension γ, which is its direct cause. Furthermore, the height is inversely proportional to tube radius—the smaller the radius r, the higher the fluid can be raised, since a smaller tube holds less mass. The height is also inversely proportional to fluid density ρ, since a larger density means a greater mass in the same volume. (See Figure 9.32.)

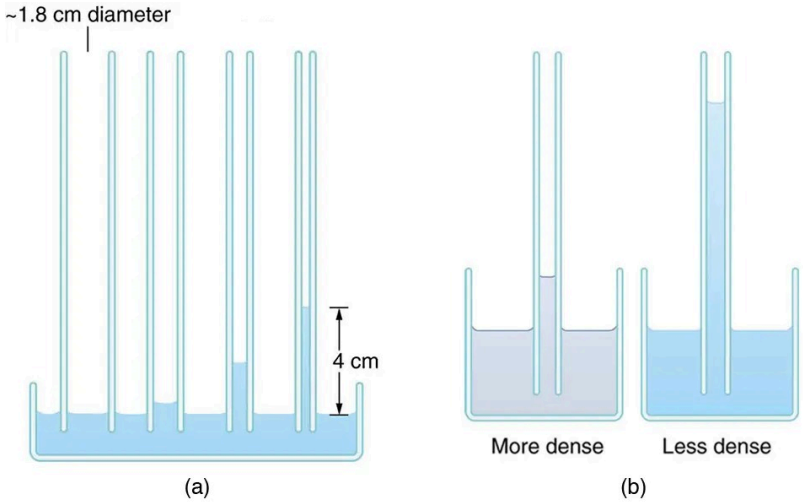


Figure 9.32 (a) Capillary action depends on the radius of a tube. The smaller the tube, the greater the height reached. The height is negligible for large-radius tubes. (b) A denser fluid in the same tube rises to a smaller height, all other factors being the same. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two diagrams labeled (a) and (b).

(a) The diagram on the left shows a series of vertical tubes with a diameter of approximately 1.8 cm. These tubes are connected at the base by a horizontal section filled with liquid. The fill height of the liquid varies in different tubes, with the tallest section being 4 cm high.

(b) The diagram on the right displays two sets of tubes labeled “More dense” and “Less dense.” In both sets, the tubes are placed in a container filled with liquid. The liquid level inside the tubes is higher in the “More dense” diagram compared to the surrounding liquid. Conversely, in the “Less dense” diagram, the liquid level inside the tubes is also higher, but it differs in height compared to the “More dense” diagram.

Example 9.12

Calculating Radius of a Capillary Tube: Capillary Action: Tree Sap

Can capillary action be solely responsible for sap rising in trees? To answer this question, calculate the radius of a capillary tube that would raise sap 100 m to the top of a giant redwood, assuming that sap's density is $1050\ \mathrm{kg/m^3}$, its contact angle is zero, and its surface tension is the same as that of water at $20.0^{\circ}\ C$.

Strategy

The height to which a liquid will rise as a result of capillary action is given by $h = \frac{2\gamma\ \cos\theta}{\rho \mathrm{gr}}$, and every quantity is known except for r.

Solution

Solving for r and substituting known values produces

$$\begin{aligned} r &= \frac{2\gamma\ \cos\theta}{\rho \mathrm{gh}} \\ &= \frac{2\left(0.0728\ \mathrm{N/m}\right)\cos\left(0^{\circ}\right)}{\left(1050 \mathrm{kg/m^3}\right)\left(9.80 \mathrm{m/s^2}\right)\left(100\ \mathrm{m}\right)} \\ &= 1.41 \times 10^{-7} \mathrm{m}. \end{aligned}$$

9.52

Discussion

This result is unreasonable. Sap in trees moves through the *xylem*, which forms tubes with radii as small as $2.5 \times 10^{-5} \mathrm{m}$. This value is about 180 times as large as the radius

found necessary here to raise sap $100\ \text{m}$. This means that capillary action alone cannot be solely responsible for sap getting to the tops of trees.

How *does* sap get to the tops of tall trees? (Recall that a column of water can only rise to a height of 10 m when there is a vacuum at the top—see Example 9.5.) The question has not been completely resolved, but it appears that it is pulled up like a chain held together by cohesive forces. As each molecule of sap enters a leaf and evaporates (a process called transpiration), the entire chain is pulled up a notch. So a negative pressure created by water evaporation must be present to pull the sap up through the xylem vessels. In most situations, *fluids can push but can exert only negligible pull*, because the cohesive forces seem to be too small to hold the molecules tightly together. But in this case, the cohesive force of water molecules provides a very strong pull. Figure 9.33 shows one device for studying negative pressure. Some experiments have demonstrated that negative pressures sufficient to pull sap to the tops of the tallest trees *can* be achieved.

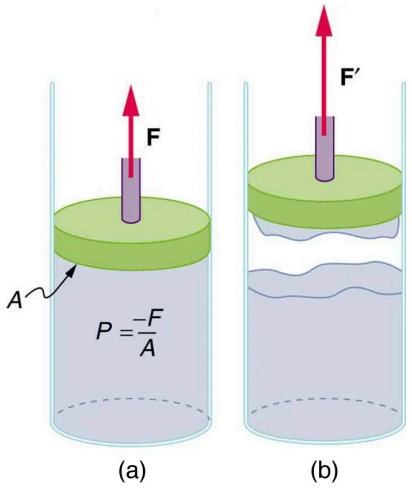


Figure 9.33 (a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure $P = -F/A$*. (b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two diagrams, labeled (a) and (b), showing a piston in a cylindrical container filled with a fluid.

Diagram (a):
– The piston is pressing down on the fluid in the container.

– An upward force, labeled “F,” is indicated by a red arrow pointing upward from the piston.
– The area of the piston is labeled “A.”
– A formula is shown: $P = \frac{F}{A}$, representing pressure as the force per unit area.

Diagram (b):
– The piston is moved upward, allowing the fluid to expand and form an irregular surface with waves.
– The upward force is labeled “F′” and indicated with a red arrow pointing upward.

The overall concept is related to the effect of force and area on pressure and how movement of the piston affects the fluid within the container.

“11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action” from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

9.9 PRESSURES IN THE BODY

Learning Objectives

By the end of this section, you will be able to:

- Explain the concept of pressure the in human body.
- Explain systolic and diastolic blood pressures.
- Describe pressures in the eye, lungs, spinal column, bladder, and skeletal system.

Pressure in the Body

Next to taking a person's temperature and weight, measuring blood pressure is the most common of all medical examinations. Control of high blood pressure is largely responsible for the significant decreases in heart attack and stroke fatalities achieved in the last three decades. The pressures in various parts of the body can be measured and often provide valuable medical indicators. In this section, we consider a few examples together with some of the physics that accompanies them.

Table 9.5 lists some of the measured pressures in mm Hg, the units most commonly quoted.

Table 9.5 Typical Pressures in Humans

Body system	Gauge pressure in mm Hg
Blood pressures in large arteries (resting)	
Maximum (systolic)	100–140
Minimum (diastolic)	60–90
Blood pressure in large veins	4–15
Eye	12–24
Brain and spinal fluid (lying down)	5–12
Bladder	
While filling	0–25
When full	100–150
Chest cavity between lungs and ribs	−8 to −4
Inside lungs	−2 to +3
Digestive tract	
Esophagus	−2
Stomach	0–20
Intestines	10–20
Middle ear	1

Blood Pressure

Common arterial blood pressure measurements typically produce values of 120 mm Hg and 80 mm Hg, respectively, for systolic and diastolic pressures. Both pressures have health implications. When systolic pressure is chronically high, the risk of stroke and heart attack is increased. If, however, it is too low, fainting is a problem. Systolic pressure increases dramatically during exercise to increase blood flow and returns to normal afterward. This change produces no ill effects and, in fact, may be beneficial to the tone of the circulatory system. Diastolic pressure can be an indicator of fluid balance. When low, it may indicate that a person is hemorrhaging internally and needs a transfusion. Conversely, high diastolic pressure indicates a ballooning of the blood vessels, which may be due to the transfusion of too much fluid into the circulatory system. High diastolic pressure is also an indication that blood vessels are not dilating properly to pass blood through. This can seriously strain the heart in its attempt to pump blood.

Blood leaves the heart at about 120 mm Hg but its pressure continues to decrease (to almost 0) as it goes from the aorta to smaller arteries to small veins (see Figure 9.34). The pressure differences in the circulation system

are caused by blood flow through the system as well as the position of the person. For a person standing up, the pressure in the feet will be larger than at the heart due to the weight of the blood $(P = h\rho g)$. If we assume that the distance between the heart and the feet of a person in an upright position is 1.4 m, then the increase in pressure in the feet relative to that in the heart (for a static column of blood) is given by

$$\begin{aligned} \Delta P &= \Delta h \rho g \\ &= (1.4m)\left(1050\ \text{kg/m}^3\right)\left(9.80 m/\text{s}^2\right) \\ &= 1.4 \times 10^4 \text{Pa} \\ &= 108\ \text{mm Hg}. \end{aligned}$$

9.53

Increase in Pressure in the Feet of a Person

$$\begin{aligned} \Delta P &= \Delta h \rho g \\ &= (1.4m)\left(1050\ \text{kg/m}^3\right)\left(9.80 m/\text{s}^2\right) \\ &= 1.4 \times 10^4 \text{Pa} \\ &= 108\ \text{mm Hg}. \end{aligned}$$

9.54

Standing a long time can lead to an accumulation of blood in the legs and swelling. This is the reason why soldiers who are required to stand still for long periods of time have been known to faint. Elastic bandages around the calf can help prevent this accumulation and can also help provide increased pressure to enable the veins to send blood back up to the heart. For similar reasons, doctors recommend tight stockings for long-haul flights.

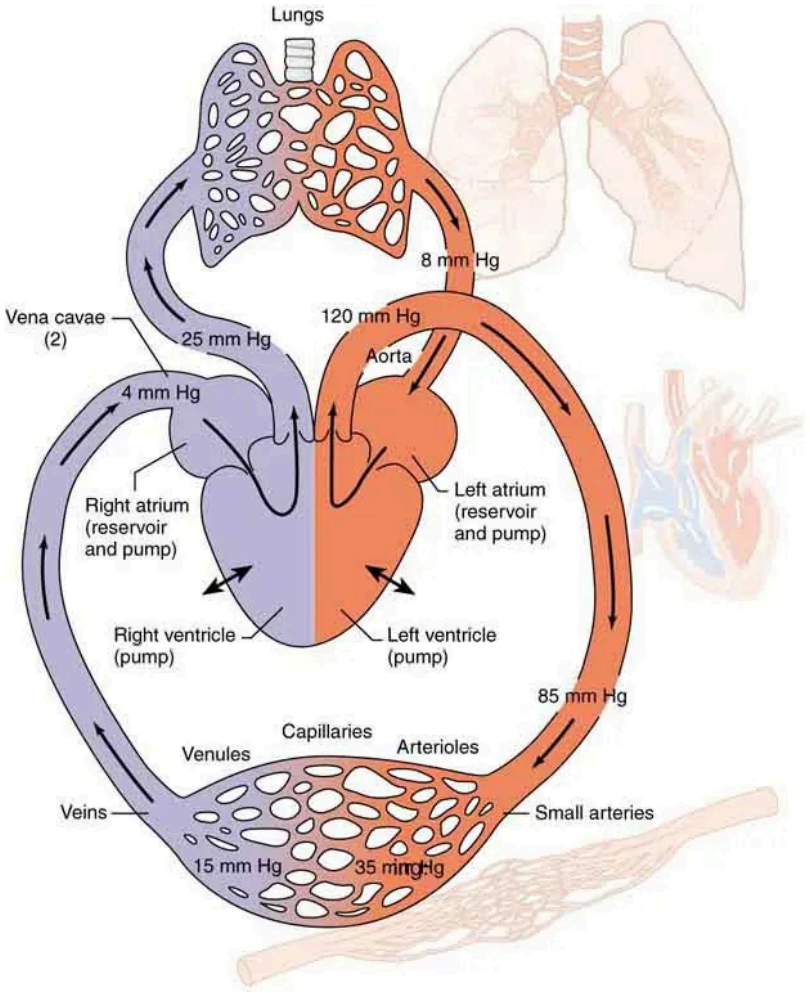
Blood pressure may also be measured in the major veins, the heart chambers, arteries to the brain, and the lungs. But these pressures are usually only monitored during surgery or for patients in intensive care since the measurements are invasive. To obtain these pressure measurements, qualified health care workers thread thin tubes, called catheters, into appropriate locations to transmit pressures to external measuring devices.

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body (Figure 9.34). Right-heart failure, for example, results in a rise in the pressure in the vena cavae and a drop in pressure in the arteries to the lungs. Left-heart failure results in a rise

in the pressure entering the left side of the heart and a drop in aortal pressure. Implications of these and other pressures on flow in the circulatory system will be discussed in more detail in Fluid Dynamics and Its Biological and Medical Applications.

Two Pumps of the Heart

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body.



Fluid Dynamics and Its Biological and Medical Applications

Figure 9.34 Schematic of the circulatory system showing typical pressures. The two pumps in the heart increase pressure and that pressure is reduced as the blood flows through the body. Long-term deviations from these pressures have medical implications discussed in some detail in the . Only aortal or arterial blood pressure can be measured noninvasively. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of the human cardiovascular system, illustrating the flow of blood through the heart, lungs, and body. The heart is at the center, divided into two sides: the right side is shaded purple, and the left side is shaded red. Key areas are labeled with the blood pressure in millimeters of mercury (mm Hg) for various parts of the circulatory system.

On the right side, the pathway is as follows:

- Blood enters the right atrium, labeled as a “reservoir and pump,” at 4 mm Hg.
- Blood then moves to the right ventricle, labeled as a “pump.” It travels through the pulmonary arteries to the lungs, where the pressure is marked as 25 mm Hg.
- After oxygenation in the lungs, the blood returns to the left side of the heart via the pulmonary veins at 8 mm Hg.

On the left side, the pathway is as follows:

- Blood from the lungs enters the left atrium, labeled as a “reservoir and pump.”
- Blood flows into the left ventricle, labeled as a “pump,” and then is pushed into the aorta at 120 mm Hg.
- The red-shaded area routes blood through small arteries and arterioles, marked with pressures of 85 mm Hg and 35 mm Hg, respectively.
- This blood travels through the capillaries, venules, and veins, returning to the right atrium with a pressure of 15 mm Hg.

The diagram includes labels like “vena cavae,” “capillaries,” “venules,” “arterioles,” “small arteries,” and “veins,” indicating various components of the circulatory system involved in the transportation of blood.

Pressure in the Eye

The shape of the eye is maintained by fluid pressure, called intraocular pressure, which is normally in the range of 12.0 to 24.0 mm Hg. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma. The net pressure can become as great as 85.0 mm Hg, an abnormally large pressure that can permanently damage the optic nerve. To get an idea of the force involved, suppose the back of the eye has an area of 6.0cm^2, and the net pressure is 85.0 mm Hg. Force is given by $F = \text{PA}$. To get F in newtons, we convert the area to m^2 ($1\ \text{m}^2 = 10^4\text{cm}^2$). Then we calculate as follows:

$$\begin{aligned} F &= h\rho\text{gA} \\ &= \left(85.0 \times 10^{-3}\text{m}\right)\left(13.6 \times 10^3\text{kg/m}^3\right)\left(9.80\text{m/s}^2\right)\left(6.0 \times 10^{-4}\text{m}^2\right) \\ &= 6.8N. \end{aligned}$$

9.55

Eye Pressure

The shape of the eye is maintained by fluid pressure, called intraocular pressure. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma. The force is calculated as

$$\begin{aligned} F &= h\rho \mathrm{gA} \\ &= \left(85.0 \times 10^{-3}\mathrm{m}\right)\left(13.6 \times 10^{3}\mathrm{kg/m^3}\right)\left(9.80\mathrm{m/s^2}\right)\left(6.0 \times 10^{-4}\mathrm{m^2}\right) \\ &= 6.8N. \end{aligned}$$

9.56

This force is the weight of about a 680-g mass. A mass of 680 g resting on the eye (imagine 1.5 lb resting on your eye) would be sufficient to cause it damage. (A normal force here would be the weight of about 120 g, less than one-quarter of our initial value.)

People over 40 years of age are at greatest risk of developing glaucoma and should have their intraocular pressure tested routinely. Most measurements involve exerting a force on the (anesthetized) eye over some area (a pressure) and observing the eye's response. A noncontact approach uses a puff of air and a measurement is made of the force needed to indent the eye (Figure 9.35). If the intraocular pressure is high, the eye will deform less and rebound more vigorously than normal. Excessive intraocular pressures can be detected reliably and sometimes controlled effectively.

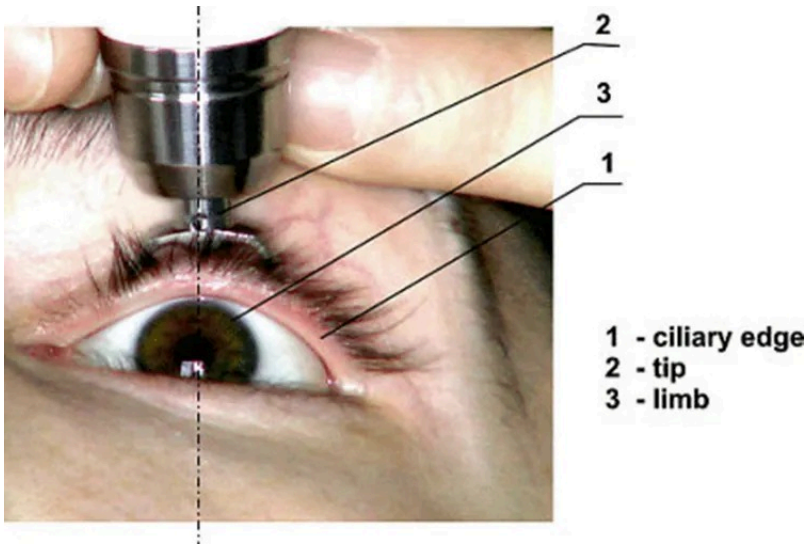


Figure 9.35 The intraocular eye pressure can be read with a tonometer. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts an eye undergoing a procedure. A tool is being operated near the eye. The hand holding the tool is gently pulling the upper eyelid upwards, revealing the sclera and the upper portion of the iris. Several lines point to specific parts of the eye with labels:

Number	Label
1	Ciliary edge
2	Tip
3	Limb

The eye is open wide and the instrument appears to be performing a task near the eyelashes and the upper part of the eyeball. The image is focused on the anatomy and tool placement.

Example 9.13

Calculating Gauge Pressure and Depth: Damage to the Eardrum

Suppose a 3.00-N force can rupture an eardrum. (a) If the eardrum has an area of 1.00cm^2, calculate the maximum tolerable gauge pressure on the eardrum in newtons per meter squared and convert it to millimeters of mercury. (b) At what depth in freshwater would this person's eardrum rupture, assuming the gauge pressure in the middle ear is zero?

Strategy for (a)

The pressure can be found directly from its definition since we know the force and area. We are looking for the gauge pressure.

Solution for (a)

$$\begin{aligned} P_{\text{g}} &= F/A \\ &= 3.00\text{N}/\left(1.00 \times 10^{-4}\text{m}^2\right) \\ &= 3.00 \times 10^4\text{N}/\text{m}^2. \end{aligned}$$

9.57

We now need to convert this to units of mm Hg:

$$P_{\text{g}} = 3.0 \times 10^4\text{N}/\text{m}^2 \left(\frac{1.0mmHg}{133\text{N}/\text{m}^2}\right) = 226\ \text{mm Hg}.$$

9.58

Strategy for (b)

Here we will use the fact that the water pressure varies linearly with depth h below the surface.

Solution for (b)

$P = h\rho g$ and therefore $h = P/\rho g$. Using the value above for P, we have

$$h = \frac{3.0 \times 10^4 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)\,(9.80 m/\text{s}^2)} = 3.06 m.$$

9.59

Discussion

Similarly, increased pressure exerted upon the eardrum from the middle ear can arise when an infection causes a fluid buildup.

Pressure Associated with the Lungs

The pressure inside the lungs increases and decreases with each breath. The pressure drops to below atmospheric pressure (negative gauge pressure) when you inhale, causing air to flow into the lungs. It increases above atmospheric pressure (positive gauge pressure) when you exhale, forcing air out.

Lung pressure is controlled by several mechanisms. Muscle action in the diaphragm and rib cage is necessary for inhalation; this muscle action increases the volume of the lungs thereby reducing the pressure within them Figure 9.36. Surface tension in the alveoli creates a positive pressure opposing inhalation. (See 9.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action.) You can exhale without muscle action by letting surface tension in the alveoli create its own positive pressure. Muscle action can add to this positive pressure to produce forced exhalation, such as when you blow up a balloon, blow out a candle, or cough.

The lungs, in fact, would collapse due to the surface tension in the alveoli, if they were not attached to the inside of the chest wall by liquid adhesion. The gauge pressure in the liquid attaching the lungs to the inside of the chest wall is thus negative, ranging from -4 to $-8 mmHg$ during exhalation and inhalation, respectively. If air is allowed to enter the chest cavity, it breaks the attachment, and one or both lungs may

collapse. Suction is applied to the chest cavity of surgery patients and trauma victims to reestablish negative pressure and inflate the lungs.

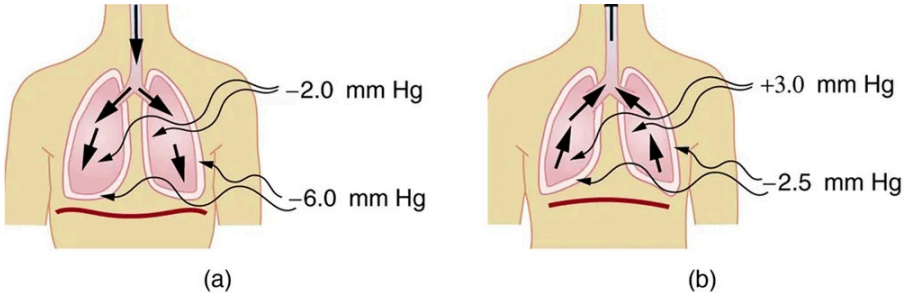


Figure 9.36 (a) During inhalation, muscles expand the chest, and the diaphragm moves downward, reducing pressure inside the lungs to less than atmospheric (negative gauge pressure). Pressure between the lungs and chest wall is even lower to overcome the positive pressure created by surface tension in the lungs. (b) During gentle exhalation, the muscles simply relax and surface tension in the alveoli creates a positive pressure inside the lungs, forcing air out. Pressure between the chest wall and lungs remains negative to keep them attached to the chest wall, but it is less negative than during inhalation. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b) that depict the pressure differences in the thoracic cavity during the respiratory cycle, highlighting inhalation and exhalation processes.

(a) *Inspiration:*

- The diagram shows a human torso with lungs, arrows indicating the flow of air into the lungs.
- Pressure annotations are shown with -2.0 mm Hg inside the lungs and -6.0 mm Hg in the pleural cavity, illustrating negative pressure facilitating airflow into the lungs.
- The diaphragm is shown in a flattened position indicating contraction.

(b) *Expiration:*

- The diagram shows a human torso with lungs, arrows indicating the flow of air out of the lungs.
- Pressure annotations are shown with +3.0 mm Hg inside the lungs and -2.5 mm Hg in the pleural cavity, illustrating positive pressure aiding in expelling air from the lungs.
- The diaphragm is shown in a relaxed, dome-shaped position.

Other Pressures in the Body

Spinal Column and Skull

Normally, there is a 5- to12-mm Hg pressure in the fluid surrounding the brain and filling the spinal column. This cerebrospinal fluid serves many purposes, one of which is to supply flotation to the brain. The buoyant force supplied by the fluid nearly equals the weight of the brain, since their densities are nearly equal. If there is a loss of fluid, the brain rests on the inside of the skull, causing severe headaches, constricted blood flow, and serious damage. Spinal fluid pressure is measured by means of a needle inserted between vertebrae that transmits the pressure to a suitable measuring device.

Bladder Pressure

This bodily pressure is one of which we are often aware. In fact, there is a relationship between our awareness of this pressure and a subsequent increase in it. Bladder pressure climbs steadily from zero to about 25 mm Hg as the bladder fills to its normal capacity of 500cm^3. This pressure triggers the micturition reflex, which stimulates the feeling of needing to urinate. What is more, it also causes muscles around the bladder to contract, raising the pressure to over 100 mm Hg, accentuating the sensation. Coughing, straining, tensing in cold weather, wearing tight clothes, and experiencing simple nervous tension all can increase bladder pressure and trigger this reflex. So can the weight of a pregnant woman's fetus, especially if it is kicking vigorously or pushing down with its head! Bladder pressure can be measured by a catheter or by inserting a needle through the bladder wall and transmitting the pressure to an appropriate measuring device. One hazard of high bladder pressure (sometimes created by an obstruction), is that such pressure can force urine back into the kidneys, causing potentially severe damage.

Pressures in the Skeletal System

These pressures are the largest in the body, due both to the high values of initial force, and the small areas to which this force is applied, such as in the joints.. For example, when a person lifts an object improperly, a force of 5000 N may be created between vertebrae in the spine, and this may be applied to an area as small as 10cm^2. The pressure created is

$P = F/A = (5000\text{ N})/(10^{-3}\text{m}^2) = 5.0 \times 10^6\text{N/m}^2$ or about 50 atm! This pressure can damage both the spinal discs (the cartilage between vertebrae), as well as the bony vertebrae themselves. Even under normal circumstances, forces between vertebrae in the spine are large enough to create pressures of several atmospheres. Most causes of excessive pressure in the skeletal system can be avoided by lifting properly and avoiding extreme physical activity. (See 7.6 Forces and Torques in Muscles and Joints.)

There are many other interesting and medically significant pressures in the body. For example, pressure caused by various muscle actions drives food and waste through the digestive system. Stomach pressure behaves much like bladder pressure and is tied to the sensation of hunger. Pressure in the relaxed esophagus is normally negative because pressure in the chest cavity is normally negative. Positive pressure in the stomach may thus force acid into the esophagus, causing "heartburn." Pressure in the middle ear can result in significant force on the eardrum if it differs greatly from atmospheric pressure, such as while scuba diving. The decrease in external pressure is also noticeable during plane flights (due to a decrease in the weight of air above relative to that at the Earth's surface). The Eustachian tubes connect the middle ear to the throat and allow us to equalize pressure in the middle ear to avoid an imbalance of force on the eardrum.

Many pressures in the human body are associated with the flow of fluids. Fluid flow will be discussed in detail in the Fluid Dynamics and Its Biological and Medical Applications.

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GLOSSARY

absolute pressure
: the sum of gauge pressure and atmospheric pressure

adhesive forces
: the attractive forces between molecules of different types

Archimedes' principle
: the buoyant force on an object equals the weight of the fluid it displaces

buoyant force
: the net upward force on any object in any fluid

capillary action
: the tendency of a fluid to be raised or lowered in a narrow tube

cohesive forces
: the attractive forces between molecules of the same type

contact angle
: the angle θ between the tangent to the liquid surface and the surface

density
: the mass per unit volume of a substance or object

diastolic pressure
: minimum arterial blood pressure; indicator for the fluid balance

diastolic pressure
: the minimum blood pressure in the artery

fluids
: liquids and gases; a fluid is a state of matter that yields to shearing forces

gauge pressure
: the pressure relative to atmospheric pressure

glaucoma
: condition caused by the buildup of fluid pressure in the eye

intraocular pressure
: fluid pressure in the eye

micturition reflex
: stimulates the feeling of needing to urinate, triggered by bladder pressure

Pascal's Principle
: a change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid

and to the walls of its container

pressure

the force per unit area perpendicular to the force, over which the force acts

pressure

the weight of the fluid divided by the area supporting it

specific gravity

the ratio of the density of an object to a fluid (usually water)

surface tension

the cohesive forces between molecules which cause the surface of a liquid to contract to the smallest possible surface area

systolic pressure

maximum arterial blood pressure; indicator for the blood flow

systolic pressure

the maximum blood pressure in the artery

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CHAPTER SUMMARY

9.1 What Is a Fluid?

- A fluid is a state of matter that yields to sideways or shearing forces. Liquids and gases are both fluids. Fluid statics is the physics of stationary fluids.

9.2 Density

- Density is the mass per unit volume of a substance or object. In equation form, density is defined as $\rho = \frac{m}{V}$.
- The SI unit of density is $\mathrm{kg/m^3}$.

9.3 Pressure

- Pressure is the force per unit perpendicular area over which the force is applied. In equation form, pressure is defined as $P = \frac{F}{A}$.
- The SI unit of pressure is pascal and $1\ \mathrm{Pa} = 1\mathrm{N/m^2}$.

9.4 Variation of Pressure with Depth in a Fluid

- Pressure is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container): $P = \frac{\mathrm{mg}}{A}$.
- Pressure due to the weight of a liquid is given by $P = h\rho g$ where P is the pressure, h is the height of the liquid, ρ is the density of the liquid, and g is the acceleration due to gravity.

9.5 Pascal's Principle

- Pressure is force per unit area.
- A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.
- A hydraulic system is an enclosed fluid system used to exert forces.

9.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

- Gauge pressure is the pressure relative to atmospheric pressure.
- Absolute pressure is the sum of gauge pressure and atmospheric pressure.
- Aneroid gauge measures pressure using a bellows-and-spring arrangement connected to the pointer of a calibrated scale.
- Open-tube manometers have U-shaped tubes and one end is always open. It is used to measure pressure.
- A mercury barometer is a device that measures atmospheric pressure.

9.7 Archimedes' Principle

- Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.
- Archimedes' principle states that the buoyant force on an object equals the weight of the fluid it displaces.
- Specific gravity is the ratio of the density of an object to a fluid (usually water).

9.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

- Attractive forces between molecules of the same type are called cohesive forces.
- Attractive forces between molecules of different types are called adhesive forces.
- Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.
- Capillary action is the tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube which is due to the relative strength of cohesive and adhesive forces.

9.9 Pressures in the Body

- Measuring blood pressure is among the most common of all medical examinations.
- The pressures in various parts of the body can be measured and often provide valuable medical indicators.
- The shape of the eye is maintained by fluid pressure, called intraocular pressure.
- When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma.
- Some of the other pressures in the body are spinal and skull pressures, bladder pressure, pressures in the skeletal system.

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CONCEPTUAL QUESTIONS

Conceptual Questions

9.1 What Is a Fluid?

1. What physical characteristic distinguishes a fluid from a solid?
2. Which of the following substances are fluids at room temperature: air, mercury, water, glass?
3. Why are gases easier to compress than liquids and solids?
4. How do gases differ from liquids?

9.2 Density

5. Approximately how does the density of air vary with altitude?
6. Give an example in which density is used to identify the substance composing an object. Would information in addition to average density be needed to identify the substances in an object composed of more than one material?
7. Figure 9.37 shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.



Figure 9.37 Image from OpenStax College Physics 2e, CC-BY 4.0

9.3 Pressure

8. How is pressure related to the sharpness of a knife and its ability to cut?
9. Why does a dull hypodermic needle hurt more than a sharp one?
10. The outward force on one end of an air tank was calculated in Example 9.2. How is this force balanced? (The tank does not accelerate, so the force must be balanced.)
11. Why is force exerted by static fluids always perpendicular to a surface?
12. In a remote location near the North Pole, an iceberg floats in a lake. Next to the lake (assume it is not frozen) sits a comparably sized glacier sitting on land. If both chunks of ice should melt due to rising global temperatures (and the melted ice all goes into the lake), which ice chunk would give the greatest increase in the level of the lake water, if any?
13. How do jogging on soft ground and wearing padded shoes reduce the pressures to which the feet and legs are subjected?
14. Toe dancing (as in ballet) is much harder on toes than normal dancing or walking. Explain in terms of pressure.
15. How do you convert pressure units like millimeters of mercury, centimeters of water, and inches of mercury into units like newtons per meter squared without resorting to a table of pressure conversion factors?

9.4 Variation of Pressure with Depth in a Fluid

16. Atmospheric pressure exerts a large force (equal to the weight of the atmosphere above your body—about 10 tons) on the top of your body when you are lying on the beach sunbathing. Why are you able to get up?
17. Why does atmospheric pressure decrease more rapidly than linearly with altitude?
18. What are two reasons why mercury rather than water is used in barometers?
19. Figure 9.38 shows how sandbags placed around a leak outside a river levee can effectively stop the flow of water under the levee. Explain how the small amount of water inside the column formed by the sandbags is able to balance the much larger body of water behind the levee.

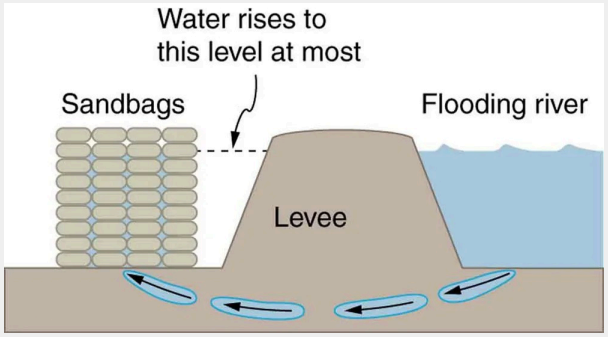


Figure 9.38 Because the river level is very high, it has started to leak under the levee. Sandbags are placed around the leak, and the water held by them rises until it is the same level as the river, at which point the water there stops rising. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram depicting a cross-section of a levee system designed to prevent flooding. On the left side, there are stacked sandbags labeled as "Sandbags." To the right of the sandbags, there is an earthen structure labeled as "Levee," which acts as a barrier between the land and a "Flooding river" depicted on the far right side. Above the levee, a dashed horizontal line with an arrow points downward, indicating "Water rises to this level at most," showing the expected maximum water level. Below the surface, there are blue curved arrows illustrating the movement of water under the levee and sandbags, indicating potential seepage.

20. Why is it difficult to swim under water in the Great Salt Lake?
21. Is there a net force on a dam due to atmospheric pressure? Explain your answer.
22. Does atmospheric pressure add to the gas pressure in a rigid tank? In a toy balloon? When, in general, does atmospheric pressure not affect the total pressure in a fluid?
23. You can break a strong wine bottle by pounding a cork into it with your fist, but the cork must press directly against the liquid filling the bottle—there can be no air between the cork and liquid. Explain why the bottle breaks, and why it will not if there is air between the cork and liquid.

9.5 Pascal's Principle

24. Suppose the pedal cylinder in a hydraulic system is at a greater height than the wheel cylinder. Explain how this will affect the force produced at the wheel cylinder.

9.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

25. Explain why the fluid reaches equal levels on either side of a manometer if both sides are open to the atmosphere, even if the tubes are of different diameters.
26. Figure 9.15 shows how a common measurement of arterial blood pressure is made. Is there any effect on the measured pressure if the manometer is lowered? What is the effect of raising the arm above the shoulder? What is the effect of placing the cuff on the upper leg with the person standing? Explain your answers in terms of pressure created by the weight of a fluid.
27. Considering the magnitude of typical arterial blood pressures, why are mercury rather than water manometers used for these measurements?

9.7 Archimedes' Principle

28. More force is required to pull the plug in a full bathtub than when it is empty. Does this contradict Archimedes' principle? Explain your answer.
29. Do fluids exert buoyant forces in a "weightless" environment, such as in the space shuttle? Explain your answer.
30. Will the same ship float higher in salt water than in freshwater? Explain your answer.
31. Marbles dropped into a partially filled bathtub sink to the bottom. Part of their weight is supported by buoyant force, yet the downward force on the bottom of the tub increases by exactly the weight of the marbles. Explain why.

9.8 Cohesion and Adhesion in Liquids: Surface Tension and

Capillary Action

32. The density of oil is less than that of water, yet a loaded oil tanker sits lower in the water than an empty one. Why?
33. Is surface tension due to cohesive or adhesive forces, or both?
34. Is capillary action due to cohesive or adhesive forces, or both?
35. Birds such as ducks, geese, and swans have greater densities than water, yet they are able to sit on its surface. Explain this ability, noting that water does not wet their feathers and that they cannot sit on soapy water.
36. Water beads up on an oily sunbather, but not on her neighbor, whose skin is not oiled. Explain in terms of cohesive and adhesive forces.
37. Could capillary action be used to move fluids in a "weightless" environment, such as in an orbiting space probe?
38. What effect does capillary action have on the reading of a manometer with uniform diameter? Explain your answer.
39. Pressure between the inside chest wall and the outside of the lungs normally remains negative. Explain how pressure inside the lungs can become positive (to cause exhalation) without muscle action.

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PROBLEMS & EXERCISES

Problems & Exercises

9.2 Density

1. Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?
2. Mercury is commonly supplied in flasks containing 34.5 kg (about 76 lb). What is the volume in liters of this much mercury?
3. (a) What is the mass of a deep breath of air having a volume of 2.00 L? (b) Discuss the effect taking such a breath has on your body's volume and density.
4. A straightforward method of finding the density of an object is to measure its mass and then measure its volume by submerging it in a graduated cylinder. What is the density of a 240-g rock that displaces 89.0cm^3 of water? (Note that the accuracy and practical applications of this technique are more limited than a variety of others that are based on Archimedes' principle.)
5. Suppose you have a coffee mug with a circular cross section and vertical sides (uniform radius). What is its inside radius if it holds 375 g of coffee when filled to a depth of 7.50 cm? Assume coffee has the same density as water.
6. (a) A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.
7. A trash compactor can reduce the volume of its contents to 0.350 their original value. Neglecting the mass of air expelled, by what factor is the density of the rubbish increased?
8. A 2.50-kg steel gasoline can holds 20.0 L of gasoline when full. What is the average density of the full gas can, taking into account the volume occupied by steel as well as by gasoline?
9. What is the density of 18.0-karat gold that is a mixture of 18 parts gold, 5 parts silver, and 1 part copper? (These values are parts by mass, not volume.) Assume

that this is a simple mixture having an average density equal to the weighted densities of its constituents.

10. There is relatively little empty space between atoms in solids and liquids, so that the average density of an atom is about the same as matter on a macroscopic scale—approximately 10^3kg/m^3. The nucleus of an atom has a radius about 10^{-5} that of the atom and contains nearly all the mass of the entire atom. (a) What is the approximate density of a nucleus? (b) One remnant of a supernova, called a neutron star, can have the density of a nucleus. What would be the radius of a neutron star with a mass 10 times that of our Sun (the radius of the Sun is $7 \times 10^8 \text{m}$)?

9.3 Pressure

11. As a woman walks, her entire weight is momentarily placed on one heel of her high-heeled shoes. Calculate the pressure exerted on the floor by the heel if it has an area of 1.50cm^2 and the woman's mass is 55.0 kg. Express the pressure in Pa. (In the early days of commercial flight, women were not allowed to wear high-heeled shoes because aircraft floors were too thin to withstand such large pressures.)
12. The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle 0.200 mm in radius, what pressure is exerted on the record in N/m^2?
13. Nail tips exert tremendous pressures when they are hit by hammers because they exert a large force over a small area. What force must be exerted on a nail with a circular tip of 1.00 mm diameter to create a pressure of $3.00 \times 10^9 \text{N/m}^2$?(This high pressure is possible because the hammer striking the nail is brought to rest in such a short distance.)

9.4 Variation of Pressure with Depth in a Fluid

14. What depth of mercury creates a pressure of 1.00 atm?
15. The greatest ocean depths on the Earth are found in the Marianas Trench near the

Philippines. Calculate the pressure due to the ocean at the bottom of this trench, given its depth is 11.0 km and assuming the density of seawater is constant all the way down.

16. Verify that the SI unit of $h\rho g$ is $\mathrm{N/m^2}$.
17. Water towers store water above the level of consumers for times of heavy use, eliminating the need for high-speed pumps. How high above a user must the water level be to create a gauge pressure of $3.00 \times 10^5 \mathrm{N/m^2}$?
18. The aqueous humor in a person's eye is exerting a force of 0.300 N on the $1.10 - \mathrm{cm^2}$ area of the cornea. (a) What pressure is this in mm Hg? (b) Is this value within the normal range for pressures in the eye?
19. How much force is exerted on one side of an 8.50 cm by 11.0 cm sheet of paper by the atmosphere? How can the paper withstand such a force?
20. What pressure is exerted on the bottom of a 0.500-m-wide by 0.900-m-long gas tank that can hold 50.0 kg of gasoline by the weight of the gasoline in it when it is full?
21. Calculate the average pressure exerted on the palm of a shot-putter's hand by the shot if the area of contact is $50.0 \mathrm{cm^2}$ and he exerts a force of 800 N on it. Express the pressure in $\mathrm{N/m^2}$ and compare it with the $1.00 \times 10^6 \mathrm{Pa}$ pressures sometimes encountered in the skeletal system.
22. The left side of the heart creates a pressure of 120 mm Hg by exerting a force directly on the blood over an effective area of $15.0 \mathrm{cm^2}$. What force does it exert to accomplish this?
23. Show that the total force on a rectangular dam due to the water behind it increases with the square of the water depth. In particular, show that this force is given by $F = \rho g h^2 L/2$, where ρ is the density of water, h is its depth at the dam, and L is the length of the dam. You may assume the face of the dam is vertical. (Hint: Calculate the average pressure exerted and multiply this by the area in contact with the water. (See Figure 11.39.)

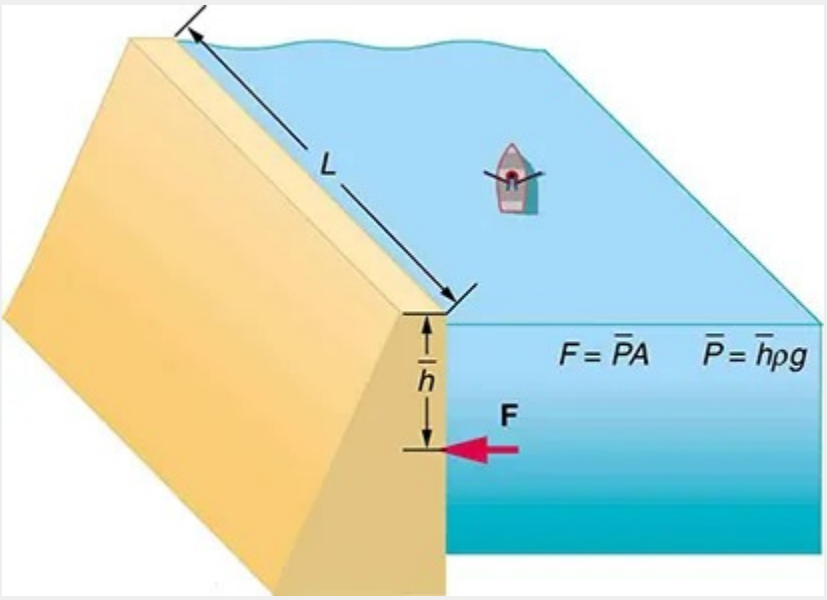


Figure 9.39 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a schematic illustration of a dam holding back water. The dam is shown in a cross-sectional view, depicting it as a triangular or trapezoidal shape with a sloped face. Water is on the right side of the dam, with a boat floating on the water surface.

Several physical notations and formulas are included in the image:

- *L*: This denotes the length of the dam along its slope.
- *h*: Represents the height of the water level against the dam.
- *F*: An arrow pointing towards the dam, indicating the force exerted by the water on the dam.
- The formula *F = \overline{P} A*: Represents the force calculation, where *\overline{P}* is the pressure on the dam and *A* is the area.
- The formula *\overline{P} = h\rho g*: Describes the pressure calculation, where *h* is the height of the water, *\rho* is the density of the water, and *g* is the acceleration due to gravity.

9.5 Pascal's Principle

24. How much pressure is transmitted in the hydraulic system considered in Example 9.6? Express your answer in pascals and in atmospheres.
25. What force must be exerted on the pedal cylinder of a hydraulic lift to support the weight of a 2000-kg car (a large car) resting on the wheel cylinder? The pedal cylinder has a 2.00-cm diameter and the wheel has a 24.0-cm diameter.
26. A crass host pours the remnants of several bottles of wine into a jug after a party. He then inserts a cork with a 2.00-cm diameter into the bottle, placing it in direct contact with the wine. He is amazed when he pounds the cork into place and the bottom of the jug (with a 14.0-cm diameter) breaks away. Calculate the extra force exerted against the bottom if he pounded the cork with a 120-N force.
27. A certain hydraulic system is designed to exert a force 100 times as large as the one put into it. (a) What must be the ratio of the area of the wheel cylinder to the area of the pedal cylinder? (b) What must be the ratio of their diameters? (c) By what factor is the distance through which the output force moves reduced relative to the distance through which the input force moves? Assume no losses to friction.
28. (a) Verify that work input equals work output for a hydraulic system assuming no losses to friction. Do this by showing that the distance the output force moves is reduced by the same factor that the output force is increased. Assume the volume of the fluid is constant. (b) What effect would friction within the fluid and between components in the system have on the output force? How would this depend on whether or not the fluid is moving?

9.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

29. Find the gauge and absolute pressures in the balloon and peanut jar shown in Figure 9.14, assuming the manometer connected to the balloon uses water whereas the manometer connected to the jar contains mercury. Express in units of centimeters of water for the balloon and millimeters of mercury for the jar,

taking $h = 0.0500\ \text{m}$ for each.

30. (a) Convert normal blood pressure readings of 120 over 80 mm Hg to newtons per meter squared using the relationship for pressure due to the weight of a fluid $(P = h\rho g)$ rather than a conversion factor. (b) Discuss why blood pressures for an infant could be smaller than those for an adult. Specifically, consider the smaller height to which blood must be pumped.
31. How tall must a water-filled manometer be to measure blood pressures as high as 300 mm Hg?
32. Pressure cookers have been around for more than 300 years, although their use has strongly declined in recent years (early models had a nasty habit of exploding). How much force must the latches holding the lid onto a pressure cooker be able to withstand if the circular lid is $25.0\ \text{cm}$ in diameter and the gauge pressure inside is 300 atm? Neglect the weight of the lid
33. Suppose you measure a standing person's blood pressure by placing the cuff on his leg 0.500 m below the heart. Calculate the pressure you would observe (in units of mm Hg) if the pressure at the heart were 120 over 80 mm Hg. Assume that there is no loss of pressure due to resistance in the circulatory system (a reasonable assumption, since major arteries are large).
34. A submarine is stranded on the bottom of the ocean with its hatch 25.0 m below the surface. Calculate the force needed to open the hatch from the inside, given it is circular and 0.450 m in diameter. Air pressure inside the submarine is 1.00 atm.
35. Assuming bicycle tires are perfectly flexible and support the weight of bicycle and rider by pressure alone, calculate the total area of the tires in contact with the ground. The bicycle plus rider has a mass of 80.0 kg, and the gauge pressure in the tires is $3.50 \times 10^5 \text{Pa}$.

9.7 Archimedes' Principle

36. What fraction of ice is submerged when it floats in freshwater, given the density of water at 0°C is very close to $1000\ \text{kg/m}^3$?
37. Logs sometimes float vertically in a lake because one end has become water-logged and denser than the other. What is the average density of a uniform-

diameter log that floats with 20.0% of its length above water?

38. Find the density of a fluid in which a hydrometer having a density of $0.750\ \text{g/mL}$ floats with 92.0% of its volume submerged.
39. If your body has a density of $995\ \text{kg/m}^3$, what fraction of you will be submerged when floating gently in: (a) freshwater? (b) salt water, which has a density of $1027\ \text{kg/m}^3$?
40. Bird bones have air pockets in them to reduce their weight—this also gives them an average density significantly less than that of the bones of other animals. Suppose an ornithologist weighs a bird bone in air and in water and finds its mass is $45.0\ \text{g}$ and its apparent mass when submerged is $3.60g$ (the bone is watertight). (a) What mass of water is displaced? (b) What is the volume of the bone? (c) What is its average density?
41. A rock with a mass of 540 g in air is found to have an apparent mass of 342 g when submerged in water. (a) What mass of water is displaced? (b) What is the volume of the rock? (c) What is its average density? Is this consistent with the value for granite?
42. Archimedes' principle can be used to calculate the density of a fluid as well as that of a solid. Suppose a chunk of iron with a mass of 390.0 g in air is found to have an apparent mass of 350.5 g when completely submerged in an unknown liquid. (a) What mass of fluid does the iron displace? (b) What is the volume of iron, using its density as given in Table 9.1 (c) Calculate the fluid's density and identify it.
43. In an immersion measurement of a woman's density, she is found to have a mass of 62.0 kg in air and an apparent mass of 0.0850 kg when completely submerged with lungs empty. (a) What mass of water does she displace? (b) What is her volume? (c) Calculate her density. (d) If her lung capacity is 1.75 L, is she able to float without treading water with her lungs filled with air?
44. Some fish have a density slightly less than that of water and must exert a force (swim) to stay submerged. What force must an 85.0-kg grouper exert to stay submerged in salt water if its body density is 1015kg/m^3?
45. (a) Calculate the buoyant force on a 2.00-L helium balloon. (b) Given the mass of the rubber in the balloon is 1.50 g, what is the net vertical force on the balloon if it

is let go? You can neglect the volume of the rubber.

46. (a) What is the density of a woman who floats in freshwater with 4.00% of her volume above the surface? This could be measured by placing her in a tank with marks on the side to measure how much water she displaces when floating and when held under water (briefly). (b) What percent of her volume is above the surface when she floats in seawater?
47. A certain man has a mass of 80 kg and a density of $955 \text{kg}/\text{m}^3$ (excluding the air in his lungs). (a) Calculate his volume. (b) Find the buoyant force air exerts on him. (c) What is the ratio of the buoyant force to his weight?
48. A simple compass can be made by placing a small bar magnet on a cork floating in water. (a) What fraction of a plain cork will be submerged when floating in water? (b) If the cork has a mass of 10.0 g and a 20.0-g magnet is placed on it, what fraction of the cork will be submerged? (c) Will the bar magnet and cork float in ethyl alcohol?
49. What fraction of an iron anchor's weight will be supported by buoyant force when submerged in saltwater?
50. Scurrilous con artists have been known to represent gold-plated tungsten ingots as pure gold and sell them at prices much below gold value but deservedly far above the cost of tungsten. With what accuracy must you be able to measure the mass of such an ingot in and out of water to tell that it is almost pure tungsten rather than pure gold?
51. A twin-sized air mattress used for camping has dimensions of 100 cm by 200 cm by 15 cm when blown up. The weight of the mattress is 2 kg. How heavy a person could the air mattress hold if it is placed in freshwater?
52. Referring to Figure 9.19, prove that the buoyant force on the cylinder is equal to the weight of the fluid displaced (Archimedes' principle). You may assume that the buoyant force is $F_2 - F_1$ and that the ends of the cylinder have equal areas A. Note that the volume of the cylinder (and that of the fluid it displaces) equals $(h_2 - h_1)A$.
53. (a) A 75.0-kg man floats in freshwater with 3.00% of his volume above water when his lungs are empty, and 5.00% of his volume above water when his lungs

are full. Calculate the volume of air he inhales—called his lung capacity—in liters. (b) Does this lung volume seem reasonable?

9.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

54. What is the pressure inside an alveolus having a radius of $2.50 \times 10^{-4}\text{m}$ if the surface tension of the fluid-lined wall is the same as for soapy water? You may assume the pressure is the same as that created by a spherical bubble.
55. (a) The pressure inside an alveolus with a 2.00×10^{-4}-m radius is $1.40 \times 10^{3}\text{Pa}$, due to its fluid-lined walls. Assuming the alveolus acts like a spherical bubble, what is the surface tension of the fluid? (b) Identify the likely fluid. (You may need to extrapolate between values in Table 9.3.)
56. What is the gauge pressure in millimeters of mercury inside a soap bubble 0.100 m in diameter?
57. Calculate the force on the slide wire in Figure 9.26 if it is 3.50 cm long and the fluid is ethyl alcohol.
58. Figure 9.32(a) shows the effect of tube radius on the height to which capillary action can raise a fluid. (a) Calculate the height h for water in a glass tube with a radius of 0.900 cm—a rather large tube like the one on the left. (b) What is the radius of the glass tube on the right if it raises water to 4.00 cm?
59. We stated in Example 9.12 that a xylem tube is of radius $2.50 \times 10^{-5}\text{m}$. Verify that such a tube raises sap less than a meter by finding h for it, making the same assumptions that sap's density is $1050\text{kg}/\text{m}^3$, its contact angle is zero, and its surface tension is the same as that of water at $20.0^{\circ}\ C$.
60. What fluid is in the device shown in Figure 9.26 if the force is $3.16 \times 10^{-3}\text{N}$ and the length of the wire is 2.50 cm? Calculate the surface tension γ and find a likely match from Table 9.3.
61. If the gauge pressure inside a rubber balloon with a 10.0-cm radius is 1.50 cm of water, what is the effective surface tension of the balloon?
62. Calculate the gauge pressures inside 2.00-cm-radius bubbles of water, alcohol, and soapy water. Which liquid forms the most stable bubbles, neglecting any

effects of evaporation?

63. Suppose water is raised by capillary action to a height of 5.00 cm in a glass tube. (a) To what height will it be raised in a paraffin tube of the same radius? (b) In a silver tube of the same radius?
64. Calculate the contact angle θ for olive oil if capillary action raises it to a height of 7.07 cm in a glass tube with a radius of 0.100 mm. Is this value consistent with that for most organic liquids?
65. When two soap bubbles touch, the larger is inflated by the smaller until they form a single bubble. (a) What is the gauge pressure inside a soap bubble with a 1.50-cm radius? (b) Inside a 4.00-cm-radius soap bubble? (c) Inside the single bubble they form if no air is lost when they touch?
66. Calculate the ratio of the heights to which water and mercury are raised by capillary action in the same glass tube.
67. What is the ratio of heights to which ethyl alcohol and water are raised by capillary action in the same glass tube?

9.9 Pressures in the Body

68. During forced exhalation, such as when blowing up a balloon, the diaphragm and chest muscles create a pressure of 60.0 mm Hg between the lungs and chest wall. What force in newtons does this pressure create on the 600cm^2 surface area of the diaphragm?
69. You can chew through very tough objects with your incisors because they exert a large force on the small area of a pointed tooth. What pressure in pascals can you create by exerting a force of $500\ \text{N}$ with your tooth on an area of 1.00mm^2?
70. One way to force air into an unconscious person's lungs is to squeeze on a balloon appropriately connected to the subject. What force must you exert on the balloon with your hands to create a gauge pressure of 4.00 cm water, assuming you squeeze on an effective area of 50.0cm^2?
71. Heroes in movies hide beneath water and breathe through a hollow reed (villains never catch on to this trick). In practice, you cannot inhale in this manner if your lungs are more than 60.0 cm below the surface. What is the maximum negative

gauge pressure you can create in your lungs on dry land, assuming you can achieve $-3.00cm$ water pressure with your lungs 60.0 cm below the surface?

72. Gauge pressure in the fluid surrounding an infant's brain may rise as high as 85.0 mm Hg (5 to 12 mm Hg is normal), creating an outward force large enough to make the skull grow abnormally large. (a) Calculate this outward force in newtons on each side of an infant's skull if the effective area of each side is 70.0cm^2. (b) What is the net force acting on the skull?
73. A full-term fetus typically has a mass of 3.50 kg. (a) What pressure does the weight of such a fetus create if it rests on the mother's bladder, supported on an area of 90.0cm^2? (b) Convert this pressure to millimeters of mercury and determine if it alone is great enough to trigger the micturition reflex (it will add to any pressure already existing in the bladder).
74. If the pressure in the esophagus is $-2.00mmHg$ while that in the stomach is $+20.0\text{ mm Hg}$, to what height could stomach fluid rise in the esophagus, assuming a density of 1.10 g/mL? (This movement will not occur if the muscle closing the lower end of the esophagus is working properly.)
75. Pressure in the spinal fluid is measured as shown in Figure 9.40. If the pressure in the spinal fluid is 10.0 mm Hg: (a) What is the reading of the water manometer in cm water? (b) What is the reading if the person sits up, placing the top of the fluid 60 cm above the tap? The fluid density is 1.05 g/mL.

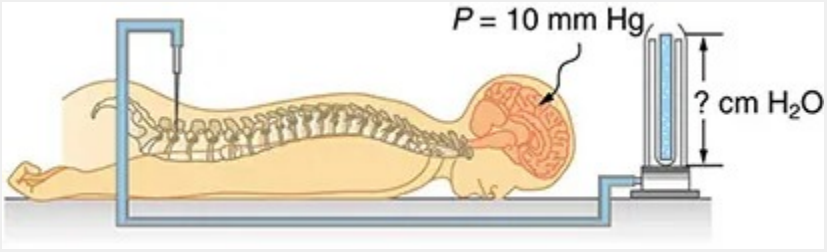


Figure 9.40 A water manometer used to measure pressure in the spinal fluid. The height of the fluid in the manometer is measured relative to the spinal column, and the manometer is open to the atmosphere. The measured pressure will be considerably greater if the person sits up. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a diagram of a person lying horizontally, with a focus on the spine and brain. Next to the person's head, there's a labeled point indicating a pressure measurement of "P = 10 mm Hg." A U-shaped tube filled with fluid is connected to the body, showing the flow of fluid. The right side of the tube has a question mark before "cm H_2O," indicating a conversion or measurement in centimeters of water pressure. The setup appears to illustrate a principle related to cerebrospinal fluid pressure or a similar fluid pressure measurement scenario.

76. Calculate the maximum force in newtons exerted by the blood on an aneurysm, or ballooning, in a major artery, given the maximum blood pressure for this person is 150 mm Hg and the effective area of the aneurysm is 20.0cm^2. Note that this force is great enough to cause further enlargement and subsequently greater force on the ever-thinner vessel wall.
77. During heavy lifting, a disk between spinal vertebrae is subjected to a 5000-N compressional force. (a) What pressure is created, assuming that the disk has a uniform circular cross section 2.00 cm in radius? (b) What deformation is produced if the disk is 0.800 cm thick and has a Young's modulus of $1.5 \times 10^9 \text{N/m}^2$?
78. When a person sits erect, increasing the vertical position of their brain by 36.0 cm, the heart must continue to pump blood to the brain at the same rate. (a) What is the gain in gravitational potential energy for 100 mL of blood raised 36.0 cm? (b) What is the drop in pressure, neglecting any losses due to friction? (c) Discuss how the gain in gravitational potential energy and the decrease in pressure are related.
79. (a) How high will water rise in a glass capillary tube with a 0.500-mm radius? (b) How much gravitational potential energy does the water gain? (c) Discuss possible sources of this energy.
80. A negative pressure of 25.0 atm can sometimes be achieved with the device in Figure 9.41 before the water separates. (a) To what height could such a negative gauge pressure raise water? (b) How much would a steel wire of the same diameter and length as this capillary stretch if suspended from above?

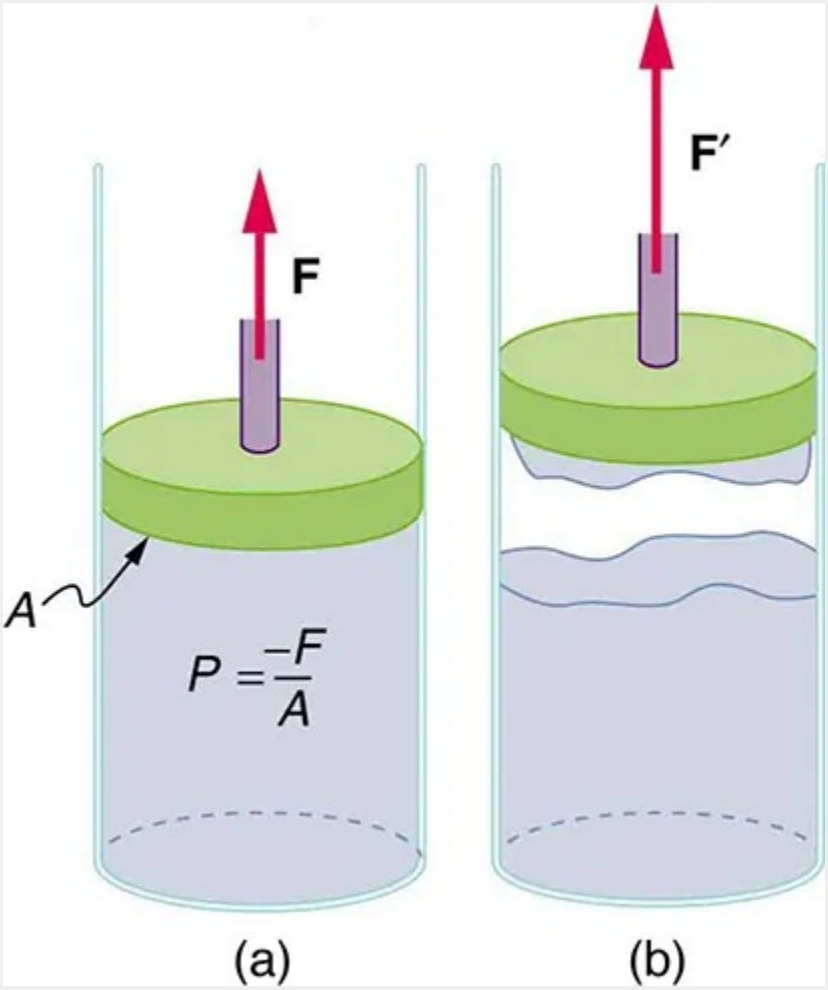


Figure 9.41 (a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure $P = -F/A$ *(b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two cylindrical containers labeled (a) and (b), each containing a piston.

1. Container (a):
– The piston is shown at the top with a downward red arrow labeled “F” indicating force being applied downwards.
– The piston is circular with an area labeled “A”.

– Inside the container, there's a mathematical expression: $P = \frac{-F}{A}$.
– The container appears empty and transparent, showing only these elements.

2. Container (b):
– The piston is also at the top but this time with an upward red arrow labeled "F'" indicating force being applied upwards.
– The interior of the container shows a wavy horizontal line, suggesting the presence of a fluid or similar medium.
– The piston is in contact with the fluid surface.

This image illustrates a fundamental principle in physics, likely related to pressure and force on a piston in different scenarios.

81. Suppose you hit a steel nail with a 0.500-kg hammer, initially moving at $15.0\ \mathrm{m/s}$ and brought to rest in 2.80 mm. (a) What average force is exerted on the nail? (b) How much is the nail compressed if it is 2.50 mm in diameter and 6.00-cm long? (c) What pressure is created on the 1.00-mm-diameter tip of the nail?
82. Calculate the pressure due to the ocean at the bottom of the Marianas Trench near the Philippines, given its depth is $11.0\ \mathrm{km}$ and assuming the density of sea water is constant all the way down. (b) Calculate the percent decrease in volume of sea water due to such a pressure, assuming its bulk modulus is the same as water and is constant. (c) What would be the percent increase in its density? Is the assumption of constant density valid? Will the actual pressure be greater or smaller than that calculated under this assumption?
83. The hydraulic system of a backhoe is used to lift a load as shown in Figure 9.42. (a) Calculate the force F the secondary cylinder must exert to support the 400-kg load and the 150-kg brace and shovel. (b) What is the pressure in the hydraulic fluid if the secondary cylinder is 2.50 cm in diameter? (c) What force would you have to exert on a lever with a mechanical advantage of 5.00 acting on a primary cylinder 0.800 cm in diameter to create this pressure?

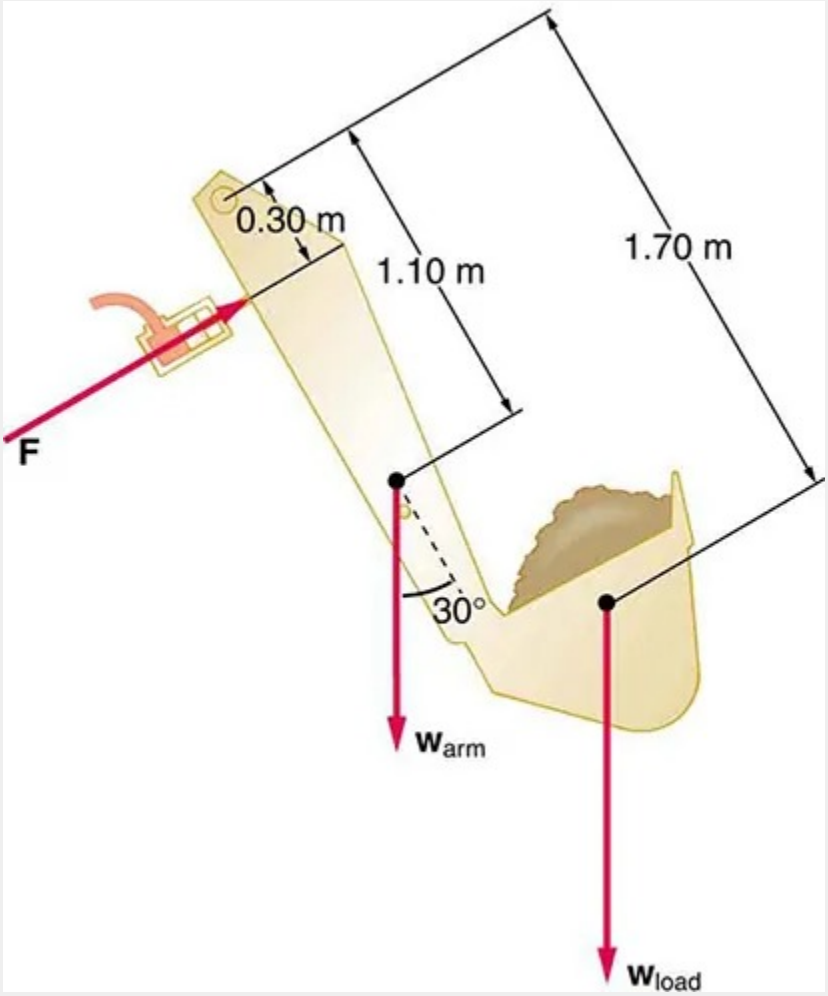


Figure 9.42 Hydraulic and mechanical lever systems are used in heavy machinery such as this back hoe. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a diagram of a mechanical arm with dimensions and forces labeled.

– There is an arm extending from a pivot point. The arm has two sections:
– The first section is 0.30 meters long.
– The second section is 1.10 meters long, angled at 30 degrees downwards from the horizontal.

– The arm then connects to a bucket-like load at the end:
– The distance from the pivot point to the end of the load is 1.70 meters.

– Two force vectors are indicated:
– F is applied horizontally at the pivot point.
– W_{arm} is vertical, acting downward at the joint between the two arm sections.
– W_{load} is vertical, acting downward at the center of the load.

The image depicts these forces and measurements in a mechanical context, likely dealing with balance or structural forces.

84. Some miners wish to remove water from a mine shaft. A pipe is lowered to the water 90 m below, and a negative pressure is applied to raise the water. (a) Calculate the pressure needed to raise the water. (b) What is unreasonable about this pressure? (c) What is unreasonable about the premise?
85. You are pumping up a bicycle tire with a hand pump, the piston of which has a 2.00-cm radius.
86. (a) What force in newtons must you exert to create a pressure of $6.90 \times 10^5 \text{Pa}$ (b) What is unreasonable about this (a) result? (c) Which premises are unreasonable or inconsistent?
87. Consider a group of people trying to stay afloat after their boat strikes a log in a lake. Construct a problem in which you calculate the number of people that can cling to the log and keep their heads out of the water. Among the variables to be considered are the size and density of the log, and what is needed to keep a person's head and arms above water without swimming or treading water.
88. The alveoli in emphysema victims are damaged and effectively form larger sacs. Construct a problem in which you calculate the loss of pressure due to surface tension in the alveoli because of their larger average diameters. (Part of the lung's ability to expel air results from pressure created by surface tension in the alveoli.) Among the things to consider are the normal surface tension of the fluid lining the alveoli, the average alveolar radius in normal individuals and its average in emphysema sufferers.

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FLUID DYNAMICS AND ITS BIOLOGICAL AND MEDICAL APPLICATIONS



Figure 10.1 Many fluids are flowing in this scene. Water from the hose and smoke from the fire are visible flows. Less visible are the flow of air and the flow of fluids on the ground and within the people fighting the fire. Firefighters by Andrew Magill, CC-BY 2.0

Chapter Outline

10.1 Flow Rate and Its Relation to Velocity
10.2 Bernoulli's Equation
10.3 The Most General Applications of Bernoulli's Equation
10.4 Viscosity and Laminar Flow; Poiseuille's Law
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

We have dealt with many situations in which fluids are static. But by their very definition, fluids flow. Examples come easily—a column of smoke rises from a camp fire, water streams from a fire hose, blood courses through your veins. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does the body regulate blood flow? The physics of fluids in motion—fluid dynamics—allows us to answer these and many other questions.
Access multimedia content

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10.1 FLOW RATE AND ITS RELATION TO VELOCITY

Learning Objectives

By the end of this section, you will be able to:

- Calculate flow rate.
- Define units of volume.
- Describe incompressible fluids.
- Explain the consequences of the equation of continuity.

Flow rate Q is defined to be the volume of fluid passing by some location through an area during a period of time, as seen in Figure 10.2. In symbols, this can be written as

$$Q = \frac{V}{t},$$

10.1

where V is the volume and t is the elapsed time.

The SI unit for flow rate is m^3/s, but a number of other units for Q are in common use. For example, the heart of a resting adult pumps blood at a rate of 5.00 liters per minute (L/min). Note that a liter (L) is 1/1000 of a cubic meter or 1000 cubic centimeters ($10^{-3}\mathrm{m}^3$ or $10^3\mathrm{cm}^3$). In this text we shall use whatever metric units are most convenient for a given situation.

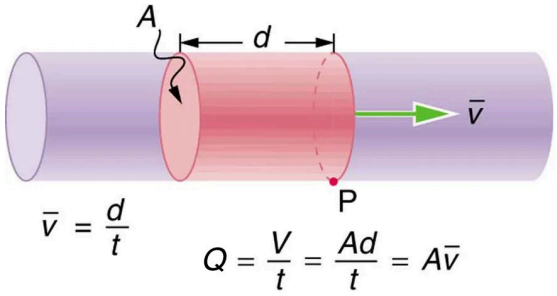


Figure 10.2 Flow rate is the volume of fluid per unit time flowing past a point through the area A*. Here the shaded cylinder of fluid flows past point* P *in a uniform pipe in time* t*. The volume of the cylinder is* Ad *and the average velocity is* $\bar{v} = d/t$ *so that the flow rate is* $Q = \mathrm{Ad}/t = A\bar{v}$*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image illustrates fluid flow through a cylindrical pipe. The pipe is shown in a longitudinal section, revealing the interior space. A shaded section of the cylinder is highlighted to represent the volume of fluid moving through the pipe.

Key elements in the image include:

- A section of the pipe is shaded red, with an arrow labeled *d* indicating the distance or length of the fluid section.
- An arrow labeled *A* is pointing to the cross-sectional area of the pipe where the fluid is moving.
- A green arrow labeled $\bar{v}$ extends horizontally along the center of the pipe to signify the average velocity of the fluid.
- A point *P* is marked at the front edge of the shaded section.

Equations displayed at the bottom of the image describe the relationships between various fluid flow parameters:

- $\bar{v} = d / t$ describes the average velocity as the distance divided by time.
- $Q = V / t = (A * d) / t = A * \bar{v}$ represents the flow rate, defined as volume over time, or as the product of area and average velocity.

Example 10.1

Calculating Volume from Flow Rate: The Heart Pumps a Lot of Blood in a Lifetime

How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

Strategy

Time and flow rate Q are given, and so the volume V can be calculated from the definition of flow rate.

Solution

Solving $Q = V/t$ for volume gives

$$V = \mathrm{Qt}.$$

10.2

Substituting known values yields

$$\begin{aligned} V &= \left(\frac{5.00\mathrm{L}}{1\ \mathrm{min}}\right)(75\mathrm{y})\left(\frac{1\mathrm{m}^3}{10^3\mathrm{L}}\right)\left(5.26\times10^5\frac{\mathrm{min}}{\mathrm{y}}\right) \\ &= 2.0\times10^5\ \mathrm{m}^3. \end{aligned}$$

10.3

Discussion

This amount is about 200,000 tons of blood. For comparison, this value is equivalent to about 200 times the volume of water contained in a 6-lane 50-m lap pool.

Flow rate and velocity are related, but quite different, physical quantities. To make the distinction clear, think

about the flow rate of a river. The greater the velocity of the water, the greater the flow rate of the river. But flow rate also depends on the size of the river. A rapid mountain stream carries far less water than the Amazon River in Brazil, for example. The precise relationship between flow rate Q and velocity $\bar{v}$ is

$$Q = A\bar{v},$$
10.4

where A is the cross-sectional area and $\bar{v}$ is the average velocity. This equation seems logical enough. The relationship tells us that flow rate is directly proportional to both the magnitude of the average velocity (hereafter referred to as the speed) and the size of a river, pipe, or other conduit. The larger the conduit, the greater its cross-sectional area. Figure 10.2 illustrates how this relationship is obtained. The shaded cylinder has a volume

$$V = \mathrm{Ad},$$
10.5

which flows past the point P in a time t. Dividing both sides of this relationship by t gives

$$\frac{V}{t} = \frac{\mathrm{Ad}}{t}.$$
10.6

We note that $Q = V/t$ and the average speed is

$\bar{v} = d/t$. Thus the equation becomes $Q = A\bar{v}$.

Figure 10.3 shows an incompressible fluid flowing along a pipe of decreasing radius. Because the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow. In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase. This logic can be extended to say that the flow rate must be the same at all points along the pipe. In particular, for points 1 and 2,

$$\left.\begin{aligned} Q_1 &= Q_2 \\ A_1(\bar{v})_1 &= A_2(\bar{v})_2 \end{aligned}\right\}.$$
10.7

This is called the equation of continuity and is valid for any incompressible fluid. The consequences of the equation of continuity can be observed when water flows from a hose into a narrow spray nozzle: it emerges with a large speed—that is the purpose of the nozzle. Conversely, when a river empties into one end of a reservoir, the water slows considerably, perhaps picking up speed again when it leaves the other end of the

reservoir. In other words, speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases.

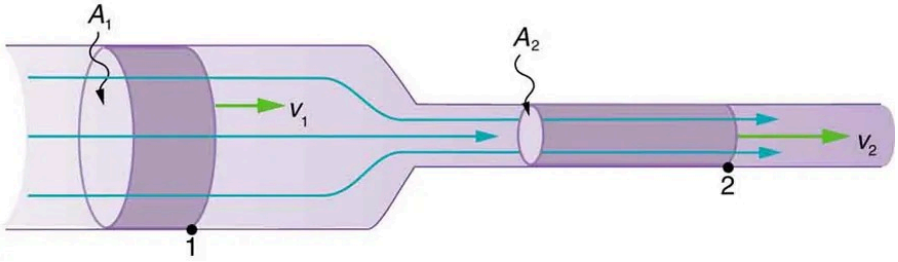


Figure 10.3 When a tube narrows, the same volume occupies a greater length. For the same volume to pass points 1 and 2 in a given time, the speed must be greater at point 2. The process is exactly reversible. If the fluid flows in the opposite direction, its speed will decrease when the tube widens. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of fluid flow through a pipe that changes in diameter. The pipe begins with a wide section on the left labeled as A_1 and narrows down to a smaller section on the right labeled as A_2. Inside the pipe, there are lines indicating the streamline flow of fluid. There is a green arrow labeled v_1 pointing rightwards in the wider section, representing the initial velocity of the fluid. In the narrower section, another green arrow labeled v_2 also points to the right, indicating the velocity of the fluid after the constriction. Points labeled as 1 and 2 correspond to the sections A_1 and A_2, respectively, likely representing different pressure or velocity measurement points.

Since liquids are essentially incompressible, the equation of continuity is valid for all liquids. However, gases are compressible, and so the equation must be applied with caution to gases if they are subjected to compression or expansion.

Example 10.2

Calculating Fluid Speed: Speed Increases When a Tube Narrows

A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

Strategy

We can use the relationship between flow rate and speed to find both velocities. We will use the subscript 1 for the hose and 2 for the nozzle.

Solution for (a)

First, we solve $Q = A\bar{v}$ for v_1 and note that the cross-sectional area is $A = \pi r^2$, yielding

$$\bar{v}_1 = \frac{Q}{A_1} = \frac{Q}{\pi r_1^2}.$$

10.8

Substituting known values and making appropriate unit conversions yields

$$\bar{v}_1 = \frac{(0.500\text{L/s})\,(10^{-3}\text{m}^3/\text{L})}{\pi\,(9.00 \times 10^{-3}\text{m})^2} = 1.96\text{m/s}.$$

10.9

Solution for (b)

We could repeat this calculation to find the speed in the nozzle $\bar{v}_2$, but we will use the

equation of continuity to give a somewhat different insight. Using the equation which states

$$A_1\bar{v}_1 = A_2\bar{v}_2,$$

10.10

solving for $\bar{v}_2$ and substituting πr^2 for the cross-sectional area yields

$$\bar{v}_2 = \frac{A_1}{A_2}\bar{v}_1 = \frac{\pi r_1^2}{\pi r_2^2}\bar{v}_1 = \frac{r_{1^2}}{r_{2^2}}\bar{v}_1.$$

10.11

Substituting known values,

$$\bar{v}_2 = \frac{(0.900\text{cm})^2}{(0.250\text{cm})^2}1.96\text{m/s} = 25.5\text{ m/s}.$$

10.12

Discussion

A speed of 1.96 m/s is about right for water emerging from a nozzleless hose. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube.

The solution to the last part of the example shows that speed is inversely proportional to the *square* of the radius of the tube, making for large effects when radius varies. We can blow out a candle at quite a distance, for example, by pursing our lips, whereas blowing on a candle with our mouth wide open is quite ineffective.

In many situations, including in the cardiovascular system, branching of the flow occurs. The blood is pumped from the heart into arteries that subdivide into smaller arteries (arterioles) which branch into very fine vessels called capillaries. In this situation, continuity of flow is maintained but it is the *sum* of the flow rates in each of the branches in any portion along the tube that is maintained. The equation of continuity in a more general form becomes

$$n_1A_1\bar{v}_1 = n_2A_2\bar{v}_2,$$

10.13

where n_1 and n_2 are the number of branches in each of the sections along the tube.

Example 10.3

Calculating Flow Speed and Vessel Diameter: Branching in the Cardiovascular System

The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body. (a) Calculate the average speed of the blood in the aorta if the flow rate is 5.0 L/min. The aorta has a radius of 10 mm. (b) Blood also flows through smaller blood vessels known as capillaries. When the rate of blood flow in the aorta is 5.0 L/min, the speed of blood in the capillaries is about 0.33 mm/s. Given that the average diameter of a capillary is $8.0\mu\text{m}$, calculate the number of capillaries in the blood circulatory system.

Strategy

We can use $Q = A\bar{v}$ to calculate the speed of flow in the aorta and then use the general form of the equation of continuity to calculate the number of capillaries as all of the other variables are known.

Solution for (a)

The flow rate is given by $Q = A\bar{v}$ or $\bar{v} = \dfrac{Q}{\pi r^2}$ for a cylindrical vessel.

Substituting the known values (converted to units of meters and seconds) gives

$$\bar{v} = \frac{(5.0\text{L/min})\left(10^{-3}\text{m}^3/\text{L}\right)(1\text{min}/60\text{ s})}{\pi(0.010\text{ m})^2}$$
$$= 0.27\text{ m/s}.$$

10.14

Solution for (b)

Using $n_1 A_1 \bar{v}_1 = n_2 A_2 \bar{v}_1$, assigning the subscript 1 to the aorta and 2 to the capillaries, and solving for n_2 (the number of capillaries) gives $n_2 = \frac{n_1 A_1 \bar{v}_1}{A_2 \bar{v}_2}$. Converting all quantities to units of meters and seconds and substituting into the equation above gives

$$n_2 = \frac{(1)\,(\pi)\left(10\ \times 10^{-3}\mathrm{m}\right)^2 (0.27 m/s)}{(\pi)\left(4.0 \times 10^{-6}\mathrm{m}\right)^2 \left(0.33 \times 10^{-3}\mathrm{m/s}\right)}$$
$$= 5.0 \times 10^{9}\mathrm{capillaries}\ .$$

10.15

Discussion

Note that the speed of flow in the capillaries is considerably reduced relative to the speed in the aorta due to the significant increase in the total cross-sectional area at the capillaries. This low speed is to allow sufficient time for effective exchange to occur although it is equally important for the flow not to become stationary in order to avoid the possibility of clotting. Does this large number of capillaries in the body seem reasonable? In active muscle, one finds about 200 capillaries per mm^3, or about 200×10^6 per 1 kg of muscle. For 20 kg of muscle, this amounts to about 4×10^9 capillaries.

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10.2 BERNOULLI'S EQUATION

Learning Objectives

By the end of this section, you will be able to:

- Explain the terms in Bernoulli's equation.
- Explain how Bernoulli's equation is related to conservation of energy.
- Explain how to derive Bernoulli's principle from Bernoulli's equation.
- Calculate with Bernoulli's principle.
- List some applications of Bernoulli's principle.

When a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases. Where does that change in kinetic energy come from? The increased kinetic energy comes from the net work done on the fluid to push it into the channel and the work done on the fluid by the gravitational force, if the fluid changes vertical position. Recall the work-energy theorem,

$$W_{\text{net}} = \frac{1}{2}\text{mv}^2 - \frac{1}{2}\text{mv}_0^2.$$

10.16

There is a pressure difference when the channel narrows. This pressure difference results in a net force on the fluid: recall that pressure times area equals force. The net work done increases the fluid's kinetic energy. As a result, the *pressure will drop in a rapidly-moving fluid*, whether or not the fluid is confined to a tube.

There are a number of common examples of pressure dropping in rapidly-moving fluids. Shower curtains have a disagreeable habit of bulging into the shower stall when the shower is on. The high-velocity stream of water and air creates a region of lower pressure inside the shower, and standard atmospheric pressure on the other side. The pressure difference results in a net force inward pushing the curtain in. You may also have

noticed that when passing a truck on the highway, your car tends to veer toward it. The reason is the same—the high velocity of the air between the car and the truck creates a region of lower pressure, and the vehicles are pushed together by greater pressure on the outside. (See Figure 10.4.) This effect was observed as far back as the mid-1800s, when it was found that trains passing in opposite directions tipped precariously toward one another.

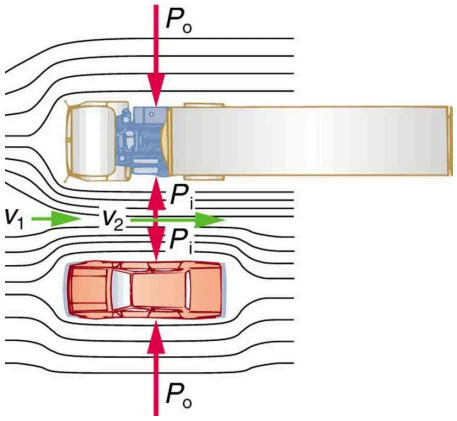


Figure 10.4 An overhead view of a car passing a truck on a highway. Air passing between the vehicles flows in a narrower channel and must increase its speed (v_2 is greater than v_1), causing the pressure between them to drop (P_i is less than P_o). Greater pressure on the outside pushes the car and truck together. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates air flow and pressure around a large truck and a smaller car positioned parallel to each other. Streamlines represent the air flow:

– The truck is situated at the top, seen from a top-down view, with the trailer extending to the right.
– Below the truck, there's a car, also viewed from above.
– Streamlines curve around both vehicles, indicating aerodynamic flow.
– Arrows labeled with "P_o" point vertically outwards from both the truck and car, representing the external pressure.
– A pair of arrows, labeled "P_i" and pointing towards each other, indicate the internal pressure between the vehicles.
– Green arrows labeled "v_1" and "v_2" point horizontally, representing velocity vectors; "v_1" is on the left, directed towards the gap between the vehicles, and "v_2" is inside the gap between the truck and car, pointing towards the car.

The diagram visually conveys the principle of air pressure and velocity changes between two moving vehicles.

Making Connections: Take-Home Investigation with a Sheet of Paper

Hold the short edge of a sheet of paper parallel to your mouth with one hand on each side of your mouth. The page should slant downward over your hands. Blow over the top of the page. Describe what happens and explain the reason for this behavior.

Bernoulli's Equation

The relationship between pressure and velocity in fluids is described quantitatively by Bernoulli's equation, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli's equation states that for an incompressible, frictionless fluid, the following sum is constant:

$$P + \frac{1}{2}\rho v^2 + \rho \mathrm{gh} = \text{constant},$$

10.17

where P is the absolute pressure, ρ is the fluid density, v is the velocity of the fluid, h is the height above some reference point, and g is the acceleration due to gravity. If we follow a small volume of fluid along its path, various quantities in the sum may change, but the total remains constant. Let the subscripts 1 and 2 refer to any two points along the path that the bit of fluid follows; Bernoulli's equation becomes

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2.$$

10.18

Bernoulli's equation is a form of the conservation of energy principle. Note that the second and third terms are the kinetic and potential energy with m replaced by ρ. In fact, each term in the equation has units of energy per unit volume. We can prove this for the second term by substituting $\rho = m/V$ into it and gathering terms:

$$\frac{1}{2}\rho v^2 = \frac{\frac{1}{2}\mathrm{mv}^2}{V} = \frac{\mathrm{KE}}{V}.$$

10.19

So $\frac{1}{2}\rho v^2$ is the kinetic energy per unit volume. Making the same substitution into the third term in the equation, we find

$$\rho gh = \frac{mgh}{V} = \frac{\text{PE}_\text{g}}{V},$$
10.20

so ρgh is the gravitational potential energy per unit volume. Note that pressure P has units of energy per unit volume, too. Since $P = F/A$, its units are N/m^2. If we multiply these by m/m, we obtain $\text{N} \cdot \text{m/m}^3 = \text{J/m}^3$, or energy per unit volume. Bernoulli's equation is, in fact, just a convenient statement of conservation of energy for an incompressible fluid in the absence of friction.

Making Connections: Conservation of Energy

Conservation of energy applied to fluid flow produces Bernoulli's equation. The net work done by the fluid's pressure results in changes in the fluid's KE and PE_g per unit volume. If other forms of energy are involved in fluid flow, Bernoulli's equation can be modified to take these forms into account. Such forms of energy include thermal energy dissipated because of fluid viscosity.

The general form of Bernoulli's equation has three terms in it, and it is broadly applicable. To understand it better, we will look at a number of specific situations that simplify and illustrate its use and meaning.

Bernoulli's Equation for Static Fluids

Let us first consider the very simple situation where the fluid is static—that is, $v_1 = v_2 = 0$. Bernoulli's equation in that case is

$$P_1 + \rho gh_1 = P_2 + \rho gh_2.$$
10.21

We can further simplify the equation by taking $h_2 = 0$ (we can always choose some height to be zero, just as

we often have done for other situations involving the gravitational force, and take all other heights to be relative to this). In that case, we get

$$P_2 = P_1 + \rho g h_1.$$
10.22

This equation tells us that, in static fluids, pressure increases with depth. As we go from point 1 to point 2 in the fluid, the depth increases by h_1, and consequently, P_2 is greater than P_1 by an amount $\rho g h_1$. In the very simplest case, P_1 is zero at the top of the fluid, and we get the familiar relationship $P = \rho g h$. (Recall that $P = \rho g h$ and $\Delta \text{PE}_\text{g} = \text{mgh}$.) Bernoulli's equation includes the fact that the pressure due to the weight of a fluid is ρgh. Although we introduce Bernoulli's equation for fluid flow, it includes much of what we studied for static fluids in the preceding chapter.

Bernoulli's Principle—Bernoulli's Equation at Constant Depth

Another important situation is one in which the fluid moves but its depth is constant—that is, $h_1 = h_2$. Under that condition, Bernoulli's equation becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$
10.23

Situations in which fluid flows at a constant depth are so important that this equation is often called Bernoulli's principle. It is Bernoulli's equation for fluids at constant depth. (Note again that this applies to a small volume of fluid as we follow it along its path.) As we have just discussed, pressure drops as speed increases in a moving fluid. We can see this from Bernoulli's principle. For example, if v_2 is greater than v_1 in the equation, then P_2 must be less than P_1 for the equality to hold.

Example 10.4

Calculating Pressure: Pressure Drops as a Fluid Speeds Up

In Example 10.2, we found that the speed of water in a hose increased from 1.96 m/s to 25.5 m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is $1.01 \times 10^5 \text{N/m}^2$ (atmospheric, as it must be) and assuming level, frictionless flow.

Strategy

Level flow means constant depth, so Bernoulli's principle applies. We use the subscript 1 for values in the hose and 2 for those in the nozzle. We are thus asked to find P_1.

Solution

Solving Bernoulli's principle for P_1 yields

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho\,(v_2^2 - v_1^2).$$

10.24

Substituting known values,

$$\begin{aligned} P_1 &= 1.01 \times 10^5 \text{ N/m}^2 \\ &\quad + \frac{1}{2}(10^3 \text{ kg/m}^3)\left[(25.5 \text{ m/s})^2 - (1.96 \text{ m/s})^2\right] \\ &= 4.24 \times 10^5 \text{ N/m}^2. \end{aligned}$$

10.25

Discussion

This absolute pressure in the hose is greater than in the nozzle, as expected since v is

greater in the nozzle. The pressure P_2 in the nozzle must be atmospheric since it emerges into the atmosphere without other changes in conditions.

Applications of Bernoulli's Principle

There are a number of devices and situations in which fluid flows at a constant height and, thus, can be analyzed with Bernoulli's principle.

Entrainment

People have long put the Bernoulli principle to work by using reduced pressure in high-velocity fluids to move things about. With a higher pressure on the outside, the high-velocity fluid forces other fluids into the stream. This process is called *entrainment*. Entrainment devices have been in use since ancient times, particularly as pumps to raise water small heights, as in draining swamps, fields, or other low-lying areas. Some other devices that use the concept of entrainment are shown in Figure 10.5.

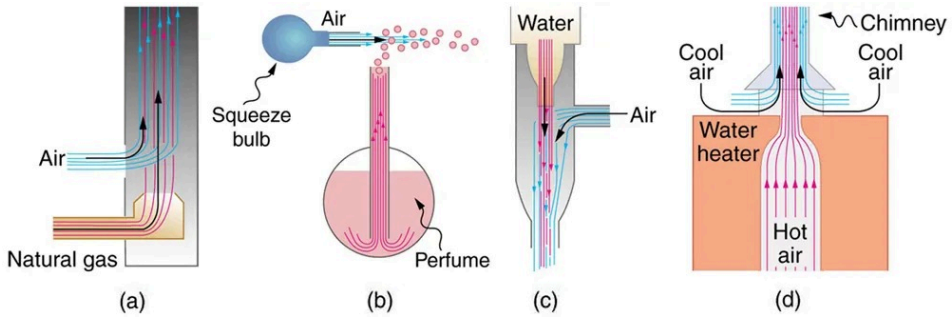


Figure 10.5 Examples of entrainment devices that use increased fluid speed to create low pressures, which then entrain one fluid into another. (a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids. (c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel. (d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains four diagrams labeled (a) to (d) that illustrate different examples of air and fluid flow dynamics:

1. Diagram (a):
– Displays a vertical tube with natural gas entering from the bottom.
– Air flows in from the sides, mixing with the natural gas and moving upwards.

2. Diagram (b):
– Features a squeeze bulb on the left which forces air through a vertical tube.
– Air carries perfume from a container upwards, dispersing it into the air.

3. Diagram (c):
– Shows a setup with water at the base and a vertical tube.
– Air flows in from the sides, mixes with the water, and moves upwards through the tube.

4. Diagram (d):
– Presents a water heater with hot air rising through a chimney in the center.
– Cool air enters from the sides, mixes with the hot air, and flows upwards through the chimney.

Wings and Sails

The airplane wing is a beautiful example of Bernoulli's principle in action. Figure 10.6(a) shows the characteristic shape of a wing. The wing is tilted upward at a small angle and the upper surface is longer, causing air to flow faster over it. The pressure on top of the wing is therefore reduced, creating a net upward force or lift. (Wings can also gain lift by pushing air downward, utilizing the conservation of momentum principle. The deflected air molecules result in an upward force on the wing — Newton's third law.) Sails also have the characteristic shape of a wing. (See Figure 10.6(b).) The pressure on the front side of the sail, P_{front}, is lower than the pressure on the back of the sail, P_{back}. This results in a forward force and even allows you to sail into the wind.

Making Connections: Take-Home Investigation with Two Strips of Paper

For a good illustration of Bernoulli's principle, make two strips of paper, each about 15 cm long and 4 cm wide. Hold the small end of one strip up to your lips and let it drape over your finger. Blow across the paper. What happens? Now hold two strips of paper up to your lips, separated by your fingers. Blow between the strips. What happens?

Velocity measurement

Figure 10.7 shows two devices that measure fluid velocity based on Bernoulli's principle. The manometer in Figure 10.7(a) is connected to two tubes that are small enough not to appreciably disturb the flow. The tube facing the oncoming fluid creates a dead spot having zero velocity ($v_1 = 0$) in front of it, while fluid passing the other tube has velocity v_2. This means that Bernoulli's principle as stated in $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ becomes

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2. \tag{10.26}$$

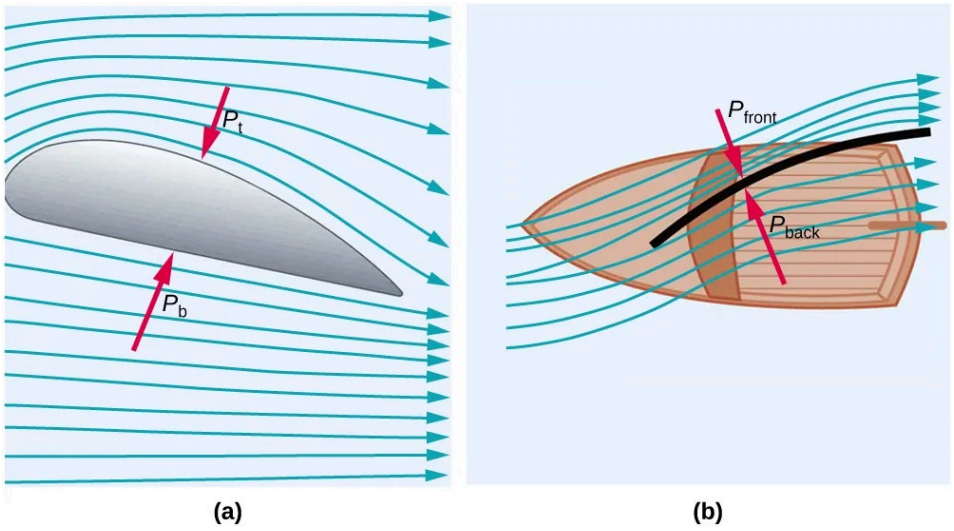


Figure 10.6 (a) The Bernoulli principle helps explain lift generated by a wing. (b) Sails use the same technique to generate part of their thrust. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two side-by-side diagrams labeled as (a) and (b).

Diagram (a):
– It shows a cross-sectional view of an airfoil, which is an aerodynamic shape similar to that of an airplane wing.
– Blue lines with arrows around the airfoil indicate the flow of air. The arrows show the direction of airflow.
– Two red arrows labeled as "P_t" and "P_b" indicate pressures on the top and bottom surfaces of the airfoil, respectively.

Diagram (b):
– It depicts a rowboat viewed from above, illustrating the effect of airflow on a curved sail.
– Blue lines with arrows suggest the direction and flow of air over the curved sail.
– Two red arrows labeled as "P_{front}" and "P_{back}" indicate the pressures at the front and back of the sail.

Thus pressure P_2 over the second opening is reduced by $\frac{1}{2}\rho v_2^2$, and so the fluid in the manometer rises by h on the side connected to the second opening, where

$$h \propto \frac{1}{2}\rho v_2^2.$$

10.27

(Recall that the symbol $\propto$ means "proportional to.") Solving for v_2, we see that

$v_2 \propto \sqrt{h}$.
10.28

Figure 10.7(b) shows a version of this device that is in common use for measuring various fluid velocities; such devices are frequently used as air speed indicators in aircraft.

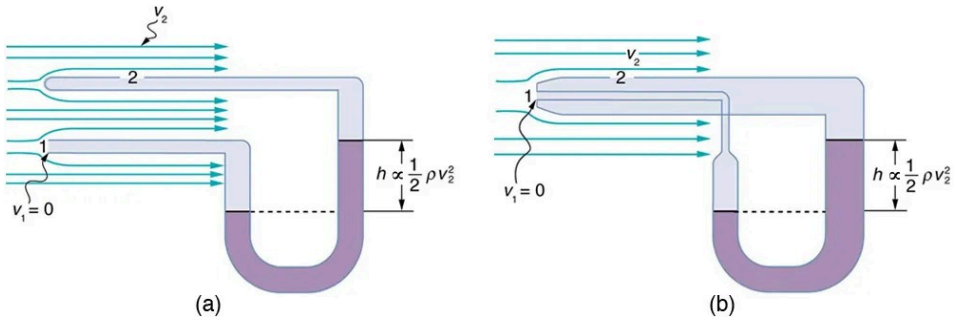


Figure 10.7 Measurement of fluid speed based on Bernoulli's principle. (a) A manometer is connected to two tubes that are close together and small enough not to disturb the flow. Tube 1 is open at the end facing the flow. A dead spot having zero speed is created there. Tube 2 has an opening on the side, and so the fluid has a speed v across the opening; thus, pressure there drops. The difference in pressure at the manometer is $\frac{1}{2}\rho v_2^2$, and so h is proportional to $\frac{1}{2}\rho v_2^2$. (b) This type of velocity measuring device is a Prandtl tube, also known as a pitot tube. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams, labeled (a) and (b), illustrating fluid flow through pipes with a U-tube manometer.

Diagram (a):
– Depicts a pipe with two sections labeled 1 and 2.
– Section 1 is horizontal, and section 2 curves downward into the U-tube manometer.
– Fluid enters section 1 with velocity $v_1 = 0$ and exits section 2 with velocity v_2.
– Streamlines are shown with arrows indicating the direction of flow.
– The U-tube manometer is half-filled with a liquid, with one side of the manometer aligned with the pipe outlet.
– The height difference h in the manometer fluid level is indicated to be proportional to $\frac{1}{2}\rho v_2^2$, where ρ is the fluid density.

Diagram (b):
– Illustrates a similar scenario to diagram (a).

– The pipe configuration is slightly different, with section 1 initially curving upwards before going horizontal.
– Section 2 exits horizontally into the U-tube manometer.
– The manometer setup and height difference h are shown similarly to diagram (a), with the same proportional relationship to $\frac{1}{2}\rho v_2^2$.
– Streamlines indicate the flow direction with arrows.

These diagrams demonstrate the principle that the height difference in the manometer is related to the velocity of the fluid in the pipe, illustrating Bernoulli's principle in action.

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10.3 THE MOST GENERAL APPLICATIONS OF BERNOULLI'S EQUATION

Learning Objectives

By the end of this section, you will be able to:

- Calculate using Torricelli's theorem.
- Calculate power in fluid flow.

Torricelli's Theorem

Figure 10.8 shows water gushing from a large tube through a dam. What is its speed as it emerges? Interestingly, if resistance is negligible, the speed is just what it would be if the water fell a distance h from the surface of the reservoir; the water's speed is independent of the size of the opening. Let us check this out. Bernoulli's equation must be used since the depth is not constant. We consider water flowing from the surface (point 1) to the tube's outlet (point 2). Bernoulli's equation as stated in previously is

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2.$$

10.29

Both P_1 and P_2 equal atmospheric pressure (P_1 is atmospheric pressure because it is the pressure at the top of the reservoir. P_2 must be atmospheric pressure, since the emerging water is surrounded by the atmosphere and cannot have a pressure different from atmospheric pressure.) and subtract out of the equation, leaving

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2.$$

10.30

Solving this equation for v_2^2, noting that the density ρ cancels (because the fluid is incompressible), yields

$$v_2^2 = v_1^2 + 2g\,(h_1 - h_2\,).$$
10.31

We let $h = h_1 - h_2$; the equation then becomes

$$v_2^2 = v_1^2 + 2\mathrm{gh}$$
10.32

where h is the height dropped by the water. This is simply a kinematic equation for any object falling a distance h with negligible resistance. In fluids, this last equation is called *Torricelli's theorem*. Note that the result is independent of the velocity's direction, just as we found when applying conservation of energy to falling objects.

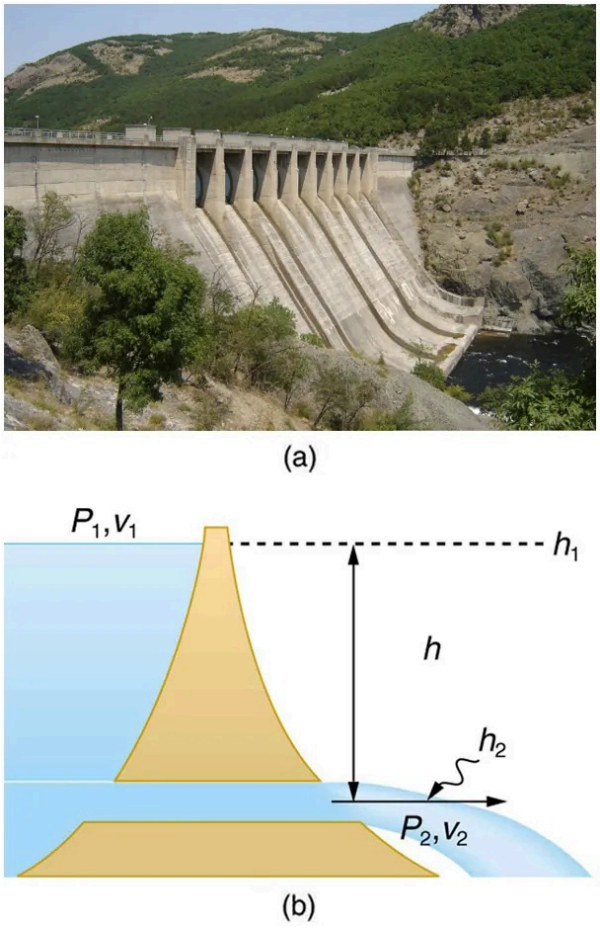


Figure 10.8 (a) Water gushes from the base of the Studen Kladenetz dam in Bulgaria. (credit: Kiril Kapustin; http://www.ImagesFromBulgaria.com) (b) In the absence of significant resistance, water flows from the reservoir with the same speed it would have if it fell the distance h without friction. This is an example of Torricelli's theorem. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two parts labeled (a) and (b).

(a) This is a photograph of a large dam situated in a mountainous area. The dam is made of concrete and features several spillways, which allow water to flow through. The surrounding landscape includes green hills and trees, and some rocky outcroppings are visible in the background. There is a body of water behind the dam, and the foreground features vegetation and trees.

(b) This is a schematic diagram illustrating the flow of water over a dam. It shows two water levels with corresponding pressures and velocities, labeled as *P1, V1* at the higher level and *P2, V2* at the lower level. The diagram includes two heights: *h* representing the total height of the dam, *h1* indicating the water level difference from the top to the surface, and *h2* relating to the drop past the dam. The diagram uses arrows and labels to convey the changes in pressure and flow velocity as water moves from higher to lower levels.

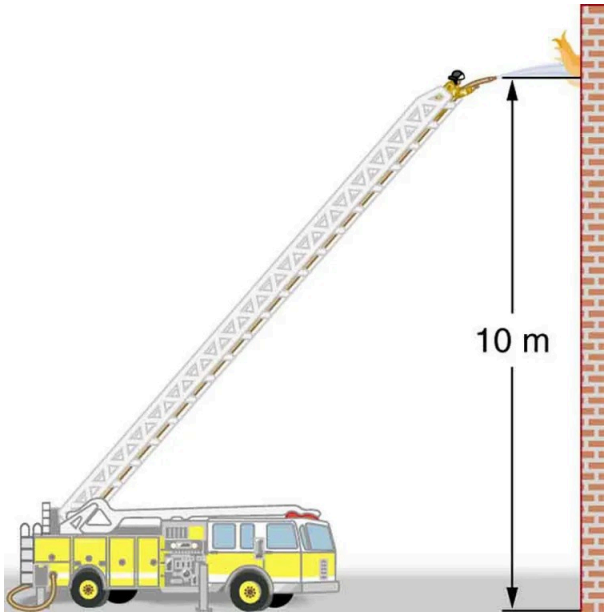


Figure 10.9 Pressure in the nozzle of this fire hose is less than at ground level for two reasons: the water has to go uphill to get to the nozzle, and speed increases in the nozzle. In spite of its lowered pressure, the water can exert a large force on anything it strikes, by virtue of its kinetic energy. Pressure in the water stream becomes equal to atmospheric pressure once it emerges into the air. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram featuring a yellow fire truck with an extended ladder. The ladder is reaching towards a brick building on the right side of the image. At the top of the ladder, a firefighter is leaning forward to rescue a cat that is perched on the edge of the building's roof. There is a vertical measurement indicating a height of

10 meters from the ground to the top of the building where the cat is situated. The ladder forms an angled extension from the truck to the building.

All preceding applications of Bernoulli's equation involved simplifying conditions, such as constant height or constant pressure. The next example is a more general application of Bernoulli's equation in which pressure, velocity, and height all change. (See Figure 10.9.)

Example 10.5

Calculating Pressure: A Fire Hose Nozzle

Fire hoses used in major structure fires have inside diameters of 6.40 cm. Suppose such a hose carries a flow of 40.0 L/s starting at a gauge pressure of $1.62 \times 10^6 \mathrm{N/m^2}$. The hose goes 10.0 m up a ladder to a nozzle having an inside diameter of 3.00 cm. Assuming negligible resistance, what is the initial water pressure at the base of the hose?

Strategy

Here we must use Bernoulli's equation to solve for the pressure, since depth is not constant.

Solution

Bernoulli's equation states

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho \mathrm{g h}_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho \mathrm{g h}_2,$$

10.33

where the subscripts 1 and 2 refer to the initial conditions at ground level and the final conditions inside the nozzle, respectively. We must first find the speeds v_1 and v_2. Since $Q = A_1 v_1$, we get

$$v_1 = \frac{Q}{A_1} = \frac{40.0 \times 10^{-3} \mathrm{m^3/s}}{\pi \left(3.20 \times 10^{-2} \mathrm{m}\right)^2} = 12.434 \mathrm{m/s}.$$

10.34

Similarly, we find

$$v_2 = 56.588 \text{ m/s}.$$

10.35

(This rather large speed is helpful in reaching the fire.) Now, taking h_1 to be zero, we solve Bernoulli's equation for P_2:

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) - \rho g h_2.$$

10.36

In the proposed solution, $P_2 = 0$, so

$$\begin{aligned} P_1 - P_2 &= P_1 \\ &= \frac{1}{2}(1000\,\text{kg/m}^3)\left[(12.434\,\text{m/s})^2 - (56.588\,\text{m/s})^2\right] \\ &\quad - (1000\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(10.0\,\text{m}) \\ &\approx 1.62 \times 10^6\,\text{N/m}^2. \end{aligned}$$

10.37

Discussion

This value is a gauge pressure, since the initial pressure was given as a gauge pressure. Thus the nozzle pressure is very close to atmospheric pressure, as it must because the water exits into the atmosphere without changes in its conditions.

Power in Fluid Flow

Power is the *rate* at which work is done or energy in any form is used or supplied. To see the relationship of power to fluid flow, consider Bernoulli's equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}.$$

10.38

All three terms have units of energy per unit volume, as discussed in the previous section. Now, considering units, if we multiply energy per unit volume by flow rate (volume per unit time), we get units of power. That is, $(E/V)(V/t) = E/t$. This means that if we multiply Bernoulli's equation by flow rate Q, we get power. In equation form, this is

$$\left(P + \frac{1}{2}\rho v^2 + \rho g h\right)Q = \text{power}.$$

10.39

Each term has a clear physical meaning. For example, PQ is the power supplied to a fluid, perhaps by a pump, to give it its pressure P. Similarly, $\frac{1}{2}\rho v^2 Q$ is the power supplied to a fluid to give it its kinetic energy. And $\rho \mathrm{ghQ}$ is the power going to gravitational potential energy.

Making Connections: Power

Power is defined as the rate of energy transferred, or E/t. Fluid flow involves several types of power. Each type of power is identified with a specific type of energy being expended or changed in form.

Example 10.6

Calculating Power in a Moving Fluid

Suppose the fire hose in the previous example is fed by a pump that receives water through a hose with a 6.40-cm diameter coming from a hydrant with a pressure of $0.700 \times 10^6 \mathrm{N/m^2}$. What power does the pump supply to the water?

Strategy

Here we must consider energy forms as well as how they relate to fluid flow. Since the input and output hoses have the same diameters and are at the same height, the pump does not change the speed of the water nor its height, and so the water's kinetic energy and gravitational potential energy are unchanged. That means the pump only supplies power to increase water pressure by $0.92 \times 10^6 \mathrm{N/m^2}$ (from $0.700 \times 10^6 \mathrm{N/m^2}$ to $1.62 \times 10^6 \mathrm{N/m^2}$).

Solution

As discussed above, the power associated with pressure is

$$\begin{aligned} \text{power} &= \mathrm{PQ} \\ &= \left(0.920 \times 10^6 \mathrm{N/m^2}\right)\left(40.0 \times 10^{-3} \mathrm{m^3/s}\right). \\ &= 3.68 \times 10^4 \mathrm{W} = 36.8 \mathrm{kW}. \end{aligned}$$

10.40

Discussion

Such a substantial amount of power requires a large pump, such as is found on some fire trucks. (This kilowatt value converts to about 50 hp.) The pump in this example increases only the water's pressure. If a pump—such as the heart—directly increases velocity and

height as well as pressure, we would have to calculate all three terms to find the power it supplies.

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10.4 VISCOSITY AND LAMINAR FLOW; POISEUILLE'S LAW

Learning Objectives

By the end of this section, you will be able to:

- Define laminar flow and turbulent flow.
- Explain what viscosity is.
- Calculate flow and resistance with Poiseuille's law.
- Explain how pressure drops due to resistance.

Laminar Flow and Viscosity

When you pour yourself a glass of juice, the liquid flows freely and quickly. But when you pour syrup on your pancakes, that liquid flows slowly and sticks to the pitcher. The difference is fluid friction, both within the fluid itself and between the fluid and its surroundings. We call this property of fluids *viscosity*. Juice has low viscosity, whereas syrup has high viscosity. In the previous sections we have considered ideal fluids with little or no viscosity. In this section, we will investigate what factors, including viscosity, affect the rate of fluid flow.

The precise definition of viscosity is based on *laminar*, or nonturbulent, flow. Before we can define viscosity, then, we need to define laminar flow and turbulent flow. Figure 10.10 shows both types of flow. Laminar flow is characterized by the smooth flow of the fluid in layers that do not mix. Turbulent flow, or turbulence, is characterized by eddies and swirls that mix layers of fluid together.



Figure 10.10 Smoke rises smoothly for a while and then begins to form swirls and eddies. The smooth flow is called laminar flow, whereas the swirls and eddies typify turbulent flow. If you watch the smoke (being careful not to breathe on it), you will notice that it rises more rapidly when flowing smoothly than after it becomes turbulent, implying that turbulence poses more resistance to flow. Image from OpenStax College Physics 2e, CC-BY 4.0

Figure 10.11 shows schematically how laminar and turbulent flow differ. Layers flow without mixing when flow is laminar. When there is turbulence, the layers mix, and there are significant velocities in directions other than the overall direction of flow. The lines that are shown in many illustrations are the paths followed by small volumes of fluids. These are called *streamlines*. Streamlines are smooth and continuous when flow is laminar, but break up and mix when flow is turbulent. Turbulence has two main causes. First, any obstruction or sharp corner, such as in a faucet, creates turbulence by imparting velocities perpendicular to the flow. Second, high speeds cause turbulence. The drag both between adjacent layers of fluid and between the fluid and its surroundings forms swirls and eddies, if the speed is great enough. We shall concentrate on laminar flow for the remainder of this section, leaving certain aspects of turbulence for later sections.

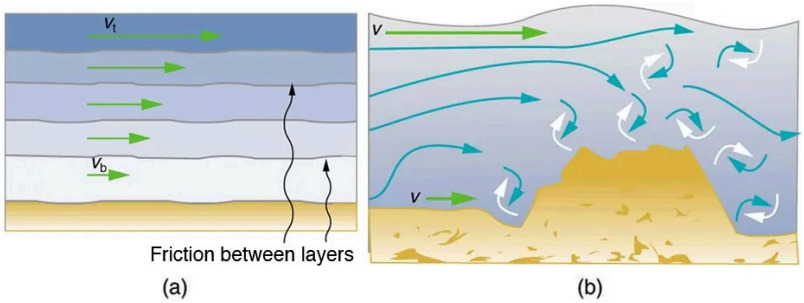


Figure 10.11 (a) Laminar flow occurs in layers without mixing. Notice that viscosity causes drag between layers as well as with the fixed surface. (b) An obstruction in the vessel produces turbulence. Turbulent flow mixes the fluid. There is more interaction, greater heating, and more resistance than in laminar flow. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two illustrations side by side labeled (a) and (b), depicting fluid dynamics concepts.

Illustration (a): This is a layered fluid flow diagram. It features parallel horizontal layers moving from left to right. The top layer is labeled V_t and the bottom layer is labeled V_b, indicating different velocities. Green arrows illustrate the direction of flow. The phrase "Friction between layers" is pointing to wavy black lines, suggesting interaction or shear between the layers.

Illustration (b): This diagram shows fluid flow over a rough, uneven surface at the bottom. The flow has wavy lines starting at the left and moving towards the right, with green arrows at the beginning of the flow, transitioning to blue arrows as the flow moves over the rough surface. The flow appears to become turbulent, with swirls represented by semicircular arrows, indicating chaotic motion. The bottom of the flow shows a yellow, rough terrain.

Making Connections: Take-Home Experiment: Go Down to the River

Try dropping simultaneously two sticks into a flowing river, one near the edge of the river and one near the middle. Which one travels faster? Why?

Figure 10.12 shows how viscosity is measured for a fluid. Two parallel plates have the specific fluid between them. The bottom plate is held fixed, while the top plate is moved to the right, dragging fluid with it. The layer (or lamina) of fluid in contact with either plate does not move relative to the plate, and so the top layer moves at v while the bottom layer remains at rest. Each successive layer from the top down exerts a force on the one below it, trying to drag it along, producing a continuous variation in speed from v to 0 as shown. Care is taken to insure that the flow is laminar; that is, the layers do not mix. The motion in Figure 10.12 is like a continuous shearing motion. Fluids have zero shear strength, but the *rate* at which they are sheared is related to the same geometrical factors A and L as is shear deformation for solids.

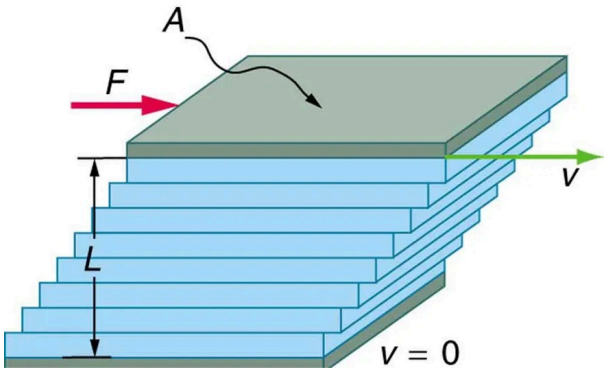


Figure 10.12 The graphic shows laminar flow of fluid between two plates of area A. The bottom plate is fixed. When the top plate is pushed to the right, it drags the fluid along with it. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram illustrating the concept of shear stress in a fluid. It shows a stack of horizontal layers that represent fluid sheets, with each one sliding over the one below it. At the top, there is a rectangular block labeled "A." An arrow labeled "F" points to the right, indicating a force applied to the top layer. Another arrow labeled "v" points horizontally to the right, indicating the velocity of fluid layers in that direction. The bottom layer is grounded with a label "v = 0," indicating no velocity. A vertical arrow labeled "L" points downward on the left side, possibly indicating the thickness or distance between layers. The entire setup shows fluid motion under the influence of an applied force, demonstrating how different layers can move at various speeds due to the force exerted on them.

A force F is required to keep the top plate in Figure 10.12 moving at a constant velocity v, and experiments have shown that this force depends on four factors. First, F is directly proportional to v (until the speed is so high that turbulence occurs—then a much larger force is needed, and it has a more complicated dependence on v). Second, F is proportional to the area A of the plate. This relationship seems reasonable, since A is directly proportional to the amount of fluid being moved. Third, F is inversely proportional to the distance

between the plates L. This relationship is also reasonable; L is like a lever arm, and the greater the lever arm, the less force that is needed. Fourth, F is directly proportional to *the coefficient of viscosity*, η. The greater the viscosity, the greater the force required. These dependencies are combined into the equation

$$F = \eta \frac{\mathrm{vA}}{L},$$
10.41

which gives us a working definition of fluid viscosity η. Solving for η gives

$$\eta = \frac{\mathrm{FL}}{\mathrm{vA}},$$
10.42

which defines viscosity in terms of how it is measured. The SI unit of viscosity is $\mathrm{N \cdot m/\left[\,(m/s\,)m^2\,\right] = (N/m^2\,)s \text{ or } Pa \cdot s}$. Table 10.1 lists the coefficients of viscosity for various fluids.

Viscosity varies from one fluid to another by several orders of magnitude. As you might expect, the viscosities of gases are much less than those of liquids, and these viscosities are often temperature dependent. The viscosity of blood can be reduced by aspirin consumption, allowing it to flow more easily around the body. (When used over the long term in low doses, aspirin can help prevent heart attacks, and reduce the risk of blood clotting.)

Laminar Flow Confined to Tubes—Poiseuille's Law

What causes flow? The answer, not surprisingly, is pressure difference. In fact, there is a very simple relationship between horizontal flow and pressure. Flow rate Q is in the direction from high to low pressure. The greater the pressure differential between two points, the greater the flow rate. This relationship can be stated as

$$Q = \frac{P_2 - P_1}{R},$$
10.43

where P_1 and P_2 are the pressures at two points, such as at either end of a tube, and R is the resistance to flow. The resistance R includes everything, except pressure, that affects flow rate. For example, R is greater for a long tube than for a short one. The greater the viscosity of a fluid, the greater the value of R. Turbulence greatly increases R, whereas increasing the diameter of a tube decreases R.

If viscosity is zero, the fluid is frictionless and the resistance to flow is also zero. Comparing frictionless flow in

a tube to viscous flow, as in Figure 10.13, we see that for a viscous fluid, speed is greatest at midstream because of drag at the boundaries. We can see the effect of viscosity in a Bunsen burner flame, even though the viscosity of natural gas is small.

The resistance R to laminar flow of an incompressible fluid having viscosity η through a horizontal tube of uniform radius r and length l, such as the one in Figure 10.14, is given by

$$R = \frac{8\eta l}{\pi r^4}.$$

10.44

This equation is called Poiseuille's law for resistance after the French scientist J. L. Poiseuille (1799–1869), who derived it in an attempt to understand the flow of blood, an often turbulent fluid.

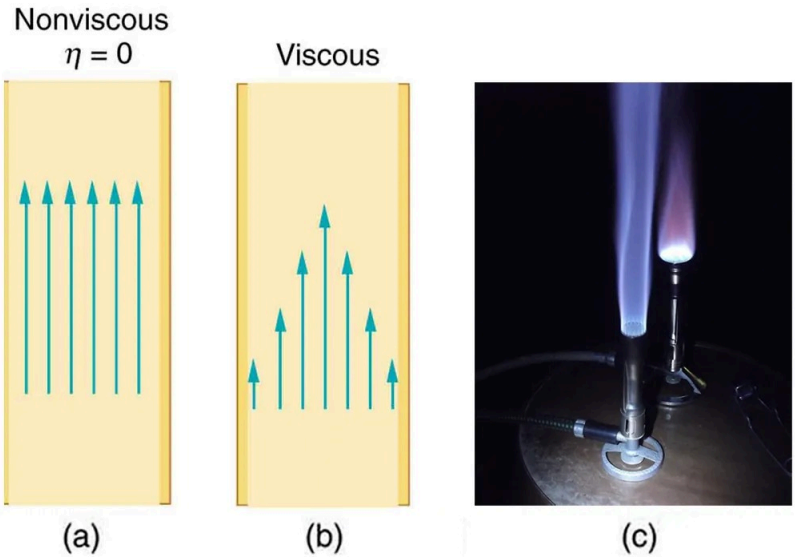


Figure 10.13 (a) If fluid flow in a tube has negligible resistance, the speed is the same all across the tube. (b) When a viscous fluid flows through a tube, its speed at the walls is zero, increasing steadily to its maximum at the center of the tube. (c) The shape of the Bunsen burner flame is due to the velocity profile across the tube. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains three parts labeled (a), (b), and (c).

Part (a): A diagram titled "Nonviscous $\eta = 0$" featuring a rectangular section with straight, vertical lines representing flow paths. The lines are evenly spaced and parallel, indicating a uniform flow with no viscosity.

Part (b): A diagram titled "Viscous" showing a similar rectangular section, but with curved lines representing

flow paths. The lines are wider at the edges and curved toward the center, illustrating how viscosity causes variation in flow speed, with faster flow in the center.

Part (c): A photograph showing a flame produced by a Bunsen burner in a dark environment. The flame is bright at the base and tapers off to a faint bluish color as it ascends, indicating gas combustion under laboratory conditions.

Let us examine Poiseuille's expression for R to see if it makes good intuitive sense. We see that resistance is directly proportional to both fluid viscosity η and the length l of a tube. After all, both of these directly affect the amount of friction encountered—the greater either is, the greater the resistance and the smaller the flow. The radius r of a tube affects the resistance, which again makes sense, because the greater the radius, the greater the flow (all other factors remaining the same). But it is surprising that r is raised to the *fourth* power in Poiseuille's law. This exponent means that any change in the radius of a tube has a very large effect on resistance. For example, doubling the radius of a tube decreases resistance by a factor of $2^4 = 16$.

Taken together, $Q = \dfrac{P_2 - P_1}{R}$ and $R = \dfrac{8\eta l}{\pi r^4}$ give the following expression for flow rate:

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

10.45

This equation describes laminar flow through a tube. It is sometimes called Poiseuille's law for laminar flow, or simply Poiseuille's law.

Example 10.7

Using Flow Rate: Plaque Deposits Reduce Blood Flow

Suppose the flow rate of blood in a coronary artery has been reduced to half its normal value by plaque deposits. By what factor has the radius of the artery been reduced, assuming no turbulence occurs?

Strategy

Assuming laminar flow, Poiseuille's law states that

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

10.46

We need to compare the artery radius before and after the flow rate reduction.

Solution

With a constant pressure difference assumed and the same length and viscosity, along the artery we have

$$\frac{Q_1}{r_1^4} = \frac{Q_2}{r_2^4}.$$

10.47

So, given that $Q_2 = 0.5Q_1$, we find that $r_2^4 = 0.(5r)_1^4$.

Therefore, $r_2 = (0.5)^{0.25} r_1 = 0.841 r_1$, a decrease in the artery radius of 16%.

Discussion

This decrease in radius is surprisingly small for this situation. To restore the blood flow in

spite of this buildup would require an increase in the pressure difference $(P_2 - P_1)$ of a factor of two, with subsequent strain on the heart.

Table 10.1 Coefficients of Viscosity of Various Fluids

Fluid	Temperature (ºC)	Viscosity $\eta(\text{mPa} \cdot \text{s})$
Gases		
Air	0	0.0171
	20	0.0181
	40	0.0190
	100	0.0218
Ammonia	20	0.00974
Carbon dioxide	20	0.0147
Helium	20	0.0196
Hydrogen	0	0.0090
Mercury	20	0.0450
Oxygen	20	0.0203
Steam	100	0.0130
Liquids		
Water	0	1.792
	20	1.002
	37	0.6947
	40	0.653
	100	0.282
Whole blood[1]	20	3.015
	37	2.084
Blood plasma[2]	20	1.810
	37	1.257
Ethyl alcohol	20	1.20
Methanol	20	0.584
Oil (heavy machine)	20	660
Oil (motor, SAE 10)	30	200
Oil (olive)	20	138
Glycerin	20	1500

Fluid	Temperature (ºC)	Viscosity $\eta(\mathrm{mPa \cdot s})$
Honey	20	2000–10000
Maple Syrup	20	2000–3000
Milk	20	3.0
Oil (Corn)	20	65

The circulatory system provides many examples of Poiseuille's law in action—with blood flow regulated by changes in vessel size and blood pressure. Blood vessels are not rigid but elastic. Adjustments to blood flow are primarily made by varying the size of the vessels, since the resistance is so sensitive to the radius. During vigorous exercise, blood vessels are selectively dilated to important muscles and organs and blood pressure increases. This creates both greater overall blood flow and increased flow to specific areas. Conversely, decreases in vessel radii, perhaps from plaques in the arteries, can greatly reduce blood flow. If a vessel's radius is reduced by only 5% (to 0.95 of its original value), the flow rate is reduced to about $(0.95)^4 = 0.81$ of its original value. A 19% decrease in flow is caused by a 5% decrease in radius. The body may compensate by increasing blood pressure by 19%, but this presents hazards to the heart and any vessel that has weakened walls. Another example comes from automobile engine oil. If you have a car with an oil pressure gauge, you may notice that oil pressure is high when the engine is cold. Motor oil has greater viscosity when cold than when warm, and so pressure must be greater to pump the same amount of cold oil.

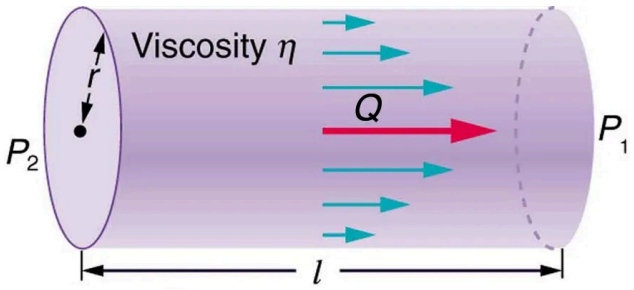


Figure 10.14 Poiseuille's law applies to laminar flow of an incompressible fluid of viscosity η through a tube of length l and radius r. The direction of flow is from greater to lower pressure. Flow rate Q is directly proportional to the pressure difference $P_2 - P_1$, and inversely proportional to the length l of the tube and viscosity η of the fluid. Flow rate increases with r^4, the fourth power of the radius. Image from OpenStax College Physics 2e, CC-BY 4.0

1. The ratios of the viscosities of blood to water are nearly constant between 0°C and 37°C.

2. See note on Whole Blood

Image Description

The image illustrates fluid flow through a cylindrical pipe, representing concepts in fluid dynamics. The cylinder is shown as a long, purple tube with its length extending horizontally. At the left end of the cylinder, the pressure is labeled as P_2, and at the right end, it's labeled as P_1. The flow direction is from right to left, indicating that P_1 is greater than P_2.

Inside the cylinder, multiple arrows of varying lengths are drawn parallel to the axis, pointing in the flow direction. A larger red arrow labeled as Q represents the overall flow rate, while smaller blue arrows illustrate velocity distribution across the pipe's cross-section.

The viscosity of the fluid is noted by the Greek letter eta (η). The radius of the pipe is marked with a line and labeled as r, starting from the center to the wall of the cylinder. The length of the pipe is denoted as l, spanning from one end to the other.

Example 10.8

What Pressure Produces This Flow Rate?

An intravenous (IV) system is supplying saline solution to a patient at the rate of $0.120 \text{cm}^3/\text{s}$ through a needle of radius 0.150 mm and length 2.50 cm. What pressure is needed at the entrance of the needle to cause this flow, assuming the viscosity of the saline solution to be the same as that of water? The gauge pressure of the blood in the patient's vein is 8.00 mm Hg. (Assume that the temperature is 20° C.)

Strategy

Assuming laminar flow, Poiseuille's law applies. This is given by

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l},$$

10.48

where P_2 is the pressure at the entrance of the needle and P_1 is the pressure in the vein. The only unknown is P_2.

Solution

Solving for P_2 yields

$$P_2 = \frac{8\eta l}{\pi r^4} Q + P_1.$$

10.49

P_1 is given as 8.00 mm Hg, which converts to $1.066 \times 10^3 \text{N/m}^2$. Substituting this and the other known values yields

$$P_2 = \left[\frac{8\,(1.00 \times 10^{-3}\text{N} \cdot \text{s/m}^2\,)\,(2.50 \times 10^{-2}\text{m}\,)}{\pi\,(0.150 \times 10^{-3}\text{m})^4}\right] \times (1.20 \times 10^{-7}\text{m}^3/\text{s}\,) + 1.066 \times 10^3\text{N/m}^2$$

$$= 1.62 \times 10^4\text{N/m}^2.$$

10.50

Discussion

This pressure could be supplied by an IV bottle with the surface of the saline solution 1.61 m above the entrance to the needle (this is left for you to solve in this chapter's Problems and Exercises), assuming that there is negligible pressure drop in the tubing leading to the needle.

Flow and Resistance as Causes of Pressure Drops

You may have noticed that water pressure in your home might be lower than normal on hot summer days when there is more use. This pressure drop occurs in the water main before it reaches your home. Let us consider flow through the water main as illustrated in Figure 10.15. We can understand why the pressure P_1 to the home drops during times of heavy use by rearranging

$$Q = \frac{P_2 - P_1}{R}$$

10.51

to

$$P_2 - P_1 = RQ,$$

10.52

where, in this case, P_2 is the pressure at the water works and R is the resistance of the water main. During times of heavy use, the flow rate Q is large. This means that $P_2 - P_1$ must also be large. Thus P_1 must decrease. It is correct to think of flow and resistance as causing the pressure to drop from P_2 to P_1. $P_2 - P_1 = RQ$ is valid for both laminar and turbulent flows.

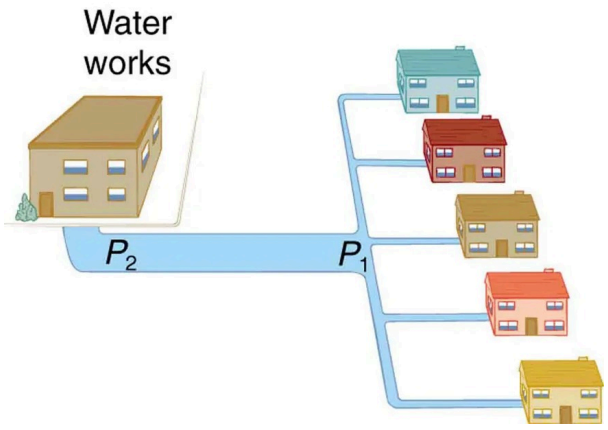


Figure 10.15 During times of heavy use, there is a significant pressure drop in a water main, and P_1 *supplied to users is significantly less than* P_2 *created at the water works. If the flow is very small, then the pressure drop is negligible, and* $P_2 \approx P_1$*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a simplified diagram of a water distribution system. On the left, there is a large building labeled "Water works," representing the water supply center. From this building, a main pipe extends to the right, labeled "P2," which then splits into several smaller pipes labeled "P1." These smaller pipes branch out to connect to several houses on the right side of the image. There are six houses in total, each of a different color: teal, red, brown, orange, and yellow. The houses are arranged in two rows, with three on the top and three on the bottom. The pipes are illustrated in blue, and the system shows how water is distributed from the water works to the individual houses.

We can use $P_2 - P_1 = RQ$ to analyze pressure drops occurring in more complex systems in which the tube radius is not the same everywhere. Resistance will be much greater in narrow places, such as an obstructed coronary artery. For a given flow rate Q, the pressure drop will be greatest where the tube is most narrow. This is how water faucets control flow. Additionally, R is greatly increased by turbulence, and a constriction that creates turbulence greatly reduces the pressure downstream. Plaque in an artery reduces pressure and hence flow, both by its resistance and by the turbulence it creates.

Figure 10.16 is a schematic of the human circulatory system, showing average blood pressures in its major parts for an adult at rest. Pressure created by the heart's two pumps, the right and left ventricles, is reduced by the resistance of the blood vessels as the blood flows through them. The left ventricle increases arterial blood pressure that drives the flow of blood through all parts of the body except the lungs. The right ventricle receives the lower pressure blood from two major veins and pumps it through the lungs for gas exchange with atmospheric gases – the disposal of carbon dioxide from the blood and the replenishment of oxygen. Only one major organ is shown schematically, with typical branching of arteries to ever smaller vessels, the smallest

of which are the capillaries, and rejoining of small veins into larger ones. Similar branching takes place in a variety of organs in the body, and the circulatory system has considerable flexibility in flow regulation to these organs by the dilation and constriction of the arteries leading to them and the capillaries within them. The sensitivity of flow to tube radius makes this flexibility possible over a large range of flow rates.

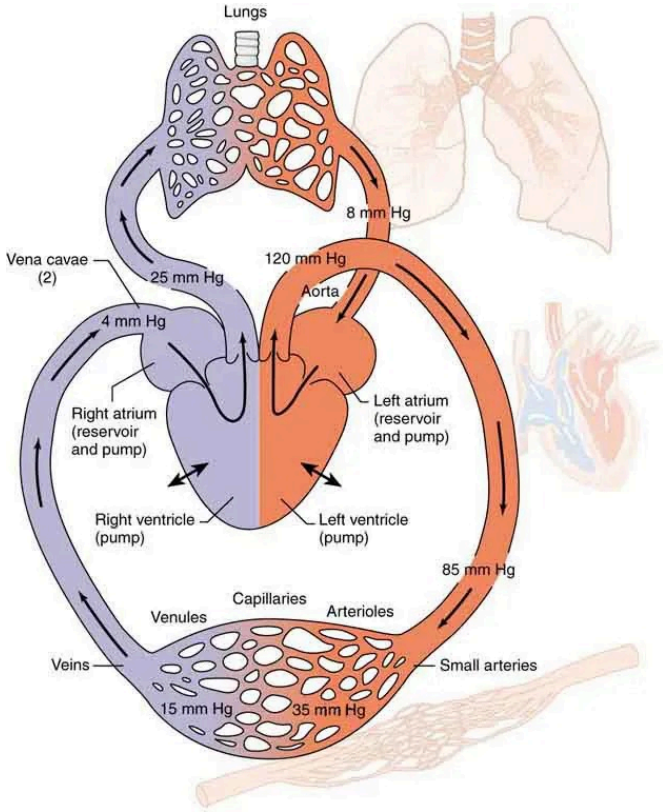


Figure 10.16 Schematic of the circulatory system. Pressure difference is created by the two pumps in the heart and is reduced by resistance in the vessels. Branching of vessels into capillaries allows blood to reach individual cells and exchange substances, such as oxygen and waste products, with them. The system has an impressive ability to regulate flow to individual organs, accomplished largely by varying vessel diameters. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image is a diagram of the human circulatory system, illustrating the flow of blood through the heart, lungs, and the body. It is divided into two sides: the left side (depicted in red) and the right side (depicted in blue).

The left side of the heart collects oxygen-rich blood from the lungs and pumps it to the body, while the right side collects deoxygenated blood from the body and pumps it to the lungs for oxygenation.

The system is labeled with various components and blood pressures at different points:

- *Lungs:* Shown above the heart.
- *Arteries:* Highlighted in red, carrying blood away from the heart.
- *Veins:* Highlighted in blue, carrying blood to the heart.
- *Right Atrium:* Acts as a reservoir and pump, receiving deoxygenated blood from the body.
- *Right Ventricle:* Pumps deoxygenated blood to the lungs.
- *Left Atrium:* Acts as a reservoir and pump, receiving oxygenated blood from the lungs.
- *Left Ventricle:* Pumps oxygenated blood to the body.
- *Vena Cavae:* Two large veins bringing deoxygenated blood back to the heart, labeled with a pressure of 2 and 25 mm Hg.
- *Aorta:* The main artery, labeled with a pressure of 120 mm Hg.
- *Arterioles and Small Arteries:* Also highlighted in red, leading to capillaries, with pressures labeled at 85 mm Hg.
- *Capillaries:* Network where oxygen and nutrients are exchanged, with pressures labeled at 35 mm Hg.
- *Venules:* Small veins leading from capillaries, labeled with a pressure of 15 mm Hg.

The diagram visually represents how blood circulates through this closed-loop system, emphasizing the differences between oxygenated and deoxygenated pathways and their associated pressures.

Each branching of larger vessels into smaller vessels increases the total cross-sectional area of the tubes through which the blood flows. For example, an artery with a cross section of 1cm^2 may branch into 20 smaller arteries, each with cross sections of 0.5cm^2, with a total of 10cm^2. In that manner, the resistance of the branchings is reduced so that pressure is not entirely lost. Moreover, because $Q = A\bar{v}$ and A increases through branching, the average velocity of the blood in the smaller vessels is reduced. The blood velocity in the aorta ($\text{diameter} = 1\text{cm}$) is about 25 cm/s, while in the capillaries ($20\ \mu\text{m}$ in diameter) the velocity is about 1 mm/s. This reduced velocity allows the blood to exchange substances with the cells in the capillaries and alveoli in particular.

"12.4 Viscosity and Laminar Flow; Poiseuille's Law" from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

GLOSSARY

active transport
the process in which a living membrane expends energy to move substances across

Bernoulli's equation
the equation resulting from applying conservation of energy to an incompressible frictionless fluid: $P + 1/2pv^2 + pgh = \text{constant}$, through the fluid

Bernoulli's principle
Bernoulli's equation applied at constant depth: $P_1 + 1/2pv_1^2 = P_2 + 1/2pv_2^2$

dialysis
the transport of any molecule other than water through a semipermeable membrane from a region of high concentration to one of low concentration

diffusion
the movement of substances due to random thermal molecular motion

flow rate
abbreviated Q, it is the volume V that flows past a particular point during a time t, or $Q = V/t$

fluid dynamics
the physics of fluids in motion

laminar
a type of fluid flow in which layers do not mix

liter
a unit of volume, equal to $10^{-3}\ \text{m}^3$

osmosis
the transport of water through a semipermeable membrane from a region of high concentration to one of low concentration

osmotic pressure
the back pressure which stops the osmotic process if one solution is pure water

Poiseuille's law
the rate of laminar flow of an incompressible fluid in a tube: $Q = (P_2 - P_1)\pi r^4/8\eta l$

Poiseuille's law for resistance
the resistance to laminar flow of an incompressible fluid in a tube: $R = 8\eta l/\pi r^4$

relative osmotic pressure
the back pressure which stops the osmotic process if neither solution is pure water

reverse dialysis
: the process that occurs when back pressure is sufficient to reverse the normal direction of dialysis through membranes

reverse osmosis
: the process that occurs when back pressure is sufficient to reverse the normal direction of osmosis through membranes

Reynolds number
: a dimensionless parameter that can reveal whether a particular flow is laminar or turbulent

semipermeable
: a type of membrane that allows only certain small molecules to pass through

terminal speed
: the speed at which the viscous drag of an object falling in a viscous fluid is equal to the other forces acting on the object (such as gravity), so that the acceleration of the object is zero

turbulence
: fluid flow in which layers mix together via eddies and swirls

viscosity
: the friction in a fluid, defined in terms of the friction between layers

viscous drag
: a resistance force exerted on a moving object, with a nontrivial dependence on velocity

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CHAPTER SUMMARY

10.1 Flow Rate and Its Relation to Velocity

- Flow rate Q is defined to be the volume V flowing past a point in time t, or $Q = \frac{V}{t}$ where
  V is volume and t is time.
- The SI unit of volume is m^3.
- Another common unit is the liter (L), which is $10^{-3}\mathrm{m}^3$.
- Flow rate and velocity are related by $Q = A\bar{v}$ where A is the cross-sectional area of the flow and $\bar{v}$ is its average velocity.
- For incompressible fluids, flow rate at various points is constant. That is, \left\begin{matrix} Q_{1} = Q_{2} \\ A_{1} \left(\bar{v}\right)_{1} = A_{2} \left(\bar{v}\right)_{2} \\ n_{1} A_{1} \left(\bar{v}\right)_{1} = n_{2} A_{2} \left(\bar{v}\right)_{2} \end{matrix}\right .

10.2 Bernoulli's Equation

- Bernoulli's equation states that the sum on each side of the following equation is constant, or the same at any two points in an incompressible frictionless fluid:
  $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho \mathrm{gh}_2.$
- Bernoulli's principle is Bernoulli's equation applied to situations in which depth is constant. The terms involving depth (or height *h*) subtract out, yielding
  $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$
- Bernoulli's principle has many applications, including entrainment, wings and sails, and velocity measurement.

10.3 The Most General Applications of Bernoulli's Equation

- Power in fluid flow is given by the equation
  $\left(P_1 + \frac{1}{2}\rho v^2 + \rho \mathrm{gh}\right) Q = \mathrm{power},$ where the first term is power associated with pressure, the second is power associated with velocity, and the third is power associated with height.

10.4 Viscosity and Laminar Flow; Poiseuille's Law

- Laminar flow is characterized by smooth flow of the fluid in layers that do not mix.
- Turbulence is characterized by eddies and swirls that mix layers of fluid together.
- Fluid viscosity η is due to friction within a fluid. Representative values are given in Table 10.1. Viscosity has units of $(\mathrm{N/m^2})\mathrm{s}$ or $\mathrm{Pa \cdot s}$.
- Flow is proportional to pressure difference and inversely proportional to resistance:
  $Q = \frac{P_2 - P_1}{R}$.
- For laminar flow in a tube, Poiseuille's law for resistance states that
  $R = \frac{8\eta l}{\pi r^4}$.
- Poiseuille's law for flow in a tube is
  $Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}$.
- The pressure drop caused by flow and resistance is given by
  $P_2 - P_1 = RQ$.

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CONCEPTUAL QUESTIONS

Conceptual Questions

10.1 Flow Rate and Its Relation to Velocity

1. What is the difference between flow rate and fluid velocity? How are they related?
2. Many figures in the text show streamlines. Explain why fluid velocity is greatest where streamlines are closest together. (Hint: Consider the relationship between fluid velocity and the cross-sectional area through which it flows.)
3. Identify some substances that are incompressible and some that are not.

10.2 Bernoulli's Equation

4. You can squirt water a considerably greater distance by placing your thumb over the end of a garden hose and then releasing, than by leaving it completely uncovered. Explain how this works.
5. Water is shot nearly vertically upward in a decorative fountain and the stream is observed to broaden as it rises. Conversely, a stream of water falling straight down from a faucet narrows. Explain why, and discuss whether surface tension enhances or reduces the effect in each case.
6. Look back to Figure 10.4. Answer the following two questions. Why is P_{o} less than atmospheric? Why is P_{o} greater than P_{i}?
7. Give an example of entrainment not mentioned in the text.
8. Many entrainment devices have a constriction, called a Venturi, such as shown in Figure 10.24. How does this bolster entrainment?

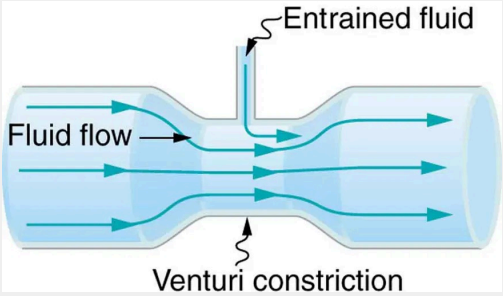


Figure 10.24 A tube with a narrow segment designed to enhance entrainment is called a Venturi. These are very commonly used in carburetors and aspirators. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains a diagram illustrating the Venturi effect in a fluid flow system. It shows a horizontal tube that narrows in the middle, forming a Venturi constriction. Arrows within the tube indicate the direction of fluid flow from left to right. At the constriction, an upward vertical tube is connected, labeled as "Entrained fluid," indicating where another fluid is drawn into the main flow. The narrowing part of the tube is labeled as "Venturi constriction," highlighting the point where the fluid velocity increases and pressure decreases, causing the entrainment. The flow is represented with blue arrows, and the background of the tube is shaded in blue to represent the fluid.

9. Some chimney pipes have a T-shape, with a crosspiece on top that helps draw up gases whenever there is even a slight breeze. Explain how this works in terms of Bernoulli's principle.
10. Is there a limit to the height to which an entrainment device can raise a fluid? Explain your answer.
11. Why is it preferable for airplanes to take off into the wind rather than with the wind?
12. Roofs are sometimes pushed off vertically during a tropical cyclone, and buildings sometimes explode outward when hit by a tornado. Use Bernoulli's principle to explain these phenomena.
13. Why does a sailboat need a keel?
14. It is dangerous to stand close to railroad tracks when a rapidly moving commuter

train passes. Explain why atmospheric pressure would push you toward the moving train.

15. What assumptions are made in Bernoulli's equation? Are these assumptions reasonably valid in the example of water flowing in a hose?
16. A perfume bottle or atomizer sprays a fluid that is in the bottle. (Figure 10.25.) How does the fluid rise up in the vertical tube in the bottle?



Figure 10.25 Atomizer: perfume bottle with tube to carry perfume up through the bottle. Image from OpenStax College Physics 2e, CC-BY 4.0

17. If you lower the window on a car while moving, an empty plastic bag can sometimes fly out the window. Why does this happen?

10.3 The Most General Applications of Bernoulli's Equation

18. Based on Bernoulli's equation, what are three forms of energy in a fluid? (Note that these forms are conservative, unlike heat transfer and other dissipative forms not included in Bernoulli's equation.)
19. Water that has emerged from a hose into the atmosphere has a gauge pressure of zero. Why? When you put your hand in front of the emerging stream you feel a force, yet the water's gauge pressure is zero. Explain where the force comes from in terms of energy.
20. The old rubber boot shown in Figure 10.26 has two leaks. To what maximum

height can the water squirt from Leak 1? How does the velocity of water emerging from Leak 2 differ from that of leak 1? Explain your responses in terms of energy.



Figure 10.26 Water emerges from two leaks in an old boot. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of a single brown boot with clear water inside, nearly reaching the top. The boot has two labeled leaks where water is visibly spraying out. “Leak 1” is located near the front at the toe, with water spraying outward. “Leak 2” is towards the back, near the heel, also releasing a stream of water. The overall impression is that the boot is not watertight and is leaking at two points.

21. Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

10.4 Viscosity and Laminar Flow; Poiseuille’s Law

22. Explain why the viscosity of a liquid decreases with temperature—that is, how might increased temperature reduce the effects of cohesive forces in a liquid? Also explain why the viscosity of a gas increases with temperature—that is, how does increased gas temperature create more collisions between atoms and molecules?
23. When paddling a canoe upstream, it is wisest to travel as near to the shore as

possible. When canoeing downstream, it may be best to stay near the middle. Explain why.

24. Why does flow decrease in your shower when someone flushes the toilet?
25. Plumbing usually includes air-filled tubes near water faucets, as shown in Figure 10.27. Explain why they are needed and how they work.

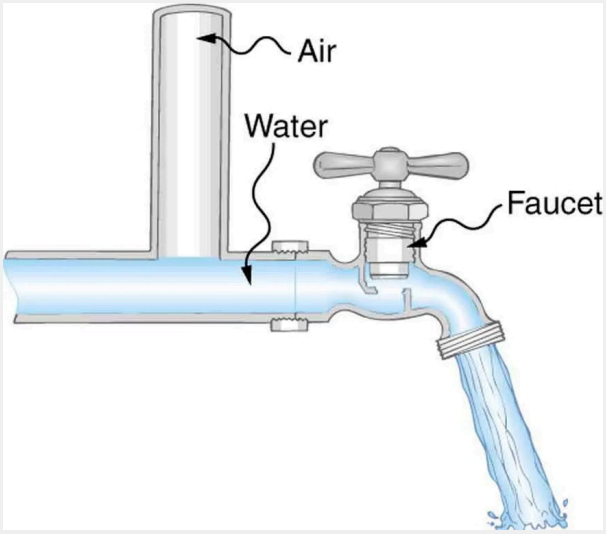


Figure 10.27 The vertical tube near the water tap remains full of air and serves a useful purpose. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the concept of a faucet connected to a pipe system. The diagram has labeled parts showing the water and air flow within the pipes:

– A vertical section of the pipe labeled “Air,” indicating the flow of air entering the system from above.
– A horizontal cross-section of the pipe, through which water flows, is featured. The water flow is labeled “Water.”
– A faucet is attached to the pipe, labeled “Faucet.” The faucet is open, allowing water to pour out.

The diagram displays the interaction between air and water within the plumbing system, illustrating water flow from the faucet when turned on.

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PROBLEMS & EXERCISES

Problems & Exercises

10.1 Flow Rate and Its Relation to Velocity

1. What is the average flow rate in cm^3/s of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?
2. The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to cm^3/s. (b) What is this rate in m^3/s?
3. Blood is pumped from the heart at a rate of 5.0 L/min into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta.
4. Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.
5. The Huka Falls on the Waikato River is one of New Zealand's most visited natural tourist attractions (see Figure 10.29). On average the river has a flow rate of about 300,000 L/s. At the gorge, the river narrows to 20 m wide and averages 20 m deep. (a) What is the average speed of the river in the gorge? (b) What is the average speed of the water in the river downstream of the falls when it widens to 60 m and its depth increases to an average of 40 m?

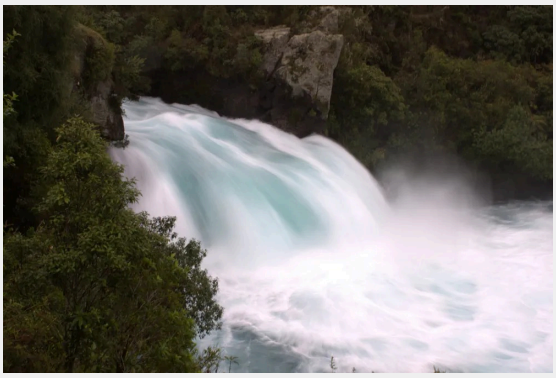


Figure 10.29 The Huka Falls in Taupo, New Zealand, demonstrate flow rate. Image from OpenStax College Physics 2e, CC-BY 4.0

6. A major artery with a cross-sectional area of 1.00cm^2 branches into 18 smaller arteries, each with an average cross-sectional area of 0.400cm^2. By what factor is the average velocity of the blood reduced when it passes into these branches?
7. (a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is 10.0cm^2, what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is $10.0\mu\text{m}$?
8. The human circulation system has approximately 1×10^9 capillary vessels. Each vessel has a diameter of about $8\mu\text{m}$. Assuming cardiac output is 5 L/min, determine the average velocity of blood flow through each capillary vessel.
9. (a) Estimate the time it would take to fill a private swimming pool with a capacity of 80,000 L using a garden hose delivering 60 L/min. (b) How long would it take to fill if you could divert a moderate size river, flowing at $5000\text{m}^3/\text{s}$, into it?
10. The flow rate of blood through a $2.00 \times 10^{-6} - \text{m}$ -radius capillary is $3.80 \times 10^{-9}\text{cm}^3/\text{s}$. (a) What is the speed of the blood flow? (This small speed allows time for diffusion of materials to and from the blood.) (b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of $90.0\text{cm}^3/\text{s}$? (The large number obtained is an overestimate, but it is still reasonable.)
11. (a) What is the fluid speed in a fire hose with a 9.00-cm diameter carrying 80.0 L of water per second? (b) What is the flow rate in cubic meters per second? (c) Would your answers be different if salt water replaced the fresh water in the fire hose?
12. The main uptake air duct of a forced air gas heater is 0.300 m in diameter. What is the average speed of air in the duct if it carries a volume equal to that of the house's interior every 15 min? The inside volume of the house is equivalent to a rectangular solid 13.0 m wide by 20.0 m long by 2.75 m high.
13. Water is moving at a velocity of 2.00 m/s through a hose with an internal diameter of 1.60 cm. (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m/s. What is the nozzle's inside diameter?
14. Prove that the speed of an incompressible fluid through a constriction, such as in a

Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)

15. Water emerges straight down from a faucet with a 1.80-cm diameter at a speed of 0.500 m/s. (Because of the construction of the faucet, there is no variation in speed across the stream.) (a) What is the flow rate in cm^3/s? (b) What is the diameter of the stream 0.200 m below the faucet? Neglect any effects due to surface tension.
16. **Unreasonable Results:** A mountain stream is 10.0 m wide and averages 2.00 m in depth. During the spring runoff, the flow in the stream reaches $100,000\mathrm{m}^3/\mathrm{s}$. (a) What is the average velocity of the stream under these conditions? (b) What is unreasonable about this velocity? (c) What is unreasonable or inconsistent about the premises?

10.2 Bernoulli's Equation

17. Verify that pressure has units of energy per unit volume.
18. Suppose you have a wind speed gauge like the pitot tube shown in Figure 10.7(b). By what factor must wind speed increase to double the value of h in the manometer? Is this independent of the moving fluid and the fluid in the manometer?
19. If the pressure reading of your pitot tube is 15.0 mm Hg at a speed of 200 km/h, what will it be at 700 km/h at the same altitude?
20. Calculate the maximum height to which water could be squirted with the hose in Example 10.2 example if it: (a) Emerges from the nozzle. (b) Emerges with the nozzle removed, assuming the same flow rate.
21. Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of $220\mathrm{m}^2$? Typical air density in Boulder is $1.14\mathrm{kg}/\mathrm{m}^3$, and the corresponding atmospheric pressure is $8.89 \times 10^4\mathrm{N}/\mathrm{m}^2$. (Bernoulli's principle as stated in the text assumes laminar flow. Using the principle here produces only an

approximate result, because there is significant turbulence.)

22. (a) Calculate the approximate force on a square meter of sail, given the horizontal velocity of the wind is 6.00 m/s parallel to its front surface and 3.50 m/s along its back surface. Take the density of air to be $1.29\ \mathrm{kg/m^3}$. (The calculation, based on Bernoulli's principle, is approximate due to the effects of turbulence.) (b) Discuss whether this force is great enough to be effective for propelling a sailboat.
23. (a) What is the pressure drop due to the Bernoulli effect as water goes into a 3.00-cm-diameter nozzle from a 9.00-cm-diameter fire hose while carrying a flow of 40.0 L/s? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.)
24. (a) Using Bernoulli's equation, show that the measured fluid speed v for a pitot tube, like the one in Figure 10.7(b), is given by $v = \left(\frac{2\rho' gh}{\rho}\right)^{1/2}$, where h is the height of the manometer fluid, ρ' is the density of the manometer fluid, ρ is the density of the moving fluid, and g is the acceleration due to gravity. (Note that v is indeed proportional to the square root of h, as stated in the text.) (b) Calculate v for moving air if a mercury manometer's h is 0.200 m.

10.3 The Most General Applications of Bernoulli's Equation

25. Hoover Dam on the Colorado River is the highest dam in the United States at 221 m, with a maximum output of 1300 MW. The dam generates electricity with water taken from a depth of 150 m and an average flow rate of $650\mathrm{m^3/s}$. (a) Calculate the power in this flow. (b) What is the ratio of this power to the facility's average of 680 MW?
26. A frequently quoted rule of thumb in aircraft design is that wings should produce about 1000 N of lift per square meter of wing. (The fact that a wing has a top and bottom surface does not double its area.) (a) At takeoff, an aircraft travels at 60.0 m/s, so that the air speed relative to the bottom of the wing is 60.0 m/s. Given the sea level density of air to be $1.29\mathrm{kg/m^3}$, how fast must it move over the upper surface to create the ideal lift? (b) How fast must air move over the upper surface at a cruising speed of 245 m/s and at an altitude where air density is one-fourth

that at sea level? (Note that this is not all of the aircraft's lift—some comes from the body of the plane, some from engine thrust, and so on. Furthermore, Bernoulli's principle gives an approximate answer because flow over the wing creates turbulence.)

27. The left ventricle of a resting adult's heart pumps blood at a flow rate of $83.0 \text{cm}^3/\text{s}$, increasing its pressure by 110 mm Hg, its speed from zero to 30.0 cm/s, and its height by 5.00 cm. (All numbers are averaged over the entire heartbeat.) Calculate the total power output of the left ventricle. Note that most of the power is used to increase blood pressure.
28. A sump pump (used to drain water from the basement of houses built below the water table) is draining a flooded basement at the rate of 0.750 L/s, with an output pressure of $3.00 \times 10^5 \text{N}/\text{m}^2$. (a) The water enters a hose with a 3.00-cm inside diameter and rises 2.50 m above the pump. What is its pressure at this point? (b) The hose goes over the foundation wall, losing 0.500 m in height, and widens to 4.00 cm in diameter. What is the pressure now? You may neglect frictional losses in both parts of the problem.

10.4 Viscosity and Laminar Flow; Poiseuille's Law

29. (a) Calculate the retarding force due to the viscosity of the air layer between a cart and a level air track given the following information—air temperature is $20^\circ\ \text{C}$, the cart is moving at 0.400 m/s, its surface area is $2.50 \times 10^{-2}\text{m}^2$, and the thickness of the air layer is $6.00 \times 10^{-5}\text{m}$. (b) What is the ratio of this force to the weight of the 0.300-kg cart?
30. What force is needed to pull one microscope slide over another at a speed of 1.00 cm/s, if there is a 0.500-mm-thick layer of $20^\circ\ \text{C}$ water between them and the contact area is 8.00cm^2?
31. A glucose solution being administered with an IV has a flow rate of $4.00\text{cm}^3/\text{min}$. What will the new flow rate be if the glucose is replaced by whole blood having the same density but a viscosity 2.50 times that of the glucose? All other factors remain constant.
32. The pressure drop along a length of artery is 100 Pa, the radius is 10 mm, and the

flow is laminar. The average speed of the blood is 15 mm/s. (a) What is the net force on the blood in this section of artery? (b) What is the power expended maintaining the flow?

33. A small artery has a length of $1.1 \times 10^{-3}\text{m}$ and a radius of $2.5 \times 10^{-5}\text{m}$. If the pressure drop across the artery is 1.3 kPa, what is the flow rate through the artery? (Assume that the temperature is $37^{\circ}\ \text{C}$.)
34. Fluid originally flows through a tube at a rate of $100\text{cm}^3/\text{s}$. To illustrate the sensitivity of flow rate to various factors, calculate the new flow rate for the following changes with all other factors remaining the same as in the original conditions. (a) Pressure difference increases by a factor of 1.50. (b) A new fluid with 3.00 times greater viscosity is substituted. (c) The tube is replaced by one having 4.00 times the length. (d) Another tube is used with a radius 0.100 times the original. (e) Yet another tube is substituted with a radius 0.100 times the original and half the length, and the pressure difference is increased by a factor of 1.50.
35. The arterioles (small arteries) leading to an organ, constrict in order to decrease flow to the organ. To shut down an organ, blood flow is reduced naturally to 1.00% of its original value. By what factor did the radii of the arterioles constrict? Penguins do this when they stand on ice to reduce the blood flow to their feet.
36. Angioplasty is a technique in which arteries partially blocked with plaque are dilated to increase blood flow. By what factor must the radius of an artery be increased in order to increase blood flow by a factor of 10?
37. (a) Suppose a blood vessel's radius is decreased to 90.0% of its original value by plaque deposits and the body compensates by increasing the pressure difference along the vessel to keep the flow rate constant. By what factor must the pressure difference increase? (b) If turbulence is created by the obstruction, what additional effect would it have on the flow rate?
38. A spherical particle falling at a terminal speed in a liquid must have the gravitational force balanced by the drag force and the buoyant force. The buoyant force is equal to the weight of the displaced fluid, while the drag force is assumed to be given by Stokes Law, $F_s = 6\pi r\eta v$. Show that the terminal speed is given by

$v = \frac{(2R)^2 g}{9\eta}(\rho_s - \rho_1)$, where R is the radius of the sphere, ρ_s is its density, and ρ_1 is the density of the fluid and η the coefficient of viscosity.

39. Using the equation of the previous problem, find the viscosity of motor oil in which a steel ball of radius 0.8 mm falls with a terminal speed of 4.32 cm/s. The densities of the ball and the oil are 7.86 and 0.88 g/mL, respectively.
40. A skydiver will reach a terminal velocity when the air drag equals their weight. For a skydiver with high speed and a large body, turbulence is a factor. The drag force then is approximately proportional to the square of the velocity. Taking the drag force to be $F_D = \frac{1}{2}\rho A v^2$ and setting this equal to the person's weight, find the terminal speed for a person falling "spread eagle." Find both a formula and a number for v_t, with assumptions as to size.
41. A layer of oil 1.50 mm thick is placed between two microscope slides. Researchers find that a force of $5.50 \times 10^{-4}\text{N}$ is required to glide one over the other at a speed of 1.00 cm/s when their contact area is 6.00cm^2. What is the oil's viscosity? What type of oil might it be?
42. (a) Verify that a 19.0% decrease in laminar flow through a tube is caused by a 5.00% decrease in radius, assuming that all other factors remain constant, as stated in the text. (b) What increase in flow is obtained from a 5.00% increase in radius, again assuming all other factors remain constant?
43. Example 10.8 dealt with the flow of saline solution in an IV system. (a) Verify that a pressure of $1.62 \times 10^4 \text{N/m}^2$ is created at a depth of 1.61 m in a saline solution, assuming its density to be that of sea water. (b) Calculate the new flow rate if the height of the saline solution is decreased to 1.50 m. (c) At what height would the direction of flow be reversed? (This reversal can be a problem when patients stand up.)
44. When physicians diagnose arterial blockages, they quote the reduction in flow rate. If the flow rate in an artery has been reduced to 10.0% of its normal value by a blood clot and the average pressure difference has increased by 20.0%, by what factor has the clot reduced the radius of the artery?
45. During a marathon race, a runner's blood flow increases to 10.0 times her resting

rate. Her blood's viscosity has dropped to 95.0% of its normal value, and the blood pressure difference across the circulatory system has increased by 50.0%. By what factor has the average radii of her blood vessels increased?

46. Water supplied to a house by a water main has a pressure of $3.00 \times 10^5 \mathrm{N/m^2}$ early on a summer day when neighborhood use is low. This pressure produces a flow of 20.0 L/min through a garden hose. Later in the day, pressure at the exit of the water main and entrance to the house drops, and a flow of only 8.00 L/min is obtained through the same hose. (a) What pressure is now being supplied to the house, assuming resistance is constant? (b) By what factor did the flow rate in the water main increase in order to cause this decrease in delivered pressure? The pressure at the entrance of the water main is $5.00 \times 10^5 \mathrm{N/m^2}$, and the original flow rate was 200 L/min. (c) How many more users are there, assuming each would consume 20.0 L/min in the morning?
47. An oil gusher shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. Neglecting air resistance but not the resistance of the pipe, and assuming laminar flow, calculate the gauge pressure at the entrance of the 50.0-m-long vertical pipe. Take the density of the oil to be $900 \mathrm{kg/m^3}$ and its viscosity to be $1.00\ (\mathrm{N/m^2}) \cdot \mathrm{s}$ (or $1.00 \mathrm{Pa} \cdot \mathrm{s}$). Note that you must take into account the pressure due to the 50.0-m column of oil in the pipe.
48. Concrete is pumped from a cement mixer to the place it is being laid, instead of being carried in wheelbarrows. The flow rate is 200.0 L/min through a 50.0-m-long, 8.00-cm-diameter hose, and the pressure at the pump is $8.00 \times 10^6 \mathrm{N/m^2}$. (a) Calculate the resistance of the hose. (b) What is the viscosity of the concrete, assuming the flow is laminar? (c) How much power is being supplied, assuming the point of use is at the same level as the pump? You may neglect the power supplied to increase the concrete's velocity.
49. **Construct Your Own Problem:** Consider a coronary artery constricted by arteriosclerosis. Construct a problem in which you calculate the amount by which the diameter of the artery is decreased, based on an assessment of the decrease in flow rate.
50. Consider a river that spreads out in a delta region on its way to the sea. Construct a problem in which you calculate the average speed at which water moves in the

delta region, based on the speed at which it was moving up river. Among the things to consider are the size and flow rate of the river before it spreads out and its size once it has spread out. You can construct the problem for the river spreading out into one large river or into multiple smaller rivers.

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TEMPERATURE AND THERMAL EXPANSION



Figure 11.1 Molecules dissolved in surfaces undergo a process of outgassing, or release into the space around them. In a car, this leads to the famous "new car smell." In precision instruments like telescopes, this leads to major distortions due to contamination. NASA engineer Nithin Abraham created a coating to absorb these contaminants. Here she is pictured securing a coated panel beneath the Johnson Space Center's Chamber A, where it was used during testing of the James Webb Space Telescope. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

11.1 Temperature
11.2 Thermal Expansion of Solids and Liquids
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

Heat is something familiar to each of us. We feel the warmth of the summer Sun, the chill of a clear summer night, the heat of coffee after a winter stroll, and the cooling effect of our sweat. Manifestations of heat transfer—the movement of heat energy from one place or material to another—are apparent throughout the universe. Heat from beneath Earth's surface is brought to the surface in flows of incandescent lava. The Sun warms Earth's surface and is the source of much of the energy we find on it. Rising levels of atmospheric carbon dioxide threaten to trap more of the Sun's energy, perhaps fundamentally altering the ecosphere. In space, supernovas explode, briefly radiating more heat than an entire galaxy does.

What is heat? How do we define it? How is it related to temperature? What are heat's effects? How is it related to other forms of energy and to work? We will find that, in spite of the richness of the phenomena, there is a small set of underlying physical principles that unite the subjects and tie them to other fields.



Figure 11.2 In a typical thermometer like this one, the alcohol, with a red dye, expands more rapidly than the glass containing it. When the thermometer's temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature. Image from OpenStax College Physics 2e, CC-BY 4.0

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11.1 TEMPERATURE

Learning Objectives

By the end of this section, you will be able to:

- Define temperature.
- Convert temperatures between the Celsius, Fahrenheit, and Kelvin scales.
- Define thermal equilibrium.
- State the zeroth law of thermodynamics.

The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. Temperature is operationally defined to be what we measure with a thermometer. (Many physical quantities are defined solely in terms of how they are measured. We shall see later how temperature is related to the kinetic energies of atoms and molecules, a more physical explanation.) Two accurate thermometers, one placed in hot water and the other in cold water, will show the hot water to have a higher temperature. If they are then placed in the tepid water, both will give identical readings (within measurement uncertainties). In this section, we discuss temperature, its measurement by thermometers, and its relationship to thermal equilibrium. Again, temperature is the quantity measured by a thermometer.

Misconception Alert: Human Perception vs. Reality

On a cold winter morning, a wooden bench feels warmer than a metal bench. Both benches are in thermal equilibrium with the outside air, and are thus the same temperature. They *feel* different because of the difference in the way that they conduct heat away from your skin. The metal conducts heat away from your body faster than the wood does (see more about conductivity in 12.5 Conduction). This is just one example demonstrating that the human sense of hot and cold is not determined by temperature alone.

Another factor that affects our perception of temperature is humidity. Most people feel much hotter on hot, humid days than on hot, dry days. This is because on humid days, sweat does not evaporate from the skin and in our lungs as efficiently as it does on dry days. It is the evaporation of sweat (or water from a sprinkler or pool) as well as water in the lungs that cools us off.

Any physical property that depends on temperature, and whose response to temperature is reproducible, can be used as the basis of a thermometer. Because many physical properties depend on temperature, the variety of thermometers is remarkable. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer, the old mercury thermometer, and the bimetallic strip (Figure 11.4). Other properties used to measure temperature include electrical resistance and color, as shown in Figure 11.5, and the emission of infrared radiation, as shown in Figure 11.6.



Figure 11.3 Alcohol thermometer with red dye. In a typical thermometer like this one, the alcohol, with a red dye, expands more rapidly than the glass containing it. When the thermometer's temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature. Image from OpenStax College Physics 2e, CC-BY 4.0

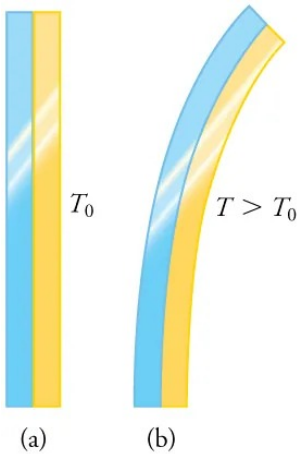


Figure 11.4 The curvature of a bimetallic strip depends on temperature. (a) The strip is straight at the starting temperature, where its two components have the same length. (b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two side-by-side illustrations labeled (a) and (b).

– Illustration (a) shows two vertical strips, one blue and one yellow, placed side by side. They are straight and aligned vertically. The label "T_0" is placed next to these strips, indicating an initial temperature.

– Illustration (b) shows the same blue and yellow strips, but they are now curved to the right. The blue strip is on the outside of the curve, and the yellow strip is on the inside. The temperature label "$T > T_0$" suggests that the temperature has increased compared to illustration (a).

These illustrations demonstrate the bending of two materials when heated.



Figure 11.5 Each of the six squares on this plastic (liquid crystal) thermometer contains a film of a different heat-sensitive liquid crystal material. Below $95^{\circ}\ \mathrm{F}$*, all six squares are black. When the plastic thermometer is exposed to temperature that increases to* $95^{\circ}\ \mathrm{F}$*, the first liquid crystal square changes color. When the temperature increases above* $96.8^{\circ}\ \mathrm{F}$ *the second liquid crystal square also changes color, and so forth. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is of a forehead temperature indicator strip. It is designed to display body temperature with a scale for both Fahrenheit and Celsius.

The strip features six black panels, each showing a temperature reading. To the left, there is a plus symbol in a red circle. Next to it, on the right, are columns of temperature readings. The numbers are white, printed on black rectangles. Each panel has two numbers: one in Fahrenheit (°F) and one in Celsius (°C).

The numbers from left to right are as follows:

°F	**95**	**96.8**	**98.6**	**100.4**	**102.2**	**104**
°C	35	36	37	38	39	40

On the right side of the strip, the text “forehead temperature indicator” is printed in red.


EXHAUST
EXHAUST

Figure 11.6 Fireman Jason Ormand uses a pyrometer to check the temperature of an aircraft carrier's ventilation system. Infrared radiation (whose emission varies with temperature) from the vent is measured and a temperature readout is quickly produced. Infrared measurements are also frequently used as a measure of body temperature. These modern thermometers, placed in the ear canal, are more accurate than alcohol thermometers placed under the tongue or in the armpit. Image by U.S. Navy photo by Photographer's Mate 3rd Class Lamel J. Hinton (RELEASED), Wikimedia Commons License

Temperature Scales

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The Celsius scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at $0^{\circ}\ \mathrm{C}$ and the boiling point at $100^{\circ}\ \mathrm{C}$. Its unit is the degree Celsius$(^{\circ}\ \mathrm{C})$. On the Fahrenheit scale (still the most frequently used in the United States), the freezing point of water is at $32^{\circ}\ \mathrm{F}$ and the boiling point is at $212^{\circ}\ \mathrm{F}$. The unit of temperature on this scale is the degree Fahrenheit$(^{\circ}\ \mathrm{F})$. Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Only 100 Celsius degrees span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale $180/100 = 9/5.$

The Kelvin scale is the temperature scale that is commonly used in science. It is an *absolute temperature* scale defined to have 0 K at the lowest possible temperature, called absolute zero. The official temperature unit on this scale is the *kelvin*, which is abbreviated K, and is not accompanied by a degree sign. The freezing and boiling points of water are 273.15 K and 373.15 K, respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.

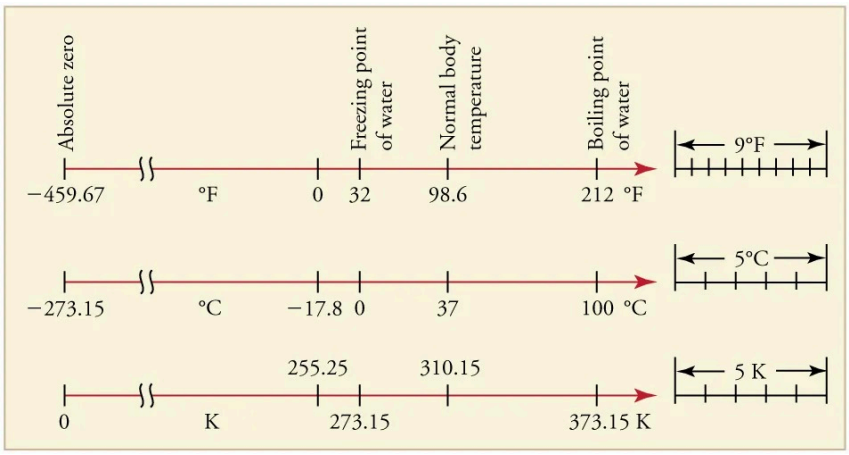


Figure 11.7 Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image presents three horizontal temperature scales for Fahrenheit, Celsius, and Kelvin, starting from absolute zero and highlighting key temperature points.

The first scale is in Fahrenheit:

- Absolute zero: -459.67 °F
- Freezing point of water: 32 °F
- Normal body temperature: 98.6 °F
- Boiling point of water: 212 °F

The interval on the right shows a difference of 9 °F.

The second scale is in Celsius:

- Absolute zero: -273.15 °C
- Freezing point of water: 0 °C
- Normal body temperature: 37 °C
- Boiling point of water: 100 °C

The interval on the right shows a difference of 5 °C.

The third scale is in Kelvin:

- Absolute zero: 0 K
- Freezing point of water: 273.15 K
- Normal body temperature: 310.15 K
- Boiling point of water: 373.15 K

The interval on the right shows a difference of 5 K.

The relationships between the three common temperature scales is shown in Figure 11.7. Temperatures on these scales can be converted using the equations in Table 11.1.

Table 11.1 Temperature Conversions

To convert from . . .	Use this equation . . .	Also written as . . .
Celsius to Fahrenheit	$T\,(^\circ\,\mathrm{F}) = \frac{9}{5}T\,(^\circ\,\mathrm{C}) + 32$	$T_{^\circ\,\mathrm{F}} = \frac{9}{5}T_{^\circ\,\mathrm{C}} + 32$
Fahrenheit to Celsius	$T\,(^\circ\,\mathrm{C}) = \frac{5}{9}(T\,(^\circ\,\mathrm{F}) - 32)$	$T_{^\circ\,\mathrm{C}} = \frac{5}{9}\left(T_{^\circ\,\mathrm{F}} - 32\right)$
Celsius to Kelvin	$T\,(\mathrm{K}) = T\,(^\circ\,\mathrm{C}) + 273.15$	$T_\mathrm{K} = T_{^\circ\,\mathrm{C}} + 273.15$
Kelvin to Celsius	$T\,(^\circ\,\mathrm{C}) = T\,(\mathrm{K}) - 273.15$	$T_{^\circ\,\mathrm{C}} = T_\mathrm{K} - 273.15$
Fahrenheit to Kelvin	$T\,(\mathrm{K}) = \frac{5}{9}(T\,(^\circ\,\mathrm{F}) - 32) + 273.15$	$T_\mathrm{K} = \frac{5}{9}\left(T_{^\circ\,\mathrm{F}} - 32\right) + 273.15$
Kelvin to Fahrenheit	$T\,(^\circ\,\mathrm{F}) = \frac{9}{5}(T\,(\mathrm{K}) - 273.15) + 32$	$T_{^\circ\,\mathrm{F}} = \frac{9}{5}(T_\mathrm{K} - 273.15) + 32$

Notice that the conversions between Fahrenheit and Kelvin look quite complicated. In fact, they are simple combinations of the conversions between Fahrenheit and Celsius, and the conversions between Celsius and Kelvin.

Example 11.1

Converting between Temperature Scales: Room Temperature

"Room temperature" is generally defined to be $25^{\circ}\ \mathrm{C}$. (a) What is room temperature in $^{\circ}\ \mathrm{F}$? (b) What is it in K?

Strategy

To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

Solution for (a)

1. Choose the right equation. To convert from $^{\circ}\ \mathrm{C}$ to $^{\circ}\ \mathrm{F}$, use the equation

$$T_{^{\circ}\mathrm{F}} = \frac{9}{5}T_{^{\circ}\mathrm{C}} + 32.$$

11.1

2. Plug the known value into the equation and solve:

$$T_{^{\circ}\mathrm{F}} = \frac{9}{5}25^{\circ}\ \mathrm{C} + 32 = 77^{\circ}\ \mathrm{F}.$$

11.2

Solution for (b)

1. Choose the right equation. To convert from $^{\circ}\ \mathrm{C}$ to K, use the equation

$$T_{\mathrm{K}} = T_{^{\circ}\mathrm{C}} + 273.15.$$

11.3

2. Plug the known value into the equation and solve:

$$T_{\mathrm{K}} = 25^{\circ}\ \mathrm{C} + 273.15 = 298\mathrm{K}.$$

11.4

Example 11.2

Converting between Temperature Scales: the Reaumur Scale

The Reaumur scale is a temperature scale that was used widely in Europe in the 18th and 19th centuries. On the Reaumur temperature scale, the freezing point of water is $0^{\circ}\ \mathrm{R}$ and the boiling temperature is $80^{\circ}\ \mathrm{R}$. If "room temperature" is $25^{\circ}\ \mathrm{C}$ on the Celsius scale, what is it on the Reaumur scale?

Strategy

To answer this question, we must compare the Reaumur scale to the Celsius scale. The difference between the freezing point and boiling point of water on the Reaumur scale is $80^{\circ}\ \mathrm{R}$. On the Celsius scale it is $100^{\circ}\ \mathrm{C}$. Therefore $100^{\circ}\ \mathrm{C} = 80^{\circ}\ \mathrm{R}$. Both scales start at 0° for freezing, so we can derive a simple formula to convert between temperatures on the two scales.

Solution

1. Derive a formula to convert from one scale to the other:

$$T_{^{\circ}\mathrm{R}} = \frac{0.8^{\circ}\ \mathrm{R}}{^{\circ}\ \mathrm{C}} \times T_{^{\circ}\mathrm{C}}.$$

11.5

2. Plug the known value into the equation and solve:

$$T_{^{\circ}\mathrm{R}} = \frac{0.8^{\circ}\ \mathrm{R}}{^{\circ}\ \mathrm{C}} \times 25^{\circ}\ \mathrm{C} = 20^{\circ}\ \mathrm{R}.$$

11.6

Temperature Ranges in the Universe

Figure 11.9 shows the wide range of temperatures found in the universe. Human beings have been known to survive with body temperatures within a small range, from $24^{\circ}\ \mathrm{C}$ to $44^{\circ}\ \mathrm{C}$ ($75^{\circ}\ \mathrm{F}$ to $111^{\circ}\ \mathrm{F}$). The average normal body temperature is usually given as $37.0^{\circ}\ \mathrm{C}$ ($98.6^{\circ}\ \mathrm{F}$), and variations in this temperature can indicate a medical condition: a fever, an infection, a tumor, or circulatory problems (see Figure 11.8).

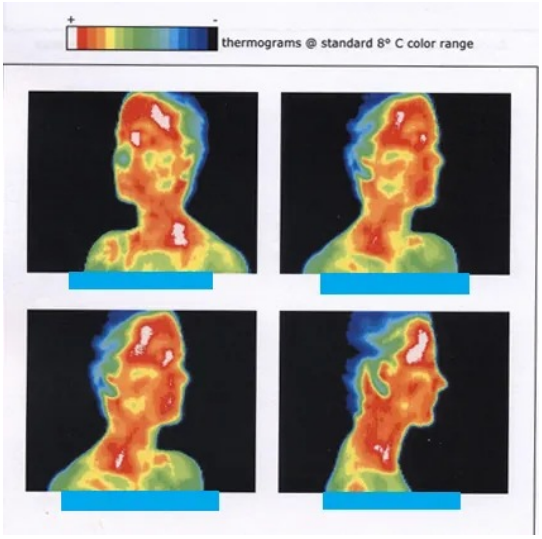


Figure 11.8 This image of radiation from a person's body (an infrared thermograph) shows the location of temperature abnormalities in the upper body. Dark blue corresponds to cold areas and red to white corresponds to hot areas. An elevated temperature might be an indication of malignant tissue (a cancerous tumor in the breast, for example), while a depressed temperature might be due to a decline in blood flow from a clot. In this case, the abnormalities are caused by a condition called hyperhidrosis. Image from OpenStax College Physics 2e, CC-BY 4.0

The lowest temperatures ever recorded have been measured during laboratory experiments: $4.5 \times 10^{-10}\ \mathrm{K}$ at the Massachusetts Institute of Technology (USA), and $1.0 \times 10^{-10}\ \mathrm{K}$ at Helsinki University of Technology (Finland). In comparison, the coldest recorded place on Earth's surface is Vostok, Antarctica at 183 K ($-89^{\circ}\ \mathrm{C}$), and the coldest place (outside the lab) known in the universe is the Boomerang Nebula, with a temperature of 1 K.

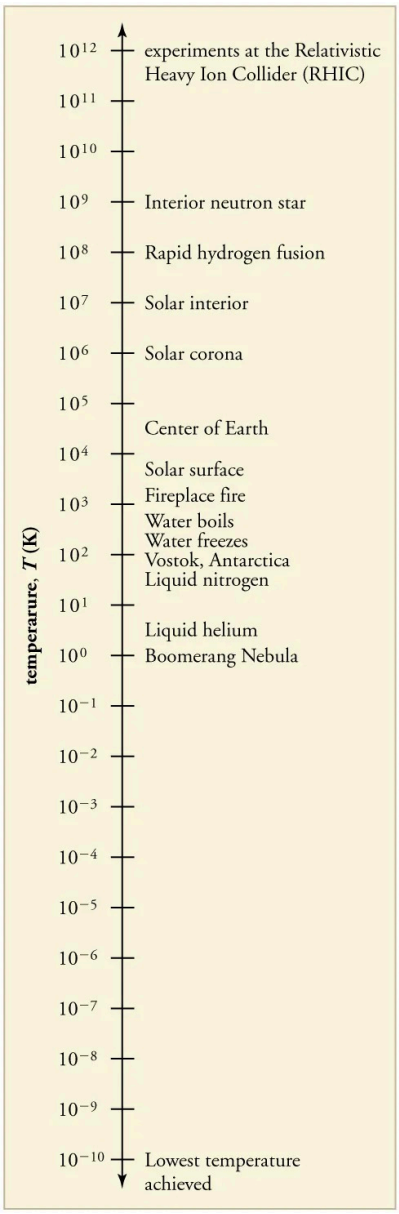


Figure 11.9 Each increment on this logarithmic scale indicates an increase by a factor of ten, and thus illustrates the tremendous range of temperatures in nature. Note that zero on a logarithmic scale would occur off the bottom of the page at infinity. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a vertical scale representing a range of temperatures measured in Kelvin (K), with various phenomena or locations associated with each temperature level. The scale ranges from 10^{12} K at the top to 10^{-10} K at the bottom. Below is an HTML table representing the information from the scale:

Temperature (K)	Phenomenon/Location
10^{12}	Experiments at the Relativistic Heavy Ion Collider (RHIC)
10^{11}	
10^{10}	
10^{9}	Interior neutron star
10^{8}	Rapid hydrogen fusion
10^{7}	Solar interior
10^{6}	Solar corona
10^{5}	
10^{4}	Center of Earth
10^{3}	Solar surface, Fireplace fire
10^{2}	Water boils, Water freezes, Vostok, Antarctica
10^{1}	Liquid nitrogen
10^{0}	Liquid helium, Boomerang Nebula
10^{-1}	
10^{-2}	
10^{-3}	
10^{-4}	
10^{-5}	
10^{-6}	
10^{-7}	
10^{-8}	
10^{-9}	
10^{-10}	Lowest temperature achieved

Making Connections: Absolute Zero

What is absolute zero? The concept of absolute zero arises from the behavior of gases. Figure 11.10 shows how the pressure of gases at a constant volume decreases as temperature decreases. Various scientists have noted that the pressures of gases extrapolate to zero at the same temperature, $-273.15^{\circ}\ \mathrm{C}$. This extrapolation suggests that there is a lowest temperature. Other thermodynamic relationships lead to the same conclusion. This lowest temperature is called *absolute zero*. At absolute zero, molecular motion is at the minimum energy possible. Today we know that most gases first liquefy and then freeze, and it is not actually possible to reach absolute zero. The numerical value of absolute zero temperature is $-273.15^{\circ}\ \mathrm{C}$ or 0 K.

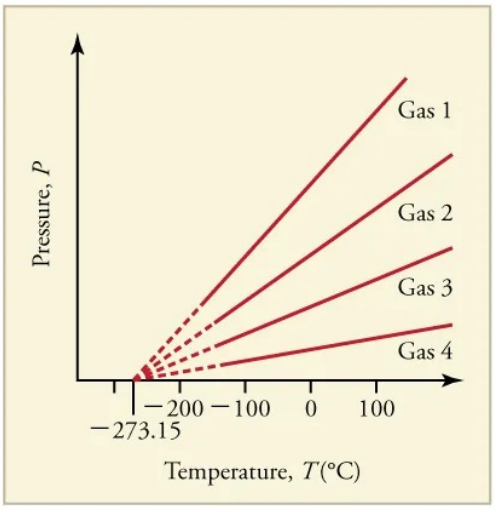


Figure 11.10 Graph of pressure versus temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph that illustrates the relationship between pressure and temperature for four different

gases. The x-axis is labeled “Temperature, T (°C)” and ranges from approximately -273.15°C to 100°C. The y-axis is labeled “Pressure, P” and indicates increasing pressure but does not have specific values marked.

Four lines representing different gases are plotted on the graph:

- *Gas 1*: The line is the farthest to the right, indicating a steeper increase in pressure with temperature compared to the other gases.
- *Gas 2*: The line is slightly less steep than Gas 1.
- *Gas 3*: The line is even less steep, indicating a moderate increase in pressure with temperature.
- *Gas 4*: The line is the least steep, indicating the least change in pressure with temperature among the four gases.

The lines extend from near the bottom left corner, with all of them converging towards the y-axis as the temperature approaches -273.15°C.

Thermal Equilibrium and the Zeroth Law of Thermodynamics

Thermometers actually take their *own* temperature, not the temperature of the object they are measuring. This raises the question of how we can be certain that a thermometer measures the temperature of the object with which it is in contact. It is based on the fact that any two systems placed in *thermal contact* (meaning heat transfer can occur between them) but isolated from the rest of the environment, will reach the same temperature. That is, heat will flow from the hotter object to the cooler one until they have exactly the same temperature. The objects are then in thermal equilibrium, and no further heat flow or changes in temperature will occur. Thus, if enough time is allowed for this transfer of heat to run its course, the temperature a thermometer registers *does* represent the system with which it is in thermal equilibrium. Thermal equilibrium is established when two bodies are in contact with each other and can freely exchange energy. A single object is said to be in internal thermal equilibrium when no heat is flowing within the object and therefore its temperature is uniform.

Furthermore, experimentation has shown that if two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system C, then A is also in thermal equilibrium with C. This conclusion may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the zeroth law of thermodynamics.

The Zeroth Law of Thermodynamics

If two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

This law was postulated in the 1930s, after the first and second laws of thermodynamics had been developed and named. It is called the *zeroth law* because it comes logically before the first and second laws of thermodynamics. Suppose, for example, a cold metal block and a hot metal block are both placed on a metal plate at room temperature. Eventually the cold block and the plate will be in thermal equilibrium. In addition, the hot block and the plate will be in thermal equilibrium. By the zeroth law, we can conclude that the cold block and the hot block are also in thermal equilibrium. Similarly, two thermometers placed in thermal contact, one with one block and the second with the other block, will also be in thermal equilibrium and therefore read the same temperature. This establishes temperature as a meaningful concept.

Check Your Understanding

Does the temperature of a body in internal thermal equilibrium depend on its size?

Click for Solution

Solution

No, the system can be divided into smaller parts. Since no heat is flowing, each part is in equilibrium with the adjacent part, and according to the zeroth law, this means all parts are in equilibrium with each other and are at the same temperature. Quantities that are independent of size are called intensive quantities. Temperature is an intensive quantity.

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11.2 THERMAL EXPANSION OF SOLIDS AND LIQUIDS

Learning Objectives

By the end of this section, you will be able to:

- Define and describe thermal expansion.
- Calculate the linear expansion of an object given its initial length, change in temperature, and coefficient of linear expansion.
- Calculate the volume expansion of an object given its initial volume, change in temperature, and coefficient of volume expansion.
- Calculate thermal stress on an object given its original volume, temperature change, volume change, and bulk modulus.



Figure 11.11 Thermal expansion joints like these in the Auckland Harbour Bridge in New Zealand allow bridges to change length without buckling. Image by Ingolfson, Wikimedia Commons License

The expansion of alcohol in a thermometer is one of many commonly encountered examples of thermal expansion, the change in size or volume of a given mass with temperature. Hot air rises because air expands when its temperature increases, which causes the hot air's density to be smaller than the density of surrounding air. As a result, the buoyant (upward) force on a given mass of air increases when the air is heated, resulting in a net upward force on that air mass. The same happens in all liquids and gases, driving natural heat transfer upwards in homes, oceans, and weather systems. Solids also undergo thermal expansion. Railroad tracks and bridges, for example, have expansion joints to allow them to freely expand and contract with temperature changes.

What are the basic properties of thermal expansion? First, thermal expansion is clearly related to temperature change. The greater the temperature change, the more a bimetallic strip will bend. Second, it depends on the material. In a thermometer, for example, the expansion of alcohol is much greater than the expansion of the glass containing it.

What is the underlying cause of thermal expansion? An increase in temperature implies an increase in the kinetic energy of the individual atoms. In a solid, unlike in a gas, the atoms or molecules are closely packed together, but their kinetic energy (in the form of small, rapid vibrations) pushes neighboring atoms or molecules apart from each other. This neighbor-to-neighbor pushing results in a slightly greater distance, on average, between neighbors, and adds up to a larger size for the whole body. For most substances under ordinary conditions, there is no preferred direction, and an increase in temperature will increase the solid's size by a certain fraction in each dimension.

Linear Thermal Expansion—Thermal Expansion in One Dimension

The change in length ΔL is proportional to length L. The dependence of thermal expansion on temperature, substance, and length is summarized in the equation

$$\Delta L = \alpha L \Delta T,$$

11.7

where ΔL is the change in length L, ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.

Table 11.2 lists representative values of the coefficient of linear expansion, which may have units of $1/^{\circ}\text{C}$ or 1/K. Because the size of a kelvin and a degree Celsius are the same, both α and ΔT can be expressed in units of kelvins or degrees Celsius. The equation $\Delta L = \alpha L \Delta T$ is accurate for small changes in temperature and can be used for large changes in temperature if an average value of α is used.

Table 11.2 Thermal Expansion Coefficients at $20^{\circ}\ \mathrm{C}$[1]

Material	Coefficient of linear expansion $\alpha\,(1/^{\circ}\ \mathrm{C})$	Coefficient of volume expansion $\beta\,(1/^{\circ}\ \mathrm{C})$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	51×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or Steel	12×10^{-6}	35×10^{-6}
Invar (Nickel-iron alloy)	0.9×10^{-6}	2.7×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Silver	18×10^{-6}	54×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete, Brick (approximate)	12×10^{-6}	36×10^{-6}
Marble (average)	7×10^{-6}	2.1×10^{-5}
Liquids		
Ether		1650×10^{-6}
Ethyl alcohol		1100×10^{-6}
Petrol		950×10^{-6}
Glycerin		500×10^{-6}
Mercury		180×10^{-6}
Water		210×10^{-6}
Gases		
Air and most other gases at atmospheric pressure		3400×10^{-6}

1. Values for liquids and gases are approximate.

Example 11.3

Calculating Linear Thermal Expansion: The Golden Gate Bridge

The main span of San Francisco's Golden Gate Bridge is 1275 m long at its coldest. The bridge is exposed to temperatures ranging from $-15^{\circ}\ \mathrm{C}$ to $40^{\circ}\ \mathrm{C}$. What is its change in length between these temperatures? Assume that the bridge is made entirely of steel.

Strategy

Use the equation for linear thermal expansion $\Delta L = \alpha L \Delta T$ to calculate the change in length , ΔL. Use the coefficient of linear expansion, α, for steel from Table 11.2, and note that the change in temperature, ΔT, is $55^{\circ}\ \mathrm{C}$.

Solution

Plug all of the known values into the equation to solve for ΔL.

$$\begin{aligned}\Delta L &= \alpha L \Delta T \\ &= \left(\frac{12 \times 10^{-6}}{^{\circ}\ \mathrm{C}}\right)(1275\ \mathrm{m})\,(55^{\circ}\ \mathrm{C}) \\ &= 0.84\ \mathrm{m}.\end{aligned}$$

11.8

Discussion

Although not large compared with the length of the bridge, this change in length is observable. It is generally spread over many expansion joints so that the expansion at each joint is small.

Thermal Expansion in Two and Three Dimensions

Objects expand in all dimensions, as illustrated in Figure 11.12. That is, their areas and volumes, as well as their lengths, increase with temperature. Holes also get larger with temperature. If you cut a hole in a metal plate, the remaining material will expand exactly as it would if the plug was still in place. The plug would get bigger, and so the hole must get bigger too. (Think of the ring of neighboring atoms or molecules on the wall of the hole as pushing each other farther apart as temperature increases. Obviously, the ring of neighbors must get slightly larger, so the hole gets slightly larger).

Thermal Expansion in Two Dimensions

For small temperature changes, the change in area ΔA is given by

$$\Delta A = 2\alpha A \Delta T,$$

11.9

where ΔA is the change in area A, ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.

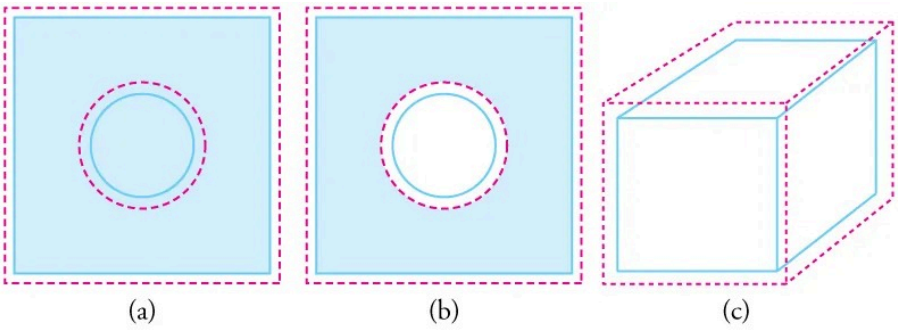


Figure 11.12 In general, objects expand in all directions as temperature increases. In these drawings, the original boundaries of the objects are shown with solid lines, and the expanded boundaries with dashed lines. (a) Area increases because both length and width increase. The area of a circular plug also increases. (b) If the plug is removed, the hole it leaves becomes larger with increasing temperature, just as if the expanding plug were still in place. (c) Volume also increases, because all three dimensions increase. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains three diagrams labeled (a), (b), and (c), showcasing geometric shapes in light blue with dashed pink outlines.

– Diagram (a): A light blue square with a centered circle cutout. The square has a solid outline, and there's a dashed pink square around it. The circle appears transparent or has a different fill, suggesting it's a solid hole.

– Diagram (b): Similar to (a), but the circle in the center appears white, indicating a complete cutout through the square. The dashed pink square outline is present.

– Diagram (c): A 3D representation of a cube with visible edges in light blue. The cube is transparent or open on one side, and there are dashed pink lines indicating its projection or borders of an extended shape.

These diagrams visually illustrate different geometric properties, focusing on dimensions and projections.

Thermal Expansion in Three Dimensions

The change in volume ΔV is very nearly $\Delta V = 3\alpha V \Delta T$. This equation is usually written as

$$\Delta V = \beta V \Delta T,$$

11.10

where β is the coefficient of volume expansion and $\beta \approx 3\alpha$. Note that the values of β in Table 11.2 are almost exactly equal to 3α.

In general, objects will expand with increasing temperature. Water is the most important exception to this rule. Water expands with increasing temperature (its density *decreases*) when it is at temperatures greater than $4^\circ\ \mathrm{C}\ (40^\circ\ \mathrm{F})$. However, it expands with *decreasing* temperature when it is between $+4^\circ\ \mathrm{C}$ and $0^\circ\ \mathrm{C}$ $(40^\circ\ \mathrm{F}$ to $32^\circ\ \mathrm{F})$. Water is densest at $+4^\circ\ \mathrm{C}$. (See Figure 11.13.) Perhaps the most striking effect of this phenomenon is the freezing of water in a pond. When water near the surface cools down to $4^\circ\ \mathrm{C}$ it is denser than the remaining water and thus will sink to the bottom. This "turnover" results in a layer of warmer water near the surface, which is then cooled. Eventually the pond has a uniform temperature of $4^\circ\ \mathrm{C}$. If the temperature in the surface layer drops below $4^\circ\ \mathrm{C}$, the water is less dense than the water below, and thus stays near the top. As a result, the pond surface can completely freeze over. The ice on top of liquid water provides an insulating layer from winter's harsh exterior air temperatures. Fish and other aquatic life can survive in $4^\circ\ \mathrm{C}$ water beneath ice, due to this unusual characteristic of water. It also produces circulation of water in the pond that is necessary for a healthy ecosystem of the body of water.

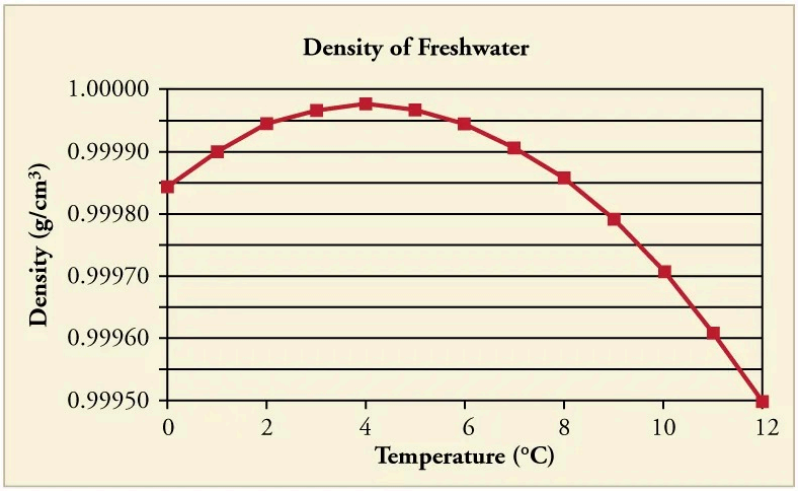


Figure 11.13 The density of water as a function of temperature. Note that the thermal expansion is actually very small. The maximum density at $+4^{\circ}\ \mathrm{C}$ *is only 0.0075% greater than the density at* $2^{\circ}\ \mathrm{C}$*, and 0.012% greater than that at* $0^{\circ}\ \mathrm{C}$*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a graph depicting the relationship between the density of freshwater and temperature. The title of the graph is "Density of Freshwater." The y-axis represents the density in grams per cubic centimeter (g/cm^3), ranging from 0.99950 to 1.00000. The x-axis represents the temperature in degrees Celsius (°C), ranging from 0°C to 12°C.

The graph is a line chart with red squares marking data points. The line starts from the lower density at 0°C, rises to a peak density of approximately 1.00000 g/cm^3 around 4°C, and then declines as the temperature approaches 12°C. The behavior of the line suggests that freshwater density increases to a maximum at around 4°C and then decreases as temperature increases beyond this point.

Making Connections: Real-World Connections—Filling the Tank

Differences in the thermal expansion of materials can lead to interesting effects at the gas station. One example is the dripping of gasoline from a freshly filled tank on a hot day. Gasoline starts out at the temperature of the ground under the gas station, which is cooler than the air temperature above. The gasoline cools the steel tank when it is filled. Both gasoline and steel tank expand as they warm to air temperature, but gasoline expands much more than steel, and so it may overflow.

This difference in expansion can also cause problems when interpreting the gasoline gauge. The actual amount (mass) of gasoline left in the tank when the gauge hits "empty" is a lot less in the summer than in the winter. The gasoline has the same volume as it does in the winter when the "add fuel" light goes on, but because the gasoline has expanded, there is less mass. If you are used to getting another 40 miles on "empty" in the winter, beware—you may only get 38 miles in the summer.

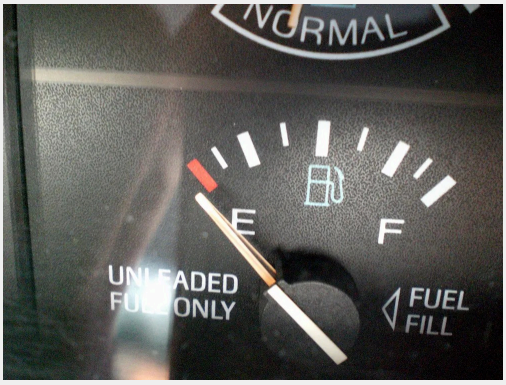


Figure 11.14 Because the gas expands more than the gas tank with increasing temperature, you can't drive as many miles on "empty" in the summer as you can in the winter. Image from OpenStax College Physics 2e, CC-BY 4.0

Example 11.4

Calculating Thermal Expansion: Gas vs. Gas Tank

Suppose your 60.0-L (15.9-gal) steel gasoline tank is full of gas, so both the tank and the gasoline have a temperature of $15.0^{\circ}\ \mathrm{C}$. How much gasoline has spilled by the time they warm to $35.0^{\circ}\ \mathrm{C}$?

Strategy

The tank and gasoline increase in volume, but the gasoline increases more, so the amount spilled is the difference in their volume changes. (The gasoline tank can be treated as solid steel.) We can use the equation for volume expansion to calculate the change in volume of the gasoline and of the tank.

Solution

1. Use the equation for volume expansion to calculate the increase in volume of the steel tank:

$$(\Delta V)_{\mathrm{s}} = \beta_{\mathrm{s}} V_{\mathrm{s}} \Delta T.$$

11.11

2. The increase in volume of the gasoline is given by this equation:

$$(\Delta V)_{\mathrm{gas}} = \beta_{\mathrm{gas}} V_{\mathrm{gas}} \Delta T.$$

11.12

3. Find the difference in volume to determine the amount spilled as

$$V_{\mathrm{spill}} = (\Delta V)_{\mathrm{gas}} - (\Delta V)_{\mathrm{s}}.$$

11.13

Alternatively, we can combine these three equations into a single equation. (Note that the original volumes are equal.)

$$\begin{aligned} V_{\text{spill}} &= (\beta_{\text{gas}} - \beta_{\text{s}})V\Delta T \\ &= \left[(950 - 35) \times 10^{-6}/^{\circ}\text{ C}\right](60.0\text{L})\,(20.0^{\circ}\text{ C}) \\ &= 1.10\text{L}. \end{aligned}$$

11.14

Discussion

This amount is significant, particularly for a 60.0-L tank. The effect is so striking because the gasoline and steel expand quickly. The rate of change in thermal properties is discussed in Heat and Heat Transfer Methods.

If you try to cap the tank tightly to prevent overflow, you will find that it leaks anyway, either around the cap or by bursting the tank. Tightly constricting the expanding gas is equivalent to compressing it, and both liquids and solids resist being compressed with extremely large forces. To avoid rupturing rigid containers, these containers have air gaps, which allow them to expand and contract without stressing them.

Thermal Stress

Thermal stress is created by thermal expansion or contraction. Thermal stress can be destructive, such as when expanding gasoline ruptures a tank. It can also be useful: for example, when two parts are joined together by heating one in manufacturing, then slipping it over the other and allowing the combination to cool. Thermal stress can explain many phenomena, such as the weathering of rocks and pavement by the expansion of ice when it freezes.

Example 11.5

Calculating Thermal Stress: Gas Pressure

What pressure would be created in the gasoline tank considered in Example 11.4, if the gasoline increases in temperature from $15.0^{\circ}\ \mathrm{C}$ to $35.0^{\circ}\ \mathrm{C}$ without being allowed to expand? Assume that the bulk modulus B for gasoline is $1.00 \times 10^9\ \mathrm{N/m^2}$.

Strategy

To solve this problem, we must use the following equation, which relates a change in volume ΔV to pressure:

$$\Delta V = \frac{1}{B}\frac{F}{A}V_0,$$

11.15

where F/A is pressure, V_0 is the original volume, and B is the bulk modulus of the material involved. We will use the amount spilled in Example 11.4 as the change in volume, ΔV. To estimate the pressure when the gas is heated at a constant volume, we can calculate the change in volume that would occur if the gas were allowed to expand at a fixed pressure, and then calculate the additional pressure needed to reduce the volume to its original level.

Solution

1. Rearrange the equation for calculating pressure:

$$P = \frac{F}{A} = \frac{\Delta V}{V_0}B.$$

11.16

2. Insert the known values. The bulk modulus for gasoline is $B = 1.00 \times 10^9\ \mathrm{N/m^2}$. In

the previous example, the change in volume $\Delta V = 1.10\ \text{L}$ is the amount that would spill. Here, $V_0 = 60.0\ \text{L}$ is the original volume of the gasoline. Substituting these values into the equation, we obtain

$$P = \frac{1.10\ \text{L}}{60.0\ \text{L}}\left(1.00 \times 10^9 \text{Pa}\right) = 1.83 \times 10^7 \text{Pa}.$$

11.17

Discussion

This pressure is about $2500 \text{lb}/\text{in}^2$, *much* more than a gasoline tank can handle.

Forces and pressures created by thermal stress are typically as great as that in the example above. Railroad tracks and roadways can buckle on hot days if they lack sufficient expansion joints. (See Figure 11.15.) Power lines sag more in the summer than in the winter, and will snap in cold weather if there is insufficient slack. Cracks open and close in plaster walls as a house warms and cools. Glass cooking pans will crack if cooled rapidly or unevenly, because of differential contraction and the stresses it creates. (Pyrex® is less susceptible because of its small coefficient of thermal expansion.) Nuclear reactor pressure vessels are threatened by overly rapid cooling, and although none have failed, several have been cooled faster than considered desirable. Biological cells are ruptured when foods are frozen, detracting from their taste. Repeated thawing and freezing accentuate the damage. Even the oceans can be affected. A significant portion of the rise in sea level that is resulting from global warming is due to the thermal expansion of sea water.



Figure 11.15 Thermal stress contributes to the formation of potholes. Image by Editor5807, Wikimedia Commons License

Metal is regularly used in the human body for hip and knee implants. Most implants need to be replaced over time because, among other things, metal does not bond with bone. Researchers are trying to find better metal coatings that would allow metal-to-bone bonding. One challenge is to find a coating that has an expansion coefficient similar to that of metal. If the expansion coefficients are too different, the thermal stresses during the manufacturing process lead to cracks at the coating-metal interface.

Another example of thermal stress is found in the mouth. Dental fillings can expand differently from tooth enamel, causing pain when eating ice cream or having a hot drink. Cracks might occur in the filling. Metal fillings (gold, silver, etc.) are being replaced by composite fillings (porcelain), which have smaller coefficients of expansion closer to those of teeth.

Check Your Understanding

Two blocks, A and B, are made of the same material. Block A has dimensions $l \times w \times h = L \times 2L \times L$ and Block B has dimensions $2L \times 2L \times 2L$. If the temperature changes, what is (a) the change in ratio of the volumes (V_B/V_A) of the two blocks, (b) the change in the ratio of the cross-sectional areas $l_B \times w_B / l_A \times w_A$, and (c) the change in the ratio of the heights (hB/hA) h_B/h_A of the two blocks?

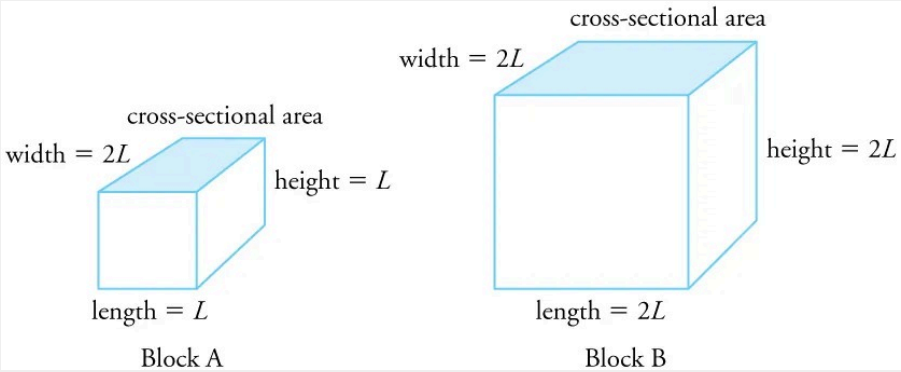


Figure 11.16 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows two three-dimensional rectangular blocks, labeled Block A and Block B. Both blocks are shown with their dimensions and cross-sectional areas marked.

Block A:

- Shape: Rectangular prism
- Length: L
- Width: 2L
- Height: L
- Cross-sectional area: Top face shown

Block B:

- Shape: Rectangular prism
- Length: 2L
- Width: 2L
- Height: 2L
- Cross-sectional area: Top face shown

Click for Solution

Solution

(a) The change in volume is proportional to the original volume. Block A has a volume of $L \times 2L \times L = (2L)^3$. Block B has a volume of $2L \times 2L \times 2L = (8L)^3$, which is 4 times that of Block A. Thus the change in volume of Block B should be 4 times the change in volume of Block A.

(b) The change in area is proportional to the area. The cross-sectional area of Block A is $L \times 2L = (2L)^2$, while that of Block B is $2L \times 2L = (4L)^2$. Because cross-sectional area of Block B is twice that of Block A, the change in the cross-sectional area of Block B is twice that of Block A.

(c) The change in height is proportional to the original height. Because the original height of Block B is twice that of A, the change in the height of Block B is twice that of Block A.

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GLOSSARY

absolute zero
the lowest possible temperature; the temperature at which all molecular motion ceases

Celsius scale
temperature scale in which the freezing point of water is $0^\circ\ \mathrm{C}$ and the boiling point of water is $100^\circ\ \mathrm{C}$

coefficient of linear expansion
α, the change in length, per unit length, per $1^\circ\ \mathrm{C}$ change in temperature; a constant used in the calculation of linear expansion; the coefficient of linear expansion depends on the material and to some degree on the temperature of the material

coefficient of volume expansion
β, the change in volume, per unit volume, per $1^\circ\ \mathrm{C}$ change in temperature

degree Celsius
unit on the Celsius temperature scale

degree Fahrenheit
unit on the Fahrenheit temperature scale

Fahrenheit scale
temperature scale in which the freezing point of water is $32^\circ\ \mathrm{F}$ and the boiling point of water is $212^\circ\ \mathrm{F}$

Kelvin scale
temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

temperature
the quantity measured by a thermometer

thermal equilibrium
the condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

thermal expansion
the change in size or volume of an object with change in temperature

thermal stress
stress caused by thermal expansion or contraction

zeroth law of thermodynamics
law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

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CHAPTER SUMMARY

11.1 Temperature

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted to temperatures on another scale using the following equations:

$$T_{^\circ\mathrm{F}} = \frac{9}{5}T_{^\circ\mathrm{C}} + 32$$

$$T_{^\circ\mathrm{C}} = \frac{5}{9}\left(T_{^\circ\mathrm{F}} - 32\right)$$

$$T_{\mathrm{K}} = T_{^\circ\mathrm{C}} + 273.15$$

$$T_{^\circ\mathrm{C}} = T_{\mathrm{K}} - 273.15$$

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

11.2 Thermal Expansion of Solids and Liquids

- Thermal expansion is the increase, or decrease, of the size (length, area, or volume) of a body due to a change in temperature.
- Thermal expansion is large for gases, and relatively small, but not negligible, for liquids and solids.
- Linear thermal expansion is
$$\Delta L = \alpha L \Delta T,$$
where ΔL is the change in length L, ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.
- The change in area due to thermal expansion is
$$\Delta A = 2\alpha A \Delta T,$$
where ΔA is the change in area.
- The change in volume due to thermal expansion is
$$\Delta V = \beta V \Delta T,$$
where β is the coefficient of volume expansion and $\beta \approx 3\alpha$. Thermal stress is created when thermal expansion is constrained.

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CONCEPTUAL QUESTIONS

Conceptual Questions

11.1 Temperature

1. What does it mean to say that two systems are in thermal equilibrium?
2. Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.
3. When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes down slightly before going up. Explain why.
4. If you add boiling water to a cup at room temperature, what would you expect the final equilibrium temperature of the unit to be? You will need to include the surroundings as part of the system. Consider the zeroth law of thermodynamics.

11.2 Thermal Expansion of Solids and Liquids

5. Thermal stresses caused by uneven cooling can easily break glass cookware. Explain why Pyrex®, a glass with a small coefficient of linear expansion, is less susceptible.
6. Water expands significantly when it freezes: a volume increase of about 9% occurs. As a result of this expansion and because of the formation and growth of crystals as water freezes, anywhere from 10% to 30% of biological cells are burst when animal or plant material is frozen. Discuss the implications of this cell damage for the prospect of preserving human bodies by freezing so that they can be thawed at some future date when it is hoped that all diseases are curable.
7. One method of getting a tight fit, say of a metal peg in a hole in a metal block, is to manufacture the peg slightly larger than the hole. The peg is then inserted when at a different temperature than the block. Should the block be hotter or colder than the peg during insertion? Explain your answer.
8. Does it really help to run hot water over a tight metal lid on a glass jar before trying to open it? Explain your answer.

9. Liquids and solids expand with increasing temperature, because the kinetic energy of a body's atoms and molecules increases. Explain why some materials shrink with increasing temperature.

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PROBLEMS & EXERCISES

Problems and Exercises

11.1 Temperature

1. What is the Fahrenheit temperature of a person with a $39.0^{\circ}\ \mathrm{C}$ fever?
2. Frost damage to most plants occurs at temperatures of $28.0^{\circ}\ \mathrm{F}$ or lower. What is this temperature on the Kelvin scale?
3. To conserve energy, room temperatures are kept at $68.0^{\circ}\ \mathrm{F}$ in the winter and $78.0^{\circ}\ \mathrm{F}$ in the summer. What are these temperatures on the Celsius scale?
4. A tungsten light bulb filament may operate at 2900 K. What is its Fahrenheit temperature? What is this on the Celsius scale?
5. The surface temperature of the Sun is about 5750 K. What is this temperature on the Fahrenheit scale?
6. One of the hottest temperatures ever recorded on the surface of Earth was $134^{\circ}\ \mathrm{F}$ in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?
7. (a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a $40.0^{\circ}\ \mathrm{F}$ decrease in temperature? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.
8. (a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value? (b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?

11.2 Thermal Expansion of Solids and Liquids

9. The height of the Washington Monument is measured to be 170 m on a day when the temperature is $35.0^{\circ}\ \mathrm{C}$. What will its height be on a day when the temperature falls to $-10.0^{\circ}\ \mathrm{C}$? Although the monument is made of limestone, assume that its thermal coefficient of expansion is the same as marble's.

10. How much taller does the Eiffel Tower become at the end of a day when the temperature has increased by $15^{\circ}\ \mathrm{C}$? Its original height is 321 m and you can assume it is made of steel.
11. What is the change in length of a 3.00-cm-long column of mercury if its temperature changes from $37.0^{\circ}\ \mathrm{C}$ to $40.0^{\circ}\ \mathrm{C}$, assuming the mercury is unconstrained?
12. How large an expansion gap should be left between steel railroad rails if they may reach a maximum temperature $35.0^{\circ}\ \mathrm{C}$ greater than when they were laid? Their original length is 10.0 m.
13. You are looking to purchase a small piece of land in Hong Kong. The price is "only" $60,000 per square meter! The land title says the dimensions are $20\ \mathrm{m} \times 30\ \mathrm{m}$. By how much would the total price change if you measured the parcel with a steel tape measure on a day when the temperature was $20^{\circ}\ \mathrm{C}$ above normal?
14. Global warming will produce rising sea levels partly due to melting ice caps but also due to the expansion of water as average ocean temperatures rise. To get some idea of the size of this effect, calculate the change in length of a column of water 1.00 km high for a temperature increase of $1.00^{\circ}\ \mathrm{C}$. Note that this calculation is only approximate because ocean warming is not uniform with depth.
15. Show that 60.0 L of gasoline originally at $15.0^{\circ}\ \mathrm{C}$ will expand to 61.1 L when it warms to $35.0^{\circ}\ \mathrm{C}$, as claimed in Example 11.4.
16. (a) Suppose a meter stick made of steel and one made of invar (an alloy of iron and nickel) are the same length at $0^{\circ}\ \mathrm{C}$. What is their difference in length at $22.0^{\circ}\ \mathrm{C}$? (b) Repeat the calculation for two 30.0-m-long surveyor's tapes.
17. (a) If a 500-mL glass beaker is filled to the brim with ethyl alcohol at a temperature of $5.00^{\circ}\ \mathrm{C}$, how much will overflow when its temperature reaches $22.0^{\circ}\ \mathrm{C}$? (b) How much less water would overflow under the same conditions?
18. Most automobiles have a coolant reservoir to catch radiator fluid that may overflow when the engine is hot. A radiator is made of copper and is filled to its 16.0-L capacity when at $10.0^{\circ}\ \mathrm{C}$. What volume of radiator fluid will overflow when the radiator and fluid reach their $95.0^{\circ}\ \mathrm{C}$ operating temperature, given that the fluid's volume coefficient of expansion is $\beta = 400 \times 10^{-6}/^{\circ}\ \mathrm{C}$? Note that this coefficient is approximate, because most car radiators have operating

temperatures of greater than $95.0^{\circ}\ \mathrm{C}$.

19. A physicist makes a cup of instant coffee and notices that, as the coffee cools, its level drops 3.00 mm in the glass cup. Show that this decrease cannot be due to thermal contraction by calculating the decrease in level if the $350\ \mathrm{cm}^3$ of coffee is in a 7.00-cm-diameter cup and decreases in temperature from $95.0^{\circ}\ \mathrm{C}$to $45.0^{\circ}\ \mathrm{C}$. (Most of the drop in level is actually due to escaping bubbles of air.)
20. (a) The density of water at $0^{\circ}\ \mathrm{C}$ is very nearly $1000\ \mathrm{kg/m}^3$ (it is actually $999.84\ \mathrm{kg/m}^3$), whereas the density of ice at $0^{\circ}\ \mathrm{C}$ is $917\ \mathrm{kg/m}^3$. Calculate the pressure necessary to keep ice from expanding when it freezes, neglecting the effect such a large pressure would have on the freezing temperature. (This problem gives you only an indication of how large the forces associated with freezing water might be.) (b) What are the implications of this result for biological cells that are frozen?
21. Show that $\beta \approx 3\alpha$, by calculating the change in volume ΔV of a cube with sides of length L.

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HEAT AND HEAT TRANSFER METHODS



Figure 12.1 Eunice Newton Foote was the first to determine the relationship between carbon dioxide, water vapor, and the potential for global heating. She designed and conducted a number of experiments to uncover the ability of different gases to trap heat, describing what would later be referred to as greenhouse gases. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

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Energy can exist in many forms and heat is one of the most intriguing. Heat is often hidden, as it only exists when in transit, and is transferred by a number of distinctly different methods. Heat transfer touches every aspect of our lives and helps us understand how the universe functions. It explains the chill we feel on a clear breezy night, or why Earth's core has yet to cool. This chapter defines and explores heat transfer, its effects, and the methods by which heat is transferred. These topics are fundamental, as well as practical, and will often be referred to in the chapters ahead.

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12.1 HEAT

Learning Objectives

By the end of this section, you will be able to:

- Define heat as transfer of energy.

In Work, Energy, and Energy Resources, we defined work as force times distance and learned that work done on an object changes its kinetic energy. We also saw in Temperature and Thermal Expansion that temperature is proportional to the (average) kinetic energy of atoms and molecules. We say that a thermal system has a certain internal energy: its internal energy is higher if the temperature is higher. If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter to the colder object until equilibrium is reached and the bodies reach thermal equilibrium (i.e., they are at the same temperature). No work is done by either object, because no force acts through a distance. The transfer of energy is caused by the temperature difference, and ceases once the temperatures are equal. These observations lead to the following definition of heat: Heat is the spontaneous transfer of energy due to a temperature difference.

As noted in Temperature and Thermal Expansion, heat is often confused with temperature. For example, we may say the heat was unbearable, when we actually mean that the temperature was high. Heat is a form of energy, whereas temperature is not. The misconception arises because we are sensitive to the flow of heat, rather than the temperature.

Owing to the fact that heat is a form of energy, it has the SI unit of *joule* (J). The *calorie* (cal) is a common unit of energy, defined as the energy needed to change the temperature of 1.00 g of water by $1.00^{\circ}\ \mathrm{C}$—specifically, between $14.5^{\circ}\ C$ and $15.5^{\circ}\ C$, since there is a slight temperature dependence. Perhaps the most common unit of heat is the kilocalorie (kcal), which is the energy needed to change the temperature of 1.00 kg of water by $1.00^{\circ}\ \mathrm{C}$. Since mass is most often specified in kilograms, kilocalorie is commonly used.

Food calories (given the notation Cal, and sometimes called "big calorie") are actually kilocalories ($1\text{kilocalorie} = 1000\text{ calories}$), a fact not easily determined from package labeling.

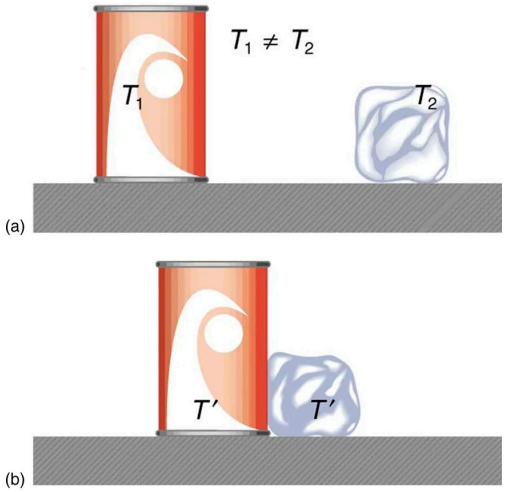


Figure 12.2 In figure (a) the soft drink and the ice have different temperatures, T_1 and T_2, and are not in thermal equilibrium. In figure (b), when the soft drink and ice are allowed to interact, energy is transferred until they reach the same temperature T', achieving equilibrium. Heat transfer occurs due to the difference in temperatures. In fact, since the soft drink and ice are both in contact with the surrounding air and bench, the equilibrium temperature will be the same for both. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is divided into two parts, labeled (a) and (b), illustrating a thermodynamic concept involving temperature.

Section (a): The illustration shows a red cylindrical container on a flat surface with a stylized design and labeled with T_1. To the right of the container, there is an irregularly shaped ice block labeled T_2. Above the ice block, the text $T_1 \neq T_2$ is present, indicating that the temperatures are different.

Section (b): This part shows a similar red cylindrical container with a design and labeled with T'. It is directly adjacent to an irregularly shaped blue ice block labeled T', both in contact on the flat surface. The illustration suggests thermal equilibrium as they share the same label.

Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work. Work can transfer energy into or out of a system. This realization helped establish the fact that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the mechanical equivalent of heat—*the work needed to*

produce the same effects as heat transfer. In terms of the units used for these two terms, the best modern value for this equivalence is

$$1.000\text{kcal} = 4186\text{J}.$$
12.1

We consider this equation as the conversion between two different units of energy, defining the 15 °C calorie. A slightly different but frequently seen conversion, 4184 J/kcal, uses the properties of water at 1 °C instead. This is known as the thermochemical calorie.

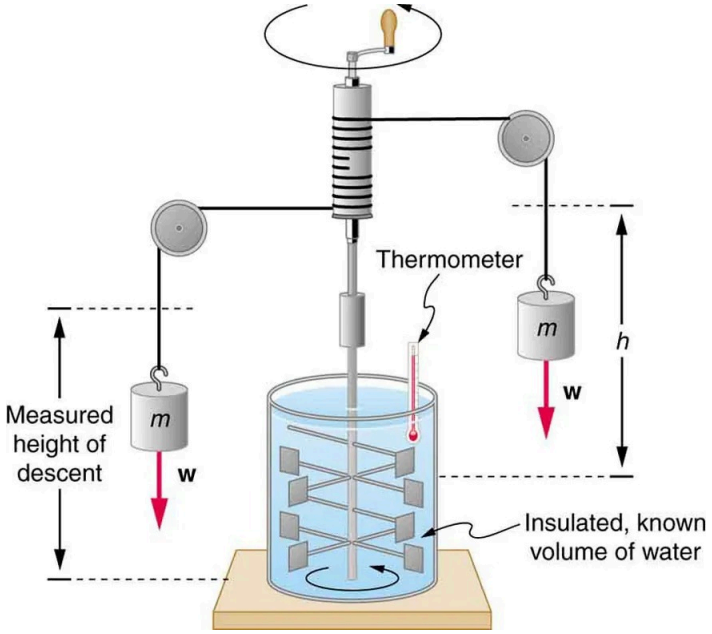


Figure 12.3 Schematic depiction of Joule's experiment that established the equivalence of heat and work. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a schematic diagram of a mechanical apparatus used to demonstrate the conversion of mechanical energy to thermal energy, often known as Joule's experiment. The apparatus includes:

– A hand crank at the top used to rotate a mechanism.
– A vertical gear system with horizontal and vertical components extending from the hand crank.
– Two weights, labeled "m" for mass, hanging from each side of the apparatus by ropes. These are connected to pulleys. The weights represent forces acting downwards with arrows marked "w."
– The left weight shows a measured height of descent, indicating how far it falls.
– In the center, a cylindrical container is filled with an insulated, known volume of water.

– Inside the water container, a series of paddles are connected to a vertical rod attached to the gear system.
– A thermometer is also submerged in the water to measure temperature changes during the process.

The function of the apparatus is to convert the potential energy of the descending weights into kinetic energy, which in turn stirs the water to increase its temperature.

The figure above shows one of Joule's most famous experimental setups for demonstrating the mechanical equivalent of heat. It demonstrated that work and heat can produce the same effects, and helped establish the principle of conservation of energy. Gravitational potential energy (PE) (work done by the gravitational force) is converted into kinetic energy (KE), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. His contributions to the field of thermodynamics were so significant that the SI unit of energy was named after him.

Heat added or removed from a system changes its internal energy and thus its temperature. Such a temperature increase is observed while cooking. However, adding heat does not necessarily increase the temperature. An example is melting of ice; that is, when a substance changes from one phase to another. Work done on the system or by the system can also change the internal energy of the system. Joule demonstrated that the temperature of a system can be increased by stirring. If an ice cube is rubbed against a rough surface, work is done by the frictional force. A system has a well-defined internal energy, but we cannot say that it has a certain "heat content" or "work content". We use the phrase "heat transfer" to emphasize its nature.

Check Your Understanding

Two samples (A and B) of the same substance are kept in a lab. Someone adds 10 kilojoules (kJ) of heat to one sample, while 10 kJ of work is done on the other sample. How can you tell to which sample the heat was added?

Click for Solution

Solution

Heat and work both change the internal energy of the substance. However, the properties of the sample only depend on the internal energy so that it is impossible to tell whether heat was added to sample A or B.

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12.2 TEMPERATURE CHANGE AND HEAT CAPACITY

Learning Objectives

By the end of this section, you will be able to:

- Observe heat transfer and change in temperature and mass.
- Calculate final temperature after heat transfer between two objects.

One of the major effects of heat transfer is temperature change: heating increases the temperature while cooling decreases it. We assume that there is no phase change and that no work is done on or by the system. Experiments show that the transferred heat depends on three factors—the change in temperature, the mass of the system, and the substance and phase of the substance.

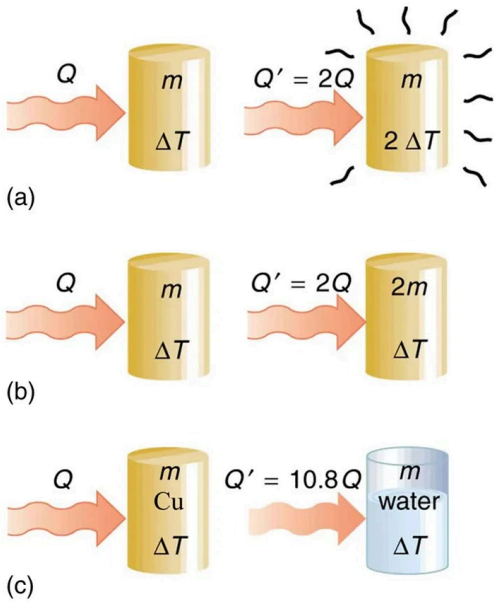


Figure 12.4 The heat Q transferred to cause a temperature change depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved. (a) The amount of heat transferred is directly proportional to the temperature change. To double the temperature change of a mass m, you need to add twice the heat. (b) The amount of heat transferred is also directly proportional to the mass. To cause an equivalent temperature change in a doubled mass, you need to add twice the heat. (c) The amount of heat transferred depends on the substance and its phase. If it takes an amount Q of heat to cause a temperature change ΔT in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams labeled (a), (b), and (c), each illustrating a different thermal energy transfer scenario involving a cylindrical object. Here's the description of each:

(a) The diagram shows a cylinder with mass "m" and temperature change "ΔT". To the left, an arrow labeled "Q" points toward the cylinder, indicating heat input. To the right, an arrow labeled "Q' = 2Q" points away from the cylinder, leading to an identical cylinder but with temperature change "2ΔT" and emitting heat shown with wavy lines.

(b) This diagram features the same initial cylinder with mass "m" and temperature change "ΔT", with an arrow labeled "Q" pointing toward it. An arrow labeled "Q' = 2Q" points to a second cylinder with double the mass "2m" but the same temperature change "ΔT".

(c) The initial cylinder with specific mass "m" and temperature change "ΔT" is made of copper, indicated by "Cu". An arrow labeled "Q" points toward it. An arrow labeled "Q' = 10.8Q" points to a cylindrical container filled with water, having the same mass "m" and temperature change "ΔT".

The dependence on temperature change and mass are easily understood. Owing to the fact that the (average) kinetic energy of an atom or molecule is proportional to the absolute temperature, the internal energy of a system is proportional to the absolute temperature and the number of atoms or molecules. Owing to the fact that the transferred heat is equal to the change in the internal energy, the heat is proportional to the mass of the substance and the temperature change. The transferred heat also depends on the substance so that, for example, the heat necessary to raise the temperature is less for alcohol than for water. For the same substance, the transferred heat also depends on the phase (gas, liquid, or solid).

Heat Transfer and Temperature Change

The quantitative relationship between heat transfer and temperature change contains all three factors:

$$Q = \mathrm{mc}\Delta T,$$

12.2

where Q is the symbol for heat transfer, m is the mass of the substance, and ΔT is the change in temperature. The symbol c stands for specific heat and depends on the material and phase. The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by $1.00^{\circ}\ \mathrm{C}$. The specific heat c is a property of the substance; its SI unit is $\mathrm{J/\left(kg \cdot K\right)}$ or $\mathrm{J/\left(kg \cdot {}^{\circ}\ C\right)}$. Recall that the temperature change (ΔT) is the same in units of kelvin and degrees Celsius. If heat transfer is measured in kilocalories, then *the unit of specific heat* is $\mathrm{kcal/\left(kg \cdot {}^{\circ}\ C\right)}$.

Values of specific heat must generally be looked up in tables, because there is no simple way to calculate them. In general, the specific heat also depends on the temperature. Table 12.1 lists representative values of specific heat for various substances. Except for gases, the temperature and volume dependence of the specific heat of most substances is weak. We see from this table that the specific heat of water is five times that of glass and ten

times that of iron, which means that it takes five times as much heat to raise the temperature of water the same amount as for glass and ten times as much heat to raise the temperature of water as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

Example 12.1

Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from $20.0^{\circ}\ \mathrm{C}$ to $80.0^{\circ}\ \mathrm{C}$. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

Strategy

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum. The specific heat values for water and aluminum are given in Table 12.1.

Solution

Because water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference:

$$\Delta T = T_{\mathrm{f}} - T_{\mathrm{i}} = 60.0^{\circ}\ \mathrm{C}. \tag{12.3}$$

2. Calculate the mass of water. Because the density of water is $1000\mathrm{kg}/\mathrm{m}^3$, one liter of water has a mass of 1 kg, and the mass of 0.250 liters of water is $m_w = 0.250\mathrm{kg}$.
3. Calculate the heat transferred to the water. Use the specific heat of water in Table 12.1:

$$\begin{aligned} Q_w &= m_w c_w \Delta T \\ &= (0.250\text{kg})\,(4186\text{J}/(\text{kg}\cdot^{\text{o}}\text{C}))\,(60.0^{\text{o}}\text{C}) \\ &= 62.8kJ. \end{aligned}$$

12.4

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in Table 12.1:

$$\begin{aligned} Q_{\text{Al}} &= m_{\text{Al}} c_{\text{Al}} \Delta T \\ &= (0.500\text{ kg})\,(900\text{ J}/(\text{kg}\cdot^{\text{o}}\text{C}))\,(60.0^{\text{o}}\text{C}) \\ &= 27.0 \times 10^4\text{J} \\ &= 27.0\text{ kJ}. \end{aligned}$$

12.5

5. Compare the percentage of heat going into the pan versus that going into the water. First, find the total transferred heat:

$$\begin{aligned} Q_{\text{Total}} &= Q_{\text{W}} + Q_{\text{Al}} \\ &= 62.8\text{kJ} + 27.0\text{kJ} \\ &= 89.8\text{kJ}. \end{aligned}$$

12.6

Thus, the amount of heat going into heating the pan is

$$\frac{27.0\text{kJ}}{89.8\text{kJ}} \times 100\% = 30.1\%,$$

12.7

and the amount going into heating the water is

$$\frac{62.8\text{kJ}}{89.8\text{kJ}} \times 100\% = 69.9\%.$$

12.8

Discussion

In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific

heat of water is over four times greater than that of aluminum. Therefore, it takes a bit more than twice the heat to achieve the given temperature change for the water as compared to the aluminum pan.



Figure 12.5 The smoking brakes on this truck are a visible evidence of the mechanical equivalent of heat. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts an illustrated truck traveling uphill on a steep slope. The truck is orange with a striped cargo container on its back, slanted to match the incline of the hill. Smoke is coming out from the exhaust pipe. On both sides of the hill, there are green trees with brown trunks. The background features blue sky and white mountain outlines, creating a scenic and adventurous atmosphere.

Example 12.2

Calculating the Temperature Increase from the Work Done on a Substance: Truck Brakes Overheat on Downhill Runs

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. The problem is that the mass of the truck is large compared with that of the brake material absorbing the energy, and the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of $800\ \mathrm{J/(kg \cdot ^\circ C)}$ if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

Strategy

If the brakes are not applied, gravitational potential energy is converted into kinetic energy. When brakes are applied, gravitational potential energy is converted into internal energy of the brake material. We first calculate the gravitational potential energy (Mgh) that the entire truck loses in its descent and then find the temperature increase produced in the brake material alone.

Solution

1. Calculate the change in gravitational potential energy as the truck goes downhill

$$\begin{aligned}\mathrm{Mgh} &= (10,000\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(75.0\ \mathrm{m}) \\ &= 7.35 \times 10^6 \mathrm{J}.\end{aligned}$$

12.9

2. Calculate the temperature from the heat transferred using $Q = 0.1\text{Mgh}$, since only 10% of the energy calculated in step 1 is retained as heat, and

$$\Delta T = \frac{Q}{\text{mc}},$$

12.10

where m is the mass of the brake material. Insert the values $m = 100\ \text{kg}$ and $c = 800\ \text{J}/(\text{kg} \cdot {}^{\text{o}}\ \text{C})$ to find

$$\Delta T = \frac{\left(7.35 \times 10^{6}\text{J}\right)}{(100\ \text{kg})\ (800\ \text{J}/(\text{kg} \cdot {}^{\text{o}}\ \text{C}))} = 9.2^{\text{o}}\ \text{C}.$$

12.11

Discussion

This same idea underlies the recent hybrid technology of cars, where mechanical energy (gravitational potential energy) is converted by the brakes into electrical energy (battery).

Table 12.1 Specific Heats[1] of Various Substances

Substances	**Specific heat (*c*)**	
Solids	J/(kg·ºC)	kcal/(kg°C)[2]
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, -50°C to 0°C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.4
Liquids		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
Gases[3]	$C_v\ (C_p)$	$C_v\ (C_p)$
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)
Oxygen	651 (913)	0.156 (0.218)
Steam (100°C)	1520 (2020)	0.363 (0.482)

Note that Example 12.2 is an illustration of the mechanical equivalent of heat. Alternatively, the temperature increase could be produced by a blow torch instead of mechanically.

1. The values for solids and liquids are at constant volume and at $25^{\circ}\ \mathrm{C}$, except as noted.
2. These values are identical in units of $\mathrm{cal/g}\cdot^{\circ}\ C$.
3. c_{v} at constant volume and at $20.0^{\circ}\ C$, except as noted, and at 1.00 atm average pressure. Values in parentheses are c_{p} at a constant pressure of 1.00 atm.

Example 12.3

Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan

Suppose you pour 0.250 kg of $20.0^{\circ}\ \mathrm{C}$ water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of $150^{\circ}\ \mathrm{C}$. Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium a short time later?

Strategy

The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings. Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water. Heat transfer then restores thermal equilibrium once the water and pan are in contact. Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water. The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved. The heat exchange can be written as $\mid Q_{\mathrm{hot}} \mid = Q_{\mathrm{cold}}$.

Solution

1. Use the equation for heat transfer $Q = \mathrm{mc}\Delta T$ to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:

$$Q_{\mathrm{hot}} = m_{\mathrm{Al}} c_{\mathrm{Al}} \left(T_{\mathrm{f}} - 150^{\circ}\ \mathrm{C}\right).$$

12.12

2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:

$$Q_{\text{cold}} = m_W c_W \left(T_{\text{f}} - 20.0^{\text{o}}\ \text{C}\right).$$

12.13

3. Note that $Q_{\text{hot}}0$ and $Q_{\text{cold}} > 0$ and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:

$$\begin{aligned} Q_{\text{cold}} + Q_{\text{hot}} &= 0, \\ Q_{\text{cold}} &= -\text{Q}_{\text{hot}}, \\ m_W c_W \left(T_{\text{f}} - 20.0^{\text{o}}\ \text{C}\right) &= -m_{Al} c_{Al} \left(T_{\text{f}} - 150^{\text{o}}\ \text{C}.\right) \end{aligned}$$

12.14

4. Bring all terms involving T_{f} on the left hand side and all other terms on the right hand side. Solve for T_{f},

$$T_{\text{f}} = \frac{m_{\text{Al}} c_{\text{Al}} \left(150^{\text{o}}\ \text{C}\right) + m_W c_W \left(20.0^{\text{o}}\ \text{C}\right)}{m_{\text{Al}} c_{\text{Al}} + m_W c_W},$$

12.15

and insert the numerical values:

$$\begin{aligned} T_{\text{f}} &= \frac{(0.500\ \text{kg})\left(900\ \text{J/(kg}\cdot^{\text{o}}\ \text{C})\right)(150^{\text{o}}\ \text{C}) + (0.250\ \text{kg})\left(4186\ \text{J/(kg}\cdot^{\text{o}}\ \text{C})\right)(20.0^{\text{o}}\ \text{C})}{(0.500\ \text{kg})\left(900\ \text{J/(kg}\cdot^{\text{o}}\ \text{C})\right) + (0.250\ \text{kg})\left(4186\ \text{J/(kg}\cdot^{\text{o}}\ \text{C})\right)} \\ &= \frac{88430\ \text{J}}{1496.5\ \text{J/}^{\text{o}}\ \text{C}} \\ &= 59.1^{\text{o}}\ \text{C}. \end{aligned}$$

12.16

Discussion

This is a typical *calorimetry* problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached. Why is the final temperature so much closer to $20.0^{\text{o}}\ \text{C}$ than $150^{\text{o}}\ \text{C}$? The reason is that water has a greater specific heat than most common substances and thus

undergoes a small temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

Take-Home Experiment: Temperature Change of Land and Water

What heats faster, land or water?

To study differences in heat capacity:

- Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can achieve approximately equal masses by using 50% more water by volume.)
- Heat both (using an oven or a heat lamp) for the same amount of time.
- Record the final temperature of the two masses.
- Now bring both jars to the same temperature by heating for a longer period of time.
- Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

Which sample cools off the fastest? This activity replicates the phenomena responsible for land breezes and sea breezes.

Check Your Understanding

If 25 kJ is necessary to raise the temperature of a block from $25^{\circ}\ \mathrm{C}$ to $30^{\circ}\ \mathrm{C}$, how much heat is necessary to heat the block from $45^{\circ}\ \mathrm{C}$ to $50^{\circ}\ \mathrm{C}$?

Click for Solution

Solution

The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

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12.3 PHASE CHANGE AND LATENT HEAT

Learning Objectives

By the end of this section, you will be able to:

- Examine heat transfer.
- Calculate final temperature from heat transfer.

So far we have discussed temperature change due to heat transfer. No temperature change occurs from heat transfer if ice melts and becomes liquid water (i.e., during a phase change). For example, consider water dripping from icicles melting on a roof warmed by the Sun. Conversely, water freezes in an ice tray cooled by lower-temperature surroundings.



Figure 12.6 Heat from the air transfers to the ice causing it to melt. Image from OpenStax College Physics 2e, CC-BY 4.0

Energy is required to melt a solid because the cohesive bonds between the molecules in the solid must be

broken apart such that, in the liquid, the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Similarly, energy is needed to vaporize a liquid, because molecules in a liquid interact with each other via attractive forces. There is no temperature change until a phase change is complete. The temperature of a cup of soda initially at $0^{\circ}\ \mathrm{C}$ stays at $0^{\circ}\ \mathrm{C}$ until all the ice has melted. Conversely, energy is released during freezing and condensation, usually in the form of thermal energy. Work is done by cohesive forces when molecules are brought together. The corresponding energy must be given off (dissipated) to allow them to stay together Figure 12.7.

The energy involved in a phase change depends on two major factors: the number and strength of bonds or force pairs. The number of bonds is proportional to the number of molecules and thus to the mass of the sample. The strength of forces depends on the type of molecules. The heat Q required to change the phase of a sample of mass m is given by

$$Q = mL_{\mathrm{f}}\ (\mathrm{melting/freezing}),$$
12.17

$$Q = \mathrm{mL_v}\ (\mathrm{vaporization/condensation}),$$
12.18

where the latent heat of fusion, L_{f}, and latent heat of vaporization, L_{v}, are material constants that are determined experimentally. See (Table 12.2).

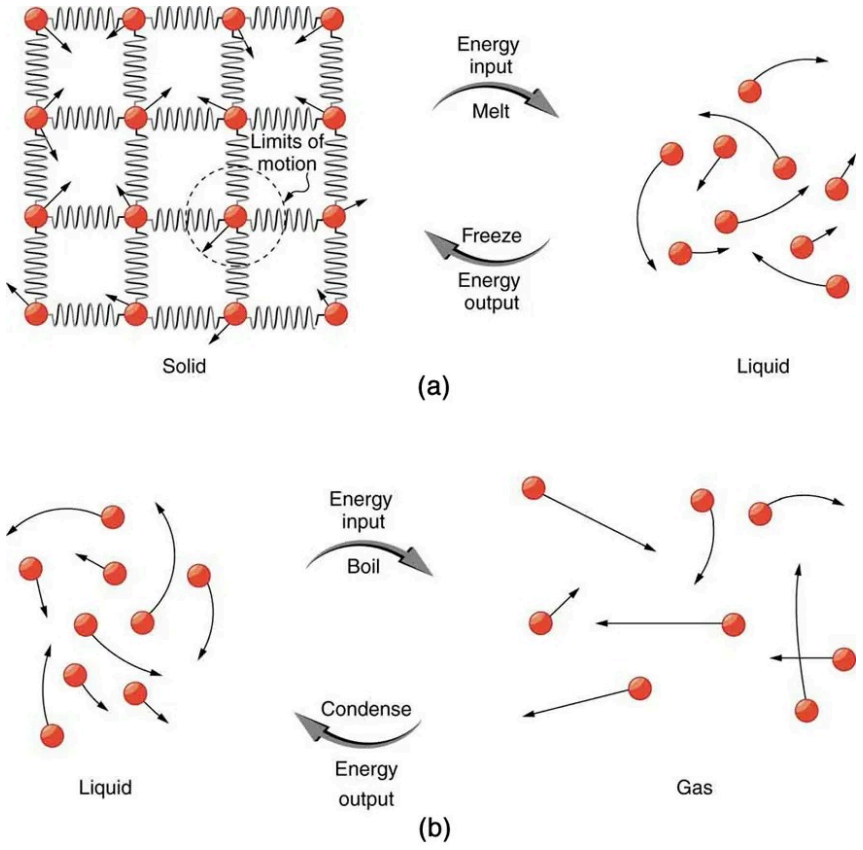


Figure 12.7 (a) Energy is required to partially overcome the attractive forces between molecules in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Molecules are separated by large distances when going from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is complete. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image contains two diagrams labeled (a) and (b) illustrating the changes in states of matter.

(a) On the left side of the diagram, there is a depiction of a solid, consisting of red spheres arranged in a rigid, organized grid structure connected by springs, indicating limited motion. Arrows suggest slight potential for movement. On the right side, it shows a liquid state with red spheres more freely dispersed, arrows indicating more significant motion between them. In the center, arrows labeled “Energy input” and “Melt” show the transition from solid to liquid, while arrows labeled “Freeze” and “Energy output” show the transition from liquid back to solid.

(b) The bottom diagram is of a liquid on the left side, with red spheres depicted as more free-form in movement, shown by multiple direction arrows. On the right side, it illustrates a gas state, where the red

spheres are even more spaced out with longer arrows indicating rapid and random motion. In the center, arrows labeled “Energy input” and “Boil” show the transition from liquid to gas, while arrows labeled “Condense” and “Energy output” indicate the transition from gas back to liquid.

Latent heat is measured in units of J/kg. Both L_{f} and L_{v} depend on the substance, particularly on the strength of its molecular forces as noted earlier. L_{f} and L_{v} are collectively called latent heat coefficients. They are *latent*, or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system; so, in effect, the energy is hidden. Table 12.2 lists representative values of L_{f} and L_{v}, together with melting and boiling points.

The table shows that significant amounts of energy are involved in phase changes. Let us look, for example, at how much energy is needed to melt a kilogram of ice at $0^{\text{o}}\ \text{C}$ to produce a kilogram of water at $0\,^{\circ}\text{C}$. Using the equation for a change in temperature and the value for water from Table 12.2, we find that $Q = \text{mL}_{\text{f}} = (1.0\text{kg})\,(334\text{kJ/kg}) = 334\text{kJ}$ is the energy to melt a kilogram of ice. This is a lot of energy as it represents the same amount of energy needed to raise the temperature of 1 kg of liquid water from $0^{\text{o}}\ \text{C}$ to $79.8^{\text{o}}\ \text{C}$. Even more energy is required to vaporize water; it would take 2256 kJ to change 1 kg of liquid water at the normal boiling point ($100^{\text{o}}\ \text{C}$ at atmospheric pressure) to steam (water vapor). This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes without a phase change.

Table 12.2 Heats of Fusion and Vaporization[1]

		L_f			L_v	
Substance	**Melting point (ºC)**	**kJ/kg**	**kcal/kg**	**Boiling point (°C)**	**kJ/kg**	**kcal/kg**
Helium	−269.7	5.23	1.25	−268.9	20.9	4.99
Hydrogen	−259.3	58.6	14.0	−252.9	452	108
Nitrogen	−210.0	25.5	6.09	−195.8	201	48.0
Oxygen	−218.8	13.8	3.30	−183.0	213	50.9
Ethanol	−114	104	24.9	78.3	854	204
Ammonia	−75		108	−33.4	1370	327
Mercury	−38.9	11.8	2.82	357	272	65.0
Water	0.00	334	79.8	100.0	2256[2]	540[3]
Water				37		580
Sulfur	119	38.1	9.10	444.6	326	77.9
Lead	327	24.5	5.85	1750	871	208
Antimony	631	165	39.4	1440	561	134
Aluminum	660	380	90	2450	11400	2720
Silver	961	88.3	21.1	2193	2336	558
Gold	1063	64.5	15.4	2660	1578	377
Copper	1083	134	32.0	2595	5069	1211
Uranium	1133	84	20	3900	1900	454
Tungsten	3410	184	44	5900	4810	1150

Phase changes can have a tremendous stabilizing effect even on temperatures that are not near the melting and boiling points, because evaporation and condensation (conversion of a gas into a liquid state) occur even at temperatures below the boiling point. Take, for example, the fact that air temperatures in humid climates rarely go above $35.0^{\circ}\ \mathrm{C}$, which is because most heat transfer goes into evaporating water into the air. Similarly, temperatures in humid weather rarely fall below the dew point because enormous heat is released

1. Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm), except where noted otherwise.
2. At $37.0^{\circ}\ \mathrm{C}$ (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg
3. At $37.0^{\circ}\ \mathrm{C}$ (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg

when water vapor condenses.

We examine the effects of phase change more precisely by considering adding heat into a sample of ice at $-20^{\circ}\ \mathrm{C}$ (Figure 12.8). The temperature of the ice rises linearly, absorbing heat at a constant rate of $0.50\ \mathrm{cal/g \cdot ^{\circ}\ C}$ until it reaches $0^{\circ}\ \mathrm{C}$. Once at this temperature, the ice begins to melt until all the ice has melted, absorbing 79.8 cal/g of heat. The temperature remains constant at $0^{\circ}\ \mathrm{C}$ during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of $1.00\ \mathrm{cal/g \cdot ^{\circ}\ C}$. At $100^{\circ}\ \mathrm{C}$, the water begins to boil and the temperature again remains constant while the water absorbs 539 cal/g of heat during this phase change. When all the liquid has become steam vapor, the temperature rises again, absorbing heat at a rate of $0.482\ \mathrm{cal/g \cdot ^{\circ}\ C}$.

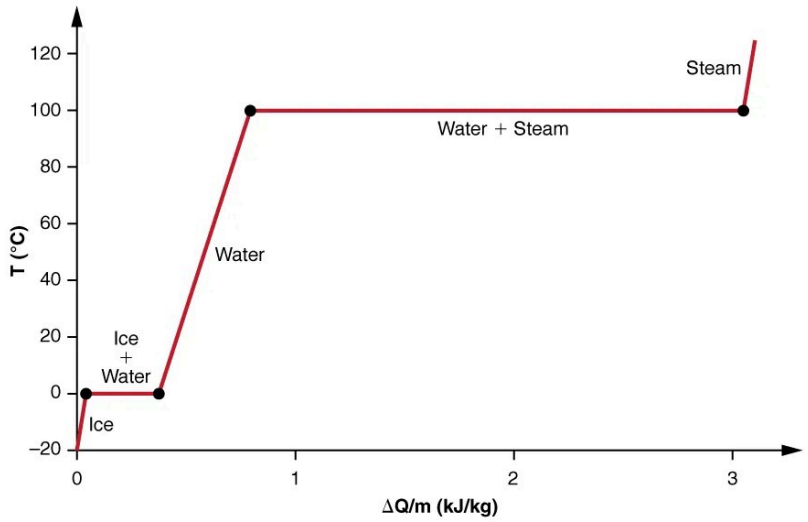


Figure 12.8 A graph of temperature versus energy added. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in of the system. The long stretches of constant temperature values at $0^{\circ}\ \mathrm{C}$ *and* $100^{\circ}\ \mathrm{C}$ *reflect the large latent heat of melting and vaporization, respectively. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a graph showing the phase changes of water with respect to energy input. The x-axis is labeled as ΔQ/m (kJ/kg), representing the energy per unit mass, and the y-axis is labeled as T (°C), representing temperature in degrees Celsius. The graph has several distinct regions:

- **Ice:** The temperature rises from -20°C to 0°C as energy is added, represented by an upward sloping line.
- **Ice + Water:** At 0°C, the line becomes horizontal, indicating a phase change where ice is turning into water without a change in temperature.
- **Water:** The temperature increases from 0°C to 100°C with more energy, shown as another upward

sloping line.

- **Water + Steam:** At 100°C, the line is horizontal again, representing the phase change from water to steam.
- **Steam:** Finally, the temperature rises beyond 100°C as steam is heated further, indicated by a short upward sloping section.

Water can evaporate at temperatures below the boiling point. More energy is required than at the boiling point, because the kinetic energy of water molecules at temperatures below $100^{\circ}\ \mathrm{C}$ is less than that at $100^{\circ}\ \mathrm{C}$, hence less energy is available from random thermal motions. Take, for example, the fact that, at body temperature, perspiration from the skin requires a heat input of 2428 kJ/kg, which is about 10 percent higher than the latent heat of vaporization at $100^{\circ}\ \mathrm{C}$. This heat comes from the skin, and thus provides an effective cooling mechanism in hot weather. High humidity inhibits evaporation, so that body temperature might rise, leaving unevaporated sweat on your brow.

Example 12.4

Calculate Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Three ice cubes are used to chill a soda at $20^{\circ}\ \mathrm{C}$ with mass $m_{soda} = 0.25\ \mathrm{kg}$. The ice is at $0^{\circ}\ \mathrm{C}$ and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored. Assume the soda has the same heat capacity as water. Find the final temperature when all ice has melted.

Strategy

The ice cubes are at the melting temperature of $0^{\circ}\ \mathrm{C}$. Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first the phase change occurs and solid (ice) transforms into liquid water at the melting temperature, then the temperature of this water rises. Melting yields water at $0^{\circ}\ \mathrm{C}$, so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium,

$$Q_{\text{ice}} = -Q_{\text{soda}}.$$

12.19

The heat transferred to the ice is $Q_{\text{ice}} = m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{W}}\left(T_{\text{f}} - 0^{\circ}\ \mathrm{C}\right)$. The heat given off by the soda is $Q_{\text{soda}} = m_{\text{soda}}c_{\text{W}}\left(T_{\text{f}} - 20^{\circ}\ \mathrm{C}\right)$. Since no heat is lost, $Q_{\text{ice}} = -Q_{\text{soda}}$, so that

$$m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{W}\left(T_{\text{f}} - 0^{\circ}\ \mathrm{C}\right) = -m_{\text{soda}}c_{W}\left(T_{\text{f}} - 20^{\circ}\ \mathrm{C}\right).$$

12.20

Bring all terms involving T_{f} on the left-hand-side and all other terms on the right-hand-side. Solve for the unknown quantity T_{f}:

$$T_{\mathrm{f}} = \frac{m_{\mathrm{soda}} c_W \left(20^{\circ}\ \mathrm{C}\right) - m_{\mathrm{ice}} L_{\mathrm{f}}}{\left(m_{soda} + m_{\mathrm{ice}}\right) c_W}.$$

12.21

Solution

1. Identify the known quantities. The mass of ice is $m_{\mathrm{ice}} = 3 \times 6.0\mathrm{g} = 0.018\ \mathrm{kg}$ and the mass of soda is $m_{\mathrm{soda}} = 0.25\ \mathrm{kg}$.
2. Calculate the terms in the numerator:

   $$\begin{aligned} m_{\mathrm{soda}} c_W \left(20^{\circ}\ \mathrm{C}\right) &= \left(0.25\ \mathrm{kg}\right)\left(4186\ \mathrm{J/(kg \cdot ^{\circ}\ C)}\right)\left(20^{\circ}\ \mathrm{C}\right) \\ &= 20,930\ \mathrm{J} \end{aligned}$$

   12.22

   and

   $$m_{\mathrm{ice}} L_{\mathrm{f}} = \left(0.018\ \mathrm{kg}\right)\left(334,000\ \mathrm{J/kg}\right) = 6012\ \mathrm{J}.$$

   12.23
3. Calculate the denominator:

   $$\begin{aligned} \left(m_{\mathrm{soda}} + m_{\mathrm{ice}}\right) c_{\mathrm{W}} &= \left(0.25\ \mathrm{kg}\ + 0.018\ \mathrm{kg}\right)\left(4186\ \mathrm{J/(kg \cdot ^{\circ}\ C)}\right) \\ &= 1122\ \mathrm{J/^{\circ}\ C}. \end{aligned}$$

   12.24
4. Calculate the final temperature:

   $$T_{\mathrm{f}} = \frac{20,930\ \mathrm{J} - 6012\ \mathrm{J}}{1122\ \mathrm{J/^{\circ}\ C}} = 13^{\circ}\ \mathrm{C}.$$

   12.25

Discussion

This example illustrates the enormous energies involved during a phase change. The mass of ice is about 7 percent the mass of water but leads to a noticeable change in the temperature of soda. Although we assumed that the ice was at the freezing temperature, this is incorrect: the typical temperature is $-6^{\circ}\ \mathrm{C}$. However, this correction

gives a final temperature that is essentially identical to the result we found. Can you explain why?

We have seen that vaporization requires heat transfer to a liquid from the surroundings, so that energy is released by the surroundings. Condensation is the reverse process, increasing the temperature of the surroundings. This increase may seem surprising, since we associate condensation with cold objects—the glass in the figure, for example. However, energy must be removed from the condensing molecules to make a vapor condense. The energy is exactly the same as that required to make the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = \mathrm{mL_v}$.

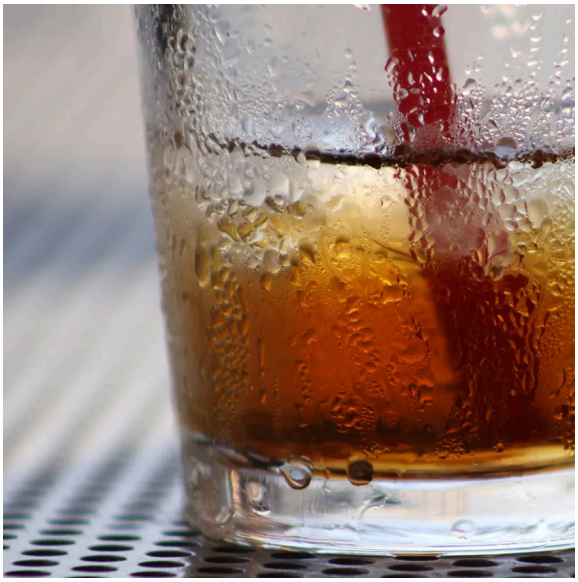


Figure 12.9 Condensation forms on this glass of iced tea because the temperature of the nearby air is reduced to below the dew point. The rate at which water molecules join together exceeds the rate at which they separate, and so water condenses. Energy is released when the water condenses, speeding the melting of the ice in the glass. Image from OpenStax College Physics 2e, CC-BY 4.0

Real-World Application

Energy is also released when a liquid freezes. This phenomenon is used by fruit growers in Florida to protect oranges when the temperature is close to the freezing point $(0^{\circ}\ \mathrm{C})$. Growers spray water on the plants in orchards so that the water freezes and heat is released to the growing oranges on the trees. This prevents the temperature inside the orange from dropping below freezing, which would damage the fruit.



Figure 12.10 The ice on these trees released large amounts of energy when it froze, helping to prevent the temperature of the trees from dropping below $0^{\circ}\ \mathrm{C}$*. Water is intentionally sprayed on orchards to help prevent hard frosts. Image by Haneburger, Wikimedia Commons License*

Sublimation is the transition from solid to vapor phase. You may have noticed that snow can disappear into

thin air without a trace of liquid water, or the disappearance of ice cubes in a freezer. The reverse is also true: Frost can form on very cold windows without going through the liquid stage. A popular effect is the making of "smoke" from dry ice, which is solid carbon dioxide. Sublimation occurs because the equilibrium vapor pressure of solids is not zero. Certain air fresheners use the sublimation of a solid to inject a perfume into the room. Moth balls are a slightly toxic example of a phenol (an organic compound) that sublimates, while some solids, such as osmium tetroxide, are so toxic that they must be kept in sealed containers to prevent human exposure to their sublimation-produced vapors.

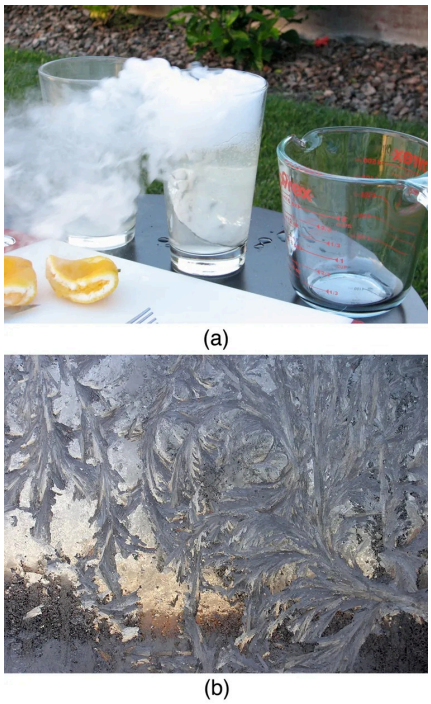


Figure 12.11 Direct transitions between solid and vapor are common, sometimes useful, and even beautiful. (a) Dry ice sublimates directly to carbon dioxide gas. The visible vapor is made of water droplets. (credit: Windell Oskay) (b) Frost forms patterns on a very cold window, an example of a solid formed directly from a vapor. Image from OpenStax College Physics 2e, CC-BY 4.0

All phase transitions involve heat. In the case of direct solid-vapor transitions, the energy required is given by the equation $Q = mL_{\text{s}}$, where L_{s} is the heat of sublimation, which is the energy required to change 1.00 kg of a substance from the solid phase to the vapor phase. L_{s} is analogous to L_{f} and L_{v}, and its value depends on the substance. Sublimation requires energy input, so that dry ice is an effective coolant, whereas the reverse process (i.e., frosting) releases energy. The amount of energy required for sublimation is of the same order of magnitude as that for other phase transitions.

The material presented in this section and the preceding section allows us to calculate any number of effects related to temperature and phase change. In each case, it is necessary to identify which temperature and phase

changes are taking place and then to apply the appropriate equation. Keep in mind that heat transfer and work can cause both temperature and phase changes.

Problem-Solving Strategies for the Effects of Heat Transfer

1. *Examine the situation to determine that there is a change in the temperature or phase. Is there heat transfer into or out of the system?* When the presence or absence of a phase change is not obvious, you may wish to first solve the problem as if there were no phase changes, and examine the temperature change obtained. If it is sufficient to take you past a boiling or melting point, you should then go back and do the problem in steps—temperature change, phase change, subsequent temperature change, and so on.
2. *Identify and list all objects that change temperature and phase.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful.
4. *Make a list of what is given or what can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).* If there is a temperature change, the transferred heat depends on the specific heat (see Table 12.1) whereas, for a phase change, the transferred heat depends on the latent heat. See Table 12.2.
6. *Substitute the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.* You will need to do this in steps if there is more than one stage to the process (such as a temperature change followed by a phase change).
7. *Check the answer to see if it is reasonable: Does it make sense?* As an example, be certain that the temperature change does not also cause a phase change that you have not taken into account.

Check Your Understanding

Why does snow remain on mountain slopes even when daytime temperatures are higher than the freezing temperature?

Click for Solution

Solution

Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be accumulated from the air, even if the air is above $0^{\circ}\ \mathrm{C}$. The warmer the air is, the faster this heat exchange occurs and the faster the snow melts.

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12.4 HEAT TRANSFER METHODS

Learning Objectives

By the end of this section, you will be able to:

- Discuss the different methods of heat transfer.

Equally as interesting as the effects of heat transfer on a system are the methods by which this occurs. Whenever there is a temperature difference, heat transfer occurs. Heat transfer may occur rapidly, such as through a cooking pan, or slowly, such as through the walls of a picnic ice chest. We can control rates of heat transfer by choosing materials (such as thick wool clothing for the winter), controlling air movement (such as the use of weather stripping around doors), or by choice of color (such as a white roof to reflect summer sunlight). So many processes involve heat transfer, so that it is hard to imagine a situation where no heat transfer occurs. Yet every process involving heat transfer takes place by only three methods:

1. Conduction is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction.
2. Convection is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air furnace and in weather systems, for example.
3. Heat transfer by radiation occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of the Earth by the Sun. A less obvious example is thermal radiation from the human body.

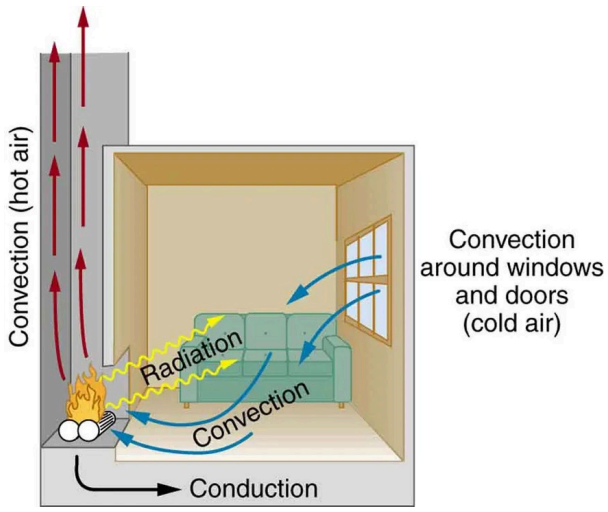


Figure 12.12 In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image illustrates different methods of heat transfer within a room. It includes a cutaway view of a room with a fireplace on the left side. Above the fireplace, flames emit wavy yellow lines labeled “Radiation” towards a couch. Blue arrows show “Convection” of air currents moving from the fireplace to the window, with an annotation indicating “Convection around windows and doors (cold air).” Red arrows labeled “Convection (hot air)” rise upwards on the left side of the wall above the fireplace. A black arrow along the floor labeled “Conduction” shows heat transfer through the floor.

We examine these methods in some detail in the three following modules. Each method has unique and interesting characteristics, but all three do have one thing in common: they transfer heat solely because of a temperature difference Figure 12.12.

Check Your Understanding

Name an example from daily life (different from the text) for each mechanism of heat transfer.
Click for Solution

Solution

Conduction: Heat transfers into your hands as you hold a hot cup of coffee.
Convection: Heat transfers as the barista "steams" cold milk to make hot *cocoa*.
Radiation: Reheating a cold cup of coffee in a microwave oven.

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12.5 CONDUCTION

Learning Objectives

By the end of this section, you will be able to:

- Calculate thermal conductivity.
- Observe conduction of heat in collisions.
- Study thermal conductivities of common substances.



Figure 12.13 Insulation is used to limit the conduction of heat from the inside to the outside (in winters) and from the outside to the inside (in summers). Image from OpenStax College Physics 2e, CC-BY 4.0

Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor. This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the

same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. Figure 12.14 shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the molecule with greater kinetic energy to the molecule with less kinetic energy occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference $\Delta T = T_{\text{hot}} - T_{\text{cold}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross-sectional area. If you touch a cold wall with your palm, your hand cools faster than if you just touch it with your fingertip.

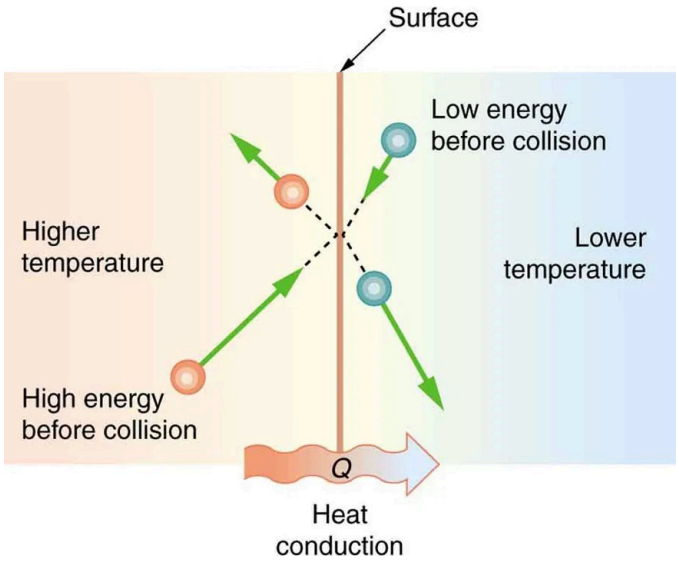


Figure 12.14 The molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a molecule in the lower temperature region (right side) has low energy before collision, but its energy increases after colliding with the contact surface. In contrast, a molecule in the higher temperature region (left side) has high energy before collision, but its energy decreases after colliding with the contact surface. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates the concept of heat conduction across a surface. It is divided into two halves: the left side represents a higher temperature area, while the right side represents a lower temperature area.

On the left side:

- The background is shaded in red, indicating higher temperature.
- A red particle is shown, labeled as "High energy before collision," with a green arrow pointing to the right, indicating its movement towards the surface.

On the right side:

- The background is shaded in blue, indicating lower temperature.
- A blue particle is shown, labeled as "Low energy before collision," with a green arrow pointing to the left, indicating its movement towards the surface.

The particles are shown on either side of a vertical line labeled "Surface." The green arrows originating from the particles cross paths and change direction after apparently colliding with the surface.

At the bottom, an arrow labeled "Q" represents the flow of heat from the higher temperature side to the lower temperature side, labeled as "Heat conduction."

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose that T_2 is greater than T_1, so that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.

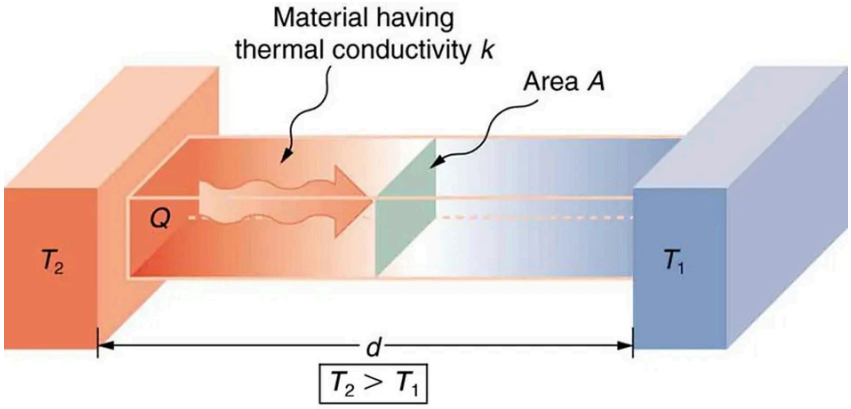


Figure 12.15 Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber. The temperature of the material is T_2 on the left and T_1 on the right, where T_2 is greater than T_1. The rate of heat transfer by conduction is directly proportional to the surface area A, the temperature difference $T_2 - T_1$, and the substance's conductivity k. The rate of heat transfer is inversely proportional to the thickness d. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a concept in thermal conductivity. There are two rectangular blocks, one on the left colored in red labeled T_2 and one on the right colored in blue labeled T_1. The red block (T_2) is warmer than the blue block (T_1), as indicated by the label $T_2 > T_1$.

Between these blocks, there is a rectangular section representing a material with thermal conductivity k. The section shows a gradient from red to blue indicating the transfer of heat. An arrow labeled Q points from the warmer block (T_2) towards the cooler block (T_1), indicating the direction of heat transfer. The area through which heat is transferred is labeled *Area A*.

The distance between the two blocks is labeled d. The diagram effectively demonstrates the flow of heat from a higher temperature region to a lower temperature region through a medium with defined properties.

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The rate of conductive heat transfer through a slab of material, such as the one in Figure 12.15, is given by

$$\frac{Q}{t} = \frac{\mathrm{kA}\,(T_2 - T_1)}{d}, \tag{12.26}$$

where Q/t is the rate of heat transfer in watts or kilocalories per second, k is the thermal conductivity of the material, A and d are its surface area and thickness, as shown in Figure 12.15, and $(T_2 - T_1)$ is the temperature difference across the slab. Table 12.3 gives representative values of thermal conductivity.

Example 12.5

Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of $0.950\ \text{m}^2$ and walls with an average thickness of 2.50 cm. The box contains ice, water, and canned beverages at $0^\circ\ \text{C}$. The inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at $35.0^\circ\ \text{C}$?

Strategy

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

Solution

1. Identify the knowns.

$$\begin{aligned} A &= 0.950\text{m}^2; \\ d &= 2.50\text{cm} = 0.0250m; \\ T_1 &= 0^\circ\ \text{C}; \\ T_2 &= 35.0^\circ\ \text{C}, \\ t &= 1\ \text{day} = 24\ \text{hours} = 86,400\ \text{s}. \end{aligned}$$

12.27

2. Identify the unknowns. We need to solve for the mass of the ice, m. We will also need to solve for the net heat transferred to melt the ice, Q.
3. Determine which equations to use. The rate of heat transfer by conduction is given by

$$\frac{Q}{t} = \frac{\mathrm{kA}\left(T_2 - T_1\right)}{d}.$$

12.28

4. The heat is used to melt the ice: $Q = \mathrm{mL_f}$.
5. Insert the known values:

$$\frac{Q}{t} = \frac{(0.010\ \mathrm{J/s \cdot m \cdot ^\circ C})\left(0.950\ \mathrm{m^2}\right)(35.0^\circ\ \mathrm{C} - 0^\circ\ \mathrm{C})}{0.0250\ \mathrm{m}} = 13.3\ \mathrm{J/s}.$$

12.29

6. Multiply the rate of heat transfer by the time ($1\ \mathrm{day} = 86,400\ \mathrm{s}$):

$$\begin{aligned} Q &= (Q/t)t \\ &= (13.3\ \mathrm{J/s})\ (86,400\ \mathrm{s}) \\ &= 1.15 \times 10^6\ \mathrm{J}. \end{aligned}$$

12.30

7. Set this equal to the heat transferred to melt the ice: $Q = \mathrm{mL_f}$. Solve for the mass m:

$$m = \frac{Q}{L_\mathrm{f}} = \frac{1.15 \times 10^6\ \mathrm{J}}{334\ \times 10^3\ \mathrm{J/kg}} = 3.44\mathrm{kg}.$$

12.31

Discussion

The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (7–10 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages.

Inspecting the conductivities in Table 12.3 shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose-down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of air's poor thermal conductivity.

Table 12.3 Thermal Conductivities of Common Substances[1]

Substance	**Thermal conductivity** $\mathbf{k\ (J/s \cdot m \cdot^{\circ} C)}$
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.08–0.16
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042
Wool	0.04
Down feathers	0.025
Air	0.023
Styrofoam	0.010

A combination of material and thickness is often manipulated to develop good insulators—the smaller the conductivity k and the larger the thickness d, the better. The ratio of d/k will thus be large for a good insulator. The ratio d/k is called the R factor. The rate of conductive heat transfer is inversely proportional to R. The larger the value of R, the better the insulation. R factors are most commonly quoted for household insulation, refrigerators, and the like—unfortunately, it is still in non-metric units of $ft^2 \cdot^{\circ}F \cdot h/Btu$,

1. At temperatures near 0ºC.

although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 °F). A couple of representative values are an R factor of 11 for 3.5-in-thick fiberglass batts (pieces) of insulation and an R factor of 19 for 6.5-in-thick fiberglass batts. Walls are usually insulated with 3.5-in batts, while ceilings are usually insulated with 6.5-in batts. In cold climates, thicker batts may be used in ceilings and walls.

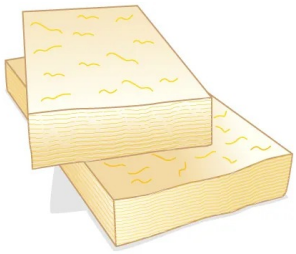


Figure 12.16 The fiberglass batt is used for insulation of walls and ceilings to prevent heat transfer between the inside of the building and the outside environment. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows an illustration of two thick, rectangular blocks that resemble stacks of sliced ramen noodles. Each block has several thin lines running horizontally, which imitate the layers of noodles. The top surfaces are slightly uneven with irregular squiggly yellow lines, giving the appearance of dried instant noodle blocks. The blocks are stacked slightly askew, with one resting partially on top of the other, casting a slight shadow below. The color palette consists mainly of beige and yellow tones.

Note that in Table 12.3, the best thermal conductors—silver, copper, gold, and aluminum—are also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made from good conductors.

Example 12.6

Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The sauce pan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

Strategy

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference.

$$T_2 - T_1 = \frac{Q}{t}\left(\frac{d}{\mathrm{kA}}\right).$$

12.32

Solution

1. Identify the knowns and convert them to the SI units.The thickness of the pan, $d = 0.800\ \mathrm{cm} = 8.0 \times 10^{-3}\ \mathrm{m}$, the area of the pan, $A = \pi\,(0.14/2)^2\ \mathrm{m}^2 = 1.54 \times 10^{-2}\ \mathrm{m}^2$, and the thermal conductivity, $k = 220\ \mathrm{J/s \cdot m \cdot {}^\circ C}$.
2. Calculate the necessary heat of vaporization of 1 g of water:

$$\begin{aligned} Q &= \mathrm{mL_v} \\ &= \left(1.00 \times 10^{-3}\ \mathrm{kg}\right)\left(2256 \times 10^{3}\ \mathrm{J/kg}\right) \\ &= 2256 J. \end{aligned}$$

12.33

3. Calculate the rate of heat transfer given that 1 g of water melts in one second:

$$Q/t = 2256 \text{ J/s or } 2.26 \text{ kW}.$$

12.34

4. Insert the knowns into the equation and solve for the temperature difference:

$$\begin{aligned} T_2 - T_1 &= \frac{Q}{t}\left(\frac{d}{\text{kA}}\right) \\ &= (2256 \text{ J/s})\frac{8.00 \times 10^{-3}\text{m}}{(220 \text{ J/s} \cdot \text{m} \cdot^{\circ} \text{C})\left(1.54 \times 10^{-2} \text{ m}^2\right)} \\ &= 5.33^{\circ} C. \end{aligned}$$

12.35

Discussion

The value for the heat transfer $Q/t = 2.26\text{kW or } 2256 \text{ J/s}$ is typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly 100° C because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of 2.26 kW into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.

Check Your Understanding

How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

Click for Solution

Solution

Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled $\left(A_{\text{final}} = (2d)^2 = 4d^2 = 4A_{initial}\right)$. The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:

$$\left(\frac{Q}{t}\right)_{\text{final}} = \frac{\text{kA}_{\text{final}}\left(T_2 - T_1\right)}{d_{\text{final}}} = \frac{k\left(4\text{A}_{\text{initial}}\right)\left(T_2 - T_1\right)}{2d_{\text{initial}}} = 2\frac{\text{kA}_{\text{initial}}\left(T_2 - T_1\right)}{d_{\text{initial}}} = 2\left(\frac{Q}{t}\right)_{\text{initial}}.$$

12.36

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12.6 CONVECTION

Learning Objectives

By the end of this section, you will be able to:

- Discuss the method of heat transfer by convection.

Convection is driven by large-scale flow of matter. In the case of Earth, the atmospheric circulation is caused by the flow of hot air from the tropics to the poles, and the flow of cold air from the poles toward the tropics. (Note that Earth's rotation causes the observed easterly flow of air in the northern hemisphere). Car engines are kept cool by the flow of water in the cooling system, with the water pump maintaining a flow of cool water to the pistons. The circulatory system is used the body: when the body overheats, the blood vessels in the skin expand (dilate), which increases the blood flow to the skin where it can be cooled by sweating. These vessels become smaller when it is cold outside and larger when it is hot (so more fluid flows, and more energy is transferred).

The body also loses a significant fraction of its heat through the breathing process.

While convection is usually more complicated than conduction, we can describe convection and do some straightforward, realistic calculations of its effects. Natural convection is driven by buoyant forces: hot air rises because density decreases as temperature increases. The house in Figure 12.17 is kept warm in this manner, as is the pot of water on the stove in Figure 12.18. Ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another. Both are examples of natural convection.

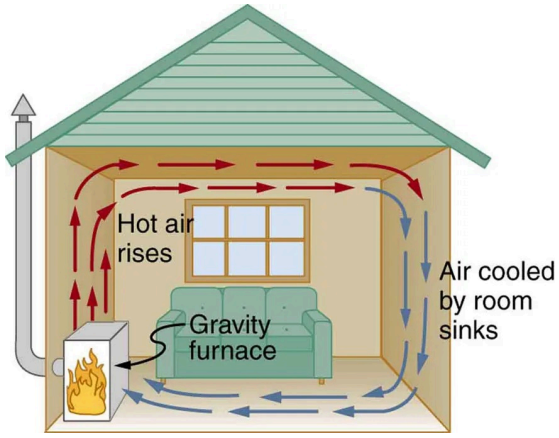


Figure 12.17 Air heated by the so-called gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system using natural convection, like this one, can be quite efficient in uniformly heating a home. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the operation of a gravity furnace system within a house. The interior is shown with a cross-section view.

– The furnace, depicted with a flame icon, is located at the bottom left of the house.
– Red arrows indicate the warm air rising from the furnace, flowing upwards and then horizontally across the ceiling.
– The label “Hot air rises” is placed near these arrows.
– As the air travels across the room, it begins to cool down and sinks, shown by blue arrows.
– The label “Air cooled by room sinks” is positioned near these blue arrows.
– A window is visible on the wall opposite the furnace.
– A green couch is placed against the same wall as the window.
– The blue arrows complete the cycle by returning the cooled air to the furnace at the floor level.
– The overall circulation shows how warm air is distributed by the gravity furnace throughout the room.

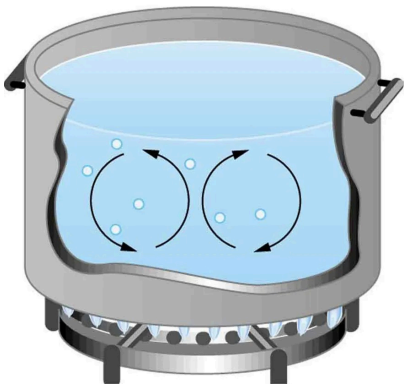


Figure 12.18 Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process keeps repeating. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a cross-section of a pot or kettle placed on a gas stove. Inside the pot, water is depicted as boiling, with circular arrows indicating the convection currents. Small bubbles are visible in the water, illustrating the boiling process. The pot itself is gray, with handles on the sides, and sits securely on a stove burner. The flame from the gas burner is blue and is depicted as providing heat to the bottom of the pot, causing the water to boil.

Take-Home Experiment: Convection Rolls in a Heated Pan

Take two small pots of water and use an eye dropper to place a drop of food coloring near the bottom of each. Leave one on a bench top and heat the other over a stovetop. Watch how the color spreads and how long it takes the color to reach the top. Watch how convective loops form.

Example 12.7

Calculating Heat Transfer by Convection: Convection of Air Through the Walls of a House

Most houses are not airtight: air goes in and out around doors and windows, through cracks and crevices, following wiring to switches and outlets, and so on. The air in a typical house is completely replaced in less than an hour. Suppose that a moderately-sized house has inside dimensions $12.0\text{m} \times 18.0\text{m} \times 3.00\text{m}$ high, and that all air is replaced in 30.0 min. Calculate the heat transfer per unit time in watts needed to warm the incoming cold air by 10.0° C, thus replacing the heat transferred by convection alone.

Strategy

Heat is used to raise the temperature of air so that $Q = \text{mc}\Delta T$. The rate of heat transfer is then Q/t, where t is the time for air turnover. We are given that ΔT is 10.0° C, but we must still find values for the mass of air and its specific heat before we can calculate Q. The specific heat of air is a weighted average of the specific heats of nitrogen and oxygen, which gives $c = c_{\text{p}} \cong 1000\text{ J/kg}\cdot^{\circ}\text{ C}$ from Table 12.1 (note that the specific heat at constant pressure must be used for this process).

Solution

1. Determine the mass of air from its density and the given volume of the house. The density is given from the density ρ and the volume

$$\begin{aligned} m &= \rho\text{V} \\ &= \left(1.29\text{kg/m}^3\right)\left(12.0\text{m} \times 18.0\text{m} \times 3.00\text{m}\right) \\ &= 836\text{kg}. \end{aligned}$$

12.37

2. Calculate the heat transferred from the change in air temperature: $Q = \mathrm{mc}\Delta T$ so that

$$\begin{aligned} Q &= (836\mathrm{kg})\,(1000\ \mathrm{J/kg}\cdot^{\circ}\ \mathrm{C})\,(10.0^{\circ}\ \mathrm{C}) \\ &= 8.36 \times 10^{6}\mathrm{J}. \end{aligned}$$

12.38

3. Calculate the heat transfer from the heat Q and the turnover time t. Since air is turned over in $t = 0.500\mathrm{h} = 1800\mathrm{s}$, the heat transferred per unit time is

$$\frac{Q}{t} = \frac{8.36 \times 10^{6}\mathrm{J}}{1800\ \mathrm{s}} = 4.64\mathrm{kW}.$$

12.39

Discussion

This rate of heat transfer is equal to the power consumed by about forty-six 100-W light bulbs. Newly constructed homes are designed for a turnover time of 2 hours or more, rather than 30 minutes for the house of this example. Weather stripping, caulking, and improved window seals are commonly employed. More extreme measures are sometimes taken in very cold (or hot) climates to achieve a tight standard of more than 6 hours for one air turnover. Still longer turnover times are unhealthy, because a minimum amount of fresh air is necessary to supply oxygen for breathing and to dilute household pollutants. The term used for the process by which outside air leaks into the house from cracks around windows, doors, and the foundation is called “air infiltration.”

A cold wind is much more chilling than still cold air, because convection combines with conduction in the body to increase the rate at which energy is transferred away from the body. The table below gives approximate wind-chill factors, which are the temperatures of still air that produce the same rate of cooling as air of a given temperature and speed. Wind-chill factors are a dramatic reminder of convection’s ability to transfer heat faster than conduction. For example, a 15.0 m/s wind at $0^{\circ}\ \mathrm{C}$ has the chilling equivalent of still air at about $-18^{\circ}\ \mathrm{C}$.

Table 12.4 Wind-Chill Factors

Moving air temperature	Wind speed (m/s)				
(° C)	2	5	10	15	20
5	3	−1	−8	−10	−12
2	0	−7	−12	−16	−18
0	−2	−9	−15	−18	−20
−5	−7	−15	−22	−26	−29
−10	−12	−21	−29	−34	−36
−20	−23	−34	−44	−50	−52
−40	−44	−59	−73	−82	−84

Although air can transfer heat rapidly by convection, it is a poor conductor and thus a good insulator. The amount of available space for airflow determines whether air acts as an insulator or conductor. The space between the inside and outside walls of a house, for example, is about 9 cm (3.5 in) —large enough for convection to work effectively. The addition of wall insulation prevents airflow, so heat loss (or gain) is decreased. Similarly, the gap between the two panes of a double-paned window is about 1 cm, which prevents convection and takes advantage of air's low conductivity to prevent greater loss. Fur, fiber, and fiberglass also take advantage of the low conductivity of air by trapping it in spaces too small to support convection, as shown in the figure. Fur and feathers are lightweight and thus ideal for the protection of animals.

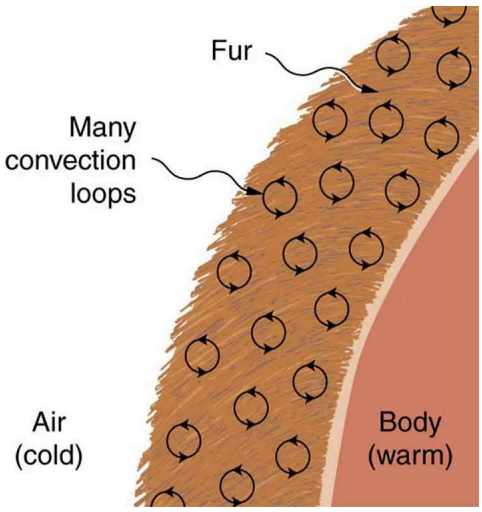


Figure 12.19 Fur is filled with air, breaking it up into many small pockets. Convection is very slow here, because the loops are so small. The low conductivity of air makes fur a very good lightweight insulator. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the concept of heat retention through fur, depicting a cross-sectional view. On the right side, there is a labeled section representing a "Body" which is described as "warm." A layer of "Fur" is shown covering the body, with arrows within the fur indicating "Many convection loops." These loops are depicted as circular arrows showing the flow of heat. The left side of the image shows "Air" which is labeled as "cold." The diagram suggests that the fur and convection loops help in retaining the body's warmth by minimizing the exposure to the cold air outside.

Some interesting phenomena happen *when convection is accompanied by a phase change*. It allows us to cool off by sweating, even if the temperature of the surrounding air exceeds body temperature. Heat from the skin is required for sweat to evaporate from the skin, but without air flow, the air becomes saturated and evaporation stops. Air flow caused by convection replaces the saturated air by dry air and evaporation continues.

Example 12.8

Calculate the Flow of Mass during Convection: Sweat-Heat Transfer away from the Body

The average person produces heat at the rate of about 120 W when at rest. At what rate must water evaporate from the body to get rid of all this energy? (This evaporation might occur when a person is sitting in the shade and surrounding temperatures are the same as skin temperature, eliminating heat transfer by other methods.)

Strategy

Energy is needed for a phase change ($Q = \mathrm{mL_v}$). Thus, the energy loss per unit time is

$$\frac{Q}{t} = \frac{\mathrm{mL_v}}{t} = 120 \ \mathrm{W} = 120 \ \mathrm{J/s}.$$

12.40

We divide both sides of the equation by L_v to find that the mass evaporated per unit time is

$$\frac{m}{t} = \frac{120 \ \mathrm{J/s}}{L_\mathrm{v}}.$$

12.41

Solution

(1) Insert the value of the latent heat from Table 12.2, $L_\mathrm{v} = 2430 \ \mathrm{kJ/kg} = 2430 \ \mathrm{J/g}$. This yields

$$\frac{m}{t} = \frac{120 \ \mathrm{J/s}}{2430 \ \mathrm{J/g}} = 0.0494 \ \mathrm{g/s} = 2.96 \ \mathrm{g/min}.$$

12.42

Discussion

Evaporating about 3 g/min seems reasonable. This would be about 180 g (about 7 oz) per hour. If the air is very dry, the sweat may evaporate without even being noticed. A significant amount of evaporation also takes place in the lungs and breathing passages.

Another important example of the combination of phase change and convection occurs when water evaporates from the oceans. Heat is removed from the ocean when water evaporates. If the water vapor condenses in liquid droplets as clouds form, heat is released in the atmosphere. Thus, there is an overall transfer of heat from the ocean to the atmosphere. This process is the driving power behind thunderheads, those great cumulus clouds that rise as much as 20.0 km into the stratosphere. Water vapor carried in by convection condenses, releasing tremendous amounts of energy. This energy causes the air to expand and rise, where it is colder. More condensation occurs in these colder regions, which in turn drives the cloud even higher. Such a mechanism is called positive feedback, since the process reinforces and accelerates itself. These systems sometimes produce violent storms, with lightning and hail, and constitute the mechanism driving hurricanes.



Figure 12.20 Cumulus clouds are caused by water vapor that rises because of convection. The rise of clouds is driven by a positive feedback mechanism. Image from OpenStax College Physics 2e, CC-BY 4.0



Figure 12.21 Convection accompanied by a phase change releases the energy needed to drive this thunderhead into the stratosphere. Image from OpenStax College Physics 2e, CC-BY 4.0



Figure 12.22 The phase change that occurs when this iceberg melts involves tremendous heat transfer. Image from OpenStax College Physics 2e, CC-BY 4.0

The movement of icebergs is another example of convection accompanied by a phase change. Suppose an iceberg drifts from Greenland into warmer Atlantic waters. Heat is removed from the warm ocean water when the ice melts and heat is released to the land mass when the iceberg forms on Greenland.

Check Your Understanding

Explain why using a fan in the summer feels refreshing!

Click for Solution

Solution

Using a fan increases the flow of air: warm air near your body is replaced by cooler air from elsewhere. Convection increases the rate of heat transfer so that moving air "feels" cooler than still air.

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12.7 RADIATION

Learning Objectives

By the end of this section, you will be able to:

- Discuss heat transfer by radiation.
- Explain the power of different materials.

You can feel the heat transfer from a fire and from the Sun. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. The space between the Earth and the Sun is largely empty, without any possibility of heat transfer by convection or conduction. In these examples, heat is transferred by radiation. That is, the hot body emits electromagnetic waves that are absorbed by our skin: no medium is required for electromagnetic waves to propagate. Different names are used for electromagnetic waves of different wavelengths: radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays.



Figure 12.23 Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light, although dramatic, transfers relatively little thermal energy. Convection transfers energy away from the observers as hot air rises, while conduction is negligibly slow here. Skin is very sensitive to infrared radiation, so that you can sense the presence of a fire without looking at it directly. Image from OpenStax College Physics 2e, CC-BY 4.0

The energy of electromagnetic radiation depends on the wavelength (color) and varies over a wide range: a smaller wavelength (or higher frequency) corresponds to a higher energy. Because more heat is radiated at higher temperatures, a temperature change is accompanied by a color change. Take, for example, an electrical element on a stove, which glows from red to orange, while the higher-temperature steel in a blast furnace glows from yellow to white. The radiation you feel is mostly infrared, which corresponds to a lower temperature than that of the electrical element and the steel. The radiated energy depends on its intensity, which is represented in the figure below by the height of the distribution.

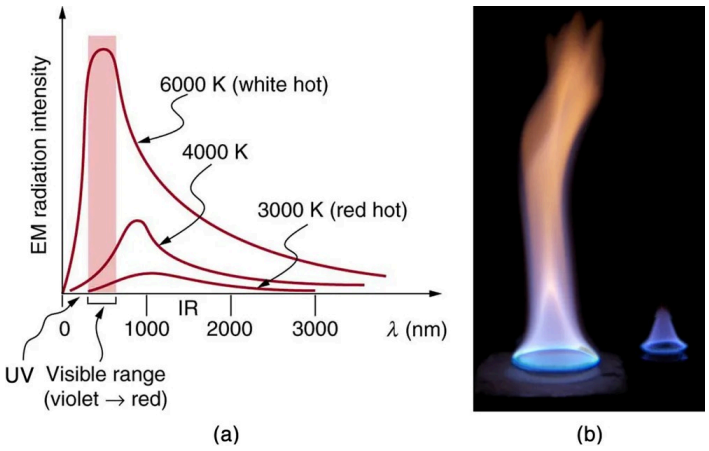


Figure 12.24 (a) A graph of the spectra of electromagnetic waves emitted from an ideal radiator at three different temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The shaded portion denotes the visible part of the spectrum. It is apparent that the shift toward the ultraviolet with temperature makes the visible appearance shift from red to white to blue as temperature increases. (b) The upper part of the flame is orange due to black body radiation. (The lower part is blue due to the chemical composition of the flame, alcohol in this case.) Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is composed of two parts:

Left Side (a): Graph Description
– The graph displays the electromagnetic (EM) radiation intensity against wavelength in nanometers (nm).
– The x-axis represents the wavelength ranging from 0 to over 3000 nm, with labels indicating UV, visible, and IR ranges.
– The y-axis represents the EM radiation intensity.
– Three curves on the graph correspond to different temperatures:
– 3000 K labeled as “red hot.”
– 4000 K.
– 6000 K labeled as “white hot.”
– The visible range is highlighted with a shaded red area, covering wavelengths where colors transition from violet to red.

Right Side (b): Flame Image
– The image shows two flames: one larger and one smaller.
– The flames have a blue base transitioning to orange at the tips, illustrating the colors related to temperature differences.

This visual representation connects temperature, color, and radiation intensity in both graphical and physical examples through flames.

All objects absorb and emit electromagnetic radiation. The rate of heat transfer by radiation is largely determined by the color of the object. Black is the most effective, and white is the least effective. People living in hot climates generally avoid wearing black clothing, for instance. Similarly, black asphalt in a parking lot will be hotter than adjacent gray sidewalk on a summer day, because black absorbs better than gray. The reverse is also true—black radiates better than gray. Thus, on a clear summer night, the asphalt will be colder than the gray sidewalk, because black radiates the energy more rapidly than gray. An *ideal radiator* is the same color as an *ideal absorber*, and captures all the radiation that falls on it. In contrast, white is a poor absorber and is also a poor radiator. A white object reflects all radiation, like a mirror. (A perfect, polished white surface is mirror-like in appearance, and a crushed mirror looks white.)

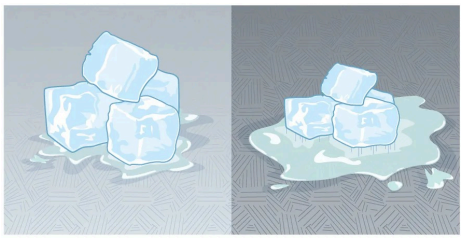


Figure 12.25 This illustration shows that the darker pavement is hotter than the lighter pavement (much more of the ice on the right has melted), although both have been in the sunlight for the same time. The thermal conductivities of the pavements are the same. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is divided into two sections.

On the left side, there are three ice cubes stacked on one another. The cubes are slightly melting, creating a small puddle beneath them. The background features a light pattern resembling a geometric design with a light gray color.

On the right side, the same three ice cubes appear more melted than in the left section, with a larger puddle of water surrounding them. The geometric pattern in the background is darker gray, making the ice cubes and melted water more prominent.

The visual contrast between the two sides emphasizes the difference in the state of the ice cubes—less melted on the left and more melted on the right.

Gray objects have a uniform ability to absorb all parts of the electromagnetic spectrum. Colored objects

behave in similar but more complex ways, which gives them a particular color in the visible range and may make them special in other ranges of the nonvisible spectrum. Take, for example, the strong absorption of infrared radiation by the skin, which allows us to be very sensitive to it.

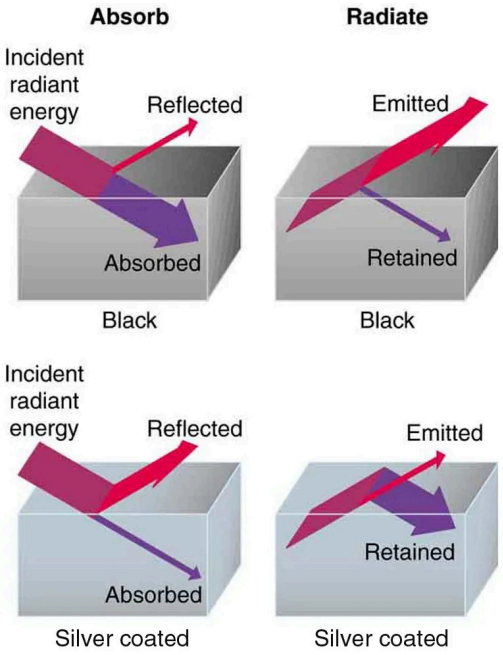


Figure 12.26 A black object is a good absorber and a good radiator, while a white (or silver) object is a poor absorber and a poor radiator. It is as if radiation from the inside is reflected back into the silver object, whereas radiation from the inside of the black object is "absorbed" when it hits the surface and finds itself on the outside and is strongly emitted. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of four diagrams showing how different surfaces absorb and radiate energy. Each diagram represents a box with arrows indicating the path of radiant energy. The image compares black and silver-coated surfaces under two conditions: absorption and radiation.

	Absorb	**Radiate**
Black	A box is shown with incident radiant energy hitting it. A large red arrow indicates that some of the energy is reflected, while a purple arrow shows that the remaining energy is absorbed.	A similar box with a large red arrow showing emitted energy leaving the box. A purple arrow indicates that some energy is retained within the box.
Silver coated	A box is shown with incident radiant energy. A large red arrow shows the majority of energy being reflected, while a small purple arrow indicates a lesser amount of energy being absorbed.	A similar box where energy is emitted as indicated by a red arrow. A purple arrow shows that most of the energy is retained within the box.

The rate of heat transfer by emitted radiation is determined by the Stefan-Boltzmann law of radiation:

$$\frac{Q}{t} = \sigma e A T^4,$$
12.43

where $\sigma = 5.67 \times 10^{-8} \text{J/s} \cdot \text{m}^2 \cdot \text{K}^4$ is the Stefan-Boltzmann constant, A is the surface area of the object, and T is its absolute temperature in kelvin. The symbol e stands for the emissivity of the object, which is a measure of how well it radiates. An ideal jet-black (or black body) radiator has $e = 1$, whereas a perfect reflector has $e = 0$. Real objects fall between these two values. Take, for example, tungsten light bulb filaments which have an e of about 0.5, and carbon black (a material used in printer toner), which has the (greatest known) emissivity of about 0.99.

The radiation rate is directly proportional to the *fourth power* of the absolute temperature—a remarkably strong temperature dependence. Furthermore, the radiated heat is proportional to the surface area of the object. If you knock apart the coals of a fire, there is a noticeable increase in radiation due to an increase in radiating surface area.



Figure 12.27 A thermograph of part of a building shows temperature variations, indicating where heat transfer to the outside is most severe. Windows are a major region of heat transfer to the outside of homes. Image from OpenStax College Physics 2e, CC-BY 4.0

Skin is a remarkably good absorber and emitter of infrared radiation, having an emissivity of 0.97 in the infrared spectrum. Thus, we are all nearly (jet) black in the infrared, in spite of the obvious variations in skin color. This high infrared emissivity is why we can so easily feel radiation on our skin. It is also the basis for the use of night scopes used by law enforcement and the military to detect human beings. Even small temperature variations can be detected because of the T^4 dependence. Images, called *thermographs*, can be used medically

to detect regions of abnormally high temperature in the body, perhaps indicative of disease. Similar techniques can be used to detect heat leaks in homes Figure 12.27, optimize performance of blast furnaces, improve comfort levels in work environments, and even remotely map the Earth's temperature profile.

All objects emit and absorb radiation. The *net* rate of heat transfer by radiation (absorption minus emission) is related to both the temperature of the object and the temperature of its surroundings. Assuming that an object with a temperature T_1 is surrounded by an environment with uniform temperature T_2, the net rate of heat transfer by radiation is

$$\frac{Q_{\text{net}}}{t} = \sigma e A \left(T_2^4 - T_1^4\right), \tag{12.44}$$

where e is the emissivity of the object alone. In other words, it does not matter whether the surroundings are white, gray, or black; the balance of radiation into and out of the object depends on how well it emits and absorbs radiation. When $T_2 > T_1$, the quantity Q_{net}/t is positive; that is, the net heat transfer is from hot to cold.

Take-Home Experiment: Temperature in the Sun

Place a thermometer out in the sunshine and shield it from direct sunlight using an aluminum foil. What is the reading? Now remove the shield, and note what the thermometer reads. Take a handkerchief soaked in nail polish remover, wrap it around the thermometer and place it in the sunshine. What does the thermometer read?

Example 12.9

Calculate the Net Heat Transfer of a Person: Heat Transfer by Radiation

What is the rate of heat transfer by radiation, with an unclothed person standing in a dark room whose ambient temperature is $22.0^{\circ}\ \mathrm{C}$. The person has a normal skin temperature of $33.0^{\circ}\ \mathrm{C}$ and a surface area of $1.50\mathrm{m}^2$. The emissivity of skin is 0.97 in the infrared, where the radiation takes place.

Strategy

We can solve this by using the equation for the rate of radiative heat transfer.

Solution

Insert the temperatures values $T_2 = 295\ \mathrm{K}$ and $T_1 = 306\ \mathrm{K}$, so that

$$\begin{aligned}\frac{Q}{t} &= \sigma e A\left(T_2^4 - T_1^4\right)\\ &= \left(5.67 \times 10^{-8}\ \mathrm{J/s \cdot\ m^2 \cdot\ K^4}\right)(0.97)\left(1.50\ \mathrm{m^2}\right)\\ &\quad \times \left[(295\ \mathrm{K})^4 - (306\ \mathrm{K})^4\right]\\ &= -99\ \mathrm{J/s} = -99\ \mathrm{W}.\end{aligned}$$

12.45

Discussion

This value is a significant rate of heat transfer to the environment (note the minus sign), considering that a person at rest may produce energy at the rate of 125 W and that conduction and convection will also be transferring energy to the environment. Indeed, we would probably expect this person to feel cold. Clothing significantly reduces heat

transfer to the environment by many methods, because clothing slows down both conduction and convection, and has a lower emissivity (especially if it is white) than skin.

The Earth receives almost all its energy from radiation of the Sun and reflects some of it back into outer space. Because the Sun is hotter than the Earth, the net energy flux is from the Sun to the Earth. However, the rate of energy transfer is less than the equation for the radiative heat transfer would predict because the Sun does not fill the sky. The average emissivity (e) of the Earth is about 0.65, but the calculation of this value is complicated by the fact that the highly reflective cloud coverage varies greatly from day to day. There is a negative feedback (one in which a change produces an effect that opposes that change) between clouds and heat transfer; greater temperatures evaporate more water to form more clouds, which reflect more radiation back into space, reducing the temperature. The often mentioned greenhouse effect is directly related to the variation of the Earth's emissivity with radiation type (see the figure given below). The greenhouse effect is a natural phenomenon responsible for providing temperatures suitable for life on Earth. The Earth's relatively constant temperature is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by carbon dioxide (CO_2) and water (H_2O) in the atmosphere and then re-radiated back to the Earth or into outer space. Re-radiation back to the Earth maintains its surface temperature about $40^{\circ}\ C$ higher than it would be if there was no atmosphere, similar to the way glass increases temperatures in a greenhouse.

The greenhouse effect and its causes were first predicted by Eunice Newton Foote after she designed and conducted experiments on heating of different gases. After filling flasks with carbon dioxide, hydrogen, and regular air, then also modifying moisture, she placed them in the sun and carefully measured their heating and, especially, their heat retention. She discovered that the CO_2 flask gained the most temperature and held it the longest. After subsequent research, her paper "Circumstances affecting the Heat of the Sun's Rays" included conclusions that an atmosphere consisting of more carbon dioxide would be hotter resulting from the gas trapping radiation.

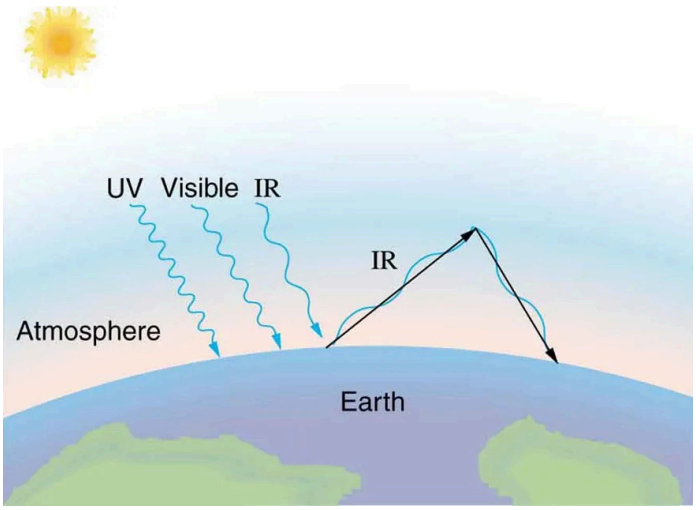


Figure 12.28 The greenhouse effect is a name given to the trapping of energy in the Earth's atmosphere by a process similar to that used in greenhouses. The atmosphere, like window glass, is transparent to incoming visible radiation and most of the Sun's infrared. These wavelengths are absorbed by the Earth and re-emitted as infrared. Since Earth's temperature is much lower than that of the Sun, the infrared radiated by the Earth has a much longer wavelength. The atmosphere, like glass, traps these longer infrared rays, keeping the Earth warmer than it would otherwise be. The amount of trapping depends on concentrations of trace gases like carbon dioxide, and a change in the concentration of these gases is believed to affect the Earth's surface temperature. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image is a diagram illustrating the interaction of different types of solar radiation with Earth's atmosphere and surface. At the top left, a sun is depicted, symbolizing the source of energy. There are three arrows pointing downwards towards Earth's atmosphere, labeled as "UV" (Ultraviolet), "Visible," and "IR" (Infrared), representing different types of radiation.

The Earth's atmosphere is labeled near the top center of the image, and below it is a representation of Earth, labeled as "Earth," with some land masses visible in green.

After passing through the atmosphere, the arrows representing "IR" (Infrared) radiation are shown bouncing back into the atmosphere, illustrating the process of infrared radiation being absorbed by the Earth's surface and then radiated back towards the atmosphere.

The greenhouse effect is also central to the discussion of global warming due to emission of carbon dioxide and methane (and other so-called greenhouse gases) into the Earth's atmosphere from industrial production and farming. Changes in global climate could lead to more intense storms, precipitation changes (affecting agriculture), reduction in rain forest biodiversity, and rising sea levels.

Heating and cooling are often significant contributors to energy use in individual homes. Mária Telkes, a Hungarian-born American scientist, was among the foremost developers of solar energy applications in industrial and community use. After inventing a widely deployed solar seawater distiller used on World War II life rafts, she partnered with architect Eleanor Raymond to design the first modern home to be completely heated by solar power. Air warmed on rooftop collectors transferred heat to salts, which stored the heat for later use. Telkes later worked with the Department of Energy to develop the first solar-electrically powered home. Current research efforts into developing environmentally friendly homes quite often focus on reducing conventional heating and cooling through better building materials, strategically positioning windows to optimize radiation gain from the Sun, and opening spaces to allow convection. It is possible to build a zero-energy house that allows for comfortable living in most parts of the United States with hot and humid summers and cold winters.

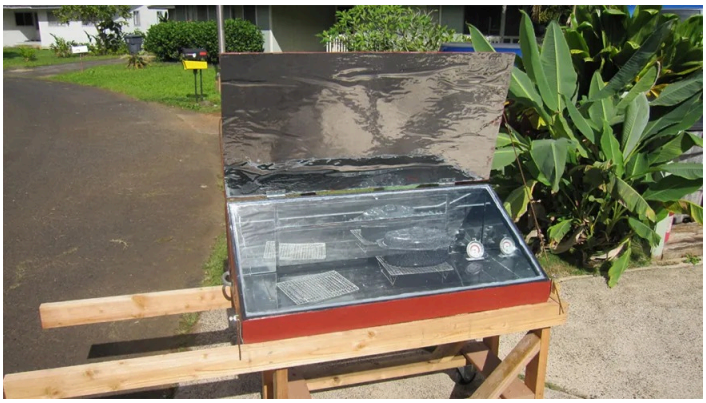


Figure 12.29 This simple but effective solar cooker uses the greenhouse effect and reflective material to trap and retain solar energy. Made of inexpensive, durable materials, it saves money and labor, and is of particular economic value in countries with lower energy availability. Mária Telkes, a pioneering innovator in solar applications, developed some of the earliest models of these cookers in the 1950s. Image from OpenStax College Physics 2e, CC-BY 4.0

Conversely, dark space is very cold, about $3\text{K}(-454^\circ\ \text{F})$, so that the Earth radiates energy into the dark sky. Owing to the fact that clouds have lower emissivity than either oceans or land masses, they reflect some of the radiation back to the surface, greatly reducing heat transfer into dark space, just as they greatly reduce heat transfer into the atmosphere during the day. The rate of heat transfer from soil and grasses can be so rapid that frost may occur on clear summer evenings, even in warm latitudes.

Check Your Understanding

What is the change in the rate of the radiated heat by a body at the temperature $T_1 = 20^\circ\ \mathrm{C}$ compared to when the body is at the temperature $T_2 = 40^\circ\ \mathrm{C}$?

Click for Solution

Solution

The radiated heat is proportional to the fourth power of the *absolute temperature*. Because $T_1 = 293\ \mathrm{K}$ and $T_2 = 313\ \mathrm{K}$, the rate of heat transfer increases by about 30 percent of the original rate.

Career Connection: Energy Conservation Consultation

The cost of energy is generally believed to remain very high for the foreseeable future. Thus, passive control of heat loss in both commercial and domestic housing will become increasingly important. Energy consultants measure and analyze the flow of energy into and out of houses and ensure that a healthy exchange of air is maintained inside the house. The job prospects for an energy consultant are strong.

Problem-Solving Strategies for the Methods of Heat Transfer

1. *Examine the situation to determine what type of heat transfer is involved.*
2. *Identify the type(s) of heat transfer—conduction, convection, or radiation.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is very useful.*
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).*
6. For conduction, equation $\frac{Q}{t} = \frac{\mathrm{kA}\left(T_2 - T_1\right)}{d}$ is appropriate. Table 12.3 lists thermal conductivities. For convection, determine the amount of matter moved and use equation $Q = \mathrm{mc}\Delta T$, to calculate the heat transfer involved in the temperature change of the fluid. If a phase change accompanies convection, equation $Q = \mathrm{mL_f}$ or $Q = \mathrm{mL_v}$ is appropriate to find the heat transfer involved in the phase change. Table 12.2 lists information relevant to phase change. For radiation, equation $\frac{Q_{\mathrm{net}}}{t} = \sigma e A\left(T_2^4 - T_1^4\right)$ gives the net heat transfer rate.
7. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.*
8. *Check the answer to see if it is reasonable. Does it make sense?*

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GLOSSARY

conduction
heat transfer through stationary matter by physical contact

convection
heat transfer by the macroscopic movement of fluid

emissivity
measure of how well an object radiates

greenhouse effect
warming of the Earth that is due to gases such as carbon dioxide and methane that absorb infrared radiation from the Earth's surface and reradiate it in all directions, thus sending a fraction of it back toward the surface of the Earth

heat
the spontaneous transfer of energy due to a temperature difference

heat of sublimation
the energy required to change a substance from the solid phase to the vapor phase

kilocalorie
$1\text{kilocalorie} = 1000\text{calories}$

latent heat coefficient
a physical constant equal to the amount of heat transferred for every 1 kg of a substance during the change in phase of the substance

mechanical equivalent of heat
the work needed to produce the same effects as heat transfer

net rate of heat transfer by radiation
is
$$\frac{Q_{\text{net}}}{t} = \sigma e A \left(T_2^4 - T_1^4\right)$$

***R* factor**
the ratio of thickness to the conductivity of a material

radiation
energy transferred by electromagnetic waves directly as a result of a temperature difference

radiation
heat transfer which occurs when microwaves, infrared radiation, visible light, or other electromagnetic radiation is emitted or absorbed

rate of conductive heat transfer

rate of heat transfer from one material to another

specific heat

the amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 ºC

Stefan-Boltzmann law of radiation

$\frac{Q}{t} = \sigma e A T^4$, where σ is the Stefan-Boltzmann constant, A is the surface area of the object, T is the absolute temperature, and e is the emissivity

sublimation

the transition from the solid phase to the vapor phase

thermal conductivity

the property of a material's ability to conduct heat

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CONCEPTUAL QUESTIONS

Conceptual Questions

12.1 Heat

1. How is heat transfer related to temperature?
2. Describe a situation in which heat transfer occurs. What are the resulting forms of energy?
3. When heat transfers into a system, is the energy stored as heat? Explain briefly.

12.2 Temperature Change and Heat Capacity

4. What three factors affect the heat transfer that is necessary to change an object's temperature?
5. The brakes in a car increase in temperature by ΔT when bringing the car to rest from a speed v. How much greater would ΔT be if the car initially had twice the speed? You may assume the car to stop sufficiently fast so that no heat transfers out of the brakes.

12.3 Phase Change and Latent Heat

6. Heat transfer can cause temperature and phase changes. What else can cause these changes?
7. How does the latent heat of fusion of water help slow the decrease of air temperatures, perhaps preventing temperatures from falling significantly below $0^{\circ}\ \mathrm{C}$, in the vicinity of large bodies of water?
8. What is the temperature of ice right after it is formed by freezing water?
9. If you place $0^{\circ}\ \mathrm{C}$ ice into $0^{\circ}\ \mathrm{C}$ water in an insulated container, what will happen? Will some ice melt, will more water freeze, or will neither take place?
10. What effect does condensation on a glass of ice water have on the rate at which the ice melts? Will the condensation speed up the melting process or slow it

down?

11. In very humid climates where there are numerous bodies of water, such as in Florida, it is unusual for temperatures to rise above about $35^{\circ}\ \mathrm{C}\ (95^{\circ}\ \mathrm{F})$. In deserts, however, temperatures can rise far above this. Explain how the evaporation of water helps limit high temperatures in humid climates.
12. In winters, it is often warmer in San Francisco than in nearby Sacramento, 150 km inland. In summers, it is nearly always hotter in Sacramento. Explain how the bodies of water surrounding San Francisco moderate its extreme temperatures.
13. Putting a lid on a boiling pot greatly reduces the heat transfer necessary to keep it boiling. Explain why.
14. Freeze-dried foods have been dehydrated in a vacuum. During the process, the food freezes and must be heated to facilitate dehydration. Explain both how the vacuum speeds up dehydration and why the food freezes as a result.
15. When still air cools by radiating at night, it is unusual for temperatures to fall below the dew point. Explain why.
16. In a physics classroom demonstration, an instructor inflates a balloon by mouth and then cools it in liquid nitrogen. When cold, the shrunken balloon has a small amount of light blue liquid in it, as well as some snow-like crystals. As it warms up, the liquid boils, and part of the crystals sublimate, with some crystals lingering for awhile and then producing a liquid. Identify the blue liquid and the two solids in the cold balloon. Justify your identifications using data from Table 12.2.

12.4 Heat Transfer Methods

17. What are the main methods of heat transfer from the hot core of Earth to its surface? From Earth's surface to outer space?
18. When our bodies get too warm, they respond by sweating and increasing blood circulation to the surface to transfer thermal energy away from the core. What effect will this have on a person in a $40.0^{\circ}\ \mathrm{C}$ hot tub?
19. Figure 12.30 shows a cut-away drawing of a thermos bottle (also known as a Dewar flask), which is a device designed specifically to slow down all forms of heat transfer. Explain the functions of the various parts, such as the vacuum, the

silvering of the walls, the thin-walled long glass neck, the rubber support, the air layer, and the stopper.

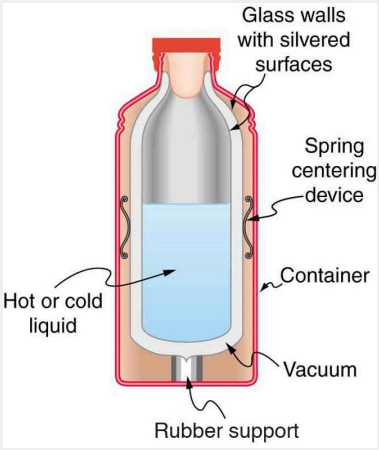


Figure 12.30 The construction of a thermos bottle is designed to inhibit all methods of heat transfer. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a labeled diagram of a thermos. It shows the following components:

- *Glass walls with silvered surfaces:* The outer part of the inner bottle, depicted in gray.
- *Spring centering device:* Springs depicted as black zigzag lines on either side of the inner bottle.
- *Container:* The outer bottle case shown in red.
- *Hot or cold liquid:* A blue liquid inside the inner bottle, representing the contents of the thermos.
- *Vacuum:* The space between the glass walls and the container, represented as a blank gap.
- *Rubber support:* A rubber part at the bottom of the inner bottle, depicted in gray.

12.5 Conduction

20. Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimeters away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?
21. Loose-fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.



Figure 12.31 A jellabiya is worn by many men in Egypt. Image by Zerida, Wikimedia Commons License

12.6 Convection

22. One way to make a fireplace more energy efficient is to have an external air supply for the combustion of its fuel. Another is to have room air circulate around the outside of the fire box and back into the room. Detail the methods of heat transfer involved in each.
23. On cold, clear nights horses will sleep under the cover of large trees. How does this help them keep warm?

12.7 Radiation

24. When watching a daytime circus in a large, dark-colored tent, you sense significant heat transfer from the tent. Explain why this occurs.
25. Satellites designed to observe the radiation from cold (3 K) dark space have sensors that are shaded from the Sun, Earth, and Moon and that are cooled to very low temperatures. Why must the sensors be at low temperature?
26. Why are cloudy nights generally warmer than clear ones?
27. Why are thermometers that are used in weather stations shielded from the sunshine? What does a thermometer measure if it is shielded from the sunshine and also if it is not?
28. On average, would Earth be warmer or cooler without the atmosphere? Explain your answer.

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CHAPTER SUMMARY

12.1 Heat

- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- Any energy unit can be used for heat transfer, and the most common are kilocalorie (kcal) and joule (J).
- Kilocalorie is defined to be the energy needed to change the temperature of 1.00 kg of water between $14.5^{\circ}\ C$ and $15.5^{\circ}\ C$.
- The mechanical equivalent of this heat transfer is $1.00\ \text{kcal} = 4186\ \text{J}$.

12.2 Temperature Change and Heat Capacity

- The transfer of heat Q that leads to a change ΔT in the temperature of a body with mass m is $Q = \text{mc}\Delta T$, where c is the specific heat of the material. This relationship can also be considered as the definition of specific heat.

12.3 Phase Change and Latent Heat

- Most substances can exist either in solid, liquid, and gas forms, which are referred to as “phases.”
- Phase changes occur at fixed temperatures for a given substance at a given pressure, and these temperatures are called boiling and freezing (or melting) points.
- During phase changes, heat absorbed or released is given by:

$$Q = \mathrm{mL},$$

where L is the latent heat coefficient.

12.4 Heat Transfer Methods

- Heat is transferred by three different methods: conduction, convection, and radiation.

12.5 Conduction

- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- The rate of heat transfer Q/t (energy per unit time) is proportional to the temperature difference $T_2 - T_1$ and the contact area A and inversely proportional to the distance d between the objects: $\frac{Q}{t} = \frac{\text{kA}\,(T_2 - T_1)}{d}$.

12.6 Convection

- Convection is heat transfer by the macroscopic movement of mass. Convection can be natural or forced and generally transfers thermal energy faster than conduction. Table 14.4 gives wind-chill factors, indicating that moving air has the same chilling effect of much colder stationary air. *Convection that occurs along with a phase change* can transfer energy from cold regions to warm ones.

12.7 Radiation

- Radiation is the rate of heat transfer through the emission or absorption of electromagnetic waves.
- The rate of heat transfer depends on the surface area and the fourth power of the absolute temperature:

$$\frac{Q}{t} = \sigma e A T^4,$$

where $\sigma = 5.67 \times 10^{-8} \text{J/s} \cdot \text{m}^2 \cdot \text{K}^4$ is the Stefan-Boltzmann constant and e is the emissivity of the body. For a black body, $e = 1$ whereas a shiny white or perfect reflector has $e = 0$, with real objects having values of e between 1 and 0. The net rate of heat transfer by radiation is

$$\frac{Q_{\text{net}}}{t} = \sigma e A \left(T_2^4 - T_1^4\right)$$

where T_1 is the temperature of an object surrounded by an environment with uniform temperature T_2 and e is the emissivity of the *object*.

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PROBLEMS & EXERCISES

Problems & Exercises

12.2 Temperature Change and Heat Capacity

1. On a hot day, the temperature of an 80,000-L swimming pool increases by $1.50^{\circ}\ \mathrm{C}$. What is the net heat transfer during this heating? Ignore any complications, such as loss of water by evaporation.
2. Show that $1\mathrm{cal/g}\cdot^{\circ}\ \mathrm{C} = 1\mathrm{kcal/kg}\cdot^{\circ}\ \mathrm{C}$.
3. To sterilize a 50.0-g glass baby bottle, we must raise its temperature from $22.0^{\circ}\ \mathrm{C}$ to $95.0^{\circ}\ \mathrm{C}$. How much heat transfer is required?
4. The same heat transfer into identical masses of different substances produces different temperature changes. Calculate the final temperature when 1.00 kcal of heat transfers into 1.00 kg of the following, originally at $20.0^{\circ}\ \mathrm{C}$: (a) water; (b) concrete; (c) steel; and (d) mercury.
5. Rubbing your hands together warms them by converting work into thermal energy. If a woman rubs her hands back and forth for a total of 20 rubs, at a distance of 7.50 cm per rub, and with an average frictional force of 40.0 N, what is the temperature increase? The mass of tissues warmed is only 0.100 kg, mostly in the palms and fingers.
6. A 0.250-kg block of a pure material is heated from $20.0^{\circ}\ \mathrm{C}$ to $65.0^{\circ}\ \mathrm{C}$ by the addition of 4.35 kJ of energy. Calculate its specific heat and identify the substance of which it is most likely composed.
7. Suppose identical amounts of heat transfer into different masses of copper and water, causing identical changes in temperature. What is the ratio of the mass of copper to water?
8. (a) The number of kilocalories in food is determined by calorimetry techniques in which the food is burned and the amount of heat transfer is measured. How many kilocalories per gram are there in a 5.00-g peanut if the energy from burning it is transferred to 0.500 kg of water held in a 0.100-kg aluminum cup,

causing a $54.9^{\circ}\ \mathrm{C}$ temperature increase? (b) Compare your answer to labeling information found on a package of peanuts and comment on whether the values are consistent.

9. Following vigorous exercise, the body temperature of an 80.0-kg person is $40.0^{\circ}\ \mathrm{C}$. At what rate in watts must the person transfer thermal energy to reduce the the body temperature to $37.0^{\circ}\ \mathrm{C}$ in 30.0 min, assuming the body continues to produce energy at the rate of 150 W?
$(1\ \text{watt}\ =\ 1\ \text{joule/second or}\ 1\ \mathrm{W}\ =\ 1\ \mathrm{J/s})$.
10. Even when shut down after a period of normal use, a large commercial nuclear reactor transfers thermal energy at the rate of 150 MW by the radioactive decay of fission products. This heat transfer causes a rapid increase in temperature if the cooling system fails (1 watt = 1 joule/second or 1 W = 1 J/s and 1 MW = 1 megawatt). (a) Calculate the rate of temperature increase in degrees Celsius per second $(^{\circ}\ \mathrm{C/s}\ )$ if the mass of the reactor core is $1.60 \times 10^{5}\mathrm{kg}$ and it has an average specific heat of $0.3349\ \mathrm{kJ/kg^{\circ}\ \cdot C}$. (b) How long would it take to obtain a temperature increase of $2000^{\circ}\ \mathrm{C}$, which could cause some metals holding the radioactive materials to melt? (The initial rate of temperature increase would be greater than that calculated here because the heat transfer is concentrated in a smaller mass. Later, however, the temperature increase would slow down because the $5 \times 10^{5} - \mathrm{kg}$ steel containment vessel would also begin to heat up.)

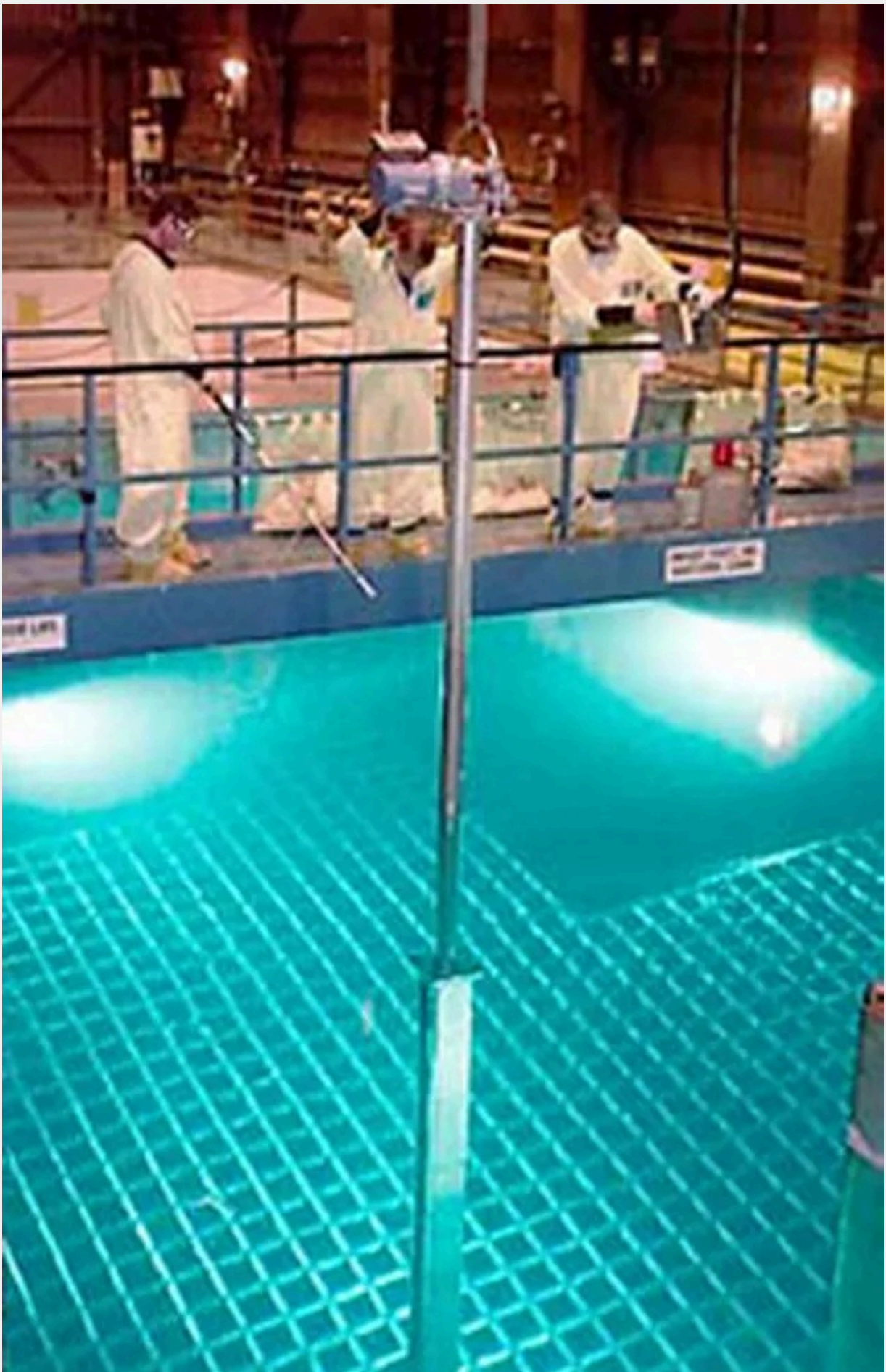


Figure 12.32 Radioactive spent-fuel pool at a nuclear power plant. Spent fuel stays hot for a long time. Image by DOE, Wikimedia Commons License

12.3 Phase Change and Latent Heat

11. How much heat transfer (in kilocalories) is required to thaw a 0.450-kg package of frozen vegetables originally at $0^{\circ}\ \mathrm{C}$ if their heat of fusion is the same as that of water?
12. A bag containing $0^{\circ}\ \mathrm{C}$ ice is much more effective in absorbing energy than one containing the same amount of $0^{\circ}\ \mathrm{C}$ water. How much heat transfer is necessary to raise the temperature of 0.800 kg of water from $0^{\circ}\ \mathrm{C}$ to $30.0^{\circ}\ \mathrm{C}$?
    How much heat transfer is required to first melt 0.800 kg of $0^{\circ}\ \mathrm{C}$ ice and then raise its temperature?
    Explain how your answer supports the contention that the ice is more effective.
13. (a) How much heat transfer is required to raise the temperature of a 0.750-kg aluminum pot containing 2.50 kg of water from $30.0^{\circ}\ \mathrm{C}$ to the boiling point and then boil away 0.750 kg of water? (b) How long does this take if the rate of heat transfer is 500 W $1\mathrm{watt} = 1\ \mathrm{joule/second}\ (1\ \mathrm{W} = 1\ \mathrm{J/s})$?
14. The formation of condensation on a glass of ice water causes the ice to melt faster than it would otherwise. If 8.00 g of condensation forms on a glass containing both water and 200 g of ice, how many grams of the ice will melt as a result? Assume no other heat transfer occurs.
15. On a trip, you notice that a 3.50-kg bag of ice lasts an average of one day in your cooler. What is the average power in watts entering the ice if it starts at $0^{\circ}\ \mathrm{C}$ and completely melts to $0^{\circ}\ \mathrm{C}$ water in exactly one day
    $1\ \mathrm{watt} = 1\ \mathrm{joule/second}\ (1\ \mathrm{W} = 1\ \mathrm{J/s})$?
16. On a certain dry sunny day, a swimming pool's temperature would rise by $1.50^{\circ}\ \mathrm{C}$ if not for evaporation. What fraction of the water must evaporate to carry away precisely enough energy to keep the temperature constant?
17. (a) How much heat transfer is necessary to raise the temperature of a 0.200-kg piece of ice from $-20.0^{\circ}\ \mathrm{C}$ to $130^{\circ}\ \mathrm{C}$, including the energy needed for phase changes? (b) How much time is required for each stage, assuming a constant 20.0

kJ/s rate of heat transfer? (c) Make a graph of temperature versus time for this process.

18. In 1986, a gargantuan iceberg broke away from the Ross Ice Shelf in Antarctica. It was approximately a rectangle 160 km long, 40.0 km wide, and 250 m thick. (a) What is the mass of this iceberg, given that the density of ice is $917\ \mathrm{kg/m^3}$? (b) How much heat transfer (in joules) is needed to melt it? (c) How many years would it take sunlight alone to melt ice this thick, if the ice absorbs an average of $100\ \mathrm{W/m^2}$, 12.00 h per day?
19. How many grams of coffee must evaporate from 350 g of coffee in a 100-g glass cup to cool the coffee from $95.0^{\circ}\ \mathrm{C}$ to $45.0^{\circ}\ \mathrm{C}$? You may assume the coffee has the same thermal properties as water and that the average heat of vaporization is 2340 kJ/kg (560 cal/g). (You may neglect the change in mass of the coffee as it cools, which will give you an answer that is slightly larger than correct.)
20. (a) It is difficult to extinguish a fire on a crude oil tanker, because each liter of crude oil releases $2.80 \times 10^7\ \mathrm{J}$ of energy when burned. To illustrate this difficulty, calculate the number of liters of water that must be expended to absorb the energy released by burning 1.00 L of crude oil, if the water has its temperature raised from $20.0^{\circ}\ \mathrm{C}$ to $100^{\circ}\ \mathrm{C}$, it boils, and the resulting steam is raised to $300^{\circ}\ \mathrm{C}$. (b) Discuss additional complications caused by the fact that crude oil has a smaller density than water.
21. The energy released from condensation in thunderstorms can be very large. Calculate the energy released into the atmosphere for a small storm of radius 1 km, assuming that 1.0 cm of rain is precipitated uniformly over this area.
22. To help prevent frost damage, 4.00 kg of $0^{\circ}\ \mathrm{C}$ water is sprayed onto a fruit tree. (a) How much heat transfer occurs as the water freezes? (b) How much would the temperature of the 200-kg tree decrease if this amount of heat transferred from the tree? Take the specific heat to be $3.35\ \mathrm{kJ/kg \cdot ^{\circ}\ C}$, and assume that no phase change occurs.
23. A 0.250-kg aluminum bowl holding 0.800 kg of soup at $25.0^{\circ}\ \mathrm{C}$ is placed in a freezer. What is the final temperature if 377 kJ of energy is transferred from the bowl and soup, assuming the soup's thermal properties are the same as that of water? Explicitly show how you follow the steps in

Problem-Solving Strategies for the Effects of Heat Transfer.

24. A 0.0500-kg ice cube at $-30.0^{\circ}\ \mathrm{C}$ is placed in 0.400 kg of $35.0^{\circ}\ \mathrm{C}$ water in a very well-insulated container. What is the final temperature?
25. If you pour 0.0100 kg of $20.0^{\circ}\ \mathrm{C}$ water onto a 1.20-kg block of ice (which is initially at $-15.0^{\circ}\ \mathrm{C}$), what is the final temperature? You may assume that the water cools so rapidly that effects of the surroundings are negligible.
26. Indigenous people sometimes cook in watertight baskets by placing hot rocks into water to bring it to a boil. What mass of $500^{\circ}\ \mathrm{C}$ rock must be placed in 4.00 kg of $15.0^{\circ}\ \mathrm{C}$ water to bring its temperature to $100^{\circ}\ \mathrm{C}$, if 0.0250 kg of water escapes as vapor from the initial sizzle? You may neglect the effects of the surroundings and take the average specific heat of the rocks to be that of granite.
27. What would be the final temperature of the pan and water in Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan if 0.260 kg of water was placed in the pan and 0.0100 kg of the water evaporated immediately, leaving the remainder to come to a common temperature with the pan?
28. In some countries, liquid nitrogen is used on dairy trucks instead of mechanical refrigerators. A 3.00-hour delivery trip requires 200 L of liquid nitrogen, which has a density of $808\ \mathrm{kg/m^3}$. (a) Calculate the heat transfer necessary to evaporate this amount of liquid nitrogen and raise its temperature to $3.00^{\circ}\ \mathrm{C}$. (Use c_p and assume it is constant over the temperature range.) This value is the amount of cooling the liquid nitrogen supplies. (b) What is this heat transfer rate in kilowatt-hours? (c) Compare the amount of cooling obtained from melting an identical mass of $0^{\circ}\ \mathrm{C}$ ice with that from evaporating the liquid nitrogen.
29. Some gun fanciers make their own bullets, which involves melting and casting the lead slugs. How much heat transfer is needed to raise the temperature and melt 0.500 kg of lead, starting from $25.0^{\circ}\ \mathrm{C}$?

12.5 Conduction

30. (a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool.

Assume there are no windows or doors. The surface area of the walls is 120m^2 and their inside surface is at 18.0°C, while their outside surface is at 5.00°C. (b) How many 1-kW room heaters would be needed to balance the heat transfer due to conduction?

31. The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through a $3.00-\text{m}^2$ window that is 0.635 cm thick (1/4 in) if the temperatures of the inner and outer surfaces are 5.00°C and -10.0°C, respectively. This rapid rate will not be maintained—the inner surface will cool, and even result in frost formation.
32. Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature is 37.0°C, the skin temperature is 34.0°C, the thickness of the tissues between averages 1.00 cm, and the surface area is 1.40m^2.
33. Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area of 80.0cm^2 with each foot. Both the ceramic and the carpet are 2.00 cm thick and are 10.0°C on their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at 33.0°C? For ceramic, use the thermal conductivity for glass.
34. A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?
35. (a) A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00-mm-thick callus with a conductivity at the low end of the range for wood and its density is 300 kg/m^3. The area of contact is 25.0 cm^2, the temperature of the coals is 700°C, and the time in contact is 1.00 s. (b) What temperature increase is produced in the 25.0 cm^3 of tissue affected? (c) What effect do you think this will have on the tissue, keeping in mind that a callus is made of dead cells?
36. (a) What is the rate of heat conduction through the 3.00-cm-thick fur of a large animal having a $1.40-\text{m}^2$ surface area? Assume that the animal's skin

temperature is $32.0^{\circ}\ \mathrm{C}$, that the air temperature is $-5.00^{\circ}\ \mathrm{C}$, and that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?

37. A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed in $-1.00^{\circ}\ \mathrm{C}$ water. The walrus's internal core temperature is $37.0^{\circ}\ \mathrm{C}$, and it has a surface area of $2.00\mathrm{m}^2$. What is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?



Figure 12.33 Walrus on ice. Image from OpenStax College Physics 2e, CC-BY 4.0

38. Compare the rate of heat conduction through a 13.0-cm-thick wall that has an area of $10.0\ \mathrm{m}^2$ and a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area of $2.00\ \mathrm{m}^2$, assuming the same temperature difference across each.
39. Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is $1.40\ \mathrm{m}^2$?
40. Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in Example 12.6, what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

41. One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.
42. (a) Calculate the rate of heat conduction through a double-paned window that has a $1.50 - \text{m}^2$ area and is made of two panes of 0.800-cm-thick glass separated by a 1.00-cm air gap. The inside surface temperature is $15.0^\circ\ \text{C}$, while that on the outside is $-10.0^\circ\ \text{C}$. (Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.) (b) Calculate the rate of heat conduction through a 1.60-cm-thick window of the same area and with the same temperatures. Compare your answer with that for part (a).
43. Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years.) Suppose you wish to install the extra insulation in Exercise 12.41. If energy cost \$1.00 per million joules and the insulation was \$4.00 per square meter, then calculate the simple payback time. Take the average ΔT for the 120 day heating season to be $15.0^\circ\ \text{C}$.
44. For the human body, what is the rate of heat transfer by conduction through the body's tissue with the following conditions: the tissue thickness is 3.00 cm, the change in temperature is $2.00^\circ\ \text{C}$, and the skin area is $1.50\ \text{m}^2$. How does this compare with the average heat transfer rate to the body resulting from an energy intake of about 2400 kcal per day? (No exercise is included.)

12.6 Convection

45. At what wind speed does $-10^\circ\ \text{C}$ air cause the same chill factor as still air at $-29^\circ\ \text{C}$?

46. At what temperature does still air cause the same chill factor as $-5^{\circ}\ \mathrm{C}$ air moving at 15 m/s?
47. The "steam" above a freshly made cup of instant coffee is really water vapor droplets condensing after evaporating from the hot coffee. What is the final temperature of 250 g of hot coffee initially at $90.0^{\circ}\ \mathrm{C}$ if 2.00 g evaporates from it? The coffee is in a Styrofoam cup, so other methods of heat transfer can be neglected.
48. (a) How many kilograms of water must evaporate from a 60.0-kg woman to lower her body temperature by $0.750^{\circ}\ \mathrm{C}$? (b) Is this a reasonable amount of water to evaporate in the form of perspiration, assuming the relative humidity of the surrounding air is low?
49. On a hot dry day, evaporation from a lake has just enough heat transfer to balance the $1.00\ \mathrm{kW/m^2}$ of incoming heat from the Sun. What mass of water evaporates in 1.00 h from each square meter? Explicitly show how you follow the steps in the Problem-Solving Strategies for the Effects of Heat Transfer.
50. One winter day, the climate control system of a large university classroom building malfunctions. As a result, $500\ \mathrm{m^3}$ of excess cold air is brought in each minute. At what rate in kilowatts must heat transfer occur to warm this air by $10.0^{\circ}\ \mathrm{C}$ (that is, to bring the air to room temperature)?
51. The Kilauea volcano in Hawaii is the world's most active, disgorging about $5\ \times 10^5\mathrm{m^3}$ of $1200^{\circ}\ \mathrm{C}$ lava per day. What is the rate of heat transfer out of Earth by convection if this lava has a density of $2700\mathrm{kg/m^3}$ and eventually cools to $30^{\circ}\ \mathrm{C}$? Assume that the specific heat of lava is the same as that of granite.



Figure 12.34 Lava flow on Kilauea volcano in Hawaii. Image from OpenStax College Physics 2e, CC-BY 4.0

52. During heavy exercise, the body pumps 2.00 L of blood per minute to the surface, where it is cooled by $2.00^{\circ}\ \mathrm{C}$. What is the rate of heat transfer from this forced convection alone, assuming blood has the same specific heat as water and its density is $1050\ \mathrm{kg/m^3}$?
53. A person inhales and exhales 2.00 L of $37.0^{\circ}\ \mathrm{C}$ air, evaporating $4.00 \times 10^{-2}\ \mathrm{g}$ of water from the lungs and breathing passages with each breath. (a) How much heat transfer occurs due to evaporation in each breath? (b) What is the rate of heat transfer in watts if the person is breathing at a moderate rate of 18.0 breaths per minute? (c) If the inhaled air had a temperature of $20.0^{\circ}\ \mathrm{C}$, what is the rate of heat transfer for warming the air? (d) Discuss the total rate of heat transfer as it relates to typical metabolic rates. Will this breathing be a major form of heat transfer for this person?
54. A glass coffee pot has a circular bottom with a 9.00-cm diameter in contact with a heating element that keeps the coffee warm with a continuous heat transfer rate of 50.0 W (a) What is the temperature of the bottom of the pot, if it is 3.00 mm thick and the inside temperature is $60.0^{\circ}\ \mathrm{C}$? (b) If the temperature of the coffee remains constant and all of the heat transfer is removed by evaporation, how many grams per minute evaporate? Take the heat of vaporization to be 2340 kJ/

kg.

12.7 Radiation

55. At what net rate does heat radiate from a $275-\mathrm{m}^2$ black roof on a night when the roof's temperature is $30.0^{\circ}\ \mathrm{C}$ and the surrounding temperature is $15.0^{\circ}\ \mathrm{C}$? The emissivity of the roof is 0.900.
56. (a) Cherry-red embers in a fireplace are at $850^{\circ}\ \mathrm{C}$ and have an exposed area of $0.200\ \mathrm{m}^2$ and an emissivity of 0.980. The surrounding room has a temperature of $18.0^{\circ}\ \mathrm{C}$. If 50% of the radiant energy enters the room, what is the net rate of radiant heat transfer in kilowatts? (b) Does your answer support the contention that most of the heat transfer into a room by a fireplace comes from infrared radiation?
57. Radiation makes it impossible to stand close to a hot lava flow. Calculate the rate of heat transfer by radiation from $1.00\ \mathrm{m}^2$ of $1200^{\circ}\ \mathrm{C}$ fresh lava into $30.0^{\circ}\ \mathrm{C}$ surroundings, assuming lava's emissivity is 1.00.
58. (a) Calculate the rate of heat transfer by radiation from a car radiator at $110^{\circ}\mathrm{C}$ into a $50.0^{\circ}\ \mathrm{C}$ environment, if the radiator has an emissivity of 0.750 and a $1.20-m^2$ surface area. (b) Is this a significant fraction of the heat transfer by an automobile engine? To answer this, assume a horsepower of $200\mathrm{hp}\ (1.5\mathrm{kW})$ and the efficiency of automobile engines as 25%.
59. Find the net rate of heat transfer by radiation from a skier standing in the shade, given the following. She is completely clothed in white (head to foot, including a ski mask), the clothes have an emissivity of 0.200 and a surface temperature of $10.0^{\circ}\ \mathrm{C}$, the surroundings are at $-15.0^{\circ}\ \mathrm{C}$, and her surface area is $1.60\mathrm{m}^2$.
60. Suppose you walk into a sauna that has an ambient temperature of $50.0^{\circ}\ \mathrm{C}$. (a) Calculate the rate of heat transfer to you by radiation given your skin temperature is $37.0^{\circ}\ \mathrm{C}$, the emissivity of skin is 0.98, and the surface area of your body is $1.50\mathrm{m}^2$. (b) If all other forms of heat transfer are balanced (the net heat transfer is zero), at what rate will your body temperature increase if your mass is 75.0 kg?
61. Thermography is a technique for measuring radiant heat and detecting variations in surface temperatures that may be medically, environmentally, or militarily

meaningful.(a) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of $34.0^{\circ}\ \mathrm{C}$ compared with that at $33.0^{\circ}\ \mathrm{C}$, such as on a person's skin? (b) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of $34.0^{\circ}\ \mathrm{C}$ compared with that at $20.0^{\circ}\ \mathrm{C}$, such as for warm and cool automobile hoods?

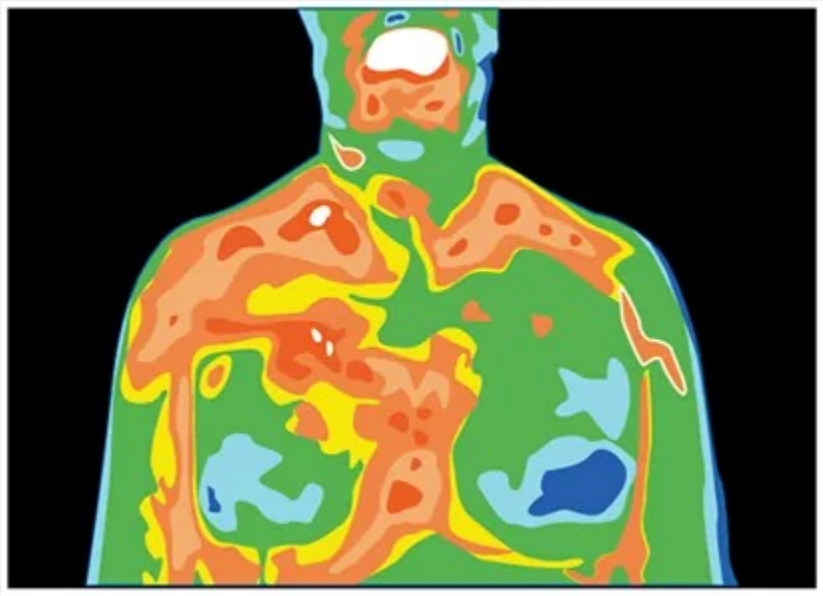


Figure 12.35 Artist's rendition of a thermograph of a patient's upper body, showing the distribution of heat represented by different colors. Image from OpenStax College Physics 2e, CC-BY 4.0

62. The Sun radiates like a perfect black body with an emissivity of exactly 1. (a) Calculate the surface temperature of the Sun, given that it is a sphere with a $7.00 \times 10^{8} - \mathrm{m}$ radius that radiates $3.80 \times 10^{26}\ \mathrm{W}$ into 3-K space. (b) How much power does the Sun radiate per square meter of its surface? (c) How much power in watts per square meter is that value at the distance of Earth, $1.50 \times 10^{11}\ \mathrm{m}$ away? (This number is called the solar constant.)
63. A large body of lava from a volcano has stopped flowing and is slowly cooling. The interior of the lava is at $1200^{\circ}\ \mathrm{C}$, its surface is at $450^{\circ}\ \mathrm{C}$, and the surroundings are at $27.0^{\circ}\ \mathrm{C}$. (a) Calculate the rate at which energy is transferred by radiation from $1.00\ \mathrm{m}^2$ of surface lava into the surroundings, assuming the emissivity is

1.00. (b) Suppose heat conduction to the surface occurs at the same rate. What is the thickness of the lava between the $450^{\circ}\ \mathrm{C}$ surface and the $1200^{\circ}\ \mathrm{C}$ interior, assuming that the lava's conductivity is the same as that of brick?

64. Calculate the temperature the entire sky would have to be in order to transfer energy by radiation at $1000\mathrm{W/m^2}$ —about the rate at which the Sun radiates when it is directly overhead on a clear day. This value is the effective temperature of the sky, a kind of average that takes account of the fact that the Sun occupies only a small part of the sky but is much hotter than the rest. Assume that the body receiving the energy has a temperature of $27.0^{\circ}\ \mathrm{C}$.

65. (a) A shirtless rider under a circus tent feels the heat radiating from the sunlit portion of the tent. Calculate the temperature of the tent canvas based on the following information: The shirtless rider's skin temperature is $34.0^{\circ}\ \mathrm{C}$ and has an emissivity of 0.970. The exposed area of skin is $0.400\ \mathrm{m^2}$. He receives radiation at the rate of 20.0 W—half what you would calculate if the entire region behind him was hot. The rest of the surroundings are at $34.0^{\circ}\ \mathrm{C}$. (b) Discuss how this situation would change if the sunlit side of the tent was nearly pure white and if the rider was covered by a white tunic.

66. **Integrated Concepts:** One $30.0^{\circ}\ \mathrm{C}$ day the relative humidity is 75.0%, and that evening the temperature drops to $20.0^{\circ}\ \mathrm{C}$, well below the dew point. (a) How many grams of water condense from each cubic meter of air? (b) How much heat transfer occurs by this condensation? (c) What temperature increase could this cause in dry air?

67. **Integrated Concepts:** Large meteors sometimes strike the Earth, converting most of their kinetic energy into thermal energy. (a) What is the kinetic energy of a $10^9\ \mathrm{kg}$ meteor moving at 25.0 km/s? (b) If this meteor lands in a deep ocean and 80% of its kinetic energy goes into heating water, how many kilograms of water could it raise by $5.0^{\circ}\ \mathrm{C}$? (c) Discuss how the energy of the meteor is more likely to be deposited in the ocean and the likely effects of that energy.

68. **Integrated Concepts:** Frozen waste from airplane toilets has sometimes been accidentally ejected at high altitude. Ordinarily it breaks up and disperses over a large area, but sometimes it holds together and strikes the ground. Calculate the mass of $0^{\circ}\ \mathrm{C}$ ice that can be melted by the conversion of kinetic and gravitational

potential energy when a $20.0\ \text{kg}$ piece of frozen waste is released at 12.0 km altitude while moving at 250 m/s and strikes the ground at 100 m/s (since less than 20.0 kg melts, a significant mess results).

69. **Integrated Concepts:** (a) A large electrical power facility produces 1600 MW of "waste heat," which is dissipated to the environment in cooling towers by warming air flowing through the towers by $5.00^{\circ}\ \text{C}$. What is the necessary flow rate of air in m^3/s? (b) Is your result consistent with the large cooling towers used by many large electrical power plants?
70. **Integrated Concepts:** (a) Suppose you start a workout on a Stairmaster, producing power at the same rate as climbing 116 stairs per minute. Assuming your mass is 76.0 kg and your efficiency is 20.0%, how long will it take for your body temperature to rise $1.00^{\circ}\ \text{C}$ if all other forms of heat transfer in and out of your body are balanced? (b) Is this consistent with your experience in getting warm while exercising?
71. **Integrated Concepts:** A 76.0-kg person suffering from hypothermia comes indoors and shivers vigorously. How long does it take the heat transfer to increase the person's body temperature by $2.00^{\circ}\ \text{C}$ if all other forms of heat transfer are balanced?
72. **Integrated Concepts:** In certain large geographic regions, the underlying rock is hot. Wells can be drilled and water circulated through the rock for heat transfer for the generation of electricity. (a) Calculate the heat transfer that can be extracted by cooling $1.00\ \text{km}^3$ of granite by $100^{\circ}\ \text{C}$. (b) How long will this take if heat is transferred at a rate of 300 MW, assuming no heat transfers back into the 1.00 km of rock by its surroundings?
73. **Integrated Concepts:** Heat transfers from your lungs and breathing passages by evaporating water. (a) Calculate the maximum number of grams of water that can be evaporated when you inhale 1.50 L of $37^{\circ}\ \text{C}$ air with an original relative humidity of 40.0%. (Assume that body temperature is also $37^{\circ}\ \text{C}$.) (b) How many joules of energy are required to evaporate this amount? (c) What is the rate of heat transfer in watts from this method, if you breathe at a normal resting rate of 10.0 breaths per minute?
74. **Integrated Concepts:** (a) What is the temperature increase of water falling 55.0

m over Niagara Falls? (b) What fraction must evaporate to keep the temperature constant?

75. **Integrated Concepts:** Hot air rises because it has expanded. It then displaces a greater volume of cold air, which increases the buoyant force on it. (a) Calculate the ratio of the buoyant force to the weight of $50.0^{\circ}\ \mathrm{C}$ air surrounded by $20.0^{\circ}\ \mathrm{C}$ air. (b) What energy is needed to cause $1.00\ \mathrm{m}^3$ of air to go from $20.0^{\circ}\ \mathrm{C}$ to $50.0^{\circ}\ \mathrm{C}$? (c) What gravitational potential energy is gained by this volume of air if it rises 1.00 m? Will this cause a significant cooling of the air?
76. **Unreasonable Results:** (a) What is the temperature increase of an 80.0 kg person who consumes 2500 kcal of food in one day with 95.0% of the energy transferred as heat to the body? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?
77. **Unreasonable Results:** An Arctic inventor surrounded by ice thinks it would be much less mechanically complex to cool a car engine by melting ice on it than by having a water-cooled system with a radiator, water pump, antifreeze, and so on. (a) If 80.0% of the energy in 1.00 gal of gasoline is converted into "waste heat" in a car engine, how many kilograms of $0^{\circ}\ \mathrm{C}$ ice could it melt? (b) Is this a reasonable amount of ice to carry around to cool the engine for 1.00 gal of gasoline consumption? (c) What premises or assumptions are unreasonable?
78. **Unreasonable Results:** (a) Calculate the rate of heat transfer by conduction through a window with an area of $1.00\ \mathrm{m}^2$ that is 0.750 cm thick, if its inner surface is at $22.0^{\circ}\ \mathrm{C}$ and its outer surface is at $35.0^{\circ}\ \mathrm{C}$. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?
79. **Unreasonable Results:** A meteorite 1.20 cm in diameter is so hot immediately after penetrating the atmosphere that it radiates 20.0 kW of power. (a) What is its temperature, if the surroundings are at $20.0^{\circ}\ \mathrm{C}$ and it has an emissivity of 0.800? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?
80. **Construct Your Own Problem:** Consider a new model of commercial airplane having its brakes tested as a part of the initial flight permission procedure. The airplane is brought to takeoff speed and then stopped with the brakes alone. Construct a problem in which you calculate the temperature increase of the brakes

during this process. You may assume most of the kinetic energy of the airplane is converted to thermal energy in the brakes and surrounding materials, and that little escapes. Note that the brakes are expected to become so hot in this procedure that they ignite and, in order to pass the test, the airplane must be able to withstand the fire for some time without a general conflagration.

81. **Construct Your Own Problem:** Consider a person outdoors on a cold night. Construct a problem in which you calculate the rate of heat transfer from the person by all three heat transfer methods. Make the initial circumstances such that at rest the person will have a net heat transfer and then decide how much physical activity of a chosen type is necessary to balance the rate of heat transfer. Among the things to consider are the size of the person, type of clothing, initial metabolic rate, sky conditions, amount of water evaporated, and volume of air breathed. Of course, there are many other factors to consider and your instructor may wish to guide you in the assumptions made as well as the detail of analysis and method of presenting your results.

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OSCILLATORY MOTION AND WAVES



Figure 13.1 There are at least four types of waves in this picture—only the water waves are evident. There are also sound waves, light waves, and waves on the guitar strings. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

13.1 Hooke's Law: Stress and Strain Revisited
13.2 Period and Frequency in Oscillations
13.3 Simple Harmonic Motion: A Special Periodic Motion
13.4 The Simple Pendulum
13.5 Energy and the Simple Harmonic Oscillator
13.6 Uniform Circular Motion and Simple Harmonic Motion
13.7 Damped Harmonic Motion
13.8 Forced Oscillations and Resonance
13.9 Waves
13.10 Superposition and Interference
13.11 Energy in Waves: Intensity
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

What do an ocean buoy, a child in a swing, the cone inside a speaker, a guitar, atoms in a crystal, the motion of chest cavities, and the beating of hearts all have in common? They all oscillate—-that is, they move back and forth between two points. Many systems oscillate, and they have certain characteristics in common. All oscillations involve force and energy. You push a child in a swing to get the motion started. The energy of atoms vibrating in a crystal can be increased with heat. You put energy into a guitar string when you pluck it.

Some oscillations create waves. A guitar creates sound waves. You can make water waves in a swimming pool by slapping the water with your hand. You can no doubt think of other types of waves. Some, such as water waves, are visible. Some, such as sound waves, are not. But *every wave is a disturbance that moves from its source and carries energy*. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves.

By studying oscillatory motion and waves, we shall find that a small number of underlying principles describe all of them and that wave phenomena are more common than you have ever imagined. We begin by studying

the type of force that underlies the simplest oscillations and waves. We will then expand our exploration of oscillatory motion and waves to include concepts such as simple harmonic motion, uniform circular motion, and damped harmonic motion. Finally, we will explore what happens when two or more waves share the same space, in the phenomena known as superposition and interference.
Access multimedia content

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13.1 HOOKE'S LAW: STRESS AND STRAIN REVISITED

Learning Objectives

By the end of this section, you will be able to:

- Explain Newton's third law of motion with respect to stress and deformation.
- Describe the restoration of force and displacement.
- Calculate the energy in Hooke's Law of deformation, and the stored energy in a spring.

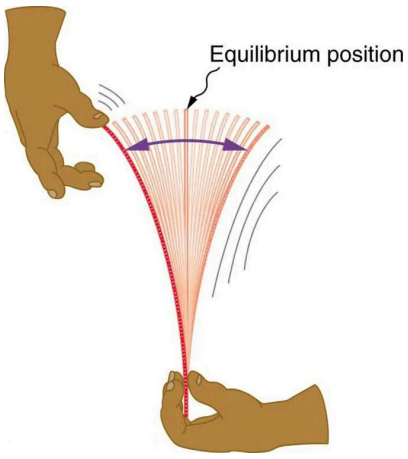


Figure 13.2 When displaced from its vertical equilibrium position, this plastic ruler oscillates back and forth because of the restoring force opposing displacement. When the ruler is on the left, there is a force to the right, and vice versa. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts an illustration of a hand holding a flexible object, like a ruler, being bent. The top hand appears poised above the ruler, while the bottom hand holds it tightly and bends it backwards, creating tension. The ruler bends in an arc, exhibiting several curved lines that indicate its motion and vibration. There are arrows showing the direction of motion towards an "equilibrium position" label, which is central to the bending arc. The arrangement suggests oscillation or vibration as the ruler is released and moves back and forth around the equilibrium position. The background is plain, emphasizing the focus on the hands and the ruler.

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in Figure 13.2. The deformation of the ruler creates a force in the opposite direction, known as a restoring force. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.

The simplest oscillations occur when the restoring force is directly proportional to displacement. When stress and strain were covered in Newton's Third Law of Motion, the name was given to this relationship between force and displacement was Hooke's law:

$$F = -\mathrm{kx}.$$
13.1

Here, F is the restoring force, x is the displacement from equilibrium or deformation, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.

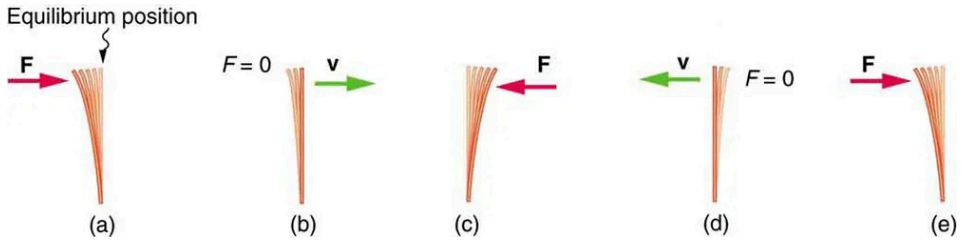


Figure 13.3 (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates the stages of a plucked object, likely a cantilever or beam, undergoing oscillation from an initial displacement due to a force, labeled F. There are five distinct positions, labeled from (a) to (e):

- *(a)*: The object is bent to the left by a force (F) shown with a pink arrow pointing left. The object is displaced from its equilibrium position, which is marked above.
- *(b)*: The force (F) is zero, and the object is returning to its equilibrium position. A green arrow indicates the velocity (v) going to the right.
- *(c)*: The object passes through the equilibrium position. A force (F) is applied from the right with a pink arrow.
- *(d)*: The object continues to the right, and the velocity (v) is depicted with a green arrow pointing left. The force (F) is zero.
- *(e)*: The object bends to the right under the influence of a force (F) shown with a pink arrow pointing right.

The force constant k is related to the rigidity (or stiffness) of a system—the larger the force constant, the greater the restoring force, and the stiffer the system. The units of k are newtons per meter (N/m). For example, k is directly related to Young's modulus when we stretch a string. Figure 13.4 shows a graph of the absolute value of the restoring force versus the displacement for a system that can be described by Hooke's law—a simple spring in this case. The slope of the graph equals the force constant k in newtons per meter. A common physics laboratory exercise is to measure restoring forces created by springs, determine if they follow Hooke's law, and calculate their force constants if they do.

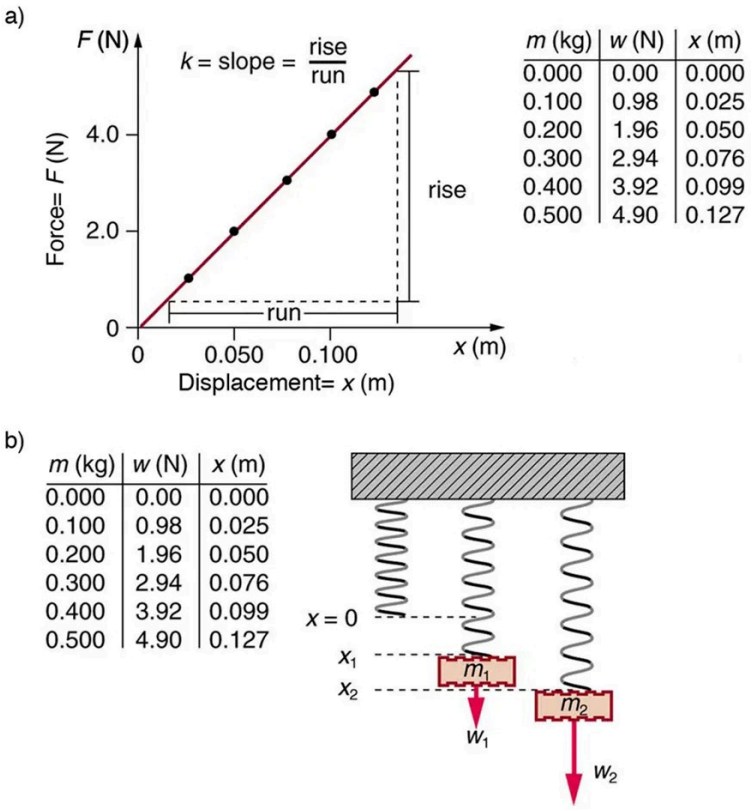


Figure 13.4 (a) A graph of absolute value of the restoring force versus displacement is displayed. The fact that the graph is a straight line means that the system obeys Hooke's law. The slope of the graph is the force constant k*. (b) The data in the graph were generated by measuring the displacement of a spring from equilibrium while supporting various weights. The restoring force equals the weight supported, if the mass is stationary. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two parts labeled a) and b). Below is the description for each part.

a) This section depicts a line graph and a table. The graph shows the relationship between Force (F) in Newtons plotted on the y-axis and Displacement (x) in meters on the x-axis. A red line with plotted points rises diagonally, indicating a linear relationship. Text annotations explain that the slope (k) is calculated as the rise divided by the run. Next to the graph is a table with three columns representing mass (m), weight (w), and displacement (x).

m (kg)	w (N)	x (m)
0.000	0.00	0.000
0.100	0.98	0.025
0.200	1.96	0.050
0.300	2.94	0.076
0.400	3.92	0.099
0.500	4.90	0.127

b) The section includes a similar table as in part a) and a diagram of two vertical springs fixed to a horizontal surface. The first spring is labeled with mass m1 and weight W1, with displacement levels x1 and x2 marked. The second spring, which is identical in form to the first, is similarly labeled with mass m2 and weight W2. Each spring extends under a mass, visualized as rectangles hanging from the spring, with forces marked by downward arrows.

Example 13.1

How Stiff Are Car Springs?



Figure 13.5 The mass of a car increases due to the introduction of a passenger. This affects the displacement of the car on its suspension system. Image by Brian Snelson, Wikimedia Commons License

What is the force constant for the suspension system of a car that settles 1.20 cm when an 80.0-kg person gets in?

Strategy

Consider the car to be in its equilibrium position $x = 0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $x = -1.20 \times 10^{-2}\text{m}$. At that point, the springs supply a restoring force F equal to the person's weight $w = \text{mg} = (80.0\ \text{kg})\left(9.80\text{m/s}^2\right) = 784\text{N}$. We take this force to be F in Hooke's law. Knowing F and x, we can then solve the force constant k.

Solution

1. Solve Hooke's law, $F = -\text{kx}$, for k:

$$k = -\frac{F}{x}.$$

13.2

Substitute known values and solve k:

$$k = -\frac{784\ \text{N}}{-1.20 \times 10^{-2}\ \text{m}} = 6.53 \times 10^{4}\ \text{N/m}.$$

13.3

Discussion

Note that F and x have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in if it were not for damping (due to

frictional forces) provided by shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Energy in Hooke's Law of Deformation

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $\mathrm{PE_s} = \frac{1}{2}kx^2$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke's law. Hence,

$$\mathrm{PE_{el}} = \frac{1}{2}kx^2,$$

13.4

where $\mathrm{PE_{el}}$ is the elastic potential energy stored in any deformed system that obeys Hooke's law and has a displacement x from equilibrium and a force constant k.

It is possible to find the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app}. The applied force is exactly opposite to the restoring force (action-reaction), and so $F_{\mathrm{app}} = \mathrm{kx}$. Figure 13.6 shows a graph of the applied force versus deformation x for a system that can be described by Hooke's law. Work done on the system is force multiplied by distance, which equals the area under the curve or $(1/2)kx^2$(Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx, so that the average force is $(1/2)kx$, the distance moved is x, and thus $W = F_{\mathrm{app}}d = [(1/2)\mathrm{kx}](x) = (1/2)kx^2$ (Method B in the figure).

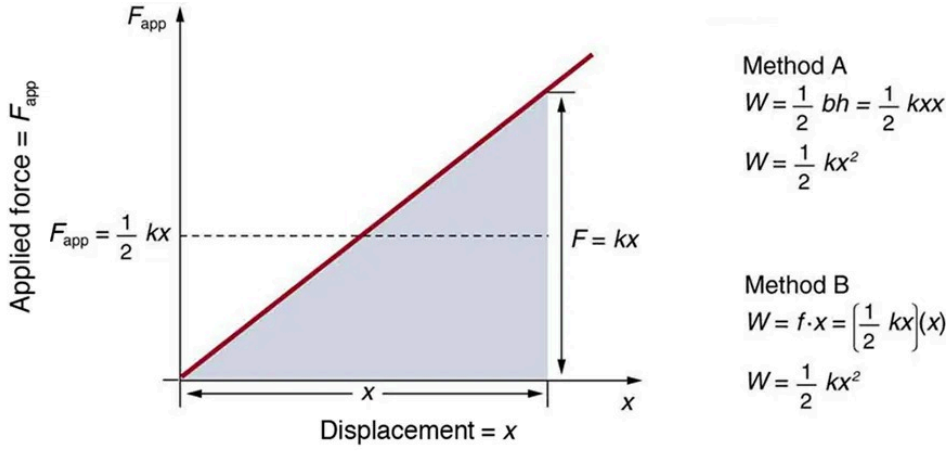


Figure 13.6 A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $W = (1/2)kx^2$. *Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a graph and mathematical formulas illustrating the concept of work done in physics regarding applied force and displacement.

Graph Description:

- **Axes:** The graph has two axes; the vertical axis is labeled "Applied force = F_{app}" and the horizontal axis is labeled "Displacement = x".
- **Line and Area:** There is a red line extending from the origin, showing a linear increase in force with displacement. A shaded triangular area lies under this line, bounded by the x-axis and the vertical dashed line labeled "F = kx".
- **Label:** On the vertical axis, a point is marked with the label "F_{app} = 1/2 kx".
- **Triangle:** The shaded triangle represents the work done, calculated as the area under the force-displacement curve.

Formulas on the Right:

- Two methods for calculating work are shown:
- *Method A:*
  - W = 1/2 bh = 1/2 kxx
  - W = 1/2 kx^2

- *Method B:*
  - $W = f \cdot x = [1/2\ kx](x)$
  - $W = 1/2\ kx^2$

Example 13.2

Calculating Stored Energy: A Toy Gun Spring

We can use a toy gun's spring mechanism to ask and answer two simple questions: (a) How much energy is stored in the spring of a toy gun that has a force constant of 50.0 N/m and is compressed 0.150 m? (b) If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?

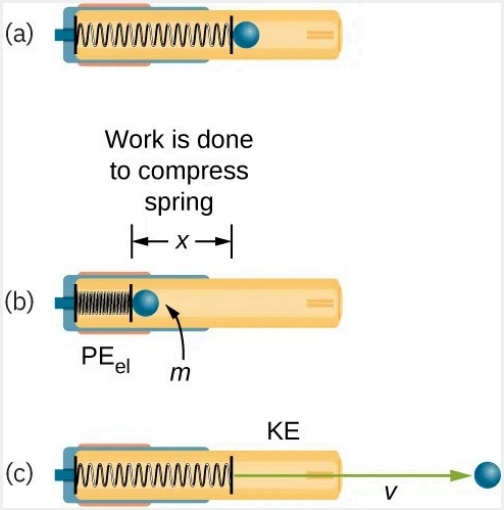


Figure 13.7 (a) In this image of the gun, the spring is uncompressed before being cocked. (b) The spring has been compressed a distance x, and the projectile is in place. (c) When released, the spring converts elastic potential energy $\mathrm{PE}_{\mathrm{el}}$ into kinetic energy. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the conversion of potential energy to kinetic energy using a compressed spring and a ball. It consists of three parts:

- **(a)** A spring inside a cylindrical tube is shown in a relaxed state, with a blue ball resting against the uncompressed end of the spring.

- **(b)** The spring is now compressed, and the ball is closer to the compression point. An arrow labeled "Work is done to compress spring" is pointing down with a label "*x*", representing the compression distance. The compressed spring has a label "PE_{el}" indicating elastic potential energy, and the ball is labeled with "*m*" for mass.
- **(c)** The spring is released, pushing the ball out of the tube. The ball is moving to the right with velocity "*v*", and an arrow labeled "*KE*" represents kinetic energy.

Strategy for a

(a): The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.

Solution for a

Entering the given values for k and x yields

$$\begin{aligned}\mathrm{PE_{el}} &= \frac{1}{2}kx^2 = \frac{1}{2}(50.0\ \mathrm{N/m})(0.150\ \mathrm{m})^2 = 0.563\mathrm{N \cdot m} \\ &= 0.563\mathrm{J}\end{aligned}$$

13.5

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile's speed.

Solution for b

1. Identify known quantities:

   $$\mathrm{KE_f} = \mathrm{PE_{el}}\ or 1/2mv^2 = (1/2)kx^2 = \mathrm{PE}_{el} = 0.563\mathrm{J}$$

   13.6
2. Solve for v:

$$v = \left[\frac{2\text{PE}_{\text{el}}}{m}\right]^{1/2} = \left[\frac{2\,(0.563\text{ J})}{0.002\text{ kg}}\right]^{1/2} = 23.7(\text{J/kg})^{1/2}$$

13.7

3. Convert units: 23.7 m/s

Discussion

(a) and (b): This projectile speed is impressive for a toy gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do, especially because the darts in many of these guns are made of soft material with a rubber tip. Yet, the speed of the dart is great enough for it to travel an acceptable distance.

Check Your Understanding

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Click for Solution

Solution

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

Check Your Understanding

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Click for Solution

Solution

It was stored in the object as potential energy.

"16.1 Hooke's Law: Stress and Strain Revisited" from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

13.2 PERIOD AND FREQUENCY IN OSCILLATIONS

Learning Objectives

By the end of this section, you will be able to:

- Observe the vibrations of a guitar string.
- Determine the frequency of oscillations.

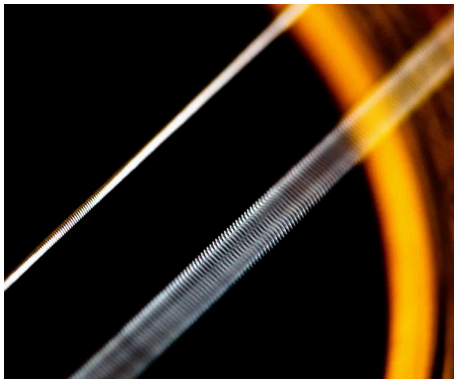


Figure 13.8 The strings on this guitar vibrate at regular time intervals. Image from OpenStax College Physics 2e, CC-BY 4.0

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define periodic motion to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the period T. Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event

whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. Frequency f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

$$f = \frac{1}{T}.$$

13.8

The SI unit for frequency is the *cycle per second*, which is defined to be a *hertz* (Hz):

$$1 \text{ Hz } = 1\frac{\text{cycle}}{\text{s}} \text{or } 1 \text{ Hz } = \frac{1}{\text{s}}$$

13.9

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

Example 13.3

Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let's try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 μs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period /[latex]T[/latex] is given and we are asked to find frequency f. In question (b), the frequency f is given and we are asked to find the period T.

Solution a

1. Substitute $0.\ 400\ \mu\ \text{s}$ for T in $f = \frac{1}{T}$:

$$f = \frac{1}{T} = \frac{1}{0.400 \times 10^{-6} s}.$$

13.10

Solve to find

$$f = 2.50 \times\ 10^{6} \text{Hz}.$$

13.11

Discussion a

The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.

Solution b

1. Identify the known values:The time for one complete oscillation is the period T:

$$f = \frac{1}{T}.$$

13.12

2. Solve for T:

$$T = \frac{1}{f}.$$

13.13

3. Substitute the given value for the frequency into the resulting expression:

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{264\ \text{Hz}} \\ &= \frac{1}{264\ \text{cycles/s}} \\ &= 3.79 \times 10^{-3}\text{s} \\ &= 3.79\ \text{ms}. \end{aligned}$$

13.14

Discussion b

The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).

Check Your Understanding

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

Click for Solution

Solution

I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

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13.3 SIMPLE HARMONIC MOTION: A SPECIAL PERIODIC MOTION

Learning Objectives

By the end of this section, you will be able to:

- Describe a simple harmonic oscillator.
- Explain the link between simple harmonic motion and waves.

The oscillations of a system in which the net force can be described by Hooke's law are of special importance, because they are very common. They are also the simplest oscillatory systems. Simple Harmonic Motion (SHM) is the name given to oscillatory motion for a system where the net force can be described by Hooke's law, and such a system is called a simple harmonic oscillator. If the net force can be described by Hooke's law and there is no *damping* (by friction or other non-conservative forces), then a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown for an object on a spring in Figure 13.9. The maximum displacement from equilibrium is called the amplitude X. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters; whereas for sound oscillations, they have units of pressure (and other types of oscillations have yet other units). Because amplitude is the maximum displacement, it is related to the energy in the oscillation.

Take-Home Experiment: SHM and the Marble

Find a bowl or basin that is shaped like a hemisphere on the inside. Place a marble inside the bowl and tilt the bowl periodically so the marble rolls from the bottom of the bowl to equally high points on the sides of the bowl. Get a feel for the force required to maintain this periodic motion. What is the restoring force and what role does the force you apply play in the simple harmonic motion (SHM) of the marble?

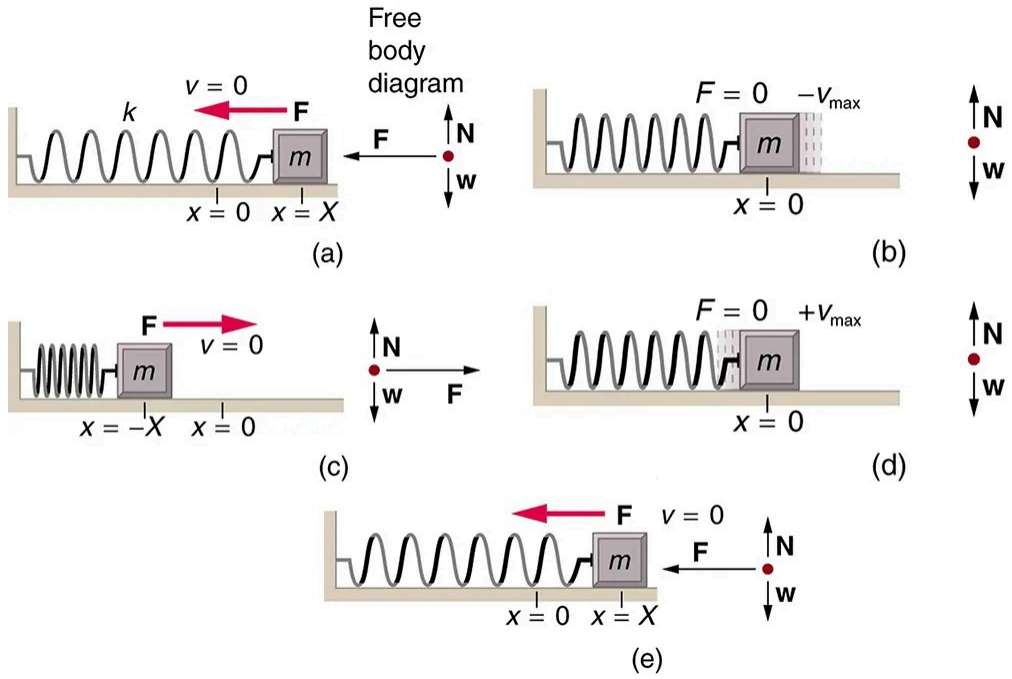


Figure 13.9 An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of five diagrams labeled (a) through (e), illustrating the motion of a block attached to a spring, showing the forces and velocities at different positions.

- **(a)** A block with mass m is at position $x = X$, compressed against a spring with spring constant k. The velocity v is 0, and a leftward force F is shown by an arrow. A free body diagram nearby shows forces: N (normal), w (weight), and F.
- **(b)** The block is at equilibrium position $x = 0$. Velocity is $-v_{max}$ (maximum leftward). No external force F is applied, as indicated by $F = 0$. A free body diagram shows N and w.
- **(c)** The block is at position $x = -X$, with no velocity (v = 0). A rightward force F, shown by an arrow, acts on the block. A free body diagram shows N, w, and F.
- **(d)** The block is again at equilibrium position $x = 0$, moving with velocity $+v_{max}$ (maximum rightward). No external force F is applied. A free body diagram is similar to (b).
- **(e)** Similar to (a), the block is compressed at $x = X$ with velocity $v = 0$. A leftward force F is shown, and the free body diagram displays N, w, and F.

What is so significant about simple harmonic motion? One special thing is that the period T and frequency f of a simple harmonic oscillator are independent of amplitude. The string of a guitar, for example, will oscillate with the same frequency whether plucked gently or hard. Because the period is constant, a simple harmonic oscillator can be used as a clock.

Two important factors do affect the period of a simple harmonic oscillator. The period is related to how stiff the system is. A very stiff object has a large force constant k, which causes the system to have a smaller period. For example, you can adjust a diving board's stiffness—the stiffer it is, the faster it vibrates, and the shorter its period. Period also depends on the mass of the oscillating system. The more massive the system is, the longer the period. For example, a heavy person on a diving board bounces up and down more slowly than a light one.

In fact, the mass m and the force constant k are the *only* factors that affect the period and frequency of simple harmonic motion.

Period of Simple Harmonic Oscillator

The *period of a simple harmonic oscillator* is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

13.15

and, because $f = 1/T$, the *frequency of a simple harmonic oscillator* is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

13.16

Note that neither T nor f has any dependence on amplitude.

Take-Home Experiment: Mass and Ruler Oscillations

Find two identical wooden or plastic rulers. Tape one end of each ruler firmly to the edge of a table so that the length of each ruler that protrudes from the table is the same. On the free end of one ruler tape a heavy object such as a few large coins. Pluck the ends of the rulers at the same time and observe which one undergoes more cycles in a time period, and measure the period of oscillation of each of the rulers.

Example 13.4

Calculate the Frequency and Period of Oscillations: Bad Shock Absorbers in a Car

If the shock absorbers in a car go bad, then the car will oscillate at the least provocation, such as when going over bumps in the road and after stopping (See Figure 16.10). Calculate the frequency and period of these oscillations for such a car if the car's mass (including its load) is 900 kg and the force constant (k) of the suspension system is $6.53 \times 10^4 \text{N/m}$.

Strategy

The frequency of the car's oscillations will be that of a simple harmonic oscillator as given in the equation $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. The mass and the force constant are both given.

Solution

1. Enter the known values of *k* and *m*:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{6.53 \times 10^4 \text{N/m}}{900 \text{ kg}}}.$$

13.17

2. Calculate the frequency:

$$\frac{1}{2\pi}\sqrt{72.6/\text{s}^2} = 1.3656/\text{s}^1 \approx 1.36/\text{s}^1 = 1.36\text{Hz}.$$

13.18

3. You could use $T = 2\pi\sqrt{\frac{m}{k}}$ to calculate the period, but it is simpler to use the

relationship $T = 1/f$ and substitute the value just found for f:

$$T = \frac{1}{f} = \frac{1}{1.356 \text{ Hz}} = 0.738 \text{ s}.$$

13.19

Discussion

The values of T and f both seem about right for a bouncing car. You can observe these oscillations if you push down hard on the end of a car and let go.

The Link between Simple Harmonic Motion and Waves

If a time-exposure photograph of the bouncing car were taken as it drove by, the headlight would make a wavelike streak, as shown in Figure 13.10. Similarly, Figure 13.11 shows an object bouncing on a spring as it leaves a wavelike "trace" of its position on a moving strip of paper. Both waves are sine functions. All simple harmonic motion is intimately related to sine and cosine waves.

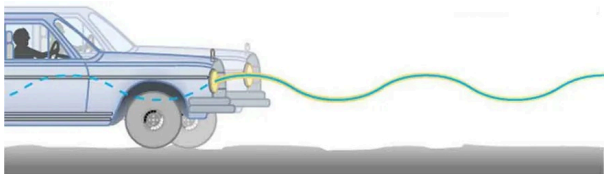


Figure 13.10 The bouncing car makes a wavelike motion. If the restoring force in the suspension system can be described only by Hooke's law, then the wave is a sine function. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a side view of a car moving over a bumpy surface. The car is drawn in a simple, outlined style, showing a driver inside. The focus is on the car's front suspension and wheels, which are reacting to road undulations. A blue dashed line represents the path of the wheel over the uneven ground, while a yellow curved line shows the motion trajectory of the car's suspension response above the wheels. The background is plain to emphasize the car's dynamics.

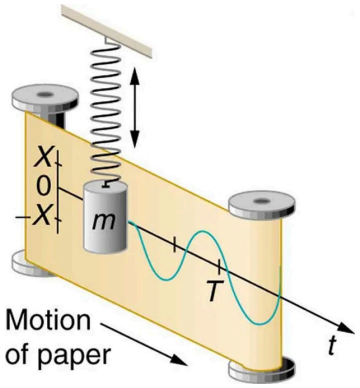


Figure 13.11 The vertical position of an object bouncing on a spring is recorded on a strip of moving paper, leaving a sine wave. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image illustrates a setup featuring a mechanical oscillator with a mass, labeled “m,” attached to a vertical spring. Above the spring, there is a horizontal line with an arrow indicating up and down motion. The mass is positioned in the center of a moving strip of paper, which is rolled onto two cylindrical spools on either side.

The paper moves in a leftward direction, as indicated by an arrow labeled “Motion of paper.” As the mass oscillates vertically, it traces a sinusoidal wave on the paper. The wave is annotated with “X0” at two points on the vertical axis and “T” on the horizontal axis, representing specific measurements related to the motion.

The time, labeled “t,” is indicated along the horizontal path of the paper that moves through the system.

The displacement as a function of time t in any simple harmonic motion—that is, one in which the net restoring force can be described by Hooke’s law, is given by

$$x\,(t) = X\cos\frac{2\pi t}{T},$$

13.20

where X is amplitude. At $t = 0$, the initial position is $x_0 = X$, and the displacement oscillates back and forth with a period T. (When $t = T$, we get $x = X$ again because $\cos 2\pi = 1$.). Furthermore, from this expression for x, the velocity v as a function of time is given by:

$$v\,(t\,) = -v_{\max}\sin\left(\frac{2\pi t}{T}\right),$$

13.21

where $v_{\text{max}} = 2\pi X/T = X\sqrt{k/m}$. The object has zero velocity at maximum displacement—for example, $v = 0$ when $t = 0$, and at that time $x = X$. The minus sign in the first equation for $v\,(t\,)$ gives the correct direction for the velocity. Just after the start of the motion, for instance, the velocity is negative because the system is moving back toward the equilibrium point. Finally, we can get an expression for acceleration using Newton's second law. [Then we have $x\,(t\,)$, $v\,(t\,)$, t, and $a\,(t\,)$, the quantities needed for kinematics and a description of simple harmonic motion.] According to Newton's second law, the acceleration is $a = F/m = \text{kx}/m$. So, $a\,(t\,)$ is also a cosine function:

$$a\,(t\,) = -\frac{\text{kX}}{m}\,\cos\,\frac{2\pi t}{T}.$$

13.22

Hence, $a\,(t\,)$ is directly proportional to and in the opposite direction to $x\,(t\,)$.

Figure 13.12 shows the simple harmonic motion of an object on a spring and presents graphs of $x\,(t\,)$, $v\,(t)$, and $a\,(t\,)$ versus time.

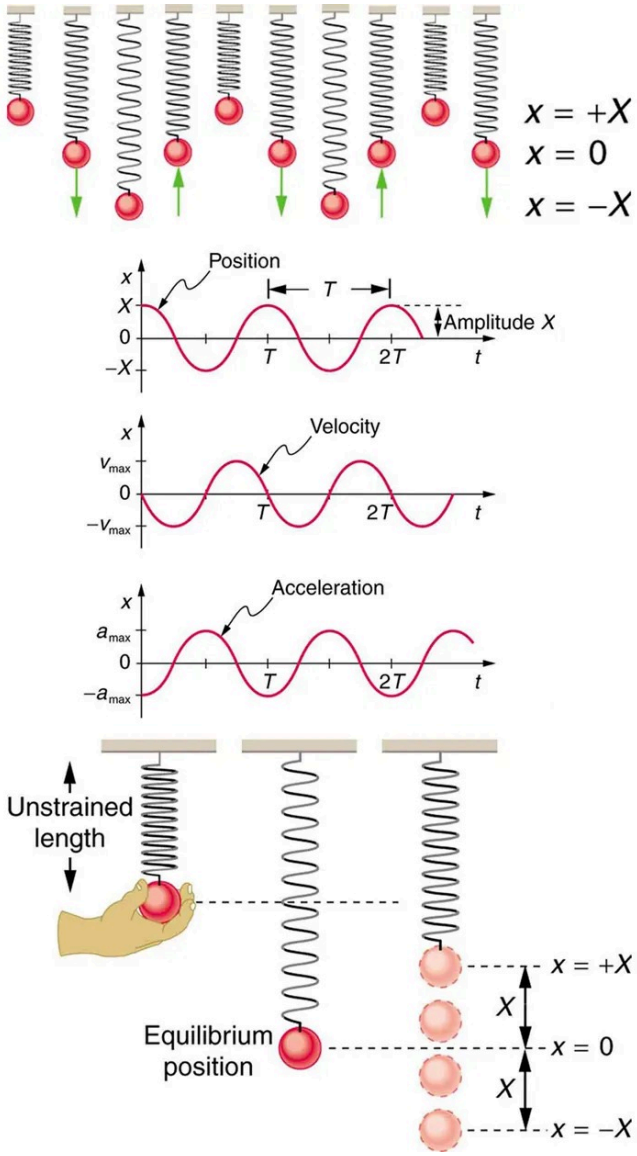


Figure 13.12 Graphs of $x(t)$, $v(t)$, *and* $a(t)$ *versus* t *for the motion of an object on a spring. The net force on the object can be described by Hooke's law, and so the object undergoes simple harmonic motion. Note that the initial position has the vertical displacement at its maximum value* X*;* v *is initially zero and then negative as the object moves down; and the initial acceleration is negative, back toward the equilibrium position and becomes zero at that point. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image illustrates a mass-spring system undergoing simple harmonic motion. It is divided into several parts:

1. Top Section:
– A series of diagrams show a red ball attached to a vertical spring. The ball moves up and down, with arrows

indicating the direction of motion.
– Three positions are labeled: x = +X (maximum positive displacement), x = 0 (equilibrium), and x = -X (maximum negative displacement).

2. Middle Section:
– Position Graph: A sine wave shows the position (x) over time (t), with amplitude marked as X. The wave oscillates between +X and -X.
– Velocity Graph: Another sine wave displays the velocity (v) over time, oscillating between maximum positive (vmax) and negative velocity (-vmax).
– Acceleration Graph: A sine wave for acceleration (a) over time, shifting between maximum positive (amax) and negative acceleration (-amax).

3. Bottom Section:
– Depicts the spring at its unstrained length when not under tension. A hand stretches the spring.
– The equilibrium position is shown, with the ball at rest in the middle.
– The ball is again shown at positions x = +X and x = -X, both an equal distance X from the equilibrium position.

This image visually represents the concepts of position, velocity, and acceleration in simple harmonic motion using a mass-spring system.

The most important point here is that these equations are mathematically straightforward and are valid for all simple harmonic motion. They are very useful in visualizing waves associated with simple harmonic motion, including visualizing how waves add with one another.

Check Your Understanding

Suppose you pluck a banjo string. You hear a single note that starts out loud and slowly quiets over time. Describe what happens to the sound waves in terms of period, frequency and amplitude as the sound decreases in volume.

Click for Solution

Solution

Frequency and period remain essentially unchanged. Only amplitude decreases as volume decreases.

Check Your Understanding

A babysitter is pushing a child on a swing. At the point where the swing reaches X, where would the corresponding point on a wave of this motion be located?

Click for Solution

Solution

X is the maximum deformation, which corresponds to the amplitude of the wave. The point on the wave would either be at the very top or the very bottom of the curve.

PhET Explorations

Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

Access multimedia content

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13.4 THE SIMPLE PENDULUM

Learning Objectives

By the end of this section, you will be able to:

- Measure acceleration due to gravity.

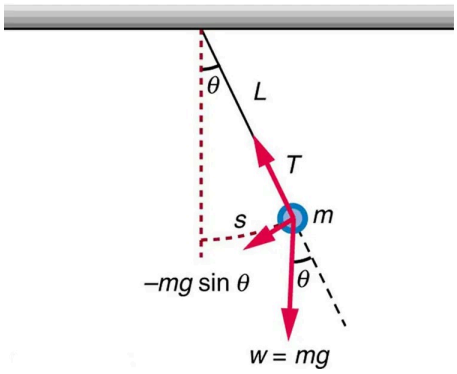


Figure 13.13 A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s*, the length of the arc. Also shown are the forces on the bob, which result in a net force of* $-\text{mg}\sin\theta$ *toward the equilibrium position—that is, a restoring force. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a physics diagram illustrating the forces acting on a swinging pendulum. It shows the following elements:

- A point mass labeled *m* attached to a string.

- The string is of length L and is inclined at an angle θ from the vertical.
- A red arrow pointing along the string representing the tension force T.
- A blue circle at the pivot point where the string is attached.
- A dashed line representing the vertical from which the angle θ is measured.
- A red vertical arrow pointing downward labeled $w = mg$, representing the weight of the mass.
- A horizontal dashed red line from the pivot point creating a right triangle with the vertical direction, showing $mg \sin \theta$ opposite the angle θ.
- A curved dashed line labeled s representing the arc length from the equilibrium position.

Pendulums are in common usage. Some have crucial uses, such as in clocks; some are for fun, such as a child's swing; and some are just there, such as the sinker on a fishing line. For small displacements, a pendulum is a simple harmonic oscillator. A simple pendulum is defined to have an object that has a small mass, also known as the pendulum bob, which is suspended from a light wire or string, such as shown in Figure 13.13. Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.

We begin by defining the displacement to be the arc length s. We see from Figure 13.13 that the net force on the bob is tangent to the arc and equals $-\text{mgsin}\theta$. (The weight mg has components $\text{mgcos}\theta$ along the string and $\text{mgsin}\theta$ tangent to the arc.) Tension in the string exactly cancels the component $\text{mgcos}\theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position at $\theta = 0$.

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about 15°), $\text{sin}\theta \approx \theta$($\text{sin}\theta$ and θ differ by about 1% or less at smaller angles). Thus, for angles less than about 15° , the restoring force F is

$$F \approx -\text{mg}\theta.$$
13.23

The displacement s is directly proportional to θ. When θ is expressed in radians, the arc length in a circle is related to its radius (L in this instance) by:

$$s = L\theta,$$
13.24

so that

$$\theta = \frac{s}{L}.$$
13.25

For small angles, then, the expression for the restoring force is:

$$F \approx -\frac{\mathrm{mg}}{L}s$$
13.26

This expression is of the form:

$$F = -\mathrm{kx},$$
13.27

where the force constant is given by $k = \mathrm{mg}/L$ and the displacement is given by $x = s$. For angles less than about 15° , the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about 15° . For the simple pendulum:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\mathrm{mg}/L}}.$$
13.28

Thus,

$$T = 2\pi\sqrt{\frac{L}{g}}$$
13.29

for the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. As with simple harmonic oscillators, the period T for a pendulum is nearly independent of amplitude, especially if θ is less than about 15° . Even simple pendulum clocks can be finely adjusted and accurate.

Note the dependence of T on g. If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity. Consider the following example.

Example 13.5

Measuring Acceleration due to Gravity: The Period of a Pendulum

What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find g given the period T and the length L of a pendulum. We can solve $T = 2\pi\sqrt{\frac{L}{g}}$ for g, assuming only that the angle of deflection is less than 15° .

Solution

1. Square $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for g:

   $$g = 4\pi^2\frac{L}{T^2}.$$

   13.30
2. Substitute known values into the new equation:

   $$g = 4\pi^2\frac{0.75000\ \text{m}}{(1.7357\ \text{s})^2}.$$

   13.31
3. Calculate to find g:

   $$g = 9.8281\text{m/s}^2.$$

   13.32

Discussion

This method for determining g can be very accurate. This is why length and period are given to five digits in this example. For the precision of the approximation $\sin\theta \approx \theta$ to be better than the precision of the pendulum length and period, the maximum displacement angle should be kept below about 0.5° .

Making Career Connections

Knowing g can be important in geological exploration; for example, a map of g over large geographical regions aids the study of plate tectonics and helps in the search for oil fields and large mineral deposits.

Take Home Experiment: Determining g

Use a simple pendulum to determine the acceleration due to gravity g in your own locale. Cut a piece of a string or dental floss so that it is about 1 m long. Attach a small object of high density to the end of the string (for example, a metal nut or a car key). Starting at an angle of less than 10° , allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch. Calculate g. How accurate is this measurement? How might it be improved?

Check Your Understanding

An engineer builds two simple pendula. Both are suspended from small wires secured to the ceiling of a room. Each pendulum hovers 2 cm above the floor. Pendulum 1 has a bob with a mass of 10kg. Pendulum 2 has a bob with a mass of $100\ \text{kg}$. Describe how the motion of the pendula will differ if the bobs are both displaced by 12° .

Click for Solution

Solution

The movement of the pendula will not differ at all because the mass of the bob has no effect on the motion of a simple pendulum. The pendula are only affected by the period (which is related to the pendulum's length) and by the acceleration due to gravity.

PhET Explorations

Pendulum Lab

Play with one or two pendulums and discover how the period of a simple pendulum depends on the length of the string, the mass of the pendulum bob, and the amplitude of the swing. It's easy to measure the period using the photogate timer. You can vary friction and the strength of gravity. Use the pendulum to find the value of g on planet X. Notice the anharmonic behavior at large amplitude.

Access multimedia content

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13.5 ENERGY AND THE SIMPLE HARMONIC OSCILLATOR

Learning Objectives

By the end of this section, you will be able to:

- Determine the maximum speed of an oscillating system.

To study the energy of a simple harmonic oscillator, we first consider all the forms of energy it can have We know from 13.1 Hooke's Law: Stress and Strain Revisited that the energy stored in the deformation of a simple harmonic oscillator is a form of potential energy given by:

$$\mathrm{PE_{el}} = \frac{1}{2}kx^2.$$

13.33

Because a simple harmonic oscillator has no dissipative forces, the other important form of energy is kinetic energy KE. Conservation of energy for these two forms is:

$$\mathrm{KE} + \mathrm{PE_{el}} = \mathrm{constant}$$

13.34

or

$$\frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{kx}^2 = \mathrm{constant}.$$

13.35

This statement of conservation of energy is valid for *all* simple harmonic oscillators, including ones where the gravitational force plays a role

Namely, for a simple pendulum we replace the velocity with $v = L\omega$, the spring constant with $k = \mathrm{mg}/L$, and the displacement term with $x = L\theta$. Thus

$$\frac{1}{2}\mathrm{mL}^2\omega^2 + \frac{1}{2}\mathrm{mgL}\theta^2 = \text{constant}.$$
13.36

In the case of undamped simple harmonic motion, the energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates. So for the simple example of an object on a frictionless surface attached to a spring, as shown again in Figure 13.14, the motion starts with all of the energy stored in the spring. As the object starts to move, the elastic potential energy is converted to kinetic energy, becoming entirely kinetic energy at the equilibrium position. It is then converted back into elastic potential energy by the spring, the velocity becomes zero when the kinetic energy is completely converted, and so on. This concept provides extra insight here and in later applications of simple harmonic motion, such as alternating current circuits.

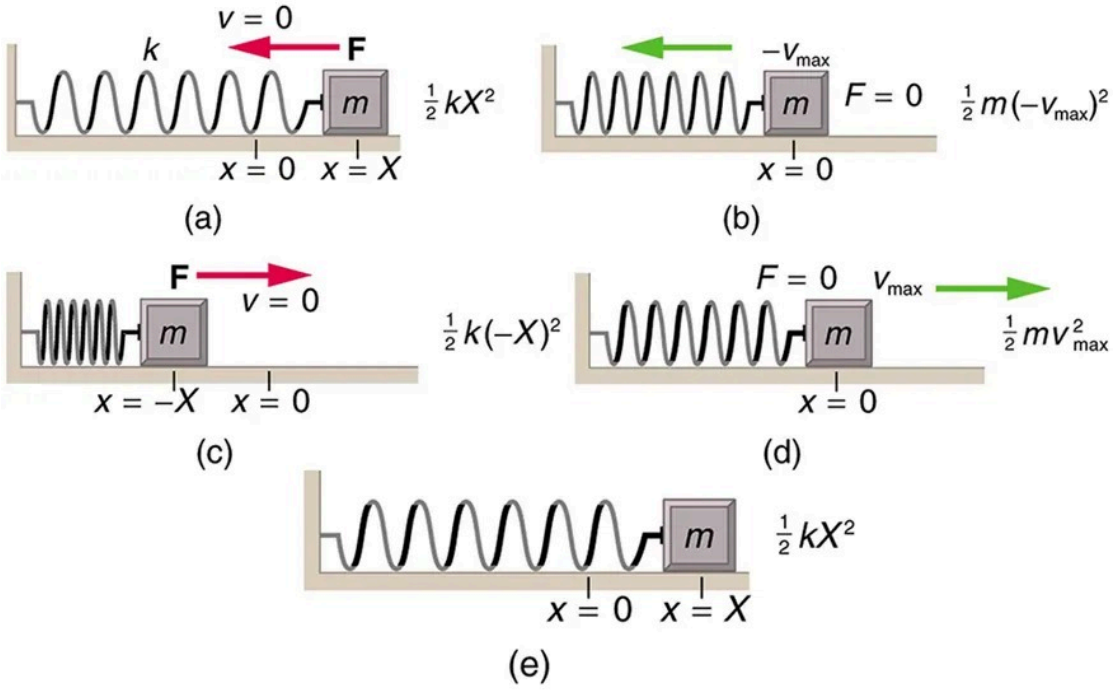


Figure 13.14 The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of five diagrams labeled (a) to (e) depicting a spring-mass system in various states of motion. Here's a detailed description:

– Diagram (a):
– A block with mass "m" is attached to a spring with constant "k".
– The system is at position "x = 0", with the spring compressed to "x = X".
– The block is exerted by a force "F" to the left, with velocity "v = 0".
– The potential energy is represented as $\frac{1}{2}kX^2$.

– Diagram (b):
– The spring is compressed to a point where the block is at maximum velocity $-v_{\text{max}}$.
– The spring is relaxed, with no external force ($F = 0$).
– The kinetic energy is represented as $\frac{1}{2}m(-v_{\text{max}})^2$.

– Diagram (c):
– The spring is compressed to the left at position "x = -X".
– A force "F" to the right acts on the block, with velocity "v = 0".
– The potential energy is represented as $\frac{1}{2}k(-X)^2$.

– Diagram (d):
– The spring is extended with the block moving to the right at maximum velocity "v_{\text{max}}".
– No external force is present ($F = 0$).
– The kinetic energy is represented as $\frac{1}{2}mv_{\text{max}}^2$.

– Diagram (e):
– The spring is fully relaxed at position "x = 0".
– Potential energy is depicted as $\frac{1}{2}kX^2$.

All diagrams illustrate the conservation of energy principle in a spring-mass system, showing potential and kinetic energy states at different positions.

The conservation of energy principle can be used to derive an expression for velocity v. If we start our simple harmonic motion with zero velocity and maximum displacement ($x = X$), then the total energy is

$$\frac{1}{2}\mathrm{kX}^2.$$

13.37

This total energy is constant and is shifted back and forth between kinetic energy and potential energy, at most times being shared by each. The conservation of energy for this system in equation form is thus:

$$\frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{kx}^2 = \frac{1}{2}\mathrm{kX}^2.$$
13.38

Solving this equation for v yields:

$$v = \pm\sqrt{\frac{k}{m}(X^2 - x^2)}.$$
13.39

Manipulating this expression algebraically gives:

$$v = \pm\sqrt{\frac{k}{m}}X\sqrt{1 - \frac{x^2}{X^2}}$$
13.40

and so

$$v = \pm v_{\mathrm{max}}\sqrt{1 - \frac{x^2}{X^2}},$$
13.41

where

$$v_{\mathrm{max}} = \sqrt{\frac{k}{m}}X.$$
13.42

From this expression, we see that the velocity is a maximum (v_{max}) at $x = 0$, as stated earlier in $v\left(t\right) = -v_{\mathrm{max}}\mathrm{sin}\frac{2\pi t}{T}$. Notice that the maximum velocity depends on three factors. Maximum velocity is directly proportional to amplitude. As you might guess, the greater the maximum displacement the greater the maximum velocity. Maximum velocity is also greater for stiffer systems, because they exert greater force for the same displacement. This observation is seen in the expression for v_{max}; it is proportional to the square root of the force constant k. Finally, the maximum velocity is smaller for objects that have larger masses, because the maximum velocity is inversely proportional to the square root of m. For a given force, objects that have large masses accelerate more slowly.

A similar calculation for the simple pendulum produces a similar result, namely:

$$\omega_{\max} = \sqrt{\frac{g}{L}}\theta_{\max}.$$

13.43

Example 13.6

Determine the Maximum Speed of an Oscillating System: A Bumpy Road

Suppose that a car is 900 kg and has a suspension system that has a force constant $k = 6.53 \times 10^4 \text{N/m}$. The car hits a bump and bounces with an amplitude of 0.100 m. What is its maximum vertical velocity if you assume no damping occurs?

Strategy

We can use the expression for v_{max} given in $v_{\text{max}} = \sqrt{\frac{k}{m}} X$ to determine the maximum vertical velocity. The variables m and k are given in the problem statement, and the maximum displacement X is 0.100 m.

Solution

1. Identify known.
2. Substitute known values into $v_{\text{max}} = \sqrt{\frac{k}{m}} X$:

$$v_{\text{max}} = \sqrt{\frac{6.53 \times 10^4 \text{N/m}}{900 \text{ kg}}} (0.100 \text{ m}).$$

13.44

3. Calculate to find $v_{\text{max}} = 0.852 \text{ m/s}$.

Discussion

This answer seems reasonable for a bouncing car. There are other ways to use

conservation of energy to find v_{max}. We could use it directly, as was done in the example featured in 13.1 Hooke's Law: Stress and Strain Revisited.

The small vertical displacement y of an oscillating simple pendulum, starting from its equilibrium position, is given as

$$y(t) = a \sin \omega t,$$

13.45

where a is the amplitude, ω is the angular velocity and t is the time taken. Substituting $\omega = \frac{2\pi}{T}$, we have

$$y(t) = a\sin\left(\frac{2\pi t}{T}\right).$$

13.46

Thus, the displacement of pendulum is a function of time as shown above.

Also the velocity of the pendulum is given by

$$v(t) = \frac{2a\pi}{T}\cos\left(\frac{2\pi t}{T}\right),$$

13.47

so the motion of the pendulum is a function of time.

Check Your Understanding

Why does it hurt more if your hand is snapped with a ruler than with a loose spring, even if the displacement of each system is equal?

Click for Solution

Solution

The ruler is a stiffer system, which carries greater force for the same amount of displacement. The ruler snaps your hand with greater force, which hurts more.

Check Your Understanding

You are observing a simple harmonic oscillator. Identify one way you could decrease the maximum velocity of the system.

Click for Solution

Solution

You could increase the mass of the object that is oscillating.

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13.6 UNIFORM CIRCULAR MOTION AND SIMPLE HARMONIC MOTION

Learning Objectives

By the end of this section, you will be able to:

- Compare simple harmonic motion with uniform circular motion.



Figure 13.15 The horses on this merry-go-round exhibit uniform circular motion. Image from OpenStax College Physics 2e, CC-BY 4.0

There is an easy way to produce simple harmonic motion by using uniform circular motion. Figure 13.16 shows one way of using this method. A ball is attached to a uniformly rotating vertical turntable, and its shadow is projected on the floor as shown. The shadow undergoes simple harmonic motion. Hooke's law usually describes uniform circular motions (ω constant) rather than systems that have large visible displacements. So observing the projection of uniform circular motion, as in Figure 13.16, is often easier than observing a precise large-scale simple harmonic oscillator. If studied in sufficient depth, simple harmonic

motion produced in this manner can give considerable insight into many aspects of oscillations and waves and is very useful mathematically. In our brief treatment, we shall indicate some of the major features of this relationship and how they might be useful.

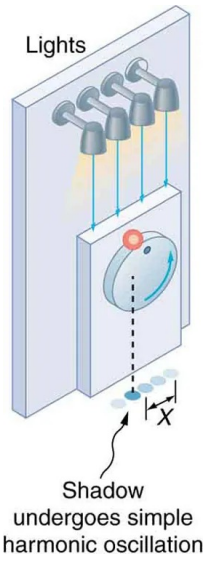


Figure 13.16 The shadow of a ball rotating at constant angular velocity ω on a turntable goes back and forth in precise simple harmonic motion. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a side view of a simple experimental setup demonstrating simple harmonic oscillation. It consists of a fixed wall with a set of five lights mounted on it, shining downwards onto a rotating disk or wheel situated on a lower ledge. The wheel has a marked point highlighted in red, which is shown on the top of the wheel. As the wheel rotates, the shadow of the red point moves back and forth in a linear path on the wall, indicated by a dashed line and labeled 'X'. This shadow's movement along the line is described as undergoing simple harmonic oscillation. The image visually explains the relationship between circular motion and simple harmonic motion, as indicated by blue arrows on the rotating wheel and along the shadow path.

Figure 13.17 shows the basic relationship between uniform circular motion and simple harmonic motion. The point P travels around the circle at constant angular velocity ω. The point P is analogous to an object on the merry-go-round. The projection of the position of P onto a fixed axis undergoes simple harmonic motion and is analogous to the shadow of the object. At the time shown in the figure, the projection has position x and moves to the left with velocity v. The velocity of the point P around the circle equals $(\bar{v})_{\max}$.The projection of $(\bar{v})_{\max}$ on the x-axis is the velocity v of the simple harmonic motion along the x-axis.

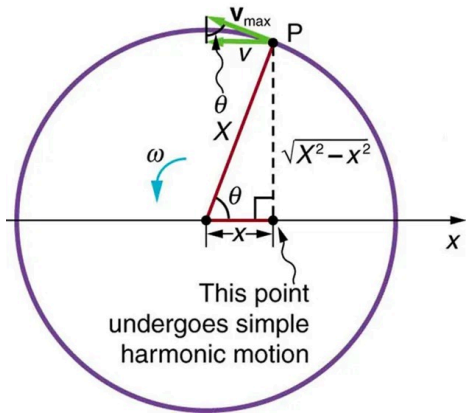


Figure 13.17 A point P moving on a circular path with a constant angular velocity ω is undergoing uniform circular motion. Its projection on the x-axis undergoes simple harmonic motion. Also shown is the velocity of this point around the circle, $(\overline{v})_{\max}$, and its projection, which is v. Note that these velocities form a similar triangle to the displacement triangle. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram explaining simple harmonic motion through a circle. The main features are as follows:

- A purple circle with center and radius marked.
- Point P on the circumference of the circle. It is connected to the center by a red line segment, labeled with angle θ and radius X.
- The horizontal axis is labeled x.
- A vertical dashed line drops from point P to intersect the horizontal axis at another point, with the segment labeled $\sqrt{(X^2 - x^2)}$.
- A horizontal segment represents the displacement x.
- Near the center of the circle, there's an arrow labeled ω, indicating angular velocity.
- The velocity vector v is shown at an angle from point P, with maximum velocity labeled as $v_{\max}$.

The text states, "This point undergoes simple harmonic motion."

To see that the projection undergoes simple harmonic motion, note that its position x is given by

$$x = X\cos\theta,$$

13.48

where $\theta = \omega t$, ω is the constant angular velocity, and X is the radius of the circular path. Thus,

$$x = X\cos\omega t.$$

13.49

The angular velocity ω is in radians per unit time; in this case 2π radians is the time for one revolution T. That is, $\omega = 2\pi/T$. Substituting this expression for ω, we see that the position x is given by:

$$x\,(t\,) = X\cos\left(\frac{2\pi t}{T}\right).$$

13.50

This expression is the same one we had for the position of a simple harmonic oscillator in 13.3 Simple Harmonic Motion: A Special Periodic Motion. If we make a graph of position versus time as in Figure 13.18, we see again the wavelike character (typical of simple harmonic motion) of the projection of uniform circular motion onto the x-axis.

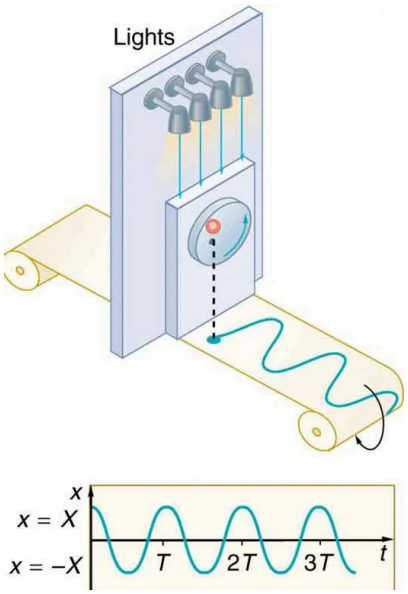


Figure 13.18 The position of the projection of uniform circular motion performs simple harmonic motion, as this wavelike graph of x versus t indicates. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two parts: a diagram of a mechanical setup and a graph showing wave patterns.

Mechanical Setup Diagram:
– A vertical panel labeled “Lights” with five attached lamps is depicted.
– Below the lights, a cylindrical device on the panel has a red dot at the top and an arrow pointing upward.

– A flat surface, resembling a paper strip, unwinds from two spools on either side of the panel.
– On this strip, a wavy line is drawn horizontally, indicating a wave traveling across the strip.

Graph of Wave Patterns:
– The graph is divided into sections marked “T,” “2T,” and “3T” along the horizontal axis, labeled as “t.”
– The vertical axis is labeled “x” with points marked as “x = X” at the top and “x = -X” at the bottom.
– A sine wave pattern is shown, with peaks and troughs repeating across the labeled sections.

This illustrates the concept of wave propagation and the role of a mechanical system in generating or analyzing waves.

Now let us use Figure 13.17 to do some further analysis of uniform circular motion as it relates to simple harmonic motion. The triangle formed by the velocities in the figure and the triangle formed by the displacements (X, x, and $\sqrt{X^2 - x^2}$) are similar right triangles. Taking ratios of similar sides, we see that

$$\frac{v}{v_{\max}} = \frac{\sqrt{X^2 - x^2}}{X} = \sqrt{1 - \frac{x^2}{X^2}}.$$
13.51

We can solve this equation for the speed v or

$$v = v_{\max}\sqrt{1 - \frac{x^2}{X^2}}.$$
13.52

This expression for the speed of a simple harmonic oscillator is exactly the same as the equation obtained from conservation of energy considerations in 13.5 Energy and the Simple Harmonic Oscillator. You can begin to see that it is possible to get all of the characteristics of simple harmonic motion from an analysis of the projection of uniform circular motion.

Finally, let us consider the period T of the motion of the projection. This period is the time it takes the point P to complete one revolution. That time is the circumference of the circle $2\pi X$ divided by the velocity around the circle, $v_{\max}$. Thus, the period T is

$$T = \frac{2\pi X}{v_{\max}}.$$
13.53

We know from conservation of energy considerations that

$$v_{\text{max}} = \sqrt{\frac{k}{m}}X.$$
13.54

Solving this equation for X/v_{max} gives

$$\frac{X}{v_{\text{max}}} = \sqrt{\frac{m}{k}}.$$
13.55

Substituting this expression into the equation for T yields

$$T = 2\pi\sqrt{\frac{m}{k}}.$$
13.56

Thus, the period of the motion is the same as for a simple harmonic oscillator. We have determined the period for any simple harmonic oscillator using the relationship between uniform circular motion and simple harmonic motion.

Some modules occasionally refer to the connection between uniform circular motion and simple harmonic motion. Moreover, if you carry your study of physics and its applications to greater depths, you will find this relationship useful. It can, for example, help to analyze how waves add when they are superimposed.

Check Your Understanding

Identify an object that undergoes uniform circular motion. Describe how you could trace the simple harmonic motion of this object as a wave.

Click for Solution

Solution

A record player undergoes uniform circular motion. You could attach dowel rod to one point on the outside edge of the turntable and attach a pen to the other end of the dowel. As the record player turns, the pen will move. You can drag a long piece of paper under the pen, capturing its motion as a wave.

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13.7 DAMPED HARMONIC MOTION

Learning Objectives

By the end of this section, you will be able to:

- Compare and discuss underdamped and overdamped oscillating systems.
- Explain critically damped system.



Figure 13.19 In order to counteract dampening forces, this mom needs to keep pushing the swing. Image from OpenStax College Physics 2e, CC-BY 4.0

A guitar string stops oscillating a few seconds after being plucked. To keep a child happy on a swing, you must keep pushing. Although we can often make friction and other non-conservative forces negligibly small, completely undamped motion is rare. In fact, we may even want to damp oscillations, such as with car shock absorbers.

For a system that has a small amount of damping, the period and frequency are nearly the same as for simple harmonic motion, but the amplitude gradually decreases as shown in Figure 13.20. This occurs because the non-conservative damping force removes energy from the system, usually in the form of thermal energy. In general, energy removal by non-conservative forces is described as

$$W_{\text{nc}} = \Delta(\text{KE} + \text{PE}),$$
13.57

where W_{nc} is work done by a non-conservative force (here the damping force). For a damped harmonic oscillator, W_{nc} is negative because it removes mechanical energy (KE + PE) from the system.

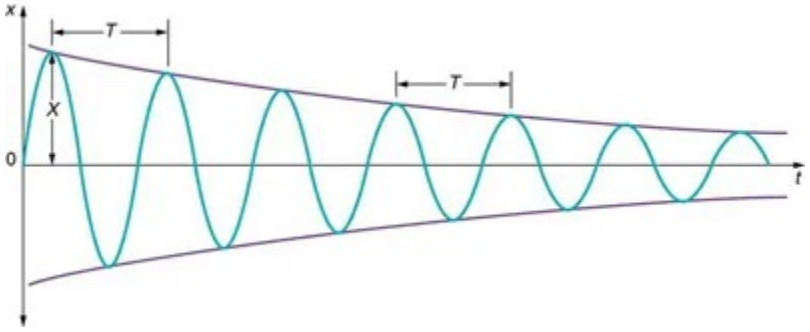


Figure 13.20 In this graph of displacement versus time for a harmonic oscillator with a small amount of damping, the amplitude slowly decreases, but the period and frequency are nearly the same as if the system were completely undamped. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a graph that represents a damped wave. The horizontal axis is labeled 't', possibly indicating time, while the vertical axis is labeled 'x'. The wave begins with a larger amplitude on the left, which gradually decreases as it moves to the right. Over the wave, two points are marked, with a horizontal distance between them labeled 'T', which could indicate the period of the wave. The wave oscillates between two lines that converge toward the right side of the graph, indicating the damping effect. The amplitude 'X' at the start of the wave near the vertical axis is highlighted.

If you gradually *increase* the amount of damping in a system, the period and frequency begin to be affected, because damping opposes and hence slows the back and forth motion. (The net force is smaller in both directions.) If there is very large damping, the system does not even oscillate—it slowly moves toward equilibrium. Figure 13.21 shows the displacement of a harmonic oscillator for different amounts of damping. When we want to damp out oscillations, such as in the suspension of a car, we may want the system to return to equilibrium as quickly as possible Critical damping is defined as the condition in which the damping of an oscillator results in it returning as quickly as possible to its equilibrium position. The critically damped system may overshoot the equilibrium position, but if it does, it will do so only once. Critical damping is represented by Curve A in Figure 13.21. With less-than critical damping, the system will return to equilibrium faster but will overshoot and cross over one or more times. Such a system is underdamped; its displacement is represented by the curve in Figure 13.20. Curve B in Figure 13.21 represents an overdamped system. As with

critical damping, it too may overshoot the equilibrium position, but will reach equilibrium over a longer period of time.

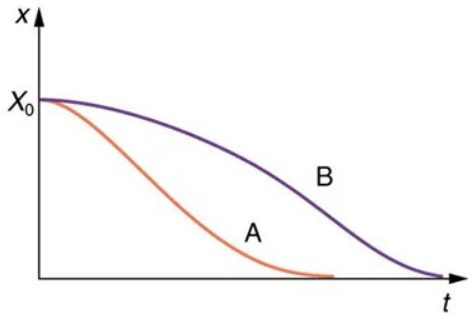


Figure 13.21 Displacement versus time for a critically damped harmonic oscillator (A) and an overdamped harmonic oscillator (B). The critically damped oscillator returns to equilibrium at $X = 0$ *in the smallest time possible without overshooting. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a graph with labeled axes and two curves. The horizontal axis is labeled "t" and represents time, while the vertical axis is labeled "x" and represents a variable. The point where the axes intersect is labeled "X_0". There are two curves on the graph:

- The first curve is orange and labeled "A". It starts at point "X_0" on the vertical axis and decreases sharply as it moves to the right along the horizontal axis.
- The second curve is purple and labeled "B". It also starts at point "X_0" on the vertical axis but decreases more gradually compared to curve "A" as it moves to the right along the horizontal axis.

The two curves demonstrate different rates of decline over time, with curve "A" having a steeper descent than curve "B".

Critical damping is often desired, because such a system returns to equilibrium rapidly and remains at equilibrium as well. In addition, a constant force applied to a critically damped system moves the system to a new equilibrium position in the shortest time possible without overshooting or oscillating about the new position. For example, when you stand on bathroom scales that have a needle gauge, the needle moves to its equilibrium position without oscillating. It would be quite inconvenient if the needle oscillated about the new equilibrium position for a long time before settling. Damping forces can vary greatly in character. Friction, for example, is sometimes independent of velocity (as assumed in most places in this text). But many damping forces depend on velocity—sometimes in complex ways, sometimes simply being proportional to velocity.

Example 13.7

Damping an Oscillatory Motion: Friction on an Object Connected to a Spring

Damping oscillatory motion is important in many systems, and the ability to control the damping is even more so. This is generally attained using non-conservative forces such as the friction between surfaces, and viscosity for objects moving through fluids. The following example considers friction. Suppose a 0.200-kg object is connected to a spring as shown in Figure 13.22, but there is simple friction between the object and the surface, and the coefficient of friction μ_k is equal to 0.0800. (a) What is the frictional force between the surfaces? (b) What total distance does the object travel if it is released 0.100 m from equilibrium, starting at $v = 0$? The force constant of the spring is $k = 50.0 N/m$.

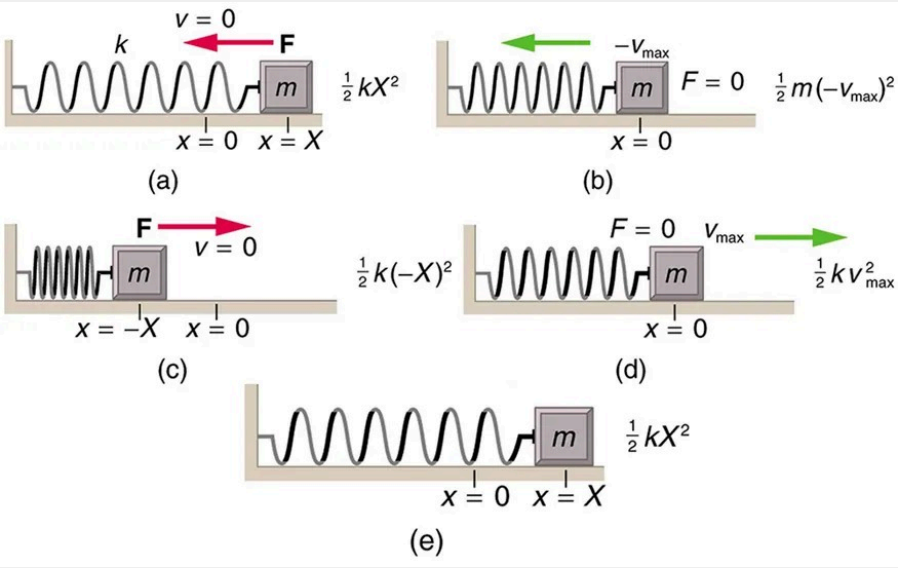


Figure 13.22 The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts five diagrams labeled (a) to (e), showing a mass-spring system in different stages of oscillation. Each diagram includes a spring constant (k), a mass (m), positions ($x = 0$, $x = X$, and $x = -X$), velocity (v), and force (F). Here is a description of each diagram:

(a) The mass is at position $x = X$ with velocity $v = 0$. An external force F is applied to the right. The potential energy is $1/2\ kX^2$.

(b) The mass is at position $x = 0$. Velocity is $-v_{max}$ to the left, and $F = 0$. The kinetic energy is $1/2\ m(-v_{max})^2$.

(c) The mass is at position $x = -X$ with velocity $v = 0$. An external force F is applied to the right. The potential energy is $1/2\ k(-X)^2$.

(d) The mass returns to position $x = 0$. Velocity is v_{max} to the right, and $F = 0$. The kinetic energy is $1/2\ k\ v_{max}^2$.

(e) The mass is back at position $x = X$. Energy is stored as potential energy $1/2\ kX^2$. The diagram shows equilibrium without motion.

Strategy

This problem requires you to integrate your knowledge of various concepts regarding waves, oscillations, and damping. To solve an integrated concept problem, you must first identify the physical principles involved. Part (a) is about the frictional force. This is a topic involving the application of Newton's Laws. Part (b) requires an understanding of work and conservation of energy, as well as some understanding of horizontal oscillatory systems.

Now that we have identified the principles we must apply in order to solve the problems, we need to identify the knowns and unknowns for each part of the question, as well as the quantity that is constant in Part (a) and Part (b) of the question.

Solution a

1. Choose the proper equation: Friction is $f = \mu_k \mathrm{mg}$.

2. Identify the known values.
3. Enter the known values into the equation:

$$f = (0.0800)(0.200\ \text{kg})(9.80\ \text{m/s}^2).$$

13.58

4. Calculate and convert units:

$$f = 0.157\ \text{N}.$$

Discussion a

The force here is small because the system and the coefficients are small.

Solution b

Identify the known:

- The system involves elastic potential energy as the spring compresses and expands, friction that is related to the work done, and the kinetic energy as the body speeds up and slows down.
- Energy is not conserved as the mass oscillates because friction is a non-conservative force.
- The motion is horizontal, so gravitational potential energy does not need to be considered.
- Because the motion starts from rest, the energy in the system is initially $\text{PE}_{el,i} = (1/2)\text{kX}^2$. This energy is removed by work done by friction $W_{\text{nc}} = -\text{fd}$, where d is the total distance traveled and $f = \mu_{\text{k}}\text{mg}$ is the force of friction. When the system stops moving, the friction force will balance the force exerted by the spring, so $\text{PE}_{\text{el,f}} = (1/2)\text{kx}^2$ where x is the final position and is given by

$$\begin{aligned} F_{\text{el}} &= f \\ \text{kx} &= \mu_{\text{k}}\text{mg} \\ x &= \frac{\mu_{\text{k}}\text{mg}}{k}. \end{aligned}$$

13.59

1. By equating the work done to the energy removed, solve for the distance d.
2. The work done by the non-conservative forces equals the initial, stored elastic potential energy. Identify the correct equation to use:

$$\begin{aligned} \mathrm{W}_{\mathrm{nc}} &= \Delta(\mathrm{KE} + \mathrm{PE}) \\ &= \mathrm{PE}_{\mathrm{el,f}} - \mathrm{PE}_{\mathrm{el,i}} \\ &= \frac{1}{2}k\left(\left(\frac{\mu_k mg}{k}\right)^2 - X^2\right). \end{aligned}$$

13.60

3. Recall that $W_{\mathrm{nc}} = -\mathrm{fd}$.
4. Enter the friction as $f = \mu_{\mathrm{k}}\mathrm{mg}$ into $W_{\mathrm{nc}} = -\mathrm{fd}$, thus

$$W_{\mathrm{nc}} = (-\mu)_{\mathrm{k}}\mathrm{mgd}.$$

13.61

5. Combine these two equations to find

$$\frac{1}{2}k\left(\left(\frac{\mu_k \mathrm{mg}}{k}\right)^2 - X^2\right) = -\mu_{\mathrm{k}}\,\mathrm{mgd}\,.$$

13.62

6. Solve the equation for d:

$$d = \frac{\mathrm{k}}{(2\mu)_{\mathrm{k}}\mathrm{mg}}(X^2 - \left(\left(\frac{\mu_{\mathrm{k}}\mathrm{mg}}{k}\right)\right)^2).$$

13.63

7. Enter the known values into the resulting equation:

$$\begin{aligned} d = &\frac{50.0 N/m}{2\,(0.0800\,)\,(0.200\mathrm{kg}\,)\,(9.80\mathrm{m/s^2}\,)} \\ &\times (((0.100\mathrm{m}\,))^2 - \left(\left(\frac{(0.0800\,)\,(0.200\mathrm{kg}\,)\,(9.80\mathrm{m/s^2}\,)}{50.0\mathrm{N/m}}\right)\right)^2). \end{aligned}$$

13.64

8. Calculate d and convert units:

$$d = 1.59\text{m}.$$

13.65

Discussion b

This is the total distance traveled back and forth across $x = 0$, which is the undamped equilibrium position. The number of oscillations about the equilibrium position will be more than $d/X = (1.\ 59\ \text{m}\ )/(0.\ 100\ \text{m}\ ) = 15\ .\ 9$ because the amplitude of the oscillations is decreasing with time. At the end of the motion, this system will not return to $x = 0$ for this type of damping force, because static friction will exceed the restoring force. This system is underdamped. In contrast, an overdamped system with a simple constant damping force would not cross the equilibrium position $x = 0$ a single time. For example, if this system had a damping force 20 times greater, it would only move 0.0484 m toward the equilibrium position from its original 0.100-m position.

This worked example illustrates how to apply problem-solving strategies to situations that integrate the different concepts you have learned. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknowns using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life.

Check Your Understanding

Why are completely undamped harmonic oscillators so rare?

Click for Solution

Solution

Friction often comes into play whenever an object is moving. Friction causes damping in a harmonic oscillator.

Check Your Understanding

Describe the difference between overdamping, underdamping, and critical damping.

Click for Solution

Solution

An overdamped system moves slowly toward equilibrium. An underdamped system moves quickly to equilibrium, but will oscillate about the equilibrium point as it does so. A critically damped system moves as quickly as possible toward equilibrium without oscillating about the equilibrium.

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13.8 FORCED OSCILLATIONS AND RESONANCE

Learning Objectives

By the end of this section, you will be able to:

- Observe resonance of a paddle ball on a string.
- Observe amplitude of a damped harmonic oscillator.



Figure 13.23 You can cause the strings in a piano to vibrate simply by producing sound waves from your voice. Image by Matt Billings, Wikimedia Commons License

Sit in front of a piano sometime and sing a loud brief note at it with the dampers off its strings. It will sing the same note back at you—the strings, having the same frequencies as your voice, are resonating in response to the forces from the sound waves that you sent to them. Your voice and a piano's strings is a good example of

the fact that objects—in this case, piano strings—can be forced to oscillate but oscillate best at their natural frequency. In this section, we shall briefly explore applying a *periodic driving force* acting on a simple harmonic oscillator. The driving force puts energy into the system at a certain frequency, not necessarily the same as the natural frequency of the system. The natural frequency is the frequency at which a system would oscillate if there were no driving and no damping force.

Most of us have played with toys involving an object supported on an elastic band, something like the paddle ball suspended from a finger in Figure 13.24. Imagine the finger in the figure is your finger. At first you hold your finger steady, and the ball bounces up and down with a small amount of damping. If you move your finger up and down slowly, the ball will follow along without bouncing much on its own. As you increase the frequency at which you move your finger up and down, the ball will respond by oscillating with increasing amplitude. When you drive the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as you drive it. The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance. A system being driven at its natural frequency is said to resonate. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller, until the oscillations nearly disappear and your finger simply moves up and down with little effect on the ball.

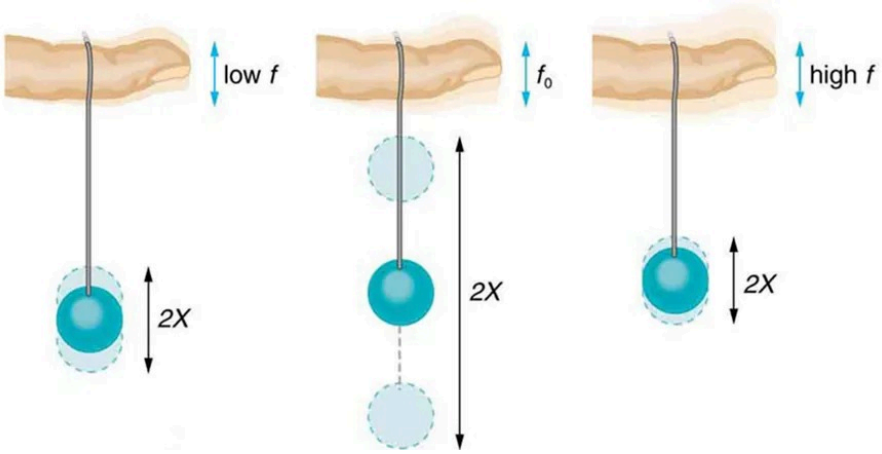


Figure 13.24 The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency f_0 *of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows a diagram with three sections, each illustrating a different vibration frequency of a circular object attached to a string suspended from a horizontal arm.

Left Section (Low Frequency): A circular blue object, labeled with a low frequency (f), swings in a vertical path, moving downward by a distance labeled as 2X.

Middle Section (Natural Frequency): A similar circular blue object is in the middle position, labeled with natural frequency (f_0), also swinging by a distance 2X, with additional faint circles indicating possible positions.

Right Section (High Frequency): The third section features a circular blue object, labeled with a high frequency (f), swinging by the same distance 2X.

Each section visually depicts how the object oscillates around an equilibrium position with varying frequencies while maintaining the same amplitude (2X).

Figure 13.25 shows a graph of the amplitude of a damped harmonic oscillator as a function of the frequency of the periodic force driving it. There are three curves on the graph, each representing a different amount of damping. All three curves peak at the point where the frequency of the driving force equals the natural frequency of the harmonic oscillator. The highest peak, or greatest response, is for the least amount of damping, because less energy is removed by the damping force.

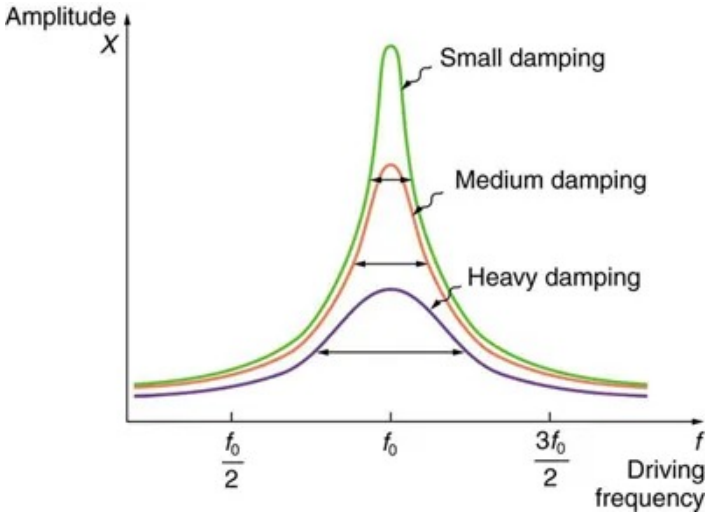


Figure 13.25 Amplitude of a harmonic oscillator as a function of the frequency of the driving force. The curves represent the same oscillator with the same natural frequency but with different amounts of damping. Resonance occurs when the driving frequency equals the natural frequency, and the greatest response is for the least amount of damping. The narrowest response is also for the least damping. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph depicting the relationship between amplitude and driving frequency for different levels of damping in a system. The graph has the following key features:

- The y-axis is labeled "Amplitude X" and the x-axis is labeled "Driving frequency f".
- There are three curves on the graph, each representing a different level of damping: small, medium, and heavy.
- The curve with the smallest damping (green) has the highest peak, indicating a larger amplitude at resonance.
- The medium damping curve (red) has a lower peak than the small damping curve.
- The heavy damping curve (purple) has the lowest peak, indicating the smallest amplitude at resonance.
- Frequencies marked on the x-axis include $f_0/2$, f_0, and $3f_0/2$.

It is interesting that the widths of the resonance curves shown in Figure 13.25 depend on damping: the less the damping, the narrower the resonance. The message is that if you want a driven oscillator to resonate at a very specific frequency, you need as little damping as possible. Little damping is the case for piano strings and many other musical instruments. Conversely, if you want small-amplitude oscillations, such as in a car's suspension system, then you want heavy damping. Heavy damping reduces the amplitude, but the tradeoff is that the system responds at more frequencies.

These features of driven harmonic oscillators apply to a huge variety of systems. When you tune a radio, for example, you are adjusting its resonant frequency so that it only oscillates to the desired station's broadcast (driving) frequency. The more selective the radio is in discriminating between stations, the smaller its damping. Magnetic resonance imaging (MRI) is a widely used medical diagnostic tool in which atomic nuclei (mostly hydrogen nuclei) are made to resonate by incoming radio waves (on the order of 100 MHz). A child on a swing is driven by a parent at the swing's natural frequency to achieve maximum amplitude. In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance. Speed bumps and gravel roads prove that even a car's suspension system is not immune to resonance. In spite of finely engineered shock absorbers, which ordinarily convert mechanical energy to thermal energy almost as fast as it comes in, speed bumps still cause a large-amplitude oscillation. On gravel roads that are corrugated, you may have noticed that if you travel at the "wrong" speed, the bumps are very noticeable whereas at other speeds you may hardly feel the bumps at all. Figure 13.26 shows a photograph of a famous example (the Tacoma Narrows Bridge) of the destructive effects of a driven harmonic oscillation. The Millennium Bridge in London was closed for a short period of time for the same reason while inspections were carried out.

In our bodies, the chest cavity is a clear example of a system at resonance. The diaphragm and chest wall drive the oscillations of the chest cavity which result in the lungs inflating and deflating. The system is critically damped and the muscular diaphragm oscillates at the resonant value for the system, making it highly efficient.



Figure 13.26 In 1940, the Tacoma Narrows Bridge in Washington state collapsed. Heavy cross winds drove the bridge into oscillations at its resonant frequency. Damping decreased when support cables broke loose and started to slip over the towers, allowing increasingly greater amplitudes until the structure failed Image *from* OpenStax College Physics 2e, *CC-BY 4.0*

Check Your Understanding

A famous magic trick involves a performer singing a note toward a crystal glass until the glass shatters. Explain why the trick works in terms of resonance and natural frequency.

Click for Solution

Solution

The performer must be singing a note that corresponds to the natural frequency of the glass. As the sound wave is directed at the glass, the glass responds by resonating at the same frequency as the sound wave. With enough energy introduced into the system, the glass begins to vibrate and eventually shatters.

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13.9 WAVES

Learning Objectives

By the end of this section, you will be able to:

- State the characteristics of a wave.
- Calculate the velocity of wave propagation.



Figure 13.27 Waves in the ocean behave similarly to all other types of waves. Image by Steve Jurvetson from Menlo Park, USA, Wikimedia Commons License

What do we mean when we say something is a wave? The most intuitive and easiest wave to imagine is the familiar water wave. More precisely, a wave is a disturbance that propagates, or moves from the place it was created. For water waves, the disturbance is in the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air

pressure, perhaps created by the oscillating cone inside a speaker. For earthquakes, there are several types of disturbances, including disturbance of Earth's surface and pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Visualizing water waves is useful because there is more to it than just a mental image. Water waves exhibit characteristics common to all waves, such as amplitude, period, frequency and energy. All wave characteristics can be described by a small set of underlying principles.

A wave is a disturbance that propagates, or moves from the place it was created. The simplest waves repeat themselves for several cycles and are associated with simple harmonic motion. Let us start by considering the simplified water wave in Figure 13.28. The wave is an up and down disturbance of the water surface. It causes a sea gull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The time for one complete up and down motion is the wave's period T. The wave's frequency is $f = 1/T$, as usual. The wave itself moves to the right in the figure. This movement of the wave is actually the disturbance moving to the right, not the water itself (or the bird would move to the right). We define wave velocity v_{w} to be the speed at which the disturbance moves. Wave velocity is sometimes also called the *propagation velocity or propagation speed,* because the disturbance propagates from one location to another.

Misconception Alert

Many people think that water waves push water from one direction to another. In fact, the particles of water tend to stay in one location, save for moving up and down due to the energy in the wave. The energy moves forward through the water, but the water stays in one place. If you feel yourself pushed in an ocean, what you feel is the energy of the wave, not a rush of water.

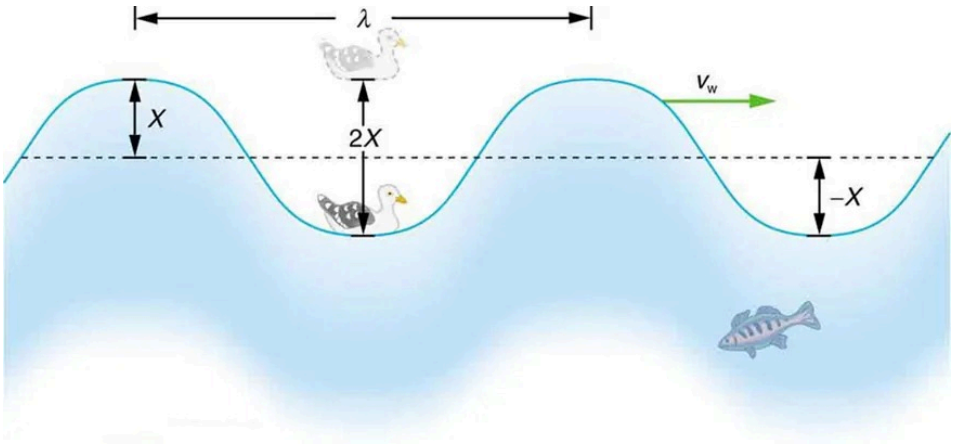


Figure 13.28 An idealized ocean wave passes under a sea gull that bobs up and down in simple harmonic motion. The wave has a wavelength λ, which is the distance between adjacent identical parts of the wave. The up and down disturbance of the surface propagates parallel to the surface at a speed v_w. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a conceptual diagram of water waves with two ducks and a fish. The diagram illustrates wave properties:

- A sinusoidal wave representing water with two peaks and two troughs. The wave is moving to the right as indicated by a green arrow labeled v_w.
- Two ducks are positioned along the wave. One duck is at the crest of the wave, and another is at the trough.
- A fish is depicted underwater below one of the wave's troughs.
- The wave's amplitude is represented by vertical double-headed arrows: one labeled as *X* going from the equilibrium line to the crest, and another labeled *-X* from the equilibrium line to the trough.
- A horizontal double-headed arrow labeled λ *(lambda)* denotes the wavelength, the distance between two consecutive crests.
- A vertical arrow labeled *2X* extends from the crest to the trough.

The water wave in the figure also has a length associated with it, called its wavelength λ, the distance between adjacent identical parts of a wave. (λ is the distance parallel to the direction of propagation.) The speed of propagation v_w is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

$$v_w = \frac{\lambda}{T}$$

13.66

or

$$v_{\text{w}} = f\lambda.$$
13.67

This fundamental relationship holds for all types of waves. For water waves, v_{w} is the speed of a surface wave; for sound, v_{w} is the speed of sound; and for visible light, v_{w} is the speed of light, for example.

Take-Home Experiment: Waves in a Bowl

Fill a large bowl or basin with water and wait for the water to settle so there are no ripples. Gently drop a cork into the middle of the bowl. Estimate the wavelength and period of oscillation of the water wave that propagates away from the cork. Remove the cork from the bowl and wait for the water to settle again. Gently drop the cork at a height that is different from the first drop. Does the wavelength depend upon how high above the water the cork is dropped?

Example 13.8

Calculate the Velocity of Wave Propagation: Gull in the Ocean

Calculate the wave velocity of the ocean wave in Figure 13.28 if the distance between wave crests is 10.0 m and the time for a sea gull to bob up and down is 5.00 s.

Strategy

We are asked to find v_{w}. The given information tells us that $\lambda = 10.0\text{m}$ and $T = 5.00\text{s}$. Therefore, we can use $v_{\text{w}} = \frac{\lambda}{T}$ to find the wave velocity.

Solution

1. Enter the known values into $v_{\text{w}} = \frac{\lambda}{T}$:

   $$v_{\text{w}} = \frac{10.0\ \text{m}}{5.00\ \text{s}}.$$

   13.68
2. Solve for v_{w} to find $v_{\text{w}} = 2.00\ \text{m/s}.$

Discussion

This slow speed seems reasonable for an ocean wave. Note that the wave moves to the right in the figure at this speed, not the varying speed at which the sea gull moves up and down.

Transverse and Longitudinal Waves

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in

Figure 13.29 propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a transverse wave or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation. In contrast, in a longitudinal wave or compressional wave, the disturbance is parallel to the direction of propagation. Figure 13.30 shows an example of a longitudinal wave. The size of the disturbance is its amplitude X and is completely independent of the speed of propagation v_{w}.

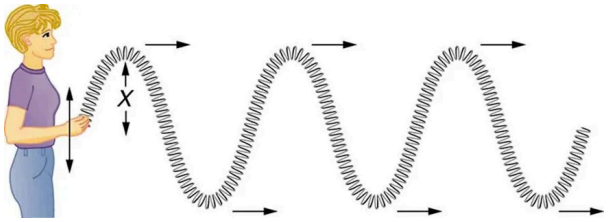


Figure 13.29 In this example of a transverse wave, the wave propagates horizontally, and the disturbance in the cord is in the vertical direction. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a person holding a long, coiled spring or slinky from their right hand. The person is on the left side of the image, wearing a purple shirt and blue jeans. The spring extends horizontally across the image, forming several wave-like curves along its length. Arrows are shown along the length of the spring, indicating the direction of wave propagation from left to right. A vertical arrow near the person's hand indicates the direction of their motion as they move the spring up and down. The waves travel horizontally away from where the spring is being held. A marking labeled "X" signifies a specific point on the wave where a vertical cut is shown, highlighting the vertical oscillation at that point.

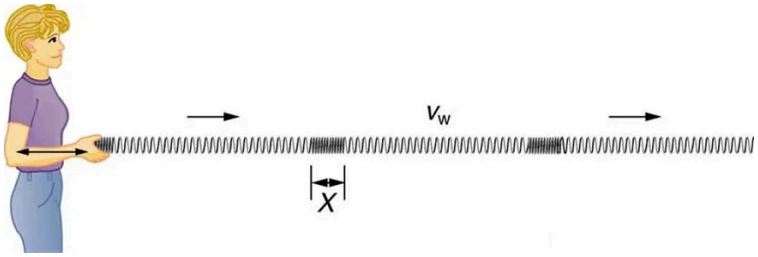


Figure 13.30 In this example of a longitudinal wave, the wave propagates horizontally, and the disturbance in the cord is also in the horizontal direction. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a person standing on the left, holding a stretched spring or slinky horizontally. The person is shown releasing a pulse down the spring to the right. The spring is compressed and forms a series of waves moving from left to right, indicating the direction of the wave's travel.

An arrow on the left side near the person's hands points right, indicating the direction of the motion applied to create the wave. A similar arrow near the middle right of the spring points in the same rightward direction, showing the propagation direction of the wave through the spring.

Above the spring, a label "V_w" indicates the wave's velocity. On the left side, a labeled distance "x" is marked between two compressions of the wave. The background is plain white, and the focus is entirely on the person and the spring.

Waves may be transverse, longitudinal, or *a combination of the two*. (Water waves are actually a combination of transverse and longitudinal. The simplified water wave illustrated in Figure 13.28 shows no longitudinal motion of the bird.) The waves on the strings of musical instruments are transverse—so are electromagnetic waves, such as visible light.

Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do not have appreciable shear strength, and thus the sound waves in them must be longitudinal or compressional. Sound in solids can be both longitudinal and transverse.

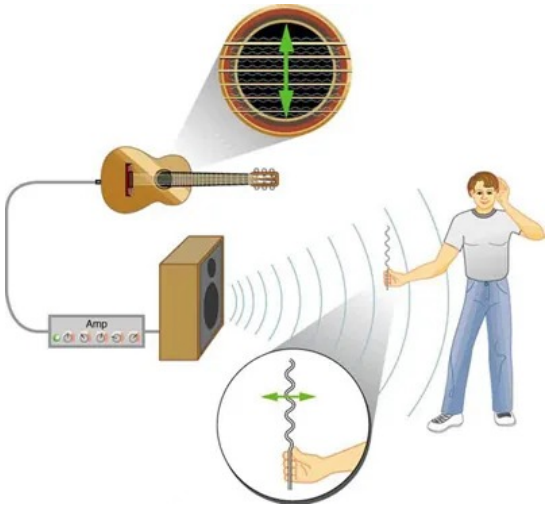


Figure 13.31 The wave on a guitar string is transverse. The sound wave rattles a sheet of paper in a direction that shows the sound wave is longitudinal. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates the process of sound amplification from a guitar to a listener. On the left side, there's an acoustic guitar. Behind the strings, there is a circular inset that highlights the vibrations of the strings with green arrows indicating the direction of the vibrations. Below the guitar, there's a device labeled 'Amp' connected to a speaker. The speaker is emitting sound waves depicted by concentric arcs that grow larger as they move away from the speaker.

On the right side, there's a person holding one hand to their ear, signifying listening, with the other hand extended outward. A magnified circular inset near the person's hand shows a sound wave with green arrows, indicating the direction of the wave's vibration. The overall illustration explains how sound travels from the guitar, through an amplifier and speaker, reaching the listener as amplified sound waves.

Earthquake waves under Earth's surface also have both longitudinal and transverse components (called compressional or P-waves and shear or S-waves, respectively). These components have important individual characteristics—they propagate at different speeds, for example. Earthquakes also have surface waves that are similar to surface waves on water.

Check Your Understanding

Why is it important to differentiate between longitudinal and transverse waves?

Click for Solution

Solution

In the different types of waves, energy can propagate in a different direction relative to the motion of the wave. This is important to understand how different types of waves affect the materials around them.

PhET Explorations

Wave on a String

Watch a string vibrate in slow motion. Wiggle the end of the string and make waves, or adjust the frequency and amplitude of an oscillator. Adjust the damping and tension. The end can be fixed, loose, or open.

Access multimedia content

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13.10 SUPERPOSITION AND INTERFERENCE

Learning Objectives

By the end of this section, you will be able to:

- Explain standing waves.
- Describe the mathematical representation of overtones and beat frequency.



Figure 13.32 These waves result from the superposition of several waves from different sources, producing a complex pattern. Image by waterborough, Wikimedia Commons License

Most waves do not look very simple. They look more like the waves in Figure 13.32 than like the simple water wave considered in Waves. (Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape). Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called superposition. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves—that is, their amplitudes add. Figure 13.33 and Figure 13.34 illustrate superposition in two special cases, both of which produce simple results.

Figure 13.33 shows two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure constructive interference. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

Figure 13.34 shows two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure destructive interference. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.

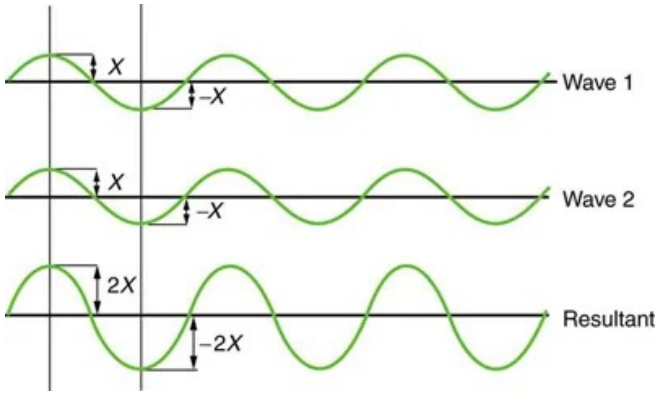


Figure 13.33 Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows three wave diagrams aligned vertically, each labeled accordingly:

- *Wave 1:* A sinusoidal wave is depicted in green, moving left to right. The wave oscillates above and below a central horizontal line. Two vertical lines intersect the wave, and distances marked as “X” and “-X” indicate the amplitude above and below the horizontal axis.
- *Wave 2:* Similar to Wave 1, this is another sinusoidal wave in green, with the same amplitude. It is slightly offset from Wave 1, beginning the first peak at the same point where Wave 1 begins its trough.

Amplitudes are marked with the same "X" and "-X" labels as above.

- *Resultant:* The resultant wave is displayed below Waves 1 and 2. It shows the combined effect of the two waves, forming a wave with double the amplitude, marked as "2X" and "-2X". This wave shows constructive interference, where peaks and troughs align to create a wave with larger amplitude.

The overall illustration demonstrates how two coherent waves with identical frequencies and amplitudes, when combined, can result in constructive interference to produce a new wave with greater amplitude.

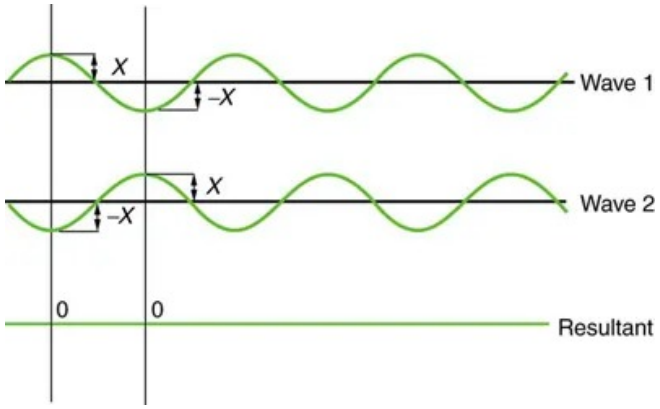


Figure 13.34 Pure destructive interference of two identical waves produces zero amplitude, or complete cancellation. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a diagram of two waves, labeled "Wave 1" and "Wave 2," and their resulting combination, labeled "Resultant." Both waves are sinusoidal, meaning they have a smooth, wave-like shape with peaks and troughs repeating at regular intervals.

Each wave is depicted with a horizontal line as the baseline. "Wave 1" is at the top and "Wave 2" is directly below it. The waves have the same amplitude and frequency, but they are out of phase. In "Wave 1," a point on the wave labeled "X" is above the baseline, while in the corresponding position in "Wave 2," labeled "-X," the wave is below the baseline. Similarly, there are other points labeled with "X" and "-X" to show the symmetrical nature of the waves' displacement about the baseline.

The "Resultant" wave is shown at the bottom of the diagram on a separate horizontal baseline. It appears as a straight line along the baseline because the two waves have opposite phases at the given points, meaning they cancel each other out completely.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference

and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot and quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive. These examples are of waves that are similar.

An example of the superposition of two dissimilar waves is shown in Figure 13.35. Here again, the disturbances add and subtract, producing a more complicated looking wave.

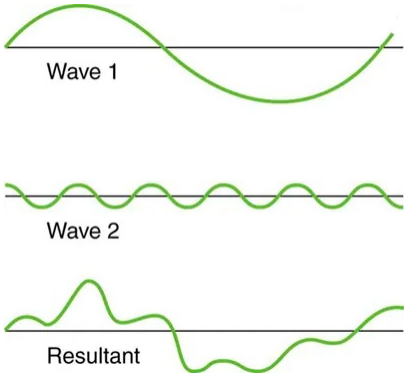


Figure 13.35 Superposition of non-identical waves exhibits both constructive and destructive interference. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image displays three graphs depicting wave patterns. Each wave is represented by a green line oscillating along a black horizontal axis:

1. *Wave 1*: A smooth, large amplitude sinusoidal wave that evenly oscillates above and below the horizontal axis.

2. *Wave 2*: A smaller amplitude wave with more frequent oscillations, resulting in a wavy pattern with several peaks and troughs closely spaced on the axis.

3. *Resultant*: A combination of the patterns from Wave 1 and Wave 2. It creates a complex wave pattern that varies in amplitude and shows irregular oscillations due to the combination of the two previous waves.

Standing Waves

Sometimes waves do not seem to move; rather, they just vibrate in place. Unmoving waves can be seen on the

surface of a glass of milk in a refrigerator, for example. Vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. These waves are formed by the superposition of two or more moving waves, such as illustrated in Figure 13.36 for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a standing wave. Waves on the glass of milk are one example of standing waves. There are other standing waves, such as on guitar strings and in organ pipes. With the glass of milk, the two waves that produce standing waves may come from reflections from the side of the glass.

A closer look at earthquakes provides evidence for conditions appropriate for resonance, standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.

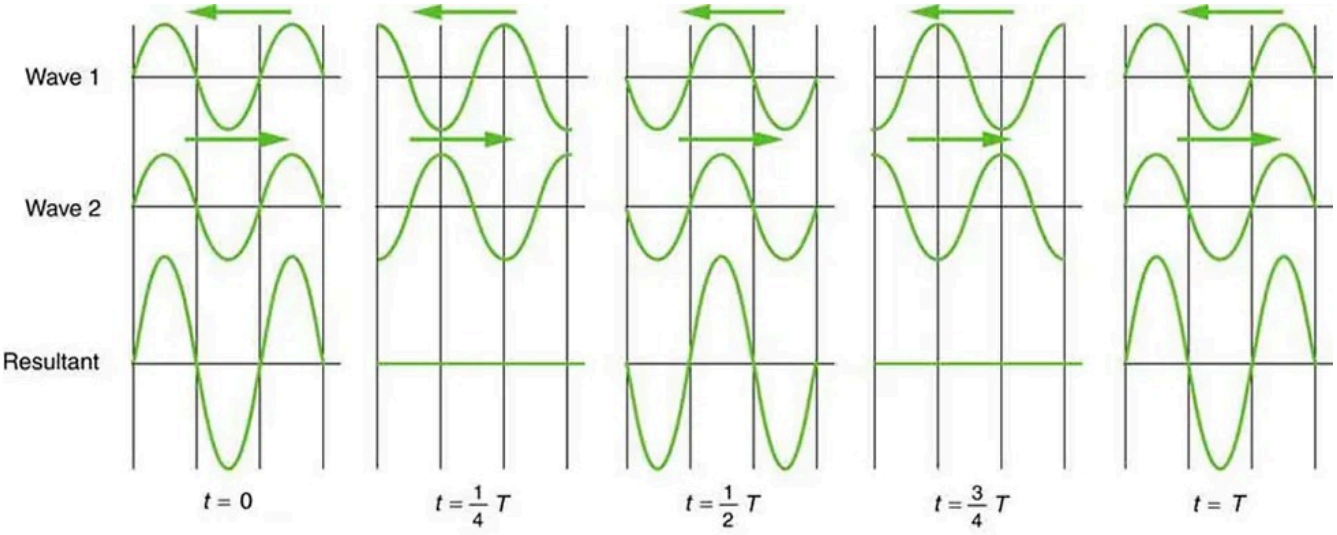


Figure 13.36 Standing wave created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternately constructive and destructive interference. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows the interference of two waves and their resultant over time. There are five columns, each

representing different time intervals. Each column has three rows depicting Wave 1, Wave 2, and the Resultant wave. The waves are shown as sinusoidal curves.

Time	Wave 1	Wave 2	Resultant
t = 0	Sinusoidal wave moving left	Sinusoidal wave moving right	Wave pattern with reduced amplitude
t = 1/4 T	Sinusoidal wave moving left	Sinusoidal wave moving right	Wave pattern with increased amplitude
t = 1/2 T	Sinusoidal wave moving left	Sinusoidal wave moving right	Wave pattern with reduced amplitude
t = 3/4 T	Sinusoidal wave moving left	Sinusoidal wave moving right	Wave pattern with increased amplitude
t = T	Sinusoidal wave moving left	Sinusoidal wave moving right	Wave pattern with reduced amplitude

Green arrows indicate the direction of wave movement. Wave 1 generally moves left while Wave 2 moves right. The resultant wave shows the combination of these two waves at each time interval.

Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. Figure 13.37 and Figure 13.38 show three standing waves that can be created on a string that is fixed at both ends. Nodes are the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word antinode is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_{w} of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

The lowest frequency, called the fundamental frequency, is thus for the longest wavelength, which is seen to be $\lambda_1 = 2L$. Therefore, the fundamental frequency is $f_1 = v_{\text{w}}/\lambda_1 = v_{\text{w}}/2L$. In this case, the overtones or harmonics are multiples of the fundamental frequency. As seen in Figure 13.38, the first harmonic can easily be calculated since $\lambda_2 = L$. Thus, $f_2 = v_{\text{w}}/\lambda_2 = v_{\text{w}}/2L = (2f)_1$. Similarly, $f_3 = (3f)_1$, and so on. All of these frequencies can be changed by adjusting the tension in the string. The greater the tension, the greater v_{w} is and the higher the frequencies. This observation is familiar to anyone who has ever observed a string instrument being tuned. We will see in later chapters that standing waves are crucial to many resonance phenomena, such as in sounding boxes on string instruments.

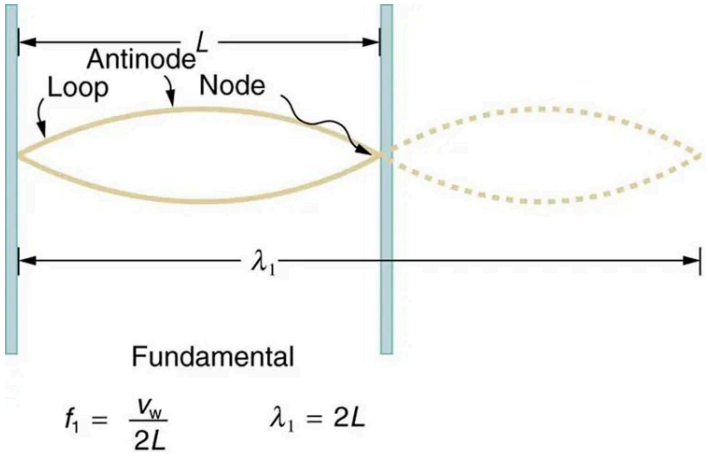


Figure 13.37 The figure shows a string oscillating at its fundamental frequency. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a fundamental mode of vibration on a string that is fixed at both ends. A horizontal line represents the string, with two vertical lines indicating rigid supports at the ends. The string shows one complete loop, indicating a standing wave.

– The left part of the wave is solid, while the right part is a faded extension, suggesting the continuation of the wave pattern.
– Labels are present:
– "Antinode" points to the middle of the loop where the amplitude is maximum.
– "Node" points to the ends of the loop where the displacement is zero.
– "Loop" refers to the entire visible wave pattern between two nodes.
– The distance between the nodes is marked as "L," and the wavelength of the wave is shown as "$\lambda_1 = 2L$."
– Additional text below reads:
– "Fundamental"
– The formula for the fundamental frequency: $f_1 = \frac{v_w}{2L}$

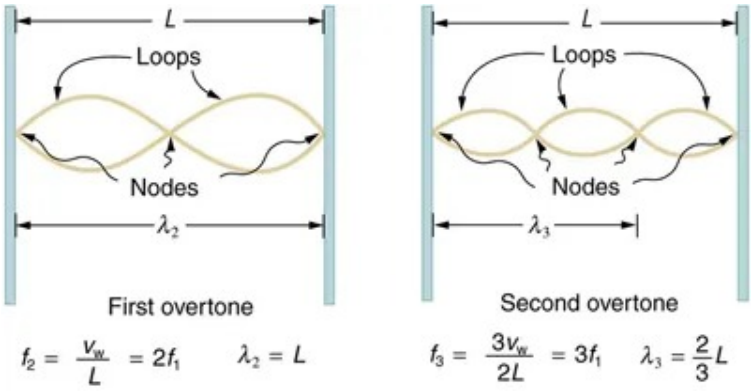


Figure 13.38 First and second overtones are shown. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows two diagrams illustrating wave patterns for the first and second overtones in a resonating system, likely involving strings or air columns.

Left Diagram: First Overtone

- A standing wave with two loops and one node is depicted between two fixed points.
- The distance between the nodes is labeled as L.
- The wavelength is labeled as $\lambda_2 = L$.
- Accompanying formula: $f_2 = v_w/L = 2f_1$.

Right Diagram: Second Overtone

- A standing wave with three loops and two nodes is depicted between the same fixed points.
- The wavelength is labeled as $\lambda_3 = 2/3\ L$.
- Accompanying formula: $f_3 = 3v_w/2L = 3f_1$.

Beats

Striking two adjacent keys on a piano produces a warbling combination usually considered to be unpleasant. The superposition of two waves of similar but not identical frequencies is the culprit. Another example is often noticeable in jet aircraft, particularly the two-engine variety, while taxiing. The combined sound of the engines goes up and down in loudness. This varying loudness happens because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. Figure 16.39 illustrates this graphically.

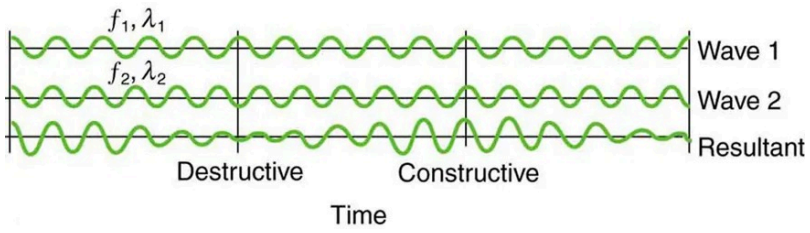


Figure 13.39 Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating wave interference. It consists of three horizontal rows of waveforms, labeled as Wave 1, Wave 2, and Resultant on the right side.

- *Wave 1* and *Wave 2*: Two separate waves are shown at the top. Each wave is labeled with frequency and wavelength symbols, f_1, λ_1 for Wave 1 and f_2, λ_2 for Wave 2. Both have a similar sinusoidal pattern, but may differ slightly in frequency or wavelength.
- *Resultant*: The third waveform shows the resulting wave from the interference of Wave 1 and Wave 2.

The diagram is divided into two sections by vertical lines, labeled as *Destructive* and *Constructive* interference. In the destructive interference section, the resultant wave has a reduced amplitude due to the out-of-phase interaction of Wave 1 and Wave 2. In the constructive interference section, the resultant wave has an increased amplitude due to the in-phase interaction of the two waves.

The horizontal axis is labeled as *Time*, suggesting that these changes occur over time.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or *beats*, with a frequency called the beat frequency. We can determine the beat frequency by adding two waves together mathematically. Note that a wave can be represented at one point in space as

$$x = X\cos\left(\frac{2\pi t}{T}\right) = X\cos\left(2\pi \mathrm{ft}\right),$$

13.69

where $f = 1/T$ is the frequency of the wave. Adding two waves that have different frequencies but identical amplitudes produces a resultant

$$x = x_1 + x_2.$$
13.70

More specifically,

$$x = X\cos\left(2\pi f_1 t\right) + X\cos\left(2\pi f_2 t\right).$$
13.71

Using a trigonometric identity, it can be shown that

$$x = 2X\cos\left(\pi f_{\mathrm{B}} t\right)\cos\left(2\pi f_{\mathrm{ave}} t\right),$$
13.72

where

$$f_B = \mid f_1 - f_2 \mid$$
13.73

is the beat frequency, and f_{ave} is the average of f_1 and f_2. These results mean that the resultant wave has twice the amplitude and the average frequency of the two superimposed waves, but it also fluctuates in overall amplitude at the beat frequency f_{B}. The first cosine term in the expression effectively causes the amplitude to go up and down. The second cosine term is the wave with frequency f_{ave}. This result is valid for all types of waves. However, if it is a sound wave, providing the two frequencies are similar, then what we hear is an average frequency that gets louder and softer (or warbles) at the beat frequency.

Making Career Connections

Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). For example, if the tuning fork has a 256Hz frequency and two beats per second are heard, then the other frequency is either 254 or 258Hz. Most keys hit multiple strings, and these strings are actually adjusted until they have nearly the same frequency and give a slow beat for richness. Twelve-string guitars and mandolins are also tuned using beats.

While beats may sometimes be annoying in audible sounds, we will find that beats have many applications. Observing beats is a very useful way to compare similar frequencies. There are applications of beats as apparently disparate as in ultrasonic imaging and radar speed traps.

Check Your Understanding

Imagine you are holding one end of a jump rope, and your friend holds the other. If your friend holds her end still, you can move your end up and down, creating a transverse wave. If your friend then begins to move her end up and down, generating a wave in the opposite direction, what resultant wave forms would you expect to see in the jump rope?

Click for Solution

Solution

The rope would alternate between having waves with amplitudes two times the original amplitude and reaching equilibrium with no amplitude at all. The wavelengths will result in both constructive and destructive interference

Check Your Understanding

Define nodes and antinodes.

Click for Solution

Solution

Nodes are areas of wave interference where there is no motion. Antinodes are areas of wave interference where the motion is at its maximum point.

Check Your Understanding

You hook up a stereo system. When you test the system, you notice that in one corner of the room, the sounds seem dull. In another area, the sounds seem excessively loud. Describe how the sound moving about the room could result in these effects.

Click for Solution

Solution

With multiple speakers putting out sounds into the room, and these sounds bouncing off walls, there is bound to be some wave interference. In the dull areas, the interference is probably mostly destructive. In the louder areas, the interference is probably mostly constructive.

PhET Explorations

Wave Interference

Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.

Access multimedia content

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13.11 ENERGY IN WAVES: INTENSITY

Learning Objectives

By the end of this section, you will be able to:

- Calculate the intensity and the power of rays and waves.



Figure 13.40 The destructive effect of an earthquake is palpable evidence of the energy carried in these waves. The Richter scale rating of earthquakes is related to both their amplitude and the energy they carry. Image by Petty Officer 2nd Class Candice Villarreal, USN, Wikimedia Commons License

All waves carry energy. The energy of some waves can be directly observed. Earthquakes can shake whole cities to the ground, performing the work of thousands of wrecking balls.

Loud sounds pulverize nerve cells in the inner ear, causing permanent hearing loss. Ultrasound is used for deep-heat treatment of muscle strains. A laser beam can burn away a malignancy. Water waves chew up beaches.

The amount of energy in a wave is related to its amplitude. Large-amplitude earthquakes produce large ground displacements. Loud sounds have higher pressure amplitudes and come from larger-amplitude source vibrations than soft sounds. Large ocean breakers churn up the shore more than small ones. More quantitatively, a wave is a displacement that is resisted by a restoring force. The larger the displacement x, the larger the force $F = \mathrm{kx}$ needed to create it. Because work W is related to force multiplied by distance (Fx) and energy is put into the wave by the work done to create it, the energy in a wave is related to amplitude. In fact, a wave's energy is directly proportional to its amplitude squared because

$$W \propto \mathrm{Fx} = \mathrm{kx}^2.$$
13.74

The energy effects of a wave depend on time as well as amplitude. For example, the longer deep-heat ultrasound is applied, the more energy it transfers. Waves can also be concentrated or spread out. Sunlight, for example, can be focused to burn wood. Earthquakes spread out, so they do less damage the farther they get from the source. In both cases, changing the area the waves cover has important effects. All these pertinent factors are included in the definition of intensity I as power per unit area:

$$I = \frac{P}{A}$$
13.75

where P is the power carried by the wave through area A. The definition of intensity is valid for any energy in transit, including that carried by waves. The SI unit for intensity is watts per square meter $(\mathrm{W/m^2})$. For example, infrared and visible energy from the Sun impinge on Earth at an intensity of $1300 \mathrm{W/m^2}$ just above the atmosphere. There are other intensity-related units in use, too. The most common is the decibel. For example, a 90 decibel sound level corresponds to an intensity of $10^{-3} \mathrm{W/m^2}$. (This quantity is not much power per unit area considering that 90 decibels is a relatively high sound level. Decibels will be discussed in some detail in a later chapter.

Example 13.9

Calculating intensity and power: How much energy is in a ray of sunlight?

The average intensity of sunlight on Earth's surface is about $700\text{W}/\text{m}^2$.

(a) Calculate the amount of energy that falls on a solar collector having an area of 0.500m^2 in 4.00h.

(b) What intensity would such sunlight have if concentrated by a magnifying glass onto an area 200 times smaller than its own?

Strategy a

Because power is energy per unit time or $P = \frac{E}{t}$, the definition of intensity can be written as $I = \frac{P}{A} = \frac{E/t}{A}$, and this equation can be solved for E with the given information.

Solution a

1. Begin with the equation that states the definition of intensity:

   $$I = \frac{P}{A}.$$

   13.76
2. Replace P with its equivalent E/t:

   $$I = \frac{E/t}{A}.$$

   13.77
3. Solve for E:

$$E = \mathrm{IAt}.$$

13.78

4. Substitute known values into the equation:

$$E = \left(700\ \mathrm{W/m^2}\right)\left(0.500\ \mathrm{m^2}\right)\left[(4.00\ \mathrm{h})\,(3600\ \mathrm{s/h})\right].$$

13.79

5. Calculate to find E and convert units:

$$5.04 \times 10^6 \mathrm{J},$$

13.80

Discussion a

The energy falling on the solar collector in 4 h in part is enough to be useful—for example, for heating a significant amount of water.

Strategy b

Taking a ratio of new intensity to old intensity and using primes for the new quantities, we will find that it depends on the ratio of the areas. All other quantities will cancel.

Solution b

1. Take the ratio of intensities, which yields:

$$\frac{I'}{I} = \frac{P'/A'}{P/A} = \frac{A}{A'} (\text{The powers cancel because} P' = P).$$

13.81

2. Identify the knowns:

$$A = 200A',$$

13.82

$$\frac{I'}{I} = 200.$$

13.83

3. Substitute known quantities:

$$I' = 200I = 200\left(700\text{W/m}^2\right).$$

13.84

4. Calculate to find I':

$$I' = 1.40 \times 10^5 \text{W/m}^2.$$

13.85

Discussion b

Decreasing the area increases the intensity considerably. The intensity of the concentrated sunlight could even start a fire.

Example 13.10

Determine the combined intensity of two waves: Perfect constructive interference

If two identical waves are spatially separated, each having an intensity of 1.00 W/m^2, interfere perfectly constructively in a particular location, what is the intensity of the wave at this particular location?

Strategy

We know from 13.10 Superposition and Interference that when two identical waves, which have equal amplitudes X, interfere perfectly constructively, the resulting wave has an amplitude of $2X$. Because a wave's intensity is proportional to amplitude squared, the intensity of the resulting wave is four times as great as in the individual waves.

Solution

1. Recall that intensity is proportional to amplitude squared.
2. Calculate the new amplitude:

$$I\prime \propto (X\prime)^2 = (2X)^2 = (4X)^2.$$

13.86

3. Recall that the intensity of the old amplitude was:

$$I \propto X^2.$$

13.87

4. Take the ratio of new intensity to the old intensity. This gives:

$$\frac{I\prime}{I} = 4.$$

13.88

5. Calculate to find $I\prime$:

$$I\prime = 4I = 4.00\text{W/m}^2.$$

13.89

Discussion

The intensity goes up by a factor of 4 when the amplitude doubles. This answer is a little disquieting. The two individual waves each have intensities of 1.00W/m^2, yet their sum has an intensity of 4.00W/m^2, which may appear to violate conservation of energy. This violation, of course, cannot happen. What does happen is intriguing. The area over which the intensity is 4.00W/m^2 is much less than the area covered by the two waves before they interfered. There are other areas where the intensity is zero. The addition of waves is not as simple as our first look in 13.10 Superposition and Interference suggested. We actually get a pattern of both constructive interference and destructive interference whenever two waves are added. For example, if we have two stereo speakers putting out 1.00W/m^2 each, there will be places in the room where the intensity is 4.00W/m^2, other places where the intensity is zero, and others in between. Figure 13.41 shows what this interference might look like. We will pursue interference patterns elsewhere in this text.

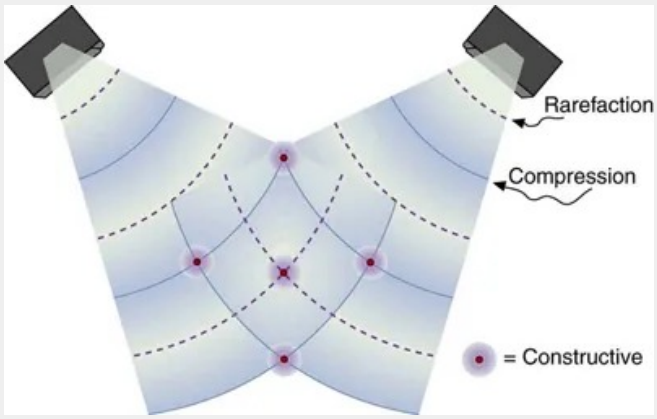


Figure 13.41 These stereo speakers produce both constructive interference and destructive interference in the room, a property common to the superposition of all types of waves. The shading is proportional to intensity. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image illustrates the concept of wave interference, showing two speakers emitting sound waves. The speakers are positioned at the upper corners, each tilted slightly towards the center. Wavy lines represent the sound waves emitted by the speakers, forming overlapping patterns across the image.

The waves are color-coded: light blue areas represent areas of rarefaction where the waves are less dense, and dark blue areas indicate compression where the waves are denser. These patterns create a series of concentric arcs emanating from each speaker.

In the central section of the image, there are red dots placed at intersections where waves from both speakers meet constructively, marked as “constructive” in the legend. These intersections signify points where the sound waves reinforce each other, resulting in amplified sound at those locations.

Check Your Understanding

Which measurement of a wave is most important when determining the wave's intensity?

Click for Solution

Solution

Amplitude, because a wave's energy is directly proportional to its amplitude squared.

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GLOSSARY

amplitude
: the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position

antinode
: the location of maximum amplitude in standing waves

beat frequency
: the frequency of the amplitude fluctuations of a wave

constructive interference
: when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

critical damping
: the condition in which the damping of an oscillator causes it to return as quickly as possible to its equilibrium position without oscillating back and forth about this position

deformation
: displacement from equilibrium

destructive interference
: when two identical waves arrive at the same point exactly out of phase; that is, precisely aligned crest to trough

elastic potential energy
: potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring

force constant
: a constant related to the rigidity of a system: the larger the force constant, the more rigid the system; the force constant is represented by k

frequency
: number of events per unit of time

fundamental frequency
: the lowest frequency of a periodic waveform

intensity
: power per unit area

longitudinal wave
: a wave in which the disturbance is parallel to the direction of propagation

natural frequency
: the frequency at which a system would oscillate if there were no driving and no damping forces

nodes
: the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

oscillate
: moving back and forth regularly between two points

over damping
: the condition in which damping of an oscillator causes it to return to equilibrium without oscillating; oscillator moves more slowly toward equilibrium than in the critically damped system

overtones
: multiples of the fundamental frequency of a sound

period
: time it takes to complete one oscillation

periodic motion
: motion that repeats itself at regular time intervals

resonance
: the phenomenon of driving a system with a frequency equal to the system's natural frequency

resonate
: a system being driven at its natural frequency

restoring force
: force acting in opposition to the force caused by a deformation

simple harmonic motion
: the oscillatory motion in a system where the net force can be described by Hooke's law

simple harmonic oscillator
: a device that implements Hooke's law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support such as a wall

simple pendulum
: an object with a small mass suspended from a light wire or string

superposition
: the phenomenon that occurs when two or more waves arrive at the same point

transverse wave
: a wave in which the disturbance is perpendicular to the direction of propagation

under damping
: the condition in which damping of an oscillator causes it to return to equilibrium with the amplitude gradually decreasing to zero; system returns to equilibrium faster but overshoots and crosses the equilibrium position one or more times

wave

a disturbance that moves from its source and carries energy

wave velocity

the speed at which the disturbance moves. Also called the propagation velocity or propagation speed

wavelength

the distance between adjacent identical parts of a wave

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CONCEPTUAL QUESTIONS

Conceptual Questions

13.1 Hooke's Law: Stress and Strain Revisited

1. Describe a system in which elastic potential energy is stored.

13.3 Simple Harmonic Motion: A Special Periodic Motion

2. What conditions must be met to produce simple harmonic motion?
3. (a) If frequency is not constant for some oscillation, can the oscillation be simple harmonic motion? (b) Can you think of any examples of harmonic motion where the frequency may depend on the amplitude?
4. Give an example of a simple harmonic oscillator, specifically noting how its frequency is independent of amplitude.
5. Explain why you expect an object made of a stiff material to vibrate at a higher frequency than a similar object made of a spongy material.
6. As you pass a freight truck with a trailer on a highway, you notice that its trailer is bouncing up and down slowly. Is it more likely that the trailer is heavily loaded or nearly empty? Explain your answer.
7. Some people modify cars to be much closer to the ground than when manufactured. Should they install stiffer springs? Explain your answer.

13.4 The Simple Pendulum

8. Pendulum clocks are made to run at the correct rate by adjusting the pendulum's length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you, will you have to lengthen or shorten the pendulum to keep the correct time, other factors remaining constant? Explain your answer.

13.5 Energy and the Simple Harmonic Oscillator

9. Explain in terms of energy how dissipative forces such as friction reduce the amplitude of a harmonic oscillator. Also explain how a driving mechanism can compensate. (A pendulum clock is such a system.)

13.7 Damped Harmonic Motion

10. Give an example of a damped harmonic oscillator. (They are more common than undamped or simple harmonic oscillators.)
11. How would a car bounce after a bump under each of these conditions?
    overdamping
    underdamping
    critical damping
12. Most harmonic oscillators are damped and, if undriven, eventually come to a stop. How is this observation related to the second law of thermodynamics?

13.8 Forced Oscillations and Resonance

13. Why are soldiers in general ordered to "route step" (walk out of step) across a bridge?

13.9 Waves

14. Give one example of a transverse wave and another of a longitudinal wave, being careful to note the relative directions of the disturbance and wave propagation in each.
15. What is the difference between propagation speed and the frequency of a wave? Does one or both affect wavelength? If so, how?

13.10 Superposition and Interference

16. Speakers in stereo systems have two color-coded terminals to indicate how to

hook up the wires. If the wires are reversed, the speaker moves in a direction opposite that of a properly connected speaker. Explain why it is important to have both speakers connected the same way.

13.11 Energy in Waves: Intensity

17. Two identical waves undergo pure constructive interference. Is the resultant intensity twice that of the individual waves? Explain your answer.
18. Circular water waves decrease in amplitude as they move away from where a rock is dropped. Explain why.

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CHAPTER SUMMARY

13.1 Hooke's Law: Stress and Strain Revisited

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations and waves are related to systems that can be described by Hooke's law:

$$F = -\mathrm{kx},$$

where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.

- Elastic potential energy $\mathrm{PE}_{\mathrm{el}}$ stored in the deformation of a system that can be described by Hooke's law is given by $\mathrm{PE}_{\mathrm{el}} = (1/2)kx^2$.

13.2 Period and Frequency in Oscillations

- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T.
- The number of oscillations per unit time is the frequency f.
- These quantities are related by $f = \dfrac{1}{T}$.

13.3 Simple Harmonic Motion: A Special Periodic Motion

- Simple harmonic motion is oscillatory motion for a system that can be described only by Hooke's law. Such a system is also called a simple harmonic oscillator.
- Maximum displacement is the amplitude X. The period T and frequency f of a simple harmonic oscillator are given by $T = 2\pi\sqrt{\frac{m}{k}}$ and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where m is the mass of the system.
- Displacement in simple harmonic motion as a function of time is given by $x\,(t\,) = X\cos\frac{2\pi t}{T}$.
- The velocity is given by $v\,(t\,) = -v_{\max}\sin\frac{2\pi\ \mathrm{t}}{T}$, where $v_{\max} = \sqrt{k/m}X$.
- The acceleration is found to be $a\,(t\,) = -\frac{kX}{m}\ \cos\ \frac{2\pi t}{T}$.

13.4 The Simple Pendulum

- A mass m suspended by a wire of length L is a simple pendulum and undergoes simple harmonic motion for amplitudes less than about 15° .The period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}},$$

where L is the length of the string and g is the acceleration due to gravity.

13.5 Energy and the Simple Harmonic Oscillator

- Energy in the simple harmonic oscillator is shared between elastic potential energy and kinetic energy, with the total being constant:
  $$\frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{kx}^2 = \text{constant}.$$
- Maximum velocity depends on three factors: it is directly proportional to amplitude, it is greater for stiffer systems, and it is smaller for objects that have larger masses:
  $$v_{\max} = \sqrt{\frac{k}{m}}X.$$

13.6 Uniform Circular Motion and Simple Harmonic Motion

A projection of uniform circular motion undergoes simple harmonic oscillation.

13.7 Damped Harmonic Motion

- Damped harmonic oscillators have non-conservative forces that dissipate their energy.
- Critical damping returns the system to equilibrium as fast as possible without overshooting.
- An underdamped system will oscillate through the equilibrium position.
- An overdamped system moves more slowly toward equilibrium than one that is critically damped.

13.8 Forced Oscillations and Resonance

- A system's natural frequency is the frequency at which the system will oscillate if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- The less damping a system has, the higher the amplitude of the forced oscillations near resonance. The more damping a system has, the broader response it has to varying driving frequencies.

13.9 Waves

- A wave is a disturbance that moves from the point of creation with a wave velocity v_{w}.
- A wave has a wavelength λ, which is the distance between adjacent identical parts of the wave.
- Wave velocity and wavelength are related to the wave's frequency and period by $v_{\text{w}} = \frac{\lambda}{T}$ or $v_{\text{w}} = f\lambda$.
- A transverse wave has a disturbance perpendicular to its direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

13.10 Superposition and Interference

- Superposition is the combination of two waves at the same location.
- Constructive interference occurs when two identical waves are superimposed in phase.
- Destructive interference occurs when two identical waves are superimposed exactly out of phase.
- A standing wave is one in which two waves superimpose to produce a wave that varies in amplitude but does not propagate.
- Nodes are points of no motion in standing waves.
- An antinode is the location of maximum amplitude of a standing wave.
- Waves on a string are resonant standing waves with a fundamental frequency and can occur at higher multiples of the fundamental, called overtones or harmonics.
- Beats occur when waves of similar frequencies f_1 and f_2 are superimposed. The resulting amplitude oscillates with a beat frequency given by $f_{\text{B}} = | f_1 - f_2 |$.

13.11 Energy in Waves: Intensity

Intensity is defined to be the power per unit area: $I = \frac{P}{A}$ and has units of W/m^2.

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PROBLEMS & EXERCISES

Problems & Exercises

13.1 Hooke's Law: Stress and Strain Revisited

1. Fish are hung on a spring scale to determine their mass (most fishermen feel no obligation to truthfully report the mass). (a) What is the force constant of the spring in such a scale if it the spring stretches 8.00 cm for a 10.0 kg load? (b) What is the mass of a fish that stretches the spring 5.50 cm? (c) How far apart are the half-kilogram marks on the scale?
2. It is weigh-in time for the local under-85-kg rugby team. The bathroom scale used to assess eligibility can be described by Hooke's law and is depressed 0.75 cm by its maximum load of 120 kg. (a) What is the spring's effective spring constant? (b) A player stands on the scales and depresses it by 0.48 cm. Is he eligible to play on this under-85 kg team?
3. One type of BB gun uses a spring-driven plunger to blow the BB from its barrel. (a) Calculate the force constant of its plunger's spring if you must compress it 0.150 m to drive the 0.0500-kg plunger to a top speed of 20.0 m/s. (b) What force must be exerted to compress the spring?
4. (a) The springs of a pickup truck act like a single spring with a force constant of $1.30 \times 10^5 \text{N/m}$. By how much will the truck be depressed by its maximum load of 1000 kg?(b) If the pickup truck has four identical springs, what is the force constant of each?
5. When an 80.0-kg man stands on a pogo stick, the spring is compressed 0.120 m. (a) What is the force constant of the spring? (b) Will the spring be compressed more when he hops down the road?
6. A spring has a length of 0.200 m when a 0.300-kg mass hangs from it, and a length of 0.750 m when a 1.95-kg mass hangs from it. (a) What is the force constant of the spring? (b) What is the unloaded length of the spring?

13.2 Period and Frequency in Oscillations

7. What is the period of 60.0Hz electrical power?
8. If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?
9. Find the frequency of a tuning fork that takes $2.50 \times 10^{-3}\text{s}$ to complete one oscillation.
10. A stroboscope is set to flash every $8.00 \times 10^{-5}\text{s}$. What is the frequency of the flashes?
11. A tire has a tread pattern with a crevice every 2.00 cm. Each crevice makes a single vibration as the tire moves. What is the frequency of these vibrations if the car moves at 30.0 m/s?
12. **Engineering Application:** Each piston of an engine makes a sharp sound every other revolution of the engine. (a) How fast is a race car going if its eight-cylinder engine emits a sound of frequency 750 Hz, given that the engine makes 2000 revolutions per kilometer? (b) At how many revolutions per minute is the engine rotating?

13.3 Simple Harmonic Motion: A Special Periodic Motion

13. A type of cuckoo clock keeps time by having a mass bouncing on a spring, usually something cute like a cherub in a chair. What force constant is needed to produce a period of 0.500 s for a 0.0150-kg mass?
14. If the spring constant of a simple harmonic oscillator is doubled, by what factor will the mass of the system need to change in order for the frequency of the motion to remain the same?
15. A 0.500-kg mass suspended from a spring oscillates with a period of 1.50 s. How much mass must be added to the object to change the period to 2.00 s?
16. By how much leeway (both percentage and mass) would you have in the selection of the mass of the object in the previous problem if you did not wish the new period to be greater than 2.01 s or less than 1.99 s?
17. Suppose you attach the object with mass m to a vertical spring originally at rest,

and let it bounce up and down. You release the object from rest at the spring's original rest length. (a) Show that the spring exerts an upward force of $2.00mg$ on the object at its lowest point. (b) If the spring has a force constant of 10.0N/m and a 0.25-kg-mass object is set in motion as described, find the amplitude of the oscillations. (c) Find the maximum velocity.

18. A diver on a diving board is undergoing simple harmonic motion. Her mass is 55.0 kg and the period of her motion is 0.800 s. The next diver is a male whose period of simple harmonic oscillation is 1.05 s. What is his mass if the mass of the board is negligible?
19. Suppose a diving board with no one on it bounces up and down in a simple harmonic motion with a frequency of 4.00 Hz. The board has an effective mass of 10.0 kg. What is the frequency of the simple harmonic motion of a 75.0-kg diver on the board?



Figure 13.42 This child's toy relies on springs to keep infants entertained. Image from OpenStax College Physics 2e, CC-BY 4.0

20. The device pictured in Figure 13.42 entertains infants while keeping them from wandering. The child bounces in a harness suspended from a door frame by a spring. (a) If the spring stretches 0.250 m while supporting an 8.00-kg child, what is its spring constant? (b) What is the time for one complete bounce of this child?

(c) What is the child's maximum velocity if the amplitude of her bounce is 0.200 m?

21. A 90.0-kg skydiver hanging from a parachute bounces up and down with a period of 1.50 s. What is the new period of oscillation when a second skydiver, whose mass is 60.0 kg, hangs from the legs of the first, as seen in Figure 13.43.



Figure 13.43 The oscillations of one skydiver are about to be affected by a second skydiver. Image by The U.S. Army from Washington, DC, Wikimedia Commons License

13.4 The Simple Pendulum

As usual, the acceleration due to gravity in these problems is taken to be $g = 9.80\text{m/s}^2$, unless otherwise specified.

22. What is the length of a pendulum that has a period of 0.500 s?
23. Some people think a pendulum with a period of 1.00 s can be driven with "mental energy" or psycho kinetically, because its period is the same as an average heartbeat. True or not, what is the length of such a pendulum?
24. What is the period of a 1.00-m-long pendulum?
25. How long does it take a child on a swing to complete one swing if her center of gravity is 4.00 m below the pivot?
26. The pendulum on a cuckoo clock is 5.00 cm long. What is its frequency?
27. Two parakeets sit on a swing with their combined center of mass 10.0 cm below the pivot. At what frequency do they swing?
28. (a) A pendulum that has a period of 3.00000 s and that is located where the acceleration due to gravity is 9.79m/s^2 is moved to a location where the acceleration due to gravity is 9.82m/s^2. What is its new period? (b) Explain why so many digits are needed in the value for the period, based on the relation between the period and the acceleration due to gravity.
29. A pendulum with a period of 2.00000 s in one location $(g = 9.80\text{m/s}^2)$ is moved to a new location where the period is now 1.99796 s. What is the acceleration due to gravity at its new location?
30. (a) What is the effect on the period of a pendulum if you double its length? (b) What is the effect on the period of a pendulum if you decrease its length by 5.00%?
31. Find the ratio of the new/old periods of a pendulum if the pendulum were transported from Earth to the Moon, where the acceleration due to gravity is 1.63m/s^2.
32. At what rate will a pendulum clock run on the Moon, where the acceleration due to gravity is 1.63m/s^2, if it keeps time accurately on Earth? That is, find the time (in hours) it takes the clock's minute hand, which on Earth makes one revolution

per hour, to make one revolution on the Moon.

33. Suppose the length of a clock's pendulum is changed by 1.000%, exactly at noon one day. What time will it read 24.00 hours later, assuming it the pendulum has kept perfect time before the change? Note that there are two answers, and perform the calculation to four-digit precision.
34. If a pendulum-driven clock gains 5.00 s/day, what fractional change in pendulum length must be made for it to keep perfect time?

13.5 Energy and the Simple Harmonic Oscillator

35. The length of nylon rope from which a mountain climber is suspended has a force constant of $1.40 \times 10^4 \text{N/m}$ (a) What is the frequency at which he bounces, given his mass plus and the mass of his equipment are 90.0 kg? Ignore the change in gravitational potential energy after the cord begins to stretch. (b) How much would this rope stretch to break the climber's fall if he free-falls 2.00 m before the rope runs out of slack? Hint: Use conservation of energy. (c) Repeat both parts of this problem in the situation where twice this length of nylon rope is used.
36. **Engineering Application:** Near the top of the Citigroup Center building in New York City, there is an object with mass of $4.00 \times 10^5 \text{kg}$ on springs that have adjustable force constants. Its function is to dampen wind-driven oscillations of the building by oscillating at the same frequency as the building is being driven—the driving force is transferred to the object, which oscillates instead of the entire building. (a) What effective force constant should the springs have to make the object oscillate with a period of 2.00 s? (b) What energy is stored in the springs for a 2.00-m displacement from equilibrium?

13.6 Uniform Circular Motion and Simple Harmonic Motion

37. (a)What is the maximum velocity of an 85.0-kg person bouncing on a bathroom scale having a force constant of $1.50 \times 10^6 \text{N/m}$, if the amplitude of the bounce is 0.200 cm? (b)What is the maximum energy stored in the spring?
38. A novelty clock has a 0.0100-kg mass object bouncing on a spring that has a force

constant of 1.25 N/m. What is the maximum velocity of the object if the object bounces 3.00 cm above and below its equilibrium position? (b) How many joules of kinetic energy does the object have at its maximum velocity?

39. At what positions is the speed of a simple harmonic oscillator half its maximum? That is, what values of x/X give $v = \pm v_{\max}/2$, where X is the amplitude of the motion?
40. A ladybug sits 12.0 cm from the center of a Beatles music album spinning at 33.33 rpm. What is the maximum velocity of its shadow on the wall behind the turntable, if illuminated parallel to the record by the parallel rays of the setting Sun?

13.7 Damped Harmonic Motion

41. The amplitude of a lightly damped oscillator decreases by 3.0% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

13.8 Forced Oscillations and Resonance

42. How much energy must the shock absorbers of a 1200-kg car dissipate in order to damp a bounce that initially has a velocity of 0.800 m/s at the equilibrium position? Assume the car returns to its original vertical position.
43. If a car has a suspension system with a force constant of $5.00 \times 10^4 \text{N/m}$, how much energy must the car's shocks remove to dampen an oscillation starting with a maximum displacement of 0.0750 m?
44. (a) How much will a spring that has a force constant of 40.0 N/m be stretched by an object with a mass of 0.500 kg when hung motionless from the spring? (b) Calculate the decrease in gravitational potential energy of the 0.500-kg object when it descends this distance. (c) Part of this gravitational energy goes into the spring. Calculate the energy stored in the spring by this stretch, and compare it with the gravitational potential energy. Explain where the rest of the energy might go.

45. Suppose you have a 0.750-kg object on a horizontal surface connected to a spring that has a force constant of 150 N/m. There is simple friction between the object and surface with a static coefficient of friction $\mu_s = 0.100$. (a) How far can the spring be stretched without moving the mass? (b) If the object is set into oscillation with an amplitude twice the distance found in part (a), and the kinetic coefficient of friction is $\mu_k = 0.0850$, what total distance does it travel before stopping? Assume it starts at the maximum amplitude.
46. **Engineering Application:** A suspension bridge oscillates with an effective force constant of $1.00 \times 10^8 \text{N/m}$. (a) How much energy is needed to make it oscillate with an amplitude of 0.100 m? (b) If soldiers march across the bridge with a cadence equal to the bridge's natural frequency and impart $1.00 \times 10^4 \text{J}$ of energy each second, how long does it take for the bridge's oscillations to go from 0.100 m to 0.500 m amplitude?

13.9 Waves

47. Storms in the South Pacific can create waves that travel all the way to the California coast, which are 12,000 km away. How long does it take them if they travel at 15.0 m/s?
48. Waves on a swimming pool propagate at 0.750 m/s. You splash the water at one end of the pool and observe the wave go to the opposite end, reflect, and return in 30.0 s. How far away is the other end of the pool?
49. Wind gusts create ripples on the ocean that have a wavelength of 5.00 cm and propagate at 2.00 m/s. What is their frequency?
50. How many times a minute does a boat bob up and down on ocean waves that have a wavelength of 40.0 m and a propagation speed of 5.00 m/s?
51. Scouts at a camp shake the rope bridge they have just crossed and observe the wave crests to be 8.00 m apart. If they shake the bridge twice per second, what is the propagation speed of the waves?
52. What is the wavelength of the waves you create in a swimming pool if you splash your hand at a rate of 2.00 Hz and the waves propagate at 0.800 m/s?
53. What is the wavelength of an earthquake that shakes you with a frequency of

10.0 Hz and gets to another city 84.0 km away in 12.0 s?

54. Radio waves transmitted through space at $3.00 \times 10^8 \mathrm{m/s}$ by the Voyager spacecraft have a wavelength of 0.120 m. What is their frequency?
55. Your ear is capable of differentiating sounds that arrive at the ear just 1.00 ms apart. What is the minimum distance between two speakers that produce sounds that arrive at noticeably different times on a day when the speed of sound is 340 m/s?
56. (a) Seismographs measure the arrival times of earthquakes with a precision of 0.100 s. To get the distance to the epicenter of the quake, they compare the arrival times of S- and P-waves, which travel at different speeds. Figure 13.44) If S- and P-waves travel at 4.00 and 7.20 km/s, respectively, in the region considered, how precisely can the distance to the source of the earthquake be determined? (b) Seismic waves from underground detonations of nuclear bombs can be used to locate the test site and detect violations of test bans. Discuss whether your answer to (a) implies a serious limit to such detection. (Note also that the uncertainty is greater if there is an uncertainty in the propagation speeds of the S- and P-waves.)



Figure 13.44 A seismograph as described in above problem. Image by Oleg Alexandrov, Wikimedia Commons License

13.10 Superposition and Interference

57. A car has two horns, one emitting a frequency of 199 Hz and the other emitting a frequency of 203 Hz. What beat frequency do they produce?
58. The middle-C hammer of a piano hits two strings, producing beats of 1.50 Hz. One of the strings is tuned to 260.00 Hz. What frequencies could the other string have?
59. Two tuning forks having frequencies of 460 and 464 Hz are struck simultaneously. What average frequency will you hear, and what will the beat frequency be?
60. Twin jet engines on an airplane are producing an average sound frequency of 4100 Hz with a beat frequency of 0.500 Hz. What are their individual frequencies?
61. A wave traveling on a Slinky® that is stretched to 4 m takes 2.4 s to travel the

length of the Slinky and back again. (a) What is the speed of the wave? (b) Using the same Slinky stretched to the same length, a standing wave is created which consists of three antinodes and four nodes. At what frequency must the Slinky be oscillating?

62. Three adjacent keys on a piano (F, F-sharp, and G) are struck simultaneously, producing frequencies of 349, 370, and 392 Hz. What beat frequencies are produced by this discordant combination?

13.11 Energy in Waves: Intensity

63. **Medical Application:** Ultrasound of intensity $1.50 \times 10^2 \mathrm{W/m^2}$ is produced by the rectangular head of a medical imaging device measuring 3.00 by 5.00 cm. What is its power output?
64. The low-frequency speaker of a stereo set has a surface area of $0.\ 05\ \mathrm{m}^2$ and produces 1W of acoustical power. What is the intensity at the speaker? If the speaker projects sound uniformly in all directions, at what distance from the speaker is the intensity $0.\ 1\mathrm{W/m}^2$?
65. To increase intensity of a wave by a factor of 50, by what factor should the amplitude be increased?
66. **Engineering Application:** A device called an insolation meter is used to measure the intensity of sunlight has an area of 100 cm2 and registers 6.50 W. What is the intensity in $\mathrm{W/m}^2$?
67. **Astronomy Application:** Energy from the Sun arrives at the top of the Earth's atmosphere with an intensity of $1.30\ \mathrm{kW/m}^2$.
    How long does it take for $1.8 \times 10^9 \mathrm{J}$ to arrive on an area of $1.\ 00\mathrm{m}^2$?
68. Suppose you have a device that extracts energy from ocean breakers in direct proportion to their intensity. If the device produces 10.0 kW of power on a day when the breakers are 1.20 m high, how much will it produce when they are 0.600 m high?
69. **Engineering Application:** (a) A photovoltaic array of (solar cells) is 10.0% efficient in gathering solar energy and converting it to electricity. If the average

intensity of sunlight on one day is $700 \mathrm{W/m}^2$, what area should your array have to gather energy at the rate of 100 W? (b) What is the maximum cost of the array if it must pay for itself in two years of operation averaging 10.0 hours per day? Assume that it earns money at the rate of 9.00 ¢ per kilowatt-hour.

70. A microphone receiving a pure sound tone feeds an oscilloscope, producing a wave on its screen. If the sound intensity is originally $2.00 \times 10^{-5} \mathrm{W/m}^2$, but is turned up until the amplitude increases by 30.0%, what is the new intensity?
71. **Medical Application:** (a) What is the intensity in $\mathrm{W/m}^2$ of a laser beam used to burn away cancerous tissue that, when 90.0% absorbed, puts 500 J of energy into a circular spot 2.00 mm in diameter in 4.00 s? (b) Discuss how this intensity compares to the average intensity of sunlight (about $700 \ \mathrm{W/m}^2$) and the implications that would have if the laser beam entered your eye. Note how the amount of damage depends on the time duration of the exposure.

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PHYSICS OF HEARING



Figure 14.1 This tree fell some time ago. When it fell, atoms in the air were disturbed. Physicists would call this disturbance sound whether someone was around to hear it or not. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

14.1 Sound
14.2 Speed of Sound, Frequency, and Wavelength
14.3 Sound Intensity and Sound Level
14.4 Doppler Effect and Sonic Booms
14.5 Sound Interference and Resonance: Standing Waves in Air Columns
14.6 Hearing
14.7 Ultrasound
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

If a tree falls in the forest and no one is there to hear it, does it make a sound? The answer to this old philosophical question depends on how you define sound. If sound only exists when someone is around to perceive it, then there was no sound. However, if we define sound in terms of physics; that is, a disturbance of the atoms in matter transmitted from its origin outward (in other words, a wave), then there *was* a sound, even if nobody was around to hear it.

Such a wave is the physical phenomenon we call *sound.* Its perception is hearing. Both the physical phenomenon and its perception are interesting and will be considered in this text. We shall explore both sound and hearing; they are related, but are not the same thing. We will also explore the many practical uses of sound waves, such as in medical imaging.
Access multimedia content

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14.1 SOUND

Learning Objectives

By the end of this section, you will be able to:

- Define sound and hearing.
- Describe sound as a longitudinal wave.



Figure 14.2 This glass has been shattered by a high-intensity sound wave of the same frequency as the resonant frequency of the glass. While the sound is not visible, the effects of the sound prove its existence. Image from OpenStax College Physics 2e, CC-BY 4.0

Sound can be used as a familiar illustration of waves. Because hearing is one of our most important senses, it is interesting to see how the physical properties of sound correspond to our perceptions of it. Hearing is the perception of sound, just as vision is the perception of visible light. But sound has important applications beyond hearing. Ultrasound, for example, is not heard but can be employed to form medical images and is also used in treatment.

The physical phenomenon of sound is defined to be a disturbance of matter that is transmitted from its

source outward. Sound is a wave. On the atomic scale, it is a disturbance of atoms that is far more ordered than their thermal motions. In many instances, sound is a periodic wave, and the atoms undergo simple harmonic motion. In this text, we shall explore such periodic sound waves.

A vibrating string produces a sound wave as illustrated in Figure 14.3, Figure 14.4, and Figure 14.5. As the string oscillates back and forth, it transfers energy to the air, mostly as thermal energy created by turbulence. But a small part of the string's energy goes into compressing and expanding the surrounding air, creating slightly higher and lower local pressures. These compressions (high pressure regions) and rarefactions (low pressure regions) move out as longitudinal pressure waves having the same frequency as the string—they are the disturbance that is a sound wave. (Sound waves in air and most fluids are longitudinal, because fluids have almost no shear strength. These are the sound waves we will study in this chapter. Although sound waves in solids can be both transverse and longitudinal, those are beyond the scope of our discussion.) Figure 14.5 shows a graph of gauge pressure versus distance from the vibrating string.

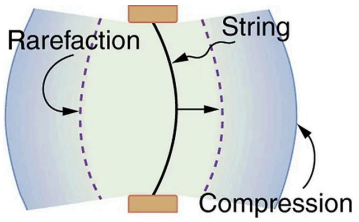


Figure 14.3 A vibrating string moving to the right compresses the air in front of it and expands the air behind it. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the concepts of sound waves, focusing on rarefaction and compression. In the center, there is a straight black line labeled "String," which represents the medium through which the wave travels, such as air or a string on a musical instrument. The string is flanked by two orange rectangles at the top and bottom, likely representing fixed points or sources of vibration.

To the left of the string, there is a section labeled "Rarefaction," which is depicted with dashed purple lines curving outward, symbolizing areas where particles are spread apart.

On the right side of the string, the area is labeled "Compression," shown with another set of dashed purple lines curving inward, indicating areas where particles are close together. Arrows point towards or away from the string in both rarefaction and compression areas to illustrate the movement of particles.

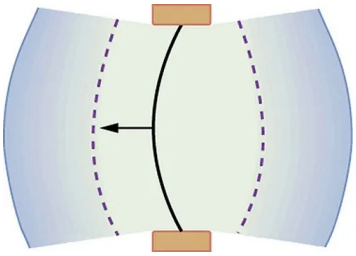


Figure 14.4 As the string moves to the left, it creates another compression and rarefaction as the ones on the right move away from the string. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of a concave lens, illustrating the bending of light. It features two light blue curved surfaces, representing the lens, with orange-brown rectangles on the top and bottom, symbolizing the lens holders. A solid black arrow is shown entering from the center of the lens, curving towards the thicker part, indicative of light refracting. On either side of the central arrow, there are two dashed purple lines showing the original path of light before bending. The overall shape of the lens is concave, as demonstrated by the inward curve of its surfaces.

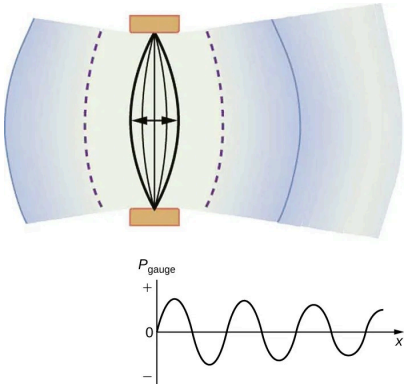


Figure 14.5 After many vibrations, there are a series of compressions and rarefactions moving out from the string as a sound wave. The graph shows gauge pressure versus distance from the source. Pressures vary only slightly from atmospheric for ordinary sounds. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two main sections. The upper part illustrates an hourglass-shaped diagram with a central oval formed by multiple curved black lines that converge towards the ends of the oval. These lines are surrounded by a light blue shade. Curved arrows point inward from either side toward the center of the oval, and two horizontal beige rectangles are located at the top and bottom of the diagram. Additionally, there are

two sets of dashed purple lines indicating the flow around the oval, with blue blurry edges giving a sense of movement or pressure.

The lower section contains a graph showing a sinusoidal wave. The horizontal axis, labeled "x," represents the displacement, and the vertical axis labeled "P_{gauge}" represents pressure with positive, zero, and negative values. The wave oscillates above and below the central zero line.

The amplitude of a sound wave decreases with distance from its source, because the energy of the wave is spread over a larger and larger area. But it is also absorbed by objects, such as the eardrum in Figure 14.6, and converted to thermal energy by the viscosity of air. In addition, during each compression a little heat transfers to the air and during each rarefaction even less heat transfers from the air, so that the heat transfer reduces the organized disturbance into random thermal motions. Whether the heat transfer from compression to rarefaction is significant depends on how far apart they are—that is, it depends on wavelength. Wavelength, frequency, amplitude, and speed of propagation are important for sound, as they are for all waves.

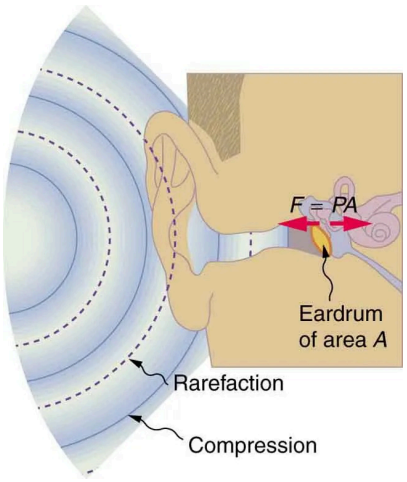


Figure 14.6 Sound wave compressions and rarefactions travel up the ear canal and force the eardrum to vibrate. There is a net force on the eardrum, since the sound wave pressures differ from the atmospheric pressure found behind the eardrum. A complicated mechanism converts the vibrations to nerve impulses, which are perceived by the person. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a cross-section of a human ear receiving sound waves. On the left side, concentric circles represent sound waves approaching the ear, with areas labeled "Compression" and "Rarefaction" showing the different phases of the sound wave. The right side depicts the anatomy of the ear, focusing on the eardrum. A close-up view shows an arrow labeled `F = PA`, indicating the force (F) on the eardrum is equal to the

pressure (P) times the area (A). The eardrum is marked as “Eardrum of area A.” The sound waves are shown reaching the eardrum, demonstrating how they transmit sound energy into the ear.

PhET Explorations

Wave Interference

Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.

Access multimedia content

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14.2 SPEED OF SOUND, FREQUENCY, AND WAVELENGTH

Learning Objectives

By the end of this section, you will be able to:

- Define pitch.
- Describe the relationship between the speed of sound, its frequency, and its wavelength.
- Describe the effects on the speed of sound as it travels through various media.
- Describe the effects of temperature on the speed of sound.



Figure 14.7 When a firework explodes, the light energy is perceived before the sound energy. Sound travels more slowly than light does. Image from OpenStax College Physics 2e, CC-BY 4.0

Sound, like all waves, travels at a certain speed and has the properties of frequency and wavelength. You can observe direct evidence of the speed of sound while watching a fireworks display. The flash of an explosion is seen well before its sound is heard, implying both that sound travels at a finite speed and that it is much slower

than light. You can also directly sense the frequency of a sound. Perception of frequency is called pitch. The wavelength of sound is not directly sensed, but indirect evidence is found in the correlation of the size of musical instruments with their pitch. Small instruments, such as a piccolo, typically make high-pitch sounds, while large instruments, such as a tuba, typically make low-pitch sounds. High pitch means small wavelength, and the size of a musical instrument is directly related to the wavelengths of sound it produces. So a small instrument creates short-wavelength sounds. Similar arguments hold that a large instrument creates long-wavelength sounds.

The relationship of the speed of sound, its frequency, and wavelength is the same as for all waves:

$$v_{\text{w}} = f\lambda,$$
14.1

where v_w is the speed of sound, f is its frequency, and λ is its wavelength. The wavelength of a sound is the distance between adjacent identical parts of a wave—for example, between adjacent compressions as illustrated in Figure 14.8. The frequency is the same as that of the source and is the number of waves that pass a point per unit time.

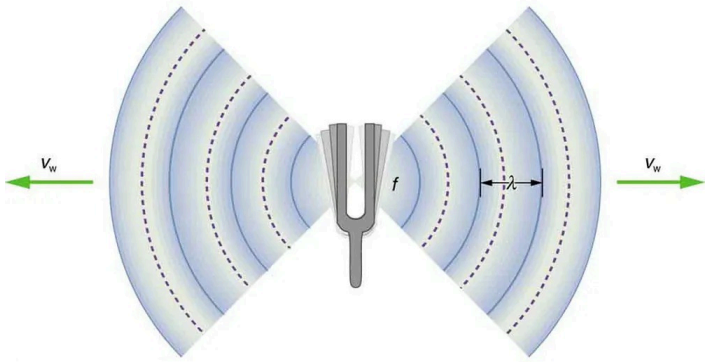


Figure 14.8 A sound wave emanates from a source vibrating at a frequency f, propagates at v_{w}, and has a wavelength λ. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram illustrating wave propagation from a tuning fork. The tuning fork is placed in the center, with wavefronts emanating from it. The waves are shown as concentric arcs on both sides of the fork, indicating the direction of sound wave travel. There are arrows labeled “Vw” on both sides, pointing in opposite directions, representing wave velocity. The distance between two consecutive wavefronts on the right side is marked as “λ,” indicating the wavelength. The tuning fork is labeled “f,” denoting frequency. The background of the wavefronts is shaded with a gradient of blue, becoming lighter as the distance from the tuning fork increases.

Table 14.1 makes it apparent that the speed of sound varies greatly in different media. The speed of sound in a medium is determined by a combination of the medium's rigidity (or compressibility in gases) and its density. The more rigid (or less compressible) the medium, the faster the speed of sound. For materials that have similar rigidities, sound will travel faster through the one with the lower density because the sound energy is more easily transferred from particle to particle. The speed of sound in air is low, because air is compressible. Because liquids and solids are relatively rigid and very difficult to compress, the speed of sound in such media is generally greater than in gases.

Table 14.1 Speed of Sound in Various Media

Medium	**v_w(m/s)**
Gases at 0° C	
Air	331
Carbon dioxide	259
Oxygen	316
Helium	965
Hydrogen	1290
Liquids at 20° C	
Ethanol	1160
Mercury	1450
Water, fresh	1480
Sea water	1540
Human tissue	1540
Solids (longitudinal or bulk)	
Vulcanized rubber	54
Polyethylene	920
Marble	3810
Glass, Pyrex	5640
Lead	1960
Aluminum	5120
Steel	5960

Earthquakes, essentially sound waves in Earth's crust, are an interesting example of how the speed of sound

depends on the rigidity of the medium. Earthquakes have both longitudinal and transverse components, and these travel at different speeds. The bulk modulus of granite is greater than its shear modulus. For that reason, the speed of longitudinal or pressure waves (P-waves) in earthquakes in granite is significantly higher than the speed of transverse or shear waves (S-waves). Both components of earthquakes travel slower in less rigid material, such as sediments. P-waves have speeds of 4 to 7 km/s, and S-waves correspondingly range in speed from 2 to 5 km/s, both being faster in more rigid material. The P-wave gets progressively farther ahead of the S-wave as they travel through Earth's crust. The time between the P- and S-waves is routinely used to determine the distance to their source, the epicenter of the earthquake. The time and nature of these wave differences also provides the evidence for the nature of Earth's core. Through careful analysis of seismographic records of large earthquakes whose waves could be clearly detected around the world, Richard Dixon Oldham established that waves passing through the center of the Earth behaved as if they were moving through a different medium: a liquid. Later on, Inge Lehmann used more precise observations (partly based on a better coordinated network of seismographs she helped set up) to better define the nature of the core: that it was a solid inner core surrounded by a liquid outer core.

The speed of sound is affected by temperature in a given medium. For air at sea level, the speed of sound is given by

$$v_{\text{w}} = (331 \text{ m/s})\sqrt{\frac{T}{273\text{K}}},$$

14.2

where the temperature (denoted as T) is in units of kelvin. The speed of sound in gases is related to the average speed of particles in the gas, v_{rms}, and that

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}},$$

14.3

where k is the Boltzmann constant ($1.38 \times 10^{-23}\text{J/K}$) and m is the mass of each (identical) particle in the gas. So, it is reasonable that the speed of sound in air and other gases should depend on the square root of temperature. While not negligible, this is not a strong dependence. At 0° C, the speed of sound is 331 m/s, whereas at 20.0^{o} C it is 343 m/s, less than a 4% increase. Figure 14.9 shows a use of the speed of sound by a bat to sense distances. Echoes are also used in medical imaging.

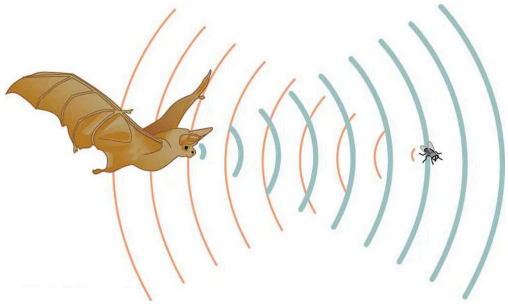


Figure 14.9 A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

An illustration depicting a bat using echolocation to locate an insect. The bat is shown flying on the left side of the image. Sound waves are emanating from the bat in the form of curved lines, colored in orange, moving outward towards an insect on the right. The insect, drawn as a small black creature with wings, is on the far right. The sound waves that reflect back from the insect to the bat are represented by blue curved lines, illustrating the process of echo detection.

One of the more important properties of sound is that its speed is nearly independent of frequency. This independence is certainly true in open air for sounds in the audible range of 20 to 20,000 Hz. If this independence were not true, you would certainly notice it for music played by a marching band in a football stadium, for example. Suppose that high-frequency sounds traveled faster—then the farther you were from the band, the more the sound from the low-pitch instruments would lag that from the high-pitch ones. But the music from all instruments arrives in cadence independent of distance, and so all frequencies must travel at nearly the same speed. Recall that

$$v_{\text{w}} = f\lambda.$$
14.4

In a given medium under fixed conditions, v_{w} is constant, so that there is a relationship between f and λ; the higher the frequency, the smaller the wavelength. See Figure 17.10 and consider the following example.

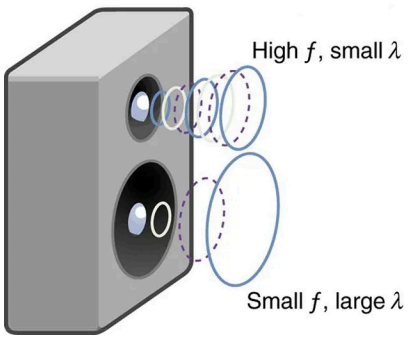


Figure 14.10 Because they travel at the same speed in a given medium, low-frequency sounds must have a greater wavelength than high-frequency sounds. Here, the lower-frequency sounds are emitted by the large speaker, called a woofer, while the higher-frequency sounds are emitted by the small speaker, called a tweeter. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a speaker-like object with two circular components on its front face, each resembling a speaker driver. The upper driver is labeled "High f, small λ," indicating high frequency and small wavelength. The lower driver is labeled "Small f, large λ," indicating low frequency and large wavelength. Concentric circles, emanating from each driver, represent sound waves. The circles for the high frequency (upper) driver are smaller and closer together, while those for the low frequency (lower) driver are larger and spaced further apart.

Example 14.1

Calculating Wavelengths: What Are the Wavelengths of Audible Sounds?

Calculate the wavelengths of sounds at the extremes of the audible range, 20 and 20,000 Hz, in $30.0^{\circ}\ \mathrm{C}$ air. (Assume that the frequency values are accurate to two significant figures.)

Strategy

To find wavelength from frequency, we can use $v_{\mathrm{w}} = f\lambda$.

Solution

1. Identify knowns. The value for v_{w}, is given by

$$v_{\mathrm{w}} = (331\mathrm{m/s})\sqrt{\frac{T}{273\mathrm{K}}}.$$

14.5

2. Convert the temperature into kelvin and then enter the temperature into the equation

$$v_{\mathrm{w}} = (331\mathrm{m/s})\sqrt{\frac{303\ \mathrm{K}}{273\mathrm{K}}} = 348.7\mathrm{m/s}.$$

14.6

3. Solve the relationship between speed and wavelength for λ:

$$\lambda = \frac{v_w}{f}.$$

14.7

4. Enter the speed and the minimum frequency to give the maximum wavelength:

$$\lambda_{\text{max}} = \frac{348.7\text{m/s}}{20\text{ Hz}} = 17\text{m}.$$

14.8

5. Enter the speed and the maximum frequency to give the minimum wavelength:

$$\lambda_{\text{min}} = \frac{348.7\text{m/s}}{20,000\text{ Hz}} = 0.017\text{m} = 1.7\text{ cm}.$$

14.9

Discussion

Because the product of f multiplied by λ equals a constant, the smaller f is, the larger λ must be, and vice versa.

The speed of sound can change when sound travels from one medium to another. However, the frequency usually remains the same because it is like a driven oscillation and has the frequency of the original source. If v_{w} changes and f remains the same, then the wavelength λ must change. That is, because $v_{\text{w}} = f\lambda$, the higher the speed of a sound, the greater its wavelength for a given frequency.

Making Connections: Take-Home Investigation—Voice as a Sound Wave

Suspend a sheet of paper so that the top edge of the paper is fixed and the bottom edge is free to move. You could tape the top edge of the paper to the edge of a table. Gently blow near the edge of the bottom of the sheet and note how the sheet moves. Speak softly and then louder such that the sounds hit the edge of the bottom of the paper, and note how the sheet moves. Explain the effects.

Check Your Understanding

Imagine you observe two fireworks explode. You hear the explosion of one as soon as you see it. However, you see the other firework for several milliseconds before you hear the explosion. Explain why this is so.

Click for Solution

Solution

Sound and light both travel at definite speeds. The speed of sound is slower than the speed of light. The first firework is probably very close by, so the speed difference is not noticeable. The second firework is farther away, so the light arrives at your eyes noticeably sooner than the sound wave arrives at your ears.

Check Your Understanding

You observe two musical instruments that you cannot identify. One plays high-pitch sounds and the other plays low-pitch sounds. How could you determine which is which without hearing either of them play?

Click for Solution

Solution

Compare their sizes. High-pitch instruments are generally smaller than low-pitch instruments because they generate a smaller wavelength.

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14.3 SOUND INTENSITY AND SOUND LEVEL

Learning Objectives

By the end of this section, you will be able to:

- Define intensity, sound intensity, and sound pressure level.
- Calculate sound intensity levels in decibels (dB).



Figure 14.11 Noise on crowded roadways like this one in Delhi makes it hard to hear others unless they shout. Image by NOMAD, Wikimedia Commons License

In a quiet forest, you can sometimes hear a single leaf fall to the ground. After settling into bed, you may hear your blood pulsing through your ears. But when a passing motorist has his stereo turned up, you cannot even hear what the person next to you in your car is saying. We are all very familiar with the loudness of sounds and aware that they are related to how energetically the source is vibrating. In cartoons depicting a screaming person (or an animal making a loud noise), the cartoonist often shows an open mouth with a vibrating uvula, the hanging tissue at the back of the mouth, to suggest a loud sound coming from the throat Figure 14.12. High noise exposure is hazardous to hearing, and it is common for musicians to have hearing losses that are sufficiently severe that they interfere with the musicians' abilities to perform. The relevant physical quantity is sound intensity, a concept that is valid for all sounds whether or not they are in the audible range.

Intensity is defined to be the power per unit area carried by a wave. Power is the rate at which energy is transferred by the wave. In equation form, intensity I is

$$I = \frac{P}{A},$$
14.10

where P is the power through an area A. The SI unit for I is W/m^2. The intensity of a sound wave is related to its amplitude squared by the following relationship:

$$I = \frac{((\Delta p))^2}{2\rho v_w}.$$
14.11

Here Δp is the pressure variation or pressure amplitude (half the difference between the maximum and minimum pressure in the sound wave) in units of pascals (Pa) or N/m^2. (We are using a lower case p for pressure to distinguish it from power, denoted by P above.) The energy (as kinetic energy $\frac{mv^2}{2}$) of an oscillating element of air due to a traveling sound wave is proportional to its amplitude squared. In this equation,
ρ is the density of the material in which the sound wave travels, in units of kg/m^3, and v_w is the speed of sound in the medium, in units of m/s. The pressure variation is proportional to the amplitude of the oscillation, and so I varies as $(\Delta p)^2$ (Figure 14.12). This relationship is consistent with the fact that the sound wave is produced by some vibration; the greater its pressure amplitude, the more the air is compressed in the sound it creates.

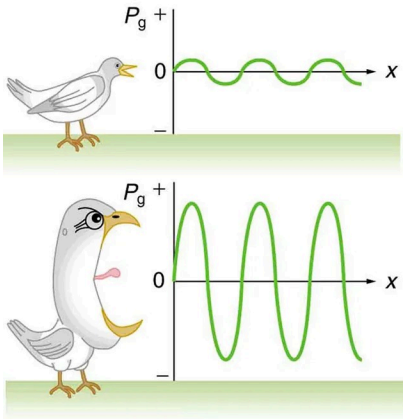


Figure 14.12 Graphs of the gauge pressures in two sound waves of different intensities. The more intense sound is produced by a source that has larger-amplitude oscillations and has greater pressure maxima and minima. Because pressures are higher in the greater-intensity sound, it can exert larger forces on the objects it encounters. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two cartoon illustrations of birds with graphs next to them, depicting sound waves.

Top illustration: A small bird is shown with its beak slightly open. To the right, a graph displays a sine wave with smaller amplitude oscillations, suggesting a quieter sound.

Bottom illustration: A larger bird is shown with its beak wide open, indicating it is making a louder noise. The accompanying graph shows a sine wave with larger amplitude oscillations, suggesting a louder sound.

Both graphs have labels: on the vertical axis "P_g" with "+" at the top and "-" at the bottom; "0" is marked in the middle. The horizontal axis is labeled "x."

Sound intensity levels are quoted in decibels (dB) much more often than sound intensities in watts per meter squared. Decibels are the unit of choice in the scientific literature as well as in the popular media. The reasons for this choice of units are related to how we perceive sounds. How our ears perceive sound can be more accurately described by the logarithm of the intensity rather than directly to the intensity. The sound intensity level β in decibels of a sound having an intensity I in watts per meter squared is defined to be

$$\beta\,(\text{dB}) = 10\log_{10}\left(\frac{I}{I_0}\right),$$

14.12

where $I_0 = 10^{-12}\text{W}/\text{m}^2$ is a reference intensity. In particular, I_0 is the lowest or threshold intensity of sound a person with normal hearing can perceive at a frequency of 1000 Hz. Sound intensity level is not the same as intensity. Because β is defined in terms of a ratio, it is a unitless quantity telling you the *level* of the sound relative to a fixed standard ($10^{-12}\text{W}/\text{m}^2$, in this case). The units of decibels (dB) are used to indicate this ratio is multiplied by 10 in its definition. The bel, upon which the decibel is based, is named for Alexander Graham Bell, the inventor of the telephone.

Table 14.2 Sound Intensity Levels and Intensities

Sound intensity level β (dB)	Intensity I(W/m^2)	Example/effect
0	1×10^{-12}	Threshold of hearing at 1000 Hz
10	1×10^{-11}	Rustle of leaves
20	1×10^{-10}	Whisper at 1 m distance
30	1×10^{-9}	Quiet home
40	1×10^{-8}	Average home
50	1×10^{-7}	Average office, soft music
60	1×10^{-6}	Normal conversation
70	1×10^{-5}	Noisy office, busy traffic
80	1×10^{-4}	Loud radio, classroom lecture
90	1×10^{-3}	Inside a heavy truck; damage from prolonged exposure[1]
100	1×10^{-2}	Noisy factory, siren at 30 m; damage from 8 h per day exposure
110	1×10^{-1}	Damage from 30 min per day exposure
120	1	Loud rock concert, pneumatic chipper at 2 m; threshold of pain
140	1×10^{2}	Jet airplane at 30 m; severe pain, damage in seconds
160	1×10^{4}	Bursting of eardrums

The decibel level of a sound having the threshold intensity of 10^{-12}W/m^2 is $\beta = 0\text{dB}$, because $\log_{10}1 = 0$. That is, the threshold of hearing is 0 decibels. Table 14.2 gives levels in decibels and intensities in watts per meter squared for some familiar sounds.

One of the more striking things about the intensities in Table 14.2 is that the intensity in watts per meter squared is quite small for most sounds. The ear is sensitive to as little as a trillionth of a watt per meter squared—even more impressive when you realize that the area of the eardrum is only about $1cm^2$, so that only 10^{-16} W falls on it at the threshold of hearing! Air molecules in a sound wave of this intensity vibrate over a distance of less than one molecular diameter, and the gauge pressures involved are less than 10^{-9} atm.

Another impressive feature of the sounds in Table 14.2 is their numerical range. Sound intensity varies by a

1. Several government agencies and health-related professional associations recommend that 85 dB not be exceeded for 8-hour daily exposures in the absence of hearing protection.

factor of 10^{12} from threshold to a sound that causes damage in seconds. You are unaware of this tremendous range in sound intensity because how your ears respond can be described approximately as the logarithm of intensity. Thus, sound intensity levels in decibels fit your experience better than intensities in watts per meter squared. The decibel scale is also easier to relate to because most people are more accustomed to dealing with numbers such as 0, 53, or 120 than numbers such as 1.00×10^{-11}.

One more observation readily verified by examining Table 14.2 or using $I = \left(\frac{(\Delta p)}{2\rho \mathrm{v_w}}\right)^2$ is that each factor of 10 in intensity corresponds to 10 dB. For example, a 90 dB sound compared with a 60 dB sound is 30 dB greater, or three factors of 10 (that is, 10^3 times) as intense. Another example is that if one sound is 10^7 as intense as another, it is 70 dB higher. See Table 14.3.

Table 14.3 Ratios of Intensities and Corresponding Differences in Sound Intensity Levels

I_2/I_1	$\beta_2 - \beta_1$
2.0	3.0 dB
5.0	7.0 dB
10.0	10.0 dB

Example 14.2

Calculating Sound Intensity Levels: Sound Waves

Calculate the sound intensity level in decibels for a sound wave traveling in air at $0^{\circ}\ C$ and having a pressure amplitude of 0.656 Pa.

Strategy

We are given Δp, so we can calculate I using the equation

$$I = ((\Delta p))^2/((2\rho v_w))^2.$$

Using I, we can calculate β straight from its definition in $\beta\,(\text{dB}) = 10\log_{10}(I/I_0)$.

Solution

(1) Identify knowns:

Sound travels at 331 m/s in air at $0^{\circ}\ C$.

Air has a density of $1.29 kg/m^3$ at atmospheric pressure and $0^{\circ}\ C$.

(2) Enter these values and the pressure amplitude into $I = ((\Delta p))^2/((2\rho v)_w)$:

$$\begin{aligned} I &= \frac{((\Delta p))^2}{2\rho v_w} \\ &= \frac{(0.656 Pa)^2}{2\,(1.29\ \text{kg/m}^3)\,(331\ \text{m/s})} \\ &= 5.04 \times 10^{-4}\ \text{W/m}^2. \end{aligned}$$

14.13

(3) Enter the value for I and the known value for I_0 into $\beta\,(\text{dB}) = 10\log_{10}(I/I_0)$. Calculate to find the sound intensity level in decibels:

$$10log_{10}\left(5.04\times10^{8}\right)=10\left(8.70\right)dB$$
$$=87dB.$$

14.14

Discussion

This 87 dB sound has an intensity five times as great as an 80 dB sound. So a factor of five in intensity corresponds to a difference of 7 dB in sound intensity level. This value is true for any intensities differing by a factor of five.

Example 14.3

Change Intensity Levels of a Sound: What Happens to the Decibel Level?

Show that if one sound is twice as intense as another, it has a sound level about 3 dB higher.

Strategy

You are given that the ratio of two intensities is 2 to 1, and are then asked to find the difference in their sound levels in decibels. You can solve this problem using of the properties of logarithms.

Solution

(1) Identify knowns:

The ratio of the two intensities is 2 to 1, or:

$$\frac{I_2}{I_1} = 2.00.$$

14.15

We wish to show that the difference in sound levels is about 3 dB. That is, we want to show:

$$\beta_2 - \beta_1 = 3 \text{ dB}.$$

14.16

Note that:

$$\log_{10} b - \log_{10} a = \log_{10} \left(\frac{b}{a} \right).$$

14.17

(2) Use the definition of β to get:

$$\begin{aligned}\beta_2 - \beta_1 &= 10 \log_{10}\left(\frac{I_2}{I_1}\right) \\ &= 10\log_{10}2.00 \\ &= 10\,(0.\,301)\text{dB}.\end{aligned}$$

14.18

Thus,

$$\beta_2 - \beta_1 = 3.\,01\ \text{dB}.$$

14.19

Discussion

This means that the two sound intensity levels differ by 3.01 dB, or about 3 dB, as advertised. Note that because only the ratio I_2/I_1 is given (and not the actual intensities), this result is true for any intensities that differ by a factor of two. For example, a 56.0 dB sound is twice as intense as a 53.0 dB sound, a 97.0 dB sound is half as intense as a 100 dB sound, and so on.

It should be noted at this point that there is another decibel scale in use, called the sound pressure level, based on the ratio of the pressure amplitude to a reference pressure. This scale is used particularly in applications where sound travels in water. It is beyond the scope of most introductory texts to treat this scale because it is not commonly used for sounds in air, but it is important to note that very different decibel levels may be encountered when sound pressure levels are quoted. For example, ocean noise pollution produced by ships may be as great as 200 dB expressed in the sound pressure level, where the more familiar sound intensity level we use here would be something under 140 dB for the same sound.

Take-Home Investigation: Feeling Sound

Find a CD player and a CD that has rock music. Place the player on a light table, insert the CD into the player, and start playing the CD. Place your hand gently on the table next to the speakers. Increase the volume and note the level when the table just begins to vibrate as the rock music plays. Increase the reading on the volume control until it doubles. What has happened to the vibrations?

Check Your Understanding

Describe how amplitude is related to the loudness of a sound.

Click for Solution

Solution

Amplitude is directly proportional to the experience of loudness. As amplitude increases, loudness increases.

Check Your Understanding

Identify common sounds at the levels of 10 dB, 50 dB, and 100 dB.

Click for Solution

Solution

10 dB: Running fingers through your hair.

50 dB: Inside a quiet home with no television or radio.

100 dB: Take-off of a jet plane.

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14.4 DOPPLER EFFECT AND SONIC BOOMS

Learning Objectives

By the end of this section, you will be able to:

- Define Doppler effect, Doppler shift, and sonic boom.
- Calculate the frequency of a sound heard by someone observing Doppler shift.
- Describe the sounds produced by objects moving faster than the speed of sound.

The characteristic sound of a motorcycle buzzing by is an example of the Doppler effect. The high-pitch scream shifts dramatically to a lower-pitch roar as the motorcycle passes by a stationary observer. The closer the motorcycle brushes by, the more abrupt the shift. The faster the motorcycle moves, the greater the shift. We also hear this characteristic shift in frequency for passing race cars, airplanes, and trains. It is so familiar that it is used to imply motion and children often mimic it in play.

The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer. Although less familiar, this effect is easily noticed for a stationary source and moving observer. For example, if you ride a train past a stationary warning bell, you will hear the bell's frequency shift from high to low as you pass by. The actual change in frequency due to relative motion of source and observer is called a Doppler shift. The Doppler effect and Doppler shift are named for the Austrian physicist and mathematician Christian Johann Doppler (1803–1853), who did experiments with both moving sources and moving observers. Doppler, for example, had musicians play on a moving open train car and also play standing next to the train tracks as a train passed by. Their music was observed both on and off the train, and changes in frequency were measured.

What causes the Doppler shift? Figure 14.13, Figure 14.14, and Figure 14.15 compare sound waves emitted by stationary and moving sources in a stationary air mass. Each disturbance spreads out spherically from the point where the sound was emitted. If the source is stationary, then all of the spheres representing the air

compressions in the sound wave centered on the same point, and the stationary observers on either side see the same wavelength and frequency as emitted by the source, as in Figure 14.13. If the source is moving, as in Figure 14.14, then the situation is different. Each compression of the air moves out in a sphere from the point where it was emitted, but the point of emission moves. This moving emission point causes the air compressions to be closer together on one side and farther apart on the other. Thus, the wavelength is shorter in the direction the source is moving (on the right in Figure 14.14), and longer in the opposite direction (on the left in Figure 14.14). Finally, if the observers move, as in Figure 17.15, the frequency at which they receive the compressions changes. The observer moving toward the source receives them at a higher frequency, and the person moving away from the source receives them at a lower frequency.

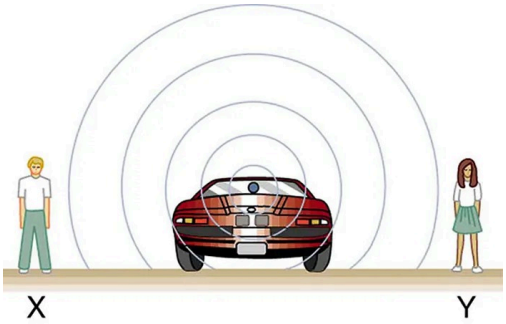


Figure 14.13 Sounds emitted by a source spread out in spherical waves. Because the source, observers, and air are stationary, the wavelength and frequency are the same in all directions and to all observers. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a red car viewed from the rear, positioned on a flat surface. Around the car, several concentric circles are drawn, indicating the propagation of sound waves. On the left side of the car, a person labeled “X” is standing, while on the right side, another person labeled “Y” is standing. The circles suggest the difference in sound perception or intensity between the two individuals due to their positions relative to the car.

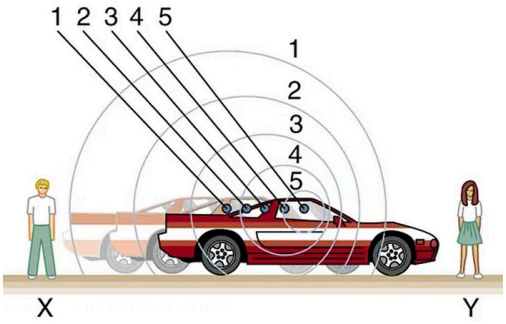


Figure 14.14 Sounds emitted by a source moving to the right spread out from the points at which they were emitted. The wavelength is reduced and, consequently, the frequency is increased in the direction of motion, so that the observer on the right hears a higher-pitch sound. The opposite is true for the observer on the left, where the wavelength is increased and the frequency is reduced. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a red sports car moving from left to right across the image, with two people standing on either side of the path of the car. On the left side (labeled as "X"), there is a person standing facing the car with a neutral expression. On the right side (labeled as "Y"), another person stands facing the car with a neutral expression.

Above the car, there are five numbered lines (1 to 5) illustrating a series of concentric circles that increase in size, radiating from a pivot on the rear of the car. These circles are labeled from 1 to 5, representing the car's positions as it moves from X to Y. Each position shows the car at a slightly forward point along its path.



Figure 14.15 The same effect is produced when the observers move relative to the source. Motion toward the source increases frequency as the observer on the right passes through more wave crests than she would if stationary. Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests than he would if stationary. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a visual representation of the Doppler effect using a scenario with a car and two people. At the center, there is a red car emitting circular waves, which represent sound waves. On the left side, labeled "X," there is a person running towards the car, shown with multiple faint outlines to indicate motion. On the right side, labeled "Y," there is another person running away from the car, also depicted with multiple faint outlines to show movement. The waves appear compressed on the left side and elongated on the right side, demonstrating the change in frequency perceived by the two individuals due to their motion relative to the sound source.

We know that wavelength and frequency are related by $v_w = f\lambda$, where v_w is the fixed speed of sound. The sound moves in a medium and has the same speed v_w in that medium whether the source is moving or not. Thus f multiplied by λ is a constant. Because the observer on the right in Figure 14.14 receives a shorter wavelength, the frequency she receives must be higher. Similarly, the observer on the left receives a longer wavelength, and hence he hears a lower frequency. The same thing happens in Figure 14.15. A higher frequency is received by the observer moving toward the source, and a lower frequency is received by an observer moving away from the source. In general, then, relative motion of source and observer toward one another increases the received frequency. Relative motion apart decreases frequency. The greater the relative speed is, the greater the effect.

The Doppler Effect

The Doppler effect occurs not only for sound but for any wave when there is relative motion between the observer and the source. There are Doppler shifts in the frequency of sound, light, and water waves, for example. Doppler shifts can be used to determine velocity, such as when ultrasound is reflected from blood in a medical diagnostic. The recession of galaxies is determined by the shift in the frequencies of light received from them and has implied much about the origins of the universe. Modern physics has been profoundly affected by observations of Doppler shifts.

For a stationary observer and a moving source, the frequency f_{obs} received by the observer can be shown to be

$$f_{\text{obs}} = f_s\left(\frac{v_w}{v_w \pm v_s}\right),$$

14.20

where f_s is the frequency of the source, v_s is the speed of the source along a line joining the source and observer, and v_w is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away from the observer, producing the appropriate shifts up and down in frequency. Note that the greater the speed of the source, the greater the effect. Similarly, for a stationary source and moving observer, the frequency received by the observer f_{obs} is given by

$$f_{\text{obs}} = f_s \left(\frac{v_w \pm v_{\text{obs}}}{v_w} \right),$$

14.21

where v_{obs} is the speed of the observer along a line joining the source and observer. Here the plus sign is for motion toward the source, and the minus is for motion away from the source.

Example 14.4

Calculate Doppler Shift: A Train Horn

Suppose a train that has a 150-Hz horn is moving at 35.0 m/s in still air on a day when the speed of sound is 340 m/s.

(a) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes?

(b) What frequency is observed by the train's engineer traveling on the train?

Strategy

To find the observed frequency in (a), $f_{\text{obs}} = f_s \left(\frac{v_w}{v_w \pm v_s} \right)$, must be used because the source is moving. The minus sign is used for the approaching train, and the plus sign for the receding train. In (b), there are two Doppler shifts—one for a moving source and the other for a moving observer.

Solution for (a)

(1) Enter known values into $f_{\text{obs}} = f_s \left(\frac{v_w}{v_w - v_s} \right)$.

$$f_{\text{obs}} = f_s \left(\frac{v_w}{v_w - v_s} \right)$$
$$= (150 \text{ Hz}) \left(\frac{340 \text{ m/s}}{340 \text{ m/s} - 35.0 \text{ m/s}} \right)$$

14.22

(2) Calculate the frequency observed by a stationary person as the train approaches.

$$f_{obs} = (150 \text{ Hz}) (1.11) = 167 \text{ Hz}$$

14.23

(3) Use the same equation with the plus sign to find the frequency heard by a stationary person as the train recedes.

$$f_{\text{obs}} = f_s \left(\frac{v_w}{v_w + v_s} \right)$$
$$= (150 \text{ Hz}) \left(\frac{340 \text{ m/s}}{340 \text{ m/s} + 35.0 \text{ m/s}} \right)$$

14.24

(4) Calculate the second frequency.

$$f_{obs} = (150 \text{ Hz}) (0.907) = 136 \text{ Hz}$$

14.25

Discussion on (a)

The numbers calculated are valid when the train is far enough away that the motion is nearly along the line joining train and observer. In both cases, the shift is significant and easily noticed. Note that the shift is 17.0 Hz for motion toward and 14.0 Hz for motion away. The shifts are not symmetric.

Solution for (b)

(1) Identify knowns:

- It seems reasonable that the engineer would receive the same frequency as emitted by the horn, because the relative velocity between them is zero.
- Relative to the medium (air), the speeds are $v_s = v_{\text{obs}} = 35.0 m/s.$
- The first Doppler shift is for the moving observer; the second is for the moving source.

(2) Use the following equation:

$$f_{\text{obs}} = [f_s \left(\frac{v_w \pm v_{\text{obs}}}{v_w} \right)] \left(\frac{v_w}{v_w \pm v_s} \right).$$

14.26

The quantity in the square brackets is the Doppler-shifted frequency due to a moving observer. The factor on the right is the effect of the moving source.

(3) Because the train engineer is moving in the direction toward the horn, we must use the plus sign for v_{obs}; however, because the horn is also moving in the direction away from the engineer, we also use the plus sign for v_s. But the train is carrying both the engineer and the horn at the same velocity, so $v_s = v_{obs}$. As a result, everything but f_s cancels, yielding

$$f_{\text{obs}} = f_s.$$

14.27

Discussion for (b)

We may expect that there is no change in frequency when source and observer move together because it fits your experience. For example, there is no Doppler shift in the frequency of conversations between driver and passenger on a motorcycle. People talking when a wind moves the air between them also observe no Doppler shift in their conversation. The crucial point is that source and observer are not moving relative to each other.

Sonic Booms to Bow Wakes

What happens to the sound produced by a moving source, such as a jet airplane, that approaches or even exceeds the speed of sound? The answer to this question applies not only to sound but to all other waves as well.

Suppose a jet airplane is coming nearly straight at you, emitting a sound of frequency f_s. The greater the plane's speed v_s, the greater the Doppler shift and the greater the value observed for f_{obs}. Now, as v_s approaches the speed of sound, f_{obs} approaches infinity, because the denominator in $f_{\text{obs}} = f_s \left(\frac{v_w}{v_w \pm v_s} \right)$ approaches zero. At the speed of sound, this result means that in front of the source, each successive wave is superimposed on the previous one because the source moves forward at the speed of sound. The observer gets them all at the same instant, and so the frequency is infinite. (Before

airplanes exceeded the speed of sound, some people argued it would be impossible because such constructive superposition would produce pressures great enough to destroy the airplane.) If the source exceeds the speed of sound, no sound is received by the observer until the source has passed, so that the sounds from the approaching source are mixed with those from it when receding. This mixing appears messy, but something interesting happens—a sonic boom is created. (See Figure 14.16.)

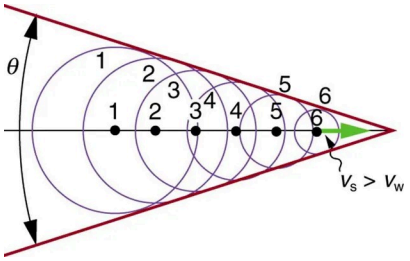


Figure 14.16 Sound waves from a source that moves faster than the speed of sound spread spherically from the point where they are emitted, but the source moves ahead of each. Constructive interference along the lines shown (actually a cone in three dimensions) creates a shock wave called a sonic boom. The faster the speed of the source, the smaller the angle θ*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

This image illustrates the concept of a sonic boom using the visual of a circular wavefront pattern. A series of six black dots are placed in a straight line on a horizontal axis, each labeled with numbers from 1 to 6. Around each dot, there are semicircles formed by purple lines that represent sound waves emanating from each point. The semicircles become smaller and more compressed towards the right, indicating movement.

A triangular shape formed by red lines encompasses the wavefronts and points to the right, with its tip near a small green triangle. This overall triangular shape signifies the Mach cone, with the angle within the triangle labeled with the Greek letter theta (θ). An arrow points to the right, indicating the direction of movement, while text beside it states "$v_s > v_w$," indicating that the speed of the source (v_s) is greater than the speed of sound in that medium (v_w).

There is constructive interference along the lines shown (a cone in three dimensions) from similar sound waves arriving there simultaneously. This superposition forms a disturbance called a sonic boom, a constructive interference of sound created by an object moving faster than sound. Inside the cone, the interference is mostly destructive, and so the sound intensity there is much less than on the shock wave. An aircraft creates two sonic booms, one from its nose and one from its tail. (See Figure 14.17.) During television coverage of space shuttle landings, two distinct booms could often be heard. These were separated by exactly the time it would take the shuttle to pass by a point. Observers on the ground often do not see the aircraft creating the sonic boom, because it has passed by before the shock wave reaches them, as seen in Figure 14.17.

If the aircraft flies close by at low altitude, pressures in the sonic boom can be destructive and break windows as well as rattle nerves. Because of how destructive sonic booms can be, supersonic flights are banned over populated areas of the United States.

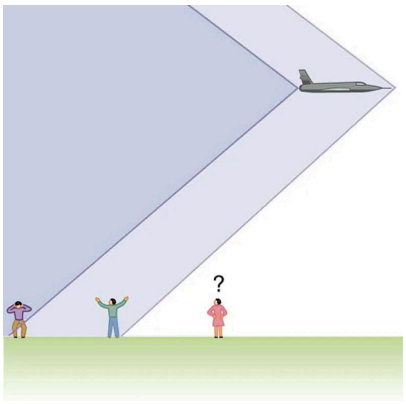


Figure 14.17 Two sonic booms, created by the nose and tail of an aircraft, are observed on the ground after the plane has passed by. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a conceptual illustration depicting a fighter jet flying at an angle over the horizon. It creates a large triangular shadow across the ground. In the foreground, three small figures are shown on a green surface: one is holding hands up to their ears, another has their arms raised, and the third is standing with a question mark above their head, indicating confusion. The large triangular shadow covers part of the ground where these figures stand, illustrating the concept of sound waves or sonic boom created by the jet.

Sonic booms are one example of a broader phenomenon called bow wakes. A bow wake, such as the one in Figure 14.18, is created when the wave source moves faster than the wave propagation speed. Water waves spread out in circles from the point where created, and the bow wake is the familiar V-shaped wake trailing the source. A more exotic bow wake is created when a subatomic particle travels through a medium faster than the speed of light travels in that medium. (In a vacuum, the maximum speed of light will be $c = 3.00 \times 10^8 \text{m/s}$; in the medium of water, the speed of light is closer to $0.75c$. If the particle creates light in its passage, that light spreads on a cone with an angle indicative of the speed of the particle, as illustrated in Figure 14.19. Such a bow wake is called Cerenkov radiation and is commonly observed in particle physics.

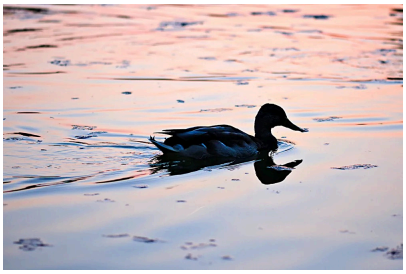


Figure 14.18 Bow wake created by a duck. Constructive interference produces the rather structured wake, while there is relatively little wave action inside the wake, where interference is mostly destructive. Image from OpenStax College Physics 2e, CC-BY 4.0

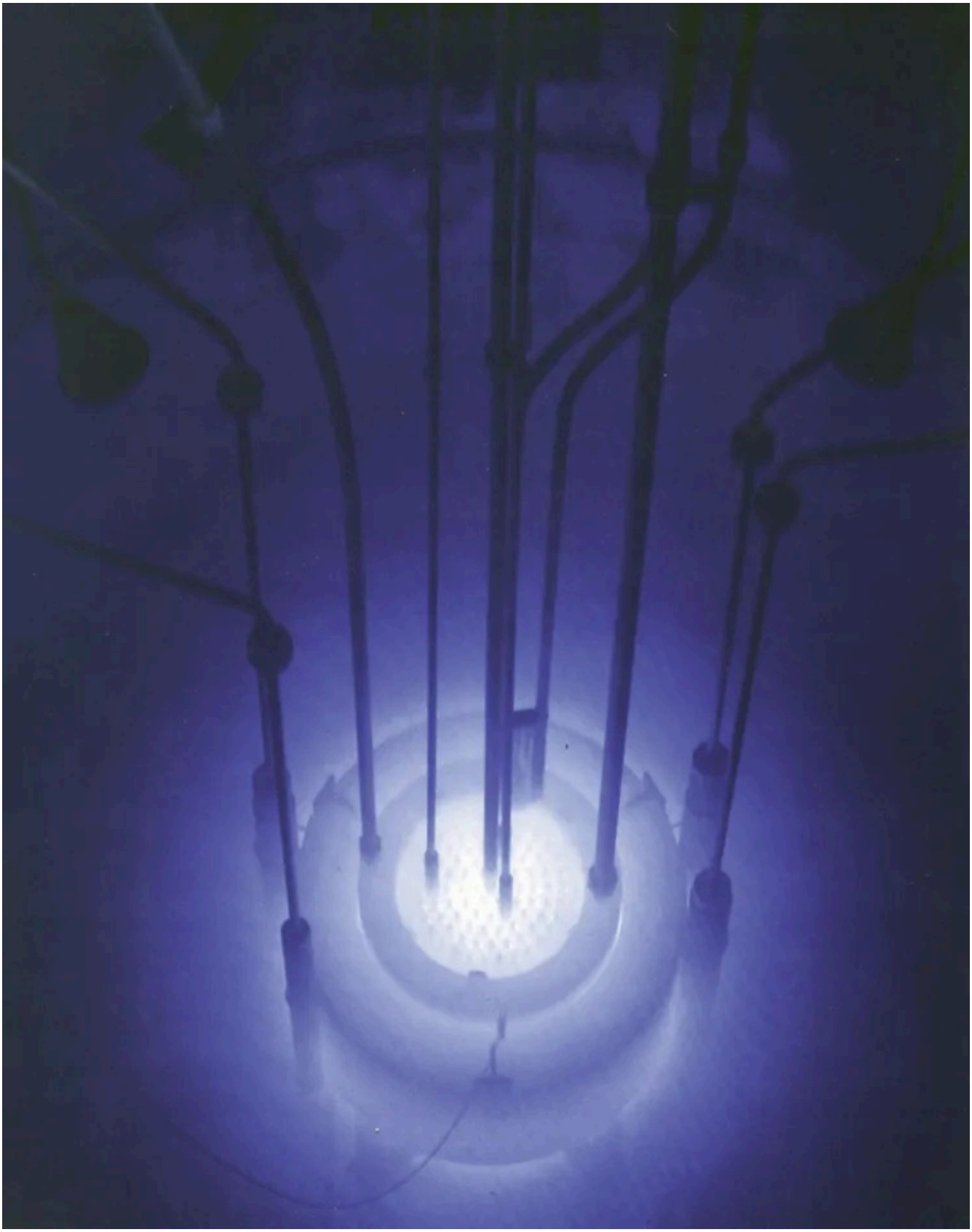


Figure 14.19 The blue glow in this research reactor pool is Cerenkov radiation caused by subatomic particles traveling faster than the speed of light in water. Image by United States

Nuclear Regulatory Commission, Wikimedia Commons License

Doppler shifts and sonic booms are interesting sound phenomena that occur in all types of waves. They can be of considerable use. For example, the Doppler shift in ultrasound can be used to measure blood velocity, while police use the Doppler shift in radar (a microwave) to measure car velocities. In meteorology, the Doppler shift is used to track the motion of storm clouds; such "Doppler Radar" can give velocity and direction and rain or snow potential of imposing weather fronts. In astronomy, we can examine the light emitted from distant galaxies and determine their speed relative to ours. As galaxies move away from us, their light is shifted to a lower frequency, and so to a longer wavelength—the so-called red shift. Such information from galaxies far, far away has allowed us to estimate the age of the universe (from the Big Bang) as about 14 billion years.

Check Your Understanding

Why did scientist Christian Doppler observe musicians both on a moving train and also from a stationary point not on the train?

Click for Solution

Solution

Doppler needed to compare the perception of sound when the observer is stationary and the sound source moves, as well as when the sound source and the observer are both in motion.

Check Your Understanding

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

Click for Solution

Solution

If I am driving and I hear Doppler shift in an ambulance siren, I would be able to tell when it was getting closer and also if it has passed by. This would help me to know whether I needed to pull over and let the ambulance through.

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14.5 SOUND INTERFERENCE AND RESONANCE: STANDING WAVES IN AIR COLUMNS

Learning Objectives

By the end of this section, you will be able to:

- Define antinode, node, fundamental, overtones, and harmonics.
- Identify instances of sound interference in everyday situations.
- Describe how sound interference occurring inside open and closed tubes changes the characteristics of the sound, and how this applies to sounds produced by musical instruments.
- Calculate the length of a tube using sound wave measurements.



Figure 14.20 Some types of headphones use the phenomena of constructive and destructive interference to cancel out outside noises. Image from OpenStax College Physics 2e, CC-BY 4.0

Interference is the hallmark of waves, all of which exhibit constructive and destructive interference exactly analogous to that seen for water waves. In fact, one way to prove something "is a wave" is to observe interference effects. So, sound being a wave, we expect it to exhibit interference; we have already mentioned a few such effects, such as the beats from two similar notes played simultaneously.

Figure 14.21 shows a clever use of sound interference to cancel noise. Larger-scale applications of active noise reduction by destructive interference are contemplated for entire passenger compartments in commercial aircraft. To obtain destructive interference, a fast electronic analysis is performed, and a second sound is introduced with its maxima and minima exactly reversed from the incoming noise. Sound waves in fluids are pressure waves and consistent with Pascal's principle; pressures from two different sources add and subtract like simple numbers; that is, positive and negative gauge pressures add to a much smaller pressure, producing a lower-intensity sound. Although completely destructive interference is possible only under the simplest conditions, it is possible to reduce noise levels by 30 dB or more using this technique.

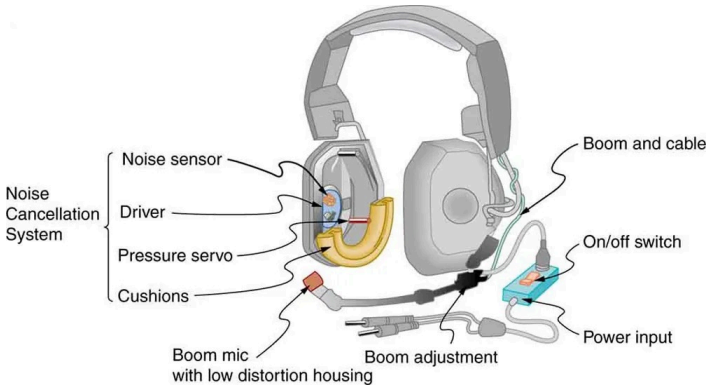


Figure 14.21 Headphones designed to cancel noise with destructive interference create a sound wave exactly opposite to the incoming sound. These headphones can be more effective than the simple passive attenuation used in most ear protection. Such headphones were used on the record-setting, around the world nonstop flight of the Voyager aircraft to protect the pilots' hearing from engine noise. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image is a labeled diagram of a pair of headphones with noise-cancelling features. The key components highlighted in the diagram include:

- **Noise Cancellation System:**
  - *Noise Sensor:* A component that detects external ambient noise.
  - *Driver:* A part responsible for producing sound.
  - *Pressure Servo:* Manages pressure differences in the earcup.

    - *Cushions:* Soft padding for comfort and sound isolation.
- **Boom Mic:** A microphone with low distortion housing for clear audio capture.
- **Boom and Cable:** Connects the boom mic to the headphones.
- **On/Off Switch:** A switch to power the headphones on or off.
- **Power Input:** The connection point for charging or powering the device.
- **Boom Adjustment:** Allows for positional adjustment of the boom mic.

Where else can we observe sound interference? All sound resonances, such as in musical instruments, are due to constructive and destructive interference. Only the resonant frequencies interfere constructively to form standing waves, while others interfere destructively and are absent. From the toot made by blowing over a bottle, to the characteristic flavor of a violin's sounding box, to the recognizability of a great singer's voice, resonance and standing waves play a vital role.

Interference

Interference is such a fundamental aspect of waves that observing interference is proof that something is a wave. The wave nature of light was established by experiments showing interference. Similarly, when electrons scattered from crystals exhibited interference, their wave nature was confirmed to be exactly as predicted by symmetry with certain wave characteristics of light.

Suppose we hold a tuning fork near the end of a tube that is closed at the other end, as shown in Figure 14.22, Figure 14.23, Figure 14.24, and Figure 14.25. If the tuning fork has just the right frequency, the air column in the tube resonates loudly, but at most frequencies it vibrates very little. This observation just means that the air column has only certain natural frequencies. The figures show how a resonance at the lowest of these natural frequencies is formed. A disturbance travels down the tube at the speed of sound and bounces off the closed end. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly half a cycle later, and it interferes constructively with the continuing sound produced by the tuning fork. The incoming and reflected sounds form a standing wave in the tube as shown.

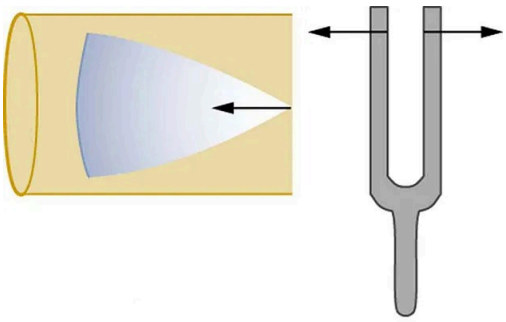


Figure 14.22 Resonance of air in a tube closed at one end, caused by a tuning fork. A disturbance moves down the tube. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two separate illustrations. On the left, there is a cylindrical object with a curved surface shaded in a gradient from light blue to white, and an arrow pointing from right to left, indicating motion or direction inside the cylinder. On the right, there is a tuning fork with its two prongs positioned vertically. Arrows point outward from each prong, indicating opposite directions of movement. The tuning fork is depicted in a simple, gray color.

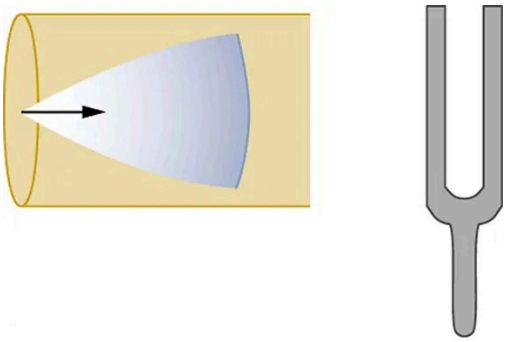


Figure 14.23 Resonance of air in a tube closed at one end, caused by a tuning fork. The disturbance reflects from the closed end of the tube. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two distinct illustrations.

On the left, there is a cylindrical shape filled with a yellow color. Inside the cylinder, there is a cone-shaped representation in white and light blue, pointing from the left side towards the center of the cylinder. An arrow is drawn inside the cone, indicating a rightward or forward movement inside the cylinder.

On the right, there is an illustration of a tuning fork. It is depicted as a simple U-shaped metallic object with two prongs extending upwards from a handle. The tuning fork is a solid gray color and positioned vertically.

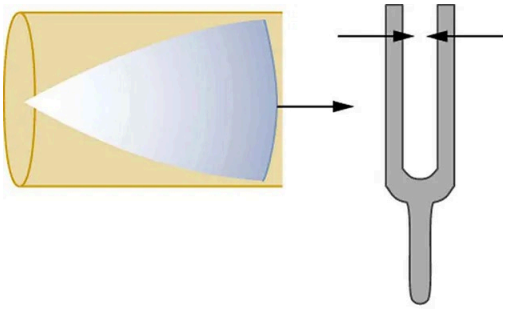


Figure 14.24 Resonance of air in a tube closed at one end, caused by a tuning fork. If the length of the tube L is just right, the disturbance gets back to the tuning fork half a cycle later and interferes constructively with the continuing sound from the tuning fork. This interference forms a standing wave, and the air column resonates. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram illustrating sound wave propagation. On the left side, there is a cylindrical shape representing an enclosed space where sound waves are released. Within this cylinder, a triangular sound wave is shown, with the broad end starting inside the cylinder and tapering to a point. The triangle is shaded in white to blue gradients, indicating the direction and intensity of the sound waves moving toward the right.

To the right of the cylinder, there's a graphic of a tuning fork with two prongs. Arrows from the triangular sound wave point directly at the tuning fork, indicating the interaction between the sound waves and the tuning fork. The tuning fork is shown in a gray color.

This diagram illustrates how sound waves can activate or interact with a tuning fork.

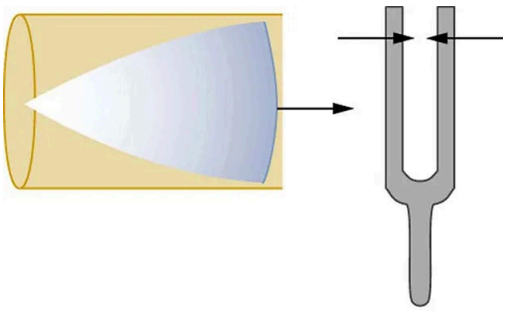


Figure 14.25 Resonance of air in a tube closed at one end, caused by a tuning fork. A graph of air displacement along the length of the tube shows none at the closed end, where the motion is constrained, and a maximum at the open end. This standing wave has one-fourth of its wavelength in the tube, so that $\lambda = 4L$. *Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image shows a diagram comparing two sections. On the left side, there is a cylindrical shape with one side having a cone-shaped recess. The shaded area inside the recess gives a sense of depth. An arrow points from this shape to a similar line on the right. On the right side, there is a U-shaped metal fork, resembling a tuning fork. Two arrows indicate the prongs of the fork, pointing to the narrow opening between them.

The standing wave formed in the tube has its maximum air displacement (an antinode) at the open end, where motion is unconstrained, and no displacement (a node) at the closed end, where air movement is halted. The distance from a node to an antinode is one-fourth of a wavelength, and this equals the length of the tube; thus, $\lambda = 4L$. This same resonance can be produced by a vibration introduced at or near the closed end of the tube, as shown in Figure 17.26. It is best to consider this a natural vibration of the air column independently of how it is induced.

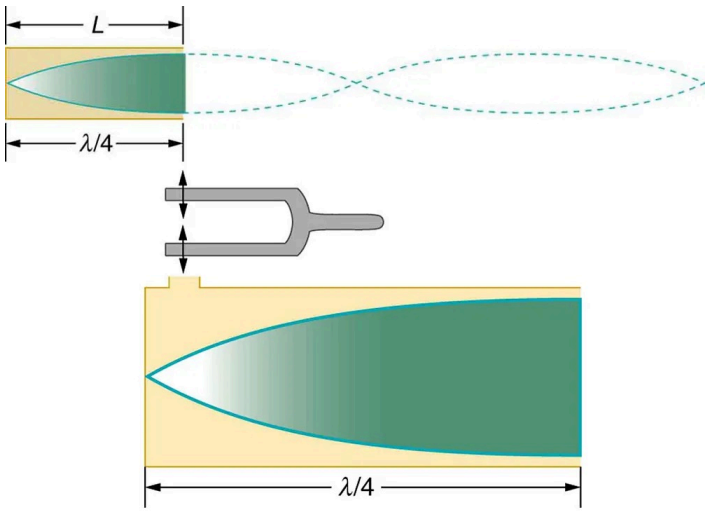


Figure 14.26 The same standing wave is created in the tube by a vibration introduced near its closed end. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram related to acoustics or wave physics. It features two main sections:

1. Top Section:
– There is a rectangular shape with a tapered gradient, transitioning from a narrow white point to a wider green area.
– The length of the rectangle is labeled "L".
– A segment within the rectangle is marked as "λ/4", representing a quarter of the wavelength.
– Dashed lines extend from the rectangle outward, forming an elliptical, symmetrical pattern, suggesting wave propagation.

2. Bottom Section:
– There is an elongated rectangular shape with one end tapering to a point, similar to the top section.
– The length of the rectangle is also labeled "λ/4".
– A tuning fork is depicted above the tapered end, indicating a connection to sound or vibration.
– The rectangular shape is encased in a beige-colored outline.

This diagram appears to illustrate the concept of wave propagation and resonance using a tuning fork setup and quarter-wavelength notations.

Given that maximum air displacements are possible at the open end and none at the closed end, there are other, shorter wavelengths that can resonate in the tube, such as the one shown in Figure 14.27. Here the standing wave has three-fourths of its wavelength in the tube, or $L = (3/4)\lambda'$, so that $\lambda' = 4L/3$.

Continuing this process reveals a whole series of shorter-wavelength and higher-frequency sounds that resonate in the tube. We use specific terms for the resonances in any system. The lowest resonant frequency is called the fundamental, while all higher resonant frequencies are called overtones. All resonant frequencies are integral multiples of the fundamental, and they are collectively called harmonics. The fundamental is the first harmonic, the first overtone is the second harmonic, and so on. Figure 14.28 shows the fundamental and the first three overtones (the first four harmonics) in a tube closed at one end.

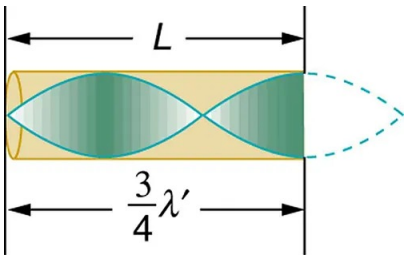


Figure 14.27 Another resonance for a tube closed at one end. This has maximum air displacements at the open end, and none at the closed end. The wavelength is shorter, with three-fourths λ' *equaling the length of the tube, so that* $\lambda' = 4L/3$*. This higher-frequency vibration is the first overtone. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a diagram of a cylindrical tube with a standing wave inside. The tube is shown from a side view, with a yellowish color indicating its boundaries. Inside the tube, there is a waveform drawn in shades of green transitioning from darker to lighter tones, indicating wave amplitude.

The wave is represented as having two symmetrical nodes inside the tube, marked by points where the wave meets the tube wall. Just outside the right end of the tube, there is a continuation of the wave shown with a dashed line, suggesting it extends beyond the boundary.

Across the top of the tube, an arrow labeled “L” points from one end of the tube to the other, indicating the length of the tube. Below the tube, there is a fraction labeled “3/4 λ′” with an arrow pointing across the same length as “L,” suggesting that the tube’s length is three-quarters of the wavelength λ’.

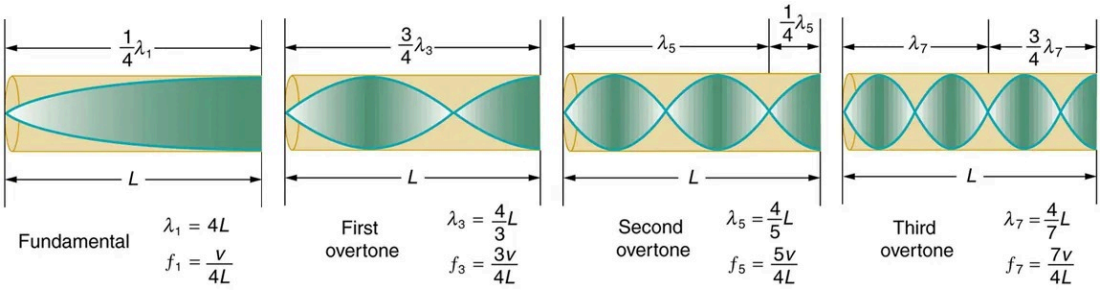


Figure 14.28 The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image displays four diagrams representing the resonant modes in a tube closed at one end, showing the fundamental frequency and three overtones. Each diagram includes waves illustrated within the tube, demonstrating different harmonic patterns.

Mode	Diagram	Length	Wavelength (λ)	Frequency (f)
Fundamental	Single peak	L	$\lambda_1 = 4L$	$f_1 = v / 4L$
First Overtone	Two peaks	L	$\lambda_3 = 4L/3$	$f_3 = 3v / 4L$
Second Overtone	Three peaks	L	$\lambda_5 = 4L/5$	$f_5 = 5v / 4L$
Third Overtone	Four peaks	L	$\lambda_7 = 4L/7$	$f_7 = 7v / 4L$

Each diagram has a labeled distance in terms of the wavelength λ and length L of the tube, indicating how the wave fits within the tube. The diagrams include mathematical relationships for λ and f related to the tube's length L and wave speed v.

The fundamental and overtones can be present simultaneously in a variety of combinations. For example, middle C on a trumpet has a sound distinctively different from middle C on a clarinet, both instruments being modified versions of a tube closed at one end. The fundamental frequency is the same (and usually the most intense), but the overtones and their mix of intensities are different and subject to shading by the musician. This mix is what gives various musical instruments (and human voices) their distinctive characteristics, whether they have air columns, strings, sounding boxes, or drumheads. In fact, much of our speech is determined by shaping the cavity formed by the throat and mouth and positioning the tongue to adjust the fundamental and combination of overtones. Simple resonant cavities can be made to resonate with the sound of the vowels, for example. (See Figure 14.29.) In males, at puberty, the larynx grows and the shape

of the resonant cavity changes giving rise to the difference in predominant frequencies in speech between different sexes.

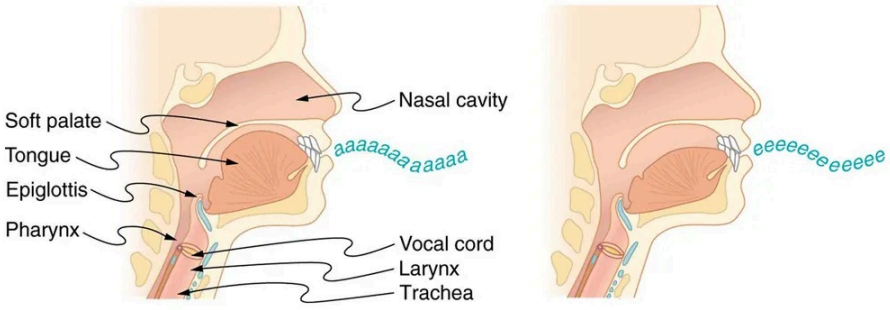


Figure 14.29 The throat and mouth form an air column closed at one end that resonates in response to vibrations in the voice box. The spectrum of overtones and their intensities vary with mouth shaping and tongue position to form different sounds. The voice box can be replaced with a mechanical vibrator, and understandable speech is still possible. Variations in basic shapes make different voices recognizable. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image consists of two side-view diagrams of the human vocal tract, labeled to show various anatomical parts involved in sound production. Each diagram highlights the structures that contribute to different vowel sounds:

On the left:

- *Nasal cavity*: Uppermost area, shown leading to the nose.
- *Soft palate*: Located at the back of the roof of the mouth, appears just above the tongue.
- *Tongue*: Positioned centrally, shown directing towards the soft palate.
- *Epiglottis*: Found behind the tongue, near the entrance to the pharynx.
- *Pharynx*: A vertical cavity through which air passes, extending to the vocal cords.

An arrow and text “aaaaaa” indicate the direction of airflow to produce the vowel sound as in “aaa.”

On the right:

- *Nasal cavity*: Similar positioning to the left diagram.
- *Vocal cord*: Located within the larynx at the throat level.
- *Larynx*: Situated before the trachea, housing the vocal cords.
- *Trachea*: The windpipe extends downward from the larynx.

An arrow and text “eeeeee” illustrate the airflow direction to produce the vowel sound as in “eee.”

Now let us look for a pattern in the resonant frequencies for a simple tube that is closed at one end. The fundamental has $\lambda = 4L$, and frequency is related to wavelength and the speed of sound as given by:

$$v_w = f\lambda.$$

14.28

Solving for f in this equation gives

$$f = \frac{v_w}{\lambda} = \frac{v_w}{4L},$$
14.29

where v_w is the speed of sound in air. Similarly, the first overtone has $\lambda' = 4L/3$ (see Figure 14.28), so that

$$f' = 3\frac{v_w}{4L} = 3f.$$
14.30

Because $f' = 3f$, we call the first overtone the third harmonic. Continuing this process, we see a pattern that can be generalized in a single expression. The resonant frequencies of a tube closed at one end are

$$f_n = n\frac{v_w}{4L}, n = 1, 3, 5,$$
14.31

where f_1 is the fundamental, f_3 is the first overtone, and so on. It is interesting that the resonant frequencies depend on the speed of sound and, hence, on temperature. This dependence is the reason why musicians commonly bring their wind instruments to room temperature before playing them together with other types of instruments.

Example 14.5

Find the Length of a Tube with a 128 Hz Fundamental

(a) What length should a tube closed at one end have on a day when the air temperature, is $22.0^{\circ}\ C$, if its fundamental frequency is to be 128 Hz (C below middle C)?

(b) What is the frequency of its fourth overtone?

Strategy

The length L can be found from the relationship in $f_n = n\frac{v_w}{4L}$, but we will first need to find the speed of sound v_w.

Solution for (a)

(1) Identify knowns:

- the fundamental frequency is 128 Hz
- the air temperature is $22.0^{\circ}\ C$

(2) Use $f_n = n\frac{v_w}{4L}$ to find the fundamental frequency ($n = 1$).

$$f_1 = \frac{v_w}{4L}$$

14.32

(3) Solve this equation for length.

$$L = \frac{v_w}{(4f)_1}$$

14.33

(4) Find the speed of sound using $v_w = (331\ \text{m/s})\sqrt{\frac{T}{273\ \text{K}}}$.

$$v_w = (331 \text{ m/s})\sqrt{\frac{295 \text{ K}}{273 \text{ K}}} = 344 \text{ m/s}$$

14.34

(5) Enter the values of the speed of sound and frequency into the expression for L.

$$L = \frac{v_w}{(4f)_1} = \frac{344 \text{ m/s}}{4\,(128 \text{ Hz})} = 0.672 \text{ m}$$

14.35

Discussion on (a)

Many wind instruments are modified tubes that have finger holes, valves, and other devices for changing the length of the resonating air column and hence, the frequency of the note played. Horns producing very low frequencies, such as tubas, require tubes so long that they are coiled into loops.

Solution for (b)

(1) Identify knowns:

- the first overtone has $n = 3$
- the second overtone has $n = 5$
- the third overtone has $n = 7$
- the fourth overtone has $n = 9$

(2) Enter the value for the fourth overtone into $f_n = n\frac{v_w}{4L}$.

$$f_9 = 9\frac{v_w}{4L} = (9f)_1 = 1.15 \text{ kHz}$$

14.36

Discussion on (b)

Whether this overtone occurs in a simple tube or a musical instrument depends on how

it is stimulated to vibrate and the details of its shape. The trombone, for example, is usually not played to produce its fundamental frequency and typically only the overtones are used.

Another type of tube is one that is *open* at both ends. Examples are some organ pipes, flutes, and oboes. The resonances of tubes open at both ends can be analyzed in a very similar fashion to those for tubes closed at one end. The air columns in tubes open at both ends have maximum air displacements at both ends, as illustrated in Figure 14.30. Standing waves form as shown.

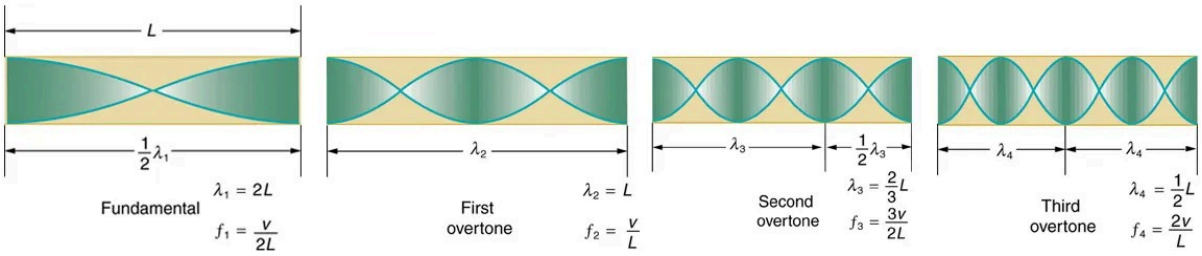


Figure 14.30 The resonant frequencies of a tube open at both ends are shown, including the fundamental and the first three overtones. In all cases the maximum air displacements occur at both ends of the tube, giving it different natural frequencies than a tube closed at one end. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates the harmonics of a vibrating string, showing different modes of standing waves and corresponding wavelengths and frequencies for a string fixed at both ends. The diagram is divided into four sections, each labeled as follows:

Section	Description
Fundamental	The string has one complete wave along its length, labeled as L. The wavelength is $\lambda_1 = 2L$ and the frequency is $f_1 = v/2L$.
First Overtone	The string displays two complete waves. The wavelength is $\lambda_2 = L$ and the frequency is $f_2 = v/L$.
Second Overtone	The string shows three complete waves. The wavelength is $\lambda_3 = 2/3\ L$ and the frequency is $f_3 = 3v/2L$.
Third Overtone	The string presents four complete waves. The wavelength is $\lambda_4 = 1/2\ L$ and the frequency is $f_4 = 2v/L$.

Each section visually represents the standing wave pattern with the nodes and antinodes shown on a horizontal string.

Based on the fact that a tube open at both ends has maximum air displacements at both ends, and using Figure 17.30 as a guide, we can see that the resonant frequencies of a tube open at both ends are:

$$f_n = n\frac{v_w}{2L}, n = 1, 2, 3\ldots,$$

14.37

where f_1 is the fundamental, f_2 is the first overtone, f_3 is the second overtone, and so on. Note that a tube open at both ends has a fundamental frequency twice what it would have if closed at one end. It also has a different spectrum of overtones than a tube closed at one end. So if you had two tubes with the same fundamental frequency but one was open at both ends and the other was closed at one end, they would sound different when played because they have different overtones. A note sounds richer when played on an instrument that includes many overtones in addition to the dominant frequency being played. Organists manually select extra harmonics to adjust the timbre of the notes they are playing. Organs often have multiple keyboards so that the overtone spectrum can be set differently for notes being played by the left hand, by the right hand, and by the feet.

Real-World Applications: Resonance in Everyday Systems

Resonance occurs in many different systems, including strings, air columns, and atoms. Resonance is the driven or forced oscillation of a system at its natural frequency. At resonance, energy is transferred rapidly to the oscillating system, and the amplitude of its oscillations grows until the system can no longer be described by Hooke's law. An example of this is the distorted sound intentionally produced in certain types of rock music.

Wind instruments use resonance in air columns to amplify tones made by lips or vibrating reeds. Other instruments also use air resonance in clever ways to amplify sound. Figure 14.31 shows a violin and a guitar, both of which have sounding boxes but with different shapes, resulting in different overtone structures. The

vibrating string creates a sound that resonates in the sounding box, greatly amplifying the sound and creating overtones that give the instrument its characteristic flavor. The more complex the shape of the sounding box, the greater its ability to resonate over a wide range of frequencies. The marimba, like the one shown in Figure 17.32 uses pots or gourds below the wooden slats to amplify their tones. The resonance of the pot can be adjusted by adding water.



Figure 14.31 String instruments such as violins and guitars use resonance in their sounding boxes to amplify and enrich the sound created by their vibrating strings. The bridge and supports couple the string vibrations to the sounding boxes and air within. Image from OpenStax College Physics 2e, CC-BY 4.0



Figure 14.32 Resonance has been used in musical instruments since prehistoric times. This marimba uses gourds as resonance chambers to amplify its sound. Image from OpenStax College Physics 2e, CC-BY 4.0

We have emphasized sound applications in our discussions of resonance and standing waves, but these ideas apply to any system that has wave characteristics. Vibrating strings, for example, are actually resonating and have fundamentals and overtones similar to those for air columns. More subtle are the resonances in atoms due to the wave character of their electrons. Their orbitals can be viewed as standing waves, which have a fundamental (ground state) and overtones (excited states). It is fascinating that wave characteristics apply to such a wide range of physical systems.

Check Your Understanding

Describe how noise-canceling headphones differ from standard headphones used to block outside sounds.

Click for Solution

Solution

Regular headphones only block sound waves with a physical barrier. Noise-canceling headphones use destructive interference to reduce the loudness of outside sounds.

Check Your Understanding

How is it possible to use a standing wave's node and antinode to determine the length of a closed-end tube?

Click for Solution

Solution

When the tube resonates at its natural frequency, the wave's node is located at the closed end of the tube, and the antinode is located at the open end. The length of the tube is equal to one-fourth of the wavelength of this wave. Thus, if we know the wavelength of the wave, we can determine the length of the tube.

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14.6 HEARING

Learning Objectives

By the end of this section, you will be able to:

- Define hearing, pitch, loudness, timbre, note, tone, phon, ultrasound, and infrasound.
- Compare loudness to frequency and intensity of a sound.
- Identify structures of the inner ear and explain how they relate to sound perception.



Figure 14.33 Hearing allows these vocalists, the band, and the fans to enjoy music. Image from OpenStax College Physics 2e, CC-BY 4.0

The human ear has a tremendous range and sensitivity. It can give us a wealth of simple information—such as pitch, loudness, and direction. And from its input we can detect musical quality and nuances of voiced emotion. How is our hearing related to the physical qualities of sound, and how does the hearing mechanism work?

Hearing is the perception of sound. (Perception is commonly defined to be awareness through the senses, a

typically circular definition of higher-level processes in living organisms.) Normal human hearing encompasses frequencies from 20 to 20,000 Hz, an impressive range. Sounds below 20 Hz are called infrasound, whereas those above 20,000 Hz are ultrasound. Neither is perceived by the ear, although infrasound can sometimes be felt as vibrations. When we do hear low-frequency vibrations, such as the sounds of a diving board, we hear the individual vibrations only because there are higher-frequency sounds in each. Other animals have hearing ranges different from that of humans. Dogs can hear sounds as high as 30,000 Hz, whereas bats and dolphins can hear up to 100,000-Hz sounds. You may have noticed that dogs respond to the sound of a dog whistle which produces sound out of the range of human hearing. Elephants are known to respond to frequencies below 20 Hz.

The perception of frequency is called pitch. Most of us have excellent relative pitch, which means that we can tell whether one sound has a different frequency from another. Typically, we can discriminate between two sounds if their frequencies differ by 0.3% or more. For example, 500.0 and 501.5 Hz are noticeably different. Pitch perception is directly related to frequency and is not greatly affected by other physical quantities such as intensity. Musical notes are particular sounds that can be produced by most instruments and in Western music have particular names. Combinations of notes constitute music. Some people can identify musical notes, such as A-sharp, C, or E-flat, just by listening to them. This uncommon ability is called perfect pitch.

The ear is remarkably sensitive to low-intensity sounds. The lowest audible intensity or threshold is about $10^{-12}\mathrm{W/m^2}$ or 0 dB. Sounds as much as 10^{12} more intense can be briefly tolerated. Very few measuring devices are capable of observations over a range of a trillion. The perception of intensity is called loudness. At a given frequency, it is possible to discern differences of about 1 dB, and a change of 3 dB is easily noticed. But loudness is not related to intensity alone. Frequency has a major effect on how loud a sound seems. The ear has its maximum sensitivity to frequencies in the range of 2000 to 5000 Hz, so that sounds in this range are perceived as being louder than, say, those at 500 or 10,000 Hz, even when they all have the same intensity. Sounds near the high- and low-frequency extremes of the hearing range seem even less loud, because the ear is even less sensitive at those frequencies. Table 14.4 gives the dependence of certain human hearing perceptions on physical quantities.

Table 14.4 Sound Perceptions

Perception	Physical quantity
Pitch	Frequency
Loudness	Intensity and Frequency
Timbre	Number and relative intensity of multiple frequencies. Subtle craftsmanship leads to non-linear effects and more detail.
Note	Basic unit of music with specific names, combined to generate tunes
Tone	Number and relative intensity of multiple frequencies.

When a violin plays middle C, there is no mistaking it for a piano playing the same note. The reason is that each instrument produces a distinctive set of frequencies and intensities. We call our perception of these combinations of frequencies and intensities tone quality, or more commonly the timbre of the sound. It is more difficult to correlate timbre perception to physical quantities than it is for loudness or pitch perception. Timbre is more subjective. Terms such as dull, brilliant, warm, cold, pure, and rich are employed to describe the timbre of a sound. So the consideration of timbre takes us into the realm of perceptual psychology, where higher-level processes in the brain are dominant. This is true for other perceptions of sound, such as music and noise. We shall not delve further into them; rather, we will concentrate on the question of loudness perception.

A unit called a phon is used to express loudness numerically. Phons differ from decibels because the phon is a unit of loudness perception, whereas the decibel is a unit of physical intensity. Figure 14.34 shows the relationship of loudness to intensity (or intensity level) and frequency for persons with normal hearing. The curved lines are equal-loudness curves. Each curve is labeled with its loudness in phons. Any sound along a given curve will be perceived as equally loud by the average person. The curves were determined by having large numbers of people compare the loudness of sounds at different frequencies and sound intensity levels. At a frequency of 1000 Hz, phons are taken to be numerically equal to decibels. The following example helps illustrate how to use the graph:

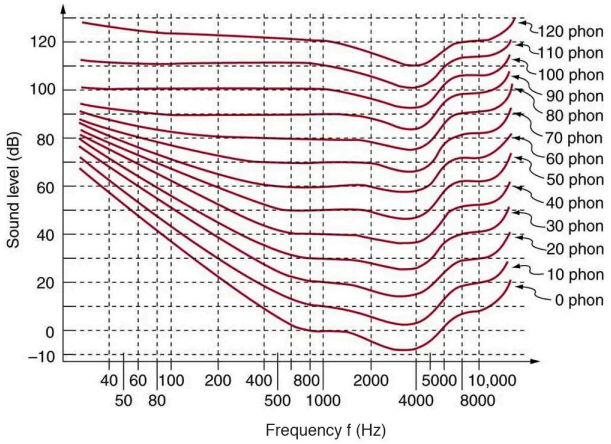


Figure 14.34 The relationship of loudness in phons to intensity level (in decibels) and intensity (in watts per meter squared) for persons with normal hearing. The curved lines are equal-loudness curves—all sounds on a given curve are perceived as equally loud. Phons and decibels are defined to be the same at 1000 Hz. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image is a graph showing equal-loudness contours, which illustrate the sound level in decibels (dB) required to produce a perception of equal loudness at different frequencies measured in Hertz (Hz). The

horizontal axis represents frequency ranging from 20 Hz to 10,000 Hz, while the vertical axis represents sound level ranging from -10 dB to 120 dB.

The graph includes multiple contour lines labeled from 0 phon to 120 phon, representing perceived loudness levels. The contours vary in shape, generally rising at low frequencies and higher frequencies, indicating variations in human hearing sensitivity across frequencies. Each line is marked with its corresponding phon value on the right side of the graph.

The contours demonstrate how the human ear perceives different frequencies at various loudness levels, highlighting that more sound level is needed at lower frequencies to be perceived as equally loud compared to higher frequencies.

Example 14.6

Measuring Loudness: Loudness Versus Intensity Level and Frequency

(a) What is the loudness in phons of a 100-Hz sound that has an intensity level of 80 dB? (b) What is the intensity level in decibels of a 4000-Hz sound having a loudness of 70 phons? (c) At what intensity level will an 8000-Hz sound have the same loudness as a 200-Hz sound at 60 dB?

Strategy for (a)

The graph in Figure 14.34 should be referenced in order to solve this example. To find the loudness of a given sound, you must know its frequency and intensity level and locate that point on the square grid, then interpolate between loudness curves to get the loudness in phons.

Solution for (a)

(1) Identify knowns:

- The square grid of the graph relating phons and decibels is a plot of intensity level versus frequency—both physical quantities.
- 100 Hz at 80 dB lies halfway between the curves marked 70 and 80 phons.

(2) Find the loudness: 75 phons.

Strategy for (b)

The graph in Figure 14.34 should be referenced in order to solve this example. To find the intensity level of a sound, you must have its frequency and loudness. Once that point is located, the intensity level can be determined from the vertical axis.

Solution for (b)

(1) Identify knowns:

- Values are given to be 4000 Hz at 70 phons.

(2) Follow the 70-phon curve until it reaches 4000 Hz. At that point, it is below the 70 dB line at about 67 dB.

(3) Find the intensity level:

67 dB

Strategy for (c)

The graph in Figure 14.34 should be referenced in order to solve this example.

Solution for (c)

(1) Locate the point for a 200 Hz and 60 dB sound.

(2) Find the loudness: This point lies just slightly above the 50-phon curve, and so its loudness is 51 phons.

(3) Look for the 51-phon level is at 8000 Hz: 63 dB.

Discussion

These answers, like all information extracted from Figure 14.34, have uncertainties of several phons or several decibels, partly due to difficulties in interpolation, but mostly related to uncertainties in the equal-loudness curves.

Further examination of the graph in Figure 14.34 reveals some interesting facts about human hearing. First, sounds below the 0-phon curve are not perceived by most people. So, for example, a 60 Hz sound at 40 dB is inaudible. The 0-phon curve represents the threshold of normal hearing. We can hear some sounds at intensity levels below 0 dB. For example, a 3-dB, 5000-Hz sound is audible, because it lies above the 0-phon curve. The loudness curves all have dips in them between about 2000 and 5000 Hz. These dips mean the ear is

most sensitive to frequencies in that range. For example, a 15-dB sound at 4000 Hz has a loudness of 20 phons, the same as a 20-dB sound at 1000 Hz. The curves rise at both extremes of the frequency range, indicating that a greater-intensity level sound is needed at those frequencies to be perceived to be as loud as at middle frequencies. For example, a sound at 10,000 Hz must have an intensity level of 30 dB to seem as loud as a 20 dB sound at 1000 Hz. Sounds above 120 phons are painful as well as damaging.

We do not often utilize our full range of hearing. This is particularly true for frequencies above 8000 Hz, which are rare in the environment and are unnecessary for understanding conversation or appreciating music. In fact, people who have lost the ability to hear such high frequencies are usually unaware of their loss until tested. The shaded region in Figure 14.35 is the frequency and intensity region where most conversational sounds fall. The curved lines indicate what effect hearing losses of 40 and 60 phons will have. A 40-phon hearing loss at all frequencies still allows a person to understand conversation, although it will seem very quiet. A person with a 60-phon loss at all frequencies will hear only the lowest frequencies and will not be able to understand speech unless it is much louder than normal. Even so, speech may seem indistinct, because higher frequencies are not as well perceived. The conversational speech region also has a gender component, in that female voices are usually characterized by higher frequencies. So the person with a 60-phon hearing impediment might have difficulty understanding the normal conversation of a woman.

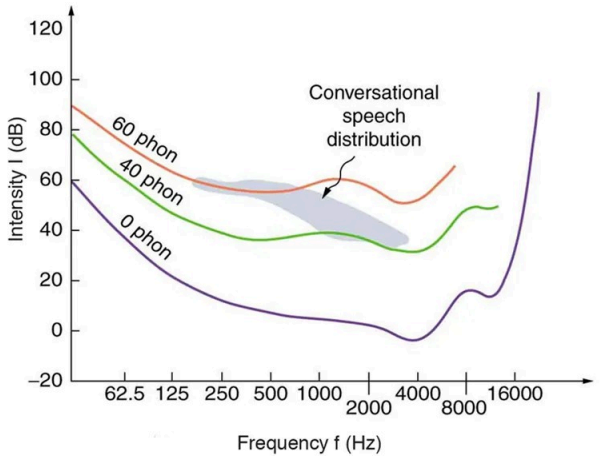


Figure 14.35 The shaded region represents frequencies and intensity levels found in normal conversational speech. The 0-phon line represents the normal hearing threshold, while those at 40 and 60 represent thresholds for people with 40- and 60-phon hearing losses, respectively. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph showing the relationship between frequency (in Hz) and intensity (in dB) with three curves labeled as 0 phon, 40 phon, and 60 phon. The x-axis represents frequency on a logarithmic scale, ranging from 62.5 Hz to 16000 Hz. The y-axis represents intensity, ranging from -20 dB to 120 dB.

The curves trace the threshold of hearing at various phon levels:

- The 0 phon curve, in purple, begins at higher dB levels for lower frequencies and slopes downward before rising steeply at higher frequencies.
- The 40 phon curve, in green, follows a similar pattern but at higher dB levels than the 0 phon curve.
- The 60 phon curve, in orange, also follows the same pattern but is displaced upward higher along the dB axis.

A shaded area labeled "Conversational speech distribution" is present between approximately 200 Hz and 4000 Hz on the frequency axis, indicating the range where conversational speech typically occurs within these phon levels.

Hearing tests are performed over a range of frequencies, usually from 250 to 8000 Hz, and can be displayed graphically in an audiogram like that in Figure 14.36. The hearing threshold is measured in dB *relative to the normal threshold*, so that normal hearing registers as 0 dB at all frequencies. Hearing loss caused by noise typically shows a dip near the 4000 Hz frequency, irrespective of the frequency that caused the loss and often affects both ears. The most common form of hearing loss comes with age and is called *presbycusis*—literally elder ear. Such loss is increasingly severe at higher frequencies, and interferes with music appreciation and speech recognition.

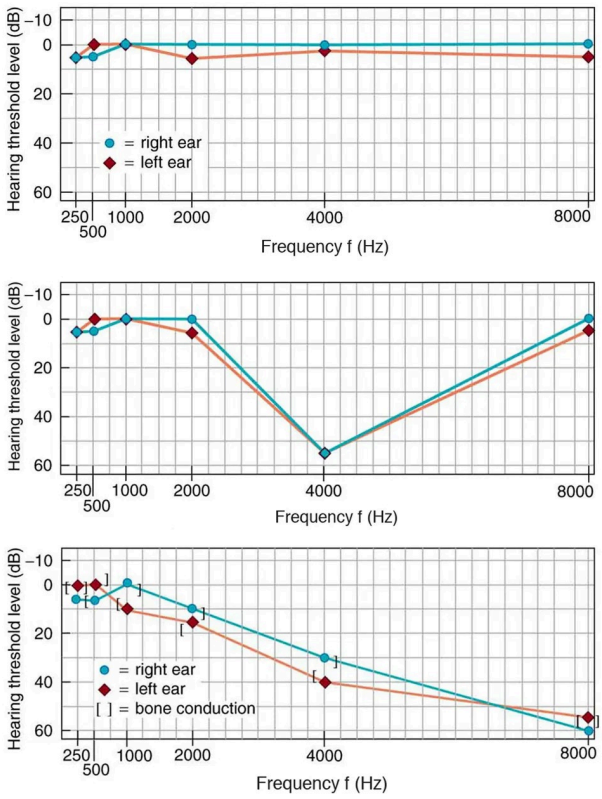


Figure 14.36 Audiograms showing the threshold in intensity level versus frequency for three different individuals. Intensity level is measured relative to the normal threshold. The top left graph is that of a person with normal hearing. The graph to its right has a dip at 4000 Hz and is that of a child who suffered hearing loss due to a cap gun. The third graph is typical of presbycusis, the progressive loss of higher frequency hearing with age. Tests performed by bone conduction (brackets) can distinguish nerve damage from middle ear damage. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image contains three audiogram charts displaying hearing threshold levels measured in decibels (dB) across a range of frequencies in hertz (Hz) for both the right and left ears.

First Chart: This chart plots hearing thresholds from 250 Hz to 8000 Hz with two lines representing the right ear (blue circles) and left ear (red diamonds). Hearing levels are between -10 dB and 0 dB for both ears, with minimal differences between the two.

Second Chart: Similar to the first, this chart presents hearing thresholds from 250 Hz to 8000 Hz for the right ear (blue circles) and left ear (red diamonds). Both ears have a V-shaped pattern, with lower threshold levels at 250-2000 Hz, a peak at 4000 Hz, and a return to lower thresholds at 8000 Hz.

Third Chart: This chart also covers 250 Hz to 8000 Hz, with right ear (blue circles and brackets for bone

conduction) and left ear (red diamonds). There is a gradual increase in threshold levels for both ears, particularly noticeable in bone conduction measurements, with higher thresholds towards 8000 Hz.

All charts are annotated with legends that describe symbols: blue circles for the right ear, red diamonds for the left ear, and brackets indicating bone conduction in the third chart.

The Hearing Mechanism

The hearing mechanism involves some interesting physics. The sound wave that impinges upon our ear is a pressure wave. The ear is a transducer that converts sound waves into electrical nerve impulses in a manner much more sophisticated than, but analogous to, a microphone. Figure 14.37 shows the gross anatomy of the ear with its division into three parts: the outer ear or ear canal; the middle ear, which runs from the eardrum to the cochlea; and the inner ear, which is the cochlea itself. The body part normally referred to as the ear is technically called the pinna.

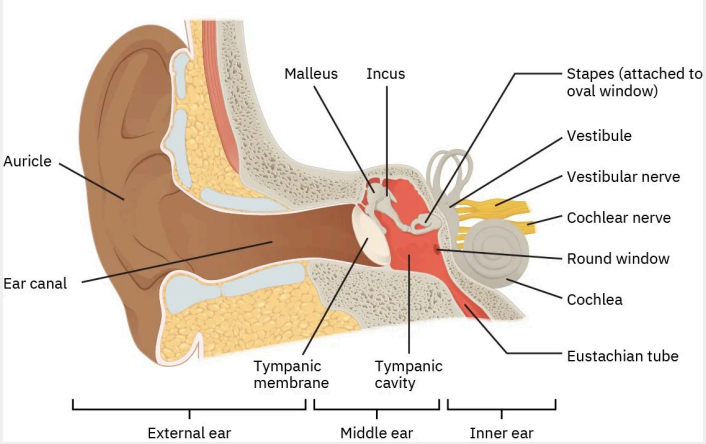


Figure 14.37 The illustration shows the gross anatomy of the human ear. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image is an anatomical diagram of the human ear, showing a cross-section of its three main parts: the external ear, the middle ear, and the inner ear.

- **External Ear:**
  - *Auricle* – The outermost visible part.
  - *Ear Canal* – The passage leading to the eardrum.

- **Middle Ear:**
    - *Tympanic Membrane* (eardrum) – Separates the external ear from the middle ear.
    - *Tympanic Cavity* – Air-filled cavity behind the tympanic membrane.
    - *Malleus* – A small bone attached to the tympanic membrane.
    - *Incus* – The bone connecting the malleus to the stapes.
    - *Stapes* (attached to oval window) – The bone that connects to the inner ear.
- **Inner Ear:**
    - *Vestibule* – Central part of the inner ear.
    - *Vestibular Nerve* – Transmits balance information to the brain.
    - *Cochlear Nerve* – Transmits sound signals to the brain.
    - *Round Window* – A membrane near the cochlea.
    - *Cochlea* – Spiral-shaped organ responsible for hearing.
    - *Eustachian Tube* – Connects the middle ear to the throat.

The diagram also indicates the location of each part within the three sections of the ear: external, middle, and inner.

The outer ear, or ear canal, carries sound to the recessed protected eardrum. The air column in the ear canal resonates and is partially responsible for the sensitivity of the ear to sounds in the 2000 to 5000 Hz range. The middle ear converts sound into mechanical vibrations and applies these vibrations to the cochlea. The lever system of the middle ear takes the force exerted on the eardrum by sound pressure variations, amplifies it and transmits it to the inner ear via the oval window, creating pressure waves in the cochlea approximately 40 times greater than those impinging on the eardrum. (See Figure 14.38.) Two muscles in the middle ear (not shown) protect the inner ear from very intense sounds. They react to intense sound in a few milliseconds and reduce the force transmitted to the cochlea. This protective reaction can also be triggered by your own voice, so that humming while shooting a gun, for example, can reduce noise damage.

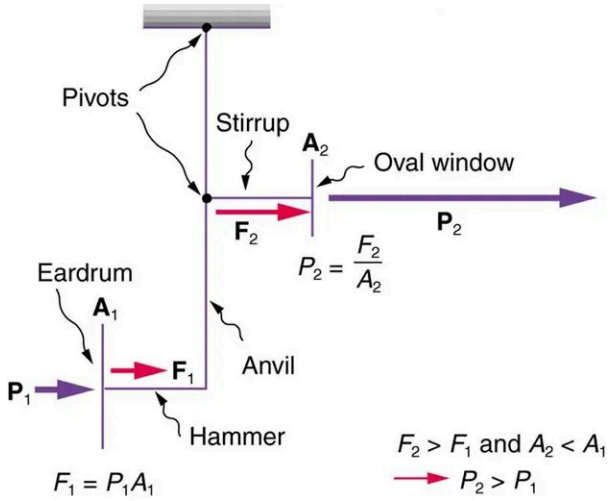


Figure 14.38 This schematic shows the middle ear's system for converting sound pressure into force, increasing that force through a lever system, and applying the increased force to a small area of the cochlea, thereby creating a pressure about 40 times that in the original sound wave. A protective muscle reaction to intense sounds greatly reduces the mechanical advantage of the lever system. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram showing the process of sound transmission through the middle ear. It illustrates how the mechanics of the ear amplify sound pressure from the eardrum to the oval window. Here's a description of each part and the flow of action:

- **Eardrum**: Labeled with 'A_1' and 'P_1', where 'F_1' is the force applied. The eardrum receives the initial sound pressure 'P_1'.
- **Hammer**: Connected to the eardrum and attached to the next component, the Anvil.
- **Anvil**: Transfers the vibrations from the hammer to the stirrup.
- **Stirrup**: Labeled with 'A_2' and 'F_2', showing a greater force than 'F_1'.
- **Oval Window**: Receives the sound transmitted from the stirrup with pressure 'P_2'.

The diagram further provides the relationship between forces and areas:

'$F_1 = P_1A_1$' and '$P_2 = F_2/A_2$.'

Additionally, it highlights the relationships:

- '$F_2 > F_1$' and '$A_2 < A_1$'.
- '$P_2 > P_1$'.

The system of "Pivots" indicates points of leverage facilitating the transmission dynamics.

Figure 14.39 shows the middle and inner ear in greater detail. Pressure waves moving through the cochlea cause the tectorial membrane to vibrate, rubbing cilia (called hair cells), which stimulate nerves that send electrical signals to the brain. The membrane resonates at different positions for different frequencies, with high frequencies stimulating nerves at the near end and low frequencies at the far end. The complete operation of the cochlea is still not understood, but several mechanisms for sending information to the brain are known to be involved. For sounds below about 1000 Hz, the nerves send signals at the same frequency as the sound. For frequencies greater than about 1000 Hz, the nerves signal frequency by position. There is a structure to the cilia, and there are connections between nerve cells that perform signal processing before information is sent to the brain. Intensity information is partly indicated by the number of nerve signals and by volleys of signals. The brain processes the cochlear nerve signals to provide additional information such as source direction (based on time and intensity comparisons of sounds from both ears). Higher-level processing produces many nuances, such as music appreciation.

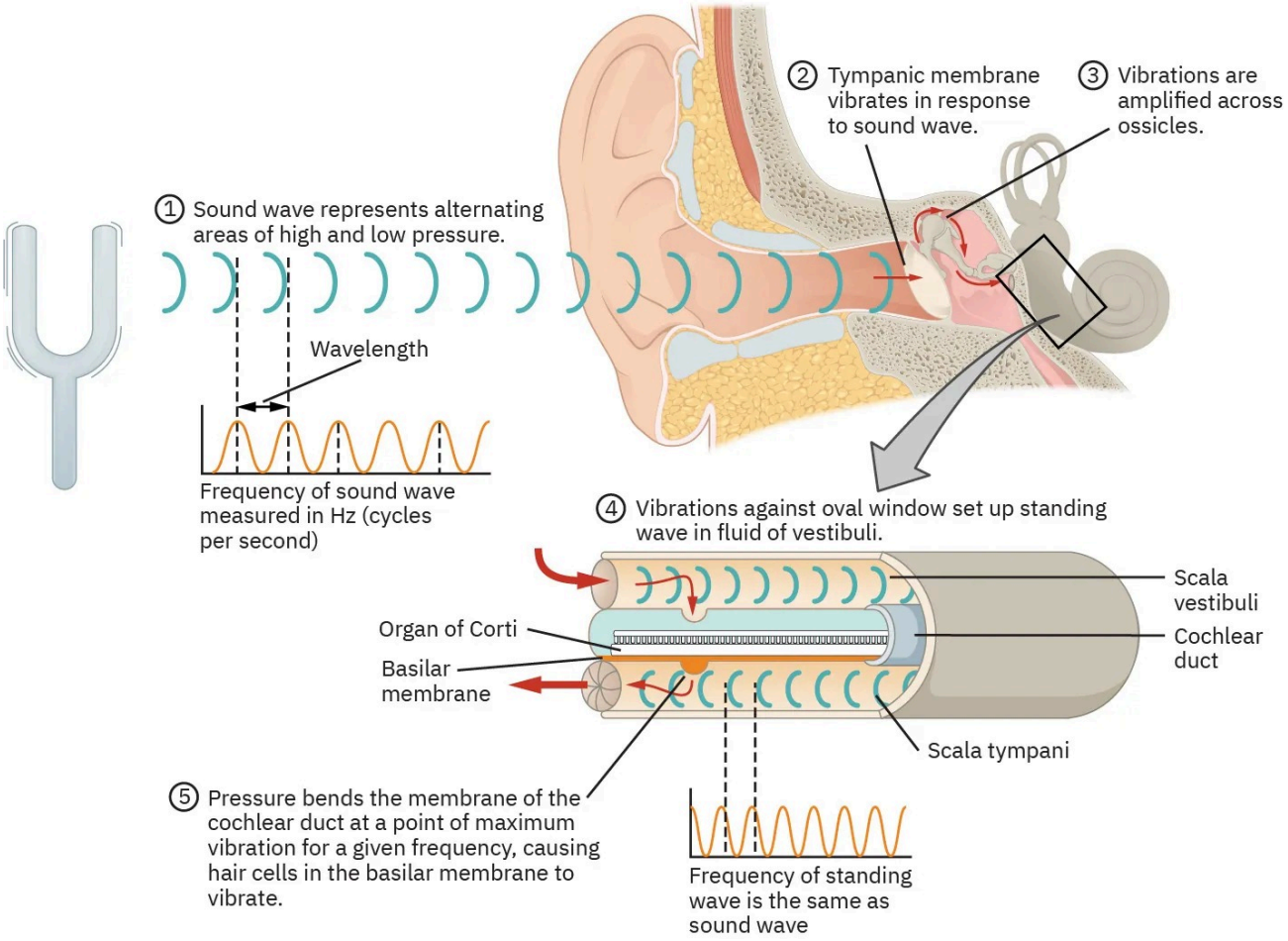


Figure 14.39 The inner ear, or cochlea, is a coiled tube about 3 mm in diameter and 3 cm in length if uncoiled. When the oval window is forced inward, as shown, a pressure wave travels through the perilymph in the direction of the arrows, stimulating nerves at the base of cilia in the organ of Corti. Image by OpenStax, Wikimedia Commons License

Image Description

This image illustrates the process of hearing through several labeled steps:

1. A sound wave, shown as alternating areas of high and low pressure, approaches the ear. The accompanying graph shows the frequency of the sound wave measured in Hz (cycles per second), identifying the wavelength.
2. The tympanic membrane (eardrum) vibrates in response to the incoming sound wave.
3. The vibrations from the tympanic membrane are amplified as they pass across the ossicles (tiny bones in the middle ear).
4. These vibrations against the oval window create a standing wave in the fluid of the vestibuli, a part of the inner ear.

5. The pressure bends the membrane of the cochlear duct at a point of maximum vibration for a given frequency. This causes hair cells in the basilar membrane to vibrate. The frequency of the standing wave is the same as the original sound wave.

Labels on the diagram indicate specific parts of the ear involved in this process: the scala vestibuli, cochlear duct, organ of Corti, basilar membrane, and scala tympani.

Hearing losses can occur because of problems in the middle or inner ear. Conductive losses in the middle ear can be partially overcome by sending sound vibrations to the cochlea through the skull. Hearing aids for this purpose usually press against the bone behind the ear, rather than simply amplifying the sound sent into the ear canal as many hearing aids do. Damage to the nerves in the cochlea is not repairable, but amplification can partially compensate. There is a risk that amplification will produce further damage. Another common failure in the cochlea is damage or loss of the cilia but with nerves remaining functional. Cochlear implants that stimulate the nerves directly are now available and widely accepted. Over 100,000 implants are in use, in about equal numbers of adults and children.

The cochlear implant was pioneered in Melbourne, Australia, by Graeme Clark in the 1970s for his deaf father. The implant consists of three external components and two internal components. The external components are a microphone for picking up sound and converting it into an electrical signal, a speech processor to select certain frequencies and a transmitter to transfer the signal to the internal components through electromagnetic induction. The internal components consist of a receiver/transmitter secured in the bone beneath the skin, which converts the signals into electric impulses and sends them through an internal cable to the cochlea and an array of about 24 electrodes wound through the cochlea. These electrodes in turn send the impulses directly into the brain. The electrodes basically emulate the cilia.

Check Your Understanding

Are ultrasound and infrasound imperceptible to all hearing organisms? Explain your answer.

Click for Solution

Solution

No, the range of perceptible sound is based in the range of human hearing. Many other organisms perceive either infrasound or ultrasound.

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14.7 ULTRASOUND

Learning Objectives

By the end of this section, you will be able to:

- Define acoustic impedance and intensity reflection coefficient.
- Describe medical and other uses of ultrasound technology.
- Calculate acoustic impedance using density values and the speed of ultrasound.
- Calculate the velocity of a moving object using Doppler-shifted ultrasound.



Figure 14.40 Ultrasound is used in medicine to painlessly and noninvasively monitor patient health and diagnose a wide range of disorders. Image from OpenStax College Physics 2e, CC-BY 4.0

Any sound with a frequency above 20,000 Hz (or 20 kHz)—that is, above the highest audible frequency—is defined to be ultrasound. In practice, it is possible to create ultrasound frequencies up to more than a gigahertz. (Higher frequencies are difficult to create; furthermore, they propagate poorly because they are very strongly absorbed.) Ultrasound has a tremendous number of applications, which range from burglar alarms

to use in cleaning delicate objects to the guidance systems of bats. We begin our discussion of ultrasound with some of its applications in medicine, in which it is used extensively both for diagnosis and for therapy.

Characteristics of Ultrasound

The characteristics of ultrasound, such as frequency and intensity, are wave properties common to all types of waves. Ultrasound also has a wavelength that limits the fineness of detail it can detect. This characteristic is true of all waves. We can never observe details significantly smaller than the wavelength of our probe; for example, we will never see individual atoms with visible light, because the atoms are so small compared with the wavelength of light.

Ultrasound in Medical Therapy

Ultrasound, like any wave, carries energy that can be absorbed by the medium carrying it, producing effects that vary with intensity. When focused to intensities of 10^3 to $10^5\ \mathrm{W/m^2}$, ultrasound can be used to shatter gallstones or pulverize cancerous tissue in surgical procedures. (See Figure 14.41.) Intensities this great can damage individual cells, variously causing their protoplasm to stream inside them, altering their permeability, or rupturing their walls through *cavitation*. Cavitation is the creation of vapor cavities in a fluid—the longitudinal vibrations in ultrasound alternatively compress and expand the medium, and at sufficient amplitudes the expansion separates molecules. Most cavitation damage is done when the cavities collapse, producing even greater shock pressures.

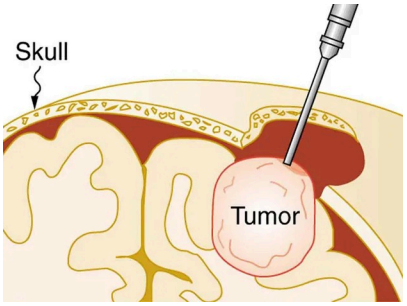


Figure 14.41 The tip of this small probe oscillates at 23 kHz with such a large amplitude that it pulverizes tissue on contact. The debris is then aspirated. The speed of the tip may exceed the speed of sound in tissue, thus creating shock waves and cavitation, rather than a smooth simple harmonic oscillator–type wave. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustrated cross-section of part of the brain and skull, highlighting the presence of a tumor. The skull is labeled and depicted as a thick outer layer. Beneath the skull, the brain is visible with its intricate folds. A tumor is labeled, shown pushing against the brain tissue. A surgical tool is illustrated as approaching the tumor, indicating a medical procedure. The style is schematic, with each anatomical part clearly distinguished by different colors and labels for educational purposes.

Most of the energy carried by high-intensity ultrasound in tissue is converted to thermal energy. In fact, intensities of 10^3 to 10^4W/m^2 are commonly used for deep-heat treatments called ultrasound diathermy. Frequencies of 0.8 to 1 MHz are typical. In both athletics and physical therapy, ultrasound diathermy is most often applied to injured or overworked muscles to relieve pain and improve flexibility. Skill is needed by the therapist to avoid "bone burns" and other tissue damage caused by overheating and cavitation, sometimes made worse by reflection and focusing of the ultrasound by joint and bone tissue.

In some instances, you may encounter a different decibel scale, called the sound *pressure* level, when ultrasound travels in water or in human and other biological tissues. We shall not use the scale here, but it is notable that numbers for sound pressure levels range 60 to 70 dB higher than you would quote for β, the sound intensity level used in this text. Should you encounter a sound pressure level of 220 decibels, then, it is not an astronomically high intensity, but equivalent to about 155 dB—high enough to destroy tissue, but not as unreasonably high as it might seem at first.

Ultrasound in Medical Diagnostics

When used for imaging, ultrasonic waves are emitted from a transducer, a crystal exhibiting the piezoelectric effect (the expansion and contraction of a substance when a voltage is applied across it, causing a vibration of

the crystal). These high-frequency vibrations are transmitted into any tissue in contact with the transducer. Similarly, if a pressure is applied to the crystal (in the form of a wave reflected off tissue layers), a voltage is produced which can be recorded. The crystal therefore acts as both a transmitter and a receiver of sound. Ultrasound is also partially absorbed by tissue on its path, both on its journey away from the transducer and on its return journey. From the time between when the original signal is sent and when the reflections from various boundaries between media are received, (as well as a measure of the intensity loss of the signal), the nature and position of each boundary between tissues and organs may be deduced.

Reflections at boundaries between two different media occur because of differences in a characteristic known as the acoustic impedance Z of each substance. Impedance is defined as

$$Z = \rho \mathrm{v},$$
14.38

where ρ is the density of the medium (in kg/m^3) and v is the speed of sound through the medium (in m/s). The units for Z are therefore $kg/(m^2 \cdot \mathrm{s})$.

Table 14.5 shows the density and speed of sound through various media (including various soft tissues) and the associated acoustic impedances. Note that the acoustic impedances for soft tissue do not vary much but that there is a big difference between the acoustic impedance of soft tissue and air and also between soft tissue and bone.

Table 14.5 The Ultrasound Properties of Various Media, Including Soft Tissue Found in the Body

Medium	**Density (kg/m^3)**	**Speed of Ultrasound (m/s)**	**Acoustic Impedance** $(\mathrm{kg}/(\mathrm{m}^2 \cdot \mathrm{s}))$
Air	1.3	330	429
Water	1000	1500	1.5×10^6
Blood	1060	1570	1.66×10^6
Fat	925	1450	1.34×10^6
Muscle (average)	1075	1590	1.70×10^6
Bone (varies)	1400–1900	4080	5.7×10^6 to 7.8×10^6
Barium titanate (transducer material)	5600	5500	30.8×10^6

At the boundary between media of different acoustic impedances, some of the wave energy is reflected and some is transmitted. The greater the *difference* in acoustic impedance between the two media, the greater the reflection and the smaller the transmission.

The intensity reflection coefficient a is defined as the ratio of the intensity of the reflected wave relative to the incident (transmitted) wave. This statement can be written mathematically as

$$a = \frac{(Z_2 - Z_1)^2}{(Z_1 + Z_2)^2},$$

14.39

where Z_1 and Z_2 are the acoustic impedances of the two media making up the boundary. A reflection coefficient of zero (corresponding to total transmission and no reflection) occurs when the acoustic impedances of the two media are the same. An impedance "match" (no reflection) provides an efficient coupling of sound energy from one medium to another. The image formed in an ultrasound is made by tracking reflections (as shown in Figure 14.42) and mapping the intensity of the reflected sound waves in a two-dimensional plane.

Example 14.7

Calculate Acoustic Impedance and Intensity Reflection Coefficient: Ultrasound and Fat Tissue

(a) Using the values for density and the speed of ultrasound given in Table 14.5, show that the acoustic impedance of fat tissue is indeed $1.34 \times 10^6 kg/(m^2 \cdot s)$.

(b) Calculate the intensity reflection coefficient of ultrasound when going from fat to muscle tissue.

Strategy for (a)

The acoustic impedance can be calculated using $Z = \rho \mathrm{v}$ and the values for ρ and v found in Table 14.5.

Solution for (a)

(1) Substitute known values from Table 14.5 into $Z = \rho \mathrm{v}$.

$$Z = \rho v = \left(925 \text{ kg/m}^3\right)\left(1450 \text{ m/s}\right)$$

14.40

(2) Calculate to find the acoustic impedance of fat tissue.

$$1.34 \times 10^6 kg/(m^2 \cdot s)$$

14.41

This value is the same as the value given for the acoustic impedance of fat tissue.

Strategy for (b)

The intensity reflection coefficient for any boundary between two media is given by $a = \frac{(Z_2 - Z_1)^2}{(Z_1 + Z_2)^2}$, and the acoustic impedance of muscle is given in Table 14.5.

Solution for (b)

Substitute known values into $a = \frac{(Z_2 - Z_1)^2}{(Z_1 + Z_2)^2}$ to find the intensity reflection coefficient:

$$\begin{aligned} a &= \frac{(Z_2 - Z_1)^2}{(Z_1 + Z_2)^2} \\ &= \frac{\left(1.34 \times 10^6 \text{kg}/(\text{m}^2 \cdot s) - 1.70 \times 10^6 \text{kg}/(\text{m}^2 \cdot s)\right)^2}{\left(1.70 \times 10^6 \text{kg}/(\text{m}^2 \cdot s) + 1.34 \times 10^6 \text{kg}/(\text{m}^2 \cdot s)\right)^2} \\ &= 0.014 \end{aligned}$$

14.42

Discussion

This result means that only 1.4% of the incident intensity is reflected, with the remaining being transmitted.

The applications of ultrasound in medical diagnostics have produced untold benefits with no known risks. Diagnostic intensities are too low (about $10^{-2}\text{W}/\text{m}^2$) to cause thermal damage. More significantly, ultrasound has been in use for several decades and detailed follow-up studies do not show evidence of ill effects, quite unlike the case for x-rays.

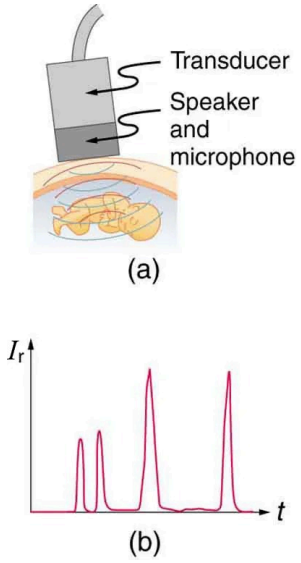


Figure 14.42 (a) An ultrasound speaker doubles as a microphone. Brief bleeps are broadcast, and echoes are recorded from various depths. (b) Graph of echo intensity versus time. The time for echoes to return is directly proportional to the distance of the reflector, yielding this information noninvasively. Image from OpenStax College Physics 2e, CC-BY 4.0

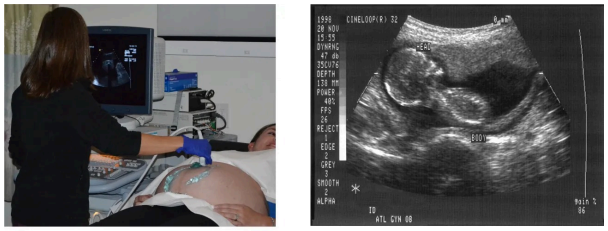
Image Description

The image consists of two parts labeled (a) and (b).

(a) This section shows a diagram of an ultrasound device used for examining a fetus. The device includes a transducer, speaker, and microphone positioned on a woman's abdomen, with an image of a baby in the womb below it. Arrows point to the transducer and the speaker and microphone.

(b) This section displays a graph with an x-axis labeled "t" and a y-axis labeled "I_r". The graph features multiple peaks, indicating varied intensity readings over time.

The most common ultrasound applications produce an image like that shown in Figure 14.43. The speaker-microphone broadcasts a directional beam, sweeping the beam across the area of interest. This is accomplished by having multiple ultrasound sources in the probe's head, which are phased to interfere constructively in a given, adjustable direction. Echoes are measured as a function of position as well as depth. A computer constructs an image that reveals the shape and density of internal structures.



(a) (b)

Figure 14.43 (a) An ultrasonic image is produced by sweeping the ultrasonic beam across the area of interest, in this case the person's abdomen. Data are recorded and analyzed in a computer, providing a two-dimensional image (credit: COD Newsroom, Flickr). (b) Ultrasound image of 12-week-old fetus. Image from OpenStax College Physics 2e, CC-BY 4.0

How much detail can ultrasound reveal? The image in Figure 14.43 is typical of low-cost systems, but that in Figure 14.44 shows the remarkable detail possible with more advanced systems, including 3D imaging. Ultrasound today is commonly used in prenatal care. Such imaging can be used to see if the fetus is developing at a normal rate, and help in the determination of serious problems early in the pregnancy. Ultrasound is also in wide use to image the chambers of the heart and the flow of blood within the beating heart, using the Doppler effect (echocardiology).

Whenever a wave is used as a probe, it is very difficult to detect details smaller than its wavelength λ. Indeed, current technology cannot do quite this well. Abdominal scans may use a 7-MHz frequency, and the speed of sound in tissue is about 1540 m/s—so the wavelength limit to detail would be $\lambda = \frac{v_w}{f} = \frac{1540\ \text{m/s}}{7 \times 10^6 \text{Hz}} = 0.22\ \text{mm}$. In practice, 1-mm detail is attainable, which is sufficient for many purposes. Higher-frequency ultrasound would allow greater detail, but it does not penetrate as well as lower frequencies do. The accepted rule of thumb is that you can effectively scan to a depth of about 500λ into tissue. For 7 MHz, this penetration limit is $500 \times 0.22mm$, which is 0.11 m. Higher frequencies may be employed in smaller organs, such as the eye, but are not practical for looking deep into the body.



Figure 14.44 A 3D ultrasound image of a fetus. Image from OpenStax College Physics 2e, CC-BY 4.0

In addition to shape information, ultrasonic scans can produce density information superior to that found in X-rays, because the intensity of a reflected sound is related to changes in density. Sound is most strongly reflected at places where density changes are greatest.

Another major use of ultrasound in medical diagnostics is to detect motion and determine velocity through the Doppler shift of an echo, known as Doppler-shifted ultrasound. This technique is used to monitor fetal heartbeat, measure blood velocity, and detect occlusions in blood vessels, for example. (See Figure 14.45.) The magnitude of the Doppler shift in an echo is directly proportional to the velocity of whatever reflects the sound. Because an echo is involved, there is actually a double shift. The first occurs because the reflector (say a fetal heart) is a moving observer and receives a Doppler-shifted frequency. The reflector then acts as a moving source, producing a second Doppler shift.

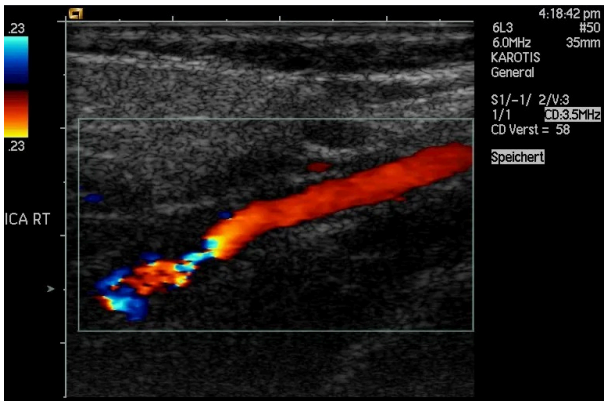


Figure 14.45 This Doppler-shifted ultrasonic image of a partially occluded artery uses color to indicate velocity. The highest velocities are in red, while the lowest are blue. The blood must move faster through the constriction to carry the same flow. Image from OpenStax College Physics 2e, CC-BY 4.0

A clever technique is used to measure the Doppler shift in an echo. The frequency of the echoed sound is superimposed on the broadcast frequency, producing beats. The beat frequency is $F_B = | f_1 - f_2 |$, and so it is directly proportional to the Doppler shift ($f_1 - f_2$) and hence, the reflector's velocity. The advantage in this technique is that the Doppler shift is small (because the reflector's velocity is small), so that great accuracy would be needed to measure the shift directly. But measuring the beat frequency is easy, and it is not affected if the broadcast frequency varies somewhat. Furthermore, the beat frequency is in the audible range and can be amplified for audio feedback to the medical observer.

Uses for Doppler-Shifted Radar

Doppler-shifted radar echoes are used to measure wind velocities in storms as well as aircraft and automobile speeds. The principle is the same as for Doppler-shifted ultrasound. There is evidence that bats and dolphins may also sense the velocity of an object (such as prey) reflecting their ultrasound signals by observing its Doppler shift.

Example 14.8

Calculate Velocity of Blood: Doppler-Shifted Ultrasound

Ultrasound that has a frequency of 2.50 MHz is sent toward blood in an artery that is moving toward the source at 20.0 cm/s, as illustrated in Figure 14.46. Use the speed of sound in human tissue as 1540 m/s. (Assume that the frequency of 2.50 MHz is accurate to seven significant figures.)

a. What frequency does the blood receive?
b. What frequency returns to the source?
c. What beat frequency is produced if the source and returning frequencies are mixed?

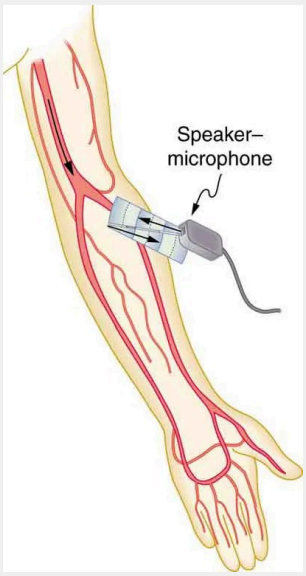


Figure 14.46 Ultrasound is partly reflected by blood cells and plasma back toward the speaker-microphone. Because the cells are moving, two Doppler shifts are produced—one for blood as a moving observer, and the other for the reflected sound coming from a moving source. The magnitude of the shift is directly proportional to blood velocity. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a medical illustration showing an arm with a transparent section revealing the veins and arteries running through it. There is a device labeled "Speaker-microphone" attached to the arm, connected by a wire. The speaker-microphone appears to be placed over the major artery of the arm, possibly to detect blood flow or other vascular signals. The veins and arteries are shown in red, while the rest of the arm is depicted in a pale skin tone. The illustration highlights the use of a medical device on the arm for vascular monitoring.

Strategy

The first two questions can be answered using $f_{\text{obs}} = f_s \left(\frac{v_w}{v_w \pm v_s} \right)$ and

$f_{\text{obs}} = f_s \left(\frac{v_w \pm v_{\text{obs}}}{v_w} \right)$ for the Doppler shift. The last question asks for beat frequency, which is the difference between the original and returning frequencies.

Solution for (a)

(1) Identify knowns:

- The blood is a moving observer, and so the frequency it receives is given by

  $$f_{\text{obs}} = f_s \left(\frac{v_w \pm v_{\text{obs}}}{v_w} \right).$$

  14.43
- v_b is the blood velocity (v_{obs} here) and the plus sign is chosen because the motion is toward the source.

(2) Enter the given values into the equation.

$$f_{\text{obs}} = (2,500,000 \text{ Hz}) \left(\frac{1540 \text{ m/s} + 0.2 \text{ m/s}}{1540 \text{ m/s}} \right)$$

14.44

(3) Calculate to find the frequency: 2,500,325 Hz.

Solution for (b)

(1) Identify knowns:

- The blood acts as a moving source.
- The microphone acts as a stationary observer.
- The frequency leaving the blood is 2,500,325 Hz, but it is shifted upward as given by

  $$f_{\text{obs}} = f_s \left(\frac{v_w}{v_w - v_b} \right).$$

  14.45

f_{obs} is the frequency received by the speaker-microphone.

- The source velocity is v_b.
- The minus sign is used because the motion is toward the observer.

The minus sign is used because the motion is toward the observer.

(2) Enter the given values into the equation:

$$f_{\text{obs}} = (2,500,325\ \text{Hz}) \left(\frac{1540\ \text{m/s}}{1540\ \text{m/s}\ - 0.200\ \text{m/s}} \right)$$

14.46

(3) Calculate to find the frequency returning to the source: 2,500,649 Hz.

Solution for (c)

(1) Identify knowns:

- The beat frequency is simply the absolute value of the difference between f_s and f_{obs}, as stated in:

  $$f_B = | f_{\text{obs}} - f_s |.$$

  14.47

(2) Substitute known values:

$$| 2,500,649\text{Hz} - 2,500,000\text{Hz} |$$

14.48

(3) Calculate to find the beat frequency: 649 Hz.

Discussion

The Doppler shifts are quite small compared with the original frequency of 2.50 MHz. It is far easier to measure the beat frequency than it is to measure the echo frequency with an accuracy great enough to see shifts of a few hundred hertz out of a couple of megahertz. Furthermore, variations in the source frequency do not greatly affect the

beat frequency, because both f_s and f_{obs}would increase or decrease. Those changes subtract out in $f_B = \left| f_{\text{obs}} - f_s \right|$.

Industrial and Other Applications of Ultrasound

Industrial, retail, and research applications of ultrasound are common. A few are discussed here. Ultrasonic cleaners have many uses. Jewelry, machined parts, and other objects that have odd shapes and crevices are immersed in a cleaning fluid that is agitated with ultrasound typically about 40 kHz in frequency. The intensity is great enough to cause cavitation, which is responsible for most of the cleansing action. Because cavitation-produced shock pressures are large and well transmitted in a fluid, they reach into small crevices where even a low-surface-tension cleaning fluid might not penetrate.

Sonar is a familiar application of ultrasound. Sonar typically employs ultrasonic frequencies in the range from 30.0 to 100 kHz. Bats, dolphins, submarines, and even some birds use ultrasonic sonar. Echoes are analyzed to give distance and size information both for guidance and finding prey. In most sonar applications, the sound reflects quite well because the objects of interest have significantly different density than the medium in which they travel. When the Doppler shift is observed, velocity information can also be obtained. Submarine sonar can be used to obtain such information, and there is evidence that some bats also sense velocity from their echoes.

Similarly, there are a range of relatively inexpensive devices that measure distance by timing ultrasonic echoes. Many cameras, for example, use such information to focus automatically. Some doors open when their ultrasonic ranging devices detect a nearby object, and certain home security lights turn on when their ultrasonic rangers observe motion. Ultrasonic “measuring tapes” also exist to measure such things as room dimensions. Sinks in public restrooms are sometimes automated with ultrasound devices to turn faucets on and off when people wash their hands. These devices reduce the spread of germs and can conserve water.

Ultrasound is used for nondestructive testing in industry and by the military. Because ultrasound reflects well from any large change in density, it can reveal cracks and voids

in solids, such as aircraft wings, that are too small to be seen with x-rays. For similar reasons, ultrasound is also good for measuring the thickness of coatings, particularly where there are several layers involved.

Basic research in solid state physics employs ultrasound. Its attenuation is related to a number of physical characteristics, making it a useful probe. Among these characteristics are structural changes such as those found in liquid crystals, the transition of a material to a superconducting phase, as well as density and other properties.

These examples of the uses of ultrasound are meant to whet the appetites of the curious, as well as to illustrate the underlying physics of ultrasound. There are many more applications, as you can easily discover for yourself.

Check Your Understanding

Why is it possible to use ultrasound both to observe a fetus in the womb and also to destroy cancerous tumors in the body?

Click for Solution

Solution

Ultrasound can be used medically at different intensities. Lower intensities do not cause damage and are used for medical imaging. Higher intensities can pulverize and destroy targeted substances in the body, such as tumors.

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GLOSSARY

acoustic impedance
: property of medium that makes the propagation of sound waves more difficult

antinode
: point of maximum displacement

bow wake
: V-shaped disturbance created when the wave source moves faster than the wave propagation speed

Doppler effect
: an alteration in the observed frequency of a sound due to motion of either the source or the observer

Doppler shift
: the actual change in frequency due to relative motion of source and observer

Doppler-shifted ultrasound
: a medical technique to detect motion and determine velocity through the Doppler shift of an echo

fundamental
: the lowest-frequency resonance

harmonics
: the term used to refer collectively to the fundamental and its overtones

hearing
: the perception of sound

infrasound
: sounds below 20 Hz

intensity
: the power per unit area carried by a wave

intensity reflection coefficient
: a measure of the ratio of the intensity of the wave reflected off a boundary between two media relative to the intensity of the incident wave

loudness
: the perception of sound intensity

node
: point of zero displacement

note
: basic unit of music with specific names, combined to generate tunes

overtones
: all resonant frequencies higher than the fundamental

phon
: the numerical unit of loudness

pitch
: the perception of the frequency of a sound

sonic boom
: a constructive interference of sound created by an object moving faster than sound

sound
: a disturbance of matter that is transmitted from its source outward

sound intensity level
: a unitless quantity telling you the level of the sound relative to a fixed standard

sound pressure level
: the ratio of the pressure amplitude to a reference pressure

timbre
: number and relative intensity of multiple sound frequencies

tone
: number and relative intensity of multiple sound frequencies

ultrasound
: sounds above 20,000 Hz

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CHAPTER SUMMARY

14.1 Sound

- Sound is a disturbance of matter that is transmitted from its source outward.
- Sound is one type of wave.
- Hearing is the perception of sound.

14.2 Speed of Sound, Frequency, and Wavelength

The relationship of the speed of sound v_w, its frequency f, and its wavelength λ is given by

$$v_{\mathrm{w}} = f\lambda,$$

which is the same relationship given for all waves.

In air, the speed of sound is related to air temperature T by

$$v_{\mathrm{w}} = (331\mathrm{m/s})\sqrt{\frac{T}{273\mathrm{K}}}.$$

v_{w} is the same for all frequencies and wavelengths.

14.3 Sound Intensity and Sound Level

- Intensity is the same for a sound wave as was defined for all waves; it is
$$I = \frac{P}{A},$$
where P is the power crossing area A. The SI unit for I is watts per meter squared. The intensity of a sound wave is also related to the pressure amplitude Δp
$$I = \frac{((\Delta p))^2}{2\rho \mathrm{v}_w},$$
where ρ is the density of the medium in which the sound wave travels and v_w is the speed of sound in the medium.
- Sound intensity level in units of decibels (dB) is
$$\beta\,(\mathrm{dB}) = 10\log_{10}\left(\frac{I}{I_0}\right),$$
where $I_0 = 10^{-12} W/\mathrm{m}^2$ is the threshold intensity of hearing.

14.4 Doppler Effect and Sonic Booms

- The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer.
- The actual change in frequency is called the Doppler shift.
- A sonic boom is constructive interference of sound created by an object moving faster than sound.
- A sonic boom is a type of bow wake created when any wave source moves faster than the wave propagation speed.
- For a stationary observer and a moving source, the observed frequency f_{obs} is:

$$f_{\text{obs}} = f_s \left(\frac{v_w}{v_w \pm v_s} \right),$$

where f_s is the frequency of the source, v_s is the speed of the source, and v_w is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away.

- For a stationary source and moving observer, the observed frequency is:

$$f_{\text{obs}} = f_s \left(\frac{v_w \pm v_{\text{obs}}}{v_w} \right),$$

where v_{obs} is the speed of the observer.

14.5 Sound Interference and Resonance: Standing Waves in Air Columns

- Sound interference and resonance have the same properties as defined for all waves.
- In air columns, the lowest-frequency resonance is called the fundamental, whereas all higher resonant frequencies are called overtones. Collectively, they are called harmonics.
- The resonant frequencies of a tube closed at one end are:

$$f_n = n\frac{v_w}{4L}, \quad n = 1, 3, 5\ldots,$$

  f_1 is the fundamental and L is the length of the tube.
- The resonant frequencies of a tube open at both ends are:

$$f_n = n\frac{v_w}{2L}, \quad n = 1, 2, 3\ldots$$

14.6 Hearing

- The range of audible frequencies is 20 to 20,000 Hz.
- Those sounds above 20,000 Hz are ultrasound, whereas those below 20 Hz are infrasound.
- The perception of frequency is pitch.
- The perception of intensity is loudness.
- Loudness has units of phons.

14.7 Ultrasound

- The acoustic impedance is defined as:
  $Z = \rho v$,
  ρ is the density of a medium through which the sound travels and v is the speed of sound through that medium.
- The intensity reflection coefficient a, a measure of the ratio of the intensity of the wave reflected off a boundary between two media relative to the intensity of the incident wave, is given by
  $$a = \frac{(Z_2 - Z_1)^2}{(Z_1 + Z_2)^2}.$$
- The intensity reflection coefficient is a unitless quantity.

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CONCEPTUAL QUESTIONS

Conceptual Questions

14.2 Speed of Sound, Frequency, and Wavelength

1. How do sound vibrations of atoms differ from thermal motion?
2. When sound passes from one medium to another where its propagation speed is different, does its frequency or wavelength change? Explain your answer briefly.

14.3 Sound Intensity and Sound Level

3. Six members of a synchronized swim team wear earplugs to protect themselves against water pressure at depths, but they can still hear the music and perform the combinations in the water perfectly. One day, they were asked to leave the pool so the dive team could practice a few dives, and they tried to practice on a mat, but seemed to have a lot more difficulty. Why might this be?
4. A community is concerned about a plan to bring train service to their downtown from the town's outskirts. The current sound intensity level, even though the rail yard is blocks away, is 70 dB downtown. The mayor assures the public that there will be a difference of only 30 dB in sound in the downtown area. Should the townspeople be concerned? Why?

14.4 Doppler Effect and Sonic Booms

5. Is the Doppler shift real or just a sensory illusion?
6. Due to efficiency considerations related to its bow wake, the supersonic transport aircraft must maintain a cruising speed that is a constant ratio to the speed of sound (a constant Mach number). If the aircraft flies from warm air into colder air, should it increase or decrease its speed? Explain your answer.
7. When you hear a sonic boom, you often cannot see the plane that made it. Why is that?

14.5 Sound Interference and Resonance: Standing Waves in Air Columns

8. How does an unamplified guitar produce sounds so much more intense than those of a plucked string held taut by a simple stick?
9. You are given two wind instruments of identical length. One is open at both ends, whereas the other is closed at one end. Which is able to produce the lowest frequency?
10. What is the difference between an overtone and a harmonic? Are all harmonics overtones? Are all overtones harmonics?

14.6 Hearing

11. Why can a hearing test show that your threshold of hearing is 0 dB at 250 Hz, when Figure 14.35 implies that no one can hear such a frequency at less than 20 dB?

14.7 Ultrasound

12. If audible sound follows a rule of thumb similar to that for ultrasound, in terms of its absorption, would you expect the high or low frequencies from your neighbor's stereo to penetrate into your house? How does this expectation compare with your experience?
13. Elephants and whales are known to use infrasound to communicate over very large distances. What are the advantages of infrasound for long distance communication?
14. It is more difficult to obtain a high-resolution ultrasound image in the abdominal region of someone who is overweight than for someone who has a slight build. Explain why this statement is accurate.
15. Suppose you read that 210-dB ultrasound is being used to pulverize cancerous tumors. You calculate the intensity in watts per centimeter squared and find it is unreasonably high ($10^5 \text{W}/\text{cm}^2$). What is a possible explanation?

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PROBLEMS & EXERCISES

Problems & Exercises

14.2 Speed of Sound, Frequency, and Wavelength

1. When poked by a spear, an operatic soprano lets out a 1200-Hz shriek. What is its wavelength if the speed of sound is 345 m/s?
2. What frequency sound has a 0.10-m wavelength when the speed of sound is 340 m/s?
3. Calculate the speed of sound on a day when a 1500 Hz frequency has a wavelength of 0.221 m.
4. (a) What is the speed of sound in a medium where a 100-kHz frequency produces a 5.96-cm wavelength? (b) Which substance in Table 14.1 is this likely to be?
5. Show that the speed of sound in $20.0^{\circ}\ \text{C}$ air is 343 m/s, as claimed in the text.
6. Air temperature in the Sahara Desert can reach $56.0^{\circ}\ \text{C}$ (about $134^{\circ}\ \text{F}$). What is the speed of sound in air at that temperature?
7. Dolphins make sounds in air and water. What is the ratio of the wavelength of a sound in air to its wavelength in seawater? Assume air temperature is $20.0^{\circ}\ \text{C}$.
8. A sonar echo returns to a submarine 1.20 s after being emitted. What is the distance to the object creating the echo? (Assume that the submarine is in the ocean, not in fresh water.)
9. (a) If a submarine's sonar can measure echo times with a precision of 0.0100 s, what is the smallest difference in distances it can detect? (Assume that the submarine is in the ocean, not in fresh water.) (b) Discuss the limits this time resolution imposes on the ability of the sonar system to detect the size and shape of the object creating the echo.
10. A physicist at a fireworks display times the lag between seeing an explosion and hearing its sound, and finds it to be 0.400 s. (a) How far away is the explosion if air temperature is $24.0^{\circ}\ \text{C}$ and if you neglect the time taken for light to reach the

physicist? (b) Calculate the distance to the explosion taking the speed of light into account. Note that this distance is negligibly greater.

11. Suppose a bat uses sound echoes to locate its insect prey, 3.00 m away. (See Figure 14.9.) (a) Calculate the echo times for temperatures of $5.00^{\circ}\ \mathrm{C}$ and $35.0^{\circ}\ \mathrm{C}$. (b) What percent uncertainty does this cause for the bat in locating the insect? (c) Discuss the significance of this uncertainty and whether it could cause difficulties for the bat. (In practice, the bat continues to use sound as it closes in, eliminating most of any difficulties imposed by this and other effects, such as motion of the prey.)

14.3 Sound Intensity and Sound Level

12. What is the intensity in watts per meter squared of 85.0-dB sound?
13. The warning tag on a lawn mower states that it produces noise at a level of 91.0 dB. What is this in watts per meter squared?
14. A sound wave traveling in $20^{\circ}\ C$ air has a pressure amplitude of 0.5 Pa. What is the intensity of the wave?
15. What intensity level does the sound in the preceding problem correspond to?
16. What sound intensity level in dB is produced by earphones that create an intensity of $4.00 \times 10^{-2}\mathrm{W/m^2}$?
17. Show that an intensity of $10^{-12}W/m^2$ is the same as $10^{-16}W/cm^2$.
18. (a) What is the decibel level of a sound that is twice as intense as a 90.0-dB sound? (b) What is the decibel level of a sound that is one-fifth as intense as a 90.0-dB sound?
19. (a) What is the intensity of a sound that has a level 7.00 dB lower than a $4.00 \times 10^{-9}W/m^2$ sound? (b) What is the intensity of a sound that is 3.00 dB higher than a $4.00 \times 10^{-9}W/m^2$ sound?
20. (a) How much more intense is a sound that has a level 17.0 dB higher than another? (b) If one sound has a level 23.0 dB less than another, what is the ratio of their intensities?
21. People with good hearing can perceive sounds as low in level as $-8.00dB$ at a

frequency of 3000 Hz. What is the intensity of this sound in watts per meter squared?

22. If a large housefly 3.0 m away from you makes a noise of 40.0 dB, what is the noise level of 1000 flies at that distance, assuming interference has a negligible effect?
23. Ten cars in a circle at a boom box competition produce a 120-dB sound intensity level at the center of the circle. What is the average sound intensity level produced there by each stereo, assuming interference effects can be neglected?
24. The amplitude of a sound wave is measured in terms of its maximum gauge pressure. By what factor does the amplitude of a sound wave increase if the sound intensity level goes up by 40.0 dB?
25. If a sound intensity level of 0 dB at 1000 Hz corresponds to a maximum gauge pressure (sound amplitude) of 10^{-9}atm, what is the maximum gauge pressure in a 60-dB sound? What is the maximum gauge pressure in a 120-dB sound?
26. An 8-hour exposure to a sound intensity level of 90.0 dB may cause hearing damage. What energy in joules falls on a 0.800-cm-diameter eardrum so exposed?
27. (a) Ear trumpets were never very common, but they did aid people with hearing losses by gathering sound over a large area and concentrating it on the smaller area of the eardrum. What decibel increase does an ear trumpet produce if its sound gathering area is $900cm^2$ and the area of the eardrum is $0.500cm^2$, but the trumpet only has an efficiency of 5.00% in transmitting the sound to the eardrum? (b) Comment on the usefulness of the decibel increase found in part (a).
28. Sound is more effectively transmitted into a stethoscope by direct contact than through the air, and it is further intensified by being concentrated on the smaller area of the eardrum. It is reasonable to assume that sound is transmitted into a stethoscope 100 times as effectively compared with transmission though the air. What, then, is the gain in decibels produced by a stethoscope that has a sound gathering area of $15.0cm^2$, and concentrates the sound onto two eardrums with a total area of $0.900cm^2$ with an efficiency of 40.0%?
29. Loudspeakers can produce intense sounds with surprisingly small energy input in spite of their low efficiencies. Calculate the power input needed to produce a

90.0-dB sound intensity level for a 12.0-cm-diameter speaker that has an efficiency of 1.00%. (This value is the sound intensity level right at the speaker.)

14.4 Doppler Effect and Sonic Booms

30. (a) What frequency is received by a person watching an oncoming ambulance moving at 110 km/h and emitting a steady 800-Hz sound from its siren? The speed of sound on this day is 345 m/s. (b) What frequency does she receive after the ambulance has passed?
31. (a) At an air show a jet flies directly toward the stands at a speed of 1200 km/h, emitting a frequency of 3500 Hz, on a day when the speed of sound is 342 m/s. What frequency is received by the observers? (b) What frequency do they receive as the plane flies directly away from them?
32. What frequency is received by a mouse just before being dispatched by a hawk flying at it at 25.0 m/s and emitting a screech of frequency 3500 Hz? Take the speed of sound to be 331 m/s.
33. A spectator at a parade receives an 888-Hz tone from an oncoming trumpeter who is playing an 880-Hz note. At what speed is the musician approaching if the speed of sound is 338 m/s?
34. A commuter train blows its 200-Hz horn as it approaches a crossing. The speed of sound is 335 m/s. (a) An observer waiting at the crossing receives a frequency of 208 Hz. What is the speed of the train? (b) What frequency does the observer receive as the train moves away?
35. Can you perceive the shift in frequency produced when you pull a tuning fork toward you at 10.0 m/s on a day when the speed of sound is 344 m/s? To answer this question, calculate the factor by which the frequency shifts and see if it is greater than 0.300%.
36. Two eagles fly directly toward one another, the first at 15.0 m/s and the second at 20.0 m/s. Both screech, the first one emitting a frequency of 3200 Hz and the second one emitting a frequency of 3800 Hz. What frequencies do they receive if the speed of sound is 330 m/s?
37. What is the minimum speed at which a source must travel toward you for you to

be able to hear that its frequency is Doppler shifted? That is, what speed produces a shift of 0.300% on a day when the speed of sound is 331 m/s?

14.5 Sound Interference and Resonance: Standing Waves in Air Columns

38. A "showy" custom-built car has two brass horns that are supposed to produce the same frequency but actually emit 263.8 and 264.5 Hz. What beat frequency is produced?
39. What beat frequencies will be present: (a) If the musical notes A and C are played together (frequencies of 220 and 264 Hz)? (b) If D and F are played together (frequencies of 297 and 352 Hz)? (c) If all four are played together?
40. What beat frequencies result if a piano hammer hits three strings that emit frequencies of 127.8, 128.1, and 128.3 Hz?
41. A piano tuner hears a beat every 2.00 s when listening to a 264.0-Hz tuning fork and a single piano string. What are the two possible frequencies of the string?
42. (a) What is the fundamental frequency of a 0.672-m-long tube, open at both ends, on a day when the speed of sound is 344 m/s? (b) What is the frequency of its second harmonic?
43. If a wind instrument, such as a tuba, has a fundamental frequency of 32.0 Hz, what are its first three overtones? It is closed at one end. (The overtones of a real tuba are more complex than this example, because it is a tapered tube.)
44. What are the first three overtones of a bassoon that has a fundamental frequency of 90.0 Hz? It is open at both ends. (The overtones of a real bassoon are more complex than this example, because its double reed makes it act more like a tube closed at one end.)
45. How long must a flute be in order to have a fundamental frequency of 262 Hz (this frequency corresponds to middle C on the evenly tempered chromatic scale) on a day when air temperature is $20.0^{\circ}\ C$? It is open at both ends.
46. What length should an oboe have to produce a fundamental frequency of 110 Hz on a day when the speed of sound is 343 m/s? It is open at both ends.
47. What is the length of a tube that has a fundamental frequency of 176 Hz and a

first overtone of 352 Hz if the speed of sound is 343 m/s?

48. (a) Find the length of an organ pipe closed at one end that produces a fundamental frequency of 256 Hz when air temperature is $18.0^{\circ}\ C$. (b) What is its fundamental frequency at $25.0^{\circ}\ C$?
49. By what fraction will the frequencies produced by a wind instrument change when air temperature goes from $10.0^{\circ}\ C$ to $30.0^{\circ}\ C$? That is, find the ratio of the frequencies at those temperatures.
50. The ear canal resonates like a tube closed at one end. (See Figure 14.37.) If ear canals range in length from 1.80 to 2.60 cm in an average population, what is the range of fundamental resonant frequencies? Take air temperature to be $37.0^{\circ}\ C$, which is the same as body temperature. How does this result correlate with the intensity versus frequency graph (Figure 14.35 of the human ear?
51. Calculate the first overtone in an ear canal, which resonates like a 2.40-cm-long tube closed at one end, by taking air temperature to be $37.0^{\circ}\ C$. Is the ear particularly sensitive to such a frequency? (The resonances of the ear canal are complicated by its nonuniform shape, which we shall ignore.)
52. A crude approximation of voice production is to consider the breathing passages and mouth to be a resonating tube closed at one end. (See Figure 14.29.) (a) What is the fundamental frequency if the tube is 0.240-m long, by taking air temperature to be $37.0^{\circ}\ C$? (b) What would this frequency become if the person replaced the air with helium? Assume the same temperature dependence for helium as for air.
53. (a) Students in a physics lab are asked to find the length of an air column in a tube closed at one end that has a fundamental frequency of 256 Hz. They hold the tube vertically and fill it with water to the top, then lower the water while a 256-Hz tuning fork is rung and listen for the first resonance. What is the air temperature if the resonance occurs for a length of 0.336 m? (b) At what length will they observe the second resonance (first overtone)?
54. What frequencies will a 1.80-m-long tube produce in the audible range at $20.0^{\circ}\ C$ if: (a) The tube is closed at one end? (b) It is open at both ends?

14.6 Hearing

55. The factor of 10^{-12} in the range of intensities to which the ear can respond, from threshold to that causing damage after brief exposure, is truly remarkable. If you could measure distances over the same range with a single instrument and the smallest distance you could measure was 1 mm, what would the largest be?
56. The frequencies to which the ear responds vary by a factor of 10^3. Suppose the speedometer on your car measured speeds differing by the same factor of 10^3, and the greatest speed it reads is 90.0 mi/h. What would be the slowest nonzero speed it could read?
57. What are the closest frequencies to 500 Hz that an average person can clearly distinguish as being different in frequency from 500 Hz? The sounds are not present simultaneously.
58. Can the average person tell that a 2002-Hz sound has a different frequency than a 1999-Hz sound without playing them simultaneously?
59. If your radio is producing an average sound intensity level of 85 dB, what is the next lowest sound intensity level that is clearly less intense?
60. Can you tell that your roommate turned up the sound on the TV if its average sound intensity level goes from 70 to 73 dB?
61. Based on the graph in Figure 14.34, what is the threshold of hearing in decibels for frequencies of 60, 400, 1000, 4000, and 15,000 Hz? Note that many AC electrical appliances produce 60 Hz, music is commonly 400 Hz, a reference frequency is 1000 Hz, your maximum sensitivity is near 4000 Hz, and many older TVs produce a 15,750 Hz whine.
62. What sound intensity levels must sounds of frequencies 60, 3000, and 8000 Hz have in order to have the same loudness as a 40-dB sound of frequency 1000 Hz (that is, to have a loudness of 40 phons)?
63. What is the approximate sound intensity level in decibels of a 600-Hz tone if it has a loudness of 20 phons? If it has a loudness of 70 phons?
64. (a) What are the loudnesses in phons of sounds having frequencies of 200, 1000, 5000, and 10,000 Hz, if they are all at the same 60.0-dB sound intensity level? (b) If they are all at 110 dB? (c) If they are all at 20.0 dB?

65. Suppose a person has a 50-dB hearing loss at all frequencies. By how many factors of 10 will low-intensity sounds need to be amplified to seem normal to this person? Note that smaller amplification is appropriate for more intense sounds to avoid further hearing damage.
66. If a woman needs an amplification of 5.0×10^{12} times the threshold intensity to enable her to hear at all frequencies, what is her overall hearing loss in dB? Note that smaller amplification is appropriate for more intense sounds to avoid further damage to her hearing from levels above 90 dB.
67. (a) What is the intensity in watts per meter squared of a just barely audible 200-Hz sound? (b) What is the intensity in watts per meter squared of a barely audible 4000-Hz sound?
68. (a) Find the intensity in watts per meter squared of a 60.0-Hz sound having a loudness of 60 phons. (b) Find the intensity in watts per meter squared of a 10,000-Hz sound having a loudness of 60 phons.
69. A person has a hearing threshold 10 dB above normal at 100 Hz and 50 dB above normal at 4000 Hz. How much more intense must a 100-Hz tone be than a 4000-Hz tone if they are both barely audible to this person?
70. A child has a hearing loss of 60 dB near 5000 Hz, due to noise exposure, and normal hearing elsewhere. How much more intense is a 5000-Hz tone than a 400-Hz tone if they are both barely audible to the child?
71. What is the ratio of intensities of two sounds of identical frequency if the first is just barely discernible as louder to a person than the second?

14.7 Ultrasound

Unless otherwise indicated, for problems in this section, assume that the speed of sound through human tissues is 1540 m/s.

72. What is the sound intensity level in decibels of ultrasound of intensity $10^5 \mathrm{W/m^2}$, used to pulverize tissue during surgery?
73. Is 155-dB ultrasound in the range of intensities used for deep heating? Calculate the intensity of this ultrasound and compare this intensity with values quoted in

the text.

74. Find the sound intensity level in decibels of $2.00 \times 10^{-2}\text{W/m}^2$ultrasound used in medical diagnostics.
75. The time delay between transmission and the arrival of the reflected wave of a signal using ultrasound traveling through a piece of fat tissue was 0.13 ms. At what depth did this reflection occur?
76. In the clinical use of ultrasound, transducers are always coupled to the skin by a thin layer of gel or oil, replacing the air that would otherwise exist between the transducer and the skin. (a) Using the values of acoustic impedance given in Table 14.5 calculate the intensity reflection coefficient between transducer material and air. (b) Calculate the intensity reflection coefficient between transducer material and gel (assuming for this problem that its acoustic impedance is identical to that of water). (c) Based on the results of your calculations, explain why the gel is used.
77. (a) Calculate the minimum frequency of ultrasound that will allow you to see details as small as 0.250 mm in human tissue. (b) What is the effective depth to which this sound is effective as a diagnostic probe?
78. (a) Find the size of the smallest detail observable in human tissue with 20.0-MHz ultrasound. (b) Is its effective penetration depth great enough to examine the entire eye (about 3.00 cm is needed)? (c) What is the wavelength of such ultrasound in $0^{\circ}\ C$ air?
79. (a) Echo times are measured by diagnostic ultrasound scanners to determine distances to reflecting surfaces in a patient. What is the difference in echo times for tissues that are 3.50 and 3.60 cm beneath the surface? (This difference is the minimum resolving time for the scanner to see details as small as 0.100 cm, or 1.00 mm. Discrimination of smaller time differences is needed to see smaller details.) (b) Discuss whether the period T of this ultrasound must be smaller than the minimum time resolution. If so, what is the minimum frequency of the ultrasound and is that out of the normal range for diagnostic ultrasound?
80. (a) How far apart are two layers of tissue that produce echoes having round-trip times (used to measure distances) that differ by $0.750\mu s$? (b) What minimum frequency must the ultrasound have to see detail this small?

81. (a) A bat uses ultrasound to find its way among trees. If this bat can detect echoes 1.00 ms apart, what minimum distance between objects can it detect? (b) Could this distance explain the difficulty that bats have finding an open door when they accidentally get into a house?
82. A dolphin is able to tell in the dark that the ultrasound echoes received from two sharks come from two different objects only if the sharks are separated by 3.50 m, one being that much farther away than the other. (a) If the ultrasound has a frequency of 100 kHz, show this ability is not limited by its wavelength. (b) If this ability is due to the dolphin's ability to detect the arrival times of echoes, what is the minimum time difference the dolphin can perceive?
83. A diagnostic ultrasound echo is reflected from moving blood and returns with a frequency 500 Hz higher than its original 2.00 MHz. What is the velocity of the blood? (Assume that the frequency of 2.00 MHz is accurate to seven significant figures and 500 Hz is accurate to three significant figures.)
84. Ultrasound reflected from an oncoming bloodstream that is moving at 30.0 cm/s is mixed with the original frequency of 2.50 MHz to produce beats. What is the beat frequency? (Assume that the frequency of 2.50 MHz is accurate to seven significant figures.)

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ELECTRIC CHARGE AND ELECTRIC FIELD



Figure 15.1 Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

15.1 Static Electricity and Charge: Conservation of Charge
15.2 Conductors and Insulators
15.3 Coulomb's Law
15.4 Electric Field: Concept of a Field Revisited
15.5 Electric Field Lines: Multiple Charges
15.6 Electric Forces in Biology
15.7 Conductors and Electric Fields in Static Equilibrium
15.8 Applications of Electrostatics
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to many schoolchildren. (See Figure 15.2.) In this experiment, Franklin demonstrated a connection between lightning and static electricity. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find particularly appealing.

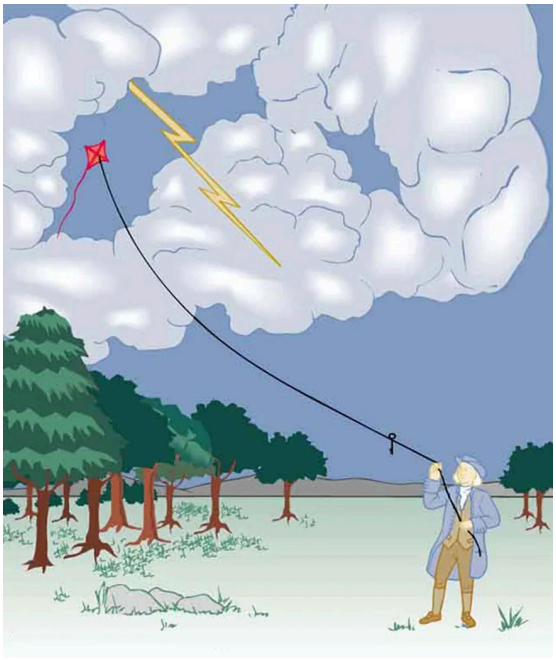


Figure 15.2 When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force. Image from OpenStax College Physics 2e, CC-BY 4.0

Much has been written about Franklin. His experiments were only part of the life of a man who was a scientist, inventor, revolutionary, statesman, and writer. Franklin's experiments were not performed in isolation, nor were they the only ones to reveal connections.

For example, the Italian scientist Luigi Galvani (1737–1798) performed a series of experiments in which static electricity was used to stimulate contractions of leg muscles of dead frogs, an effect already known in humans subjected to static discharges. But Galvani also found that if he joined two metal wires (say copper and zinc) end to end and touched the other ends to muscles, he produced the same effect in frogs as static discharge. Alessandro Volta (1745–1827), partly inspired by Galvani's work, experimented with various combinations of metals and developed the battery.

During the same era, other scientists made progress in discovering fundamental connections. The periodic table was developed as the systematic properties of the elements were discovered. This influenced the development and refinement of the concept of atoms as the basis of matter. Such submicroscopic descriptions of matter also help explain a great deal more.

Atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the electromagnetic force. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism.

All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

This chapter begins the study of electromagnetic phenomena at a fundamental level. The next several chapters will cover static electricity, moving electricity, and magnetism—collectively known as electromagnetism. In this chapter, we begin with the study of electric phenomena due to charges that are at least temporarily stationary, called electrostatics, or static electricity.
Access multimedia content

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15.1 STATIC ELECTRICITY AND CHARGE: CONSERVATION OF CHARGE

Learning Objectives

By the end of this section, you will be able to:

- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.



Figure 15.3 Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins. When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge. At the same time, the silk, having lost electrons, becomes positively charged. Image by Sebakoamber, Wikimedia Commons License

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it to attract bits of straw (see Figure 15.3). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across

the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with a conductive strip of aluminum foil on the bottoms to avoid creating sparks which may ignite flammable anesthesia gases combined with the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of electric charge? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge "positive", and the other type "negative." For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. Figure 15.4 shows how these simple materials can be used to explore the nature of the force between charges.

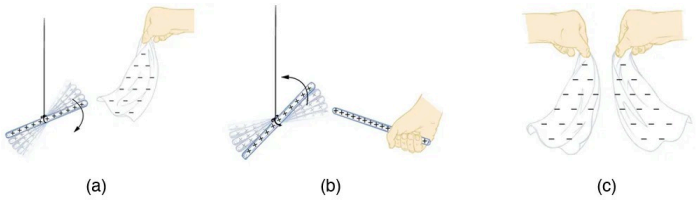


Figure 15.4 A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged. (a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a sequence of three illustrations labeled (a), (b), and (c), demonstrating a process involving hands, a rod, and a piece of fabric with arrows indicating motion.

(a) A hand is holding the end of a fabric with horizontal lines. The fabric is being drawn away from a vertical rod with a circular band around it, shown with a downward arrow portraying rotational movement.

(b) Two hands are holding the rod horizontally. One hand is positioned at each end of the rod. An upward arrow indicates rotational movement around the rod.

(c) Two hands are each holding the end of separate pieces of fabric. The fabrics have horizontal lines and are spread apart.

More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

Figure 15.5 shows a simple model of an atom with negative electrons orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged protons. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.

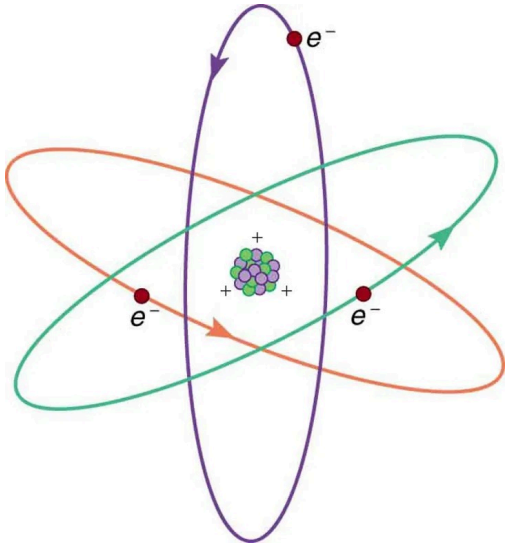


Figure 15.5 This simplified (and not to scale) view of an atom is called the planetary model of the atom. Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual negative and positive charges. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a simplified diagram of an atom. At the center, there is a cluster representing the nucleus, which consists of small circles indicating protons and neutrons. The protons are marked with a positive sign. Surrounding the nucleus are three elliptical orbits, each with an electron (depicted as a small, negatively charged particle) on it. The electrons are marked with an "e^-" symbol. Each orbit is shown with a different color, including green, orange, and purple, indicating the paths of the electrons around the nucleus. Arrows on each orbit line indicate the direction of the electrons' movement.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

$$| q_e | = 1.60 \times 10^{-19} \text{C}.$$
15.1

The symbol q is commonly used for charge and the subscript e indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

$$1.00\ \text{C} \times \frac{1\text{proton}}{1.60 \times 10^{-19}\text{C}} = 6.25 \times 10^{18}\text{protons}.$$

15.2

Similarly, 6.25×10^{18} electrons have a combined charge of −1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than $|q_e|$ (see Things Great and Small: The Submicroscopic Origin of Charge), and all observed charges are integral multiples of $|q_e|$.

Things Great and Small: The Submicroscopic Origin of Charge

With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See Figure 15.6.) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

Figure 15.6 shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.

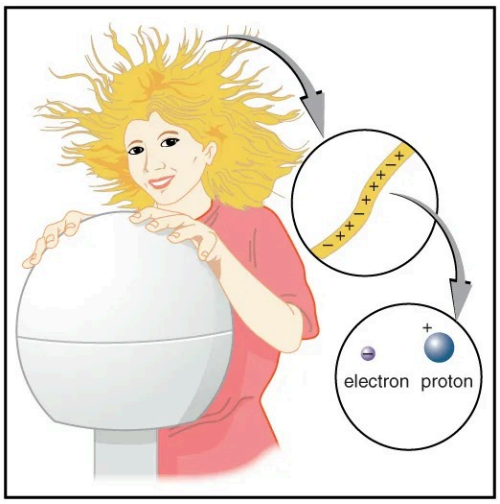


Figure 15.6 When this person touches a Van de Graaff generator, some electrons are attracted to the generator, resulting in an excess of positive charge, causing her hair to stand on end. The charges in one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a person with yellow hair standing next to a Van de Graaff generator, represented by a large, shiny sphere. The person's hair is standing on end, which demonstrates the effect of static electricity generated by the device. To the right, there are two inset diagrams:

- The first inset shows a strip with black "+" and "-" symbols, indicating the presence of positive and negative charges.
- The second inset contains two particles: one labeled as "electron" with a negative charge symbol (represented by a small purple sphere) and the other labeled as "proton" with a positive charge symbol (represented by a larger blue sphere).

The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in Figure 15.7. Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either $-\frac{1}{3}$ or $+\frac{2}{3}$. There are continuing

attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.

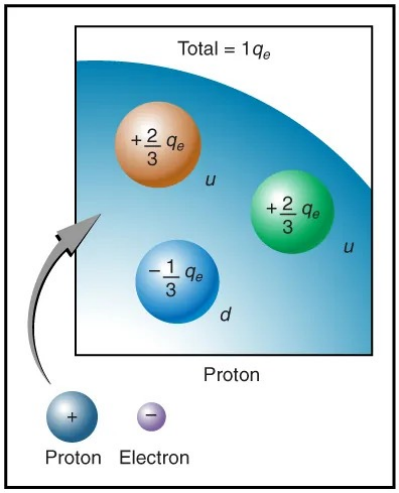


Figure 15.7 Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton: $-\frac{1}{3}q_e + \frac{2}{3}q_e + \frac{2}{3}q_e = (+1q)_e$. *Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image illustrates the composition of a proton in terms of quarks. It presents a proton containing three quarks, labeled as 'u', 'u', and 'd'. Each quark is shown with its charge:

- Two 'u' (up) quarks are depicted with a charge of $^{+2}/_{3}\, q_e$ each.
- One 'd' (down) quark is depicted with a charge of $^{-1}/_{3}\, q_e$.

The total charge of the proton is shown as $1\, q_e$, which is the sum of the charges of the quarks. An arrow points to the proton image, indicating it consists of these quarks. At the bottom left, symbols represent the proton with a '+' charge and an electron with a '-' charge.

Separation of Charge in Atoms

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See Figure 15.8.) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical

interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.

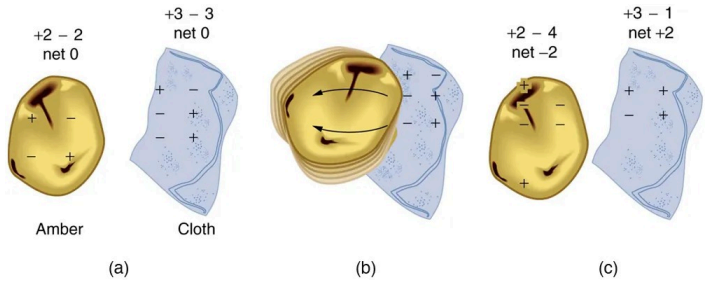


Figure 15.8 When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams labeled (a), (b), and (c), illustrating interactions between an amber object and a cloth.

Description of each diagram:

(a) The amber object has a charge notation of "+2 – 2," resulting in a "net 0" charge. The adjacent cloth has a charge notation of "+3 – 3," also resulting in a "net 0" charge.

(b) The amber object is rubbed against the cloth, indicated by curved arrows. This interaction shows movement and exchange of charges.

(c) After rubbing, the amber object is labeled as having "+2 – 4," resulting in a "net -2" charge. The cloth has "+3 – 1," resulting in a "net +2" charge.

Objects:

- Amber object: Presented in a yellowish-brown color with positive (+) and negative (-) charge symbols.
- Cloth: Represented in a light blue color with positive (+) and negative (-) charge symbols.

Description of interaction:

Initially, both objects are neutral in charge (net 0). During rubbing, charges are exchanged resulting in the amber object gaining a negative net charge and the cloth gaining a positive net charge.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the law of conservation of charge.

Law of Conservation of Charge

Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass, Δm, can be created from energy in the amount $\Delta m = \frac{E}{c^2}$. Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are "matter-antimatter" counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See Figure 15.9.) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy E, again obeying the relationship $\Delta m = \frac{E}{c^2}$. Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

Making Connections: Conservation Laws

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.

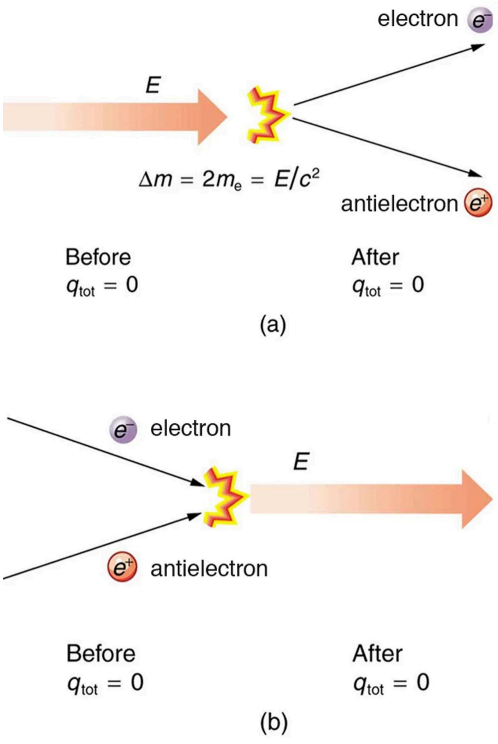


Figure 15.9 (a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. (m_e is the electron's mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two diagrams labeled (a) and (b), demonstrating concepts from particle physics related to energy and particle interactions.

Diagram (a):

- A large orange arrow pointing right represents energy (E) moving to the right.
- The arrow ends in a yellow and red jagged explosion, indicating a process where energy is converted.
- From this explosion, two smaller arrows emerge to the upper right and lower right, labeled as "electron" (e^-) and "antielectron" (e^+), respectively.
- A formula is shown: $\Delta m = 2m_e = E/c^2$, indicating the relationship between energy and mass.
- Text "Before, $q_{tot} = 0$" and "After, $q_{tot} = 0$" states the total charge remains zero before and after the interaction.

Diagram (b):

- An electron and an antielectron, represented by smaller arrows labeled as "electron" (e^-) and "antielectron" (e^+), approach each other from the left and the right towards a similar explosion.
- From the explosion, a large orange arrow representing energy (E) moves to the right, the same as in the first part.
- Text "Before, $q_{tot} = 0$" and "After, $q_{tot} = 0$" reaffirms that the total charge is zero both before and after the interaction.

Both diagrams illustrate processes where energy and particles interact while conserving the total charge.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

PhET Explorations

Balloons and Static Electricity

Why does a balloon stick to your sweater? Rub a balloon on a sweater, then let go of the balloon and it flies over and sticks to the sweater. View the charges in the sweater, balloons, and the wall.

Access multimedia content

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15.2 CONDUCTORS AND INSULATORS

Learning Objectives

By the end of this section, you will be able to:

- Define conductor and insulator, explain the difference, and give examples of each.
- Describe three methods for charging an object.
- Explain what happens to an electric force as you move farther from the source.
- Define polarization.



Figure 15.10 This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. Image by Evan-Amos, Wikimedia Commons License

Some substances, such as metals and salty water, allow charges to move through them with relative ease. Some of the electrons in metals and similar conductors are not bound to individual atoms or sites in the material. These free electrons can move through the material much as air moves through loose sand. Any substance that has free electrons and allows charge to move relatively freely through it is called a conductor. The moving electrons may collide with fixed atoms and molecules, losing some energy, but they can move in a conductor. Superconductors allow the movement of charge without any loss of energy. Salty water and other similar conducting materials contain free ions that can move through them. An ion is an atom or molecule having a positive or negative (nonzero) total charge. In other words, the total number of electrons is not equal to the total number of protons.

Other substances, such as glass, do not allow charges to move through them. These are called insulators. Electrons and ions in insulators are bound in the structure and cannot move easily—as much as 10^{23} times more slowly than in conductors. Pure water and dry table salt are insulators, for example, whereas molten salt and salty water are conductors.

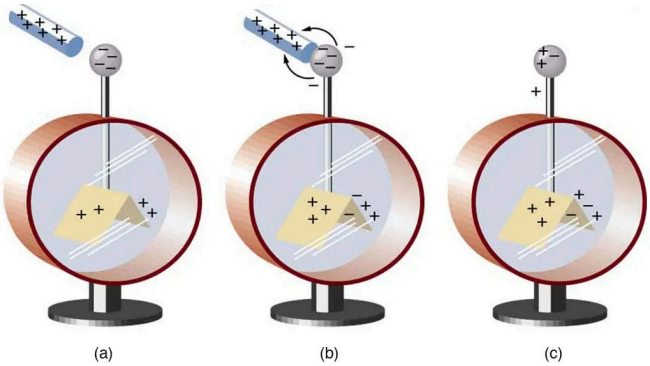


Figure 15.11 An electroscope is a favorite instrument in physics demonstrations and student laboratories. It is typically made with gold foil leaves hung from a (conducting) metal stem and is insulated from the room air in a glass-walled container. (a) A positively charged glass rod is brought near the tip of the electroscope, attracting electrons to the top and leaving a net positive charge on the leaves. Like charges in the light flexible gold leaves repel, separating them. (b) When the rod is touched against the ball, electrons are attracted and transferred, reducing the net charge on the glass rod but leaving the electroscope positively charged. (c) The excess charges are evenly distributed in the stem and leaves of the electroscope once the glass rod is removed. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams labeled (a), (b), and (c), each illustrating the process of charging a gold leaf electroscope through induction:

(a) An electroscope with a glass dome covering a metal rod. Inside, a gold leaf is attached to the rod. A charged rod with positive and negative symbols approaches from the left, showing the presence of positive charge (labeled with plus signs) near the electroscope.

(b) The charged rod is closer to the metal ball on top of the electroscope, inducing a negative charge on the side of the metal ball closest to the rod, shown with negative symbols. This causes the gold leaves to repel as indicated by the plus symbols on both leaves inside the dome.

(c) The charged rod is removed, and the electroscope remains charged. A positive charge is seen on the electroscope ball, and the gold leaves remain spread, indicating the presence of some charges, shown by plus and minus symbols inside the dome.

Charging by Contact

Figure 15.11 shows an electroscope being charged by touching it with a positively charged glass rod. Because the glass rod is an insulator, it must actually touch the electroscope to transfer charge to or from it. (Note that

the extra positive charges reside on the surface of the glass rod as a result of rubbing it with silk before starting the experiment.) Since only electrons move in metals, we see that they are attracted to the top of the electroscope. There, some are transferred to the positive rod by touch, leaving the electroscope with a net positive charge.

Electrostatic repulsion in the leaves of the charged electroscope separates them. The electrostatic force has a horizontal component that results in the leaves moving apart as well as a vertical component that is balanced by the gravitational force. Similarly, the electroscope can be negatively charged by contact with a negatively charged object.

Charging by Induction

It is not necessary to transfer excess charge directly to an object in order to charge it. Figure 15.12 shows a method of induction wherein a charge is created in a nearby object, without direct contact. Here we see two neutral metal spheres in contact with one another but insulated from the rest of the world. A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

This is an example of induced polarization of neutral objects. Polarization is the separation of charges in an object that remains neutral. If the spheres are now separated (before the rod is pulled away), each sphere will have a net charge. Note that the object closest to the charged rod receives an opposite charge when charged by induction. Note also that no charge is removed from the charged rod, so that this process can be repeated without depleting the supply of excess charge.

Another method of charging by induction is shown in Figure 15.13. The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since the earth is large and most ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge is achieved when charging by induction and the charged rod loses none of its excess charge.

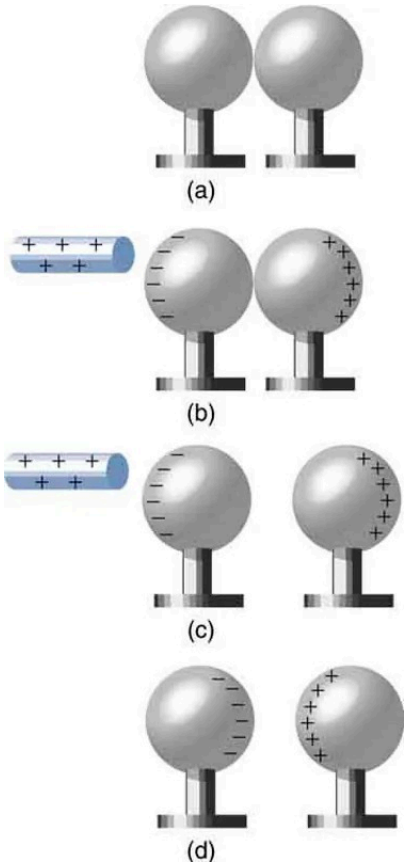


Figure 15.12 Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The spheres are separated before the rod is removed, thus separating negative and positive charge. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of four diagrams labeled (a) through (d), depicting the process of charging by induction using metallic spheres:

(a) Two metallic spheres are shown side by side, uncharged and neutral. They are identical with no markings.

(b) A cylindrical rod with positive charges (indicated by plus signs) approaches the left sphere. As a result, the left sphere shows negative charges on its outward side (indicated by minus signs), while the right sphere shows positive charges on its outward side (plus signs).

(c) The rod remains near the left sphere. The distribution of charges is consistent with the previous step: negative charges on the side of the left sphere and positive charges on the side of the right sphere.

(d) The rod is removed. Both spheres maintain their charge distribution: the left sphere has negative charges, and the right sphere has positive charges on the side facing the removed rod.

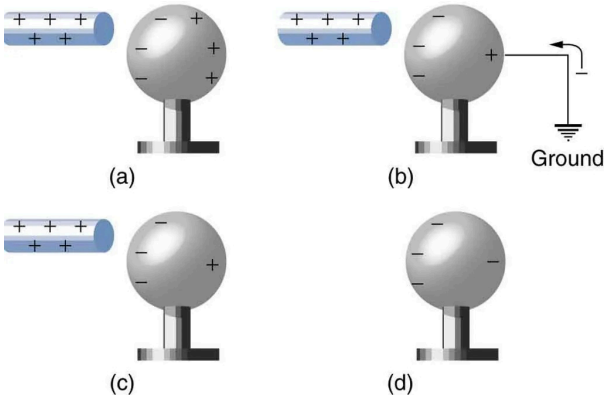


Figure 15.13 Charging by induction, using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from the earth's ample supply. (c) The ground connection is broken. (d) The positive rod is removed, leaving the sphere with an induced negative charge. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image consists of four diagrams labeled (a) to (d), illustrating the process of charging by induction using a metal sphere and a charged rod.

(a): A metal sphere is fixed on a stand. A charged rod with positive charges is shown close to the sphere on the left side. The sphere shows an uneven distribution of charges, with negative charges on the side closest to the rod and positive charges on the far side.

(b): The charged rod remains close to the sphere. A grounding wire is shown connecting the sphere to the ground, indicated by an arrow and a ground symbol. Negative charges remain near the rod, and some positive charges are shown being removed through the grounding wire.

(c): The grounding wire is removed. The charged rod is still close to the sphere, and the sphere has a distribution of charges with negative charges on the left near the rod and positive charges on the right.

(d): The charged rod is now removed from the vicinity of the sphere. The sphere retains a uniform distribution of negative charges, indicating it is left with a net negative charge after the rod's removal.

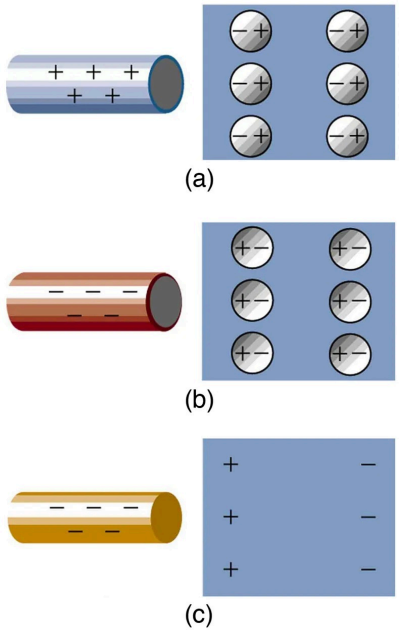


Figure 15.14 Both positive and negative objects attract a neutral object by polarizing its molecules. (a) A positive object brought near a neutral insulator polarizes its molecules. There is a slight shift in the distribution of the electrons orbiting the molecule, with unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three sections labeled (a), (b), and (c), each illustrating different arrangements of charges in cylinders and their surrounding environments.

1. **(a)**
    - On the left, there is a light blue cylindrical object with several positive signs along its surface, indicating positive charges.
    - On the right, a blue square area contains six circles, each with a negative sign on the left and a positive sign on the right, symbolizing the presence of both negative and positive charges.
2. **(b)**
    - On the left, a brown cylindrical object with alternating positive and negative signs along its surface indicates a mix of positive and negative charges.

- On the right, a blue square area containing six circles, each with a positive sign on the left and a negative sign on the right, again showing both negative and positive charges within the circles.

3. **(c)**

- On the left, a yellow cylindrical object has alternating positive and negative signs along its surface, similar to section (b).
- On the right, a blue square area contains three plus signs on the left and three minus signs on the right, indicating separated positive and negative charges.

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. Figure 15.14 shows how the polarization of atoms and molecules in neutral objects results in their attraction to a charged object.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

Check Your Understanding

Can you explain the attraction of water to the charged rod in the figure below?

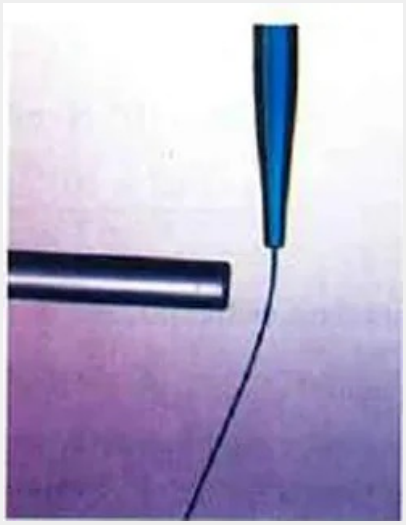


Figure 15.15 Image from OpenStax College Physics 2e, CC-BY 4.0

Click for Solution

Solution

Water molecules are polarized, giving them slightly positive and slightly negative sides. This makes water even more susceptible to a charged rod's attraction. In addition, tap water contains dissolved ions (positive and negative charges). As the water flows downward, due to the force of gravity, the charged conductor exerts a net attraction to the opposite charges in the stream of water, pulling it closer.

PhET Explorations

John Travoltage

Make sparks fly with John Travoltage. Wiggle Johnnie's foot and he picks up charges from the carpet. Bring his hand close to the door knob and get rid of the excess charge.

Access multimedia content

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15.3 COULOMB'S LAW

Learning Objectives

By the end of this section, you will be able to:

- State Coulomb's law in terms of how the electrostatic force changes with the distance between two objects.
- Calculate the electrostatic force between two charged point forces, such as electrons or protons.
- Compare the electrostatic force to the gravitational attraction for a proton and an electron; for a human and the Earth.

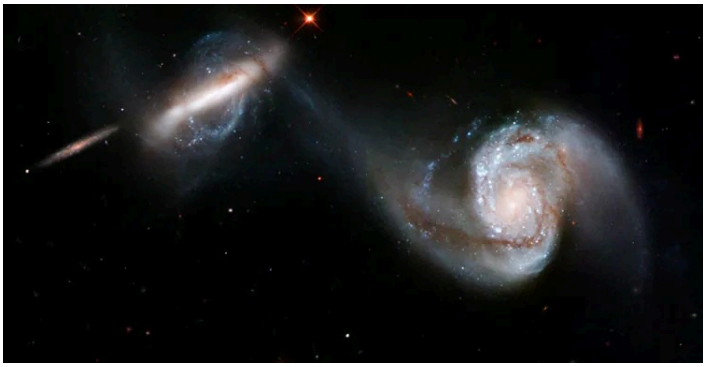


Figure 15.16 This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. Image from OpenStax College Physics 2e, CC-BY 4.0

Through the work of scientists in the late 18th century, the main features of the electrostatic force—the

existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called Coulomb's law after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

Coulomb's Law

$$F = k\frac{|q_1 q_2|}{r^2}.$$

15.3

Coulomb's law calculates the magnitude of the force F between two point charges, q_1 and q_2, separated by a distance r. In SI units, the constant k is equal to

$$k = 8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \approx 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}.$$

15.4

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See Figure 15.17.)

Although the formula for Coulomb's law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared $\left(F \propto 1/r^2\right)$ to an accuracy of 1 part in 10^{16}. No exceptions have ever been found, even at the small distances within the atom.

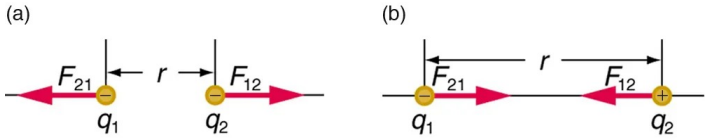


Figure 15.17 The magnitude of the electrostatic force F between point charges q_1 and q_2 separated by a distance r is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on q_1 is equal in magnitude and opposite in direction to the force it exerts on q_2. (a) Like charges. (b) Unlike charges. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two diagrams labeled (a) and (b), illustrating force interactions between charges.

Diagram (a):
– Two negatively charged particles are shown, labeled q_1 and q_2.
– The particles are separated by a distance labeled r.
– An arrow labeled F_{21} points leftward from q_2 towards q_1, indicating the force exerted on q_1.
– An arrow labeled F_{12} points rightward from q_1 towards q_2, indicating the force exerted on q_2.
– Both force arrows are depicted in red and positioned parallel to each other.

Diagram (b):
– The charges q_1 and q_2 are depicted again, but this time q_1 is negative and q_2 is positive.
– The distance between the charges is labeled r.
– An arrow labeled F_{21} points leftward from q_2 towards q_1, representing the force on q_1.
– An arrow labeled F_{12} points rightward from q_1 towards q_2, representing the force on q_2.
– The forces are indicated by red arrows and positioned parallel to each other, just as in diagram (a).

Example 15.1

How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by $0.530 \times 10^{-10}\text{m}$ with the gravitational force between them. This distance is their average separation in a hydrogen atom.

Strategy

To compare the two forces, we first compute the electrostatic force using Coulomb's law, $F = k\frac{|q_1 q_2|}{r^2}$. We then calculate the gravitational force using Newton's universal law of gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

Solution

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

$$F = k\frac{|q_1 q_2|}{r^2}$$

15.5

$$= \left(8.99 \times 10^{9}\text{N} \cdot \text{m}^2/\text{C}^2\right) \times \frac{\left(1.60 \times 10^{-19}\text{C}\right)\left(1.60 \times 10^{-19}\text{C}\right)}{\left(0.530 \times 10^{-10}\text{m}\right)^2}$$

15.6

Thus the Coulomb force is

$$F = 8.19 \times 10^{-8}\text{N}.$$

15.7

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an acceleration of $8.99 \times 10^{22} \mathrm{m/s^2}$ (verification is left as an end-of-section problem).The gravitational force is given by Newton's law of gravitation as:

$$F_G = G\frac{mM}{r^2},$$

15.8

where $G = 6.67 \times 10^{-11} \mathrm{N \cdot m^2/kg^2}$. Here m and M represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

$$\begin{aligned} F_G &= (6.67 \times 10^{-11} \mathrm{N \cdot m^2/kg^2}) \\ &\quad \times \frac{(9.11 \times 10^{-31} \mathrm{kg})(1.67 \times 10^{-27} \mathrm{kg})}{(0.530 \times 10^{-10} \mathrm{m})^2} \\ &= 3.61 \times 10^{-47} \mathrm{N} \end{aligned}$$

15.9

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

$$\frac{F}{F_G} = 2.27 \times 10^{39}.$$

15.10

Discussion

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive Coulomb forces nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

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15.4 ELECTRIC FIELD: CONCEPT OF A FIELD REVISITED

Learning Objectives

By the end of this section, you will be able to:

- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force (F) on a test charge and electrical field strength (E).

Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the Coulomb force. Action at a distance is a force between objects that are not close enough for their atoms to "touch." That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a force field. The force field carries the force to another object (called a test object) some distance away.

Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb's law, $F = k|q_1 q_2|/r^2$, its magnitude is given by the equation

$F = k|qQ|/r^2$, for a point charge (a particle having a charge Q) acting on a test charge q at a distance r (see Figure 15.18). Both the magnitude and direction of the Coulomb force field depend on Q and the test charge q.

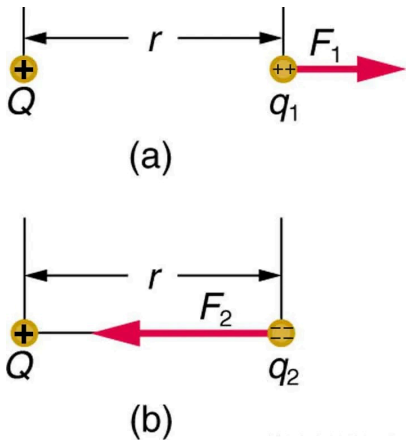


Figure 15.18 The Coulomb force field due to a positive charge Q is shown acting on two different charges. Both charges are the same distance from Q. (a) Since q_1 is positive, the force F_1 acting on it is repulsive. (b) The charge q_2 is negative and greater in magnitude than q_1, and so the force F_2 acting on it is attractive and stronger than F_1. The Coulomb force field is thus not unique at any point in space, because it depends on the test charges q_1 and q_2 as well as the charge Q. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two diagrams illustrating the interaction between electric charges.

(a) The first diagram shows two positive charges represented by circles with plus signs. The charge on the left is labeled as "Q," and the one on the right is labeled as "q_1." An arrow pointing to the right indicates the force "F_1" acting on "q_1." Both charges are separated by a distance labeled "r."

(b) The second diagram depicts a positive charge "Q" on the left and a negative charge "q_2" on the right, represented by a circle with a negative sign. An arrow pointing to the left indicates the force "F_2" acting on "q_2." Both charges are separated by a distance labeled "r."

Both diagrams demonstrate the direction of force between the charges based on their polarity, with like charges repelling and opposite charges attracting.

To simplify things, we would prefer to have a field that depends only on Q and not on the test charge q. The

electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field E is defined to be the ratio of the Coulomb force to the test charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$

15.11

where $\mathbf{F}$ is the electrostatic force (or Coulomb force) exerted on a positive test charge q. It is understood that $\mathbf{E}$ is in the same direction as $\mathbf{F}$. It is also assumed that q is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge q is simply obtained by multiplying charge times electric field, or $\mathbf{F} = q\mathbf{E}$. Consider the electric field due to a point charge Q. According to Coulomb's law, the force it exerts on a test charge q is $F = k|qQ|/r^2$. Thus the magnitude of the electric field, E, for a point charge is

$$E = \left|\frac{F}{q}\right| = k\left|\frac{\mathrm{qQ}}{qr^2}\right| = k\frac{|Q|}{r^2}.$$

15.12

Since the test charge cancels, we see that

$$E = k\frac{|Q|}{r^2}.$$

15.13

The electric field is thus seen to depend only on the charge Q and the distance r; it is completely independent of the test charge q.

Example 15.2

Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

Strategy

We can find the electric field created by a point charge by using the equation $E = \mathrm{kQ}/r^2$.

Solution

Here $Q = 2.00 \times 10^{-9}$ C and $r = 5.00 \times 10^{-3}$ m. Entering those values into the above equation gives

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= (8.99 \times 10^{9}\mathrm{N \cdot m^2/C^2}) \times \frac{(2.00 \times 10^{-9}\mathrm{C})}{(5.00 \times 10^{-3}\mathrm{m})^2} \\ &= 7.19 \times 10^{5}\mathrm{N/C}. \end{aligned}$$

15.14

Discussion

This electric field strength is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q.

Example 15.3

Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of $-0.250\mu\text{C}$?

Strategy

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field $\mathbf{E} = \mathbf{F}/q$ rearranged to $\mathbf{F} = q\mathbf{E}$.

Solution

The magnitude of the force on a charge $q = -0.250\mu\text{C}$ exerted by a field of strength $E = 7.20 \times 10^5$ N/C is thus,

$$\begin{aligned} F &= -\text{qE} \\ &= (0.250 \times 10^{-6}\text{C})\,(7.20 \times 10^5\text{N/C}) \\ &= 0.180\ \text{N}. \end{aligned}$$

15.15

Because q is negative, the force is directed opposite to the direction of the field.

Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

PhET Explorations

Electric Field of Dreams

Play ball! Add charges to the Field of Dreams and see how they react to the electric field. Turn on a background electric field and adjust the direction and magnitude.

Click to view content.

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15.5 ELECTRIC FIELD LINES: MULTIPLE CHARGES

Learning Objectives

By the end of this section, you will be able to:

- Calculate the total force (magnitude and direction) exerted on a test charge from more than one charge
- Describe an electric field diagram of a positive point charge; of a negative point charge with twice the magnitude of positive charge
- Draw the electric field lines between two points of the same charge; between two points of opposite charge.

Drawings using lines to represent electric fields around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all vectors, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

Figure 15.19 shows two pictorial representations of the same electric field created by a positive point charge Q. Figure 15.19 (b) shows the standard representation using continuous lines. Figure 15.19 (a) shows numerous individual arrows with each arrow representing the force on a test charge q. Field lines are essentially a map of infinitesimal force vectors.

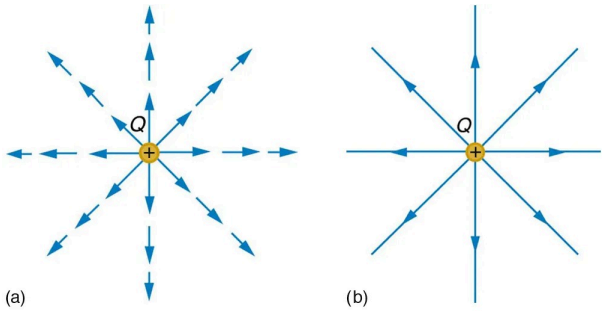


Figure 15.19 Two equivalent representations of the electric field due to a positive charge Q. (a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point as the electric field. The closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two separate diagrams labeled (a) and (b).

Diagram (a):

- A central yellow circle marked with a plus sign (+) representing a positive charge (Q).
- Several blue arrows radiate outward from the central circle evenly in all directions, indicating the electric field lines due to a positive charge.

Diagram (b):

- A similar central yellow circle marked with a plus sign (+) representing a positive charge (Q).
- Several blue arrows point inward towards the central circle from all directions, indicating the electric field lines due to a negative charge.

Note that the electric field is defined for a positive test charge q, so that the field lines point away from a positive charge and toward a negative charge. (See Figure 15.20.) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is $E = k|Q|/r^2$ and area is proportional to r^2. This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.

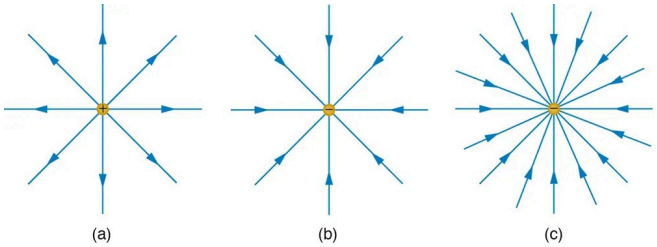


Figure 15.20 The electric field surrounding three different point charges. (a) A positive charge. (b) A negative charge of equal magnitude. (c) A larger negative charge. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams labeled (a), (b), and (c), each depicting electric field lines around a charge.

(a) The first diagram shows eight blue arrows radiating outward from a central yellow circle labeled with a plus sign, indicating a positive charge. The arrows are evenly distributed around the circle.

(b) The second diagram also has eight blue arrows, but they are pointing inward toward a central yellow circle labeled with a minus sign, indicating a negative charge. The arrows are evenly distributed around the circle.

(c) The third diagram shows sixteen blue arrows, all pointing inward toward a central yellow circle labeled with a minus sign. The arrows are more densely packed than in the second diagram, indicating a stronger negative charge.

In many situations, there are multiple charges. The total electric field created by multiple charges is the vector sum of the individual fields created by each charge. The following example shows how to add electric field vectors.

Example 15.4

Adding Electric Fields

Find the magnitude and direction of the total electric field due to the two point charges, q_1 and q_2, at the origin of the coordinate system as shown in Figure 15.21.

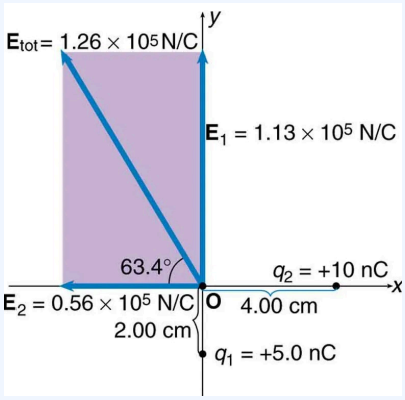


Figure 15.21 The electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ at the origin O add to $\mathbf{E}_{\text{tot}}$. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of electric fields and charges. It shows a coordinate plane with an x-axis and y-axis. On the x-axis, there are two charges labeled q_1 = *+5.0 nC* located at the origin (point O) and q_2 = *+10 nC* located 4.00 cm to the right. The distance between charge q_1 and the reference point on the x-axis is 2.00 cm.

Three vectors are shown: E_1, E_2, and E_{tot}. The vector E_1 = *1.13 × 10^5 N/C* points upward along the y-axis, starting from the origin. The vector E_2 = *0.56 × 10^5 N/C* points left along the negative x-axis, also starting from the origin. The resultant vector E_{tot} = *1.26 × 10^5 N/C* points diagonally upwards and to the left, forming an angle of 63.4 degrees from the horizontal leftward direction. The shaded area between E_1 and E_{tot} forms a right triangle.

Strategy

Since the electric field is a vector (having magnitude and direction), we add electric fields with the same vector techniques used for other types of vectors. We first must find the electric field due to each charge at the point of interest, which is the origin of the coordinate system (O) in this instance. We pretend that there is a positive test charge, q, at point O, which allows us to determine the direction of the fields $\mathbf{E}_1$ and $\mathbf{E}_2$. Once those fields are found, the total field can be determined using vector addition.

Solution

The electric field strength at the origin due to q_1 is labeled E_1 and is calculated:

$$\begin{aligned} E_1 &= k\frac{q_1}{r_1^2} \\ &= \left(8.99 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2\right)\frac{\left(5.00 \times 10^{-9}\text{C}\right)}{\left(2.00 \times 10^{-2}\text{m}\right)^2} \\ &= 1.124 \times 10^5 \text{N/C}. \end{aligned}$$

15.16

Similarly, E_2 is

$$\begin{aligned} E_2 &= k\frac{q_2}{r_2^2} \\ &= \left(8.99 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2\right)\frac{\left(10.0 \times 10^{-9}\text{C}\right)}{\left(4.00 \times 10^{-2}\text{m}\right)^2} \\ &= 0.5619 \times 10^5 \text{N/C}. \end{aligned}$$

15.17

Four digits have been retained in this solution to illustrate that E_1 is exactly twice the magnitude of E_2. Now arrows are drawn to represent the magnitudes and directions of $\mathbf{E}_1$ and $\mathbf{E}_2$. (See Figure 15.21.) The direction of the electric field is that of the force on a positive charge so both arrows point directly away from the positive charges that create them. The arrow for $\mathbf{E}_1$ is exactly twice the length of that for $\mathbf{E}_2$

. The arrows form a right triangle in this case and can be added using the Pythagorean theorem. The magnitude of the total field E_{tot} is

{\small\begin{eqnarray*}E_{\text{tot}} &=& \left(\right. E_{1}^{2} + E_{2}^{2} \left.\right)^{\text{1}/\text{2}} \\ &=& {\scriptsize\left(\left\{\right. \left(\right. \text{1}.\text{124} \times \text{10}^{5} \text{N}/\text{C} \left.\right)^{2} + \left(\right. \text{0}.\text{5619} \times \text{10}^{5} \text{N}/\text{C} \left.\right)^{2} \left.\right\}\right)^{\text{1}/\text{2}}} \\ &=& \text{1}.\text{26} \times \text{10}^{5} \text{N}/\text{C}.\end{eqnarray*}

15.18
The direction is

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{E_1}{E_2}\right) \\ &= \tan^{-1}\left(\frac{1.124 \times 10^5 \text{N/C}}{0.5619 \times 10^5 \text{N/C}}\right) \\ &= 63.4^\circ\ ,\end{aligned}$$

15.19
or 63.4° above the *x*-axis.

Discussion

In cases where the electric field vectors to be added are not perpendicular, vector components or graphical techniques can be used. The total electric field found in this example is the total electric field at only one point in space. To find the total electric field due to these two charges over an entire region, the same technique must be repeated for each point in the region. This impossibly lengthy task (there are an infinite number of points in space) can be avoided by calculating the total field at representative points and using some of the unifying features noted next.

Figure 15.22 shows how the electric field from two point charges can be drawn by finding the total field at representative points and drawing electric field lines consistent with those points. While the electric fields from multiple charges are more complex than those of single charges, some simple features are easily noticed.

For example, the field is weaker between like charges, as shown by the lines being farther apart in that region. (This is because the fields from each charge exert opposing forces on any charge placed between them.) (See Figure 15.22 and Figure 15.23(a).) Furthermore, at a great distance from two like charges, the field becomes identical to the field from a single, larger charge.

Figure 15.23(b) shows the electric field of two unlike charges. The field is stronger between the charges. In that region, the fields from each charge are in the same direction, and so their strengths add. The field of two unlike charges is weak at large distances, because the fields of the individual charges are in opposite directions and so their strengths subtract. At very large distances, the field of two unlike charges looks like that of a smaller single charge.

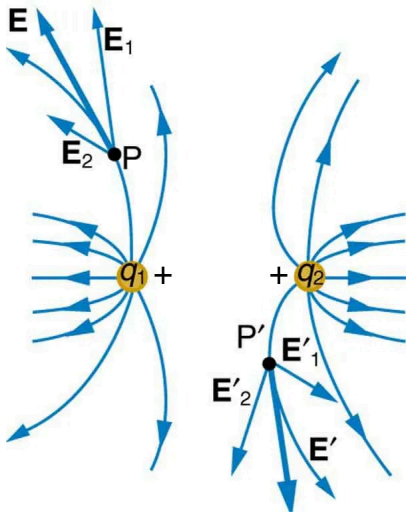


Figure 15.22 Two positive point charges q_1 and q_2 produce the resultant electric field shown. The field is calculated at representative points and then smooth field lines drawn following the rules outlined in the text. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image depicts two electric field diagrams for positive point charges.

– On the left side, there is a positive charge labeled “q1.” Blue arrows emanate outward from q1, representing the electric field lines. Two lines converge at a point labeled “P,” where two electric field vectors, “E1” and “E2,” are shown. These represent the components of the electric field at point P. “E” is the resultant electric field at that point.

– On the right side, there is another positive charge labeled “q2.” Similarly, blue arrows emanate outward from q2. They converge at a point labeled “P’,” where vectors “E’1” and “E’2” depict the components of the electric field. The resultant electric field at this point is denoted as “E’.”

Both diagrams visually illustrate how electric fields are vectors that have both magnitude and direction, originating from positive charges and demonstrating how fields interact at certain points.

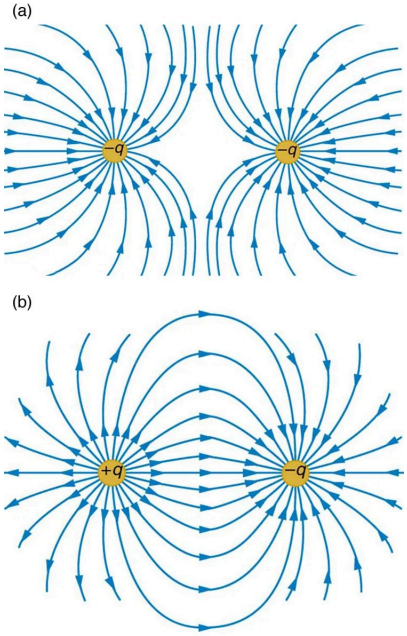


Figure 15.23 (a) Two negative charges produce the fields shown. It is very similar to the field produced by two positive charges, except that the directions are reversed. The field is clearly weaker between the charges. The individual forces on a test charge in that region are in opposite directions. (b) Two opposite charges produce the field shown, which is stronger in the region between the charges. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b), illustrating electric field lines associated with charges.

Diagram (a):

This shows two identical negative charges, both labeled *-q*, depicted as small yellow circles. Blue arrows represent the electric field lines, which radiate outwards from each negative charge. The field lines are dispersed, pointing outward in all directions from each charge. There is no interaction or overlap between the field lines of the two charges.

Diagram (b):

This shows a positive charge labeled *+q* on the left and a negative charge labeled *-q* on the right, also depicted as yellow circles. Blue arrows represent the electric field lines, which begin at the positive charge and end at the

negative charge, illustrating attraction. The field lines are symmetrical around the charges and curve between them, showing a clear interaction as they connect the positive and negative charges.

We use electric field lines to visualize and analyze electric fields (the lines are a pictorial tool, not a physical entity in themselves). The properties of electric field lines for any charge distribution can be summarized as follows:

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
3. The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is tangent to the field line at any point in space.
5. Field lines can never cross.

The last property means that the field is unique at any point. The field line represents the direction of the field; so if they crossed, the field would have two directions at that location (an impossibility if the field is unique).

PhET Explorations

Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

Access multimedia content

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15.6 ELECTRIC FORCES IN BIOLOGY

Learning Objectives

By the end of this section, you will be able to:

- Describe how a water molecule is polar.
- Explain electrostatic screening by a water molecule within a living cell.

Classical electrostatics has an important role to play in modern molecular biology. Large molecules such as proteins, nucleic acids, and so on—so important to life—are usually electrically charged. DNA itself is highly charged; it is the electrostatic force that not only holds the molecule together but gives the molecule structure and strength. Figure 15.24 is a schematic of the DNA double helix.

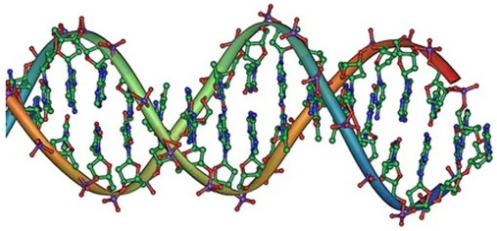


Figure 15.24 DNA is a highly charged molecule. The DNA double helix shows the two coiled strands each containing a row of nitrogenous bases, which "code" the genetic information needed by a living organism. The strands are connected by bonds between pairs of bases. While pairing combinations between certain bases are fixed (C-G and A-T), the sequence of nucleotides in the strand varies. Image from OpenStax College Physics 2e, CC-BY 4.0

The four nucleotide bases are given the symbols A (adenine), C (cytosine), G (guanine), and T (thymine). The order of the four bases varies in each strand, but the pairing between bases is always the same. C and G

are always paired and A and T are always paired, which helps to preserve the order of bases in cell division (mitosis) so as to pass on the correct genetic information. Since the Coulomb force drops with distance ($F \propto 1/r^2$), the distances between the base pairs must be small enough that the electrostatic force is sufficient to hold them together.

DNA is a highly charged molecule, with about $2q_e$ (fundamental charge) per 0.3×10^{-9} m. The distance separating the two strands that make up the DNA structure is about 1 nm, while the distance separating the individual atoms within each base is about 0.3 nm.

One might wonder why electrostatic forces do not play a larger role in biology than they do if we have so many charged molecules. The reason is that the electrostatic force is "diluted" due to screening between molecules. This is due to the presence of other charges in the cell.

Polarity of Water Molecules

The best example of this charge screening is the water molecule, represented as H_2O. Water is a strongly polar molecule. Its 10 electrons (8 from the oxygen atom and 2 from the two hydrogen atoms) tend to remain closer to the oxygen nucleus than the hydrogen nuclei. This creates two centers of equal and opposite charges—what is called a dipole, as illustrated in Figure 15.25. The magnitude of the dipole is called the dipole moment.

These two centers of charge will terminate some of the electric field lines coming from a free charge, as on a DNA molecule. This results in a reduction in the strength of the Coulomb interaction. One might say that screening makes the Coulomb force a short range force rather than long range.

Cell Membranes

Other ions of importance in biology that can reduce or screen Coulomb interactions are Na^+, and K^+, and Cl^-. These ions are located both inside and outside of living cells. The movement of these ions through cell membranes is crucial to the motion of nerve impulses through nerve axons.

Recent studies of electrostatics in biology seem to show that electric fields in cells can be extended over larger distances, in spite of screening, by "microtubules" within the cell. These microtubules are hollow tubes composed of proteins that guide the movement of chromosomes when cells divide, the motion of other organisms within the cell, and provide mechanisms for motion of some cells (as motors).

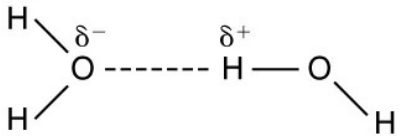


Figure 15.25 This schematic shows water (H_2O) as a polar molecule. Unequal sharing of electrons between the oxygen (O) and hydrogen (H) atoms leads to a net separation of positive and negative charge—forming a dipole. The symbols δ^- and δ^+ indicate that the oxygen side of the H_2O molecule tends to be more negative, while the hydrogen ends tend to be more positive. This leads to an attraction of opposite charges between molecules. Image from OpenStax College Physics 2e, CC-BY 4.0

You are likely familiar with the role of electrical signals in nerve conduction and the importance of charges in cardiac and related activity. Changes in electrical properties are also essential in core biological processes. Ernest Everett Just, whose expertise in understanding and handling egg cells led to a number of critical experimental discoveries, investigated the role of the cell membrane in reproductive fertilization. In one key experiment, Just established that the egg membrane undergoes a depolarizing "wave of negativity" the moment it fuses with a sperm cell. This change in charge is now known as the "fast block" that ensures that only one sperm cell fuses with an egg cell and is critical for embryonic development.

Bioelectricity and Wound Healing

Just as electrical forces drive activities in healthy cells and systems, they are also critical in damaged ones. Scientists have long known that injuries or infections are managed by the body through various responses, including increased white blood cell concentrations, swelling, and tissue repair. For example, human cells damaged by wounds heal through a complex process. But what triggers it?

Physicists and biologists working together at Vanderbilt University used an ultra-precise laser to uncover the processes organisms use to repair damage. Lead researchers Andrea Page-McCaw and Shane Hutson and study author Erica Shannon discovered that immediately upon damage, cells release calcium ions and eventually other molecules, driving an electrochemical response that initiates the healing process. Shannon notes that different types of damage lead to different chemical releases, demonstrating how organisms may initiate specific responses to best address the injury.

While far more research is required to understand the triggering and response method, other research indicates that bioelectricity is highly involved in wound healing. Several studies have indicated that precise and low-level electrical stimulation of wounds (such as those from surgeries) leads to faster healing. While the mechanisms are not fully understood, electrical stimulation is a growing area of research and practice in medicine.

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15.7 CONDUCTORS AND ELECTRIC FIELDS IN STATIC EQUILIBRIUM

Learning Objectives

By the end of this section, you will be able to:

- List the three properties of a conductor in electrostatic equilibrium.
- Explain the effect of an electric field on free charges in a conductor.
- Explain why no electric field may exist inside a conductor.
- Describe the electric field surrounding Earth.
- Explain what happens to an electric field applied to an irregular conductor.
- Describe how a lightning rod works.
- Explain how a metal car may protect passengers inside from the dangerous electric fields caused by a downed line touching the car.

Conductors contain free charges that move easily. When excess charge is placed on a conductor or the conductor is put into a static electric field, charges in the conductor quickly respond to reach a steady state called electrostatic equilibrium.

Figure 15.26 shows the effect of an electric field on free charges in a conductor. The free charges move until the field is perpendicular to the conductor's surface. There can be no component of the field parallel to the surface in electrostatic equilibrium, since, if there were, it would produce further movement of charge. A positive free charge is shown, but free charges can be either positive or negative and are, in fact, negative in metals. The motion of a positive charge is equivalent to the motion of a negative charge in the opposite direction.

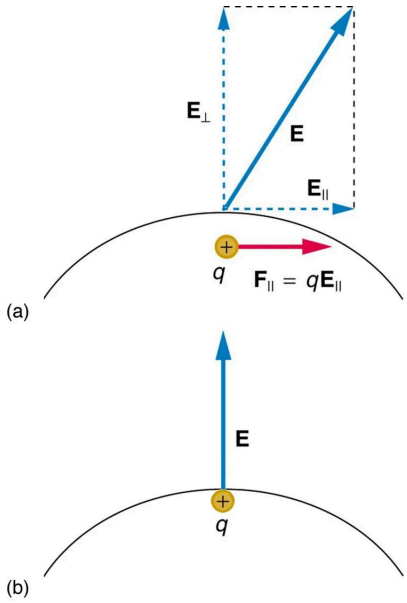


Figure 15.26 When an electric field $\mathbf{E}$ *is applied to a conductor, free charges inside the conductor move until the field is perpendicular to the surface. (a) The electric field is a vector quantity, with both parallel and perpendicular components. The parallel component (*$\mathbf{E}_{||}$*) exerts a force (*$\mathbf{F}_{||}$*) on the free charge* q*, which moves the charge until* $\mathbf{F}_{||} = 0$*. (b) The resulting field is perpendicular to the surface. The free charge has been brought to the conductor's surface, leaving electrostatic forces in equilibrium. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two diagrams labeled (a) and (b), illustrating the interactions between an electric field and a charge on a curved surface.

(a) The first diagram shows a curved horizontal surface with a positive charge, labeled q, sitting on it. A blue vector labeled E extends upwards and to the right, originating from the charge. This vector represents the electric field. The vector is decomposed into two components: $E_{\perp}$, a vertical blue dashed vector extending directly upwards, and $E_{\parallel}$, a horizontal blue dashed vector extending to the right. There is also a red vector labeled $F_{\parallel} = qE_{\parallel}$ pointing horizontally to the right from the charge, representing the force parallel to the surface.

(b) The second diagram also shows the curved horizontal surface with a positive charge, labeled q, on it. A blue vector labeled E extends directly upwards from the charge, representing the electric field. Unlike the first diagram, there is no decomposition of the vector or additional forces shown.

A conductor placed in an electric field will be polarized. Figure 15.27 shows the result of placing a neutral

conductor in an originally uniform electric field. The field becomes stronger near the conductor but entirely disappears inside it.

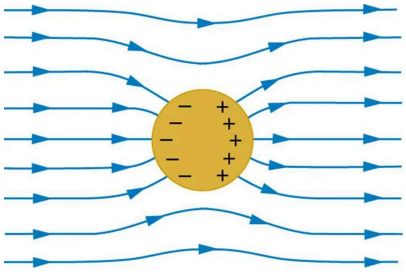


Figure 15.27 This illustration shows a spherical conductor in static equilibrium with an originally uniform electric field. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin again on excess positive charge on the opposite side. No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any field until it was neutralized. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains a diagram illustrating the flow of lines around a circular object. The object is represented as a yellow circle in the center of the image. Inside the circle, there are symbols depicting charges: three negative signs (-) on the left side and three positive signs (+) on the right side.

Blue arrows, representing lines of flow or force, move horizontally from the left side of the image to the right. As they encounter the yellow circle, the arrows bend around it, indicating that the flow is being influenced by the object. The bending of the arrows signifies that the flow is being deflected and redirected by the forces within the circle.

Misconception Alert: Electric Field inside a Conductor

Excess charges placed on a spherical conductor repel and move until they are evenly distributed, as shown in Figure 15.28. Excess charge is forced to the surface until the field inside the conductor is zero. Outside the conductor, the field is exactly the same as if the conductor were replaced by a point charge at its center equal to the excess charge.

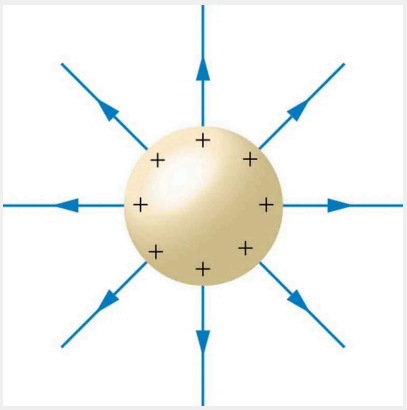


Figure 15.28 The mutual repulsion of excess positive charges on a spherical conductor distributes them uniformly on its surface. The resulting electric field is perpendicular to the surface and zero inside. Outside the conductor, the field is identical to that of a point charge at the center equal to the excess charge. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a yellow sphere with multiple plus signs scattered across its surface, indicating a positive charge. Blue arrows, evenly spaced around the sphere, point outward from the sphere. These arrows represent electric field lines emanating from the positively charged sphere, illustrating the direction of the electric field.

Properties of a Conductor in Electrostatic Equilibrium

1. The electric field is zero inside a conductor.
2. Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
3. Any excess charge resides entirely on the surface or surfaces of a conductor.

The properties of a conductor are consistent with the situations already discussed and can be used to analyze any conductor in electrostatic equilibrium. This can lead to some interesting new insights, such as described below.

How can a very uniform electric field be created? Consider a system of two metal plates with opposite charges on them, as shown in Figure 15.29. The properties of conductors in electrostatic equilibrium indicate that the electric field between the plates will be uniform in strength and direction. Except near the edges, the excess charges distribute themselves uniformly, producing field lines that are uniformly spaced (hence uniform in strength) and perpendicular to the surfaces (hence uniform in direction, since the plates are flat). The edge effects are less important when the plates are close together.

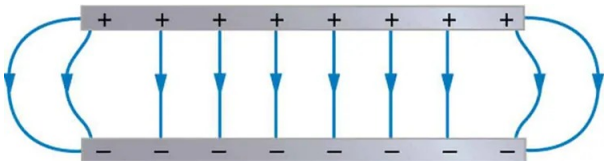


Figure 15.29 Two metal plates with equal, but opposite, excess charges. The field between them is uniform in strength and direction except near the edges. One use of such a field is to produce uniform acceleration of charges between the plates, such as in the electron gun of a TV tube. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts an electric field between two parallel plates.

– The top plate is labeled with several positive "+" signs, indicating it is positively charged.
– The bottom plate is labeled with several negative "-" signs, indicating it is negatively charged.

– Blue arrows originate from the positive plate and point directly downward to the negative plate, illustrating the direction of the electric field lines between the plates.
– There are additional blue arrows arching from the edges of the positive plate to the edges of the negative plate, showing the fringe effect of the electric field outside the main area between the plates.

The image visually represents the concept of uniform electric fields between parallel plate capacitors and the fringing of field lines at the edges.

Earth's Electric Field

A near uniform electric field of approximately 150 N/C, directed downward, surrounds Earth, with the magnitude increasing slightly as we get closer to the surface. What causes the electric field? At around 100 km above the surface of Earth we have a layer of charged particles, called the ionosphere. The ionosphere is responsible for a range of phenomena including the electric field surrounding Earth. In fair weather the ionosphere is positive and the Earth largely negative, maintaining the electric field (Figure 15.30(a)).

In storm conditions clouds form and localized electric fields can be larger and reversed in direction (Figure 15.30(b)). The exact charge distributions depend on the local conditions, and variations of Figure 15.30(b) are possible.

If the electric field is sufficiently large, the insulating properties of the surrounding material break down and it becomes conducting. For air this occurs at around 3×10^6 N/C. Air ionizes ions and electrons recombine, and we get discharge in the form of lightning sparks and corona discharge.



Figure 15.30 Earth's electric field. (a) Fair weather field. Earth and the ionosphere (a layer of charged particles) are both conductors. They produce a uniform electric field of about 150 N/C. (credit: D. H. Parks) (b) Storm fields. In the presence of storm clouds, the local electric fields can be larger. At very high fields, the insulating properties of the air break down and lightning can occur. Image from OpenStax College Physics 2e, CC-BY 4.0

Electric Fields on Uneven Surfaces

So far we have considered excess charges on a smooth, symmetrical conductor surface. What happens if a conductor has sharp corners or is pointed? Excess charges on a nonuniform conductor become concentrated at the sharpest points. Additionally, excess charge may move on or off the conductor at the sharpest points.

To see how and why this happens, consider the charged conductor in Figure 15.31. The electrostatic repulsion of like charges is most effective in moving them apart on the flattest surface, and so they become least concentrated there. This is because the forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surfaces are different. The component parallel to the surface is greatest on the flattest surface and, hence, more effective in moving the charge.

The same effect is produced on a conductor by an externally applied electric field, as seen in Figure 15.31 (c). Since the field lines must be perpendicular to the surface, more of them are concentrated on the most curved parts.

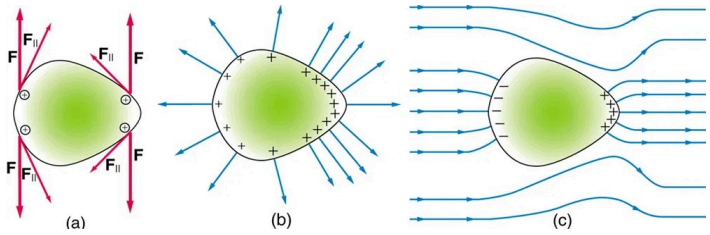


Figure 15.31 Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature. (a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is $\mathbf{F}_{||}$ *that moves the charges apart once they have reached the surface. (b)* $\mathbf{F}_{||}$ *is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c) An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of three diagrams labeled as (a), (b), and (c), each illustrating different physical concepts using arrows and a central green shape.

Diagram (a): This diagram shows a green irregular shape in the center with four small plus signs on its surface. Several red arrows labeled “F” point upwards along the sides of the shape, while “F‖” arrows point inwards towards the shape. The arrows illustrate forces acting on the shape.

Diagram (b): This diagram has the same central green shape, but is surrounded by several blue arrows pointing outwards, indicating expansion or radiation from the shape. The surface of the shape is dotted with plus signs.

Diagram (c): This diagram also contains the central green shape. Blue arrows flow in parallel lines from left to right, going around the shape. These arrows depict a stream or flow of particles moving past the shape, with some curving slightly as they pass.

Applications of Conductors

On a very sharply curved surface, such as shown in Figure 15.32, the charges are so concentrated at the point that the resulting electric field can be great enough to remove them from the surface. This can be useful.

Lightning rods work best when they are most pointed. The large charges created in storm clouds induce an opposite charge on a building that can result in a lightning bolt hitting the building. The induced charge is bled away continually by a lightning rod, preventing the more dramatic lightning strike.

Of course, we sometimes wish to prevent the transfer of charge rather than to facilitate it. In that case, the conductor should be very smooth and have as large a radius of curvature as possible. (See Figure 15.33.) Smooth surfaces are used on high-voltage transmission lines, for example, to avoid leakage of charge into the air.

Another device that makes use of some of these principles is a Faraday cage. This is a metal shield that encloses a volume. All electrical charges will reside on the outside surface of this shield, and there will be no electrical field inside. A Faraday cage is used to prohibit stray electrical fields in the environment from interfering with sensitive measurements, such as the electrical signals inside a nerve cell.

During electrical storms if you are driving a car, it is best to stay inside the car as its metal body acts as a Faraday cage with zero electrical field inside. If in the vicinity of a lightning strike, its effect is felt on the outside of the car and the inside is unaffected, provided you remain totally inside. This is also true if an active (“hot”) electrical wire was broken (in a storm or an accident) and fell on your car.

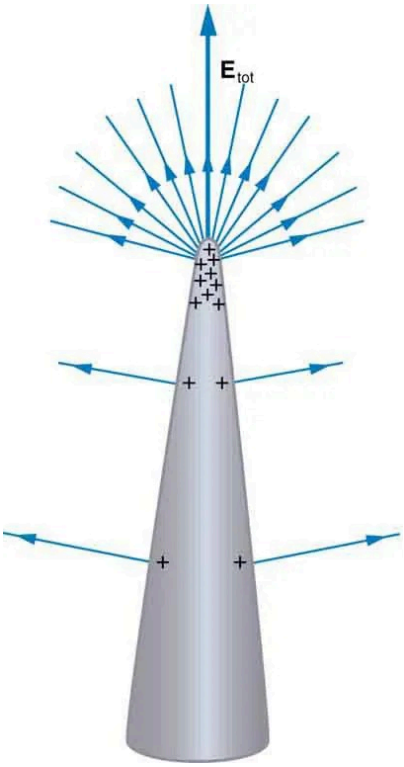


Figure 15.32 A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor. Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a cone with a metallic appearance, vertically oriented with its pointed end upwards. At the tip, there are multiple small plus signs indicating positive charges. Blue arrows, which represent electric field lines, emanate outward from the tip of the cone. These lines point upward, to the sides, and in all directions away from the tip, illustrating the electric field distribution. The label "E_{tot}" is placed near the top of the cone, indicating the total electric field direction predominantly moving upward. The illustration aims to show how the electric field lines spread out from a charged pointed conductor.



Figure 15.33 (a) A lightning rod is pointed to facilitate the transfer of charge. (credit: Romaine, Wikimedia Commons) (b) This Van de Graaff generator has a smooth surface with a large radius of curvature to prevent the transfer of charge and allow a large voltage to be generated. The mutual repulsion of like charges is evident in the person's hair while touching the metal sphere. Image from OpenStax College Physics 2e, CC-BY 4.0

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15.8 APPLICATIONS OF ELECTROSTATICS

Learning Objectives

By the end of this section, you will be able to:

The study of electrostatics has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

- Name several real-world applications of the study of electrostatics.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. Figure 15.34 shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.

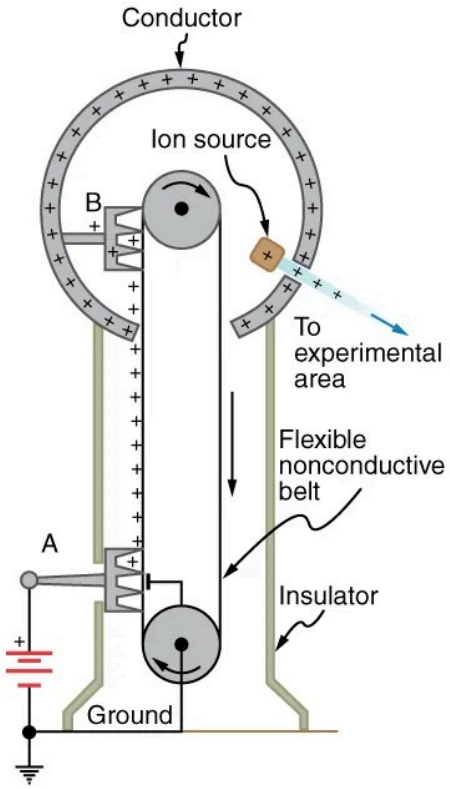


Figure 15.34 Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image is a diagram of a particle accelerator mechanism. It consists of the following components:

- **Conductor:** A circular section located at the top, labeled with plus signs indicating positive charge.
- **Ion Source:** Positioned inside the circular conductor, it releases particles towards an "experimental area" indicated by an arrow.
- **Flexible Nonconductive Belt:** Runs vertically through the center, connecting the top and bottom parts of the apparatus.
- **Insulator:** Located on either side of the belt, providing isolation.
- **Components A and B:** Placed along the belt; A is connected to a power supply and ground, while B is situated within the circular conductor.
- **Ground:** A grounding symbol is present at the bottom, connected to component A.

Arrows depict the movement direction of the belt and the path of emitted ions towards the experimental area.

Take-Home Experiment: Electrostatics and Humidity

Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

Xerography

Most copy machines use an electrostatic process called xerography—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in Figure 15.35.

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a photoconductor. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is grounded so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner within the fibers of the paper.

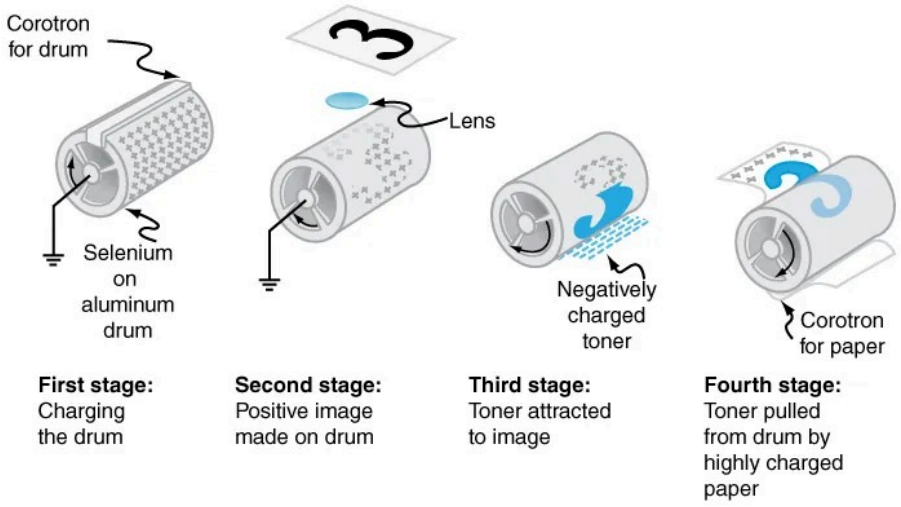


Figure 15.35 Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates the stages of a photocopying process involving an aluminum drum with a selenium coating. There are four main stages described visually:

First stage: The image shows a cylinder labeled as a "Corotron for drum" above it. This is used to charge the aluminum drum with selenium coating.

Second stage: A light source, identified as a "Lens," projects a positive image onto the drum surface, which is now electrostatically charged.

Third stage: Negatively charged toner particles are depicted moving towards and adhering to the positively charged image areas on the drum.

Fourth stage: The toner image transfers from the drum to a sheet of paper. The paper is charged by a "Corotron for paper," attracting the toner to the paper surface.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in Figure 15.36. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with

which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and may contain a computer more powerful than the one giving them the raw data to be printed.

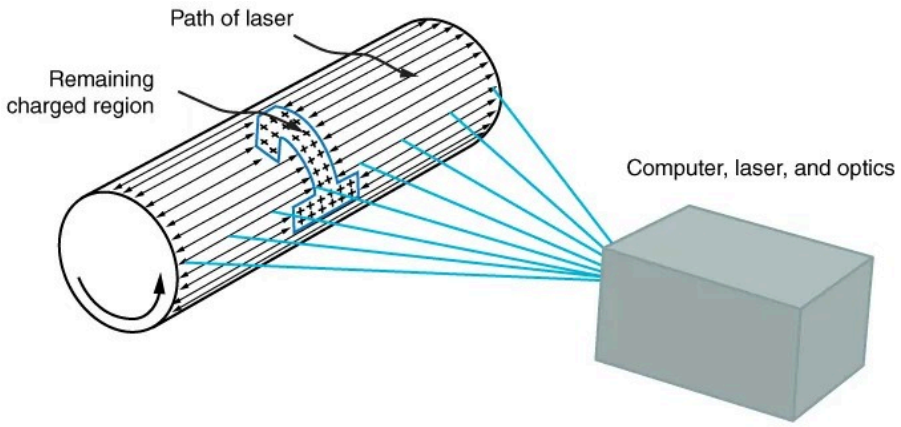


Figure 15.36 In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positive charge image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a schematic diagram of a laser printing process. It shows a cylindrical drum with lines representing its rotation and sections labeled “Path of laser” and “Remaining charged region.” The drum has an arrow indicating its rotational direction. A series of blue lines are depicted moving from the drum toward a rectangular block labeled “Computer, laser, and optics.” This suggests the interaction between the laser, represented by the blue lines, and the drum in the printing process.

Ink Jet Printers and Electrostatic Painting

The ink jet printer, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge. (See Figure 15.37.)

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)

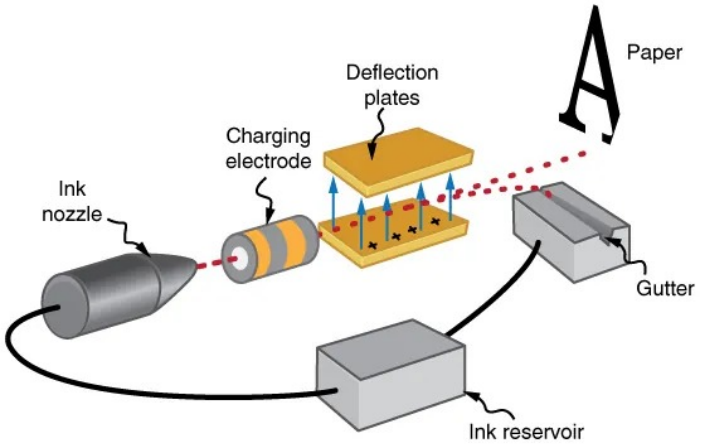


Figure 15.37 The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the working mechanism of a continuous inkjet printer. The flow of ink begins from an "Ink reservoir" marked as a rectangular box at the bottom of the diagram. A line connects it to an "Ink nozzle" which appears as a cylindrical shape on the left. From the nozzle, red dotted lines represent the stream of ink droplets moving towards the right.

Next, the droplets pass through a "Charging electrode," designed as a cylindrical component encircling the ink stream, indicating that the droplets are being electrically charged. Following this, the droplets are directed through "Deflection plates," represented as two horizontal plates with arrows indicating electromagnetic forces that guide the direction of the charged droplets.

From the deflection plates, the ink droplets continue forwards, with some being deflected downwards towards a "Gutter," depicted as a box that recycles unused ink back to the reservoir. The remaining droplets continue onto "Paper," printing the letter "A" as demonstrated on the top right of the image.

Electrostatic painting employs electrostatic charge to spray paint onto odd-shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach those hard-to-get at places, applying an even coat in a controlled manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles. (See Figure 15.38.)

Large electrostatic precipitators are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.

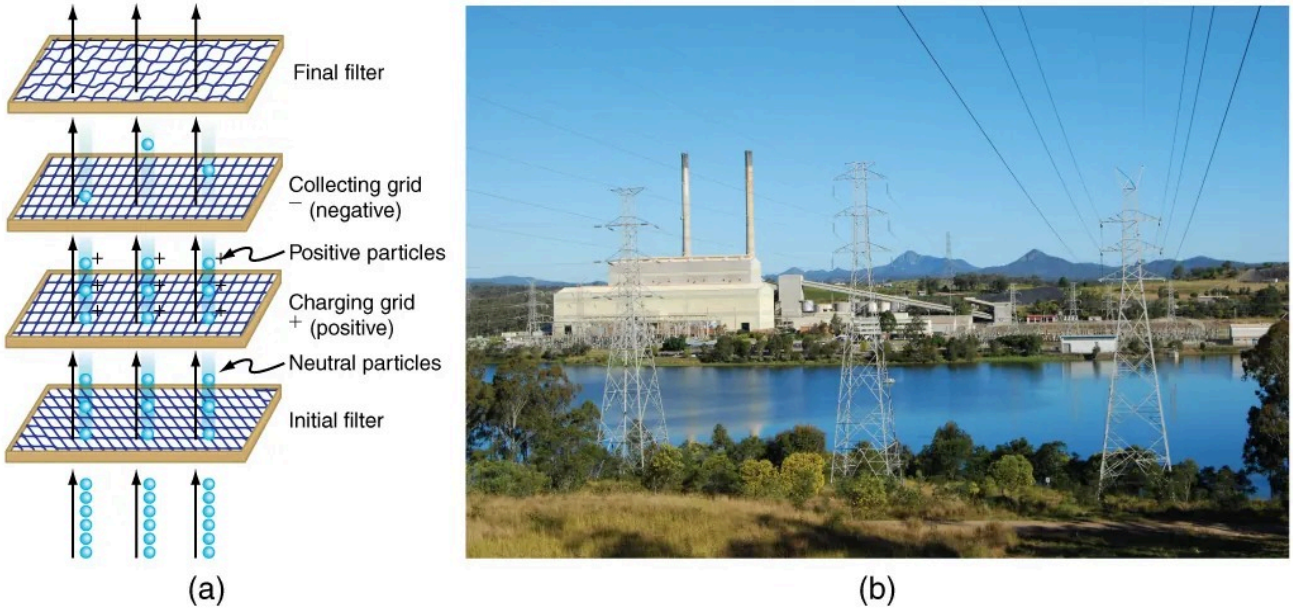


Figure 15.38 (a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two parts:

Part (a): This section is a schematic diagram illustrating a multi-stage filtration process. It consists of four horizontal layers:

- **Initial filter:** Positioned at the bottom, it shows neutral particles entering the filter.

- **Charging grid (+):** This layer positively charges the particles, indicated by plus symbols.
- **Collecting grid (negative):** This negatively charged grid attracts the positively charged particles, effectively collecting them.
- **Final filter:** Located at the top, it illustrates the filtered air exiting upward, with arrows indicating clean air passing through.

Part (b): A photograph of a power plant in a scenic landscape. The image shows:

- A large industrial building with two tall chimneys in the center.
- Several high-voltage power lines and towers in the foreground.
- A body of water in front of the industrial complex, reflecting the building and landscape.
- Mountains and a clear blue sky in the background, providing a natural setting for the power plant.

Problem-Solving Strategies for Electrostatics

1. Examine the situation to determine if static electricity is involved. This may concern separated stationary charges, the forces among them, and the electric fields they create.
2. Identify the system of interest. This includes noting the number, locations, and types of charges involved.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful. Determine whether the Coulomb force is to be considered directly—if so, it may be useful to draw a free-body diagram, using electric field lines.
4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is important to distinguish the Coulomb force F from the electric field E, for example.
5. Solve the appropriate equation for the quantity to be determined (the unknown) or draw the field lines as requested.
6. Examine the answer to see if it is reasonable: Does it make sense? Are units correct and the numbers involved reasonable?

Integrated Concepts

The Integrated Concepts exercises for this module involve concepts such as electric charges, electric fields, and several other topics. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. The electric field exerts force on charges, for example, and hence the relevance of Dynamics: Force and Newton's Laws of Motion. The following topics are involved in some or all of the problems labeled "Integrated Concepts":

- Kinematics
- Two-Dimensional Kinematics
- Dynamics: Force and Newton's Laws of Motion

- Uniform Circular Motion and Gravitation
- Statics and Torque
- Fluid Statics

The following worked example illustrates how this strategy is applied to an Integrated Concept problem:

Example 15.5

Acceleration of a Charged Drop of Gasoline

If steps are not taken to ground a gasoline pump, static electricity can be placed on gasoline when filling your car's tank. Suppose a tiny drop of gasoline has a mass of $4.00 \times 10^{-15}\text{kg}$ and is given a positive charge of $3.20 \times 10^{-19}\text{C}$. (a) Find the weight of the drop. (b) Calculate the electric force on the drop if there is an upward electric field of strength $3.00 \times 10^{5}\text{N/C}$ due to other static electricity in the vicinity. (c) Calculate the drop's acceleration.

Strategy

To solve an integrated concept problem, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example asks for weight. This is a topic of dynamics and is defined in Dynamics: Force and Newton's Laws of Motion. Part (b) deals with electric force on a charge, a topic of Electric Charge and Electric Field. Part (c) asks for acceleration, knowing forces and mass. These are part of Newton's laws, also found in Dynamics: Force and Newton's Laws of Motion.

The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

Solution for (a)

Weight is mass times the acceleration due to gravity, as first expressed in

$$w = \text{mg}.$$

15.20

Entering the given mass and the average acceleration due to gravity yields

$$w = (4.00 \times 10^{-15}\text{kg})(9.80\text{m/s}^2)$$
$$= 3.92 \times 10^{-14}\text{N}.$$

15.21

Discussion for (a)

This is a small weight, consistent with the small mass of the drop.

Solution for (b)

The force an electric field exerts on a charge is given by rearranging the following equation:

$$F = \text{qE}.$$

15.22

Here we are given the charge ($3.20 \times 10^{-19}\text{C}$ is twice the fundamental unit of charge) and the electric field strength, and so the electric force is found to be

$$F = (3.20 \times 10^{-19}\text{C})(3.00 \times 10^{5}\text{N/C})$$
$$= 9.60 \times 10^{-14}\text{N}.$$

15.23

Discussion for (b)

While this is a small force, it is greater than the weight of the drop.

Solution for (c)

The acceleration can be found using Newton's second law, provided we can identify all of the external forces acting on the drop. We assume only the drop's weight and the electric force are significant. Since the drop has a positive charge and the electric field is given to be upward, the electric force is upward. We thus have a one-dimensional (vertical direction) problem, and we can state Newton's second law as

$$a = \frac{F_{\text{net}}}{m}.$$

15.24

where $F_{\text{net}} = F - w$. Entering this and the known values into the expression for Newton's second law yields

$$\begin{aligned} a &= \frac{F - w}{m} \\ &= \frac{9.60 \times 10^{-14}\text{N} - 3.92 \times 10^{-14}\text{N}}{4.00 \times 10^{-15}\text{kg}} \\ &= 14.2\text{m/s}^2. \end{aligned}$$

15.25

Discussion for (c)

This is an upward acceleration great enough to carry the drop to places where you might not wish to have gasoline.

This worked example illustrates how to apply problem-solving strategies to situations that include topics in different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Unreasonable Results

The Unreasonable Results exercises for this module have results that are unreasonable because some premise is unreasonable or because certain of the premises are inconsistent with one another. Physical principles applied correctly then produce unreasonable results. The purpose of these problems is to give practice in assessing whether nature is being accurately described, and if it is not to trace the source of difficulty.

Problem-Solving Strategy

To determine if an answer is reasonable, and to determine the cause if it is not, do the following.

1. Solve the problem using strategies as outlined above. Use the format followed in the worked examples in the text to solve the problem as usual.
2. Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, and so on?
3. If the answer is unreasonable, look for what specifically could cause the identified difficulty. Usually, the manner in which the answer is unreasonable is an indication of the difficulty. For example, an extremely large Coulomb force could be due to the assumption of an excessively large separated charge.

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GLOSSARY

conductor
a material that allows electrons to move separately from their atomic orbits

conductor
an object with properties that allow charges to move about freely within it

Coulomb force
another term for the electrostatic force

Coulomb interaction
the interaction between two charged particles generated by the Coulomb forces they exert on one another

Coulomb's law
the mathematical equation calculating the electrostatic force vector between two charged particles

dipole
a molecule's lack of symmetrical charge distribution, causing one side to be more positive and another to be more negative

electric charge
a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

electric field
a three-dimensional map of the electric force extended out into space from a point charge

electric field lines
a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

electromagnetic force
one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

electron
a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

electrostatic equilibrium
an electrostatically balanced state in which all free electrical charges have stopped moving about

electrostatic force
the amount and direction of attraction or repulsion between two charged bodies

electrostatic precipitators
: filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

electrostatic repulsion
: the phenomenon of two objects with like charges repelling each other

electrostatics
: the study of electric forces that are static or slow-moving

Faraday cage
: a metal shield which prevents electric charge from penetrating its surface

field
: a map of the amount and direction of a force acting on other objects, extending out into space

free charge
: an electrical charge (either positive or negative) which can move about separately from its base molecule

free electron
: an electron that is free to move away from its atomic orbit

grounded
: connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object

grounded
: when a conductor is connected to the Earth, allowing charge to freely flow to and from Earth's unlimited reservoir

induction
: the process by which an electrically charged object brought near a neutral object creates a charge in that object

ink-jet printer
: small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

insulator
: a material that holds electrons securely within their atomic orbits

ionosphere
: a layer of charged particles located around 100 km above the surface of Earth, which is responsible for a range of phenomena including the electric field surrounding Earth

laser printer
: uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image

law of conservation of charge
: states that whenever a charge is created, an equal amount of charge with the opposite sign is created

simultaneously

photoconductor

a substance that is an insulator until it is exposed to light, when it becomes a conductor

point charge

A charged particle, designated Q, generating an electric field

polar molecule

a molecule with an asymmetrical distribution of positive and negative charge

polarization

slight shifting of positive and negative charges to opposite sides of an atom or molecule

polarized

a state in which the positive and negative charges within an object have collected in separate locations

proton

a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

screening

the dilution or blocking of an electrostatic force on a charged object by the presence of other charges nearby

static electricity

a buildup of electric charge on the surface of an object

test charge

A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

Van de Graaff generator

a machine that produces a large amount of excess charge, used for experiments with high voltage

vector

a quantity with both magnitude and direction

vector addition

mathematical combination of two or more vectors, including their magnitudes, directions, and positions

xerography

a dry copying process based on electrostatics

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CHAPTER SUMMARY

15.1 Static Electricity and Charge: Conservation of Charge

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge $|\ q_e\ |$ is
  $|\ q_e\ | = 1.60 \times 10^{-19}\text{C}.$
- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.
- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

15.2 Conductors and Insulators

- Polarization is the separation of positive and negative charges in a neutral object.
- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge within its atomic structure.
- Objects with like charges repel each other, while those with unlike charges attract each other.
- A conducting object is said to be grounded if it is connected to the Earth through a conductor. Grounding allows transfer of charge to and from the earth's large reservoir.
- Objects can be charged by contact with another charged object and obtain the same sign charge.
- If an object is temporarily grounded, it can be charged by induction, and obtains the opposite sign charge.
- Polarized objects have their positive and negative charges concentrated in different areas, giving them a non-symmetrical charge.
- Polar molecules have an inherent separation of charge.

15.3 Coulomb's Law

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb's law gives the magnitude of the force between point charges. It is

$$F = k\frac{|q_1 q_2|}{r^2},$$

where q_1 and q_2 are two point charges separated by a distance r, and $k \approx 8.99 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2$

- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

15.4 Electric Field: Concept of a Field Revisited

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance r depends on the charge of both charges, as well as the distance between the two.
- The electric field $\mathbf{E}$ is defined to be
$$\mathbf{E} = \frac{\mathbf{F}}{q,}$$
where $\mathbf{F}$ is the Coulomb or electrostatic force exerted on a small positive test charge q. $\mathbf{E}$ has units of N/C.
- The magnitude of the electric field $\mathbf{E}$ created by a point charge Q is
$$\mathbf{E} = k\frac{|Q|}{r^2}.$$
where r is the distance from Q. The electric field $\mathbf{E}$ is a vector and fields due to multiple charges add like vectors.

15.5 Electric Field Lines: Multiple Charges

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

15.6 Electric Forces in Biology

- Many molecules in living organisms, such as DNA, carry a charge.
- An uneven distribution of the positive and negative charges within a polar molecule produces a dipole.
- The effect of a Coulomb field generated by a charged object may be reduced or blocked by other nearby charged objects.
- Biological systems contain water, and because water molecules are polar, they have a strong effect on other molecules in living systems.

15.7 Conductors and Electric Fields in Static Equilibrium

- A conductor allows free charges to move about within it.
- The electrical forces around a conductor will cause free charges to move around inside the conductor until static equilibrium is reached.
- Any excess charge will collect along the surface of a conductor.
- Conductors with sharp corners or points will collect more charge at those points.
- A lightning rod is a conductor with sharply pointed ends that collect excess charge on the building caused by an electrical storm and allow it to dissipate back into the air.
- Electrical storms result when the electrical field of Earth's surface in certain locations becomes more strongly charged, due to changes in the insulating effect of the air.
- A Faraday cage acts like a shield around an object, preventing electric charge from penetrating inside.

15.8 Applications of Electrostatics

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

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CONCEPTUAL QUESTIONS

Conceptual Questions

15.1 Static Electricity and Charge: Conservation of Charge

1. There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?
2. Why do most objects tend to contain nearly equal numbers of positive and negative charges?

15.2 Conductors and Insulators

3. An eccentric inventor attempts to levitate by first placing a large negative charge on himself and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on himself, his clothes fly off. Explain.
4. If you have charged an electroscope by contact with a positively charged object, describe how you could use it to determine the charge of other objects. Specifically, what would the leaves of the electroscope do if other charged objects were brought near its knob?
5. When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.
6. Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)
7. Describe how a positively charged object can be used to give another object a negative charge. What is the name of this process?
8. What is grounding? What effect does it have on a charged conductor? On a charged insulator?

15.3 Coulomb's Law

9. Figure 15.39 shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.

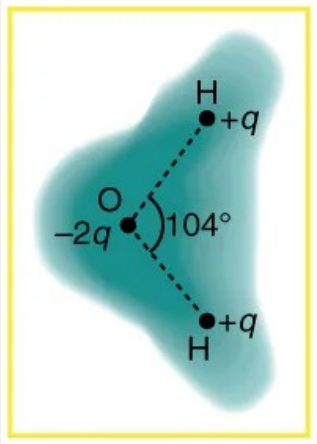


Figure 15.39 Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a polar molecule. It is more easily affected by electrostatic forces than molecules with uniform charge distributions. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a molecule of water (H^2O) with a stylized representation. It shows an oxygen atom (O) at the center with a partial negative charge indicated as -2q, and two hydrogen atoms (H) each with a partial positive charge indicated as +q. The hydrogen atoms are connected to the oxygen atom by dashed lines representing covalent bonds. The bond angle between the two hydrogen atoms is labeled as 104 degrees, illustrating the bent molecular shape of water. The whole structure is shown within a shaded area that represents the approximate electron cloud distribution.

10. Using Figure 15.39, explain, in terms of Coulomb's law, why a polar molecule (such

as in Figure 15.39) is attracted by both positive and negative charges.

11. Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

15.4 Electric Field: Concept of a Field Revisited

12. Why must the test charge q in the definition of the electric field be vanishingly small?
13. Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

15.5 Electric Field Lines: Multiple Charges

14. Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)
15. Figure 15.40 shows an electric field extending over three regions, labeled I, II, and III. Answer the following questions. (a) Are there any isolated charges? If so, in what region and what are their signs? (b) Where is the field strongest? (c) Where is it weakest? (d) Where is the field the most uniform?

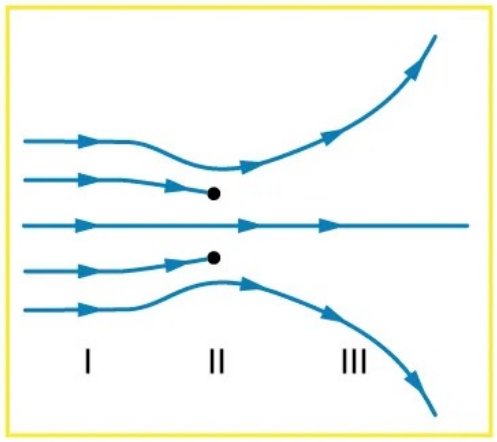


Figure 15.40 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram with a series of blue arrows illustrating fluid or traffic flow. The arrows are organized into three distinct sections, labeled I, II, and III below the lines.

– On the left side, several straight horizontal arrows approach from a distance.
– As the arrows move rightward, they encounter black dots. Some arrows curve around these dots while others pass through them.
– In section I, the arrows diverge, splitting into different paths.
– In section II, the arrows converge and align into a single path.
– In section III, the arrows diverge again, similar to section I but with a downward curve.

This diagram may represent a concept of flow dynamics, traffic patterns, or a similar process involving movement and directionality.

15.6 Electric Forces in Biology

16. A cell membrane is a thin layer enveloping a cell. The thickness of the membrane is much less than the size of the cell. In a static situation the membrane has a charge distribution of -2.5×10^{-6}C/m 2 on its inner surface and $+2.5 \times 10^{-6}$ C/m2 on its outer surface. Draw a diagram of the cell and the surrounding cell membrane. Include on this diagram the charge distribution and the corresponding electric field. Is there any electric field inside the cell? Is there any electric field

outside the cell?

15.7 Conductors and Electric Fields in Static Equilibrium

17. Is the object in Figure 15.41 a conductor or an insulator? Justify your answer.

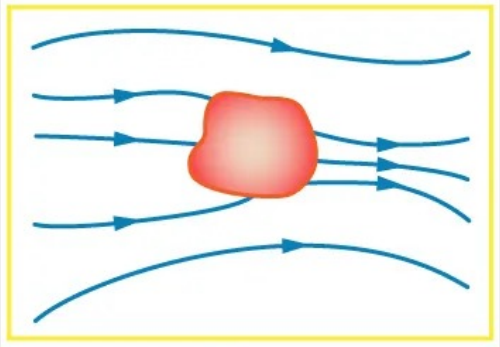


Figure 15.41 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains a conceptual diagram illustrating fluid dynamics around an object. There is a reddish, irregularly-shaped object in the center. Blue arrows, representing fluid flow lines, are curving around the object. The arrows are longer and straighter before reaching the object and then they curve around it, becoming closer together behind the object, indicating the flow's continuity and deflection due to the obstruction. The background is white and there is a yellow border around the image.

18. If the electric field lines in the figure above were perpendicular to the object, would it necessarily be a conductor? Explain.
19. The discussion of the electric field between two parallel conducting plates, in this module states that edge effects are less important if the plates are close together. What does close mean? That is, is the actual plate separation crucial, or is the ratio of plate separation to plate area crucial?
20. Would the self-created electric field at the end of a pointed conductor, such as a lightning rod, remove positive or negative charge from the conductor? Would the same sign charge be removed from a neutral pointed conductor by the application of a similar externally created electric field? (The answers to both questions have

implications for charge transfer utilizing points.)

21. Why is a golfer with a metal club over her shoulder vulnerable to lightning in an open fairway? Would she be any safer under a tree?
22. Can the belt of a Van de Graaff accelerator be a conductor? Explain.
23. Are you relatively safe from lightning inside an automobile? Give two reasons.
24. Discuss pros and cons of a lightning rod being grounded versus simply being attached to a building.
25. Using the symmetry of the arrangement, show that the net Coulomb force on the charge q at the center of the square below (Figure 15.42) is zero if the charges on the four corners are exactly equal.

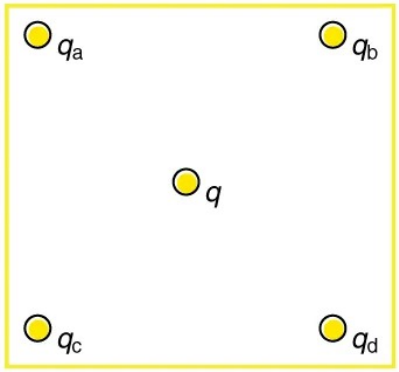


Figure 15.42 Four point charges q_a, q_b, q_c, and q_d lie on the corners of a square and q is located at its center. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image displays a geometrical diagram consisting of five circles, each filled with yellow color and outlined in black, positioned on a white background with a thin yellow border. Four of these circles are located near the corners of the square-shaped area, and one is at the center.

- Top left corner: Circle labeled as "q_a"
- Top right corner: Circle labeled as "q_b"
- Bottom left corner: Circle labeled as "q_c"
- Bottom right corner: Circle labeled as "q_d"
- Center of the square: Circle labeled as "q"

26. (a) Using the symmetry of the arrangement, show that the electric field at the center of the square in Figure 15.42 is zero if the charges on the four corners are exactly equal. (b) Show that this is also true for any combination of charges in which $q_a = q_d$ and $q_b = q_c$
27. (a) What is the direction of the total Coulomb force on q in Figure 15.42 if q is negative, $q_a = q_c$ and both are negative, and $q_b = q_d$ and both are positive? (b) What is the direction of the electric field at the center of the square in this situation?
28. Considering Figure 15.42, suppose that $q_a = q_d$ and $q_b = q_c$. First show that q is in static equilibrium. (You may neglect the gravitational force.) Then discuss whether the equilibrium is stable or unstable, noting that this may depend on the signs of the charges and the direction of displacement of q from the center of the square.
29. If $q_a = 0$ in Figure 15.42, under what conditions will there be no net Coulomb force on q?
30. In regions of low humidity, one develops a special "grip" when opening car doors, or touching metal door knobs. This involves placing as much of the hand on the device as possible, not just the ends of one's fingers. Discuss the induced charge and explain why this is done.
31. Tollbooth stations on roadways and bridges usually have a piece of wire stuck in the pavement before them that will touch a car as it approaches. Why is this done?
32. Suppose a person carries an excess charge. To maintain their charged status can they be standing on ground wearing just any pair of shoes? How would you discharge them? What are the consequences if they simply walk away?

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PROBLEMS & EXERCISES

Problems & Exercises

15.1 Static Electricity and Charge: Conservation of Charge

1. Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of -2.00nC (b) How many electrons must be removed from a neutral object to leave a net charge of $0.500\mu\text{C}$?
2. If 1.80×10^{20} electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?
3. To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?
4. A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $|\ q_e\ |$ is this?

15.2 Conductors and Insulators

5. Suppose a speck of dust in an electrostatic precipitator has 1.0000×10^{12} protons in it and has a net charge of –5.00 nC (a very large charge for a small speck). How many electrons does it have?
6. An amoeba has 1.00×10^{16} protons and a net charge of 0.300 pC. (a) How many fewer electrons are there than protons? (b) If you paired them up, what fraction of the protons would have no electrons?
7. A 50.0 g ball of copper has a net charge of $2.00\mu\text{C}$. What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)
8. What net charge would you place on a 100 g piece of sulfur if you put an extra electron on 1 in 10^{12} of its atoms? (Sulfur has an atomic mass of 32.1.)
9. How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

15.3 Coulomb's Law

10. What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of – 30.0 nC?
11. (a) How strong is the attractive force between a glass rod with a $0.700\mu\text{C}$ charge and a silk cloth with a $-0.600\mu\text{C}$ charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.
12. Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?
13. Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?
14. How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?
15. If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.
16. A test charge of $+2\mu\text{C}$ is placed halfway between a charge of $+6\mu\text{C}$ and another of $+4\mu\text{C}$ separated by 10 cm. (a) What is the magnitude of the force on the test charge? (b) What is the direction of this force (away from or toward the $+6\mu\text{C}$ charge)?
17. Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.
18. (a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.
19. Suppose you have a total charge q_{tot} that you can split in any manner. Once split,

the separation distance is fixed. How do you split the charge to achieve the greatest force?

20. (a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.
21. (a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?
22. At what distance is the electrostatic force between two protons equal to the weight of one proton?
23. A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.
24. (a) Two point charges totaling $8.00\mu\text{C}$ exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each? (b) What is the charge on each if the force is attractive?
25. Point charges of $5.00\mu\text{C}$ and $-3.00\mu\text{C}$ are placed 0.250 m apart. (a) Where can a third charge be placed so that the net force on it is zero? (b) What if both charges are positive?
26. Two point charges q_1 and q_2 are $3.00m$ apart, and their total charge is $20\mu\text{C}$. (a) If the force of repulsion between them is 0.075N, what are magnitudes of the two charges? (b) If one charge attracts the other with a force of 0.525N, what are the magnitudes of the two charges? Note that you may need to solve a quadratic equation to reach your answer.

15.4 Electric Field: Concept of a Field Revisited

27. What is the magnitude and direction of an electric field that exerts a $2.00 \times 10^{-5}\text{N}$ upward force on a $-1.75\mu\text{C}$ charge?
28. What is the magnitude and direction of the force exerted on a $3.50\mu\text{C}$ charge by a 250 N/C electric field that points due east?
29. Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).
30. (a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m? (b) How large is the field at 10.0 m?
31. Calculate the initial (from rest) acceleration of a proton in a $5.00 \times 10^{6}\text{N/C}$ electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.
32. (a) Find the magnitude and direction of an electric field that exerts a $4.80 \times 10^{-17}\text{N}$ westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?

15.5 Electric Field Lines: Multiple Charges

33. (a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$.
34. Sketch the electric field lines a long distance from the charge distributions shown in Figure 15.23 (a) and (b)
35. Figure 15.43 shows the electric field lines near two charges q_1 and q_2. What is the ratio of their magnitudes? (b) Sketch the electric field lines a long distance from the charges shown in the figure.

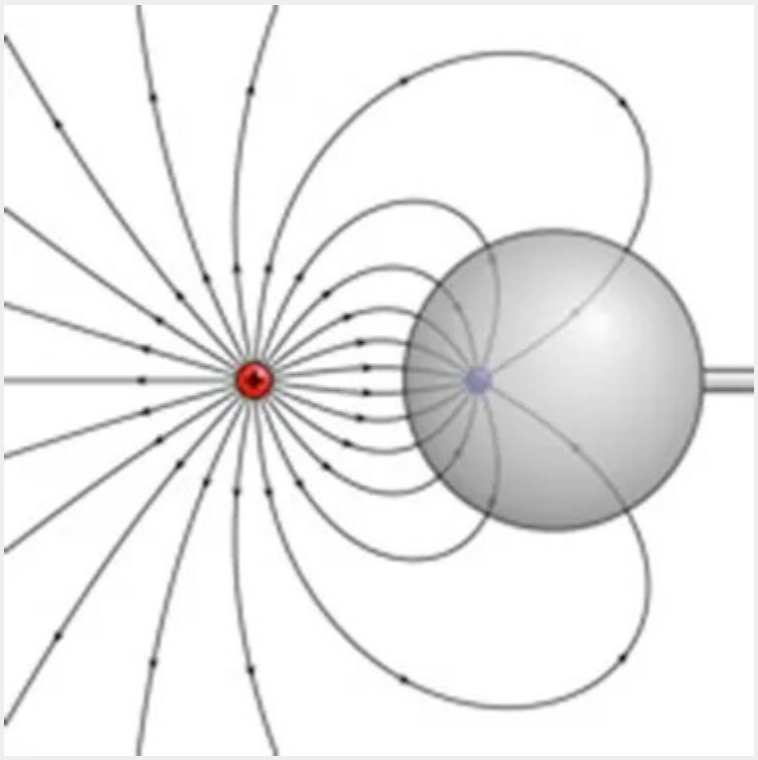


Figure 15.43 The electric field near two charges. Image by Geek3, Wikimedia Commons License

Image Description

The image depicts a diagram showing magnetic field lines. On the left side, there is a small red circle with a tiny black dot at its center, representing a magnetic pole. Emanating from this circle are numerous curved black lines that extend outward in all directions. These lines curve around a large gray sphere on the right side of the image. The gray sphere has a smaller purple circle on its left side, possibly indicating another magnetic pole on the sphere. The lines illustrate the magnetic flux and their interaction with the sphere, showing how the field lines are distorted around the sphere. No text or labels are present in the image.

36. Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See Figure 15.43 for a similar situation).

15.7 Conductors and Electric Fields in Static Equilibrium

37. Sketch the electric field lines in the vicinity of the conductor in Figure 15.44 given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?



Figure 15.44 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a single, elongated, oval-shaped object with a smooth surface. The object is light gray in color. It is horizontally oriented and appears within a yellow rectangular border. The lighting creates a slight reflection on the top surface of the object, indicating a glossy texture.

38. Sketch the electric field lines in the vicinity of the conductor in Figure 15.45 given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?



Figure 15.45 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

A blue, elongated, oval-shaped object resembling a 3D geometric form that tapers at both ends, similar to an inflated cushion or pill. It is centered on a plain white background, creating a stark contrast with its smooth, shiny surface.

39. Sketch the electric field between the two conducting plates shown in Figure 15.46,

given the top plate is positive and an equal amount of negative charge is on the bottom plate. Be certain to indicate the distribution of charge on the plates.

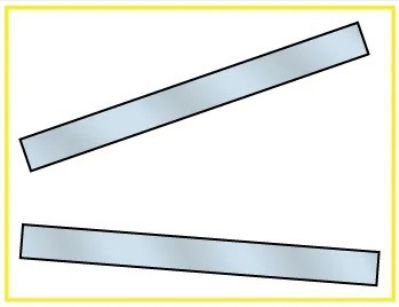


Figure 15.46 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows two long, narrow rectangles positioned diagonally on a white background. The rectangles have a slight gradient suggesting they are metallic or reflective, with black outlines. The top rectangle is angled slightly upwards from left to right, while the bottom rectangle is angled slightly downwards from left to right. The entire image is bordered by a thin yellow line.

40. Sketch the electric field lines in the vicinity of the charged insulator in Figure 15.47 noting its nonuniform charge distribution.

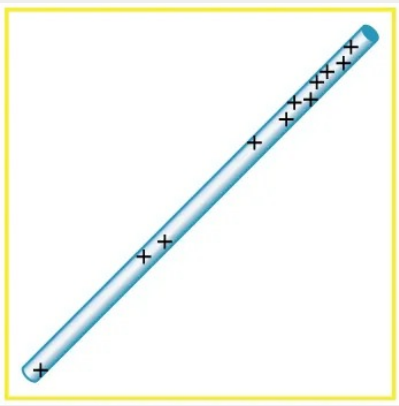


Figure 15.47 A charged insulating rod such as might be used in a classroom demonstration. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a long, cylindrical object, resembling a light blue rod, positioned diagonally across the image. The rod has a series of black crosses evenly spaced along its length. The entire image is bordered by a thin, yellow line.

41. What is the force on the charge located at $x = 8.00cm$ in Figure 15.48(a) given that $q = 1.00\mu\text{C}$?

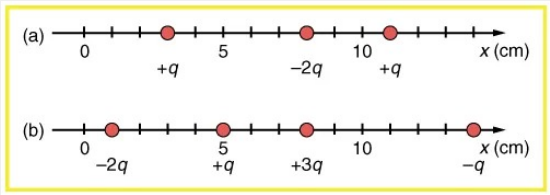


Figure 15.48 (a) Point charges located at 3.00, 8.00, and 11.0 cm along the x-axis. (b) Point charges located at 1.00, 5.00, 8.00, and 14.0 cm along the x-axis. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b), each having a straight horizontal line with evenly spaced tick marks along the x-axis, marked in centimeters (cm). Both diagrams include red dots positioned at specific intervals, representing charges with magnitude and sign.

Diagram (a):

- The horizontal axis is labeled *x (cm)*.
- There are four red dots placed at the following positions:
    - At 0 cm, labeled with charge +q.
    - At 5 cm.
    - At 10 cm, labeled with charge -2q.
    - Beyond 10 cm, labeled with charge +q.

Diagram (b):

- The horizontal axis is labeled *x (cm)*.
- There are four red dots placed at the following positions:

- At 0 cm, labeled with charge -2q.
- At 5 cm, labeled with charge +q.
- At 10 cm, labeled with charge +3q.
- Beyond 10 cm, labeled with charge -q.

42. (a) Find the total electric field at $x = 1.00cm$ in Figure 15.48(b) given that $q = 5.00nC$. (b)Find the total electric field at $x = 11.00cm$ in Figure 15.48(b) given that $q = 5.00nC$. (c) If the charges are allowed to move and eventually be brought to rest by friction, what will the final charge configuration be? (That is, will there be a single charge, double charge, etc., and what will its value(s) be?)
43. (a) Find the electric field at $x = 5.00cm$ in Figure 15.48(a), given that $q = 1.00\mu\text{C}$. (b) At what position between 3.00 and 8.00 cm is the total electric field the same as that for $-2q$ alone? (c) Can the electric field be zero anywhere between 0.00 and 8.00 cm? (d) At very large positive or negative values of x, the electric field approaches zero in both (a) and (b). In which does it most rapidly approach zero and why? (e) At what position to the right of 11.0 cm is the total electric field zero, other than at infinity? (Hint: A graphing calculator can yield considerable insight in this problem.)
44. (a) Find the total Coulomb force on a charge of 2.00 nC located at $x = 4.00cm$ in Figure 15.48 (b), given that $q = 1.00\mu\text{C}$. (b) Find the x-position at which the electric field is zero in Figure 15.48 (b).
45. Using the symmetry of the arrangement, determine the direction of the force on q in the figure below, given that $q_a = q_b = +7.50\mu\text{C}$ and $q_c = q_d = -7.50\mu\text{C}$. (b) Calculate the magnitude of the force on the charge q, given that the square is 10.0 cm on a side and $q = 2.00\mu\text{C}$.

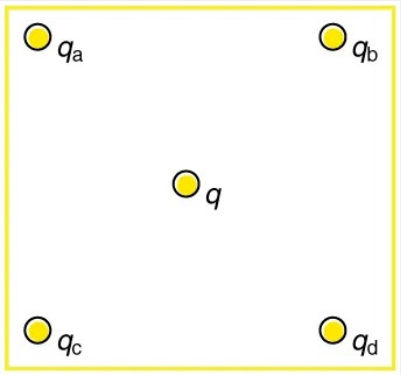


Figure 15.49 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram showing five yellow circles enclosed within a yellow square outline. Four circles are positioned at each corner of the square, labeled as "q_a" in the top-left, "q_b" in the top-right, "q_c" in the bottom-left, and "q_d" in the bottom-right. There is one circle located at the center of the square labeled as "q." Each circle has a black outline.

46. (a) Using the symmetry of the arrangement, determine the direction of the electric field at the center of the square in Figure 15.49, given that $q_a = q_b = -1.00\mu\text{C}$ and $q_c = q_d = +1.00\mu\text{C}$. (b) Calculate the magnitude of the electric field at the location of q, given that the square is 5.00 cm on a side.
47. Find the electric field at the location of q_a in Figure 15.49 given that $q_b = q_c = q_d = +2.00\text{nC}$, $q = -1.00\text{nC}$, and the square is 20.0 cm on a side.
48. Find the total Coulomb force on the charge q in Figure 15.49, given that $q = 1.00\mu\text{C}$, $q_a = 2.00\mu\text{C}$, $q_b = -3.00\mu\text{C}$, $q_c = -4.00\mu\text{C}$, and $q_d = +1.00\mu\text{C}$. The square is 50.0 cm on a side.
49. (a) Find the electric field at the location of q_a in Figure 15.50, given that $q_\text{b} = +10.00\mu\text{C}$ and $q_\text{c} = -5.00\mu\text{C}$. (b) What is the force on q_a, given that $q_\text{a} = +1.50\text{nC}$?

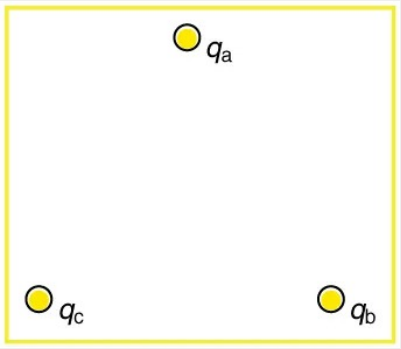


Figure 15.50 Point charges located at the corners of an equilateral triangle 25.0 cm on a side. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a simple diagram consisting of three small yellow circles, each labeled with a different subscript letter. These circles are distributed at the corners of an imaginary triangle inside the larger white square with a thin yellow border.

- The top circle is labeled "q_a".
- The bottom right circle is labeled "q_b".
- The bottom left circle is labeled "q_c".

50. (a) Find the electric field at the center of the triangular configuration of charges in Figure 15.50, given that $q_a = +2.50\text{nC}$, $q_b = -8.00\text{nC}$, and $q_c = +1.50\text{nC}$. (b) Is there any combination of charges, other than $q_a = q_b = q_c$, that will produce a zero strength electric field at the center of the triangular configuration?

15.8 Applications of Electrostatics

51. (a) What is the electric field 5.00 m from the center of the terminal of a Van de Graaff with a 3.00 mC charge, noting that the field is equivalent to that of a point charge at the center of the terminal? (b) At this distance, what force does the field exert on a $2.00\mu\text{C}$ charge on the Van de Graaff's belt?
52. (a) What is the direction and magnitude of an electric field that supports the weight of a free electron near the surface of Earth? (b) Discuss what the small value for this field implies regarding the relative strength of the gravitational and

electrostatic forces.

53. A simple and common technique for accelerating electrons is shown in Figure 15.51, where there is a uniform electric field between two plates. Electrons are released, usually from a hot filament, near the negative plate, and there is a small hole in the positive plate that allows the electrons to continue moving. (a) Calculate the acceleration of the electron if the field strength is $2.50 \times 10^4 \mathrm{N/C}$. (b) Explain why the electron will not be pulled back to the positive plate once it moves through the hole.

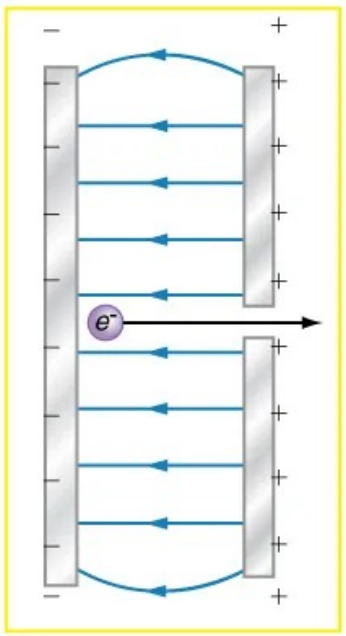


Figure 15.51 Parallel conducting plates with opposite charges on them create a relatively uniform electric field used to accelerate electrons to the right. Those that go through the hole can be used to make a TV or computer screen glow or to produce X-rays. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a diagram of an electric field between two parallel metal plates. The setup includes:

– Two vertical silver-colored plates on each side with small positive and negative signs indicating polarity. The left plate is positive, and the right plate is negative.

– Blue arrows are pointing straight from the positive plate to the negative plate, indicating the direction of the electric field.
– A single electron, marked with "e⁻" inside a purple circle, is positioned between the plates. An arrow next to the electron shows its movement direction towards the right.
– The entire setup is enclosed within a yellow border.

This diagram illustrates the force acting on an electron in an electric field generated by parallel plate capacitors.

54. Earth has a net charge that produces an electric field of approximately 150 N/C downward at its surface. (a) What is the magnitude and sign of the excess charge, noting the electric field of a conducting sphere is equivalent to a point charge at its center? (b) What acceleration will the field produce on a free electron near Earth's surface? (c) What mass object with a single extra electron will have its weight supported by this field?
55. Point charges of $25.0\mu\text{C}$ and $45.0\mu\text{C}$ are placed 0.500 m apart. (a) At what point along the line between them is the electric field zero? (b) What is the electric field halfway between them?
56. What can you say about two charges q_1 and q_2, if the electric field one-fourth of the way from q_1 to q_2 is zero?
57. **Integrated Concepts:** Calculate the angular velocity ω of an electron orbiting a proton in the hydrogen atom, given the radius of the orbit is $0.530 \times 10^{-10}\text{m}$. You may assume that the proton is stationary and the centripetal force is supplied by Coulomb attraction.
58. **Integrated Concepts:** An electron has an initial velocity of $5.00 \times 10^{6}\text{m/s}$ in a uniform $2.00 \times 10^{5}\text{N/C}$ strength electric field. The field accelerates the electron in the direction opposite to its initial velocity. (a) What is the direction of the electric field? (b) How far does the electron travel before coming to rest? (c) How long does it take the electron to come to rest? (d) What is the electron's velocity when it returns to its starting point?
59. **Integrated Concepts:** The practical limit to an electric field in air is about $3.00 \times 10^{6}\text{N/C}$. Above this strength, sparking takes place because air begins to ionize and charges flow, reducing the field. (a) Calculate the distance a free proton

must travel in this field to reach 3.00% of the speed of light, starting from rest. (b) Is this practical in air, or must it occur in a vacuum?

60. **Integrated Concepts:** A 5.00 g charged insulating ball hangs on a 30.0 cm long string in a uniform horizontal electric field as shown in Figure 15.52. Given the charge on the ball is $1.00\mu\text{C}$, find the strength of the field.

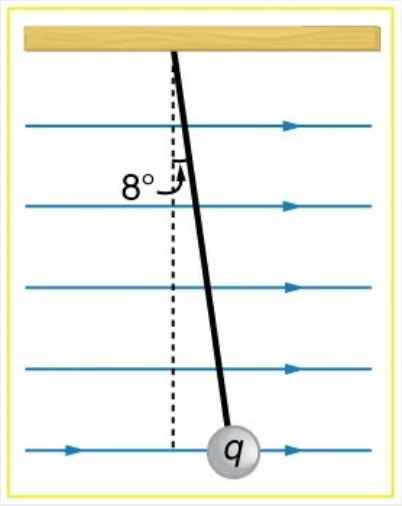


Figure 15.52 A horizontal electric field causes the charged ball to hang at an angle of 8.00° *. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a pendulum with a weight, labeled “q,” suspended from a horizontal wooden beam. The pendulum is shown at an angle of 8 degrees from the vertical, indicated by a dashed line representing the vertical position and a black line representing the pendulum. Several horizontal blue arrows point to the right, representing a uniform force, such as an electric field, acting on the pendulum. The background has lines evenly spaced horizontally across the image.

61. **Integrated Concepts:** Figure 15.53 shows an electron passing between two charged metal plates that create an 100 N/C vertical electric field perpendicular to the electron’s original horizontal velocity. (These can be used to change the electron’s direction, such as in an oscilloscope.) The initial speed of the electron is $3.00 \times 10^6 \text{m/s}$, and the horizontal distance it travels in the uniform field is 4.00

cm. (a) What is its vertical deflection? (b) What is the vertical component of its final velocity? (c) At what angle does it exit? Neglect any edge effects.

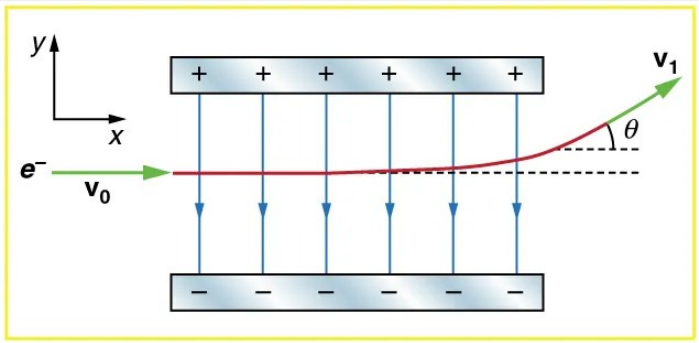


Figure 15.53 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram illustrating the trajectory of an electron, represented by e^-, moving initially at velocity v_0 towards the right. The path of the electron is marked by a red curve indicating its trajectory under the influence of an electric field. The field is represented by vertical blue arrows pointing downwards, indicating the direction of the electric force between two parallel plates. The upper plate has a positive charge, marked with plus signs, and the lower plate has a negative charge, marked with minus signs.

The electron's path curves upward as it exits the field, with its final velocity represented by the vector v_1. The angle between the initial horizontal line and the final direction of the electron is labeled as θ. An *x*-*y* coordinate system is shown at the top left corner for reference, with the *x*-axis pointing to the right and the *y*-axis pointing upward.

62. **Integrated Concepts:** The classic Millikan oil drop experiment was the first to obtain an accurate measurement of the charge on an electron. In it, oil drops were suspended against the gravitational force by a vertical electric field. (See Figure 15.54.) Given the oil drop to be $1.00\mu\text{m}$ in radius and have a density of $920kg/m^3$: (a) Find the weight of the drop. (b) If the drop has a single excess electron, find the electric field strength needed to balance its weight.

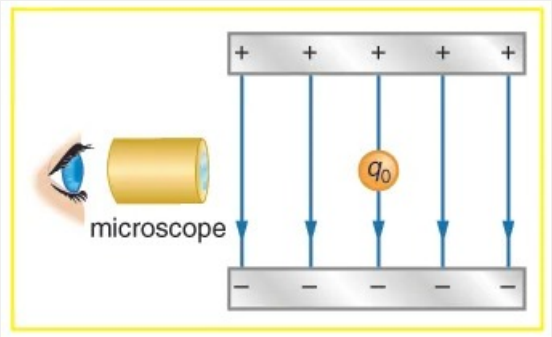


Figure 15.54 In the Millikan oil drop experiment, small drops can be suspended in an electric field by the force exerted on a single excess electron. Classically, this experiment was used to determine the electron charge q_e by measuring the electric field and mass of the drop. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of an experiment setup. On the left side, there is a graphic of an eye looking through a microscope. To the right of the microscope, there is an arrangement consisting of two horizontal plates, one above the other. The upper plate is marked with positive (+) signs, while the lower plate is marked with negative (-) signs, representing a uniform electric field. Vertical arrows point downward between the plates, indicating the direction of the electric field. In the center, there is a small circle labeled "q_0," representing a point charge, placed between the plates. The setup is enclosed within a yellow border.

63. **Integrated Concepts:** (a) In Figure 15.55, four equal charges q lie on the corners of a square. A fifth charge Q is on a mass m directly above the center of the square, at a height equal to the length d of one side of the square. Determine the magnitude of q in terms of Q, m, and d, if the Coulomb force is to equal the weight of m. (b) Is this equilibrium stable or unstable? Discuss.

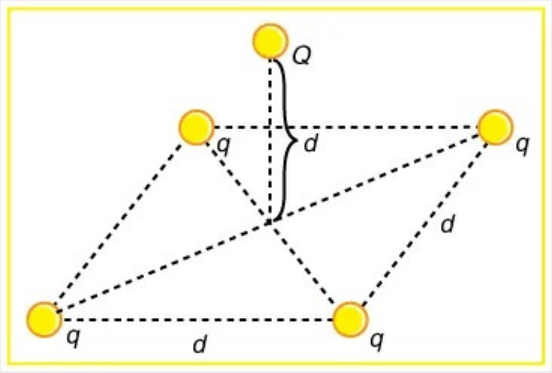


Figure 15.55 Four equal charges on the corners of a horizontal square support the weight of a fifth charge located directly above the center of the square. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a geometric configuration of five points, each represented by a yellow circle, connected by dashed lines forming a network. The points are arranged as follows:

- Four points are positioned at the vertices of a quadrilateral (a four-sided shape).
- One point is labeled "Q" and is located above the center of the quadrilateral.

The connections are labeled with distances:

- The distance from the central point "Q" to the top vertex of the quadrilateral is labeled "d."
- The distances connecting the other vertices of the quadrilateral to each other are labeled "q" and "d" as well.

The entire configuration is enclosed within a rectangle outlined with a yellow border.

64. **Unreasonable Results:** (a) Calculate the electric field strength near a 10.0 cm diameter conducting sphere that has 1.00 C of excess charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?
65. **Unreasonable Results:** (a) Two 0.500 g raindrops in a thunderhead are 1.00 cm apart when they each acquire 1.00 mC charges. Find their acceleration. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

66. **Unreasonable Results:** A wrecking yard inventor wants to pick up cars by charging a 0.400 m diameter ball and inducing an equal and opposite charge on the car. If a car has a 1000 kg mass and the ball is to be able to lift it from a distance of 1.00 m: (a) What minimum charge must be used? (b) What is the electric field near the surface of the ball? (c) Why are these results unreasonable? (d) Which premise or assumption is responsible?
67. **Construct Your Own Problem:** Consider two insulating balls with evenly distributed equal and opposite charges on their surfaces, held with a certain distance between the centers of the balls. Construct a problem in which you calculate the electric field (magnitude and direction) due to the balls at various points along a line running through the centers of the balls and extending to infinity on either side. Choose interesting points and comment on the meaning of the field at those points. For example, at what points might the field be just that due to one ball and where does the field become negligibly small? Among the things to be considered are the magnitudes of the charges and the distance between the centers of the balls. Your instructor may wish for you to consider the electric field off axis or for a more complex array of charges, such as those in a water molecule.
68. **Construct Your Own Problem:** Consider identical spherical conducting space ships in deep space where gravitational fields from other bodies are negligible compared to the gravitational attraction between the ships. Construct a problem in which you place identical excess charges on the space ships to exactly counter their gravitational attraction. Calculate the amount of excess charge needed. Examine whether that charge depends on the distance between the centers of the ships, the masses of the ships, or any other factors. Discuss whether this would be an easy, difficult, or even impossible thing to do in practice.
69. **Critical Thinking:** A frictionless circular track with a diameter of 4.00 m has two spheres of 10.0 kg each anchored very near it. The track is oriented with a diameter along the x-axis. The anchored spheres are just outside the track. One of the anchored spheres is just left of the origin at a distance of $1.00 \times 10^{-15}\text{m}$ from it, and the other is on the x-axis just right of the track at a distance of $1.00 \times 10^{-15}\text{m}$ past $(4, 0)$. Each sphere carries a charge of $2.00 \times 10^{-9}\ \text{C}$.

(Note: The slight distance from the track is so the particles cannot collide, and does not enter into the calculations for three significant figures.) (a) A third sphere has a mass of 10.00 kg and carries a charge of -1.00×10^{-9} C. It is free to move on the track. It is initially placed an equal length from the two anchored spheres at $(2.00, 2.00)$. What is the total force on the third sphere? (Be certain to include gravity between the spheres.) (b) Will the third sphere move? (c) Label the first two spheres A and B, and the movable sphere C. Now start sphere C at $(1.00, 2.00)$. What are the forces on sphere C? (d) Will sphere C now move? (e) If it does, where will it stop moving and reverse course? If it does not move, you may skip this part. (f) How many equilibrium points are there on the track at which the movable sphere will remain stationary? Where are they?

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ELECTRIC CURRENT, RESISTANCE, AND OHM'S LAW



Figure 16.1 Electric energy in massive quantities is transmitted from this hydroelectric facility, the Srisailam power station located along the Krishna River in India, by the movement of charge—that is, by electric current. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

16.1 Current
16.2 Ohm's Law: Resistance and Simple Circuits
16.3 Resistance and Resistivity
16.4 Electric Power and Energy
16.5 Alternating Current versus Direct Current
16.6 Electric Hazards and the Human Body
16.7 Nerve Conduction–Electrocardiograms
16.8 Resistors in Series and Parallel
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

The flicker of numbers on a handheld calculator, nerve impulses carrying signals of vision to the brain, an ultrasound device sending a signal to a computer screen, the brain sending a message for a baby to twitch its toes, an electric train pulling its load over a mountain pass, a hydroelectric plant sending energy to metropolitan and rural users—these and many other examples of electricity involve *electric current, the movement of charge*. Humankind has indeed harnessed electricity, the basis of technology, to improve our quality of life. Whereas the previous two chapters concentrated on static electricity and the fundamental force underlying its behavior, the next few chapters will be devoted to electric and magnetic phenomena involving current. In addition to exploring applications of electricity, we shall gain new insights into nature—in particular, the fact that all magnetism results from electric current.
Access multimedia content

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16.1 CURRENT

Learning Objectives

By the end of this section, you will be able to:

- Define electric current, ampere, and drift velocity
- Describe the direction of charge flow in conventional current.
- Use drift velocity to calculate current and vice versa.

Electric Current

Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, electric current I is defined to be

$$I = \frac{\Delta Q}{\Delta t},$$

16.1

where ΔQ is the amount of charge passing through a given area in time Δt. (As in previous chapters, initial time is often taken to be zero, in which case $\Delta t = t$.) (See Figure 16.2.) The SI unit for current is the ampere (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \Delta Q/\Delta t$, we see that an ampere is one coulomb per second:

$$1\ \text{A} = 1\ \text{C/s}$$

16.2

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.

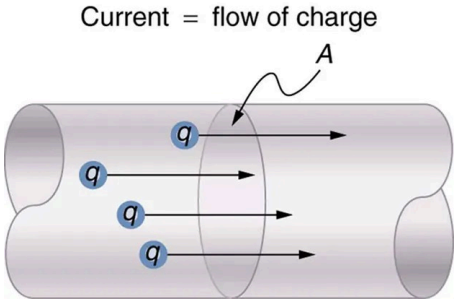


Figure 16.2 The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates the concept of electric current as the flow of charge. It features a cylindrical pipe-like structure with multiple blue circles inside, each labeled with "q," representing charges. These charges are moving in a straight line through the cylinder, depicted by black arrows pointing in the direction of the flow. The letter "A" is used to indicate an area perpendicular to the direction of current flow. The text "Current = flow of charge" is displayed at the top of the image.

Example 16.1

Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Strategy

We can use the definition of current in the equation $I = \Delta Q/\Delta t$ to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

Solution for (a)

Entering the given values for charge and time into the definition of current gives

$$I = \frac{\Delta Q}{\Delta t} = \frac{720\ \text{C}}{4.00\ \text{s}} = 180\ \text{C/s} = 180\ \text{A}.$$

16.3

Discussion for (a)

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large because large frictional forces need to be overcome when setting something in motion.

Solution for (b)

Solving the relationship $I = \Delta Q/\Delta t$ for time Δt, and entering the known values for charge and current gives

$$\Delta t = \frac{\Delta Q}{I} = \frac{1.00\ \text{C}}{0.300 \times\ 10^{-3}\ \text{C/s}}$$
$$= 3.33\ \times\ 10^{3}\ \text{s}.$$

16.4

Discussion for (b)

This time is slightly less than an hour. The small current used by the hand-held calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It's because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

Figure 16.3 shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in Figure 16.3 (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.

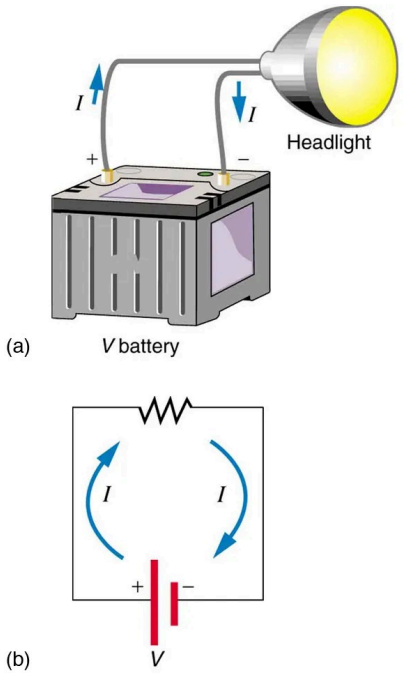


Figure 16.3 (a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide variety of similar circuits. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is divided into two labeled parts depicting an electric circuit with a battery and a headlight.

(a) The first part shows a three-dimensional illustration of a battery connected to a headlight. The battery is marked "V battery" and has a positive (+) and negative (-) terminal. Two wires connect the battery to the headlight, forming a closed circuit. Arrows labeled "I" indicate the direction of the electric current flowing from the positive terminal through the headlight (which is illuminated) and back to the negative terminal.

(b) The second part shows a schematic diagram of the circuit. It features a rectangle representing the circuit with a battery symbol at the bottom (marked with + and – signs) connected to a resistor at the top. Arrows labeled "I" inside the rectangle show the current direction circulating through the circuit. The battery is labeled with "V" indicating voltage.

Note that the direction of current flow in Figure 16.3 is from positive to negative. *The direction of conventional current is the direction that positive charge would flow*. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This

is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. Figure 16.4 illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in Figure 16.4. Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?

Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.

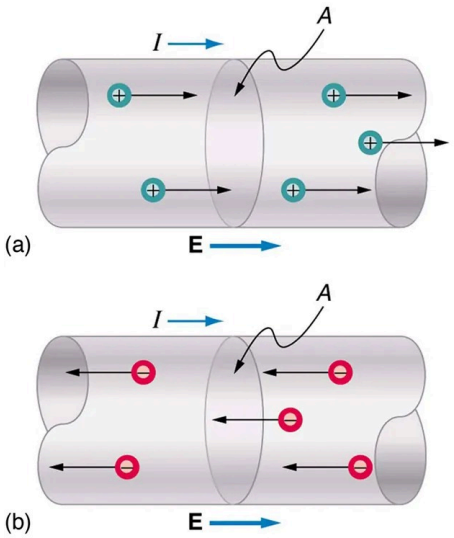


Figure 16.4 Current I is the rate at which charge moves through an area A, such as the cross-section of a wire. Conventional current is defined to move in the direction of the electric field. (a) Positive charges move in the direction of the electric field and the same direction as conventional current. (b) Negative charges move in the direction opposite to the electric field. Conventional current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b) depicting the movement of charges through cylindrical tubes.

(a) The top diagram shows a cylindrical tube with positive charges (indicated by circles with a plus sign inside). The charges move to the right, as indicated by arrows. The direction of the electric field is also to the right, represented by a bold arrow labeled "E." A similar arrow labeled "I" shows the direction of the current, pointing to the right. The tube is divided in the middle by a section labeled "A."

(b) The bottom diagram resembles the top diagram but features negative charges (indicated by circles with a minus sign inside). These charges move to the left, as indicated by arrows. The electric field direction, shown by a bold arrow labeled "E," still points to the right, while the current direction, labeled "I," aligns with the positive charges and points to the right. The same middle section is labeled "A."

Example 16.2

Calculating the Number of Electrons that Move through a Calculator

If the 0.300-mA current through the calculator mentioned in the Example 16.1 example is carried by electrons, how many electrons per second pass through it?

Strategy

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite, $I_{\text{electrons}} = -0.300 \times 10^{-3}\text{C/s}$ .Since each electron (e^-) has a charge of $-1.60 \times 10^{-19}\text{C}$, we can convert the current in coulombs per second to electrons per second.

Solution

Starting with the definition of current, we have

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} = \frac{-0.300 \times 10^{-3}\ \text{C}}{\text{s}}.$$

16.5

We divide this by the charge per electron, so that

$$\begin{aligned}\frac{e^-}{\text{s}} &= \frac{-0.300 \times 10^{-3}\ \text{C}}{\text{s}} \times \frac{1e^-}{-1.60 \times 10^{-19}\ \text{C}} \\ &= 1.88 \times 10^{15}\frac{e^-}{\text{s}}.\end{aligned}$$

16.6

Discussion

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

Drift Velocity

Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of 10^8m/s, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move *much* more slowly on average, typically drifting at speeds on the order of 10^{-4}m/s. How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in Figure 16.5, the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. The density of charge in a system cannot easily be increased, and so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this rapidly moving signal or shock wave is a rapidly propagating change in electric field.

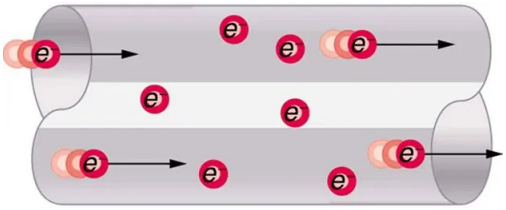


Figure 16.5 When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of an electrical wire or conductor in the shape of a cylinder. The cylinder is shown as being hollow, with arrows inside pointing from left to right, indicating the direction of electron flow. Scattered inside the cylinder are several symbols of electrons, represented by the letter "e" within circles, moving in the same direction as the arrows. The diagram illustrates the concept of electron flow in a conductor, where electrons move through the wire from one end to the other. The outer walls of the cylinder have a subtle gradient, possibly to indicate the material's boundaries or the inner surface of the wire.

Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. Figure 16.6 shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electric field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The drift velocity v_d is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.

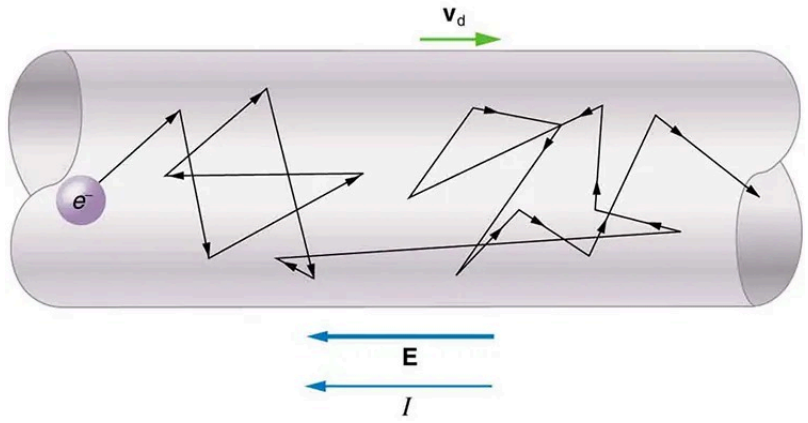


Figure 16.6 Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of the free charges is called the drift velocity, v_d, and it is in the direction opposite to the electric field for electrons. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a cylindrical conductor with various arrows and symbols illustrating the movement and behavior of an electron within it. Inside the cylinder, there is a small purple sphere labeled with an "e^-" representing an electron. The electron's pathway is shown by a series of black zigzagging arrows, suggesting a

random or scattered movement. Despite this random motion, there is a green arrow labeled "v_d" indicating the electron's overall drift velocity in the direction of the arrow. Below the cylinder, two blue arrows are shown pointing in opposite directions. The arrow labeled "E" points to the right, indicating the direction of the electric field, while the other arrow labeled "I" points to the left, indicating the direction of the conventional current flow.

Conduction of Electricity and Heat

Good electrical conductors are often good heat conductors, too. This is because large numbers of free electrons can carry electrical current and can transport thermal energy.

The free-electron collisions transfer energy to the atoms of the conductor. The electric field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed, therefore) of the electrons. The work is transferred to the conductor's atoms, possibly increasing temperature. Thus a continuous power input is required to keep a current flowing. An exception, of course, is found in superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings. In contrast, the supply of energy can be useful, such as in a lightbulb filament. The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

Making Connections: Take-Home Investigation—Filament Observations

Find a lightbulb with a filament. Look carefully at the filament and describe its structure. To what points is the filament connected?

We can obtain an expression for the relationship between current and drift velocity by considering the

number of free charges in a segment of wire, as illustrated in Figure 16.7. *The number of free charges per unit volume* is given the symbol n and depends on the material. The shaded segment has a volume Ax, so that the number of free charges in it is nAx. The charge ΔQ in this segment is thus qnAx, where q is the amount of charge on each carrier. (Recall that for electrons, q is $-1.60 \times 10^{-19}\mathrm{C}$.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time Δt, the current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{\mathrm{qnAx}}{\Delta t}.$$
16.7

Note that $x/\Delta t$ is the magnitude of the drift velocity, v_d, since the charges move an average distance x in a time Δt. Rearranging terms gives

$$I = nqAv_\mathrm{d},$$
16.8

where I is the current through a wire of cross-sectional area A made of a material with a free charge density n. The carriers of the current each have charge q and move with a drift velocity of magnitude v_d.

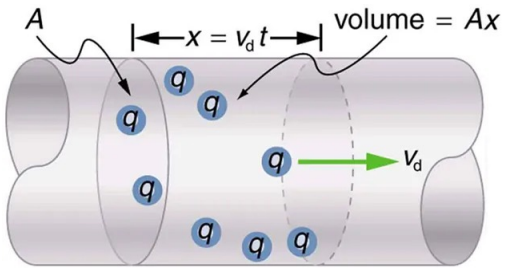


Figure 16.7 All the charges in the shaded volume of this wire move out in a time t, having a drift velocity of magnitude $v_\mathrm{d} = x/t$. See text for further discussion. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a cylindrical section of a pipe that represents the flow of charge, often used to illustrate concepts in electromagnetism.

– The cylinder is oriented horizontally and contains several small circles with the letter “q” inside, representing charge carriers.
– A dashed boundary is drawn along a section of the cylinder, indicating a specific volume.
– An arrow labeled “V_d” points to the right along the cylinder’s axis, showing the direction of the drift velocity of the charge carriers.
– On the left end of the cylinder, the letter “A” is labeled, corresponding to the cross-sectional area of the

cylinder.
– A horizontal bracket spans part of the cylinder's length, where "$x = V_d t$" is labeled, indicating the displacement of the carriers over time.
– Above the cylinder, "volume = A x" is labeled, referring to the calculation of the volume the charge carriers occupy within the marked section.

Overall, this diagram helps visualize the movement of charge within a conductor, emphasizing the relationships between area, drift velocity, and volume.

Note that simple drift velocity is not the entire story. The speed of an electron is much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons? Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as much as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a "sea" of electrons. These free electrons respond by accelerating when an electric field is applied. Of course as they move they collide with the atoms in the lattice and other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

Example 16.3

Calculating Drift Velocity in a Common Wire

Calculate the drift velocity of electrons in a 12-gauge copper wire (which has a diameter of 2.053 mm) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20 A.) The density of copper is $8.80 \times 10^3 \text{kg/m}^3$.

Strategy

We can calculate the drift velocity using the equation $I = nqAv_\text{d}$. The current

$$I = 20.0A$$

is given, and

$$q = -1.60 \times 10^{-19}\text{C}$$

is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $A = \pi r^2$, where r is one-half the given diameter, 2.053 mm. We are given the density of copper, $8.80 \times 10^3 \text{kg/m}^3$, and the periodic table shows that the atomic mass of copper is 63.54 g/mol. We can use these two quantities along with Avogadro's number, $6.02 \times 10^{23} \text{atoms/mol}$, to determine n, the number of free electrons per cubic meter.

Solution

First, calculate the density of free electrons in copper. There is one free electron per

copper atom. Therefore, is the same as the number of copper atoms per m^3. We can now find n as follows:

$$n = \frac{1\,e^-}{\text{atom}} \times \frac{6.\ 02\ \times 10^{\ 23}\,\text{atoms}}{\text{mol}} \times \frac{1mol}{63\ .\ 54\ \text{g}} \times \frac{1000\ \text{g}}{\text{kg}} \times \frac{8.80\ \times 10^{\ 3}\text{kg}}{1\ \text{m}^{\ 3}}$$

$$= 8\ .\ 342\ \times 10^{\ 28}\,e^-/\text{m}^{\ 3}.$$

16.9

The cross-sectional area of the wire is

$$A = \pi r^2$$

$$= \pi\left(\frac{2.053 \times 10^{\ -3}\text{m}}{2}\right)^2$$

$$= 3.310\ \times 10^{\ -6}\,\text{m}^{\ 2}.$$

16.10

Rearranging $I = nqAv_\text{d}$ to isolate drift velocity gives

$$v_\text{d} = \frac{I}{nqA}$$

$$= \frac{20.0\ \text{A}}{(8.\ 342\ \times 10^{\ 28}/\text{m}^{\ 3})(-1\ .\ 60\ \times 10^{\ -19}\,\text{C}\ )(3.\ 310\ \times 10^{\ -6}\,\text{m}^{\ 2})}$$

$$= -4\ .\ 53\ \times 10^{\ -4}\,\text{m/s}.$$

16.11

Discussion

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of 10^{-4}m/s) confirms that the signal moves on the order of 10^{12} times faster (about 10^{8}m/s) than the charges that carry it.

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16.2 OHM'S LAW: RESISTANCE AND SIMPLE CIRCUITS

Learning Objectives

By the end of this section, you will be able to:

- Explain the origin of Ohm's law.
- Calculate voltages, currents, or resistances with Ohm's law.
- Explain what an ohmic material is.
- Describe a simple circuit.

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

Ohm's Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V.$$
16.12

This important relationship is known as Ohm's law. It can be viewed as a cause-and-effect relationship, with

voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called resistance R. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

$$I \propto \frac{1}{R}.$$
16.13

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

$$I = \frac{V}{R}.$$
16.14

This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called ohmic. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I. An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an ohm and is given the symbol Ω (upper case Greek omega). Rearranging $I = \mathrm{V/R}$ gives $R = \mathrm{V/I}$, and so the units of resistance are 1 ohm = 1 volt per ampere:

$$1\ \Omega = \ 1\ \frac{V}{A}.$$
16.15

Figure 16.8 shows the schematic for a simple circuit. A simple circuit has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R.

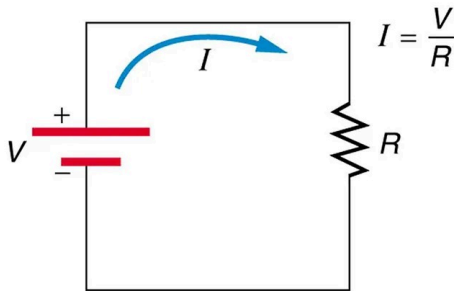


Figure 16.8 A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a simple electrical circuit diagram showcasing Ohm's Law. The diagram includes:

- A battery represented by two parallel lines with the positive side on top, labeled with a plus symbol, and the negative side on the bottom, labeled with a minus symbol. The voltage is indicated by the letter "V".
- A resistor, illustrated as a zigzag line, labeled with the letter "R".
- An arrow indicating the direction of current flow, represented by "I", flowing from the positive to the negative side of the battery.
- An equation shown to the right of the circuit, stating Ohm's Law: $I = V/R$, where "I" is current, "V" is voltage, and "R" is resistance.

Example 16.4

Calculating Resistance: An Automobile Headlight

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Strategy

We can rearrange Ohm's law as stated by $I = \mathrm{V}/\mathrm{R}$ and use it to find the resistance.

Solution

Rearranging $I = \mathrm{V}/\mathrm{R}$ and substituting known values gives

$$R = \frac{V}{I} = \frac{12.0\ \mathrm{V}}{2.50\ \mathrm{A}} = 4.80\ \Omega.$$

16.16

Discussion

This is a relatively small resistance, but it is larger than the cold resistance of the headlight. As we shall see in 16.3 Resistance and Resistivity, resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\Omega$ or more. A dry person may have a hand-to-foot resistance of $10^{5}\Omega$, whereas the resistance of the human heart is about $10^{3}\Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\Omega$, and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed, as will be seen in 16.3 Resistance and Resistivity.

Additional insight is gained by solving $I = \mathrm{V}/\mathrm{R}$ for V, yielding

$$V = \mathrm{IR}.$$
16.17

This expression for V can be interpreted as the *voltage drop across a resistor produced by the flow of current* I. The phrase IR *drop* is often used for this voltage. For instance, the headlight in Example 16.4 has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $\mathrm{PE} = q\Delta V$, and the same q flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See Figure 16.9.)

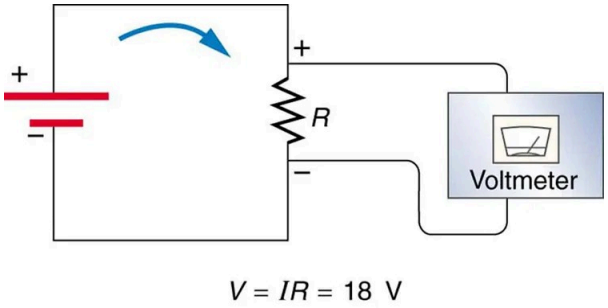


Figure 16.9 The voltage drop across a resistor in a simple circuit equals the voltage output of the battery. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows an electrical circuit diagram with a battery, a resistor symbolized as *R*, and a voltmeter. The battery is on the left side, marked with positive and negative signs, indicating the direction of current flow, which is shown by a blue arrow. The current flows from the positive terminal of the battery, through the resistor, and back to the negative terminal. The voltmeter is connected in parallel with the resistor on the right side of the circuit to measure the voltage across it. Below the circuit, the equation $V = IR = 18\ V$ is displayed, representing Ohm's law, where V is the voltage, I is the current, and R is the resistance.

PhET Explorations

Making Connections: Conservation of Energy

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

PhET Explorations

Ohm's Law

See how the equation form of Ohm's law relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.

Access multimedia content

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16.3 RESISTANCE AND RESISTIVITY

Learning Objectives

By the end of this section, you will be able to:

- Explain the concept of resistivity.
- Use resistivity to calculate the resistance of specified configurations of material.
- Use the thermal coefficient of resistivity to calculate the change of resistance with temperature.

Material and Shape Dependence of Resistance

The resistance of an object depends on its shape and the material of which it is composed. The cylindrical resistor in Figure 16.10 is easy to analyze, and, by so doing, we can gain insight into the resistance of more complicated shapes. As you might expect, the cylinder's electric resistance R is directly proportional to its length L, similar to the resistance of a pipe to fluid flow. The longer the cylinder, the more collisions charges will make with its atoms. The greater the diameter of the cylinder, the more current it can carry (again similar to the flow of fluid through a pipe). In fact, R is inversely proportional to the cylinder's cross-sectional area A .

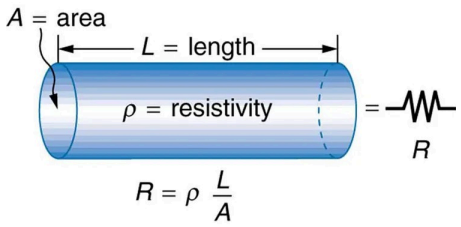


Figure 16.10 A uniform cylinder of length L and cross-sectional area A. Its resistance to the flow of current is similar to the resistance posed by a pipe to fluid flow. The longer the cylinder, the greater its resistance. The larger its cross-sectional area A, the smaller its resistance. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a conceptual illustration of electrical resistivity in a cylindrical conductor. The following elements are shown in the image:

- A blue-colored cylinder representing a wire or conductor, which is oriented horizontally.
- Labels with arrows pointing to different parts of the cylinder:
  - *A = area*: Arrow pointing to the cross-section of the cylinder, indicating its circular surface area.
  - *L = length*: Arrow pointing parallel to the cylinder's long axis, indicating its length.
  - *ρ = resistivity*: Text near the cylinder, indicating the material property known as resistivity.
- An electrical resistor symbol is shown on the right end of the cylinder, labeled as *R*.
- Below the cylinder, the formula *R = ρ (L/A)* is presented, indicating the relationship between resistance (R), resistivity (ρ), length (L), and area (A).

For a given shape, the resistance depends on the material of which the object is composed. Different materials offer different resistance to the flow of charge. We define the resistivity ρ of a substance so that the **resistance** R of an object is directly proportional to ρ. Resistivity ρ is an *intrinsic* property of a material, independent of its shape or size. The resistance R of a uniform cylinder of length L, of cross-sectional area A, and made of a material with resistivity ρ, is

$$R = \frac{\rho L}{A}.$$

16.18

Table 16.1 gives representative values of ρ. The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivities. Conductors have the smallest resistivities, and insulators have the largest; semiconductors have intermediate resistivities. Conductors have varying but large free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than

conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as will be explored in later chapters.

Table 16.1 Resistivities ρ of Various materials at $20^{\circ}\ \mathrm{C}$

Material	**Resistivity $\rho\,(\,\Omega\cdot\mathrm{m}\,)$**
Conductors	
Silver	1.59×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.65×10^{-8}
Tungsten	5.6×10^{-8}
Iron	9.71×10^{-8}
Platinum	10.6×10^{-8}
Steel	20×10^{-8}
Lead	22×10^{-8}
Manganin (Cu, Mn, Ni alloy)	44×10^{-8}
Constantan (Cu, Ni alloy)	49×10^{-8}
Mercury	96×10^{-8}
Nichrome (Ni, Fe, Cr alloy)	100×10^{-8}
Semiconductors[1]	
Carbon (pure)	3.5×10^{-5}
Carbon	$(3.5-60)\times10^{-5}$
Germanium (pure)	600×10^{-3}
Germanium	$(1-600)\times10^{-3}$
Silicon (pure)	2300
Silicon	$0.1\text{–}2300$
Insulators	
Amber	5×10^{14}
Glass	$10^{9}-10^{14}$
Lucite	$>10^{13}$
Mica	$10^{11}-10^{15}$
Quartz (fused)	75×10^{16}

Material	**Resistivity ρ ($\Omega \cdot \mathrm{m}$)**
Rubber (hard)	$10^{13} - 10^{16}$
Sulfur	10^{15}
Teflon	$> 10^{13}$
Wood	$10^{8} - 10^{11}$

1. Values depend strongly on amounts and types of impurities

Example 16.5

Calculating Resistor Diameter: A Headlight Filament

A car headlight filament is made of tungsten and has a cold resistance of 0.350Ω. If the filament is a cylinder 4.00 cm long (it may be coiled to save space), what is its diameter?

Strategy

We can rearrange the equation $R = \frac{\rho L}{A}$ to find the cross-sectional area A of the filament from the given information. Then its diameter can be found by assuming it has a circular cross-section.

Solution

The cross-sectional area, found by rearranging the expression for the resistance of a cylinder given in $R = \frac{\rho L}{A}$, is

$$A = \frac{\rho L}{R}.$$

16.19

Substituting the given values, and taking ρ from Table 16.1, yields

$$\begin{aligned} A &= \frac{(5.6 \times 10^{-8}\Omega \cdot \text{m})\,(4.00 \times 10^{-2}\ \text{m})}{0.350\ \Omega} \\ &= 6.40 \times 10^{-9}\ \text{m}^2. \end{aligned}$$

16.20

The area of a circle is related to its diameter D by

$$A = \frac{\pi D^2}{4}.$$

16.21

Solving for the diameter D, and substituting the value found for A, gives

$$D = 2\left(\frac{A}{\pi}\right)^{\frac{1}{2}} = 2\left(\frac{6.40 \times 10^{-9}\text{m}^2}{3.14}\right)^{\frac{1}{2}}$$
$$= 9.0 \times 10^{-5}\text{m}.$$

16.22

Discussion

The diameter is just under a tenth of a millimeter. It is quoted to only two digits, because ρ is known to only two digits.

Temperature Variation of Resistance

The resistivity of all materials depends on temperature. Some even become superconductors (zero resistivity) at very low temperatures. (See Figure 16.11.) Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal make more collisions, effectively making the resistivity higher. Over relatively small temperature changes (about $100^{\circ}\ \text{C}$ or less), resistivity ρ varies with temperature change ΔT as expressed in the following equation

$$\rho = \rho_0\,(1\ + \alpha\Delta T\,),$$

16.23

where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity. (See the values of α in Table 16.2 below.) For larger temperature changes, α may vary or a nonlinear equation may be needed to find ρ. Note that α is positive for metals, meaning their resistivity increases with temperature. Some alloys have been developed specifically to have a small temperature dependence. Manganin (which is made of copper, manganese and nickel), for example, has α close to zero (to three digits on the scale in Table 16.2), and so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard, for example.

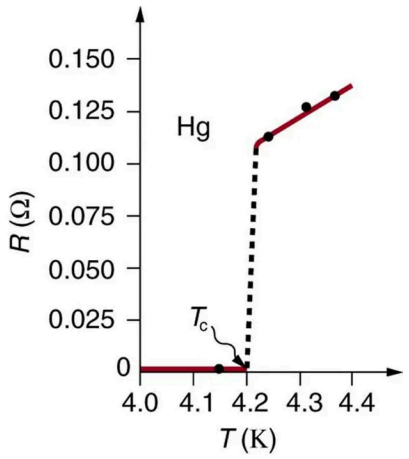


Figure 16.11 The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph showing the relationship between resistance (R) in ohms (Ω) and temperature (T) in Kelvin (K) for Mercury (Hg). The x-axis represents temperature, ranging from 4.0 K to 4.4 K. The y-axis represents resistance, ranging from 0 to 0.150 Ω.

There is a red line on the graph that illustrates the data, which is marked with black dots at key points. The graph shows a horizontal line near zero resistance from 4.0 K to approximately 4.15 K, indicating zero resistance in this temperature range.

At around 4.15 K, marked as T_c, there is a sudden upward change, where the resistance sharply increases. A dashed line is used to connect this jump. After the transition, the red line continues upward with increasing resistance as temperature increases.

Table 16.2 Temperature Coefficients of Resistivity α

Material	**Coefficient α(1/°C)**[2]
Conductors	
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.93×10^{-3}
Lead	3.9×10^{-3}
Manganin (Cu, Mn, Ni alloy)	0.000×10^{-3}
Constantan (Cu, Ni alloy)	0.002×10^{-3}
Mercury	0.89×10^{-3}
Nichrome (Ni, Fe, Cr alloy)	0.4×10^{-3}
Semiconductors	
Carbon (pure)	-0.5×10^{-3}
Germanium (pure)	-50×10^{-3}
Silicon (pure)	-70×10^{-3}

Note also that α is negative for the semiconductors listed in Table 16.2, meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing ρ with temperature is also related to the type and amount of impurities present in the semiconductors.

The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ. For a cylinder we know $R = \rho L/A$, and so, if L and A do not change greatly with temperature, R will have the same temperature dependence as ρ. (Examination of the coefficients of linear expansion shows them to be about

2. Values at 20°C.

two orders of magnitude less than typical temperature coefficients of resistivity, and so the effect of temperature on L and A is about two orders of magnitude less than on ρ.) Thus,

$$R = R_0 (1 + \alpha \Delta T)$$
16.24

is the temperature dependence of the resistance of an object, where R_0 is the original resistance and R is the resistance after a temperature change ΔT. Numerous thermometers are based on the effect of temperature on resistance. (See Figure 16.12.) One of the most common is the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.

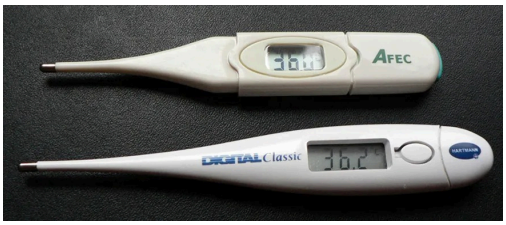


Figure 16.12 These familiar thermometers are based on the automated measurement of a thermistor's temperature-dependent resistance. Image from OpenStax College Physics 2e, CC-BY 4.0

Example 16.6

Calculating Resistance: Hot-Filament Resistance

Although caution must be used in applying $\rho = \rho_0 (1 + \alpha \Delta T)$ and $R = R_0 (1 + \alpha \Delta T)$ for temperature changes greater than $100^\circ\ \text{C}$, for tungsten the equations work reasonably well for very large temperature changes. What, then, is the resistance of the tungsten filament in the previous example if its temperature is increased from room temperature ($20^\circ\ \text{C}$) to a typical operating temperature of $2850^\circ\ \text{C}$?

Strategy

This is a straightforward application of $R = R_0 (1 + \alpha \Delta T)$, since the original resistance of the filament was given to be $R_0 = 0.350\ \Omega$, and the temperature change is $\Delta T = 2830^\circ\ \text{C}$.

Solution

The hot resistance R is obtained by entering known values into the above equation:

$$\begin{aligned} R &= R_0 (1 + \alpha \Delta T) \\ &= (0.350\ \Omega)\left[1 + (4.5 \times 10^{-3}/^\circ\ \text{C})\,(2830^\circ\ \text{C})\right] \\ &= 4.8\ \Omega. \end{aligned}$$

16.25

Discussion

This value is consistent with the headlight resistance example in 16.2 Ohm's Law: Resistance and Simple Circuits.

PhET Explorations

Resistance in a Wire

Learn about the physics of resistance in a wire. Change its resistivity, length, and area to see how they affect the wire's resistance. The sizes of the symbols in the equation change along with the diagram of a wire.

Access multimedia content

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16.4 ELECTRIC POWER AND ENERGY

Learning Objectives

By the end of this section, you will be able to:

- Calculate the power dissipated by a resistor and power supplied by a power supply.
- Calculate the cost of electricity under various circumstances.

Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for electric power? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See Figure 16.13(a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?

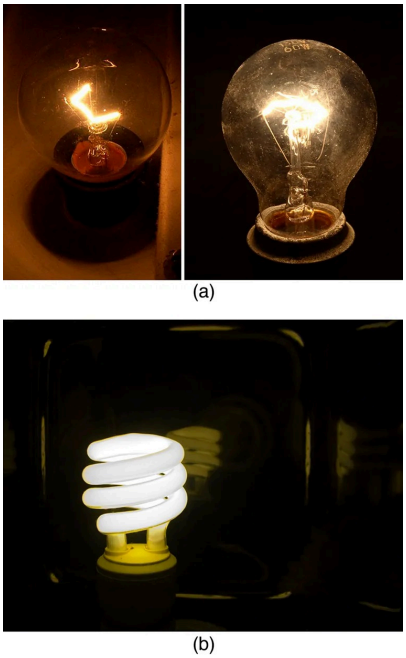


Figure 16.13 (a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why? (credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b) This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. Image from OpenStax College Physics 2e, CC-BY 4.0

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $\mathrm{PE} = \mathrm{qV}$, where q is the charge moved and V is the voltage (or more precisely, the potential difference the charge moves through). Power is the rate at which energy is moved, and so electric power is

$$P = \frac{\mathrm{PE}}{t} = \frac{\mathrm{qV}}{t}.$$
16.26

Recognizing that current is $I = q/t$ (note that $\Delta t = t$ here), the expression for power becomes

$$P = \mathrm{IV}.$$
16.27

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1\ \mathrm{A}\ \cdot \mathrm{V} = \ 1\ \mathrm{W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power

$P = \text{IV} = (20\ \text{A})(12\ \text{V}) = 240\ \text{W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1\ \text{kA} \cdot \text{V} = 1\ \text{kW}$).

To see the relationship of power to resistance, we combine Ohm's law with $P = \text{IV}$. Substituting $I = \text{V}/\text{R}$ gives $P = (V/R)V = V^2/R$. Similarly, substituting $V = \text{IR}$ gives $P = I(\text{IR}) = I^2R$. Three expressions for electric power are listed together here for convenience:

$$P = \text{IV}$$
16.28

$$P = \frac{V^2}{R}$$
16.29

$$P = I^2R.$$
16.30

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

Example 16.7

Calculating Power Dissipation and Current: Hot and Cold Power

(a) Consider the examples given in 16.2 Ohm's Law: Resistance and Simple Circuits and 16.3 Resistance and Resistivity. Then find the power dissipated by the car headlight in these examples, both when it is hot and when it is cold. (b) What current does it draw when cold?

Strategy for (a)

For the hot headlight, we know voltage and current, so we can use $P = \mathrm{IV}$ to find the power. For the cold headlight, we know the voltage and resistance, so we can use $P = V^2/R$ to find the power.

Solution for (a)

Entering the known values of current and voltage for the hot headlight, we obtain

$$P = \mathrm{IV} = (2.50\ \mathrm{A})\,(12.0\ \mathrm{V}) = \ 30.0\ \mathrm{W}.$$

16.31

The cold resistance was 0.350Ω, and so the power it uses when first switched on is

$$P = \frac{V^2}{R} = \frac{(12.0\ \mathrm{V})^2}{0.350\Omega} = \ 411\ \mathrm{W}.$$

16.32

Discussion for (a)

The 30 W dissipated by the hot headlight is typical. But the 411 W when cold is

surprisingly higher. The initial power quickly decreases as the bulb's temperature increases and its resistance increases.

Strategy and Solution for (b)

The current when the bulb is cold can be found several different ways. We rearrange one of the power equations, $P = I^2R$, and enter known values, obtaining

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{411\ \text{W}}{0.350\ \Omega}} = 34.3\ \text{A}.$$

16.33

Discussion for (b)

The cold current is remarkably higher than the steady-state value of 2.50 A, but the current will quickly decline to that value as the bulb's temperature increases. Most fuses and circuit breakers (used to limit the current in a circuit) are designed to tolerate very high currents briefly as a device comes on. In some cases, such as with electric motors, the current remains high for several seconds, necessitating special "slow blow" fuses.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E/t$, we see that

$$E = \text{Pt}$$

16.34

is the energy used by a device using power P for a time interval t. For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour ($\text{kW} \cdot \text{h}$), consistent with the relationship $E = \text{Pt}$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized

energy units such as food calories, can be converted to joules. You can prove to yourself that $1\,\text{kW}\cdot\text{h} = 3.6\times10^{6}\,\text{J}\cdot$

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See Figure 16.13(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

Making Connections: Energy, Power, and Time

The relationship $E = \text{Pt}$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

Example 16.8

Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs $1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?

Strategy

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

Solution for (a)

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

$$E = \mathrm{Pt} = (60\ \mathrm{W})\,(1000\ \mathrm{h}) = \ 60,000\ \mathrm{W}\ \cdot\ \mathrm{h}.$$

16.35

In kilowatt-hours, this is

$$E = \ 60.0\ \mathrm{kW}\ \cdot\ \mathrm{h}.$$

16.36

Now the electricity cost is

$$\mathrm{cost} = (60.0\ \mathrm{kW}\cdot\mathrm{h})\,(\$0.12/\mathrm{kW}\cdot\mathrm{h}) = \$7.20.$$

16.37

The total cost, including the cost of the bulb, will be $7.45 for 1000 hours (about one-half year at 5 hours per day).

Solution for (b)

Since the CFL uses only 15 W and not 60 W, the electricity cost will be $7.20/4 = $1.80. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or 0.1($1.50) = $0.15. Therefore, the total cost will be $1.95 for 1000 hours.

Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P = \mathrm{IV}$. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

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16.5 ALTERNATING CURRENT VERSUS DIRECT CURRENT

Learning Objectives

By the end of this section, you will be able to:

- Explain the differences and similarities between AC and DC current.
- Calculate rms voltage, current, and average power.
- Explain why AC current is used for power transmission.

Alternating Current

Most of the examples dealt with so far, and particularly those utilizing batteries, have constant voltage sources. Once the current is established, it is thus also a constant. Direct current (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Most well-known applications, however, use a time-varying voltage source. Alternating current (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating current circuit. Examples include the commercial and residential power that serves so many of our needs. Figure 16.14 shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world.

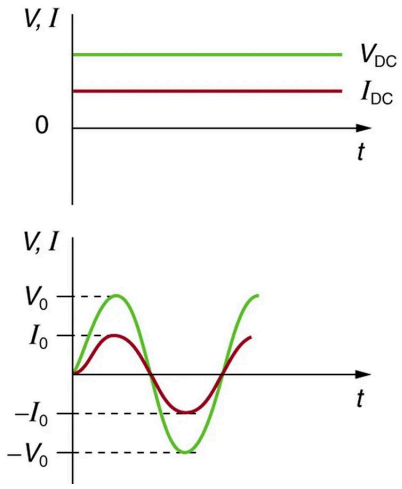


Figure 16.14 (a) DC voltage and current are constant in time, once the current is established. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two graphs depicting voltage and current over time.

The top graph shows a Direct Current (DC) representation. It has a horizontal axis labeled "t" for time, and a vertical axis labeled "V, I" for voltage and current. Two straight horizontal lines are in the graph: a green line labeled "V_{DC}" and a red line labeled "I_{DC}", both lines are parallel to the time axis, indicating constant voltage and current.

The bottom graph shows an Alternating Current (AC) representation. It also has a horizontal axis labeled "t" and a vertical axis labeled "V, I". The graph contains two sinusoidal waves: a green wave representing voltage and a red wave representing current. These waves oscillate above and below the horizontal axis. Key points along the vertical axis are marked with dashed lines and labeled as "V_0", "I_0", "$-I_0$", and "$-V_0$", indicating the peak and negative peak values of the waveform.

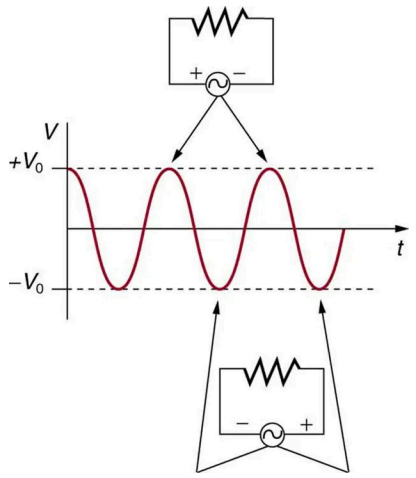


Figure 16.15 The potential difference V between the terminals of an AC voltage source fluctuates as shown. The mathematical expression for V is given by $V = V_0 \sin 2\pi ft$. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates an electrical circuit and waveform diagram. At the top, there is a diagram of an electrical circuit containing a resistor connected in series with an AC voltage source indicated by a sine wave symbol with positive and negative terminals.

Below the circuit diagram is a graph showing a sinusoidal waveform. The graph has a vertical axis labeled 'V' and a horizontal axis labeled 't'. The waveform oscillates symmetrically around the horizontal center line.

The maximum and minimum values of the waveform are marked by dashed lines labeled '+V_0' and '-V_0', respectively. Arrows point from the circuit diagram to the positive and negative peaks of the waveform.

A similar circuit diagram is repeated at the bottom, inverted in orientation, with arrows pointing upwards towards the graph.

Figure 16.15 shows a schematic of a simple circuit with an AC voltage source. The voltage between the terminals fluctuates as shown, with the AC voltage given by

$$V = V_0 \sin 2\pi ft,$$
16.38

where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz. For this simple resistance circuit, $I = \mathrm{V}/\mathrm{R}$, and so the AC current is

$$I = I_0 \sin 2\pi \mathrm{ft},$$
16.39

where I is the current at time t, and $I_0 = V_0/\mathrm{R}$ is the peak current. For this example, the voltage and current are said to be in phase, as seen in Figure 16.14(b).

Current in the resistor alternates back and forth just like the driving voltage, since $I = \mathrm{V}/\mathrm{R}$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see a stroboscopic effect evidencing AC. The fact that the light output fluctuates means that the power is fluctuating. The power supplied is $P = \mathrm{IV}$. Using the expressions for I and V above, we see that the time dependence of power is $P = I_0 V_0 \sin^2 2\pi \mathrm{ft}$, as shown in Figure 16.16.

Making Connections: Take-Home Experiment—AC/DC Lights

Wave your hand back and forth between your face and a fluorescent light bulb. Do you observe the same thing with the headlights on your car? Explain what you observe. *Warning: Do not look directly at very bright light.*

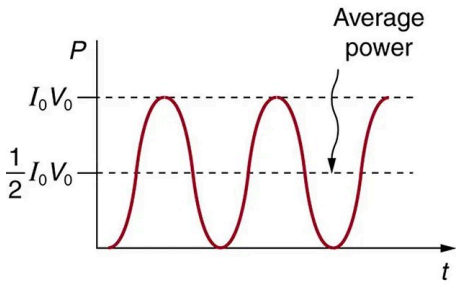


Figure 16.16 AC power as a function of time. Since the voltage and current are in phase here, their product is non-negative and fluctuates between zero and I_0V_0. Average power is $(1/2)I_0V_0$. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph showing a waveform representing power over time. The horizontal axis is labeled “t” for time, and the vertical axis is labeled “P” for power. The waveform is sinusoidal, oscillating between two levels: a higher level labeled as “I_0V_0” and a lower level labeled as “$^1/_2\, I_0V_0$“. A dashed horizontal line indicates the average power level, which is also marked on the graph. An arrow points to this average level with the text “Average power.”

We are most often concerned with average power rather than its fluctuations—that 60-W light bulb in your desk lamp has an average power consumption of 60 W, for example. As illustrated in Figure 16.16, the average power P_{ave} is

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0.$$

16.40

This is evident from the graph, since the areas above and below the $(1/2)I_0V_0$ line are equal, but it can also be proven using trigonometric identities. Similarly, we define an average or rms current I_{rms} and average or rms voltage V_{rms} to be, respectively,

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$

16.41

and

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}.$$

16.42

where rms stands for root mean square, a particular kind of average. In general, to obtain a root mean square, the particular quantity is squared, its mean (or average) is found, and the square root is taken. This is useful for AC, since the average value is zero. Now,

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}},$$

16.43

which gives

$$P_{\text{ave}} = \frac{I_0}{\sqrt{2}} \cdot \frac{V_0}{\sqrt{2}} = \frac{1}{2} I_0 V_0,$$

16.44

as stated above. It is standard practice to quote I_{rms}, V_{rms}, and P_{ave} rather than the peak values. For example, most household electricity is 120 V AC, which means that V_{rms} is 120 V. The common 10-A circuit breaker will interrupt a sustained I_{rms} greater than 10 A. Your 1.0-kW microwave oven consumes $P_{\text{ave}} = 1.0\ \text{kW}$, and so on. You can think of these rms and average values as the equivalent DC values for a simple resistive circuit.

To summarize, when dealing with AC, Ohm's law and the equations for power are completely analogous to those for DC, but rms and average values are used for AC. Thus, for AC, Ohm's law is written

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}.$$
16.45

The various expressions for AC power P_{ave} are

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}},$$
16.46

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R},$$
16.47

and

$$P_{\text{ave}} = I_{\text{rms}}^2 R.$$
16.48

Example 16.9

Peak Voltage and Power for AC

(a) What is the value of the peak voltage for 120-V AC power? (b) What is the peak power consumption rate of a 60.0-W AC light bulb?

Strategy

We are told that V_{rms} is 120 V and P_{ave} is 60.0 W. We can use $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ to find the peak voltage, and we can manipulate the definition of power to find the peak power from the given average power.

Solution for (a)

Solving the equation $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ for the peak voltage V_0 and substituting the known value for V_{rms} gives

$$V_0 = \sqrt{2}V_{\text{rms}} = 1.414\,(120\ \text{V}) = 170\ \text{V}.$$

16.49

Discussion for (a)

This means that the AC voltage swings from 170 V to $-170\ \text{V}$ and back 60 times every second. An equivalent DC voltage is a constant 120 V.

Solution for (b)

Peak power is peak current times peak voltage. Thus,

$$P_0 = I_0 V_0 = 2\left(\frac{1}{2} I_0 V_0\right) = 2P_{\text{ave}}.$$

16.50

We know the average power is 60.0 W, and so

$$P_0 = 2\,(60.0\ \text{W}) = 120\ \text{W}.$$

16.51

Discussion

So the power swings from zero to 120 W one hundred twenty times per second (twice each cycle), and the power averages 60 W.

Why Use AC for Power Distribution?

Most large power-distribution systems are AC. Moreover, the power is transmitted at much higher voltages than the 120-V AC (240 V in most parts of the world) we use in homes and on the job. Economies of scale make it cheaper to build a few very large electric power-generation plants than to build numerous small ones. This necessitates sending power long distances, and it is obviously important that energy losses en route be minimized. High voltages can be transmitted with much smaller power losses than low voltages, as we shall see. (See Figure 16.17.) For safety reasons, the voltage at the user is reduced to familiar values. The crucial factor is that it is much easier to increase and decrease AC voltages than DC, so AC is used in most large power distribution systems.



Figure 16.17 Power is distributed over large distances at high voltage to reduce power loss in the transmission lines. The voltages generated at the power plant are stepped up by passive devices called transformers (see) to 330,000 volts (or more in some places worldwide). At the point of use, the transformers reduce the voltage transmitted for safe residential and commercial use. Image from OpenStax College Physics 2e, CC-BY 4.0

Example 16.10

Power Losses Are Less for High-Voltage Transmission

(a) What current is needed to transmit 100 MW of power at 200 kV? (b) What is the power dissipated by the transmission lines if they have a resistance of 1.00Ω? (c) What percentage of the power is lost in the transmission lines?

Strategy

We are given $P_{\text{ave}} = 100\ \text{MW}$, $V_{\text{rms}} = 200\ \text{kV}$, and the resistance of the lines is $R = 1.00\Omega$. Using these givens, we can find the current flowing (from $P = \text{IV}$) and then the power dissipated in the lines ($P = I^2R$), and we take the ratio to the total power transmitted.

Solution

To find the current, we rearrange the relationship $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}$ and substitute known values. This gives

$$I_{\text{rms}} = \frac{P_{\text{ave}}}{V_{\text{rms}}} = \frac{100\ \times\ 10^6\ \text{W}}{200\ \times\ 10^3\ \text{V}} =\ 500\ \text{A}.$$

16.52

Solution

Knowing the current and given the resistance of the lines, the power dissipated in them is found from $P_{\text{ave}} = I_{\text{rms}}^2R$. Substituting the known values gives

$$P_{\text{ave}} = I_{\text{rms}}^2R = (500\ \text{A})^2\,(1.00\ \Omega\,) =\ 250\ \text{kW}.$$

16.53

Solution

The percent loss is the ratio of this lost power to the total or input power, multiplied by 100:

$$\%\ \text{loss} = \frac{250\ \text{kW}}{100\ \text{MW}} \times 100 = 0.250\ \%.$$

16.54

Discussion

One-fourth of a percent is an acceptable loss. Note that if 100 MW of power had been transmitted at 25 kV, then a current of 4000 A would have been needed. This would result in a power loss in the lines of 16.0 MW, or 16.0% rather than 0.250%. The lower the voltage, the more current is needed, and the greater the power loss in the fixed-resistance transmission lines. Of course, lower-resistance lines can be built, but this requires larger and more expensive wires. If superconducting lines could be economically produced, there would be no loss in the transmission lines at all. But, as we shall see in a later chapter, there is a limit to current in superconductors, too. In short, high voltages are more economical for transmitting power, and AC voltage is much easier to raise and lower, so that AC is used in most large-scale power distribution systems.

It is widely recognized that high voltages pose greater hazards than low voltages. But, in fact, some high voltages, such as those associated with common static electricity, can be harmless. So it is not voltage alone that determines a hazard. It is not so widely recognized that AC shocks are often more harmful than similar DC shocks. Thomas Edison thought that AC shocks were more harmful and set up a DC power-distribution system in New York City in the late 1800s. There were bitter fights, in particular between Edison and George Westinghouse and Nikola Tesla, who were advocating the use of AC in early power-distribution systems. AC has prevailed largely due to transformers and lower power losses with high-voltage transmission.

PhET Explorations

Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

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16.6 ELECTRIC HAZARDS AND THE HUMAN BODY

Learning Objectives

By the end of this section, you will be able to:

- Define thermal hazard, shock hazard, and short circuit.
- Explain what effects various levels of current have on the human body.

There are two known hazards of electricity—thermal and shock. A thermal hazard is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A shock hazard occurs when electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. This section considers these hazards and the various factors affecting them in a quantitative manner.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the short circuit, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in Figure 16.18. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a *short*. Since the resistance of the short, r, is very small, the power dissipated in the short, $P = V^2/r$, is very large. For example, if V is 120 V and r is 0.100Ω, then the power is 144 kW, *much* greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.

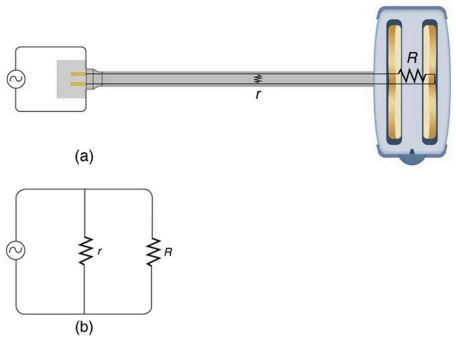


Figure 16.18 A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance r. *Since* $P = V^2/r$, *thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two diagrams, labeled (a) and (b), depicting electrical circuits:

1. Diagram (a):
– Shows a power source on the left, represented by a circle with a sine wave inside.
– Two parallel lines extend from the power source, representing a cable or wire.
– The wire encounters a small resistor labeled “r” and extends to a wider section which is an electrical plug.
– The plug is connected to a larger device on the right, which includes a prominent resistor labeled “R” inside.

2. Diagram (b):
– Displays a simplified schematic of the circuit shown in (a).
– The power source is again denoted by a circle with a sine wave.
– Two parallel lines extend from the power source, indicating wires.
– A small resistor “r” and a larger resistor “R” are shown in parallel configuration.

Overall, the image illustrates how the electrical circuit is arranged both in a real-world scenario (a) and as a simplified circuit diagram (b).

One particularly insidious aspect of a short circuit is that its resistance may actually be decreased due to the increase in temperature. This can happen if the short creates ionization. These charged atoms and molecules are free to move and, thus, lower the resistance r. Since $P = V^2/r$, the power dissipated in the short rises, possibly causing more ionization, more power, and so on. High voltages, such as the 480-V AC used in some

industrial applications, lend themselves to this hazard, because higher voltages create higher initial power production in a short.

Another serious, but less dramatic, thermal hazard occurs when wires supplying power to a user are overloaded with too great a current. As discussed in the previous section, the power dissipated in the supply wires is $P = I^2 R_{\text{w}}$, where R_{w} is the resistance of the wires and I the current flowing through them. If either I or R_{w} is too large, the wires overheat. For example, a worn appliance cord (with some of its braided wires broken) may have $R_{\text{w}} = 2.00\Omega$ rather than the 0.100Ω it should be. If 10.0 A of current passes through the cord, then $P = I^2 R_{\text{w}} = 200\ \text{W}$ is dissipated in the cord—much more than is safe. Similarly, if a wire with a $0.100 - \Omega$ resistance is meant to carry a few amps, but is instead carrying 100 A, it will severely overheat. The power dissipated in the wire will in that case be $P = 1000\ \text{W}$. Fuses and circuit breakers are used to limit excessive currents. (See Figure 16.19 and Figure 16.20.) Each device opens the circuit automatically when a sustained current exceeds safe limits.

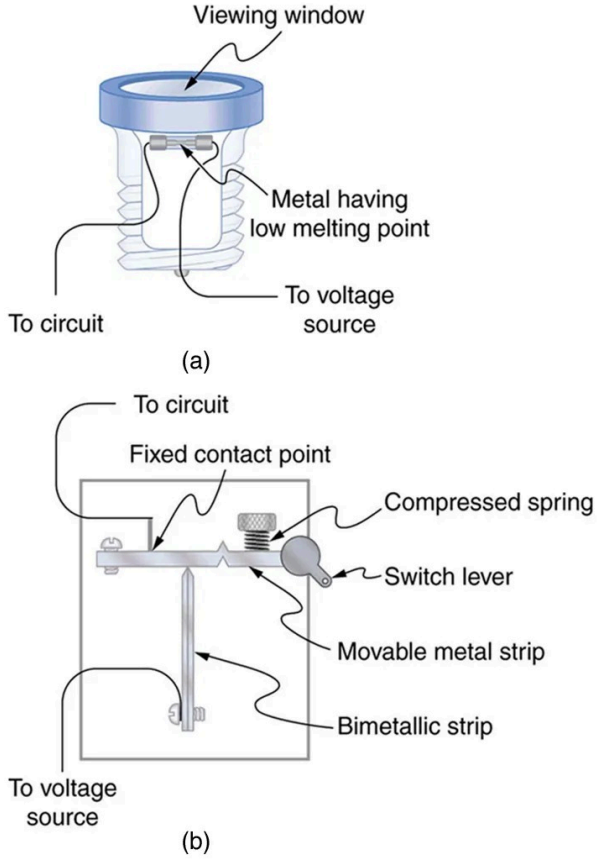


Figure 16.19 (a) A fuse has a metal strip with a low melting point that, when overheated by an excessive current, permanently breaks the connection of a circuit to a voltage source. (b) A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b), illustrating two different electrical components.

(a) The diagram shows a fuse with a transparent viewing window at the top. Inside, a metal with a low melting point is connected between components marked "To circuit" and "To voltage source." The purpose of the metal is to melt if the current is too high, breaking the circuit.

(b) The diagram shows a circuit breaker with various components labeled. A movable metal strip and a bimetallic strip are central to its function. A fixed contact point is connected to the circuit. A compressed spring and a switch lever are also present. The bimetallic strip bends under excessive heat, triggering the switch lever to disconnect the movable strip from the fixed contact point, interrupting the circuit.

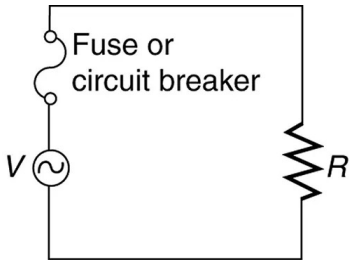


Figure 16.20 Schematic of a circuit with a fuse or circuit breaker in it. Fuses and circuit breakers act like automatic switches that open when sustained current exceeds desired limits. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a simple electrical circuit diagram. The circuit includes three main components:

- An AC voltage source, represented by a circle with a sine wave inside, labeled as "V".
- A fuse or circuit breaker, indicated by a break in the line with a note next to it saying "Fuse or circuit breaker".
- A resistor, symbolized by a jagged line, labeled as "R".

The circuit is a closed loop, with the components connected in series. The voltage source is connected to the fuse or circuit breaker, which is in turn connected to the resistor, completing the loop back to the voltage source.

Fuses and circuit breakers for typical household voltages and currents are relatively simple to produce, but those for large voltages and currents experience special problems. For example, when a circuit breaker tries to interrupt the flow of high-voltage electricity, a spark can jump across its points that ionizes the air in the gap

and allows the current to continue flowing. Large circuit breakers found in power-distribution systems employ insulating gas and even use jets of gas to blow out such sparks. Here AC is safer than DC, since AC current goes through zero 120 times per second, giving a quick opportunity to extinguish these arcs.

Shock Hazards

Electrical currents through people produce tremendously varied effects. An electrical current can be used to block back pain. The possibility of using electrical current to stimulate muscle action in paralyzed limbs, perhaps allowing paraplegics to walk, is under study. TV dramatizations in which electrical shocks are used to bring a heart attack victim out of ventricular fibrillation (a massively irregular, often fatal, beating of the heart) are more than common. Yet most electrical shock fatalities occur because a current put the heart into fibrillation. A pacemaker uses electrical shocks to stimulate the heart to beat properly. Some fatal shocks do not produce burns, but warts can be safely burned off with electric current (though freezing using liquid nitrogen is now more common). Of course, there are consistent explanations for these disparate effects. The major factors upon which the effects of electrical shock depend are

1. The amount of current I
2. The path taken by the current
3. The duration of the shock
4. The frequency f of the current ($f = 0$ for DC)

Table 16.3 gives the effects of electrical shocks as a function of current for a typical accidental shock. The effects are for a shock that passes through the trunk of the body, has a duration of 1 s, and is caused by 60-Hz power.

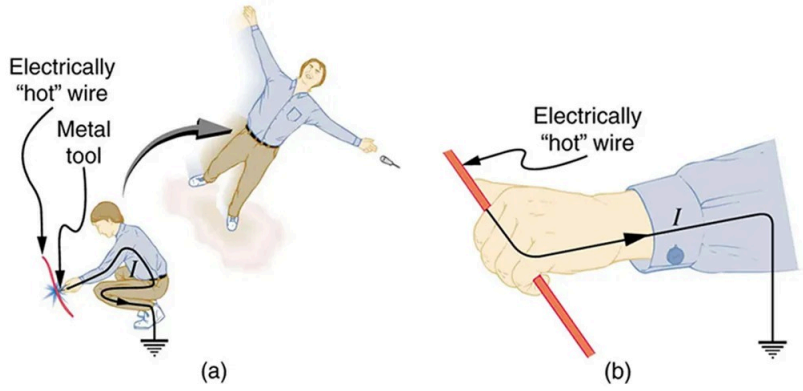


Figure 16.21 An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b), illustrating electrical hazards:

(a) A person is shown crouching and using a metal tool on an electrically "hot" wire. Electricity is depicted as passing through the person's body to the ground, indicating a risk of electrical shock. The person is then shown in a standing position, arms outstretched, suggesting they've been shocked and thrown backward.

(b) A close-up of a hand holding an electrically "hot" wire is shown. An arrow illustrates the path of electrical current flowing through the hand and arm to the ground, again indicating a potential shock hazard.

Table 16.3 Effects of Electrical Shock as a Function of Current[1]

Current (mA)	Effect
1	Threshold of sensation
5	Maximum harmless current
10–20	Onset of sustained muscular contraction; cannot let go for duration of shock; contraction of chest muscles may stop breathing during shock
50	Onset of pain
100–300+	Ventricular fibrillation possible; often fatal
300	Onset of burns depending on concentration of current
6000 (6 A)	Onset of sustained ventricular contraction and respiratory paralysis; both cease when shock ends; heartbeat may return to normal; used to defibrillate the heart

Our bodies are relatively good conductors due to the water in our bodies. Given that larger currents will flow through sections with lower resistance (to be further discussed in the next chapter), electric currents preferentially flow through paths in the human body that have a minimum resistance in a direct path to earth. The earth is a natural electron sink. Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills), or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently

1. For an average male shocked through trunk of body for 1 s by 60-Hz AC. Values for females are 60–80% of those listed.

harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 10 to 20 mA and above, the current can stimulate sustained muscular contractions much as regular nerve impulses do. People sometimes say they were knocked across the room by a shock, but what really happened was that certain muscles contracted, propelling them in a manner not of their own choosing. (See Figure 16.21(a).) More frightening, and potentially more dangerous, is the "can't let go" effect illustrated in Figure 16.21(b). The muscles that close the fingers are stronger than those that open them, so the hand closes involuntarily on the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer's hand may close about the victim's wrist. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Modern electric fences, used in animal enclosures, are now pulsed on and off to allow people who touch them to get free, rendering them less lethal than in the past.

Greater currents may affect the heart. Its electrical patterns can be disrupted, so that it beats irregularly and ineffectively in a condition called "ventricular fibrillation." This condition often lingers after the shock and is fatal due to a lack of blood circulation. The threshold for ventricular fibrillation is between 100 and 300 mA. At about 300 mA and above, the shock can cause burns, depending on the concentration of current—the more concentrated, the greater the likelihood of burns.

Very large currents cause the heart and diaphragm to contract for the duration of the shock. Both the heart and breathing stop. Interestingly, both often return to normal following the shock. The electrical patterns on the heart are completely erased in a manner that the heart can start afresh with normal beating, as opposed to the permanent disruption caused by smaller currents that can put the heart into ventricular fibrillation. The latter is something like scribbling on a blackboard, whereas the former completely erases it. TV dramatizations of electric shock used to bring a heart attack victim out of ventricular fibrillation also show large paddles. These are used to spread out current passed through the victim to reduce the likelihood of burns.

Current is the major factor determining shock severity (given that other conditions such as path, duration, and frequency are fixed, such as in the table and preceding discussion). A larger voltage is more hazardous, but since $I = \mathrm{V}/\mathrm{R}$, the severity of the shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance of about $200\ \mathrm{k}\Omega$. If he comes into contact with 120-V AC, a current $I = (120\ \mathrm{V})/(200\ \mathrm{k}\ \Omega) = 0.6\ \mathrm{mA}$ passes harmlessly through him. The same person soaking wet may have a resistance of $10.0\ \mathrm{k}\Omega$ and the same 120 V will produce a current of 12 mA—above the "can't let go" threshold and potentially dangerous.

Most of the body's resistance is in its dry skin. When wet, salts go into ion form, lowering the resistance significantly. The interior of the body has a much lower resistance than dry skin because of all the ionic solutions and fluids it contains. If skin resistance is bypassed, such as by an intravenous infusion, a catheter, or exposed pacemaker leads, a person is rendered microshock sensitive. In this condition, currents about 1/1000

those listed in Table 16.3 produce similar effects. During open-heart surgery, currents as small as $20\ \mu\text{A}$ can be used to still the heart. Stringent electrical safety requirements in hospitals, particularly in surgery and intensive care, are related to the doubly disadvantaged microshock-sensitive patient. The break in the skin has reduced his resistance, and so the same voltage causes a greater current, and a much smaller current has a greater effect.

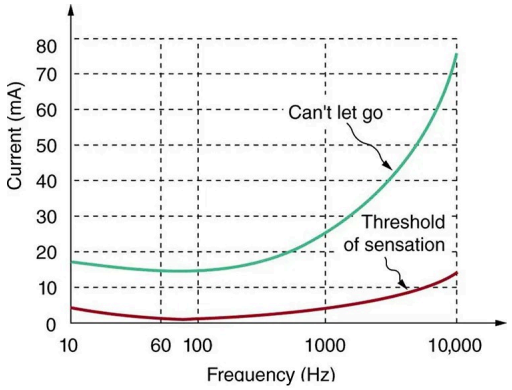


Figure 16.22 Graph of average values for the threshold of sensation and the "can't let go" current as a function of frequency. The lower the value, the more sensitive the body is at that frequency. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a graph showing the relationship between frequency (in Hertz) and current (in milliamperes). The x-axis represents frequency, ranging from 10 Hz to 10,000 Hz, and the y-axis represents current, ranging from 0 mA to 80 mA.

There are two curves plotted on the graph:

- *Threshold of sensation:* This is a lower, red curve indicating the current levels at which sensation begins to be felt. It starts just above 0 mA and gradually increases as frequency increases.
- *Can't let go:* This is a higher, green curve indicating the current levels at which a person cannot voluntarily let go. It starts at around 20 mA and increases sharply, reaching higher values as the frequency increases.

Both curves demonstrate a positive correlation between frequency and current, with the "Can't let go" curve being significantly higher than the "Threshold of sensation" curve across the frequency spectrum.

Factors other than current that affect the severity of a shock are its path, duration, and AC frequency. Path has obvious consequences. For example, the heart is unaffected by an electric shock through the brain, such as

may be used to treat manic depression. And it is a general truth that the longer the duration of a shock, the greater its effects. Figure 16.22 presents a graph that illustrates the effects of frequency on a shock. The curves show the minimum current for two different effects, as a function of frequency. The lower the current needed, the more sensitive the body is at that frequency. Ironically, the body is most sensitive to frequencies near the 50- or 60-Hz frequencies in common use. The body is slightly less sensitive for DC ($f = 0$), mildly confirming Edison's claims that AC presents a greater hazard. At higher and higher frequencies, the body becomes progressively less sensitive to any effects that involve nerves. This is related to the maximum rates at which nerves can fire or be stimulated. At very high frequencies, electrical current travels only on the surface of a person. Thus a wart can be burned off with very high frequency current without causing the heart to stop. (Do not try this at home with 60-Hz AC!) Some of the spectacular demonstrations of electricity, in which high-voltage arcs are passed through the air and over people's bodies, employ high frequencies and low currents. (See Figure 16.23.)

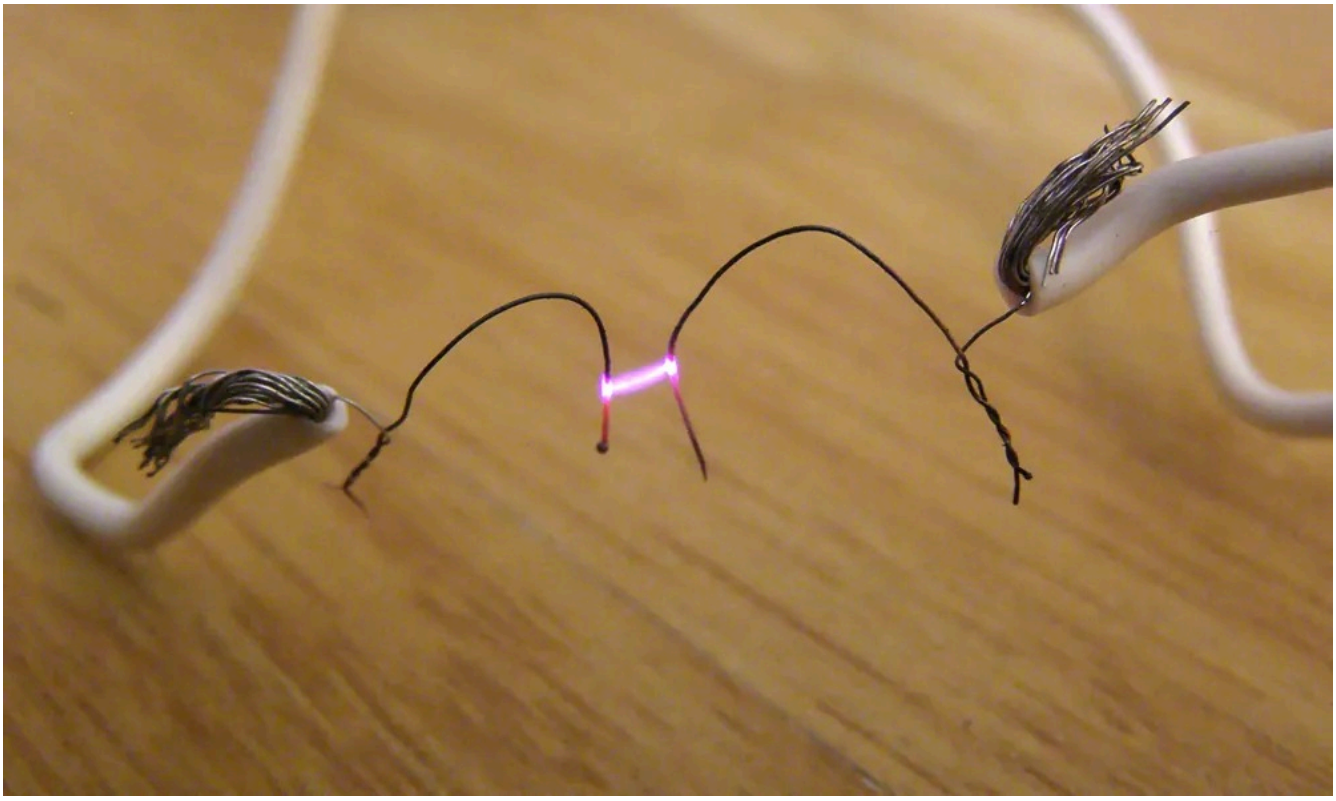


Figure 16.23 Is this electric arc dangerous? The answer depends on the AC frequency and the power involved. Image by Khimich Alex, Wikimedia Commons License

Image Description

The image shows a close-up of two white wires with exposed, twisted ends positioned towards each other. Between the wires is a small, glowing pink spark of electricity, indicating a short arc.

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16.7 NERVE CONDUCTION–ELECTROCARDIOGRAMS

Learning Objectives

By the end of this section, you will be able to:

- Explain the process by which electric signals are transmitted along a neuron.
- Explain the effects myelin sheaths have on signal propagation.
- Explain what the features of an ECG signal indicate.

Nerve Conduction

Electric currents in the vastly complex system of billions of nerves in our body allow us to sense the world, control parts of our body, and think. These are representative of the three major functions of nerves. First, nerves carry messages from our sensory organs and others to the central nervous system, consisting of the brain and spinal cord. Second, nerves carry messages from the central nervous system to muscles and other organs. Third, nerves transmit and process signals within the central nervous system. The sheer number of nerve cells and the incredibly greater number of connections between them makes this system the subtle wonder that it is. Nerve conduction is a general term for electrical signals carried by nerve cells. It is one aspect of bioelectricity, or electrical effects in and created by biological systems.

Nerve cells, properly called *neurons*, look different from other cells—they have tendrils, some of them many centimeters long, connecting them with other cells. (See Figure 16.24.) Signals arrive at the cell body across *synapses* or through *dendrites*, stimulating the neuron to generate its own signal, sent along its long *axon* to other nerve or muscle cells. Signals may arrive from many other locations and be transmitted to yet others, conditioning the synapses by use, giving the system its complexity and its ability to learn.

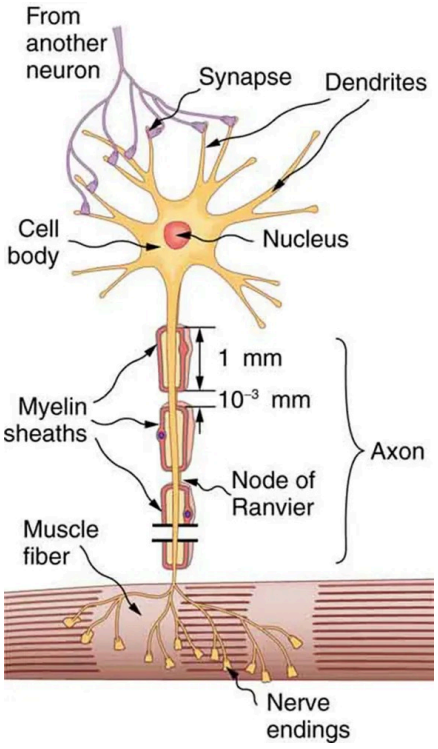


Figure 16.24 A neuron with its dendrites and long axon. Signals in the form of electric currents reach the cell body through dendrites and across synapses, stimulating the neuron to generate its own signal sent down the axon. The number of interconnections can be far greater than shown here. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a labeled diagram of a neuron, illustrating its key components and their functions. Here is a description of the elements within the diagram:

- **From another neuron:** This indicates connections from a neighboring neuron.
- **Dendrites:** Branch-like structures that receive signals from other neurons.
- **Cell body:** The central part of the neuron containing the nucleus.
- **Nucleus:** Located within the cell body, housing the cell's genetic material.
- **Synapse:** The junction where signals are transmitted between neurons.
- **Axon:** A long, slender projection that conducts electrical impulses away from the cell body.
- **Myelin sheaths:** Insulating layers surrounding the axon to enhance signal transmission speed.
- **Node of Ranvier:** Gaps in the myelin sheath where ion exchange occurs, helping in rapid signal conduction.
- **Muscle fiber:** Located at the end of the neuron, representing the connection to muscles.
- **Nerve endings:** Terminal points where the neuron communicates with other cells or tissues.

The diagram also includes scale markers showing distances along the axon:

- 1 mm: This label applies to the length of the axon being depicted.
- 10^{-3} mm: This smaller scale is marked under a segment of the axon, illustrating the micro-scale of some neuron components.

The method by which these electric currents are generated and transmitted is more complex than the simple movement of free charges in a conductor, but it can be understood with principles already discussed in this text. The most important of these are the Coulomb force and diffusion.

Figure 16.25 illustrates how a voltage (potential difference) is created across the cell membrane of a neuron in its resting state. This thin membrane separates electrically neutral fluids having differing concentrations of ions, the most important varieties being Na^+, K^+, and Cl^- (these are sodium, potassium, and chlorine ions with single plus or minus charges as indicated). Free ions will diffuse from a region of high concentration to one of low concentration. But the cell membrane is semipermeable, meaning that some ions may cross it while others cannot. In its resting state, the cell membrane is permeable to K^+ and Cl^-, and impermeable to Na^+. Diffusion of K^+ and Cl^- thus creates the layers of positive and negative charge on the outside and inside of the membrane. The Coulomb force prevents the ions from diffusing across in their entirety. Once the charge layer has built up, the repulsion of like charges prevents more from moving across, and the attraction of unlike charges prevents more from leaving either side. The result is two layers of charge right on the membrane, with diffusion being balanced by the Coulomb force. A tiny fraction of the charges move across and the fluids remain neutral (other ions are present), while a separation of charge and a voltage have been created across the membrane.

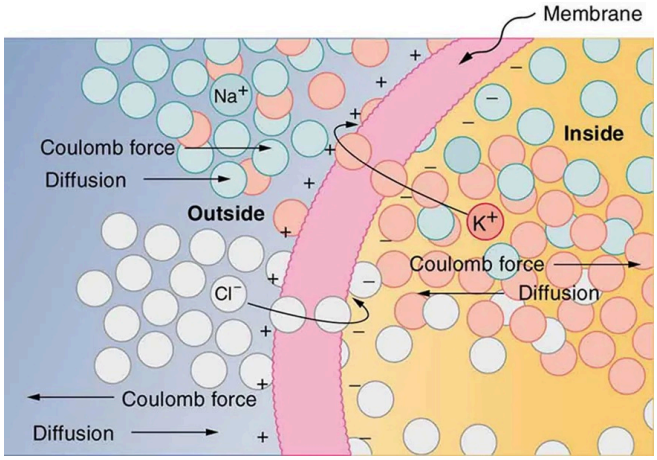


Figure 16.25 The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the K^+ *and* Cl^- *ions in the direction shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to* Na^+*. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a diagram illustrating the movement of ions across a cellular membrane, highlighting the concepts of Coulomb force and diffusion.

- The diagram is divided into two sections: "Outside" on the left (blue background) and "Inside" on the right (yellow background).
- A pink membrane separates these two areas.
- On the "Outside" section, positively charged Na^+ ions (blue) and negatively charged Cl^- ions (white) are shown. Arrows indicate the direction of Coulomb force and diffusion towards the membrane.
- On the "Inside" section, positively charged K^+ ions (red) and other molecules are depicted. Arrows show the movement direction under the influence of Coulomb force and diffusion towards the membrane and between ions.
- The membrane surface displays both positive and negative charges.

The diagram visually represents the dynamic interplay between electrical and chemical gradients across the membrane, influencing ion distribution and movement.

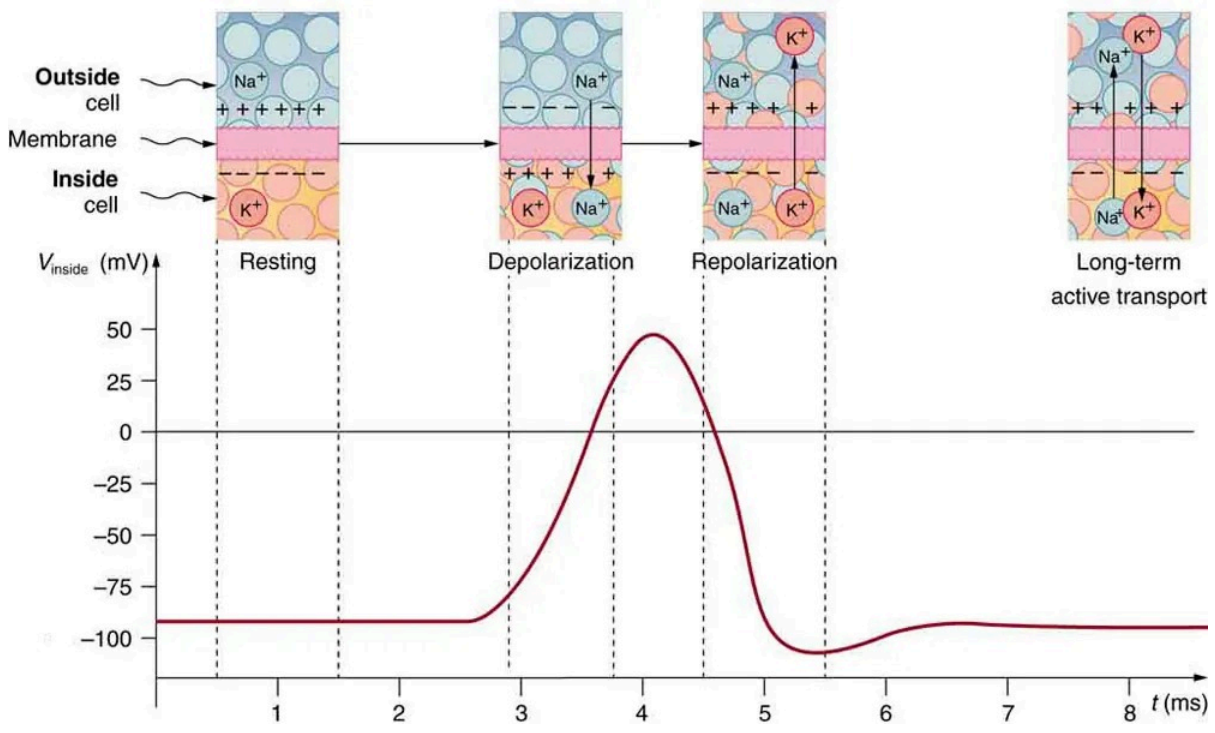


Figure 16.26 An action potential is the pulse of voltage inside a nerve cell graphed here. It is caused by movements of ions across the cell membrane as shown. Depolarization occurs when a stimulus makes the membrane permeable to Na^+ *ions. Repolarization follows as the membrane again becomes impermeable to* Na^+, *and* K^+ *moves from high to low concentration. In the long term, active transport slowly maintains the concentration differences, but the cell may fire hundreds of times in rapid succession without seriously depleting them. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image illustrates the process of an action potential in a cell with a focus on the changes in membrane potential over time, depicted in a graph. The x-axis represents time in milliseconds (ms), ranging from 0 to 8 ms, while the y-axis represents membrane potential (V_{inside}) in millivolts (mV), ranging from -100 to 50 mV.

There are four stages shown through both diagrams and a graph:

1. **Resting Potential:** The cell is at rest with a membrane potential of approximately -70 mV. The schematic shows sodium (Na^+) ions outside the cell and potassium (K^+) ions inside, with a negative charge inside.
2. **Depolarization:** The membrane potential rapidly rises to about 50 mV. Sodium ions move inside the cell, and the inside becomes positively charged.
3. **Repolarization:** The membrane potential returns towards -70 mV. Potassium ions move outside the

cell, restoring the negative charge inside.

4. **Long-term Active Transport:** Involves the movement of sodium and potassium ions to re-establish resting conditions. The membrane potential stabilizes around -70 mV.

The graph displays the changes in membrane potential during these phases with a distinct peak during depolarization and a trough during repolarization, reflecting the rapid changes in voltage.

The separation of charge creates a potential difference of 70 to 90 mV across the cell membrane. While this is a small voltage, the resulting electric field ($E = V/d$) across the only 8-nm-thick membrane is immense (on the order of 11 MV/m!) and has fundamental effects on its structure and permeability. Now, if the exterior of a neuron is taken to be at 0 V, then the interior has a *resting potential* of about −90 mV. Such voltages are created across the membranes of almost all types of animal cells but are largest in nerve and muscle cells. In fact, fully 25% of the energy used by cells goes toward creating and maintaining these potentials.

Electric currents along the cell membrane are created by any stimulus that changes the membrane's permeability. The membrane thus temporarily becomes permeable to Na^+, which then rushes in, driven both by diffusion and the Coulomb force. This inrush of Na^+ first neutralizes the inside membrane, or *depolarizes* it, and then makes it slightly positive. The depolarization causes the membrane to again become impermeable to Na^+, and the movement of K^+ quickly returns the cell to its resting potential, or *repolarizes* it. This sequence of events results in a voltage pulse, called the *action potential*. (See Figure 16.26.) Only small fractions of the ions move, so that the cell can fire many hundreds of times without depleting the excess concentrations of Na^+ and K^+. Eventually, the cell must replenish these ions to maintain the concentration differences that create bioelectricity. This sodium-potassium pump is an example of *active transport*, wherein cell energy is used to move ions across membranes against diffusion gradients and the Coulomb force.

The action potential is a voltage pulse at one location on a cell membrane. How does it get transmitted along the cell membrane, and in particular down an axon, as a nerve impulse? The answer is that the changing voltage and electric fields affect the permeability of the adjacent cell membrane, so that the same process takes place there. The adjacent membrane depolarizes, affecting the membrane further down, and so on, as illustrated in Figure 16.27. Thus the action potential stimulated at one location triggers a *nerve impulse* that moves slowly (about 1 m/s) along the cell membrane.

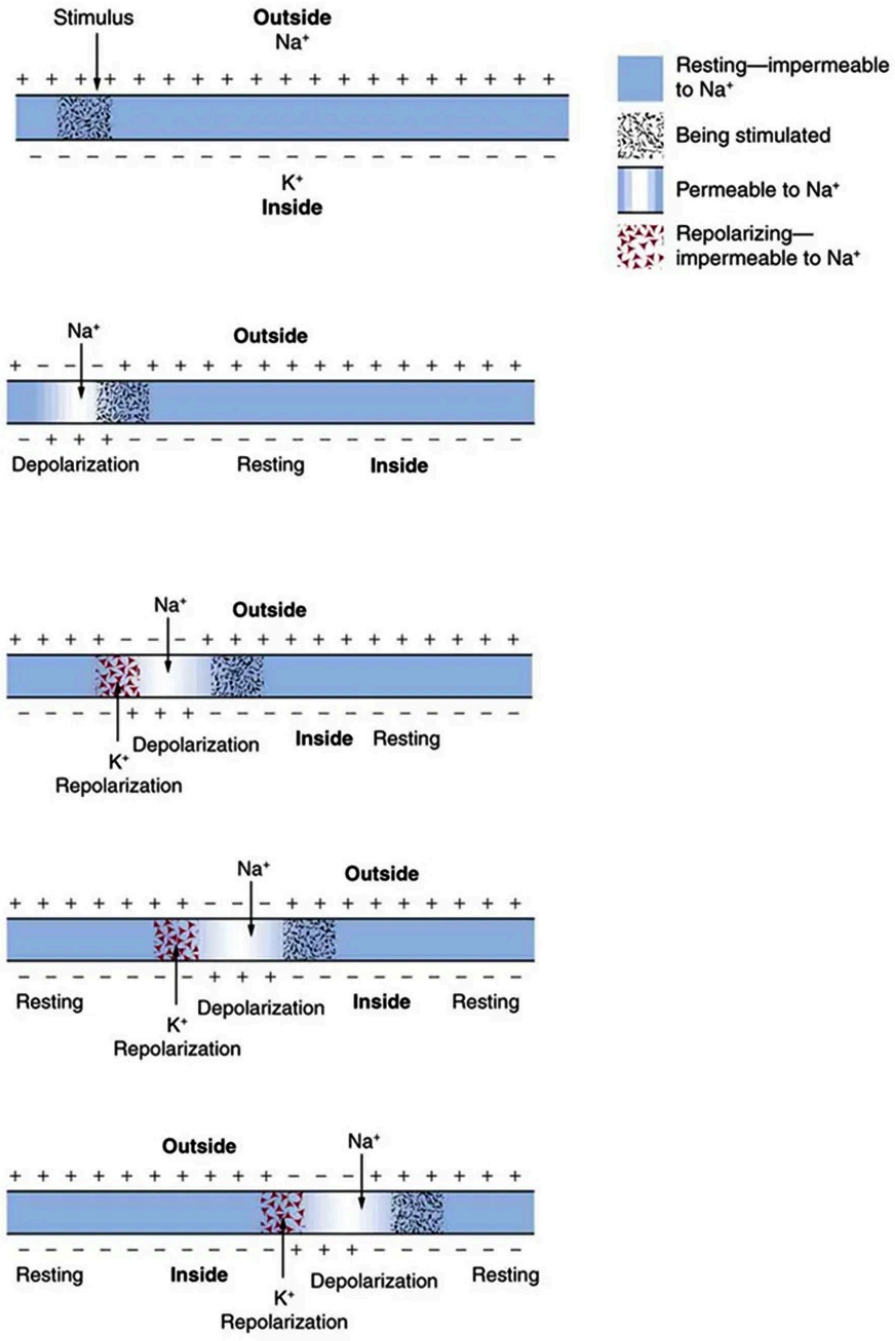


Figure 16.27 A nerve impulse is the propagation of an action potential along a cell membrane. A stimulus causes an action potential at one location, which changes the permeability of the adjacent membrane, causing an action potential there. This in turn affects the membrane further down, so that the action potential moves slowly (in electrical terms) along the cell membrane. Although the impulse is due to Na^+ *and* K^+ *going across the membrane, it is equivalent to a wave of charge moving along the outside and inside of the membrane. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of a series of diagrams illustrating the stages of nerve impulse transmission along an axon.

1. First Diagram:
– Shows a section of an axon in a resting state with a "Stimulus" label pointing to it.
– The outer surface is positive with sodium ions (Na^+), and the inside is negative with potassium ions (K^+).
– The axon is labeled "Resting—impermeable to Na^+" using a light blue color.

2. Second Diagram:
– Demonstrates depolarization where Na^+ enters the axon, making the inside positive.
– The region is labeled "Depolarization" and is marked with a pattern.
– The remaining axon is in the resting state.

3. Third Diagram:
– Depicts the beginning of repolarization with K^+ leaving the axon.
– The inner section switches back to negative.
– The diagram highlights two processes: Depolarization and Repolarization.

4. Fourth Diagram:
– Shows repolarization progressing, with K^+ continuing to exit.
– Na^+ is actively transported outside.
– The diagram represents the sequence of depolarization followed by repolarization.

5. Fifth Diagram:
– Concludes the sequence, with Na^+ and K^+ restored to their original sides.
– The axon returns to the resting state.

Legend:
– Light Blue: Resting—impermeable to Na^+
– Dotted Pattern: Being stimulated
– Vertical Lines: Permeable to Na^+
– Red Arrow Pattern: Repolarizing—impermeable to Na^+

Overall, the image illustrates the process of action potential propagation through depolarization and repolarization phases in a neuron.

Some axons, like that in Figure 16.24, are sheathed with *myelin*, consisting of fat-containing cells. Figure 16.28 shows an enlarged view of an axon having myelin sheaths characteristically separated by unmyelinated gaps (called nodes of Ranvier). This arrangement gives the axon a number of interesting properties. Since

myelin is an insulator, it prevents signals from jumping between adjacent nerves (cross talk). Additionally, the myelinated regions transmit electrical signals at a very high speed, as an ordinary conductor or resistor would. There is no action potential in the myelinated regions, so that no cell energy is used in them. There is an IR signal loss in the myelin, but the signal is regenerated in the gaps, where the voltage pulse triggers the action potential at full voltage. So a myelinated axon transmits a nerve impulse faster, with less energy consumption, and is better protected from cross talk than an unmyelinated one. Not all axons are myelinated, so that cross talk and slow signal transmission are a characteristic of the normal operation of these axons, another variable in the nervous system.

The degeneration or destruction of the myelin sheaths that surround the nerve fibers impairs signal transmission and can lead to numerous neurological effects. One of the most prominent of these diseases comes from the body's own immune system attacking the myelin in the central nervous system—multiple sclerosis. MS symptoms include fatigue, vision problems, weakness of arms and legs, loss of balance, and tingling or numbness in one's extremities (neuropathy). It is more apt to strike younger adults, especially females. Causes might come from infection, environmental or geographic affects, or genetics. At the moment there is no known cure for MS.

Most animal cells can fire or create their own action potential. Muscle cells contract when they fire and are often induced to do so by a nerve impulse. In fact, nerve and muscle cells are physiologically similar, and there are even hybrid cells, such as in the heart, that have characteristics of both nerves and muscles. Some animals, like the infamous electric eel (see Figure 16.29), use muscles ganged so that their voltages add in order to create a shock great enough to stun prey.

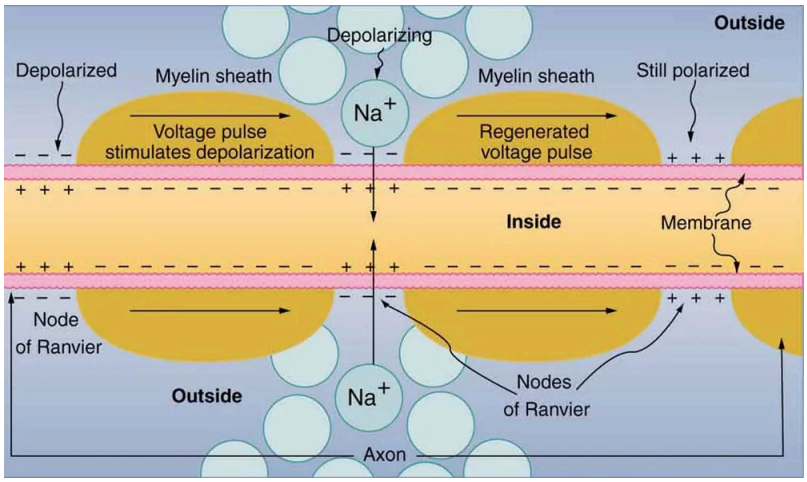


Figure 16.28 Propagation of a nerve impulse down a myelinated axon, from left to right. The signal travels very fast and without energy input in the myelinated regions, but it loses voltage. It is regenerated in the gaps. The signal moves faster than in unmyelinated axons and is insulated from signals in other nerves, limiting cross talk. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of an axon, a part of a nerve cell, showing the process of nerve impulse conduction, specifically saltatory conduction. The diagram depicts the axon with its outer membrane and nodes of Ranvier.

The axon is divided into sections with a yellow hue that have myelin sheaths, which are separated by gaps called nodes of Ranvier.

Here's a detailed description:

- **Myelin Sheath:** The axon is mostly covered by this insulating layer shown in yellow. It speeds up the transmission of electrical impulses.
- **Nodes of Ranvier:** These are the gaps between the myelin sheath, labeled on the image. The diagram shows them as exposed parts of the axon, where ion exchange occurs.
- **Voltage Pulses and Depolarization:** The diagram illustrates the movement of ions (Na^+) across the membrane at the nodes, suggesting depolarization and how a "regenerated voltage pulse" moves along the axon. The left side of the axon is shown with a label "Voltage pulse stimulates depolarization."
- **Inside and Outside Regions:** The inside of the axon is marked "Inside" and the area outside the axon is labeled "Outside." The image shows a concentration of positive charges on the outside during depolarization.
- **Direction of Nerve Impulse:** Arrows indicate the conduction of a nerve impulse along the axon, moving from the depolarized node to the next node, where the voltage is regenerated.

The diagram provides a visual explanation of how the action potential jumps from one node to the next, allowing for rapid nerve signal transmission.



Figure 16.29 An electric eel flexes its muscles to create a voltage that stuns prey. Image from OpenStax College Physics 2e, CC-BY 4.0

Electrocardiograms

Just as nerve impulses are transmitted by depolarization and repolarization of adjacent membrane, the depolarization that causes muscle contraction can also stimulate adjacent muscle cells to depolarize (fire) and contract. Thus, a depolarization wave can be sent across the heart, coordinating its rhythmic contractions and enabling it to perform its vital function of propelling blood through the circulatory system. Figure 16.30 is a simplified graphic of a depolarization wave spreading across the heart from the *sinoarterial (SA) node*, the heart's natural pacemaker.

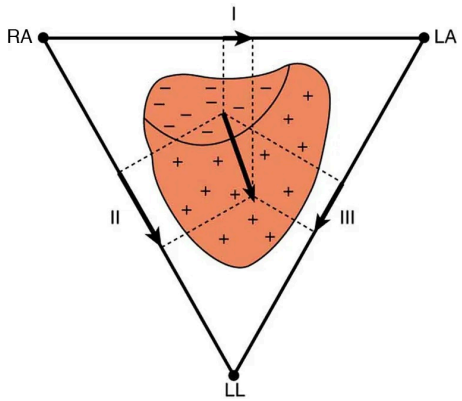


Figure 16.30 The outer surface of the heart changes from positive to negative during depolarization. This wave of depolarization is spreading from the top of the heart and is represented by a vector pointing in the direction of the wave. This vector is a voltage (potential difference) vector. Three electrodes, labeled RA, LA, and LL, are placed on the patient. Each pair (called leads I, II, and III) measures a component of the depolarization vector and is graphed in an ECG. Image from OpenStax College Physics 2e, CC-BY 4.0

An electrocardiogram (ECG) is a record of the voltages created by the wave of depolarization and subsequent repolarization in the heart. (They are also abbreviated EKG.) Voltages between pairs of electrodes placed on the chest are vector components of the voltage wave on the heart. Standard ECGs have 12 or more electrodes, but only three are shown in Figure 16.30 for clarity. Decades ago, three-electrode ECGs were performed by placing electrodes on the left and right arms and the left leg. The voltage between the right arm and the left leg is called the *lead II potential* and is the most often graphed. We shall examine the lead II potential as an indicator of heart-muscle function and see that it is coordinated with arterial blood pressure as well.

Heart function and its four-chamber action are explored in 10.4 Viscosity and Laminar Flow; Poiseuille's Law. Basically, the right and left atria receive blood from the body and lungs, respectively, and pump the blood into the ventricles. The right and left ventricles, in turn, pump blood through the lungs and the rest of the body, respectively. Depolarization of the heart muscle causes it to contract. After contraction it is repolarized to ready it for the next beat. The ECG measures components of depolarization and repolarization of the heart muscle and can yield significant information on the functioning and malfunctioning of the heart.

Figure 16.31 shows an ECG of the lead II potential and a graph of the corresponding arterial blood pressure. The major features are labeled P, Q, R, S, and T. The *P wave* is generated by the depolarization and contraction of the atria as they pump blood into the ventricles. The *QRS complex* is created by the depolarization of the ventricles as they pump blood to the lungs and body. Since the shape of the heart and the path of the depolarization wave are not simple, the QRS complex has this typical shape and time span. The lead II QRS signal also masks the repolarization of the atria, which occur at the same time. Finally, the *T wave* is generated by the repolarization of the ventricles and is followed by the next P wave in the next heartbeat. Arterial blood pressure varies with each part of the heartbeat, with systolic (maximum) pressure occurring closely after the QRS complex, which signals contraction of the ventricles.

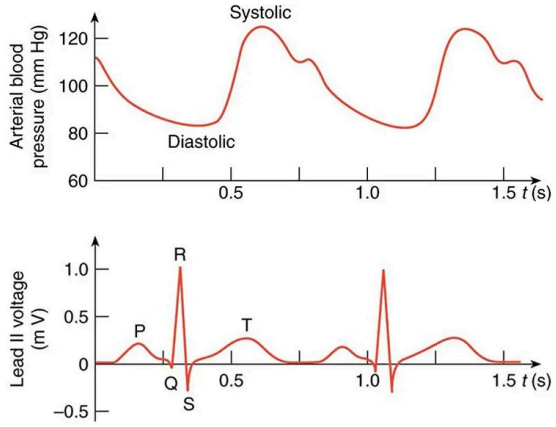


Figure 16.31 A lead II ECG with corresponding arterial blood pressure. The QRS complex is created by the depolarization and contraction of the ventricles and is followed shortly by the maximum or systolic blood pressure. See text for further description. Image from OpenStax College Physics 2e, CC-BY 4.0

Taken together, the 12 leads of a state-of-the-art ECG can yield a wealth of information about the heart. For example, regions of damaged heart tissue, called infarcts, reflect electrical waves and are apparent in one or more lead potentials. Subtle changes due to slight or gradual damage to the heart are most readily detected by comparing a recent ECG to an older one. This is particularly the case since individual heart shape, size, and orientation can cause variations in ECGs from one individual to another. ECG technology has advanced to the point where a portable ECG monitor can be incorporated into wearable devices and other small objects. See Figure 16.32.

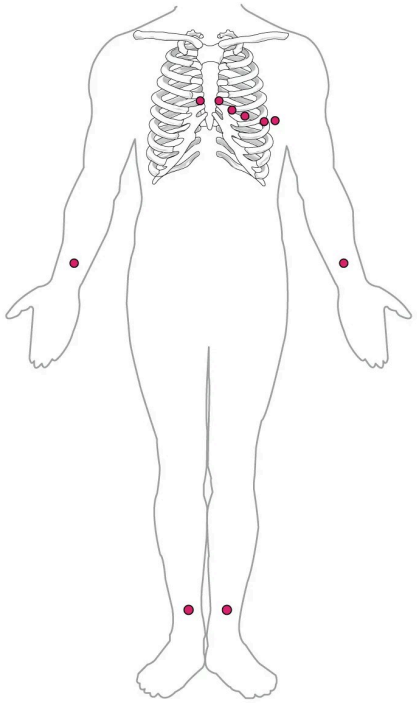


Figure 16.32 In a 12-lead ECG, six electrodes are placed on the chest, and four electrodes are placed on the limbs. Image from OpenStax College Physics 2e, CC-BY 4.0

PhET Explorations

Neuron

Construct the circuit shown and use the simulation's voltmeter and ammeter to measure the current in the circuit and the voltage across each component.

Access multimedia content

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16.8 RESISTORS IN SERIES AND PARALLEL

Learning Objectives

By the end of this section, you will be able to:

- Draw a circuit with resistors in parallel and in series.
- Calculate the voltage drop of a current across a resistor using Ohm's law.
- Contrast the way total resistance is calculated for resistors in series and in parallel.
- Explain why total resistance of a parallel circuit is less than the smallest resistance of any of the resistors in that circuit.
- Calculate total resistance of a circuit that contains a mixture of resistors connected in series and in parallel.

Most circuits have more than one component, called a resistor that limits the flow of charge in the circuit. A measure of this limit on charge flow is called resistance. The simplest combinations of resistors are the series and parallel connections illustrated in Figure 16.32. The total resistance of a combination of resistors depends on both their individual values and how they are connected.

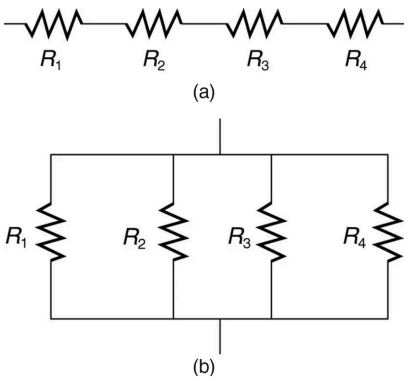


Figure 16.32 (a) A series connection of resistors. (b) A parallel connection of resistors. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows two diagrams of electrical circuits with resistors.

(a) The top diagram depicts a series circuit with four resistors arranged in a line. They are labeled as R_1, R_2, R_3, and R_4, connected one after the other.

(b) The bottom diagram shows a parallel circuit with four resistors. They are labeled as R_1, R_2, R_3, and R_4, each connected separately between two common lines.

Resistors in Series

When are resistors in series? Resistors are in series whenever the flow of charge, called the current, must flow through devices sequentially. For example, if current flows through a person holding a screwdriver and into the Earth, then R_1 in Figure 16.32(a) could be the resistance of the screwdriver's shaft, R_2 the resistance of its handle, R_3 the person's body resistance, and R_4 the resistance of her shoes.

Figure 16.33 shows resistors in series connected to a voltage source. It seems reasonable that the total resistance is the sum of the individual resistances, considering that the current has to pass through each resistor in sequence. (This fact would be an advantage to a person wishing to avoid an electrical shock, who could reduce the current by wearing high-resistance rubber-soled shoes. It could be a disadvantage if one of the resistances were a faulty high-resistance cord to an appliance that would reduce the operating current.)

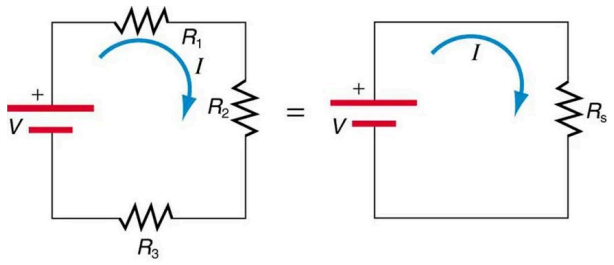


Figure 16.33 Three resistors connected in series to a battery (left) and the equivalent single or series resistance
Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two electrical circuits, shown side by side, with an equality sign between them. The left circuit consists of a voltage source labeled "V" indicated by red lines and a voltage symbol, connected in series with three resistors labeled "R_1", "R_2", and "R_3". An arrow with the label "I" indicates current direction flowing clockwise from the positive terminal through the resistors.

The right circuit shows a simplified version, with the same voltage source "V" and a single equivalent resistor labeled "R_s". A similar arrow labeled "I" demonstrates the current flowing clockwise in this simplified circuit. This illustration represents the concept of reducing series resistors to a single equivalent resistor in an electrical circuit.

To verify that resistances in series do indeed add, let us consider the loss of electrical power, called a voltage drop, in each resistor in Figure 16.33.

According to Ohm's law, the voltage drop, V, across a resistor when a current flows through it is calculated using the equation $V = \mathrm{IR}$, where I equals the current in amps (A) and R is the resistance in ohms (Ω). Another way to think of this is that V is the voltage necessary to make a current I flow through a resistance R .

So the voltage drop across R_1 is $V_1 = IR_1$, that across R_2 is $V_2 = IR_2$, and that across R_3 is $V_3 = IR_3$. The sum of these voltages equals the voltage output of the source; that is,

$$V = V_1 + V_2 + V_3.$$
16.55

This equation is based on the conservation of energy and conservation of charge. Electrical potential energy can be described by the equation $\mathrm{PE} = \mathrm{qV}$, where q is the electric charge and V is the voltage. Thus the energy supplied by the source is qV, while that dissipated by the resistors is

$$\mathrm{qV_1 + qV_2 + qV_3}.$$
16.56

Connections: Conservation Laws

The derivations of the expressions for series and parallel resistance are based on the laws of conservation of energy and conservation of charge, which state that total charge and total energy are constant in any process. These two laws are directly involved in all electrical phenomena and will be invoked repeatedly to explain both specific effects and the general behavior of electricity.

These energies must be equal, because there is no other source and no other destination for energy in the circuit. Thus, $qV = qV_1 + qV_2 + qV_3$. The charge q cancels, yielding $V = V_1 + V_2 + V_3$, as stated. (Note that the same amount of charge passes through the battery and each resistor in a given amount of time, since there is no capacitance to store charge, there is no place for charge to leak, and charge is conserved.)

Now substituting the values for the individual voltages gives

$$V = \mathrm{IR_1 + IR_2 + IR_3} = I\,(R_1 + R_2 + R_3\,).$$
16.57

Note that for the equivalent single series resistance R_s, we have

$$V = \mathrm{IR_s}.$$
16.58

This implies that the total or equivalent series resistance R_s of three resistors is $R_\mathrm{s} = R_1 + R_2 + R_3$.

This logic is valid in general for any number of resistors in series; thus, the total resistance R_s of a series connection is

$$R_\mathrm{s} = R_1 + R_2 + R_3 + \ldots,$$
16.59

as proposed. Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

Example 16.11

Calculating Resistance, Current, Voltage Drop, and Power Dissipation: Analysis of a Series Circuit

Suppose the voltage output of the battery in Figure 16.33 is 12.0V, and the resistances are $R_1 = 1.00\Omega$, $R_2 = 6.00\Omega$, and $R_3 = 13.0\Omega$. (a) What is the total resistance? (b) Find the current. (c) Calculate the voltage drop in each resistor, and show these add to equal the voltage output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance is simply the sum of the individual resistances, as given by this equation:

$$\begin{aligned} R_\text{s} &= R_1 + R_2 + R_3 \\ &= 1.00\Omega + 6.00\Omega + 13.0\ \Omega \\ &= 20.0\ \Omega. \end{aligned}$$

16.60

Strategy and Solution for (b)

The current is found using Ohm's law, $V = \text{IR}$. Entering the value of the applied voltage and the total resistance yields the current for the circuit:

$$I = \frac{V}{R_\text{s}} = \frac{12.0\ \text{V}}{20.0\ \Omega} = 0.600\ \text{A}.$$

16.61

Strategy and Solution for (c)

The voltage—or IR drop—in a resistor is given by Ohm's law. Entering the current and the value of the first resistance yields

$$V_1 = IR_1 = (0.600\ \mathrm{A})(1.0\Omega) = 0.600\ \mathrm{V}.$$

16.62

Similarly,

$$V_2 = IR_2 = (0.600\ \mathrm{A})(6.0\Omega) = 3.60\ \mathrm{V}$$

16.63

and

$$V_3 = IR_3 = (0.600\ \mathrm{A})(13.0\Omega) = 7.80\ \mathrm{V}.$$

16.64

Discussion for (c)

The three IR drops add to $12.0\mathrm{V}$, as predicted:

$$\begin{aligned} V_1 + V_2 + V_3 &= (0.600 + 3.60 + 7.80)\mathrm{V} \\ &= 12.0\ \mathrm{V}. \end{aligned}$$

16.65

Strategy and Solution for (d)

The easiest way to calculate power in watts (W) dissipated by a resistor in a DC circuit is to use Joule's law, $P = \mathrm{IV}$, where P is electric power. In this case, each resistor has the same full current flowing through it. By substituting Ohm's law $V = \mathrm{IR}$ into Joule's law, we get the power dissipated by the first resistor as

$$P_1 = I^2R_1 = (0.600\ \mathrm{A})^2(1.00\Omega) = 0.360\ \mathrm{W}.$$

16.66

Similarly,

$$P_2 = I^2 R_2 = (0.600\ \text{A})^2\,(6.00\Omega\,) = 2.16\ \text{W}$$

16.67

and

$$P_3 = I^2 R_3 = (0.600\ \text{A})^2\,(13.0\Omega\,) = 4.68\ \text{W}.$$

16.68

Discussion for (d)

Power can also be calculated using either $P = \text{IV}$ or $P = \frac{V^2}{R}$, where V is the voltage drop across the resistor (not the full voltage of the source). The same values will be obtained.

Strategy and Solution for (e)

The easiest way to calculate power output of the source is to use $P = \text{IV}$, where V is the source voltage. This gives

$$P = (0.600\ \text{A}\,)\,(12.0\ \text{V}\,) = 7.20\ \text{W}.$$

16.69

Discussion for (e)

Note, coincidentally, that the total power dissipated by the resistors is also 7.20 W, the same as the power put out by the source. That is,

$$\begin{aligned} P_1 + P_2 + P_3 &= (0.360 + 2.16 + 4.68\,)\text{W} \\ &= 7.20\ \text{W}. \end{aligned}$$

16.70

Power is energy per unit time (watts), and so conservation of energy requires the power output of the source to be equal to the total power dissipated by the resistors.

Major Features of Resistors in Series

1. Series resistances add: $R_s = R_1 + R_2 + R_3 + \ldots$.
2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it.

Resistors in Parallel

Figure 16.34 shows resistors in parallel, wired to a voltage source. Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

Each resistor draws the same current it would if it alone were connected to the voltage source (provided the voltage source is not overloaded). For example, an automobile's headlights, radio, and so on, are wired in parallel, so that they utilize the full voltage of the source and can operate completely independently. The same is true in your house, or any building. (See Figure 16.34(b).)

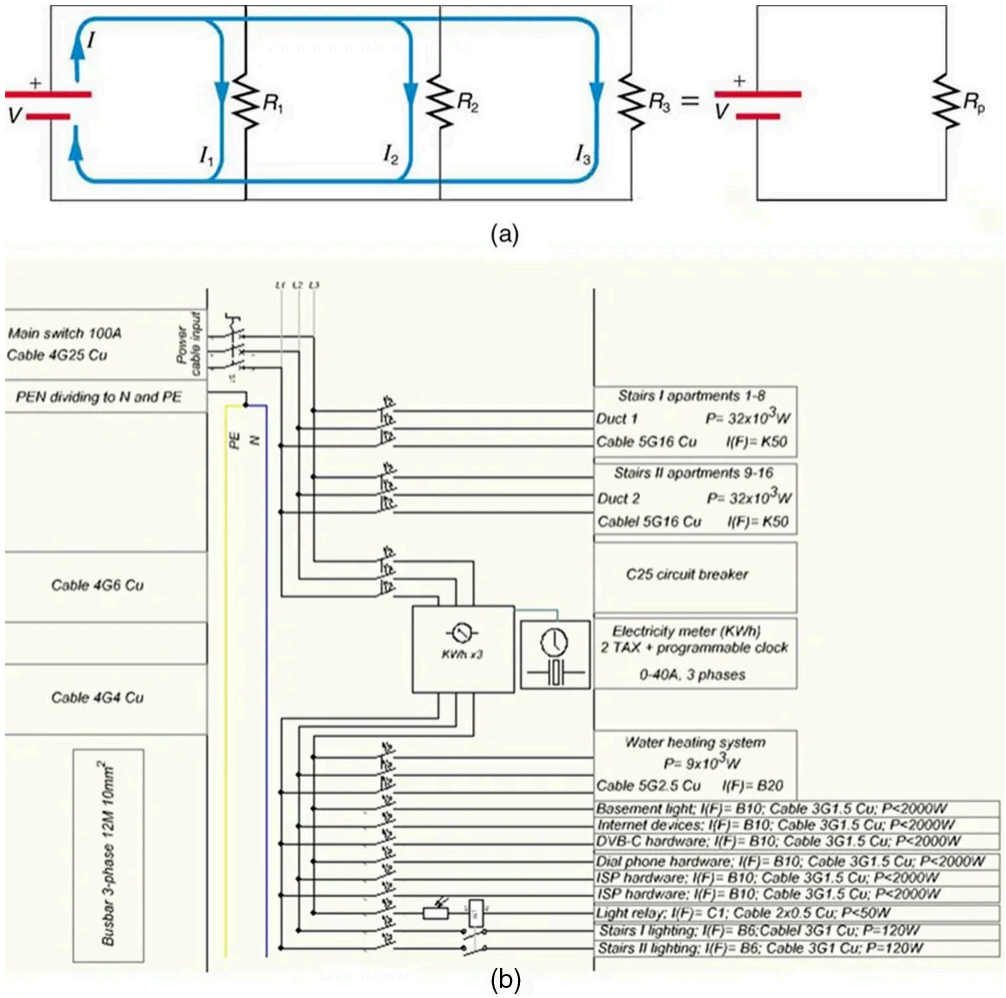


Figure 16.34 (a) Three resistors connected in parallel to a battery and the equivalent single or parallel resistance. (b) Electrical power setup in a house. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two main diagrams related to electrical circuits and systems:

1. Diagram (a):
– This is a schematic representation of a parallel resistor circuit.
– A voltage source is shown on the left with a positive and negative terminal.
– Three resistors (R1, R2, R3) are connected in parallel with arrows indicating the direction of currents (I, I1, I2, I3) flowing through each resistor.
– An equation shows that these resistors can be simplified to a single equivalent resistor (Rp).

2. Diagram (b):
– This is a more complex schematic for an electrical distribution system.

– It includes detailed labeling for various components, such as:
– Main switch 100A connected with Cable 4G25 Cu.
– A PEN conductor dividing to N (neutral) and PE (protective earth).
– Components like C25 circuit breaker and electricity meter with TAX and a programmable clock for up to 40A, 3 phases.
– Connections to different areas and systems, such as stairs, ducts, basement, internet device, and lighting, each with specific cable and power specifications.
– Specific notation for power and cable type, such as 5G16 Cu and 3G1.5 Cu, denoting the number and type of conductors and their current or power capacity.

3. Additional Details:
– The distribution panel includes busbars, water heating system with 9×10^3 W power, and connections for various hardware systems.

The diagrams are labeled as (a) and (b) for reference and include comprehensive descriptions of the electrical components and their specifications.

To find an expression for the equivalent parallel resistance R_{p}, let us consider the currents that flow and how they are related to resistance. Since each resistor in the circuit has the full voltage, the currents flowing through the individual resistors are $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, and $I_3 = \frac{V}{R_3}$. Conservation of charge implies that the total current I produced by the source is the sum of these currents:

$$I = I_1 + I_2 + I_3.$$
16.71

Substituting the expressions for the individual currents gives

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$
16.72

Note that Ohm's law for the equivalent single resistance gives

$$I = \frac{V}{R_p} = V\left(\frac{1}{R_p}\right).$$
16.73

The terms inside the parentheses in the last two equations must be equal. Generalizing to any number of resistors, the total resistance R_{p} of a parallel connection is related to the individual resistances by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{.3}} + \ldots.$$

16.74

This relationship results in a total resistance R_p that is less than the smallest of the individual resistances. (This is seen in the next example.) When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, and so the total resistance is lower.

Example 16.12

Calculating Resistance, Current, Power Dissipation, and Power Output: Analysis of a Parallel Circuit

Let the voltage output of the battery and resistances in the parallel connection in Figure 16.34 be the same as the previously considered series connection: $V = 12.0\ \text{V}$, $R_1 = 1.00\Omega$, $R_2 = 6.00\Omega$, and $R_3 = 13.0\Omega$. (a) What is the total resistance? (b) Find the total current. (c) Calculate the currents in each resistor, and show these add to equal the total current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.00\Omega} + \frac{1}{6.00\Omega} + \frac{1}{13.0\Omega}.$$

16.75

Thus,

$$\frac{1}{R_p} = \frac{1.00}{\Omega} + \frac{0.1667}{\Omega} + \frac{0.07692}{\Omega} = \frac{1.2436}{\Omega}.$$

16.76

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance R_p. This yields

$$R_\text{p} = \frac{1}{1.2436}\Omega = 0.8041\ \Omega.$$

16.77

The total resistance with the correct number of significant digits is $R_{\text{p}} = 0.804\Omega$.

Discussion for (a)

R_{p} is, as predicted, less than the smallest individual resistance.

Strategy and Solution for (b)

The total current can be found from Ohm's law, substituting R_{p} for the total resistance. This gives

$$I = \frac{V}{R_{\text{p}}} = \frac{12.0 \text{ V}}{0.8041 \ \Omega} = 14.92 \text{ A}.$$

16.78

Discussion for (b)

Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

Strategy and Solution for (c)

The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

$$I_1 = \frac{V}{R_1} = \frac{12.0 \text{ V}}{1.00 \ \Omega} = 12.0 \text{ A}.$$

16.79

Similarly,

$$I_2 = \frac{V}{R_2} = \frac{12.0 \text{ V}}{6.00 \ \Omega} = 2.00 \text{ A}$$

16.80

and

$$I_3 = \frac{V}{R_3} = \frac{12.0\ \text{V}}{13.0\ \Omega} = 0.92\ \text{A}.$$

16.81

Discussion for (c)

The total current is the sum of the individual currents:

$$I_1 + I_2 + I_3 = 14.92\ \text{A}.$$

16.82

This is consistent with conservation of charge.

Strategy and Solution for (d)

The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P = \frac{V^2}{R}$, since each resistor gets full voltage. Thus,

$$P_1 = \frac{V^2}{R_1} = \frac{(12.0\ \text{V})^2}{1.00\ \Omega} = 144\ \text{W}.$$

16.83

Similarly,

$$P_2 = \frac{V^2}{R_2} = \frac{(12.0\ \text{V})^2}{6.00\ \Omega} = 24.0\ \text{W}$$

16.84

and

$$P_3 = \frac{V^2}{R_3} = \frac{(12.0\ \text{V})^2}{13.0\ \Omega} = 11.1\ \text{W}.$$

16.85

Discussion for (d)

The power dissipated by each resistor is considerably higher in parallel than when connected in series to the same voltage source.

Strategy and Solution for (e)

The total power can also be calculated in several ways. Choosing $P = \mathrm{IV}$, and entering the total current, yields

$$P = \mathrm{IV} = (14.92\ \mathrm{A})\,(12.0\ \mathrm{V}) = 179\ \mathrm{W}.$$

16.86

Discussion for (e)

Total power dissipated by the resistors is also 179 W:

$$\begin{aligned} P_1 + P_2 + P_3 &= 144\mathrm{W} + 24.0\mathrm{W} + 11.1\mathrm{W} \\ &= 179\mathrm{W}. \end{aligned}$$

16.87
This is consistent with the law of conservation of energy.

Overall Discussion

Note that both the currents and powers in parallel connections are greater than for the same devices in series.

Major Features of Resistors in Parallel

1. Parallel resistance is found from $\frac{1}{R_{\text{p}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$, and it is smaller than any individual resistance in the combination.
2. Each resistor in parallel has the same full voltage of the source applied to it. (Power distribution systems most often use parallel connections to supply the myriad devices served with the same voltage and to allow them to operate independently.)
3. Parallel resistors do not each get the total current; they divide it.

Combinations of Series and Parallel

More complex connections of resistors are sometimes just combinations of series and parallel. These are commonly encountered, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in Figure 16.35. Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.

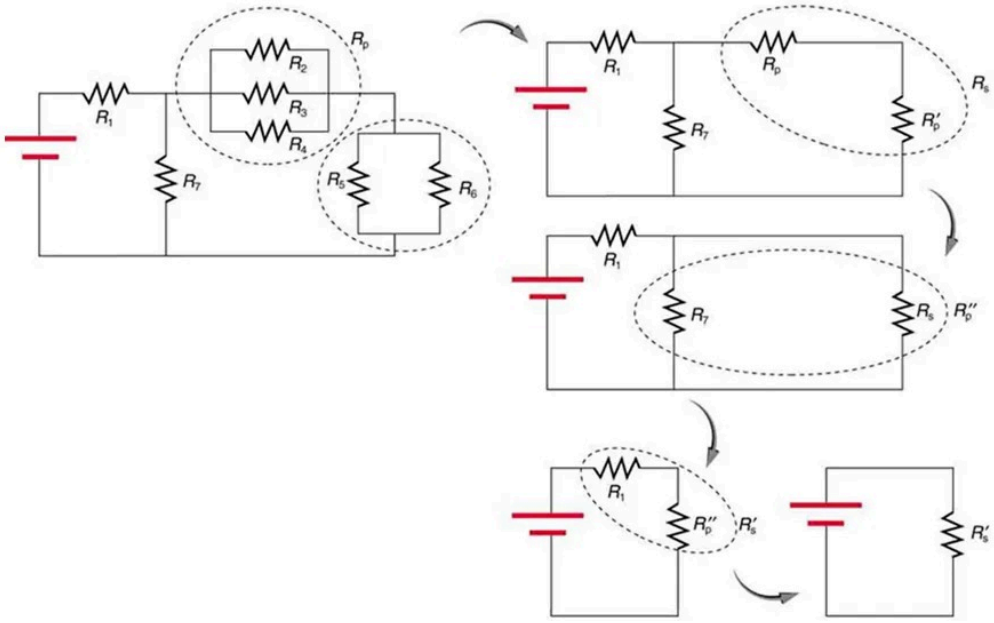


Figure 16.35 This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a series of electrical circuit diagrams demonstrating the process of simplifying a complex circuit. The steps are as follows:

1. Top Left Diagram:
– A circuit with a battery connected to various resistors, labeled R1, R7, and a group of parallel resistors Rp (comprising R2 and R3), and another set with R4 and R5. Resistor R6 is separate.

2. Top Right Diagram:
– The parallel resistors R2 and R3 have been combined into Rp, and R4 and R5 into R5′. The circuit now has R1, Rp, R7, and the combined R5 and R6 in parallel.

3. Middle Diagram:
– Resistor R6 is now combined with R5 into Rp”. The circuit is simplified further to include R1, R7, and Rp” in parallel.

4. Bottom Left Diagram:
– R1 and R7 remain in parallel with Rp’.

5. Bottom Right Diagram:
– Finally, the circuit is reduced to the battery connected to a single equivalent resistor R'eq.

Each diagram is connected with arrows, indicating the progression and simplification steps of combining resistors in parallel and series to reduce the circuit's complexity.

The simplest combination of series and parallel resistance, shown in Figure 16.36, is also the most instructive, since it is found in many applications. For example, R_1 could be the resistance of wires from a car battery to its electrical devices, which are in parallel. R_2 and R_3 could be the starter motor and a passenger compartment light. We have previously assumed that wire resistance is negligible, but, when it is not, it has important effects, as the next example indicates.

Example 16.13

Calculating Resistance, IR Drop, Current, and Power Dissipation: Combining Series and Parallel Circuits

Figure 16.36 shows the resistors from the previous two examples wired in a different way—a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3. (a) Find the total resistance. (b) What is the IR drop in R_1? (c) Find the current I_2 through R_2. (d) What power is dissipated by R_2?

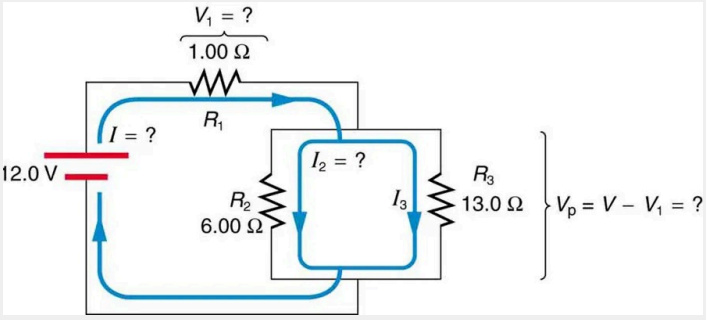


Figure 16.36 These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of an electrical circuit with two loops. The details are as follows:

- A battery on the left side labeled “12.0 V” is connected in series with three resistors: R1, R2, and R3.
- R1, at the top of the first loop, is labeled “1.00 Ω” and has its voltage drop labeled as V_1 = ?.
- R2, at the bottom of the first loop, is labeled “6.00 Ω”.
- The first loop has a current labeled as I = ?.
- The second loop is nested inside the first and shares R2.

- R3, at the right side of the second loop, is labeled “13.0 Ω”.
- The second loop has a current labeled as I_2 = ? in one section and I_3 in another section of the loop.
- To the right of R3, there is an expression $V_p = V - V_1$ = ?.

The diagram contains directional arrows indicating the direction of current flow in both loops.

Strategy and Solution for (a)

To find the total resistance, we note that R_2 and R_3 are in parallel and their combination R_p is in series with R_1. Thus the total (equivalent) resistance of this combination is

$$R_\text{tot} = R_1 + R_\text{p}.$$

16.88

First, we find R_p using the equation for resistors in parallel and entering known values:

$$\frac{1}{R_\text{p}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6.00\Omega} + \frac{1}{13.0\Omega} = \frac{0.2436}{\Omega}.$$

16.89

Inverting gives

$$R_\text{p} = \frac{1}{0.2436}\Omega = 4.11\Omega.$$

16.90

So the total resistance is

$$R_\text{tot} = R_1 + R_\text{p} = 1.00\Omega + 4.11\ \Omega = 5.11\ \Omega.$$

16.91

Discussion for (a)

The total resistance of this combination is intermediate between the pure series and

pure parallel values (20.0Ω and 0.804Ω, respectively) found for the same resistors in the two previous examples.

Strategy and Solution for (b)

To find the IR drop in R_1, we note that the full current I flows through R_1. Thus its IR drop is

$$V_1 = \mathrm{IR}_1.$$

16.92

We must find I before we can calculate V_1. The total current I is found using Ohm's law for the circuit. That is,

$$I = \frac{V}{R_{\mathrm{tot}}} = \frac{12.0\ \mathrm{V}}{5.11\ \Omega} = 2.35\mathrm{A}.$$

16.93

Entering this into the expression above, we get

$$V_1 = \mathrm{IR}_1 = (2.35\ \mathrm{A})\,(1.00\Omega) = 2.35\ \mathrm{V}.$$

16.94

Discussion for (b)

The voltage applied to R_2 and R_3 is less than the total voltage by an amount V_1. When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3.

Strategy and Solution for (c)

To find the current through R_2, we must first find the voltage applied to it. We call this voltage V_{p}, because it is applied to a parallel combination of resistors. The voltage applied to both R_2 and R_3 is reduced by the amount V_1, and so it is

$$V_{\mathrm{p}} = V - V_1 = 12.0\ \mathrm{V} - 2.35\ \mathrm{V} = 9.65\ \mathrm{V}.$$

16.95

Now the current I_2 through resistance R_2 is found using Ohm's law:

$$I_2 = \frac{V_{\text{p}}}{R_2} = \frac{9.65\ \text{V}}{6.00\ \Omega} = 1.61\ \text{A}.$$

16.96

Discussion for (c)

The current is less than the 2.00 A that flowed through R_2 when it was connected in parallel to the battery in the previous parallel circuit example.

Strategy and Solution for (d)

The power dissipated by R_2 is given by

$$P_2 = (I_2)^2 R_2 = (1.61\ \text{A})^2\,(6.00\Omega\,) = 15.5\ \text{W}.$$

16.97

Discussion for (d)

The power is less than the 24.0 W this resistor dissipated when connected in parallel to the 12.0-V source.

Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in Figure 16.37. The device represented by

R_3 has a very low resistance, and so when it is switched on, a large current flows. This increased current causes a larger IR drop in the wires represented by R_1, reducing the voltage across the light bulb (which is R_2), which then dims noticeably.

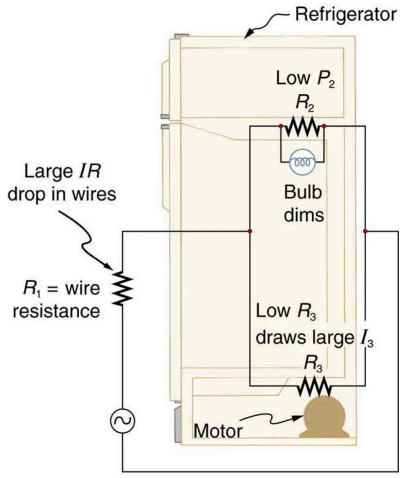


Figure 16.37 Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant IR *drop in the wires and reduces the voltage across the light. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image is a diagram of a refrigerator's electrical circuit, highlighting various components and their relationships:

- **Refrigerator:** The main object in the diagram, shown in profile.
- **Internal Light Bulb:** Positioned at the top inside the refrigerator, accompanied by a note that says "Bulb dims" due to low power (P_2) across resistance (R_2).
- **Motor:** Located at the bottom, marked by low resistance (R_3) drawing a large current (I_3).
- **Wire Resistance:** Denoted as R_1, indicating a large IR drop in the wires.
- **AC Power Source:** Represented by a sine wave symbol.

Check Your Understanding

Can any arbitrary combination of resistors be broken down into series and parallel combinations? See if you can draw a circuit diagram of resistors that cannot be broken down into combinations of series and parallel.

Click for Solution

Solution

No, there are many ways to connect resistors that are not combinations of series and parallel, including loops and junctions.

Problem-Solving Strategies for Series and Parallel Resistors

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the knowns for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel. If your problem has a combination of series and parallel, reduce it in steps by considering individual groups of series or parallel connections, as done in this module and the examples. Special note: When finding R_{p}, the reciprocal must be taken with care.
5. Check to see whether the answers are reasonable and consistent. Units and numerical results must be reasonable. Total series resistance should be greater, whereas total parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

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GLOSSARY

AC current
current that fluctuates sinusoidally with time, expressed as $I = I_0 \sin 2\pi ft$, where I is the current at time t, I_0 is the peak current, and f is the frequency in hertz

AC voltage
voltage that fluctuates sinusoidally with time, expressed as $V = V_0 \sin 2\pi ft$, where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz

alternating current
(AC) the flow of electric charge that periodically reverses direction

ammeter
an instrument that measures current

analog meter
a measuring instrument that gives a readout in the form of a needle movement over a marked gauge

ampere
(amp) the SI unit for current; 1 A = 1 C/s

bioelectricity
electrical effects in and created by biological systems

conservation laws
require that energy and charge be conserved in a system

current
the flow of charge through an electric circuit past a given point of measurement

digital meter
a measuring instrument that gives a readout in a digital form

direct current
(DC) the flow of electric charge in only one direction

drift velocity
the average velocity at which free charges flow in response to an electric field

electric current
the rate at which charge flows, $I = \Delta Q/\Delta t$

electric power
the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

electrocardiogram (ECG)
usually abbreviated ECG, a record of voltages created by depolarization and repolarization, especially in the heart

electromotive force (emf)
the potential difference of a source of electricity when no current is flowing; measured in volts

internal resistance
the amount of resistance within the voltage source

Joule's law
the relationship between potential electrical power, voltage, and resistance in an electrical circuit, given by: $P_e = \mathrm{IV}$

microshock sensitive
a condition in which a person's skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

nerve conduction
the transport of electrical signals by nerve cells

ohm
the unit of resistance, given by 1Ω = 1 V/A

Ohm's law
an empirical relation stating that the current I is proportional to the potential difference V, $\propto V$; it is often written as $I = V/R$, where R is the resistance

ohmic
a type of a material for which Ohm's law is valid

ohmmeter
an instrument that applies a voltage to a resistance, measures the current, calculates the resistance using Ohm's law, and provides a readout of this calculated resistance

parallel
the wiring of resistors or other components in an electrical circuit such that each component receives an equal voltage from the power source; often pictured in a ladder-shaped diagram, with each component on a rung of the ladder

potential difference
the difference in electric potential between two points in an electric circuit, measured in volts

resistance
the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$

resistor
a component that provides resistance to the current flowing through an electrical circuit

resistivity
an intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by ρ

rms current
the root mean square of the current, $I_{\text{rms}} = I_0/\sqrt{2}$, where I_0 is the peak current, in an AC system

rms voltage
the root mean square of the voltage, $V_{\text{rms}} = V_0/\sqrt{2}$, where V_0 is the peak voltage, in an AC system

semipermeable
property of a membrane that allows only certain types of ions to cross it

series
a sequence of resistors or other components wired into a circuit one after the other

shock hazard
when electric current passes through a person

short circuit
also known as a "short," a low-resistance path between terminals of a voltage source

simple circuit
a circuit with a single voltage source and a single resistor

temperature coefficient of resistivity
an empirical quantity, denoted by α, which describes the change in resistance or resistivity of a material with temperature

terminal voltage
the voltage measured across the terminals of a source of potential difference

thermal hazard
a hazard in which electric current causes undesired thermal effects

voltage
the electrical potential energy per unit charge; electric pressure created by a power source, such as a battery

voltage drop
the loss of electrical power as a current travels through a resistor, wire or other component

voltmeter
an instrument that measures voltage

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CHAPTER SUMMARY

16.1 Current

- Electric current I is the rate at which charge flows, given by
$$I = \frac{\Delta Q}{\Delta t},$$
where ΔQ is the amount of charge passing through an area in time Δt.
- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where $1\ \text{A} = 1\ \text{C/s}$.
- Current is the flow of free charges, such as electrons and ions.
- Drift velocity v_d is the average speed at which these charges move.
- Current I is proportional to drift velocity v_d, as expressed in the relationship $I = \text{nqAv}_\text{d}$. Here, I is the current through a wire of cross-sectional area A. The wire's material has a free-charge density n, and each carrier has charge q and a drift velocity v_d.
- Electrical signals travel at speeds about 10^{12} times greater than the drift velocity of free electrons.

16.2 Ohm's Law: Resistance and Simple Circuits

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I, voltage V, and resistance R in a simple circuit to be $I = \frac{V}{R}$.
- Resistance has units of ohms (Ω), related to volts and amperes by $1\Omega = 1\ \text{V/A}$.
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by $V = \text{IR}$.

16.3 Resistance and Resistivity

- The resistance R of a cylinder of length L and cross-sectional area A is $R = \frac{\rho L}{A}$, where ρ is the resistivity of the material.
- Values of ρ in Table 16.1 show that materials fall into three groups—*conductors, semiconductors, and insulators*.
- Temperature affects resistivity; for relatively small temperature changes ΔT, resistivity is $\rho = \rho_0 (1 + \alpha \Delta T)$, where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.
- Table 16.2 gives values for α, the temperature coefficient of resistivity.
- The resistance R of an object also varies with temperature: $R = R_0 (1 + \alpha \Delta T)$, where R_0 is the original resistance, and R is the resistance after the temperature change.

16.4 Electric Power and Energy

- Electric power P is the rate (in watts) that energy is supplied by a source or dissipated by a device.
- Three expressions for electrical power are $P = \mathrm{IV}$, $P = \dfrac{V^2}{R}$, and $P = I^2R$.
- The energy used by a device with a power P over a time t is $E = \mathrm{Pt}$.

16.5 Alternating Current versus Direct Current

- Direct current (DC) is the flow of electric current in only one direction. It refers to systems where the source voltage is constant.
- The voltage source of an alternating current (AC) system puts out $V = V_0 \sin 2\pi ft$, where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz.
- In a simple circuit, $I = V/R$ and AC current is $I = I_0 \sin 2\pi ft$, where I is the current at time t, and $I_0 = V_0/R$ is the peak current.
- The average AC power is $P_{\text{ave}} = \frac{1}{2} I_0 V_0$.
- Average (rms) current I_{rms} and average (rms) voltage V_{rms} are $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$ and $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$, where rms stands for root mean square.
- Thus, $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$.
- Ohm's law for AC is $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$.
- Expressions for the average power of an AC circuit are $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$, $P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R}$, and $P_{\text{ave}} = I_{\text{rms}}^2 R$, analogous to the expressions for DC circuits.

16.6 Electric Hazards and the Human Body

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- Table 16.3 lists shock hazards as a function of current.
- Figure 16.22 graphs the threshold current for two hazards as a function of frequency.

16.7 Nerve Conduction–Electrocardiograms

- Electric potentials in neurons and other cells are created by ionic concentration differences across semipermeable membranes.
- Stimuli change the permeability and create action potentials that propagate along neurons.
- Myelin sheaths speed this process and reduce the needed energy input.
- This process in the heart can be measured with an electrocardiogram (ECG).

16.8 Resistors in Series and Parallel

- The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances: $R_{\text{s}} = R_1 + R_2 + R_3 + \ldots$.
- Each resistor in a series circuit has the same amount of current flowing through it.
- The voltage drop, or power dissipation, across each individual resistor in a series is different, and their combined total adds up to the power source input.
- The total resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula: $\frac{1}{R_{\text{p}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$.
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

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CONCEPTUAL QUESTIONS

Conceptual Questions

16.1 Current

1. Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.
2. Car batteries are rated in ampere-hours ($\mathrm{A} \cdot \mathrm{h}$). To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?
3. If two different wires having identical cross-sectional areas carry the same current, will the drift velocity be higher or lower in the better conductor? Explain in terms of the equation $v_{\mathrm{d}} = \dfrac{I}{\mathrm{nqA}}$, by considering how the density of charge carriers n relates to whether or not a material is a good conductor.
4. Why are two conducting paths from a voltage source to an electrical device needed to operate the device?
5. In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?
6. Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

16.2 Ohm's Law: Resistance and Simple Circuits

7. The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.
8. How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

16.3 Resistance and Resistivity

9. In which of the three semiconducting materials listed in Table 16.1 do impurities supply free charges? (Hint: Examine the range of resistivity for each and determine whether the pure semiconductor has the higher or lower conductivity.)
10. Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width? (See Figure 16.38.)

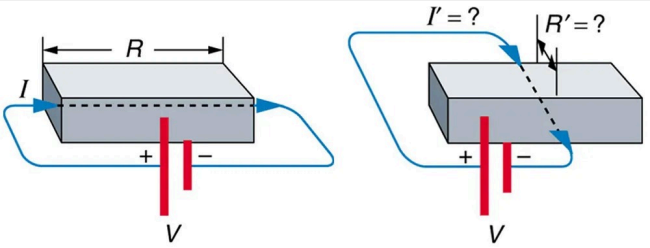


Figure 16.38 Does current taking two different paths through the same object encounter different resistance? Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams of a rectangular block representing circuits with voltage and current configurations.

Left Diagram:

– A rectangular block is shown with a current “I” entering from the left side.
– The top of the block is annotated with “R,” indicating resistance.
– Two vertical red lines with a “+” and “−” sign denote voltage “V” across the block.
– A blue wire forms a loop around the block, representing the flow of current.

Right Diagram:

– Similar to the left diagram, another rectangular block is shown.
– Current “I'” and resistance “R'” are marked with a question mark.
– The “+” and “−” voltage “V” remains the same on the red vertical lines.
– A dashed line on the top indicates a change in resistance.
– A blue wire loop illustrates the current flow.

This demonstrates a comparison between two different configurations of current and resistance across a similar medium with the same voltage applied.

11. If aluminum and copper wires of the same length have the same resistance, which has the larger diameter? Why?
12. Explain why $R = R_0 (1 + \alpha \Delta T)$ for the temperature variation of the resistance R of an object is not as accurate as $\rho = \rho_0 (1 + \alpha \Delta T)$, which gives the temperature variation of resistivity ρ.

16.4 Electric Power and Energy

13. Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?
14. The power dissipated in a resistor is given by $P = V^2/R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2 R$, which means power increases if resistance increases. Explain why there is no contradiction here.

16.5 Alternating Current versus Direct Current

15. Give an example of a use of AC power other than in the household. Similarly, give an example of a use of DC power other than that supplied by batteries.
16. Why do voltage, current, and power go through zero 120 times per second for 60-Hz AC electricity?
17. You are riding in a train, gazing into the distance through its window. As close objects streak by, you notice that the nearby fluorescent lights make dashed streaks. Explain.

16.6 Electric Hazards and the Human Body

18. Using an ohmmeter, a student measures the resistance between various points on his body. They find that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when

more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

19. What are the two major hazards of electricity?
20. Why isn't a short circuit a shock hazard?
21. What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?
22. An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?
23. Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?
24. Some devices often used in bathrooms, such as hairdryers, often have safety messages saying "Do not use when the bathtub or basin is full of water." Why is this so?
25. We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?
26. Before working on a power transmission line, experts will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?
27. Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?
28. Could a person on intravenous infusion (an IV) be microshock sensitive?
29. In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

16.7 Nerve Conduction–Electrocardiograms

30. Note that in Figure 16.25, both the concentration gradient and the Coulomb force

tend to move Na^+ ions into the cell. What prevents this?

31. Define depolarization, repolarization, and the action potential.
32. Explain the properties of myelinated nerves in terms of the insulating properties of myelin.

16.8 Resistors in Series and Parallel

33. A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in Figure 16.39 has on current when open and when closed.

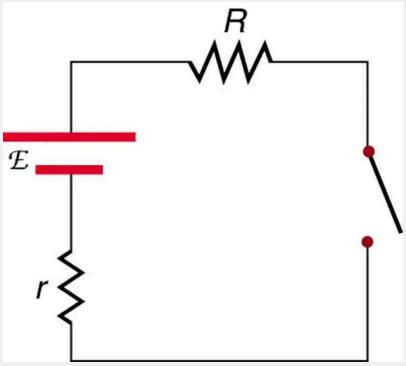


Figure 16.39 A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a simple electrical circuit diagram. The main components include:

– A voltage source on the left, represented by two parallel lines, where the longer line is marked with “E” and the shorter with “r”.
– A resistor labeled “R” is located at the top of the circuit.
– A switch is depicted on the right side, shown in an open position, represented by two small circles connected by a line which is tilted to signify the open state.
– The wires connecting these components form a rectangular loop, completing the circuit.

The diagram illustrates the basic elements of an electrical circuit with an internal

resistance “r” and an external resistor “R”, while the switch is currently open, indicating that the circuit is not complete.

34. What is the voltage across the open switch in Figure 16.39?
35. There is a voltage across an open switch, such as in Figure 16.39. Why, then, is the power dissipated by the open switch small? Why is the power dissipated by a closed switch, such as in Figure 16.39, small?
36. A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in Figure 16.40. Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)

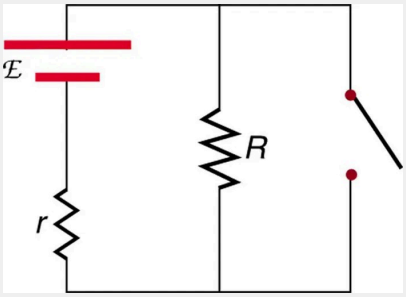


Figure 16.40 A wiring mistake put this switch in parallel with the device represented by R. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts an electrical circuit diagram consisting of a battery and resistors. On the left, there is a symbol for an ideal battery with an electromotive force labeled as $\mathscr{E}$. Below it, there is a resistor labeled *r*, representing the internal resistance of the battery. The circuit splits into two parallel branches: one contains a large zigzag line resistor labeled *R*, and the other includes a closed switch. Red dots indicate connection points in the circuit where the wires and components are joined.

38. Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.
39. Would your headlights dim when you start your car’s engine if the wires in your automobile were superconductors? (Do not neglect the battery’s internal

resistance.) Explain.

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PROBLEMS & EXERCISES

Problems & Exercises

16.1 Current

1. What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?
2. A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?
3. What is the current when a typical static charge of $0.250\mu\text{C}$ moves from your finger to a metal doorknob in $1.00\mu\text{s}$?
4. Find the current when 2.00 nC jumps between your comb and hair over a $0.500\ -\ \mu\text{s}$ time interval.
5. A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?
6. The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?
7. (a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10,000-V potential as in the figure below. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power: $P = I^2R$.)

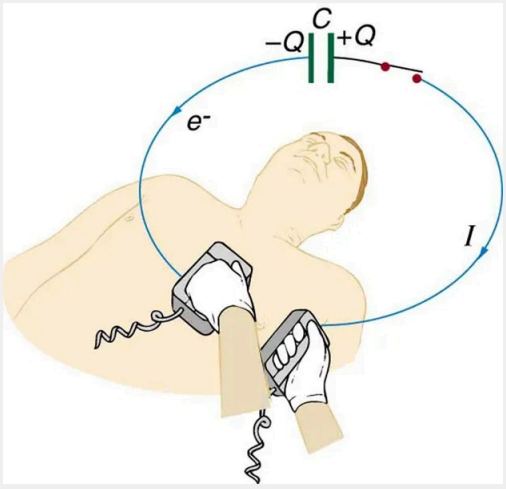


Figure 16.41 The capacitor in a defibrillation unit drives a current through the heart of a patient. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is an illustration of a person lying down, likely for a resuscitation procedure. The person is depicted from the shoulders up, with their eyes closed, suggesting they are unconscious.

At the bottom of the image, two hands wearing gloves are holding defibrillator paddles, positioned on the person's chest, indicating preparation to deliver an electric shock.

Above the person, a conceptual illustration of an electrical circuit is shown. The circuit includes symbols such as a capacitor labeled "C," with charges "-Q" and "+Q" on either side. An arrow labeled "e-" signifies the flow of electrons, while another arrow labeled "I" represents the current direction in the circuit.

Overall, the image suggests a depiction of the physics involved in using a defibrillator, with a focus on the electrical components and the current flow during the procedure.

8. During open-heart surgery, a defibrillator can be used to bring a patient out of cardiac arrest. The resistance of the path is $500\ \Omega$ and a 10.0-mA current is needed. What voltage should be applied?
9. (a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires

connected to the patient? (See figure two problems earlier.)

10. A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?
11. The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (6.02×10^{23}) of electrons at this rate?
12. Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?
13. A large cyclotron directs a beam of He^{++} nuclei onto a target with a beam current of 0.250 mA. (a) How many He^{++} nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of He^{++} nuclei strike the target?
14. Repeat the above example on Example 16.3, but for a wire made of silver and given there is one free electron per silver atom.
15. Using the results of the above example on Example 16.3, find the drift velocity in a copper wire of twice the diameter and carrying 20.0 A.
16. A 14-gauge copper wire has a diameter of 1.628 mm. What magnitude current flows when the drift velocity is 1.00 mm/s? (See above example on Example 16.3 for useful information.)
17. SPEAR, a storage ring about 72.0 m in diameter at the Stanford Linear Accelerator (closed in 2009), has a 20.0-A circulating beam of electrons that are moving at nearly the speed of light. (See Figure 16.42.) How many electrons are in the beam?

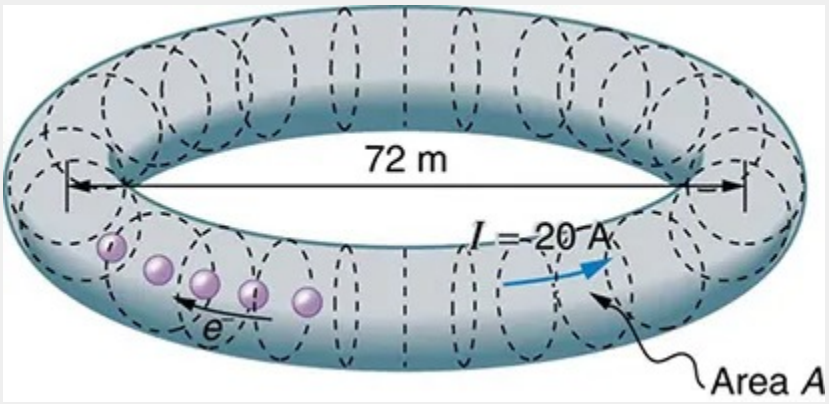


Figure 16.42 Electrons circulating in the storage ring called SPEAR constitute a 20.0-A current. Because they travel close to the speed of light, each electron completes many orbits in each second. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a large toroidal structure, resembling a donut shape, with a diameter labeled as “72 m”. The toroid is illustrated with dashed lines following its curved path, likely representing magnetic field lines. Inside the toroid, an arrow labeled “I = 20 A” indicates the direction and magnitude of an electric current flowing through the structure. On the left side, small purple circles with the label “e⁻” represent electrons. An arrow points inward toward the structure with text labeled “Area A”. The visualization suggests a magnetic or electromagnetic context, potentially explaining concepts related to magnetic fields or electron flow within a toroidal coil.

16.2 Ohm’s Law: Resistance and Simple Circuits

18. What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?
19. Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.
20. What is the effective resistance of a car’s starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?
21. How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \Omega$, given that 25.0 mA passes through it?

22. (a) Find the voltage drop in an extension cord having a $0.0600\ \Omega$ resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of 0.300Ω. What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?
23. A power transmission line is hung from metal towers with glass insulators having a resistance of $1.00 \times 10^9\Omega$. What current flows through the insulator if the voltage is 200 kV? (Some high-voltage lines are DC.)

16.3 Resistance and Resistivity

24. What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?
25. The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.
26. If the 0.010-mm diameter tungsten filament in a light bulb is to have a resistance of $0.200\ \Omega$ at $20.00^{\circ}\ \mathrm{C}$, how long should it be?
27. Find the ratio of the diameter of aluminum to copper wire, if they have the same resistance per unit length (as they might in household wiring).
28. What current flows through a 2.54-cm-diameter rod of pure silicon that is 20.0 cm long, when $1.00\times10^3\mathrm{V}$ is applied to it? (Such a rod may be used to make nuclear-particle detectors, for example.)
29. (a) To what temperature must you raise a copper wire, originally at $20.0^{\circ}\ \mathrm{C}$, to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?
30. A resistor made of Nichrome wire is used in an application where its resistance cannot change more than 1.00% from its value at $20.0^{\circ}\ \mathrm{C}$. Over what temperature range can it be used?
31. Of what material is a resistor made if its resistance is 40.0% greater at $100^{\circ}\ \mathrm{C}$ than at $20.0^{\circ}\ \mathrm{C}$?
32. An electronic device designed to operate at any temperature in the range from $-10.0^{\circ}\ \mathrm{C}$ to $55.0^{\circ}\ \mathrm{C}$ contains pure carbon resistors. By what factor does their

resistance increase over this range?

33. (a) Of what material is a wire made, if it is 25.0 m long with a 0.100 mm diameter and has a resistance of $77.7\,\Omega$ at $20.0^{\circ}\,\mathrm{C}$? (b) What is its resistance at $150^{\circ}\,\mathrm{C}$?
34. Assuming a constant temperature coefficient of resistivity, what is the maximum percent decrease in the resistance of a constantan wire starting at $20.0^{\circ}\,\mathrm{C}$?
35. A wire is drawn through a die, stretching it to four times its original length. By what factor does its resistance increase?
36. A copper wire has a resistance of $0.500\,\Omega$ at $20.0^{\circ}\,\mathrm{C}$, and an iron wire has a resistance of $0.525\,\Omega$ at the same temperature. At what temperature are their resistances equal?
37. (a) Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has $\alpha = -0.0600/^{\circ}\,\mathrm{C}$) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at $37.0^{\circ}\,\mathrm{C}$ (normal body temperature)? (b) The negative value for α may not be maintained for very high temperatures. Discuss why and whether this is the case here. (Hint: Resistance can't become negative.)
38. **Integrated Concepts:** (a) Redo Exercise 16.25 taking into account the thermal expansion of the tungsten filament. You may assume a thermal expansion coefficient of $12 \times 10^{-6}/^{\circ}\,\mathrm{C}$. (b) By what percentage does your answer differ from that in the example?
39. **Unreasonable Results:** (a) To what temperature must you raise a resistor made of constantan to double its resistance, assuming a constant temperature coefficient of resistivity? (b) To cut it in half? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable, or which premises are inconsistent?

16.4 Electric Power and Energy

40. What is the power of a $1.00 \times 10^{2}\,\mathrm{MV}$ lightning bolt having a current of $2.00 \times 10^{4}\,\mathrm{A}$?
41. What power is supplied to the starter motor of a large truck that draws 250 A of

current from a 24.0-V battery hookup?

42. A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See Figure 16.43.)



Figure 16.43 The strip of solar cells just above the keys of this calculator convert light to electricity to supply its energy needs. Image from OpenStax College Physics 2e, CC-BY 4.0

43. How many watts does a flashlight that has $6.00 \times 10^2 \text{C}$ pass through it in 0.500 h use if its voltage is 3.00 V?
44. Find the power dissipated in each of these extension cords: (a) an extension cord having a $0.0600 - \Omega$ resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of 0.300Ω.
45. Verify that the units of a volt-ampere are watts, as implied by the equation $P = \text{IV}$.
46. Show that the units $1\ \text{V}^2/\Omega = 1\ \text{W}$, as implied by the equation $P = V^2/R$.
47. Show that the units $1\ \text{A}^2 \cdot \Omega = 1\ \text{W}$, as implied by the equation $P = I^2R$.
48. Verify the energy unit equivalence that $1\ \text{kW} \cdot \text{h}\ =\ 3.60 \times 10^6\ \text{J}$.
49. Electrons in an X-ray tube are accelerated through $1.00 \times 10^2 \text{kV}$ and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA.
50. An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs $12.0\ \text{cents}/\text{kW} \cdot \text{h}$? See Figure 16.44.



Figure 16.44 On-demand electric hot water heater. Heat is supplied to water only when needed. Image from OpenStax College Physics 2e, CC-BY 4.0

51. With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At
$9.0\ \text{cents/kW} \cdot \text{h}$,
how much does this cost?
52. What would be the maximum cost of a CFL such that the total cost (investment plus operating) would be the same for both CFL and incandescent 60-W bulbs? Assume the cost of the incandescent bulb is 25 cents and that electricity costs $10\ \text{cents/kWh}$.
Calculate the cost for 1000 hours, as in the cost effectiveness of CFL example.
53. Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?
54. Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at $1.00\ \text{A} \cdot \text{h}$ and 1.58 V keep a 1.00-W flashlight bulb burning?
55. A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?
56. The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages $12.0\ \text{cents/kW} \cdot \text{h}$.
57. An old lightbulb draws only 50.0 W, rather than its original 60.0 W, due to evaporative thinning of its filament. By what factor is its diameter reduced, assuming uniform thinning along its length? Neglect any effects caused by

temperature differences.

58. 00-gauge copper wire has a diameter of 9.266 mm. Calculate the power loss in a kilometer of such wire when it carries

$1.00 \times 10^2 \text{A}$.

59. **Integrated Concepts:** Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See Figure 16.45.)

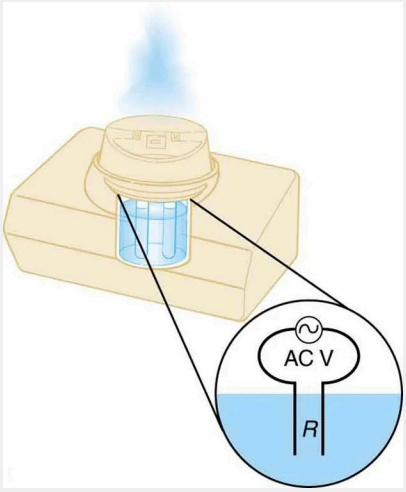


Figure 16.45 This cold vaporizer passes current directly through water, vaporizing it directly with relatively little temperature increase. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a simplified diagram of an electrical component with a focus on a compartment and a symbol. The component is likely a device, with a circular section on top of a rectangular base. There is a magnified view highlighting a part of this device. Inside the magnified section, there is a blue liquid depicted at the bottom, and an electrical symbol above it. The symbol includes a waveform line, labeled “AC V,” and a resistor symbol labeled “R.” The overall illustration suggests a focus on measuring or indicating an electrical parameter, possibly related to voltage or resistance.

60. **Integrated Concepts:** (a) What energy is dissipated by a lightning bolt having a 20,000-A current, a voltage of $1.00 \times 10^2 \text{MV}$, and a length of 1.00 ms? (b) What mass of tree sap could be raised from $18.0^\circ\ \text{C}$ to its boiling point and then evaporated by this energy, assuming sap has the same thermal characteristics as water?
61. **Integrated Concepts:** What current must be produced by a 12.0-V battery-operated bottle warmer in order to heat 75.0 g of glass, 250 g of baby formula, and $3.00 \times 10^2 \text{g}$ of aluminum from $20.0^\circ\ \text{C}$ to $90.0^\circ\ \text{C}$ in 5.00 min?
62. **Integrated Concepts:** How much time is needed for a surgical cauterizer to raise the temperature of 1.00 g of tissue from $37.0^\circ\ \text{C}$ to $100^\circ\ \text{C}$ and then boil away 0.500 g of water, if it puts out 2.00 mA at 15.0 kV? Ignore heat transfer to the surroundings.
63. **Integrated Concepts:** Hydroelectric generators (see Figure 16.46) at Hoover Dam produce a maximum current of $8.00 \times 10^3 \text{A}$ at 250 kV. (a) What is the power output? (b) The water that powers the generators enters and leaves the system at low speed (thus its kinetic energy does not change) but loses 160 m in altitude. How many cubic meters per second are needed, assuming 85.0% efficiency?



Figure 16.46 Hydroelectric generators at the Hoover dam. Image by Unknown, Wikimedia Commons License

64. **Integrated Concepts:** (a) Assuming 95.0% efficiency for the conversion of electrical power by the motor, what current must the 12.0-V batteries of a 750-kg electric car be able to supply: (a) To accelerate from rest to 25.0 m/s in 1.00 min? (b) To climb a $2.00 \times 10^2 - \text{m}$-high hill in 2.00 min at a constant 25.0-m/s speed while exerting $5.00 \times 10^2 \text{N}$ of force to overcome air resistance and friction? (c) To travel at a constant 25.0-m/s speed, exerting a $5.00 \times 10^2 \text{N}$ force to overcome air resistance and friction? See Figure 16.47.

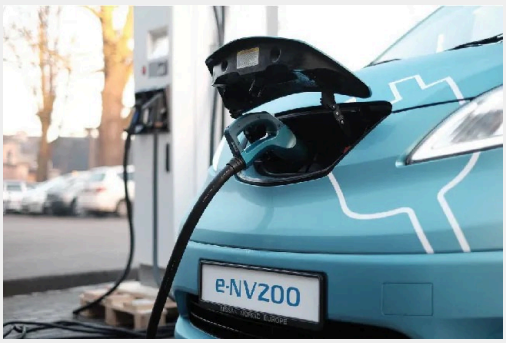


Figure 16.47 This REVAi, an electric car, gets recharged on a street in London. Image from OpenStax College Physics 2e, CC-BY 4.0

65. **Integrated Concepts:** A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is $5.30 \times 10^4\ \text{kg}$, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.
66. **Integrated Concepts:** (a) An aluminum power transmission line has a resistance of $0.0580\Omega/\text{km}$. What is its mass per kilometer? (b) What is the mass per kilometer of a copper line having the same resistance? A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.
67. **Integrated Concepts:** (a) An immersion heater utilizing 120 V can raise the temperature of a $1.00 \times 10^2 - \text{g}$ aluminum cup containing 350 g of water from $20.0^\circ\ \text{C}$ to $95.0^\circ\ \text{C}$ in 2.00 min. Find its resistance, assuming it is constant during the process. (b) A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.
68. **Integrated Concepts:** (a) What is the cost of heating a hot tub containing 1500 kg of water from $10.0^\circ\ \text{C}$ to $40.0^\circ\ \text{C}$, assuming 75.0% efficiency to account for heat transfer to the surroundings? The cost of electricity is $9\ \text{cents}/\text{kW}\cdot\text{h}$. (b) What current was used by the 220-V AC electric heater, if this took 4.00 h?
69. **Unreasonable Results:** (a) What current is needed to transmit $1.00 \times 10^2\text{MW}$

of power at 480 V? (b) What power is dissipated by the transmission lines if they have a $1.00\ \Omega$ resistance? (c) What is unreasonable about this result? (d) Which assumptions are unreasonable, or which premises are inconsistent?

70. **Unreasonable Results:** (a) What current is needed to transmit $1.00 \times 10^2 \text{MW}$ of power at 10.0 kV? (b) Find the resistance of 1.00 km of wire that would cause a 0.0100% power loss. (c) What is the diameter of a 1.00-km-long copper wire having this resistance? (d) What is unreasonable about these results? (e) Which assumptions are unreasonable, or which premises are inconsistent?
71. **Construct Your Own Problem:** Consider an electric immersion heater used to heat a cup of water to make tea. Construct a problem in which you calculate the needed resistance of the heater so that it increases the temperature of the water and cup in a reasonable amount of time. Also calculate the cost of the electrical energy used in your process. Among the things to be considered are the voltage used, the masses and heat capacities involved, heat losses, and the time over which the heating takes place. Your instructor may wish for you to consider a thermal safety switch (perhaps bimetallic) that will halt the process before damaging temperatures are reached in the immersion unit.

16.5 Alternating Current versus Direct Current

72. (a) What is the hot resistance of a 25-W light bulb that runs on 120-V AC? (b) If the bulb's operating temperature is $2700^{\circ}\ \text{C}$, what is its resistance at $2600^{\circ}\ \text{C}$?
73. Certain heavy industrial equipment uses AC power that has a peak voltage of 679 V. What is the rms voltage?
74. A certain circuit breaker trips when the rms current is 15.0 A. What is the corresponding peak current?
75. Military aircraft use 400-Hz AC power, because it is possible to design lighter-weight equipment at this higher frequency. What is the time for one complete cycle of this power?
76. A North American tourist takes his 25.0-W, 120-V AC razor to Europe, finds a special adapter, and plugs it into 240 V AC. Assuming constant resistance, what power does the razor consume as it is ruined?

77. In this problem, you will verify statements made at the end of the power losses for Example 16.10. (a) What current is needed to transmit 100 MW of power at a voltage of 25.0 kV? (b) Find the power loss in a $1.00\ -\Omega$ transmission line. (c) What percent loss does this represent?
78. A small office-building air conditioner operates on 408-V AC and consumes 50.0 kW. (a) What is its effective resistance? (b) What is the cost of running the air conditioner during a hot summer month when it is on 8.00 h per day for 30 days and electricity costs $9.00 cents/\mathrm{kW}\cdot\mathrm{h}$?
79. What is the peak power consumption of a 120-V AC microwave oven that draws 10.0 A?
80. What is the peak current through a 500-W room heater that operates on 120-V AC power?
81. Two different electrical devices have the same power consumption, but one is meant to be operated on 120-V AC and the other on 240-V AC. (a) What is the ratio of their resistances? (b) What is the ratio of their currents? (c) Assuming its resistance is unaffected, by what factor will the power increase if a 120-V AC device is connected to 240-V AC?
82. Nichrome wire is used in some radiative heaters. (a) Find the resistance needed if the average power output is to be 1.00 kW utilizing 120-V AC. (b) What length of Nichrome wire, having a cross-sectional area of $5.00\ \mathrm{mm}^2$, is needed if the operating temperature is $500^{\circ}\ \mathrm{C}$? (c) What power will it draw when first switched on?
83. Find the time after $t = 0$ when the instantaneous voltage of 60-Hz AC first reaches the following values: (a) $V_0/2$ (b) V_0 (c) 0.
84. (a) At what two times in the first period following $t = 0$ does the instantaneous voltage in 60-Hz AC equal V_{rms}? (b) $-V_{\mathrm{rms}}$?

16.6 Electric Hazards and the Human Body

85. (a) How much power is dissipated in a short circuit of 220-V AC through a resistance of 0.250Ω? (b) What current flows?
86. What voltage is involved in a 1.44-kW short circuit through a $0.100\ -\Omega$

resistance?

87. Find the current through a person and identify the likely effect on her if she touches a 120-V AC source: (a) if she is standing on a rubber mat and offers a total resistance of $300\ \text{k}\Omega$; (b) if she is standing barefoot on wet grass and has a resistance of only 4500Ω.
88. While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of 4000Ω. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?
89. Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?
90. (a) During surgery, a current as small as $20.0\ \mu\text{A}$ applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is 300Ω, what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?
91. (a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage was 120 V AC?
92. A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.
93. **Integrated Concepts:** A short circuit in a 120-V appliance cord has a $0.500\ -\ \Omega$ resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is $0.200\ \text{cal/g}\cdot^{\circ}\ \text{C}$ and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?
94. **Construct Your Own Problem:** Consider a person working in an environment where electric currents might pass through their body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, ...), and acceptable currents

(safe but sensed, safe and unfelt, ...).

16.7 Nerve Conduction–Electrocardiograms

95. **Integrated Concepts:** Use the ECG in Figure 16.31 to determine the heart rate in beats per minute assuming a constant time between beats.
96. **Integrated Concepts:** (a) Referring to Figure 16.31, find the time systolic pressure lags behind the middle of the QRS complex. (b) Discuss the reasons for the time lag.
97. **Critical Thinking:** A copper wire of length 1.25 m is in a circuit. A current of 2.50 A flows through the wire. (a) Initially, the circuit is at 20.0°C. If there is an energy loss of 3.25 MJ in the first second caused by the wire as soon as the circuit is turned on, what is the radius of the wire? (b) After the wire heats 3.000°C, how much energy is being dissipated by the circuit per second? Neglect the effects of thermal expansion since the wire is considered to be physically constrained from expanding. Assume the current is maintained. (c) At what temperature will the energy dissipation be 1.00% higher than it was at the beginning of the experiment?

16.8 Resistors in Series and Parallel

Note: Data taken from figures can be assumed to be accurate to three significant digits.

98. (a) What is the resistance of ten $275-\Omega$ resistors connected in series? (b) In parallel?
99. (a) What is the resistance of a $1.00\times10^2-\Omega$, a $2.50-\text{k}\Omega$, and a $4.00-\text{k}\Omega$ resistor connected in series? (b) In parallel?
100. What are the largest and smallest resistances you can obtain by connecting a $36.0-\Omega$, a $50.0-\Omega$, and a $700-\Omega$ resistor together?
101. An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A, 120-V circuit. (The three devices are in parallel when plugged into the same socket.). (a) What current is drawn by each device?

(b) Will this combination blow the 15-A fuse?

102. Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)
103. (a) Given a 48.0-V battery and $24.0 - \Omega$ and $96.0 - \Omega$ resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.
104. Referring to the example combining series and parallel circuits and Figure 16.36, calculate I_3 in the following two different ways: (a) from the known values of I and I_2; (b) using Ohm's law for R_3. In both parts explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.
105. Referring to Figure 16.36: (a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.
106. Refer to Figure 16.37 and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is 0.400Ω, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?
107. A 240-kV power transmission line carrying $5.00 \times 10^2 \text{A}$ is hung from grounded metal towers by ceramic insulators, each having a $1.00 \times 10^9 - \Omega$ resistance. Figure 16.48. (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this? Explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.

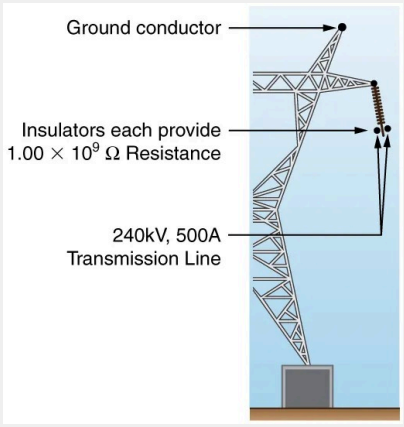


Figure 16.48 High-voltage (240-kV) transmission line carrying $5.00 \times 10^2 \text{A}$ *is hung from a grounded metal transmission tower. The row of ceramic insulators provide* $1.00 \times 10^9 \Omega$ *of resistance each. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image depicts a diagram of an electricity transmission tower. The following elements are labeled:

- **Ground conductor:** Shown at the top of the tower, represented by a black dot connected to the structure.
- **Insulators:** These are shown as cylindrical units, placed below the ground conductor, with a label indicating that each provides resistance of 1.00 × 10^9 Ω (Ohms).
- **Transmission Line:** The line is marked as carrying 240kV (kilovolts) and 500A (amperes). It is positioned further down the tower.

The tower is illustrated against a blue sky background and has a lattice structure, with a rectangular base resting on the ground.

108. Show that if two resistors R_1 and R_2 are combined and one is much greater than the other ($R_1 >> R_2$): (a) Their series resistance is very nearly equal to the greater resistance R_1. (b) Their parallel resistance is very nearly equal to smaller resistance R_2.
109. **Unreasonable Results:** Two resistors, one having a resistance of $145\ \Omega$, are

connected in parallel to produce a total resistance of 150Ω. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

110. **Unreasonable Results:** Two resistors, one having a resistance of $900\ \text{k}\Omega$, are connected in series to produce a total resistance of $0.500\ \text{M}\Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

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MAGNETISM



Figure 17.1 The magnificent spectacle of the Aurora Borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by the Earth's magnetic field, this light is produced by radiation spewed from solar storms. Image from OpenStax College Physics 2e, CC-BY 4.0

Chapter Outline

17.1 Magnets
17.2 Ferromagnets and Electromagnets
17.3 Magnetic Fields and Magnetic Field Lines
17.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field
17.5 Force on a Moving Charge in a Magnetic Field: Examples and Applications
17.6 The Hall Effect
17.7 Magnetic Force on a Current-Carrying Conductor
17.8 Torque on a Current Loop: Motors and Meters
17.9 Magnetic Fields Produced by Currents: Ampere's Law
17.10 Magnetic Force between Two Parallel Conductors
17.11 More Applications of Magnetism
Glossary
Chapter Summary
Conceptual Questions
Problems & Exercises

One evening, an Alaskan sticks a note to his refrigerator with a small magnet. Through the kitchen window, the Aurora Borealis glows in the night sky. This grand spectacle is shaped by the same force that holds the note to the refrigerator.

People have been aware of magnets and magnetism for thousands of years. The earliest records date to well before the time of Christ, particularly in a region of Asia Minor called Magnesia (the name of this region is the source of words like *magnetic*). Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. A practical application for magnets was found later, when they were employed as navigational compasses. The use of magnets in compasses resulted not only in improved long-distance sailing, but also in the names of "north" and "south" being given to the two types of magnetic poles.

Today magnetism plays many important roles in our lives. Physicists' understanding of magnetism has enabled the development of technologies that affect our everyday lives. The iPod in your purse or backpack,

for example, wouldn't have been possible without the applications of magnetism and electricity on a small scale.

The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology. The 2007 Nobel Prize in Physics went to Albert Fert from France and Peter Grunberg from Germany for this discovery of *giant magnetoresistance* and its applications to computer memory.

All electric motors, with uses as diverse as powering refrigerators, starting cars, and moving elevators, contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Hundreds of millions of dollars are spent annually on magnetic containment of fusion as a future energy source. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is used to explain atomic energy levels, cosmic rays, and charged particles trapped in the Van Allen belts. Once again, we will find all these disparate phenomena are linked by a small number of underlying physical principles.



Figure 17.2 Engineering of technology like iPods would not be possible without a deep understanding magnetism. Image from OpenStax College Physics 2e, CC-BY 4.0

Access multimedia content

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17.1 MAGNETS

Learning Objectives

By the end of this section, you will be able to:

- Describe the difference between the north and south poles of a magnet.
- Describe how magnetic poles interact with each other.

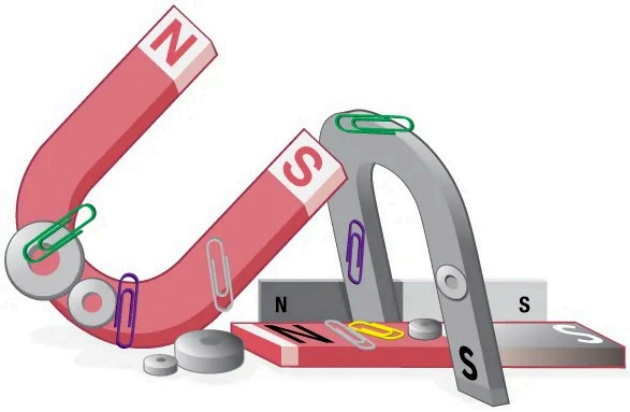


Figure 17.3 Magnets come in various shapes, sizes, and strengths. All have both a north pole and a south pole. There is never an isolated pole Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image depicts three types of magnets with metal paperclips and small metal discs attracted to them. The magnets include:

- An **U-shaped magnet** on the left, colored red with the poles labeled 'N' for North and 'S' for South.

Green and purple paperclips are attached to it, along with some small metal discs.

- A **horseshoe magnet** in the center, colored gray with the poles labeled 'N' and 'S'. A purple paperclip is attached to it, along with a green paperclip on top.
- A **rectangular bar magnet** on the right, colored red and white with black letters 'N' for North and 'S' for South. An assortment of paperclips and small metal discs are attached to it.

All magnets attract iron, such as that in a refrigerator door. However, magnets may attract or repel other magnets. Experimentation shows that all magnets have two poles. If freely suspended, one pole will point toward the north. The two poles are thus named the north magnetic pole and the south magnetic pole (or more properly, north-seeking and south-seeking poles, for the attractions in those directions).

Universal Characteristics of Magnets and Magnetic Poles

It is a universal characteristic of all magnets that *like poles repel and unlike poles attract*. (Note the similarity with electrostatics: unlike charges attract and like charges repel.)

Further experimentation shows that it is *impossible to separate north and south poles* in the manner that + and − charges can be separated.

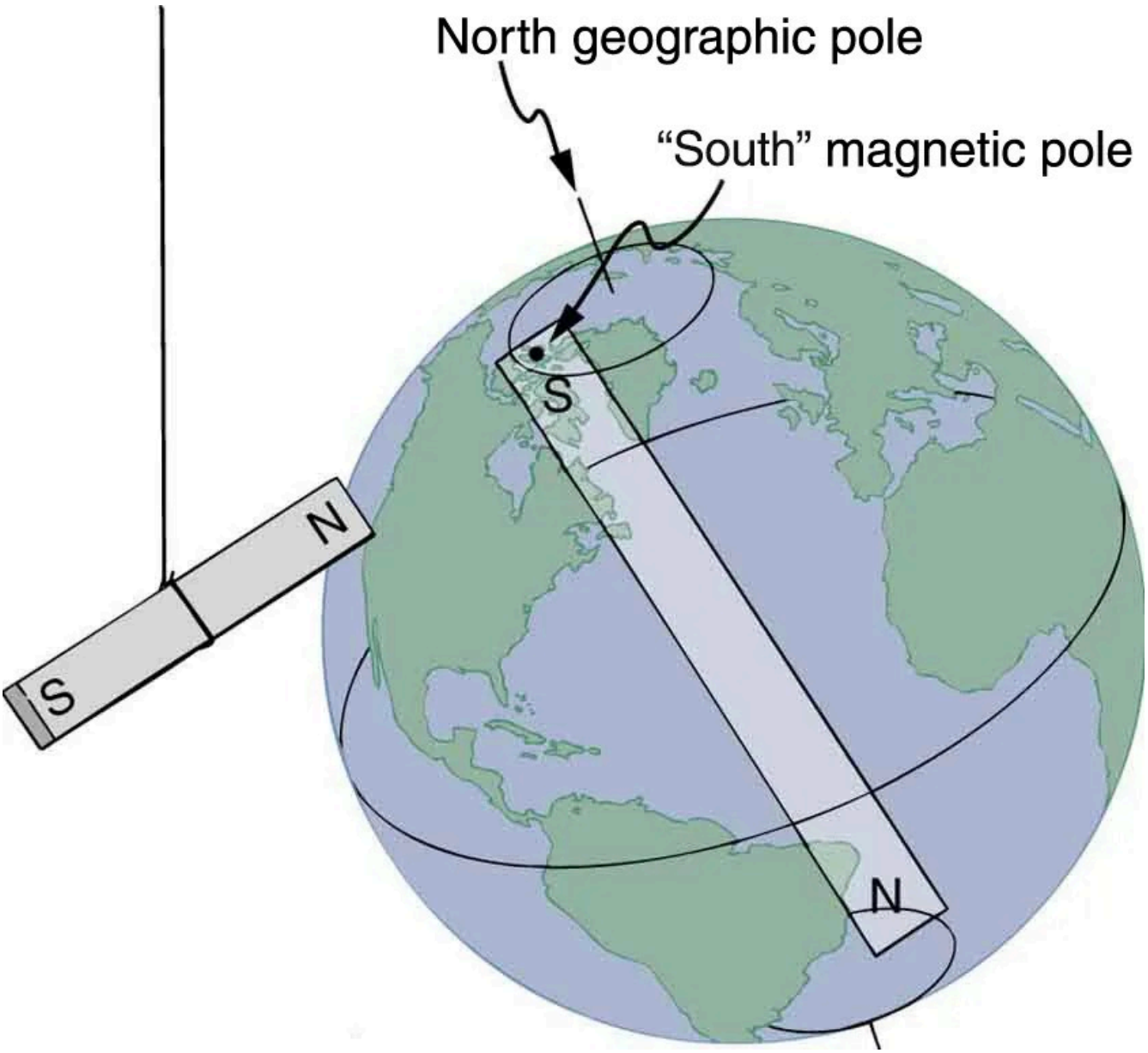


Figure 17.4 One end of a bar magnet is suspended from a thread that points toward north. The magnet's two poles are labeled N and S for north-seeking and south-seeking poles, respectively. *Image by OpenStax, Wikimedia Commons License*

Image Description

The image is an illustration depicting the Earth's magnetic field in relation to its geographic poles. The Earth is shown as a large sphere with continents visible.

- A vertical line represents the Earth's rotational axis, connecting the North and South geographic poles.
- The North geographic pole is labeled, and nearby, the "South" magnetic pole is indicated.
- A rectangular bar magnet is shown outside the Earth with its ends marked "S" (South) and "N"

(North).

- The magnet is positioned so that its South end is aligned with the North geographic pole and the Earth's "South" magnetic pole, illustrating the concept of magnetic polarity.
- Curved lines surround the Earth, representing the magnetic field lines extending between the magnet's poles.

Misconception Alert: Earth's Magnetic Poles

Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term "North Pole" has come to be used (incorrectly) for the magnetic pole that is near the North Pole. Thus, "north magnetic pole" is actually a misnomer—it should be called the south magnetic pole.

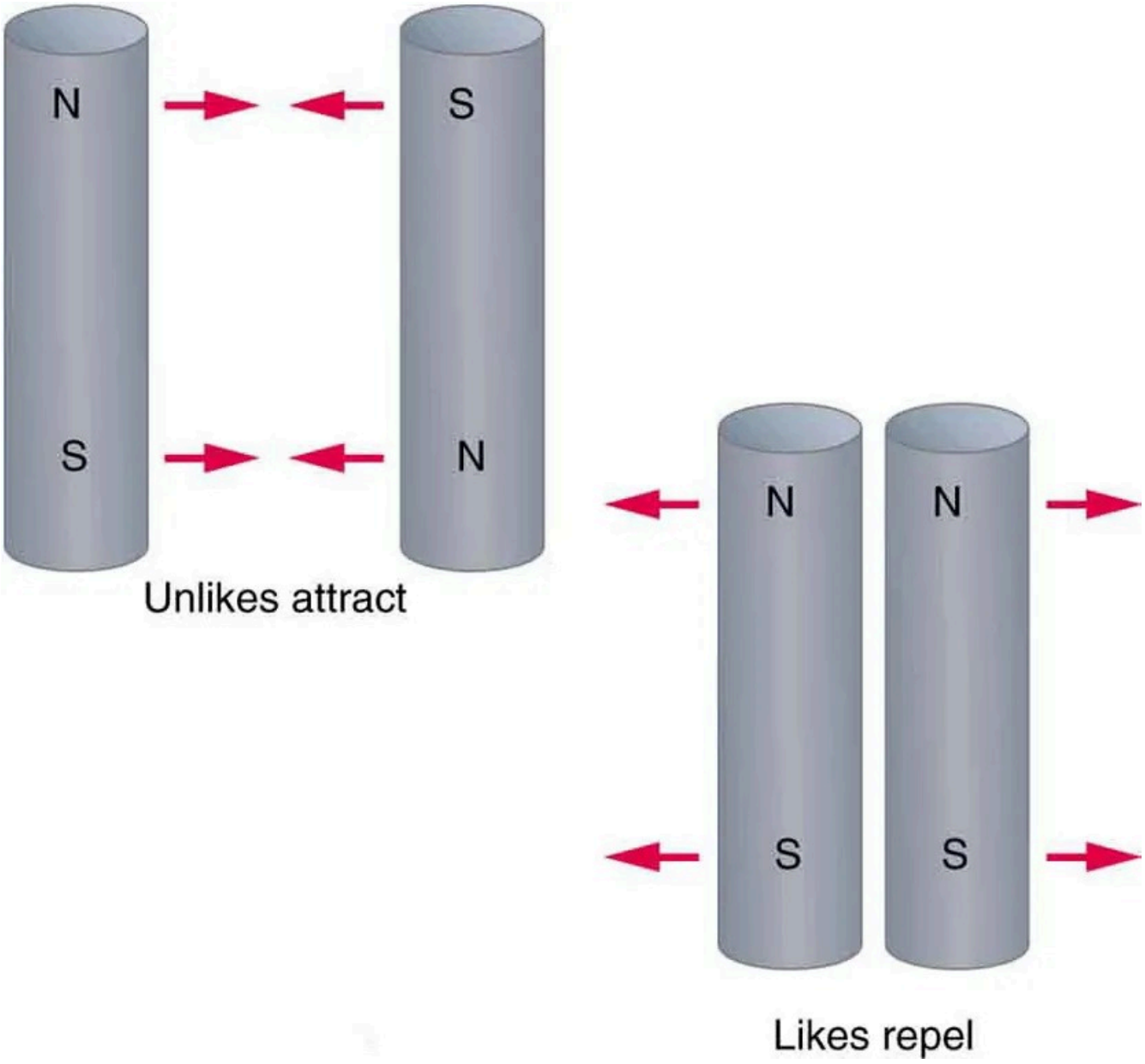


Figure 17.5 Unlike poles attract, whereas like poles repel. Image by OpenStax, Wikimedia Commons License

Image Description

The image shows two sets of magnetic bars illustrating the concepts of attraction and repulsion between magnetic poles.

The set on the left consists of two vertical magnets facing each other with opposite poles. One magnet is labeled “N” (north) on top and “S” (south) on bottom, while the other is labeled “S” on top and “N” on

bottom. Red arrows between the magnets point toward each other, indicating attraction. Below this set, the text reads "Unlikes attract."

The set on the right shows two vertical magnets positioned parallel to each other with like poles facing each other. Both magnets are labeled with "N" on top and "S" on bottom, with red arrows pointing away from each other, indicating repulsion. Below this set, the text reads "Likes repel."

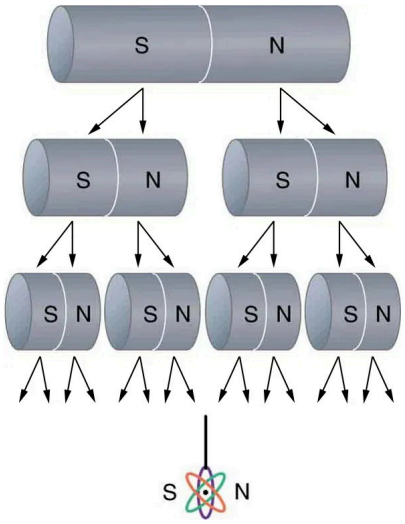


Figure 17.6 North and south poles always occur in pairs. Attempts to separate them result in more pairs of poles. If we continue to split the magnet, we will eventually get down to an iron atom with a north pole and a south pole—these, too, cannot be separated. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a sequence depicting magnetic poles within cylindrical magnets and their division into smaller sections. At the top, there is a large cylindrical magnet with its left end labeled "S" (South) and the right end labeled "N" (North), separated by a line. This cylinder is connected with arrows pointing downward to two smaller cylinders, each similarly labeled with "S" and "N".

From each of these smaller cylinders, additional arrows point downwards to even smaller cylinders, each labeled with "S" and "N". The sequence continues, with each level of cylinders smaller than the last, suggesting a division into smaller magnetic sections while maintaining the "S" and "N" polar labeling.

At the bottom of the image, a single arrow points down to an illustration of an atom with an "S" and "N" labeled, which is surrounded by orbiting paths, representing electrons.

The fact that magnetic poles always occur in pairs of north and south is true from the very large scale—for example, sunspots always occur in pairs that are north and south magnetic poles—all the way down to the

very small scale. Magnetic atoms have both a north pole and a south pole, as do many types of subatomic particles, such as electrons, protons, and neutrons.

Making Connections: Take-Home Experiment—Refrigerator Magnets

We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the door anyway? What can you say about the magnetic properties of the door next to the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

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17.2 FERROMAGNETS AND ELECTROMAGNETS

Learning Objectives

By the end of this section, you will be able to:

- Define ferromagnet.
- Describe the role of magnetic domains in magnetization.
- Explain the significance of the Curie temperature.
- Describe the relationship between electricity and magnetism.

Ferromagnets

Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called ferromagnetic, after the Latin word for iron, *ferrum*. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets; a popular one is neodymium. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be magnetized themselves—that is, they can be induced to be magnetic or made into permanent magnets.

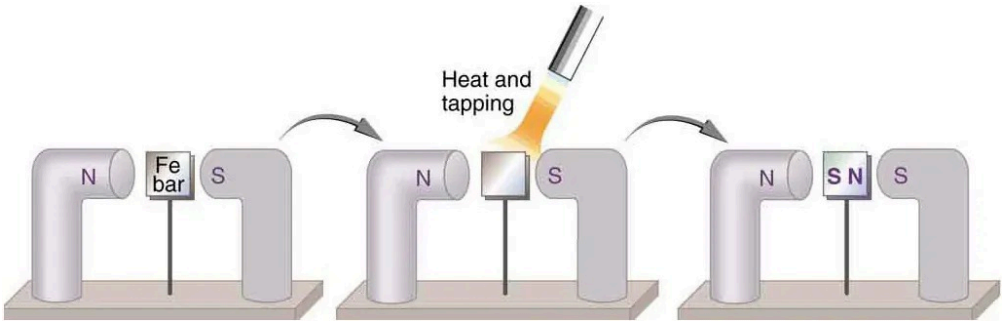


Figure 17.7 An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a three-step process involving magnets and a metal bar.

In the first panel, there is an iron (Fe) bar placed vertically on a base. It is situated between two large, bent magnets labeled 'N' for North and 'S' for South, respectively.

In the second panel, above the iron bar, there is a tool applying heat and tapping to the bar, indicating a process of heating and mechanical impact.

In the third panel, the central bar now displays magnetic poles labeled 'N' for North and 'S' for South, indicating that the iron bar has been magnetized. The bent magnets remain in the same positions.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in Figure 17.7. (This results in the attraction of the previously unmagnetized material to the magnet.) What happens on a microscopic scale is illustrated in Figure 17.8. The regions within the material called domains act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in Figure 17.8(b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.

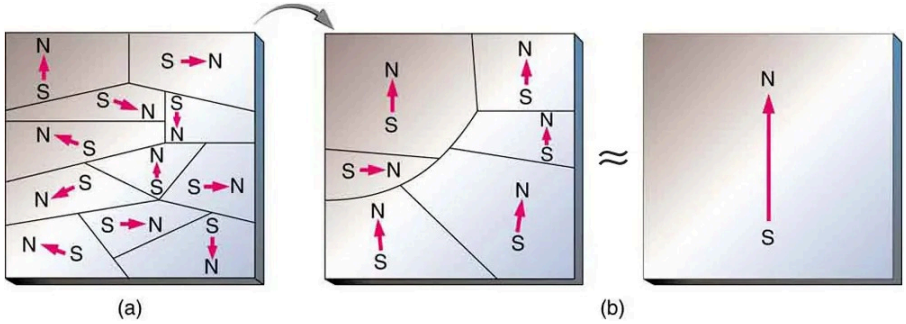


Figure 17.8 (a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams labeled (a), (b), and an unlabeled one depicting a transformation of magnetic domains:

(a) The left diagram shows a rectangular block divided into irregular sections, each with an arrow indicating the direction of magnetic poles. Some arrows point upwards with the letter 'N' (North) at the top and 'S' (South) at the bottom. Others point in various directions, showing that the magnetic domains are disordered.

(b) The middle diagram illustrates the same rectangular block with fewer divisions. The arrows now align uniformly upwards, with the letters 'N' at the top and 'S' at the bottom in each section, indicating partial alignment of magnetic domains.

The right diagram, connected to the middle one by a symbol indicating approximation or transformation, depicts a uniform block with one large arrow pointing upwards. It is labeled with 'N' at the top and 'S' at the bottom, signifying a fully aligned magnetic field throughout the block.

Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of the domains. There is a well-defined temperature for ferromagnetic materials, which is called the Curie temperature, above which they cannot be magnetized. The Curie temperature for iron is 1043 K $(770^{\circ}\ \mathrm{C})$, which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

Electromagnets

Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. Electromagnetism is the use of electric current to make magnets. These temporarily induced magnets are called electromagnets. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines (See Figure 17.9).



Figure 17.9 Instrument for magnetic resonance imaging (MRI). The device uses a superconducting cylindrical coil for the main magnetic field. The patient goes into this "tunnel" on the gurney. Image from OpenStax College Physics 2e, CC-BY 4.0

Figure 17.10 shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The patterns are similar. In fact, electromagnets and ferromagnets have the same basic characteristics—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.

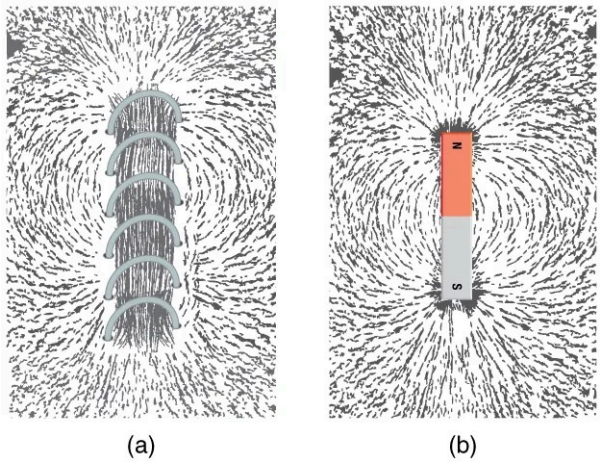


Figure 17.10 Iron filings near (a) a current-carrying coil and (b) a magnet act like tiny compass needles, showing the shape of their fields. Their response to a current-carrying coil and a permanent magnet is seen to be very similar, especially near the ends of the coil and the magnet. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is split into two sections, labeled (a) and (b), each depicting a different magnetic field representation.

(a) *Magnetic Field Around a Solenoid:* This section shows the magnetic field lines produced by a solenoid, which consists of a coil of wire. The solenoid is oriented vertically, and the field lines are densely concentrated inside the coil, indicating a strong magnetic field. Outside the solenoid, the field lines loop around, similar to the pattern of a bar magnet, forming closed curves.

(b) *Magnetic Field Around a Bar Magnet:* This section depicts the magnetic field lines of a bar magnet. The magnet is oriented vertically, with the north pole (N) at the top and the south pole (S) at the bottom. Field lines emerge from the north pole, loop around the magnet, and re-enter at the south pole, forming symmetrical, closed loops.

Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. (See Figure 17.11.) Whenever strong magnetic effects are needed, such as lifting scrap metal, or in particle accelerators, electromagnets are enhanced by ferromagnetic materials. Limits to how strong the magnets can be made are imposed by coil resistance (it will overheat and melt at sufficiently high current), and so superconducting magnets may be employed. These are still limited, because superconducting properties are destroyed by too great a magnetic field.

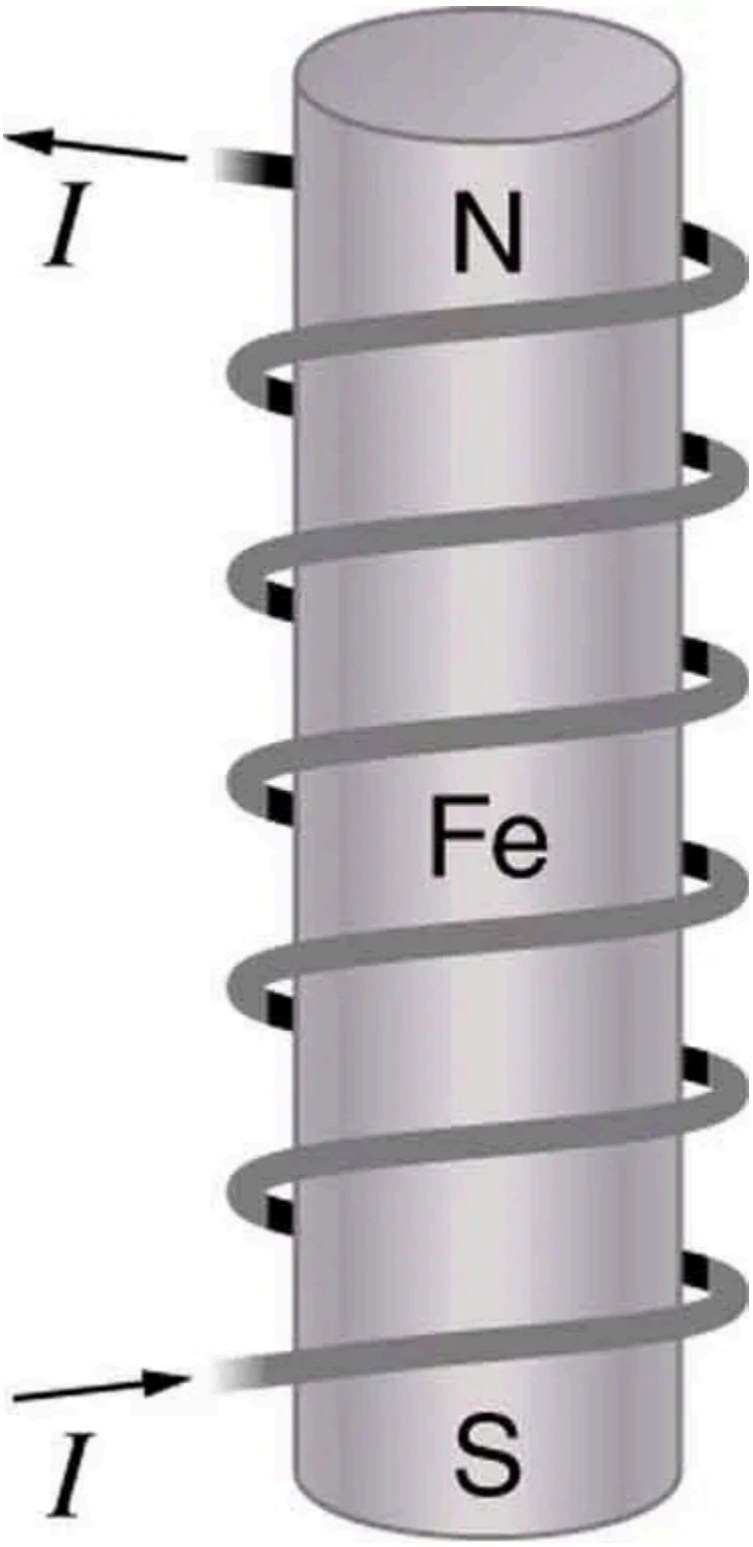


Figure 17.11 An electromagnet with a ferromagnetic core can produce very strong magnetic effects. Alignment of domains in the core produces a magnet, the poles of which are aligned with the electromagnet. Image by OpenStax, Wikimedia Commons License

Image Description

The image depicts a diagram of an electromagnet. It shows a cylindrical iron core, labeled “Fe,” with a wire coil wrapped around it. The coil is shown as a helical path spiraling down the cylinder. The ends of the coil have arrows and are labeled “I,” indicating the direction of the electric current. The top and bottom ends of the cylinder are labeled “N” and “S,” representing the north and south poles of the electromagnet. This setup demonstrates how an electric current flowing through the coil creates a magnetic field, turning the iron core into a magnet.

Figure 17.12 shows a few uses of combinations of electromagnets and ferromagnets. Ferromagnetic materials can act as memory devices, because the orientation of the magnetic fields of small domains can be reversed or erased. Magnetic information storage on videotapes and computer hard drives are among the most common applications. This property is vital in our digital world.

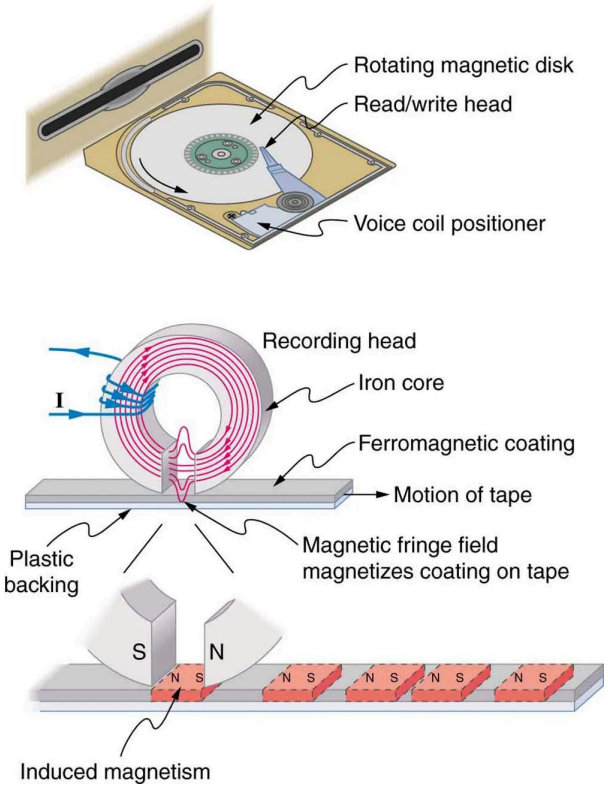


Figure 17.12 An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog (with a varying strength), such as on audiotapes. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram showing the working principles of data storage using magnetic technology. It contains three main parts:

1. **Top Section:** A schematic of a hard disk drive. The components include:

- *Rotating magnetic disk:* A circular disc where data is stored magnetically.
- *Read/write head:* The tool that reads data from and writes data to the disk.
- *Voice coil positioner:* The mechanism that precisely moves the read/write head across the disk surface.

2. **Middle Section:** A diagram illustrating a recording head in a magnetic tape recorder. Components shown are:

- *Recording head:* This includes an iron core and coils where electric current flows, producing a magnetic field.
- *Ferromagnetic coating:* A coating on the tape that gets magnetized by the recording head.
- *Motion of tape:* Indicates the direction the tape moves under the recording head.
- *Plastic backing:* The base material of the tape providing support to the ferromagnetic coating.
- *Magnetic fringe field:* The magnetic field produced by the recording head that magnetizes the coating on the tape.

3. **Bottom Section:** A representation of how magnetism is induced in a magnetic medium, showing:

- *Induced magnetism:* The creation of magnetic poles (north 'N' and south 'S') in the tape due to the magnetic field.

Current: The Source of All Magnetism

An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. But what about ferromagnets? Figure 17.13 shows models of how electric currents create magnetism at the submicroscopic level. (Note that we cannot directly observe the paths of individual electrons about atoms, and so a model or visual image, consistent with all direct observations, is made. We can directly observe the electron's orbital angular momentum, its spin momentum, and subsequent magnetic moments, all of which are explained with electric-current-creating subatomic magnetism.) Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Crucial to the statement that electric current is the source of all magnetism is the fact that it is impossible to separate north and south magnetic poles. (This is far different from the case of positive and negative charges, which are easily separated.) A current loop always produces a magnetic dipole—that is, a magnetic field that

acts like a north pole and south pole pair. Since isolated north and south magnetic poles, called magnetic monopoles, are not observed, currents are used to explain all magnetic effects. If magnetic monopoles did exist, then we would have to modify this underlying connection that all magnetism is due to electrical current. There is no known reason that magnetic monopoles should not exist—they are simply never observed—and so searches at the subnuclear level continue. If they do *not* exist, we would like to find out why not. If they *do* exist, we would like to see evidence of them.

Electric Currents and Magnetism

Electric current is the source of all magnetism.

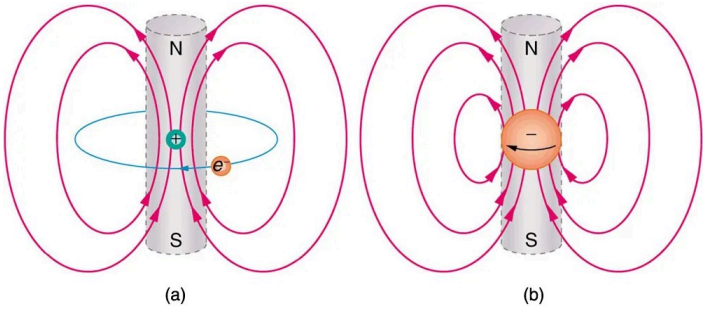


Figure 17.13 (a) In the planetary model of the atom, an electron orbits a nucleus, forming a closed-current loop and producing a magnetic field with a north pole and a south pole. (b) Electrons have spin and can be crudely pictured as rotating charge, forming a current that produces a magnetic field with a north pole and a south pole. Neither the planetary model nor the image of a spinning electron is completely consistent with modern physics. However, they do provide a useful way of understanding phenomena. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two diagrams labeled (a) and (b), illustrating magnetic fields.

Diagram (a):

- A cylindrical magnet is depicted with the top as the North (N) pole and the bottom as the South (S)

pole.

- There are curved pink arrows, representing magnetic field lines, emerging from the North pole, looping around, and entering the South pole.
- A green circle with a positive sign (+) inside is located inside the loops, symbolizing a proton.
- A smaller orange circle with an "e" representing an electron is orbiting the proton in a circular blue path around the magnet.

Diagram (b):

- Similar to diagram (a), a cylindrical magnet is displayed with the North (N) pole on top and the South (S) pole at the bottom.
- The same curved pink arrows represent magnetic field lines, emerging from the North pole and entering the South pole.
- A large orange circle with a negative sign (−) inside is located in the center, representing a spinning electron.
- An arrow inside the orange circle indicates the direction of the electron's spin.

PhET Explorations

Magnets and Electromagnets

Explore the interactions between a compass and bar magnet. Discover how you can use a battery and wire to make a magnet! Can you make it a stronger magnet? Can you make the magnetic field reverse?

Click to view content.

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17.3 MAGNETIC FIELDS AND MAGNETIC FIELD LINES

Learning Objectives

By the end of this section, you will be able to:

- Define magnetic field and describe the magnetic field lines of various magnetic fields.

Einstein is said to have been fascinated by a compass as a child, perhaps musing on how the needle felt a force without direct physical contact. His ability to think deeply and clearly about action at a distance, particularly for gravitational, electric, and magnetic forces, later enabled him to create his revolutionary theory of relativity. Since magnetic forces act at a distance, we define a magnetic field to represent magnetic forces. The pictorial representation of magnetic field lines is very useful in visualizing the strength and direction of the magnetic field. As shown in Figure 17.14, the direction of magnetic field lines is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the *B*-field.

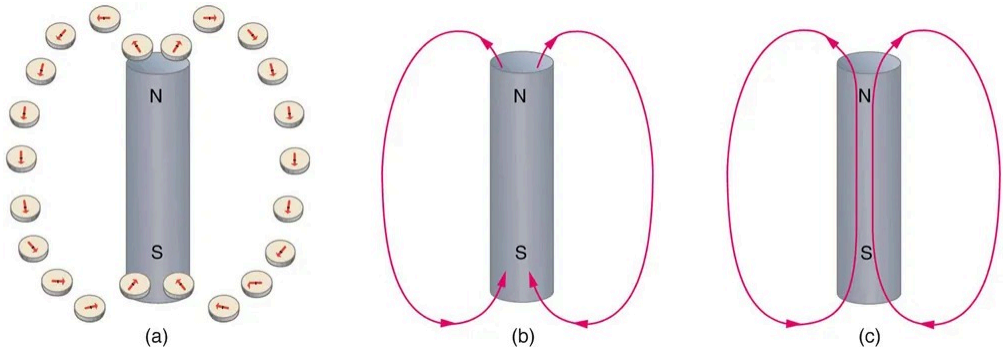


Figure 17.14 Magnetic field lines are defined to have the direction that a small compass points when placed at a location. (a) If small compasses are used to map the magnetic field around a bar magnet, they will point in the directions shown: away from the north pole of the magnet, toward the south pole of the magnet. (Recall that the Earth's north magnetic pole is really a south pole in terms of definitions of poles on a bar magnet.) (b) Connecting the arrows gives continuous magnetic field lines. The strength of the field is proportional to the closeness (or density) of the lines. (c) If the interior of the magnet could be probed, the field lines would be found to form continuous closed loops. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three illustrations labeled (a), (b), and (c), depicting magnetic fields around a cylindrical magnet:

(a) The first illustration shows a cylindrical magnet with a north (N) pole on top and a south (S) pole at the bottom. Surrounding the magnet are small circular symbols representing atoms, each containing an arrow indicating random magnetic orientation.

(b) The second illustration depicts the same cylindrical magnet with magnetic field lines represented as red arrows curving outward from the north pole and returning to the south pole.

(c) The third illustration shows the same magnet with magnetic field lines forming continuous loops, moving from the north to the south pole, indicating a more uniform magnetic field structure.

Small compasses used to test a magnetic field will not disturb it. (This is analogous to the way we tested electric fields with a small test charge. In both cases, the fields represent only the object creating them and not the probe testing them.) Figure 17.15 shows how the magnetic field appears for a current loop and a long straight wire, as could be explored with small compasses. A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of B. Note the symbols used for field into and out of the paper.

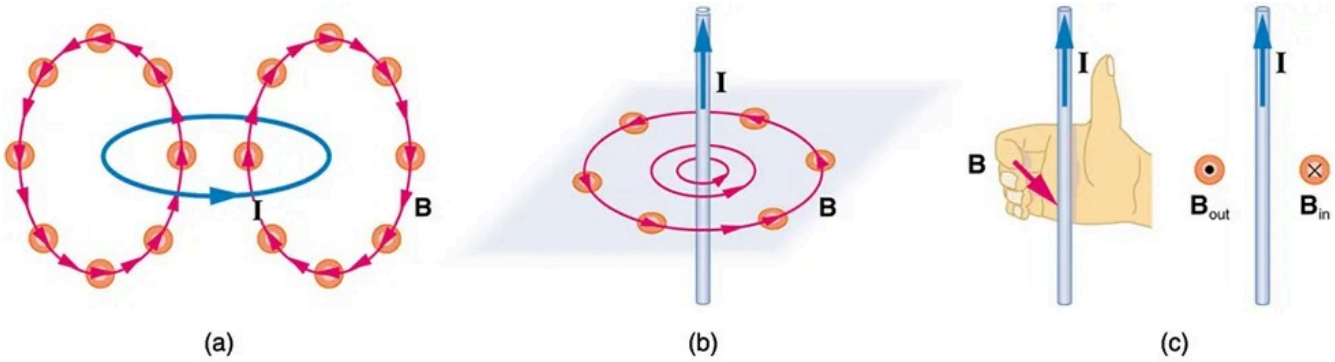


Figure 17.15 Small compasses could be used to map the fields shown here. (a) The magnetic field of a circular current loop is similar to that of a bar magnet. (b) A long and straight wire creates a field with magnetic field lines forming circular loops. (c) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note that the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image consists of three diagrams labeled (a), (b), and (c) demonstrating the magnetic fields around a conducting wire with current.

(a) The first diagram shows two loops of circular arrows with a wire running vertically through the center. The arrows indicate the direction of the magnetic field. A current labeled "I" flows upward through the wire. The magnetic field lines, labeled "B," surround the wire in circular loops, suggesting the right-hand rule.

(b) The second diagram shows a single wire oriented vertically with circular magnetic field lines around it, also following the right-hand rule. The current "I" flows upward through the wire while the magnetic field lines "B" surround the wire in a circular pattern, perpendicular to the wire.

(c) The third diagram includes two sections. On the left, a vertical wire with current "I" flowing upward is shown alongside a right hand with the thumb pointing up, indicating the direction of the current. The curled fingers represent the direction of the magnetic field "B." On the right, symbols "B_{out}" (a dot with a circle) and "B_{in}" (a cross with a circle) represent the magnetic field coming out and going into the page, respectively.

Making Connections: Concept of a Field

A field is a way of mapping forces surrounding any object that can act on another object at a distance without apparent physical connection. The field represents the object generating it. Gravitational fields map gravitational forces, electric fields map electrical forces, and magnetic fields map magnetic forces.

Extensive exploration of magnetic fields has revealed a number of hard-and-fast rules. We use magnetic field lines to represent the field (the lines are a pictorial tool, not a physical entity in and of themselves). The properties of magnetic field lines can be summarized by these rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without beginning or end. They go from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which begin and end on the positive and negative charges. If magnetic monopoles existed, then magnetic field lines would begin and end on them.

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17.4 MAGNETIC FIELD STRENGTH: FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects of magnetic fields on moving charges.
- Use the right hand rule 1 to determine the velocity of a charge, the direction of the magnetic field, and the direction of the magnetic force on a moving charge.
- Calculate the magnetic force on a moving charge.

What is the mechanism by which one magnet exerts a force on another? The answer is related to the fact that all magnetism is caused by current, the flow of charge. *Magnetic fields exert forces on moving charges*, and so they exert forces on other magnets, all of which have moving charges.

Right Hand Rule 1

The magnetic force on a moving charge is one of the most fundamental known. Magnetic force is as important as the electrostatic or Coulomb force. Yet the magnetic force is more complex, in both the number of factors that affects it and in its direction, than the relatively simple Coulomb force. The magnitude of the magnetic force F on a charge q moving at a speed v in a magnetic field of strength B is given by

$$F = \mathrm{qvBsin}\theta,$$

17.1

where θ is the angle between the directions of $\mathbf{v}$ and $\mathbf{B}$. This force is often called the Lorentz force. In fact, this is how we define the magnetic field strength B—in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnetic field strength B is called the tesla (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943). To determine how the tesla relates to other SI units, we solve $F = \mathrm{qvBsin}\theta$ for B.

$$B = \frac{F}{\mathrm{qvsin}\theta}$$

17.2

Because $\mathrm{sin}\theta$ is unitless, the tesla is

$$1\ \mathrm{T} = \frac{1\ \mathrm{N}}{C \cdot \mathrm{m/s}} = \frac{1\ \mathrm{N}}{A \cdot m}$$

17.3

(note that C/s = A).

Another smaller unit, called the gauss (G), where $1G = 10^{-4}T$, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. The Earth's magnetic field on its surface is only about $5 \times 10^{-5}T$, or 0.5 G.

The *direction* of the magnetic force $\mathbf{F}$ is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$, as determined by the right hand rule 1 (or RHR-1), which is illustrated in Figure 17.16. RHR-1 states that, to determine the direction of the magnetic force on a positive moving charge, you point the thumb of the right hand in the direction of $\mathbf{v}$, the fingers in the direction of $\mathbf{B}$, and a perpendicular to the palm points in the direction of $\mathbf{F}$. One way to remember this is that there is one velocity, and so the thumb represents it. There are many field lines, and so the fingers represent them. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge.

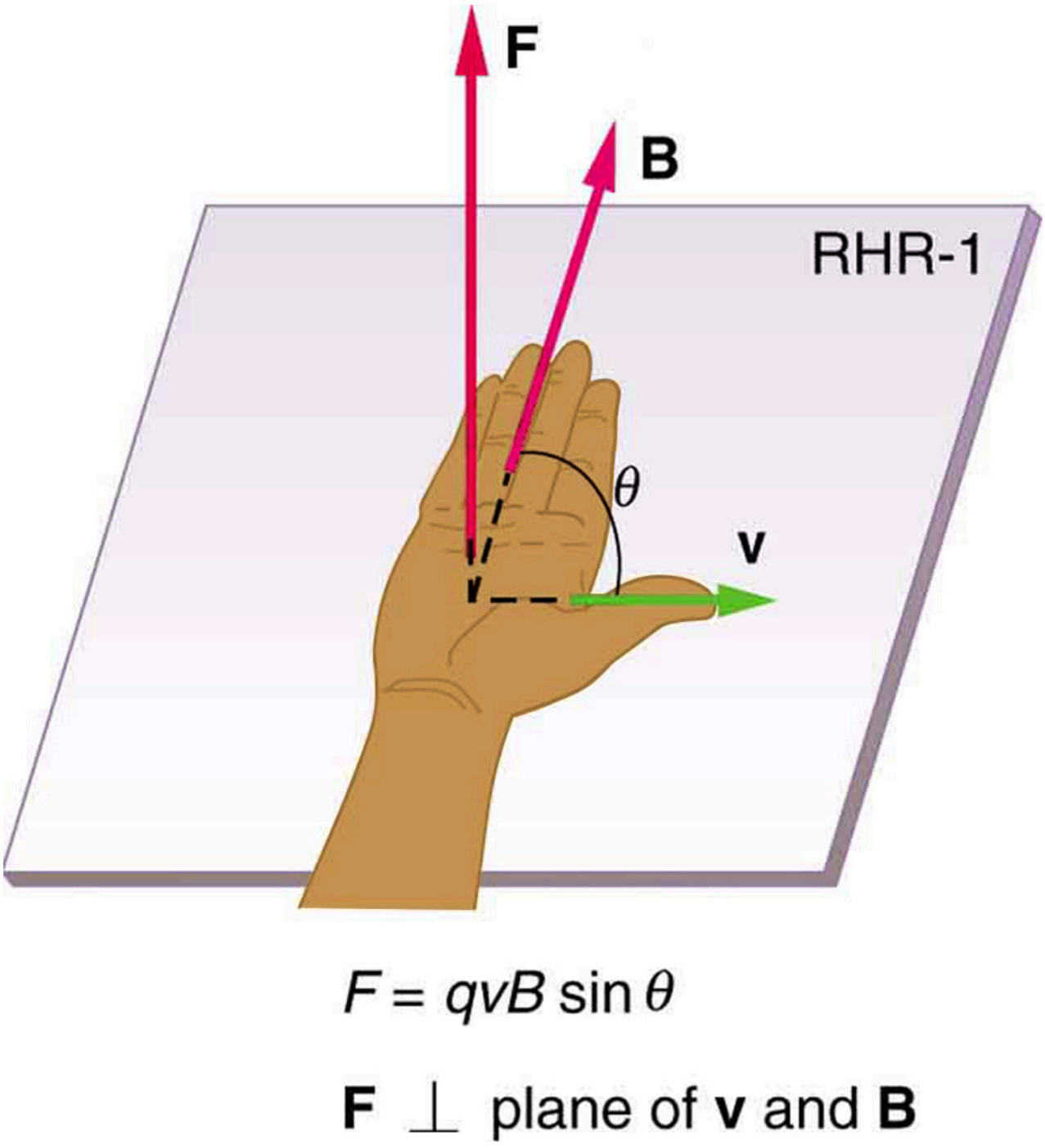


Figure 17.16 Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by $\mathbf{v}$ *and* $\mathbf{B}$ *and follows right hand rule–1 (RHR-1) as shown. The magnitude of the force is proportional to* q, v, B, *and the sine of the angle between* $\mathbf{v}$ *and* $\mathbf{B}$. *Image by OpenStax, Wikimedia Commons License*

Image Description

The image illustrates the Right-Hand Rule 1 (RHR-1) in physics. It shows a hand with fingers pointing upwards, palm facing left, and the thumb pointing to the right. Three vectors are represented:

- **F**: A red arrow pointing upwards, representing force.
- **B**: A red arrow pointing outwards from the palm, representing magnetic field.
- **v**: A green arrow aligned with the thumb pointing to the right, representing velocity.

An angle θ is depicted between vectors **v** and **B**. The caption includes the formula $F = qvB \sin \theta$, indicating the force is perpendicular to the plane of **v** and **B**.

Making Connections: Charges and Magnets

There is no magnetic force on static charges. However, there is a magnetic force on moving charges. When charges are stationary, their electric fields do not affect magnets. But, when charges move, they produce magnetic fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic fields emerges—each affects the other.

Example 17.1

Calculating Magnetic Force: Earth's Magnetic Field on a Charged Glass Rod

With the exception of compasses, you seldom see or personally experience forces due to the Earth's small magnetic field. To illustrate this, suppose that in a physics lab you rub a glass rod with silk, placing a 20-nC positive charge on it. Calculate the force on the rod due to the Earth's magnetic field, if you throw it with a horizontal velocity of 10 m/s due west in a place where the Earth's field is due north parallel to the ground. (The direction of the force is determined with right hand rule 1 as shown in Figure 17.17.)

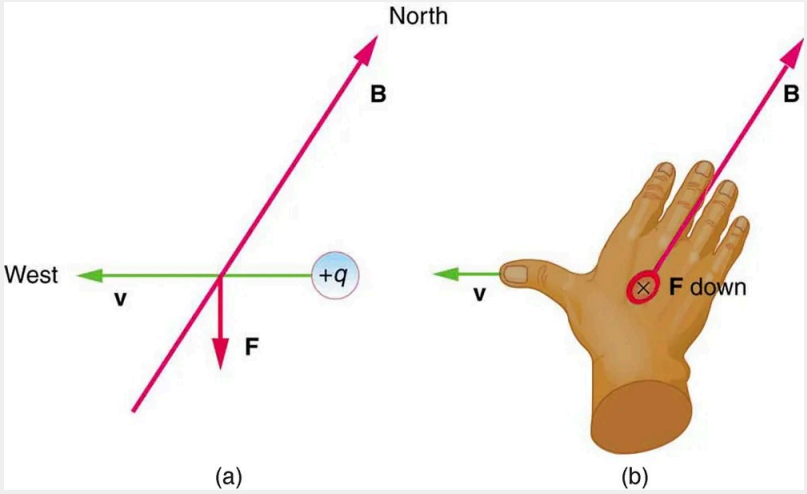


Figure 17.17 A positively charged object moving due west in a region where the Earth's magnetic field is due north experiences a force that is straight down as shown. A negative charge moving in the same direction would feel a force straight up. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled as (a) and (b), illustrating the right-hand rule for determining the direction of force on a positive charge in a magnetic field.

Diagram (a):

- A particle with a positive charge (+q) is moving to the west, shown by a green arrow labeled “v” pointing left.
- A magnetic field (B) is directed towards the north, indicated by a red arrow pointing upwards and right.
- The resultant force (F) on the charged particle is directed downwards, indicated by a red arrow pointing straight down.

Diagram (b):

- An open right hand is shown with the fingers extended.
- The thumb points west, representing the direction of velocity “v”.
- The fingers point north, representing the direction of the magnetic field “B”.
- The palm faces downward, showing the direction of the force “F down” on the positive charge, marked by a circular symbol with an “x”.

Strategy

We are given the charge, its velocity, and the magnetic field strength and direction. We can thus use the equation $F = \mathrm{qvBsin}\theta$ to find the force.

Solution

The magnetic force is

$$F = \mathrm{qvBsin}\theta.$$

17.4

We see that $\mathrm{sin}\theta = 1$, since the angle between the velocity and the direction of the field is 90° . Entering the other given quantities yields

$$\begin{aligned} F &= \left(20 \times 10^{-9} C\right)\left(10\ \mathrm{m/s}\right)\left(5 \times 10^{-5} T\right) \\ &= 1 \times 10^{-11}\left(C \cdot \mathrm{m/s}\right)\left(\frac{N}{C \cdot \mathrm{m/s}}\right) \\ &= 1 \times 10^{-11} N. \end{aligned}$$

17.5

Discussion

This force is completely negligible on any macroscopic object, consistent with experience. (It is calculated to only one digit, since the Earth's field varies with location and is given to only one digit.) The Earth's magnetic field, however, does produce very important effects, particularly on submicroscopic particles. Some of these are explored in Force on a Moving Charge in a Magnetic Field: Examples and Applications.

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17.5 FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD: EXAMPLES AND APPLICATIONS

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects of a magnetic field on a moving charge.
- Calculate the radius of curvature of the path of a charge that is moving in a magnetic field.

Magnetic force can cause a charged particle to move in a circular or spiral path. Cosmic rays are energetic charged particles in outer space, some of which approach the Earth. They can be forced into spiral paths by the Earth's magnetic field. Protons in giant accelerators are kept in a circular path by magnetic force. The bubble chamber photograph in Figure 17.18 shows charged particles moving in such curved paths. The curved paths of charged particles in magnetic fields are the basis of a number of phenomena and can even be used analytically, such as in a mass spectrometer.



Figure 17.18 Trails of bubbles are produced by high-energy charged particles moving through the superheated liquid hydrogen in this artist's rendition of a bubble chamber. There is a strong magnetic field perpendicular to the page that causes the curved paths of the particles. The radius of the path can be used to find the mass, charge, and energy of the particle. Image from OpenStax College Physics 2e, CC-BY 4.0

So does the magnetic force cause circular motion? Magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected, but not the speed. This is typical of uniform circular motion. The simplest case occurs when a charged particle moves perpendicular to a uniform B-field, such as shown in Figure 17.19. (If this takes place in a vacuum, the magnetic field is the dominant factor determining the motion.) Here, the magnetic force supplies the centripetal force $F_c = \mathrm{mv}^2/r$. Noting that $\sin\theta = 1$, we see that $F = \mathrm{qvB}$.

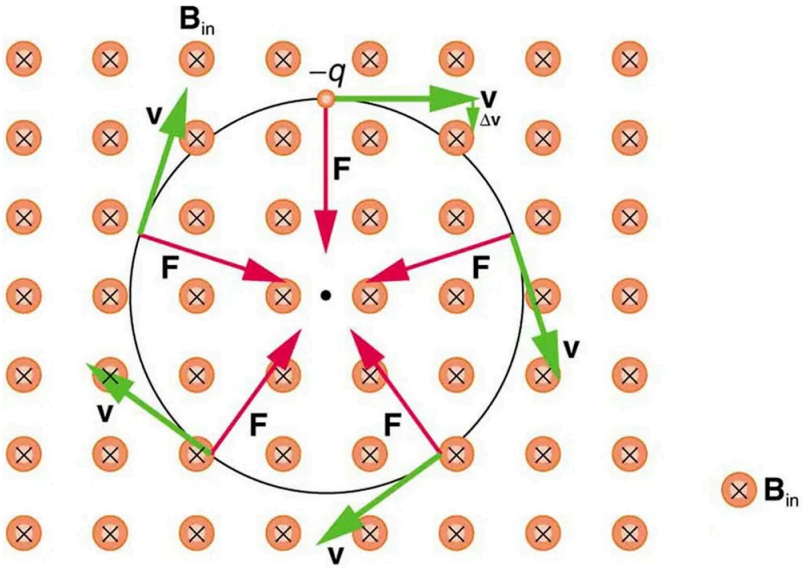


Figure 17.19 A negatively charged particle moves in the plane of the page in a region where the magnetic field is perpendicular into the page (represented by the small circles with x's—like the tails of arrows). The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude. Uniform circular motion results. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the motion of a charged particle, labeled as -q, in a uniform magnetic field. The magnetic field is represented by a series of evenly spaced orange circles with black crosses inside, indicating the direction of the magnetic field into the page, labeled as B_{in}.

The charged particle is depicted as a small orange circle near the center of the image. It is shown moving in a circular path. This path is depicted with a black circle around the charge's location, illustrating the trajectory.

Green arrows labeled as v indicate the velocity of the charged particle at various points along its circular path. A single green arrow labeled $v_{\Delta \text{v}}$ is positioned at the top near the charged particle, showing an initial or change in velocity moving to the right.

Red arrows labeled as F point radially inward towards the center of the circular path, illustrating the force acting on the charged particle. This force is magnetic in nature, responsible for the centripetal motion of the charged particle.

Because the magnetic force F supplies the centripetal force F_c, we have

$$\text{qvB} = \frac{\text{mv}^2}{r}.$$

17.6

Solving for r yields

$$r = \frac{\mathrm{mv}}{\mathrm{qB}}.$$

17.7

Here, r is the radius of curvature of the path of a charged particle with mass m and charge q, moving at a speed v perpendicular to a magnetic field of strength B. If the velocity is not perpendicular to the magnetic field, then v is the component of the velocity perpendicular to the field. The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces a spiral motion rather than a circular one.

Example 17.2

Calculating the Curvature of the Path of an Electron Moving in a Magnetic Field: A Magnet on a TV Screen

A magnet brought near an old-fashioned TV screen such as in Figure 17.20 (TV sets with cathode ray tubes instead of LCD screens) severely distorts its picture by altering the path of the electrons that make its phosphors glow. ***(Don't try this at home, as it will permanently magnetize and ruin the TV.)*** To illustrate this, calculate the radius of curvature of the path of an electron having a velocity of $6.00 \times 10^7 \mathrm{m/s}$ (corresponding to the accelerating voltage of about 10.0 kV used in some TVs) perpendicular to a magnetic field of strength $B = 0.500\ \mathrm{T}$ (obtainable with permanent magnets).

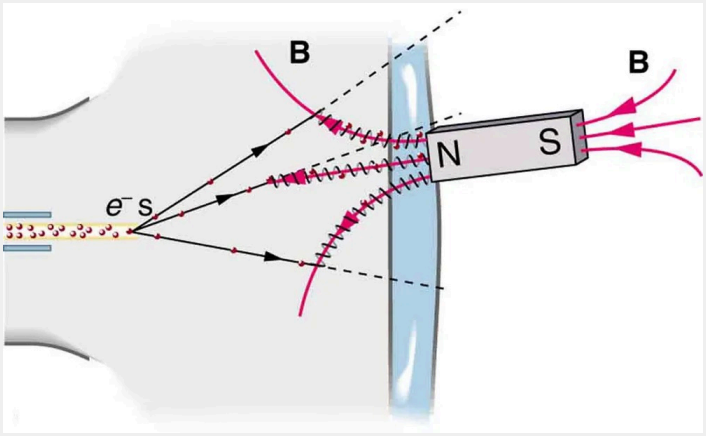


Figure 17.20 Side view showing what happens when a magnet comes in contact with a computer monitor or TV screen. Electrons moving toward the screen spiral about magnetic field lines, maintaining the component of their velocity parallel to the field lines. This distorts the image on the screen. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the behavior of electrons within a magnetic field. On

the left side, a stream of electrons, indicated by the symbol "e^-", is shown moving from left to right. The electrons are depicted as small, red spheres within a pale yellow beam.

The beam of electrons enters a region with a magnetic field represented by the letter "B". The magnetic field lines are shown as curved red arrows entering and exiting a rectangular magnet. The magnet is oriented with its north (N) and south (S) poles facing sideways, relative to the electron beam.

As the electrons enter the magnetic field, they follow curved, parabolic trajectories. Three distinct paths are illustrated with dashed black lines and arrows, depicting how the electron paths are bent differently as they pass through the magnetic field.

Strategy

We can find the radius of curvature r directly from the equation $r = \dfrac{mv}{qB}$, since all other quantities in it are given or known.

Solution

Using known values for the mass and charge of an electron, along with the given values of v and B gives us

$$r = \frac{\mathrm{mv}}{\mathrm{qB}} = \frac{\left(9.11 \times 10^{-31}\mathrm{kg}\right)\left(6.00 \times 10^{7}\mathrm{m/s}\right)}{\left(1.60 \times 10^{-19}\mathrm{C}\right)\left(0.500\mathrm{T}\right)}$$
$$= 6.83 \times 10^{-4}\mathrm{m}$$

17.8

or

$$r = 0.683\ \mathrm{mm}.$$

17.9

Discussion

The small radius indicates a large effect. The electrons in the TV picture tube are made to move in very tight circles, greatly altering their paths and distorting the image.

Figure 17.21 shows how electrons not moving perpendicular to magnetic field lines follow the field lines. The component of velocity parallel to the lines is unaffected, and so the charges spiral along the field lines. If field strength increases in the direction of motion, the field will exert a force to slow the charges, forming a kind of magnetic mirror, as shown below.

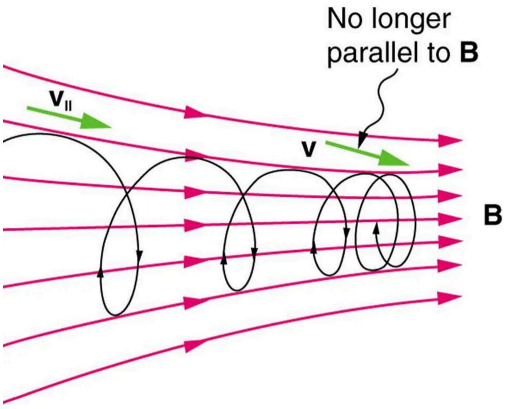


Figure 17.21 When a charged particle moves along a magnetic field line into a region where the field becomes stronger, the particle experiences a force that reduces the component of velocity parallel to the field. This force slows the motion along the field line and here reverses it, forming a "magnetic mirror." Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the movement of a charged particle in a magnetic field. The diagram shows several horizontal parallel lines with arrows pointing to the right, representing the magnetic field, labeled as "B". The magnetic field lines are in pink.

There is a series of helical or spiral paths shown in black, with arrows indicating the direction of motion of the particle along these paths. The particle is depicted as moving in a helical manner along the magnetic field lines.

There are two green arrows labeled as "V" and "$V_{||}$" indicating the velocity components of the particle. "V" represents the total velocity vector at an angle, while "$V_{||}$" represents the component of the velocity that is parallel to the magnetic field lines.

A text label indicates a transition where the velocity vector "V" is no longer parallel to the magnetic field "B".

The properties of charged particles in magnetic fields are related to such different things as the Aurora Australis or Aurora Borealis and particle accelerators. *Charged particles approaching magnetic field lines may get trapped in spiral orbits about the lines rather than crossing them*, as seen above. Some cosmic rays, for example, follow the Earth's magnetic field lines, entering the atmosphere near the magnetic poles and causing

the southern or northern lights through their ionization of molecules in the atmosphere. This glow of energized atoms and molecules is seen in Introduction to Magnetism. Those particles that approach middle latitudes must cross magnetic field lines, and many are prevented from penetrating the atmosphere. Cosmic rays are a component of background radiation; consequently, they give a higher radiation dose at the poles than at the equator.

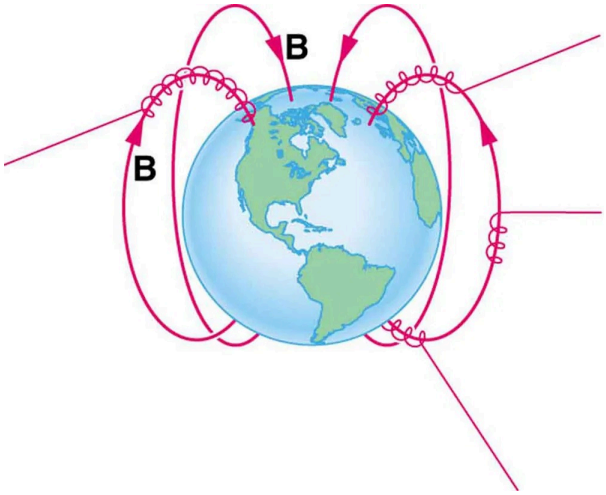


Figure 17.22 Energetic electrons and protons, components of cosmic rays, from the Sun and deep outer space often follow the Earth's magnetic field lines rather than cross them. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating Earth's magnetic field. It features the Earth in the center, depicted with blue oceans and green continents. Arrows and lines surround the Earth, representing magnetic field lines. These lines are depicted in red and form loops entering and exiting the Earth's surface, signifying magnetic poles. The letter "B" is used to label sections of the magnetic field. The field lines curve from the top of the diagram, looping down towards the equator and back up again, demonstrating the Earth's magnetic field's structure.

Some incoming charged particles become trapped in the Earth's magnetic field, forming two belts above the atmosphere known as the Van Allen radiation belts after the discoverer James A. Van Allen, an American astrophysicist. (See Figure 17.23.) Particles trapped in these belts form radiation fields (similar to nuclear radiation) so intense that piloted space flights avoid them and satellites with sensitive electronics are kept out of them. In the few minutes it took lunar missions to cross the Van Allen radiation belts, astronauts received radiation doses more than twice the allowed annual exposure for radiation workers. Other planets have similar belts, especially those having strong magnetic fields like Jupiter.

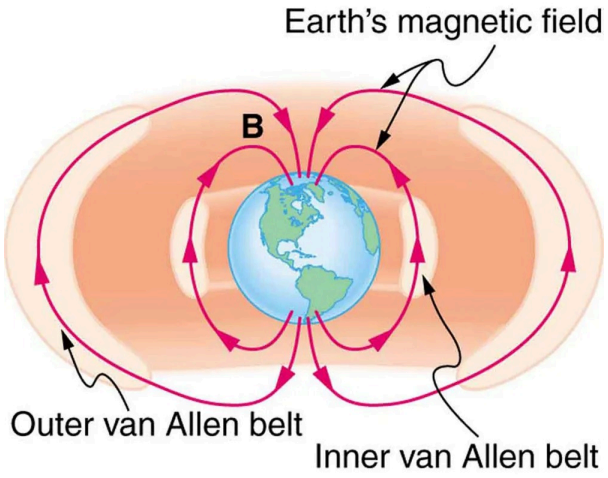


Figure 17.23 The Van Allen radiation belts are two regions in which energetic charged particles are trapped in the Earth's magnetic field. One belt lies about 300 km above the Earth's surface, the other about 16,000 km. Charged particles in these belts migrate along magnetic field lines and are partially reflected away from the poles by the stronger fields there. The charged particles that enter the atmosphere are replenished by the Sun and sources in deep outer space. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a conceptual illustration showing the Earth at the center, surrounded by its magnetic field. The lines representing the magnetic field emanate from the poles and curve around the planet, depicted with arrows to indicate direction. There are two ring-shaped regions highlighted around the Earth, which are the Van Allen belts. The belts are shown as two distinct, concentric donut-shaped areas:

- The *Inner Van Allen belt* is closer to the Earth and is indicated in the diagram with a label.
- The *Outer Van Allen belt* is farther out and also labeled accordingly.

Parts of the magnetic field lines extend outward through the belts and return to the opposite pole. The illustration includes a label at the top indicating "Earth's magnetic field" and uses curves and arrows to direct understanding of the field's flow.

Back on Earth, we have devices that employ magnetic fields to contain charged particles. Among them are the giant particle accelerators that have been used to explore the substructure of matter. (See Figure 17.24.) Magnetic fields not only control the direction of the charged particles, they also are used to focus particles into beams and overcome the repulsion of like charges in these beams.

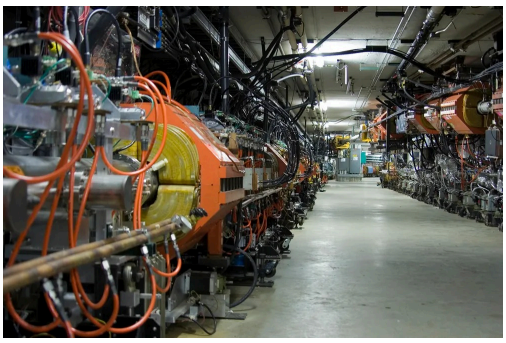


Figure 17.24 The Fermilab facility in Illinois has a large particle accelerator (the most powerful in the world until 2008) that employs magnetic fields (magnets seen here in orange) to contain and direct its beam. This and other accelerators have been in use for several decades and have allowed us to discover some of the laws underlying all matter. Image from OpenStax College Physics 2e, CC-BY 4.0

Thermonuclear fusion (like that occurring in the Sun) is a hope for a future clean energy source. One of the most promising devices is the *tokamak*, which uses magnetic fields to contain (or trap) and direct the reactive charged particles. (See Figure 17.25.) Less exotic, but more immediately practical, amplifiers in microwave ovens use a magnetic field to contain oscillating electrons. These oscillating electrons generate the microwaves sent into the oven.

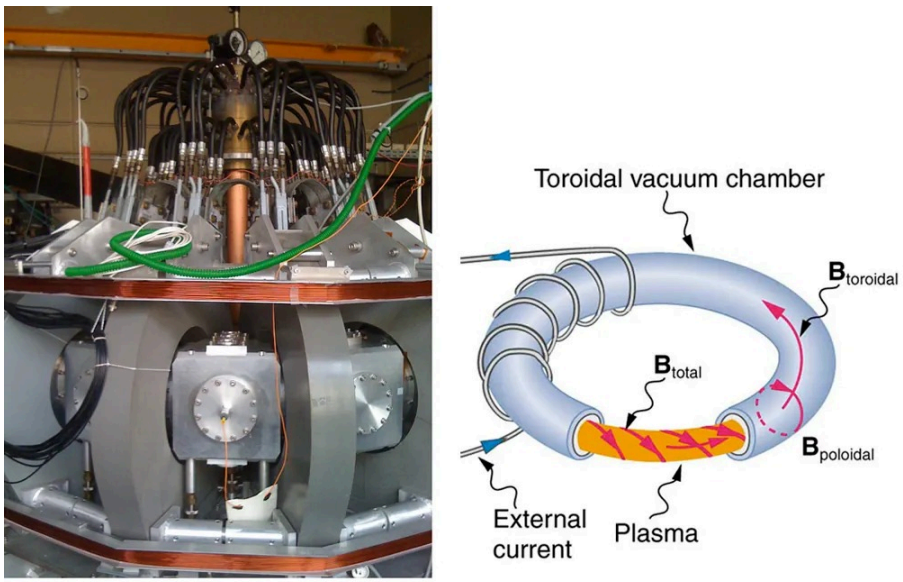


Figure 17.25 Tokamaks such as the one shown in the figure are being studied with the goal of economical production of energy by nuclear fusion. Magnetic fields in the doughnut-shaped device contain and direct the reactive charged particles. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

On the left (labeled 'a'), there is an image of an industrial machine or device comprising various metal components and wiring. Visible are numerous thick black and green cables attached to a top section with metallic attachments. Below, there are silver-toned blocks and structural elements, possibly part of a scientific apparatus or experiment setup.

On the right (labeled 'b'), there is a diagram depicting a toroidal structure, which appears to represent a cross-section of a tokamak or fusion reactor. The diagram shows a toroidal vacuum chamber with currents and magnetic fields labeled. The terms included are "Toroidal vacuum chamber," "B toroidal," "B total," "B poloidal," "External current," and "Plasma." Arrows indicate the direction of these fields and currents.

Mass spectrometers have a variety of designs, and many use magnetic fields to measure mass. The curvature of a charged particle's path in the field is related to its mass and is measured to obtain mass information. (See More Applications of Magnetism.) Historically, such techniques were employed in the first direct observations of electron charge and mass. Today, mass spectrometers (sometimes coupled with gas chromatographs) are used to determine the make-up and sequencing of large biological molecules.

"22.5 Force on a Moving Charge in a Magnetic Field: Examples and Applications" from College Physics 2e by OpenStax is licensed under a Creative Commons Attribution 4.0 International License.

17.6 THE HALL EFFECT

Learning Objectives

By the end of this section, you will be able to:

- Describe the Hall effect.
- Calculate the Hall emf across a current-carrying conductor.

We have seen effects of a magnetic field on free-moving charges. The magnetic field also affects charges moving in a conductor. One result is the Hall effect, which has important implications and applications.

Figure 17.26 shows what happens to charges moving through a conductor in a magnetic field. The field is perpendicular to the electron drift velocity and to the width of the conductor. Note that conventional current is to the right in both parts of the figure. In part (a), electrons carry the current and move to the left. In part (b), positive charges carry the current and move to the right. Moving electrons feel a magnetic force toward one side of the conductor, leaving a net positive charge on the other side. This separation of charge *creates a voltage* ϵ, known as the Hall emf, *across* the conductor. The creation of a voltage *across* a current-carrying conductor by a magnetic field is known as the Hall effect, after Edwin Hall, the American physicist who discovered it in 1879.

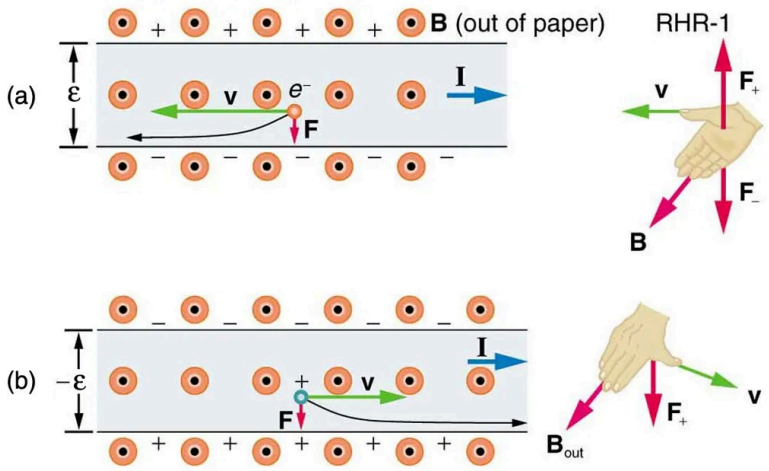


Figure 17.26 The Hall effect. (a) Electrons move to the left in this flat conductor (conventional current to the right). The magnetic field is directly out of the page, represented by circled dots; it exerts a force on the moving charges, causing a voltage ϵ*, the Hall emf, across the conductor. (b) Positive charges moving to the right (conventional current also to the right) are moved to the side, producing a Hall emf of the opposite sign,* $-\varepsilon$*. Thus, if the direction of the field and current are known, the sign of the charge carriers can be determined from the Hall effect. Image from OpenStax College Physics 2e, CC-BY 4.0*

Image Description

The image consists of two diagrams labeled as (a) and (b), illustrating the motion of charged particles in a magnetic field, with accompanying hand illustrations showing the right-hand rule (RHR-1) for determining direction of forces.

Diagram (a):

- There is a horizontal channel with upper and lower lines of positive and negative charges represented by "+" and "-" signs, respectively.
- An electron, labeled as "e^-", is moving to the left, indicated by a green arrow labeled "v".
- A red arrow labeled "F" points downward, showing the force on the electron.
- A blue arrow labeled "I" points to the right, indicating the direction of current.
- A note indicates "B (out of paper)" representing the magnetic field direction coming out of the paper.

Diagram (b):

- Similar setup with horizontal channel and charges.
- A positive charge labeled "+" is moving to the right with a green arrow labeled "v".
- A red arrow labeled "F" points upwards, indicating the force.

- A blue arrow labeled “I” also points to the right, indicating current direction.
- A note indicates “B (out of paper)” representing the magnetic field direction coming out of the paper.

Right-Hand Rule (RHR-1) Illustrations:

- Each diagram has an accompanying right-hand illustration to demonstrate the right-hand rule.
- For the electron, the thumb points in the direction of “v”, fingers point in the direction of “B”, and the palm faces the direction of force “F”.
- Similar setup for the positive charge with directions adjusted for opposite force direction.

One very important use of the Hall effect is to determine whether positive or negative charges carries the current. Note that in Figure 17.26(b), where positive charges carry the current, the Hall emf has the sign opposite to when negative charges carry the current. Historically, the Hall effect was used to show that electrons carry current in metals and it also shows that positive charges carry current in some semiconductors. The Hall effect is used today as a research tool to probe the movement of charges, their drift velocities and densities, and so on, in materials. In 1980, it was discovered that the Hall effect is quantized, an example of quantum behavior in a macroscopic object.

The Hall effect has other uses that range from the determination of blood flow rate to precision measurement of magnetic field strength. To examine these quantitatively, we need an expression for the Hall emf, ϵ, across a conductor. Consider the balance of forces on a moving charge in a situation where B, v, and l are mutually perpendicular, such as shown in Figure 17.27. Although the magnetic force moves negative charges to one side, they cannot build up without limit. The electric field caused by their separation opposes the magnetic force, $F = \mathrm{qvB}$, and the electric force, $F_e = \mathrm{qE}$, eventually grows to equal it. That is,

$$\mathrm{qE} = \mathrm{qvB}$$
17.10

or

$$E = \mathrm{vB}.$$
17.11

Note that the electric field E is uniform across the conductor because the magnetic field B is uniform, as is the conductor. For a uniform electric field, the relationship between electric field and voltage is $E = \epsilon/l$, where l is the width of the conductor and ϵ is the Hall emf. Entering this into the last expression gives

$$\frac{\epsilon}{l} = \mathrm{vB}.$$
17.12

Solving this for the Hall emf yields

$$\epsilon = \text{Blv}\ (B, v, \text{and} l, \text{mutually perpendicular}\ ),$$
17.13

where ϵ is the Hall effect voltage across a conductor of width l through which charges move at a speed v.

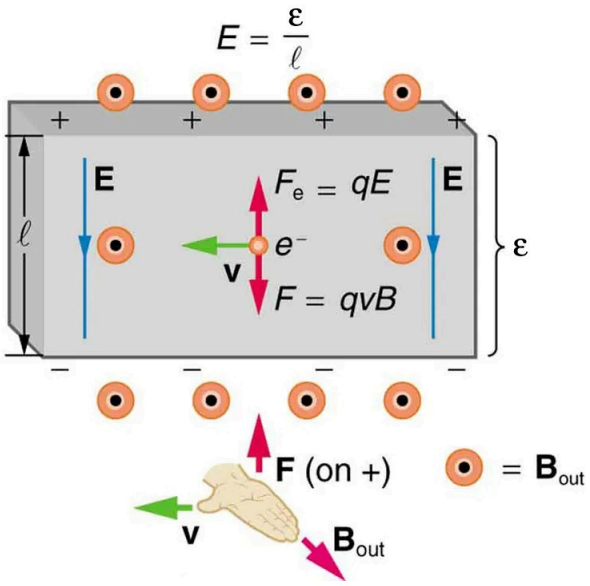


Figure 17.27 The Hall emf ϵ produces an electric force that balances the magnetic force on the moving charges. The magnetic force produces charge separation, which builds up until it is balanced by the electric force, an equilibrium that is quickly reached. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image illustrates the concept of the Hall Effect in a conductive material. It shows a rectangular conductor placed in a magnetic field. The following elements are present in the image:

- A rectangular block labeled with dimensions l (length) and ε (electromotive force), which indicates the electric field $E = \varepsilon/l$.
- Positive and negative charges are depicted on the top and bottom surfaces of the block, respectively, indicating a separation of charge due to the magnetic field.
- The force on an electron e^- is shown with a red arrow labeled F, with equations $F_e = qE$ and $F = qvB$ beside it. This depicts the electric force and magnetic force acting on a charged particle.
- The electron's velocity v is shown with a green arrow pointing to the left.
- An illustration of a hand is used to demonstrate the right-hand rule. The thumb represents the velocity v, the fingers represent the magnetic field B_{out}, and the force direction $F\ (on\ +)$ is shown with a red

arrow.

This image provides a visual representation of how an external magnetic field affects the flow of electrons in a conductor, resulting in an observable Hall voltage across the conductor.

One of the most common uses of the Hall effect is in the measurement of magnetic field strength B. Such devices, called *Hall probes*, can be made very small, allowing fine position mapping. Hall probes can also be made very accurate, usually accomplished by careful calibration. Another application of the Hall effect is to measure fluid flow in any fluid that has free charges (most do). (See Figure 17.28.) A magnetic field applied perpendicular to the flow direction produces a Hall emf ϵ as shown. Note that the sign of ϵ depends not on the sign of the charges, but only on the directions of B and v. The magnitude of the Hall emf is $\epsilon = \mathrm{Blv}$, where l is the pipe diameter, so that the average velocity v can be determined from ϵ providing the other factors are known.

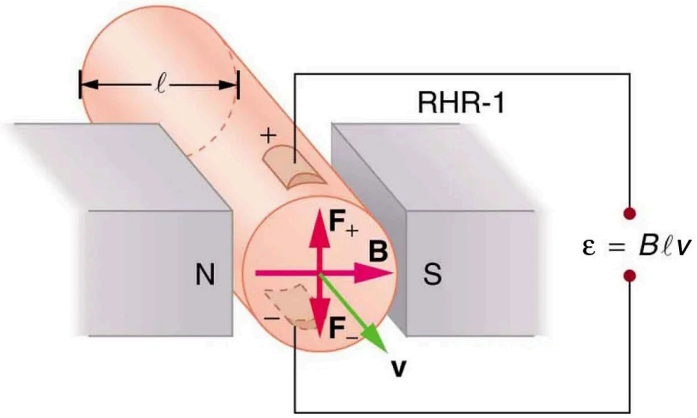


Figure 17.28 The Hall effect can be used to measure fluid flow in any fluid having free charges, such as blood. The Hall emf ϵ is measured across the tube perpendicular to the applied magnetic field and is proportional to the average velocity v. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image illustrates an electromagnetic induction scenario involving a metal rod positioned between the poles of a magnet. The rod is represented as a cylinder with a marked length indicated by the symbol "l." The poles of the magnet are labeled "N" for the North pole and "S" for the South pole, positioned on opposite sides of the rod.

A schematic circuit connects the rod to a measuring device with two red connection points. The right side of the image includes the equation ε = Bℓv, where ε (epsilon) represents the electromotive force, B represents the magnetic field strength, ℓ (ell) is the length of the rod, and v is the velocity of the rod.

Arrows indicate the direction of various forces and fields:

- A pink arrow labeled “B” points horizontally across the rod, representing the magnetic field.
- A red arrow labeled “F_+” points upward, indicating the force experienced by positive charges.
- A green arrow labeled “F_-” points downward, indicating the force on negative charges.
- A green arrow labeled “v” points to the right, showing the direction of velocity.

A label “RHR-1” indicates the application of the right-hand rule to determine the direction of these forces. Overall, the diagram explains the concept of electromagnetic induction by showing how the motion of the rod through a magnetic field induces an electric current.

Example 17.3

Calculating the Hall emf: Hall Effect for Blood Flow

A Hall effect flow probe is placed on an artery, applying a 0.100-T magnetic field across it, in a setup similar to that in Figure 17.28. What is the Hall emf, given the vessel's inside diameter is 4.00 mm and the average blood velocity is 20.0 cm/s?

Strategy

Because B, v, and l are mutually perpendicular, the equation $\epsilon = \mathrm{Blv}$ can be used to find ϵ.

Solution

Entering the given values for B, v, and l gives

$$\begin{aligned}\epsilon = \mathrm{Blv} &= (0.100\ \mathrm{T})\left(4.00 \times 10^{-3} m\right)(0.200\ \mathrm{m/s}) \\ &= 80.0\ \mu\mathrm{V}\end{aligned}$$

17.14

Discussion

This is the average voltage output. Instantaneous voltage varies with pulsating blood flow. The voltage is small in this type of measurement. ϵ is particularly difficult to measure, because there are voltages associated with heart action (ECG voltages) that are on the order of millivolts. In practice, this difficulty is overcome by applying an AC magnetic field, so that the Hall emf is AC with the same frequency. An amplifier can be very selective in picking out only the appropriate frequency, eliminating signals and noise at other frequencies.

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17.7 MAGNETIC FORCE ON A CURRENT-CARRYING CONDUCTOR

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects of a magnetic force on a current-carrying conductor.
- Calculate the magnetic force on a current-carrying conductor.

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.

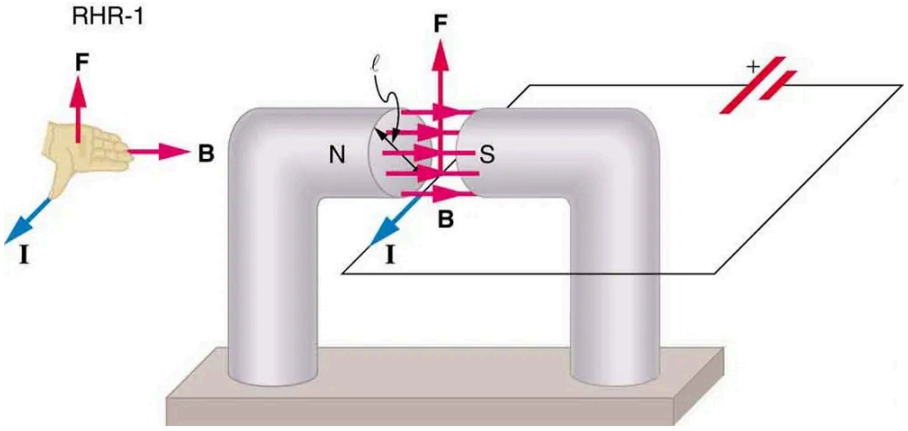


Figure 17.29 The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a metallic pipe bent into a U-shape with ends anchored on a base. The pipe represents a conductor in a magnetic field. Key elements include:

- **RHR-1:** Shown as an upward-facing left hand demonstrating the right-hand rule (RHR-1).
- **Arrows:** Various colored arrows represent different physical quantities.
  - I: Blue arrow pointing along the wire, indicating the direction of current flow.
  - F: Red arrows showing the direction of magnetic force acting on the wire.
  - B: Pink arrows pointing horizontally, indicating the direction of the magnetic field.
- **N and S:** Labels for the north (N) and south (S) poles on either side of the curved section of the pipe, indicating the orientation of the magnetic field.
- **Battery Symbol:** A symbol representing a battery is shown connected to the circuit, with positive and negative terminals marked.

The image serves as an illustration in electromagnetism to demonstrate how current, magnetic field, and force relate through the right-hand rule.

We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity v_d is given by $F = \mathrm{qv}_d B \mathrm{sin}\theta$. Taking B to be uniform over a length of wire l and zero elsewhere, the total magnetic force on the wire is then $F = (\mathrm{qv}_d B \mathrm{sin}\theta\,)\,(N\,)$, where N is the number of charge carriers in the section of wire of length l. Now, $N = \mathrm{nV}$, where n is the number of charge carriers per unit volume and V is the volume of wire in the field. Noting that $V = \mathrm{Al}$, where A is the cross-sectional area of the wire, then the force on the wire is $F = (\mathrm{qv}_d B \mathrm{sin}\theta\,)\,(\mathrm{nAl}\,)$. Gathering terms,

$$F = (\mathrm{nqAv}_d\,)\mathrm{lBsin}\theta.$$
17.15

Because $\mathrm{nqAv}_d = I$ (see Current),

$$F = \mathrm{IlBsin}\theta$$
17.16

is the equation for *magnetic force on a length l of wire carrying a current I in a uniform magnetic field B*, as shown in Figure 17.30. If we divide both sides of this expression by l, we find that the magnetic force per unit length of wire in a uniform field is $\frac{F}{l} = \mathrm{IBsin}\theta$. The direction of this force is given by RHR-1, with the

thumb in the direction of the current I. Then, with the fingers in the direction of B, a perpendicular to the palm points in the direction of F, as in Figure 17.30.

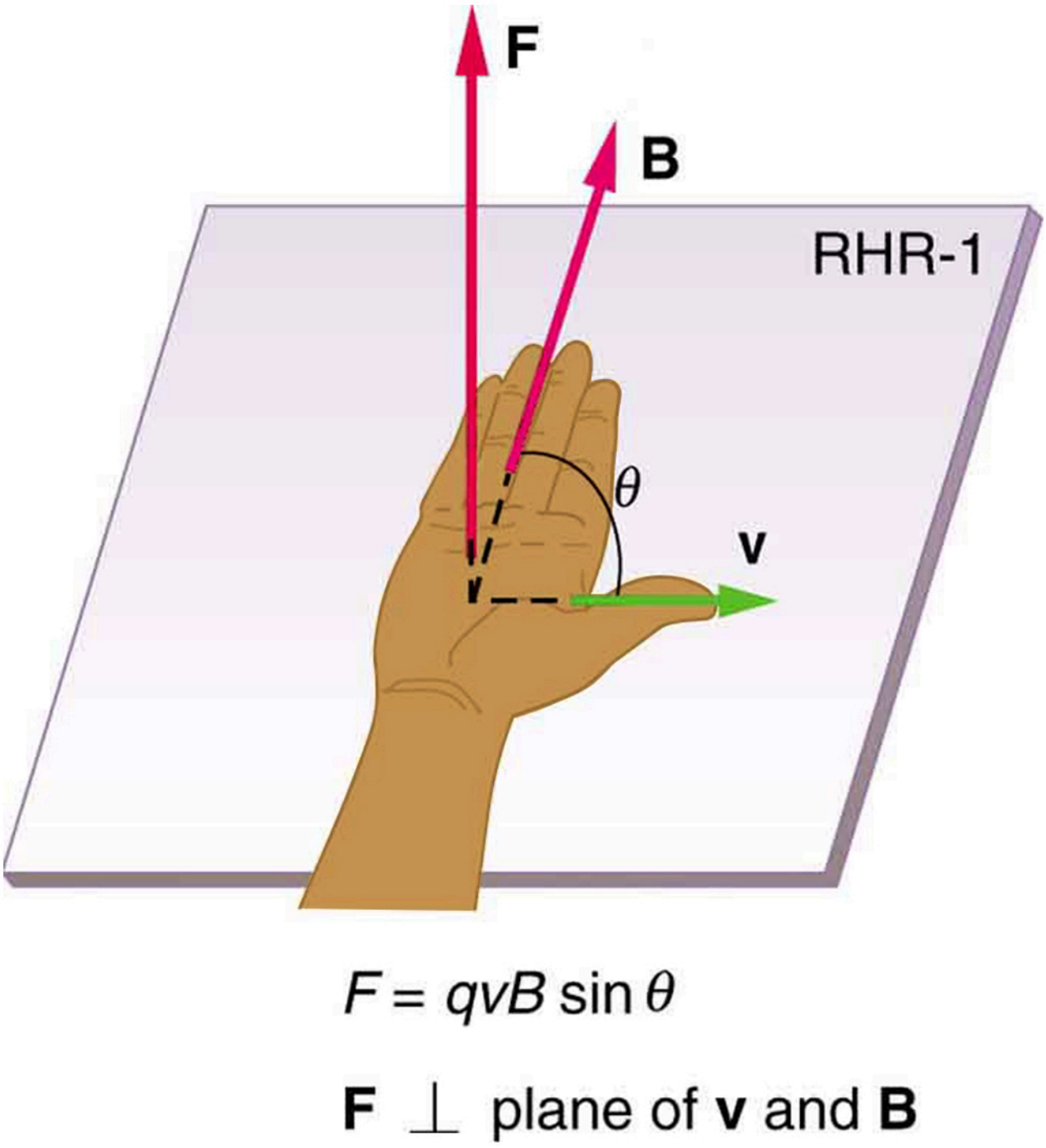


Figure 17.30 The force on a current-carrying wire in a magnetic field is $F = \mathrm{IlBsin}\theta$. Its direction is given by RHR-1. Image by OpenStax, Wikimedia Commons License

Image Description

The image illustrates the Right-Hand Rule Number 1 (RHR-1), commonly used in physics to determine the direction of the force acting on a charged particle moving through a magnetic field. In the image, a hand is positioned with the thumb, index, and middle fingers perpendicular to each other.

- *Thumb (labeled "v")*: This represents the direction of the velocity of the charged particle, pointing to the right.
- *Index finger (labeled "B")*: This indicates the direction of the magnetic field, pointing upwards and slightly towards the observer.
- *Middle finger (labeled "F")*: This shows the direction of the magnetic force exerted on the charged particle, pointing directly upward, forming a 90-degree angle with the plane created by "v" and "B".

There is a mathematical expression shown below the hand: $F = qvB \sin \theta$, which describes the magnitude of the force, where q is the charge, v is the velocity, B is the magnetic field strength, and θ is the angle between v and B. The text also mentions that F is perpendicular to the plane formed by v and B.

Example 17.4

Calculating Magnetic Force on a Current-Carrying Wire: A Strong Magnetic Field

Calculate the force on the wire shown in Figure 17.29, given $B = 1.50\ \text{T}$, $l = 5.00\ \text{cm}$, and $I = 20.0\text{A}$.

Strategy

The force can be found with the given information by using $F = \text{IlBsin}\theta$ and noting that the angle θ between I and B is 90° , so that $\text{sin}\theta = 1$.

Solution

Entering the given values into $F = \text{IlBsin}\theta$ yields

$$\begin{aligned} F &= \text{IlBsin}\theta \\ &= (20.0\ \text{A})\,(0.0500\ \text{m})\,(1.50\ \text{T})\,(1)\ . \end{aligned}$$

17.17

The units for tesla are $1\ \text{T} = \dfrac{N}{A \cdot m}$; thus,

$$F = 1.50\ \text{N}.$$

17.18

Discussion

This large magnetic field creates a significant force on a small length of wire.

Magnetic force on current-carrying conductors is used to convert electric energy to work. (Motors are a prime example—they employ loops of wire and are considered in the next section.) Magnetohydrodynamics

(MHD) is the technical name given to a clever application where magnetic force pumps fluids without moving mechanical parts. (See Figure 17.31.)

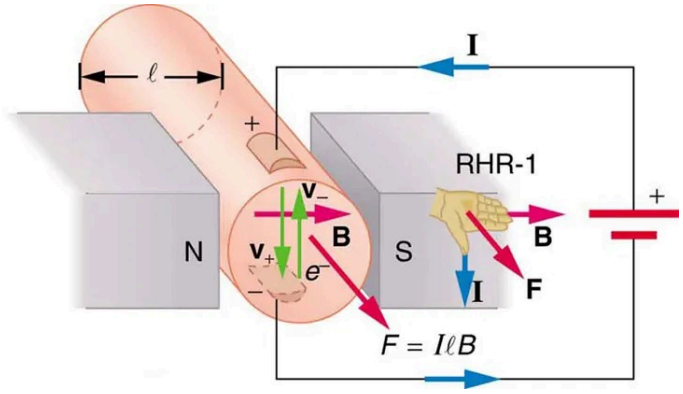


Figure 17.31 Magnetohydrodynamics. The magnetic force on the current passed through this fluid can be used as a nonmechanical pump. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram illustrating the interaction between a magnetic field and an electric current. It shows a cylindrical conductor placed between the north (N) and south (S) poles of a magnet. The magnetic field lines (B) are represented by pink arrows pointing from N to S. An electric current (I) is flowing through the conductor, represented by blue arrows.

The force (F) acting on the conductor is shown with a red arrow, perpendicular to both the magnetic field and the current. The force direction is determined using the right-hand rule (RHR-1), depicted by a right hand with fingers pointing in the direction of the current and the thumb pointing in the direction of the force.

There are additional green arrows inside the conductor, labeled as velocity (V) and electron movement (e-). The notation $F = I\ell B$ indicates the formula for calculating the magnetic force on the conductor, where I is the current, ℓ is the length of the conductor within the magnetic field, and B is the magnetic field strength.

A strong magnetic field is applied across a tube and a current is passed through the fluid at right angles to the field, resulting in a force on the fluid parallel to the tube axis as shown. The absence of moving parts makes this attractive for moving a hot, chemically active substance, such as the liquid sodium employed in some nuclear reactors. Experimental artificial hearts are testing with this technique for pumping blood, perhaps circumventing the adverse effects of mechanical pumps. (Cell membranes, however, are affected by the large fields needed in MHD, delaying its practical application in humans.) MHD propulsion for nuclear submarines has been proposed, because it could be considerably quieter than conventional propeller drives. The deterrent value of nuclear submarines is based on their ability to hide and survive a first or second nuclear

strike. As we slowly disassemble our nuclear weapons arsenals, the submarine branch will be the last to be decommissioned because of this ability (See Figure 22.32.) Existing MHD drives are heavy and inefficient—much development work is needed.

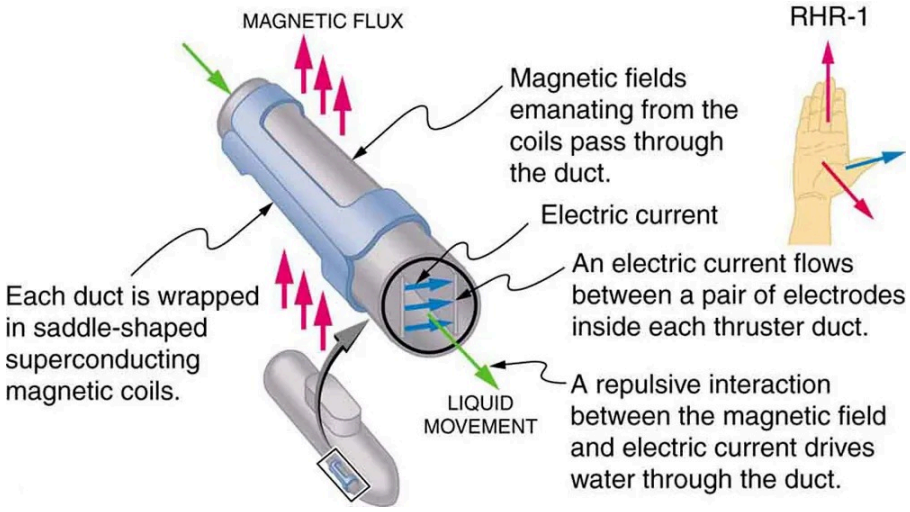


Figure 17.32 An MHD propulsion system in a nuclear submarine could produce significantly less turbulence than propellers and allow it to run more silently. The development of a silent drive submarine was dramatized in the book and the film The Hunt for Red October. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a schematic diagram of a propulsion system featuring saddle-shaped superconducting magnetic coils wrapped around a duct. The process includes the following key components and interactions:

- Magnetic coils: Each duct is wrapped in these superconducting coils, which generate magnetic fields.
- Magnetic flux: The magnetic fields, represented by pink arrows, emanate from the coils and pass through the duct.
- Electric current: An electric current flows between a pair of electrodes inside each thruster duct, indicated by blue arrows.
- Liquid movement: The interaction of the magnetic field and electric current creates a repulsive force that drives water through the duct, represented by a green arrow.

Additionally, a reference hand labeled “RHR-1” (Right-Hand Rule 1) illustrates the direction of the magnetic field, electric current, and force vector. The thumb points upwards, the index finger to the right, and the middle finger outwards, demonstrating the mutually perpendicular relationship of these vectors.

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17.8 TORQUE ON A CURRENT LOOP: MOTORS AND METERS

Learning Objectives

By the end of this section, you will be able to:

- Describe how motors and meters work in terms of torque on a current loop.
- Calculate the torque on a current-carrying loop in a magnetic field.

Motors are the most common application of magnetic force on current-carrying wires. Motors have loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process. (See Figure 17.33.)

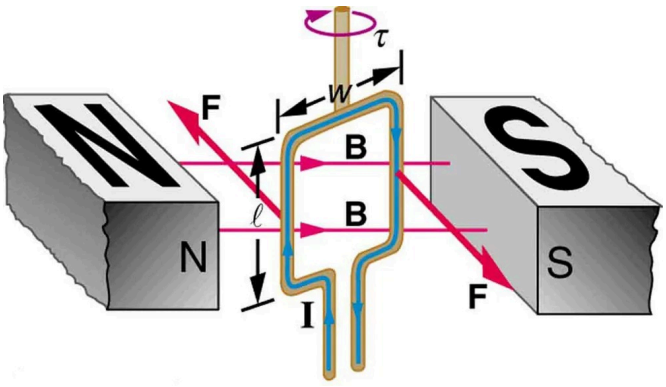


Figure 17.33 Torque on a current loop. A current-carrying loop of wire attached to a vertically rotating shaft feels magnetic forces that produce a clockwise torque as viewed from above. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram of a rectangular coil placed between two magnetic poles labeled “N” (North) and “S” (South). The coil is shown with current flowing through it, represented by arrows labeled “I.” The letters “B” indicate the magnetic field lines running horizontally across the coil.

There are forces labeled “F” acting on the coil in opposite directions on each side, illustrating the magnetic force due to the current and the magnetic field. These forces are shown with red arrows perpendicular to the magnetic field lines.

Additional arrows indicate the direction of torque, labeled “τ,” showing a rotational effect on the coil. Dimensions are marked with labels such as “ℓ” for length and “w” for width, indicating the dimensions of the coil.

Let us examine the force on each segment of the loop in Figure 17.33 to find the torques produced about the axis of the vertical shaft. (This will lead to a useful equation for the torque on the loop.) We take the magnetic field to be uniform over the rectangular loop, which has width w and height l. First, we note that the forces on the top and bottom segments are vertical and, therefore, parallel to the shaft, producing no torque. Those vertical forces are equal in magnitude and opposite in direction, so that they also produce no net force on the loop. Figure 17.34 shows views of the loop from above. Torque is defined as $\tau = \mathrm{rFsin}\theta$, where F is the force, r is the distance from the pivot that the force is applied, and θ is the angle between r and F. As seen in Figure 17.34(a), right hand rule 1 gives the forces on the sides to be equal in magnitude and opposite in direction, so that the net force is again zero. However, each force produces a clockwise torque. Since $r = w/2$, the torque on each vertical segment is $(w/2\,)F\mathrm{sin}\theta$, and the two add to give a total torque.

$$\tau = \frac{w}{2}F\mathrm{sin}\theta + \frac{w}{2}F\mathrm{sin}\theta = \mathrm{wFsin}\theta$$

17.19

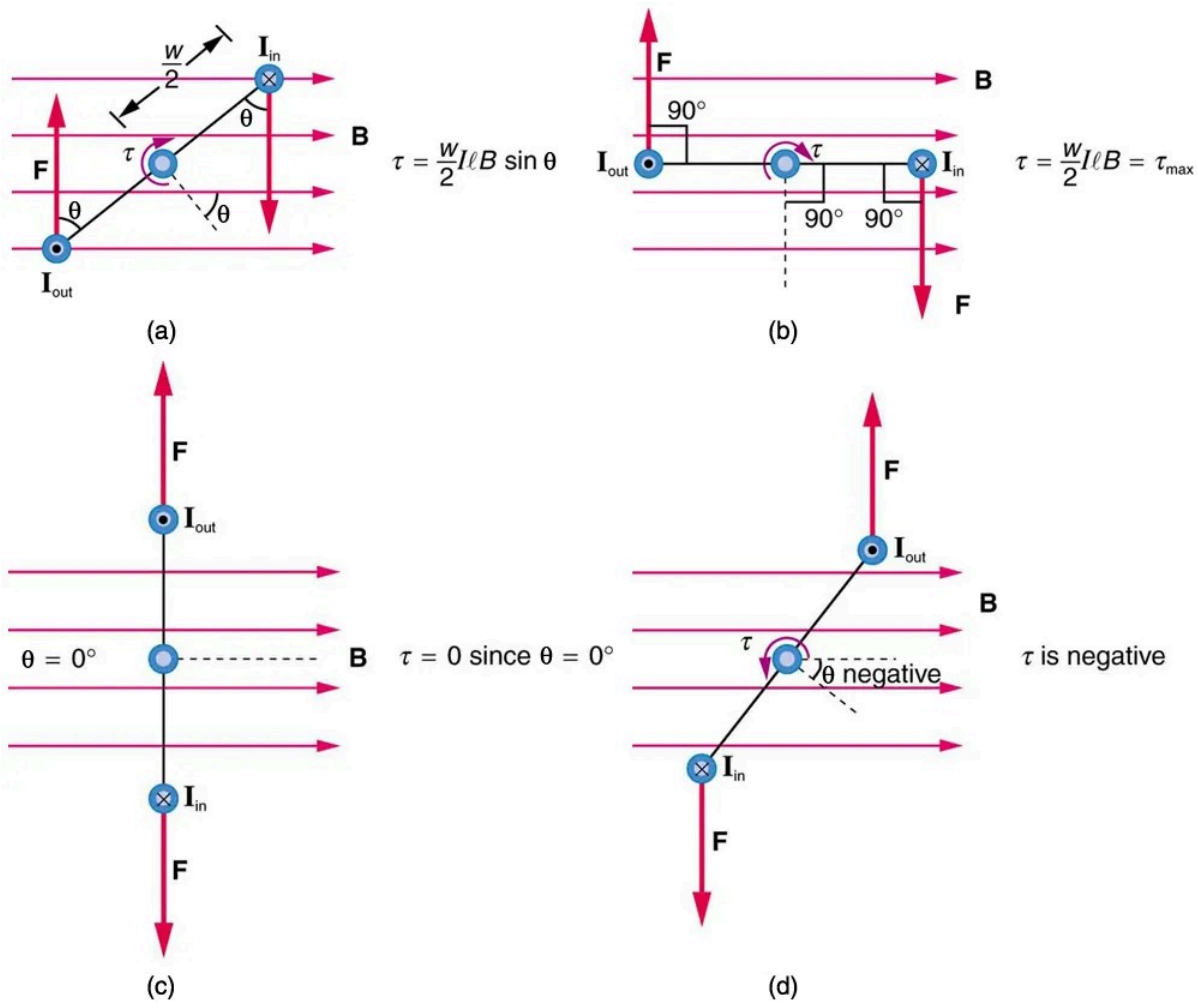


Figure 17.34 Top views of a current-carrying loop in a magnetic field. (a) The equation for torque is derived using this view. Note that the perpendicular to the loop makes an angle θ with the field that is the same as the angle between $w/2$ and $\mathbf{F}$. (b) The maximum torque occurs when θ is a right angle and $\sin\theta = 1$. (c) Zero (minimum) torque occurs when θ is zero and $\sin\theta = 0$. (d) The torque reverses once the loop rotates past $\theta = 0$. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of a series of diagrams labeled (a) to (d), demonstrating concepts of magnetic force and torque.

(a) A current loop is shown in a magnetic field. The loop is tilted with angle θ. The force (F) is directed perpendicular to the plane of the loop. The torque (τ) is calculated as τ = (w/2)IℓB sin θ, where w is the width, I is the current, ℓ is the length, B is the magnetic field, and θ is the angle.

(b) The current loop is rotated so that θ = 90°, and the forces are again perpendicular with the maximum torque $\tau = (w/2)I\ell B = \tau_{max}$.

(c) The current loop is shown with θ = 0°. The direction of the magnetic field (B) is aligned with the current, resulting in $\tau = 0$ since θ = 0°.

(d) The loop is shown with a negative angle, resulting in a negative torque. The force (F) is opposite the loop with a negative τ.

Now, each vertical segment has a length l that is perpendicular to B, so that the force on each is $F = \mathrm{IlB}$. Entering F into the expression for torque yields

$$\tau = \mathrm{wIlBsin}\theta.$$
17.20

If we have a multiple loop of N turns, we get N times the torque of one loop. Finally, note that the area of the loop is $A = \mathrm{wl}$; the expression for the torque becomes

$$\tau = \mathrm{NIABsin}\theta.$$
17.21

This is the torque on a current-carrying loop in a uniform magnetic field. This equation can be shown to be valid for a loop of any shape. The loop carries a current I, has N turns, each of area A, and the perpendicular to the loop makes an angle θ with the field B. The net force on the loop is zero.

Example 17.5

Calculating Torque on a Current-Carrying Loop in a Strong Magnetic Field

Find the maximum torque on a 100-turn square loop of a wire of 10.0 cm on a side that carries 15.0 A of current in a 2.00-T field.

Strategy

Torque on the loop can be found using $\tau = \mathrm{NIAB}\sin\theta$. Maximum torque occurs when $\theta = 90^\circ$ and $\sin\theta = 1$.

Solution

For $\sin\theta = 1$, the maximum torque is

$$\tau_{\max} = \mathrm{NIAB}.$$

17.22

Entering known values yields

$$\begin{aligned}\tau_{\max} &= (100)\,(15.0\ \mathrm{A})\left(0.100\ \mathrm{m}^2\right)(2.00\ \mathrm{T}) \\ &= 30.0\ \mathrm{N}\cdot m.\end{aligned}$$

17.23

Discussion

This torque is large enough to be useful in a motor.

The torque found in the preceding example is the maximum. As the coil rotates, the torque decreases to zero at $\theta = 0$. The torque then *reverses* its direction once the coil rotates past $\theta = 0$. (See Figure 17.34(d).) This

means that, unless we do something, the coil will oscillate back and forth about equilibrium at $\theta = 0$. To get the coil to continue rotating in the same direction, we can reverse the current as it passes through $\theta = 0$ with automatic switches called *brushes*. (See Figure 17.35.)

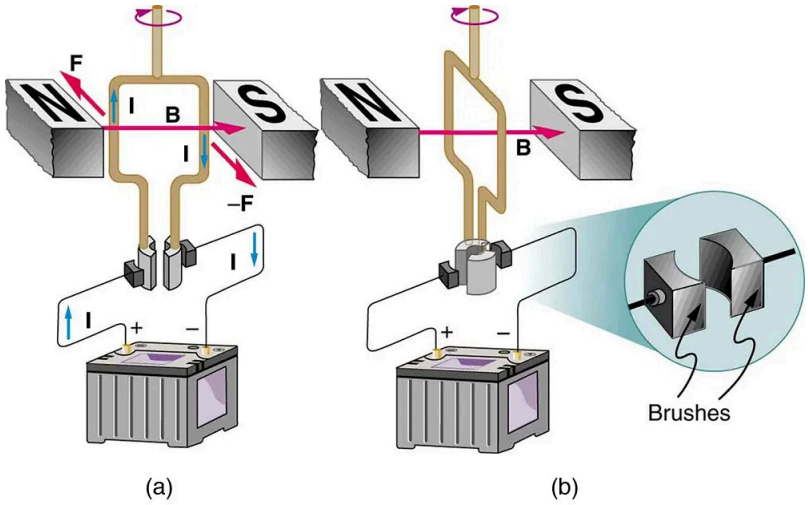


Figure 17.35 (a) As the angular momentum of the coil carries it through $\theta = 0$, the brushes reverse the current to keep the torque clockwise. (b) The coil will rotate continuously in the clockwise direction, with the current reversing each half revolution to maintain the clockwise torque. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a labeled diagram illustrating the functioning of an electric motor using the concept of electromagnetic forces. It has two parts, labeled "(a)" and "(b)":

(a) The diagram shows a rectangular loop of wire placed between the poles of a permanent magnet labeled "N" (North) and "S" (South). The loop is connected to a battery at the bottom, with wires marked showing the direction of the electrical current ("I"). The current flows in one direction along the vertical parts of the loop but is reversed when it comes out of the battery. There are arrows showing the magnetic field direction ("B") and the force ("F") experienced by the loop due to the interaction between the current and the magnetic field. The force is labeled in opposite directions on either side of the loop, illustrating the concept of torque that causes the loop to rotate.

(b) This part of the diagram illustrates the continuation of (a), showing a similar setup but with a focus on the mechanical components of the motor. The loop is connected to a split-ring commutator that reverses the current direction each half turn, allowing for continuous rotation. A zoomed-in view of the commutator and brushes is shown on the right side, highlighting how the brushes maintain contact with the rotating commutator, ensuring the current flows correctly.

Both parts emphasize the combination of electrical input, magnetic field interaction, and mechanical output (rotation), which are crucial for the functioning of an electric motor.

Meters, such as those in analog fuel gauges on a car, are another common application of magnetic torque on a current-carrying loop. Figure 17.36 shows that a meter is very similar in construction to a motor. The meter in the figure has its magnets shaped to limit the effect of θ by making B perpendicular to the loop over a large angular range. Thus the torque is proportional to I and not θ. A linear spring exerts a counter-torque that balances the current-produced torque. This makes the needle deflection proportional to I. If an exact proportionality cannot be achieved, the gauge reading can be calibrated. To produce a galvanometer for use in analog voltmeters and ammeters that have a low resistance and respond to small currents, we use a large loop area A, high magnetic field B, and low-resistance coils.

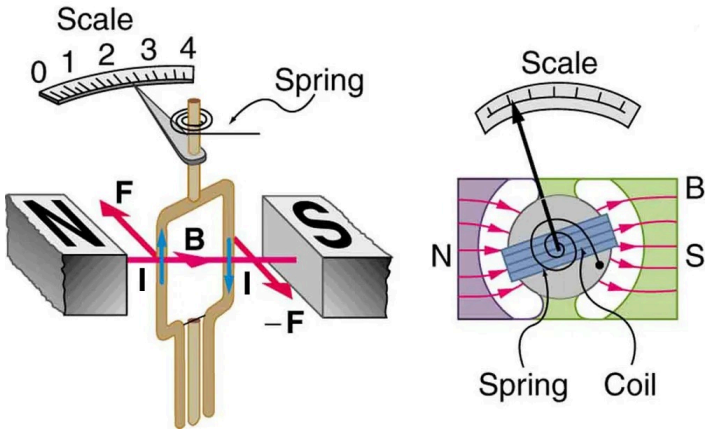


Figure 17.36 Meters are very similar to motors but only rotate through a part of a revolution. The magnetic poles of this meter are shaped to keep the component of B perpendicular to the loop constant, so that the torque does not depend on θ and the deflection against the return spring is proportional only to the current I. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains two schematic diagrams illustrating components of a galvanometer.

Left Diagram: This section shows a partial side view of a galvanometer. From left to right, there is a block labeled “N” representing the North pole of a magnet, and a block labeled “S” representing the South pole. Between these poles, a rectangular coil is depicted, labeled “B” and “I”. Red arrows point rightward through “B” from “N” to “S”, indicating the direction of magnetic force and current flow. Blue arrows labeled “I” indicate current direction within the coil. The coil is suspended on a spring, connecting it to a scale, which is labeled from 0 to 4. The scale pointer is shown pointing to a position between 2 and 3, with a spring attached near the top.

Right Diagram: This section shows a top view of a similar setup. Blocks labeled “N” and “S” are shown with a coil in between, similar to the left diagram. Curved, pink arrows within the coil represent the magnetic field lines emanating from the North to the South. A spring connected to the coil is visible, along with a scale marked from 0 to 4. The pointer indicates a position near the middle of the scale. The labels “B”, “Spring”, and “Coil” describe the components.

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17.9 MAGNETIC FIELDS PRODUCED BY CURRENTS: AMPERE'S LAW

Learning Objectives

By the end of this section, you will be able to:

- Calculate current that produces a magnetic field.
- Use the right hand rule 2 to determine the direction of current or the direction of magnetic field loops.

How much current is needed to produce a significant magnetic field, perhaps as strong as the Earth's field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted earlier that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire or a toroid (doughnut)? How is the direction of a current-created field related to the direction of the current? Answers to these questions are explored in this section, together with a brief discussion of the law governing the fields created by currents.

Magnetic Field Created by a Long Straight Current-Carrying Wire: Right Hand Rule 2

Magnetic fields have both direction and magnitude. As noted before, one way to explore the direction of a magnetic field is with compasses, as shown for a long straight current-carrying wire in Figure 17.37. Hall probes can determine the magnitude of the field. The field around a long straight wire is found to be in circular loops. The right hand rule 2 (RHR-2) emerges from this exploration and is valid for any current

segment—*point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops* created by it.

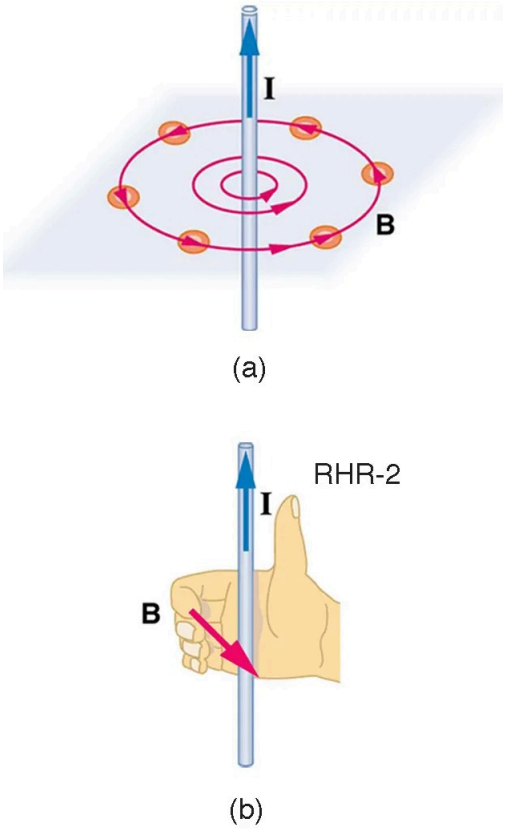


Figure 17.37 (a) Compasses placed near a long straight current-carrying wire indicate that field lines form circular loops centered on the wire. (b) Right hand rule 2 states that, if the right hand thumb points in the direction of the current, the fingers curl in the direction of the field. This rule is consistent with the field mapped for the long straight wire and is valid for any current segment. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two parts, labeled (a) and (b), illustrating the concept of the magnetic field around a current-carrying conductor.

Image (a): A vertical straight conductor is shown with a blue arrow pointing upward labeled "I" indicating the direction of electric current. Surrounding the conductor are concentric circular arrows, shown in red, illustrating the magnetic field lines. These circles are labeled as "B" to indicate the magnetic field direction.

Image (b): A similar vertical straight conductor is shown with a blue arrow labeled "I" pointing upward, indicating the direction of electric current. A right hand is positioned with the thumb pointing in the

direction of the current (upward), while the fingers curl around the conductor. The curl of the fingers indicates the direction of the magnetic field lines, marked with a red arrow labeled “B”. The text “RHR-2” is present near the hand, referring to the Right-Hand Rule, which is used to determine the direction of the magnetic field around a conductor carrying current.

The magnetic field strength (magnitude) produced by a long straight current-carrying wire is found by experiment to be

$$B = \frac{\mu_0 I}{2\pi r} (\text{long straight wire}),$$
17.24

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7} T \cdot \text{m/A}$ is the permeability of free space. (μ_0 is one of the basic constants in nature. We will see later that μ_0 is related to the speed of light.) Since the wire is very long, the magnitude of the field depends only on distance from the wire r, not on position along the wire.

Example 17.6

Calculating Current that Produces a Magnetic Field

Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth's at a distance of 5.0 cm from the wire.

Strategy

The Earth's field is about $5.0 \times 10^{-5} T$, and so here B due to the wire is taken to be $1.0 \times 10^{-4} T$. The equation $B = \frac{\mu_0 I}{2\pi r}$ can be used to find I, since all other quantities are known.

Solution

Solving for I and entering known values gives

$$\begin{aligned} I &= \frac{2\pi \mathrm{rB}}{\mu_0} \\ &= \frac{2\pi \left(5.0 \times 10^{-2} m\right)\left(1.0 \times 10^{-4} T\right)}{4\pi \times 10^{-7} T \cdot \mathrm{m/A}} \\ &= 25 \text{ A}. \end{aligned}$$

17.25

Discussion

So a moderately large current produces a significant magnetic field at a distance of 5.0 cm from a long straight wire. Note that the answer is stated to only two digits, since the Earth's field is specified to only two digits in this example.

Ampere's Law and Others

The magnetic field of a long straight wire has more implications than you might at first suspect. *Each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment.* The formal statement of the direction and magnitude of the field due to each segment is called the Biot-Savart law. Integral calculus is needed to sum the field for an arbitrary shape current. This results in a more complete law, called Ampere's law, which relates magnetic field and current in a general way. Ampere's law in turn is a part of Maxwell's equations, which give a complete theory of all electromagnetic phenomena. Considerations of how Maxwell's equations appear to different observers led to the modern theory of relativity, and the realization that electric and magnetic fields are different manifestations of the same thing. Most of this is beyond the scope of this text in both mathematical level, requiring calculus, and in the amount of space that can be devoted to it. But for the interested student, and particularly for those who continue in physics, engineering, or similar pursuits, delving into these matters further will reveal descriptions of nature that are elegant as well as profound. In this text, we shall keep the general features in mind, such as RHR-2 and the rules for magnetic field lines listed in Magnetic Fields and Magnetic Field Lines, while concentrating on the fields created in certain important situations.

Making Connections: Relativity

Hearing all we do about Einstein, we sometimes get the impression that he invented relativity out of nothing. On the contrary, one of Einstein's motivations was to solve difficulties in knowing how different observers see magnetic and electric fields.

Magnetic Field Produced by a Current-Carrying Circular Loop

The magnetic field near a current-carrying loop of wire is shown in Figure 17.38. Both the direction and the magnitude of the magnetic field produced by a current-carrying loop are complex. RHR-2 can be used to give the direction of the field near the loop, but mapping with compasses and the rules about field lines given in Magnetic Fields and Magnetic Field Lines are needed for more detail. There is a simple formula for the magnetic field strength at the center of a circular loop. It is

$$B = \frac{\mu_0 I}{2R} (\text{at center of loop}),$$

17.26

where R is the radius of the loop. This equation is very similar to that for a straight wire, but it is valid *only* at the center of a circular loop of wire. The similarity of the equations does indicate that similar field strength can be obtained at the center of a loop. One way to get a larger field is to have N loops; then, the field is $B = N\mu_0 I / (2R)$. Note that the larger the loop, the smaller the field at its center, because the current is farther away.

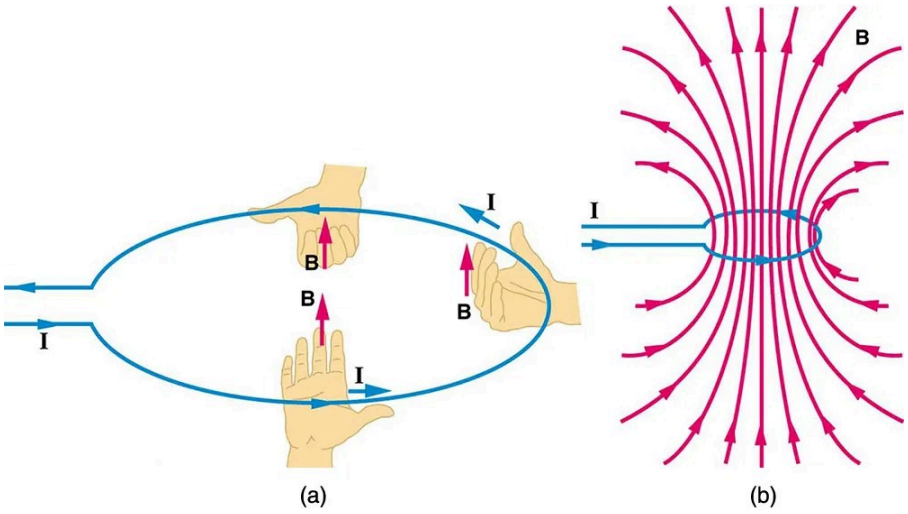


Figure 17.38 (a) RHR-2 gives the direction of the magnetic field inside and outside a current-carrying loop. (b) More detailed mapping with compasses or with a Hall probe completes the picture. The field is similar to that of a bar magnet. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two diagrams labeled (a) and (b) that demonstrate the right-hand rule and magnetic fields around a current-carrying wire.

(a) The left diagram shows a loop of blue wire carrying a current, labeled "I," directed into and out of the loop. Three hands are depicted using the right-hand rule to determine the direction of the magnetic field, labeled "B." The fingers of each hand point in the direction of the magnetic field, while the thumb indicates the direction of the current flow.

(b) The right diagram displays a blue wire with an electric current "I" flowing through it. Surrounding the wire are concentric pink magnetic field lines, labeled "B," forming circular patterns around the wire.

These diagrams illustrate the relationship between electric currents and their associated magnetic fields using the right-hand rule for directionality.

Magnetic Field Produced by a Current-Carrying Solenoid

A solenoid is a long coil of wire (with many turns or loops, as opposed to a flat loop). Because of its shape, the field inside a solenoid can be very uniform, and also very strong. The field just outside the coils is nearly zero. Figure 17.39 shows how the field looks and how its direction is given by RHR-2.

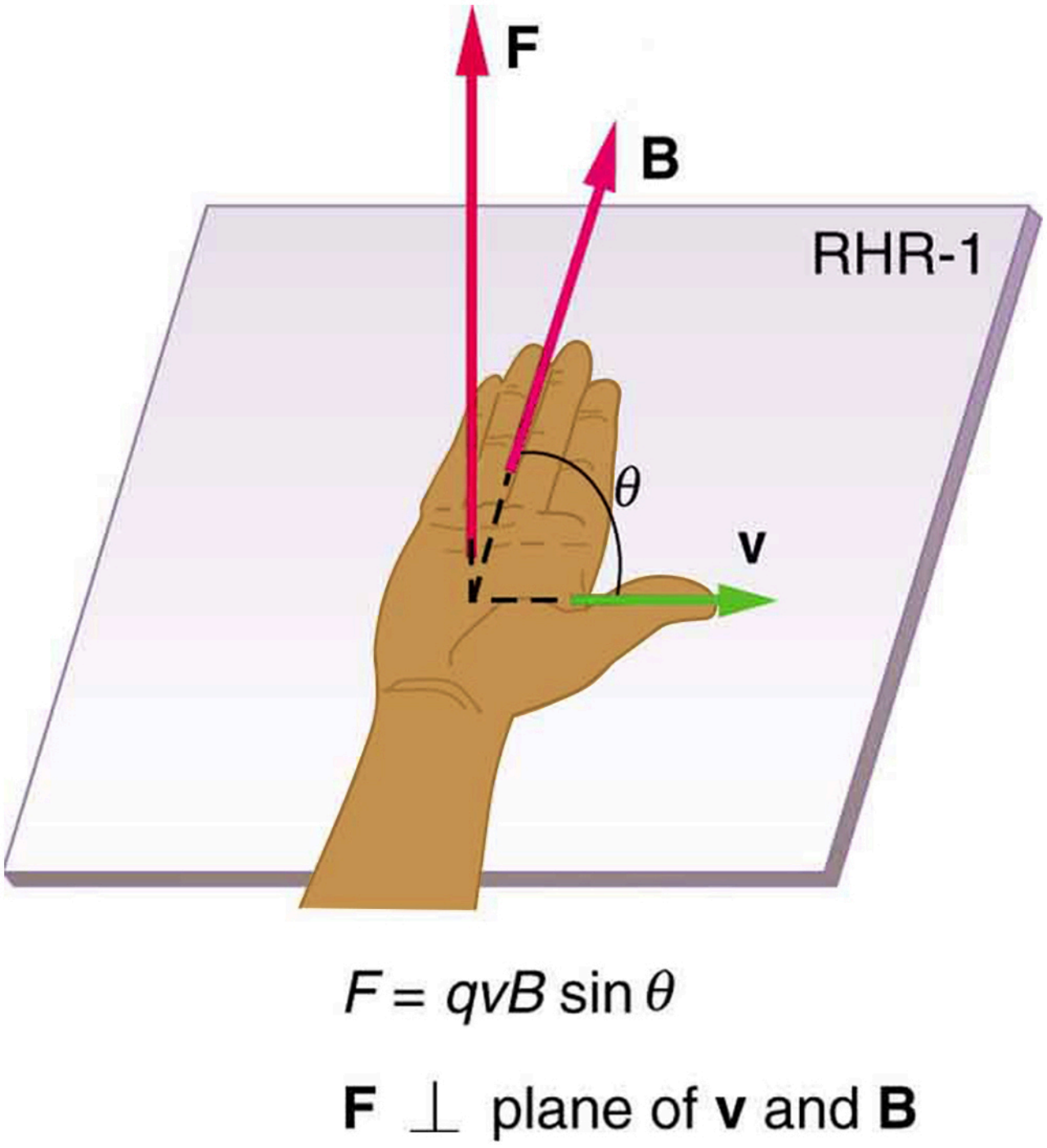


Figure 17.39 (a) Because of its shape, the field inside a solenoid of length l is remarkably uniform in magnitude and direction, as indicated by the straight and uniformly spaced field lines. The field outside the coils is nearly zero. (b) This cutaway shows the magnetic field generated by the current in the solenoid. Image by OpenStax, Wikimedia Commons License

Image Description

The image illustrates the right-hand rule for determining the direction of force in a magnetic field, labeled as

"RHR-1." A hand is shown with fingers extended and the thumb pointing perpendicular to the fingers. The fingers represent the direction of velocity, labeled as "v," shown with a green arrow. The magnetic field, labeled "B," is indicated by a magenta arrow pointing upward from the palm.

An angle, labeled "θ," is formed between the velocity "v" and the magnetic field "B." The force "F," which results from the interaction between the velocity and magnetic field, is depicted by a magenta arrow pointing upward and perpendicular to the palm. Below the diagram, the mathematical expression for the force is given: $F = qvB \sin \theta$, indicating that the force is proportional to the charge "q," the velocity "v," the magnetic field "B," and the sine of the angle "θ" between them. It notes that F is perpendicular to the plane formed by "v" and "B."

The magnetic field inside of a current-carrying solenoid is very uniform in direction and magnitude. Only near the ends does it begin to weaken and change direction. The field outside has similar complexities to flat loops and bar magnets, but the magnetic field strength inside a solenoid is simply

$$B = \mu_0 \mathrm{nI} \text{ (inside a solenoid)},$$
17.27

where n is the number of loops per unit length of the solenoid ($n = N/l$, with N being the number of loops and l the length). Note that B is the field strength anywhere in the uniform region of the interior and not just at the center. Large uniform fields spread over a large volume are possible with solenoids, as Example 17.7 implies.

Example 17.7

Calculating Field Strength inside a Solenoid

What is the field inside a 2.00-m-long solenoid that has 2000 loops and carries a 1600-A current?

Strategy

To find the field strength inside a solenoid, we use $B = \mu_0 \mathrm{nI}$. First, we note the number of loops per unit length is

$$n = \frac{N}{l} = \frac{2000}{2.00m} = 1000 \text{ m}^{-1} = 10 \text{ cm}^{-1}.$$

17.28

Solution

Substituting known values gives

$$\begin{aligned} B &= \mu_0 \mathrm{nI} \\ &= \left(4\pi \times 10^{-7} T \cdot \mathrm{m/A}\right)\left(1000 m^{-1}\right)(1600 \text{ A}) \\ &= 2.01 \text{ T}. \end{aligned}$$

17.29

Discussion

This is a large field strength that could be established over a large-diameter solenoid, such as in medical uses of magnetic resonance imaging (MRI). The very large current is an indication that the fields of this strength are not easily achieved, however. Such a large current through 1000 loops squeezed into a meter's length would produce significant heating. Higher currents can be achieved by using superconducting wires,

although this is expensive. There is an upper limit to the current, since the superconducting state is disrupted by very large magnetic fields.

There are interesting variations of the flat coil and solenoid. For example, the toroidal coil used to confine the reactive particles in tokamaks is much like a solenoid bent into a circle. The field inside a toroid is very strong but circular. Charged particles travel in circles, following the field lines, and collide with one another, perhaps inducing fusion. But the charged particles do not cross field lines and escape the toroid. A whole range of coil shapes are used to produce all sorts of magnetic field shapes. Adding ferromagnetic materials produces greater field strengths and can have a significant effect on the shape of the field. Ferromagnetic materials tend to trap magnetic fields (the field lines bend into the ferromagnetic material, leaving weaker fields outside it) and are used as shields for devices that are adversely affected by magnetic fields, including the Earth's magnetic field.

PhET Explorations

Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

Click to view content.

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17.10 MAGNETIC FORCE BETWEEN TWO PARALLEL CONDUCTORS

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects of the magnetic force between two conductors.
- Calculate the force between two parallel conductors.

You might expect that there are significant forces between current-carrying wires, since ordinary currents produce significant magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to *define* the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long straight and parallel conductors separated by a distance r can be found by applying what we have developed in preceding sections. Figure 17.40 shows the wires, their currents, the fields they create, and the subsequent forces they exert on one another. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force F_2). The field due to I_1 at a distance r is given to be

$$B_1 = \frac{\mu_0 I_1}{2\pi r}.$$

17.30

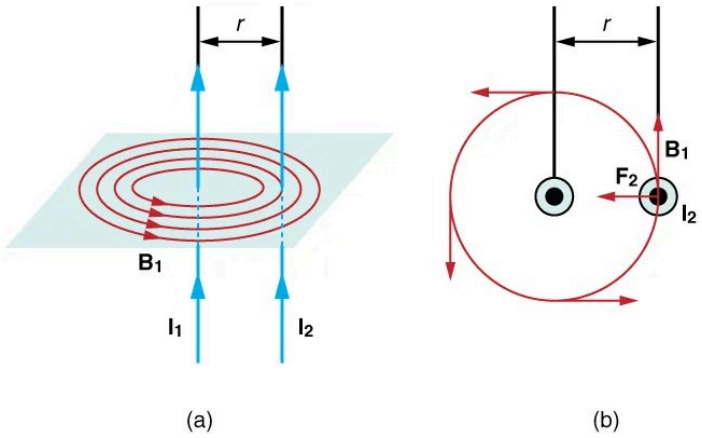


Figure 17.40 (a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by RHR-2. (b) A view from above of the two wires shown in (a), with one magnetic field line shown for each wire. RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of two labeled diagrams (a) and (b) showing the interaction between two parallel current-carrying wires and the resulting magnetic fields.

Diagram (a):
– Two vertical parallel wires are shown, labeled as I1 and I2, carrying current in the upward direction.
– The distance between the wires is labeled as “r”.
– Concentric circles around wire I1 are depicted with arrows, indicating the magnetic field B1 created by I1.
– The magnetic field lines are shown in red.

Diagram (b):
– A top view of the same setup with wire I1 on the left and wire I2 on the right.
– The distance between the wires is again labeled as “r”.
– The magnetic field B1 around I1 is shown as a circular path.
– A force F2 is shown acting on wire I2, suggesting the interaction due to I1’s magnetic field.
– Arrows indicate the direction of the magnetic field and forces.

This field is uniform along wire 2 and perpendicular to it, and so the force F_2 it exerts on wire 2 is given by $F = \mathrm{IlBsin}\theta$ with $\mathrm{sin}\theta = 1$:

$$F_2 = I_2 \mathrm{lB}_1.$$

17.31

By Newton's third law, the forces on the wires are equal in magnitude, and so we just write F for the magnitude of F_2. (Note that $F_1 = -F_2$.) Since the wires are very long, it is convenient to think in terms of F/l, the force per unit length. Substituting the expression for B_1 into the last equation and rearranging terms gives

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}.$$

17.32

F/l is the force per unit length between two parallel currents I_1 and I_2 separated by a distance r. The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the *pinch effect* in electric arcs and plasmas. The force exists whether the currents are in wires or not. In an electric arc, where currents are moving parallel to one another, there is an attraction that squeezes currents into a smaller tube. In large circuit breakers, like those used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The *operational definition of the ampere* is based on the force between current-carrying wires. Note that for parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

$$\frac{F}{l} = \frac{\left(4\pi \times 10^{-7} T \cdot \text{m/A}\right)(1A)^2}{(2\pi)\,(1m)} = 2 \times 10^{-7}\ \text{N/m}.$$

17.33

Since μ_0 is exactly $4\pi \times 10^{-7} T \cdot \text{m/A}$ by definition, and because $1\ \text{T} = 1\ \text{N}/\left(A \cdot m\right)$, the force per meter is exactly $2 \times 10^{-7} \text{N/m}$. This is the basis of the operational definition of the ampere.

The Ampere

The official definition of the ampere is:

One ampere of current through each of two parallel conductors of infinite length, separated by one meter in empty space free of other magnetic fields, causes a force of exactly $2 \times 10^{-7}\ \mathrm{N/m}$ on each conductor.

Infinite-length straight wires are impractical and so, in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is, $1C = 1A \cdot s$. For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

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17.11 MORE APPLICATIONS OF MAGNETISM

Learning Objectives

By the end of this section, you will be able to:

- Describe some applications of magnetism.

Mass Spectrometry

The curved paths followed by charged particles in magnetic fields can be put to use. A charged particle moving perpendicular to a magnetic field travels in a circular path having a radius r.

$$r = \frac{\mathrm{mv}}{\mathrm{qB}}$$

17.34

It was noted that this relationship could be used to measure the mass of charged particles such as ions. A mass spectrometer is a device that measures such masses. Most mass spectrometers use magnetic fields for this purpose, although some of them have extremely sophisticated designs. Since there are five variables in the relationship, there are many possibilities. However, if v, q, and B can be fixed, then the radius of the path r is simply proportional to the mass m of the charged particle. Let us examine one such mass spectrometer that has a relatively simple design. (See Figure 17.41.) The process begins with an ion source, a device like an electron gun. The ion source gives ions their charge, accelerates them to some velocity v, and directs a beam of them into the next stage of the spectrometer. This next region is a *velocity selector* that only allows particles with a particular value of v to get through.

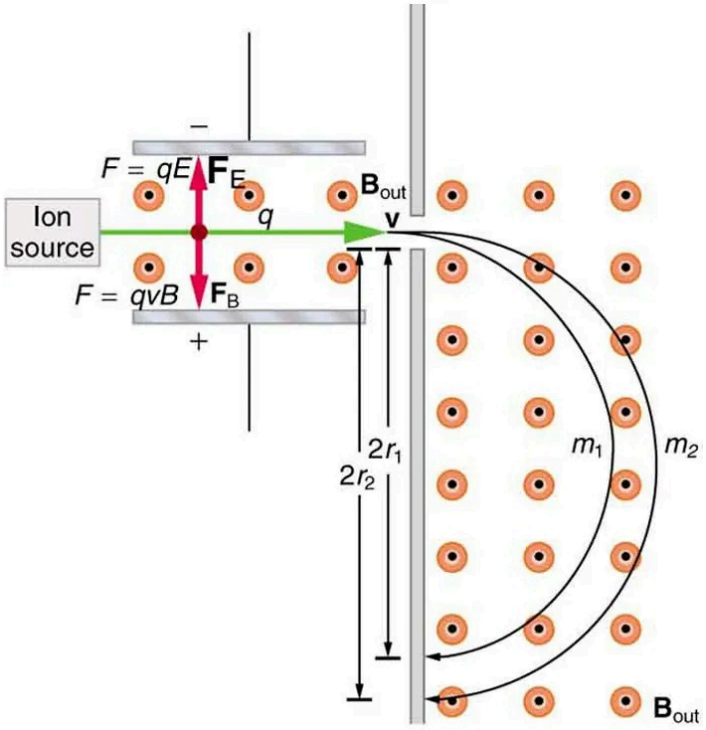


Figure 17.41 This mass spectrometer uses a velocity selector to fix v so that the radius of the path is proportional to mass. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the principles of a mass spectrometer. Here's a detailed description:

– The diagram is divided into two main sections: an ion source on the left and a detection area on the right.
– On the left, there is a labeled "Ion source" where ions are generated. The ions are represented by small circles with a dot inside.
– A green horizontal arrow originates from the ion source, indicating the direction of the ion beam.
– In the middle, two plates create an electric field. The top plate is marked with a minus sign (-) and the bottom with a plus sign (+), generating an electric field (E) vertically between them.
– A red dot with red vertical arrows between the plates represents the electric force (F_E) acting on an ion with charge q. The formula $F = qE$ is noted beside the arrows.
– Below this is the magnetic force (F_B), shown with a downward red arrow, created by the magnetic field (B), with the formula $F = qvB$ indicated beside it.
– As ions move to the right, they pass through a gap between the plates into a region labeled B_{out}.
– The ions enter a magnetic field causing them to follow curved paths. Two distinct curved paths are shown, labeled m_1 and m_2, representing ions of different masses.
– The curved paths lead to a detection region where the ions are collected.

– Two vertical lines indicate the radii of the paths, marked as $2r_1$ and $2r_2$, showing the separation based on mass.

The diagram illustrates how ions are separated based on their mass-to-charge ratio using electric and magnetic fields.

The velocity selector has both an electric field and a magnetic field, perpendicular to one another, producing forces in opposite directions on the ions. Only those ions for which the forces balance travel in a straight line into the next region. If the forces balance, then the electric force $F = \mathrm{qE}$ equals the magnetic force $F = \mathrm{qvB}$, so that $\mathrm{qE} = \mathrm{qvB}$. Noting that q cancels, we see that

$$v = \frac{E}{B}$$
17.35

is the velocity particles must have to make it through the velocity selector, and further, that v can be selected by varying E and B. In the final region, there is only a uniform magnetic field, and so the charged particles move in circular arcs with radii proportional to particle mass. The paths also depend on charge q, but since q is in multiples of electron charges, it is easy to determine and to discriminate between ions in different charge states.

Mass spectrometry today is used extensively in chemistry and biology laboratories to identify chemical and biological substances according to their mass-to-charge ratios. In medicine, mass spectrometers are used to measure the concentration of isotopes used as tracers. Usually, biological molecules such as proteins are very large, so they are broken down into smaller fragments before analyzing. Recently, large virus particles have been analyzed as a whole on mass spectrometers. Sometimes a gas chromatograph or high-performance liquid chromatograph provides an initial separation of the large molecules, which are then input into the mass spectrometer.

Cathode Ray Tubes—CRTs—and the Like

What do non-flat-screen TVs, old computer monitors, x-ray machines, and the 2-mile-long Stanford Linear Accelerator have in common? All of them accelerate electrons, making them different versions of the electron gun. Many of these devices use magnetic fields to steer the accelerated electrons. Figure 17.42 shows the construction of the type of cathode ray tube (CRT) found in some TVs, oscilloscopes, and old computer monitors. Two pairs of coils are used to steer the electrons, one vertically and the other horizontally, to their desired destination.

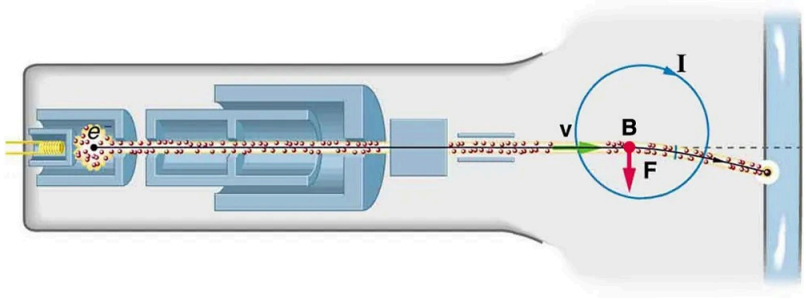


Figure 17.42 The cathode ray tube (CRT) is so named because rays of electrons originate at the cathode in the electron gun. Magnetic coils are used to steer the beam in many CRTs. In this case, the beam is moved down. Another pair of horizontal coils would steer the beam horizontally. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram illustrating the setup and function of an electron beam within a cathode ray tube, commonly used for demonstrating the concept of the Lorentz force.

– On the left side, there is a yellow element symbolizing the electron source. Electrons, represented by red circles with a negative sign, are emitted from this source.
– The electrons pass through a series of blue structures that guide and focus the beam.
– The path of the electron beam is shown as a dotted line, initially moving straight through the apparatus.
– As the beam enters a magnetic field, depicted as a blue circle, the path of the electrons curves due to the Lorentz force.
– The magnetic field is indicated by the letter "B" with a blue circular line demonstrating its direction.
– The velocity of the electrons is represented by the letter "v," and their curved path is determined by the Lorentz force, marked with the letter "F" and a red arrow pointing downward.
– The direction of current is shown with the letter "I" in the same circle as the magnetic field, following the path of the electron beam.
– The diagram ends on the right side with a representation of a screen or detector where the deflected electron beam impacts.

Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) is one of the most useful and rapidly growing medical imaging tools. It non-invasively produces two-dimensional and three-dimensional images of the body that provide important medical information with none of the hazards of x-rays. MRI is based on an effect called nuclear magnetic resonance (NMR) in which an externally applied magnetic field interacts with the nuclei of certain atoms,

particularly those of hydrogen (protons). These nuclei possess their own small magnetic fields, similar to those of electrons and the current loops discussed earlier in this chapter.

When placed in an external magnetic field, such nuclei experience a torque that pushes or aligns the nuclei into one of two new energy states—depending on the orientation of its spin (analogous to the N pole and S pole in a bar magnet). Transitions from the lower to higher energy state can be achieved by using an external radio frequency signal to "flip" the orientation of the small magnets. (This is actually a quantum mechanical process. The direction of the nuclear magnetic field is quantized as is energy in the radio waves. We will return to these topics in later chapters.) The specific frequency of the radio waves that are absorbed and reemitted depends sensitively on the type of nucleus, the chemical environment, and the external magnetic field strength. Therefore, this is a *resonance* phenomenon in which *nuclei* in a *magnetic* field act like resonators (analogous to those discussed in the treatment of sound in Oscillatory Motion and Waves) that absorb and reemit only certain frequencies. Hence, the phenomenon is named *nuclear magnetic resonance (NMR).*

NMR has been used for more than 50 years as an analytical tool. It was formulated in 1946 by F. Bloch and E. Purcell, with the 1952 Nobel Prize in Physics going to them for their work. Over the past two decades, NMR has been developed to produce detailed images in a process now called magnetic resonance imaging (MRI), a name coined to avoid the use of the word "nuclear" and the concomitant implication that nuclear radiation is involved. (It is not.) The 2003 Nobel Prize in Medicine went to P. Lauterbur and P. Mansfield for their work with MRI applications.

The largest part of the MRI unit is a superconducting magnet that creates a magnetic field, typically between 1 and 2 T in strength, over a relatively large volume. MRI images can be both highly detailed and informative about structures and organ functions. It is helpful that normal and non-normal tissues respond differently for slight changes in the magnetic field. In most medical images, the protons that are hydrogen nuclei are imaged. (About 2/3 of the atoms in the body are hydrogen.) Their location and density give a variety of medically useful information, such as organ function, the condition of tissue (as in the brain), and the shape of structures, such as vertebral disks and knee-joint surfaces. MRI can also be used to follow the movement of certain ions across membranes, yielding information on active transport, osmosis, dialysis, and other phenomena. With excellent spatial resolution, MRI can provide information about tumors, strokes, shoulder injuries, infections, etc.

An image requires position information as well as the density of a nuclear type (usually protons). By varying the magnetic field slightly over the volume to be imaged, the resonant frequency of the protons is made to vary with position. Broadcast radio frequencies are swept over an appropriate range and nuclei absorb and reemit them only if the nuclei are in a magnetic field with the correct strength. The imaging receiver gathers information through the body almost point by point, building up a tissue map. The reception of reemitted radio waves as a function of frequency thus gives position information. These "slices" or cross sections through the body are only several mm thick. The intensity of the reemitted radio waves is proportional to the

concentration of the nuclear type being flipped, as well as information on the chemical environment in that area of the body. Various techniques are available for enhancing contrast in images and for obtaining more information. Scans called T1, T2, or proton density scans rely on different relaxation mechanisms of nuclei. Relaxation refers to the time it takes for the protons to return to equilibrium after the external field is turned off. This time depends upon tissue type and status (such as inflammation).

While MRI images are superior to x rays for certain types of tissue and have none of the hazards of x rays, they do not completely supplant x-ray images. MRI is less effective than x rays for detecting breaks in bone, for example, and in imaging breast tissue, so the two diagnostic tools complement each other. MRI images are also expensive compared to simple x-ray images and tend to be used most often where they supply information not readily obtained from x rays. Another disadvantage of MRI is that the patient is totally enclosed with detectors close to the body for about 30 minutes or more, leading to claustrophobia. It is also difficult for the obese patient to be in the magnet tunnel. New "open-MRI" machines are now available in which the magnet does not completely surround the patient.

Over the last decade, the development of much faster scans, called "functional MRI" (fMRI), has allowed us to map the functioning of various regions in the brain responsible for thought and motor control. This technique measures the change in blood flow for activities (thought, experiences, action) in the brain. The nerve cells increase their consumption of oxygen when active. Blood hemoglobin releases oxygen to active nerve cells and has somewhat different magnetic properties when oxygenated than when deoxygenated. With MRI, we can measure this and detect a blood oxygen-dependent signal. Most of the brain scans today use fMRI.

Other Medical Uses of Magnetic Fields

Currents in nerve cells and the heart create magnetic fields like any other currents. These can be measured but with some difficulty since their strengths are about 10^{-6} to 10^{-8} *less* than the Earth's magnetic field. Recording of the heart's magnetic field as it beats is called a magnetocardiogram (MCG), while measurements of the brain's magnetic field is called a magnetoencephalogram (MEG). Both give information that differs from that obtained by measuring the electric fields of these organs (ECGs and EEGs), but they are not yet of sufficient importance to make these difficult measurements common.

In both of these techniques, the sensors do not touch the body. MCG can be used in fetal studies, and is probably more sensitive than echocardiography. MCG also looks at the heart's electrical activity whose voltage output is too small to be recorded by surface electrodes as in EKG. It has the potential of being a rapid scan for early diagnosis of cardiac ischemia (obstruction of blood flow to the heart) or problems with the fetus.

MEG can be used to identify abnormal electrical discharges in the brain that produce weak magnetic signals. Therefore, it looks at brain activity, not just brain structure. It has been used for studies of Alzheimer's disease

and epilepsy. Advances in instrumentation to measure very small magnetic fields have allowed these two techniques to be used more in recent years. What is used is a sensor called a SQUID, for superconducting quantum interference device. This operates at liquid helium temperatures and can measure magnetic fields thousands of times smaller than the Earth's.

Finally, there is a burgeoning market for magnetic cures in which magnets are applied in a variety of ways to the body, from magnetic bracelets to magnetic mattresses. The best that can be said for such practices is that they are apparently harmless, unless the magnets get close to the patient's computer or magnetic storage disks. Claims are made for a broad spectrum of benefits from cleansing the blood to giving the patient more energy, but clinical studies have not verified these claims, nor is there an identifiable mechanism by which such benefits might occur.

PhET Explorations

Magnet and Compass

Ever wonder how a compass worked to point you to the Arctic? Explore the interactions between a compass and bar magnet, and then add the Earth and find the surprising answer! Vary the magnet's strength, and see how things change both inside and outside. Use the field meter to measure how the magnetic field changes.

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GLOSSARY

Ampere's law
the physical law that states that the magnetic field around an electric current is proportional to the current; each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment

***B*-field**
another term for magnetic field

Biot-Savart law
a physical law that describes the magnetic field generated by an electric current in terms of a specific equation

Curie temperature
the temperature above which a ferromagnetic material cannot be magnetized

direction of magnetic field lines
the direction that the north end of a compass needle points

domains
regions within a material that behave like small bar magnets

electromagnet
an object that is temporarily magnetic when an electrical current is passed through it

electromagnetism
the use of electrical currents to induce magnetism

ferromagnetic
materials, such as iron, cobalt, nickel, and gadolinium, that exhibit strong magnetic effects

gauss
G, the unit of the magnetic field strength; $1\ \mathrm{G} = 10^{-4}T$

Hall effect
the creation of voltage across a current-carrying conductor by a magnetic field

Hall emf
the electromotive force created by a current-carrying conductor by a magnetic field, $\epsilon = \mathrm{Blv}$

Lorentz force
the force on a charge moving in a magnetic field

magnetic field
the representation of magnetic forces

magnetic field lines
: the pictorial representation of the strength and the direction of a magnetic field

magnetic field strength (magnitude) produced by a long straight current-carrying wire
: defined as $B = \frac{\mu_0 I}{2\pi r}$, where I is the current, r is the shortest distance to the wire, and μ_0 is the permeability of free space

magnetic field strength at the center of a circular loop
: defined as $B = \frac{\mu_0 I}{2R}$ where R is the radius of the loop

magnetic field strength inside a solenoid
: defined as $B = \mu_0 \mathrm{nI}$ where n is the number of loops per unit length of the solenoid ($n = N/l$, with N being the number of loops and l the length)

magnetic force
: the force on a charge produced by its motion through a magnetic field; the Lorentz force

magnetic monopoles
: an isolated magnetic pole; a south pole without a north pole, or vice versa (no magnetic monopole has ever been observed)

magnetic resonance imaging (MRI)
: a medical imaging technique that uses magnetic fields create detailed images of internal tissues and organs

magnetized
: to be turned into a magnet; to be induced to be magnetic

magnetocardiogram (MCG)
: a recording of the heart's magnetic field as it beats

magnetoencephalogram (MEG)
: a measurement of the brain's magnetic field

Maxwell's equations
: a set of four equations that describe electromagnetic phenomena

meter
: common application of magnetic torque on a current-carrying loop that is very similar in construction to a motor; by design, the torque is proportional to I and not θ, so the needle deflection is proportional to the current

motor
: loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted to mechanical work in the process

north magnetic pole
: the end or the side of a magnet that is attracted toward Earth's geographic north pole

nuclear magnetic resonance (NMR)

a phenomenon in which an externally applied magnetic field interacts with the nuclei of certain atoms

permeability of free space

the measure of the ability of a material, in this case free space, to support a magnetic field; the constant $\mu_0 = 4\pi \times 10^{-7} T \cdot \mathrm{m/A}$

right hand rule 1 (RHR-1)

the rule to determine the direction of the magnetic force on a positive moving charge: when the thumb of the right hand points in the direction of the charge's velocity $\mathbf{v}$ and the fingers point in the direction of the magnetic field $\mathbf{B}$, then the force on the charge is perpendicular and away from the palm; the force on a negative charge is perpendicular and into the palm

right hand rule 2 (RHR-2)

a rule to determine the direction of the magnetic field induced by a current-carrying wire: Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops

solenoid

a thin wire wound into a coil that produces a magnetic field when an electric current is passed through it

south magnetic pole

the end or the side of a magnet that is attracted toward Earth's geographic south pole

tesla

T, the SI unit of the magnetic field strength; $1\ \mathrm{T} = \dfrac{1\ \mathrm{N}}{A \cdot m}$

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CHAPTER SUMMARY

17.1 Magnets

- Magnetism is a subject that includes the properties of magnets, the effect of the magnetic force on moving charges and currents, and the creation of magnetic fields by currents.
- There are two types of magnetic poles, called the north magnetic pole and south magnetic pole.
- North magnetic poles are those that are attracted toward the Earth's geographic north pole.
- Like poles repel and unlike poles attract.
- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

17.2 Ferromagnets and Electromagnets

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.
- All magnetism is created by electric current.
- Ferromagnetic materials, such as iron, are those that exhibit strong magnetic effects.
- The atoms in ferromagnetic materials act like small magnets (due to currents within the atoms) and can be aligned, usually in millimeter-sized regions called domains.
- Domains can grow and align on a larger scale, producing permanent magnets. Such a material is magnetized, or induced to be magnetic.
- Above a material's Curie temperature, thermal agitation destroys the alignment of atoms, and ferromagnetism disappears.
- Electromagnets employ electric currents to make magnetic fields, often aided by induced fields in ferromagnetic materials.

17.3 Magnetic Fields and Magnetic Field Lines

Magnetic fields can be pictorially represented by magnetic field lines, the properties of which are as follows:

1. The field is tangent to the magnetic field line.
2. Field strength is proportional to the line density.
3. Field lines cannot cross.
4. Field lines are continuous loops.

17.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

- Magnetic fields exert a force on a moving charge q, the magnitude of which is
$$F = \mathrm{qvBsin}\theta,$$
where θ is the angle between the directions of v and B.
- The SI unit for magnetic field strength B is the tesla (T), which is related to other units by
$$1T = \frac{1\ \mathrm{N}}{C \cdot \mathrm{m/s}} = \frac{1\ \mathrm{N}}{A \cdot m}.$$
- The *direction* of the force on a moving charge is given by right hand rule 1 (RHR-1): Point the thumb of the right hand in the direction of v, the fingers in the direction of B, and a perpendicular to the palm points in the direction of F .
- The force is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$. Since the force is zero if $\mathbf{v}$ is parallel to $\mathbf{B}$, charged particles often follow magnetic field lines rather than cross them.

17.5 Force on a Moving Charge in a Magnetic Field: Examples and Applications

- Magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius
$$r = \frac{\mathrm{mv}}{\mathrm{qB}},$$
where v is the component of the velocity perpendicular to B for a charged particle with mass m and charge q.

17.6 The Hall Effect

- The Hall effect is the creation of voltage ϵ, known as the Hall emf, across a current-carrying conductor by a magnetic field.
- The Hall emf is given by
$$\epsilon = \text{Blv} \, (B, v, \text{and} l, \text{mutually perpendicular} \,)$$
for a conductor of width l through which charges move at a speed v.

17.7 Magnetic Force on a Current-Carrying Conductor

- The magnetic force on current-carrying conductors is given by
$$F = \text{IlBsin}\theta,$$
where I is the current, l is the length of a straight conductor in a uniform magnetic field B, and θ is the angle between I and B. The force follows RHR-1 with the thumb in the direction of I.

17.8 Torque on a Current Loop: Motors and Meters

- The torque τ on a current-carrying loop of any shape in a uniform magnetic field. is

$$\tau = \mathrm{NIABsin}\theta,$$

where N is the number of turns, I is the current, A is the area of the loop, B is the magnetic field strength, and θ is the angle between the perpendicular to the loop and the magnetic field.

17.9 Magnetic Fields Produced by Currents: Ampere's Law

- The strength of the magnetic field created by current in a long straight wire is given by
$$B = \frac{\mu_0 I}{2\pi r}(\text{long straight wire}),$$
where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\text{T} \cdot \text{m/A}$ is the permeability of free space.
- The direction of the magnetic field created by a long straight wire is given by right hand rule 2 (RHR-2): *Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops* created by it.
- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampere's law.
- The magnetic field strength at the center of a circular loop is given by
$$B = \frac{\mu_0 I}{2R}(\text{at center of loop}),$$
where R is the radius of the loop. This equation becomes $B = \mu_0 \text{nI}/(2R)$ for a flat coil of N loops. RHR-2 gives the direction of the field about the loop. A long coil is called a solenoid.
- The magnetic field strength inside a solenoid is
$$B = \mu_0 \text{nI}\,(\text{inside a solenoid}),$$
where n is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.

17.10 Magnetic Force between Two Parallel Conductors

- The force between two parallel currents I_1 and I_2, separated by a distance r, has a magnitude per unit length given by $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$.
- The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

17.11 More Applications of Magnetism

- Crossed (perpendicular) electric and magnetic fields act as a velocity filter, giving equal and opposite forces on any charge with velocity perpendicular to the fields and of magnitude $v = \frac{E}{B}$.

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CONCEPTUAL QUESTIONS

Conceptual Questions

17.1 Magnets

1. Volcanic and other such activity at the mid-Atlantic ridge extrudes material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth's magnetic field and how could the knowledge of the spreading rate be used to give its historical record?

17.3 Magnetic Fields and Magnetic Field Lines

2. Explain why the magnetic field would not be unique (that is, not have a single value) at a point in space where magnetic field lines might cross. (Consider the direction of the field at such a point.)
3. List the ways in which magnetic field lines and electric field lines are similar. For example, the field direction is tangent to the line at any point in space. Also list the ways in which they differ. For example, electric force is parallel to electric field lines, whereas magnetic force on moving charges is perpendicular to magnetic field lines.
4. Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet? Is this consistent with your experience with magnets?
5. Is the Earth's magnetic field parallel to the ground at all locations? If not, where is it parallel to the surface? Is its strength the same at all locations? If not, where is it greatest?

17.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

6. If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is necessarily zero?

17.5 Force on a Moving Charge in a Magnetic Field: Examples and Applications

7. How can the motion of a charged particle be used to distinguish between a magnetic and an electric field?
8. High-velocity charged particles can damage biological cells and are a component of radiation exposure in a variety of locations ranging from research facilities to natural background. Describe how you could use a magnetic field to shield yourself.
9. If a cosmic ray proton approaches the Earth from outer space along a line toward the center of the Earth that lies in the plane of the equator, in what direction will it be deflected by the Earth's magnetic field? What about an electron? A neutron?
10. What are the signs of the charges on the particles in Figure 17.43?

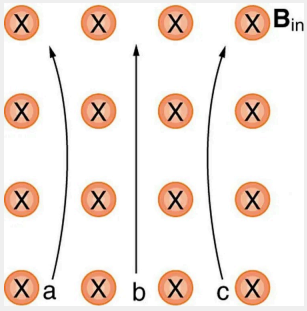


Figure 17.43 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a concept in electromagnetism with a magnetic field represented by orange circles containing black "X" marks, arranged in a grid pattern. The background is white. The circles are evenly spaced, with three rows and five columns. Three vertical

lines are present: two curved lines labeled “a” and “c” and a straight line labeled “b”. These lines are directed upwards, indicating the path of magnetic flux. The label “B_{in}” is present in the upper right corner, signifying an inward magnetic field.

11. Which of the particles in Figure 17.44 has the greatest velocity, assuming they have identical charges and masses?

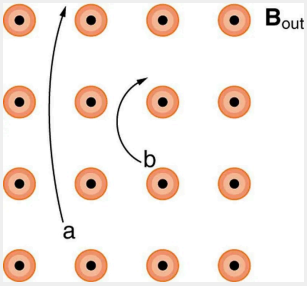


Figure 17.44 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a grid-like arrangement of circular patterns. Each pattern consists of concentric circles: a black dot in the center, surrounded by a pink ring, and another orange ring. The circles are evenly spaced, forming a structured array.

There are two curved arrows labeled “a” and “b” indicating specific directions. The arrow labeled “a” points diagonally upward from the bottom left towards the top left. The arrow labeled “b” points horizontally from left to right at the center of the image.

Additionally, there is text labeled “B_{out}” located at the top right corner of the image, suggesting an annotation or a description related to the direction of the arrows or arrangement.

12. Which of the particles in Figure 17.44 has the greatest mass, assuming all have identical charges and velocities?
13. While operating, a high-precision TV monitor is placed on its side during maintenance. The image on the monitor changes color and blurs slightly. Discuss the possible relation of these effects to the Earth’s magnetic field.

17.6 The Hall Effect

14. Discuss how the Hall effect could be used to obtain information on free charge density in a conductor. (Hint: Consider how drift velocity and current are related.)

17.7 Magnetic Force on a Current-Carrying Conductor

15. Draw a sketch of the situation in Figure 17.29 showing the direction of electrons carrying the current, and use RHR-1 to verify the direction of the force on the wire.
16. Verify that the direction of the force in an MHD drive, such as that in Figure 17.31, does not depend on the sign of the charges carrying the current across the fluid.
17. Why would a magnetohydrodynamic drive work better in ocean water than in fresh water? Also, why would superconducting magnets be desirable?
18. Which is more likely to interfere with compass readings, AC current in your refrigerator or DC current when you start your car? Explain.

17.8 Torque on a Current Loop: Motors and Meters

19. Draw a diagram and use RHR-1 to show that the forces on the top and bottom segments of the motor's current loop in Figure 17.33 are vertical and produce no torque about the axis of rotation.

17.9 Magnetic Fields Produced by Currents: Ampere's Law

20. Make a drawing and use RHR-2 to find the direction of the magnetic field of a current loop in a motor (such as in Figure 17.33). Then show that the direction of the torque on the loop is the same as produced by like poles repelling and unlike poles attracting.

17.10 Magnetic Force between Two Parallel Conductors

21. Is the force attractive or repulsive between the hot and neutral lines hung from power poles? Why?

22. If you have three parallel wires in the same plane, as in Figure 17.45, with currents in the outer two running in opposite directions, is it possible for the middle wire to be repelled by both? Attracted by both? Explain.

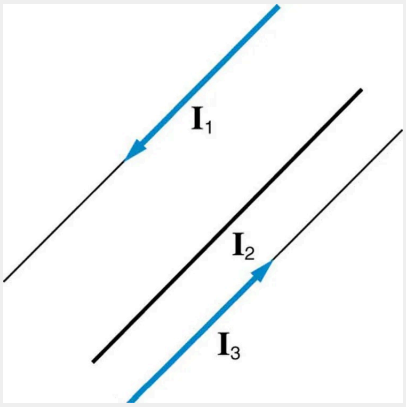


Figure 17.45 Three parallel coplanar wires with currents in the outer two in opposite directions. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows three parallel lines with arrows, representing current flow.

– The first line, labeled as "I1," is at the top, depicted with a blue arrow pointing downwards.
– The second line, labeled as "I2," is in the middle, shown as a black line without an arrow.
– The third line, labeled as "I3," is at the bottom, depicted with a blue arrow pointing upwards.

The blue arrows indicate the direction of current flow for "I1" and "I3," while "I2" has no direction shown.

23. Suppose two long straight wires run perpendicular to one another without touching. Does one exert a net force on the other? If so, what is its direction? Does one exert a net torque on the other? If so, what is its direction? Justify your responses by using the right hand rules.
24. Use the right hand rules to show that the force between the two loops in Figure 17.46 is attractive if the currents are in the same direction and repulsive if they are

in opposite directions. Is this consistent with like poles of the loops repelling and unlike poles of the loops attracting? Draw sketches to justify your answers.

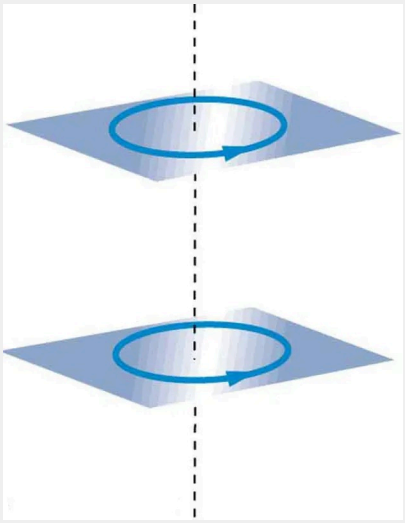


Figure 17.46 Two loops of wire carrying currents can exert forces and torques on one another. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts two parallel, blue planes positioned one above the other with a dashed vertical black line running through the center, symbolizing an axis of rotation. On each plane, there is a circular blue arrow indicating a clockwise rotation. The overall visual suggests rotational motion around the vertical axis.

25. If one of the loops in Figure 17.46 is tilted slightly relative to the other and their currents are in the same direction, what are the directions of the torques they exert on each other? Does this imply that the poles of the bar magnet-like fields they create will line up with each other if the loops are allowed to rotate?
26. Electric field lines can be shielded by the Faraday cage effect. Can we have magnetic shielding? Can we have gravitational shielding?

17.11 More Applications of Magnetism

27. Measurements of the weak and fluctuating magnetic fields associated with brain

activity are called magnetoencephalograms (MEGs). Do the brain's magnetic fields imply coordinated or uncoordinated nerve impulses? Explain.

28. Discuss the possibility that a Hall voltage would be generated on the moving heart of a patient during MRI imaging. Also discuss the same effect on the wires of a pacemaker. (The fact that patients with pacemakers are not given MRIs is significant.)
29. A patient in an MRI unit turns his head quickly to one side and experiences momentary dizziness and a strange taste in his mouth. Discuss the possible causes.
30. You are told that in a certain region there is either a uniform electric or magnetic field. What measurement or observation could you make to determine the type? (Ignore the Earth's magnetic field.)
31. An example of magnetohydrodynamics (MHD) comes from the flow of a river (salty water). This fluid interacts with the Earth's magnetic field to produce a potential difference between the two river banks. How would you go about calculating the potential difference?
32. Draw gravitational field lines between 2 masses, electric field lines between a positive and a negative charge, electric field lines between 2 positive charges and magnetic field lines around a magnet. Qualitatively describe the differences between the fields and the entities responsible for the field lines.

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PROBLEMS & EXERCISES

Problems & Exercises

17.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

1. What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in Figure 17.47?

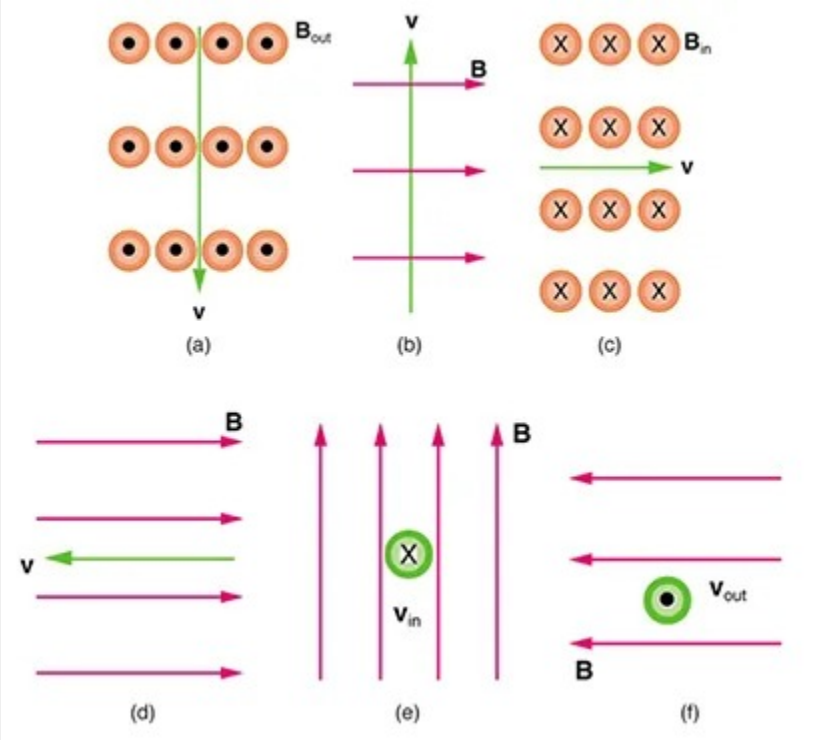


Figure 17.47 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains six diagrams labeled (a) to (f), illustrating the interaction between charged particles, velocity (v), and magnetic fields (B):

– Diagram (a): Shows a grid of circles with a dot in the center, representing a magnetic

field (B_out) directed out of the plane. Green arrows indicate velocity (v) pointing upward.

– Diagram (b): Displays several horizontal pink arrows pointing to the right, representing a magnetic field (B). A vertical green arrow in the middle represents velocity (v) pointing upward.

– Diagram (c): Consists of a grid of circles with an "X" in the center, symbolizing a magnetic field (B_in) directed into the plane. A green arrow shows velocity (v) moving to the right.

– Diagram (d): Features horizontal pink arrows pointing to the right for the magnetic field (B) and a green arrow directed leftward, depicting velocity (v).

– Diagram (e): Shows vertical pink arrows pointing upward, representing the magnetic field (B), and a circle with an "X" indicating velocity (v_in) directed into the plane.

– Diagram (f): Illustrates horizontal pink arrows directed rightward for the magnetic field (B). A circle with a dot in the center indicates velocity (v_out) directed outward from the plane.

2. Repeat Exercise 17.1 for a negative charge.
3. What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases in Figure 17.48, assuming it moves perpendicular to $\mathbf{B}$?

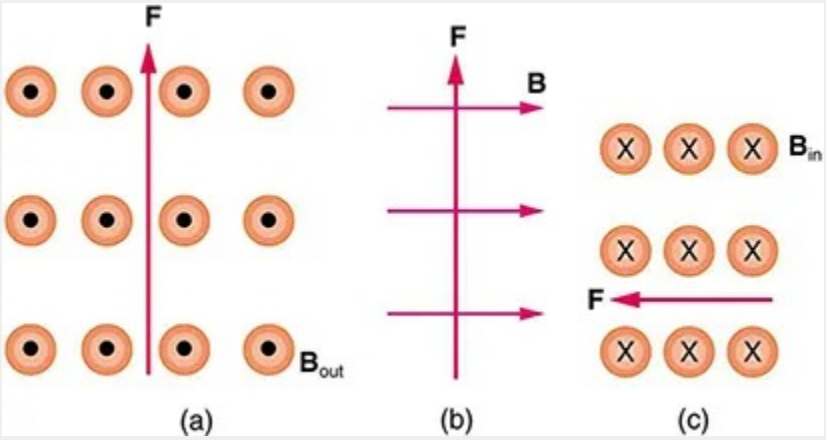


Figure 17.48 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

This image contains three diagrams labeled (a), (b), and (c), illustrating magnetic fields and forces on charged particles:

(a) Shows a grid of circles with dots at their center, indicating a magnetic field directed out of the page. A red arrow labeled “F” points upward, and a label “B_{out}” is at the bottom right, indicating the direction of the magnetic field.

(b) Displays three horizontal arrows labeled “B” pointing to the right, and one vertical arrow labeled “F” pointing upward. This illustrates the magnetic field direction and the force acting perpendicular to it.

(c) Features a grid of circles with an “X” in each, representing a magnetic field directed into the page. A red arrow labeled “F” points to the left. The label “B_{in}” is at the right, indicating the direction of the magnetic field.

4. Repeat Exercise 17.3 for a positive charge.
5. What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases in the figure below, assuming $\mathbf{B}$ is perpendicular to $\mathbf{v}$?

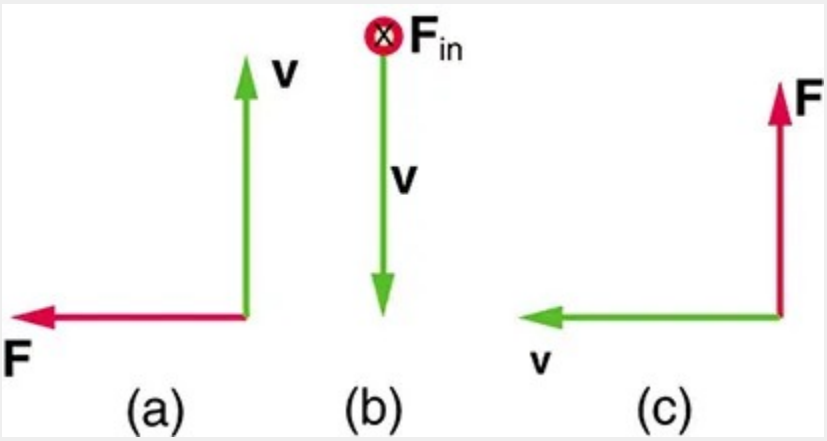


Figure 17.49 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image contains three diagrams labeled (a), (b), and (c) that illustrate different vector scenarios with forces and velocities.

(a) Features two arrows: a red arrow labeled “F” pointing to the left, and a green arrow labeled “v” pointing upwards.

(b) Features a downward green arrow labeled “v” and an upward red arrow with an encircled “x,” labeled “F_{in}.”

(c) Features two arrows: a green arrow labeled “v” pointing to the right, and a red arrow labeled “F” pointing upwards.

6. Repeat Exercise 17.5 for a negative charge.
7. What is the maximum magnitude of the force on an aluminum rod with a $0.100 - \mu\text{C}$ charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s? In what direction is the force?
8. (a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a $0.500 - \mu\text{C}$ charge and flies due west at a speed of 660 m/s over the Earth’s magnetic south pole (near Earth’s geographic north pole), where the $8.00 \times 10^{-5} - \text{T}$ magnetic field points straight down. What are the direction and the magnitude of the magnetic force on the plane? (b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.
9. (a) A cosmic ray proton moving toward the Earth at $5.00 \times 10^{7}\text{m/s}$ experiences a magnetic force of $1.70 \times 10^{-16}\text{N}$. What is the strength of the magnetic field if there is a 45° angle between it and the proton’s velocity? (b) Is the value obtained in part (a) consistent with the known strength of the Earth’s magnetic field on its surface? Discuss.
10. An electron moving at $4.00 \times 10^{3}\text{m/s}$ in a 1.25-T magnetic field experiences a magnetic force of $1.40 \times 10^{-16}\text{N}$. What angle does the velocity of the electron make with the magnetic field? There are two answers.
11. (a) A physicist performing a sensitive measurement wants to limit the magnetic force on a moving charge in her equipment to less than $1.00 \times 10^{-12}N$. What is the greatest the charge can be if it moves at a maximum speed of 30.0 m/s in the Earth’s field? (b) Discuss whether it would be difficult to limit the charge to less than the value found in (a) by comparing it with typical static electricity and noting that static is often absent.

17.5 Force on a Moving Charge in a Magnetic Field: Examples and Applications

If you need additional support for these problems, see 17.11 More Applications of Magnetism.

12. A cosmic ray electron moves at $7.50 \times 10^6 \mathrm{m/s}$ perpendicular to the Earth's magnetic field at an altitude where field strength is $1.00 \times 10^{-5} T$. What is the radius of the circular path the electron follows?
13. A proton moves at $7.50 \times 10^7 \mathrm{m/s}$ perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?
14. (a) Viewers of Star Trek hear of an antimatter drive on the Starship Enterprise. One possibility for such a futuristic energy source is to store antimatter charged particles in a vacuum chamber, circulating in a magnetic field, and then extract them as needed. Antimatter annihilates with normal matter, producing pure energy. What strength magnetic field is needed to hold antiprotons, moving at $5.00 \times 10^7 \mathrm{m/s}$ in a circular path 2.00 m in radius? Antiprotons have the same mass as protons but the opposite (negative) charge. (b) Is this field strength obtainable with today's technology or is it a futuristic possibility?
15. (a) An oxygen-16 ion with a mass of $2.66 \times 10^{-26} \mathrm{kg}$ travels at $5.00 \times 10^6 \mathrm{m/s}$ perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer.
16. What radius circular path does an electron travel if it moves at the same speed and in the same magnetic field as the proton in Exercise 17.13?
17. A velocity selector in a mass spectrometer uses a 0.100-T magnetic field. (a) What electric field strength is needed to select a speed of $4.00 \times 10^6 \mathrm{m/s}$? (b) What is the voltage between the plates if they are separated by 1.00 cm?
18. An electron in a TV CRT moves with a speed of $6.00 \times 10^7 \mathrm{m/s}$, in a direction

perpendicular to the Earth's field, which has a strength of $5.00 \times 10^{-5} T$. (a) What strength electric field must be applied perpendicular to the Earth's field to make the electron moves in a straight line? (b) If this is done between plates separated by 1.00 cm, what is the voltage applied? (Note that TVs are usually surrounded by a ferromagnetic material to shield against external magnetic fields and avoid the need for such a correction.)

19. (a) At what speed will a proton move in a circular path of the same radius as the electron in Exercise 17.12? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?
20. A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18, the mass of oxygen-16 is $2.66 \times 10^{-26} \text{kg}$, and they are singly charged and travel at $5.00 \times 10^{6} \text{m/s}$ in a 1.20-T magnetic field. What is the separation between their paths when they hit a target after traversing a semicircle?
21. (a) Triply charged uranium-235 and uranium-238 ions are being separated in a mass spectrometer. (The much rarer uranium-235 is used as reactor fuel.) The masses of the ions are $3.90 \times 10^{-25} \text{kg}$ and $3.95 \times 10^{-25} \text{kg}$, respectively, and they travel at $3.00 \times 10^{5} \text{m/s}$ in a 0.250-T field. What is the separation between their paths when they hit a target after traversing a semicircle? (b) Discuss whether this distance between their paths seems to be big enough to be practical in the separation of uranium-235 from uranium-238.

17.6 The Hall Effect

22. A large water main is 2.50 m in diameter and the average water velocity is 6.00 m/s. Find the Hall voltage produced if the pipe runs perpendicular to the Earth's $5.00 \times 10^{-5} - \text{T}$ field.
23. What Hall voltage is produced by a 0.200-T field applied across a 2.60-cm-diameter aorta when blood velocity is 60.0 cm/s?

24. (a) What is the speed of a supersonic aircraft with a 17.0-m wingspan, if it experiences a 1.60-V Hall voltage between its wing tips when in level flight over the north magnetic pole, where the Earth's field strength is $8.00 \times 10^{-5}\text{T}$? (b) Explain why very little current flows as a result of this Hall voltage.
25. A nonmechanical water meter could utilize the Hall effect by applying a magnetic field across a metal pipe and measuring the Hall voltage produced. What is the average fluid velocity in a 3.00-cm-diameter pipe, if a 0.500-T field across it creates a 60.0-mV Hall voltage?
26. Calculate the Hall voltage induced on a patient's heart while being scanned by an MRI unit. Approximate the conducting path on the heart wall by a wire 7.50 cm long that moves at 10.0 cm/s perpendicular to a 1.50-T magnetic field.
27. A Hall probe calibrated to read $1.00\ \mu\text{V}$ when placed in a 2.00-T field is placed in a 0.150-T field. What is its output voltage?
28. Using information in Example 17.6, what would the Hall voltage be if a 2.00-T field is applied across a 10-gauge copper wire (2.588 mm in diameter) carrying a 20.0-A current?
29. Show that the Hall voltage across wires made of the same material, carrying identical currents, and subjected to the same magnetic field is inversely proportional to their diameters. (Hint: Consider how drift velocity depends on wire diameter.)
30. A patient with a pacemaker is mistakenly being scanned for an MRI image. A 10.0-cm-long section of pacemaker wire moves at a speed of 10.0 cm/s perpendicular to the MRI unit's magnetic field and a 20.0-mV Hall voltage is induced. What is the magnetic field strength?

17.7 Magnetic Force on a Current-Carrying Conductor

31. What is the direction of the magnetic force on the current in each of the six cases in Figure 17.50?

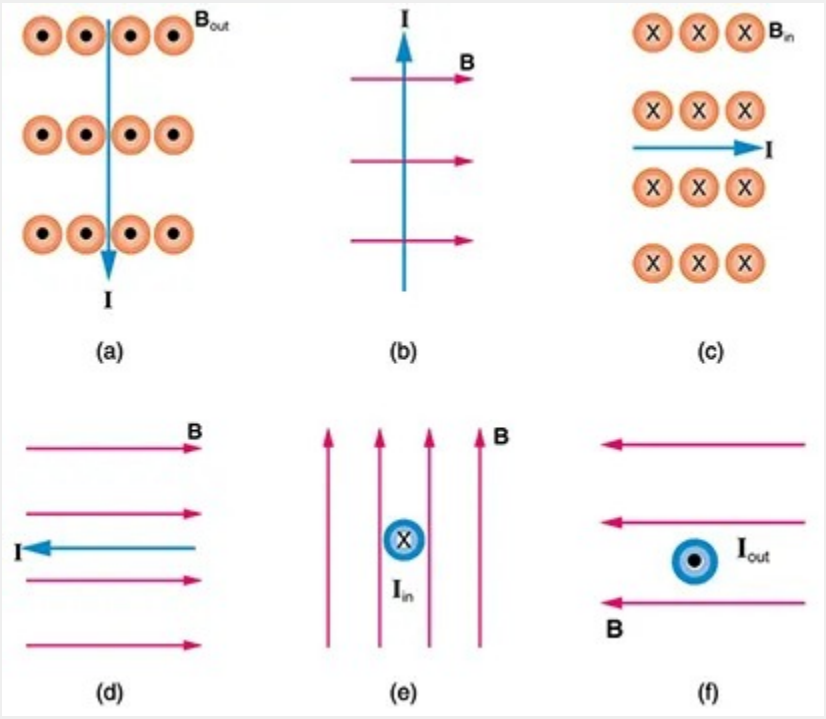


Figure 17.50 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of six diagrams labeled (a) to (f) demonstrating magnetic fields and electric currents in different configurations:

(a) A series of loops arranged vertically. A current "I" flows upward through them, and the magnetic field "B_{out}" is directed outward at the top.

(b) Straight lines representing a magnetic field "B" pointing to the right. A current "I" flows vertically upwards.

(c) A grid of circles with X through them, indicating a magnetic field "B_{in}" pointing inward. A current "I" flows horizontally to the right.

(d) Horizontal lines with arrows pointing to the right, representing a magnetic field "B." A current line is directed leftward against the field.

(e) A central circle with an "X" indicating a current "I_{in}" pointing inward, enclosed by upward lines representing a magnetic field "B."

(f) A central circle with a dot, indicating a current "I_{out}" pointing outward towards the viewer, with horizontal lines representing a magnetic field "B" pointing to the left.

32. What is the direction of a current that experiences the magnetic force shown in each of the three cases in Figure 17.51, assuming the current runs perpendicular to B?

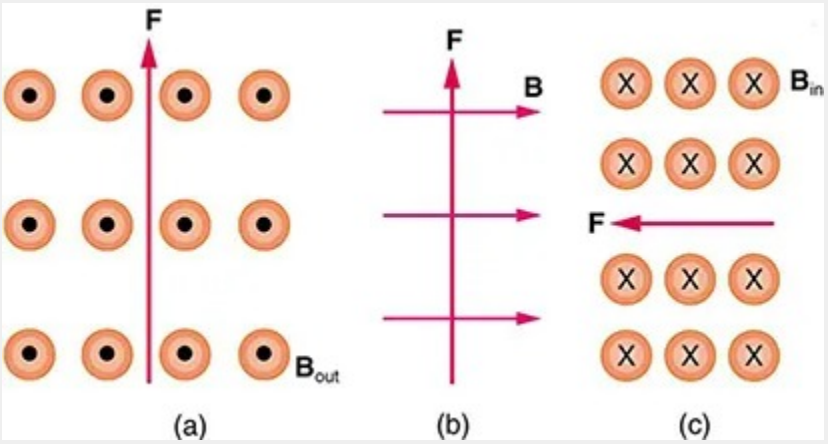


Figure 17.51 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams, labeled (a), (b), and (c), illustrating magnetic and force field interactions:

1. Diagram (a):
– A grid of nine circles arranged in three rows and three columns.
– Each circle has a dot in the center, representing the magnetic field coming out of the page.
– An arrow labeled "F" points upward through the center of the grid.
– The magnetic field is labeled "B out" at the lower right corner.

2. Diagram (b):
– A vertical arrow labeled "F" pointing upward, crossing three horizontal arrows pointing to the right.
– The horizontal arrows represent the magnetic field "B."

3. Diagram (c):

– A grid of nine circles arranged in three rows and three columns.
– Each circle contains an “X” to represent the magnetic field going into the page.
– An arrow labeled “F” points leftward across the middle of the grid.
– The magnetic field is labeled “B in” at the upper right corner.

33. What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases in Figure 22.52, assuming $\mathbf{B}$ is perpendicular to $\mathbf{I}$?

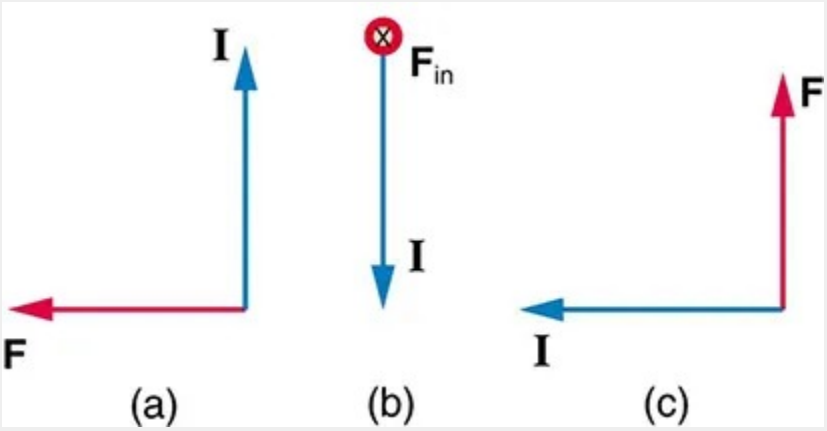


Figure 17.52 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams labeled (a), (b), and (c). Each diagram depicts vectors representing force and current, illustrated with different colored arrows.

Diagram (a): A red arrow labeled “F” points left, representing force. A blue arrow labeled “I” points upwards, representing current.

Diagram (b): A blue arrow labeled “I” points downwards, indicating current. A red circle with an “X” inside marks the position at the top of this arrow. Another arrow labeled “F_in” points downwards from this circle, suggesting the direction of an applied force.

Diagram (c): A blue arrow labeled “I” points to the right, signifying current. A red arrow labeled “F” points upwards, representing force.

34. (a) What is the force per meter on a lightning bolt at the equator that carries 20,000 A perpendicular to the Earth’s $3.00 \times 10^{-5} - \mathrm{T}$ field? (b) What is the

direction of the force if the current is straight up and the Earth's field direction is due north, parallel to the ground?

35. (a) A DC power line for a light-rail system carries 1000 A at an angle of 30.0° to the Earth's $5.00 \times 10^{-5} - \mathrm{T}$ field. What is the force on a 100-m section of this line? (b) Discuss practical concerns this presents, if any.
36. What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)
37. A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?
38. (a) A 0.750-m-long section of cable carrying current to a car starter motor makes an angle of 60° with the Earth's $5.50 \times 10^{-5}\ \mathrm{T}$ field. What is the current when the wire experiences a force of $7.00 \times 10^{-3} N$? (b) If you run the wire between the poles of a strong horseshoe magnet, subjecting 5.00 cm of it to a 1.75-T field, what force is exerted on this segment of wire?
39. (a) What is the angle between a wire carrying an 8.00-A current and the 1.20-T field it is in if 50.0 cm of the wire experiences a magnetic force of 2.40 N? (b) What is the force on the wire if it is rotated to make an angle of 90° with the field?
40. The force on the rectangular loop of wire in the magnetic field in Figure 17.53 can be used to measure field strength. The field is uniform, and the plane of the loop is perpendicular to the field. (a) What is the direction of the magnetic force on the loop? Justify the claim that the forces on the sides of the loop are equal and opposite, independent of how much of the loop is in the field and do not affect the net force on the loop. (b) If a current of 5.00 A is used, what is the force per tesla on the 20.0-cm-wide loop?

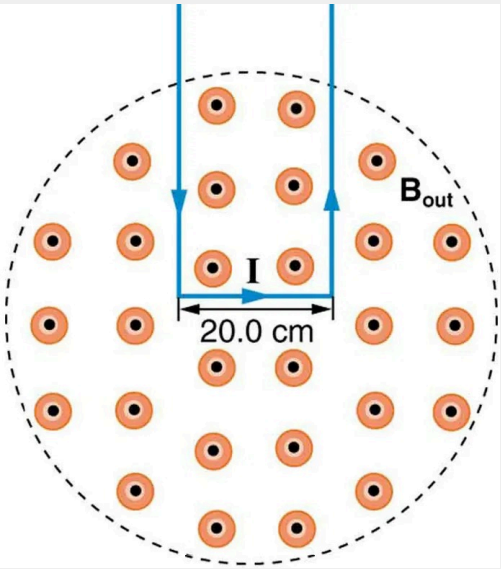


Figure 17.53 A rectangular loop of wire carrying a current is perpendicular to a magnetic field. The field is uniform in the region shown and is zero outside that region. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image is a diagram of a circular area with a dashed outline labeled "B_{out}". Inside this circle, there are evenly spaced orange rings, each containing a smaller black circle. These represent a grid-like pattern, possibly indicating particles or similar objects. In the center of the circle, two blue arrows indicate a direction in and out of the circle, labeled with "I". Beneath these arrows is a double-sided arrow labeled with a distance measurement of "20.0 cm", indicating the separation between two points within the circle.

17.8 Torque on a Current Loop: Motors and Meters

41. (a) By how many percent is the torque of a motor decreased if its permanent magnets lose 5.0% of their strength? (b) How many percent would the current need to be increased to return the torque to original values?
42. (a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field? (b) What is the torque when θ is 10.9° ?
43. Find the current through a loop needed to create a maximum torque of

$9.00\ \mathrm{N \cdot m}$. The loop has 50 square turns that are 15.0 cm on a side and is in a uniform 0.800-T magnetic field.

44. Calculate the magnetic field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of $300\ \mathrm{N \cdot m}$ if the loop is carrying 25.0 A.
45. Since the equation for torque on a current-carrying loop is $\tau = \mathrm{NIAB sin}\theta$, the units of $N \cdot m$ must equal units of $A \cdot m^2 T$. Verify this.
46. (a) At what angle θ is the torque on a current loop 90.0% of maximum? (b) 50.0% of maximum? (c) 10.0% of maximum?
47. A proton has a magnetic field due to its spin on its axis. The field is similar to that created by a circular current loop $0.650 \times 10^{-15} m$ in radius with a current of $1.05 \times 10^4 A$ (no kidding). Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)
48. (a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. The Earth's field here is due north, parallel to the ground, with a strength of $3.00 \times 10^{-5} T$. What are the direction and magnitude of the torque on the loop? (b) Does this device have any practical applications as a motor?
49. Repeat Exercise 17.41, but with the loop lying flat on the ground with its current circulating counterclockwise (when viewed from above) in a location where the Earth's field is north, but at an angle 45.0° below the horizontal and with a strength of $6.00 \times 10^{-5} T$.

17.10 Magnetic Force between Two Parallel Conductors

50. (a) The hot and neutral wires supplying DC power to a light-rail commuter train carry 800 A and are separated by 75.0 cm. What is the magnitude and direction of the force between 50.0 m of these wires? (b) Discuss the practical consequences of this force, if any.
51. The force per meter between the two wires of a jumper cable being used to start a stalled car is 0.225 N/m. (a) What is the current in the wires, given they are separated by 2.00 cm? (b) Is the force attractive or repulsive?
52. A 2.50-m segment of wire supplying current to the motor of a submerged

submarine carries 1000 A and feels a 4.00-N repulsive force from a parallel wire 5.00 cm away. What is the direction and magnitude of the current in the other wire?

53. The wire carrying 400 A to the motor of a commuter train feels an attractive force of $4.00 \times 10^{-3}\text{N/m}$ due to a parallel wire carrying 5.00 A to a headlight. (a) How far apart are the wires? (b) Are the currents in the same direction?
54. An AC appliance cord has its hot and neutral wires separated by 3.00 mm and carries a 5.00-A current. (a) What is the average force per meter between the wires in the cord? (b) What is the maximum force per meter between the wires? (c) Are the forces attractive or repulsive? (d) Do appliance cords need any special design features to compensate for these forces?
55. Figure 17.54 shows a long straight wire near a rectangular current loop. What is the direction and magnitude of the total force on the loop?

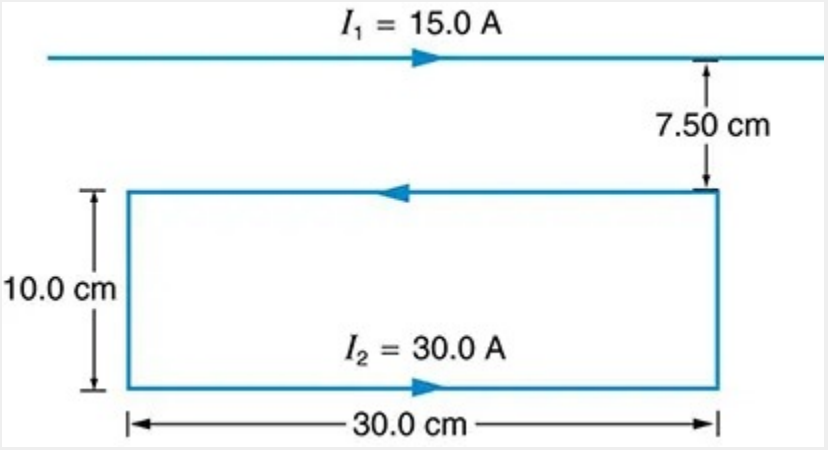


Figure 17.54 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a diagram with two blue lines and annotated measurements. The top line is horizontal, labeled with an arrow to the right, indicating a current I_1 of 15.0 A. Below it, a rectangular loop has its top side parallel to the top line. The rectangle is labeled as carrying a current I_2 of 30.0 A, indicated by arrows pointing clockwise around the loop.

- The distance between the top line and the top side of the rectangle is marked 7.50 cm.
- The height of the rectangle is marked as 10.0 cm.
- The width of the rectangle is marked as 30.0 cm.

56. Find the direction and magnitude of the force that each wire experiences in Figure 17.55(a) by, using vector addition.

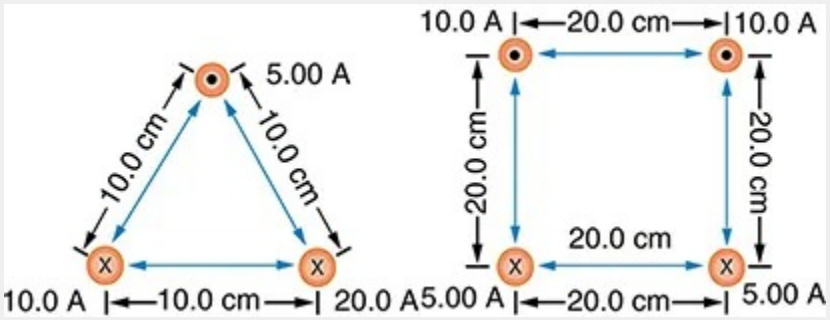


Figure 17.55 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image displays two electrical circuit diagrams featuring closed loops with resistors and ampere values:

1. **Triangle Circuit Diagram:**

- Three sides, each measuring 10.0 cm.
- Corners have small circles with an X marking, representing points of connection.
- Currents flow around the triangle with 10.0 A on two sides and 5.0 A on the third side.
- Arrows indicate the direction of current flow along each side, moving clockwise.

2. **Square Circuit Diagram:**

- Four sides, each measuring 20.0 cm.
- Corners have small circles with an X marking, representing points of connection.
- Currents flow around the square with 10.0 A on two adjacent sides and 5.0 A on the other two sides.

- Arrows indicate the direction of current flow along each side, moving clockwise.

57. Find the direction and magnitude of the force that each wire experiences in Figure 17.55(b), using vector addition.

17.11 More Applications of Magnetism

58. Indicate whether the magnetic field created in each of the three situations shown in Figure 17.56 is into or out of the page on the left and right of the current.

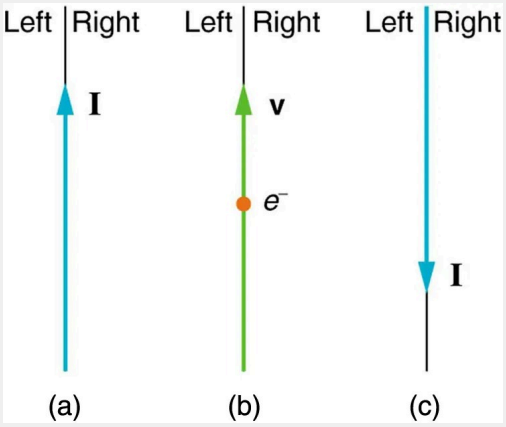


Figure 17.56 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams labeled (a), (b), and (c) from left to right. Each diagram shows a vertical line with arrows and labels. There are labels “Left” and “Right” at the top of each line, and arrows indicating directions.

Diagram (a): A blue arrow pointing upwards along a vertical line with the label “I”. The arrow is positioned below the labels “Left” and “Right”.

Diagram (b): A green arrow pointing upwards along a vertical line with the label “v”. There is a small orange dot positioned midway along the line with the label “e^-” next to it, indicating an electron.

Diagram (c): A blue arrow pointing downwards along a vertical line with the label “I”. Similar to diagram (a), the arrow is positioned below the labels “Left” and “Right”.

59.

What are the directions of the fields in the center of the loop and coils shown in Figure 17.57?

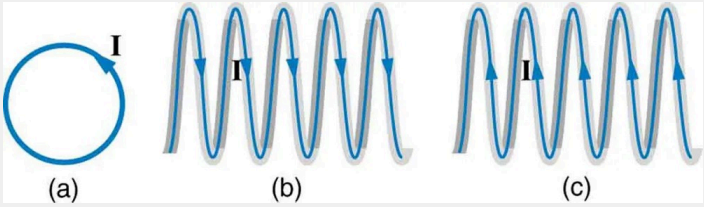


Figure 17.57 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three diagrams labeled (a), (b), and (c):

(a) A circular loop with a current labeled “I” flowing in a clockwise direction. An arrow indicates the direction of the current.

(b) A series of vertical loops forming a coil with current labeled “I” flowing downward, indicated by arrows pointing downward along the loops.

(c) A similar series of vertical loops forming a coil, but the current “I” is flowing upward, indicated by arrows pointing upward along the loops.

60.

What are the directions of the currents in the loop and coils shown in Figure 17.58?

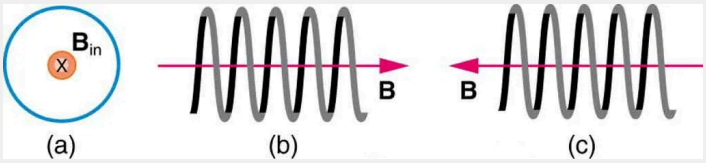


Figure 17.58 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image consists of three sections labeled as (a), (b), and (c):

– Section (a): Shows a circular diagram with a blue outer ring. Inside the circle, there is an orange circle marked with an 'X' and labeled "B_{in}".

– Section (b): Depicts a wavy black and gray line illustrating a spiral, with a magenta horizontal line through the center. An arrow labeled "B" points to the right, indicating the direction of the spiral.

– Section (c): Similar to Section (b), showing a wavy black and gray line illustrating a spiral. A magenta horizontal line runs through the center, with an arrow labeled "B" pointing to the left, indicating the opposite direction to Section (b).

61. To see why an MRI utilizes iron to increase the magnetic field created by a coil, calculate the current needed in a 400-loop-per-meter circular coil 0.660 m in radius to create a 1.20-T field (typical of an MRI instrument) at its center with no iron present. The magnetic field of a proton is approximately like that of a circular current loop $0.650 \times 10^{-15} m$ in radius carrying $1.05 \times 10^{4} A$. What is the field at the center of such a loop?
62. Inside a motor, 30.0 A passes through a 250-turn circular loop that is 10.0 cm in radius. What is the magnetic field strength created at its center?
63. Nonnuclear submarines use batteries for power when submerged. (a) Find the magnetic field 50.0 cm from a straight wire carrying 1200 A from the batteries to the drive mechanism of a submarine. (b) What is the field if the wires to and from the drive mechanism are side by side? (c) Discuss the effects this could have for a compass on the submarine that is not shielded.
64. How strong is the magnetic field inside a solenoid with 10,000 turns per meter that carries 20.0 A?
65. What current is needed in the solenoid described in Exercise 17.64 to produce a magnetic field 10^{4} times the Earth's magnetic field of $5.00 \times 10^{-5} T$?
66. How far from the starter cable of a car, carrying 150 A, must you be to experience a field less than the Earth's $(5.00 \times 10^{-5} T)$? Assume a long straight wire carries the current. (In practice, the body of your car shields the dashboard compass.)
67. Measurements affect the system being measured, such as the current loop in Figure 17.53. (a) Estimate the field the loop creates by calculating the field at the

center of a circular loop 20.0 cm in diameter carrying 5.00 A. (b) What is the smallest field strength this loop can be used to measure, if its field must alter the measured field by less than 0.0100%?

68. Figure 17.59 shows a long straight wire just touching a loop carrying a current I_1. Both lie in the same plane. (a) What direction must the current I_2 in the straight wire have to create a field at the center of the loop in the direction opposite to that created by the loop? (b) What is the ratio of I_1/I_2 that gives zero field strength at the center of the loop? (c) What is the direction of the field directly above the loop under this circumstance?

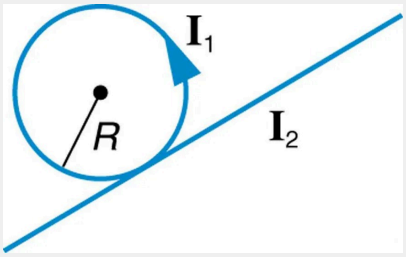


Figure 17.59 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image shows a blue circle representing a loop of wire with an arrow marking a current labeled as I_1. Inside the circle, there is a black dot at the center and a line indicating the radius, labeled as R. The circle is placed on a blue slanted line that also has a current flowing through it, labeled as I_2. The slanted line cuts through the circle, and the current within the circle appears to be directed clockwise.

69. Find the magnitude and direction of the magnetic field at the point equidistant from the wires in Figure 17.55(a), using the rules of vector addition to sum the contributions from each wire.
70. Find the magnitude and direction of the magnetic field at the point equidistant from the wires in Figure 17.55(b), using the rules of vector addition to sum the contributions from each wire.
71. What current is needed in the top wire in Figure 17.55(a) to produce a field of zero at the point equidistant from the wires, if the currents in the bottom two wires

are both 10.0 A into the page?

72. Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300,000 V.
73. **Integrated Concepts:** (a) A pendulum is set up so that its bob (a thin copper disk) swings between the poles of a permanent magnet as shown in Figure 17.60. What is the magnitude and direction of the magnetic force on the bob at the lowest point in its path, if it has a positive $0.250\ \mu\text{C}$ charge and is released from a height of 30.0 cm above its lowest point? The magnetic field strength is 1.50 T. (b) What is the acceleration of the bob at the bottom of its swing if its mass is 30.0 grams and it is hung from a flexible string? Be certain to include a free-body diagram as part of your analysis.

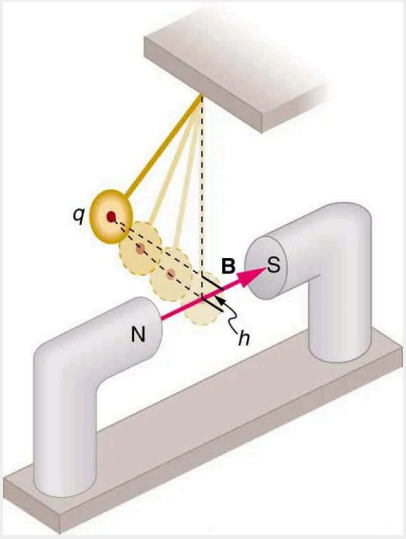


Figure 17.60 Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image illustrates a simplified diagram of a magnetic field experiment. It features the following elements:

– Two large, L-shaped structures, labeled “N” (North) and “S” (South) on the top of each arm, representing the poles of a magnet.
– Between these poles, an arrow marked with “B” and pointed from “N” to “S” symbolizes the direction of the magnetic field.

– A pendulum-like setup is shown above the magnetic field. A bob labeled “q” is suspended from a pivot. Its path is shown as a dotted arc trajectory, indicating swinging motion.
– The label “h” marks the vertical separation between the pendulum’s path and the field.
– The illustration is on a rectangular base platform, indicating stability for the magnetic setup.

The diagram represents the interaction between a magnetic field and a charged object influenced by an external force, such as the swinging pendulum.

74. **Integrated Concepts:** (a) What voltage will accelerate electrons to a speed of $6.00 \times 10^6 \text{m/s}$? (b) Find the radius of curvature of the path of a proton accelerated through this potential in a 0.500-T field and compare this with the radius of curvature of an electron accelerated through the same potential.
75. **Integrated Concepts:** Find the radius of curvature of the path of a 25.0-MeV proton moving perpendicularly to the 1.20-T field of a cyclotron.
76. **Integrated Concepts:** To construct a nonmechanical water meter, a 0.500-T magnetic field is placed across the supply water pipe to a home and the Hall voltage is recorded. (a) Find the flow rate in liters per second through a 3.00-cm-diameter pipe if the Hall voltage is 60.0 mV. (b) What would the Hall voltage be for the same flow rate through a 10.0-cm-diameter pipe with the same field applied?
77. **Integrated Concepts:** (a) Using the values given for an MHD drive in Exercise 17.59, and assuming the force is uniformly applied to the fluid, calculate the pressure created in N/m^2. (b) Is this a significant fraction of an atmosphere?
78. **Integrated Concepts:** (a) Calculate the maximum torque on a 50-turn, 1.50 cm radius circular current loop carrying $50\ \mu\text{A}$ in a 0.500-T field. (b) If this coil is to be used in a galvanometer that reads $50\ \mu\text{A}$ full scale, what force constant spring must be used, if it is attached 1.00 cm from the axis of rotation and is stretched by the 60° arc moved?
79. **Integrated Concepts:** A current balance used to define the ampere is designed so that the current through it is constant, as is the distance between wires. Even so, if the wires change length with temperature, the force between them will

change. What percent change in force per degree will occur if the wires are copper?

80. **Integrated Concepts:** (a) Show that the period of the circular orbit of a charged particle moving perpendicularly to a uniform magnetic field is $T = 2\pi m/(\mathrm{qB})$. (b) What is the frequency f? (c) What is the angular velocity ω? Note that these results are independent of the velocity and radius of the orbit and, hence, of the energy of the particle. (Figure 17.61.)

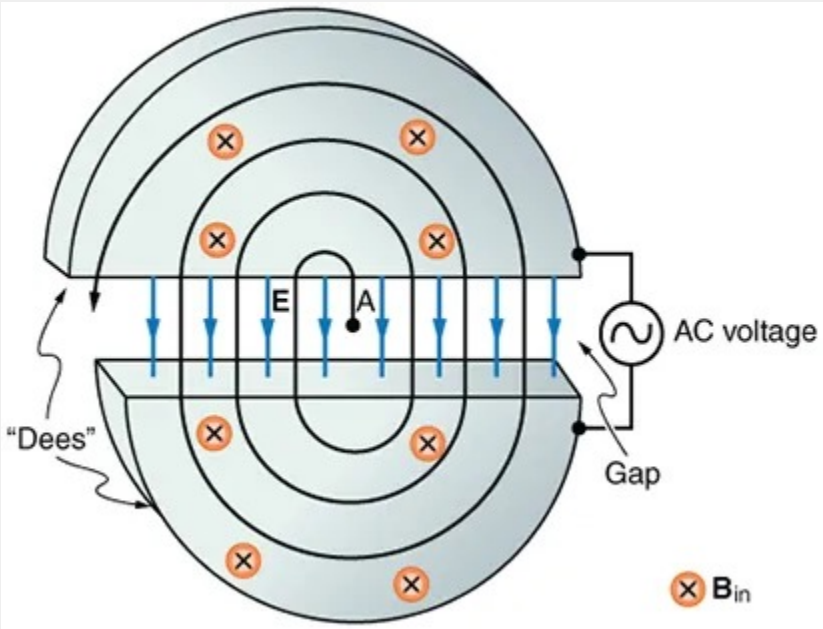


Figure 17.61 Cyclotrons accelerate charged particles orbiting in a magnetic field by placing an AC voltage on the metal Dees, between which the particles move, so that energy is added twice each orbit. The frequency is constant, since it is independent of the particle energy—the radius of the orbit simply increases with energy until the particles approach the edge and are extracted for various experiments and applications. Image from OpenStax College Physics 2e, CC-BY 4.0

Image Description

The image depicts a schematic diagram of a cyclotron, a type of particle accelerator. It consists of two large, semi-circular regions referred to as “Dees” because of their ‘D’

shape, facing each other with a narrow gap in between. The space within the "Dees" is shown as a series of concentric circles, indicating paths for the particles.

Inside the diagram, orange circles with an 'X' represent the magnetic field denoted as B_{in}, which is directed into the page. Blue arrows point vertically between the "Dees," across the central gap, indicating the electric field direction labeled as E.

To the right of the diagram, an AC voltage source is connected across the gap. It is depicted with a sine wave symbol, and the term "AC voltage" is labeled beside it, indicating that alternating current is applied across the gap.

The center point is marked with an 'A', showing the starting location where particles are injected and then spiraled outward as they gain energy.

81. **Integrated Concepts:** A cyclotron accelerates charged particles as shown in Figure 17.61. Using the results of the previous problem, calculate the frequency of the accelerating voltage needed for a proton in a 1.20-T field.
82. **Integrated Concepts:** (a) A 0.140-kg baseball, pitched at 40.0 m/s horizontally and perpendicular to the Earth's horizontal $5.00 \times 10^{-5} T$ field, has a 100-nC charge on it. What distance is it deflected from its path by the magnetic force, after traveling 30.0 m horizontally? (b) Would you suggest this as a secret technique for a pitcher to throw curve balls?
83. **Integrated Concepts:** (a) What is the direction of the force on a wire carrying a current due east in a location where the Earth's field is due north? Both are parallel to the ground. (b) Calculate the force per meter if the wire carries 20.0 A and the field strength is $3.00 \times 10^{-5} T$. (c) What diameter copper wire would have its weight supported by this force? (d) Calculate the resistance per meter and the voltage per meter needed.
84. **Integrated Concepts:** One long straight wire is to be held directly above another by repulsion between their currents. The lower wire carries 100 A and the wire 7.50 cm above it is 10-gauge (2.588 mm diameter) copper wire. (a) What current must flow in the upper wire, neglecting the Earth's field? (b) What is the smallest current if the Earth's $3.00 \times 10^{-5} T$ field is parallel to the ground and is not neglected? (c) Is the supported wire in a stable or unstable equilibrium if displaced vertically? If displaced horizontally?

85. **Unreasonable Results:** (a) Find the charge on a baseball, thrown at 35.0 m/s perpendicular to the Earth's $5.00 \times 10^{-5} T$ field, that experiences a 1.00-N magnetic force. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?
86. **Unreasonable Results:** A charged particle having mass $6.64 \times 10^{-27} \text{kg}$ (that of a helium atom) moving at $8.70 \times 10^{5} \text{m/s}$ perpendicular to a 1.50-T magnetic field travels in a circular path of radius 16.0 mm. (a) What is the charge of the particle? (b) What is unreasonable about this result? (c) Which assumptions are responsible?
87. **Unreasonable Results:** An inventor wants to generate 120-V power by moving a 1.00-m-long wire perpendicular to the Earth's $5.00 \times 10^{-5} T$ field. (a) Find the speed with which the wire must move. (b) What is unreasonable about this result? (c) Which assumption is responsible?
88. **Unreasonable Results:** Frustrated by the small Hall voltage obtained in blood flow measurements, a medical physicist decides to increase the applied magnetic field strength to get a 0.500-V output for blood moving at 30.0 cm/s in a 1.50-cm-diameter vessel. (a) What magnetic field strength is needed? (b) What is unreasonable about this result? (c) Which premise is responsible?
89. **Unreasonable Results:** A surveyor 100 m from a long straight 200-kV DC power line suspects that its magnetic field may equal that of the Earth and affect compass readings. (a) Calculate the current in the wire needed to create a $5.00 \times 10^{-5} T$ field at this distance. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?
90. **Construct Your Own Problem:** Consider a mass separator that applies a magnetic field perpendicular to the velocity of ions and separates the ions based on the radius of curvature of their paths in the field. Construct a problem in which you calculate the magnetic field strength needed to separate two ions that differ in mass, but not charge, and have the same initial velocity. Among the things to consider are the types of ions, the velocities they can be given before entering the magnetic field, and a reasonable value for the radius of curvature of the paths they follow. In addition, calculate the separation distance between the ions at the point where they are detected.

91. **Construct Your Own Problem:** Consider using the torque on a current-carrying coil in a magnetic field to detect relatively small magnetic fields (less than the field of the Earth, for example). Construct a problem in which you calculate the maximum torque on a current-carrying loop in a magnetic field. Among the things to be considered are the size of the coil, the number of loops it has, the current you pass through the coil, and the size of the field you wish to detect. Discuss whether the torque produced is large enough to be effectively measured. Your instructor may also wish for you to consider the effects, if any, of the field produced by the coil on the surroundings that could affect detection of the small field.
92. **Critical Thinking:** A magnetic field of $2.00 \times 10^{-7}\text{T}$ in the positive x-direction exists. Three electrons pass through the origin, one at a time, each with a velocity of $5.00 \times 10^{7}\text{m/s}$. The first electron passes the origin along the x-axis, the second electron along the y-axis, and the third at an angle of 45° with respect to both the y- and z-axes in the yz-plane. (a) What is the force acting on each electron? (b) What is the shape of the path of each electron? (c) When, if ever, will each electron return to the origin?

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APPENDIX A: USEFUL INFORMATION

This appendix is broken into several tables.

- Table A1, Important Constants
- Table A2, Submicroscopic Masses
- Table A3, Solar System Data
- Table A4, Metric Prefixes for Powers of Ten and Their Symbols
- Table A5, The Greek Alphabet
- Table A6, SI units
- Table A7, Selected British Units
- Table A8, Other Units
- Table A9, Useful Formulae

Symbol	Meaning	Best Value
cc">cc"> c	Speed of light in vacuum	2.99792458×108m/s2.99792458×108m/s">2.99792458×108m/s $2.99792458 \times 10^{8}\,\mathrm{m/s}$
GG">GG"> G	Gravitational constant	6.67408(31)×10−11N⋅m2/kg26.67408(31)×10−11N·m2/kg2">6.67408(31)×10−11N·m2/kg2"> $6.67408(31) \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$
NANA">NANA"> N_A	Avogadro's number	6.02214076×10236.02214076×1023">6.02214076×10236.02214 $6.02214076 \times 10^{23}$
kk">kk"> k	Boltzmann's constant	1.380649×10−23J/K1.380649×10−23J/K">1.380649×1 $1.380649 \times 10^{-23}\,\mathrm{J/K}$
RR">RR"> R	Gas constant	8.3144621(75)J/mol⋅K8.3144621(75)J/mol·K">8.3144621(75)J/mo $8.3144621(75)\,\mathrm{J/mol \cdot K}$
σσ">σσ"> σ	Stefan-Boltzmann constant	5.670373(21)×10−8W/m2⋅K5.670373(21)×10−8W/m2⋅K5.670373(21)×10−8W/m2·K"> $5.670373(21) \times 10^{-8}\,\mathrm{W/m^2 \cdot K}$
kk">kk"> k	Coulomb force constant	8.987551788...×109N⋅m2/C28.987551788...×109N·m2/C2" C28.987551788...×109N·m2/C2"> $8.987551788... \times 10^{9}\,\mathrm{N \cdot m^2/C^2}$

Symbol	Meaning	Best Value
qeqe″>qeqe″> q_e	Charge on electron	−1.602176634×10−19C−1.602176634×10−19C″>&# $-1.602176634 \times 10^{-19}\,\mathrm{C}$
ε0ε0″>ε0ε0″> ε_0	Permittivity of free space	8.854187817...×10−12C2/N⋅m28.854187817...×10− N⋅m28.854187817...×10−12C2/N·m2″> $8.854187817... \times 10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2$
μ0μ0″>μ0μ0″> μ_0	Permeability of free space	4π×10−7T⋅m/A4π×10−7T·m/A″>4&#x3C0 $4\pi \times 10^{-7}\,\mathrm{T}\cdot\mathrm{m}/\mathrm{A}$
hh″>hh″> h	Planck's constant	6.62607015×10−34J⋅s6.62607015×10−34J·s″>6.6260 $6.62607015 \times 10^{-34}\,\mathrm{J}\cdot\mathrm{s}$

Table A2 Submicroscopic Masses

Symbol	Meaning	Best Value	Approximate Value
meme"> m_e	Electron mass	9.10938291(40)×10−31kg9.10938291(40)×10−31kg"> $9.10938291(40) \times 10^{-31}\mathrm{kg}$	9.11×10−31kg9.11 $9.11 \times 10^{-31}\mathrm{kg}$
mpmp"> m_p	Proton mass	1.672621777(74)×10−27kg1.672621777(74)×10−27kg"> $1.672621777(74) \times 10^{-27}\mathrm{kg}$	1.6726×10−27kg1 $1.6726 \times 10^{-27}\mathrm{kg}$
mnmn"> m_n	Neutron mass	1.674927351(74)×10−27kg1.674927351(74)×10−27kg"> $1.674927351(74) \times 10^{-27}\mathrm{kg}$	1.6749×10−27kg1 $1.6749 \times 10^{-27}\mathrm{kg}$
uu"> u	Atomic mass unit	1.660538921(73)×10−27kg1.660538921(73)×10−27kg"> $1.660538921(73) \times 10^{-27}\mathrm{kg}$	1.6605×10−27kg1 $1.6605 \times 10^{-27}\mathrm{kg}$

Table A3 Solar System Data

Sun	mass	1.99×1030kg1.99×1030kg"> $1.99 \times 10^{30}\text{kg}$
	average radius	6.96×108m6.96×108m"> $6.96 \times 10^{8}\text{m}$
	Earth-sun distance (average)	1.496×1011m1.496×1011 m"> $1.496 \times 10^{11}\text{m}$
Earth	mass	5.9736×1024kg5.9736×1024kg"> $5.9736 \times 10^{24}\text{kg}$
	average radius	6.376×106m6.376×106m"> $6.376 \times 10^{6}\text{m}$
	orbital period	3.16×107s3.16×107 s "> $3.16 \times 10^{7}\text{s}$
Moon	mass	7.35×1022kg7.35×1022kg"> $7.35 \times 10^{22}\text{kg}$

	average radius	1.74×106m1.74×106m"> $1.74 \times 10^6 \text{m}$
	orbital period (average)	2.36×106s2.36×106 s"> $2.36 \times 10^6 \text{s}$
	Earth-moon distance (average)	3.84×108m3.84×108 m"> $3.84 \times 10^8 \text{m}$

Table A4 Metric Prefixes for Powers of Ten and Their Symbols

Prefix	Symbol	Value	Prefix	Symbol	Value
tera	T	10121012">10121012"> 10^{12}	deci	d	10−110−1">10−110−1"> 10^{-1}
giga	G	109109">109109"> 10^{9}	centi	c	10−210−2">10−210−2"> 10^{-2}
mega	M	106106">106106"> 10^{6}	milli	m	10−310−3">10−310−3"> 10^{-3}
kilo	k	103103">103103"> 10^{3}	micro	μμ"> μ	10−610−6">10−610−6"> 10^{-6}
hecto	h	102102">102102"> 10^{2}	nano	n	10−910−9">10−910−9"> 10^{-9}
deka	da	101101">101101"> 10^{1}	pico	p	10−1210−12">10−1210−12"> 10^{-12}
—	—	100(=1)100(=1)">100(=1)100(=1)"> $10^{0}(=1)$	femto	f	10−1510−15">10−1510−15"> 10^{-15}

Table A5 T

Alpha	ΑA">ΑA"> Α	αα">αα"> α	Eta	ΗH">ΗH"> Η	ηη">ηη"> η
Beta	ΒB">ΒB"> Β	ββ">ββ"> β	Theta	ΘΘ">ΘΘ"> Θ	θθ">θθ"> θ
Gamma	ΓΓ">ΓΓ"> Γ	γγ">γγ"> γ	Iota	ΙI">ΙI"> Ι	ιι">ιι"> ι
Delta	ΔΔ">ΔΔ"> Δ	δδ">δδ"> δ	Kappa	ΚK">ΚK"> Κ	κκ">κκ"> κ
Epsilon	ΕE">ΕE"> Ε	εε">εε"> ε	Lambda	ΛΛ">ΛΛ"> Λ	λλ">λλ"> λ
Zeta	ΖZ">ΖZ"> Ζ	ζζ">ζζ"> ζ	Mu	ΜM">ΜM"> Μ	μμ">μμ" μ

Table A6 SI Units

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	N=kg⋅m/s2N=kg·m/s2"> $\mathrm{N} = \mathrm{kg} \cdot \mathrm{m/s^2}$	newton
	Energy	J=kg⋅m2/s2J=kg·m2/s2"> $\mathrm{J} = \mathrm{kg} \cdot \mathrm{m^2/s^2}$	joule
	Power	W=J/sW=J/s"> $\mathrm{W} = \mathrm{J/s}$	watt
	Pressure	Pa=N/m2Pa=N/m2"> $\mathrm{Pa} = \mathrm{N/m^2}$	pascal
	Frequency	Hz=1/sHz=1/s"> $\mathrm{Hz} = 1/\mathrm{s}$	hertz
	Electronic potential	V=J/CV=J/C"> $\mathrm{V} = \mathrm{J/C}$	volt

	Entity	Abbreviation	Name
	Capacitance	F=C/VF=C/V"> $\mathrm{F} = \mathrm{C}/\mathrm{V}$	farad
	Charge	C=s⋅AC=s·A"> $\mathrm{C} = \mathrm{s} \cdot \mathrm{A}$	coulomb
	Resistance	Ω=V/AΩ=V/A"> $\Omega = \mathrm{V}/\mathrm{A}$	ohm
	Magnetic field	T=N/A⋅mT=N/A·m"> $\mathrm{T} = \mathrm{N}/(\mathrm{A} \cdot \mathrm{m})$	tesla
	Nuclear decay rate	Bq=1/sBq=1/s"> $\mathrm{Bq} = 1/\mathrm{s}$	becquerel

Table A7 Selected British Units

Length	1inch(in.)=2.54cm(exactly)1inch (in.)=2.54 cm (exactly)">1inch(in.)=2.54cm(exactly)1inch (in.)=2.54 cm (exactly)"> $1\text{ inch (in.)} = 2.54\text{ cm (exactly)}$
	1foot(ft)=0.3048m1 foot (ft)=0.3048 m">1foot(ft)=0.3048m1 foot (ft)=0.3048 m"> $1\text{ foot (ft)} = 0.3048\text{ m}$
	1mile(mi)=1.609km1 mile (mi)=1.609km">1mile(mi)=1.609km1 mile (mi)=1.609km"> $1\text{ mile (mi)} = 1.609\text{ km}$
Force	1pound(lb)=4.448N1 pound (lb)=4.448 N">1pound(lb)=4.448N1 pound (lb)=4.448 N"> $1\text{ pound (lb)} = 4.448\text{ N}$
Energy	1British thermal unit(Btu)=1.055×103J1 British thermal unit (Btu)=1.055×103 J">1British thermal unit(Btu)=1.055×103J1 British thermal unit (Btu)=1.055×103 J"> $1\text{ British thermal unit (Btu)} = 1.055 \times 10^3\text{ J}$
Power	1horsepower(hp)=746W1 horsepower (hp)=746 W">1horsepower(hp)=746W1 horsepower (hp)=746 W"> $1\text{ horsepower (hp)} = 746\text{ W}$
Pressure	1lb/in2=6.895×103Pa1lb/in2=6.895×103 Pa">1lb/in2=6.895×103Pa1lb/in2=6.895×103 Pa"> $1\text{ lb/in}^2 = 6.895 \times 10^3\text{ Pa}$

Table A8 Other Units

Length	1light year(ly)=9.46×1015m1 light year(ly)=9.46×1015 m">1light year(ly)=9.46×1015m1 light year(ly)=9.46×1015 m"> $1\ \text{light year (ly)} = 9.46 \times 10^{15}\ \text{m}$
	1astronomical unit(au)=1.50×1011m1 astronomical unit(au)=1.50×1011 m">1astronomical unit(au)=1.50×1011m1 astronomical unit(au)=1.50×1011 m"> $1\ \text{astronomical unit (au)} = 1.50 \times 10^{11}\ \text{m}$
	1nautical mile=1.852km1 nautical mile=1.852 km">1nautical mile=1.852km1 nautical mile=1.852 km"> $1\ \text{nautical mile} = 1.852\ \text{km}$
	1angstrom(Å)=10−10m1 angstrom(Å)=10−10 m">1angstrom(Å)=10−10m1 angstrom(Å)=10−10 m"> $1\ \text{angstrom(Å)} = 10^{-10}\ \text{m}$
Area	1acre(ac)=4.05×103m21 acre(ac)=4.05×103 m2">1acre(ac)=4.05×103m21 acre(ac)=4.05×103 m2"> $1\ \text{acre (ac)} = 4.05 \times 10^{3}\ \text{m}^2$
	1square foot(ft2)=9.29×10−2m21square foot(ft2)=9.29×10−2 m2">1square foot(ft2)=9.29×10−2m21square foot(ft2)=9.29×10−2 m2"> $1\ \text{square foot (ft}^2\text{)} = 9.29 \times 10^{-2}\ \text{m}^2$
	1barn(b)=10−28m21 barn(b)=10−28 m2">1barn(b)=10−28m21 barn(b)=10−28 m2"> $1\ \text{barn}\ (b) = 10^{-28}\ \text{m}^2$

Volume	1liter(L)=10−3m31 liter(L)=10−3 m3″>1liter(L)=10−3m31 liter(L)=10−3 m3″> $1 \text{ liter } (L) = 10^{-3} \text{ m}^3$
	1U.S. gallon(gal)=3.785×10−3m31 U.S. gallon(gal)=3.785×10−3 m3″>1U.S. gallon(gal)=3.785×10−3m31 U.S. gallon(gal)=3.785×10−3 m3″> $1 \text{ U.S. gallon (gal)} = 3.785 \times 10^{-3} \text{ m}^3$
Mass	1solar mass=1.99×1030kg1 solar mass=1.99×1030 kg″>1solar mass=1.99×1030kg1 solar mass=1.99×1030 kg″> $1 \text{ solar mass } = 1.99 \times 10^{30} \text{ kg}$
	1metric ton=103kg1 metric ton=103 kg″>1metric ton=103kg1 metric ton=103 kg″> $1 \text{ metric ton} = 10^{3} \text{ kg}$
	1atomic mass unit(u)=1.6605×10−27kg1 atomic mass unit(u)=1.6605×10−27 kg″>1atomic mass unit(u)=1.6605×10−27kg1 atomic mass unit(u)=1.6605×10−27 kg″> $1 \text{ atomic mass unit } (u) = 1.6605 \times 10^{-27} \text{ kg}$
Time	1year(y)=3.16×107s1 year(y)=3.16×107 s″>1year(y)=3.16×107s1 year(y)=3.16×107 s″> $1 \text{ year } (y) = 3.16 \times 10^{7} \text{ s}$
	1day(d)=86,400s1 day(d)=86,400 s″>1day(d)=86,400s1 day(d)=86,400 s″> $1 \text{ day } (d) = 86{,}400 \text{ s}$

Speed	1mile per hour(mph)=1.609km/h1 mile per hour(mph)=1.609km/h">1mile per hour(mph)=1.609km/h1 mile per hour(mph)=1.609km/h"> $1\ \text{mile per hour (mph)} = 1.609\ \text{km/h}$
	1nautical mile per hour(naut)=1.852km/h1 nautical mile per hour(naut)=1.852km/h">1nautical mile per hour(naut)=1.852km/h1 nautical mile per hour(naut)=1.852km/h"> $1\ \text{nautical mile per hour (naut)} = 1.852\ \text{km/h}$
Angle	1degree(°)=1.745×10−2rad1 degree(°)=1.745×10−2 rad">1degree(°)=1.745×10−2rad1 degree(°)=1.745×10−2 rad"> $1\ \text{degree}\ (^\circ) = 1.745 \times 10^{-2}\ \text{rad}$
	1minute of arc(′)=1/60degree1 minute of arc(′)=1/60 degree">1minute of arc(′)=1/60degree1 minute of arc(′)=1/60 degree"> $1\ \text{minute of arc}\ (') = 1/60\ \text{degree}$
	1second of arc(″)=1/60minute of arc1 second of arc(″)=1/60 minute of arc">1second of arc(″)=1/60minute of arc1 second of arc(″)=1/60 minute of arc"> $1\ \text{second of arc}\ (") = 1/60\ \text{minute of arc}$
	1grad=1.571×10−2rad1 grad=1.571×10−2 rad">1grad=1.571×10−2rad1 grad=1.571×10−2 rad"> $1\ \text{grad} = 1.571 \times 10^{-2}\ \text{rad}$
Energy	1kiloton TNT(kT)=4.2×1012J1 kiloton TNT(kT)=4.2×1012 J">1kiloton TNT(kT)=4.2×1012J1 kiloton TNT(kT)=4.2×1012 J"> $1\ \text{kiloton TNT (kT)} = 4.2 \times 10^{12}\ \text{J}$

	1kilowatt hour(kW⋅h)=3.60×106J1 kilowatt hour(kW·h)=3.60×106 J">1kilowatt hour(kW⋅h)=3.60×106J1 kilowatt hour(kW·h)=3.60×106 J"> $1\text{ kilowatt hour (kW}\cdot h) = 3.60 \times 10^6\text{ J}$
	1food calorie(kcal)=4186J1 food calorie(kcal)=4186 J">1food calorie(kcal)=4186J1 food calorie(kcal)=4186 J"> $1\text{ food calorie (kcal)} = 4186\text{ J}$
	1calorie(cal)=4.186J1 calorie(cal)=4.186 J">1calorie(cal)=4.186J1 calorie(cal)=4.186 J"> $1\text{ calorie (cal)} = 4.186\text{ J}$
	1electron volt(eV)=1.60×10−19J1 electron volt(eV)=1.60×10−19 J">1electron volt(eV)=1.60×10−19J1 electron volt(eV)=1.60×10−19 J"> $1\text{ electron volt (eV)} = 1.60 \times 10^{-19}\text{ J}$
Pressure	1atmosphere(atm)=1.013×105Pa1 atmosphere(atm)=1.013×105 Pa">1atmosphere(atm)=1.013×105Pa1 atmosphere(atm)=1.013×105 Pa"> $1\text{ atmosphere (atm)} = 1.013 \times 10^5\text{ Pa}$
	1millimeter of mercury(mmHg)=133.3Pa1 millimeter of mercury(mmHg)=133.3 Pa">1millimeter of mercury(mmHg)=133.3Pa1 millimeter of mercury(mmHg)=133.3 Pa"> $1\text{ millimeter of mercury (mm Hg)} = 133.3\text{ Pa}$
	1torricelli(torr)=1mmHg=133.3Pa1 torricelli(torr)=1mmHg =133.3 Pa">1torricelli(torr)=1mmHg=133.3Pa1 torricelli(torr)=1mmHg =133.3 Pa"> $1\text{ torricelli (torr)} = 1\text{ mm Hg} = 133.3\text{ Pa}$

Nuclear decay rate	1curie(Ci)=3.70×1010Bq1 curie(Ci)=3.70×1010 Bq">1curie(Ci)=3.70×1010Bq1 curie(Ci)=3.70×1010 Bq"> $1\ \text{curie (Ci)} = 3.70 \times 10^{10}\ \text{Bq}$

Table A9 Useful Formulae

Circumference of a circle with radius rr"> r or diameter dd"> d	C=2πr=πdC=2πr=πd">C=2πr=πdC=2πr=πd"> $C = 2\pi\,\mathrm{r} = \pi\,\mathrm{d}$
Area of a circle with radius rr"> r or diameter dd"> d	A=πr2=πd2/4A=πr2=πd2/4">A=πr2=πd2/4A=πr2=πd2/4"> $A = \pi\,\mathrm{r}^2 = \pi\,\mathrm{d}^2/4$
Area of a sphere with radius rr"> r	A=4πr2A=4πr2">A=4πr2A=4πr2"> $A = 4\pi\,\mathrm{r}^2$
Volume of a sphere with radius rr"> r	V=4/3πr3V=4/3πr3">V=4/3πr3V=4/3πr3"> $V = (4/3)\left(\pi\,\mathrm{r}^3\right)$

APPENDIX B: GLOSSARY OF KEY SYMBOLS AND NOTATION

In this glossary, key symbols and notation are briefly defined.

Table C1

Symbol	Definition
any symbol¯any symbol¯"> $\overline{\text{any symbol}}$	average (indicated by a bar over a symbol—e.g., v¯v¯"> $\overline{v}$ is average velocity)
°C °C"> °C	Celsius degree
°F °F"> °F	Fahrenheit degree
////"> //	parallel
⊥⊥"> ⊥	perpendicular
∝∝"> ∝	proportional to
± ±"> ±	plus or minus

Symbol	Definition
00″> 0	zero as a subscript denotes an initial value
αα″> α	alpha rays
αα″> α	angular acceleration
αα″> α	temperature coefficient(s) of resistivity
ββ″> β	beta rays
ββ″> β	sound level
ββ″> β	volume coefficient of expansion
β−β−″> β^{-}	electron emitted in nuclear beta decay

Symbol	Definition
β+β+"> β^+	positron decay
γγ"> γ	gamma rays
γγ"> γ	surface tension
γ=1/1−v2/c2γ=1/1−v2/c2"> $\gamma = 1/\sqrt{1 - v^2/c^2}$	a constant used in relativity
ΔΔ"> Δ	change in whatever quantity follows
δδ"> δ	uncertainty in whatever quantity follows
ΔEΔE"> $\Delta \mathrm{E}$	change in energy between the initial and final orbits of an electron in an atom

Symbol	Definition
ΔEΔE"> ΔE	uncertainty in energy
ΔmΔm"> Δm	difference in mass between initial and final products
ΔNΔN"> ΔN	number of decays that occur
ΔpΔp"> Δp	change in momentum
ΔpΔp"> Δp	uncertainty in momentum
ΔPEgΔPEg"> ΔPE_g	change in gravitational potential energy
ΔθΔθ"> $\Delta \theta$	rotation angle
ΔsΔs"> Δs	distance traveled along a circular path

Symbol	Definition
ΔtΔt″> Δt	uncertainty in time
Δt0Δt0″> Δt_0	proper time as measured by an observer at rest relative to the process
ΔVΔV″> ΔV	potential difference
ΔxΔx″> Δx	uncertainty in position
ε0ε0″> ε_0	permittivity of free space
ηη″> η	viscosity
θθ″> θ	angle between the force vector and the displacement vector

Symbol	Definition
θθ"> θ	angle between two lines
θθ"> θ	contact angle
θθ"> θ	direction of the resultant
θbθb"> θ_b	Brewster's angle
θcθc"> θ_c	critical angle
κκ"> κ	dielectric constant
λλ"> λ	decay constant of a nuclide
λλ"> λ	wavelength

Symbol	Definition
λnλn"> λ_n	wavelength in a medium
μ0μ0"> μ_0	permeability of free space
μkμk"> μ_k	coefficient of kinetic friction
μsμs"> μ_s	coefficient of static friction
veve"> v_e	electron neutrino
π+π+"> π^+	positive pion
π−π−"> π^-	negative pion
π0π0"> π^0	neutral pion

Symbol	Definition
ρρ"> ρ	density
ρcρc"> ρ_c	critical density, the density needed to just halt universal expansion
ρflρfl"> ρ_{fl}	fluid density
ρ¯objρ¯obj"> $\overline{\rho}_{obj}$	average density of an object
ρ/ρw ρ/ρw"> ρ/ρ_w	specific gravity
ττ"> τ	characteristic time constant for a resistance and inductance (RL)(RL)"> (RL) or resistance and capacitance (RC)(RC)"> (RC) circuit
ττ"> τ	characteristic time for a resistor and capacitor (RC)(RC)"> (RC) circuit

Symbol	Definition
ττ"> τ	torque
Υϒ"> Υ	upsilon meson
ΦΦ"> Φ	magnetic flux
ϕϕ"> ϕ	phase angle
ΩΩ"> Ω	ohm (unit)
ωω"> ω	angular velocity
AA"> A	ampere (current unit)
AA"> A	area

Symbol	Definition
AA″> A	cross-sectional area
AA″> A	total number of nucleons
aa″> a	acceleration
aBaB″> a_B	Bohr radius
acac″> a_c	centripetal acceleration
atat″> a_t	tangential acceleration
ACAC″> AC	alternating current
AMAM″> AM	amplitude modulation

Symbol	Definition
atmatm″> atm	atmosphere
BB″> B	baryon number
BB″> B	blue quark color
B¯B‾″> $\overline{B}$	antiblue (yellow) antiquark color
bb″> b	quark flavor bottom or beauty
BB″> B	bulk modulus
BB″> B	magnetic field strength
BintBint″> $\mathrm{B}_{\mathrm{int}}$	electron’s intrinsic magnetic field

Symbol	Definition
BorbBorb″> B_{orb}	orbital magnetic field
BEBE″> BE	binding energy of a nucleus—it is the energy required to completely disassemble it into separate protons and neutrons
BE/A BE/A″> BE/A	binding energy per nucleon
BqBq″> Bq	becquerel—one decay per second
CC″> C	capacitance (amount of charge stored per volt)
CC″> C	coulomb (a fundamental SI unit of charge)
CpCp″> C_p	total capacitance in parallel

Symbol	Definition
CsCs″> C_{s}	total capacitance in series
CGCG″> CG	center of gravity
CMCM″> CM	center of mass
cc″> c	quark flavor charm
cc″> c	specific heat
cc″> c	speed of light
CalCal″> Cal	kilocalorie
calcal″> cal	calorie

Symbol	Definition
COPhpCOPhp″> $\mathrm{COP}_{\mathrm{hp}}$	heat pump’s coefficient of performance
COPrefCOPref″> $\mathrm{COP}_{\mathrm{ref}}$	coefficient of performance for refrigerators and air conditioners
cosθ cosθ″> $\cos\theta$	cosine
cotθ cotθ″> $\cot\theta$	cotangent
cscθ cscθ″> $\csc\theta$	cosecant
DD″> D	diffusion constant
dd″> d	displacement
dd″> d	quark flavor down

Symbol	Definition
dBdB"> dB	decibel
didi"> d_{i}	distance of an image from the center of a lens
dodo"> d_{o}	distance of an object from the center of a lens
DCDC"> DC	direct current
EE"> E	electric field strength
εε"> ε	emf (voltage) or Hall electromotive force
emfemf"> emf	electromotive force
EE"> E	energy of a single photon

Symbol	Definition
EE″> E	nuclear reaction energy
EE″> E	relativistic total energy
EE″> E	total energy
E0E0″> E_0	ground state energy for hydrogen
E0E0″> E_0	rest energy
ECEC″> EC	electron capture
EcapEcap″> E_{cap}	energy stored in a capacitor
EffEff″> Eff	efficiency—the useful work output divided by the energy input

Symbol	Definition
EffCEffC″> $\mathrm{Eff}_{\mathrm{C}}$	Carnot efficiency
EinEin″> E_{in}	energy consumed (food digested in humans)
EindEind″> E_{ind}	energy stored in an inductor
EoutEout″> E_{out}	energy output
ee″> e	emissivity of an object
e+e+″> e^{+}	antielectron or positron
eVeV″> eV	electron volt
FF″> F	farad (unit of capacitance, a coulomb per volt)

Symbol	Definition
FF″> $\mathbf{F}$	focal point of a lens
FF″> $\mathbf{F}$	force
FF″> F	magnitude of a force
FF″> F	restoring force
FBFB″> F_{B}	buoyant force
FcFc″> F_{c}	centripetal force
FiFi″> F_{i}	force input
FnetFnet″> $\mathbf{F}_{\mathrm{net}}$	net force

Symbol	Definition
FoFo″> F_o	force output
FMFM″> FM	frequency modulation
ff″> f	focal length
ff′> f	frequency
f0f0″> f_0	resonant frequency of a resistance, inductance, and capacitance (RLC)(RLC)″> (RLC) series circuit
f0f0″> f_0	threshold frequency for a particular material (photoelectric effect)
f1f1″> f_1	fundamental

Symbol	Definition
f2f2″> f_2	first overtone
f3f3″> f_3	second overtone
fBfB″> f_B	beat frequency
fkfk″> f_k	magnitude of kinetic friction
fsfs″> f_s	magnitude of static friction
GG″> G	gravitational constant
GG″> G	green quark color

Symbol	Definition
G¯G¯"> $\overline{G}$	antigreen (magenta) antiquark color
gg"> g	acceleration due to gravity
gg"> g	gluons (carrier particles for strong nuclear force)
hh"> h	change in vertical position
hh"> h	height above some reference point
hh"> h	maximum height of a projectile
hh"> h	Planck's constant
hfhf"> hf	photon energy

Symbol	Definition
hihi″> h_{i}	height of the image
hoho″> h_{o}	height of the object
II″> I	electric current
II″> I	intensity
II″> I	intensity of a transmitted wave
II″> I	moment of inertia (also called rotational inertia)
I0I0″> I_0	intensity of a polarized wave before passing through a filter
IaveIave″> I_{ave}	average intensity for a continuous sinusoidal electromagnetic wave

Symbol	Definition
IrmsIrms″> I_{rms}	average current
JJ″> J	joule
J/Ψ J/Ψ″> J/Ψ	Joules/psi meson
KK″> K	kelvin
kk″> k	Boltzmann constant
kk″> k	force constant of a spring
KαKα″> K_{α}	x rays created when an electron falls into an n=1n=1″> $n = 1$ shell vacancy from the n=3n=3″> $n = 3$ shell
KβKβ″> K_{β}	x rays created when an electron falls into an n=2n=2″> $n = 2$ shell vacancy from the n=3n=3″> $n = 3$ shell

Symbol	Definition
kcalkcal"> kcal	kilocalorie
KEKE"> KE	translational kinetic energy
KE+PE KE + PE"> $\mathrm{KE} + \mathrm{PE}$	mechanical energy
KEeKEe"> KE_e	kinetic energy of an ejected electron
KErelKErel"> $\mathrm{KE}_{\mathrm{rel}}$	relativistic kinetic energy
KErotKErot"> $\mathrm{KE}_{\mathrm{rot}}$	rotational kinetic energy
KE¯KE¯"> $\overline{\mathrm{KE}}$	thermal energy
kgkg"> kg	kilogram (a fundamental SI unit of mass)

Symbol	Definition
LL"> L	angular momentum
LL"> L	liter
LL"> L	magnitude of angular momentum
LL"> L	self-inductance
ℓℓ"> ℓ	angular momentum quantum number
LαLα"> L_{α}	x rays created when an electron falls into an n=2n=2"> $n = 2$ shell
LeLe"> L_{e}	electron total family number
LμLμ"> L_{μ}	muon family total number

Symbol	Definition
LτLτ"> L_τ	tau family total number
LfLf"> L_f	heat of fusion
LfandLvLf andLv"> L_f and L_v	latent heat coefficients
LorbLorb"> L_orb	orbital angular momentum
LsLs"> L_s	heat of sublimation
LvLv"> L_v	heat of vaporization
LzLz"> L_z	z – component of the angular momentum
MM"> M	angular magnification

Symbol	Definition
MM″> M	mutual inductance
mm″> m	indicates metastable state
mm″> m	magnification
mm″> m	mass
mm″> m	mass of an object as measured by a person at rest relative to the object
mm″> m	meter (a fundamental SI unit of length)
mm″> m	order of interference
mm″> m	overall magnification (product of the individual magnifications)

Symbol	Definition
mAXmA X″> $m\left({}^{A}\mathrm{X}\right)$	atomic mass of a nuclide
MAMA″> MA	mechanical advantage
meme″> m_{e}	magnification of the eyepiece
meme″> m_e	mass of the electron
mℓmℓ″> m_ℓ	angular momentum projection quantum number
mnmn″> m_n	mass of a neutron
momo″> m_{o}	magnification of the objective lens

Symbol	Definition
molmol″> mol	mole
mpmp″> m_p	mass of a proton
msms″> m_s	spin projection quantum number
NN″> N	magnitude of the normal force
NN″> N	newton
NN″> $\mathbf{N}$	normal force
NN″> N	number of neutrons
nn″> n	index of refraction

Symbol	Definition
nn"> n	number of free charges per unit volume
NANA"> N_{A}	Avogadro's number
NrNr"> N_{r}	Reynolds number
N⋅m N·m"> $\mathrm{N \cdot m}$	newton-meter (work-energy unit)
N⋅m N·m"> $\mathrm{N \cdot m}$	newtons times meters (SI unit of torque)
OEOE"> OE	other energy
PP"> P	power
PP"> P	power of a lens

Symbol	Definition
PP"> P	pressure
pp"> $\mathbf{p}$	momentum
pp"> p	momentum magnitude
pp"> p	relativistic momentum
ptotptot"> $\mathbf{p}_{\text{tot}}$	total momentum
ptot'ptot'"> $\mathbf{p}_{\text{tot}}^{\prime}$	total momentum some time later
PabsPabs"> P_{abs}	absolute pressure
PatmPatm"> P_{atm}	atmospheric pressure

Symbol	Definition
PatmPatm″> P_{atm}	standard atmospheric pressure
PEPE″> PE	potential energy
PElPEl″> PE_{el}	elastic potential energy
PElecPElec″> PE_{elec}	electric potential energy
PEsPEs″> PE_{s}	potential energy of a spring
PgPg″> P_{g}	gauge pressure
PinPin″> P_{in}	power consumption or input

Symbol	Definition
PoutPout″> P_{out}	useful power output going into useful work or a desired, form of energy
QQ″> Q	latent heat
QQ″> Q	net heat transferred into a system
QQ″> Q	flow rate—volume per unit time flowing past a point
+Q +Q″> $+Q$	positive charge
−Q −Q″> $-Q$	negative charge
qq″> q	electron charge

Symbol	Definition
qpqp″> q_p	charge of a proton
qq″> q	test charge
QFQF″> QF	quality factor
RR″> R	activity, the rate of decay
RR″> R	radius of curvature of a spherical mirror
RR″> R	red quark color
R¯R¯″> $\overline{R}$	antired (cyan) quark color
RR″> R	resistance

Symbol	Definition
RR"> R	resultant or total displacement
RR"> R	Rydberg constant
RR"> R	universal gas constant
rr"> r	distance from pivot point to the point where a force is applied
rr"> r	internal resistance
r⊥r⊥"> $r_{\perp}$	perpendicular lever arm
rr"> r	radius of a nucleus
rr"> r	radius of curvature

Symbol	Definition
rr"> r	resistivity
r or radr or rad"> r or rad	radiation dose unit
remrem"> rem	roentgen equivalent man
radrad"> rad	radian
RBERBE"> RBE	relative biological effectiveness
RCRC"> RC	resistor and capacitor circuit
rmsrms"> rms	root mean square
rnrn"> r_n	radius of the nth H-atom orbit

Symbol	Definition
RpRp″> R_p	total resistance of a parallel connection
RsRs″> R_s	total resistance of a series connection
RsRs″> R_s	Schwarzschild radius
SS″> S	entropy
SS″> $\mathbf{S}$	intrinsic spin (intrinsic angular momentum)
SS″> S	magnitude of the intrinsic (internal) spin angular momentum
SS″> S	shear modulus
SS″> S	strangeness quantum number

Symbol	Definition
ss″> s	quark flavor strange
ss″> s	second (fundamental SI unit of time)
ss″> s	spin quantum number
ss″> $\mathbf{s}$	total displacement
secθ secθ″> $\sec\theta$	secant
sinθ sinθ″> $\sin\theta$	sine
szsz″> s_z	z-component of spin angular momentum
TT″> T	period—time to complete one oscillation

Symbol	Definition
TT"> T	temperature
TcTc"> T_c	critical temperature—temperature below which a material becomes a superconductor
TT"> T	tension
TT"> T	tesla (magnetic field strength B)
tt"> t	quark flavor top or truth
tt"> t	time
t1/2t1/2"> $t_{1/2}$	half-life—the time in which half of the original nuclei decay
tanθ tanθ"> $\tan\theta$	tangent

Symbol	Definition
UU″> U	internal energy
uu″> u	quark flavor up
uu″> u	unified atomic mass unit
uu″> **u**	velocity of an object relative to an observer
u′u′″> **u**‘	velocity relative to another observer
VV″> V	electric potential
VV″> V	terminal voltage
VV″> V	volt (unit)

Symbol	Definition
VV"> V	volume
vv"> $\mathbf{v}$	relative velocity between two observers
vv"> v	speed of light in a material
vv"> $\mathbf{v}$	velocity
v¯v¯"> $\overline{\mathbf{v}}$	average fluid velocity
VB−VA VB−VA"> $V_\mathrm{B} - V_\mathrm{A}$	change in potential
vdvd"> $\mathbf{v}_\mathrm{d}$	drift velocity
VpVp"> V_p	transformer input voltage

Symbol	Definition
VrmsVrms"> V_{rms}	rms voltage
VsVs"> V_{s}	transformer output voltage
vtotvtot"> $\mathbf{v}_{\text{tot}}$	total velocity
vwvw"> v_{w}	propagation speed of sound or other wave
vwvw"> $\mathbf{v}_{\text{w}}$	wave velocity
WW"> W	work
WW"> W	net work done by a system
WW"> W	watt

Symbol	Definition
ww"> w	weight
wflwfl"> w_{fl}	weight of the fluid displaced by an object
WcWc"> W_{c}	total work done by all conservative forces
WncWnc"> W_{nc}	total work done by all nonconservative forces
WoutWout"> W_{out}	useful work output
XX"> X	amplitude
XX"> X	symbol for an element
AZXNAZXN"> $^{Z}_{A}X_{N}$	notation for a particular nuclide

Symbol	Definition
xx″> x	deformation or displacement from equilibrium
xx″> x	displacement of a spring from its undeformed position
xx″> x	horizontal axis
XCXC″> X_{C}	capacitive reactance
XLXL″> X_{L}	inductive reactance
xrmsxrms″> x_{rms}	root mean square diffusion distance
yy″> y	vertical axis
YY″> Y	elastic modulus or Young's modulus

Symbol	Definition
ZZ"> Z	atomic number (number of protons in a nucleus)
ZZ"> Z	impedance

ANSWER KEY

Chapter 1

Problems & Exercises

1.

(a) 27.8 m/s

(b) 62.1 mph

3.

$$\frac{1.0\ \text{m}}{s} = \frac{1.0\ \text{m}}{s} \times \frac{3600\ \text{s}}{1\ \text{hr}} \times \frac{1\ km}{1000\ \text{m}}$$

= 3.6 km/h.

5.

length: 377 ft; 4.53×10^3 in. width: 280 ft; 3.3×10^3 in.

7.

8.847 km

9.

(a) 1.3×10^{-9} m

(b) 40 km/My

11.

2 kg

13.

(a) 85 to 95 km/h

(b) 53 to 59 mi/h

15.

(a) 7.6×10^7 beats

(b) 7.57×10^7 beats

(c) 7.57×10^7 beats

17.

(a) 3

(b) 3

(c) 3

19.

(a) 2.2%

(b) 59 to 61 km/h

21.

80 ± 3 beats/min

23.

2.8 h

25.

11.4 ± 0.6 cm^3

27.

12.06 ± 0.04 m^2

29.

Sample answer: 2×10^9
heartbeats

31.

Sample answer: 2×10^{31}
if an average human lifetime is taken to be about 70 years.

33.

Sample answer: 50 atoms

35.

Sample answers:

(a) 10^{12} cells/hummingbird

(b) 10^{16} cells/human

Chapter 2

Problems & Exercises

1.

(a) 7 m

(b) 7 m

(c) +7 *m*

3.

a. $8\ \text{m} + 2\ \text{m} + 3\ \text{m} = 13\ \text{m}$

b. $9\ \text{m}$

c. $\Delta x = 11\ \text{m} - 2\ \text{m} = 9\ \text{m}$

5.

(a) 3.0×10^4 m/s

(b) 0 m/s

7.

2×10^7 years

9.

34.689 m/s = 124.88 km/h

11.

(a) 40.0 km/h

(b) 34.3 km/h, 25º S of E.

(c)average speed = 3.20 km/h

13.

384,000 km

15.

(a) 6.61×10^{15} rev/s

(b) 0 m/s

16.

4.29 m/s^2

18.

(a) 1.43 s

(b) −2.50 m/s^2

20.

(a) 10.8 m/s

(b)

image

Image Description

The image is a graph titled “Position vs. Time.” It depicts a curve showing how position changes over time. The horizontal axis is labeled “Time (s)” and spans from 0 to 6 seconds, with intervals marked at each second. The vertical axis is labeled “Position (m)” and ranges from 0 to 60 meters, with intervals marked every 10 meters.

The graph is a curve that begins at the origin (0, 0) and rises steadily, illustrating an acceleration. Data points marked by diamonds are plotted along the curve at various points, indicating that the object’s position increases more rapidly as time progresses.

21.

38.9 m/s (about 87 miles per hour)

23.

(a) 16.5 s

(b) 13.5 s

(c) $-2.68\ m/s^2$

25.

(a) 20.0 m

(b) −1.00 m/s

(c) This result does not really make sense. If the runner starts at 9.00 m/s and decelerates at 2.00 m/s^2, then she will have stopped after 4.50 s. If she continues to decelerate, she will be running backwards.

27.

0.799 m

29.

(a) 28.0 m/s

(b) 50.9 s

(c) 7.68 km to accelerate and 713 m to decelerate

31.

(a) 51.4 m

(b) 17.1 s

33.

(a) $-80.4\ m/s^2$

(b) 9.33×10^{-2} s

35.

(a) 7.7 m/s

(b) $-15 \times 10^2\ m/s^2$. This is

about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

37.

(a) 32.6 m/s^2

(b) 162 m/s

(c) $v > v_{max}$,
because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at 32.6 m/s^2
during the last few meters, but substantially less, and the final velocity would be less than 162 m/s.

39.

104 s

40.

(a) v = 12.2 m/s; a = 4.07 m/s^2

(b) v = 11.2 m/s

41.

(a) y_1 = 6.28 m;
v_1 = 10.1 m/s

(b) y_2 = 10.1 m;
v_2 = 5.20 m/s

(c) y_3 = 11.5 m;
v_3 = 0.300 m/s

(d) y_4 = 10.4 m;
v_4 = −4.60 m/s

43.

v_0 = 4.95
m/s

45.

(a) a = −9.80
m/s^2; v_0 = 13.0 m/s; y_0 = 0 m

(b) v = 0m/s. Unknown is
distance y to top of
trajectory, where velocity is zero. Use equation $v^2 = v_0^2 + 2a(y - y_0)$
because it contains all known values except for y, so we can solve for y. Solving for y
gives

$$v^2 - v_0^2 = 2a\,(y - y_0)$$

$$\frac{v^2 - v_0^2}{2a} = y - y_0$$

$$y = y_0 + \frac{v^2 - v_0^2}{2a} = 0\text{m} + \frac{(0\text{m/s})^2 - (13.0\text{m/s})^2}{2(-9.80\text{m/s})^2} = 8.62\text{m}$$

Dolphins measure about 2 meters long and can jump several times their
length out of the water, so this is a reasonable result.

(c) 2.65 s

image

Image Description

The image is a diagram illustrating a projectile motion scenario. It shows a side view of a surface with two distinct levels. The left side is a raised platform with a vertical drop to the lower level on the right. An arrow, labeled with an “S”, begins from the top of the platform, arcs upward, and then downward to the lower level, indicating the trajectory of a projectile. There are two labels: “y_0 = ?” near the top of the platform, prompting the initial vertical position, and “y = 0 m” on the lower level, indicating the final vertical position is at ground level.

47.

(a) 8.26 m

(b) 0.717 s

49.

1.91 s

51.

(a) 94.0 m

(b) 3.13 s

53.

(a) -70.0 m/s (downward)

(b) 6.10 s

55.

(a) 19.6 m

(b) 18.5 m

57.

(a) 305 m

(b) 262 m, -29.2 m/s

(c) 8.91 s

59.

(a) 115 m/s

(b) 5.0 m/s^2

61.

$$v = \frac{(11.7 - 6.95) \times 10^3 \text{ m}}{(40.0 - 20.0) \text{ s}} = 238 \text{ m/s}$$

63.

image

Image Description

The image is a graph titled “Position vs. Time” showing the relationship between time in seconds (x-axis) and position in kilometers (y-axis). The x-axis ranges from 0 to 25 seconds, while the y-axis ranges from 4.68 to 4.82 kilometers. A red line with two data points illustrates the position increasing over time. Initially, the line is relatively flat from 0 to about 9 seconds, indicating a slower change in position. From 9 to 21 seconds, the line curves upward, suggesting an acceleration in the change of position.

65.

(a) 6 m/s

(b) 12 m/s

(c) 3 m/s^2

(d) 10 s

67.

(a) Car A is traveling faster at the checkpoint because it must go past the speed of car B to reach the same distance.

(b) i. Yes, the equation is consistent with the answer because the speed of car A is only a constant away from the correct answer. ii. Yes, the equation makes sense because $V = 2V_0$.

(c)
imagegn: center;”>

Image Description

The image is a graph with two lines representing different functions. The graph has an X-axis labeled as *x* and a Y-axis labeled as *V*. There are two important lines on the graph:

- A horizontal blue line labeled as V_B, which represents the constant value V_0 and runs parallel to the X-axis.
- A slanted maroon line labeled as V_A, which starts at the origin (0,0) and increases linearly to reach $2V_0$ at t_A.

The Y-axis is marked at two points: V_0 and $2V_0$. There is also a dashed line extending horizontally from the point where the slanted line reaches $2V_0$ down to the X-axis at t_A. The background is a light beige color.

Chapter 3

Problems & Exercises

1.

(a) 480 m

(b) 379 m, 18.4º east of north

3.

north component 3.21 km, east component 3.83 km

5.

19.5 m, 4.65º south of west

7.

(a) 26.6 m, 65.1º north of east

(b) 26.6 m, 65.1º south of west

9.

52.9 m, 90.1º with respect to the
x-axis.

11.

x-component 4.41 m/s

y-component 5.07 m/s

13.

(a) 1.56 km

(b) 120 m east

15.

North-component 87.0 km, east-component 87.0 km

17.

30.8 m, 35.8 west of north

19.

(a) 30.8 m, 54.2^o south of west

(b) 30.8 m, 54.2^o north of east

21.

18.4 km south, then 26.2 km west(b) 31.5 km at 45.0^o south of west, then 5.56 km at 45.0^o west of north

23.

7.34 km, 63.5^o south of east

25.

$$x = 1.30\text{m} \times 10^2$$

$$y = 30.9\text{m}$$

27.

(a) 3.50 s

(b) 28.6 m/s (c) 34.3 m/s

(d) 44.7 m/s, 50.2^o below horizontal

29.

(a) 18.4^o

(b) The arrow will go over the branch.

$$R = \frac{v_0^2}{\sin 2\theta_0 g}$$

31.

$\text{For } \theta = 45^\circ, \quad R = \frac{v_0^2}{g}$

R = 91.8 m for v_0 = 30 m/s; R = 163 m for v_0 = 40 m/s; R = 255 m for v_0 = 50 m/s.

33.

(a) 560 m/s

(b) 8.00×10^3 m

(c) 80.0 m. This error is not significant because it is only 1% of the answer in part (b).

35.

1.50 m, assuming launch angle of 45°

37.

θ = 6.1^o

yes, the ball lands at 5.3 m from the net

39.

(a) −0.486 m

(b) The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

41.

4.23 m. No, the owl is not lucky; he misses the nest.

43.

No, the maximum range (neglecting air resistance) is about 92 m.

45.

15.0 m/s

47.

(a) 24.2 m/s

(b) The ball travels a total of 57.4 m with the brief gust of wind.

49.

$$y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta)t - \frac{1}{2}gt^2,$$

so that $t = \dfrac{2\,(v_0 \sin\theta)}{g}$

$x - x_0 = v_{0x}t = (v_0 \cos\theta)t = R$,
and substituting for t
gives:

$$R = v_0 \cos\theta \left(\frac{2v_0 \sin\theta}{g}\right) = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

since $2 \sin\theta \cos\theta = \sin 2\theta$,
the range is:

$$R = \frac{{v_0}^2 \sin 2\theta}{g}.$$

52.

(a) 35.8 *km*, 45º south of east

(b) 5.53 m/s, 45º south of east

(c) 56.1 *km*, 45º south of east

54.

(a) 0.70 m/s faster

(b) Second runner wins

(c) 4.17 m

56.

17.0 m/s, 22.1^o

58.

(a) 230 m/s, 8.0^o south of west

(b) The wind should make the plane travel slower and more to the south, which is what was calculated.

60.

(a) 63.5 m/s

(b) 29.6 m/s

62.

6.68 m/s, 53.3^o south of west

64.

(a) $H_{\text{average}} = 14.9 \frac{\text{km/s}}{\text{Mly}}$

(b) 20.2 billion years

66.

1.72 m/s, 42.3^o north of east

71.

(a) Since $\sin^2 50 > \sin^2 40$,
then B reaches the greatest height.

(b) i. Yes, it is consistent because $\sin 50 > \sin 40$.
ii. No, it does not make sense because y is proportional to $\sin^2\theta$.

(c)
image

Image Description

The image is a graph illustrating two lines labeled A and B on a coordinate system. The horizontal axis is labeled “y,” and the vertical axis is labeled “V.”

The line labeled “A” is shown in a purple color and starts at a point labeled “V_{0A}” on the vertical axis, sloping downwards towards the horizontal axis.

The line labeled “B” is shown in a blue color and starts at a higher point labeled “V_{0B}” on the vertical axis, also sloping downwards towards the horizontal axis. It intersects the horizontal axis farther along than line A.

Chapter 4

Problems & Exercises

1.

265 N

3.

13.3 m/s^2

7.

(a) 12 m/s^2.

(b) The acceleration is not one-fourth of what it was with all rockets burning because the frictional force is still as large as it was with all rockets burning.

9.

(a) The system is the child in the wagon plus the wagon.

(b)

image

Image Description

The image shows a diagram of forces acting on an object represented by a small circle labeled “m” at the center. There are four arrows originating from this circle in different directions:

- An upward arrow labeled “N” indicating a normal force.
- A downward arrow labeled “w” depicting weight.
- A rightward arrow labeled “F_2” showing an applied force.
- A leftward arrow labeled “F_1” indicating another applied force, with an additional smaller arrow labeled “f” pointing in the left direction representing

friction.

(c) $a = 0.130\ m/s^2$ in the direction of the second child's push.

(d) $a = 0.00\ m/s^2$

11.

(a) 3.68×10^3 N . This force is 5.00 times greater than his weight.

(b) 3750 N; 11.3º above horizontal

13.

1.5×10^3 N, 150 kg, 150 kg

15.

Force on shell: 2.64×10^7 N

Force exerted on ship = -2.64×10^7 N, by Newton's third law

17.

(a) $0.11\ m/s^2$
(b) 1.2×10^4 N

19.

(a) 7.84×10^{-4} N

(b) 1.89×10^{-3} N. This is 2.41 times the tension in the vertical strand.

21.

Newton's second law applied in vertical direction gives

$F_Y = F - 2T \sin \theta = 0$

$F = 2T \sin \theta$

$$T = \frac{F}{2 \sin \theta}.$$

image

Image Description

The image is a physics diagram representing forces acting on a mass. In the center, there is a circle labeled with the letter “m” to denote mass. There are three arrows indicating forces acting on this mass:

- An upward arrow labeled “T” representing tension.
- A downward arrow labeled “f” representing force, likely friction or another opposing force.
- A downward arrow labeled “mg” representing the gravitational force, where “m” is mass and “g” is the acceleration due to gravity.

23.

Using the free-body diagram:

$F_{\text{net}} = T - f - mg = \text{ma}$,

so that

$$a = \frac{T - f - \text{mg}}{m} = \frac{1.250 \times 10^7\ \text{N} - 4.50 \times 10^6\ N - (5.00 \times 10^5\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{5.00 \times 10^5\ \text{kg}} = 6.20\ \text{m/s}^2$$

.

25.

1. Use Newton’s laws of motion.

   image

   Image Description

The image is a physics diagram illustrating forces acting on a mass. In the center, there is a circle labeled with the letter “m,” representing mass. There are two arrows extending vertically from this circle:

- An upward arrow on the left side is labeled “F,” indicating a force applied upwards.
- A downward arrow on the left side is labeled “w,” representing the weight acting downwards.

To the right of the circle, there is another upward arrow labeled “a,” representing acceleration in the upward direction.

2. Given : $a = 4.00g = (4.00)(9.80\ m/s^2) = 39.2\ m/s^2$; $m = 70.0$ kg,

- Find: F.

3. $\sum F = +F - w = ma$, so that $F = ma + w = ma + mg = m(a + g)$.

- $F = (70.0\ kg)[(39.2\ m/s^2) + (9.80\ m/s^2)] = 3.43 \times 10^3 N$. The force exerted by the high-jumper is actually down on the ground, but F is up from the ground and makes him jump.

4. This result is reasonable, since it is quite possible for a person to exert a force of the magnitude of 10^3 N.

27.

(a) 4.41×10^5 N

(b) 1.50×10^5 N

29.

(a) 910 N

(b) 1.11×10^3 N

31.

a = 0.139 m/s, θ = 12.4° north of east

33.

1. Use Newton's laws since we are looking for forces.
2. Draw a free-body diagram: image

   Image Description
   The image depicts a diagram with three arrows forming angles with a horizontal dashed line. The central arrow points vertically downward, creating a vertical line which intersects the horizontal dashed line. Two additional arrows extend from the intersection point, pointing downward and outward at symmetrical angles. These two arrows are each labeled with "15°", indicating that they form a 15-degree angle with the horizontal dashed line. The overall formation resembles an inverted and narrow 'V' shape with the central vertical arrow splitting the 'V'.

3. The tension is given as T = 25.0 N. Find F_{app}. Using Newton's laws gives: $\Sigma F_y = 0$, so that applied force is due to the y-components of the two tensions: $F_{app} = 2T\sin\theta = 2(25.0 \text{ N})\sin(15^\circ) = 12.9$ N

   - The x-components of the tension cancel. $\sum F_x = 0$.

4. This seems reasonable, since the applied tensions should be greater than the force applied to the tooth.

40.

$10.2\ m/s^2$, 4.67º from
vertical
image

Image Description
The image depicts a free body diagram of an object with mass labeled as “m” at the center. Three forces are acting on the object:

- **Tension T1:** Represented by an upward and leftward pointing dashed arrow inclined at 15 degrees from the vertical axis.
- **Tension T2:** Represented by a rightwards pointing dashed arrow inclined at 10 degrees from the horizontal axis.
- **Weight w:** Represented by a downward pointing solid arrow along the vertical axis.

42.

$T_1 = 736\ N$

$T_2 = 194\ N$

44.

(a) $7.43\ m/s$

(b) 2.97 m

46.

(a) $4.20\ m/s$

(b) $29.4\ m/s^2$

(c) $4.31 \times 10^3\ N$

48.

(a) 47.1 m/s

(b) $2.47 \times 10^3\ m/s^2$

(c) 6.18×10^3 N . The
average force is 252 times the shell's weight.

52.

(a) 1×10^{-13}

(b) 1×10^{-11}

54.

10^2

55.

(a) Box A travels faster at the finishing distance since a greater force with equal mass results in a greater acceleration. Also, a greater acceleration over the same distance results in a greater final speed.

(b) i. Yes, it is consistent because a greater force results in a greater final speed. ii. It does not make sense because $V = \sqrt{2(f/m)x} = \mathrm{K}\sqrt{(F)}$

.

(c)
image

Image Description

The image shows a plot with a downward sloping curve on a two-dimensional graph. The x-axis is labeled with the letter "F" and the y-axis is labeled with the letter "x". The curve starts high on the y-axis and slopes downward as it moves to the right along the x-axis, indicating an inverse relationship. The equation *x ∝ 1/F* is written below the graph, suggesting that x is inversely proportional to F.

56.

5.00 *N*

59.

(a) 588 N

(b) 1.96 m/s^2

61.

(a) 3.29 m/s^2

(b) 3.52 m/s^2

(c) 980 N; 945 N

65.

1.83 m/s^2

69.

(a) 4.20 m/s^2

(b) 2.74 m/s^2

(c) –0.195 m/s^2

71.

(a) 1.03×10^6 *N*

(b) 3.48×10^5 *N*

73.

(a) 51.0 *N*

(b) 0.720 m/s^2

Chapter 5

Problems & Exercises

1.

723 km

3.

5×10^7 rotations

5.

117 rad/s

7.

76.2 rad/s

728 rpm

8.

(a) 33.3 rad/s

(b) 500 N

(c) 40.8 m

10.

12.9 rev/min

12.

4×10^{21} m

14.

a) 3.47×10^4 m/s^2, 3.55×10^3 *g*

b) 51.1 m/s

16.

a) 31.4 rad/s

b) 118 m/s

c) 384 m/s

d)The centripetal acceleration felt by Olympic skaters is 12 times larger than the acceleration due to gravity. That's quite a lot of acceleration in itself. The centripetal acceleration felt by Button's nose was 39.2 times larger than the acceleration due to gravity. It is no wonder that he ruptured small blood vessels in his spins.

18.

a) 0.524 km/s

b) 29.7 km/s

20.

(a) 1.35×10^3 rpm

(b) 8.47×10^3 m/s^2

(c) 8.47×10^{-12} N

(d) 865

21.

(a) 16.6 m/s

(b) 19.6 m/s^2

(c)

image

Image Description

The image depicts a simple diagram featuring a rectangle with a vertical line passing through its center. The line extends beyond the rectangle's top and bottom edges. At the top end of the line is an arrow pointing upward labeled "N" for North. At the

bottom end, another arrow points downward labeled “W” for West. The rectangle seems to represent some form of directional layout.

(d) 1.76×10^3 N or
3.00 w, that is, the normal force (upward) is three
times her weight.

(e) This answer seems reasonable, since she feels like she’s being
forced into the chair MUCH stronger than just by gravity.

22.

a) 40.5 m/s^2

b) 905 N

c) The force in part (b) is very large. The acceleration in part (a)
is too much, about 4 g.

d) The speed of the swing is too large. At the given velocity at the
bottom of the swing, there is enough kinetic energy to send the child
all the way over the top, ignoring friction.

23.

a) 483 N

b) 17.4 N

c) 2.24 times her weight, 0.0807 times her weight

25.

4.14°

27.

a) 24.6 m

b) 36.6 m/s^2

c) $a_C = 3.73\ g$.

This does not seem too large, but it is clear that bobsledders feel a lot of force on them going through sharply banked turns.

29.

a) 2.56 rad/s

b) 5.71^o

30.

a) 16.2 m/s

b) 0.234

32.

a) 1.84

b) A coefficient of friction this much greater than 1 is unreasonable

.

c) The assumed speed is too great for the tight curve.

33.

a) $g = \frac{GM}{r^2} \rightarrow M = \frac{r^2 g}{G} = 5.95296 \times 10^{24} \text{kg}$.

b) This is slightly, but measurably, smaller than the accepted value, differing in the third significant digit. The equation we used to find *g* assumes a sphere, but we already noted that the radius at the pole is smaller than at the equator. The calculation gives the mass for a sphere of the radius we used—and ignores the additional mass outside that radius.

35.

a) 1.62 m/s^2

b) 3.75 m/s^2

37.

a) 3.42×10^{-5} m/s^2

b) 3.34×10^{-5} m/s^2

The values are nearly identical. One would expect the gravitational force to be the same as the centripetal force at the core of the system.

39.

a) 7.01×10^{-7} N

b) 1.35×10^{-6} N, 0.521

41.

a) 1.66×10^{-10} m/s^2

b) 2.17×10^{5} m/s

42.

a) 2.937×10^{17} kg

b) 4.91×10^{-8}

of the Earth's mass.

c) The mass of the mountain and its fraction of the Earth's mass are too great.

d) The gravitational force assumed to be exerted by the mountain is too great.

Chapter 6

Problems & Exercises

1.

3.00 J = 7.17×10^{-4} kcal

3.

(a) 5.92×10^{5} J

(b) -5.88×10^{5} J

(c) The net force is zero.

5.

3.14×10^{3} J

7.

(a) −700 J

(b) 0

(c) 700 J

(d) 38.6 N

(e) 0

9.

1/250

11.

1.1×10^{10} J

13.

2.8×10^{3} N

15.

102 N

16.

(a) 1.96×10^{16} J

(b) The ratio of gravitational potential energy in the lake to the energy stored in the bomb is 0.52. That is, the energy stored in the lake is approximately half that in a 9-megaton fusion bomb.

18.

(a) 1.8 J

(b) 8.6 J

20.

$$v_f = \sqrt{2gh + {v_0}^2} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(-0.180\ \text{m}) + (2.00\ \text{m/s})^2} = 0.687\ \text{m/s}$$

22.

7.81×10^5 N/m

24.

9.46 m/s

26.

4×10^4 molecules

27.

Equating ΔPE_g and ΔKE, we obtain

$$v = \sqrt{2gh + {v_0}^2} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(20.0\ \text{m}) + (15.0\ \text{m/s})^2} = 24.8\ \text{m/s}$$

29.

(a) 25×10^6 years

(b) This is much, much longer than human time scales.

30.

2×10^{-10}

32.

(a) 40

(b) 8 million

34.

$149

36.

(a) 208 W

(b) 141 s

38.

(a) 3.20 s

(b) 4.04 s

40.

(a) 9.46×10^{7} J

(b) 2.54 y

42.

Identify knowns: $m = 950$ kg, slope angle $\theta = 2.00^{o}$, $v = 3.00$ m/s, $f = 600$ N

Identify unknowns: power P of the car, force F that car applies to road

Solve for unknown:

$$P = \frac{W}{t} = \frac{\mathrm{Fd}}{t} = F\left(\frac{d}{t}\right) = \mathrm{Fv},$$

where *F* is parallel to the
incline and must oppose the resistive forces and the force of gravity:

$F = f + w = 600\ \mathrm{N} + mg \sin \theta$

Insert this into the expression for power and solve:

$$P = (f + mg \sin \theta)v$$

$$= \left[600\mathrm{N} + (950\mathrm{kg})\left(9.80\mathrm{m/s^2}\right)\sin 2^\circ\right](30.0\mathrm{m/s})$$

$$= 2.77 \times 10^4\mathrm{W}$$

About 28 kW (or about 37 hp) is reasonable for a car to climb a gentle incline.

44.

(a) 9.5 min

(b) 69 flights of stairs

46.

641 W, 0.860 hp

48.

31 g

50.

14.3%

52.

(a) 3.21×10^4 N

(b) 2.35×10^3 N

(c) Ratio of net force to weight of person is 41.0 in part (a); 3.00 in part (b)

54.

(a) 108 kJ

(b) 599 W

56.

(a) 144 J

(b) 288 W

58.

(a) 2.50×10^{12} J

(b) 2.52%

(c) 1.4×10^4 kg (14 metric tons)

60.

(a) 294 N

(b) 118 J

(c) 49.0 W

62.

(a) 0.500 m/s^2

(b) 62.5 *N*

(c) Assuming the acceleration of the swimmer decreases linearly with time over the 5.00 s interval, the frictional force must therefore be increasing linearly with time, since $f = F - ma$. If the acceleration decreases linearly with time, the velocity will contain a

term dependent on time squared (t^2). Therefore, the water resistance will not depend linearly on the velocity.

64.

(a) 16.1×10^3 *N*

(b) 3.22×10^5 *J*

(c) 5.66 m/s

(d) 4.00 kJ

66.

(a) 4.65×10^3 kcal

(b) 38.8 kcal/min

(c) This power output is higher than the highest value on Table 7.5, which is about 35 kcal/min (corresponding to 2415 watts) for sprinting.

(d) It would be impossible to maintain this power output for 2 hours (imagine sprinting for 2 hours!).

69.

(a) 4.32 m/s

(b) 3.47×10^3 *N*

(c) 8.93 kW

70.

(a) Box A has the greatest speed since it has the greatest work done; work equals change in kinetic energy and thus fastest speed.

(b) i. Yes, this equation is consistent. An increased force or distance increases the work done, and thus increases the change in kinesthetic energy. ii. No, this equation does not make sense.

(c)
image

Image Description

The image is a graph on a light beige background illustrating the equation $w = kx^2$. The graph features two axes: the horizontal axis labeled as x and the vertical axis labeled as w. The curve on the graph starts at the origin (0,0) and curves upward, showing an exponential growth pattern consistent with a quadratic function. To the right of the curve, the equation $w = kx^2$ is displayed, emphasizing the relationship between w and x in the context of the graph.

Chapter 7

Problems & Exercises

1.

a) 46.8 N·m

b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force's application from the hinges. (Children don't have a tougher time opening a door because they push lower than adults, they have a tougher time because they don't push far enough from the hinges.)

3.

23.3 N

5.

Given:

$m_1 = 26.0\text{kg}, m_2 = 32.0\text{kg}, m_\text{s} = 12.0\text{kg},$

$r_1 = 1.60\text{m}, r_\text{s} = 0.160\text{m}, \text{find (a) } r_2, \text{(b) } F_\text{p}$

a) Since children are balancing:

$$\text{net } \tau_\text{cw} = -\text{net } \tau_\text{ccw}$$

$$\Rightarrow w_1 r_1 + m_\text{s} g r_\text{s} = w_2 r_2$$

So, solving for r_2 gives:

$$r_2 = \frac{w_1 r_1 + m_\text{s} g r_\text{s}}{w_2} = \frac{m_1 g r_1 + m_\text{s} g r_\text{s}}{m_2 g} = \frac{m_1 r_1 + m_\text{s} r_\text{s}}{m_2}$$

$$= \frac{(26.0\text{kg})\,(1.60\text{m}) + (12.0\text{kg})\,(0.160\text{m})}{32.0\text{kg}}$$

$$= 1.36\text{m}$$

b) Since the children are not moving:

$$\text{net } F = 0 = F_{\text{p}} - w_1 - w_2 - w_{\text{s}}$$

$$\Rightarrow F_{\text{p}} = w_1 + w_2 + w_{\text{s}}$$

So that

$$F_{/textp} = (26.0\text{kg} + 32.0\text{kg} + 12.0\text{kg})\left(9.80\text{m/s}^2\right)$$

$$= 686\text{N}$$

6.

$F_{\text{wall}} = 1.43 \times 10^3$ N

8.

a) 2.55×10^3 N, 16.3º to the
left of vertical (i.e., toward the wall)

b) 0.292

10.

$F_{\text{B}} = 2.12 \times 10^4$ N

12.

a) 0.167, or about one-sixth of the weight is supported by the
opposite shore.

b) $F = 2.0 \times 10^4$ N, straight
up.

14.

a) 21.6 N

b) 21.6 N

16.

350 N directly upwards

19.

25

50 N

21.

a) MA = 18.5

b) F_i = 29.1
N

c) 510 N downward

23.

1.3×10^3 N

25.

a) T = 299 N

b) 897 N upward

26.

$$F_B = 470\text{N};\ r_1 = 4.00\text{cm};\ w_a = 2.50\text{kg};\ r_2 = 16.0\text{cm};\ w_b = 4.00\text{kg};\ r_3 = 38.0\text{cm}$$

$$F_E = w_a\left(\frac{r_2}{r_1} - 1\right) + w_b\left(\frac{r_3}{r_1} - 1\right)$$

$$= (2.50\text{kg})\left(9.80\text{m/s}^2\right)\left(\frac{16.0\text{cm}}{4.0\text{cm}} - 1\right) + (4.00\text{kg})\left(9.80/textm/s^2\right)\left(\frac{38.0\text{cm}}{4.00\text{cm}} - 1\right)$$

$$= 407\text{N}$$

$1.1 \times 10^3\text{N}$

28.

$\theta = 190^\circ$ ccw from positive x axis

30.

F_V = 97 N, θ = 59°

32.

(a) 25 N downward

(b) 75 N upward

33.

(a) $F_A = 2.21 \times 10^3$ N
upward

(b) $F_B = 2.94 \times 10^3$ N
downward

35.

(a) $F_{\text{teeth on bullet}} = 1.2 \times 10^2$ N upward

(b) $F_J = 84$ N
downward

37.

(a) 147 N downward

(b) 1680 N, 3.4 times her weight

(c) 118 J

(d) 49.0 W

39.

a) $\bar{x}_2 = 2.33 \text{ m}$

b) The seesaw is 3.0 m long, and hence, there is only 1.50 m of board on the other side of the pivot. The second child is off the board.

c) The position of the first child must be shortened, i.e. brought closer to the pivot.

Chapter 8

Problems & Exercises

1.

ω = 0.737 rev/s

3.

(a) $-0.26\ rad/s^2$

(b) 27 rev

5.

(a) $80\ rad/s^2$

(b) 1.0 rev

7.

(a) 45.7 s

(b) 116 rev

9.

a) $600\ rad/s^2$

b) $450\ rad/s^s$

c) $21.0\ m/s^s$

10.

(a) 0.338 s

(b) 0.0403 rev

(c) 0.313 s

12.

$0.50\ kg \cdot m^2$

14.

(a) 50.4 $N \cdot$ m

(b) 17.1 rad/s^2

(c) 17.0 rad/s^2

16.

3.96 × 10^{18} s

or 1.26 × 10^{11} y

18.

$$I_{end} = I_{center} + m\left(\frac{1}{2}\right)^2$$

$$\text{Thus, } I_{center} = I_{end} - \frac{1}{4}ml^2 = \frac{1}{3}ml^2 - \frac{1}{4}ml^2 = \frac{1}{12}ml^2$$

19.

(a) 2.0 ms

(b) The time interval is too short.

(c) The moment of inertia is much too small, by one to two orders of magnitude. A torque of 500 N $\cdot$ m is reasonable.

20.

(a) 17,500 rpm

(b) This angular velocity is very high for a disk of this size and mass. The radial acceleration at the edge of the disk is > 50,000 gs.

(c) Flywheel mass and radius should both be much greater, allowing for a lower spin rate (angular velocity).

21.

(a) 185 J

(b) 0.0785 rev

(c) $W = 9.81$ N

23.

(a) 2.57×10^{29} J

(b) $KE_{rot} = 2.65 \times 10^{33}$ J

25.

$KE_{rot} = 434$ J

27.

(a) 128 rad/s

(b) 19.9 m

29.

(a) 10.4 rad/s^2

(b) net $W = 6.11$ J

34.

(a) 1.49 kJ

(b) 2.52×10^4 N

36.

(a) 2.66×10^{40} kg · m^2/s

(b) 7.07×10^{33} kg · m^2/s

The angular momentum of the Earth in its orbit around the Sun is 3.77×10^6 times larger than
the angular momentum of the Earth around its axis.

38.

$22.5\ \text{kg} \cdot \text{m}^2/\text{s}$

40.

25.3 rpm

Chapter 9

Problems & Exercises

1.

1.610 cm^3

3.

(a) 2.58 g

(b) The volume of your body increases by the volume of air you inhale. The average density of your body decreases when you take a deep breath, because the density of air is substantially smaller than the average density of the body before you took the deep breath.

4.

2.70 g/cm^3

6.

(a) 0.163 m

(b) Equivalent to 19.4 gallons, which is reasonable

8.

7.9×10^2 kg/m^3

9.

15.8 g/cm^3

10.

(a) 10^{18} kg/m^3

(b) 2×10^4 m

11.

3.59×10^6 Pa; or 521 lb/in^2

13.

2.36×10^3 N

14.

0.760 m

16.

$$(hpg)_{\text{units}} = (\text{m})\left(\text{kg/m}^3\right)\left(\text{m/s}^2\right) = \left(\text{kg} \cdot \text{m}^2\right)/\left(\text{m}^3 \cdot 2^2\right)$$

$$= \left(\text{kg} \cdot \text{m/s}^2\right)\left(1/\text{m}^2\right)$$

$$= \text{N/m}^2$$

18.

(a) 20.5 mm Hg

(b) The range of pressures in the eye is 12–24 mm Hg, so the result in part (a) is within that range

20.

1.09×10^3 N/m^2

22.

24.0 N

24.

2.55×10^7 Pa; or 251 atm

26.

5.76×10^3 N extra force

28.

(a) $V = d_{\text{i}}A_{\text{i}} = d_{\text{o}}A_{\text{o}} \Rightarrow d_{\text{o}} = d_{\text{i}}\left(\frac{A_{\text{i}}}{A_{\text{o}}}\right).$

Now, using equation:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_{\text{o}} = F_{\text{i}}\left(\frac{A_{\text{o}}}{A_{\text{i}}}\right).$$

Finally,

$$W_{\text{o}} = F_{\text{o}}d_{\text{o}} = \left(\frac{F_{\text{i}}A_{\text{o}}}{A_{\text{i}}}\right)\left(\frac{d_{\text{i}}A_{\text{i}}}{A_{\text{o}}}\right) = F_{\text{i}}d_{\text{i}} = W_{\text{i}}.$$

In other words, the work output equals the work input.

(b) If the system is not moving, friction would not play a role. With friction, we know there are losses, so that $W_{\text{out}} = W_{\text{in}} - W_{\text{f}}$; therefore, the work output is less than the work input. In other words, with friction, you need to push harder on the input piston than was calculated for the nonfriction case.

29.

Balloon:

$$P_{\text{g}} = 5.00\text{cm H}_2\text{O},$$

$$P_{\text{abs}} = 1.035 \times 10^3\text{cm H}_2\text{O}.$$

Jar:

$$P_{\text{g}} = -50.0\text{mm Hg},$$

$$P_{\text{abs}} = 710\text{mm Hg}.$$

31.

4.08 m

33.

$$\Delta P = 38.7\text{mm Hg},$$

$$\text{Leg blood pressure} = \frac{159}{119}$$

35.

22.4 cm^2

36.

91.7%

38.

815 kg/m^3

40.

(a) 41.4 g

(b) 41.4 cm^3

(c) 1.09 g/cm^3

42.

(a) 39.5 g

(b) 50 cm^3

(c) 0.79 g/cm^3

It is ethyl alcohol.

44.

8.21 N

46.

(a) 960 kg/m^3

(b) 6.34%

She indeed floats more in seawater.

48.

(a) 0.24

(b) 0.68

(c) Yes, the cork will float because $\rho_{obj} < \rho_{ethyl\ alcohol}$(0.678 g/cm^3 < 0.79 g/cm^3)

50.

The difference is 0.006%.

52.

$F_{net} = F_2 - F_1 = P_2A - P_1A = (P_2 - P_1)A$

$= (h_2\rho_{fl}g - h_1\rho_{fl}g)A$

$= (h_2 - h_1)\rho_{fl}gA$

where ρ_{fl} =
density of fluid. Therefore,

$F_{net} = (h_2 - h_1)A\rho_{fl}g = V_{fl}\rho_{fl}g = m_{fl}g = w_{fl}$

where is w_{fl} the
weight of the fluid displaced.

54.

592 N/m^2

56.

2.23×10^{-2} mm Hg

58.

(a) 1.65×10^{-3} m

(b) 3.71×10^{-4} m

60.

6.32×10^{-2} N/m

Based on the values in table, the fluid is probably glycerin.

62.

$P_{\mathrm{w}} = 14.6\ \mathrm{N/m^2},$

$P_{\mathrm{a}} = 4.46\ \mathrm{N/m^2},$

$P_{\mathrm{sw}} = 7.40\ \mathrm{N/m^2}.$

Alcohol forms the most stable bubble, since the absolute pressure inside is closest to atmospheric pressure.

64.

5.1^o

This is near the value of $\theta = 0^o$ for most organic liquids.

66.

−2.78

The ratio is negative because water is raised whereas mercury is lowered.

68.

479 N

70.

1.96 N

71.

−63.0 cm H_2O

73.

(a) 3.81×10^3 N/m^2

(b) 28.7 mm Hg, which is sufficient to trigger micturition reflex

75.

(a) 13.6 m water

(b) 76.5 cm water

77.

(a) 3.98×10^{6} Pa

(b) 2.1×10^{-3} cm

79.

(a) 2.97 cm

(b) 3.39×10^{-6} J

(c) Work is done by the surface tension force through an effective distance $h/2$ to raise the
column of water.

81.

(a) 2.01×10^{4} N

(b) 1.17×10^{-3} m

(c) 2.56×10^{10} N/m^2

83.

(a) 1.38×10^{4} N

(b) 2.81×10^{7} N/m^2

(c) 283 N

85.

(a) 867 N

(b) This is too much force to exert with a hand pump.

(c) The assumed radius of the pump is too large; it would be nearly two inches in diameter—too large for a pump or even a master cylinder. The pressure is reasonable for bicycle tires.

Chapter 10

Problems & Exercises

1.

2.78 cm^3/s

3.

27 cm/s

5.

(a) 0.75 m/s

(b) 0.13 m/s

7.

(a) 40.0 cm^2

(b) 5.09×10^7

9.

(a) 22 h

(b) 0.016 s

11.

(a) 12.6 m/s

(b) 0.0800 m^3/s

(c) No, independent of density.

13.

(a) 0.402 L/s

(b) 0.584 cm

15.

(a) 127 cm^3/s

(b) 0.890 cm

17.

$$P = \frac{\text{Force}}{\text{Area}},$$

$$(P)_{\text{units}} = \text{N}/\text{m}^2 = \text{N} \cdot \text{m}/\text{m}^3 = \text{J}/\text{m}^3$$

$$= \text{energy}/\text{volume}$$

19.

184 mm Hg

21.

2.54×10^5 N

23.

(a) 1.58×10^6 N/m^2

(b) 163 m

25.

(a) 9.56×10^8 W

(b) 1.41

27.

1.26 W

29.

(a) 3.02×10^{-3} N

(b) 1.03×10^{-3}

31.

1.60 cm^3/min

33.

$8.7 \times 10^{-11}\ m^3/s$

35.

0.316

37.

(a) 1.52

(b) Turbulence will decrease the flow rate of the blood, which would require an even larger increase in the pressure difference, leading to higher blood pressure.

39.

225 mPa · s

41.

0.138 Pa · s,

or

Olive oil.

43.

(a) $1.62 \times 10^4\ N/m^2$

(b) $0.111\ cm^3/s$

(c)10.6 cm

45.

1.59

47.

$2.95 \times 10^6\ N/m^2$(gauge pressure)

Chapter 11

Problems & Exercises

1.

102ºF

3.

20.0ºC and 25.6ºC

5.

9890ºF

7.

(a) 22.2ºC

$$\Delta T\,(^{\circ}\mathrm{F}) = T_2\,(^{\circ}\mathrm{F}) - T_1\,(^{\circ}\mathrm{F})$$

(b)
$$= \frac{9}{2}T_2\,(^{\circ}\mathrm{C}) + 32.0^{\circ} - \left(\frac{9}{5}T_1\,(^{\circ}\mathrm{C}) + 32.0^{\circ}\right)$$

$$= \frac{9}{5}(T_2\,(^{\circ}\mathrm{C}) - T_1\,(^{\circ}\mathrm{C})) = \frac{9}{5}\Delta T\,(^{\circ}\mathrm{C})$$

9.

169.98 m

11.

5.4×10^{-6} m

13.

Because the area gets smaller, the price of the land DECREASES by ~$17,000.

15.

$$V = V_0 + \Delta V = V_0 \left(1 + \beta \Delta T\right)$$
$$= (60.00\text{L}) \left[1 + \left(950 \times 10^{-6}/^\circ\text{C}\right) (35.0^\circ\text{C}) (35.0^\circ\text{C} - 15.0^\circ\text{C})\right]$$
$$= 61.1\text{L}$$

17.

(a) 9.35 mL

(b) 7.56 mL

19.

0.832 mm

21.

We know how the length changes with temperature: $\Delta L = \alpha L_0 \Delta T$.
Also we know that the volume of a cube is related to its length by $V = L^3$, so the final volume is then $V = V_0 + \Delta V = (L_0 + \Delta L)^3$.
Substituting for ΔL gives

$V = (L_0 + \alpha L_0 \Delta T)^3 = L_0^3 (1 + \alpha \Delta T)^3$.

Now, because $\alpha \Delta T$
is small, we can use the binomial expansion:

$V \approx L_0^3 (1 + 3\alpha \Delta T) = L_0^3 + 3\alpha L_0^3 \Delta T$.

So writing the length terms in terms of volumes gives $V = V_0 + \Delta V \approx V_0 + 3\alpha V_0 \Delta T$, and so

$\Delta V = \beta V_0 \Delta T \approx 3\alpha V_0 \Delta T$, or $\beta \approx 3\alpha$.

Chapter 12

Problems & Exercises

1.

5.02×10^8 J

3.

3.07×10^3 J

5.

0.171°C

7.

10.8

9.

617 W

11.

35.9 kcal

13.

(a) 591 kcal

(b) 4.94×10^3 s

15.

13.5 W

17.

(a) 148 kcal

(b) 0.418 s, 3.34 s, 4.19 s, 22.6 s, 0.456 s

19.

33.0 g

20.

(a) 9.67 L

(b) Crude oil is less dense than water, so it floats on top of the water, thereby exposing it to the oxygen in the air, which it uses to burn. Also, if the water is under the oil, it is less efficient in absorbing the heat generated by the oil.

22.

a) 319 kcal

b) 2.00°C

24.

20.6°C

26.

4.38 kg

28.

(a) 1.57×10^4 kcal

(b) 18.3 kW · h

(c) 1.29×10^4 kcal

30.

(a) 1.01×10^3 W

(b) One

32.

84.0 W

34.

2.59 kg

36.

(a) 39.7 W

(b) 820 kcal

38.

35 to 1, window to wall

40.

1.05×10^3 K

42.

(a) 83 W

(b) 24 times that of a double pane window.

44.

20.0 W, 17.2% of 2400 kcal per day

45.

10 m/s

47.

85.7°C

49.

1.48 kg

51.

2×10^4 MW

53.

(a) 97.2 J

(b) 29.2 W

(c) 9.49 W

(d) The total rate of heat loss would be 29.2 *W* + 9.49 *W* = 38.7 *W*.
While sleeping, our body consumes 83 W of power, while sitting it consumes 120 to 210 W. Therefore, the total rate of heat loss from breathing will not be a major form of heat loss for this person.

55.

−21.7 kW Note that the negative
answer implies heat loss to the surroundings.

57.

−266 kW

59.

−36.0 W

61.

(a) 1.31%

(b) 20.5%

63.

(a) −15.0 kW

(b) 4.2 cm

65.

(a) 48.5°C

(b) A pure white object reflects more of the radiant energy that hits it, so a white tent would prevent more of the sunlight from heating up the inside of the tent, and the white tunic would prevent that heat which entered the tent from heating the rider. Therefore, with a white tent, the temperature would be lower than 48.5°C, and the rate of radiant heat transferred to the rider would be less than 20.0 W.

67.

(a) 3×10^{17} J

(b) 1×10^{13} kg

(c) When a large meteor hits the ocean, it causes great tidal waves, dissipating large amount of its energy in the form of kinetic energy of the water.

69.

(a) 3.44×10^{5} m^3/s

(b) This is equivalent to 12 million cubic feet of air per second.
That is tremendous. This is too large to be dissipated by heating the air by only 5 °C. Many of these cooling towers use the circulation of cooler air over warmer water to increase the rate of evaporation. This would allow much smaller amounts of air necessary to remove such a large amount of heat because evaporation removes larger quantities of heat than was considered in part (a).

71.

20.9 min

73.

(a) 3.96×10^{-2} g

(b) 96.2 J

(c) 16.0 W

75.

(a) 1.102

(b) 2.79×10^{4} J

(c) 12.6 J. This will not cause a significant cooling of the air because it is much less than the energy found in part (b), which is the energy required to warm the air from 20.0 °C to 50.0 °C.

76.

(a) 36°C

(b) Any temperature increase greater than about 3°C would be unreasonably large. In this case the final temperature of the person would rise to 73°C (163°F).

(c) The assumption of 95% heat retention is unreasonable.

78.

(a) 1.46 kW

(b) Very high power loss through a window. An electric heater of this power can keep an entire room warm.

(c) The surface temperatures of the window do not differ by as great an amount as assumed. The inner surface will be warmer, and the outer surface will be cooler.

Chapter 13

Problems & Exercises

1.

(a) 1.23×10^3 N/m

(b) 6.88 kg

(c) 4.00 mm

3.

(a) 889 N/m

(b) 133 N

5.

(a) 6.53×10^3 N/m

(b) Yes

7.

16.7 ms

8.

0.400 *s*/beats

9.

400 Hz

10.

12,500 Hz

11.

1.50 kHz

12.

(a) 93.8 m/s

(b) 11.3×10^3 rev/min

13.

2.37 N/m

15.

0.389 kg

18.

94.7 kg

21.

1.94 s

22.

6.21 cm

24.

2.01 s

26.

2.23 Hz

28.

(a) 2.99541 s

(b) Since the period is related to the square root of the acceleration of gravity, when the acceleration changes by 1% the period
changes by $(0.01)^2$ = 0.01% so it is necessary to have at least 4 digits after the decimal to see the changes.

30.

(a) Period increases by a factor of 1.41 ($\sqrt{2}$)

(b) Period decreases to 97.5% of old period

32.

Slow by a factor of 2.45

34.

length must increase by 0.0116%.

35.

(a) 1.99 Hz

(b) 50.2 cm

(c) 1.40 Hz, 71.0 cm

36.

(a) 3.95×10^6 N/m

(b) 7.90×10^6 J

37.

a). 0.266 m/s

b). 3.00 J

39.

$$\pm\frac{\sqrt{3}}{2}$$

42.

384 J

44.

(a). 0.123 m

(b). −0.600 J

(c). 0.300 J. The rest of the energy may go into heat caused by friction and other damping forces.

46.

(a) 5.00×10^5 J

(b) 1.20×10^3 s

47.

t = 9.26 d

49.

f = 40.0 Hz

51.

v_w = 16.0 m/s

53.

λ = 700 m

55.

d = 34.0 cm

57.

f = 4 Hz

59.

462 Hz,

4 Hz

61.

(a) 3.33 m/s

(b) 1.25 Hz

63.

0.225 W

65.

7.07

67.

16.0 d

68.

2.50 kW

70.

3.38×10^{-5} W/m^2

Chapter 14

Problems & Exercises

1.

0.288 m

3.

332 m/s

5.

$$v_{\text{w}} = (331\text{m/s})\sqrt{\frac{T}{273\text{K}}} = (331\text{m/s})\sqrt{\frac{293\text{K}}{273\text{K}}}$$

$$= 343\text{m/s}$$

7.

0.223

9.

(a) 7.70 m

(b) This means that sonar is good for spotting and locating large objects, but it isn't able to resolve smaller objects, or detect the detailed shapes of objects. Objects like ships or large pieces of airplanes can be found by sonar, while smaller pieces must be found by other means.

11.

(a) 18.0 ms, 17.1 ms

(b) 5.00%

(c) This uncertainty could definitely cause difficulties for the bat, if it didn't continue to use sound as it closed in on its prey. A 5%

uncertainty could be the difference between catching the prey around the neck or around the chest, which means that it could miss grabbing its prey.

12.

3.16×10^{-4} W/m^2

14.

3.04×10^{-4} W/m^2

16.

106 dB

18.

(a) 93 dB

(b) 83 dB

20.

(a) 50.1

(b) 5.01×10^{-3} or $\frac{1}{200}$

22.

70.0 dB

24.

100

26.

1.45×10^{-3} J

28.

28.2 dB

30.

(a) 878 Hz

(b) 735 Hz

32.

3.79×10^3 Hz

34.

(a) 12.9 m/s

(b) 193 Hz

36.

First eagle hears 4.23×10^3 Hz

Second eagle hears 3.56×10^3 Hz

38.

0.7 Hz

40.

0.3 Hz, 0.2 Hz, 0.5 Hz

42.

(a) 256 Hz

(b) 512 Hz

44.

180 Hz, 270 Hz, 360 Hz

46.

1.56 m

48.

(a) 0.334 m

(b) 259 Hz

50.

3.39 to 4.90 kHz

52.

(a) 367 Hz

(b) 1.07 kHz

54.

(a) $f_n = n(47.6\ Hz)$, $n = 1, 3, 5, \ldots, 419$

(b) $f_n = n(95.3\ Hz)$, $n = 1, 2, 3, \ldots, 210$

55.

1×10^6 km

57.

498.5 or 501.5 Hz

59.

82 dB

61.

approximately 48, 9, 0, –7, and 20 dB, respectively

63.

(a) 23 dB

(b) 70 dB

65.

Five factors of 10

67.

(a) 2×10^{-10} W/m^2

(b) 2×10^{-13} W/m^2

69.

2.5

71.

1.26

72.

170 dB

74.

103 dB

76.

(a) 1.00

(b) 0.823

(c) Gel is used to facilitate the transmission of the ultrasound between the transducer and the patient's body.

78.

(a) 77.0 *μm*

(b) Effective penetration depth = 3.85 cm, which is enough to examine the eye.

(c) 16.6 *μm*

80.

(a) 5.78×10^{-4} m

(b) 2.67×10^{6} Hz

82.

(a) $v_w = 1540\ m/s = f\lambda \Rightarrow \lambda = \dfrac{1540\ m/s}{100 \times 10^3\ \text{Hz}} = 0.0154\ m\ <\ 3.50\ m.$

Because the wavelength is much shorter than the distance in question, the wavelength is not the limiting factor.

(b) 4.55 ms

84.

974 Hz

(Note: extra digits were retained in order to show the difference.)

Chapter 15

Problems & Exercises

1.

(a) 1.25×10^{10}

(b) 3.13×10^{12}

3.

-600 C

5.

1.03×10^{12}

7.

9.09×10^{-13}

9.

1.48×10^{8} C

11.

(a) 0.263 N

(b) If the charges are distributed over some area, there will be a concentration of charge along the side closest to the oppositely charged object. This effect will increase the net force.

13.

The separation decreased by a factor of 5.

17.

$$F = k\frac{|q_1 q_2|}{r^2} = ma \Rightarrow a = \frac{kq^2}{mr^2}$$

$$= \frac{\left(9.00 \times 10^9\ \mathrm{N \cdot m^2/C^2}\right)\left(1.60 \times 10^{-19}\mathrm{m}\right)^2}{\left(1.67 \times 10^{-27}\mathrm{kg}\right)\left(2.00 \times 10^{-9}\mathrm{m}\right)^2}$$

$$= 3.45 \times 10^{16}\mathrm{m/s^2}$$

18.

(a) 3.2

(b) If the distance increases by 3.2, then the force will decrease by a factor of 10 ; if the distance decreases by 3.2, then the force will increase by a factor of 10. Either way, the force changes by a factor of 10.

20.

(a) 1.04×10^{-9} C

(b) This charge is approximately 1 nC, which is consistent with the magnitude of charge typical for static electricity

23.

1.02×10^{-11}

25.

a. 0.859 m beyond negative charge on line connecting two charges
b. 0.109 m from lesser charge on line connecting two charges

28.

8.75×10^{-4} N

30.

(a) 6.94×10^{-8} C

(b) 6.25 N/C

32.

(a) 300 N/C (east)

(b) 4.80×10^{-17} N (east)

42.

(a) 2.12×10^{5} N/C

(b) one charge of $+q$

44.

(a) 0.252 N to the left

(b) $x = 6.07\ cm$

46.

(a)The electric field at the center of the square will be straight up, since q_a and q_b are positive and q_c and q_d are negative and all have the same magnitude.

(b) 2.04×10^{7} N/C (upward)

48.

0.102 N, in the $-y$ direction

50.

(a) $\vec{E} = 4.36 \times 10^{3}\ \mathrm{N/C},\ 35.0^{\circ}$, below the horizontal.

(b) No

52.

(a) 5.58×10^{-11} N/C

(b)the coulomb force is extraordinarily stronger than gravity

54.

(a) -6.76×10^{5} C

(b) 2.63×10^{13} m/s^2 (upward)

(c) 2.45×10^{-18} kg

56.

The charge q_2 is
9 times greater than q_1.

69.

(a) The forces are balanced in the x-direction, so the net force is vertical. It is composed of the sum of the vertical components of the Coulomb force and the gravitational force.

$$\begin{aligned}
\mathrm{F}_y &= -2\left(\frac{kq_1q_2}{r^2} + G\frac{m_1m_2}{r^2}\right)\cos 45^\circ \\
&= -2\left(\frac{8.99\times10^9\left(1.00\times10^{-9}\right)\left(2.00\times10^{-9}\right)}{8} + 6.67\times10^{-11}\frac{(10.0)(10.0)}{8}\right)\cos 45^\circ \\
&= -4.36\times10^{-9}\mathrm{N}
\end{aligned}$$

(b) No, it is in a metastable position. Since it cannot move horizontally, it cannot traverse any part of the track.

(c)

$$\begin{aligned}
\mathrm{F}_x &= -\left(\frac{kq_1q_2}{4} + G\frac{m_1m_2}{4}\right)\frac{1}{2} + \left(\frac{kq_1q_2}{12} + G\frac{m_1m_2}{12}\right)\frac{\sqrt{2}}{2} \\
&= -\left(\frac{8.99\times10^9\left(-1.00\times10^{-9}\right)\left(2.00\times10^{-9}\right)}{4}\frac{1}{2} - 6.67\times10^{-11}\frac{(10.0)(10.0)}{4}\frac{1}{2}\right) \\
&\quad + \left(\frac{8.99\times10^9\left(-1.00\times10^{-9}\right)\left(2.00\times10^{-9}\right)}{12}\frac{\sqrt{3}}{2} - 6.67\times10^{-11}\frac{(10.0)(10.0)}{12}\frac{\sqrt{3}}{2}\right)\mathrm{N} \\
&= 2.60\times10^{-9}\mathrm{N}
\end{aligned}$$

$$\begin{aligned}
\mathrm{F}_y &= -\left(\frac{kq_1q_2}{4} + G\frac{m_1m_2}{4}\right)\frac{\sqrt{3}}{2} - \left(\frac{kq_1q_2}{12} + G\frac{m_1m_2}{12}\right)\frac{1}{2} \\
&= -2.08 \times 10^{-9}\mathrm{N} - \left(\frac{8.99 \times 10^{9}\left(1.00 \times 10^{-9}\right)\left(2.00 \times 10^{-9}\right)}{12}\frac{\sqrt{3}}{2} - 6.67 \times 10^{-11}\frac{(10.0)\,(10.0)}{4}\frac{\sqrt{3}}{2}\right) \\
&\quad - \left(\frac{8.99 \times 10^{9}\left(10.0^{-9}\right)}{12}\frac{1}{2} + 6.67 \times 10^{-11}\frac{(10.0)\,(10.0)}{12}\frac{1}{2}\right)\mathrm{N} \\
&= -6.36 \times 10^{-9}\mathrm{N}
\end{aligned}$$

(d) Yes.

(e) It will reach $(1, -\sqrt{3})$ before it will change direction.

(f) The metastable positions where there is no component in one direction are (0, 0), (4, 0), (2, 2), and (2, −2). They number 4.

Chapter 16

Problems & Exercises

1.

0.278 mA

3.

0.250 A

5.

1.50ms

7.

(a) 1.67kΩ

(b) If a 50 times larger resistance existed, keeping the current about the same, the power would be increased by a factor of about 50 (based on the equation $P = I^2R$), causing much more energy to be transferred to the skin, which could cause serious burns. The gel used reduces the resistance, and therefore reduces the power transferred to the skin.

9.

(a) 0.120 C

(b) 7.50×10^{17}electrons

11.

96.3 s

13.

(a) 7.81×10^{14} He^{++} nuclei/s

(b) 4.00×10^{3} s

(c) 7.71×10^{8} s

15.

-1.13×10^{-4} m/s

17.

9.42×10^{13} electrons

18.

0.833 A

20.

$7.33 \times 10^{-2}\ \Omega$

22.

(a) 0.300 V

(b) 1.50 V

(c) The voltage supplied to whatever appliance is being used is reduced because the total voltage drop from the wall to the final output of the appliance is fixed. Thus, if the voltage drop across the extension cord is large, the voltage drop across the appliance is significantly decreased, so the power output by the appliance can be significantly decreased, reducing the ability of the appliance to work properly.

24.

0.104 Ω

26.

2.81×10^{-4} m

28.

1.10×10^{-3} A

30.

−5°C to 45°C

32.

1.03

34.

0.06%

36.

−17°C

38.

(a) 4.7 Ω (total)

(b) 3.0% decrease

40.

2.00×10^{12} W

44.

(a) 1.50 W

(b) 7.50 W

46.

$$\frac{V^2}{\Omega} = \frac{V^2}{\mathrm{V/A}} = \mathrm{AV} = \left(\frac{C}{s}\right)\left(\frac{J}{C}\right) = \frac{J}{s} = 1\ \mathrm{W}$$

48.

$$1\ \mathrm{kW \cdot h} = \left(\frac{1 \times 10^3\ \mathrm{J}}{1\ \mathrm{s}}\right)(1\ h)\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right) = 3.60 \times 10^6\ \mathrm{J}$$

50.

$438/y

52.

$6.25

54.

1.58 h

56.

$3.94 billion/year

58.

25.5 W

60.

(a) 2.00×10^9 J

(b) 769 kg

62.

45.0 s

64.

(a) 343 A

(b) 2.17×10^3 A

(c) 1.10×10^3 A

66.

(a) 1.23×10^3 kg

(b) 2.64×10^3 kg

69.

(a) 2.08×10^5 A

(b) 4.33×10^4 MW

(c) The transmission lines dissipate more power than they are supposed to transmit.

(d) A voltage of 480 V is unreasonably low for a transmission voltage. Long-distance transmission lines are kept at much higher voltages (often hundreds of kilovolts) to reduce power losses.

73.

480 V

75.

2.50 ms

77.

(a) 4.00 kA

(b) 16.0 MW

(c) 16.0%

79.

2.40 kW

81.

(a) 4.0

(b) 0.50

(c) 4.0

83.

(a) 1.39 ms

(b) 4.17 ms

(c) 8.33 ms

85.

(a) 194 kW

(b) 880 A

87.

(a) 0.400 mA, no effect

(b) 26.7 mA, muscular contraction for duration of the shock (can't let go)

89.

$1.20 \times 10^5\ \Omega$

91.

(a) 1.00 Ω

(b) 14.4 kW

93.

Temperature increases 860° C. It is very likely to be damaging.

95.

80 beats/minute

97.

(a)

$$\frac{3.25 \times 10^{-3}}{1}\text{j/s} = Power = I^2R = \frac{1}{r^2}\frac{(2.50)^2\left(1.72 \times 10^{-8}\right)(1.25)}{\pi}\text{jm}^2/\text{s}$$

$$r = \sqrt{\frac{(2.50)^2\left(1.72 \times 10^{-8}\right)(1.25)}{0.00325\pi}} = 3.63\ \text{mm}$$

(b) $Power = \frac{(2.50)^2\left(1.74 \times 10^{-8}\right)(1.25)}{\left(3.63 \times 10^{-3}\right)^2\pi}\text{j/s}$, so 3.28 j of energy is expended in 1 s. Since α is specified in the chart to only two significant figures, round to 3.3 j.

(c) Since the energy dissipation is directly proportional to the

resistivity, the increase of 0.01% in energy dissipation is when the resistivity increases 1%. Or, 1.01 = 1 + 0.0039ΔT. T = 23° C.

98.

(a) 2.75 kΩ

(b) 27.5 Ω

100.

(a) 786 Ω

(b) 20.3 Ω

102.

29.6 W

104.

(a) 0.74 A

(b) 0.742 A

106.

(a) 60.8 W

(b) 3.18 kW

108.

(a) $R_{\mathrm{s}} = R_1 + R_2 \Rightarrow R_{\mathrm{s}} \approx R_1 \, (R_1 >> R_2)$

(b) $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{R_1 + R_2}{R_1 R_2}$,

so that

$$R_{\mathrm{p}} = \frac{R_1 R_2}{R_1 + R_2} \approx \frac{R_1 R_2}{R_1} = R_2 \, (R_1 >> R_2)$$

110.

(a) −400 kΩ

(b) Resistance cannot be negative.

(c) Series resistance is said to be less than one of the resistors, but it must be greater than any of the resistors.

Chapter 17

Problems & Exercises

1.

(a) Left (West)

(b) Into the page

(c) Up (North)

(d) No force

(e) Right (East)

(f) Down (South)

3.

(a) East (right)

(b) Into page

(c) South (down)

5.

(a) Into page

(b) West (left)

(c) Out of page

7.

7.50×10^{-7} N
perpendicular to both the magnetic field lines and the velocity

9.

(a) 3.01×10^{-5} T

(b) This is slightly less then the magnetic field strength of 5×10^{-5} T at the surface of

the
Earth, so it is consistent.

11.

(a) 6.67×10^{-10} C
(taking the Earth's field to be 5.00×10^{-5} T)

(b) Less than typical static, therefore difficult

12.

4.27 m

14.

(a) 0.261 T

(b) This strength is definitely obtainable with today's technology. Magnetic field strengths of 0.500 T are obtainable with permanent magnets.

16.

4.36×10^{-4} m

18.

(a) 3.00 kV/m

(b) 30.0 V

20.

0.173 m

22.

7.50×10^{-4} *V*

24.

(a) 1.18×10^{3} m/s

(b) Once established, the Hall emf pushes charges one direction and

the magnetic force acts in the opposite direction resulting in no net force on the charges. Therefore, no current flows in the direction of the Hall emf. This is the same as in a current-carrying conductor—current does not flow in the direction of the Hall emf.

26.

11.3 mV

28.

1.16 μV

30.

2.00 T

31.

(a) west (left)

(b) into page

(c) north (up)

(d) no force

(e) east (right)

(f) south (down)

33.

(a) into page

(b) west (left)

(c) out of page

35.

(a) 2.50 N

(b) This is about half a pound of force per 100 m of wire, which is

much less than the weight of the wire itself. Therefore, it does not cause any special concerns.

37.

1.80 T

39.

(a) 30°

(b) 4.80 N

41.

(a) τ decreases by 5.00% if B decreases by 5.00%

(b) 5.26% increase

43.

10.0 A

45.

$$A \cdot m^2 \cdot T = A \cdot m^2 \left(\frac{N}{A \cdot m} \right) = N \cdot m.$$

47.

$3.48 \times 10^{-26}\ N \cdot m$

49.

(a) 0.666 N · *m* west

(b) This is not a very significant torque, so practical use would be limited. Also, the current would need to be alternated to make the loop rotate (otherwise it would oscillate).

50.

(a) 8.53 N, repulsive

(b) This force is repulsive and therefore there is never a risk that the two wires will touch and short circuit.

52.

400 A in the opposite direction

54.

(a) 1.67×10^{-3} N/m

(b) 3.33×10^{-3} N/m

(c) Repulsive

(d) No, these are very small forces

56.

(a) Top wire: 2.65×10^{-4} N/m s, 10.9^{o} to left of up

(b) Lower left wire: 3.61×10^{-4} N/m, 13.9^{o} down from right

(c) Lower right wire: 3.46×10^{-4} N/m, 30.0^{o} down from left

58.

(a) right-into page, left-out of page

(b) right-out of page, left-into page

(c) right-out of page, left-into page

60.

(a) clockwise

(b) clockwise as seen from the left

(c) clockwise as seen from the right

61.

1.01×10^{13} T

63.

(a) 4.80×10^{-4} *T*

(b) Zero

(c) If the wires are not paired, the field is about 10 times stronger than Earth's magnetic field and so could severely disrupt the use of a compass.

65.

39.8 A

67.

(a) 3.14×10^{-5} *T*

(b) 0.314 T

69.

7.55×10^{-5} *T*,
23.4°

71.

10.0 A

73.

(a) 9.09×10^{-7} *N* upward

(b) 3.03×10^{-5} m/s^2

75.

60.2 cm

77.

(a) 1.02×10^{3} N/m^2

(b) Not a significant fraction of an atmosphere

79.

17.0×10^{-4}%/°C

81.

18.3 MHz

83.

(a) Straight up

(b) 6.00×10^{-4} N/m

(c) 94.1 μm

(d)2.47 Ω/m, 49.4 V/m

85.

(a) 571 C

(b) Impossible to have such a large separated charge on such a small object.

(c) The 1.00-N force is much too great to be realistic in the Earth's field.

87.

(a) 2.40×10^{6} m/s

(b) The speed is too high to be practical ≤ 1% speed of light

(c) The assumption that you could reasonably generate such a voltage with a single wire in the Earth's field is unreasonable

89.

(a) 25.0 kA

(b) This current is unreasonably high. It implies a total power delivery in the line of 50.0×10^9 W, which is much too high for standard transmission lines.

(c)100 meters is a long distance to obtain the required field strength. Also coaxial cables are used for transmission lines so that there is virtually no field for DC power lines, because of cancellation

from opposing currents. The surveyor's concerns are not a problem for his magnetic field measurements.

92.

(a) $F = qvB\sin\theta$
Since $\sin\theta = 0$, $F = 0$ for the first electron.
For the second and third electrons, $\sin\theta = 1$.
So, $F = (1.60 \times 10^{-19}\text{C})(5.00 \times 10^{7}\text{m/s})(2.00 \times 10^{-7}\text{T}) = 1.60 \times 10^{-18}\text{N}$.

(b) The first electron moves in a straight line. The other two electrons take a circular path.

(c) The first electron will never return to the origin. The other two will return.

$$r = \frac{\left(9.11 \times 10^{-11}\right)\left(5.00 \times 10^{7}\right)}{\left(1.60 \times 10^{-19}\right)\left(2.00 \times 10^{-7}\right)}\text{m}$$

$$v = \frac{r}{t}$$

$$t = \frac{\left(9.11 \times 10^{-11}\right)}{\left(1.60 \times 10^{-19}\right)\left(2.00 \times 10^{-7}\right)}\text{s} = 2.85 \times 10^{15}\text{s}$$

VERSION HISTORY

This page provides a record of changes made to the open textbook since its initial publication. If the change is minor, the version number increases by 0.1. If the change involves substantial updates, the version number increases to the next full number.

Version	Date	Change	Affected Web Rage
1.0	August 25, 2025	Publication	N/A