Tutorial problems for cons. of L.

1. (a) $Wd = Fd = \frac{1}{2} I \omega^2$

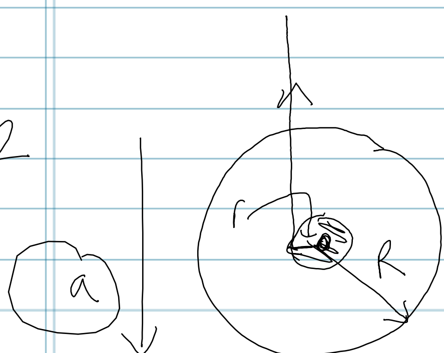
$$\omega = \sqrt{\frac{2Fd}{I}} = \boxed{8.16 \text{ rad/s}}$$

(b) $Fd = \frac{1}{2} I \omega^2 + \frac{1}{2} m V^2$ $\qquad V = r\omega$

$$= \frac{1}{2} I \omega^2 + \frac{1}{2} m (r^2 \omega^2)$$

$$\omega = \sqrt{\frac{2Fd}{I + mr^2}} = \boxed{8 \text{ rad/s}}$$

2.



Balance forces $\quad Ma = Mg - T$

$$T = M(g-a)$$

Torques about CoM $\Rightarrow$ $\quad Tr = I\alpha \qquad \alpha = \frac{a}{r}$

$$= I\frac{a}{r}$$

$$Tr^2 = Ia$$

$$Tr^2 = \frac{MR^2}{2} a$$

$$M(g-a)r^2 = \frac{MR^2 a}{2}$$

$$\cancel{M}gr^2 - \cancel{M}ar^2 = \frac{\cancel{M}R^2 a}{2}$$

$$gr^2 - ar^2 = \frac{R^2 a}{2}$$

$$a\left(\frac{R^2}{2} + r^2\right) = gr^2$$

$$a\left(\frac{R^2}{2r^2} + 1\right) = g$$

$$\boxed{a = \frac{g}{\left(\frac{R^2}{2r^2} + 1\right)}}$$

3. Bowling ball.

Point when slipping stops $\Rightarrow$ $\boxed{v = r\omega}$

Skidding $\longrightarrow$ rolling + sliding $\longrightarrow$ pure rolling
(sliding, no ~~to~~ rotation)

$\boxed{v = r\omega.}$

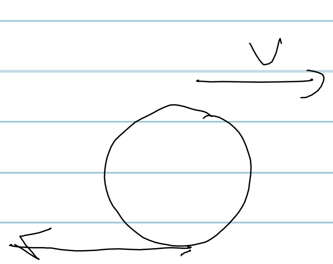
Friction: – slows ball down $v \downarrow$

– exerts a torque $\omega \uparrow$

① Expression for v (using kinematics)

② — " —— ω (using torques)

③ $v = r\omega$

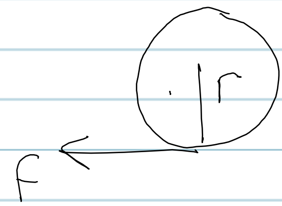
① 

$F_F = \mu_k mg$

$F_F = ma = -\mu_k mg$ $\quad a = -\mu_k g$

$v = v_0 - \mu_k g t$

(2) F_F exerts a torque.



$$F_F r = I\alpha$$

$$\mu_k m g r = \frac{2}{5} m r^2 \alpha .$$

$$\alpha = \frac{5}{2}\frac{\mu_k g}{r}$$

ω of any point on ball

$$\omega = \alpha t$$

$$= \frac{5}{2}\frac{\mu_k g}{r} t$$

a) Combine $v = r\omega$.

$$v_0 - \mu_k g t = \cancel{r}\,\frac{5}{2}\frac{\mu_k g t}{\cancel{r}} \rightarrow v_0 = \frac{7}{2}\mu_k g t$$

$$t = \frac{2}{7}\frac{v_0}{\mu_k g} = \underline{0.87 s}$$

b) At this time $v = v_0 - \cancel{\mu_k g}\,\frac{2}{7}\frac{v_0}{\cancel{\mu_k g}}$

$$\boxed{v = \frac{5}{7} v_0}$$

c) $v^2 = u^2 + 2aS$

$$S = \frac{v^2 - u^2}{2a} = \frac{\left(\frac{5}{7}\right)^2 v_0^2 - v_0^2}{2\mu_k g} = \boxed{6.75\,m}$$