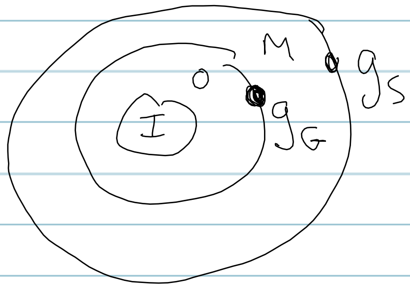
Gravity

1. Gutenberg.

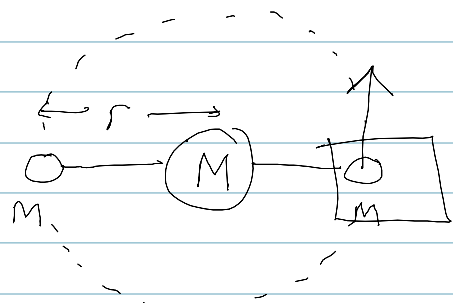


$$g_S = \frac{G\left(M_I + M_O + M_M\right) \times M_E}{r_E^{\,2}}$$

$$g_G = \frac{G\left(M_I + M_O\right) M_E}{r_G^{\,2}}$$

$$g_G / g_S = 1.01$$

2.



$$F_G = \frac{GMm}{r^2} + \frac{Gm^2}{4r^2}$$

$$= \frac{mv^2}{r}$$

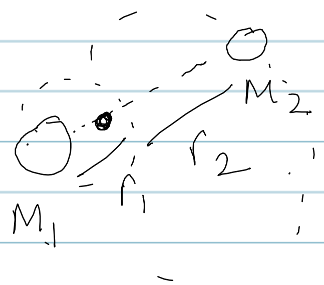
$$\frac{v^2}{r} = \frac{GM}{r^2} + \frac{Gm}{4r^2} \qquad v^2 = \frac{4\pi^2 r^2}{T^2}$$

$$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} + \frac{Gm}{4r^2}$$

$$= \frac{G}{r^2}\left(M + \frac{m}{4}\right)$$

$$\boxed{T^2 = \frac{4\pi^2}{G\left(M + \frac{m}{4}\right)} r^3}$$

3) Central body actually moves.

a) 

a) No, always opposite of CoM.
No, periods have to be equal
↳ F_G points along centre line.

$$\left.\begin{aligned} v_1 &= \frac{2\pi r_1}{T} \\ v_2 &= \frac{2\pi r_2}{T} \end{aligned}\right\} \quad \boxed{\frac{v_2}{v_1} = \frac{r_2}{r_1}} \leftarrow$$

b) $F_{G,1} = F_{G,2}$
$\downarrow \qquad \downarrow$
$F_{C,1} = F_{C,2}$

$$\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

$$\frac{m_1}{\cancel{r_1}} \frac{\cancel{v_2^2}\, r_1^{\cancel{2}}}{r_2^{\cancel{2}}} = \frac{m_2}{\cancel{r_2}} \cancel{v_2^2}$$

$$\boxed{\frac{m_1}{m_2} = \frac{r_2}{r_1}}$$

c) $$\frac{G\cancel{m_1}m_2}{(r_1+r_2)^2} = \frac{\cancel{m_1}v_1^2}{r_1} = \frac{2\pi r_1^2}{T^2 r_1}$$

$$T^2 = \frac{4\pi^2 (r_1 + r_2)^2}{G} \qquad \frac{r_1}{m_2}$$

Aside $\frac{m_1}{m_2} = \frac{r_2}{r_1}$ $\qquad \frac{a}{b} = \frac{c}{d}$ $\qquad c = fa \quad f = d/b$

$$\frac{a+c}{b+d} = \frac{a+fa}{b+fb} = \frac{(1+f)}{(1+f)} = \frac{a}{b}$$

$$\cancel{\frac{m_1}{m_2}} = \quad \frac{m_1}{r_2} = \frac{m_2}{r_1} \Rightarrow \frac{m_1 + m_2}{r_1 + r_2}$$

$$\frac{r_1}{m_2} = \frac{r_1 + r_2}{m_1 + m_2}$$

$$T^2 = \frac{4\pi^2 (r_1 + r_2)^3}{G(m_1 + m_2)}.$$

d) $m_m = 7.34 \times 10^{22}$ kg

$r_1 + r_2 = 3.84 \times 10^8$ m.

$\boxed{r_1 = 4700 \text{ km}}$ $\qquad r_E = 6371$ km.

4 Escape velocity.

$$r = kt^{p} \qquad t = \frac{r^{1/p}}{k^{1/p}}$$

$$v = \frac{dr}{dt} = kpt^{(p-1)}$$

$$a = \frac{dv}{dt} = kp(p-1)\,t^{(p-2)}$$

$$\cancel{m}a = \frac{GM\cancel{m}}{r^2}$$

$$kp(p-1)\,\underbrace{t^{(p-2)}} = \frac{GM}{k^2 \underbrace{t^{2p}}} = \frac{GM}{k^2}\,t^{-2p}$$

$$p - 2 = -2p$$

$$3p = 2$$

$$\boxed{p = 2/3}$$

$$a = \frac{GM}{r^2} \Rightarrow \frac{2}{3}k \cdot -\frac{1}{3}\,\cancel{t^{-4/3}} = \frac{GM}{k^2\,\cancel{t^{4/3}}}$$

$$-\frac{2}{9}k^3 = GM \Rightarrow \boxed{k = \left(\frac{9GM}{2}\right)^{1/3}}$$

b) $\underline{v = kpt^{-1/3}} \longrightarrow 0$

c) t when $r = r_E$.

$$r = Rt^p$$

$$r = \left(\frac{9GM}{2}\right)^{1/3} t^{2/3}$$

$$r_E = \left(\frac{9GM}{2}\right)^{1/3} t^{2/3}$$

Subs. into $\underline{v =} kpt^{-1/3}$

$$\boxed{v = 1.1 \times 10^4 \text{ m/s}}$$