

Tutorial problems for cons. of  $L$ .

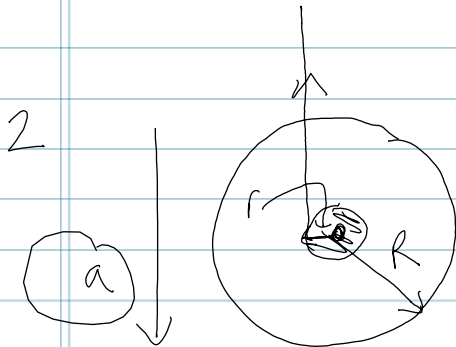
1. (a)  $Wd = Fd = \frac{1}{2} I \omega^2$

$$\omega = \sqrt{\frac{2Fd}{I}} = \boxed{8.16 \text{ rad/s}}$$

(b)  $Fd = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$   $v = r\omega$

$$= \frac{1}{2} I \omega^2 + \frac{1}{2} m (r^2 \omega^2)$$

$$\omega = \sqrt{\frac{2Fd}{I + mr^2}} = \boxed{8 \text{ rad/s}}$$



Balance forces

$$Ma = Mg - T$$

$$T = M(g - a)$$

Torques about CoM  $\Rightarrow$

$$Tr = I \alpha$$

$$= I \frac{a}{r}$$

$$\alpha = \frac{a}{r}$$

$$Tr^2 = \frac{Ia}{\frac{2}{R^2}} a$$

$$M(g-a)r^2 = \frac{MR^2a}{2}$$

$$\cancel{M}gr^2 - \cancel{M}ar^2 = \frac{\cancel{M}R^2a}{2}$$

$$gr^2 - ar^2 = \frac{R^2a}{2}$$

$$a \left( \frac{R^2}{2} + r^2 \right) = gr^2$$

$$a \left( \frac{R^2}{2r^2} + 1 \right) = g$$

$$a = \frac{g}{\left(\frac{R^2}{2r^2} + 1\right)}$$

### 3. Bowling ball.

Point when slipping stops  $\Rightarrow$   $v = r\omega$

skidding  $\rightarrow$  rolling + sliding  $\rightarrow$  pure rolling  
 (sliding, no ~~no~~ rotation)

$$\begin{array}{c} \uparrow \\ \boxed{v = r\omega} \end{array}$$

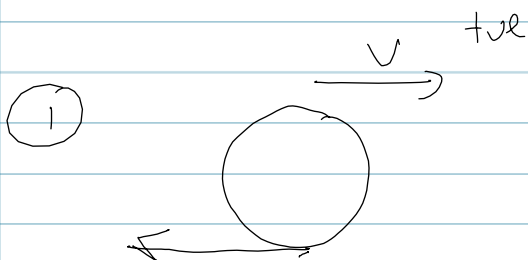
friction: — slows ball down  $v \downarrow$

— exerts a torque  $\omega \uparrow$

① Expression for  $v$  (using kinematics)

②  $\rightarrow$   $\omega$  (using torques)

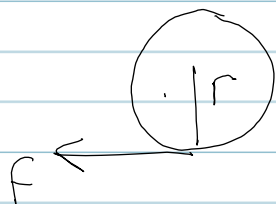
③  $v = r\omega$



$$F_f = ma = -\mu_k mg \quad a = -\mu_k g$$

$$v = v_0 - \mu_k g t$$

②  $F_f$  exerts a torque.



$$F_f r = I \alpha$$

$$\mu_k m g r = \frac{2}{5} m r^2 \alpha$$

$$\alpha = \frac{5}{2} \frac{\mu_k g}{r}$$

$\omega$  of any point on ball

$$\omega = \alpha t$$

$$= \frac{5}{2} \mu_k g t$$

a) Combine  $v = r \omega$ .

$$v_0 - \mu_k g t = r \frac{5}{2} \frac{\mu_k g t}{r} \quad \rightarrow \quad v_0 = \frac{7}{2} \mu_k g t$$

$$t = \cancel{\frac{2}{7}} \left( \frac{2}{7} \right) \frac{v_0}{\mu_k g} = \underline{0.87 \text{ s}}$$

b) At this time  $v = v_0 - \cancel{\mu_k g} \frac{2}{7} \frac{v_0}{\cancel{\mu_k g}}$

$$\boxed{v = \frac{5}{7} v_0}$$

$$c) \quad v^2 = u^2 + 2aS$$

$$S = \frac{v^2 - u^2}{2a} = \frac{\left(\frac{5}{7}\right)^2 v_0^2 - v_0^2}{2\mu_k g} = \boxed{6.75 \text{ m}}$$