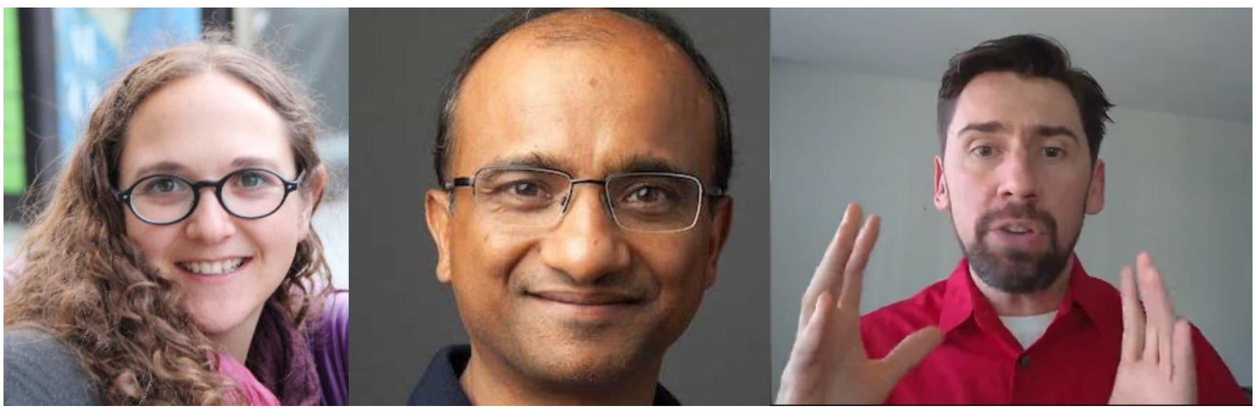
GAMES: A Gentle Introduction to Math, Excel, and Stats



Course 1: PreCalculus and Introductory Calculus

Who is this course for?

This course is for a diverse set of learners in business, economics, and other social sciences including (a) those who may lack a strong enough math and statistics background from high school, (b) learners seeking to refresh their math and statistics knowledge in preparation for upper years in university and college, (c) learners wishing to re-skill or up-skill by switching majors or adding a minor and lacking the necessary math and statistics tools.

What does this course cover?

The course has two segments.

Segment 1 covers precalculus algebra. These topics are necessary for performing mathematical calculations and higher-level mathematics. It can be useful to think of algebra as a language with its own definitions and rules. Topics are introduced assuming the learner has forgotten all their high school math. We begin by relating algebraic expressions to real world examples from the social sciences. These examples demonstrate the utility of mathematics to motivate the learner as more technical topics are introduced. Sets and real numbers are explored, followed by exponents, factors, and fractions. These mathematical concepts are illustrated using two-dimensional graphs, and their applications are demonstrated with real-world examples whenever possible. Functions and summation notation are important tools for relating real-world phenomena to mathematics in a way that is useful for analysis and simplifies communication.

Segment 2 introduces differential calculus which is perhaps the most commonly used mathematical concept in business and the social sciences. We begin by introducing the concept of limits: the value of a function as the input variable moves closer and closer to some particular number. Limits help understand the concept of differentiation which is in turn used to calculate rates of change for expressions more complicated than a straight line. We develop the important idea that on a graph, differentiation equates to the slope of a curve. We then explore three useful applications of differential calculus: implicit differentiation, linear approximation, and optimization.

Each module relies on at least some of the topics covered in a previous module, so we suggest that the learner work through each module in sequence. A learner who has not mastered the content in part 1 is likely to be challenged to develop even a basic understanding of the content in part 2.

Course 1 Learning objectives

By following the lectures and attempting the embedded and practice questions, the learner should be able to:

- Model real-world phenomena as an equation and inequality.
- Demonstrate an understanding of the basics of limits and summation notation.
- Create a two-dimensional diagram to represent the mathematical relationship between two variables, both by hand and using Excel or another spreadsheet program.
- Identify all points of interest on a diagram including the axes labels, slopes, and intercepts.
- Identify expressions that are implicit functions and apply implicit differentiation to find the slope of these expressions.
- Use linear approximation to approximate the level of a variable and its rate of change.
- Convert a word problem into a mathematical optimization problem and solve this optimization problem.

Course Resources

Students have access to the following course resources:

- Close-captioned lecture videos
- Embedded questions (with solutions) in the lecture videos to encourage retention and provide immediate feedback

- Full lecture transcripts
- Blank and annotated lecture slides
- Practice problems, including some computations with spreadsheet software such as Excel / Google sheets (provided in both .xls and .ods formats)
- Solutions to the practice problems, including Excel computations (provided in both .xls and .ods formats)

Module 1: Variables and Algebraic Expressions in the Social Sciences

Most areas in the social sciences now use algebraic expressions to succinctly model and analyze real-life situations. In this module, using examples from a variety of contexts, we introduce the idea of depicting a given real-world situation using an algebraic expression involving some essential variables. Subsequent analysis of such expressions, using mathematical tools (together with spreadsheet software such as Excel or Google Sheets) can help develop deep insights into the problem and the impact of policies and other changes. Examples in this module are drawn from politics, business, geography, sports analytics and modelling the spread of infections.

By the end of the module, the student should be able to:

- Formulate some common situations using an algebraic expression / formula
- Analyze an algebraic expression to answer questions relevant to the social sciences
- Use Excel or Google Sheets to computationally analyze and compare between different real-world situations and policies

Module 2: Sets & Numbering Systems

This module introduces a simple but essential element of most mathematical problems: sets. Before any function or algebraic expression is defined, it is important to identify the set over which the definition holds. Starting with real numbers and integers, the module develops the various ways of writing sets, both finite and infinite. The latter half of the module focusses on the essential concepts in combining sets, namely their union and intersection.

By the end of the module, the student should be able to:

- Succinctly write a set using appropriate notation
- Explain finite and infinite sets
- Identify the union and intersection of given sets

Module 3: Exponents, Factors and Fractions

This module continues to develop your ability to manipulate algebraic expressions by introducing more advanced operators such as Exponents and understanding their interaction with real numbers and fractions. Fractions are common in the social sciences and business and successfully manipulating their properties is a prerequisite for your future success. Exponents act as a more succinct instruction to multiply while factors allow for solving many common algebraic expressions with more than one solution. While this module is predominantly a technical one, the usefulness of the skills cannot be overestimated. An application applying fractions to an ESG (environmental, social, governance) investment portfolio is considered.

By the end of this module, the student should be able to:

- Manipulate exponents with a common base.
- Memorize the three most common factor identities and apply them to solving common algebraic expressions.
- Memorize the rules for fractions and apply them to finding common denominators and understanding investment portfolios

Module 4: Fractions and Systems

The module begins by considering fractions in the exponent which combines two skills developed previously in module 3. The concept of a function is next introduced. A function is a useful concept that is analogous to a machine that takes inputs to produce outputs. Building on simple examples like a vending machine, we then add compositions of functions - functions within other functions. We consider an application from economics using a Laffer Curve which combines tax policy with labor incentives to demonstrate optimal (revenue maximizing) taxation policy. The remainder of the module focuses on an important technical skill: manipulating a system of equations. A variety of techniques to solve systems of equations are presented including the substitution and elimination methods.

By the end of this module, the student should be able to:

- Apply the rules of fractions to fractions in the exponent.
- Define a function and identify real-world examples of functions.
- Solve a composition of functions model such as the Laffer Curve by hand and with Excel or another spreadsheet program.
- Solve a system of equation using both the substitution and elimination methods.

Module 5: Graphs and the Euclidean Plane

In this module, we focus on the graphical representation of two variables. Two-dimensional representations are extremely powerful methods for communicating findings. Even in the most sophisticated and highly regarded academic paper, one can usually find a two-dimensional plot.

By the end of this module, the learner should be able to

- identify points in Euclidean space
- Draw mathematical functions on a properly labeled two-dimensional graph
- Contrast a linear function with a nonlinear function

Module 6: Functions in Practice

A more mathematical definition of functions is necessary to create useful models of the real world that can be algebraically rearranged to produce important insights into human behaviour. We introduce discrete and continuous functions while explaining the concept of the domain and range. Also included is a simple test to determine whether a mathematical relationship is a function, called the **vertical line test**. This understanding of functions and inverse functions is applied to models from economics and business. Models are solved by hand and using Excel or another spreadsheet program.

By the end of this module, the student should be able to:

- Define a function in mathematical terms and identify mathematical relationships as either a function or not a function.
- Use set notation to limit the domain and range of a function or mathematical relationship.
- Define the conditions under which a specific function has an inverse function.
- Manipulate and solve models with composite functions by hand and with Excel or another spreadsheet program.

Module 7: Summation Notation, Series and Sequences

The summation $\sum$ notation is one of the most widely used notations in the social sciences. However, many students often struggle with it. In this module, we focus on developing familiarity with the meaning and usage of the summation notation. Furthermore, we introduce students to two of the most common sums: the arithmetic and geometric series. As an application, we develop the formula for the valuation of annuities and perpetuities. The module also covers doing summations and using some in-built functions in Excel or Google Sheets, and the limits of some common sequences.

By the end of the module, the student should be able to:

- Interpret summation notation and develop summation notation for a given situation
- Sum an arithmetic or geometric series
- Use in-built functions in Excel or Google Sheets for some common mathematical computations

Module 8: Limits

This module introduces students to limits, the idea of investigating where a function or process is headed towards as one continues to change a variable in a particular way. Understanding limits is central to understanding the subsequent important topic of differentiation. The concept is also useful for social scientists in understanding where a social process is headed towards and where it may end up. The module helps students develop the basic idea of limits in general, and also the tools for computing limits of the most common types of functions.

By the end of the module, the student should be able to:

- Develop an understanding of the idea of the limit of a function.
- Compute the limits for common types of functions.

- Apply the properties of limits to solve problems.

Module 9: Introduction to Derivatives

This module introduces one of the most commonly used mathematical concepts in business, economics and other social science subjects, namely the concept of a derivative. Starting from the rate of change of a line and average velocity, it moves to the idea of instantaneous velocity and tangent of a curve to establish the concept of derivative of a function. The module then introduces the formal definition of a derivative and uses the definition to establish derivatives of some simple functions. It also exposes students to the related idea of non-differentiability.

By the end of the module, the student should be able to:

- Develop an understanding of the idea of derivative of a function.
- Know the formal definition of a derivative and be able to use the definition
- Understand differentiability and non-differentiability of a function

Module 10: Derivative Computations

Additional rules are developed that allow the learner to find the derivative of more complicated expressions and functions. The product rule simplifies the process of taking the derivative when two or more functions are multiplied with each other. The chain rule allows the learner to take the derivative even when functions are embedded in other functions, called composite functions. Combining the two rules allows the learner to find the derivative even for very long and complicated expressions. We conclude with an introduction of higher-order derivatives such as the second derivative which is the rate of change of the rate of change.

By the end of this module, the student should be able to:

- Apply the product rule and chain rule to find the derivative of simple algebraic expressions and functions.
- Combine the product rule and chain rule to find the derivative of more complex expressions and functions.
- Explain how the second derivative is derivative of the derivative or the rate of change in the rate of change.

Module 11: Applied Differential Calculus

The tools of single variable calculus are extended to implicit functions where the value of y depends implicitly on x. A variation on the previous methods of computing derivatives of implicit differentiation is introduced. Implicit differentiation allows for analyzing algebraic expressions that are too complicated for one variable to be arranged and isolated on a side of the expression. To start, the technique is applied to the equation of a circle followed by more complicated expressions. Next, we introduce linear approximations and how the derivative can be used to approximate change. We conclude the module with an example from finance looking at the price of fixed income bonds using Excel or another spreadsheet program.

By the end of this module, the student should be able to:

- Identify an implicit function and take its derivative.
- Graph the implicit function on a diagram and identify the tangent line (slope).
- Use a linear approximation to estimate the change in one variable from the change in another variable.
- Demonstrate your understanding of linear approximation by analyzing the change in the price of a bond from a change in interest rates using Excel or another spreadsheet program.

Module 12: Optimization

Finding the optimal use of a scarce resource is fundamental to making smart decisions. The decision-making process becomes more challenging when the choice is made over a continuous set such as how much to spend, what price to charge, or how much product to create. Business, economics, and the broader social sciences often encounter these types of quantitative problems. We introduce methods to find extreme values: the maximum and minimum of a function over a continuous domain. Real-world examples are explored by hand and with Excel or another spreadsheet program. The intuition underlying single variable optimization can be extended to multiple variables.

By the end of this module, the student should be able to:

- Determine if a global maximum or global minimum exists and to identify where this point occurs using Excel or another spreadsheet program,
- Differentiate an equation for the purpose of optimization, solve for its critical points and evaluate whether the critical point is a maximum, a minimum, or neither.
- Apply the above optimization techniques to common problems from economics and business by hand and using Excel or another spreadsheet program.

This project is made possible with funding by the Government of Ontario and through eCampusOntario's support of the Virtual Learning Strategy. To learn more about the Virtual Learning Strategy visit: https://vls.ecampusontario.ca/.

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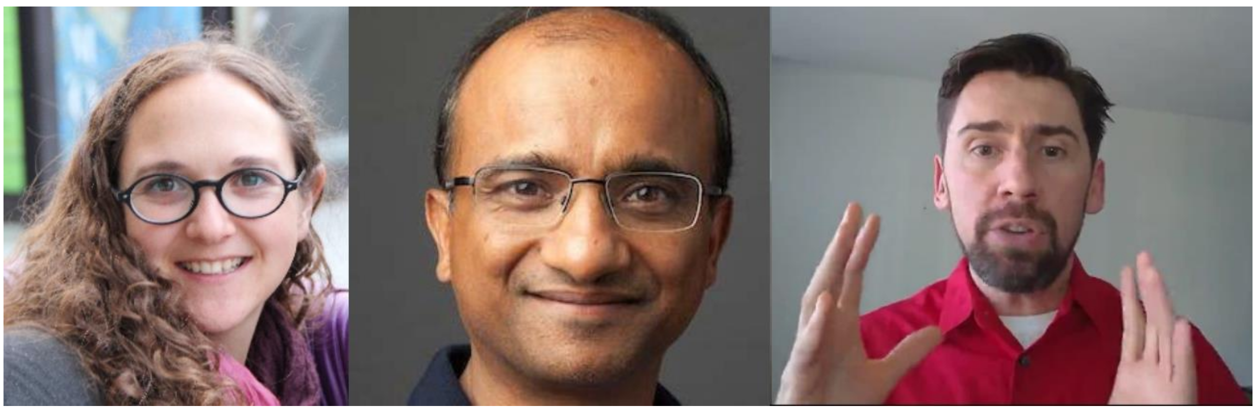

Course 2: Calculus, Probabilities, and Statistics

Who is this course for?

This course is for a diverse set of learners in business, economics and other social sciences including (a) those who may lack a strong enough math and statistics background from high school, (b) learners seeking to refresh their math and statistics knowledge in preparation for upper years in university and college, (c) learners wishing to re-skill or up-skill by switching majors or adding a minor and lacking the necessary math and statistics tools.

What does this course cover?

The course has three primary segments.

Segment 1 covers some advanced topics related to the material in course 1: integration, logarithms and exponential functions and their relation with modelling growth, multivariable calculus and unconstrained optimization with many variables. Optimization and multivariable functions are common in many areas of the social sciences; this part of the course will help prepare learners in understanding these concepts and analyzing them with mathematical rigor.

Segment 2 introduces learners to the basic idea of probability, the relation between events, and the important concepts of conditional, marginal and total probability, and Bayes' rule. The idea of probability distributions across random variables is then developed, along with their expectation and variance. The segment also introduces learners to some of the most common probability distributions, such as the Binomial and the Normal distribution. Examples from the social sciences and the use of Excel / Google sheets enhance the applicability of the concepts for learners.

Segment 3 focuses on some of the most essential statistical tools for learners, an important area given the ever-increasing usage of data in all disciplines. One is introduced to the most common tools for visually representing data, and then exposed to the various descriptive measures for data such as the average, the standard deviation, covariance and correlation. Learners are also introduced to linear regression and the basics of statistical inference such as estimation, confidence intervals for estimates, and hypothesis testing. Real-life data examples and the use of Excel / Google sheets for statistical computation makes the segment of immediate practical use.

There is a natural progression in terms of the segments. Some parts of segment 2 on probability draw from segment 1 (especially the concepts of integration), while parts of segment 3 on statistics build on the basics of probability distributions to develop understanding of the important concepts in statistical inference.

Learning objectives

By following the lectures and attempting the embedded and practice questions, the learner should be able to:

- Develop an understanding of the basics of integration, multivariable calculus, and unconstrained optimization with many variables.
- Comprehend the core concepts in probability, including dependent and independent events, conditional and marginal probability, and Bayes' rule.
- Develop an understanding of expectation, variance and covariance of random variables, and be introduced to some of the most common probability distributions such as the Binomial, the Poisson and the Normal distributions.
- Represent statistical data visually as well as in terms of descriptive statistics, using spreadsheet software such as Excel / Google sheets.
- Develop an understanding of linear regression and the basics of statistical inference, such as estimation, generation of confidence intervals, testing of hypotheses and significance of regression coefficients.

Course resources

Learners have access to the following course resources:

- Close-captioned lecture videos
- Embedded questions (with solutions) in the lecture videos to encourage retention and provide immediate feedback
- Full lecture transcripts

- Blank and annotated lecture slides
- Practice problems, including some statistical problems with real-world data (data-sets provided in both .xls and .ods formats)
- Solutions to the practice questions, including Excel computations (provided in both .xls and .ods formats)

Module 1: Integration 1

This module introduces students to integration, a somewhat advanced concept in calculus. It builds intuition for the basic idea through the use of Riemann sums and calculating the area under a curve using left and right end points. Taking the limit of the Riemann sums then establishes the concept of a definite integral. Having developed the concept for this fundamental idea, the module ends by talking about the practical issues of integrability and net area.

By the end of the module, the student should be able to:

- Develop an understanding of the concept of integration.
- Relate how integration is related to the area under a curve.
- Understand integrability and net area

Module 2: Integration 2

This module continues with integration, now introducing it as an antiderivative i.e. the opposite of differentiation. The basis for this is the Fundamental Theorem of Calculus. Developing this two-way connection between integration and differentiation leads one to the idea of an indefinite integral, and the relationship between definite and indefinite integrals. The second half of the module focusses on properties of integrals and some of the common techniques for computing integrals.

By the end of the module, the student should be able to:

- Develop an understanding of the concept of integration as an antiderivative.
- Understand indefinite integrals.
- Understand properties of integrals and techniques for integration

Module 3: Logarithms and Exponential Functions and Growth of Functions

This module introduces students to the idea of logarithms and exponential functions. Having established the basic idea, it then delves into an understanding of the connection of these functions with growth. The module then studies growth rates of exponential functions and compares the consequences of exponential versus linear growth. The module also looks at some practical examples of exponential growth and ends by introducing the useful idea of the logarithmic scale.

By the end of the module, the student should be able to:

- Understand the concept of an exponential function.

- Develop an understanding of exponential growth and decay and their consequences in real-world examples.
- Distinguish between exponential and linear growth

Module 4: Multivariable Calculus

This module introduces students to multivariable calculus. This is a major area of calculus and this module covers only some of the basic ideas that will be useful for students in the social sciences. The module begins by introducing functions of many variables with some graphs of 2-variable functions. Having established the basic idea, we then develop the important concept of partial derivatives. The second half of the module focuses on local and global extrema for multivariable functions and on the useful methodology for finding extrema.

By the end of the module, the student should be able to:

- Explain how a function of many variables differs from a single variable function.
- Compute partial derivatives
- Develop an understanding of local and global extrema for multivariable functions and the methodology for finding extrema.

Module 5: Basics of Probability

Uncertainty pervades human decision-making, the focus of much of the social sciences. In this module, we introduce the basic idea of probability, which provides a formal language for analyzing uncertainty. Starting from the main axioms, we develop how probabilities of different events can be calculated. The important idea of independence of events and its implication for calculating probabilities of compound events is also covered in the module. A diverse set of examples introduce the student to calculating probabilities in a variety of contexts.

By the end of the module, the student should be able to:

- Calculate probabilities of some simple and compound events
- Develop an understanding of the independence of events and its implications
- Develop an understanding of the interpretation of probability

Module 6: Conditional Probability and Bayes' Rule

This module focuses on developing an understanding of the relationship between events. Using probability trees, we show how probabilities of combination of events can be calculated. The module then introduces the related ideas of Conditional Probability, Marginal and Total Probability and demonstrates them in the context of two-way frequency tables. The module ends by introducing students to the idea of reverse conditioning and the important concept of Bayes' rule. Examples from politics and stock markets are used to demonstrate some of the concepts.

By the end of the module, the student should be able to:

- Calculate probabilities of dependent and independent events

- Calculate Conditional, Marginal and Total Probability
- Develop an understanding of reverse conditioning and Bayes' rule

Module 7: Discrete Random Variables

This module begins by revisiting the important concept of Bayes' rule with examples from medical testing and credit histories. We then introduce discrete random variables and develop an understanding of their probability distribution, in particular their p.m.f. and c.d.f. and the relationship between the two. The important concepts of expectation and variance of a discrete random variable are subsequently studied. The module ends by introducing students to three common discrete distributions: the Bernoulli, the Binomial and the Poisson distribution, and how their probabilities can be computed using Excel/Google sheets.

By the end of the module, the student should be able to:

- Develop an understanding of the p.m.f. and c.d.f. of a discrete random variable
- Develop an understanding of the expectation and variance of a discrete random variable
- Understand the Bernoulli, the Binomial and the Poisson distributions, and use Excel/Google sheets to calculate probabilities for these distributions

Module 8: Continuous Random Variables

This module begins by covering a residual topic from the previous module: Covariance and Correlation between two random variables. We then introduce continuous random variables and develop an understanding of their probability distribution, in particular their p.d.f. and c.d.f. The expectation and variance of a continuous random variable are subsequently studied. The final segment of the module introduces students to two common continuous distributions: the Uniform and the Normal distribution. It delves into the important properties of the Normal distribution in preparation for their use in the Statistics modules later.

By the end of the module, the student should be able to:

- Understand Covariance and Correlation between two random variables
- Develop an understanding of the p.d.f. and c.d.f. of a continuous random variable
- Develop an understanding of the expectation and variance of a continuous random variable
- Understand properties of the Uniform and the Normal Distribution

Module 9: Introduction to Statistics and Data Visualization

The analysis of data is an integral part of most areas in the social sciences. The widespread availability of data and related statistical software has only increased its importance. This module begins by introducing students to the various types of data and variables that are usually encountered in the social sciences. We then focus on some of the common tools that are used to visually represent different types of data: Bar/Column charts, Pie charts, Time trends, Scatter plots and Histograms. Using real data from a diverse set of areas, we demonstrate how Excel/Google sheets can be used to generate the particular type of chart or plot that is most appropriate for the given situation.

By the end of the module, the student should be able to:

- Understand the various types of data and variables that are common in the social sciences
- Develop an understanding of the appropriate visual representation tool to use for a given situation
- Generate Bar/Column charts, Pie charts, Time trends, Scatter plots and Histograms using Excel/Google sheets

Module 10: Average and Variability

A data series can have dozens, hundreds, or even millions of observations. People in business and the social sciences need simple statistics to describe the data. The most common are measures of central tendency which suggest the most common or most likely values in the data. Measures of central tendency are also known as averages: the median, the mean, and the mode. Another common descriptive statistic relates to variability or dispersion - how spread out is the data. Common measures of variability include the range, IQR, variance, and standard deviation. These descriptive statistics are useful tools for describing data, and they are also used as the basis upon which to perform more advanced statistical testing and forecasting.

By the end of this module, the student should be able to:

- Compute the median, the mean, and the mode, range, IQR, variance, and standard deviation by hand and using real-world and simulated data.
- Create a stem-and-leaf plot Excel or another spreadsheet program and use it to understand the distribution of values in a series.
- Create a frequency PDF and CDF Table by hand and using Excel or another spreadsheet program.

Module 11: Correlation and Linear Regression

Business leaders and social scientists need to understand the relationship between two or more variables. For example, determining whether a larger sales force produces more sales or whether increasing income inequality leads to more crime. Correlation coefficients can capture the relationship between two variables in a single statistic. This module explains how to calculate a correlation coefficient and describes the conditions under which the correlation coefficient is accurate. The concept of correlation is built upon to introduce simple linear regression - a simple but useful technique for forecasting. Lastly, a measure of error called the residual is introduced and used to verify whether the conditions for an accurate simple linear regression have been satisfied.

By the end of this module, the student should be able to:

- Calculate the correlation coefficient and use it to identify the direction and strength of the relationship between two variables.

- Explain the conditions under which the correlation coefficient and simple linear regression are accurate measures of the true underlying relationship between two variables.
- Graphically analyze the residuals from linear regression to determine if the conditions for an accurate prediction are satisfied.
- Calculate correlation, covariance, and a simple linear regression by hand and using Excel or another spreadsheet program.

Module 12: Basics of Statistical Inference

This module introduces students to the idea of drawing inference about the entire population based on observations on a small part of the population. It is a vast area of statistics, and this long-ish module covers only some of the basic ideas of statistical inference. Using insights from the Central Limit Theorem, we develop methods of estimation of population parameters, their standard errors and confidence intervals, and the testing of hypotheses. The module ends by connecting these ideas with the statistical significance of regression coefficients, including some examples from academic research papers.

By the end of the module, the student should be able to:

- Compute an estimate, its standard error and confidence intervals
- Test hypothesis for a single population
- Develop an understanding of the statistical significance of regression coefficients

This project is made possible with funding by the Government of Ontario and through eCampusOntario's support of the Virtual Learning Strategy. To learn more about the Virtual Learning Strategy visit: https://vls.ecampusontario.ca/.